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Spatial Flexibility Provision from Geographically Dispersed Data Centers Enabling Coordinated Operation of Multi-local Flexibility Markets

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Abstract—In local flexibility markets (LFMs), distribution system operators (DSOs) can obtain flexibility from distributed energy resource aggregators (DERAs) to address operation problems of distribution systems (DSs). However, in a single LFM, the supply and demand of flexibility may not be balanced, leading DSOs to face significant economic challenges. In this paper, leveraging the geographically coupled flexibility (GCF) provided by geographically dispersed data centers (DCs), a coordinated operation framework for multi-LFMs is proposed. This framework enables wireless connections between multi-LFMs without geographical limitations, only requiring interactions between DSOs and their local DC. To clear the multi-LFMs, an optimization model is established with the objective of minimizing total operation costs of multi-LFMs. The optimization model is solved by ADMM to protect the data privacy. A settlement method for GCF is proposed to allocate the total cost fairly. Furthermore, the uncertain parameters are addressed using distributionally robust chance constraints based on Wasserstein metric. Three DCs, which are connected to three DSs adapted from IEEE 15-bus, 33-bus, and 69-bus DSs, respectively, are used in case studies. The results demonstrate that the GCF enables effective integration and utilization of DERAs' flexibility across the three LFMs, facilitating the balance between flexibility supply and demand.

Index Terms—Data center, distributed energy resource aggregators, distributed optimization, distributionally robust chance constraints, geographically coupled flexibility, multi-local flexibility market, power system flexibility, spatial flexibility.

NOMENCLATURE

Abbreviations

DC	Data center
DCO	DC operator
DER	Distributed energy resource
DERA	DER aggregator
DS	Distribution system

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DSO	Distribution system operator
LFM	Local flexibility market
GCF	Geographically coupled flexibility
FS	Front-end server
Sets	
K	Set of DSs and DCs
F	Set of FSs
$\mathcal{N}$	Set of DS buses
$\mathcal{L}$	Set of DS branches
$\mathcal{N}_d$	Set of DERAs
Indices	
k	Indexes of DSs and DCs
j	Indexes of DS buses
ℓ	Indexes of DS branches
n_d	Indexes of DERAs
Parameters	
$\mathbf{1}_N$	N -dimensional all-ones vector
$\mathbf{p}_d^{bl}, \mathbf{q}_d^{bl} \in \mathbb{R}^{N_d \times 1}$	Active and reactive baseline power of DERAs
$\bar{\mathbf{p}}_d^+, \bar{\mathbf{p}}_d^- \in \mathbb{R}^{N_d \times 1}$	Upward and downward flexibility capacity of DERAs
$\mathbf{p}_{dc}^{bl}, \mathbf{q}_{dc}^{bl} \in \mathbb{R}^{K \times 1}$	Active and reactive baseline power of DCs
$\mathbf{r}, \mathbf{x} \in \mathbb{R}^{L \times 1}$	Resistance and reactance of DS branches
$\bar{\mathbf{s}}_b \in \mathbb{R}^{L \times 1}$	Capacity of DS branches
V_0	Square voltage magnitude at substation bus
$\underline{V}, \bar{V}$	Lower and upper limitation of buses voltage magnitude
$C_{k,s}, C_{k,c}$	Load shedding and generation curtailment price
$C_{k,d}^+, C_{k,d}^-$	Upward and downward flexibility price of DERAs
$C_{dc,k}^+, C_{dc,k}^-$	Upward and downward flexibility price of DC
$\mathbf{p}_{peak}, \mathbf{p}_{idle} \in \mathbb{R}^{K \times 1}$	Peak and idle power of DC servers
$\mathbf{pue} \in \mathbb{R}^{K \times 1}$	PUE coefficient of DCs
$\zeta \in \mathbb{R}^{K \times 1}$	Power consumption parameter of DCs
$\mathbf{t}_D \in \mathbb{R}^{1 \times K}$	Workload processing time requirements
$\mathbf{t}_d \in \mathbb{R}^{F \times K}$	Workload transfer time from FSs to DCs
$\boldsymbol{\mu} \in \mathbb{R}^{K \times 1}$	Server rates of DCs
$\bar{\mathbf{s}} \in \mathbb{R}^{K \times 1}$	Servers number of DCs
Variables	
$\mathbf{p}_b, \mathbf{q}_b \in \mathbb{R}^{L \times 1}$	Active and relative power on DS branches
$\mathbf{p}_{net}, \mathbf{q}_{net} \in \mathbb{R}^{N \times 1}$	Net injected active and reactive power of DS buses

$\mathbf{I} \in \mathbb{R}^{L \times 1}$	Square of DS branch current
$\mathbf{v} \in \mathbb{R}^{N \times 1}$	Square voltage magnitude of DS buses
$\mathbf{p}_d^+, \mathbf{p}_d^- \in \mathbb{R}^{N_d \times 1}$	Upward and downward flexibility of DERAs active power
$\mathbf{q}_d^+, \mathbf{q}_d^- \in \mathbb{R}^{N_d \times 1}$	Upward and downward flexibility of DERAs reactive power
$\hat{\mathbf{p}}_{dc}^+, \mathbf{p}_{dc}^+ \in \mathbb{R}^{K \times 1}$	Upward flexibility of DC active power
$\hat{\mathbf{p}}_{dc}^-, \mathbf{p}_{dc}^- \in \mathbb{R}^{K \times 1}$	Downward flexibility of DC active power
$\hat{\mathbf{q}}_{dc}^+, \hat{\mathbf{q}}_{dc}^- \in \mathbb{R}^{K \times 1}$	Upward and downward flexibility of DC reactive power
$\mathbf{p}_s, \mathbf{q}_s \in \mathbb{R}^{N \times 1}$	Active and reactive load shedding of DS
$\mathbf{p}_c, \mathbf{q}_c \in \mathbb{R}^{N \times 1}$	Active and reactive generation curtailment of DS
$\alpha_{fle} \in \mathbb{R}^{F \times K}$	Percentage of flexible workloads sent from FSs to DCs
$\mathbf{w} \in \mathbb{R}^{F \times K}$	Workload dispatched from the FSs to DCs
$\mathbf{s} \in \mathbb{R}^{F \times K}$	Number of active servers in DCs
$\mathbf{p}_{dc} \in \mathbb{R}^{K \times 1}$	Power consumption of DCs
<i>Uncertain parameters</i>	
$\tilde{\mathbf{p}}, \tilde{\mathbf{q}}$	Active and reactive net load of DS buses
$\tilde{\mathbf{w}}_{fix} \in \mathbb{R}^{F \times K}$	Fixed workload sent from FSs to DCs
$\tilde{\mathbf{w}}_{fle} \in \mathbb{R}^{F \times 1}$	Flexible workloads of FSs

I. INTRODUCTION

A. Background and Motivations

With the electrification of transportation and heating sectors and the decarbonization of the power system, distribution systems (DSs) have experienced a rapid increase in loads and an integration of renewable energy sources. This may lead to operation issues in DSs, such as network congestion and voltage violations. These problems can be managed by distribution system operators (DSOs), who are responsible for the safe operation of DSs, by implementing network reinforcement [1]. However, such reinforcement imposes a significant economic burden and presents obstacles to the low-carbon transformation of the power system. In DSs, there are several types of distributed energy resources (DERs) with operation flexibility, such as distributed generators, electric vehicles, air conditioners, and heat pumps. DERs can adjust their operational state to modify the exchange power with the DS, generating upward and downward flexibility, which corresponds to reducing and increasing power from the DS to the DERs, respectively. The DSO can utilize the upward and downward flexibility provided by DERs to address or alleviate operational issues, thereby avoiding or delaying the need for costly network reinforcement [2].

