On the dynamics of pure remanufacturing systems

Abstract:
Pure remanufacturing systems and their respective supply chains are increasingly common in many industries. However, the scientific community has paid little attention to such systems, and thus their dynamics are still not well understood. To address this gap, we build an archetype by generalising the real-world closed-loop supply chain of pure remanufacturers across different industries. Through control engineering, we study its mathematical properties and their impact on performance. We benchmark it against two extensively studied models: traditional manufacturing systems and hybrid manufacturing-remanufacturing systems. We show the proportional order-up-to policy to be a promising approach to streamline the operations of pure remanufacturers, and we highlight critical trade-offs that need to be recognised by decision-makers to improve their control. Through simulation, we identify effective management strategies for these closed-loop supply chains, enabling remanufacturers to improve customer satisfaction while reducing inventory and capacity requirements. Key drivers for enhancing closed-loop supply chain dynamics include increasing pre-evaluation accuracy, reducing remanufacturing lead times, and improving the quality of reverse logistics operations. Additionally, synchronised tuning of the different inventory controllers becomes imperative for optimising overall performance.

Keywords:
Closed-loop supply chain; Inventory control; Remanufacturing; Supply chain dynamics.

1. Introduction

Original equipment manufacturers (OEMs) have been pursuing the economic value associated with remanufacturing since the 1940s (Zhang and Chen, 2015). This mostly refers to increased profit margins through decreased cost of inputs, improved customer-supplier relationships, and wider control of the supply chain (Charter and Gray, 2008). Increasing environmental concerns and new legislation for product end-of-life management have more recently acted as catalysts for further developments in remanufacturing, as it considerably reduces raw material and energy consumption, CO2 emissions, and waste (Genovese et al., 2023). Reports have also underscored the key social value of remanufacturing, for example, in creating local job opportunities (European Remanufacturing Network, 2015). To leverage these combined benefits of remanufacturing, a growing number of supply chains need to undergo increasingly urgent transition from traditional open-loop (linear, make-use-dispose) to closed-loop (circular, make-use-reuse) forms, incorporating the collection and processing of used products (cores).

This transition is however not straightforward. A fundamental challenge relates to production and inventory management (Guide et al., 2003). While both open- and closed-loop supply chains are driven by customer demand, the latter are also strongly influenced by the core acquisition process (Behret and Korugan, 2009). Consider this simple example: to manufacture a bicycle, we adhere to a fixed bill of materials that deterministically instructs us to use two wheels for each bicycle. If we were to remanufacture a bicycle, however, the number of (new) wheels required is stochastic and conditional to the quality of the returned core. In both cases, we need to forecast the rate (timing and quantity) of demand, while the efficient management of remanufacturing also requires forecasting the rate and quality (condition) of returns. This twofold (demand + returns) uncertainty cannot be well managed by existing models designed for the linear economy. New models that effectively integrate the forward and reverse flows of materials become then essential (Goltsos et al., 2019b). Similarly, the understanding of these closed-loop supply chains' dynamics needs to be progressed if they are to ever mirror the efficiencies attained by traditional operations over the decades (Goltsos et al., 2019a).

1.1. Context and motivation

In practice, remanufacturing may display different forms. In some cases, manufacturing (from raw materials) and remanufacturing (from cores) are integrated within the same business structure (Benkherouf et al., 2014). These are commonly referred to as *hybrid manufacturing-remanufacturing systems*, although this term has not been used uniformly in the literature. It is either used in terms of sharing resources (Aras et al., 2006), i.e., *hybrid production*, and/or jointly servicing end demand (Toktay et al., 2000), i.e., *hybrid service*.

The first may be the case of a production line that normally operates as an OEM, but every, say, fourth week gets reconfigured to remanufacture used products (e.g., Tang and Teunter, 2006). The second is common when the assumption of perfect substitution holds, i.e., when remanufactured products are indistinguishable from new (OEM) products in terms of customer perception (Souza, 2013). The consideration of hybrid systems is thus linked to whether the market (customers) distinguish between remanufactured and new products in any shape or form; that is, if the circularly treated cores can readily contribute towards servicing the demand for new products.

However, remanufacturing operations are separate from manufacturing operations in many industrial settings, using distinct resources and serving a different market. This points to the industrial relevance of what are often referred to as *pure remanufacturing systems* (e.g., Sun et al., 2013; Sarkar et al., 2017; Piñeyro and Viera, 2022). In such cases, remanufacturing may be performed by a third-party, a common option in many industries (Abbey et al., 2018), often under service contracts (as in Goltsos et al., 2019b).

Even for companies that engage in both manufacturing and remanufacturing, these two lines generally operate separately and satisfy different customers (Guide and van Wassenhove, 2009) or business models (Ferguson et al., 2009). From a product lifecycle perspective, pure systems can also be prevalent during the decline phase of products: when returns surpass demand, OEM production can potentially stop, and the demand can be met through (pure) remanufacturing in its entirety (see Östlin et al., 2009).

A fundamental distinction between hybrid and pure remanufacturing systems lies in the integration point between forward and reverse material flows. In hybrid (service) systems, (manufactured and remanufactured) products integrate at the serviceable inventory, which directly fulfils customer demand. In such cases, the manufacturing orders and serviceable inventory must be carefully managed to synchronise incoming flows, while the recoverable inventory is often handled with passive, push-based (remanufacture upon arrival) policies.

In contrast, in pure remanufacturing systems, the integration occurs upstream at the recoverable inventory, where returned and externally sourced cores are pooled, waiting to be pulled by remanufacturing orders. This then becomes the critical control point, requiring active and tailored management to ensure operational stability and secure the downstream supply of serviceable stock. This structural difference has significant managerial implications for supply chain design and system dynamics.

As we will discuss in the following section, in the closed-loop supply chain and circular operations management literature, hybrid (service) systems have been much more widely investigated than pure remanufacturing systems. Such systems are however not highly common in practice. They can be found in some specific industries such as spare parts (Souza, 2013), while pure remanufacturing is becoming increasingly popular. Wei et al. (2015, p.15) noted the following interesting paradox: *"Hybrid remanufacturing system[s] ha[ve] received relatively more attention than non-hybrid remanufacturing system[s in the literature], even though non-hybrid remanufacturing system[s are] more common in practice".* At the same time, the assumption of perfect substitution is prevalent in the remanufacturing literature to the point it is most often taken for granted (Goltsos et al., 2019a), despite not always being necessarily very realistic, and thus "*can reduce modelling efforts to elegant solutions addressing non-existent problems*" (Guide and van Wassenhove, 2009, p. 17).

Our research is thus motivated by: (i) the contemporary industrial relevance of pure remanufacturing systems; (ii) the limited existing knowledge regarding the performance of their supply chains; and (iii) the crucial role such understanding plays in advancing the transition toward a more circular economy. We look at this issue from the perspective of *supply chain dynamics* (Towill, 1991; Dejonckheere et al., 2003; Framinan, 2022). This discipline explores the interaction between the different elements that define a supply chain structure, including flows, delays, feedback loops, and decision rules.

It does so by considering the time-varying behaviour of the variables that define the properties of the system as a whole, and hence its performance. By integrating complementary and related priorities into the same analysis (e.g., the trade-off between inventory investment and service level, and the need for smoothing supply chain operations), this perspective can offer broad and comprehensive insights into the management of pure remanufacturing systems. This would in turn facilitate the design and implementation of more effective and efficient closed-loop supply chains in practical applications.

1.2. Purpose and impact

So far, several papers have contributed to a better understanding of the dynamic behaviour and performance of closed-loop supply chains based on hybrid systems (e.g., Tang and Naim, 2004; Hosoda et al., 2015; Ponte et al., 2019; Cannella et al., 2021; Huang et al., 2023). However, as will become more apparent in Section 2, the closed-loop supply chain dynamics literature has not sufficiently addressed pure remanufacturing systems, if at all. At the same time, the general pure remanufacturing literature has not investigated supply chain dynamics considerations.

It is not known then to what extent the dynamics, behaviour, and performance of pure systems resemble and/or differ from the those of traditional and hybrid supply chains. In fact, a supply chain dynamics archetype for exploring pure remanufacturing systems has not yet even been established.

To address these shortcomings, we:

(i) create a generalised mathematical model of a pure remanufacturer's closed-loop supply chain and establish a system archetype for benchmarking and future research;

(ii) investigate the stability, dynamic behaviour, and operational performance of this closed-loop supply chain through both analytical and numerical techniques;

(iii) examine effective management strategies for managing the dynamics and optimising the performance of pure remanufacturing systems;

(iv) benchmark its dynamic behaviour, considering both effectiveness (service level) and efficiency (operational costs), against the well-established understanding of the dynamics of traditional as well as hybrid closed-loop supply chain archetypes.

To conduct our analysis, we draw inspiration from real-world pure remanufacturers across diverse industries, providing an empirical foundation for addressing the problem under consideration. In line with a well-established stream of the closed-loop supply chain dynamics literature (e.g., Tang and Naim, 2004; Zhou et al., 2017; Ponte et al., 2021), we leverage concepts, principles, and techniques from control engineering. Specifically, within the broader family of control-theoretic approaches applied to supply chain analysis, we employ Laplace transforms, block diagram representations, transfer functions, and unit-step response analysis.

This control-theoretic framework is well aligned with our objectives, as it enables a transparent and rigorous representation of the structure, logic, and feedback mechanisms that shape the dynamics of inventory systems. In particular, it facilitates the systematic analysis of how (production and purchase) orders and (recoverable and serviceable) inventories respond to changes in demand and system parameters. This, in turn, allows us to derive clear insights into system stability (as a prerequisite), transient behaviour, and closed-loop supply chain performance, thereby supporting a broad understanding of its dynamic characteristics under realistic operational conditions.

We identify high-order systems with multiple but real poles, characterising the system response with the lack of oscillatory behaviours, and several zeros, which also determine the supply chain dynamics and performance. We explore the impact of all system (physical and decision) parameters and identify meaningful differences in relation to the traditional and hybrid systems. Understanding its mathematical properties facilitates impactful managerial decision-making (Syntetos and Nikolopoulos, 2024). In this sense, our findings yield key managerial insights, with implications for economic, environmental, and social sustainability. They provide support for expediting the transition towards circular economies in contemporary societies by facilitating the implementation of high-performance remanufacturing supply chains.

In this way, the study allows us to offer operationalised suggestions that should result in higher customer satisfaction along with substantial reductions in inventory and capacity requirements (see e.g., Scarf et al., 2023). Importantly, our findings highlight the proportional order-up-to (POUT) policy as a valuable strategy to enhance the operations of pure remanufacturers. We also identify essential trade-offs that decision-makers must acknowledge to enhance the control of their inventories by appropriately regulating the inventory controllers of this replenishment policy.

From this perspective, our study provides prescriptive insights for the effective synchronised tuning of the controllers governing both recoverable and serviceable inventories. Furthermore, it delves into the lead-time complexity of these supply chains, marked by multiple lead times that influence their dynamics, emphasising how uninformed decisions may result in ineffective investments. Our results also highlight the need to enhance the accuracy of pre-evaluation processes and improve the quality of reverse logistics operations as key drivers for enhancing the operational performance of these closed-loop supply chains.

To summarise, the purpose of this study is to investigate the dynamic behaviour of pure remanufacturing systems subject to demand and returns. Rather than seeking to develop an optimal control policy, our objective is to explore how such systems perform when managed under the industry-relevant POUT inventory policy. Thus, our contribution lies in characterising how structural and operational parameters, such as lead times, return proportions, and remanufacturing yields, shape the system's stability and transient response, and thus its performance.

We focus on pure remanufacturing systems, recognising their growing relevance in circular economy contexts and their distinct behaviour compared to well-established hybrid manufacturing–remanufacturing systems. Consequently, the insights obtained are prescriptive rather than predictive: they inform decision-making by identifying the key levers that influence performance under the studied conditions and set the field for further research.

The remainder of this paper is structured as follows. Section 2 discusses the pertinent literature. Section 3 addresses the development of the archetype. Sections 4 and 5 conduct the static and dynamic analyses, respectively, based on control-theoretic developments and step-response analyses. Section 6 presents a simulation study to explore effective management strategies. Section 7 presents the conclusions, relating our findings to previous studies on traditional supply chain and hybrid remanufacturing systems, and next steps of research.

2. Background

The goal of supply chains is to deliver the highest possible customer value at the lowest possible cost (traditionally economic, but increasingly environmental and social). Supply chain actors need then to manage the interaction of the different elements of these systems, across multiple levels of decomposition, from the actor level to the policy level, including forecasting techniques, inventory and production control policies, information sharing mechanisms, and so on. To do so, management needs to understand the dynamic behaviour over time of the materials and information flows that collectively define their performance (Towill, 1991). In this section, we first briefly discuss the main theory related to the dynamics of traditional supply chains (Section 2.1). We then review relevant closed-loop supply chain dynamics theory based on hybrid manufacturing-remanufacturing systems (Section 2.2). Later, we focus on existing knowledge about pure remanufacturing systems (Section 2.3). Finally, we identify the research gaps we consider in this work and outline our contribution to the literature (Section 2.4).

2.1. The dynamics of (open-loop) supply chains

The discipline of supply chain dynamics traces its origins back to the works of Jay W. Forrester at MIT, on what he named industrial dynamics (Forrester, 1958). He formalised the amplification phenomenon of order variances across the supply chain, known now as the bullwhip effect (e.g., Najafi and Farahani, 2014; Wang and Disney, 2016; Weisz, 2023). However, it was not until the 1990s that significant new advancements took place in this area through the works of Lee et al. (1997) and Chen et al. (2000), among others. Metters (1997) showed that the inability to manage the dynamics of supply chains results in severe economic losses for all actors involved.

Since then, many authors have focused on understanding how different conditions and factors affect the dynamics of traditional (open-loop) supply chains (Figure 1), and thus their performance. They have used a wide variety of methodological approaches, which Wang and Disney (2016) categorised into four different types: empirical, experimental, analytical, and simulation-based.

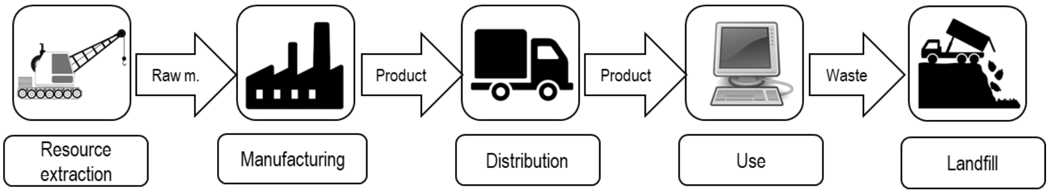


Figure 1: Traditional supply chain. Such supply chain dynamic modelling has elevated our understanding of the interaction between the different elements. While important and relevant, it does not (nor it ever intended to) study the pertinent interaction between sales and returns in circular economic contexts.

Analytical and simulation studies, like the present, abstract the real-world problem into a mathematical model. This allows researchers to quantify supply chain performance, investigate complex cause-effect relationships, and offer prescriptive guidelines for professionals, and have thus become the most popular approaches to supply chain dynamics (Wang and Disney, 2016). Authors have widely explored replenishment policies, such as the industrially popular order-up-to (OUT) policy (e.g., Dejonckheere et al., 2003), lead times (e.g., Chen et al., 2000), and supply chain structures (e.g., Dominguez et al., 2015), and their impact on the dynamic behaviour and operational performance of these systems.

Authors have also explored various solutions for enhancing the dynamics of these supply chains (i.e., improving customer satisfaction, reducing inventory levels, and mitigating bullwhip levels). These include information transparency (e.g., Chen et al., 2000), wider collaborative strategies, such as vendor-managed inventory (VMI) (e.g., Disney and Towill, 2003b), and demand forecasting improvements (e.g., Zhang, 2004). In terms of inventory control, the POUT policy has often been recommended, as it can reduce bullwhip whilst maintaining high customer service (Disney et al., 2007). This incorporates a proportional controller into the OUT policy, regulating how fast the system responds to deviations from the target inventory position (e.g., Disney, 2022).

The above works collectively motivate our choice of methodologies and policies as well as the metrics and perspectives we use in subsequent sections, allowing comparisons between the three archetypes (linear, hybrid, pure).

2.2. The dynamics of (hybrid) closed-loop supply chains

Tang and Naim (2004) are often credited with being the first to study the dynamics of closed-loop supply chains (Goltsos et al., 2019a). They investigated a hybrid service manufacturing-remanufacturing system (Figure 2); specifically, they considered the case of perfect substitution. Many authors have since studied the behaviour of closed-loop structures based on hybrid production systems, with some variations in the assumptions, by predominantly exploiting mathematical modelling techniques (both analytical and simulation-based), such as Zhou and Disney (2006), Hosoda et al. (2015), Zhou et al. (2017), Cannella et al. (2021), and Huang et al. (2023), among others.

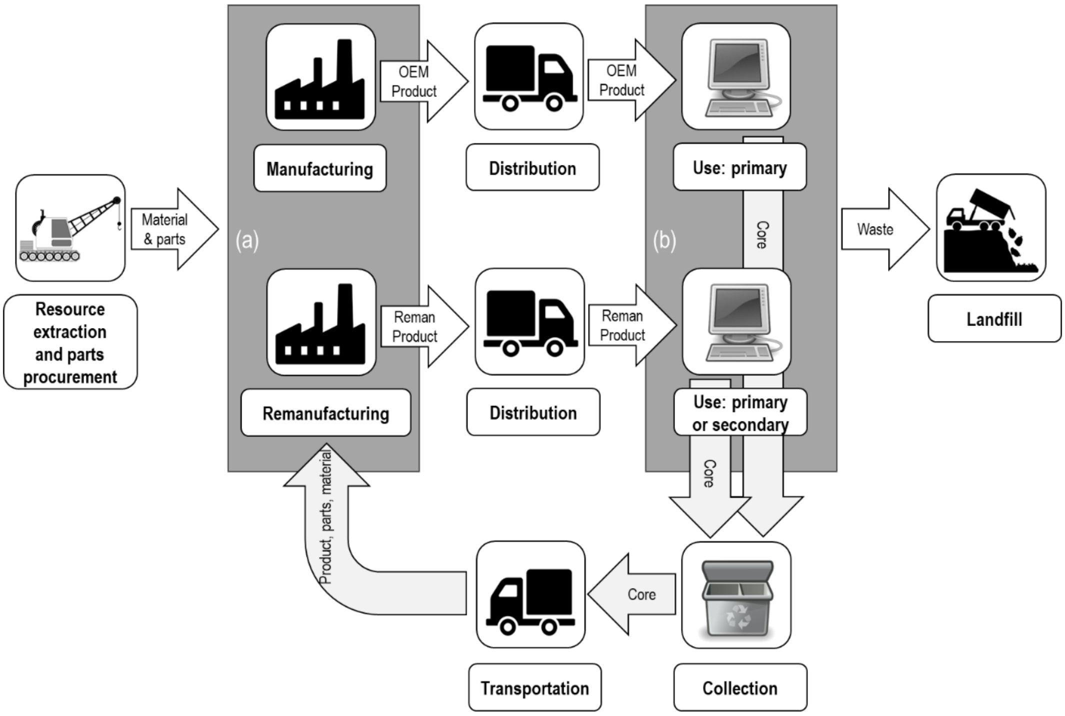


Figure 2: Closed-loop supply chain based on a hybrid manufacturing-remanufacturing system. Generally, some integration between manufacturing and remanufacturing processes (hybrid production, assumption of resource sharing) or joint service of a market (hybrid service; assumption of perfect substitution) is required for a system to be considered hybrid. However, the assumption of resource sharing does not commonly hold in practice, while the assumption of perfect substitution is more often than not unrealistic.

