Heterogeneous agent-based model, artificial stock market, and secondary priority rules

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Highlight
·The pro-rata rule improves liquidity and price efficiency.
·The equal-sharing priority rule is ill-suited to stock markets.
·An agent-based model compares alternative secondary priority rule designs.

Abstract
This study utilizes a heterogeneous agent-based artificial stock market to examine how secondary priority rules (SPRs) within order precedence affect stock market quality. We analyze the effectiveness of three SPRs: the time, pro-rata, and equal-sharing priority rules. We use the time priority rule as a benchmark and consider investors' trading strategies. The pro-rata priority rule can further promote market quality by achieving more active trading, higher liquidity, and more efficient asset pricing. In contrast, the equal-sharing priority rule impairs market quality. Our results provide new evidence on the relationship between SPRs and stock market quality.

Keywords: Agent-based modeling; Genetic programming; Heterogeneity; Market quality; Secondary priority rule
JEL Classification: D83, D84, G12, G17

1. Introduction
Order precedence rules govern the way market and limit orders are matched on a trading platform, and therefore directly affect trading activity, market quality, and other aspects of the financial market microstructure (Ahn, Kang, and Ryu, 2008, 2010; Biais, Glosten, and Spatt, 2005; Hu et al., 2024; Lee, Ryu, and Yang, 2021; Madhavan, 2000; Ryu, 2016; Ryu, Webb, and Yu, 2022; Yu, Kim, and Ryu, 2024). Whether to use market or limit orders, traders need

to weigh the immediacy of market orders against the execution uncertainty and potential price improvement of limit orders; and the key determinants are the order precedence and the rules they need to follow (Domowitz, 1993; Goettler, Parlour, and Rajan, 2005; Roşu, 2009). In particular, order precedence rules, divided into the first priority rules and the secondary priority rules (SPRs), determine how orders are to be prioritized for execution. When market orders are matched with standing limit orders, most stock markets worldwide adopt the first priority rule — price priority, which ensures execution at the best available price. However, when multiple limit orders exist at the best price, an SPR is required to allocate the execution quantity among limit orders resting at the best price (the execution price is pinned down by price priority). The time priority rule is the most prevalent SPR in financial markets, which is also known as "first in, first out" (FIFO) based on its matching mechanism. The time priority rule matches new incoming market orders with the limit order that posted the best price earliest. Besides matching the market order to a single limit order, in some financial markets, market orders are allocated among all limit orders with the best price. The pro-rata priority rule and the equal-sharing priority rule are representatives of this type of SPR. In particular, when a market order is placed, the pro-rata priority rule fills limit orders proportionally to their quantity in relation to the total quantity of limit orders at the best price, while the equal-sharing priority rule allocates the newly entered market order equally to each limit order at the best price.

Based on the matching principle of each SPR, the time priority rule, pro-rata priority rule, and equal-sharing priority rule all have their respective advantages and disadvantages. The most prominent advantage of the time priority rule lies in its simplicity, which requires less computational power. Thus, the high transactional speed and efficiency of the time priority rule make it the preferred choice in financial markets (John and Millum, 2020). Nevertheless, this rule does have certain drawbacks. The time priority rule constrains market participation since only the order that is first to quote the best price gains the opportunity to trade. Given the substantial amount of trading volume each day, the abundance of orders intensifies competition among traders. Consequently, prioritizing placement in the order queue often favors network speed over well-considered trading strategies. Pro-rata and equal-sharing priority rules could potentially address these challenges. By sharing the allocation among orders at the best price, these two SPRs could promote investor participation and provide sufficient time for consideration. However, due to the constraints of the sophisticated system, the pro-rata priority rule and the equal-sharing priority rule are generally adopted by relatively inactive futures markets or are embedded in the time priority rule.

The enhancement of computational power alleviates the restrictions of the complex system, enabling stock markets to reform their SPRs. The New York Stock Exchange (NYSE) was the first to undertake this attempt. Considering the constraints of the time priority rule and the advantages of the pro-rata priority rule, NYSE adopts "parity" as the SPR, which can be regarded as a combination of the time and pro-rata priority rules. Parity allocates market orders proportionally among the earliest orders on the electronic limit order books, designated market makers, and floor brokers at the most competitive price (Battalio, Jennings, and McDonald, 2021).

According to the "New York Stock Exchange broker systems and parity/priority allocation model[1]", the NYSE claims that the implementation of parity could "promote deep liquidity and superior market quality by sharing the allocation among those who post the best price, rather than how quickly they place the order, thereby delivering better fill rates, lower execution costs, and the ability to share executions at the same price." The adoption of parity by the NYSE highlights the importance of the SPR, signaling its substantial impact on market quality. This deviation from the time priority rule aligns with the notion that the latter may not be the optimal choice for an SPR in stock markets. However, while the NYSE's view supports this idea, it contrasts with the practice of other stock markets. The absence of reforms regarding SPRs within stock markets hinders empirical studies aimed at exploring the effectiveness of various designs of SPRs.

