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On the impact of supply uncertainty on inventory decisions: EOQ-based models

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Abstract

Inventory management is influenced by both demand and supply uncertainty, with the latter often manifesting as yield variability, where delivered quantities deviate from orders. This study develops a unified framework to analyze four key sources of variability in order quantities: (1) random yield, where deliveries differ from the order quantity; (2) random quality, where received items include defects identified through post-delivery inspection; (3) random capacity, where supplier limitations prevent full order fulfillment; and (4) supply disruption, where the order fails to materialize. Within an *EOQ* context and under an (s, S) policy, we derive a common long-run average cost formulation for all four uncertainty types and determine the cost-minimizing inventory policies. We conduct both theoretical and numerical comparisons, examining impacts on total cost, optimal policy parameters, and replenishment intervals. Beyond providing a unified perspective on yield-related supply uncertainty, our findings offer guidance for supplier evaluation and selection.

Keywords: Inventory, Supply uncertainty, Random yield, Imperfect quality, Random capacity, Supply disruptions

1. Introduction

Inventory systems almost invariably operate under supply uncertainty, which can impact performance across multiple dimensions. This uncertainty typically arises from variability in order quantities and/or delivery lead times. The former is the focus of this study and is mainly caused by i) random yield, ii) imperfect quality products, iii) random supplier capacity, and iv) supply disruptions. Despite a substantial body of related literature, to the best of our knowledge, this is the first time that the different types of yield uncertainty are systematically compared with respect to their impact on inventory system performance.

Random yield refers to the unpredictable variability in the quantity of output produced due to production inefficiencies, defective raw materials, or unpredictable processes. Random yield leads to waste, reprocessing costs, and inconsistent production rates. In high-technology manufacturing, such as semiconductors and pharmaceuticals, the appearance of random yield is common (Berling and Sonntag [2]). Fluctuating yields complicate inventory decisions, leading to increased safety stock requirements, overordering to compensate for yield variability, and supplier diversification to reduce dependency, as potential mitigation strategies. However, each strategy has specific trade-offs, such as higher holding costs, product obsolescence, and higher coordination costs.

Imperfect quality products are items that fail to meet the desired specifications or quality standards. This can be due to production errors, material defects, or human mistakes. The production of imperfect-quality items is commonly expected in ultra-precision manufacturing industries e.g., automotive, aerospace, defense, and shipbuilding (Kang et al. [39], Nakhaeinejad [22]). When dealing with imperfect quality items, the risk that some goods may not meet the required standards should be accounted for. This uncertainty unavoidably impacts inventory decisions. Specifically, to ensure product quality, additional inspection processes or rework procedures may need to be implemented, leading to delays and added costs in sorting out defective items. Furthermore, higher safety stock may be required to compensate for defects, leading to increased holding costs due to excess inventory. In industries where imperfect quality is common, return or replacement policies are often demanded, especially if defects are discovered after delivery, resulting in logistics costs related to returns, replacements, and customer satisfaction efforts. Imperfect quality can also lead to stockouts if defective products are returned or discarded, which can hurt sales and damage the firm's reputation.

Random supplier capacity refers to the uncertain ability of suppliers to deliver the required quantity of raw materials or components. Factors affecting this could include labor shortages, equipment breakdowns, or logistical challenges. Managing random supplier capacity is a common challenge in industries such as electronics and automotive manufacturing, with firms facing the need to optimize sourcing decisions between reliable and unreliable suppliers to maintain stability under uncertain conditions (Chen and Tan [5]). When uncertainty around a supplier's capacity arises, inventory management decisions are adapted through increased safety stock, supplier diversification, flexible order quantities, or paying for guaranteed capacity, which ties up capital and increases holding costs.

Finally, supply disruptions occur when the supply chain is interrupted due to factors like political instability, natural disasters, pandemics, or transportation issues. These disruptions lead to delays or shortages of products and raw material. Inventory management adaptations to mitigate supply disruption include increased safety stock, advanced ordering, diversification of suppliers, aggregating inventory from different suppliers or regions, coordination with logistics providers. Addressing supply disruption risks before launching new programs is a critical issue for firms operating under just-in-time frameworks, particularly those with a small supplier base and low inventory levels, as highlighted by Sanci et al. [30] in collaboration with Ford Motor Company. Obviously, any adaptation could lead to higher holding costs, risk of overstocking, higher transportation, administrative, logistical and quality control expenses.

In this context, we can observe that the above-mentioned types of uncertainties arise from different causes but share several interrelations. Random yield and imperfect quality are closely related, as random yield refers to the quantity of usable output (which may be greater or less than the ordered quantity), while imperfect quality products refer to variations in quality within that output. Random supplier capacity is closely associated with random yield and imperfect quality, as unreliable material supply can reduce production efficiency and degrade product quality. A cascading effect occurs when a supply disruption reduces supplier capacity, which in turn affects yield and increases the likelihood of quality issues.

In this paper, we consider an inventory system operating under a reorder point order-up-to level (s, S) policy with constant demand, ensuring that uncertainty arises only from the supply side. In this context, we aim to mathematically examine the interconnections between the aforementioned supply uncertainties through a unified representation of the underlying process, along with the corresponding long-run average profit. We then seek to compare how each type of uncertainty affects decisions and costs. Understanding this impact should lead to better supplier selection and assessment decisions.

The remainder of the paper is as follows: In section 2, we provide a literature review of systems with the aforementioned supply uncertainties under constant demand along with a detailed discussion of our contribution. Section 3 introduces the notation and underlying assumptions. Section 4 presents the objective function in a unified manner for all types of uncertainties considered, along with its specification for each type. Section 5 provides optimal policy variables for all supply uncertainty types. Numerical comparisons are given in Section 6. Section 7 concludes with the main findings and suggestions for future research. Lastly, all proofs appear in the Appendix.

2. Research background and contribution

In this section, we first discuss the research background, organized around the four types of uncertainly considered in our work, followed by our contribution to the field of inventory management.

2.1. Research background

In the Economic Order Quantity (*EOQ*) framework, one of the earliest contributions addressing random yield was by Silver [32], who modified the classical *EOQ* model to incorporate yield uncertainty. Assuming that the received quantity is a random proportion of the ordered quantity, with a constant mean and a variance that may depend on the order quantity, he investigated the impact of random yield on cost and order quantity. Kalro and Gohil [12] extended this model in order to include both complete and partial backorders by also considering that the standard deviation of the quantity received is either independent of the quantity ordered or dependent on it. Perry et al. [26], analyzed the particular case of exponentially distributed yields in Silver's [32] model. A review of single-item continuous review inventory models under random yield can be found in the work of Yano and Lee [40].

Salameh and Jaber [29] first modified the traditional *EOQ* model to consider imperfect quality items that require full inspection. They assumed that, upon receiving an order, a screening process with a finite rate begins, to distinguish the perfect quality items from the imperfect ones. While the former are used to meet the customer demand, the latter are sold at the end of the process as a single batch at a lower price to a secondary market. Cárdenas-Barron [3] corrected an error in Salameh and Jaber's proposed order quantity, while Maddah and Jaber [19] revisited it using renewal theory, resulting in simpler expressions for both long-term average profit and optimal order quantity. Their findings indicate that the optimal order quantity increases with a higher screening rate, which facilitates faster removal of imperfect quality items. Additionally, the optimal order quantity decreases with greater variability in the proportion of imperfect quality items.

The model of Salameh and Jaber [29], has inspired extensive research and various extensions (see Khan et al. [15]), including backordered shortages (e.g., Eroglu and Ozdemir [6]), inspection errors (e.g., Khan et al. [16], Hsu [10]) or inspection errors with planned backorders (e.g., Hsu and Hsu [9]). A common assumption in these models is that, despite the substantial time required for inspection, no unplanned shortages occur during this process. This assumption rests on the idea that the quantity of perfect quality items at least meets the total demand throughout the screening period. However, Papachristos and Konstantaras [23] showed that this assumption does not guarantee the avoidance of unplanned shortages. Vörös [37] also revised the Salameh and Jaber [29] model, relaxing the assumption that demand is fully met at the end of the screening process. However, this approach does not effectively prevent unplanned shortages. To address this issue, Maddah et al. [20] proposed a policy that maintains inventory availability throughout the screening period by placing an order when the inventory level equals the total demand during the screening period. Karakatsoulis and Skouri [13], proposed an improved (r, Q) inventory model for imperfect quality items, optimizing both reorder point and order quantity to minimize the average total cost while assuming that the number of perfect quality items at least equals the demand during the screening. They account for both cases that shortages cannot occur and that unplanned shortages with backlogging costs occur during the screening process. Their study, assuming that only imperfect quality items are detected until the detection of all the imperfect ones, identifies a lower reorder point that prevents shortages and reduces the average inventory, minimizing total costs.

With regard to supplier capacity, Wang and Gerchak [38] incorporated capacity variability into both the classical EOQ model with constant demand

and a continuous review order quantity/reorder point model with backlogging, showing that capacity uncertainty generally increases optimal order quantities and reorder points, with tractable solutions in the exponential case. Hariga and Haouari [7] focused more narrowly on the EOQ framework, demonstrating that under random supplier capacity, the cost function is unimodal and pseudo-convex, and that the classical EOQ solution performs nearly optimally across a range of capacity distributions. Extending this line of research, Moon et al. [21] integrated variable capacity with random yields, developed multi-item EOQ formulations subject to budget constraints, and applied a distribution-free approach to the (Q, r) model, enabling robust solutions when demand distributions are only partially known.