According to guidelines from the Council of European energy regulators [3], market-based procurement can be implemented to utilize the flexibility of DERs at the distribution level. Local flexibility markets (LFMs), as a form of market-based procurement, are recommended as a promising approach [4]. LFMs enhance distribution-level flexibility by providing DSOs with a structured mechanism to

resolve operational constraints in DSs, such as network congestion and voltage violation issues, through the activation of geographically targeted flexibility. By enabling competition among flexibility providers, LFMs can lower the cost of solving operational challenges in DSs compared to traditional network reinforcements, while also facilitating the integration of variable renewable energy by compensating for short-term fluctuations and uncertainties. This localized and transparent approach allows DSOs to procure services only when and where they are needed, thereby improving operational efficiency, reliability, and resilience. In LFMs, DSOs can purchase necessary upward and downward flexibility from DERs.

In Europe, several projects have been developed to facilitate flexibility trading between a DSO and DERs in an LFM, employing various time scales and settlement methods. For instance, in the Great Britain, according to the National Grid guidance for electricity distribution flexibility service providers, DSOs can procure four types of flexibility services across long term (annually) and short term (weekly) timescales [5]. Among these services, SU-SPP is a scheduled constraint management service with fixed delivery periods. The SU-SEP service is used to manage peak demand loading on the network and to proactively reduce network stress. The SAOU-DA service has been developed to support the network during specific fault conditions, such as maintenance work. The OU-15 service supports power restoration following rare fault events. In Denmark, the EcoGrid 2.0 project organized an LFM involving 800 customers who used residential heating units to provide flexibility. The flexibility products were traded one to twelve months in advance [6]. These projects effectively reduce peak load, alleviate operational stress on the distribution network, address operational challenges faced by the distribution system, and provide load support during fault periods. A considerable amount of literature has also extensively investigated the operation framework of LFMs [7]. Within these frameworks, smaller capacity DERs on the consumer side, such as batteries [8], and electrical water heaters [9], can participate in LFM through DER aggregators (DERAs) during the day-ahead [9][10] or intraday phases [10]–[12].

Data centers (DCs), which host high-performance IT equipment for processing workloads, possess operation flexibility and is an emerging type of DER in DSs. Moreover, with the continuous demand for cloud computing and online services, alongside the digitalization of various industries, there is a growing need for DCs. Typically, a DC operator (DCO) manages several geographically dispersed DCs, which are connected to power systems in different regions. By dispatching workloads between different DCs, the load profiles of these DCs can be adjusted, a concept known as spatial flexibility of DC loads. In previous research, considering the spatial flexibility of DC loads, studies have investigated economic dispatch problems of energy systems [13], generation and expansion planning problems of transmission networks [14], planning issues of multi-energy

systems [15], energy management problems of microgrids [16], demand response problem of retailers [17], and unit commitment problems of power systems [18]. These studies highlight the significant role of spatial flexibility from DCs in the power system.

Additionally, some other research focused on utilizing the spatial flexibility of DC loads at the distribution level. In [19], a post-event restoration method was proposed for distribution networks with DCs as critical loads. In [20], branch current margin and node voltage margin terms were included in the objective function. Through spatial temporal workload scheduling, the operational flexibility of the DS was improved, preventing network congestion and voltage violation issues. Ref. [21] proposed a low-carbon planning approach for the distribution network, considering the geographically dispatchable loads of DCs. In [22], a coordinated supply restoration method of DCs in a distribution network with spatial-temporal regulation was proposed to minimize data-dropping loss of DCs. However, there is currently a lack of literature exploring how the spatial flexibility of geographically dispersed DCs can be leveraged in LFMs.

From the perspective of LFMs, previous studies have mainly focused on the operation of individual LFMs [7]. Ref. [12] proposed an iterative auction algorithm based on a dual-decomposition technique to clear a single LFM, while considering the network constraints of the distribution system. In [23], a central clearing algorithm for a single LFM was developed using a two-stage stochastic model. Nevertheless, the supply and demand of flexibility may not always be balanced within a single LFM. In different LFMs, the supply and demand of flexibility are heterogeneous. For instance, insufficient flexibility supply in an LFM inevitably results in load shedding, generation curtailment, or necessitates network reinforcement, posing significant economic challenges for DSO. Meanwhile, cheaper surplus flexibility from DERAs may be available in other LFM markets. Therefore, exploring a potential coordinated operation method among multi-LFMs can enhance the effective utilization of flexibility resources and reduce the operation costs of the LFMs.

From the perspective of DCs, spatial flexibility is generated by dispatching workloads across geographically distributed DCs. This results in only relatively minor additional electricity consumption costs for the DCO in the retail market, without impacting the quality of service, and does not require additional capital costs. Moreover, DCs are highly automated facilities equipped with inherent communication and control infrastructure, making them naturally qualified for participation in LFMs without aggregation due to their larger capacities. Hence, the utilization of spatial flexibility of DC loads is of great practical significance, especially in the context of significant operation challenges faced by DSOs and the surge of DC loads.

B. Problem Statement and Contributions

In the rest of this paper, the term 'workload' specifically refers to delay-sensitive workloads, such as real-time video

streaming, online gaming, or web searches, that are transmitted from front-end servers (FSs) to several geographically dispersed DCs for immediate processing. Typically, a DCO develops a workload dispatch plan aimed at minimizing operation costs. This plan resulted in the baseline power consumption of the DCs. The DCO can modify the dispatch plan, altering the DC's power consumption and thereby generating upward or downward flexibility. Since the total amount of processed workloads remains constant, when modifying the dispatch plan, a reduction in workload at one DC (leading to decreased power consumption and generating upward flexibility) will inevitably be accompanied by an increase in workload at other DCs (resulting in increased power consumption and generating downward flexibility). Vice versa, an increase in workload at one DC will lead to a decrease in workload at other DCs. Thus, the flexibility provided by these geographically dispersed DCs is interrelated and is referred to as geographically coupled flexibility (GCF).

This paper focuses on a scenario where several geographically dispersed DCs are connected to different DSs. In each DS, the DSO organizes the LFM to obtain upward and downward flexibility from DERAs, aiming to address network congestion and voltage violation problems. Simultaneously, the DCO provides GCF, enabling the coordinated operation of these LFMs to facilitate the complementary utilization of flexibility from DERAs across the heterogeneous LFMs. In this way, multi-LFMs can operate in coordination wirelessly without geographic limitations. This differs from the study in [24], which coordinated the flexibility resources of several geographically adjacent DSs interconnected by physical lines within an LFM.

To the best of our knowledge, this is the first study to investigate the coordinated operation method of multi-LFMs and explore the role of spatial flexibility of a DCO in LFMs. The main contributions of this paper are as follows:

(1) A coordinated operation framework for multi-LFMs is proposed, leveraging the GCF from DCs. Within this framework, each DSO only needs to interact with its local DC. A coordinated operation optimization model is established to minimize the total operation costs of the multi-LFMs, which is used to clear the multi-LFMs.

(2) A distributionally robust chance constraint method based on the Wasserstein metric is introduced to address uncertainties of net load of DS buses and workloads of the DCO. A procedure for identifying effective constraints in the DS operation model has been incorporated, resulting in a significant reduction in the scale of the chance-constrained model and enhanced solution efficiency. The alternating direction method of multipliers (ADMM) algorithm is employed to solve the model in a distributed manner, facilitating only limited data exchange between the DSO and its local DC during the clearing process.

