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Emissions Taxes Versus Tradeable Permits With Many Countries

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ABSTRACT

Tradeable permits and emissions taxes are compared in a multi-country global emissions game with perfectly competitive firms and a trans-boundary production externality. In a one-shot game, comparing welfare under the Nash equilibria, it is shown that tradeable permits are superior to emissions taxes. In an infinitely-repeated game, comparing the discount factors required to sustain a global International Environmental Agreement (IEA), it is shown that it is easier to sustain cooperation with tradeable permits than with emissions taxes when the number of countries is sufficiently large. In a coalition-formation game, the number of countries in a stable IEA is two with tradeable permits, but may be all countries in the world with emissions taxes.

JEL Classification: C72, F53, H23, Q58

1 | Introduction

In response to the Paris Agreement, which was adopted by 195 countries in 2015, many jurisdictions have introduced emissions trading systems (tradeable permits) to restrict their greenhouse gas emissions.[1] A perennial question in environmental economics is whether using prices (emissions taxes) or quantities (tradeable permits) to control pollution will be equivalent, and this question has led to a vast literature. One strand of this literature has considered how the choice of policy instrument (taxes or permits) affects strategic interaction between countries in non-cooperative and cooperative global environmental games. Cooperative games, such as infinitely-repeated and coalition-formation games, are often used to address the sustainability and stability of an International Environmental Agreement (IEA), such as the Paris Agreement. This paper will use a novel model with a global environmental externality to compare tradeable permits and emissions taxes in three different games: a one-shot game, an infinitely-repeated game, and a coalition-formation game. In the one-shot game, welfare of all countries is higher in the Nash equilibrium (NE) if they use tradeable permits rather than emissions taxes, and choosing tradeable permits is a dominant strategy in a two-stage extension of this game. In the infinitely-repeated game, the critical discount factor required to sustain cooperation in an IEA is lower with tradeable permits than with emissions taxes if the number of countries in the world is sufficiently large. In the coalition-formation game, the number of countries in a stable IEA is two with tradeable permits, but may be all the countries in the world with emissions taxes, even if the total number of countries in the world is large.

With perfect competition and no uncertainty, tradeable permits and emissions taxes will be equivalent but, in a seminal article, Weitzman (1974) showed that they will not be equivalent when there is uncertainty. Some of the subsequent literature is surveyed by Sterner and Robinson (2018), and more recent contributions to the literature include Pizer and Prest (2020) and Karp and Traeger (2024). The topic of this paper is to consider how strategic interaction between governments in an international setting, rather than uncertainty, affects the equivalence of emissions taxes and tradeable permits. Hence, this brief survey will concentrate upon that aspect of the literature. In a one-shot game, using a model with a global emissions externality and international trade, Hoel (2005) argues that the most likely outcome is a NE in differentiated emissions taxes, but that there may also be a NE in tradeable permits that Pareto dominates the NE in emissions taxes. Kiyono and Ishikawa (2004) demonstrate that tradeable permits and emissions taxes are *unilaterally* equivalent in the sense that they are equivalent given the policy choice of other countries. They then show that tradeable permits and emissions taxes are not *strategically* equivalent as emissions in the NE in tradeable permits will be different from those in the NE in emissions taxes. This analysis is extended by Kiyono and Ishikawa (2013) who consider a two-stage game where governments choose the policy instrument in the first stage and then the levels of these policy instruments in the second stage. They find that many equilibria are possible in this two stage game including NE where one country uses emissions taxes while the other country uses tradeable permits. Harstad et al. (2022) find that tradeable permits and emissions taxes are strategically equivalent in terms of welfare, which is due to the functional form of their welfare function.

Turning to cooperative games, there are two approaches to the sustainability and stability of an IEA: infinitely-repeated games and coalition formation games.[2] In an infinitely repeated game, where a global IEA can be sustained using Nash-reversion trigger strategies, Harstad et al. (2022) argue that a price agreement (emissions taxes) dominates a quantity agreement (tradeable permits), although the result is due to the investment stage in their game. Starting with Barrett (1994), many authors have used the stability concept that D'Aspremont et al. (1983) applied to cartels to analyse the stability of an IEA. Eichner and Pethig (2015) use this approach to compare how the instruments affect the formation of an IEA. They show that a global IEA may be stable (self-enforcing) with emissions taxes but not with tradeable permits.

Section 2 uses the global emissions game from Collie (2022) to analyse the situation when countries use tradeable permits in a one-shot game, an infinitely-repeated game, and in coalition-formation game. The global emissions externality is modelled as a production externality rather than a consumption externality, which fits with the findings of WHO, WMO (2025) that with climate change there is a 2–3% decrease in labour productivity for every 1° increase in temperature above 20°C. Section 3 analyses the situation when countries use emissions taxes in the same games. Emission taxes are analysed using the same method employed by Vives (1985) to compare Bertrand and Cournot equilibria. Countries are modelled as setting the quantity of emissions (rather than emissions taxes) subject to equivalence constraints given by the other countries setting emissions taxes. In this way, the analysis is all undertaken in terms of the quantities of emissions which makes comparisons between policy instruments much easier. Section 4 compares the outcome of these games with tradeable permits and emissions taxes, and also considers a two-stage game where countries choose the type of instrument in the first stage. The conclusions are presented in section 5.

2 | Global Emissions Externality Game With Tradeable Permits

Consider the complete-information game used as an example in Collie (2022), where there is a global (transboundary) emissions externality and each country has its own closed emissions trading system that restricts the quantity of emissions in that country using tradeable permits. The $J \geq 2$ countries are all identical with the same labour endowment and the same production function, and they are labelled $j = 1, \ldots, J$. In each country, a perfectly competitive industry produces the consumption good, Y_j, using the country's labour endowment, L, and a quantity of emissions, E_j with a constant returns to scale technology. The labour endowment can be regarded as a composite factor including other factors such as capital and land, although it turns out that the labour endowment has little impact on the analysis. The quantity of emissions represents each country's use of the environment in the production of the consumption good. Since there is only one consumption good, there is no possibility of trade in this model. The consumption good acts as the numeraire good with its price set to unity. In each country, the perfectly-competitive industry consists of a large number F of small firms that take the price of the consumption good and the global quantity of emissions as given. The production function of a firm is given by the modified Cobb-Douglas production function that exhibits constant returns to scale to the factors employed by the firm:

$$y_{jf} = \exp\left[-\beta\left(E_j + \textstyle\sum_{i \neq j} E_i\right)\right] A E_{jf}^{\alpha} L_{jf}^{1-\alpha} \quad j = 1, \ldots, J \quad f = 1, \ldots, F \tag{1}$$

where E_{jf} is the emissions and L_{jf} is the labour employed by the fth firm in the jth country, $A > 0$ is a technology parameter, and $\alpha \in (0, 1)$ is the share of emissions in production. The exponential function represents the externality effect on production of global emissions, where the parameter $\beta > 0$ determines the size of this externality effect. Mitigation of emissions occurs by firms substituting labour for emissions in the production process in response to environmental policy (tradeable permits or emissions taxes).

