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3D topological modeling and multi-agent movement simulation for viral infection risk analysis

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ABSTRACT

This paper introduces an integrated method that combines computer-aided modelling of indoor environments, multi-agent movement simulation and airborne viral transmission modelling to analyse how spatial design and occupant behaviour affect disease spread. Using TopologicPy, interior spaces are represented as connected networks that support navigation-graph generation and agent movement based on schedules, walking speeds and activities. Agents move incrementally along shortest paths, while the system calculates precise inter-agent distances and respects architectural constraints such as walls and doorways. Viral aerosol concentrations are modelled via a reaction-diffusion equation, and infection risk is estimated using an extended Wells–Riley model. By capturing detailed spatio-temporal and topological interactions, the framework offers realistic infection-risk assessments. The resulting tool serves as a rapid decision-support system for policymakers, facility managers and designers, enabling evaluation of mitigation strategies and informing future building design. A comparative study of cellular and open-plan offices demonstrates the method's capabilities.

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Computer-aided modelling; spatial design and analysis; topological analysis; navigation graphs; multi-agent movement simulation; aerosol distribution; viral transmission; infection risk

1. Introduction

The COVID-19 pandemic has prompted researchers to study how the configuration of indoor spaces and the activities taking place within them affect the risk of viral transmission (Lau et al. 2022; Lu et al. 2020; Morawska and Cao 2020; Riley, Murphy, and Riley 1978; Tong, Li, and Kang 2022). Lessons learned from the pandemic have prompted new architectural design guidelines for infection prevention and control (Emmanuel, Osondu, and Kalu 2020).

Historically, major disasters have often led to significant changes in building regulations. For instance, the Great Fire of London in 1666, which destroyed a large portion of the city, prompted the implementation of stricter fire safety standards, including the use of brick and stone in construction instead of wood, and the creation of wider streets to prevent the rapid spread of future fires (Alagna 2004). Another example is the 1906 San Francisco earthquake and subsequent fire, which led to improvements to the water supply infrastructure and the development of more stringent building codes to improve structural integrity and earthquake resistance (Cutcliffe 2000).

Similar to earlier outbreaks such as the spread of measles (Riley, Murphy, and Riley 1978), the COVID-19 outbreak has revealed the link between building attributes and infection risk (Tong, Li, and Kang 2022). This highlights the urgent need to analyse building designs and develop decision-support tools for policymakers, stakeholders, architects, and engineers to help them design and implement changes that mitigate infection risk. More specifically, there is a need for simulation models that balance the requirements for accuracy and speed of execution, take into consideration complex geometries, simulate people movement according to daily schedules and event locations, and visualize scenarios and the resulting infection risk in a manner easily accessible to stakeholders.

Modelling airborne infection risk in indoor environments is inherently complex due to the dynamic interplay between spatial configuration, human behaviour, and environmental factors such as ventilation and occupancy patterns. Traditional tools often focus on isolated aspects, such as airflow dynamics through computational fluid dynamics (CFD) or people movement through crowd simulation but rarely integrate these dimensions in a way that reflects the day-to-day functioning of occupied spaces. The challenge lies in capturing this complexity at a resolution that is both meaningful and computationally manageable, while remaining accessible to designers and decision-makers without specialized technical expertise. Furthermore, each building is unique in its layout, use, and operation, making generalized solutions difficult to apply in practice. These factors help explain why there has been limited progress in creating practical tools that support early-stage design decisions or ongoing building operations in the context of infection risk.

To address these needs, this research aims to create an accessible web-based decision-support tool for optimizing building designs to enhance safety and health in response to current and future pandemics. The tool is intended for use during the design stage, where architects and engineers can evaluate and compare alternative spatial layouts, circulation patterns, and room configurations to minimize infection risk. It can also be applied during

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the operational phase of a building's lifecycle. Facility managers and policymakers can use the platform to simulate daily activities, assess occupancy schedules, evaluate ventilation strategies, and implement targeted non-pharmaceutical interventions (NPIs) to improve performance under changing conditions.

To achieve this aim, the study integrates computer-aided three-dimensional (3D) modelling of spaces, navigational graphs, agent-based simulation, and mathematical models of viral concentration and infection risk. The underlying hypothesis is that this integrated approach which combines realistic representations of indoor environments, multi-agent movement simulation, and spatio-temporal mathematical models, enables more accurate analysis of infection risk, and more effectively supports informed decision-making in building design.

2. Related work

The work in this paper lies at the intersection of non-manifold topological modelling, spatial graph and space syntax analysis, multi-agent simulation, and mathematical modelling of airborne viral transmission. In this section, we briefly review prominent related literature within these strands and clarify how the present paper extends and connects them.

Various methodologies and theoretical frameworks inform the analysis of indoor space design and human movement in the context of viral transmission. By examining these diverse yet interconnected fields, the aim is to provide a robust foundation for understanding and mitigating the risk associated with viral transmission in indoor environments. Related areas of research include non-manifold topological modelling (Weiler 1986), dual graphs (Whitney 1931), space syntax (Hillier and Hanson 1984), multi-agent simulation (Hewitt, Bishop, and Steiger 1973), and viral transmission modelling (Bernoulli and Chapelle 1760; Riley, Murphy, and Riley 1978; Wells 1955). It is important to note, that although viral transmission modelling plays a significant role in formulating the theoretical underpinnings and quantitative predictions of this research, a detailed exposition is beyond the primary scope of this paper and is provided in a companion study (Xue et al. 2024). The present paper focuses on the spatial, topological, and multi-agent simulation infrastructure upon which the companion paper by Xue et al. builds its research on disease transmission modelling, policy evaluation, and decision support. The companion paper details the VIRIS epidemiological simulator, including reaction – diffusion modelling of airborne transmission, spatiotemporal infection risk estimation, non-pharmaceutical intervention (NPI) evaluation, real-world validation against a superspreader event, and the deployment of a public-facing web application. The two papers are therefore complementary but non-overlapping in scope: the present paper is methodological and architectural – computational in focus, whereas the companion paper is epidemiological and application-driven.

2.1. Non-manifold topological representations

Most 3D modelling software relies on an underlying modelling 'engine' to provide needed computations to compose and edit 3D geometry. These engines are usually classified as either manifold or non-manifold (Chatzivasileiadi et al. 2018). Manifold Topology (MT), or more precisely 2-manifold topology, refers to bodies without internal voids that can be fabricated from a single block of material. A key property of manifold objects is that a sphere drawn cantered on any point of the object's surface will be divided into two distinct regions: one inside and one outside the object. This clear division is a fundamental characteristic that makes manifold bodies manufacturable and physically realizable.

However, manifold modelling lacks the direct ability to derive adjacency information or the presence of shared entities as each manifold object exists independently of other objects. While commercial 3D modelling software systems that rely on manifold engines are readily available for sophisticated geometric modelling, they fall short as extendable simulation engines as they do not usually model topological connections within indoor spaces, and do not simulate people movement, making it challenging to meet the outlined needs.

In contrast, Non-Manifold Topology (NMT) is defined mathematically as cell complexes within Euclidean space, representing spatial relationships between geometric entities (Masuda 1993). Practically, NMT allows any combination of vertices, edges, surfaces, and volumes to coexist within a single logical body, where multiple cells can share faces, faces can meet at an edge, and multiple edges can converge at a vertex. Coincident vertices, edges, and faces are merged.

While NMT results in configurations that cannot be physically realizable, this characteristic provides the direct ability to find adjacencies between regions or spaces, enabling the construction of 'dual' navigational graphs that can be easily traversed and searched for the shortest paths (Dijkstra 1959). A dual graph is a graph that represents the spatial or topological adjacency between elements of a geometric structure. Each region (e.g. a room or volume) of the original graph is represented as a node in the dual graph. An edge exists between two nodes in the dual graph if their corresponding regions in the original graph share a common boundary. These links can have various weights or attributes assigned to them. This network can be employed for indoor navigation, where paths between two points can be determined using graph algorithms. For example, non-manifold topology and dual graphs have been used to build an emergency response tool that accurately computes egress paths in case of an emergency (Boguslawski et al. 2016).

The present paper builds directly on this line of work by embedding a non-manifold representation within the TopologicPy environment and using it to derive adjacency and dual graphs, and generate explicit navigation graphs that respect walls, doors, and furniture. Unlike most prior NMT applications, which focus on distance to egress or static connectivity, we use NMT as the backbone of a full movement-simulation pipeline, connecting geometric/topological structure to time-dependent agent movement, and to infection-risk analysis in the companion paper.

2.2. Graph theory and space syntax

Airborne viral transmission in indoor environments requires the simulation of how people within that environment move and interact with each other. A person's physical proximity and duration of interaction with another infected person directly affects their risk of contracting an airborne virus. Whitehead and Eldars, for example, empirically mapped the movements of nurses within a theatre suite for one day's duty creating a 'string' diagram showing the routes taken and the heaviest concentrations of movements (Whitehead and Eldars 1965). Their approach aggregates the movements over time to arrive at the heaviest trafficked areas. In contrast, the approach in this study calculates viral concentration at each simulation time step taking into consideration the location of agents at that time step.