(3) Due to the unique geographical coupling characteristic of GCF, certain LFMs may experience increased operating costs as a result of passively accepting the flexibility provided by data centers. Therefore, a settlement method for the GCF

ends at bus j , and $M_{ij} = 0$ otherwise. The reduced incidence matrix of $\mathcal{G}$ is denoted as $\mathbf{M} \in \mathbb{R}^{L \times N}$, which is obtained by removing the first column of $\bar{\mathbf{M}}$. The mapping matrix between the DERAs indices and the buses is denoted by $\mathbf{C}_d \in \mathbb{R}^{N \times N_d}$, which is defined as follows: the entry at the j th row, n_d th column of $\mathbf{C}_d$ is 1, if DERA n_d is connected to bus j , and 0 otherwise. Similarly, the mapping matrix vector between the DC indices and the buses is denoted as $\mathbf{C}_{dc} \in \mathbb{R}^{N \times 1}$.

Based on Distflow power flow model, the compact matrix-vector formulation of the LFM operation model is shown as follows:

$$\mathbf{p}_b = \mathbf{M}^{-T} (\mathbf{p}_{net} - \text{diag}(\mathbf{r}) \cdot \mathbf{I}) \quad (1)$$

$$\mathbf{q}_b = \mathbf{M}^{-T} (\mathbf{q}_{net} - \text{diag}(\mathbf{x}) \cdot \mathbf{I}) \quad (2)$$

$$\mathbf{v} = \mathbf{v}_0 \mathbf{1}_N - \mathbf{M}^{-1} (\text{diag}(\mathbf{r})^2 + \text{diag}(\mathbf{x})^2) \mathbf{I} + 2\mathbf{M}^{-1} \begin{pmatrix} \text{diag}(\mathbf{r}) [\mathbf{M}^{-T} (\mathbf{p}_{net} - \text{diag}(\mathbf{r}) \mathbf{I})] \\ + \text{diag}(\mathbf{x}) [\mathbf{M}^{-T} (\mathbf{q}_{net} - \text{diag}(\mathbf{x}) \mathbf{I})] \end{pmatrix} \quad (3)$$

$$\mathbf{I} = (\mathbf{p}_b^2 + \mathbf{q}_b^2) \oslash \mathbf{v} \quad (4)$$

$$\mathbf{p}_{net} = -\mathbf{C}_d (\mathbf{p}_d^{bl} - \mathbf{p}_d^+ + \mathbf{p}_d^-) - \mathbf{C}_{dc} (\mathbf{p}_{dc}^{bl} - \mathbf{p}_{dc}^+ + \mathbf{p}_{dc}^-) - \tilde{\mathbf{p}}_l + \mathbf{p}_s - \mathbf{p}_c \quad (5)$$

$$\mathbf{q}_{net} = -\mathbf{C}_d (\mathbf{q}_d^{bl} - \mathbf{q}_d^+ + \mathbf{q}_d^-) - \mathbf{C}_{dc} (\mathbf{q}_{dc}^{bl} - \mathbf{q}_{dc}^+ + \mathbf{q}_{dc}^-) - \tilde{\mathbf{q}}_l + \mathbf{q}_s - \mathbf{q}_c \quad (6)$$

$$\mathbf{p}_s, \mathbf{p}_c, \mathbf{q}_s, \mathbf{q}_c \geq 0 \quad (7)$$

$$0 \leq \mathbf{p}_d^+ \leq \bar{\mathbf{p}}_d^+, 0 \leq \mathbf{p}_d^- \leq \bar{\mathbf{p}}_d^- \quad (8)$$

where $\text{diag}(\cdot)$ returns a diagonal matrix with the entries of the argument on its diagonal. Symbols $\odot$ and $\oslash$ denote the Hadamard multiplication and division, respectively. Eq. (1) represents the relationship between active power on branches and buses net inject power. Similarly, Eq. (2) represents the relationship between reactive power on branches and buses net inject power. Eq. (3) represents the relationship between buses voltage magnitude and net inject power. Eq. (4) represents the relationship between current on branches and active and reactive power on branches. The active and reactive net power injections of buses are calculated by Eqs. (5) and (6), respectively. Eq. (7) are the constraints of load shedding and generation curtailment. Eq. (8) indicate that the upward and downward flexibility obtained from DERAs should be less than bid capacities.

The model is nonlinear due to the squared term in Eq. (4). Here, Eq. (4) is linearized using a Taylor series approximation at an operating point $\mathbf{s}_0 = \{\mathbf{p}_{b,0}, \mathbf{q}_{b,0}, \mathbf{v}_0\}$ [26]. The approximate linear equation is shown as follows:

$$\mathbf{I} = (\mathbf{p}_{b,0}^2 + \mathbf{q}_{b,0}^2) \oslash \mathbf{v}_0 - (\mathbf{v} - \mathbf{v}_0) \odot (\mathbf{p}_{b,0}^2 + \mathbf{q}_{b,0}^2) \oslash \mathbf{v}_0^2 + 2 \left[\mathbf{p}_{b,0} \odot (\mathbf{p}_b - \mathbf{p}_{b,0}) + \mathbf{q}_{b,0} \odot (\mathbf{q}_b - \mathbf{q}_{b,0}) \right] \oslash \mathbf{v}_0 \quad (9)$$

$$= 2\mathbf{v}_0 \odot (\mathbf{p}_{b,0} \odot \mathbf{p}_b + \mathbf{q}_{b,0} \odot \mathbf{q}_b) - \mathbf{v} \odot (\mathbf{p}_{b,0}^2 + \mathbf{q}_{b,0}^2) \oslash \mathbf{v}_0^2$$

Then, for convenience, the active and reactive power on

branches $\mathbf{p}_b$, $\mathbf{q}_b$ and voltage magnitude $\mathbf{v}$ of buses are expressed as linear functions of the buses net injected power $\mathbf{p}_{net}$, $\mathbf{q}_{net}$ and the operating point $\mathbf{s}_0$, as shown follows:

$$\mathbf{p}_b(\ell) = \mathbf{g}_{p,\ell}(\mathbf{p}_{net}, \mathbf{q}_{net}, \mathbf{s}_0), \forall \ell \in \mathcal{L} \quad (10)$$

$$\mathbf{q}_b(\ell) = \mathbf{g}_{q,\ell}(\mathbf{p}_{net}, \mathbf{q}_{net}, \mathbf{s}_0), \forall \ell \in \mathcal{L} \quad (11)$$

$$\mathbf{v}(j) = \mathbf{h}_j(\mathbf{p}_{net}, \mathbf{q}_{net}, \mathbf{s}_0), \forall j \in \mathcal{N} \quad (12)$$

The branch power flows should remain in safety margins as described in the following:

$$\|[\mathbf{p}_b(\ell); \mathbf{q}_b(\ell)]\|_2 \leq \bar{\mathbf{s}}_b(\ell), \forall \ell \in \mathcal{L} \quad (13)$$

where $\bar{\mathbf{s}}_b$ is the apparent power flow capacity of DS branches.