A closed emissions trading system may allocate tradeable permits using an auction or by grandfathering based upon previous emissions.[3] In either case, the opportunity cost of emissions for the firms is given by the price of the tradeable permits. Suppose that the wage is ω_j and that the price of tradeable permits is ε_j in the jth country, then cost minimisation by the firms implies that the marginal product of each factor is equal to its price:

$$\begin{aligned} \varepsilon_j &= \frac{\partial y_{jf}}{\partial E_{jf}} = \exp\left[-\beta\left(E_j + \textstyle\sum_{i \neq j} E_i\right)\right] \alpha A \left(\frac{E_{jf}}{L_{jf}}\right)^{\alpha - 1} \\ \omega_j &= \frac{\partial y_{jf}}{\partial L_{jf}} = \exp\left[-\beta\left(E_j + \textstyle\sum_{i \neq j} E_i\right)\right] (1 - \alpha) A \left(\frac{E_{jf}}{L_{jf}}\right)^{\alpha} \\ \Rightarrow \quad \frac{E_{jf}}{L_{jf}} &= \frac{\alpha}{1 - \alpha} \frac{\omega_j}{\varepsilon_j} \end{aligned} \tag{2}$$

With a competitive market for tradeable permits (and labour) all firms in the country will face the same relative factor price ω_j/ε_j and hence tradeable permits will be allocated efficiently between firms in the country with all firms using the same factor intensity $E_{jf}/L_{jf} = E_j/L$. Hence, with constant returns to scale, a perfectly competitive industry can be modelled as a representative firm that takes prices and the quantity of global emissions as given, with an aggregate production function that gives the jth country's total output of the consumption good, $Y_j = \sum_{f=1}^{F} y_{jf}$. Since there is only one consumption good and the welfare of each country is given by its consumption, which is equal to production, the welfare of each country is given by this aggregate production function:

$$W_j(\mathbf{E}) = Y_j(\mathbf{E}) = \exp\left[-\beta\left(E_j + \sum_{i\neq j} E_i\right)\right] A E_j^{\alpha} L^{1-\alpha} \quad j = 1, \ldots, J \tag{3}$$

where $\mathbf{E} = (E_1, \ldots, E_J)'$.[4] It is straightforward to show that the welfare of each country is quasi-concave in its own emissions, since it is log-concave, and that $\partial W_j / \partial E_i < 0$, for $i \neq j$, so there is a negative externality from the emissions of other countries. An increase in global emissions reduces the productivity of labour and thereby reduces consumption and welfare in all countries, as in the WHO, WMO (2025) report.

2.1 | One-Shot Game

In the one-shot game, each country chooses its quantity of emissions E_j by issuing the requisite quantity of tradeable permits to its firms. If countries set the quantity of emissions (tradeable permits) non-cooperatively then each country will choose E_j to maximise $W_j = Y_j$ given the quantities of emissions chosen by all the other countries. Hence, differentiating (3)with respect to E_j yields the first-order conditions for a Nash equilibrium (NE) in the quantity of emissions:

$$\frac{\partial W_j}{\partial E_j} = \exp\left[-\beta\left(E_j + \sum_{i\neq j} E_i\right)\right] A L^{1-\alpha} E_j^{\alpha-1}(\alpha - \beta E_j) = 0 \quad j = 1, \ldots, J \tag{4}$$

For each country, setting the quantity of emissions equal to α/β is a dominant strategy, and hence in the NE all countries will set the quantity of emissions equal to $E_N^T = \alpha/\beta$, where the superscript T denotes tradeable permits and the subscript N denotes the NE. Substituting the NE quantity of emissions into welfare (3) yields the NE welfare of each country:

$$W_N^T = \exp[-\alpha J]\left(\frac{\alpha}{\beta}\right)^{\alpha} A L^{1-\alpha} \tag{5}$$

If the countries cooperate on the setting of tradeable permits, then they choose the quantity of emissions, E_j, to maximise their joint welfare. Since all countries are identical, it seems reasonable for the tradeable permits to be set so that the quantity of emissions is the same in all countries. Setting $E_j = E$ for all countries, $j = 1, \ldots, J$, in (3) and then differentiating with respect to E yields the first-order condition for welfare maximisation:

$$\frac{\partial W_j}{\partial E} = \exp[-\beta J E] A L^{1-\alpha} E^{\alpha-1}(\alpha - \beta J E) = 0 \tag{6}$$

Hence, when setting tradeable permits cooperatively, the cooperative quantity of emissions will be: $E_C^T = \alpha/\beta J$, where the subscript C denotes the cooperative outcome. Note that the total emissions of all countries when they cooperate are the same as the emissions of one country in the NE, $J E_C^T = \alpha/\beta = E_N^T$. Substituting $E_C^T = \alpha/\beta J$ into welfare yields the welfare of each country in the cooperative outcome:

$$W_C^T = \exp[-\alpha]\left(\frac{\alpha}{\beta J}\right)^{\alpha} A L^{1-\alpha} \tag{7}$$

If when all the other countries are setting the tradeable permits equal to the cooperative quantity of emissions, E_C^T, a country deviates, then it will set E_j to maximise its welfare given that all the other countries are setting E_C^T. This yields the quantity of emissions when a country deviates $E_D^T = \alpha/\beta$, where the subscript D denotes deviation, and is equal to E_N^T since $E_j = \alpha/\beta$ is a strictly dominant strategy so the best-reply function of the jth country does not depend upon the quantities of emissions chosen by the other countries. The corresponding welfare of a deviating country is:

$$W_D^T = \exp[-\alpha(2 - 1/J)]\left(\frac{\alpha}{\beta}\right)^{\alpha} A L^{1-\alpha} \tag{8}$$

This one-shot game with two countries is illustrated in Figure 1 with $R_1^T(E_2)$ and $R_2^T(E_1)$ being the best-reply functions of country one and two, respectively, NE^T being the NE with tradeable permits and C being the cooperative outcome.

2.2 | Infinitely-Repeated Game

Now consider an infinitely-repeated game where the one-shot game is repeated infinitely with countries having a common discount factor $\delta < 1$. Cooperation in an IEA can be sustained by using Nash-reversion trigger strategies as in Friedman (1971) if the discount factor is sufficiently close to one. Countries cooperate by setting $E_j = E_C^T$ in every round provided no country has deviated in past rounds, but if a country has deviated in past rounds then all

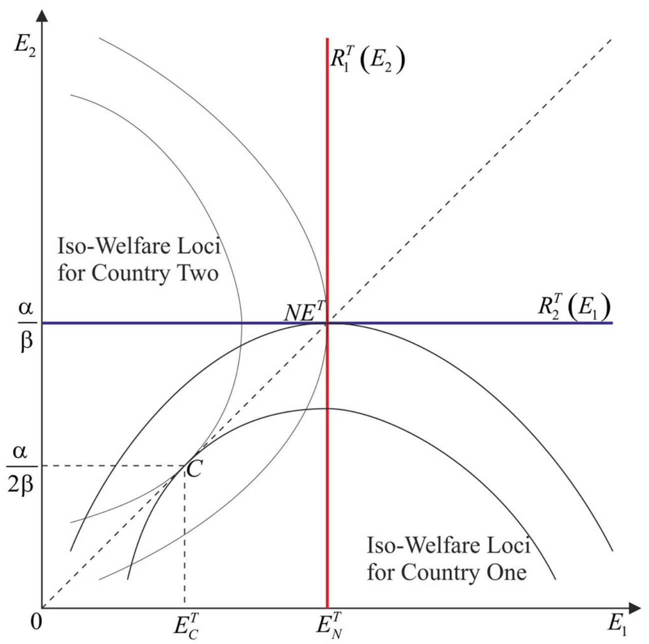

FIGURE 1 | Best-reply functions for tradeable permits.

countries set $E_j = E_N^T$ in all future rounds. Then, countries will cooperate if the net present value of welfare from cooperation exceeds that of welfare from deviation followed by the NE welfare forever thereafter: $W_C^T/(1-\delta) \geq W_D^T + \delta W_N^T/(1-\delta)$. Making this an equality and solving for the discount factor yields the critical discount factor δ_N^T. Hence, cooperation in an IEA can be sustained by Nash-reversion trigger strategies if the discount factor is greater than the critical discount factor:

$$\delta_N^T = \frac{W_D^T - W_C^T}{W_D^T - W_N^T} = \frac{1 - W_C^T/W_D^T}{1 - W_N^T/W_D^T} = \frac{1 - J^{-\alpha}\exp[\alpha - \alpha/J]}{1 - \exp[-\alpha(J-1)^2/J]} \tag{9}$$

This depends upon the share of emissions in production, α, and the number of countries, J, but it does not depend upon the size of the externality effect, β, the technology parameter, A, or the labour endowment, L. It can be shown that as the number of countries goes to infinity, then the critical discount factor goes to one.[5]

2.3 | Coalition Formation Game

An alternative approach to the modelling of cooperation is the simple game of coalition formation proposed by D'Aspremont et al. (1983) in the context of cartels but which can be applied to the formation of an IEA. Consider a static game where countries simultaneously choose whether to join a single IEA. If H countries join the IEA then they cooperate when setting the quantity of emissions whereas the remaining $J-H$ countries set their quantity of emissions non-cooperatively. Since setting the quantity of emissions $E_j = \alpha/\beta$ is a dominant strategy, the $J-H$ countries that are not members of the IEA will choose this quantity of emissions. The H countries that are members of the IEA will set their quantity of emissions to maximise their joint welfare with all members of the IEA assumed to set the same quantity of emissions since they are all identical. Hence, if each member of the IEA sets their quantity of emissions equal to E_h then the welfare of each member of the IEA is:

$$W_h = \exp[-\beta(HE_h + (J-H)\alpha/\beta)]AL^{1-\alpha}E_h^{\alpha} \tag{10}$$