Finally, disruptions in supply chains can be categorized into two main types: exogenous and endogenous (e.g., Konstantaras et al. [17], Taleizadeh et al. [35]) meaning that they can be unrelated to supplier's efficiency (e.g. natural disasters, terrorist attacks, geopolitical events, economic factors, accidents, pandemics), or they can reflect issues within the supply process itself (e.g. the supplier's inability to deliver, supplier failures, poor quality supplies). The study of inventory systems affected by supply disruptions began with Chao's [4] foundational work, where a finite state space model was developed, while using a Markov process to represent the system's states. Building on this, Parlar and Berkin [24] extended the analysis in the EOQ environment and initiated a large class of inventory models, now known as the $EOQD$ (with D for Disruptions). They assumed that the supply process alternates between periods of supply availability and supply unavailability, the durations of which are random variables with constant average rates. Two scenarios were examined: one with both periods exponentially distributed (parameters λ and μ), and another with an exponential supply availability period (parameter λ) and a deterministic unavailability period. They employed a zero-inventory ordering policy, assuming any unmet demand during a disruption was lost. This early work was refined by Berk and Arreola-Risa [1], who derived an optimal order quantity formula, aiming to cost minimization, although no closed-form solution was provided. Their numerical analysis revealed that as the availability ratio λ/μ increased, the optimal order size also increased. Snyder [33] developed a convex cost function that closely approximates the previous model and can be solved in closed form. Parlar and Perry [25] further advanced this research by adopting a continuous review order-up-to policy, where the reorder point was treated as a decision variable, allowing for complete backordering of unmet demand. Unlike the traditional EOQ models, in systems under disruption risk maintaining a positive reorder point can be beneficial. This finding suggests that holding safety stock is an effective strategy for mitigating the negative effects of disruptions. Their results indicated that both the reorder point and the order-up-to level increase as the frequency of supply availability decreases, assuming a constant recovery rate. Additionally, for a fixed order quantity, shorter average supply availability durations lead to an increased reorder point while shorter disruption durations correspond to a lower one. Karakatsoulis and Skouri [14] accounted for different limiting supply disruption probabilities and examined a periodic review model under the (S, T) policy, facing disruptions occurring at random points in time. Extending the periodic-review model of Konstantaras et al. [17], they used a continuous-time Markov chain to analyze systems with identical limiting disruption probabilities but differing disruption profiles, finding that rare, long disruptions require larger safety stocks, longer review intervals, and incur higher costs than frequent, short disruptions. For a recent review on supply disruptions, see Snyder et al. [34].

2.2. Contribution

It is evident that each type of supply uncertainty stems from different real-world situations. However, they seem to be interrelated. Therefore, it would be interesting if they could be handled in a unified way, allowing their associations to be highlighted and making comparisons as well as combinations between them possible.

In this context, we consider an inventory installation controlled by a continuous review (s, S) policy, facing a fixed-rate deterministic demand under the aforementioned types of supply uncertainty. Maintaining a constant demand prevents the introduction of additional uncertainty factors, allowing the focus to remain solely on supply-related uncertainty. Meanwhile, the (s, S) policy can help mitigate supply uncertainty by providing flexibility in order quantities and adjusting safety stock. In addition, the (s, S) policy facilitates the integration of multiple types of uncertainty into a single model while formulating the average profit per unit time in a unified manner. This enables the identification of similarities and differences between various types of supply uncertainty, which, under specific conditions, become interrelated.

To create comparable models for random yield, imperfect-quality products, and random capacity−types of uncertainty−we adopt the assumptions of Silver [32], Maddah and Jaber [19], and Hariga and Haouari [7], respectively (except for the zero-safety-stock assumption). Regarding supply disruptions, we adopt the assumptions of Berk and Arreola-Risa [1], assuming lost sales for unsatisfied demand to ensure the greatest fairness with previous models, and we further extend their model by using an (s, S) policy. By assuming different limiting supply disruption probabilities, our work also connects with the model of Karakatsoulis and Skouri [14]. To ensure comparability with the existing literature, we adopt a profit maximization criterion. From the optimization of the resulting models, it is noteworthy that a zero-safety-stock and order-up-to levels—coinciding with those derived by Silver [32], Maddah and Jaber [19], and Hariga and Haouari [7] for each associated type of uncertainty—are obtained. This implies that the zero-safety-stock assumption made in those models is not only intuitively correct but also formally proven.

3. Notation and assumptions

In this section, we present the notation and abbreviations used throughout this paper, as well as the common assumptions underlying the operation of the inventory systems considered in our work. Some additional assumptions concerning each type of supply uncertainty are introduced afterwards.

3.1. Notation

S	order-up-to level (decision variable)
s	reorder point (decision variable)
Q	the difference $Q = S - s$
t	time between two successive replenishments of inventory, with $T = E(t)$
h	holding cost (per unit per unit time)
c	purchase cost (per unit)
r	sales price (per unit)
$\hat{l}$	loss of goodwill (per unit)
l	lost sales cost (per unit), $l = \hat{l} + r - c$
g	screening cost (per unit)
D	demand rate
x	screening rate
K	fixed cost (per order)

3.2. Abbreviations

r random yield

f imperfect quality products
c random capacity
d supply disruption
v variance dependent on $S - s$
n variance independent of $S - s$
EOQ economic order quantity

3.3. Assumptions

We study a single-item inventory system controlled by a continuous review (s, S) policy. In this context, if the system's inventory level is equal to the reorder point, s, a replenishment order is released. The quantity ordered becomes available immediately, aiming to restore the inventory position at an order-up-to level S. The demand rate is constant, and shortages are not allowed. Under a reliable supply process, the received quantity coincides with the quantity ordered, i.e. $Q = S - s$. As unreliable supply is assumed, the received quantity becomes a random variable. Particularly we assume that supply is subject to one of the following types of uncertainty: a) random yield b) quality issues (imperfect quality products) c) random capacity and d) supply disruptions. Next, we list the individual assumptions for each of the types mentioned above.

Random yield

a_1 In each order, of size $Q = S - s$, the quantity received is $Y_y(s, S) = (1 - p)(S - s)$, where p is a continuous random variable with support on $[0, 1)$. Random yield could indeed lead to received quantities higher than the ordered quantity, but we make this assumption to enable a fair comparison with the following types of supply uncertainty—a common practice found in the literature (e.g., Yano and Lee [40], Lee and Yano [18]).

a_2 The number of items delivered in each cycle is independent of the number delivered in other cycles.

a_3 We consider two cases regarding $Var(p)$: $Var(p)$ is either independent of $Q = S - s$, or $Var(p) = \frac{\rho(1-\rho)}{S-s}$, where $\rho \in [0, 1)$. The latter case is motivated by the fact that each item in an order has a probability $\rho \in [0, 1)$ of being received. This suggests that $Y_y(s, S) \sim \text{Bin}(S - s, 1 - \rho)$. In this case, p is defined as $p = \frac{Y_y(s,S)}{S-s}$, so $E(p) = \rho$ and $Var(p) = \frac{\rho(1-\rho)}{S-s}$.

Imperfect quality products

In addition to the assumptions a_2-a_3 the following assumptions are also considered.

- b_1 In each batch of $S - s$ items a quantity of $p(S - s)$ imperfect quality items is included, which is detected after a 100% error-free screening process at a finite rate $x > D$. We assume that x is sufficiently large to eliminate the probability of shortages at the end of the screening process. Consequently, the number of perfect quality items is given by $Y_f(s, S) = (1-p)(S-s)$, where p is a continuous random variable with support on $[0, 1)$.

- b_2 The holding cost per unit per unit of time is the same for both perfect and imperfect quality products, as they are stored in the same warehouse (see also Salameh and Jaber [29], Maddah and Jaber [19]).

Random capacity

- c_1 The quantity received, for an order of size $Q = S - s$, is: $Y_c(s, S) = (S - s)(1 - \max\{1 - p, 0\})$, where p is a continuous random variable with support in $(0, \infty)$.

Supply disruptions

- d_1 Supply disruptions occur randomly in time and last for random durations. The times of disruption and recovery follow exponential distributions with rates λ and μ, respectively. The choice of exponential distributions for supply availability and supply disruption, aside from their mathematical tractability, is well-suited for describing the time between events that occur independently and at a constant average rate. At the scheduled replenishment time, the availability of the supply is described by the random variable p, which takes the value 1 if the supply is available and 0 if it is not.

- d_2 If unplanned shortages occur during the disruption period, unsatisfied demand is lost.

Based on Assumptions d_1, d_2 and since continuous review allows orders to be placed immediately upon recovery, the received quantity is then determined by:

$$Y_d(s,S) = \begin{cases} Dt, & 0 < t \leq \dfrac{S}{D} \\ S, & t > \dfrac{S}{D} \end{cases}$$

4. Objective function formulation

Under the continuous review policy (s, S), if the inventory position of the system is equal to (or below) the reorder point, s, a replenishment order is released. The order size aims to restore the inventory level at an order-up-to level S. However, due to supply uncertainty, the inventory level after any replenishment is within the interval $[\min\{s, \max\{S - W(s,S), 0\}\}, S - W(s,S)]$, where $W(s,S)$ is the divergence from the target order-up-to level S. The form of $W(s,S)$ for each type of uncertainty will be specified later. So, the system inventory level at any time instant τ with $\tau \in [0,t]$ can be determined as:

$$I(\tau) = S - W(s,S) - D\tau \tag{1}$$

Since the demand rate is constant, $I(\tau)$ is a random variable with a distribution depending on $W(s,S)$.