The quadratic branch power flow limit constraints are linearized using an outer approximation as shown in follows [27]:

$$-\sqrt{2}\bar{\mathbf{s}}_b(\ell) \leq \mathbf{p}_b(\ell) + \mathbf{q}_b(\ell) \leq \sqrt{2}\bar{\mathbf{s}}_b(\ell), \forall \ell \in \mathcal{L} \quad (14)$$

$$-\bar{\mathbf{s}}_b(\ell) \leq \mathbf{p}_b(\ell) \leq \bar{\mathbf{s}}_b(\ell), \forall \ell \in \mathcal{L} \quad (15)$$

$$-\bar{\mathbf{s}}_b(\ell) \leq \mathbf{q}_b(\ell) \leq \bar{\mathbf{s}}_b(\ell), \forall \ell \in \mathcal{L} \quad (16)$$

Voltage magnitude at each bus of the DS should be within a prespecified limit, which is described as follows:

$$\underline{\mathbf{v}} \leq \mathbf{v}(j), \bar{\mathbf{v}} \geq \mathbf{v}(j), \forall j \in \mathcal{N} \quad (17)$$

C. Operation Model of The Data Center Operator

The compact matrix-vector formulation of the DCO operation model is as follows [28]:

$$\alpha_{fle} \mathbf{1}_K = \mathbf{1}_F \quad (18)$$

$$\mathbf{w} = \tilde{\mathbf{w}}_{fle} \mathbf{1}_K \odot \alpha_{fle} + \tilde{\mathbf{w}}_{fix} \quad (19)$$

$$\mathbf{p}_{dc} = \mathbf{s}^T \mathbf{1}_F \odot \delta_1 + \mathbf{w}^T \mathbf{1}_F \odot \delta_2 + \boldsymbol{\varsigma} \quad (20)$$

$$\delta_1 = \mathbf{p}_{idle} + (\mathbf{p}_{ue} - \mathbf{1}_K) \odot \mathbf{p}_{peak} \quad (21)$$

$$\delta_2 = (\mathbf{p}_{peak} - \mathbf{p}_{idle}) \oslash \boldsymbol{\mu} \quad (22)$$

$$0 < \frac{1}{\mathbf{1}_F \boldsymbol{\mu}^T - \mathbf{w} \oslash \mathbf{s}} + \mathbf{t}_d \leq \mathbf{1}_F \mathbf{t}_D \quad (23)$$

$$\mathbf{s}^T \mathbf{1}_F \leq \bar{\mathbf{s}} \quad (24)$$

$$\hat{\mathbf{p}}_{dc}^+ = \max\{\mathbf{p}_{dc}^{bl} - \mathbf{p}_{dc}^-, 0\}, \hat{\mathbf{p}}_{dc}^- = \max\{\mathbf{p}_{dc} - \mathbf{p}_{dc}^{bl}, 0\} \quad (25)$$

Eq. (18) implies that the sum of percentages representing the flexible workload dispatched from one FS to different DCs should equal 1. Eq. (19) is used to calculate total workload dispatched from FSs to DCs. Eq. (20) is used to calculate the power consumption of each DC. Eqs. (21) and (22) are used to calculate the power consumption coefficient of DCs. Eq. (23) represents the quality-of-service constraints based on the M/M/1 queueing model. Eq. (24) indicates that the number of active servers in DCs should be less than the total number of servers. Eq. (25) calculates the upward and downward flexibility provided by each DC. The baseline power consumption of DCs is obtained by minimizing the operation cost of the DCO in the retail market.

D. Optimization Model for The Coordinated Operation of Multi-local Flexibility Markets

The objective of the coordinated operation optimization

model is to minimize the total operation cost of the multi-LFMs, including cost of DSOs $cost_{dso,k}^{co}$, cost of purchase flexibility from DERAs $cost_{der,k}^{co}$ and DCs $cost_{dc,k}^{co}$. The objective function is as follows:

$$\min \sum_{k \in K} cost_{lfm,k}^{co} = \sum_{k \in K} (cost_{dso,k}^{co} + cost_{der,k}^{co} + cost_{dc,k}^{co}) \quad (26)$$

$$cost_{dso,k}^{co} = c_{k,s} \mathbf{1}_{N_k}^T \mathbf{p}_{k,s} + c_{k,c} \mathbf{1}_{N_k}^T \mathbf{p}_{k,c} \quad (27)$$

$$cost_{der,k}^{co} = \sum_{d \in \mathcal{N}_{k,d}} c_{k,d}^+ p_{k,d}^+ + c_{k,d}^- p_{k,d}^- \quad (28)$$

$$cost_{dc,k}^{co} = c_{dc,k}^+ p_{dc,k}^+ + c_{dc,k}^- p_{dc,k}^- \quad (29)$$

where $cost_{lfm,k}^{co}$ is the operation cost of LFM k .

Additionally, the flexibility obtained by the DSOs from the DCO should be equal to the flexibility provided by the DCO to DSOs, as shown follows:

$$\mathbf{p}_{dc}^+ = \hat{\mathbf{p}}_{dc}^+, \mathbf{p}_{dc}^- = \hat{\mathbf{p}}_{dc}^- \quad (30)$$

To this end, the optimization model for the multi-LFMs clearing problem is established as follows:

$$\begin{aligned} \text{obj (26)} \\ \text{s.t. (5)–(12), (14)–(25), (27)–(30)} \end{aligned} \quad (31)$$

By solving the optimization model presented in Eq. (31), results for the coordinated operation of the multi-LFMs can be obtained, achieving the clearing of the multi-LFMs.

III. SOLVING METHODOLOGIES

In this section, a distributionally robust chance constraint based on the Wasserstein metric is employed to address the uncertainties of $\tilde{\mathbf{p}}_l$, $\tilde{\mathbf{q}}_l$, $\tilde{\mathbf{w}}_{fix}$, and $\tilde{\mathbf{w}}_{fle}$. Subsequently, the ADMM algorithm is used to solve the coordinated operation optimization model in a distributed manner to protect data privacy. In addition, a settlement method for the GCF provided by the DCO is developed.

A. Wasserstein Metric

To quantify the underlying randomness in the probability distributions of net loads, and workloads, Wasserstein metric is used in this paper. The Wasserstein metric of two probability distributions denoted by $\mathbb{P}_1, \mathbb{P}_2 \in \mathcal{M}(\Xi)$ is defined as follows:

$$d_w(\mathbb{P}_1, \mathbb{P}_2) := \inf \left\{ \int_{\Xi} \|\xi_1 - \xi_2\| \Pi(d\xi_1, d\xi_2) \right\} \quad (32)$$

where Π is a joint distribution of two random vectors, ξ_1 and ξ_2 , with marginal distributions $\mathbb{P}_1$ and $\mathbb{P}_2$; $\mathcal{M}(\Xi)$ is the probability space including all probability distributions $\mathbb{P}$ supported on the uncertainty set Ξ .

Based on the definition of the Wasserstein metric, an ambiguity set, which can be expressed as a Wasserstein ball of radius ε centered at the empirical distribution $\mathbb{P}_{N_{sam}}$, is established as follows [29]:

$$\mathcal{P}_{N_{sam}}(\varepsilon) = \left\{ \mathbb{P} \in \mathcal{M}(\Xi) : d_w(\mathbb{P}_{N_{sam}}, \mathbb{P}) \leq \varepsilon \right\} \quad (33)$$

where the empirical distribution $\mathbb{P}_{N_{sam}}$ is computed using N_{sam}

historical samples $\hat{\xi}_i \in [\underline{\xi}_i, \bar{\xi}_i]$ as follows:

$$\mathbb{P}_{N_{\xi}} := \sum_{i=1}^N \delta_{\hat{\xi}_i} / N_{sam} \quad (34)$$

where $\delta_{\hat{\xi}_i}$ is the unit point mass at $\hat{\xi}_i$.