Differentiating (10) with respect to E_h yields the first-order condition for joint-welfare maximisation by the IEA:

$$\frac{\partial W_h}{\partial E_h} = \exp[-\beta(HE_h + (J-H)\alpha/\beta)]AL^{1-\alpha}E_h^{\alpha-1}(\alpha - \beta HE_h) = 0 \tag{11}$$

Hence, each member of the IEA sets their quantity of emissions equal to $E_h = \alpha/H\beta$, and the welfare of each member of the IEA is then obtained by substituting this into (10), where the subscript I denotes that the country is in the IEA:

$$W_I^T(H) = \exp[-(J-H+1)\alpha]AL^{1-\alpha}\left(\frac{\alpha}{H\beta}\right)^{\alpha} \tag{12}$$

The welfare of a country that is not a member of the IEA, where the subscript O denotes that the country is outside the IEA, is:

$$W_O^T(H) = \exp[-(J-H+1)\alpha]AL^{1-\alpha}\left(\frac{\alpha}{\beta}\right)^{\alpha} \tag{13}$$

Internal stability of the IEA requires that a country that is a member of the IEA does not want to leave the IEA, which implies that $W_I^T(H) \geq W_O^T(H-1)$ and hence that:

$$I^T(H) \equiv \frac{W_I^T(H)}{W_O^T(H-1)} = \frac{\exp[\alpha]}{H^{\alpha}} \geq 1 \quad \Rightarrow \quad H \leq e \tag{14}$$

External stability in the IEA requires that a country that is not a member of the IEA does not want to join the IEA, which implies that $W_O^T(H) \geq W_I^T(H+1)$ and hence that:

$$X^T(H) \equiv \frac{W_O^T(H)}{W_I^T(H+1)} = \frac{(H+1)^{\alpha}}{\exp[\alpha]} \geq 1 \quad \Rightarrow \quad H \geq e-1 \tag{15}$$

Since $I^T(H) = 1/X^T(H-1)$, if $I^T(H) = 1$ then $X^T(H-1) = 1$. Internal stability implies that the number of IEA members is less than e, while external stability implies that the number of IEA members is greater than $e-1$, Hence, with tradeable permits, a stable IEA will have just two members even if there are hundreds of countries in the world, and this result does not depend upon any parameters of the model. The conditions for internal and external stability versus the number of IEA members are shown in Figure 2 for the case when $\alpha = 3/10$.

3 | Global Emissions Externality Game With Emissions Taxes

Now consider the same global emissions externality game, but with countries choosing emissions taxes rather than the quantity of emissions using tradeable permits.

3.1 | One-Shot Game

In this one-shot game, the strategic variable is the emissions tax, with each country choosing its emissions tax given the emissions taxes of all the other countries. However, to facilitate comparisons, the analysis can be undertaken in terms of the quantity of emissions rather than the emissions taxes. This can be done by supposing that each country is choosing the quantity of emissions subject to a unilateral equivalence constraint that implies that the other countries keep their emissions taxes constant.[6] To obtain the unilateral equivalence constraint, note that the perfectly-competitive firms in a country equate the emissions tax to their marginal product of emissions. Differentiating the aggregate production function (3) of country j with respect to E_j, but treating the exponential function as a constant since it represents the global emissions externality, yields the marginal product of emissions for a perfectly-competitive firm that is equated to the emissions tax t_j:

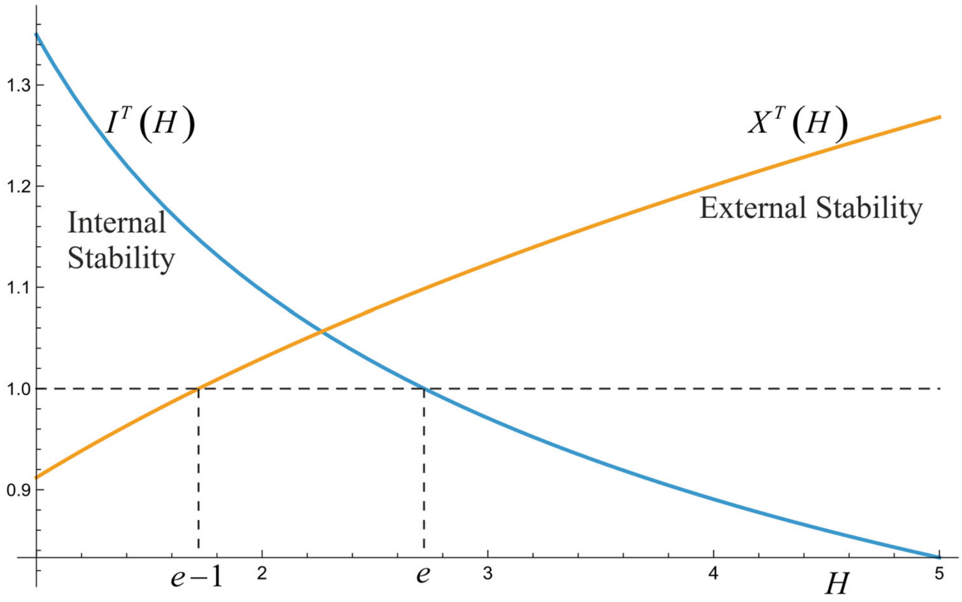

FIGURE 2 | Internal and external stability of iea with tradeable permits.

$$t_j(\mathbf{E}) = \frac{\partial Y_j}{\partial E_j} = \exp\left[-\beta\left(E_j + \sum\nolimits_{i\neq j} E_i\right)\right]\alpha A E_j^{\alpha-1} L^{1-\alpha} \tag{16}$$

The expression in (16) defines a one-to-one mapping (an equivalence constraint) between t_j and E_j which depends on the aggregate emissions of the other countries $\sum_{i\neq j} E_i$.

If countries set emissions taxes non-cooperatively then each country will choose its emissions tax to maximise its welfare given the emissions taxes set by all the other countries. Given that the game is symmetric, it can be analysed by considering the kth country that chooses E_k to maximise welfare $W_k(\mathbf{E})$ subject to the equivalence constraint that $t_j(\mathbf{E}) = \bar{t}$ where $\bar{t}$ is the emissions tax set by the jth country for all $j \neq k$. When the kth country sets E_k it has to take into account that this will affect the emissions of the other countries that have set their emissions taxes equal to $\bar{t}$ because the constraint (16) must hold. Totally differentiating the equivalence constraint, which is identical for all countries $j \neq k$ so E_j will be the same for all $j \neq k$, yields:

$$\left.\frac{dE_j}{dE_k}\right|_{t_j=\bar{t}} = -\frac{\partial t_j/\partial E_k}{\partial t_j/\partial E_j} = -\frac{\beta E_j}{1-\alpha+\beta(J-1)E_j} < 0 \tag{17}$$

When the kth country increases its emissions, while all the other countries keep their emissions taxes constant, the emissions of all the other countries decrease. The NE in emissions taxes is given by the first-order condition of the constrained optimisation problem:

$$\frac{dW_k}{dE_k} = \frac{\partial W_k}{\partial E_k} + \sum\nolimits_{j\neq k} \left.\frac{\partial W_k}{\partial E_j}\frac{dE_j}{dE_k}\right|_{t_j=\bar{t}} = 0 \tag{18}$$

Differentiating (3) and using (17) to evaluate (18), and then noting that the NE will be symmetric, $E_j = E_k$, yields the first-order condition:

$$\frac{dW_k}{dE_k} = \frac{\exp[-\beta J E_j] A E_j^{\alpha-1} L^{1-\alpha}\{\alpha(1-\alpha) - \beta(1-\alpha J)E_j\}}{1-\alpha+\beta(J-1)E_j} = 0 \tag{19}$$

If $\alpha J < 1$ or $\alpha < 1/J$ then setting the expression in curly brackets equal to zero yields emissions in the NE with emissions taxes (denoted by the superscript t): $E_N^t = \alpha(1-\alpha)/(\beta(1-\alpha J))$, which can be substituted into (3) to give welfare in the NE with emissions taxes. If $\alpha J \geq 1$ or $\alpha \geq 1/J$ then the expression in curly brackets is positive and the derivative is positive for finite quantities of emissions, but the exponential goes to zero as emissions go to infinity. As the quantity of emissions goes to infinity, output and welfare will go to zero. Hence, welfare in the NE with emissions taxes is:

$$W_N^t = \begin{cases} \exp\left[\dfrac{-\alpha(1-\alpha)J}{1-\alpha J}\right]\left(\dfrac{\alpha(1-\alpha)}{\beta(1-\alpha J)}\right)^{\alpha} A L^{1-\alpha} & \alpha < \dfrac{1}{J} \\ 0 & \alpha \geq \dfrac{1}{J} \end{cases} \tag{20}$$

With emissions taxes, when a country increases its emissions then the emissions of other countries decreases, see (17), which increases the incentive for the country to increase its emissions. When $\alpha J \geq 1$, this incentive is so large that emissions go to infinity and welfare goes to zero.