These works, invariably applied to hybrid contexts, have observed that supply chain dynamics can either benefit or suffer from incorporating the reverse flow of materials depending on certain circumstances (Goltsos et al., 2019a). For instance, Tang and Naim (2004) showed that information transparency between both (forward and reverse) product channels considerably reduces order variability (when compared to the traditional supply chain). Conversely, Ponte et al. (2019) showed severe deterioration of performance (i.e., lower customer satisfaction, higher bullwhip) when there is high uncertainty associated with the quantity of the cores. The same may occur if there are significant differences in the quality of the cores (Dominguez et al., 2020).

Information transparency and POUT policies are also valid approaches to improve the dynamics of these systems (Ponte et al., 2020; Cannella et al., 2021), along with improved returns forecasting (Goltsos et al., 2019b). An interesting finding regarding hybrid closed-loop supply chains is the lead-time paradox (Tang and Naim, 2004; Hosoda et al., 2015; Dominguez et al., 2020). This challenges the conventional wisdom where shorter lead times are considered better through improved customer satisfaction and reduced bullwhip levels. Hosoda and Disney (2018) explored this paradox in detail, and suggested making remanufacturing lead times equal to manufacturing lead times may lead to enhanced supply chain dynamics in hybrid contexts.

To what extent this paradox and specific observations about hybrid closed-loop systems also exist in pure remanufacturing systems is an open question, and a motivation for our paper.

2.3. The (unknown) dynamics of pure remanufacturing closed-loop supply chains

The operational dynamics of closed-loop supply chains centred around pure remanufacturers (Figure 3) remain unexplored. A few years ago, Goltsos et al. (2019a) found that more than 75% of papers of closed-loop supply chains explored hybrid systems, with the remaining 25% (approx.) mainly addressing recycling systems (e.g., Adenso-Diaz et al., 2012). More recent papers have followed the same line (e.g. Cannella et al., 2021). However, it is essential to note that existing literature has examined pure remanufacturing systems from other angles.

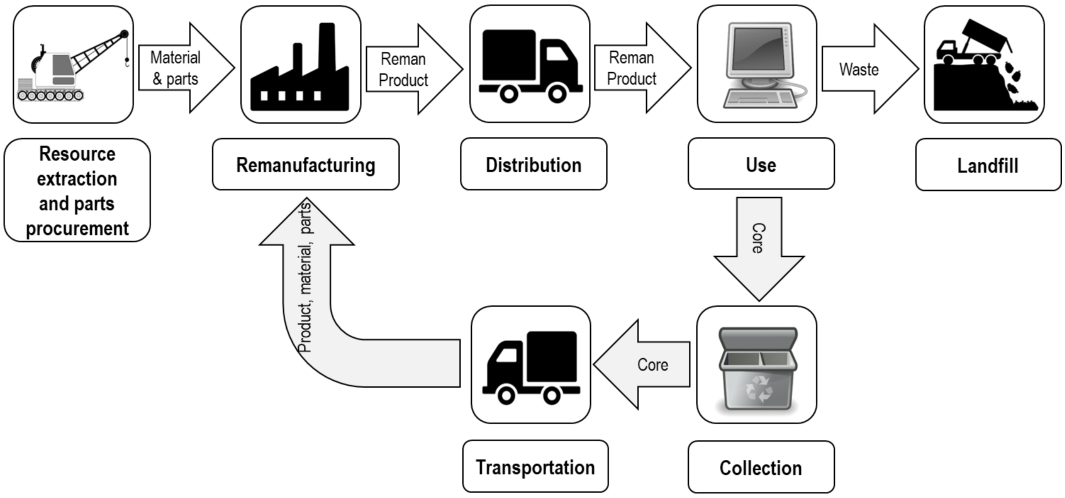


Figure 3: Closed-loop supply chain based on a pure remanufacturing system. Here an independent (from manufacturing, in terms of resources, including labour and infrastructure) remanufacturing operation serves either: (a) the demand for a secondary market of remanufactured products; or (b) in the presence of the perfect substitution assumption, a part of the demand for products (which could eventually be the entire demand, once the return volume surpasses the demand in a circular product life-cycle point of view).

Toktay et al. (2000) investigated the procurement of components for remanufactured Kodak single-use cameras and found returns forecasting to be the most important operational lever. Goltsos et al. (2019b) adapted methods by Kelle and Silver (1989) to detail the benefits of serialised information in the accuracy and utility (inventory performance) of returns forecasting methods. Ferguson et al. (2009) examined the value of introducing quality grading (i.e., pre-sorting the inventory of cores based on their condition) in pure remanufacturing operations, showing a 4% increase in profitability because of better control of the remanufacturing process. Teunter et al. (2008) found 6.5% cost reductions when comparing a pure remanufacturing setup to its hybrid counterpart (itself examined in Tang and Teunter, 2006, on the same case), attributed to lower production rates and increased scheduling flexibility.

Under these circumstances, the limited research that has been applied to pure remanufacturing systems has focused on various aspects of supply chain operations, but not on these systems' supply chain dynamics. While we are by no means diminishing the need for (or importance of) such contributions, or their scope and parameters of interest, such approaches do not readily provide a comprehensive understanding of the dynamic behaviour of these supply chains.

2.4. Summary of research gaps and our contribution

Table 1 summarises key dimensions that distinguish the widely studied hybrid (service) systems from pure remanufacturing ones, which, to the best of our knowledge, have not yet been studied through the lens of supply chain dynamics. These differences necessitate dedicated analysis of pure remanufacturing systems, particularly given their growing relevance in many sectors and their distinct dynamic behaviour within closed-loop supply chains.

Table 1. Key distinctions between hybrid and pure closed-loop supply chain systems.

Dimension / Closed-loop supply chain	**Hybrid Manufacturing–Remanufacturing System (main focus of prior works)**	**Pure Remanufacturing System (this study)**
Fulfilment of demand	Served through a combination of manufactured and remanufactured items	Served exclusively through remanufactured items
External material sourcing (stabilizing mechanism)	Involves sourcing of all material needed for manufactured products	Involves potential external core acquisition
Serviceable inventory	Key integration point of manufactured and remanufactured products – *Must be carefully managed to balance both flows*	Fed solely by remanufactured items – *Can be managed similarly to traditional inventory systems*
Recoverable inventory	Typically used to temporarily store returns – *Not always managed with an active inventory control policy*	Key integration point of returns and external sourcing (e.g., via core broker) – *Must be carefully managed to balance both flows*
Managerial focus	Balancing manufacturing and remanufacturing flows at the serviceable site	Balancing returns and external cores at the recoverable site
Relevance in practice	Only found in specific industries, such as spare parts	Common in many industries, with several variants existing
Understanding in theory	Widely investigated in the closed-loop supply chain (dynamics) literature	Very limited research in the closed-loop supply chain (dynamics) literature

We can reasonably conclude that: (a) the existing supply chain dynamics literature has not investigated pure remanufacturing and its implications; and (b) studies on pure remanufacturing have not yet considered supply chain dynamics. We use the literature presented in the preceding sections as motivation for studying the supply chain dynamics of pure remanufacturing systems, which have not yet been researched and consequently have not been juxtaposed against traditional systems. This knowledge gap is significant, considering the industrial relevance of these systems nowadays.

The present study then contributes to the literature by providing a thorough and comprehensive understanding of the operational dynamics and performance of closed-loop supply chains based on pure remanufacturing systems. To achieve this, we introduce a formal archetype designed to facilitate the investigation of pure remanufacturing systems. We compare the properties and behaviour of this archetype with well-established traditional manufacturing and hybrid manufacturing-remanufacturing systems. Based on our theoretical developments and numerical experiments, we also present a set of actionable implications for professionals, outlining mechanisms that can enhance customer satisfaction and reduce inventory and capacity requirements in the closed-loop supply chains of pure remanufacturers.

3. Supply chain model

In this section, we develop a formalised mathematical representation of a pure remanufacturer's closed-loop supply chain and establish a generalised control-theoretic archetype for the research of its dynamics. We also explain the key assumptions behind the development of this archetype. The structure and parameters of the model are designed to reflect realistic configurations observed in practice, particularly in sectors where remanufacturing-based operations are common in asset supply, such as defence sustainment and automotive. The choices are also consistent with modelling conventions established in the supply chain dynamics literature (e.g., Disney and Towill, 2003a; Zhou et al., 2017; Cannella et al., 2021). Our aim is not to recreate a specific case but to develop a generalisable archetype that enables the systematic exploration of the system behaviour.

3.1 Generic structure of the pure remanufacturing system

First, we build a structural model that represents the general operations of many remanufacturers in different industries. To do so, we draw inspiration from the case company discussed by Goltsos et al. (2019b), Qioptiq. This organisation operates in the photonics industry and serves the UK Ministry of Defence through a closed-loop supply chain. We have been further exposed to other pure remanufacturing operations, such as the automotive engine remanufacturing facility of MCT ReMan in Bristol, UK. In addition, the development of the model was facilitated by a series of conversations and meetings with the European Remanufacturing Network.

The structure of the closed-loop supply chain under consideration is displayed schematically in Figure 4. The company, a pure remanufacturer, transforms cores returned from the market into serviceable products (ready to be sold), which meet a set of standards defined by the business.

Once (if) a used product arrives at the premises of the company, it is held temporarily in an initial inventory until periodic evaluation occurs. Then, it is evaluated and if deemed repairable then sits on an inventory of recoverable items. The evaluation process can be characterised by a lead time of T_e. Repairable products are pulled from the recoverable inventory according to production plans and are then remanufactured and tested, with lead times of T_r and T_s, respectively. Finally, remanufactured products that meet the requirements to fulfil customer demand are stored in an inventory of serviceable items until they are sold. Once sold, items are utilised by users, a process characterised by the lead time T_u, after which some of them return to the pure remanufacturer's premises, a process characterised by a transportation lead time T_t.

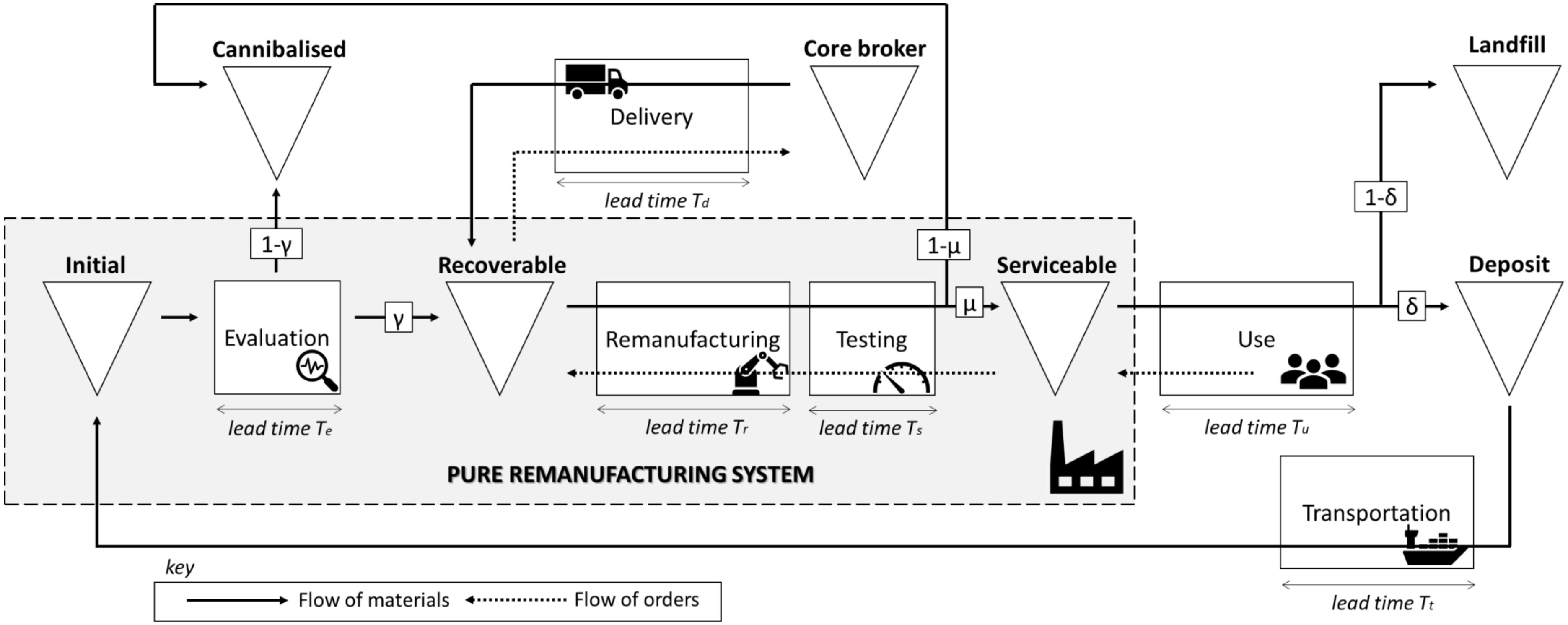


Figure 4: Generic structure of the closed-loop supply chain under consideration.

Table of notations:
Lead times - T_e: evaluation lead time; T_r: remanufacturing lead time; T_s: testing lead time; $T_{u:}$ usage lead time; $T_{d:}$ delivery lead time; T_t: transportation lead time.
Proportion rates - γ: economically viable (remanufacturable) proportion; μ: remanufacturing yield proportion (conforming products); δ: return proportion

There exist three potential exit points from the system. The first refers to items that are never returned by the customers, with δ being the probability of a used item returning (thus, $1-\delta$ represents the proportion of used products that are disposed of in landfills). The second is for beyond-economical-repair (BER) products, i.e., cores that cannot be remanufactured due to their poor condition. Here, γ is the probability of an item passing the initial BER test ($1-\gamma$ is thus the probability for a return to be written off and cannibalised for spare parts). We note γ reflects the overall quality and usability of returned cores, which can be affected by factors such as wear, usage history, and the number of prior remanufacturing cycles. Finally, the third exit point is for items that fail the post-remanufacturing quality control, with μ being the probability that a product passes the test (the remainder are also cannibalised).

To compensate for the loss of used products that permanently exit the closed-loop supply chain (e.g., due to disposal or non-viability), the system includes a replenishment mechanism through an external core broker. This actor plays a stabilising role by supplying recoverable items when internal returns are insufficient to meet remanufacturing needs. Such arrangements are common in the remanufacturing industry, particularly in sectors like automotive, where firms often rely on brokers or third-party suppliers to ensure a steady inflow of cores (e.g., Sundin and Dunbäck, 2013; Goltsos et al., 2019b). In this sense, the remanufacturer issues purchase orders to the core broker when needed, with the ordered cores received after a delivery lead time, T_d.

3.2 Control-theoretic archetype of the closed-loop supply chain

In developing the formal model of the pure remanufacturing system and its supply chain, we draw on control engineering as a well-established and versatile analytical framework for capturing, understanding, and managing supply chain dynamics, as demonstrated in seminal and subsequent works (e.g., John et al., 1994; Dejonckheere et al., 2003; Papanagnou, 2022).

Within the broad family of control-theoretic methods, we adopt a Laplace-based, block-diagram modelling approach. While alternative modelling approaches (e.g., state-space representations, and robust and stochastic control frameworks) can provide complementary insights, particularly under high levels of uncertainty or structural complexity (see Ivanov et al., 2018, for a survey), our approach prioritises analytical transparency and the interpretability of feedback-driven dynamics, enabling a direct link between supply chain structure and observed dynamic behaviour. Closely related formulations based on the Z-transform are also commonly used to explore supply chain dynamics and share the same foundations (see, e.g., Disney and Lambrecht, 2008).

Specifically, our model builds on the well-established, Laplace-based, Inventory and Order-Based Production Control System (IOBPCS) family. This framework, recognised for its theoretical flexibility and its practical relevance, provides a coherent and analytically tractable foundation for representing inventory control policies, information delays, and material flows interactions in supply chains. Thus, it has been widely employed in supply chain research over the past four decades; see Lin et al. (2017) for an overview of the model and a review of its applications.

Within this family, our supply chain archetype is grounded in the Automatic Pipeline, Variable IOBPCS (APVIOBPCS) model, which is characterised by having variable inventory and work-in-progress targets in the ordering rule (Disney and Towill, 2005). We model the dynamics associated with each of the inventories (serviceable and recoverable) by adapting an

APVIOBPCS echelon to incorporate the specific structural and dynamic characteristics of the pure remanufacturing closed-loop model, as explained in the following paragraphs.

Our design decision assumes that both inventories are managed through periodic-review, POUT replenishment policies. The choice of this inventory model is underpinned by its demonstrated ability to provide optimal efficiency trade-offs, particularly in balancing order and inventory variabilities within supply chains. In this sense, it outperforms the traditional OUT policy when considering not only inventory costs but also production costs contingent upon order variability, an assumption that aligns well with many practical settings (see, e.g., Disney et al., 2007).

For the serviceable inventory, we have modified the original APVIOBPCS model to integrate the remanufacturing yield proportion, μ. In doing so, this inventory issues periodic remanufacturing (production) orders, ro_t, which comprise three components: (i) the customer demand forecast made at the end of period t for future periods, df_t; (ii) the difference between the target inventory, or safety stock, ss_t, and the actual inventory, si_t, in period t, adjusted by the proportional controller, k_s; and (iii) the difference between the target work-in-progress, tws_t, and the actual work-in-progress, ws_t, in period t, similarly adjusted by k_s. We note that k_s is typically defined within the interval $(0,1]$, where $k_s = 1$ corresponds to the traditional OUT policy, and $k_s = 0$ results in orders that ignore the inventory and work-in-progress errors (feedback).