Based on (1), we model the average total profit, for uniformity purposes, in terms of the policy variables.

$$\begin{aligned}
\hat{\Pi}(s,S) &= \underbrace{rE(S - [S-Dt]^{+})}_{revenue\ per\ unit\ sold} \\
&- \Bigg[\underbrace{K}_{order\ cost} + \underbrace{cE(S - [S-Dt]^{+})}_{purchase\ cost} \\
&+ \underbrace{hE\left(\int_0^t [S - W(s,S) - D\omega]^{+}\, d\omega \right)}_{holding\ cost} + \underbrace{\hat{l}E[S-Dt]^{-}}_{loss\ of\ goodwill} \\
&= rE(S - [(S-Dt) + [S-Dt]^{-}]) \\
&- \Bigg[K + cE(S - [(S-Dt) + [S-Dt]^{-}]) \\
&+ hE\left(\int_0^t [S - W(s,S) - D\omega]^{+}\, d\omega \right) + \hat{l}E[S-Dt]^{-} \Bigg]
\end{aligned}$$

$$
\begin{aligned}
&= (r-c)E(Dt) - \Bigg[K + hE\bigg(\int_0^t [S - W(s,S) - D\omega]^+ \, d\omega\bigg) \\
&+ (r - c + \hat{l})E[S - Dt]^-\Bigg] \\
&= (r-c)E(Dt) \\
&- \Bigg[K + hE\bigg(\int_0^t [S - W(s,S) - D\omega]^+ \, d\omega\bigg) + lE[S - Dt]^-\Bigg] \qquad (2)
\end{aligned}
$$

If t represents the time between two successive replenishments of inventory, we denote its expected length as:

$$T = E(t) \qquad (3)$$

The expected profit per unit time is obtained by invoking the renewal-reward theorem (Ross [27]):

$$\Pi(s,S) = \frac{\hat{\Pi}(s,S)}{T} = (r-c)D - C(s,S) \qquad (4)$$

where:

$$C(s,S) = \frac{K + hE\bigg(\int_0^t [S - W(s,S) - D\omega]^+ \, d\omega\bigg) + lE[S - Dt]^-}{T} \qquad (5)$$

In this perspective, the total profit per time unit consists of two terms: one independent of the variables (s,S) and the term $C(s,S)$. Since $\underset{(s,S)}{\text{argmax}}\, \Pi(s,S) = \underset{(s,S)}{\text{argmin}}\, C(s,S)$, we henceforth intend to minimize $C(s,S)$.

4.1. Objective function formulation per type of uncertainty

In this section, we seek to specify $C(s,S)$, for each type of supply uncertainty considered.

4.1.1. Cost function under random yield

Given the underlying assumptions, we seek to determine the corresponding average cost. To do so, we need to specify expressions for $W(s,S)$ and t. Figure 1 depicts the inventory level for the random yield case. As we can see from the figure and based on Assumption a_1, the received quantity (at time $\tau = 0$) is $(1-p)(S-s)$, so:

$$W(s,S) = \begin{cases} p(S-s), \ \tau = 0, \\ \\ 0, \ 0 < \tau \leq t \end{cases} \tag{6}$$

The cycle, t, between two consecutive replenishment is $t = \frac{(1-p)(S-s)}{D}$ so,

$$T = \frac{[1-E(p)](S-s)}{D}$$

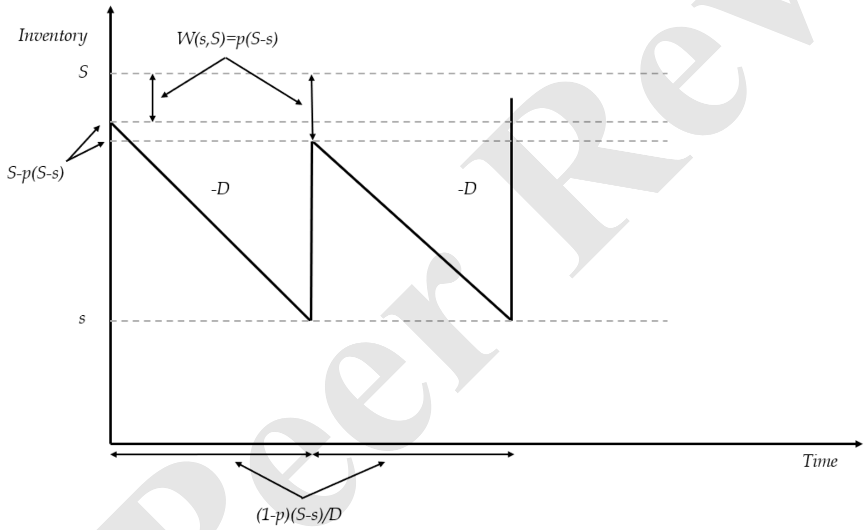

Figure 1: Sample path of the inventory evolution under random yield.

By replacing the above expressions in (5) the average cost per unit time is:

$$C_y(s,S) = \frac{KD}{(S-s)[1-E(p)]} + h(S-s)\left\{\frac{Var(p)}{2[1-E(p)]} + \frac{[1-E(p)]}{2}\right\} + hs \tag{7}$$

Using the transformation $Q = S - s$ we obtain:

$$C_y(s,Q) = \frac{KD}{Q[1-E(p)]} + hQ\left\{\frac{Var(p)}{2[1-E(p)]} + \frac{[1-E(p)]}{2}\right\} + hs \tag{8}$$

In case with $Var(p) = \dfrac{\rho(1-\rho)}{S-s}$, (7) becomes:

$$C_y(s,S) = \frac{KD}{(S-s)[1-E(p)]} + h(S-s)\frac{[1-E(p)]}{2} + hs + h\frac{E(p)}{2} \quad (9)$$

Using the transformation $Q = S - s$ we obtain:

$$C_y(s,Q) = \frac{KD}{Q[1-E(p)]} + hQ\frac{[1-E(p)]}{2} + hs + h\frac{E(p)}{2} \quad (10)$$

4.1.2. Cost function for imperfect quality products

Based on the underlying assumptions, we aim to formulate the corresponding average cost. To achieve this, we need to specify expressions for $W(s,S)$ and t. The inventory level, when imperfect quality products are included in a delivered order, is graphically represented in Figure 2. As we can see from the figure and based on Assumption b_1, the perfect quality items (which are identified at time $\tau = \frac{S-s}{x}$) are equal to $(1-p)(S-s)$, so $W(s,S)$ is:

$$W(s,S) = \begin{cases} 0, \; 0 < \tau \leq t \text{ and } \tau \neq \dfrac{(S-s)}{x}, \\ p(S-s), \; \tau = \dfrac{S-s}{x} \end{cases} \quad (11)$$

The cycle, t, between two consecutive replenishment is $t = \frac{(1-p)(S-s)}{D}$ so,

$$T = \frac{[1-E(p)](S-s)}{D}$$

By substituting the above expressions into (5) we obtain:

$$\frac{KD}{(S-s)[1-E(p)]} + hs + h(S-s)\left\{\frac{DE(p)}{x[1-E(p)]} + \frac{Var(p)}{2[1-E(p)]} + \frac{[1-E(p)]}{2}\right\}$$

As imperfect quality items are included in the order, an additional cost per unit time is considered for the screening process, $\dfrac{gD}{1-E(p)}$, leading to the following average cost per unit time:

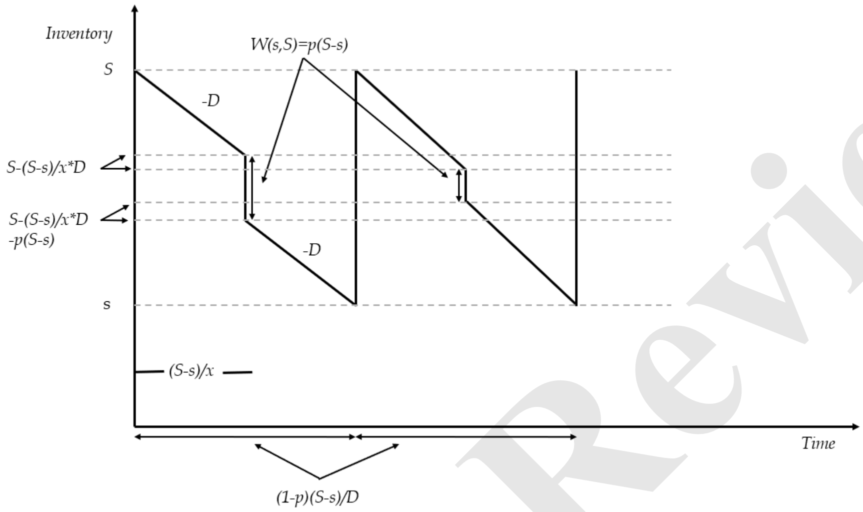

Figure 2: Sample path of the inventory evolution when imperfect quality products are included in an order.

$$C_f(s,S) = \frac{KD}{(S-s)[1-E(p)]} + \frac{gD}{1-E(p)} + hs + h(S-s)\left\{\frac{DE(p)}{x[1-E(p)]} + \frac{Var(p)}{2[1-E(p)]} + \frac{[1-E(p)]}{2}\right\} \quad (12)$$

Using the transformation $Q = S - s$ we obtain:

$$C_f(s,Q) = \frac{KD}{Q[1-E(p)]} + \frac{gD}{1-E(p)} + hs + hQ\left\{\frac{DE(p)}{x[1-E(p)]} + \frac{Var(p)}{2[1-E(p)]} + \frac{[1-E(p)]}{2}\right\} \quad (13)$$

In case of $Var(p) = \dfrac{\rho(1-\rho)}{S-s}$, (12) becomes:

$$C_f(s,S) = \frac{KD}{(S-s)[1-E(p)]} + \frac{gD}{1-E(p)} + hs + h(S-s)\left\{\frac{DE(p)}{x[1-E(p)]} + \frac{[1-E(p)]}{2}\right\} + h\frac{E(p)}{2} \quad (14)$$

Using the transformation $Q = S - s$ we obtain:

$$C_f(s,Q) = \frac{KD}{Q[1-E(p)]} + \frac{gD}{1-E(p)} + hs + hQ\left\{\frac{DE(p)}{x[1-E(p)]} + \frac{[1-E(p)]}{2}\right\} + h\frac{E(p)}{2} \quad (15)$$

4.1.3. Cost function under random capacity

Given the underlying assumptions, we aim to formulate the corresponding average cost. For this purpose, we need to specify expressions for $W(s,S)$ and t. The inventory level for the random capacity case is depicted in Figure 3. As we can see from the figure and based on Assumption c_1, the received quantity (at time $\tau = 0$) is $(S-s)\min\{p,1\}$, so $W(s,S)$ is:

$$W(s,S) = \begin{cases} (S-s)\max\{1-p,0\}, \ \tau = 0, \\ 0, \ \tau \in (0,t] \end{cases} \quad (16)$$