The situation with two countries is shown in Figure 3. Country one is maximising its welfare (represented by the iso-welfare loci) subject to the unilateral equivalence constraint that the emissions tax of country two is equal to $\bar{t} = t_2(E_1, E_2)$, which is downward sloping as shown in (17). The optimum is at II, where the equivalence constraint is tangential to the iso-welfare locus, and this gives a point on the best-reply function of country one with emissions taxes, $R_1^t(E_2)$. Further points on the best-reply function can be obtained by changing $\bar{t}$ and finding the new optimum. The same method can be used to obtain the

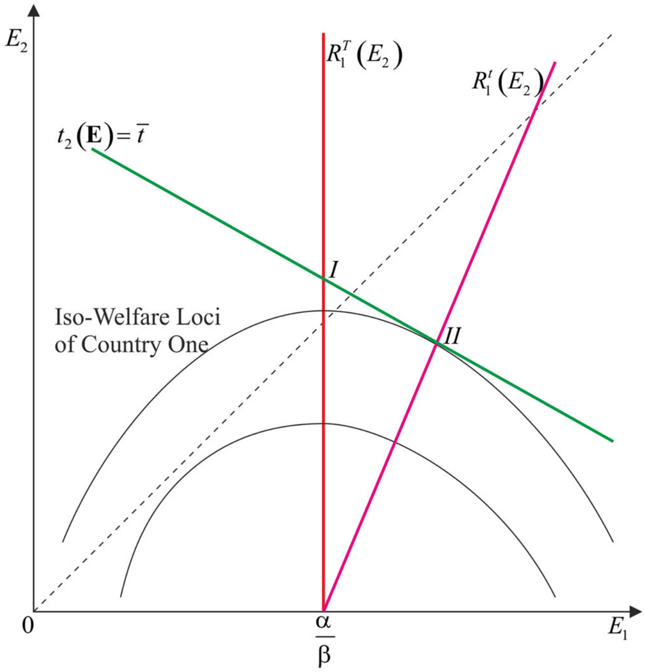

FIGURE 3 | Derivation of the best-reply function with emissions taxes.

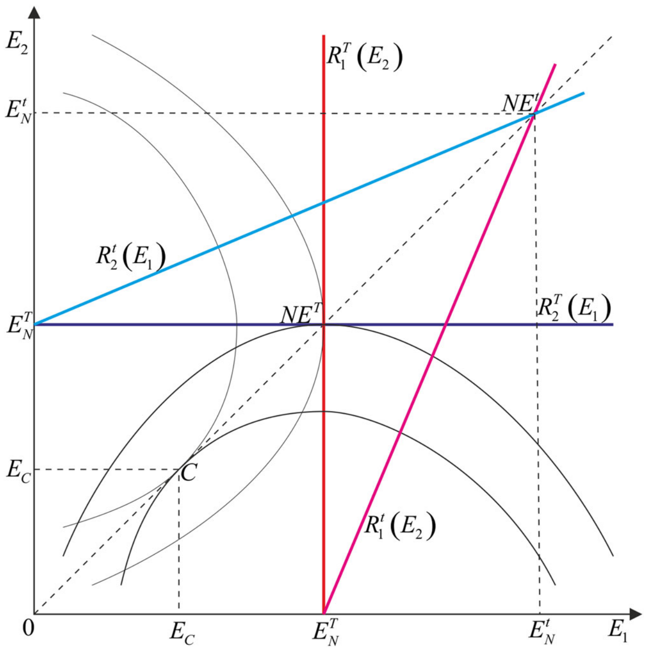

FIGURE 4 | Best-reply functions with emissions taxes.

best-reply function of country two with emissions taxes $R_2^t(E_1)$ as in Figure 4 where the NE with emissions taxes is given by the intersection of the best-reply-functions at NE^t, assuming $\alpha < 1/2$ so that there is an interior solution.

When countries cooperate on the setting of emissions taxes, the outcome in terms of the quantity of emissions will be the same as when countries set the quantity of emissions using tradeable permits. Setting the quantity of emissions in (16) equal to the cooperative quantity of emissions, $E_i = E_j = E_C^T = \alpha/\beta J$, yields the cooperative emissions tax for all the countries: $t_C = \exp[-\alpha]\alpha^\alpha(\beta J)^{1-\alpha}AL^{1-\alpha}$ and welfare will be as in (7) so $W_C^T = W_C^t$.

If when all the other countries are setting their emissions taxes equal to the cooperative emissions tax, t_C, the kth country deviates then it will set E_k to maximise its welfare given that all the other countries are setting $t_j(\mathbf{E}) = t_C$. This yields a first-order condition for welfare maximisation as in (18), which is a function of E_k and E_j for $j \neq k$. After using the constraint $t_j(\mathbf{E}) = t_C$, to obtain $E_k = (\alpha - \beta(J-1)E_j - (1-\alpha)\ln[\beta J E_j/\alpha])/\beta$ substitute this into (18), and then solve for $E_j = E_j^D = (1-\alpha)^2\Omega^*/(\beta(J-1))$, where $\Omega^* = \Omega[\alpha(J-1)/((1-\alpha)^2 J)]$ and $\Omega[\bullet]$ is the Lambert W function, see Corless et al. (1996). Hence, it follows that $\Omega^* > 0$ and increasing in its argument. Consequently, using the constraint, it can be shown that $E_k^D = (\alpha + \alpha(1-\alpha)\Omega^*)/\beta$. Substituting $E_k = E_k^D$ and $E_j = E_j^D$ into the welfare of the kth country yields the welfare from deviation with emissions taxes:

$$W_D^t = \exp[-(\alpha + (1-\alpha)\Omega^*)]\left(\frac{\alpha + \alpha(1-\alpha)\Omega^*}{\beta}\right)^\alpha AL^{1-\alpha} \tag{21}$$

3.2 | Infinitely-Repeated Game

Consider once again the infinitely-repeated game where the one-shot game is repeated infinitely with countries having a common discount factor, δ. With emissions taxes, cooperation in an IEA can be sustained using Nash-reversion trigger strategies if the discount factor is greater than the critical discount factor $\delta_N^t = \left(W_D^t - W_C^t\right)/\left(W_D^t - W_N^t\right) = \left(1 - \left(W_C^t/W_D^t\right)\right)/\left(1 - \left(W_N^t/W_D^t\right)\right)$:

$$\delta_N^t = \begin{cases} \dfrac{1 - \exp[(1-\alpha)\Omega^*](J + (1-\alpha)J\Omega^*)^{-\alpha}}{1 - \exp\left[-\dfrac{(J-1)\alpha}{1-\alpha J} + (1-\alpha)\Omega^*\right]\left(\dfrac{(1-\alpha J)(1 + (1-\alpha)\Omega^*)}{1-\alpha}\right)^{-\alpha}} & \alpha < \dfrac{1}{J} \\ 1 - \exp[(1-\alpha)\Omega^*](J + (1-\alpha)J\Omega^*)^{-\alpha} & \alpha \geq \dfrac{1}{J} \end{cases} \tag{22}$$

Since Ω^* is a function of α and J, the critical discount factor only depends on α and J, and the limit as the number of countries goes to infinity is equal to one.[7]

3.3 | Coalition Formation Game

Consider once again the alternative approach to cooperation of a coalition formation game where countries simultaneously choose whether to join a single IEA, but now with the countries setting emissions taxes rather than tradeable permits. Suppose that H countries join the IEA with all members of the IEA assumed to set the same emissions tax t_h and with the quantity of emissions of each member being E_h. For the purposes of the analysis, the remaining countries that are not members of the IEA consist of the representative country k that sets an emissions tax t_k with its quantity of emissions being E_k, and $J-H-1$ countries that set an emissions tax t_j with their quantity of emissions being E_j. In equilibrium, since all the countries that are not members of the IEA are identical, it will be the case that $t_j = t_k$ and $E_j = E_k$. The H countries that are members of the IEA set their emissions tax to maximise their joint welfare whereas the $J-H$ that are not members of the IEA set their emissions taxes non-cooperatively to maximise their own welfare.