The need to understand and map how people use and navigate the built environment has developed into the well-established field of space syntax (Hillier et al. 1976). The original definition of space syntax constitutes objects, relations, and operations with two kinds of entities: a solid entity (obstacles), and a vacant entity through which movement is possible. Given this framework, space syntax evolved to modelling and analysing the built environment, road networks, and whole cities (Porta, Crucitti, and Latora 2006). It has also moved beyond 2D representations to analysing 3D configurations (Varoudis and Psarra 2014). Through graph-theoretical approaches, space syntax measures have shown correlations with human spatial behaviour and have, thus, been used to predict the impacts of architectural and urban spaces on users.

Our work is aligned with this tradition in treating architecture as a graph-structured spatial system but differs in two important ways. First, rather than using ad hoc methods to construct axial or visibility graph or use lower resolution grid-based methods, we leverage NMT to directly to construct navigation graphs from detailed 3D geometry. We also offer an alternative method for larger and more complex plans that relies on trimmed Delaunay triangulation. Regardless of the method, the resulting graph has identical features and capabilities. We ensure that paths explicitly respect obstacles, door openings, and furniture. Second, we couple these graphs to an event – schedule – agent system and a reaction – diffusion infection model (in the companion paper), moving from static configurational analysis to dynamically simulated, time-resolved contact and exposure patterns. In this sense, our framework can be viewed as a bridge between space-syntax-style configurational analysis and epidemiological risk modelling.

2.3. Multi-agent simulation

Multi-agent simulations model the behaviour and interactions of individuals moving within a shared space. This approach helps in predicting movement patterns and contact points that could enhance or mitigate airborne viral transmission. One of the foundational works in agent-based systems is Carl Hewitt's 'Actor Model' introduced in 1973 (Hewitt, Bishop, and Steiger 1973). The Actor Model conceptualized independent entities (actors) that interact with each other through message passing. This model laid the groundwork for concurrent computation and influenced the development of agent-based systems.

In 1996, Epstein and Axtell introduced the concept of agent-based computer programming to the study of social phenomena such as transmission of culture and disease propagation (Epstein and Axtell 1996). Within the Built Environment research area, Turner and Penn studied pedestrian behaviour using agent-based systems. They found that by imbuing many agents with attributes and parameters such as destination, field of view, and location between events, they can generate movement patterns like those found in the real world (Turner and Penn 2002). Schuamann et al. also used agent-based system for pre-occupancy evaluation of architectural designs (Schaumann et al. 2019). Their model can analyse how different spatial configurations impact human spatial behaviour.

During the COVID-19 pandemic, agent-based models with stationary agents and no architectural structure (a rectangular domain) were used to compare NPIs. For example, Moore et al. and Woodhouse et al. (2022) used agent-based models to compare the infection risk in educational indoor settings (Moore et al. 2021; Woodhouse et al. 2022).

Ozcan and Haciomeroglu implemented a path-based multi-agent navigation model that uses the A∗ shortest-path algorithm to manage local trajectories while a global planner manages global decisions (Ozcan and Haciomeroglu 2015). Their system primarily focuses on agent movement between discrete spaces, such as rooms and zones, and assumes relatively coarse spatial granularity. In contrast, the approach described in this paper provides a higher-resolution spatial framework based on non-manifold topology, enabling continuous agent movement constrained by detailed architectural elements such as walls, corridors, doorways, and furniture layouts.

Balkan et al. simulated infection risk in indoor spaces using schedules and multi-agent movement (Atamer Balkan, et al. 2024). Their navigation model is based on NOMAD, a microscopic pedestrian movement model. The NOMAD model provides a detailed and realistic simulation of pedestrian behaviour by focusing on factors such as route choice, personal space preferences, and responses to surrounding conditions within a discretized environment. While the NOMAD model offers a highly detailed and realistic approach to simulating pedestrian movements, its computational intensity, and complexity of implementation, can limit its practicality in certain applications.

Hernandez-Mejia et al also used a multi-agent approach to study how architectural interventions can mitigate the spread of SARS-CoV-2 in emergency departments (Hernandez-Mejia et al. 2024). Their model, based on records from a German hospital, uses a time manager to organize patient arrivals and manage the duration of their stay in each area of the emergency department. One of the limitations of their approach is that their model of the emergency department is based on a flowchart of activities and events rather than a realistic architectural floor plan with accurate locations for agents. The use of a simplified navigation model within an accurate architectural floor plan in this study strikes a balance between spatial accuracy and computational efficiency, addressing a key challenge in implementing rapid simulations required during fast-evolving epidemic scenarios.

Zhang et al. (2023) developed a web-based multi-agent simulation framework to assess airborne transmission risk, integrating occupant behaviour and ventilation parameters within a discretized spatial model. Their framework adopts a dynamic path choice algorithm on a uniform 0.5 m × 0.5 m grid, assuming shortest-path routing. In contrast, the approach presented in this paper moves agents continuously over a navigation graph constructed from the building geometry. This graph-based method, derived from non-manifold topology, allows for computationally efficient pathfinding which requires only a shortest-path search on a precomputed graph for each event and provides greater spatial precision by avoiding artefacts introduced by grid discretization. Additionally, while the Zhang et al. model accounts for crowding and queuing behaviour, which may improve path realism under high-occupancy conditions, the method presented here focuses on detailed navigation through interior partitions, furniture, and circulation routes, enabling fine-grained assessment of movement constraints and agent proximity.

In modelling aerosol concentration, Zhang et al. discretize the indoor space using the same uniform grid and solve the diffusion process via a finite difference method. In contrast, this study treats the indoor environment as a continuous domain and applies the finite element method to solve a reaction-diffusion equation. This approach supports more accurate geometric representation of complex spatial layouts. However, Zhang et al. incorporate additional environmental details, such as the influence of ventilation layout, which are not explicitly modelled in the present study. Taken together, both approaches contribute complementary perspectives: one emphasizing environmental physics and crowd behaviour, the other offering architectural fidelity and computational efficiency for design and operational decision-making.

Ciunkiewicz et al. developed a simulation framework to forecast COVID-19 transmission in localized environments (Ciunkiewicz et al. 2022). They followed a similar approach to this study using agents, scenarios, and a universal scheduler to move agents from one position to another and calculate, at each time step, their infection risk. However, their 2D spatial map is similar to Zhang et al and is more basic than the one presented here in that it uses a chessboard-like matrix where agents can move only orthogonally or diagonally. In contrast, this study uses spatial maps that are geometrically realistic with navigation paths that do not follow a pre-determined grid and can turn corners and avoid obstacles in a realistic manner. Given the use of a fully three-dimensional spatial model, the system can determine whether barriers exist between two agents. This distinction is important to avoid incorrectly assuming that agents are in proximity when they are separated by architectural elements such as walls.

Finally, while sophisticated commercial people movement simulation engines, such as those used for modelling crowd egress during fire emergencies, are readily available (Gwynne and Kuligowski 2010), they are typically domain-specific desktop applications that are not easily integrated into an open, customizable web-based system. As such, they were deemed unsuitable for the aims of this study.

The present paper sits between highly detailed crowd simulation engines and oversimplified rectangular or grid-based domains. We propose a movement model in which agents follow shortest paths on navigation graphs derived from detailed floor plans, with schedules defined in terms of architecturally meaningful event locations (e.g. offices, meeting rooms). Our contribution is a general, architecture-centred multi-agent framework that: (i) is lightweight enough to be integrated into a fast decision-support tool; (ii) uses realistic building geometry and topological relations; and (iii) exposes movement data in a form that can be tightly coupled to airborne transmission models as detailed in the companion paper. This positions our work as a reusable, geometry-aware movement engine for a wide range of indoor risk-assessment and design-evaluation tasks.

2.4. Mathematical modelling of airborne viral transmission

Mathematical models of airborne diseases have been used to estimate infection risk since the pioneering work by (Wells 1955) and (Riley, Murphy, and Riley 1978). In the Wells-Riley model, the infectious aerosols are assumed to be well-mixed in the indoor space, neglecting spatial heterogeneity due to architectural details and the behaviour and interactions of individuals.

In this study, the concentration of infectious aerosols emitted by infectious individuals is simulated using a reaction-diffusion equation. The infection risk of individuals is determined using a spatio-temporal infection risk formula (Lau et al. 2022) which extends the Wells-Riley ansatz by incorporating also spatial information. This study's new model determines the number of moving individuals being infected and those remaining susceptible to the disease as time progresses, assuming the 'dose' of viral particles needed for a person to get infected. An explanation of the mathematical modelling details is included in a companion study which provides useful insights and guidance to policymakers and space managers through comparing and ranking a series of NPIs according to the type of indoor space(Xue et al. 2024).