By rewriting $W(s,S) = \max\{(1-p)(S-s), 0\}$ and replacing $p(S-s)$ with c to represent supplier capacity we are able to exploit results existing in the literature. Thus we get:

$$W(s,S) = \begin{cases} \max\{S-s-c,0\}, \ \tau = 0, \\ 0, \ \tau \in (0,t] \end{cases} \quad (17)$$

The random variable $Y_c(s,S)$ has a mixed distribution function, with a discrete positive mass at $S-s$ and a continuous distribution for $c < S-s$. The probability density function of $Y_c(s,S)$, $f_c(y)$, is given by:

$$f_c(y)dy = \begin{cases} 1-F(y), & y = S-s \\ f(y)dy, & y < S-s \end{cases}$$

The cycle, t, between two consecutive replenishment is $t = \dfrac{Y_c(s,S)}{D}$ so,

$$T = \frac{\int_0^{S-s} cf(c)dc + \int_{S-s}^{\infty} (S-s)f(c)dc}{D} = \frac{\int_0^{S-s} cf(c)dc + (S-s)[1-F(S-s)]}{D}$$

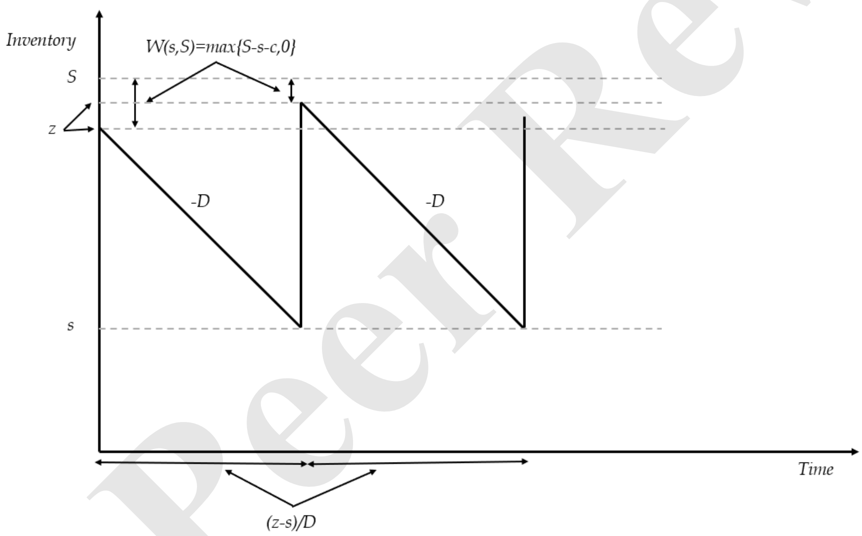

Figure 3: Sample path of the inventory evolution under random capacity.

Using the above results the average cost per unit time, resulting from (5), is:

$$C_c(s,S) = \frac{2KD + h\int_0^{S-s} c(c+2s)f(c)dc + h(S-s)[(S-s)+2s][1-F(S-s)]}{2\int_0^{S-s} cf(c)dc + 2(S-s)[1-F(S-s)]} \tag{18}$$

Using the transformation $Q = S - s$ we obtain:

$$C_c(s,Q) = \frac{2KD + h\displaystyle\int_0^Q c(c+2s)f(c)dc + hQ[Q+2s][1-F(Q)]}{2\displaystyle\int_0^Q cf(c)dc + 2Q[1-F(Q)]} \tag{19}$$

4.1.4. Cost function under supply disruptions

Given the underlying assumptions, we seek to determine the corresponding average cost. To do so, once again we need to specify expressions for $W(s,S)$ and t. Figure 4 depicts the inventory level for the supply disruptions case. As we can see from the figure and based on Assumptions d_1 and d_2, the received quantity (at time $\tau = 0$) is $\min\{Dt, S\}$, so $W(s,S)$ is:

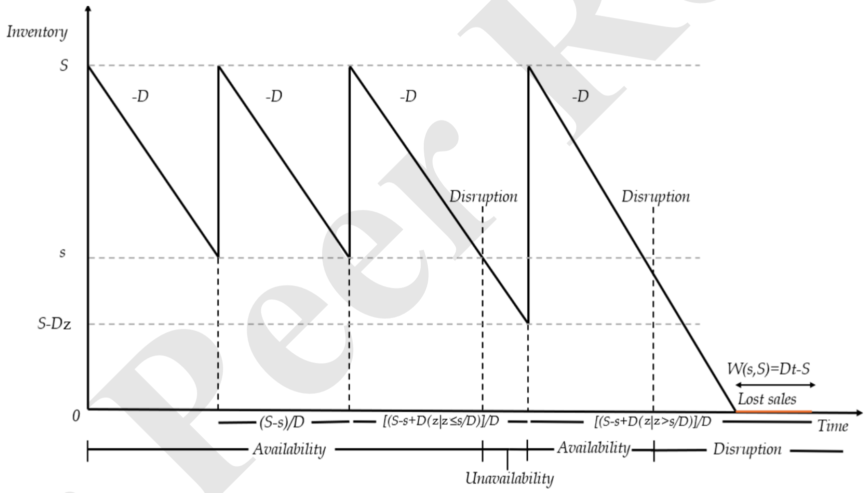

Figure 4: Sample path of inventory evolution under supply disruption.

$$W(s,S) = \begin{cases} 0, & 0 < \tau \leq \dfrac{S}{D} \\ D\tau - S, & \dfrac{S}{D} < \tau \leq t \end{cases}$$

Also, the supply availability ($P_r(p=1)$) or disruption at the time of replenishment placement ($P_r(p=0)$) must be considered. If there is no disruption

at order placement, the inventory-related cycle cost is given by $h\left(\frac{S^2-s^2}{2D}\right)$, with cycle length equal to $\frac{S-s}{D}$. In the case of a disruption, two possible scenarios regarding its duration, z, can occur: either $z \leq \frac{s}{D}$ or $z > \frac{s}{D}$ with z, being a random variable with $E(z) = 1/\mu$.
When $z \leq \frac{s}{D}$:
the holding cost (being a continuous function) is given as:

$$h\left(\frac{S^2-s^2}{2D} + \frac{(2s-Dz)z}{2}\right)$$

with a cycle length of:

$$\frac{S-s+D\left(z|z \leq \frac{s}{D}\right)}{D}$$

When $z > \frac{s}{D}$:
the holding cost equals:

$$h\frac{s^2}{2D} + h\left(\frac{S^2-s^2}{2D}\right)$$

and a lost sales cost of:

$$lD(z - \frac{s}{D})$$

is incurred.

The cycle length in this case is:

$$\frac{S-s+D\left(z|z > \frac{s}{D}\right)}{D}$$

To calculate the average total cost and cycle, the probability of disruption or availability at the time of order placement must be determined. Lemma 1, which is derived from Kolmogorov's differential equations for the two-state Continuous-Time Markov Chain (Ross [28]), provides useful results towards this direction (see also Karakatsoulis and Skouri [14]).

Lemma 1. *Let the states 1 and 0 represent supply availability and disruption, respectively. Also, let $P_{ij}(t)$ be the probability of the system being in state j at time t given that it was in state i at the beginning. Then:*

$$P_{10}(t) = \frac{\lambda}{\lambda+\mu}\left[1 - e^{-(\lambda+\mu)t}\right] \tag{20}$$

$$P_{01}(t) = \frac{\mu}{\lambda+\mu}\left[1 - e^{-(\lambda+\mu)t}\right] \tag{21}$$

with $P_{ij}(t) + P_{ii}(t) = 1, i, j \in \{0, 1\}, i \neq j.$

Given the transition probabilities, we can derive the limiting probability of disruption as:

$$P_r(p=1) = \frac{\lambda}{\lambda+\mu}$$

By using Lemma 1, the function $C_d(s, S)$ and the expected cycle length are formulated as:

$$\begin{aligned}
C_d(s,S) = \frac{1}{T}\Bigg\{&K + \left[1 - P_{10}\left(\frac{S-s}{D}\right)\right] h\frac{(S-s)[(S-s)+2s]}{2D} + P_{10}\left(\frac{S-s}{D}\right)\\
&\Bigg\{P\left(z \leq \frac{s}{D}\right) E\left[h\left(\frac{(S-s)[(S-s)+2s]}{2D}\right) + h\frac{(2s-Dz)z}{2}\Bigg| z \leq \frac{s}{D}\right]\\
&+ P\left(z > \frac{s}{D}\right) E\left[h\frac{(S-s)[(S-s)+2s]}{2D} + \frac{hs^2}{2D} + lD\left(z - \frac{s}{D}\right)\Bigg| z > \frac{s}{D}\right]\Bigg\}\Bigg\}\\
= \frac{1}{T}\Bigg\{&K + \left[1 - P_{10}\left(\frac{S-s}{D}\right)\right] h\frac{(S-s)[(S-s)+2s]}{2D} + P_{10}\left(\frac{S-s}{D}\right)\\
&\left[h\frac{(S-s)[(S-s)+2s]}{2D} + \frac{hs}{\mu} - \frac{hD}{\mu^2}\left(1 - e^{-\frac{\mu s}{D}}\right) + l\frac{D}{\mu}e^{-\frac{\mu s}{D}}\right]\Bigg\} \quad (22)
\end{aligned}$$

and

$$\begin{aligned}
T =& \left[1 - P_{10}\left(\frac{S-s}{D}\right)\right]\left(\frac{S-s}{D}\right) + P_{10}\left(\frac{S-s}{D}\right)\\
&\left[\frac{S-s}{D} + E\left(z\Big|z < \frac{s}{D}\right) P\left(z < \frac{s}{D}\right) + E\left(z\Big|z > \frac{s}{D}\right) P\left(z > \frac{s}{D}\right)\right]\\
=& \frac{S-s}{D} + P_{10}\left(\frac{S-s}{D}\right)\frac{1}{\mu}
\end{aligned}$$

The following remark outlines the relationships between different types of supply uncertainty.