First, consider the decision of the representative country k i.e. not a member of the IEA. The welfare of country k as a function of the quantities of emissions is given by:

$$W_k(\mathbf{E}) = \exp[-\beta(E_k + (J-H-1)E_j + HE_h)]AL^{1-\alpha}E_k^{\alpha} \tag{23}$$

Country k maximises its welfare given the emissions taxes set by the other non-members of the IEA and the emissions tax set by the members of the IEA. As in (16), the emissions taxes t_j and t_h are equal to the marginal products of emissions in the countries j and h, respectively:

$$\begin{aligned} t_j(\mathbf{E}) &= \exp[-\beta(E_k + (J-H-1)E_j + HE_h)]\alpha AL^{1-\alpha}{E_j}^{\alpha-1} \\ t_h(\mathbf{E}) &= \exp[-\beta(E_k + (J-H-1)E_j + HE_h)]\alpha AL^{1-\alpha}{E_h}^{\alpha-1} \end{aligned} \tag{24}$$

Country k sets its quantity of emissions to maximise its welfare (23) given the equivalence constraints (24) implied by the emissions taxes set by the other countries. Hence, the first-order condition for country k maximising welfare (23) subject to the equivalence constraints (24) is:

$$\frac{dW_k}{dE_k} = \frac{\partial W_k}{\partial E_k} + \frac{\partial W_k}{\partial E_j}\frac{dE_j}{dE_k} + \frac{\partial W_k}{\partial E_h}\frac{dE_h}{dE_k} = 0 \tag{25}$$

Totally differentiating the constraints with respect to the quantities of emissions yields the effect of an increase in the emissions of country k on the emissions of the other countries:

$$\begin{aligned} \frac{dE_j}{dE_k} &= \frac{-\beta E_j}{1-\alpha+\beta HE_h+\beta(J-H-1)E_j} < 0 \\ \frac{dE_h}{dE_k} &= \frac{-\beta E_h}{1-\alpha+\beta HE_h+\beta(J-H-1)E_j} < 0 \end{aligned} \tag{26}$$

Using (26) to evaluate (25) and noting that $E_j = E_k$ in equilibrium, yields:

$$\frac{dW_k}{dE_k} = \Lambda_k\{\alpha(1-\alpha) - (1-\alpha(J-H))\beta E_k + \alpha\beta HE_h\} = 0 \tag{27}$$

where $\Lambda_k \equiv \frac{\exp[-\beta(HE_h+(J-H)E_k)]AL^{1-\alpha}{E_k}^{\alpha-1}}{1-\alpha+\beta HE_h+\beta(J-H-1)E_k} \geq 0$.

Second, consider the decision of the representative country h that is a member of the IEA. There is no longer a need to distinguish between the $J-H$ non-members of the IEA so they will all be denoted by k with $E_j = E_k$. The welfare of country h as a function of the quantities of emissions is given by:

$$W_h(\mathbf{E}) = \exp[-\beta((J-H)E_k + HE_h)]AL^{1-\alpha}E_h^{\alpha} \tag{28}$$

The countries in the IEA maximise the welfare of the representative country h given the emissions taxes set by the non-members of the IEA. The emissions tax set by the non-members of the IEA is equal to their marginal product of emissions as in (16):

$$t_k = \exp[-\beta((J-H)E_k + HE_h)]\alpha AL^{1-\alpha}E_k^{\alpha-1} \tag{29}$$

The IEA sets the quantity of emissions for each member to maximise the welfare of the representative country h (28) given the equivalence constraint (29) implied by the emissions taxes set by the non-members of the IEA. Hence, the first-order condition for country h maximising welfare subject to the equivalence constraints is:

$$\frac{dW_h}{dE_h} = \frac{\partial W_h}{\partial E_h} + \frac{\partial W_h}{\partial E_k}\frac{dE_k}{dE_h} = 0 \tag{30}$$

Totally differentiating the constraint with respect to the quantities of emissions yields the effect of an increase in the emissions of country h on the emissions of the other countries:

$$\frac{dE_k}{dE_h} = \frac{-\beta HE_k}{1-\alpha+\beta(J-H)E_k} < 0 \tag{31}$$

Using (31) to evaluate (30) yields:

$$\begin{aligned} \frac{dW_h}{dE_h} = \Lambda_h\{\alpha(1-\alpha) + \alpha\beta(J-H)E_k - (1-\alpha) \\ \beta HE_h\} = 0 \end{aligned} \tag{32}$$

where $\Lambda_h \equiv \frac{\exp[-\beta(HE_h+(J-H)E_k)]AL^{1-\alpha}{E_h}^{\alpha-1}}{1-\alpha+\beta(J-H)E_k} \geq 0$.

Assuming an interior solution where the quantities of emissions are finite, the welfare derivatives (27) and (32) will be equal to zero and, since Λ_k and Λ_h will be positive, the expressions in curly brackets will both be equal to zero. Subtracting one from the other yields that $E_k = HE_h$, and substituting this into (27) yields that:

$$\frac{dW_k}{dE_k} = \Lambda_k\{\alpha(1-\alpha) - (1-\alpha(J-H+1))\beta E_k\} = 0 \tag{33}$$

If $\alpha < 1/(J-H+1)$ or $H > \tilde{H} \equiv J-(1-\alpha)/\alpha \leq J$ then setting the expression in curly brackets equal to zero yields the quantity of emissions of a nonmember of the IEA: $E_k = E_O^t = \alpha(1-\alpha)/\beta(1-\alpha(J-H+1))$, and that of a member of the IEA: $E_h = E_I^t = E_k/H$.[8] If $\alpha \geq 1/(J-H+1)$ or $H \leq \tilde{H}$ then the expression in curly brackets will be positive for

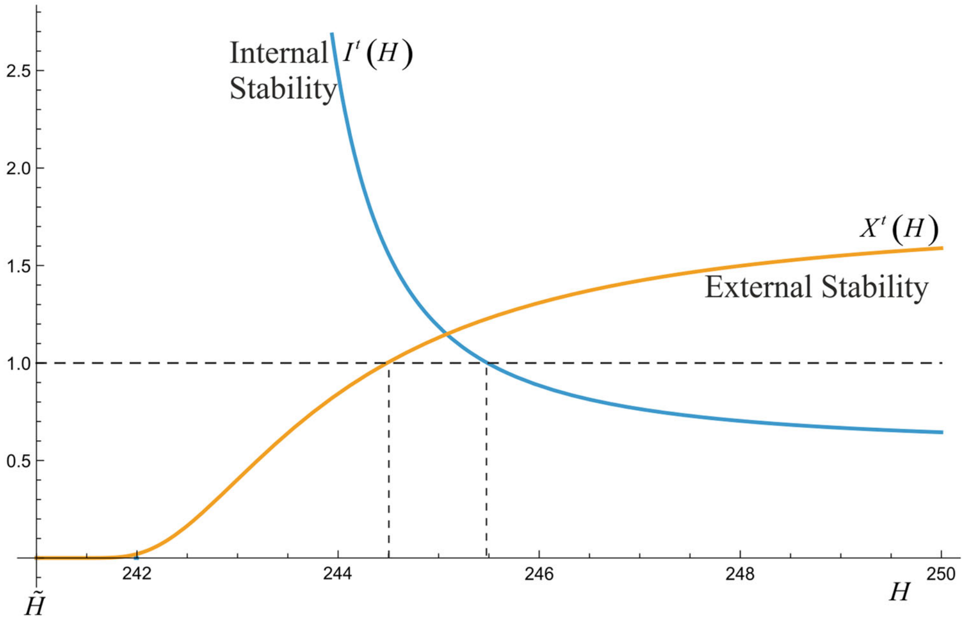

FIGURE 5 | Internal and external stability of iea with emissions taxes.