The sum of the three components must be adjusted to account for yield losses in the production process. Only a fraction, denoted by μ, of the remanufactured items are expected to pass post-remanufacturing quality control; as a result, the planned remanufacturing order quantity must be scaled accordingly (by the inverse of the estimate $\hat{\mu}$) to ensure that the desired number of conforming items is ultimately available. The replenishment rule is articulated by Equation (1). These orders result in remanufactured products that are available to satisfy the customer, after due consideration of remanufacturing and testing lead times, jointly denoted by $T_{rs} = T_r + T_s$.

The implementation of the APVIOBPCS echelon also implies adopting the following assumptions. Firstly, demand forecasts are generated using the simple exponential smoothing (SES) method (e.g., Gardner Jr, 2006), as outlined in Equation (2), where α_d defines the weight of the demand. This technique is widely employed in industry for forecasting stationary demand (Hsieh et al., 2020). Secondly, a time-varying safety stock is defined as the product of the most recent forecast and the safety factor, f_s, as presented in Equation (3). We note that f_s thus represents the fraction of forecasted demand held as buffer stock to hedge against demand uncertainty. This formulation is commonly used in practice due to its simplicity and intuitive appeal (Hoberg et al., 2007). Thirdly, the target work-in-progress is calculated by forecasting the demand of customers over the estimate of the relevant lead time, $\widehat{T_{rs}}$, as defined by Equation (4).

$$ro_t = \frac{1}{\hat{\mu}}[df_t + k_s(ss_t - si_t) + k_s(tws_t - ws_t)] \tag{1}$$

$$df_t = df_{t-1} + \alpha_d[d_t - df_{t-1}] \tag{2}$$

$$ss_t = f_s df_t \tag{3}$$

$$tws_t = \widehat{T_{rs}} df_t \tag{4}$$

For the recoverable inventory, the APVIOBPCS model also requires specific adaptations. It is reasonable to assume that the remanufacturer gives priority to the remanufacturing of products that return from the market, due to economic and/or environmental reasons. When more demand needs to be satisfied, the remanufacturer relies on the core broker to procure used products and satisfy it. Consequently, the recoverable inventory issues purchase orders to the broker based on the difference between the demand and the returns. However, it is crucial to acknowledge that some remanufactured items will not meet post-remanufacturing quality standards. This not only increases the production requirements of the remanufacturing process, but also necessitates additional products from the core broker. Thus, the 'effective' net demand, nd_t, is defined as the difference between the demand, adjusted by $\hat{\mu}$, and the actual returns, r_t, i.e., $nd_t = \left(\frac{d_t}{\hat{\mu}}\right) - r_t$.

Returns stem from past demand, and are governed by the joint probability that a used item is both returned and remanufacturable, which we denote as $\beta = \delta \cdot \gamma$ to streamline mathematical developments in this and the following sections. Additionally, returns are subject to a combined usage and transportation lead time, which we define jointly as $T_{ut} = T_u + T_t$. We assume that the evaluation lead time is negligible compared to the other lead times, i.e., $T_e = 0$.

In these circumstances the POUT replenishment policy, utilising the same rationale as previously discussed, determines purchase orders, po_t, as per Equation (5). Here, ndf_t represents the forecast of the net demand made at the end of period t for future periods, ndf_t; $sr_t - ri_t$ accounts for the gap between the safety stock and the actual position of the recoverable inventory in period t; $twr_t - wr_t$ reflects the gap between the target and actual position of the work-in-progress in period t; and k_r is the proportional controller of the POUT rule. The ordered products will be received after the delivery lead time, T_d. Finally, we note that the methods for computing the net demand forecast, the safety stock, and the target work-in-progress align with that described for the serviceable inventory and can be formalised by Equations (6), (7) and (8). In these equations, the relevant parameters include the SES parameter of the net demand forecast, α_{nd}, the safety factor of the recoverable inventory, f_r, and the estimate of the delivery lead time, $\widehat{T_d}$.

$$po_t = ndf_t + k_r(sr_t - ri_t) + k_r(twr_t - wr_t) \quad (5)$$

$$ndf_t = ndf_{t-1} + \alpha_{nd}[nd_t - ndf_{t-1}] \quad (6)$$

$$sr_t = f_r ndf_t \quad (7)$$

$$twr_t = \widehat{T_d} ndf_t \quad (8)$$

The preceding difference equations, (1)-(8), formalise the control system of the closed-loop supply chain in the time, *t*-domain. To maintain brevity, we have omitted additional difference equations that characterise the dynamics of the pure remanufacturing system (specifically those concerning inventory and work-in-progress balances and lead times), as they are well established in the literature. Interested readers are directed to the original work by John et al. (1994) and the review by Lin et al. (2017), among other relevant sources, for a more in-depth exploration of details.

By considering the above, as well as the generic structure of the closed-loop supply chain discussed in Section 3.1, we formalise the archetype through the linear block diagram presented in Figure 5. This formal representation precisely describes the relations between all the relevant variables and parameters of the supply chain in the Laplace, *s*-domain. It is equivalent to the earlier formalisation of the control system in the time, *t*-domain (Equations 1-8) through well-known Laplace transformations (adhering to standard practice, we use lowercase letters for the variables in the *t*-domain and uppercase for the *s*-domain). In developing the block diagram, we have used first-order delays to represent the lead times, following common practice (e.g., Zhou et al., 2017), while we have configured the SES mechanism through the time constants T_a (for the demand) and T_{an} (for the net demand). These can be easily linked to the conventional SES parameter, i.e., $\alpha_d \approx 1/(1+T_a)$, $\alpha_{nd} \approx 1/(1+T_{an})$, as discussed by John et al. (1994).

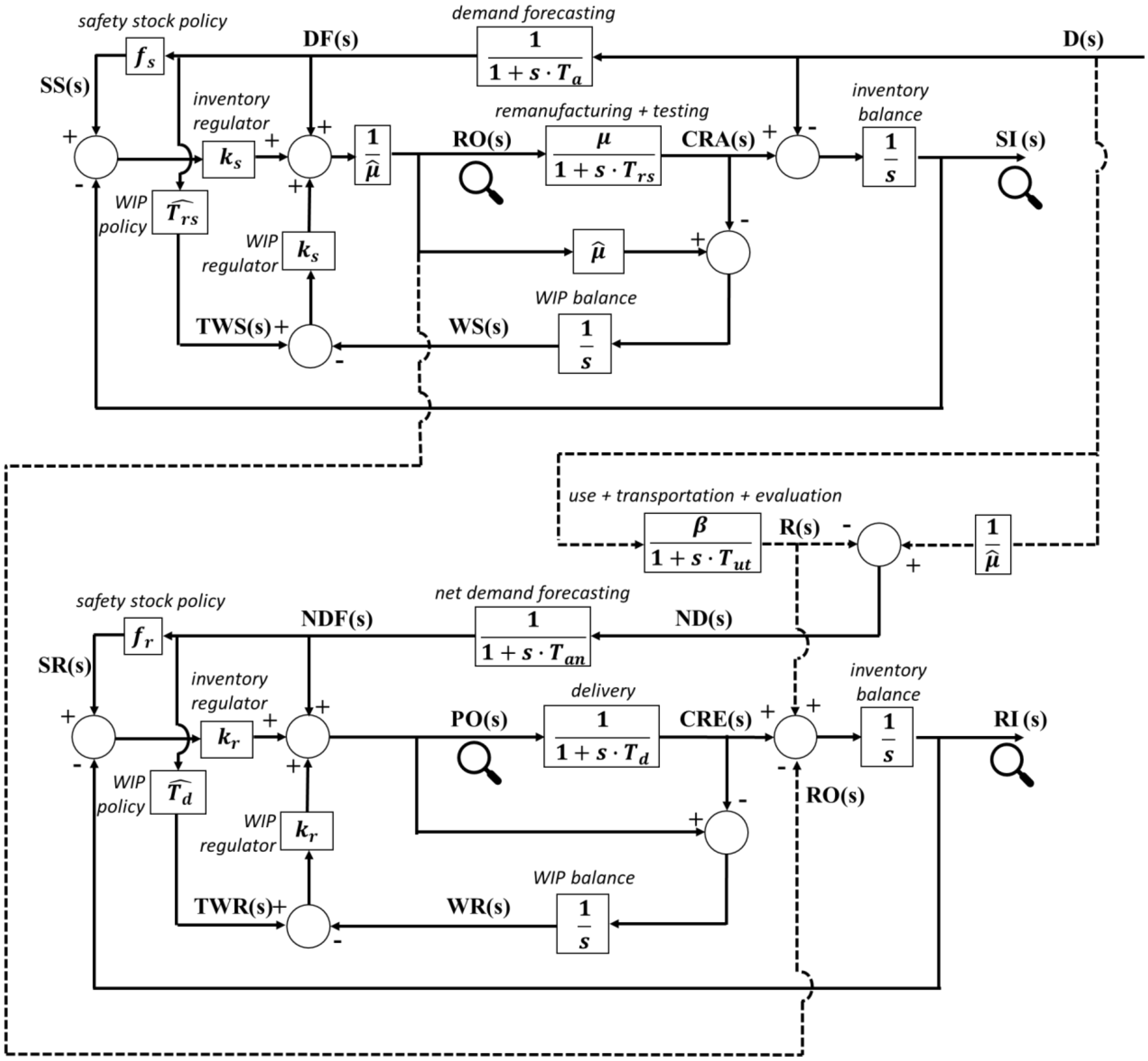


Figure 5: Control-theoretic archetype of the pure remanufacturing system and its supply chain.

Table of notations:

Variables of the serviceable inventory – CRA: completion rate; D: demand; DF: demand forecast; RO: remanufacturing orders; SI: serviceable inventory position; SS: safety stock; TWS: target work-in-progress; WS: work-in-progress.

Variables of the recoverable inventory – CRE: cores receipt: ND: net demand; NDF: net demand forecast; PO: purchase orders; R: returns; RI: recoverable inventory position; SR: safety stock; TWR: target work-in-progress; WR: work-in-progress.

Parameters are defined throughout the text. A hat/caret is used to denote estimates as required.

The block diagram of Figure 5 establishes the archetype, providing a control-engineering perspective that facilitates an in-depth exploration of the dynamic behaviour and operational performance of the closed-loop supply chain. Aligned with prior discussions, this archetype, which can be interpreted as a new member of the IOBPCS family, is founded on two echelons (defined by the recoverable and serviceable inventories) interconnected in two essential ways. First, the demand of the serviceable inventory generates the returns, and determines the net demand, for the recoverable inventory. Second, the remanufacturing orders issued by the serviceable inventory impact (decrease) the position of the recoverable inventory. Loupes in Figure 5 highlight the four key variables that define the behaviour and performance of the remanufacturing system: remanufacturing orders, RO(s); serviceable inventory, SI(s); purchase orders, PO(s); and recoverable inventory, RI(s). These will be investigated in detail in the following subsections.

This Laplace formalisation simplifies the mathematical analysis of the system behaviour through the development of transfer functions, which provide concise mathematical representations of the (complex) relationships between the relevant inputs and outputs in the *s*-domain. We have derived the four transfer functions that relate the four key variables previously defined to the demand of the supply chain using standard techniques. Interested readers are referred to Nise (2019) for an exposition on control engineering techniques. To derive these functions, we have considered that the estimates of the various parameters perfectly match their value, i.e., we assume there is no estimation error in the parameters by the decision-makers. The four relevant transfer functions, which are used for the subsequent analysis, are detailed in Appendix A.

4. Static analysis

This section provides analytical results concerning the stability and steady-state behaviour of the pure remanufacturing system. From a control-theoretic perspective, stability is a fundamental property that determines whether a system can maintain a bounded and predictable response over time following an input change or disturbance. In the context of inventory control systems, stability ensures that orders and inventory levels do not diverge uncontrollably, but instead evolve towards a steady state, making it a necessary condition for reliable and effective supply chain operation (Wang et al., 2012). Determining system stability is thus a critical first step that underpins the subsequent analysis of the dynamics and performance of our closed-loop system.

As is well known (e.g., Nise, 2019), a control system is stable if and only if all the poles of the relevant transfer functions (that is, the roots of the denominator polynomials) lie in the left half of the complex plane. Using this criterion, we derive the stability conditions reported in Table 2 through direct analysis of the transfer functions detailed in Appendix A.

Table 2. Stability conditions of the closed-loop supply chain and static gains of its transfer functions.

	Stability conditions	**Static gain**
Remanufacturing orders: $\boldsymbol{RO(s)/D(s)}$	$k_s > 0;\ T_a \geq 0$	$\frac{1}{\mu}$
Serviceable inventory: $\boldsymbol{SI(s)/D(s)}$	$k_s > 0;\ T_a \geq 0;\ T_{rs} \geq 0$	f_s

Purchase orders: $\boldsymbol{PO(s)/D(s)}$	$k_s > 0;\ k_r > 0;\ T_a \geq 0;$ $T_{an} \geq 0;\ T_{ut} \geq 0$	$\rho\left(=\frac{1}{\mu}-\beta\right)$
Recoverable inventory: $\boldsymbol{RI(s)/D(s)}$	$k_s > 0;\ k_r > 0;\ T_a \geq 0;$ $T_{an} \geq 0;\ T_d \geq 0;\ T_{ut} \geq 0$	$f_r\rho$

Inspection of Table 2 reveals that the stability of this closed-loop supply chain depends on: (i) the inventory controllers of the POUT policy, k_s and k_r; (ii) the time constants of the SES forecasts, T_a and T_{an}; and (iii) the lead times, T_{rs}, T_d, and T_{ut}. Specifically, the inventory control system is stable if, and only if, the time parameters (that is, time constants and lead times) are non-negative, i.e., $\{T_a, T_{an}, T_{rs}, T_{ut}, T_d\} \geq 0$, and the control parameters of the POUT policy are strictly positive, i.e., $\{k_s, k_r\} > 0$. It is interesting to note that the stability does not depend either on the proportions β and μ or the safety factors, f_s and f_r. That is, irrespective of their values (although, considering their definitions, it is reasonable to specify them within the intervals $\{f_s, f_r\} \geq 0$ and $0 \leq \{\beta, \mu\} \leq 1$), the closed-loop supply chain will always be stable if the previous conditions are satisfied.

As discussed earlier, the inventory controllers, characterised by k_s and k_r, define the proportion of the gap between the target and the actual (on-hand and on-order) inventory that is considered by the POUT policy, with $\{k_r, k_s\} = 0$ breaking the feedback loop. It results in the ordering rule ignoring the actual inventory position when issuing orders, which generates an unstable behaviour of the supply chain inventories (Disney and Lambrecht, 2008). Therefore, these parameters are generally recommended to be set within the interval $(0, 1]$. In addition, negative values of the time parameters have no physical and/or conceptual meaning. For these reasons, we can conclude that the closed-loop supply chain is stable for all reasonable values of its parameters.

We perform the steady-state analysis by deriving the static gains of the transfer functions that shape the behaviour of the pure remanufacturing system. The static gains provide the final value of the variables' responses when there is a unit increase in demand, representing a sudden and constant change in this input. Table 2 reports the static gains of the four relevant transfer functions. Detailed computation of these static gains is provided in Appendix B.

We first look at the orders. Here, a step in demand generates an increase in the remanufacturing orders that is proportional to $1/\mu$. The increase in the order quantity thus compensates for the fact that some products will not satisfy the standards of the post-remanufacturing quality control. For the same reasons, the increase in the purchase orders tends to be proportional to $1/\mu$; however, it also needs to consider that a portion of the used products, defined by β, will return to the supply chain and be remanufacturable. The increase in the orders thus becomes proportional to $\rho = \frac{1}{\mu} - \beta$, which we define as the *compensation proportion* for convenience. The increase in orders may be lower (if $\beta > (1/\mu) - 1$; then $\rho < 1$) or higher (if $\beta < (1/\mu) - 1$); then $\rho > 1$) than the increase in demand. Only in the ideal case that $\mu = 1$ and $\beta = 1$, the core broker is not needed ($\rho = 0$).

Finally, we look at the inventories. Table 2 shows that a step increase in demand will also result in an increase in the serviceable inventory position. This increase is proportional to f_s, as a direct consequence of the safety stock policy. Meanwhile, the increase in the position of the recoverable stock is proportional to the product of the safety factor, f_r, and the compensation proportion, ρ.

5. Dynamic analysis

Having determined the regions of stability and the static gains of the closed-loop supply chain as a preliminary diagnostic, our focus now shifts to its dynamic performance. To this end, we study the transient response of the orders and inventories of the pure remanufacturing system when it faces a unit step in demand. This is determined by the location of the transfer functions' poles and zeros (the zeros are the roots of the numerator). The analysis of such step responses enables a comprehensive understanding of the long-term behaviour of supply chains, providing valuable inputs for the design of inventory control systems and the tuning of their parameters (e.g., Towill et al., 2007). Consequently, such analysis is common practice in the closed-loop supply chain dynamics literature (e.g., Tang and Naim, 2004; Zhou et al., 2017; Ponte et al., 2019).

In this context, we first define a baseline scenario characterised by the levels of the decision (control) and physical (uncontrollable) parameters reported in Table 3. Naturally, the baseline scenario lies entirely within the region of stability, as step-response characteristics are only of practical managerial relevance for stable systems. This table also provides information on the variation levels selected for the subsequent sensitivity analysis, the aim of which is to understand the dynamic effects of varying the decision and physical parameters. The rationale behind the selection of the baseline and variation levels is explained below.

Table 3. Levels of the parameters in the baseline scenario and for the sensitivity analysis.

Notation	Definition	Baseline level	Variation levels
Decision (control) parameters			
$\boldsymbol{k_s}$	controller of the serviceable inventory	1	0.618 ($1/\varphi$); 0.382 ($1/\varphi^2$)
$\boldsymbol{f_s}$	safety factor of the serviceable inventory	1	0.5; 2
$\boldsymbol{T_a}$	time constant of the SES demand forecasts	4 ($\alpha \approx 0.2$)	2.333 ($\alpha \approx 0.3$); 9 ($\alpha \approx 0.1$)
$\boldsymbol{k_r}$	controller of the recoverable inventory	1	0.618 ($1/\varphi$); 0.382 ($1/\varphi^2$)
$\boldsymbol{f_r}$	safety factor of the recoverable inventory	1	0.5; 2
$\boldsymbol{T_{an}}$	time constant of the SES net demand forecasts	4 ($\alpha \approx 0.2$)	2.333 ($\alpha \approx 0.3$); 9 ($\alpha \approx 0.1$)
Physical (uncontrollable) parameters			
$\boldsymbol{T_{rs}}$	remanufacturing (and testing) lead time	2	1; 4
$\boldsymbol{T_d}$	delivery lead time	2	1; 4
$\boldsymbol{T_{ut}}$	return lead time (includes use and transportation)	27	8; 64
$\boldsymbol{\beta}$	remanufacturable return proportion	40%	10%; 70%
$\boldsymbol{\mu}$	remanufacturing yield proportion	93.30% (3σ)	69.15% (2σ); 99.38% (4σ)

For the inventory controllers, we use $\{k_s, k_r\} = 1$ in the baseline scenario. That is, we define traditional OUT policies in both inventories. Later, we will explore the impact of the controllers by reducing their tuning to $\{k_s, k_r\} = 1/\varphi$, where φ is the golden ratio. This adjustment has been found to provide a 'good' trade-off between order and inventory variabilities in traditional supply chains; see Disney et al. (2004). Also, we explore $\{k_s, k_r\} = 1/\varphi^2$ to see if further reducing the tuning of the inventory controller can result in additional benefits for pure remanufacturers.