B. Distributionally Robust Chance Constraints Model

The uncertainties of the DS net load and workloads are addressed using chance constraints method, ensuring that constraint violations occur with a low probability. Eqs. (14)–(17) and (23) are reformulated as chance constraints, as shown follows:

$$\mathbb{P}_{\tilde{\mathbf{p}}_l, \tilde{\mathbf{q}}_l} \left(\begin{aligned} -\sqrt{2}\bar{s}_b(\ell) &\leq \mathbf{p}_b(\ell) + \mathbf{q}_b(\ell) \leq \sqrt{2}\bar{s}_b(\ell) \\ -\bar{s}_b(\ell) &\leq \mathbf{p}_b(\ell) \leq \bar{s}_b(\ell) \\ -\bar{s}_b(\ell) &\leq \mathbf{q}_b(\ell) \leq \bar{s}_b(\ell) \end{aligned} \right) \geq 1 - \epsilon, \forall \ell \in \mathcal{L} \quad (35)$$

$$\mathbb{P}_{\tilde{\mathbf{p}}_l, \tilde{\mathbf{q}}_l} (\underline{\mathbf{v}} \leq \mathbf{v}(j), \bar{\mathbf{v}} \geq \mathbf{v}(j)) \geq 1 - \epsilon, \forall j \in \mathcal{N} \quad (36)$$

$$\mathbb{P}_{\tilde{\mathbf{w}}_{fle}, \tilde{\mathbf{w}}_{fix}} \left(0 < \frac{1}{\mathbf{1}_F \boldsymbol{\mu}_k^T - \mathbf{w}_k^T \odot \mathbf{s}_k^T} + \mathbf{t}_k^T \leq \mathbf{t}_k^D \right) \geq 1 - \epsilon, \forall k \quad (37)$$

For convenience, the chance constraints Eqs. (35)–(37) are expressed as general form as follows:

$$\mathbb{P}_{\xi} \mathbf{a}^T \xi \leq b \geq 1 - \epsilon, \xi \in \mathbb{R}^w \quad (38)$$

Since the probability distribution of uncertain parameters cannot be precisely determined, the distributionally robust chance constraint is formulated based on the ambiguity set established in Eq. (33) as follows:

$$\min_{\mathbb{P} \in \mathcal{P}} \mathbb{P}(\mathbf{a}^T \xi \leq b) \geq 1 - \epsilon \quad (39)$$

The distributionally robust chance constraint can be conservatively approximated by a constraint including the CVaR at level ϵ with respect to $\mathbb{P}$, as shown follows:

$$\max_{\mathbb{P} \in \mathcal{P}} \mathbb{P} - \text{CVaR}_{\epsilon}(\mathbf{a}^T \xi - b) \leq 0 \Rightarrow \min_{\mathbb{P} \in \mathcal{P}} \mathbb{P}(\mathbf{a}^T \xi \leq b) \geq 1 - \epsilon \quad (40)$$

Then, the left side of Eq. (40) is reformulated as follows:

$$\min_{\tau \in \mathbb{R}} \tau + \frac{1}{\epsilon} \max_{\mathbb{P} \in \mathcal{P}} \mathbb{E}^{\mathbb{P}} [\max\{\mathbf{a}^T \xi - b - \tau, 0\}] \leq 0 \quad (41)$$

By using the duality theorem, the max operator in Eq. (41) can be transformed into a min operator, enabling the Eq. (41) to be solved as linear constraints, as shown in Eq. (42) [30].

$$\begin{cases} \tau + \frac{1}{\epsilon} \left(\lambda^{\text{CVaR}} \rho + \frac{1}{N_{sam}} \sum_{i=1}^N \sigma_i^{\text{CVaR}} \right) \leq 0 \\ \mathbf{a}^T \hat{\xi}_i - b - \tau + \gamma_{i,1}^T (\mathbf{h} - \mathbf{Q} \hat{\xi}_i) \leq \sigma_i^{\text{CVaR}} \quad \forall i \in \{1, \dots, N_{sam}\} \\ \gamma_{i,2}^T (\mathbf{h} - \mathbf{Q} \hat{\xi}_i) \leq \sigma_i^{\text{CVaR}} \quad \forall i \in \{1, \dots, N_{sam}\} \\ \|\mathbf{Q}^T \gamma_{i,1} - \mathbf{a}\|_* \leq \lambda^{\text{CVaR}} \quad \forall i \in \{1, \dots, N_{sam}\} \\ \|\mathbf{Q}^T \gamma_{i,2}\|_* \leq \lambda^{\text{CVaR}} \quad \forall i \in \{1, \dots, N_{sam}\} \\ \gamma_i \geq 0 \quad \forall i \in \{1, \dots, N_{sam}\} \end{cases} \quad (42)$$

where $\lambda^{\text{CVaR}} \in \mathbb{R}$, $\sigma^{\text{CVaR}} \in \mathbb{R}^{N_{\xi}}$, $\gamma_1 \in \mathbb{R}^{2W}$, $\gamma_2 \in \mathbb{R}^{2W}$ and $\tau \in \mathbb{R}$ are auxiliary variables. $\mathbf{Q} \in \mathbb{R}^{2W \times W}$ and $\mathbf{h} \in \mathbb{R}^{2W}$ are the coefficient matrices representing the boundary constraints

of the uncertain parameters in the form $\mathbf{Q}\xi \leq \mathbf{h}$.

To this end, the distributionally robust chance-constrained optimization model of multi-LFMs is established as follows:

$$\begin{aligned} & \text{obj (26)} \\ & \text{s.t. (5)–(12), (18)–(22), (24), (25), (27)–(30), (42)} \end{aligned} \quad (43)$$

The optimization model Eq. (43) is a centralized optimization model. If an independent third-party entity is introduced to directly solve Eq. (43), it would be necessary to collect internal data and parameters from the DSO, such as node net loads, branch resistance, and reactance, as well as internal data and parameters from the DCO, such as server operating parameter and workloads. Disclosing such internal data to an independent third-party entity would compromise the data privacy of both the DSO and DCO. Therefore, based on the proposed coordinated operation framework of Multi-LFMs, Eq. (43) is solved in a distributed manner using the ADMM algorithm, without the need to introduce an independent third-party entity. In the ADMM algorithm, the optimization model Eq. (43) is decomposed into sub-problems for the DCO and each DSO by decoupling the coupling constraint in Eq. (30). In this approach, the DSO and DCO independently solve their respective subproblems and exchange only limited data in an iterative process until convergence, thereby effectively protecting data privacy. The detailed steps of the ADMM algorithm can be found in [31] and are not elaborated here.

Moreover, during the iterative process of the ADMM algorithm, applying distributionally robust chance constraints to all buses' voltage constraints and branch capacity constraints in the DSO's sub-problem would significantly increase the complexity of the optimization model. Hence, in this paper, potential buses and branches with operation problems are first identified based on the boundary values of the uncertain parameters $\tilde{p}_l$ and $\tilde{q}_l$ by each DSO when solving their respective sub-problems. Then, distributionally robust chance constraints are applied only to these identified buses and branches. The specific steps for each DSO are as follows:

Step 1: Set the $\tilde{p}_l$ and $\tilde{q}_l$ as their upper boundary values $\mathbf{p}_l^{\max}$ and $\mathbf{q}_l^{\max}$, and solve sub-optimization problem of DSO to obtain the maximum power on branches $\mathbf{p}_b^{\max}$, $\mathbf{q}_b^{\max}$, as well as the minimum voltage magnitude of buses $\mathbf{v}^{\min}$. If $\mathbf{p}_b^{\max}$, $\mathbf{q}_b^{\max}$, or $\mathbf{v}^{\min}$ reach the constraint boundary, then formulate the corresponding constraints as chance constraints.

Step 2: Set the $\tilde{p}_l$ and $\tilde{q}_l$ to their lower boundary values $\mathbf{p}_l^{\min}$ and $\mathbf{q}_l^{\min}$, and solve sub-optimization problem of DSO to obtain the minimum power on branches $\mathbf{p}_b^{\min}$, $\mathbf{q}_b^{\min}$, as well as the maximum voltage magnitude of buses $\mathbf{v}^{\max}$. If $\mathbf{p}_b^{\min}$, $\mathbf{q}_b^{\min}$, or $\mathbf{v}^{\max}$ reach the constraint boundary, then formulate the corresponding constraints as chance constraints.

Step 3: Using the distributionally robust chance constraint method, convert the chance constraints into linear constraints

(42).