3. Methodology

The proposed software system consists of several critical modules. Firstly, a geometric and topological editor is necessary for representing the geometry of indoor spaces and their connectivity, enabling both topological and spatial analysis of buildings and their internal spaces. Using this editor, a navigational graph is constructed that supports the placement and simulation of multiple agents moving through the building. An agent-specific schedule, with a series of events at varying locations in a building, provides the needed information to accurately locate the agents at each time step. Lastly, calculating spatio-temporal infection risk is made feasible through an extension of the Wells-Riley ansatz.

The integration of these modules allows us to simulate different scenarios of practical interest using diverse building typologies and mixtures of agents inhabiting and moving through them. Further, the software offers a comprehensive and accessible decision-support tool for optimizing building designs to enhance safety and health in response to current and future public health challenges.

3.1. Spatial modelling and analysis software

This study leverages topologicpy, an open-source and extendable topological 3D spatial modelling and analysis environment (Jabi 2024a; Jabi and Chatzivasileiadi 2021). Through its reliance on a non-manifold modelling engine for boundary representation, topologicpy can precisely model the required geometry, automatically build the topological connections among spaces, and answer spatial and topological queries. The topologicpy application protocol interface (API) contains nine (9) core classes

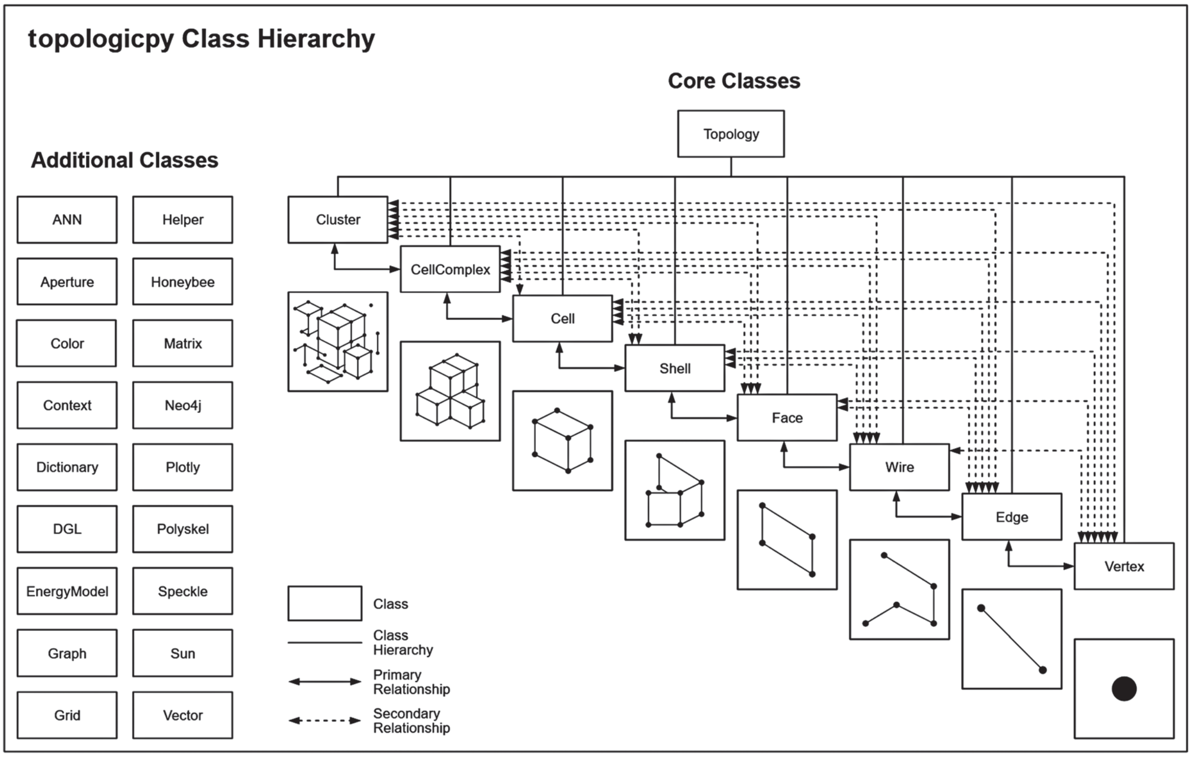

Figure 1. The topologicpy class hierarchy.

(Vertex, Edge, Wire, Face, Shell, Cell, CellComplex, Cluster, and Topology), seven (7) secondary classes (e.g. Aperture, Dictionary, Graph, Grid, Helper, Matrix, Vector), and several additional utility and specialized classes (Figure 1). Of note here is the Dictionary class that can store keys and values and be associated with any topology. Dictionaries allow us to store and retrieve attributes as and when needed. topologicpy's API is extensive, covering areas of geometric construction, spatial analysis, visualization, energy simulation, and artificial intelligence among other capabilities (Jabi 2024b). This paper focuses on the capabilities and classes that facilitate the aims of this work.

The most basic topology in topologicpy is a Vertex which represents a point in 3D space with X, Y, and Z coordinates. Two vertices can be connected by an Edge. Several Edges can be connected to form a Wire (i.e. a polygon or a non-manifold network of edges). A polygonal, planar, closed Wire can form a Face which has a direction vector and an area. Faces can have holes in them (called internal boundaries). Several Faces that share edges can be combined into a Shell (an equivalent to a mesh) which can be either open or closed. A manifold shell that is closed forms a Cell which has a volume. Several Cells that share faces can be combined into a CellComplex. A Cluster (a group) is an abstract class that has any number of other topologies in it. Finally, a Topology is an abstract super-class of all other types of topologies that contains methods that can apply to multiple types of topologies.

Topologicpy's ability to create 'primitives' such as lines, rectangles, circles, and prismatic solids were used to create the architectural models (Figure 2). Topologicpy can also import 3D models created using other 3D modelling software through standard file formats.

In addition, topologicpy provides six types of spatial and topological queries:

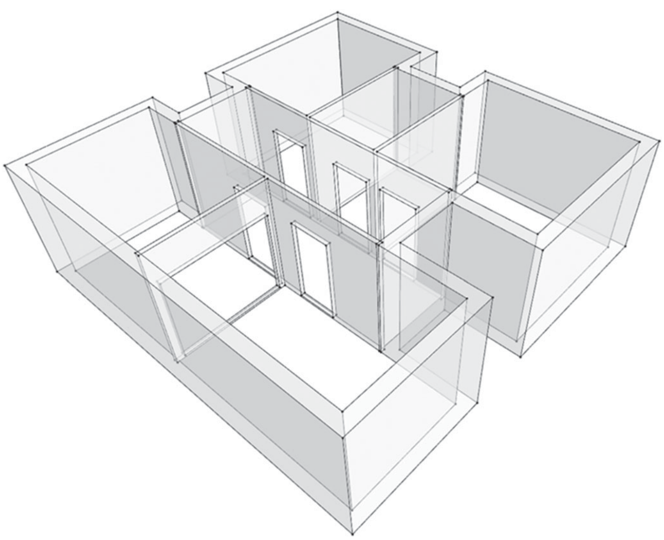

Figure 2. 3D model created in topologicpy.

1. Constituent Query: This type of query allows the user to retrieve the constituent sub-topologies that make up a topology. For example, the user can ask topologicpy to provide the faces, the edges, and the vertices of a cell.
2. Hierarchical Query: This type of query can be thought of as the reverse of a constituent query. This query allows the user to retrieve the super-topologies that have the queried topology as part of their constituent sub-topologies. For example, the user can ask an edge for the faces that it is a part of. Or they can ask a face for a list of cells that it is a part of.

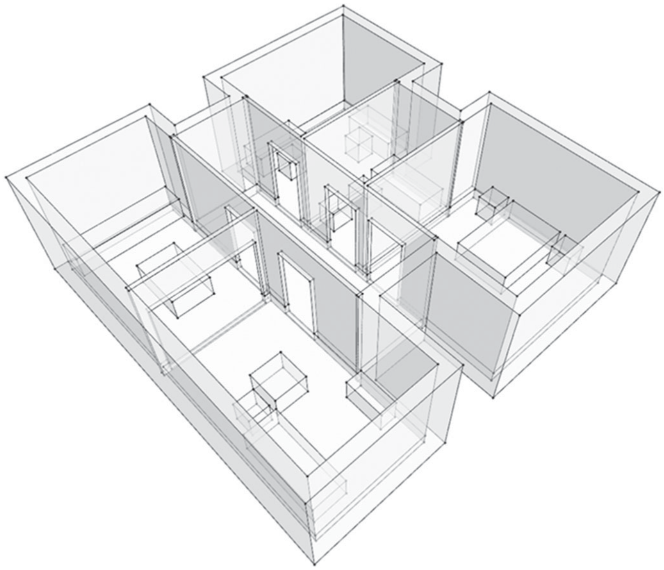

Figure 3. 3D model of an architectural indoor environment with added furniture.