To prevent stock-outs and ensure sufficient stock in both inventories, we first set a safety stock level that equals the forecasted demand, i.e., $\{f_s, f_r\} = 1$. Variations are explored with $\{f_s, f_r\} = 0.5$ and $\{f_s, f_r\} = 2$ to assess the effects of this parameter. In addition, we first use an SES constant of $\alpha \approx 0.2$, i.e., $\{T_a, T_{an}\} = 4$, which is within the typically recommended interval for α (Teunter et al., 2011). Later, we investigate $\alpha \approx 0.1$ ($\{T_a, T_{an}\} = 9$) and $\alpha \approx 0.3$ ($\{T_a, T_{an}\} = 2.333$) to analyse how different configurations in the SES forecasts affect the system performance.

For the remanufacturing lead time, the baseline scenario uses $T_{rs} = 2$, in line with prior studies (e.g., Cannella et al., 2021). Also, we use $T_d = 2$ for the delivery lead time. Later, the sensitivity analysis explores the impact of halving and doubling each, i.e., $\{T_{rs}, T_d\} = 1$ and $\{T_{rs}, T_d\} = 4$. This approach allows us to explore different scenarios (that is, remanufacturing lead times being higher, equal, or lower than delivery lead times). Given that return lead times are typically much higher than any other in closed-loop supply chains (e.g., Tang and Naim, 2004), we use $T_{ut} = 3^3 = 27$ in the baseline scenario, with $T_{ut} = 2^3 = 8$ and $T_{ut} = 4^3 = 64$ as the variation levels.

For the remanufacturable return proportion, we use $\beta = \{10\%, 40\%, 70\%\}$, where $\beta = 40\%$ characterises the baseline scenario. This defines three different levels of circularity in the closed-loop supply chain (i.e., low, moderate, and high). Last, with regards to the remanufacturing yield proportion, we use $\mu = \{69.15\%, 93.30\%, 99.38\%\}$, corresponding respectively to 2σ, 3σ, and 4σ levels on the widely-used Six Sigma scale (e.g., Adams et al., 2007). From the baseline level $\mu = 93.30\%$, we aim to examine the impact of reducing or increasing quality in the production process.

In the following subsections, we examine how all system parameters influence the four variables that determine the operational performance of the closed-loop supply chain. Appendix C presents the dynamics of the baseline scenario, which serves as the benchmark for the analysis.

Aligned with prior discussions, the upper echelon of the block diagram, related to the control of the serviceable stock, features an APVIOBPCS echelon that integrates the remanufacturing yield proportion. Consequently, both remanufacturing orders (Section 5.1) and serviceable inventory (Section 5.2) are contingent solely upon those parameters related to this subsystem, namely $\{k_s, f_s, T_a, T_{rs}, \mu\}$. While many of these effects are documented in the literature, we revisit them briefly to offer a complete depiction of the pure remanufacturing system's dynamics. In contrast, the lower echelon, controlling the recoverable inventory, is also notably affected by the parameters associated with the serviceable stock. For this reason, the behaviour of purchase orders (Section 5.3) and recoverable inventory (Section 5.4) is impacted by all 11 parameters. This is where our study contributes novelty, elucidating previously unexplored effects that enhance understanding of the closed-loop supply chain dynamics and, in turn, inform the management of pure remanufacturing systems.

5.1. Remanufacturing orders

Figure 6 shows how modifying the control and physical parameters of the serviceable inventory subsystem, $\{k_s, f_s, T_a, T_{rs}, \mu\}$, affects the unit-step response of remanufacturing orders. In each plot, the *y*-axis represents the orders, while the *x*-axis represents time periods, *t*, where a unit corresponds to one inventory review interval (e.g., a day or a week) under the POUT policy.

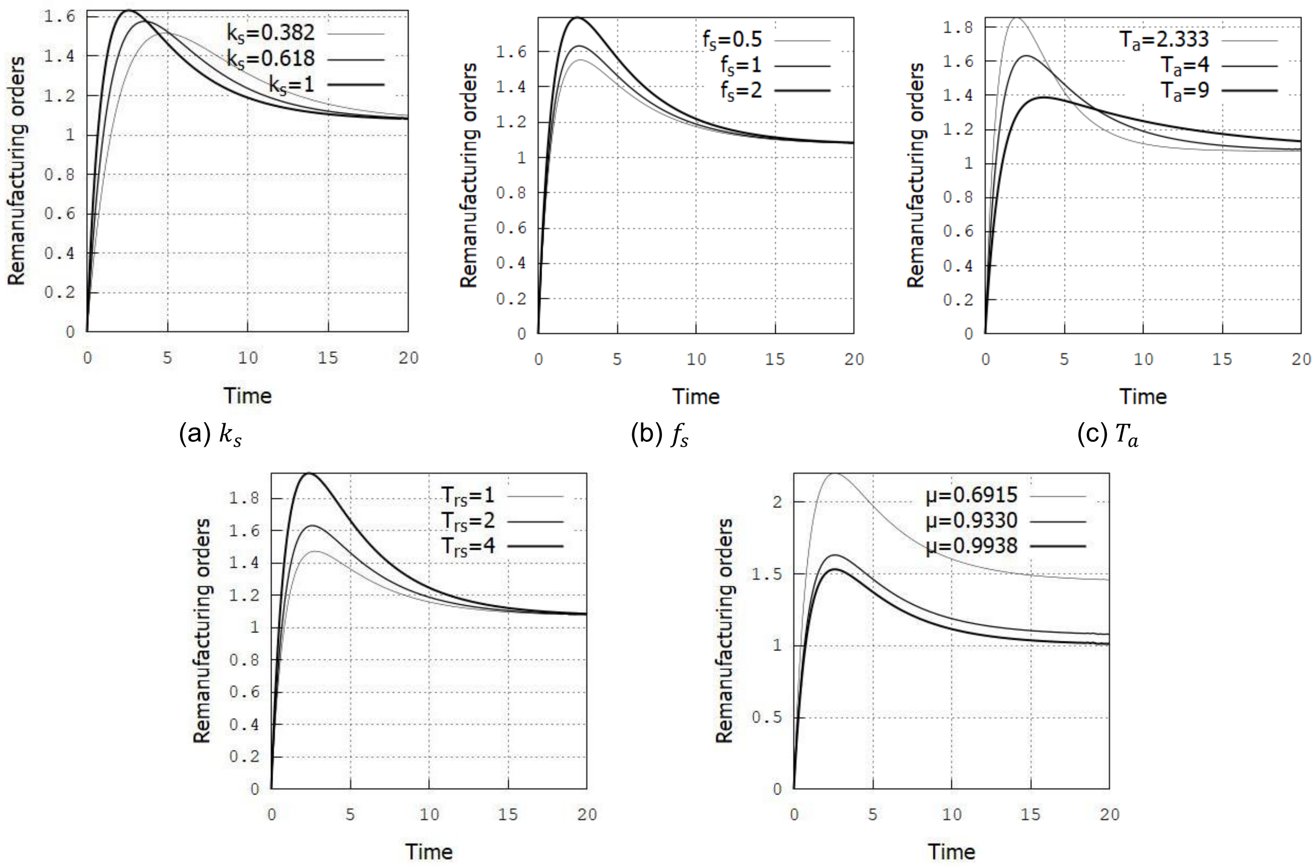
The three control parameters have different effects on the behaviour of the remanufacturing orders. First, decreasing k_s reduces the peak of the response and increases the settling time (Fig.

6(a)), which refers to the time required for the output (orders) to stabilise after the unit change in demand. Second, decreasing f_s reduces the peak but does not affect considerably the settling time (Fig. 6(b)). Third, increasing T_a reduces the peak, but it increases the settling time (Fig. 6(c)).

Therefore, implementing the POUT policy in the management of the serviceable inventory is an effective strategy to reduce the bullwhip effect in pure remanufacturing systems, as it is in traditional supply chains (e.g., Disney and Lambrecht, 2008) as well as in other types of closed-loop supply chains (e.g., Cannella et al., 2021). Also, reducing the safety factor (f_s) and lowering the value of the SES constant (i.e., increasing T_a) contribute to smoothing the dynamics of (pure) remanufacturing orders. These findings are aligned with well-known knowledge for traditional and other closed-loop supply chains (e.g., Chen et al., 2000; Wang et al., 2012).

Regarding the physical parameters, decreasing T_{rs} smooths the dynamics of these orders, as it reduces the overshoot, i.e., the initial peak in the output that exceeds the steady-state value (Fig. 6(d)); it thus mitigates bullwhip. Meanwhile, μ has a higher impact on the steady state, discussed in Section 4, than on the transient behaviour of the orders (Fig. 6(e)), while the absolute size of the peak, compared to the final value, grows as μ reduces.

The observed effects of lead time and quality yield are also in line with those of previous studies conducted on traditional supply chains (e.g., Wang and Disney, 2016). However, these findings establish a clear distinction between pure remanufacturing systems and hybrid ones, in which it is well known that longer remanufacturing lead times can enhance supply chain performance, namely, the lead-time paradox (among others, Hosoda and Disney, 2018).



(a) k_s (b) f_s (c) T_a

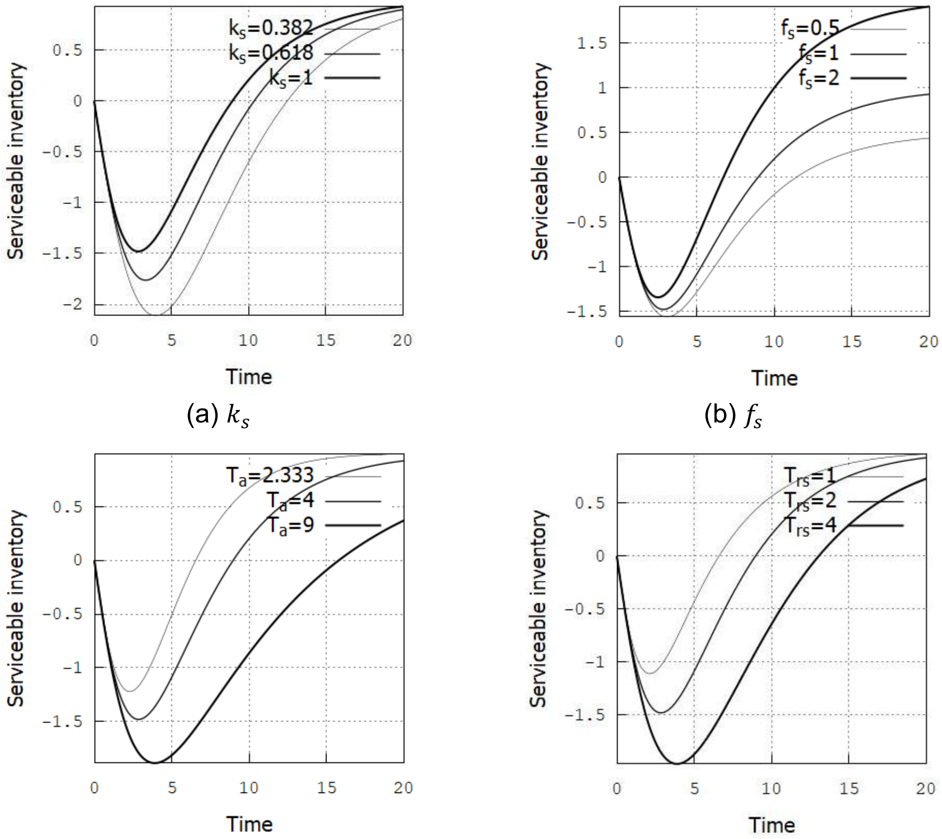
(d) T_{rs} (e) μ

Figure 6: Sensitivity analysis of the remanufacturing orders.

5.2. Serviceable inventory

In our control system, quality losses are perfectly compensated by the ordering rule. Thus, the serviceable inventory response to a unit step in demand does not depend on μ (see $SI(s)/D(s)$ in Appendix A). Figure 7 shows this response for different levels of $\{k_s, f_s, T_a, T_{rs}\}$.

First, we observe that reducing k_s tends to increase the size of the (potential) stock-out following a sudden change in demand, which is illustrated by the trough of the inventory response (Fig. 7(a)). In these conditions, the key trade-off of POUT policies emerges (e.g., Cannella et al., 2021): decreasing k_s has a positive impact on orders but may have a negative effect on inventories.



(a) k_s (b) f_s

(c) T_a (d) T_{rs}

Figure 7: Sensitivity analysis of the serviceable inventory.

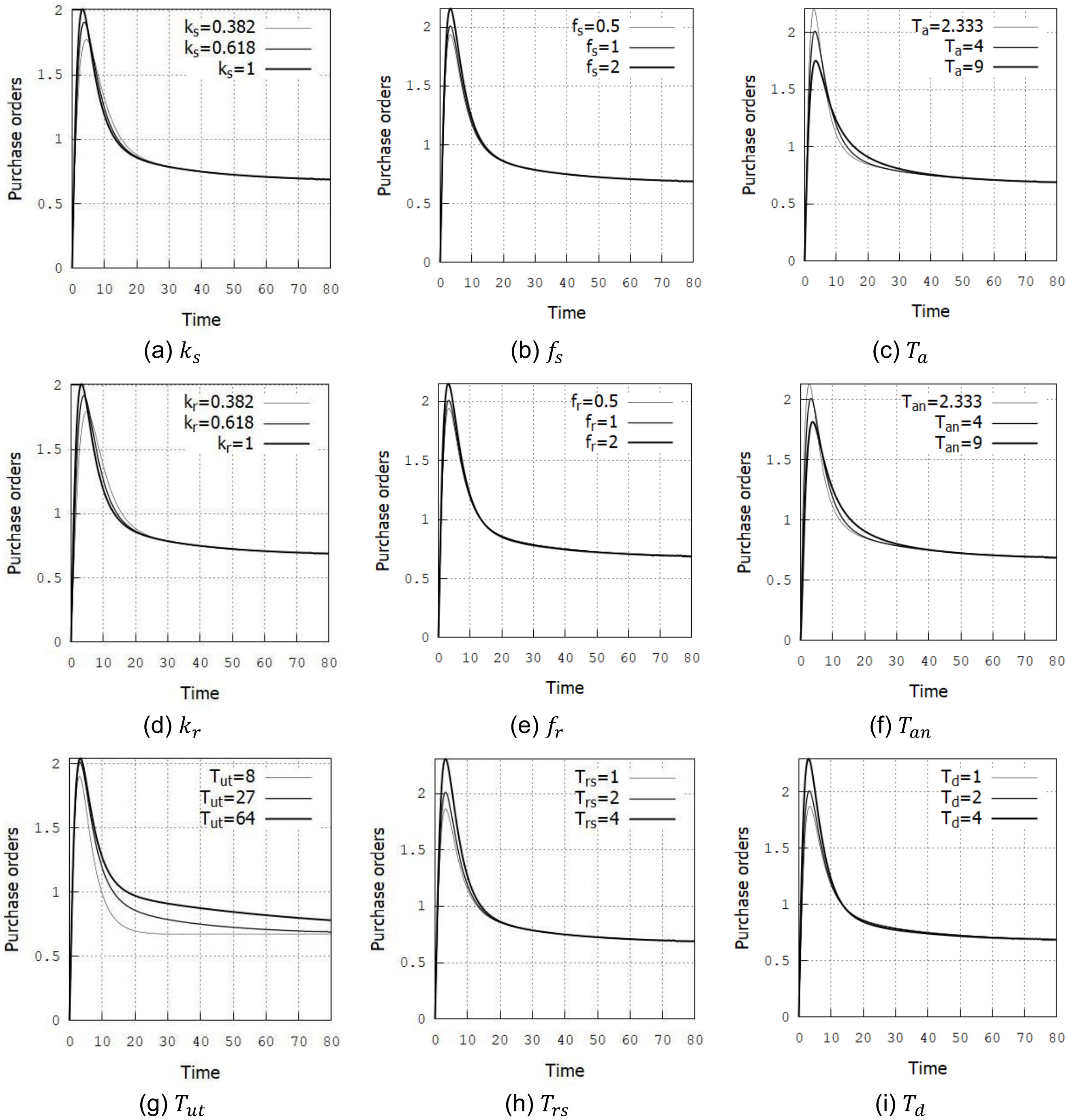
In line with the static analysis, f_s strongly affects the final value of the response: higher values of f_s result in higher serviceable stock levels (Fig. 7(b)). This positively impacts the service level at the expense of higher holding costs. As higher values of f_s also lead to increased inventory volatility (comparing the minimum of the response to its final value), the potential benefits of increasing f_s may sometimes be insufficient to offset its drawbacks. The observed impact of the safety factor is consistent with its effect in traditional supply chains (e.g., Disney and Towill, 2005).

Additionally, an increase in T_a notably prolongs the settling time of the response and amplifies the magnitude of the trough (Fig. 7(c)). These observations align with the existing literature on traditional supply chains (e.g., Hussain et al., 2012). Nonetheless, it is important to highlight that the impact of T_a on inventory performance is particularly complex, as it is significantly influenced by the regulation of the inventory controller, k_s, as shown by Disney and Towill (2003a).

Finally, we see that decreasing T_{rs} benefits the dynamics of the serviceable inventory (Fig. 7(d)). By reducing remanufacturing lead times, both the stability of orders and the efficiency of inventories thus improve. Under these circumstances, there are no indications in this pure remanufacturing setting of the lead-time paradox widely observed in the closed-loop supply chain literature for hybrid manufacturing-remanufacturing systems (e.g., Tang and Naim, 2004).