The multiple reforms on SPRs in futures markets, on the other hand, have provided opportunities to investigate the effectiveness of various designs of SPRs on financial market quality. The NYSE Liffe (London International Financial Futures Exchange) is a typical representation of these futures markets, as it carries out two famous reforms on SPRs, from time pro-rata (combination of time priority with pro-rata priority) to pro-rata in 2005, and returned to time pro-rata in 2007. Lepone and Yang (2012) find that the 2005 reform has resulted in higher trading volume and transaction costs, as well as lower market depth; while Aspris et al. (2015) claim a lower market depth, volatility, and trading activity after the reform in 2007. Some studies discuss the impacts of SPRs on bid-ask spread. Frino, Hill, and Jarnecic (2000) find no significant difference in bid-ask spreads under either time or pro-rata priority rule. In contrast, Janeček and Kabrhel (2007) suggest time priority rule might narrow the bid-ask spread compared to the pro-rata priority rule. These findings suggest certain limitations within previous studies. First, in contrast to reforming the pure time priority rule to the pure pro-rata priority rule or vice versa, almost all pre-existing reforms opt for the time pro-rata priority rule, a combination of time and pro-rata, as the reform system. Embedding a new priority rule within the original one can indeed reduce market volatility caused by reforms. However, the combined method of time and pro-rata priority rule would influence the degree of effectiveness exerted by each pure SPR. In this case, although these reforms can compare the effectiveness of the time priority rule and the pro-rata priority rule to a certain extent, it is difficult to identify the effectiveness of the pure time and pure pro-rata priority rules effectively. Second, there is no consensus regarding the impacts of the time and pro-rata priority rule on market quality as revealed by empirical studies. On the one hand, this is attributed to the different combined methods as discussed above. On the other hand, SPR is not the only variable that changes before and after reforms due to the constantly changing market environment, which implies that the empirical results might be influenced by other noise variables in the markets. Thus, the actual effects of different SPRs remain to be investigated. Third, while prior research on pro-rata as the second priority rule is grounded in futures markets, its implications for stock markets may differ fundamentally due to distinct trader ecosystems. Particularly, futures markets are often used more intensively by hedgers and

[1] https://beta.nyse.com/publicdocs/NYSE_broker_systems_and_parity_priority_allocation_model.pdf

arbitrageurs (He et al., 2023; Ryu, 2011), while stock markets tend to feature a relatively larger share of directional and liquidity-motivated trading. These differences in prevalent trading motives may affect order-submission incentives under alternative SPRs. Specifically, traders in stock markets focus more on the speed of transaction, while traders in futures markets put more emphasis on the large bid-ask spread. As a result, the traders in stock markets have an incentive to quote a more competitive price, i.e., a higher bid price and a lower ask price, to execute orders quickly. In this case, we predict that the impact of the pro-rata priority rule on traders' behavior, especially trading strategies regarding quoted prices, would differ between stock markets and futures markets. Therefore, there exists a demand for specific studies in stock markets.

Given the prevalence in usage and theoretical support in existing research, this study investigates the effectiveness of the pro-rata priority rule and equal-sharing priority rule. Using the performance of the time priority rule, the universal SPR used in stock markets, as the benchmark, this study examines the impacts of pro-rata and equal-sharing priority rules on market quality, including market liquidity, price efficiency, and market volatility. To overcome the limitations of conducting empirical studies due to insufficient variation in order precedence rules in stock markets, we adopt a heterogeneous agent-based model (HAM) to comprehensively analyze the impacts of three common SPRs (i.e., time, pro-rata, and equal-sharing) on stock market quality. The HAM provides a controlled experimental environment where the SPRs and the corresponding trading strategies investors adopt are the only setting that changes. Previous studies have already demonstrated that different SPRs could significantly influence the trading strategies of traders. Given that trader behavior is a key determinant for stock market quality, without an accurate representation of the trading strategies under each SPR, the simulated market quality may significantly deviate from reality. HAM, which is established based on the agent design and could mimic the interaction among agents, could match the requirement of mimicking the trading strategies accurately. By pre-setting the trading strategies that have already been proven in real markets to each agent under different SPRs, HAM could precisely simulate the change of market quality under the combined effects of SPRs and related trading strategies. We enhance the HAM by integrating Genetic Programming (GP), which replicates the learning process of investors. By incorporating GP, the agents in our model can adjust their strategies based on the market conditions they observe. This adaptive nature of GP reflects the learning behavior of real investors, who continuously gather information and update their strategies accordingly. By mimicking this learning process, GP enables the agents in our model to make more realistic decisions, leading to more credible experimental outcomes. Our simulation results demonstrate that the pro-rata priority rule, in comparison to the time priority rule, enhances market quality, while the equal-sharing priority rule hurts market quality. The efficacy of each SPR can be ascribed to the distinct trading strategies employed under each SPR. On the one hand, our results uphold the previous perspective identified in futures markets that the pro-rata priority rule can attain higher trading volume due to the oversizing strategy adopted by investors. On the other hand, we discover, contrary to that in futures markets, that the pro-rata priority rule can narrow the bid-ask spread. This disparity can be attributed to the distinct trader ecosystems in both markets. As the traders in stock markets are more aggressive, the degree of competition on price is more intense than in futures markets. Under the pro-rata priority rule, which reduces the priority of traders with time advantages, these traders would have a stronger incentive to update the best-quoted price to attempt to monopolize the incoming market order, thereby further narrowing the bid-ask spread.