Remark 1. *a) When* $x \to \infty$ *then* $W_f \to W_y$ *implying that a system with imperfect quality items behaves the same as one under random yield.*
b) Under random yield, when $p \to 1$ *then* $W_y \to W_d$ *at* $\frac{S}{D} < \tau \leq t$ *and the*

system under random yield behaves like one with supply disruptions. This suggests that a system under random yield operates similarly to one experiencing supply disruptions. In this case, however, the time that the system operates under supply unavailability must be considered, and the assumption about this time is crucial for deriving the cost of C_d.
c) Under random capacity, when $p \to 0$ then $W_c \to W_d$ at $\frac{S}{D} < \tau \leq t$ and the system behaves like a system with supply disruptions. Once again, the time that the system operates under supply unavailability must be considered, and the assumption about this time is crucial for deriving the cost of C_d.

5. Optimization per type of uncertainty

In this section, we aim to determine the values of the variables s and S that minimize the cost function $C(s, S)$ for each type of supply uncertainty considered. Furthermore, we derive the optimal policy variables to address supply disruptions with a complete loss of demand during stock-outs a topic that, to the best of our knowledge, is being explored for the first time in the literature. In order to facilitate the analysis we use the expressions of cost functions (8), (10), (13), (15), (19) and (22). The optimal values are in terms of s, S for all the types of supply uncertainty except for the case of supply disruption.

5.1. Optimization under random yield

From (8) we observe that cost is increasing in s, so $s = 0$. Therefore minimizing (7), with $s = 0$ leads to the optimal value for S which is equivalent to the results obtained in Silver [32]. As two cases regarding the variance of the random variable p are considered, the following proposition holds.

Proposition 1. *The globally optimal policy variables, (s_y, S_y) are:*
a) if $Var(p)$ is independent of $S - s$ then

$$(s_{yn}, S_{yn}) = \left(0, \sqrt{\frac{2KD}{hE[(1-p)^2]}}\right) \tag{23}$$

with cost:

$$C_{yn}(s_{yn}, S_{yn}) = \frac{2KD}{S_{yn}[1 - E(p)]} \tag{24}$$

b) if $Var(p) = \dfrac{\rho(1-\rho)}{S-s}$ *then*

$$(s_{yv}, S_{yv}) = \left(0, \sqrt{\frac{2KD}{h[1-E(p)]^2}}\right) \tag{25}$$

with cost:

$$C_{yv}(s_{yv}, S_{yv}) = \frac{2KD}{S_{yv}[1-E(p)]} + \frac{hE(p)}{2} \tag{26}$$

It is worthwhile to note that, from relations (23) and (25), we observe that, due to $[1-E(p)]^2 < E[(1-p)^2]$, $S_{yn} < S_{yv}$.

5.2. Optimization under imperfect quality products

From (13) we observe that cost is increasing in s, so $s = 0$. Therefore minimizing (12), with $s = 0$ leads to the optimal value for S derived in Maddah and Jaber [19] assuming that $Var(p)$ is independent of $S - s$. As two cases regarding the variance of the random variable p are considered, the following proposition holds.

Proposition 2. *The globally optimal policy variables,* (s_y, S_y) *are:*
a) if $Var(p)$ *is independent of* $S - s$ *then*

$$(s_{fn}, S_{fn}) = \left(0, \sqrt{\frac{2KD}{h\left\{E[(1-p)^2] + \dfrac{2DE(p)}{x}\right\}}}\right) \tag{27}$$

with cost:

$$C_{fn}(s_{fn}, S_{fn}) = \frac{2KD}{S_{fn}[1-E(p)]} + \frac{gD}{1-E(p)} \tag{28}$$

b) if $Var(p) = \dfrac{\rho(1-\rho)}{S-s}$ *then*

$$(s_{fv}, S_{fv}) = \left(0, \sqrt{\frac{2KD}{h\left\{\Big[1-E(p)\Big]^2 + \dfrac{2DE(p)}{x}\right\}}}\right) \tag{29}$$

with cost:

$$C_{fv}(s_{fv}, S_{fv}) = \frac{2KD}{S_{fv}[1-E(p)]} + \frac{gD}{1-E(p)} + \frac{hE(p)}{2} \tag{30}$$

Once again, from (27) and (29) we observe that $S_{fn} < S_{fv}$.

As it is apparent through equations (23)-(30) it follows that the order-up-to level for the imperfect quality items is less than the one in the random yield case. We also get that the order-up-to level decreases as $Var(p)$ increases for both types of supply uncertainty. Also, we observe that the cost for the imperfect quality items is greater than the one in the random yield case. Finally, we get that the cost increases as $Var(p)$ decreases for all the types of supply uncertainty.

5.3. Optimization under random capacity

From (19) we observe that cost is increasing in s, so $s = 0$. Therefore minimizing (18), with $s = 0$ leads to the optimal value for S, as derived in Hariga and Haouari [7] and summarized in the next proposition:
Let, S_c the solution of equation:

$$l(S) = 0$$

where:

$$l(S) = S^2 - \int_0^S (c - S)^2 f(c)dc - {S_{EOQ}}^2$$

Proposition 3. *The globally optimal policy variables,* $(0, S_c)$ *with* $C_c(s_c, S_c) = hS_c$.

Furthermore, Hariga and Haouari [7], showed that:

$$S_{EOQ} \leq S_c \leq S_u$$

with:

$$S_u = \frac{S_{EOQ}^2 + [E(c)]^2 + [Var(c)]^2}{2E(c)}$$

5.4. Optimization under supply disruptions

After the variable substitution we get that:

$$T(s, Q) = \frac{Q}{D} + P_{10}\left(\frac{Q}{D}\right)\frac{1}{\mu} \tag{31}$$

and

$$C_d(s, Q) = \frac{K + h\frac{Q(Q+2s)}{2D} + P_{10}\left(\frac{Q}{D}\right)\left[\frac{hs}{\mu} - \frac{hD}{\mu^2}\left(1 - e^{-\frac{\mu s}{D}}\right) + l\frac{D}{\mu}e^{-\frac{\mu s}{D}}\right]}{\frac{Q}{D} + P_{10}\left(\frac{Q}{D}\right)\frac{1}{\mu}} \tag{32}$$

The following proposition ensures the existence of the optimal policy.

Proposition 4. *$C_d(s, Q)$ attains a unique minimum.*

Let $(s_d, Q_d) = \underset{(s,Q)}{\operatorname{argmin}}\, C_d(s, Q)$, that is not always feasible. The following proposition provides conditions that lead to feasible solutions.

Let Q_{1d} and Q_{2d} satisfy:

$$\left[1 + e^{\frac{Q_{1d}(\lambda+\mu)}{D}}\right] h{Q_{1d}}^2(\lambda + \mu) - 2D\left[-1 + e^{\frac{Q_{1d}(\lambda+\mu)}{D}}\right][hQ_{1d} + K(\lambda + \mu)] = 0$$

and

$$\left(\frac{hQ_{2d}^2\mu(\lambda + \mu)}{2\lambda} + DhQ_{2d} - \frac{DK\mu(\lambda + \mu)}{\lambda} - lD^2\right) + \left\{-\frac{hQ_{2d}^2(\lambda + \mu)}{2} - Q_{2d}D[h + l(\lambda + \mu)] + D[lD - K(\lambda + \mu)]\right\} e^{-\frac{Q_{2d}(\lambda+\mu)}{D}} = 0.$$

Proposition 5. *The optimal values of the decision variables are given as:*

$$(s_d, Q_d) = \begin{cases} \left(-\frac{D}{\mu}\ln\left[\dfrac{h\left(\dfrac{Q_{1d}}{DP_{10}\left(\frac{Q_{1d}}{D}\right)} + \dfrac{1}{\mu}\right)}{h\dfrac{1}{\mu} + l}\right], Q_{1d}\right), \textit{if } h\frac{Q_{1d}}{D} - P_{10}\left(\frac{Q_{1d}}{D}\right) l < 0 \\ (0, Q_{2d}), \ \textit{otherwise} \end{cases}$$

We also note that once $s_d = 0$, the cost function reduces to the formulation of Berk and Arreola-Risa [1]. Since they proved that this cost function is unimodal, it follows that it has a unique minimum, which is obtained from the first-order condition.

We then provide the conditions under which it is optimal to keep safety stock (i.e. $s_d > 0$).

Let $\hat{Q}$ satisfy:

$$\frac{h\hat{Q}}{D} - \frac{\lambda}{\lambda + \mu}\left[1 - e^{-\frac{(\lambda+\mu)\hat{Q}}{D}}\right] l = 0$$

Proposition 6. *a) If $h \geq l\lambda$, then $s_d = 0$.*

b) If $h < l\lambda$ and $\begin{cases} Q \geq \hat{Q}, \ \textit{then } s_d = 0, \\ Q < \hat{Q}, \ \textit{then } s_d = s(Q_{1d}) \end{cases}$

The above proposition proves that it is not always optimal to keep safety stock, especially when the holding cost is higher than the average lost sales cost during the disruption period. Even in the opposite situation, where the holding cost is less than the average lost sales cost during the disruption period, it is not always optimal to keep safety stock, especially when the value of variable Q exceeds a certain threshold.

Since the optimal value of variable Q_{1d} cannot be found in closed form, the following proposition provides an upper and a lower bound.