5.3. Purchase orders

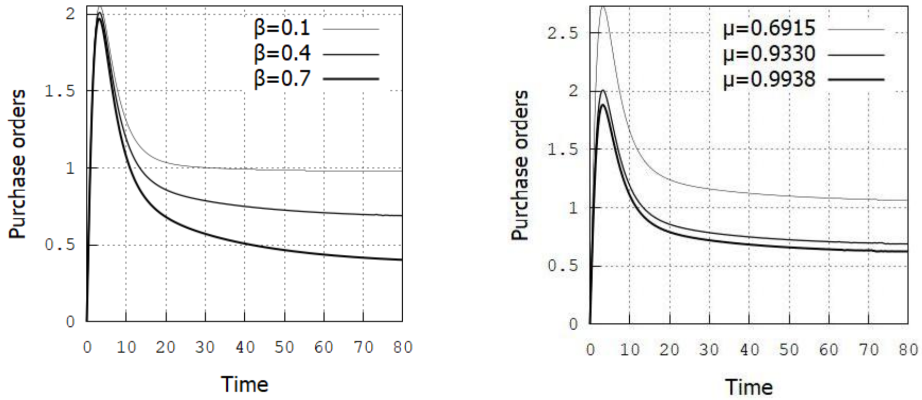
As outlined earlier, the behaviour of purchase orders depends on all 11 system parameters. Figure 8 shows their unit-step response when modifying the decision and physical parameters.



(a) k_s (b) f_s (c) T_a

(d) k_r (e) f_r (f) T_{an}

(g) T_{ut} (h) T_{rs} (i) T_d



(j) β (k) μ

Figure 8: Sensitivity analysis of the purchase orders.

First, Figures 8(a) and 8(d) indicate that decreasing both k_s and k_r contributes to a reduction in the overshoot of the response, with minimal impact on the settling time. Under these circumstances, it is interesting to highlight that not only implementing the controller at the recoverable inventory (k_r) facilitates a reduction of the variability of purchase orders, but also the serviceable inventory's controller (k_s) smooths the dynamics of these orders.

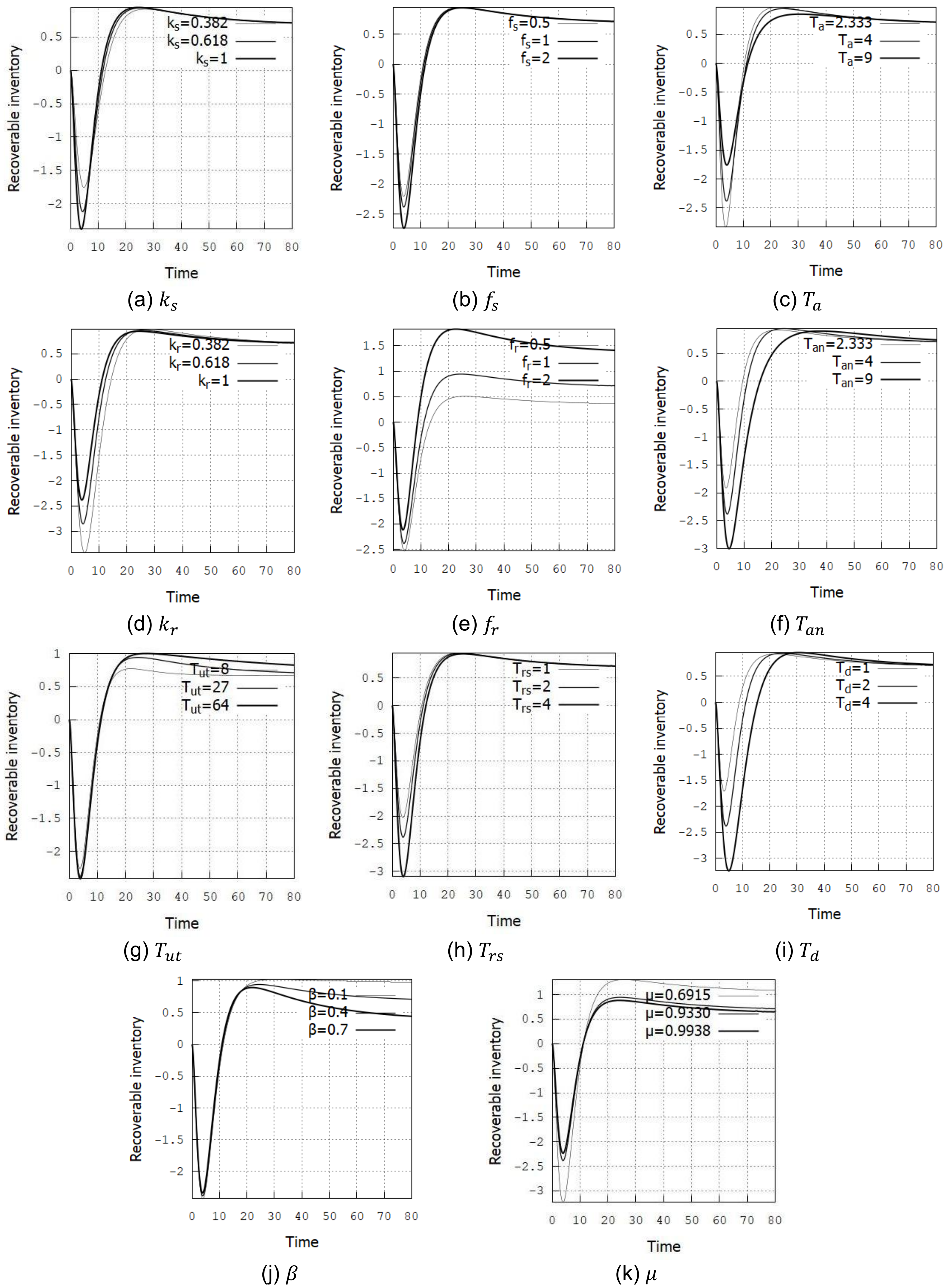
Second, Figures 8(b) and 8(e) reveal that reducing f_s and f_r allows the pure remanufacturer to reduce the variability of purchase orders. It is noteworthy that, while (in linear supply chains) safety stocks do not affect order variability when they are fixed (e.g., John et al., 1994), increasing them is problematic from a bullwhip perspective if they are proportional to forecasts. Last, Figures 8(c) and 8(f) indicate that increasing T_a and T_{an} (or, equivalently, reducing the SES constants) smooth the response of these orders, thus allowing for a reduction in their variability. In this sense, we highlight that strategically tuning the control parameters of the serviceable inventory (k_s, f_s, T_a) is particularly important due to their impact on both the remanufacturing and purchase orders.

Regarding the physical parameters, Figures 8(g), 8(h), and 8(i) show that reducing the three lead times (T_{ut}, T_{rs}, T_d) smooths the dynamics of purchase orders. However, it is crucial to emphasize the need for a well-informed reduction of lead times. For instance, reducing T_{ut} from 64 to 27 could be very costly (if feasible) while yielding only minor advantages (the peak marginally decreases). In other cases, even small reductions can lead to notable enhancements. In this sense, effective decision-making on lead-time reductions in pure remanufacturing systems necessitates a comprehensive understanding of the dominant poles of the supply chain.

Last, Figures 8(j) and 8(k) consider β and μ, which, as discussed earlier, have a substantial influence on the response's final value. We observe that, as the remanufacturable return proportion (β) increases, the maximum value of the response slightly decreases, suggesting that enhancing the circularity of the supply chain may help to mitigate bullwhip. Furthermore, we find that increasing the remanufacturing yield proportion (μ) is a valuable solution to increase supply chain stability by reducing the variability of remanufacturing orders and purchase orders.

5.4. Recoverable inventory

Finally, Figure 9 shows the unit-step responses of the recoverable inventory for varying levels of the system parameters.



(a) k_s (b) f_s (c) T_a

(d) k_r (e) f_r (f) T_{an}

(g) T_{ut} (h) T_{rs} (i) T_d

(j) β (k) μ

Figure 9: Sensitivity analysis of the recoverable inventory.

Figure 9(d) illustrates that the trough of the unit-step response becomes more prominent as k_r reduces. In other words, implementing the proportional controller in the recoverable inventory may exacerbate its dynamics, the same effect discussed for the serviceable stock in Section 5.2. In contrast, Figure 9(a) reveals that reducing k_s results in a smaller trough. This implies that using the POUT replenishment rule in the serviceable inventory facilitates effective management of the recoverable stock. Both controllers then exhibit opposite effects on this inventory.

Figures 9(b) and 9(e) indicate that decreasing both safety factors, f_s and f_r, improves the dynamics of the recoverable stock. However, the reason behind the reduction of variability is different. The former is due to the transient behaviour: the through reduces as f_s decreases; the latter is provoked by the steady state: the final value becomes less variable as f_r decreases. Meanwhile, Figure 9(c) show that increasing T_a (i.e., reducing the SES constant of the demand forecasts) smooths the response of the recoverable inventory. In contrast, Figure 9(f) show that reducing T_{an} (i.e., reducing the SES constant of the net demand forecasts) would have the opposite effect. Once more, it is evident that the parameters of the closed-loop supply chain can influence both inventories differently. Comprehending the overall effects of tuning each parameter then becomes indispensable for optimising the performance of the pure remanufacturing system.

Figures 9(g), 9(h), and 9(i) demonstrate that reducing the three lead times also plays a role in smoothing the response of this inventory. However, the magnitude of their impacts varies considerably. For instance, decreasing T_d results in a greater reduction in the trough compared to cutting T_{rs} or T_{ut}. However, it is noteworthy to emphasize that, once again, the lead time T_{ut} stands out as the parameter with the most significant influence on the settling time. Overall, a well-informed decision-making on lead-time reduction would again make a difference.

In line with the static analysis, Figure 9(j) illustrates that lower volumes of returns (i.e., smaller values of β) provoke a higher steady-state value in the response of the serviceable stock. Additionally, we observe that both the maximum and minimum response values deviate more from zero as β decreases, while the settling time shortens with decreasing β. Finally, Figure 9(k) shows that improving the quality of the remanufacturing process (that is, increasing μ) also enables a more effective management of the recoverable inventory. Indeed, we observe that this parameter exerts a substantial influence on the variability of the unit-step response.

6. Managerial insights

The findings presented and discussed in the preceding section provide evidence of the value of inventory controllers in enhancing the dynamics of the closed-loop supply chain under study. Specifically, the results demonstrate that employing POUT policies in both inventories can significantly outperform the traditional OUT policy in terms of overall supply chain performance. However, regulating these controllers is complex, as their performance depends on various internal decisions, e.g., the forecasting method and its adjustment, and external factors, e.g., lead times and return rates. This section draws managerial insights from the analysis developed in the preceding sections to inform decision-making in the configuration and operation of pure remanufacturing systems. Our insights thus provide prescriptive guidance on how decision-makers can enhance the performance of these systems through the strategic and coordinated setting of inventory controllers under the POUT policy.

To this end, and given the analytical complexity of the transfer functions involved, we conduct a series of numerical analyses through simulations. These simulations are based on the discrete-time, difference-equation version of the developed control-theoretic model. We assume that demands conform to independent and identically distributed (i.i.d.) random variables following normal distributions with a mean of $\mu = 100$ and a standard deviation of $\sigma = 30$, a level of variability consistent with typical real-world conditions, as reported by Dejonckheere et al. (2003). Each simulation runs for 2,000 periods to mitigate the impact of randomness and ensure the generation of robust insights. To facilitate the visualisation of the curves and the understanding of the key effects, we use the same demand seed for all simulations.

6.1. Performance metrics

To assess the operational performance of the supply chain across the scenarios examined, we utilise the key performance indicators outlined in Table 4. These metrics are tailored variants, adapted to our closed-loop context, of the well-known Bullwhip (Bw) and Net Stock Amplification ($NSAmp$) ratios, which are extensively employed in the supply chain literature due to their well-documented impact on cost performance in many industrial settings (Disney and Lambrecht, 2008). The first two metrics are variants of the Bw ratio, which typically compares the variance of orders to the variance of demand. Specifically, RBw focuses on remanufacturing orders, while PBw considers purchase orders. A Bw value greater than 1 indicates the existence of bullwhip effect in the supply chain. The last two metrics are variants of the $NSAmp$ ratio, which typically compares the variance of net stocks to the variance of demand. $NSSAmp$ focuses on the serviceable inventory and $NRSAmp$ focuses on the recoverable inventory. High $NSAmp$ values suggest low fill rates and/or high inventory investments. To support interpretation, Table 4 also incorporates the primary economic implications associated with each metric.

Table 4. Key operational performance indicators in the simulations.

Operational Metric	**Mathematical definition**	**Economic implications**
Remanufacturing Bullwhip Ratio	$RBw = \frac{\mathbb{V}[ro_t]}{\mathbb{V}[d_t]}$	▪ Capacity-related remanufacturing costs
Purchase Bullwhip Ratio	$PBw = \frac{\mathbb{V}[po_t]}{\mathbb{V}[d_t]}$	▪ Variability-related transportation costs ▪ Production costs of upstream members
Net Serviceable Stock Amplification	$NSSAmp = \frac{\mathbb{V}[si_t]}{\mathbb{V}[d_t]}$	▪ Shortage costs (backlog or lost sales) ▪ Inventory holding costs
Net Recoverable Stock Amplification	$NRSAmp = \frac{\mathbb{V}[ri_t]}{\mathbb{V}[d_t]}$	▪ Shortage costs (typically, backlog costs) ▪ Inventory holding costs

Notes: $\mathbb{V}[\cdot]$ is the variance operator. x_t is the discrete-time version of $X(s)$.

Each operational indicator defined in Table 4 captures a specific and distinct facet of performance within the pure remanufacturing system, with each numerator addressing a different variable. In this context, and consistent with the dynamic analysis, conflicts of interest may arise: a decision to adjust a control parameter may positively affect certain metrics, while adversely impacting others. For this reason, in conjunction with the four 'local' metrics, we introduce the 'global' metric

J, formulated as a weighted sum of the four ratios, as defined in Equation (9). The adoption of such a metric aligns with common practices in the supply chain dynamics literature (e.g., Disney et al., 2004; Ponte et al., 2020; Cannella et al., 2021), enabling researchers and practitioners to evaluate the trade-offs that may emerge.

$$J = \omega_1 RBw + \omega_2 PBw + \omega_3 NSSAmp + \omega_4 NRSAmp \quad (9)$$

Interestingly, J serves as an indicator of overall (closed-loop) supply chain costs when the weights assigned to each local metric, in our case, $\omega_1, \omega_2, \omega_3, \omega_4$, such that $\sum_i \omega_i = 1$, are adjusted in accordance with the cost structure of the company (Disney and Lambrecht, 2008).

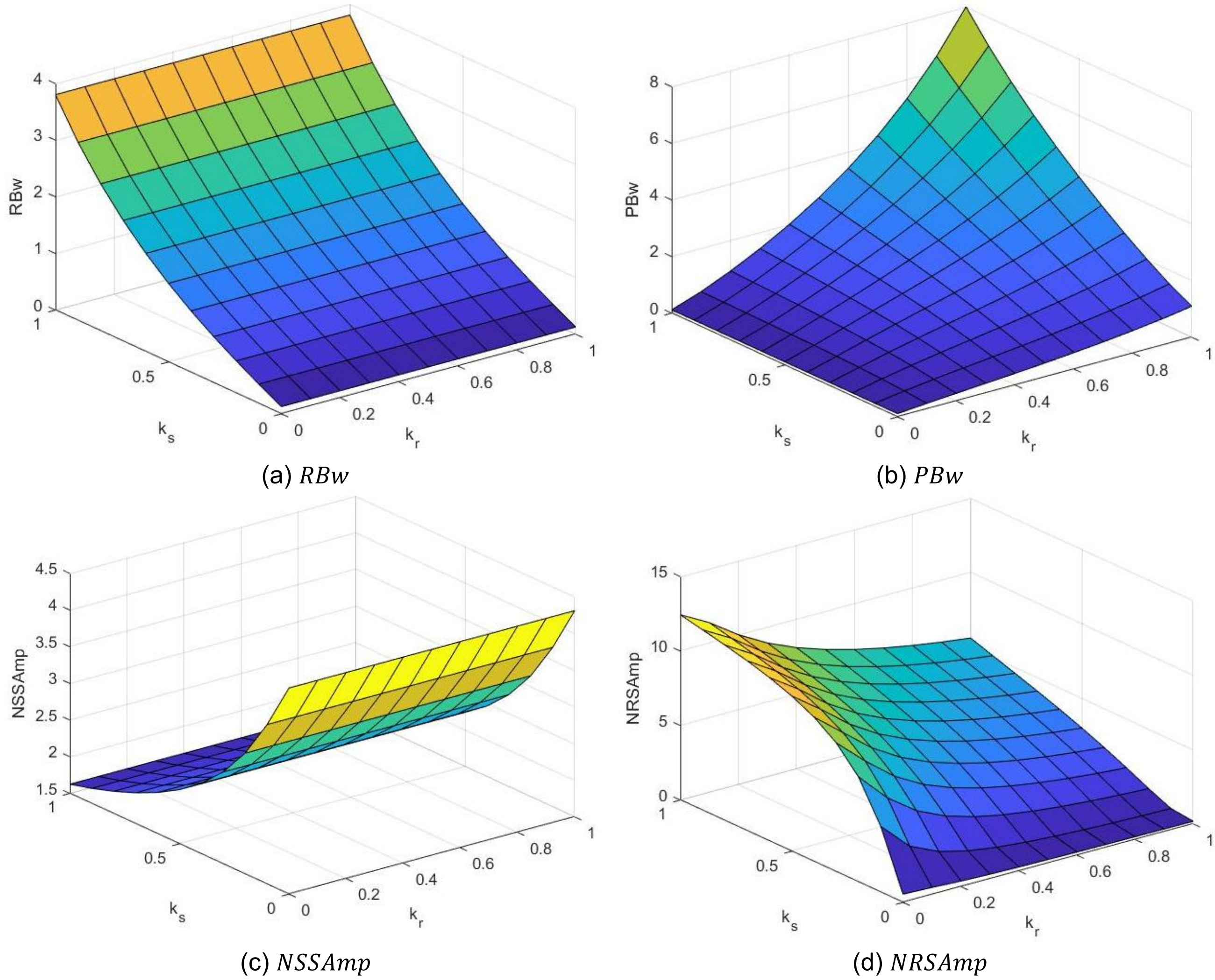
While significant differences may arise across industrial settings, it is reasonable to posit that the variability of the serviceable inventory is particularly costly in most instances. This is due to its direct exposure to customer demand; maintaining a low $NSSAmp$ is critical for both enhancing customer satisfaction and reducing supply chain (inventory) costs. Hence, for simplicity, in the subsequent analyses, we assume $\omega_1 = \omega_2 = \omega_4$ and $\omega_3 = \lambda\omega_1$, with $\lambda > 1$. Taking this into consideration, we explore two different situations: $\lambda = 2$ (i.e., $\omega_1 = \omega_2 = \omega_4 = 0.2$; $\omega_3 = 0.4$) and $\lambda = 7$ (i.e., $\omega_1 = \omega_2 = \omega_4 = 0.1$; $\omega_3 = 0.7$). The former (low λ) may typify a (re)manufacturing organisation that focuses primarily on production efficiency, such as pursuing a cost leadership strategy, while also carefully managing inventory to reduce holding costs and prevent shortages. In contrast, the latter (high λ) would characterise a company with a critical need for near-complete product availability, as seen in industries like pharmaceuticals or defence, or those needing to minimise stock variability, due to perishability or obsolescence concerns.