3. Lateral Query: This type of query is used for finding adjacent topologies. For example, a user can ask a cell to return the list of cells that share a face with that cell.
4. Containment Query: This type of query allows the user to retrieve the contents of a topology. Unlike constituent parts that make up the geometry of a topology, contents are independent topologies that are simply stored in a topology (think furniture in a room as opposed to the walls and corners of a room).
5. Intersectional Query: This type of query allows the user to identify shared elements between two topologies. For example, a user can request the list of faces, edges, or vertices shared by two neighbouring cells.
6. Associative Query: This type of query allows the user to find topologies linked to another topology through dictionary information rather than topological proximity. Each topology in topologicpy has a dictionary with varying number of keys and values stored in it. These dictionaries can contain references to other topologies (e.g. by an ID key). This is akin to building a relational database or a graph that can be traversed to retrieve the needed information.

These methods were used to decompose and analyse the 3D model and derive the needed information to create a simplified 3D model with furniture obstacles represented as simple prisms (Figure 3).

3.2. Navigation graph

The next step in the methodology is to create a navigation graph that controls the paths on which agents travel. A navigation graph is a representation of a spatial environment where nodes correspond to locations (vertices) and edges represent possible paths or connections between these locations (Afyouni, Ray, and Claramunt 2012). This graph structure is used to model the navigability of a space, allowing for efficient route planning within that environment. The API of topologicpy includes the ability to directly create a navigation path from a face with an optional list of obstacles that are converted into holes in the face. Thus, in the context of a building floor plan, walls, furniture, and other obstacles are subtracted from the floor plate to create a face with holes (Figure 4). As can be seen on the left side of Figure 4, the furniture was slightly lowered vertically to subtract it from the floor plate using topologicpy's Boolean operations. This created the needed holes in the face, as seen on the right side of Figure 4. If a buffer area between the navigable space and obstacles is required, the user can use topologicpy's Face and Wire offsetting methods to create larger holes and to offset the exterior boundary inwards (Figure 5).

The navigation graph algorithm uses parallel processing to connect a list of source vertices to a list of destination vertices on a face without traversing over holes in that face. The algorithm first checks if the distance between the source and destination vertices exceeds a specified tolerance value. If the distance is less than or equal to that value, the connection is discarded. This ensures that only connections between vertices that are significantly apart are considered. Then, an edge is created between the source and destination vertices. A Boolean intersection operation is performed between this edge and the face (the floor plan of the building with obstacles represented as holes in the face). The edge is only considered valid if the result of this Boolean operation is also an instance of an Edge class rather than a null result or a cluster of edges. A null result means the edge is either completely outside the face or completely within a hole in the face. A cluster of edges result means that the edge has been split into disjointed edges by one or more holes in the face. Filtering for results that are an instance of the Edge class ensures that the edge intersects fully with the face and is not outside its boundaries or intersecting in an invalid manner (Figure 6). The final list of valid edges is sent to the Graph class to create the navigation graph. The resulting graph can be visualized by asking it for its topology (wire) and displaying that topology (Figure 7).

For larger floor plans, the creation of the navigation graph might become prohibitively time-consuming. In that case, an alternative graph made of a pruned Delaunay triangulation might be sufficient to represent how agents move in an indoor environment (Figure 8). This process is detailed below in the illustrative case study.

3.3. Events, activities, schedules, agents

The multi-agent simulation system models events, schedules, and agents. An event is defined by a location in the building where an activity takes place for a certain duration of time. It is represented using (X, Y, Z) coordinates. A schedule is a list of events each associated with its own start time and the type of activity taking place. An agent is an occupant of the space (whether staff or visitors). In this research, an agent's level of physical and vocal activity can vary. This affects both breathing rate and aerosol emission rate, thus having a significant impact on the calculated infection risk.

The scheduling system does not allow time gaps. Thus, the duration of an event is calculated based on the subtraction of the start time of the next event from its own start time.

A daily schedule is then assigned to an agent which is represented by a user-specified geometry (e.g. a cylinder or a cone)

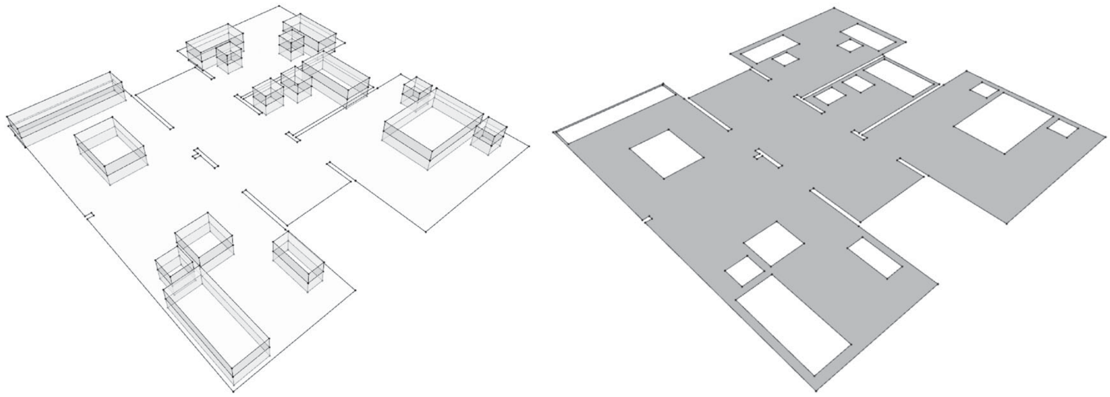

Figure 4. Derving the host face with holes from a floor plan with obstacles.

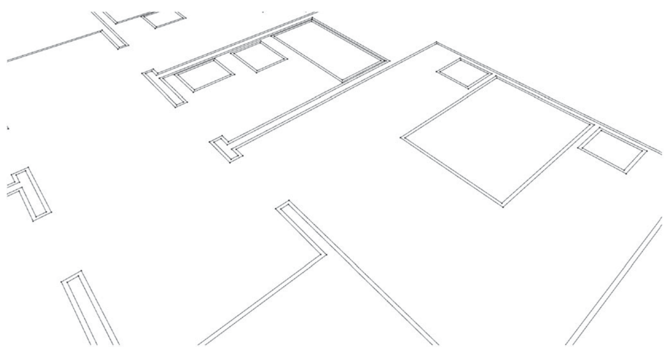

Figure 5. Offsetting the boundaries to create a buffer area between the navigable space and the obstacles.

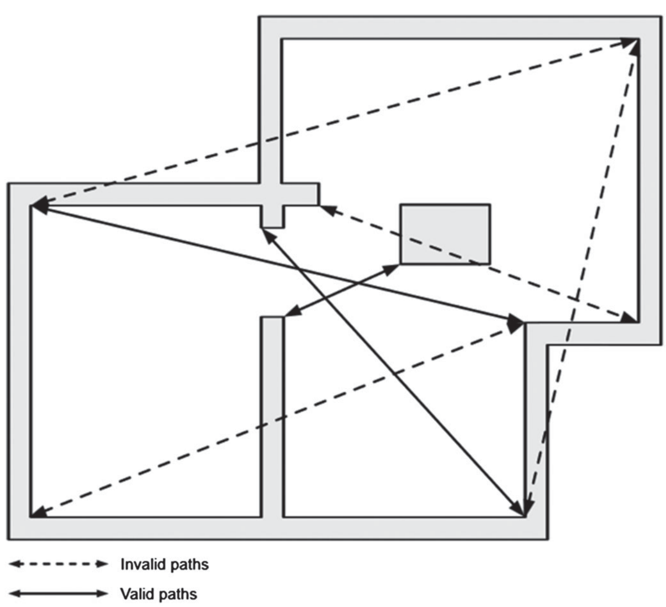

Figure 6. Example of valid and invalid paths.

with a local (X, Y, Z) origin. Each agent has attributes that are stored in its dictionary that are necessary for the simulation calculations such as:

- Walking speed: determines how fast an agent moves.
- Infection status: indicates if the agent is infectious, infected but not yet infectious, or susceptible.
- Superspreader: defines whether the agent is a superspreader or not (only applies to infectious agents).
- Mask: indicates whether a mask is worn and, if so, what type (e.g. cotton, surgical, or N95).
- Activity type: states where the agent is resting, talking, talking loudly, moderately exercising, or vigorously exercising.

Agents navigate a spatial environment following a pre-calculated navigation graph. To travel from one event to another, the system uses topologicpy's Graph class to compute the shortest path between any two points on it, using Dijkstra's algorithm (Dijkstra 1959). A path is an instance of the Wire class which can compute the position of a vertex on it given a user-specified distance from its starting point. At each time step, the agent's new position is computed as:

$$V = Wire.Vertex\mathrm{By}Distance(P,\ T \times S) \tag{1}$$

where: V is the calculated vertex position; P is the calculated shortest path; T is the current time; S is the agent's walking speed.

An agent remains at its last location until the current time step is equal to or greater than a computed time-to-leave parameter. This parameter is computed as follows:

$$T = A - \left(\frac{L}{S}\right) \tag{2}$$

where: T is the time-to-leave parameter; A is the desired arrival time at the next event; L is the path length; S is the agent's walking speed.