C. Settlement Method for The Geographically Coupled Flexibility

The objective of coordinated operations is to minimize the total operation costs of multi-LFMs. Due to the geographically coupled nature of the flexibility provided by the DCO, some LFMs may benefit from local DC flexibility, resulting in reduced operation costs. However, other LFMs might experience increased operation costs due to passively accommodating changes in DC load within the DS. To prevent such unfair situations for DSOs, it is crucial to develop a fair settlement method for the GCF provided by DCO. In this paper, a settlement method for the GCF is proposed by analyzing which LFM drives the generation of GCF.

First, after obtaining the coordinated operation results, for LFM k , the following optimization model for settlement problem is solved:

$$\begin{aligned} & \min \text{cost}_{lfm,k}^{se} = \text{cost}_{dso,k}^{se} + \text{cost}_{der,k}^{se} + \text{cost}_{dc,k}^{se} \\ & \text{s.t. } \text{cost}_{dso,k}^{se} = \mathbf{c}_{k,s} \mathbf{1}_{N_k}^T \mathbf{p}_{k,s} + \mathbf{c}_{k,c} \mathbf{1}_{N_k}^T \mathbf{p}_{k,c} \\ & \text{cost}_{der,k}^{se} = \sum_{d \in \mathcal{N}_{k,d}} \mathbf{c}_{k,d}^+ \mathbf{p}_{k,d}^+ + \mathbf{c}_{k,d}^- \mathbf{p}_{k,d}^- \\ & \text{cost}_{dc,k}^{se} = \mathbf{c}_{dc,k}^+ \mathbf{p}_{dc,k}^+ + \mathbf{c}_{dc,k}^- \mathbf{p}_{dc,k}^- \\ & 0 \leq \mathbf{p}_{dc,k}^+ \leq \mathbf{p}_{dc,k}^{+,co}, 0 \leq \mathbf{p}_{dc,k}^- \leq \mathbf{p}_{dc,k}^{-,co} \\ & (5) - (12), (42) \end{aligned} \quad (44)$$

where $\mathbf{p}_{dc,k}^{+,co}$ and $\mathbf{p}_{dc,k}^{-,co}$ are the upward and downward flexibility results provided by DC k in the coordinated operation. By solving the above optimization model, the results of $\mathbf{p}_{dc,k}^+$ and $\mathbf{p}_{dc,k}^-$ are denoted as $\mathbf{p}_{dc,k}^{+,se}$ and $\mathbf{p}_{dc,k}^{-,se}$, respectively. Since the coordinated scheduling results for LFM k are within the feasible region of the above optimization model, it follows that $\text{cost}_{lfm,k}^{se} \leq \text{cost}_{lfm,k}^{co}$.

If $\mathbf{p}_{dc,k}^{+,se} < \mathbf{p}_{dc,k}^{+,co}$ or $\mathbf{p}_{dc,k}^{-,se} < \mathbf{p}_{dc,k}^{-,co}$, it means $\mathbf{p}_{dc,k}^{+,co}$ or $\mathbf{p}_{dc,k}^{-,co}$ is influenced by other LFMs, while $\mathbf{p}_{dc,k}^{+,se}$ or $\mathbf{p}_{dc,k}^{-,se}$ is driven by LFM k . In this case, the payment in LFM k from DSO k to DC k is as follows:

$$\text{pay}_{dc,k}^{lfm,k} = \text{cost}_{dc,k}^{se} + \text{cost}_{der,k}^{se} - \text{cost}_{der,k}^{co} + \text{cost}_{dso,k}^{se} - \text{cost}_{dso,k}^{co} \quad (45)$$

Since $\text{cost}_{dc,k}^{se}$ is the cost of flexibility provided by DC k driven by LFM k , DSO k should pay to DC k for this cost, as shown in the first term on the right-hand side of Eq. (45). Additionally, the flexibility provided by DC k leads to changes in the cost of DSO k and DERAs flexibility. These changes are represented by the last four terms on the right-hand side of Eq. (45).

In the coordinated optimization, the cost of the flexibility provided by DC k is $\text{cost}_{dc,k}^{co}$. Therefore, the remaining cost $\text{pay}_{dc,k}^{lfm,other}$ should be paid by the other LFMs that drive the generation of the flexibility of DC k .

$$\begin{aligned}
\text{pay}_{dc,k}^{\text{lfm,other}} &= \text{cost}_{dc,k}^{\text{co}} - \text{pay}_{dc,k}^{\text{lfm,k}} \\
&= \text{cost}_{dc,k}^{\text{co}} + \text{cost}_{der,k}^{\text{co}} + \text{cost}_{dso,k}^{\text{co}} - \text{cost}_{dc,k}^{\text{se}} - \text{cost}_{der,k}^{\text{se}} - \text{cost}_{dso,k}^{\text{se}} \quad (46) \\
&= \text{cost}_{lfm,k}^{\text{co}} - \text{cost}_{lfm,k}^{\text{se}} > 0
\end{aligned}$$

For example, in the coordinated operation optimization results, if the flexibility provided by DC k coupled with DC q is $p_{dc,k}^{+,co,q}$ or $p_{dc,k}^{-,co,q}$, then in the LFM q , DSO q needs to pay an additional amount $\text{pay}_{dc,k}^{\text{lfm,q}}$ to DC q .

$$\text{pay}_{dc,k}^{\text{lfm,q}} = \begin{cases} \text{pay}_{dc,k}^{\text{lfm,other}} \left(p_{dc,k}^{+,co,q} / p_{dc,k}^{+,co} \right), & p_{dc,k}^{+,co} > 0 \\ \text{pay}_{dc,k}^{\text{lfm,other}} \left(p_{dc,k}^{-,co,q} / p_{dc,k}^{-,co} \right), & p_{dc,k}^{-,co} > 0 \end{cases} \quad (47)$$

If $p_{dc,k}^{+,se} = p_{dc,k}^{+,co}$ or $p_{dc,k}^{-,se} = p_{dc,k}^{-,co}$, it means the flexibility $p_{dc,k}^{+,co}$ or $p_{dc,k}^{-,co}$ provided by DC k is driven by LFM k . Hence, in LFM k , DSO k should pay $\text{cost}_{dc,k}^{\text{se}}$ to DC k to purchase this flexibility.

The overall process flowchart of the proposed solving method is presented in Fig. 2.

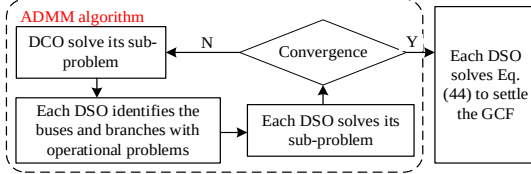


Fig. 2. Overall process flowchart of the proposed solving method.

IV. CASE STUDY

A. Case Setup

In this paper, DS1, DS2, and DS3, which are adapted from IEEE 15-buses, 33-buses, and 69-buses DSs, respectively, located in three different regions, are used in case studies. In each DS, one DC is connected. The topologies of the three DSs are shown in Fig. 3.

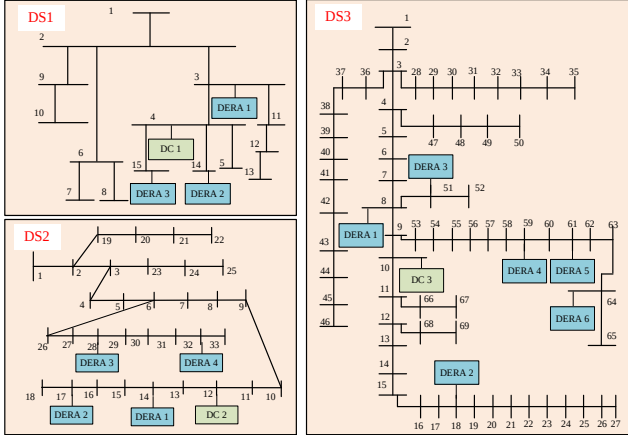


Fig. 3. The topologies of the three DSs.