6.2. Baseline scenario

Figure 10 represents how the inventory controllers, and their combination, impact the four operational metrics under study (RBw, PBw, $NSSAmp$, $NRSAmp$). In this initial analysis, the other parameters are set to the values defined previously for the baseline scenario (see Table 3).

Inspection of the graphs reveal that, regardless of k_r, decreasing k_s always leads to reduced RBw, PBw, and $NRSAmp$, while $NSSAmp$ considerably increases. Implementing the controller in the serviceable stock thus proves to be an effective solution not only for increasing the stability of remanufacturing orders but also impacts positively the stability of the recoverable inventory and the purchase orders it generates. In contrast, decreasing k_r considerably reduces PBw and increases $NRSAmp$, with no significant impact on the metrics related to the serviceable inventory ($RBw, NSSAmp$). These findings align with the insights derived from the dynamic analysis.

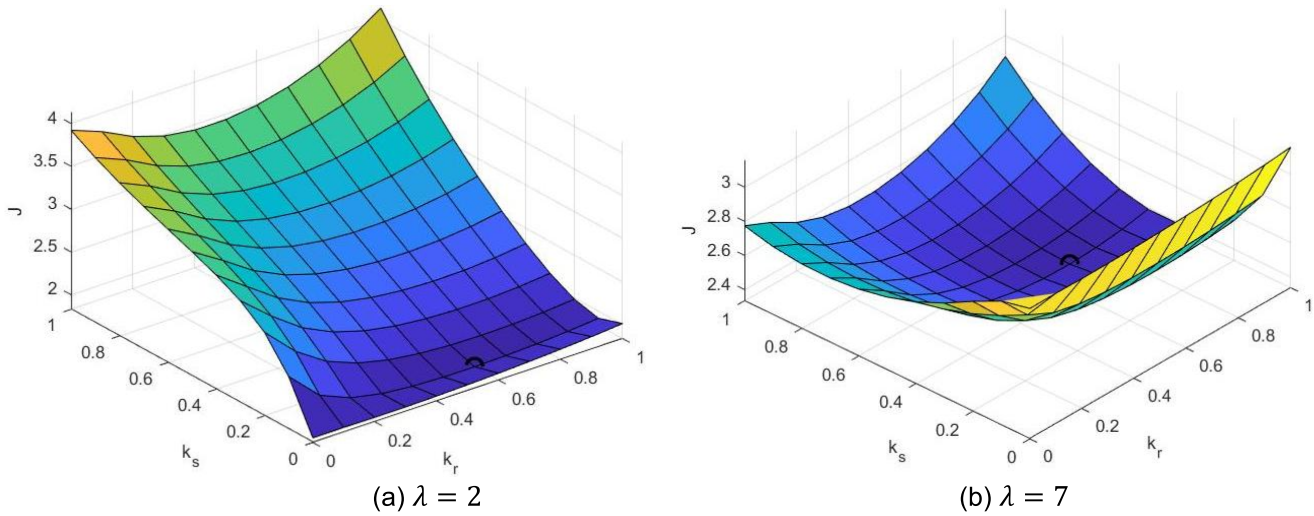
It is interesting to note that the impact of k_r on PBw and $NRSAmp$ is significantly influenced by k_s, and vice versa. In this sense, when k_s is set to low values, reducing k_r leads to a moderate decrease in PBw but a slight increase in $NRSAmp$. Conversely, if k_s is tuned to higher values, a decrease in k_r results in a substantial reduction in PBw, as well as a significant increase in $NRSAmp$. These interdependencies are crucial considerations for professionals managing pure remanufacturing systems and their closed-loop supply chains. To facilitate the trade-off analysis, Figure 11 illustrates the global metric, J, as a function of both controllers in both scenarios considered ($\lambda = 2$; $\lambda = 7$). The minimum value of J in both graphs is indicated by a black circle.



(a) RBw (b) PBw

(c) $NSSAmp$ (d) $NRSAmp$

Figure 10: Impact of the inventory controllers on the operational metrics.



(a) $\lambda = 2$ (b) $\lambda = 7$

Figure 11: Impact of the inventory controllers on J.

We observe that, for $\lambda = 2$, reducing k_s consistently yields benefits, except for very low values of this parameter ($k_s < 0.1$), indicating that the operational advantages (RBw, PBw, $NRSAmp$) generally outweigh the disadvantages ($NSSAmp$). In contrast, for $\lambda = 7$, low regulations of k_s

should be avoided due to the increase in $NSSAmp$, which is particularly costly in this situation. In this case, intermediate values of k_s provide the best, most cost-effective solutions. From the perspective of k_r, regardless of λ, optimal regulation is found at intermediate levels. With this parameter only affecting PBw and $NRSAmp$, having opposite effects on each, intermediate values demonstrate superior performance.

When analysing both controllers jointly, the best solution is achieved with $k_s = 0.1$ and $k_r = 0.6$ for $\lambda = 2$, and $k_s = 0.5$ and $k_r = 0.7$ for $\lambda = 7$. In both scenarios, the (cost) performance obtained is substantially better compared to using two OUT policies, highlighting substantial opportunities for cost reduction in the closed-loop supply chains of pure remanufacturers.

It is then evident that, despite their widespread use in industry, traditional OUT policies can be greatly improved upon in pure remanufacturing systems, akin to conventional supply chains. This is attributable to their extensive contribution to bullwhip amplification, resulting in several inefficiencies across the closed-loop supply chain. Considering this perspective, and supported by our results, we advocate for the adoption of POUT policies to enhance the dynamics and overall performance of pure remanufacturing systems.

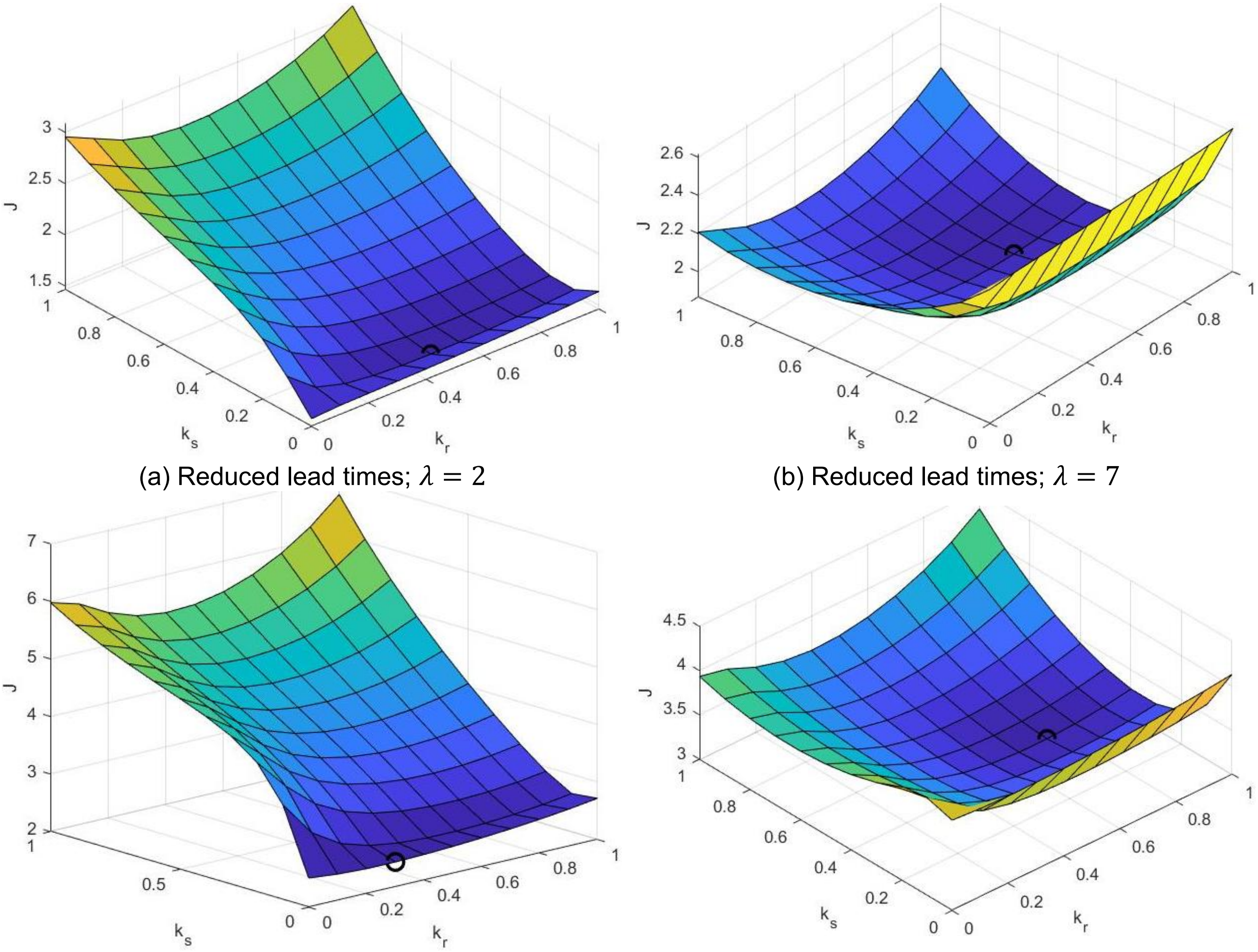
The incorporation of the POUT controller into the management of both recoverable and serviceable inventories can facilitate (much) smoother execution of closed-loop supply chain operations, yielding substantial economic savings. However, potential challenges may also arise, particularly in the serviceable inventory, where controllers can negatively impact service levels and/or increase stock levels. Therefore, appropriate fine-tuning of the inventory controllers within the POUT policy becomes crucial for optimising the operational performance of the system.

In the following subsections, we conduct sensitivity analyses to investigate how lead times and return and yield proportions impact the performance of the closed-loop supply chain, and how these parameters should affect the fine-tuning of the inventory controllers.

6.3. Lead-time effects

Supply chain managers generally recognise the negative impact of lead times on performance, but struggle to quantify this impact accurately, often underestimating the overall costs associated with lead times (de Treville et al., 2014). As a result, they find it particularly difficult to incorporate lead-time considerations into decision-making processes; for example, it is mostly unclear how lead times should affect the adjustment of inventory controllers. Intuition-based choices often lead to poor results; well-informed decisions, grounded in a comprehensive understanding of supply chain dynamics, can yield substantial improvements. The lead-time challenge is even more pronounced in closed-loop supply chains, which are characterised by their lead-time complexity. The effective integration of multiple processes, with different lead times, in the two distinct material flows, upstream and downstream, poses additional decision-making challenges.

In our closed-loop supply chain for a pure remanufacturer, we consider three different lead times: remanufacturing (T_{rs}), delivery (T_d), and return (T_{ut}). In the baseline scenario (explored in Section 6.2), we set $T_{rs} = 2$, $T_d = 2$, $T_{ut} = 27$. We now consider two additional scenarios. First, the 'reduced lead times' scenario is characterised by $T_{rs} = 1$, $T_d = 1$, $T_{ut} = 18$, representing real-world situations where lead times are particularly short, such as in local supply chains and/or highly optimised production processes. Second, the 'prolonged lead times' scenario, represented by $T_{rs} = 4,\ T_d = 4,\ T_{ut} = 36$, reflects real-world scenarios where lead times are considerably longer, such as in global supply chains and/or complex production processes. In all scenarios, T_{ut} remains the dominant lead time in order to maintain the realistic assumption that consumption and return processes typically take longer than remanufacturing and delivery operations. For the two new lead-time scenarios, Figure 12 illustrates the combined impact of both inventory controllers on the performance metric J in the two previously defined cost settings ($\lambda = 2$; $\lambda = 7$).



(a) Reduced lead times; $\lambda = 2$ (b) Reduced lead times; $\lambda = 7$

(c) Prolonged lead times; $\lambda = 2$ (d) Prolonged lead times; $\lambda = 7$

Figure 12: Impact of the inventory controllers on J in different lead-time scenarios.

A simple comparison of both lead-time scenarios in Figure 12 (and Figure 11, representing the baseline lead-time scenario) highlights the enormous cost impact of lead times in the closed-loop supply chain. The values on the *z*-axis of these figures clearly illustrate the significant economic advantages of reducing lead times. For instance, for $\lambda = 2, J = 3.087$ when OUT policies are used, and $min\{J\} = 1.484$ in the reduced lead-time scenario. Conversely, in the prolonged lead-time scenario, the use of OUT policies leads to $J = 6.679$, and $min\{J\} = 2.433$, indicating a substantial cost reduction due to shorter lead times regardless of the controllers' adjustments. Consistent with the previous dynamic analysis, the cost reduction stems from lead-time reduction positively impacting the four operational metrics under consideration (RBw, PBw, $NSSAmp$, $NRSAmp$).

Under these circumstances, reducing lead times stands out as a pivotal driver of closed-loop supply chain improvement for pure remanufacturers. This reduction increases the stability of all operations and enhances the efficiency of both inventories. Importantly, we highlight that this also applies to remanufacturing lead times, which represents a meaningful distinction from hybrid manufacturing-remanufacturing systems, where increasing remanufacturing lead times can sometimes be beneficial (e.g. Hosoda and Disney, 2018), as previously discussed.

Having highlighted the importance of reducing lead times, we emphasise that professionals must weigh the expected benefits against the costs associated with lead-time reduction, which can exhibit significant variability. From a managerial perspective, not all lead times are equally costly to reduce, nor do they affect performance to the same extent, as highlighted by our dynamic analysis. In this sense, for well-informed decision-making, it becomes imperative for managers of pure remanufacturing systems to identify how lead times are limiting the performance of their supply chains, and to accurately estimate the costs involved in lead-time reduction processes.

Examining how the lead times of the closed-loop supply chain should influence the adjustment of both inventory controllers (again, we note that a black circle indicates the minimum value of J in the graphs), we observe distinct outcomes for $\lambda = 2$ and $\lambda = 7$. For $\lambda = 2$, the lowest J is achieved when $k_s = 0.1$ and $k_r = 0.5$ with reduced lead times, and when $k_s = 0$ and $k_r = 0.3$ with prolonged lead times (compared to the baseline scenario, where $k_s = 0.1$ and $k_r = 0.6$). For $\lambda = 7$, the configuration that yields the lowest J is $k_s = 0.5$ and $k_r = 0.7$ in the scenario with reduced lead times (mirroring the baseline scenario), and $k_s = 0.4$ and $k_r = 0.7$ with prolonged lead times.

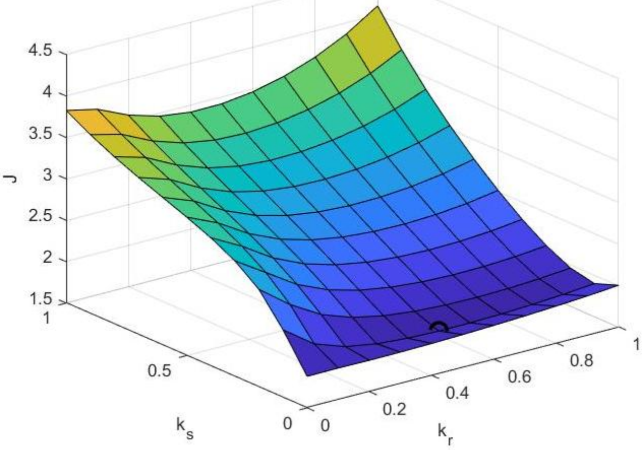
The above analysis highlights that, while lead times are critical factors influencing the performance of closed-loop supply chains, their impact on the optimal tuning of inventory controllers is complex and varies depending on the relative importance of the different sources of variability.

In high-service level settings (industries aiming for very high fill rates), where serviceable inventory variability dominates (i.e., high λ), lead times have a relatively minor influence on the optimal controller configuration. Only slight reductions in k_s may be beneficial as lead times increase, and overall tuning remains relatively stable. Conversely, in highly cost-sensitive settings (industries where reducing operational costs is a top priority), lead times have a more pronounced impact on optimal controller tuning. In these contexts, all sources of variability considerably affect supply chain performance (i.e., low λ), and reducing both k_s and k_r as lead times increase can improve performance by mitigating the adverse effects of longer remanufacturing and delivery lead times. In any case, we emphasise that the coordinated adjustment of controllers helps offset the loss of responsiveness and stability caused by longer remanufacturing and/or delivery lead times.

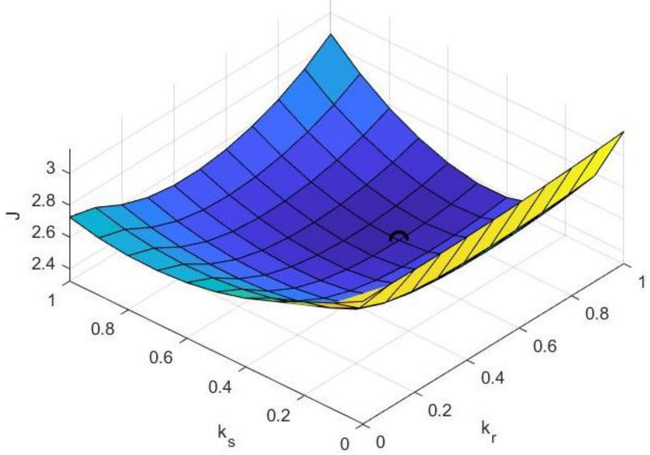
6.4. Return and yield proportion effects

In managing closed-loop supply chains of pure remanufacturers, the remanufacturable return proportion (β) and the remanufacturing yield proportion (μ) are critical determinants of customer satisfaction, operational efficiency, and sustainability. The former (β), indicating the proportion of sold products that return to the closed-loop supply chain and can be remanufactured, directly affects the acquisition costs for the remanufacturer and the overall environmental footprint of the supply chain. Higher values of β increase the availability of recoverable cores, which can lower the economic and environmental costs of supply chain operations. The latter (μ), measuring the percentage of remanufactured products that meet customer requirements, allows organisations to increase efficiency in the remanufacturing process by improving material utilisation, thereby decreasing production costs and waste generation.