3.4. Simulation and visualization

The viral aerosol concentration is determined using a reaction-diffusion equation, which is implemented via a fast finite element method (Scikit-Fem n.d.). The infection risk of an individual, as a function of time, is estimated based on an extended form of the Wells-Riley ansatz (Lau et al. 2022). The average infection risk is calculated by the sum of the infection risk of each susceptible individual, divided by the number of susceptible individuals.

The simulation results are visualized in topologicpy as a heatmap projected on the floor faces of the 3D model using

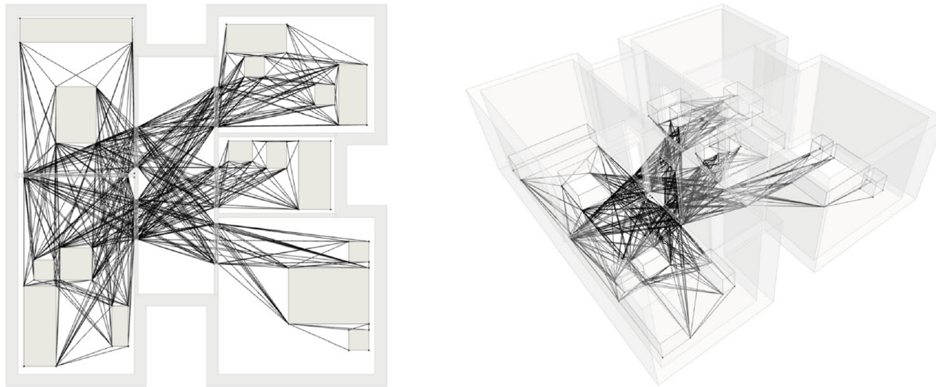

Figure 7. The created navigation graph. Plan view (left), Perspective view (right).

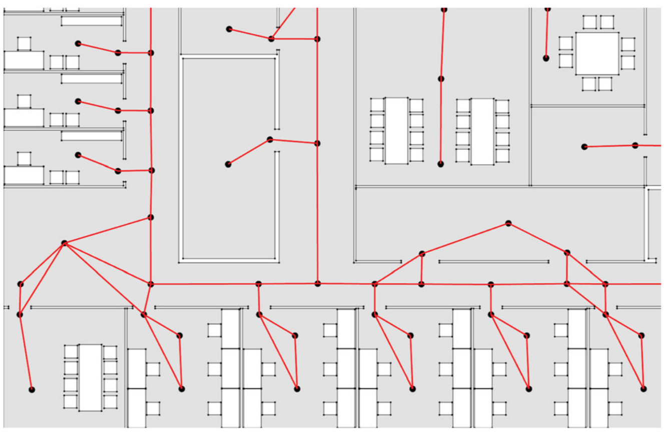

Figure 8. Example of a simplified navigation graph in an office plan using a pruned Delaunay triangulation.

the Plotly graphing library which is integrated into topologicpy (Plotly n.d.). At each timestep, the heatmap, the location and colours of the agents are updated in the 3D model as well as the simulation results. These frames can then be automatically saved and compiled into an animation (Figure 9).

4. Illustrative case study

To illustrate the features of the proposed system, two hypothetical office plan layouts were studied. The first layout emphasizes cellular offices, whereas the second layout favours an open-plan design (Figure 10). Both layouts fit into the same rectangular floor plate with an overall width of 40 m and overall length of 20 m. The layouts have the same dual core where the lifts and staircases are located. It is important to note that the layouts presented here are fictitious. Any resemblance to actual office plans is purely coincidental.

4.1. Constructing the navigation graph

The next step was to construct a navigation graph for each layout. Given the size and complexity of the floor plan, constructing a standard navigation graph, as described above, is computationally expensive. Thus, an alternative semi-automated approach is followed. The layout is populated with a set of points representing significant locations as follows: Firstly, the centre of each chair in the layout was added to the list of points. Secondly, two points were added to either side of each door opening. Thirdly, a set of boundary points were added around or near each piece of furniture to allow access. Fourthly, a point was placed near the centre of each cellular office and other defined areas. Lastly, corridors were populated with colinear points to allow navigation along their main axis. These can be junctures at which a person would decide to turn in a different direction (Figure 11).

The points were then triangulated using a Delaunay algorithm (Ruppert, 1995) that divides their convex hull into triangles, where the circumcircles of these triangles do not enclose any of the points (Figure 12). The Delaunay triangulation generates a spatial graph that provides a coherent and connected representation of navigable space. This method produces well-formed triangles that approximate interior geometry effectively, supporting realistic agent movement and interaction within the built environment.

To derive the navigation graph, each edge in the Delaunay triangulation was intersected with the walls and obstacles in the floor plan. Only edges that do not intersect any walls or obstacles are kept (Figure 13). These filtered edges and their end vertices are then used as input to create a navigation graph using the *Graph.ByVerticesEdges()* method. All information is then transferred to the multi-agent simulation software scripts.

4.2. Multi-agent simulation

The two multi-agent simulation scenarios (cellular plan, open plan) were populated with 60 agents each. Agents are assumed to be familiar with the office layout and thus always travel along the shortest paths within the navigation graph using their assigned walking speed. Both scenarios were given the same overall schedule as follows:

1. 9:00 am: All agents arrive from two lift cores and walk to their assigned desk location. Starting at 9:30 am, 20 agents visit 1–6 other offices, staying at each location for 5-10 min.
2. 11:00 am-11:30 am: 40 agents meet in conference rooms.
3. 1:00 pm-2:00 pm: Agents have lunch in three shared spaces. One group eats from 1:00 pm to 1:30 pm, and a second group eats from 1:30 pm to 2:00 pm.
4. 2:00 pm-3:00 pm: All agents work at their assigned desk.
5. 3:00 pm-3:30 pm: 40 agents meet in conference rooms.

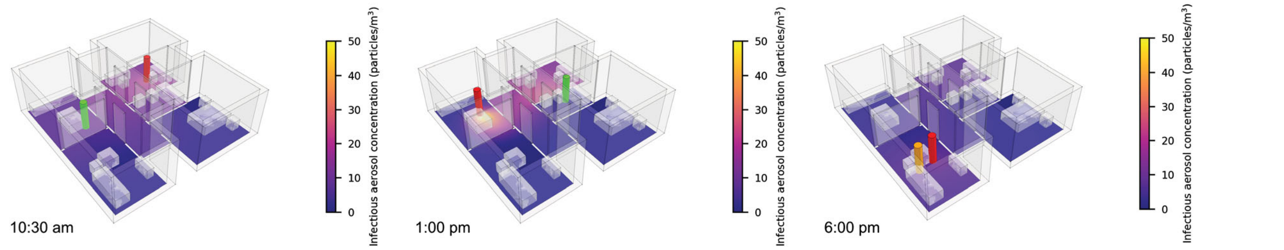

Figure 9. Simulation results showing the aerosol concentration in a household (heatmap) and agent locations at three times during the day. The infectious agent is represented by a red cylinder. The susceptible agent is represented by a green cylinder when not infected, and an orange one when infected.

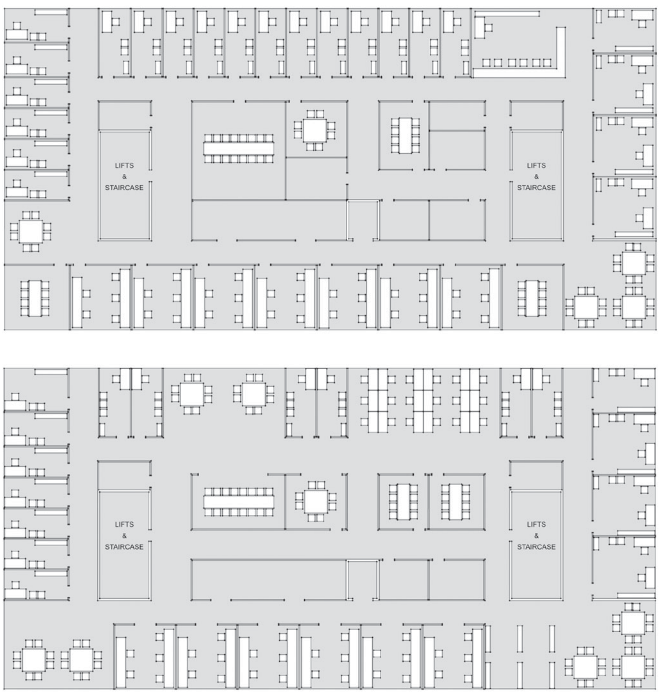

Figure 10. Two hypothetical office plan layouts. Top: cellular layout, Bottom: open-plan layout.