It is assumed that the reactive power injections of DERs and DCs are proportional to the active power injections according to the power factor $\cos\phi = 0.95$ [32]. The retail electricity prices of DS1, DS2, and DS3 are 96, 90, and 100€/MWh [33], respectively, with costs of load shedding and generation curtailment are 300 and 200 €/MWh, respectively. The flexibility bids data of DERs are shown in TABLE I.

TABLE I
FLEXIBILITY BIDS DATA OF DERs

Parameters		Baseline load (MW)	Capacity (MW)		Price (€/MWh)	
			Up	Down	Up	Down
DS1	DERA1	-0.4	0.1	0.1	40	38
	DERA2	0.3	0.2	0.2	40	35
	DERA3	-0.3	0.15	0.15	50	46
DS2	DERA1	0.2	0.1	0.1	35	38
	DERA2	-0.7	0.3	0.3	35	36
	DERA3	-0.3	0.2	0.2	32	30
	DERA4	-0.6	0.3	0.3	55	51
DS3	DERA1	0.4	0.1	0.1	25	23
	DERA2	-0.3	0.2	0.2	27	28
	DERA3	0.3	0.1	0.1	20	23
	DERA4	-0.5	0.3	0.3	20	18
	DERA5	-0.4	0.2	0.2	24	22
	DERA6	-0.5	0.2	0.2	24	21

The DCO is responsible for managing the three DCs and four FSs. The data of flexible workload at each FSs and fixed workload from each FSs to DCs, as well as the transmission delay of workload are shown in Fig. 4 [17]. The flexibility prices of the three DCs c_{dc}^{+} and c_{dc}^{-} are 37€/MWh. The operation parameters of the DCs are shown in TABLE II. The history samples of uncertain parameters are generated by using Monte Carlo simulation. The lower and upper bounds of uncertain parameters $\tilde{\xi}$ are $0.85\hat{\xi}$ and $1.15\hat{\xi}$, respectively, where $\hat{\xi}$ is the forecast value. The confidence level of chance constraints $1-\epsilon$ is 0.95, and $N_{sam} = 200$. The convergence threshold for the ADMM algorithm is 0.00001. The code is programmed in MATLAB, and the optimization models are solved using Gurobi on a laptop equipped with an Intel i7 CPU.

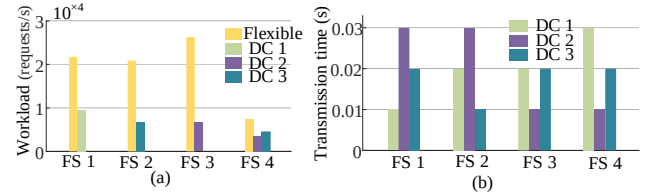


Fig. 4. (a) Flexible and fixed workloads and (b) transmission delays.

TABLE II
OPERATION PARAMETERS OF THE DCs

	P_{peak} (kW)	P_{idle} (kW)	PUE	$\bar{S}$	μ (requests/s)	t_D (s)
DC1	0.23	0.11	1.26	5000	22	0.1
DC2	0.18	0.08	1.3	8000	19	0.1
DC3	0.2	0.1	1.25	12000	20	0.1

B. Results and Analysis

For convenience of analysis, two scenarios are defined. Scenario 1 (S1): The three LFMs operate independently without GCF from the DCO. Scenario 2 (S2): Leveraging the GCF provided by the DCO, the three LFMs are operated in a coordinated manner using the method proposed in this paper.

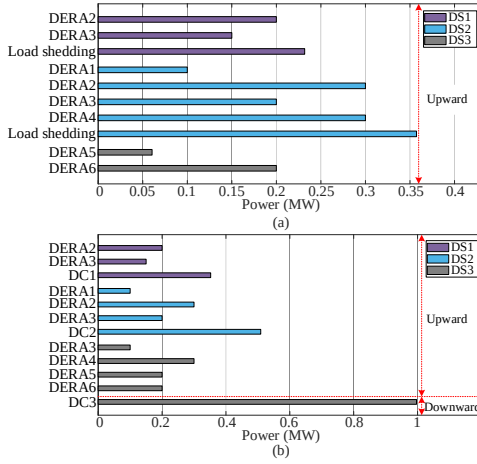


Fig. 5. The clearing results of the three LFM in (a) S1 and (b) S2.

In DS1, network congestion occurs between buses 3 and 4. Hence, in S1, DERA2 and DERA3 provide upward flexibility in LFM1 to alleviate the congestion issue in DS1, as shown in Fig. 5 (a). Additionally, load shedding occurs at buses 4, 5, and 14 in DS1 due to insufficient upward flexibility. In DS2 and DS3, an under-voltage violation issue is present. In S1, DERA1, DERA2, DERA3, and DERA4 provide upward flexibility in LFM2 to raise the voltage magnitude in DS2. Additionally, load shedding occurs at buses 11, 12, 13, and 14 in DS2, as shown in Fig. 5 (a). In LFM3, DSO3 obtains upward flexibility from DERA5 and DERA6 to resolve the under-voltage violation issue without load shedding.

In S2, DC1, DERA2, and DERA3 provide upward flexibility to DSO1 in LFM1 to address the congestion issue in DS1, as shown in Fig. 5 (b). Similarly, DC2 provides upward flexibility in LFM2 to enhance the buses voltage magnitude of DS2. Due to the geographically coupled nature of the DCO's flexibility, DC3 provides downward flexibility in LFM3. In fact, this downward flexibility is not driven by DSO3 and exacerbates the under-voltage violation issue in DS3. Consequently, DSO3 obtains additional upward flexibility from DERAs to resolve the issue. Compared to S1, there is no load shedding in LFM1 and LFM2 in S2. Therefore, in S2, by utilizing the GCF provided by the DCO, the three LFM operate in a coordinated manner, achieving effective integration of flexibility resources across the three LFM and balancing flexibility supply and demand within the three LFM. From another perspective, in this scenario, the GCF provided by the DCO can be viewed as a "bridge" connecting the three LFM, allowing LFM1 and LFM2 to obtain upward flexibility from DERAs in LFM3. Moreover, this "bridge" is wireless and not constrained by geographical location.

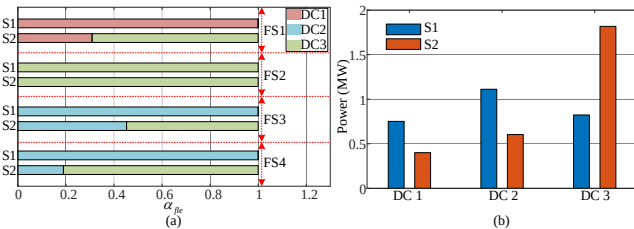


Fig. 6. (a) Dispatch results of flexible workload and (b) power consumption results of the three DCs.

As shown in Fig. 6 (a), in S2, part of flexibility workload in FS1 is transferred from DC1 to DC3 for processing, and portions of flexibility workloads in FS3 and FS4 are transferred from DC2 to DC3. This results in changes in the power consumption of the three DCs, as depicted in Fig. 6 (b). Specifically, compared to S1, in S2, the power consumption of DC1 and DC2 decreases, while the power consumption of DC3 increases. This corresponds to the upward flexibility provided by DC1 and DC2 and the downward flexibility provided by DC3, as shown in Fig. 5 (b).