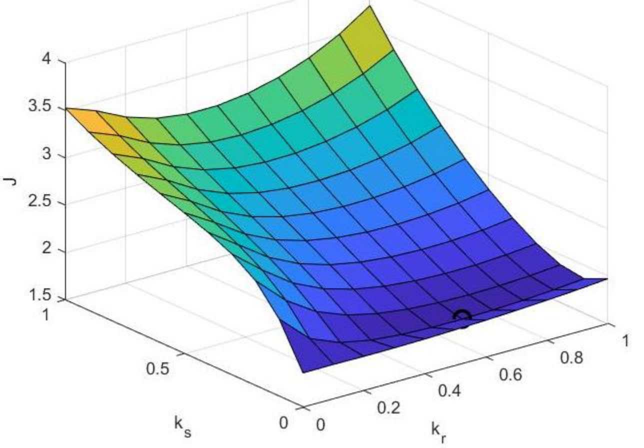
In this subsection, we examine the influence of β and μ on the operational performance of the closed-loop supply chain and the appropriate regulation of the inventory controllers (k_s, k_r). In the baseline scenario, we used $\beta = 40\%$ and $\mu = 93.30\%$ (3σ). We now consider two additional scenarios, each improving one parameter individually. First, a scenario of 'increased circularity', with $\beta = 70\%$. In practice, this may result from economic incentives to return products in good condition or increased customer environmental awareness. Second, we model a 'enhanced quality' scenario, characterised by $\mu = 99.38\%$ (4σ). This may be facilitated by highly accurate inspection of returns, the return of products in better condition, or enhanced remanufacturing operations. Figure 13 shows the relationship between J and both controllers (k_s, k_r) across both cost scenarios ($\lambda = 2$; $\lambda = 7$).



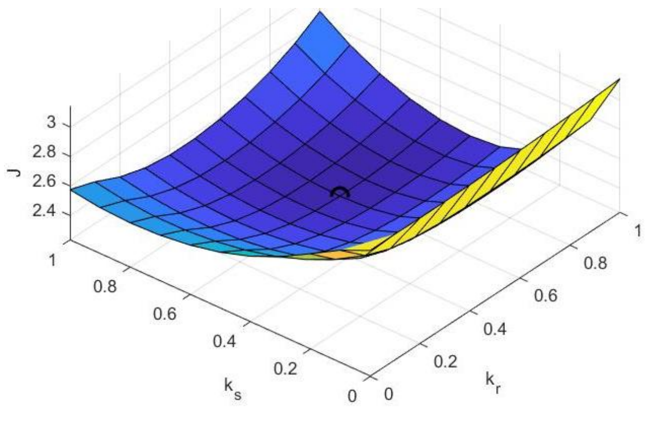
(a) Increased circularity; $\lambda = 2$



(b) Increased circularity; $\lambda = 7$



(c) Enhanced quality; $\lambda = 2$



(d) Enhanced quality; $\lambda = 7$

Figure 13: Impact of the inventory controllers on J in different scenarios of return and yield proportions.

Analysing the 'increased circularity' scenario and comparing it to the baseline setting (discussed in Section 6.2), we observe that increasing β facilitates a very slight reduction in operational costs. Specifically, $min\{J\} = \{1.830, 2.333\}$ for $\lambda = \{2,\ 7\}$, respectively, in the baseline scenario, while they marginally improve to $min\{J\} = \{1.811, 2.320\}$ in the new scenario. These minimal differences can be attributed to the fact that, in line with our dynamic analysis, β does not affect RBw and $NSSAmp$, and has relatively minor effects on PBw and $NRSAmp$. Despite the modest operational improvement, it is essential to emphasise that such an increase would reduce the environmental costs of supply chain operations and likely lower the material acquisition costs for the remanufacturer.

At this juncture, it is important to reiterate that, as discussed in our review of the literature (Section 2), researchers have found the impact of β on the performance of closed-loop supply chains based on hybrid manufacturing-remanufacturing systems to be particularly complex. While an increase in β may considerably enhance performance in certain scenarios, it can lead to undesirable dynamics and substantially reduced performance in others (Goltsos et al., 2019).

In view of our results, we conclude that augmenting the level of circularity does not exert a significant effect on the operational dynamics of pure remanufacturing systems under the conditions examined, including the modelling assumptions (e.g., the presence of a core broker to compensate for shortages in customer returns) and the experimental setting. The limited impact of β on closed-loop supply chain dynamics, compared to hybrid systems, can be partly attributed to the structural characteristics of pure remanufacturing systems, where material flow integration occurs upstream in the recoverable inventory, rather than downstream in the serviceable inventory. As a result, the integration point is more decoupled from customer-facing operations, which attenuates its influence on overall system dynamics, particularly under a POUT inventory control policy, which is known to dampen variability across the supply chain.

Consistently with this analysis, we observe that β has relatively small effects on the optimal configuration of the controllers within the POUT policy. Specifically, for $\lambda = 2$, we obtain $k_s = 0.1$ and $k_r = 0.5$, resulting in a slightly lower adjustment of the controller for the recoverable inventory compared to the baseline scenario ($k_s = 0.1, k_r = 0.6$). For $\lambda = 7$, we should set $k_s = 0.5$ and $k_r = 0.7$, which matches the best solution found for the baseline scenario. Thus, small reductions in k_r as the level of circularity increases may yield positive impacts if λ is low, but no adjustments are necessary for higher values of λ.

We now shift our focus to the ‘enhanced quality’ scenario, which can also be compared to the baseline case (Section 6.2). Here, the graphs illustrate that improving μ leads to a more significant enhancement in operations. When $\lambda = 2$, $min\{J\}$ improves from 1.830 to 1.786, representing a 2.40% reduction. For $\lambda = 7$, $min\{J\}$ improves from 2.333 to 2.234, indicating a decrease of 4.24%. This positive impact stems from μ significantly affecting RBw, PBw, and $NRSAmp$, while $NSSAmp$ remains unaffected. Specifically, in alignment with our dynamic analysis, increasing the remanufacturing yield proportion significantly reduces variability in supply chain operations and improves the stability of the recoverable inventory, offering clear economic benefits.

From this perspective, increasing μ is fundamental for enhancing the efficiency of closed-loop supply chains. When a considerable number of remanufactured cores are rejected during quality testing, not only do production costs escalate due to processing items not meeting customer demand, but it also results in heightened variability in remanufacturing and purchase orders. Furthermore, the variability of the recoverable stock increases. As a result, the costs associated with poor quality dramatically rise. In this sense, our findings emphasise the operational benefits derived from increasing the quality of remanufacturing operations as well as implementing precise (pre-)evaluation processes. Interestingly, less precise evaluations of cores tend to artificially increase β, which does not have significant operational benefits, but decrease μ, thereby adversely impacting the dynamics of the pure remanufacturing system.

Regarding the optimal configuration of the inventory controllers, our analysis also reveals distinct optimal settings for different values of λ. When $\lambda = 2$, the adjustment $\{k_s = 0.1, k_r = 0.6\}$ yields the lowest value of J, like in the baseline scenario. However, for $\lambda = 7$, the initially optimal parameters for the baseline scenario, $\{k_s = 0.5, k_r = 0.7\}$, no longer hold; instead, the optimal setting shifts to $\{k_s = 0.6, k_r = 0.6\}$ when the value of μ is increased. This underscores the benefits of recalibrating the inventory controllers to maintain optimal performance under varying conditions. Therefore, it is reasonable to slightly increase k_s and decrease k_r in scenarios where μ is particularly high, ensuring that the inventory system consistently operates at its lowest cost.

6.5. Summary of effects and general recommendations

Based on the step-response and simulation analyses, Figure 14 summarises the main effects of all system parameters on the four operational metrics. In this figure, "+" indicates that an increase (decrease) in the parameter results in an increase (decrease) in the metric, while "-" indicates that an increase (decrease) in the parameter results in a decrease (increase) in the metric. Additionally, dashed lines represent significant yet marginal effects, while solid lines indicate the control parameter that has the highest impact on each metric. In this sense, we highlight that the controller of the serviceable inventory, k_s, has the highest impact on both operational metrics directly related to this inventory: RBw and $NSSAmp$. At the same time, the controller of the recoverable inventory, k_r, is a critical parameter for controlling PBw, while, interestingly, the controller of the serviceable inventory, k_s, holds the greatest influence on $NRSAmp$ (while k_r also have a strong influence). It is important to note that the analysis reported in Figure 14 is performed for the baseline scenario and around the variation levels previously defined. However, the parameters may exhibit different effects at very specific points within the parameter space.

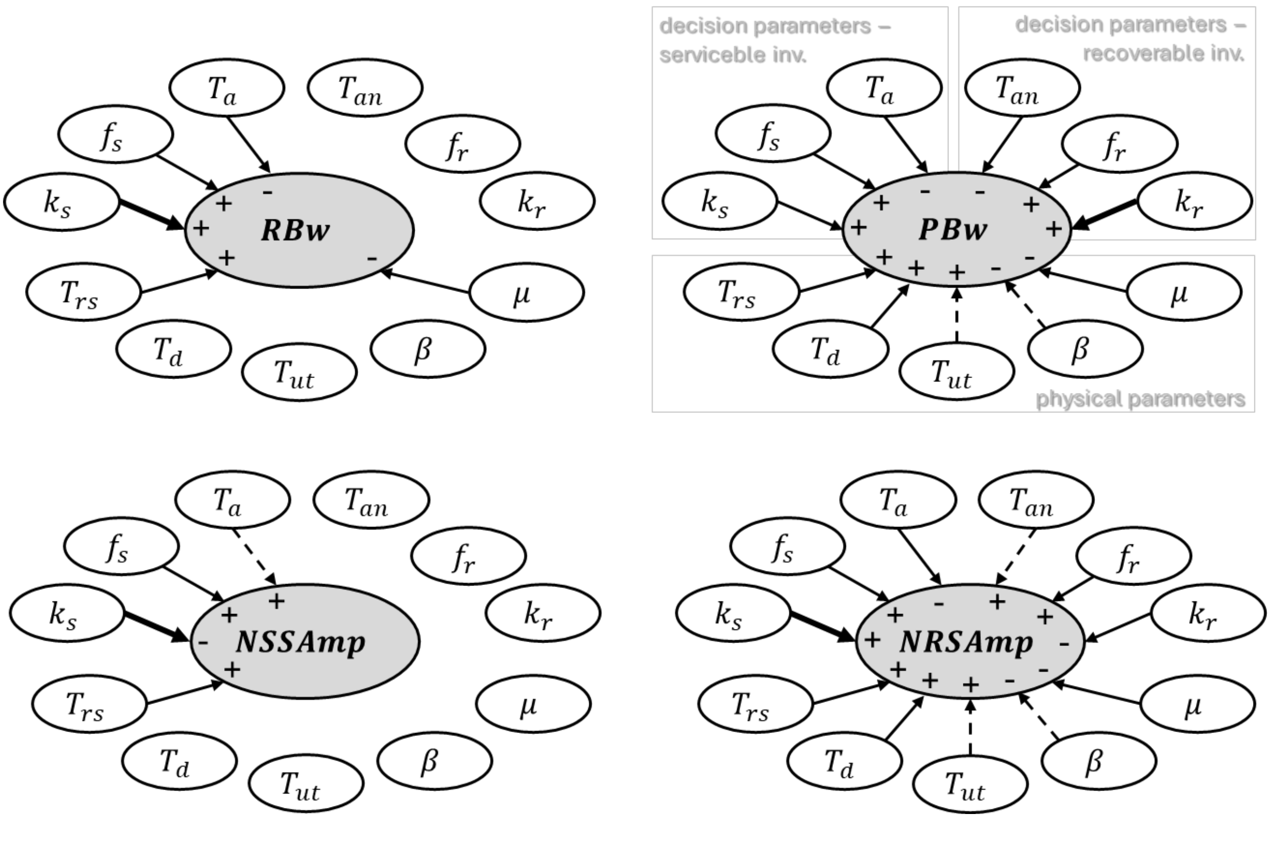


Figure 14: Impact of all system parameters on the four operational metrics. *Note*: For detailed meaning of the parameters and metrics, please refer to previous Tables 3 and 4, respectively.

Overall, our results demonstrate that pure remanufacturing systems exhibit some behavioural similarities (known effects) to traditional (manufacturing) systems and hybrid (manufacturing-remanufacturing) systems in terms of inventory control and supply chain dynamics. However, they also reveal distinct behaviours and potentially counterintuitive effects that are specific to pure remanufacturing contexts. Some of these findings, which extend the current understanding of closed-loop supply chain dynamics, are summarised below.

First, we observe that increasing return volume (circularity) does not significantly affect supply chain dynamics under the studied conditions, contrasting with the strong influence of this parameter in hybrid systems reported in the literature. Second, we find no evidence of the lead-time paradox in pure remanufacturing systems: reducing remanufacturing lead times consistently improves system performance, unlike hybrid systems where longer lead times can occasionally be beneficial. And third, we highlight that the tuning of the serviceable inventory controller affects both upstream and downstream dynamics, reinforcing the importance of coordinated tuning across subsystems.

In addition, the value and potential drawbacks of the POUT controllers underscore the need for a comprehensive assessment of their overall implications before implementation in pure remanufacturing systems. While they hold the capacity to considerably improve closed-loop supply chain performance, it is imperative to ensure the correct tuning of the inventory controllers; otherwise, dynamics may deteriorate, and performance would decrease. While each industrial setting necessitates a tailored assessment, a general recommendation, based on our analysis, may be to use intermediate levels for the controller of the recoverable inventory (e.g., the 'golden ratio' rule, $k_r = 1/\varphi \approx 0.618$), while adapting that of the serviceable inventory to the specific costs faced by the relevant actors. It may be reasonable to use moderate levels of this controller when the costs associated with the serviceable inventory are considerably high compared to other costs (e.g., $k_s = 0.5$), and lower levels if they are comparatively similar (e.g., $k_s = 0.1$).

In most real-world pure remanufacturing scenarios, these simple rules would offer considerable operational improvements, and thus cost benefits, over other alternatives, like the conventional OUT policy. This configuration of the POUT controllers can ensure efficiency across supply chain operations, while maintaining high levels of customer satisfaction. However, supply chain performance can be further elevated by adjusting the controllers in response to variations in key physical parameters, mainly, lead times and remanufacturing yield proportion. Specifically, in cases with relatively low values of λ and particularly long lead times, it may be reasonable to reduce the values of both controllers (e.g., $k_r = 0.3$; $k_s = 0.05$). Conversely, in scenarios with relatively high values of λ and particularly high remanufacturing yield proportions, tuning both controllers at similar levels may yield benefits (e.g., $k_r = 1/\varphi \approx 0.618$). These adjustments to the POUT controllers can be key for achieving near-optimal performance and addressing specific challenges that may arise in different real-world contexts.

7. Conclusion

Pure remanufacturing systems have not received sufficient attention in the closed-loop supply chain literature despite their practical relevance, leading to a poor understanding of their dynamics. To bridge this gap, we have developed a generalised model of a pure remanufacturer's closed-loop supply chain, drawing inspiration from real-world operations across different industries. Using control engineering, we have examined the mathematical properties of the resulting two-echelon inventory system (consisting of recoverable and serviceable inventories) to provide a comprehensive understanding of its stability, dynamics, and performance. We have observed that its dynamic behaviour shares some similarities with the well-studied traditional and hybrid closed-loop supply chains, but it also differs significantly in several ways.

Compared to the traditional supply chain, the integration of the traditional (downstream) and reverse (upstream) flows of materials gives rise to complex interdependencies that characterise the system's dynamics. As a result, key trade-offs appear, which need to be appropriately recognised and managed. Our results have revealed that the POUT policy emerges as a highly effective strategy to manage the recoverable and serviceable inventories of pure remanufacturing systems. This policy can significantly smooth the operations of these systems; however, it may decrease customer service. Thus, the precise regulation of its inventory controllers is crucial for gaining competitive advantages over other, industrially popular replenishment policies. Another trade-off emerges when safety stocks are set proportional to forecasts. While higher safety factors improve service levels, they not only increase inventory costs but also amplify order variability, thereby reducing the efficiency of pure remanufacturing systems.

A fundamental aspect of closed-loop supply chains is their inherent lead-time complexity. Several lead times of different natures, in both flows of materials, coexist, necessitating appropriate integration. While this complexity also applies to hybrid systems, the lead-time effects exhibit notable distinctions in pure remanufacturing systems. In the former, increasing remanufacturing times can be beneficial; in contrast, for closed-loop supply chains of pure remanufacturers, shortening every lead time (while maintaining quality standards) consistently results in operational benefits. Nevertheless, the magnitude of these benefits may considerably differ, and it is also crucial to consider the costs associated with lead-time reduction. Thus, a comprehensive understanding of lead-time dynamics empowers managers to make informed decisions that enhance customer service and reduce operational costs, while avoiding inefficient investments.

Our findings also underscore the value of directing efforts towards increasing remanufacturing yield proportions. A continuous improvement mindset, coupled with the integration of advanced technologies, acts as a crucial driver for elevating the quality of remanufacturing operations and, consequently, enhancing the overall dynamics of the closed-loop supply chain. Additionally, our results place a premium on the accuracy of pre-evaluation processes. Inexact evaluations of cores not only escalate production costs (due to processing nonconforming products), but also deteriorate the dynamics of pure remanufacturing systems.

Conversely, our observations reveal that, while increasing the return rate can enhance circularity and thus the environmental sustainability of the supply chain, it does not exert a significant impact on the effectiveness and efficiency of pure remanufacturers under the studied conditions. This contrasts with the behaviour observed in hybrid systems, while the situation may differ in cases with high uncertainty in the return of used products. In such cases, managers may need to take measures to handle these uncertainties, including accurate return forecasts and promoting information sharing, and thus harness the unique advantages offered by remanufacturing.

In managing closed-loop supply chains of pure remanufacturers, fine-tuning inventory controllers within the POUT policy is key to optimising operational performance. Our analysis provides general guidelines on adjusting these controllers for serviceable and recoverable inventories to achieve the best balance between various operational metrics. A general recommendation, based on our findings, would be to use intermediate levels for the recoverable inventory's controller (e.g., $k_r = 1/\varphi \approx 0.618$) to ensure robustness across various conditions. In addition, we suggest adjusting the serviceable inventory's controller (k_s) primarily based on the specific cost structure of the remanufacturer; from intermediate values (e.g., $k_s = 0.5$), when the variability of the serviceable inventory is much more costly, to low values (e.g., $k_s = 0.1$), when cost differences are relatively small.