Proposition 7. *$S_{EOQ} \leq Q_{1d} \leq Q_{ub}$, with* $Q_{ub} = \dfrac{D\left[1+\sqrt{1+\dfrac{2K(\lambda+\mu)^2}{Dh}}\right]}{\lambda+\mu}$

6. Numerical comparisons

6.1. Experimental set up

We studied several datasets to assess the impact of different types of supply uncertainty. We evaluated the optimal policy variables and costs under each supply uncertainty type, as well as for the classical EOQ model, which obviously represents a scenario without supply uncertainty. The results were obtained for the specific parameter values $K = 100, D = 4000$, and $h = 2$, as in Karakatsoulis and Skouri [14]. In order to describe supply uncertainty, we assume that for all types, $E(p) = q$ (i.e. the limiting probability of supply disruption), where q takes values from 0.1 to 0.7 in increments of 0.05. For random yield and imperfect-quality products, when the variance is independent of the order quantity, we assume that p follows a Beta(α, β) distribution. The parameters α and β are chosen such that $E(p) = \alpha/(\alpha + \beta) = q$ and variance equal to 0.01 so as to ensure small dispersion around q. In this context, we compute the parameters α and β from equations $\alpha/(\alpha + \beta) = q$ and $(\alpha\beta)/[(\alpha + \beta)^2(\alpha + \beta + 1)] = 0.01$ for each value of q. For random capacity, we assume that the random variable p follows a Normal distribution with a mean of $(1-q)S_{EOQ}$ (where $S_{EOQ} = 632.46$) and a standard deviation of 10. We choose this mean because in the absence of supply uncertainty, the quantity delivered would always be $Q_{EOQ}(:= S_{EOQ})$, while we set the standard deviation to be small enough compared to the order quantities. For the supply disruption, we consider two different disruption profiles. Specifically, we assume that $\lambda+\mu = 2$ and $\lambda+\mu = 9$, with the latter case indicating faster switching between supply availability and disruption

$E(p)$ $(=q)$	s_{yn}	S_{yn}	s_{yv}	S_{yv}	s_{fn}	S_{fn}	s_{fv}	S_{fv}	s_c	S_c	s_d $(\lambda+\mu=2)$	S_d $(\lambda+\mu=2)$	s_d $(\lambda+\mu=9)$	S_d $(\lambda+\mu=9)$
0.10	0	698.43	0	702.73	0	671.66	0	675.48	0	636.06	826.16	2519.68	643.13	1694.58
0.15	0	738.97	0	744.07	0	693.17	0	697.37	0	640.92	1740.85	3434.38	880.90	1932.35
0.20	0	784.47	0	790.57	0	714.59	0	719.20	0	648.37	2468.36	4161.88	1081.73	2133.17
0.25	0	835.88	0	843.27	0	735.63	0	740.66	0	658.91	3112.22	4805.75	1269.55	2321.00
0.30	0	894.43	0	903.51	0	755.93	0	761.39	0	673.23	3721.02	5414.54	1456.38	2507.83
0.35	0	961.69	0	973.01	0	775.08	0	780.97	0	692.17	4323.92	6017.45	1650.28	2701.73
0.40	0	1039.75	0	1054.09	0	792.64	0	798.94	0	716.92	4942.43	6635.96	1858.10	2909.55
0.45	0	1131.37	0	1149.92	0	808.12	0	814.80	0	749.03	5595.52	7289.04	2086.89	3138.34
0.50	0	1240.35	0	1264.91	0	821.07	0	828.08	0	790.73	6302.68	7996.20	2344.91	3396.36
0.55	0	1371.99	0	1405.46	0	831.05	0	838.32	0	845.21	7086.49	8780.01	2642.78	3694.22
0.60	0	1533.93	0	1581.14	0	837.71	0	845.15	0	917.26	7975.55	9669.07	2995.11	4046.56
0.65	0	1737.49	0	1807.02	0	840.79	0	848.32	0	1014.41	9008.68	10720.21	3423.30	4474.75
0.70	0	2000.00	0	2108.19	0	840.17	0	847.68	0	1149.22	10242.04	11935.56	3960.51	5011.96

Table 1: Optimal reorder point and order-up-to level per uncertainty.

states. The distributions were selected to ensure the same mean batch loss under each type of supply uncertainty while minimizing variance. Finally, we set $l = 22$ to be significantly higher than the holding cost, $g = 0.4$, in parallel with Maddah and Jaber [19], and $x = 12000$, assuming it is sufficiently large to eliminate the probability of any shortages at the end of the screening process. We note that in the absence of supply uncertainty, the system setup aligns with the EOQ model, for which the values are $S_{EOQ} = 632.46$ and $C_{EOQ} = 1264.9$ respectively.

6.2. Numerical results

Tables 1-6 present the results based on the setup described above. Table 1 presents the optimal policy under each type of uncertainty. We observe that an increase in the probability of any type of uncertainty (as indicated by an increase in q) leads to higher order-up-to levels. More specifically, supply disruption causes the greatest increase in the order-up-to level, followed by random yield. For values of q up to 0.55, the presence of imperfect quality items leads to a slightly larger increase in the order-up-to level than random capacity; beyond this point, the reverse effect occurs. It is also worth noting that, in the cases of random yield and imperfect quality items, the increase in the order-up-to level is greater when $Var(p) = \frac{\rho(1-\rho)}{S-s}$. In the case of supply disruption, the nominal order quantity ($Q = S - s$) remains constant as q increases. Essentially, it is the safety stock that increases. In fact, the safety stock is larger when $\lambda + \mu = 2$ compared to when $\lambda + \mu = 9$, indicating that a larger safety stock is required for systems experiencing infrequent but prolonged disruptions (i.e., lower values of $\lambda + \mu$).

Table 2 reports the average cycle lengths per type of supply uncertainty. It is observed that, for all types of uncertainty, the average cycle length de-

$E(p)$ $(=q)$	T_{yn}	T_{yv}	T_{fn}	T_{fv}	T_c	T_d $(\lambda+\mu=2)$	T_d $(\lambda+\mu=9)$
0.10	0.157	0.158	0.151	0.152	0.142	0.455	0.274
0.15	0.157	0.158	0.147	0.148	0.134	0.474	0.281
0.20	0.157	0.158	0.143	0.144	0.126	0.495	0.288
0.25	0.157	0.158	0.138	0.139	0.119	0.519	0.296
0.30	0.157	0.158	0.132	0.133	0.111	0.546	0.306
0.35	0.156	0.158	0.126	0.127	0.103	0.577	0.317
0.40	0.156	0.158	0.119	0.120	0.095	0.614	0.330
0.45	0.156	0.158	0.111	0.112	0.087	0.657	0.345
0.50	0.155	0.158	0.103	0.104	0.079	0.709	0.364
0.55	0.154	0.158	0.093	0.094	0.071	0.772	0.386
0.60	0.153	0.158	0.084	0.085	0.063	0.852	0.414
0.65	0.152	0.158	0.074	0.074	0.055	0.954	0.450
0.70	0.150	0.158	0.063	0.064	0.047	1.090	0.498

Table 2: Expected cycle length per uncertainty.

creases as the parameter q increases, with the exception of supply disruption, where the opposite trend is evident. Particularly, in the cases of random yield, imperfect quality items, and random capacity, it becomes necessary to reduce the average time between orders as q increases, in order to ensure demand fulfillment within the given interval. Conversely, in the presence of supply disruption, the average time between orders tends to increase with q, which can be attributed to the corresponding increase in safety stock. It seems, holding inventory becomes more cost-effective than placing frequent orders. This behavior is particularly pronounced in systems characterized by infrequent but prolonged disruptions (i.e., when $\lambda+\mu=2$). Furthermore, the average cycle length is shorter than the classical Economic Order Quantity (EOQ) cycle in all cases, except under supply disruption, where it is longer. For random yield and imperfect quality items, this behavior can also be demonstrated analytically. In particular, $T_{yn} = T_{EOQ}\frac{(1-E(p))}{\sqrt{E[(1-p)^2]}}$, $T_{yv} = T_{EOQ}$ -and thus constant-, $T_{fn} = T_{EOQ}\frac{(1-E(p))}{\sqrt{E[(1-p)^2]+\frac{2DE(p)}{x}}}$ and $T_{fv} = T_{EOQ}\frac{(1-E(p))}{\sqrt{[1-E(p)]^2+\frac{2DE(p)}{x}}}$.

Even when the optimal policy is determined for each type of uncertainty, the received quantity is random. Hence, Table 3 presents the average received quantities under each uncertainty type. The pattern of average cycle lengths

$E(p)$ $(= q)$	$\bar{Q}_{yn}$	$\bar{Q}_{yv}$	$\bar{Q}_{fn}$	$\bar{Q}_{fv}$	$\bar{Q}_c$	$\bar{Q}_d$ $(\lambda + \mu = 2)$	$\bar{Q}_d$ $(\lambda + \mu = 9)$
0.10	628.59	632.46	671.66	675.48	569.21	1732.94	1084.03
0.15	628.12	632.46	693.17	697.37	537.59	1798.92	1109.33
0.20	627.57	632.46	714.59	719.20	505.96	1872.72	1137.76
0.25	626.91	632.46	735.63	740.66	474.34	1955.79	1169.93
0.30	626.10	632.46	755.93	761.39	442.72	2050.01	1206.63
0.35	625.10	632.46	775.08	780.97	411.10	2157.77	1248.89
0.40	623.85	632.46	792.64	798.94	379.47	2282.23	1298.08
0.45	622.25	632.46	808.12	814.80	347.85	2427.59	1356.04
0.50	620.17	632.46	821.07	828.08	316.23	2599.60	1425.37
0.55	617.40	632.46	831.05	838.32	284.61	2806.32	1509.77
0.60	613.57	632.46	837.71	845.15	252.98	3059.46	1614.75
0.65	608.12	632.46	840.79	848.32	221.36	3376.60	1748.88
0.70	600.00	632.46	840.17	847.68	189.74	3785.55	1926.27

Table 3: Expected received quantity per uncertainty.

is mirrored in the average received quantities. For random yield and random capacity, the expected received quantity decreases as q increases, whereas under supply disruption it increases, owing to higher safety stock. The received quantity approximates the demand during the cycle, except for the case of imperfect quality items since the received quantity equals the order quantity, because the imperfect quality items that cannot be used to fulfill demand are detected after the screening process takes place. This suggests that cycle lengths allow the average order quantities to adapt, where feasible, to satisfy demand within the specified time interval. Additionally, inventory mitigation generally occurs through order-up-to levels, except under supply disruptions, where safety stock serves as the primary mechanism.

Table 4 shows that, for each type of supply uncertainty, the cost rises as q increases, with supply disruption leading to the largest increase. Furthermore, the cost is larger when $\lambda + \mu = 2$. This increase could be related to higher order-up-to levels and reorder points, leading to increased holding costs. The presence of imperfect quality items causes the next highest cost increase reasonably due to cost stems from screening requirement. The random yield appears to have a very slight effect on cost increase. The latter can be theoretically proven since $C_{yn} = C_{EOQ}\frac{\sqrt{E[(1-p)^2]}}{1-E(p)}$ and $C_{yv} = C_{EOQ} + \frac{hE(p)}{2}$.