This study makes several contributions to the existing literature. First, we expand the research on SPRs within the context of the stock market. Second, to the best of our knowledge, this is the first study employing a HAM to investigate SPRs. It addresses the gap in empirical studies caused by a lack of natural experiments by providing reliable simulation results, thereby enriching the literature in both market microstructure and HAM. Third, our results provide new insights into the relationship between the application of the pro-rata priority rule and the bid-ask spread. In contrast to the previous findings suggesting that the pro-rata priority rule might broaden the bid-ask spread in futures markets, our results suggest that the pro-rata priority rule can narrow the bid-ask spread in stock markets, which is ascribed to the distinct trader ecosystems in both markets. Finally, this research suggests that adopting pro-rata as the SPR may be more effective than the time priority rule in stock markets. Accordingly, the pro-rata priority rule can be considered by policymakers for potential SPR reform, while the findings also provide a theoretical foundation for further studies.

The remainder of this paper is structured as follows. Section 2 reviews the literature. Section 3 introduces the HAM that we employ for this study. Sections 4 and 5 report simulation results and analyze the sensitivity of our findings. Section 6 concludes.

2. Literature Review and Research Background

2.1. Three prevalent SPRs

The priority rule is defined as the system for matching market orders and limit orders in financial markets (Hersch, 2023). Generally, price acts as the primary priority rule in financial markets. Nevertheless, when multiple orders are placed at the same price level, the SPR becomes requisite. Time priority, pro-rata priority, and equal-sharing priority rules are the three most prevailing SPRs in financial markets. Specifically, under the time priority rule, the incoming market order is allocated to the earliest limit order at the best price, while under the pro-rata and equal-sharing priority rules, the incoming market order is matched with all limit orders at the best price on a pro-rata and equal basis, respectively. In cases where the supply of market orders exceeds the demand of limit orders, the demand is fulfilled first. The remaining market order is then re-allocated according to the priority rule again, until the market order is completed. Examples are provided in Table 1 to elucidate the allocation algorithm of each SPR more explicitly. Assuming there are four limit orders with sizes of 100, 300, 50, and 150, respectively, in chronological order at the best price, and two market orders of sizes 60 and 270 enter the market successively. Panel A illustrates the detailed allocation algorithm of each SPR based on an example when a market order with size 60 enters the market, while Panel B displays the changes in the sizes of limit orders when market orders with sizes 60 and 270 are submitted sequentially.

Table 1. An example of the allocation algorithm for three SPRs

Panel A: The detailed allocation algorithm of three SPRs (when a market order with size 60 enters the market)

Limit order	Size	Time priority rule		Pro-rata priority rule		Equal-sharing priority rule	
		Allocation	Size (after trading)	Allocation	Size (after trading)	Allocation	Size (after trading)
L1	100	60	100-60=40	$100 \times \frac{60}{100+300+50+150} = 10$	$100 - 10 = 90$	$\frac{60}{4} = 15$	$100 - 15 = 85$
L2	300	-	300	$300 \times \frac{60}{100+300+50+150} = 30$	$300 - 30 = 270$	$\frac{60}{4} = 15$	$300 - 15 = 285$
L3	50	-	50	$50 \times \frac{60}{100+300+50+150} = 5$	$50 - 5 = 45$	$\frac{60}{4} = 15$	$50 - 15 = 35$
L4	150	-	150	$150 \times \frac{60}{100+300+50+150} = 15$	$150 - 15 = 135$	$\frac{60}{4} = 15$	$150 - 15 = 135$

Panel B: The change of limit orders' sizes under three SPRs (when market orders with sizes 60 and 270 enter into the market in sequence)

Limit order	Size	Time priority rule		Pro-rata priority rule		Equal-sharing priority rule	
		Size (after the first market order enters)	Size (after the second market order enters)	Size (after the first market order enters)	Size (after the second market order enters)	Size (after the first market order enters)	Size (after the second market order enters)
L1	100	40	0	90	45	85	6.67
L2	300	300	70	270	135	285	206.67
L3	50	50	50	45	22.5	35	0
L4	150	150	150	135	67.5	135	56.67

Notes. Panel A displays the effects of three SPRs used to allocate a market order of size 60 when four limit orders, i.e., *L1*, *L2*, *L3*, and *L4*, are in the market. Four limit orders share the best price and are arranged chronologically. Size denotes the order size of each limit order, while allocation denotes the order size that each limit could be assigned (including the detailed algorithm) in each transaction. Panel B presents changes in the sizes of limit orders under three SPRs when market orders with sizes 60 and 270 are submitted sequentially.