While optimising each real-world application requires a tailored study based on the specific conditions affecting the closed-loop supply chain, such as cost structure, lead times, return proportion, and yield proportion, among others, this simple configuration yields significant cost benefits over other policies, such as the OUT policy, in most real-world scenarios. However, under certain conditions, it is reasonable to adjust the controllers in response to particularly long lead times and/or varying remanufacturing yield proportions.

While we have confidence in the theoretical soundness and practical relevance of our findings, our study does have limitations, mainly emerging from its scope and certain underlying assumptions, which can inspire further research to broaden the understanding of the subject. We thus conclude this article by outlining three key areas for future research that, in our view, would enhance the understanding of the dynamics and management of closed-loop supply chains.

First, our study represents the pure remanufacturer's closed-loop supply chain using a linear model. While this modelling approach enables tractable analysis, it also abstracts away certain real-world complexities. Indeed, practical settings often involve nonlinearities, such as capacity restrictions, delivery delays, or lost sales, that may considerably affect the supply chain dynamics. Therefore, conducting detailed analyses on the impact of these nonlinearities, as well as varying other modelling assumptions, would enrich our understanding of the performance of real-world pure remanufacturing systems. For example, exploring scenarios without a core broker, or with constrained broker capacity or responsiveness, could yield valuable insights into the dynamics of real-world pure remanufacturing systems. Another promising extension would be to incorporate item-level quality degradation and aging across multiple remanufacturing cycles.

Second, exploring the interactions between the pure remanufacturer's closed-loop supply chain and the OEM's supply chain, not considered in our work, would also provide valuable insights. Understanding and effectively managing these interactions, which may occur through the core broker and/or through downstream demand-side links (e.g., market competition), has the potential to yield positive effects on the sustainability and efficiency of both supply chains.

And third, investigating the dynamics of alternative closed-loop supply chain structures would also be fundamental to promoting the adoption of more sustainable production systems. In this sense, examining the behaviour of closed-loop systems based on other recovery strategies, such as reuse or recycle, insufficiently addressed by prior works, would provide useful guidance to managers seeking to adopt such circular economy practices.

Data availability statement

The analysis was conducted using the COMA (COntrol engineering for MAxima) package for Maxima (Haager, 2017) and MATLAB/Simulink. The control-theoretic developments of this article can be made available upon request.

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Appendix A - Transfer functions

This appendix provides the transfer functions that relate the four key variables that define the behaviour and performance of the pure remanufacturing system to its demand, which serve as the basis for the analytical and numerical results obtained and presented in Sections 4, 5, and 6.

First, we look at the upper echelon of the block diagram presented in Figure 5. The relationship between the remanufacturing orders and the demand is described by

$$\frac{RO(s)}{D(s)} = \frac{(f_s k_s + T_{rs} k_s + T_a k_s + 1)s + k_s}{\mu(s + k_s)(T_a s + 1)}. \quad (A1)$$

Note that this transfer function has two poles and one zero. At the same time, the relationship between the serviceable inventory and the demand is defined by

$$\frac{SI(s)}{D(s)} = -\frac{T_a T_{rs} s^2 + (T_a T_{rs} k_s + T_{rs} + T_a)s - f_s k_s}{(s + k_s)(T_a s + 1)(T_{rs} s + 1)}. \quad (A2)$$

In this case, the transfer function has three poles and two zeros.

The transfer functions linking purchase orders and recoverable inventory to supply chain demand are particularly more complex. This complexity arises from the substantial influence of the upper echelon on the lower echelon within the closed-loop supply chain's block diagram, as elucidated in Section 3. The transfer function expressing the relationship between the purchase orders and the demand, with five poles and four zeros, can be formulated as

$$\frac{PO(s)}{D(s)} = \frac{A_{po} s^4 + B_{po} s^3 + C_{po} s^2 + D_{po} s + E_{po}}{\mu(s + k_r)(s + k_s)(T_a s + 1)(T_{an} s + 1)(T_{ut} s + 1)}; \quad (A3)$$

where:

$$A_{po} = T_a T_{ut} f_r k_r + T_a T_d T_{ut} k_r + T_a T_{ut};$$

$$\begin{aligned} B_{po} = {} & T_{an} T_{ut} f_s k_r k_s + T_a T_{ut} f_r k_r k_s + T_{an} T_{rs} T_{ut} k_r k_s + T_a T_d T_{ut} k_r k_s + T_a T_{an} T_{ut} k_r k_s + T_a T_{ut} k_s \\ & + T_{ut} f_r k_r - \mu\beta T_a f_r k_r + T_a f_r k_r + T_d T_{ut} k_r + T_{an} T_{ut} k_r - \mu\beta T_a T_d k_r + T_a T_d k_r \\ & - \mu\beta T_a T_{an} k_r + T_{ut} - \mu\beta T_a + T_a; \end{aligned}$$

$$\begin{aligned} C_{po} = {} & T_{ut} f_s k_r k_s + T_{an} f_s k_r k_s + T_{ut} f_r k_r k_s - \mu\beta T_a f_r k_r k_s + T_a f_r k_r k_s + T_{rs} T_{ut} k_r k_s + T_d T_{ut} k_r k_s \\ & + T_{an} T_{ut} k_r k_s + T_a T_{ut} k_r k_s + T_{an} T_{rs} k_r k_s - \mu\beta T_a T_d k_r k_s + T_a T_d k_r k_s - \mu\beta T_a T_{an} k_r k_s \\ & + T_a T_{an} k_r k_s + T_{ut} k_s - \mu\beta T_a k_s + T_a k_s - \mu\beta f_r k_r + f_r k_r + T_{ut} k_r - \mu\beta T_d k_r + T_d k_r \\ & - \mu\beta T_{an} k_r + T_{an} k_r - \mu\beta T_a k_r - \mu\beta + 1; \end{aligned}$$

$$\begin{aligned} D_{po} = {} & f_s k_r k_s - \mu\beta f_r k_r k_s + f_r k_r k_s + T_{ut} k_r k_s + T_{rs} k_r k_s - \mu\beta T_d k_r k_s + T_d k_r k_s - \mu\beta T_{an} k_r k_s \\ & + T_{an} k_r k_s - \mu\beta T_a k_r k_s + T_a k_r k_s - \mu\beta k_s + k_s - \mu\beta k_r + k_r; \end{aligned}$$

$$E_{po} = -\mu\beta k_r k_s + k_r k_s.$$

Finally, the relationship between the recoverable inventory and the demand, with six poles and four zeros, can be expressed by

$$\frac{RI(s)}{D(s)} = -\frac{A_{ri} s^4 + B_{ri} s^3 + C_{ri} s^2 + D_{ri} s + E_{ri}}{\mu(s + k_r)(s + k_s)(T_a s + 1)(T_{an} s + 1)(T_d s + 1)(T_{ut} s + 1)}; \quad (A4)$$

where:

$$A_{ri} = T_{an}T_dT_{ut}f_sk_s + T_{an}T_dT_{rs}T_{ut}k_s + T_aT_{an}T_dT_{ut}k_s + T_{an}T_dT_{ut} - \mu\beta T_aT_{an}T_d;$$

$$\begin{aligned}B_{ri} = {} & T_{an}T_dT_{ut}f_sk_rk_s + T_{an}T_dT_{rs}T_{ut}k_rk_s + T_aT_{an}T_dT_{ut}k_rk_s + T_dT_{ut}f_sk_s + T_{an}T_{ut}f_sk_s + T_{an}T_df_sk_s \\ & + T_dT_{rs}T_{rt}k_s + T_{an}T_{rs}T_{ut}k_s + T_{an}T_dT_{ut}k_s + T_aT_dT_{ut}k_s + T_aT_{an}T_{ut}k_s + T_{an}T_dT_{rs}k_s \\ & - \mu\beta T_aT_{an}T_dk_s + T_aT_{an}T_dk_s - T_aT_{ut}f_rk_r + T_{an}T_dT_{ut}k_r - T_aT_dT_{ut}k_r - \mu\beta T_aT_{an}T_dk_r \\ & + T_dT_{ut} + T_{an}T_{ut} - T_aT_{ut} - \mu\beta T_{an}T_d + T_{an}T_d - \mu\beta T_aT_d - \mu\beta T_aT_{an};\end{aligned}$$

$$\begin{aligned}C_{ri} = {} & T_dT_{ut}f_sk_rk_s + T_{an}T_df_sk_rk_s - T_aT_{ut}f_rk_rk_s + T_dT_{rs}T_{ut}k_rk_s + T_{an}T_dT_{ut}k_rk_s + T_{an}T_dT_{rs}k_rk_s \\ & - \mu\beta T_aT_{an}T_dk_rk_s + T_aT_{an}T_dk_rk_s + T_{ute}f_sk_s + T_df_sk_s + T_{an}f_sk_s + T_{rs}T_{ut}k_s \\ & + T_dT_{ut}k_s + T_{an}T_{ut}k_s + T_dT_{rs}k_s + T_{an}T_{rs}k_s - \mu\beta T_{an}T_dk_s + T_{an}T_dk_s - \mu\beta T_aT_dk_s \\ & + T_aT_dk_s - \mu\beta T_aT_{an}k_s + T_aT_{an}k_s - T_{ute}f_rk_r + \mu\beta T_af_rk_r - T_af_rk_r - \mu\beta T_{an}T_dk_r \\ & + T_{an}T_dk_r - T_aT_dk_r - \mu\beta T_d + T_d - \mu\beta T_{an} + T_{an} - T_a;\end{aligned}$$

$$\begin{aligned}D_{ri} = {} & T_df_sk_rk_s - T_{ut}f_rk_rk_s + \mu\beta T_af_rk_rk_s - T_af_rk_rk_s + T_dT_{rs}k_rk_s - \mu\beta T_{an}T_dk_rk_s + T_{an}T_dk_rk_s \\ & + f_sk_s + T_{rs}k_s - \mu\beta T_dk_s + T_dk_s - \mu\beta T_{an}k_s + T_{an}k_s + \mu\beta f_rk_r - f_rk_r;\end{aligned}$$

$$E_{ri} = \mu\beta f_rk_rk_s - f_rk_rk_s.$$

Appendix B - Derivation of static gains

In this appendix, we obtain the static gains of the four transfer functions. Applying the Final Value Theorem (e.g., Nise, 2019), the static gain, ϑ, of a generic transfer function, $F(s)$, with input $X(s)$ and output $Y(s)$, can be easily obtained for a unit-step input signal (i.e., $X(s) = 1/s$) as follows,

$$\vartheta[F(s)] = \lim_{s\to 0} sY(s) = \lim_{s\to 0} sX(s)F(s) = \lim_{s\to 0} s\left(\frac{1}{s}\right)F(s) = \lim_{s\to 0} F(s). \tag{B1}$$

For the transfer function $\frac{RO(s)}{D(s)}$, the static gain is

$$\vartheta\left[\frac{RO(s)}{D(s)}\right] = \lim_{s\to 0}\left[\frac{RO(s)}{D(s)}\right] = \lim_{s\to 0}\left[\frac{(f_sk_s + T_{rs}k_s + T_ak_s + 1)s + k_s}{\mu(s+k_s)(T_as+1)}\right] = \frac{k_s}{\mu k_s} = \frac{1}{\mu}. \tag{B2}$$

For the transfer function $\frac{RO(s)}{D(s)}$, the static gain is

$$\begin{aligned}\vartheta\left[\frac{SI(s)}{D(s)}\right] &= \lim_{s\to 0}\left[\frac{SI(s)}{D(s)}\right] = \lim_{s\to 0}\left[-\frac{T_aT_{rs}s^2 + (T_aT_{rs}k_s + T_{rs} + T_a)s - f_sk_s}{(s+k_s)(T_as+1)(T_{rs}s+1)}\right] = \frac{-(-f_sk_s)}{k_s} \\ &= f_s.\end{aligned} \tag{B3}$$

For the transfer function $\frac{PO(s)}{D(s)}$, the static gain is

$$\begin{aligned}\vartheta\left[\frac{PO(s)}{D(s)}\right] &= \lim_{s\to 0}\left[\frac{PO(s)}{D(s)}\right] = \lim_{s\to 0}\left[\frac{A_{po}s^4 + B_{po}s^3 + C_{po}s^2 + D_{po}s + E_{po}}{\mu(s+k_r)(s+k_s)(T_as+1)(T_{an}s+1)(T_{ut}s+1)}\right] = \frac{E_{po}}{\mu k_rk_s} \\ &= \frac{-\mu\beta k_rk_s + k_rk_s}{\mu k_rk_s} = \frac{-\mu\beta + 1}{\mu} = \frac{1}{\mu} - \beta,\end{aligned} \tag{B4}$$

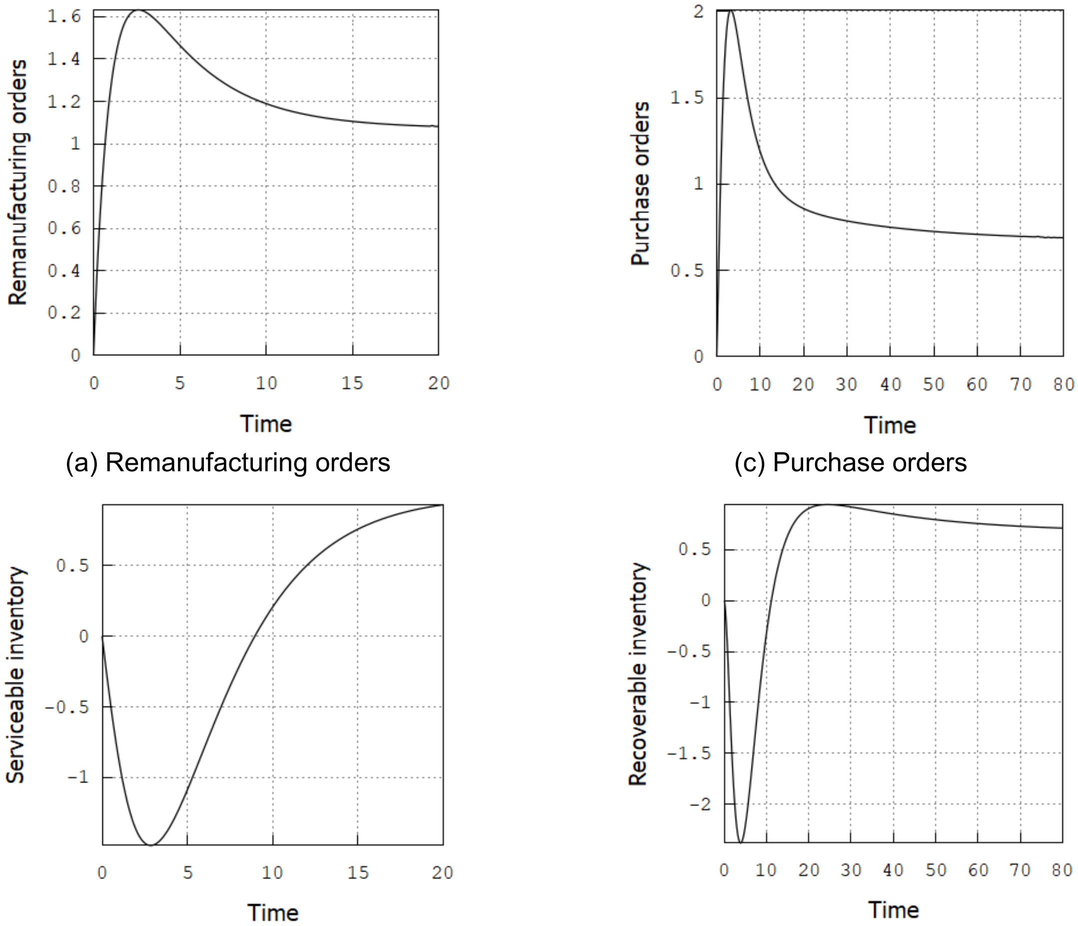
defined as the compensation proportion, ρ.

Finally, the static gain of the transfer function $\frac{RI(s)}{D(s)}$ is

$$\begin{aligned}\vartheta\left[\frac{RI(s)}{D(s)}\right] &= \lim_{s\to 0}\left[\frac{RI(s)}{D(s)}\right] \\ &= \lim_{s\to 0}\left[-\frac{A_{ri}s^4 + B_{ri}s^3 + C_{ri}s^2 + D_{ri}s + E_{ri}}{\mu(s+k_r)(s+k_s)(T_a s+1)(T_{an}s+1)(T_d s+1)(T_{ut}s+1)}\right] \\ &= \frac{-E_{ri}}{\mu k_r k_s} = \frac{-(\mu\beta f_r k_r k_s - f_r k_r k_s)}{\mu k_r k_s} = \frac{-\mu\beta f_r + f_r}{\mu} = f_r\left(\frac{1}{\mu} - \beta\right) = f_r\rho.\end{aligned} \tag{B5}$$

Appendix C - The dynamics of the baseline scenario

Figure C1 displays the unit-step responses of the remanufacturing orders (a), serviceable inventory (b), purchase orders (c), and recoverable inventory (d) of the pure remanufacturing system in the baseline scenario. We note that the control system does not suffer from oscillations, as the poles are always real (see Appendix A). However, we see initial overshoots (for orders) and undershoots (for inventories) that are induced by the presence of dominant zeros.



(a) Remanufacturing orders

(c) Purchase orders

(b) Serviceable inventory

(d) Recoverable inventory

Figure C1: Step responses of the baseline system.

Figure C1(a) reveals a considerable overshoot in the transient response of the remanufacturing orders before stabilising at $(1/\mu) \approx 1.07$. Figure C1(c) exposes that the overshoot is even more remarkable for the response of the purchase orders, which eventually reaches $\rho = \frac{1}{\mu} - \beta \approx 0.67$. Both peaks suggest that the baseline system may be significantly affected by the bullwhip effect.

Figures C1(b) and C1(d) show that there are substantial troughs in the responses of the serviceable and recoverable inventories when demand increases. After the trough, the inventory grows and stabilises at $\rho = 1$ and $f_r\rho \approx 0.67$, respectively. However, there is a (positive) peak in the response of the recoverable inventory that cannot be seen for the serviceable stock. Overall, the responses indicate that the baseline system may suffer to efficiently satisfy customer demand.

We also highlight that the responses of the purchase orders and the recoverable inventory (Figs. C1(b) and C1(d)) are considerably slower than those of the remanufacturing orders and the serviceable stock (Figs. C1(a) and C1(c)). This is due to the return lead time, significantly higher than the other lead times, generating a dominant pole in the two transfer functions defined by T_{ut}.