finite quantities of emissions, but the exponential function in Λ_k will go to zero if the quantities of emissions goes to infinity and the derivative will be equal to zero. As the quantity of emissions goes to infinity then outputs and the welfare of members and non-members of the IEA will go to zero. Substituting these quantities into the welfare of countries h and k yields the welfare of a member and that of a nonmember of the IEA as a function of the number of countries in the IEA:

$$W_I^t(H) = \begin{cases} 0 & H \leq \tilde{H} \\ \exp\left[-\dfrac{\alpha(1-\alpha)(J-H+1)}{1-\alpha(J-H+1)}\right] & H > \tilde{H} \\ \left(\dfrac{\alpha(1-\alpha)}{\beta H(1-\alpha(J-H+1))}\right)^{\alpha} AL^{1-\alpha} & \end{cases}$$
$$W_O^t(H) = \begin{cases} 0 & H \leq \tilde{H} \\ \exp\left[-\dfrac{\alpha(1-\alpha)(J-H+1)}{1-\alpha(J-H+1)}\right] & H > \tilde{H} \\ \left(\dfrac{\alpha(1-\alpha)}{\beta(1-\alpha(J-H+1))}\right)^{\alpha} AL^{1-\alpha} & \end{cases} \tag{34}$$

To determine the equilibrium of the coalition game, consider internal and external stability as a function of the number of countries in the IEA. Define the floor function $\lfloor \bullet \rfloor$ as giving the largest integer less than or equal to its argument. For $H \leq \lfloor \tilde{H} \rfloor - 1$, internal stability is satisfied since $W_I^t(H) = 0 \geq W_O^t(H-1) = 0$ and external stability is satisfied since $W_O^t(H) = 0 \geq W_I^t(H+1) = 0$. Thus, there could be a stable coalition but the welfare of all countries would be equal to zero. For $H = \lfloor \tilde{H} \rfloor$, internal stability is satisfied since $W_I^t(H) = 0 \geq W_O^t(H-1) = 0$ but external stability is not satisfied since $W_O^t(H) = 0 < W_I^t(H+1)$. For $H \geq \lfloor \tilde{H} \rfloor + 1$, there are two cases to consider. First, if $\alpha < 1/2$ which implies that $\tilde{H} + 1 < J$, internal stability requires that $I^t(H) = W_I^t(H)/W_O^t(H-1) \geq 1$ and external stability requires that $X^t(H) = W_O^t(H)/W_I^t(H+1) \geq 1$. Since $I^t(H)$ is continuous and decreasing in H for $H > \tilde{H} + 1$ and $I^t(\tilde{H}+1)$ goes to infinity, either $I^t(H^*) = 1$ for some unique $H^* \in (\tilde{H}+1, J)$ or $I^t(J) \geq 1$. By definition, if $I^t(H^*) = 1$ then $X^t(H^*-1) = 1$ and hence the size of the stable IEA is $\lfloor H^* \rfloor$ where there is internal and external stability of the coalition, and the welfare of all countries is positive. If $I^t(J) \geq 1$ then an IEA consisting of all countries is stable as there is internal stability and external stability is irrelevant when all countries are members of the IEA. Second, if $\alpha \geq 1/2$ which implies that $\tilde{H} + 1 \geq J$ then there is internal stability with all countries in the coalition since $W_I^t(J) > W_O^t(J-1) = 0$. Since external stability is not relevant if all countries are in the coalition, an IEA consisting of all countries is a stable coalition for $\alpha \geq 1/2$. Such an IEA will yield the first-best outcome for the world. Hence, if $\alpha < 1/J$ then the IEA consists of between one and J while if $1/J < \alpha < 1/2$ then it consists of between $\lfloor \tilde{H} \rfloor + 1$ and J countries whereas if $\alpha \geq 1/2$ then the IEA consists of all countries.

The conditions for internal and external stability versus the number of countries in the IEA are shown in Figure 5 for the case of 250 countries and $\alpha = 1/10$.

For the case when there are 250 countries in the world, Figure 6 shows $\tilde{H}$, which is shown in green, and a lower bound for the size of a stable IEA given by numerically solving $X^t(\underline{H}) = 1$ to get $\underline{H}$, which is shown in blue, for $\alpha \in [0, 1/2]$. It can be seen that almost all countries are members of the stable IEA for $\alpha \geq 1/10$.

4 | Tradeable Permits Versus Emissions Taxes

Having analysed the one-shot, infinitely-repeated, and coalition-formation games using this common model for both tradeable permits and emissions taxes, the outcome of the games can now be compared.

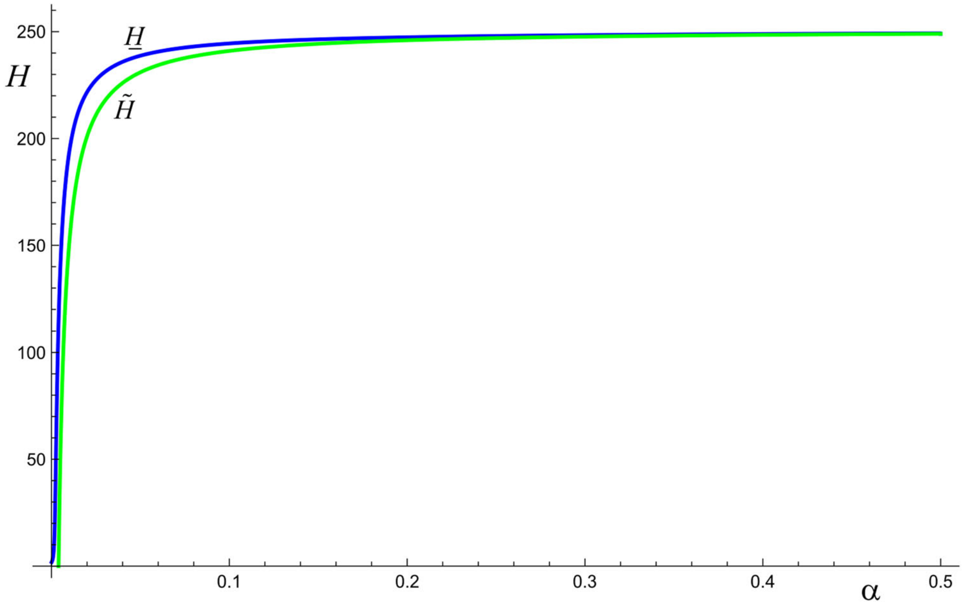

FIGURE 6 | Number of countries in iea with emissions taxes.

4.1 | One-Shot Game

For the static game, emissions are lower with tradeable permits than with emissions taxes since $E_N^T = \alpha/\beta < E_N^t = \alpha(1-\alpha)/(\beta(1-\alpha J))$ if $\alpha < 1/J$ and since E_N^t goes to infinity if $\alpha \geq 1/J$ so is obviously larger than $E_N^T = \alpha/\beta$. The two-county case is shown in Figure 4 for the case when $\alpha < 1/2$, where NE^T is the NE with tradeable permits and NE^t is the NE with emissions taxes. Clearly, welfare is higher for both countries in the NE in tradeable permits than in the NE in emissions taxes. With many countries, welfare in the NE with emissions taxes (20) relative to that with tradeable permits (5) is:

$$\frac{W_N^t}{W_N^T} = \begin{cases} \exp\left[\dfrac{-\alpha^2 J(J-1)}{1-\alpha J}\right]\left(\dfrac{1-\alpha}{1-\alpha J}\right)^{\alpha} & \alpha < \dfrac{1}{J} \\ 0 & \alpha \geq \dfrac{1}{J} \end{cases} \tag{35}$$

This leads to the following proposition:

Proposition 1. *In the static game, welfare is higher in the NE with tradeable permits than with emissions taxes, $W_N^T > W_N^t$.*

Proof: For the case when $\alpha \geq 1/J$, since $W_N^t/W_N^T = 0$ it is obvious that $W_N^T > W_N^t = 0$. For the case when $\alpha < 1/J$, taking logs and defining $z \equiv (1-\alpha J)/(1-\alpha) \in (0,1)$ yields that $\ln[W_N^t/W_N^T] = \alpha J(1 - 1/z - \ln[z]/J) < \alpha J(1 - 1/z - \ln[z]) < 0$ and the final term in brackets is negative for $z \in (0,1)$. Hence, $W_N^t/W_N^T < 1$ so obviously $W_N^T > W_N^t$.∎

This result will hold for any game where the production externality is negative and the slope of the equivalence constraint is negative. In contrast, tradeable permits and emissions taxes are welfare equivalent in Harstad et al. (2022) because their welfare function is additively separable in the emissions of the other countries, which would imply that the slope of the equivalence constraint (17) is equal to zero and hence the best-reply function is the same whether other countries use tradeable permits or emissions taxes.