The adopted SPRs would exert an influence on the trading strategies of traders, and further have an impact on the market liquidity, price efficiency, and market volatility (Field and Large, 2008). Specifically, the time priority rule creates an incentive for traders to swiftly submit their quotes to gain priority. However, under pro-rata priority and equal-sharing priority rules, the timing of orders may become less crucial. The pro-rata rule, in particular, introduces a more sophisticated matching process. Notably, limit orders may not be fulfilled regardless of when they were submitted, as the matching is contingent on the quoted size.[2] In order to fulfill their orders, traders may choose to inflate their order size under the pro-rata priority rule (Aspris, Foley, Harris, and O'Neill, 2015). Conversely, under the equal-sharing priority rule, where the execution of limit orders is solely tied to the number of orders rather than the order size, traders may split their limit orders into many smaller ones to achieve their desired trading quantity. These quoting strategies would further impact market quality. In studies exploring pro-rata rules, researchers observed that time pro-rata, in general, results in lower trading volumes, with the inverse change promoting trading activity in futures markets. Lepone and Yang (2012) further argue that, under a pro-rata rule, traders are less motivated to submit their orders promptly due to the reduced significance of order timing. Instead, they may submit "oversized" orders to gain a higher proportion, leading to a higher trading volume.

While previous studies have not reached an agreement on the change of bid-ask spread, a majority proposes that the bid-ask spread under the pro-rata priority rule tends to be larger than that under the time priority rule or time pro-rata priority rule (Cordella and Foucault, 1999). The supporters of this perspective, such as Haynes and Onur (2020), argue that the marginal value of quoting an order in a long queue is higher under the pro-rata priority rule compared to the time priority rule, as the order is unlikely to be executed under the latter rule. A similar reasoning is applied to the equal-sharing priority rule. However, some studies find empirical evidence suggesting that the performance of bid-ask spreads is similar under both time and pro-rata priority rules. This shows that the above perspective might overlook the value of the time priority rule. Even though the marginal value of new orders under the time priority rule is lower than under the pro-rata and equal-sharing priority rule, the order value remains constant after quoting in the time priority rule. However, the existing studies discussed above primarily focus on futures markets. Futures markets are characterized by a prevalence of hedgers and spread-based arbitrageurs who often act as uninformed liquidity providers. Their primary objective is to profit from the bid-ask spread or to manage risk efficiently, rather than trade swiftly on information. Under a pro-rata rule, which dilutes their order execution, these liquidity providers may see their per-trade profits compressed. To compensate, their rational response could be to quote less aggressively (widen the spread) or reduce limit order submission, potentially harming liquidity – a finding consistent with prior futures market studies.

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[2] Given the limited focus on the equal-sharing priority rule in existing research, and the common categorization of the pro-rata priority rule and equal-sharing priority rule as a single category (Weaver, 2022), discussions regarding the equal-sharing priority rule are formulated by drawing reference from studies conducted on the pro-rata priority rule.

In contrast, stock markets are dominated by speed-sensitive traders, including speculators acting on private information or valuations, for whom delays in execution can erode potential profits (Tripathi, Vipul, and Dixit, 2020). For these traders, speed of execution is the most critical factor. The pro-rata rule, by weakening the guarantee of execution via time priority, intensifies the competition for the primary source of priority: price. Consequently, stock traders have a heightened incentive to improve their quotes, i.e., submit higher bids and lower asks to jump the queue and secure a larger share of the available volume. This strategy to compete on price should, in equilibrium, compress the bid-ask spread.

Therefore, we hypothesize that due to this difference in dominant trader incentives, where futures liquidity providers may widen spreads to protect profits, and stock speculators narrow spreads to gain execution priority, the pro-rata priority rules will have opposing effects on bid-ask spreads across the two markets. We predict this rule will lead to a narrowing of the bid-ask spread in stock markets, contrary to the effects observed or theorized for futures markets. The same hypothesis applies to the equal-sharing priority rule.

While quite a few studies have explored the impacts of SPR on trading volume and bid-ask spread, research on their impacts on other aspects of market quality is relatively scarce. The impacts of the pro-rata rule on market depth remain a debated subject in the research literature. Janeček and Kabrhel (2007) find that market depth tends to be larger under the pro-rata priority rule. Conversely, Lepone and Yang (2012) and Panchapagesan (1997) argue that market depth decreases when transitioning from the time priority rule to the pro-rata priority rule. The change in bid-ask spread, trading volume, and market depth would further influence the price efficiency and market volatility. Scholars such as Amihud and Mendelson (1991), Chordia, Roll, and Subrahmanyam (2008), and Chung and Chuwonganant (2023) argue that continuous trading will help the market reach equilibrium prices, which means the larger trading volume leads to higher price efficiency. For market volatility, Lee and Rui (2002) find that there is a positive relationship between trading volume and market volatility. In addition, the market depth and bid-ask spread may all impact volatility. Thus, market volatility is a composite outcome influenced by both the consumption of trading volume, the accumulation of market depth, and the bid-ask spread.

2.2. Heterogeneous agent-based model and trading

Agent-based models are defined as “a class of computer models in which entities (referred to as agents) interact with each other and or their local environment” (Laubenbacher, Hinkelmann, and Oremland, 2013; Park, Hong, and Ryu, 2024; Park and Ryu, 2023). The models have gained widespread usage in diverse research fields, serving as a valuable complement to empirical studies. To our knowledge, no study investigates SPRs based on HAM, so this section will review the relevant HAM studies within the realm of financial research. Agent-based models have been employed to address various questions, offering insights and important theoretical and managerial implications for financial market management. In the mid to late 1990s, researchers began utilizing agent-based financial models to explain certain financial regularities (LeBaron, Arthur, and Palmer, 1999). During this period, most models have two types of agents, typically fundamentalists and chartists. Over time, the scope of agent-based financial models expands to include additional types of agents.