TABLE III
THE OPERATION COST AND GCF RESULTS AFTER CLEARING IN S2

	$P_{dc}^{+,co}$ (MW)	$P_{dc}^{-,co}$ (MW)	$cost_{dso}^{co}$ (€)	$cost_{der}^{co}$ (€)	$cost_{dc}^{co}$ (€)	$cost_{lfm}^{co}$ (€)
LFM1	0.3522	0	0	15.5	13.0313	28.5313
LFM2	0.5089	0	0	20.4	18.8299	39.2299
LFM3	0	0.9966	0	17.6	36.8759	54.4759

After the multi-LFM is cleared, the operation costs of the three LFM and the clearing results of the GCF are shown in TABLE III. The total coordinated operation cost for the three LFM is 122.3254€.

The operation costs and GCF results for the three LFM, obtained by solving the model in Eq. (44), are shown in TABLE IV.

TABLE IV
THE OPERATION COST AND GCF RESULTS OBTAINED BY SOLVING EQ. (44)

	$P_{dc}^{+,se}$ (MW)	$P_{dc}^{-,se}$ (MW)	$cost_{dso}^{se}$ (€)	$cost_{der}^{se}$ (€)	$cost_{dc}^{se}$ (€)	$cost_{lfm}^{se}$ (€)
LFM1	0.3522	0	0	15.5	13.0313	28.5313
LFM2	0.5089	0	0	20.4	18.8299	39.2299
LFM3	0	0	0	6.1641	0	6.1641

From the results in TABLE IV, it can be observed that the flexibility provided by DC1 and DC2 is entirely driven by LFM1 and LFM2, respectively, while the flexibility provided by DC3 is influenced by other LFM. Additionally, $p_{dc,3}^{-,co}$ includes 0.4216MW associated with DC1, accounting for 42.3%, and 0.575MW associated with DC2, accounting for 57.7%. Based on the results in TABLE III and IV, the settlement results for each LFM, determined using the settlement method proposed in Section 3.4 of this paper, are presented in TABLE V.

TABLE V
THE OPERATION COST OF THE THREE LFMS AFTER SETTLEMENT IN S2

Cost (€)		DSO	DERAs	DC	Total
LFM1	S1	69.5318	15.5	0	85.0318
	S2	0	15.5	33.4659	48.9659
LFM2	S1	107.196	36.9	0	144.096
	S2	0	20.4	46.7071	67.1071
LFM3	S1	0	6.1641	0	6.1641
	S2	0	17.6	-11.4359	6.1641

As shown in TABLE V, compared to S1, in S2, the operation costs of LFM1 and LFM2 decrease by 42.41% and 53.43%, respectively, due to the GCF provided by the DCO. Additionally, in S2, load shedding does not occur, and power

supply reliability is improved. Meanwhile, the increased flexibility cost of DERAs in LFM3 is offset by the settlement with DC3. As a result, LFM3 is not affected by the GCF, and its operation cost remains unchanged. Therefore, the GCF settlement method proposed in this paper effectively ensures a fair allocation of the total operation costs among the three LFMs in S2.

In retail market, the baseline electricity purchase cost of DCO is 254.5336€. Due to changes in the dispatch plan of flexible workloads, the DCO's electricity purchase costs in the retail market increased by 19.9799€. Meanwhile, in S2, by providing GCF, the DCO receive 68.7372€ in revenue from LFM1, LFM2, and LFM3 for a single time slot, i.e., 1 hour. As a result, the DCO's operation cost is reduced by 19.16% through providing GCF in the multi-LFMs.

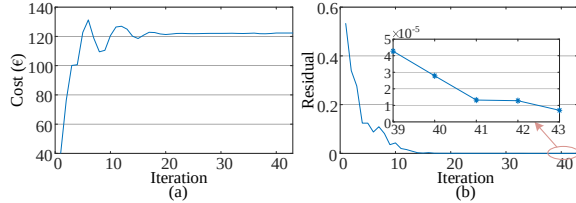


Fig. 7. (a) Total operation cost and (b) residual results in iteration process.

As shown in Fig. 7, the ADMM algorithm converges after 43 iterations. The total operation cost obtained from solving the centralized optimization model represented by Eq. (43) is 122.2184€. Thus, in comparison with the centralized solution, the ADMM algorithm ensures effective data privacy protection across different entities while maintaining a low error rate of 0.0876%, which validates the effectiveness of the ADMM algorithm.

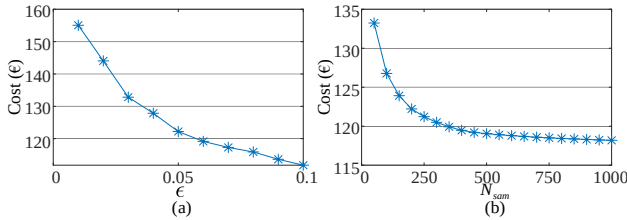


Fig. 8. Operation cost of multi-LFMs with the change of (a) ϵ and (b) N_{sam} .

As shown in Fig. 8 (a), as ϵ increases, the probability of violating the chance constraints increases, leading to a decrease in the conservativeness of the model. Consequently, the required flexibility decreases, and the total operation cost is reduced. Similarly, as N_{sam} increases, the conservativeness of the Wasserstein metric-based ambiguity set decreases, which also results in a reduction in total operation cost, as shown in Fig. 8 (b).

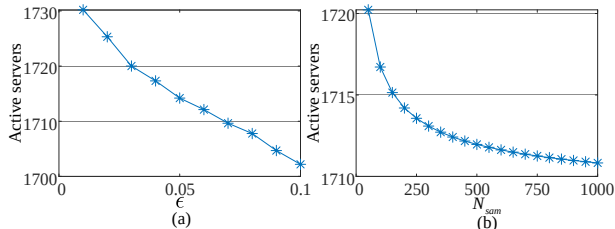


Fig. 9. Active servers of DC1 with the change of (a) ϵ and (b) N_{sam} .

In Fig. 9, as ϵ and N_{sam} increase, the conservativeness of the chance constraints related to DC quality-of-service decreases. Consequently, the number of active servers is reduced.

V. CONCLUSION

In this paper, a coordinated operation method for multi-LFMs is proposed, enabling their operation in a wireless manner and without geographical limitations, using the GCF from DCO. First, an optimization model for the coordinated operation of multi-LFMs is established, while using pay-as-bid method to compensate the flexibility from DERAs and DCO. To address data privacy concerns, the ADMM algorithm is used to solve the optimization model in a distributed manner, yielding the clearing results for the multi-LFMs. During this clearing process, only limited data needs to be exchanged between DSOs and their local DC. After clearing the multi-LFMs, a settlement method for the GCF provided by the DCO is proposed to ensure a fair allocation of the total operation costs among multi-LFMs. Similarly, during the settlement process, only interactions between DSOs and their local DC are required. Additionally, a distributionally robust chance constraint based on the Wasserstein metric is employed to address uncertain parameters.

In the case study, three LFMs are considered for coordinated operation. The clearing results show that utilizing the GCF provided by the DCO enables effective integration and utilization of DERAs flexibility across the three LFMs, facilitating the balance between flexibility supply and demand. The method proposed in this paper for settling the GCF achieves a fair distribution of total costs among the three LFMs. The operation costs of LFM1 and LFM2 are reduced by 42.41% and 53.43%, respectively, compared to operation individually, while also eliminating the load shedding of DS1 and DS2 and avoiding the need for network reinforcement for DS1 and DS2. The revenue obtained by the DCO from providing GCF in the multi-LFMs results in a 19.16% reduction in the DCO operation costs. Additionally, the results also demonstrate the effectiveness of the ADMM algorithm and the distributionally robust chance constraint method.

Future research can further investigate the temporal scheduling of data center workloads and leverage their temporal flexibility to participate in local flexibility markets.

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