4.2 | Two-Stage Game

Consider an extension of the one-shot game to a two-stage game where the countries each choose whether to use tradeable permits or emissions taxes in stage one and then choose the quantity of emissions or emissions tax in stage two. Suppose that M countries choose to use emissions taxes at stage one then these countries choose a quantity of emissions E_t in stage two while the other $J-M$ countries choose to use tradeable permits at stage one and then choose a quantity of emissions E_T in stage two. Using the same methods as in previous sections, assuming an interior solution where $\alpha < 1/M$, it is readily shown that the quantity of emissions of a country that uses an emissions tax is $E_t = \alpha(1-\alpha)/\beta(1-\alpha M)$ and for a country that uses tradeable permits is $E_T = \alpha/\beta(1-\alpha M)$ so $E_t = (1-\alpha)E_T$. If $\alpha \geq 1/M$ then the quantity of emissions of all countries goes to infinity and the welfare of all countries goes to zero. Hence, the welfare of a country that uses an emissions tax and of a country that uses a tradeable permit as functions of M are:

$$\begin{aligned} W_t(M) &= \begin{cases} \exp\left[\dfrac{-\alpha(J-\alpha M)}{1-\alpha M}\right] & \alpha < \dfrac{1}{M} \\ \left(\dfrac{\alpha(1-\alpha)}{\beta(1-\alpha M)}\right)^{\alpha} A L^{1-\alpha} & \\ 0 & \alpha \geq \dfrac{1}{M} \end{cases} \\ W_T(M) &= \begin{cases} \exp\left[\dfrac{-\alpha(J-\alpha M)}{1-\alpha M}\right] & \alpha < \dfrac{1}{M} \\ \left(\dfrac{\alpha}{\beta(1-\alpha M)}\right)^{\alpha} A L^{1-\alpha} & \\ 0 & \alpha \geq \dfrac{1}{M} \end{cases} \end{aligned} \tag{36}$$

At stage one of the game, each country can choose whether to use an emissions tax or a tradeable permit. If M countries have

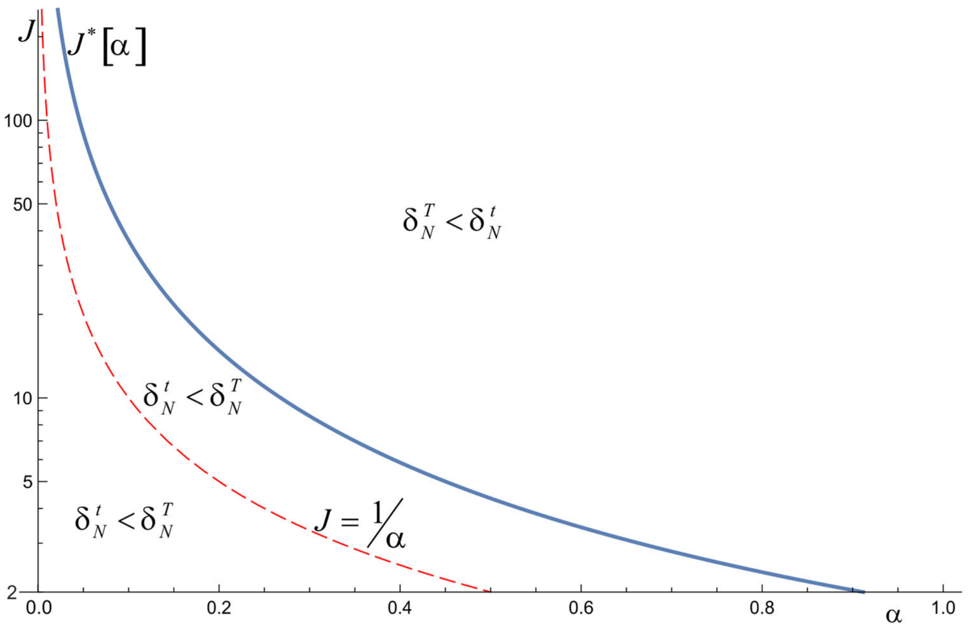

FIGURE 7 | The critical number of countries.

chosen to use emissions taxes then a country that has chosen to use emissions taxes will change to using tradeable permits if $W_T(M-1) > W_t(M)$ or in the case of an interior solution where $\alpha < 1/M$:

$$\Pi \equiv \frac{W_T(M-1)}{W_t(M)} = \exp\left[\frac{-(J-1)\alpha^2}{(1-\alpha(M-1))(1-\alpha M)}\right]\left(\frac{1-\alpha M}{(1-\alpha)(1-\alpha(M-1))}\right)^{\alpha} > 1 \tag{37}$$

This is greater than one when $M = 1$ since $\Pi = \exp[(J-1)\alpha^2/(1-\alpha)] > 1$. Taking logs and differentiating (37) yields that:

$$\frac{\partial \ln \Pi}{\partial M} = \frac{\alpha^3((2J + \alpha(M-1) - 3)(1-\alpha M) + \alpha(J-1))}{(1-\alpha(M-1))^2(1-\alpha M)^2} > 0 \tag{38}$$

Since $\Pi > 1$ when $M = 1$ and it is increasing in M, choosing to use tradeable permits is a strictly dominant strategy for all countries if there is an interior solution where $\alpha < 1/J$. If $\alpha \geq 1/J$ then emissions go to infinity and welfare goes to zero for some M so $W_T(M-1) \geq W_t(M) = 0$, and choosing to use tradeable permits is a weakly dominant strategy. This leads to the following proposition:

Proposition 2. *In the two-stage game, choosing tradeable permits rather than emissions taxes is a dominant strategy for all countries.*

All countries will choose to use tradeable permits rather than emissions taxes, and be better off in the NE in tradeable permits than in the NE in emissions taxes. This result contrasts with that of Kiyono and Ishikawa (2013) who show that there are equilibria where one country uses emissions taxes while the other country uses tradeable permits. Although there are similarities between the two models, there are also significant differences such as international trade in the polluting good and asymmetries between countries in Kiyono and Ishikawa (2013), which may explain the different results.

4.3 | Infinitely-Repeated Game

For the infinitely-repeated game, an analytical comparison of the critical discount factor with tradeable permits (9) and with emissions taxes (22) is not tractable, but the two discount factors are both functions of only α and J. Hence, without any loss of generality, numerically solving $\delta_N^T = \delta_N^t$ yields the critical value for the number of countries, $J^*[\alpha]$, where the two discount factors are equal, which is shown in Figure 7.

Proposition 3. *The critical discount factor required to sustain cooperation is lower (higher) with tradeable permits than with emissions taxes if the number of countries* $J > (<) J^*[\alpha]$.