For example, some researchers introduce noise traders or contrarian traders into the basic two-type model (Sansone and Garofalo, 2007). In the last decade, agent-based financial models have further evolved with the adoption of machine-learning techniques, such as genetic algorithms (Park and Ryu, 2022). Autonomous-agent models, also referred to as heterogeneous agent-based models (HAMs), have emerged.[3] Unlike the previous two-type and N-type models, agents in autonomous-agent models possess the ability to learn and update their trading strategies autonomously. This enhanced autonomy significantly increases the capabilities of financial agents (Kaizoji, 2004; Park, Ryu, and Webb, 2024).

The development of agent-based financial models has led to their widespread utilization in investigating policy and behavior in financial markets. Notable studies in this line of research include Chiarella's (1992) examination of speculative behavior, Lux's (1995) study of herd behavior, bubbles, and crashes in speculative markets, and Dong et al.'s (2024) analyses of the effectiveness of price limits. Chiarella et al. (2017) and Wei and Shi (2020) have employed agent-based models to explore investor sentiment in limit order markets. Other examples include Chica et al. (2017), Dai, Zhang, and Chang (2025), He, Li, and Zheng (2019), Ladley et al. (2015), Yang, Zhang, and Ye (2020), and Yeh and Yang (2014) yield a comprehensive review of the background and development of heterogeneous agent models in financial markets. These studies demonstrate the diverse applications of agent-based financial models in understanding various aspects of financial markets.

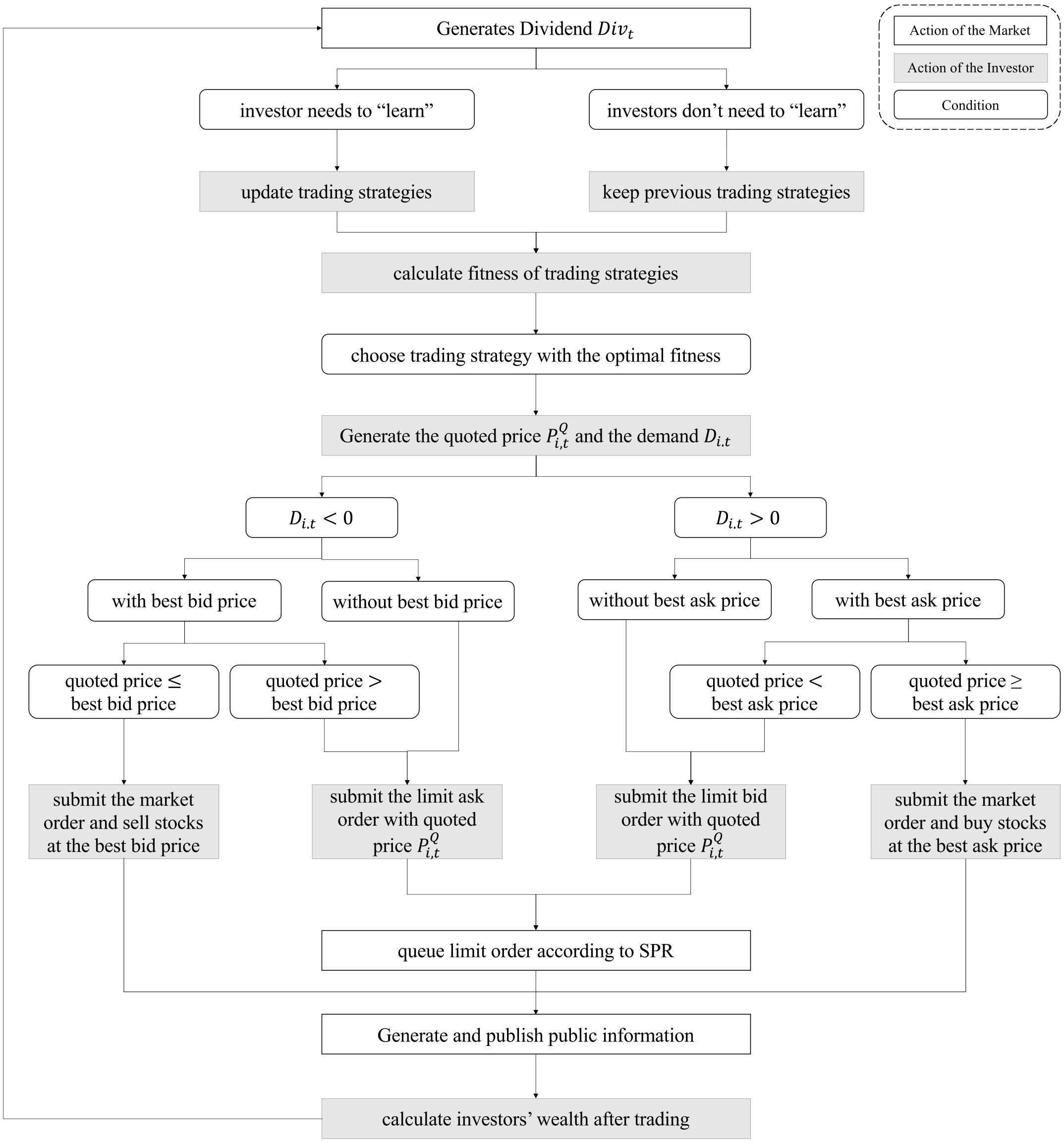
3. Methodology

3.1. Basic pricing

Extending Yang et al. (2025) and Yeh and Yang (2010), our model represents an artificial order-driven market, in line with the organization of most modern equity exchanges (Guilbaud and Pham, 2013), and consists of two types of assets. Figure 1 provides an overview of the general flowchart in HAMs. This section outlines the specific details of the model.