To further evaluate the cost impact of each supply uncertainty type, we

$E(p)$ $(=q)$	C_{yn}	C_{yv}	C_{fn}	C_{fv}	C_c	C_d $(\lambda+\mu=2)$	C_d $(\lambda+\mu=9)$
0.10	1272.7	1265.0	3101.2	3093.8	1272.1	7582.0	3607.0
0.15	1273.6	1265.1	3240.1	3232.1	1281.8	9411.4	4082.6
0.20	1274.8	1265.1	3399.4	3390.6	1296.7	10866.4	4484.2
0.25	1276.1	1265.2	3583.3	3573.7	1317.8	12154.2	4859.9
0.30	1277.8	1265.2	3797.6	3787.0	1346.5	13371.8	5233.5
0.35	1279.8	1265.3	4049.5	4037.8	1384.3	14577.6	5621.3
0.40	1282.4	1265.3	4348.8	4336.0	1433.8	15814.6	6037.0
0.45	1285.6	1265.4	4709.0	4694.7	1498.1	17120.7	6494.6
0.50	1290.0	1265.4	5148.7	5132.7	1581.5	18535.1	7010.6
0.55	1295.8	1265.5	5694.7	5676.7	1690.4	20102.7	7606.3
0.60	1303.8	1265.5	6387.5	6367.0	1834.5	21880.8	8311.0
0.65	1315.5	1265.6	7290.0	7266.5	2028.8	23947.1	9167.4
0.70	1333.3	1265.6	8507.3	8479.9	2298.4	26413.8	10241.8

Table 4: Optimal cost per uncertainty.

compare it with the EOQ cost (no uncertainty), using the indicators $\Delta_i = \frac{C_i - C_{EOQ}}{C_{EOQ}} 100\%, i \in \{yn, yv, fn, fv, c, d\}$. As shown in Table 5, long and rare supply disruptions ($\lambda + \mu = 2$) can increase costs by up to 1988.19% at a limiting probability of 0.7.

To evaluate the impact of using the EOQ policy instead of the optimal policy, we employ the indicators $E_i = \frac{C(EOQ) - C_i}{C_i} 100\%, i \in \{yn, yv, fn, fv, c, d\}$. As shown in Table 6, EOQ can result in substantial cost increases, particularly under random yield (up to 81.62%) and supply disruption (up to 381.92% when $\lambda + \mu = 9$), likely due to the required adaptation of order up to levels. By contrast, random capacity shows relatively good performance, likely due to the specific choice of capacity distribution.

Overall, our numerical comparisons show that supply disruptions are the most critical type of uncertainty in terms of cost increases compared to the basic EOQ model (representing reliable supply). This cost increase is especially significant for systems experiencing infrequent but prolonged disruptions. These findings align with common understanding, as supply disruptions directly affect availability and can halt entire operations, leading to broader consequences. For the other three types of uncertainty, the cost increase is relatively small when q is low. However, as q increases, random yield uncertainty tends to have higher cost consequences. This study

$E(p)$ $(=q)$	Δ_{yn}	Δ_{yv}	Δ_{fn}	Δ_{fv}	Δ_c	Δ_d $(\lambda+\mu=2)$	Δ_d $(\lambda+\mu=9)$
0.10	0.62	0.01	145.17	144.59	0.57	499.41	185.16
0.15	0.69	0.01	156.16	155.52	1.34	644.04	222.76
0.20	0.78	0.02	168.75	168.05	2.52	759.07	254.51
0.25	0.88	0.02	183.29	182.53	4.18	860.87	284.21
0.30	1.02	0.02	200.22	199.39	6.45	957.13	313.75
0.35	1.18	0.03	220.14	219.22	9.44	1052.46	344.40
0.40	1.38	0.03	243.80	242.79	13.35	1150.25	377.26
0.45	1.64	0.04	272.28	271.15	18.43	1253.51	413.44
0.50	1.98	0.04	307.04	305.77	25.03	1365.33	454.24
0.55	2.44	0.04	350.21	348.79	33.64	1489.26	501.33
0.60	3.08	0.05	404.97	403.36	45.03	1629.83	557.04
0.65	4.00	0.05	476.32	474.47	60.39	1793.18	624.74
0.70	5.41	0.06	572.56	570.39	81.71	1988.19	709.69

Table 5: Percentage cost increase over the classical *EOQ* model.

$E(p)$ $(=q)$	E_{yn}	E_{yv}	E_{fn}	E_{fv}	E_c	E_d $(\lambda+\mu=2)$	E_d $(\lambda+\mu=9)$
0.10	0.49	0.56	0.08	0.09	0.00	16.18	69.60
0.15	1.21	1.32	0.18	0.20	0.00	34.55	113.75
0.20	2.33	2.50	0.31	0.34	0.00	52.52	155.78
0.25	3.91	4.17	0.46	0.50	0.00	69.02	195.44
0.30	6.07	6.43	0.63	0.68	0.00	83.75	232.53
0.35	8.91	9.42	0.81	0.87	0.00	96.59	266.77
0.40	12.61	13.33	0.99	1.06	0.00	107.46	297.83
0.45	17.39	18.40	1.15	1.23	0.00	116.24	325.23
0.50	23.55	24.99	1.30	1.38	0.00	122.83	348.40
0.55	31.51	33.60	1.41	1.49	0.00	127.06	366.64
0.60	41.88	44.98	1.49	1.57	0.00	128.77	379.07
0.65	55.56	60.33	1.52	1.61	0.00	127.75	384.62
0.70	73.93	81.62	1.51	1.60	0.00	123.72	381.92

Table 6: Percentage cost increase when using the classical *EOQ* quantity.

could be valuable for products that inherently deal with multiple supply uncertainties such as seasonal consumer electronics (e.g., gaming consoles or smartphones). Our results aim to contribute to better-informed supplier assessment and selection decisions. These findings suggest that, when inventory is the sole mitigation lever, suppliers characterized by predictable, statistical uncertainties (e.g., yield variability and quality defects) are preferable. Conversely, suppliers subject to binary shocks (e.g., supply disruptions) should be avoided, as such uncertainties are the most challenging to buffer using inventory alone. Capacity-related uncertainty warrants careful evaluation; although our numerical results indicate that its cost implications are comparable to those associated with yield and defect-related uncertainties, this outcome may be attributable to the relatively low variance assumed in the analysis.

7. Conclusions

Ensuring consistent, on-time supply is the foundation of effective inventory management. Without a reliable supply, organizations risk facing stockouts or overstocking, both of which are costly. The reliability of supply directly affects the ability to meet demand and maintain a balanced inventory. Even the best systems, forecasting methods, and quality measures can fail if suppliers do not deliver on time, in full, and with perfect quality.

In this paper, we examine supply uncertainty under four distinct types, aiming to study its impact on system performance. In this context, we formulate the average cost of the system in a unified manner, allowing us to identify the differences and similarities in the processes that lead to each type of uncertainty. This approach also enables us to view one type of supply uncertainty as a limiting case of another.

We assume deterministic demand to avoid introducing additional uncertainty factors and apply a continuous review (s, S) replenishment policy. For random yield and imperfect quality items uncertainties both closely related we present closed-form expressions for policy variables (existing in the literature under the assumption $s = 0$), which are directly comparable to those in the classical EOQ model. For random capacity and supply disruptions, the existence of optimal policy variables is established, and bounds for the corresponding optimal values of the variable Q, relative to the classical EOQ model, are provided.

The proposed model, however, presents certain limitations. The study assumes that each source of supply uncertainty considered in the model—namely random yield, imperfect quality items, random capacity, and supply disruptions—has been examined in isolation. In practice, these uncertainties may occur simultaneously or be interrelated, and their joint effects warrant further investigation within an integrated modeling framework. Furthermore, the analysis focuses on a single-supplier, single-buyer setting, which does not capture the complexity of real-world supply networks, where multi-supplier or multi-echelon systems often exist and may involve backup sourcing or coordination contracts.

The previously mentioned limitations could inspire several areas for further research, aiming to enhance the model's practical relevance and make it more reflective of actual supply chain conditions. One area of research involves accommodating an advance information regime. For instance, the supplier could provide information about supply availability before the replenishment release, allowing the buyer to mitigate supply uncertainty. Supply chain visibility, information-sharing and collaboration among partners could be implemented through Industry 4.0 technologies, enabling the buyer to respond to yield randomness and significantly strengthen the resilience of the supply chain (Ismail et al. [11]). Another area could focus on adapting models to account for different planned shortage-related assumptions, such as complete or partial backorders or lost sales. A different area could concern supplier diversification, enabling differentiation from a single source of supply, while aiming to address how different types of supply uncertainty affect a buyer's sourcing strategy when constraints are present, e.g. minimum order quantity requirements (Heese [8]). Lastly, introducing uncertainty in demand and exploring its interrelation with supply, or comparing the different types of supply uncertainty with the demand uncertainty-perhaps through appropriate heuristics (Teunter et al. [36]) while investigating the different strategies that are appropriate dealing with each (Schmitt et al. [31]) could also be an interesting avenue of study.