Cooperation in an IEA is sustained when deviation by a country is deterred by the threat of reversion to the NE forever after any deviation. The welfare from deviation is increasing in the number of countries with both instruments, but it is larger with an emissions tax than with tradeable permits. Whereas welfare in the NE is decreasing in the number of countries (when $J < 1/\alpha$ for emissions taxes), but it is lower with an emissions tax than with tradeable permits. Therefore, due to these two opposing forces, the effect of the number of countries on the critical discount factor is ambiguous. When $J > 1/\alpha$, welfare in the NE with emissions taxes is zero, $W_N^t = 0$, which means that it is no longer decreasing in the number of countries, but the welfare from deviation is still increasing in the number of countries. With tradeable permits, welfare from deviation is still increasing while welfare in the NE is still decreasing in the number of countries. Hence, for a sufficiently large number of

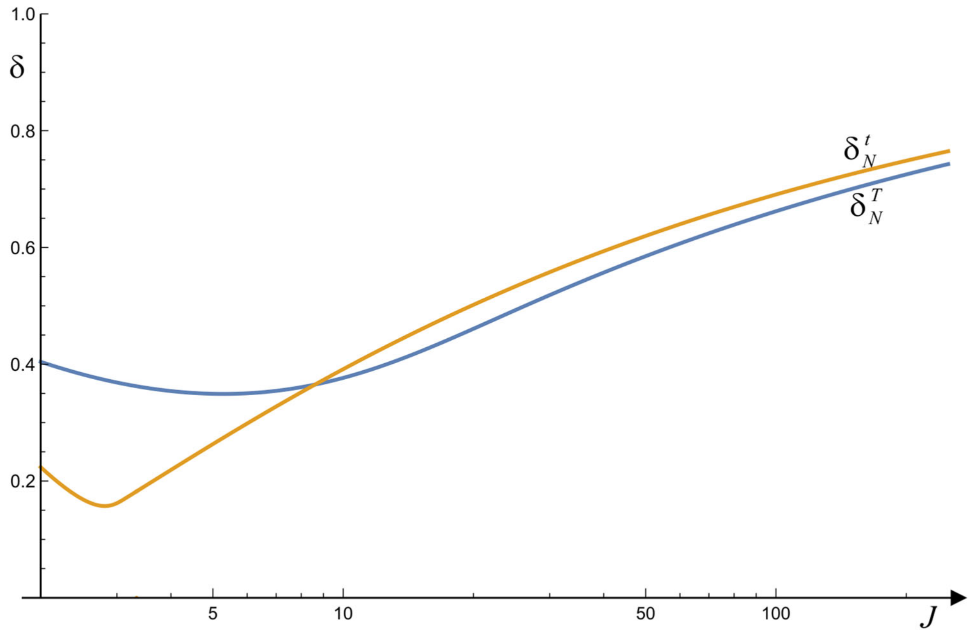

FIGURE 8 | The critical discount factor versus the number of countries.

countries, $J > J^*[\alpha]$, the critical discount factor is lower with tradeable permits than with emissions taxes.

To illustrate the results, Figure 8 shows the critical discount factors with tradeable permits and emissions taxes as functions of the number of countries, J, for the case when $\alpha = 3/10$.

4.4 | Coalition-Formation Game

For the coalition-formation game, the size of the stable IEA is just two countries when countries use tradeable permits, whereas it is between one and all countries when countries use emissions taxes. This leads to the following proposition:

Proposition 4. In the coalition-formation game, the stable IEA consists of two countries with tradeable permits whereas it consists of $H^* \in [1, J]$ countries with emissions taxes and $H^* = J$ countries if $\alpha \geq 1/2$.

With tradeable permits, welfare as a member or a nonmember of the IEA is positive regardless of the size of the IEA, whereas welfare as a member or nonmember of the IEA is zero if the number of countries is sufficiently large with emissions taxes. As a result, with emissions taxes, countries have nothing to lose from joining an IEA without a sufficiently large number of members and will continue to join the IEA until welfare is positive when the number of members is sufficiently large, $H > \tilde{H}$. The larger the share of emissions in production, α, then the stronger the incentive to increase emissions and the more members are needed in the IEA to lessen this incentive thereby avoiding emissions going to infinity. In contrast to the infinitely-repeated game, the possibility of zero welfare with emissions taxes leads to more cooperation than with tradeable permits in this coalition-formation game.

Although the welfare ranking of IEAs with tradeable permits and emissions taxes cannot be inferred simply by the number of countries in a stable IEA, if all countries are in a stable IEA then it can obtain the first-best outcome in terms of welfare. Hence, if $\alpha \geq 1/2$ then an IEA where countries use emissions taxes will yield the first-best outcome and will be superior to an IEA where countries use tradeable permits. Note that in Eichner and Pethig (2015) a stable IEA consisting of all countries is more likely the smaller the total number of countries whereas here if $\alpha \geq 1/2$ then an IEA consisting of all countries will be stable for any total number of countries in the world.

5 | Conclusions

Emissions taxes and tradeable permits have been compared using the same model of global emissions in a number of non-cooperative and cooperative games. In non-cooperative games, tradeable permits dominate emissions taxes and countries are better off in the NE in tradeable permits than the NE in emissions taxes. In infinitely-repeated games, it is easier to sustain cooperation in an IEA when countries use tradeable permits than when they use emissions taxes if the number of countries is sufficiently large. In the coalition formation game, it is possible to have a stable IEA consisting of all countries when countries use emission taxes whereas a stable IEA consists of two countries when countries use tradeable permits. In that case, the IEA when countries use emissions taxes yields the first best in terms of welfare and it is superior to a stable IEA when countries use tradeable permits.

One may question the functional form used for the welfare function with an exponential function to represent the global production externality. Obviously, the advantage of this functional form is its tractability that allows solutions to be obtained for all four games analysed for both tradeable permits and emissions taxes. An alternative functional form that could be used to represent the externality is $\left(1 - (\beta/\theta) \sum_{j=1}^{J} E_j\right)^\theta$ rather than $\exp\left[-\beta \sum_{j=1}^{J} E_j\right]$ where $\theta > 0$ with $\theta = 1$ giving a linear function and this function going to the exponential function as

θ goes to infinity. This alternative function for the externality is less tractable than the exponential function, but yields fairly similar qualitative results (available on request from the authors). For the one-shot game, the comparison between tradeable permits and emissions taxes can be generalised as it only depends upon the production externality being negative and the slope of the equivalence constraint being negative. One may also question whether the externality should be a consumption externality rather than a production externality. With a consumption externality, the analysis is unchanged if the emissions tax is expressed in a way that makes it the same in real terms as with the production externality. The assumption of perfect competition rather than imperfect competition may also be questioned. Perfect competition makes the analysis tractable as it is important for the equivalence constraint, which may not hold with imperfect competition, and this may be a topic for future research. Another possible topic for future research would be to introduce uncertainty into the model, but this would add much complexity into the analysis.

Acknowledgments

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Data Availability Statement

Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

Endnotes

[1] According to ICAP (2025), 38 emissions trading systems (ETS) are in force and a further 20 are in development covering various jurisdictions ranging from cities to a supra-national body, the European Union, with jurisdictions covering 58% of global GDP using emissions trading.

[2] See Barrett (2005) for an extensive survey of the earlier literature.

[3] With a closed emissions trading system, Böhringer and Lange (2005) show that grandfathering schemes that allocate tradeable permits proportionally based upon past emissions are first-best like auctions. There is no trade in tradeable permits between countries in a closed emissions trading system.

[4] Most of the analysis is easily extended to consider asymmetric countries that differ in terms of the labour endowment and the technology parameter with little change in the results, but at the expense of notational complexity.

[5] Usually, asymmetries in the size of countries would be expected to make sustaining co-operation more difficult in the sense that the critical discount factor would be higher. In this case, introducing asymmetries in terms of the labour endowment and the technology parameter would not affect the critical discount factor provided that the quantity of emissions is the same for all countries in the cooperative outcome, which will be the case if the countries are maximising a weighted sum of their welfare functions, where the weights are inversely related to the size of the countries. Obviously, such a cooperative outcome is Pareto efficient.

[6] This is similar to the method used by Vives (1985) to compare prices under Bertrand and Cournot oligopoly with differentiated products, and by Azacis and Collie (2025) to compare specific and ad valorem trade taxes in a trade war. Kiyono and Ishikawa (2004, 2013) call the constraint the unilateral equivalence constraint.

[7] Again, asymmetries in country size do not affect the critical discount factor if the quantity of emissions is the same for all countries in the cooperative outcome.

[8] Note that $\tilde{H}$ is less than one if $\alpha < 1/J$ and therefore in that case H is always greater than $\tilde{H}$.

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