Figure 1. Flowchart in HAMs

[3] There is an ongoing dispute regarding the definition of HAMs; one perspective suggests that all two-type, N-type, and autonomous-agent models can be considered as HAMs. However, an alternative viewpoint argues that only autonomous-agent models should be classified as HAMs. While there is no unanimous agreement on this matter, there is a consensus among researchers that autonomous-agent models are indeed categorized as HAMs.



Notes. This figure presents the general flowchart in heterogeneous agent-based models, representing the fundamental trading process in each trading period. The contents within the white rectangle, grey rectangle, and rounded white rectangle, respectively, denote the actions of the market, the actions of the investor, and the conditions.

There are two assets in the model: a risk-free asset with an interest rate r_f, where the subscript f denotes "risk-free," and a risky asset – a stock that pays a dividend Div_t. The dividend is determined by an autoregressive process (LeBaron, 2006):

$$Div_t = \alpha + \rho(Div_{t-1} - \alpha) + \mu_t, \ \ (Div_t \geq 0), \tag{1}$$

where Div_t is the dividend in trading time *t*; α is the average dividend in the whole sample period; ρ is the persistence of the dividend, which shows the influence of the most recent dividend Div_{t-1} for Div_t; and μ_t is independent and identically distributed, where $\mu_t \sim N(0, \sigma_\mu^2)$. The values of each parameter are summarized in Table 2.

Table 2. Parameters for simulation

Panel A. Basic framework	
Initial quantity of risky assets for each trader (S)	1
Initial quantity of risk-free assets for each trader (B)	200
Initial stock price	20
Number of periods (N_t)	5,000
Interest rate (r_f)	0.01
Average dividend of stock (α)	0.2
$\sigma_\mu^2 (variance\ of\ \mu_t)$	0.01
$\rho(persistence\ of\ dividend)$	0.95
$\theta_0(parameter\ of\ fitness)$	0.7
$\omega(parameter\ of\ fitness)$	15
$\lambda(degree\ of\ absolute\ risk\ aversion)$	$Random\ (0.2) + 0.5$
Panel B. Traders	
Number of traders (n)	100
Number of trading strategies for each trader $(size\ of\ GP\ population)$	2
Learning Cycle for each trader	Random between 5 and 100 trading times
Panel C. Parameters of genetic programming	
Function set	
Arithmetic & Trigonometric	$+, -, *, \div, \sqrt{}, abs, sin, cos$
Comparison	$>, <, \geq, \leq, =, \neq$
Logical	$If - Else, If, AND, OR$
Terminal set	
Lagged Price	P_{t-k} (for $k = 1, \ldots, 5$)
Lagged Price and Dividend	$P_{t-k} + D_{t-k}$ (for $k = 1, \ldots, 5$)
Market Activity	Trading Volume_{t-1}
Probability of a clone	0.1
Probability of crossover	0.7
Probability of mutation	0.2

Notes. This table reports the parameter values utilized in the basic model, encompassing three components: the parameters for building the basic framework, the parameters for setting traders, and the parameters for genetic programming.

In the model, there are n agents representing investors, each initially holding B units of the risk-free asset and S units of the risky asset. Each investor has a trading strategy pool consisting of two trading strategies used to predict the expected stock price and dividend, as well as determine the demand at each trading time. These trading strategies reflect the trading skills and understanding of the stock possessed by each investor. The trading strategies undergo updates and evolution based on Genetic Programming (GP), and their operating mechanism will be explained in further detail in Section 3.2. Then, the trading strategy j of investor i that has better fitness $f_{i,t,j}$ is selected as the optimal trading strategy at time t, so that investors can utilize it to estimate the expected stock price and dividend based on equation (2).

More specifically, by defining $E_{i,t}(.)$ As the investor i's expectation based on the information available up to trading time t, the total expected stock price and dividend $E_{i,t}(P_{t+1}+Div_{t+1})$ are calculated as follows:

$$E_{i,t}(P_{t+1}+Div_{t+1})=\begin{cases}(P_t+Div_t)\left[1+\theta_0\tanh\left(\frac{ln(1+f_{i,t,j})}{\omega}\right)\right] & if\ \ f_{i,t,j}\geq 0,\\ (P_t+Div_t)\left[1-\theta_0\tanh\left(\frac{\ln(|-1+f_{i,t,j}|)}{\omega}\right)\right] & if\ \ f_{i,t,j}<0,\end{cases}, \tag{2}$$

where θ_0 and ω are the parameters of fitness $f_{i,t,j}$. The detailed parameter values are shown in Table 2.