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8. Appendix. Proofs

Proof of Proposition 4. The first and second derivatives of $C_d(s,Q)$ with respect to s are:

$$\frac{\partial C_d(s,Q)}{\partial s} = \frac{\frac{hQ}{D} + P_{10}(\frac{Q}{D})\left(\frac{h}{\mu} - \frac{h}{\mu}e^{-\frac{\mu s}{D}} - le^{-\frac{\mu s}{D}}\right)}{\frac{Q}{D} + P_{10}(\frac{Q}{D})\frac{1}{\mu}}$$

$$\frac{\partial^2 C_d(s,Q)}{\partial s^2} = \frac{P_{10}(\frac{Q}{D})e^{-\frac{\mu s}{D}}\left(\frac{h}{D} + \frac{l\mu}{D}\right)}{\frac{Q}{D} + P_{10}(\frac{Q}{D})\frac{1}{\mu}} > 0$$

therefore from the first order condition for a minimum, $\frac{\partial C_d(s,Q)}{\partial s} = 0$, we get:

$$s_d(Q) = -\frac{D}{\mu}\ln\left[\frac{h\left(\frac{Q}{DP_{10}\left(\frac{Q}{D}\right)} + \frac{1}{\mu}\right)}{h\frac{1}{\mu} + l}\right]$$

$$= -\frac{D}{\mu}\ln\left[h\left(\frac{Q}{DP_{10}\left(\frac{Q}{D}\right)} + \frac{1}{\mu}\right)\right] + \frac{D}{\mu}\ln\left(h\frac{1}{\mu} + l\right)$$

For $s = s_d(Q)$, $C_d(s,Q)$ becomes:

$$C_d(s,Q) = \frac{KD + h\frac{Q^2}{2} + hD\frac{Q}{\mu}}{Q + P_{10}\left(\frac{Q}{D}\right)\frac{D}{\mu}} - h\frac{D}{\mu}\ln\left[\frac{h\left(\frac{Q}{DP_{10}\left(\frac{Q}{D}\right)} + \frac{1}{\mu}\right)}{h\frac{1}{\mu} + l}\right]$$

Differentiating with respect to Q, we get:

$$\frac{\partial C_d(s,Q)}{\partial Q} = \frac{e^{\frac{Q(\lambda+\mu)}{D}}\mu(\lambda+\mu)\Big(\lambda + e^{\frac{Q(\lambda+\mu)}{D}}\mu\Big)\phi(Q)}{2\Big(-1+e^{\frac{Q(\lambda+\mu)}{D}}\Big)\Big(D\lambda - e^{\frac{Q(\lambda+\mu)}{D}}\left(D\lambda + Q\mu(\lambda+\mu)\right)\Big)^2} \tag{33}$$

where:

$$\phi(Q) = \left[1+e^{\frac{Q(\lambda+\mu)}{D}}\right] hQ^2(\lambda+\mu) - 2D\left[-1+e^{\frac{Q(\lambda+\mu)}{D}}\right][hQ + K(\lambda+\mu)]$$

Instead of solving the first order condition ($\frac{\partial C_d(s,Q)}{\partial Q} = 0$), is equivalent to solve $\phi(Q) = 0$. To this end, the monotonicity of $\phi(Q)$ will be examined.

Differentiating with respect to Q, we get:

$$\frac{\partial \phi(Q)}{\partial Q} = e^{\frac{Q(\lambda+\mu)}{D}}\left[h\frac{Q^2(\lambda+\mu)^2}{D} - 2Dh - 2(\lambda+\mu)^2 K\right] + 2hQ(\lambda+\mu) + 2Dh$$

By setting $\frac{\partial \phi(Q)}{\partial Q} = 0$ we get:

$$\left[hQ^2\frac{(\lambda+\mu)^2}{D} - 2Dh - 2(\lambda+\mu)^2 K\right] + [2hQ(\lambda+\mu) + 2Dh]\, e^{-\frac{Q(\lambda+\mu)}{D}} = 0$$

Define:

$$\begin{aligned} f_1(Q) &= \left[hQ^2\frac{(\lambda+\mu)^2}{D} - 2Dh - 2(\lambda+\mu)^2 K\right] + [2hQ(\lambda+\mu) + 2Dh]\, e^{-\frac{Q(\lambda+\mu)}{D}} \\ &= \frac{d\phi(Q)}{dQ} e^{-\frac{Q(\lambda+\mu)}{D}} \end{aligned}$$

It is worth mentioning that $f_1(Q)$ and $\frac{d\phi(Q)}{dQ}$ have the exact same roots. At this stage it is more convenient to study $f_1(Q)$. For any $Q > 0$, we get:

$$\frac{df_1(Q)}{dQ} = 2h\frac{(\lambda+\mu)^2}{D}Q - \frac{(\lambda+\mu)^2}{D}2hQe^{-\frac{Q(\lambda+\mu)}{D}}$$

as well as:

$$\frac{d^2 f_1(Q)}{dQ^2} = e^{-\frac{Q(\lambda+\mu)}{D}} 2h\frac{(\lambda+\mu)^2}{D^2}\left[D\Big(-1+e^{\frac{Q(\lambda+\mu)}{D}}\Big) + Q(\lambda+\mu)\right] \tag{34}$$

$$> e^{-\frac{Q(\lambda+\mu)}{D}} 2h \frac{(\lambda+\mu)^2}{D^2} 2Q(\lambda+\mu)$$

with the latter relation being a consequence of the inequality $e^x > 1 + x$ for $x > 0$, so in that case (34) implies that $\frac{d^2 f_1(Q)}{dQ^2} > 0$, meaning that $\frac{df_1(Q)}{dQ}$ is strictly increasing. Observe also that $\frac{df_1(Q)}{dQ}\Big|_{Q=0} = 0$, and since $\frac{df_1(Q)}{dQ}$ is monotone, we have $\frac{df_1(Q)}{dQ} > 0$ for any $Q > 0$. Hence, $f_1(Q)$ is strictly increasing for $Q > 0$. In addition, since $f_1(0) < 0$ and $\lim_{Q\to\infty} f_1(Q) = \infty$, $f_1(Q) = 0$ has a unique positive solution, say $\overline{Q} > 0$, that coincides with that of $\frac{d\phi(Q)}{dQ} = 0$. The unique solution $\overline{Q}$ of $f_1(Q)$ also implies that $f_1(Q) < 0$ for $Q < \overline{Q}$ and $f_1(Q) > 0$ for $Q > \overline{Q}$, which also implies that $\frac{d\phi(Q)}{dQ} < 0$ for $Q < \overline{Q}$ and $\frac{d\phi(Q)}{dQ} > 0$ for $Q > \overline{Q}$. Moreover, since $\phi(0) = 0$, there is a unique positive value Q satisfying $\phi(Q) = 0$, which implies that C_d attains a unique minimum. □

Proof of Proposition 6.

$$s_d(Q) = -\frac{D}{\mu} \ln \left[\frac{h\left(\frac{Q}{DP_{10}\left(\frac{Q}{D}\right)} + \frac{1}{\mu}\right)}{h\frac{1}{\mu} + l} \right]$$

is positive only when

$$\frac{h\left(\frac{Q}{DP_{10}\left(\frac{Q}{D}\right)} + \frac{1}{\mu}\right)}{h\frac{1}{\mu} + l} < 1$$

Let

$$f_2(Q) = h\frac{Q}{D} - lP_{10}\left(\frac{Q}{D}\right)$$

The first and second order derivative of $f_2(Q)$ with respect to Q are:

$$\frac{df_2(Q)}{dQ} = \frac{h}{D} - \frac{\lambda l}{D} e^{\frac{-(\lambda+\mu)Q}{D}}$$

$$\frac{d^2 f_2(Q)}{dQ^2} = \frac{l\lambda(\lambda+\mu)}{D^2} e^{\frac{-(\lambda+\mu)Q}{D}} > 0$$

In case that $h \geq l\lambda$, function $f_2(Q)$ is increasing and since $f_2(0) = 0$ and $Q > 0$ then $f_2(Q) > 0$ and so:

$$\frac{h\left(\dfrac{Q}{DP_{10}\left(\frac{Q}{D}\right)} + \dfrac{1}{\mu}\right)}{h\dfrac{1}{\mu} + l} > 1$$

thus $s_d = 0$.

In case that $h < l\lambda$, there is a unique value of Q such that $\frac{df_2(Q)}{dQ} = 0$. The function $f_2(Q)$ is decreasing up to that specific value of Q and negative (since $f_2(0) = 0$) for $Q < \hat{Q}$ so:

$$\frac{h\left(\dfrac{Q}{DP_{10}\left(\frac{Q}{D}\right)} + \dfrac{1}{\mu}\right)}{h\dfrac{1}{\mu} + l} < 1$$

thus $s_d > 0$.

For $Q \geq \hat{Q}$ we get that $f_2(Q) \geq 0$ and so:

$$\frac{h\left(\dfrac{Q}{DP_{10}\left(\frac{Q}{D}\right)} + \dfrac{1}{\mu}\right)}{h\dfrac{1}{\mu} + l} \geq 1$$

thus $s_d = 0$. □

Proof of Proposition 7. As we aim to solve $\phi(Q) = 0$, we rewrite it as:

$$\underbrace{-e^{\frac{Q(\lambda+\mu)}{D}}\left[hQ^2(\lambda+\mu) - 2DhQ - 2DK(\lambda+\mu)\right]}_{\phi_1(Q)}$$
$$= \underbrace{hQ^2(\lambda+\mu) + 2DhQ + 2DK(\lambda+\mu)}_{\phi_2(Q)}$$

It is apparent that $\phi_2(Q)$ is an increasing function and has positive values for $Q > 0$.

Regarding $\phi_1(Q)$, it is easy to show that $\phi_1(Q) = 0$ has two roots, say Q_{lb} and Q_{ub}, such that $Q_{lb} < 0 < Q_{ub}$ we only consider the root:

$$Q_{ub} = \frac{D\left[1 + \sqrt{1 + \frac{2K(\lambda+\mu)^2}{Dh}}\right]}{\lambda + \mu}$$

Since $\phi(Q_{ub}) = -\phi_1(Q_{ub}) + \phi_2(Q_{ub}) > 0$, from the unimodality of $\phi(Q)$ (and consequently of $C_d(s(Q), Q)$) it follows that $Q_{1d} < Q_{ub}$.

For the lower bound, we can show that $\frac{\partial \phi(Q)}{\partial Q}\Big|_{Q=Q_{EOQ}} < 0$. Since $\phi(Q)$ is unimodal -decreasing and then increasing in Q- as shown in the proof of Proposition 4, it follows that $\phi(Q_{EOQ}) < 0$, which implies that $Q_{EOQ} < Q_{1d}$. □

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