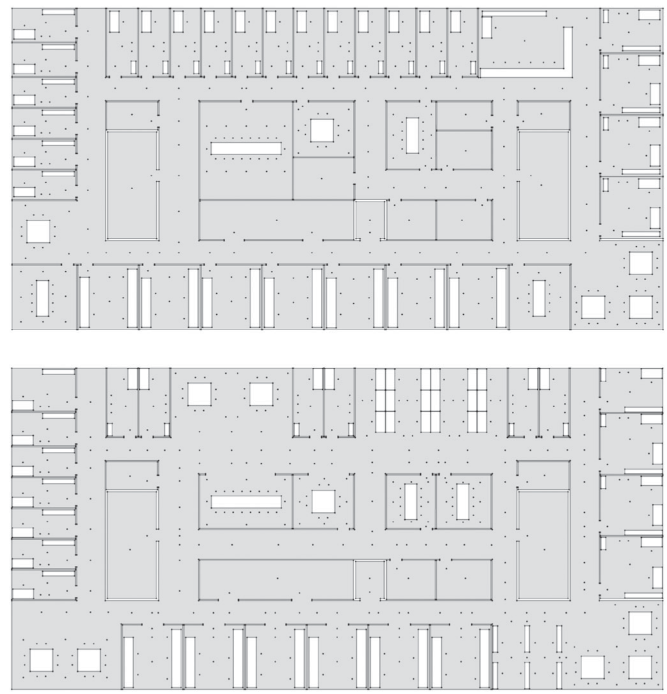

Figure 11. Significant point locations for both layouts. Top: cellular layout, Bottom: open-plan layout.

6. 3:30 pm-5:00 pm: All agents return to their assigned desk location. 20 agents are allowed to visit 1–4 other offices from 3:30 pm, staying at each location for 5-15 min.
7. 5:00 pm: All agents depart through the two lift cores.

The simulation settings were also kept the same for both scenarios as follows:

1. Ventilation: very poor, with 0.12 air changes per hour (ACH).
2. Environment: constant temperature, relative humidity, and sunlight, which result in an exponential decay rate of the viruses in infectious aerosols.
3. Walking speed: 1.5 meters per second.
4. Infection rate: 10% of the agents (i.e. 6 agents) are infectious, while 90% of these are susceptible. No infectious agent is a superspreader.
5. Agent activities: Each agent is walking when moving, talking loudly during meetings, talking when having lunch or visiting other offices, and not talking when working at their office desk. The aerosol emission rate of an infectious agent decreases in the order listed above for these four (4) activities (walking, talking loudly, talking, and resting).

5. Results

The highest concentrations of infectious aerosols were observed in meeting rooms and lunch areas, which aligns with the expected outcome due to the extended periods of proximity and interaction of the individuals in these spaces (Figure 14).

The results indicate that, for this specific case and set of inputs, the cellular office layout is slightly more effective in reducing infection risk compared to the open-plan layout between the hours of 9:00 am and 3:30 pm. This is primarily due to the presence of physical barriers such as walls and doors, which limit the dispersion of airborne particles and reduce the frequency and duration of interpersonal interactions. These spatial separations effectively isolate agents, thereby minimizing their cumulative exposure to infectious aerosols during peak occupancy periods. However, after 3:30 pm, the infection risk in the cellular layout surpasses that of the open-plan layout and remains higher until 5:00 pm (Figure 15). This shift may be

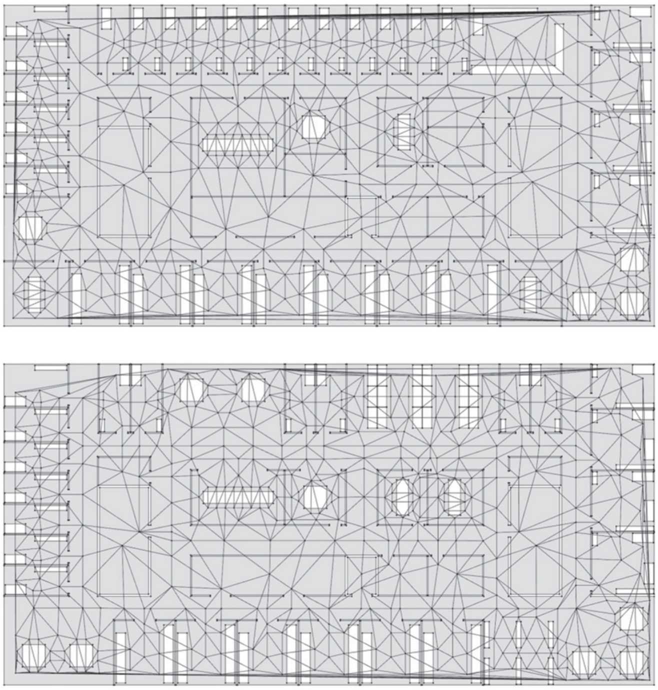

Figure 12. Delaunay triangulation for both layouts. Top: cellular layout, Bottom: open-plan layout.

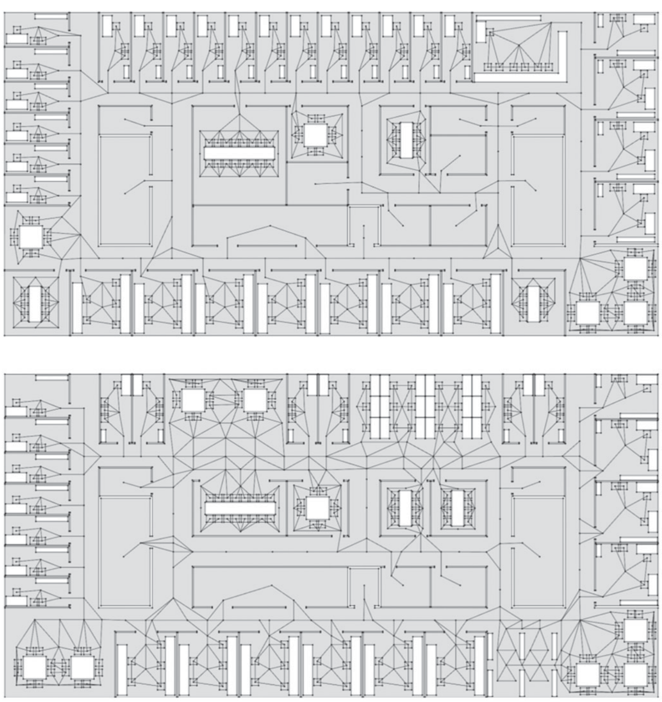

Figure 13. The final navigation graph for each layout. Top: cellular layout, Bottom: open-plan layout.

attributed to a combination of factors: reduced occupancy in the open-plan area during the late afternoon enhances air dilution and decreases the likelihood of cross-infection, while in the cellular layout, stagnant air and limited ventilation in enclosed rooms may allow residual aerosols to accumulate. Additionally, individuals in cellular offices may remain in their rooms for prolonged periods without ventilation refreshment, leading to higher localized exposure for occupants remaining in those spaces later in the day.

It is important to note, however, that these findings are based on sole case study with a predefined and arbitrary pattern of agent movement. As such, the results should not be interpreted as universally applicable. Rather, this work is intended to demonstrate the potential of the proposed simulation platform to support 'what-if' analyses and explore the impact of alternative spatial configurations, behavioural patterns, and environmental parameters on airborne infection risk.

6. Contributions

This paper makes several novel contributions to the state-of-the-art:

1. Non-manifold topological modelling of indoor environments

Instead of ad hoc methods to establish adjacency between spaces, this paper introduces a streamlined foundational non-manifold topological representation of buildings that enables explicit modelling of spatial adjacencies, shared boundaries, and connectivity between spaces, which are essential for movement simulation and spatial analysis.

2. Navigation graph construction for obstacle-aware movement

It develops robust methodologies for constructing navigation graphs directly from architectural geometry, ensuring that shortest paths respect walls, doors, furniture, and other obstacles within complex floor plans and provide accurate agent placement based on their walking speed and elapsed time.

3. Event – schedule – agent multi-agent movement model

The paper formalises a structured event-based scheduling system for agents, linking spatial locations, activity types, and time-dependent movement to enable realistic simulation of human behaviour in buildings.

4. Scalable alternatives for large and complex layouts

To balance accuracy, performance, and scalability, we introduce pruned Delaunay triangulation-based navigation graphs as a computationally efficient alternative to full obstacle-aware visibility-based graph construction for large floor plans.

5. Integrated software architecture within TopologicPy

The framework is implemented as a reusable and extensible software infrastructure within TopologicPy, establishing data structures for agents, activities, schedules, paths, and spatial queries. This framework is extendible to the third dimension which is the focus of our current research under development.