At time t, if the investor decides to invest in stock, she pays P_t to buy 1 stock, and the expected value of his portfolio is $E_{i,t}(P_{t+1}+Div_{t+1})$ at time $t+1$, while if the investor invests P_t in risk-free assets, the expected value of his portfolio is $(1+r_f)P_t$ based on the risk-free rate r_f in $t+1$. Assuming investors are rational, they will avoid negative returns. When $E_{i,t}(P_{t+1}+Div_{t+1})$ is larger than $(1+r_f)P_t$, the maximal price that the investor could quote is the present value of $E_{i,t}(P_{t+1}+Div_{t+1})$, which is $\frac{E_{i,t}(P_{t+1}+Div_{t+1})}{1+r_f}$. However, if $(1+r_f)P_t$ is larger than $E_{i,t}(P_{t+1}+Div_{t+1})$, the maximum price that an investor could quote is P_t. We can define the quoted price of investor i at trading time t $P_{i,t}^Q$ as:

$$P_{i,t}^Q=\begin{cases}P_t+X_1,\ if\ E_{i,t}(P_{t+1}+Div_{t+1})\ \geq\ (1+r_f)P_t\\ \frac{E_{i,t}(P_{t+1}+Div_{t+1})}{1+r_f}+X_2,\ if\ E_{i,t}(P_{t+1}+Div_{t+1})\ <\ (1+r_f)P_t\end{cases}, \tag{3}$$

where X_1 and X_2 are both random variables that $X_1 \sim U[0,\frac{E_{i,t}(P_{t+1}+Div_{t+1})}{1+r_f}-P_t]$ and $X_2 \sim U[0,P_t-\frac{E_{i,t}(P_{t+1}+Div_{t+1})}{1+r_f}]$. The wealth of investor i at trading time $t+1$ is defined as:

$$W_{i,t+1}=(1+r_f)W_{i,t}+[P_{t+1}+Div_{t+1}-(1+r_f)P_t]h_{i,t}, \tag{4}$$

where $h_{i,t}$ is the number of units of the stock that investor i holds at trading time t. In particular, the wealth of investors has two components: the value of risky assets, $(P_{t+1}+Div_{t+1})h_{i,t}$, and the value of the risk-free asset, $(1+r_f)(W_{i,t}-P_t h_{i,t})$.

Define λ as the degree of absolute risk aversion and $U(W_{i,t})=-\exp(-\lambda W_{i,t})$ as the utility function, and $V_{i,t}(.)$ as the investor i's forecast of variance given the information up to trading time t. R_{t+1} denotes the excess return at time $t+1$. To maximize the expected utility of investors, the optimal number of stocks $oh_{i,t}$ held by investor i at trading time t can be calculated as:

$$oh_{i,t}=\frac{E_{i,t}(P_{t+1}+Div_{t+1})-(1+r_f)P_t}{\lambda V_{i,t}(R_{t+1})}, \tag{5}$$

Therefore, the demand at this trading time $D_{i,t}$ is equal to the difference between the optimal holding $oh_{i,t}$ and the current holding $h_{i,t}$. Positive demand $(D_{i,t}>0)$ implies that the investor would like to purchase stocks, while negative demand $(D_{i,t}<0)$ means the investor would like to sell stocks.

$$D_{i,t}=oh_{i,t}-h_{i,t}, \tag{6}$$

Once all investors have generated their quoted prices and demands, they enter the market in random order to engage in trading activities. The order-matching process follows the procedure described below:

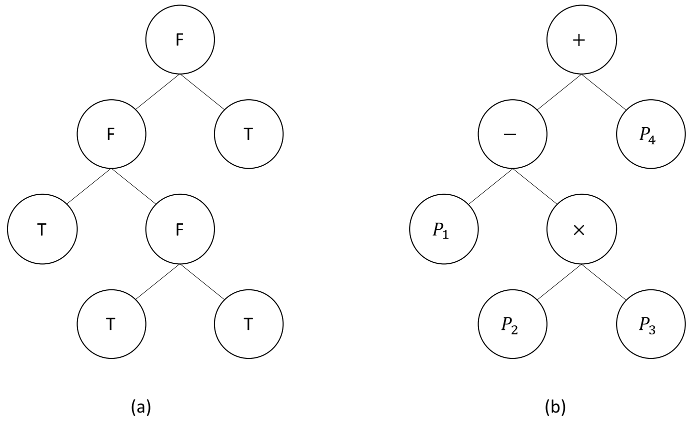
- *When investors wish to buy stock (demand > 0):*
  - *If the best ask price exists:*
    - *If the quoted price ≥ best ask price, investors will submit a market order and buy stocks at the best ask price*
    - *else(quoted price < best ask price), investors will submit a limit buy order with the quoted price*
  - *else (best ask price does not exist), investors will submit a limit buy order with the quoted price.*
- *When investors wish to sell stock (demand < 0):*
  - *If the best bid price exists:*
    - *If the quoted price ≤ the best bid price, investors will submit a market order and sell stocks at the best bid price*
    - *else (quoted price > best bid price), investors will submit a limit sell order with the quoted price*
  - *else (best bid price does not exist), investors will submit a limit sell order with the quoted price.*

At the end of each trading time, the unfulfilled limit orders will be canceled, and the last trade price will be recorded as the stock price for this trading time. In addition, the dividend in this trading time is paid to shareholders. Then the stock price, dividend, and total trading volume for this trading time are updated and disclosed to all investors publicly.