6. Architectural validation through comparative case studies

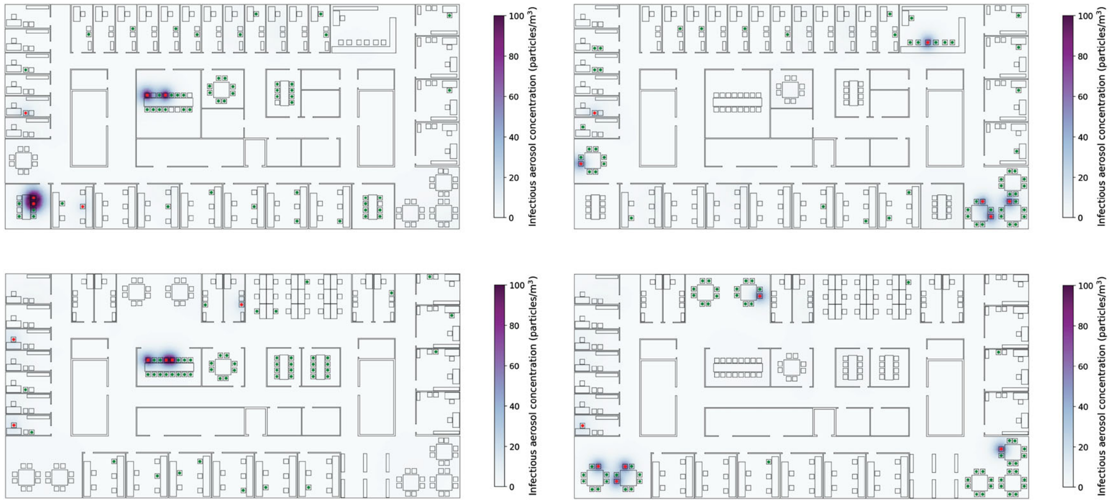

Figure 14. Areas of highest infectious aerosol concentration for each layout. Top: cellular layout, Bottom: open layout. Left: Meeting rooms. Right: Lunch areas.

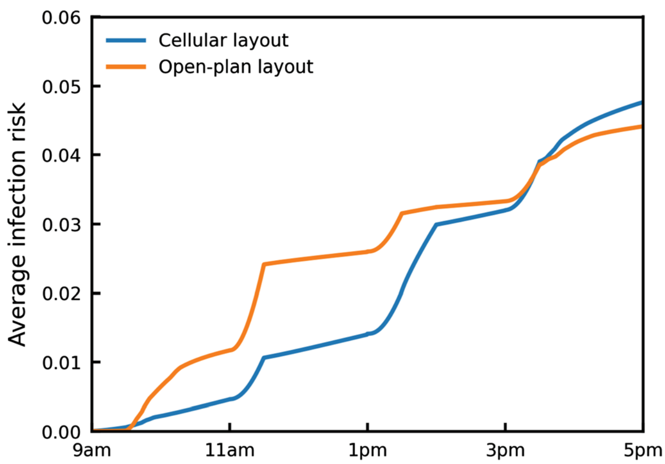

Figure 15. Average infection risk of susceptible individuals for the two layouts.

Through a controlled comparison of cellular and open-plan office layouts, the paper demonstrates how spatial configuration alone shapes movement patterns and interaction density, independent of any epidemiological model. This illustrative case study provides a methodological guide for others to conduct comparative *What-If* analysis to improve and optimise their design proposals.

7. Limitations and future work

The primary limitation of this study is that the simulation does not use real-world data to model variations in human movement but instead relies on idealized linear movements using shortest paths. While the supporting software operates in a full 3D environment, aerosol concentration modelling is performed in two dimensions to improve computational efficiency. Additionally, the model does not account for all possible environmental factors, such as natural and mechanical ventilation systems. Thus, future research should aim to incorporate more comprehensive environmental variables, such as HVAC systems, and to validate simulations with real-world data. Expanding the scope to include a wider range of building typologies and spatial configurations, as well as varying demographic and behavioural patterns, would enhance the robustness and applicability of this study.

Other crowded and confined spaces such as elevators/lifts, were not explicitly modelled in this study, as the primary focus of the simulation tool is on spatial features of the building that can be readily adjusted during the design or retrofit process, such as interior partition walls, furniture arrangements, and circulation layouts. In contrast, elements like elevators/lifts are fixed infrastructural components that are less amenable to spatial reconfiguration and typically require policy-driven interventions such as scheduling strategies, occupancy limits, or enhanced ventilation measures. These procedural and operational aspects, while important, lie beyond the current scope of this design-oriented simulation platform.

Although the simulations presented in this paper are based on illustrative scenarios, a preliminary validation of the underlying model has been conducted in a companion study (Xue et al. 2024). In that work, the simulator was assessed against a well-documented superspreader event that occurred in a Swiss courtroom during the COVID-19 pandemic. The model was able to closely reproduce the infection outcomes observed in the real event, providing an important first step in validating the simulation framework with empirical data. While this validation provides initial support for the spatio-temporal infection risk calculations, further work is required to calibrate the model across a wider range of real-world settings. Future research will focus on integrating additional case studies and observational datasets to refine and generalize the model's predictive capabilities.

Future work will focus on finalizing and deploying a web application, which is currently in the last stages of development and testing. The platform is designed to guide users through uploading 3D models, configuring events, schedules, and agents, setting key simulation parameters, and visualizing infection risk outcomes. Planned efforts will include user experience testing to evaluate usability and effectiveness across different user groups, including architects, facility managers, and public health professionals. The web app is intended to support both design-stage decision-making and operational assessments, offering a practical tool for improving indoor environmental quality.

8. Discussion

This study demonstrates that even subtle differences in spatial configuration can meaningfully influence infection risk throughout the day. The illustrative case study findings reveal that the cellular office layout is more effective in reducing infection risk than the open-plan layout during morning and early afternoon hours (9:00–15:30), likely due to the isolating effect of partition walls and restricted interaction. However, this advantage reverses after 15:30, as aerosol buildup in enclosed spaces with limited ventilation becomes a dominant factor. These findings suggest that while spatial barriers can reduce transmission risk during periods of high movement, they may exacerbate risk during longer stationary periods without adequate air exchange.

For this study, a simplified shortest-path approach to people movement was used. Compared to traditional crowd simulation tools, such as the one used by Mohareb (2011) in their emergency evacuation model, the followed approach shifts the focus from accessibility and egress efficiency to the spatio-temporal dynamics of infection risk within occupied environments. Rather than simulating larger people movement patterns with urban environments, the developed tool is designed to simulate individuals' specific daily activities and interpersonal interactions in spaces where people remain for extended periods. This allows the modelling of disease transmission more comprehensively in settings such as offices, schools, or clinics, where crowd density, agent schedules, and air stagnation all interact.

In contrast to Computational Fluid Dynamics (CFD)-based tools, which model airflow and aerosol dispersion with high precision, the approach presented here offers a computationally lighter alternative. CFD simulations typically require detailed HVAC system data, long processing times, and specialized knowledge to operate. While they are invaluable for precise airflow analysis, they are less suited to rapid, scenario-based testing across multiple building layouts and behavioural conditions. This study's method sacrifices granularity for scalability, enabling faster turnaround and accessibility for architects, designers, and policymakers seeking to explore design alternatives and behavioural interventions in early-stage decision-making.

The simulation platform presented in the paper complements existing approaches by filling a critical niche: enabling rapid 'what-if' analyses that link realistic agent behaviours with spatial configurations and airborne infection risk. While the outcomes presented are based on a single, illustrative case study with an arbitrary pattern of movement, the platform is intended to be adaptable and extensible, offering a foundation for further scenario testing and policy exploration in diverse built environments.

9. Concluding remarks

This study provides a multidisciplinary approach that integrates detailed 3D modelling of spaces, agent-based navigation modelling, and spatio-temporal modelling of the viral aerosol concentration and infection risk for susceptible individuals. The office layout case study clearly illustrates that spatial configuration can significantly influence infection risk in indoor environments. As we continue to confront complex global health threats, such innovative multidisciplinary research methodologies will be essential in developing effective strategies to prevent future epidemics.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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Data availability statement

Data and relevant code for this research work are stored in a GitHub repository: https://github.com/wassimj/VIRIS.

References

Afyouni, I., C. Ray, and C. Claramunt. 2012. "Spatial Models for Context-Aware Indoor Navigation Systems: A Survey." *Journal of Spatial Information Science* 4: 85–123. https://doi.org/10.5311/JOSIS.2012.4.73.

Alagna, M. 2004. *The Great Fire of London of 1666*. New York: Rosen Publishing Group.

Atamer Balkan,, B., Y. Chang, M. Sparnaaij, B. Wouda, D. Boschma, Y. Liu, Y. Yuan, et al. 2024. "The Multi-dimensional Challenges of Controlling Respiratory Virus Transmission in Indoor Spaces: Insights from the Linkage of a Microscopic Pedestrian Simulation and SARS-CoV-2 Transmission Model." *PLOS Computational Biology* 20 (3): e1011956. https://doi.org/10.1371/journal.pcbi.1011956.

Bernoulli, D., and D. Chapelle. 1760. "Essai d'une nouvelle analyse de la mortalité causée par la petite vérole, et des avantages de l'inoculation pour la prévenir".

Boguslawski, P., L. Mahdjoubi, V. Zverovich, and F. Fadli. 2016. "Automated Construction of Variable Density Navigable Networks in a 3D Indoor Environment for Emergency Response." *Automation in Construction* 72:115–128. https://doi.org/10.1016/J.AUTCON.2016.08.041.

Chatzivasileiadi, A., N. M. Wardhana, W. Jabi, R. Aish, and S. Lannon. 2018. "Characteristics of 3D Solid Modeling Software Libraries for Non-manifold Modeling." *Computer-Aided Design and Applications* 16 (3): 496–518. https://doi.org/10.14733/cadaps.2019.496-518.

Ciunkiewicz, P., W. Brooke, M. Rogers, and S. Yanushkevich. 2022. "Agent-Based Epidemiological Modeling of COVID-19 in Localized Environments." *Computers in Biology and Medicine* 144:105396. https://doi.org/10.1016/J.COMPBIOMED.2022.105396.

Cutcliffe, S. H. 2000. "Earthquake Resistant Building Design Codes and Safety Standards: The California Experience." *GeoJournal* 51 (3): 259–262. https://doi.org/10.1023/A:1017566714380.

Dijkstra, E. W. 1959. "A Note on Two Problems in Connexion with Graphs." *Numerische Mathematik* 1 (1): 269–271. https://doi.org/10.1007/BF01386390.

Emmanuel, U., E. D. Osondu, and K. C. Kalu. 2020. "Architectural Design Strategies for Infection Prevention and Control (IPC) in Health-Care Facilities: Towards Curbing the Spread of Covid-19." *Journal of Environmental Health Science and Engineering* 18 (2): 1699. https://doi.org/10.1007/S40201-020-00580-Y.

Epstein, J. M., and R. L. Axtell. 1996. *Growing Artificial Societies: Social Science from the Bottom Up*. Boston, MA: The MIT Press. https://doi.org/10.7551/mitpress/3374.001.0001

Gwynne, S. M. V., and E. D. Kuligowski. 2010. "Application Modes of Egress Simulation." In *Pedestrian and Evacuation Dynamics 2008*, edited by W. Klingsch, C. Rogsch, A. Schadschneider, and M. Schreckenberg. Berlin, Heidelberg: Springer. https://doi.org/10.1007/978-3-642-04504-2_34.

Hernandez-Mejia, G., S. Scheithauer, S. Blaschke, N. Kucheryava, K. Schwarz, J. Moellmann, D. V. Tomori, et al. 2024. "Architectural Interventions to Mitigate the Spread of SARS-CoV-2 in Emergency Departments." *Journal of Hospital Infection* 151: 1–10. https://doi.org/10.1016/J.JHIN.2024.05.019.

Hewitt, C., P. Bishop, and R. Steiger. 1973. "A Universal Modular ACTOR Formalism for Artificial Intelligence." Proceedings of the 3rd International Joint Conference on Artificial Intelligence, 235–245.
Hillier, B., and J. Hanson. 1984. *The Social Logic of Space*. Cambridge: Cambridge University Press. https://doi.org/10.1017/CBO9780511597237
Hillier, B., A. Leaman, P. Stansall, and M. Bedford. 1976. *Environment and Planning B: Planning and Design* 3 (1990): 147–185. https://doi.org/10.1068/B030147
Jabi, W. 2024a. "topologicpy." https://doi.org/10.5281/zenodo.11555173.
Jabi, W. 2024b. "topologicpy API Documentation." https://topologicpy.readthedocs.io/.
Jabi, W., and A. Chatzivasileiadi. 2021. "Topologic: Exploring Spatial Reasoning through Geometry, Topology, and Semantics." In *Formal Methods in Architecture*, edited by S. Eloy, D. Leite Viana, F. Morais, & J. Vieira Vaz, 277–285. London: Springer International Publishing. https://doi.org/10.1007/978-3-030-57509-0_25
Lau, Z., I. M. Griffiths, A. English, and K. Kaouri. 2022. "Predicting the Spatio-Temporal Infection Risk in Indoor Spaces Using an Efficient Airborne Transmission Model." *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 478:2259. https://doi.org/10.1098/rspa.2021.0383.
Lu, J., J. Gu, J. Gu, K. Li, C. Xu, W. Su, Z. Lai, et al. 2020. "COVID-19 Outbreak Associated with Air Conditioning in Restaurant, Guangzhou, China 2020." *Emerging Infectious Diseases* 26 (7): 1628. https://doi.org/10.3201/EID2607.200764.
Masuda, H. 1993. "Topological Operators and Boolean Operations for Complex-Based Nonmanifold Geometric Models." *Computer-Aided Design* 25 (2): 119–129. https://doi.org/10.1016/0010-4485(93)90097-8.
Mohareb, N. 2011. "Emergency Evacuation Model: Accessibility as a Starting Point." *Proceedings of the Institution of Civil Engineers - Urban Design and Planning* 164:215–224. https://doi.org/10.1680/udap.2011.164.4.215.
Moore, J. W., Z. Lau, K. Kaouri, T. C. Dale, and T. E. Woolley. 2021. "A General Computational Framework for COVID-19 Modelling with Applications to Testing Varied Interventions in Education Environments." *COVID* 1 (4): 674–703. https://doi.org/10.3390/covid1040055.
Morawska, L., and J. Cao. 2020. "Airborne Transmission of SARS-CoV-2: The World Should Face the Reality." *Environment International* 139:105730. https://doi.org/10.1016/j.envint.2020.105730.
Ozcan, C. Y., and M. Haciomeroglu. 2015. "A Path-Based Multi-agent Navigation Model." *The Visual Computer* 31 (6): 863–872. https://doi.org/10.1007/s00371-015-1110-2.
Plotly. n.d. Accessed July 10, 2024. https://plotly.com/.
Porta, S., P. Crucitti, and V. Latora. 2006. "The Network Analysis of Urban Streets: A Dual Approach." *Physica A: Statistical Mechanics and Its Applications* 369 (2): 853–866. https://doi.org/10.1016/J.PHYSA.2005.12.063.
Riley, E. C., G. Murphy, and R. L. Riley. 1978. "Airborne Spread of Measles in a Suburban Elementary School." *American Journal of Epidemiology* 107 (5): 421–432. https://doi.org/10.1093/oxfordjournals.aje.a112560.
Ruppert, J. 1995. "A Delaunay Refinement Algorithm for Quality 2-Dimensional Mesh Generation." *Journal of Algorithms* 18 (3): 548–585. https://doi.org/10.1006/JAGM.1995.1021.
Schaumann, D., N. Putievsky Pilosof, H. Sopher, J. Yahav, and Y. E. Kalay. 2019. "Simulating Multi-agent Narratives for Pre-occupancy Evaluation of Architectural Designs." *Automation in Construction* 106:102896. https://doi.org/10.1016/J.AUTCON.2019.102896.
scikit-fem. n.d. Accessed July 10, 2024. https://github.com/kinnala/scikit-fem.
Tong, H., M. Li, and J. Kang. 2022. "Relationships between Building Attributes and COVID-19 Infection in London." *Building and Environment* 225:109581. https://doi.org/10.1016/j.buildenv.2022.109581.
Turner, A., and A. Penn. 2002. "Encoding Natural Movement as an Agent-Based System: An Investigation into Human Pedestrian Behaviour in the Built Environment." *Environment and Planning B: Planning and Design* 29 (4): 473–490. https://doi.org/10.1068/b12850.
Varoudis, T., and S. Psarra. 2014. "Beyond Two Dimensions: Architecture through Three-Dimensional Visibility Graph Analysis." *The Journal of Space Syntax* 5:91–108. https://api.semanticscholar.org/CorpusID:117511113
Weiler, K. J. 1986. *Topological Structures for Geometric Modeling*. Troy, New York: Rennselaer Polytechnic Institute.
Wells, W. F. 1955. "Airborne Contagion and Air Hygiene: An Ecological Study of Droplet Infections." https://api.semanticscholar.org/CorpusID:83282393.
Whitehead, B., and M. Z. Eldars. 1965. "The Planning of Single-Storey Layouts." *Building Science* 1 (2): 127–139. https://doi.org/10.1016/0007-3628(65)90014-9.
Whitney, H. 1931. "A Theorem on Graphs." *The Annals of Mathematics* 32 (2): 378–390. https://doi.org/10.2307/1968197.
Woodhouse, M. J., W. P. Aspinall, R. S. J. Sparks, E. Brooks-Pollock, and C. Relton. 2022. "Alternative COVID-19 Mitigation Measures in School Classrooms: Analysis Using an Agent-Based Model of SARS-CoV-2 Transmission." *Royal Society Open Science* 9:8. https://doi.org/10.1098/rsos.211985.
Xue, Y., W. Jabi, T. E. Woolley, and K. Kaouri. 2024. "Modelling Indoor Airborne Transmission Combining Architectural Design and People Movement Using the VIRIS Simulator and Web App." *Scientific Reports* 14 (1): 28220. https://doi.org/10.1038/s41598-024-79525-6.
Zhang, A., Q. Zhen, C. Zheng, J. Li, Y. Zheng, Y. Du, Q. Huang, and Q. Zhang. 2023. "Assessing the Impact of Architectural and Behavioral Interventions for Controlling Indoor COVID-19 Infection Risk: An Agent-Based Approach." *Journal of Building Engineering* 74:106807. https://doi.org/10.1016/j.jobe.2023.106807.