3.2. Operating mechanism of GP

Trading strategies based on GP are represented as a hierarchical tree structure. Figure 2(a) depicts an example hierarchical tree structure, consisting of terminal points (T) and function points (F). The terminal points are endpoints and include information on historical stock prices, dividends, and trading volumes, while the function points are link points that contain 18 mathematical operators. Detailed information about the content of the terminal and function points is given in Table 2. Panel (b) of Figure 2 provides an example that indicates $(P_1 - P_2 \times P_3) + P_4$.

Figure 2. Examples of the hierarchical tree structure



Notes. This figure shows examples of the hierarchical tree structure. Panel (a) presents an instance of a hierarchical tree structure featuring terminal points (T) and function points (F), whereas Panel (b) portrays the hierarchical tree structure of a specific example $(P_1 - P_2 \times P_3) + P_4$.

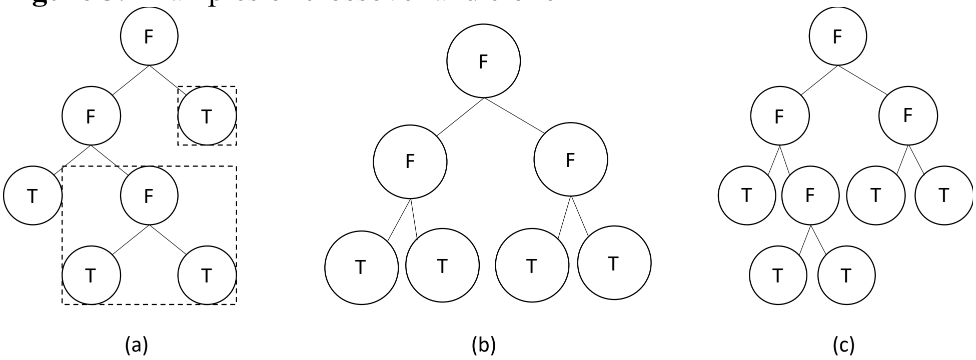
The trading strategy pool of each investor is updated via GP at the end of their learning cycle. The learning cycle adopted in this model aims to mimic the updated frequency of trading strategies of investors in stock markets. The learning cycle is presented by a certain period that differs across investors. A learning cycle of N means that this investor improves his or her trading strategies every N periods. In our model, investors have heterogeneous learning cycles that are randomly distributed between 5 and 100 trading times. More specifically, the procedure investors follow to update and choose their trading strategies at each trading time is divided into three steps:

First, if this trading time is at the end of investor i's learning cycle, the investor updates his or her trading strategy pool based on GP. Fitness is an evaluation criterion in GP to compare the effectiveness of trading strategies in HAMs. The fitness $f_{i,t,j}$ for investor i's trading strategy j in the pool, also called prediction error in this model, is defined as the difference between the expected price from the trading strategy j $P_{GP,j}$ and the stock price P_t, which can be expressed as

$$f_{i,t,j} = P_{GP,j} - P_t, \quad (7)$$

There are three genetic operations in the GP method: mutation, crossover, and cloning. The probability of adoption of each operation is defined in Table 2. Mutation involves modifying the information at a specific terminal point or function point. For example, if the original information at a terminal point is P_{t-1}, a mutation can change it to P_{t-2}. Mutations can also occur in function points, such as changing a minus sign to a plus sign. It is important to note that mutations only occur within terminal points and function points, respectively. For instance, P_{t-1} will not mutate into a minus sign. Specific information regarding terminal points and function points can be found in Table 2. Crossover refers to the exchange of two randomly selected subtrees. On the other hand, a clone indicates replicating a randomly selected subtree and using it to replace another randomly selected subtree. To clarify the operation mechanism of crossover and clone, a series of sample figures is used to illustrate their results. Figure 3(a) depicts the original tree, with dashed boxes indicating two randomly selected subtrees. If crossover occurs, the original tree (a) will transform into the new tree shown in (b). If a clone occurs and the subtree in the left dashed box is chosen to replace the subtree in the right dashed box, the original tree (a) will change to the new tree shown in (c).

Figure 3. Examples of crossover and clone



Notes. This figure presents examples of two genetic operations, namely crossover and cloning. Panel (a) depicts the original hierarchical tree structure, with dashed boxes indicating two randomly selected subtrees. Panels (b) and (c) respectively indicate the evolved hierarchical trees based on crossover and cloning.

The GP works as follows:

Generate a random number between 0 and 1 in the system:

If the random number < probability of clone

The tree structure of the original trading strategy is updated by a clone

Else

If the random number < probability of clone + probability of mutation

The tree structure is updated by mutating

Else

The tree structure is updated by crossover

Calculate the fitness of the updated trading strategy

Repeat the above procedure until all of the trading strategies in the pool are updated

Second, defining the fitness of the updated trading strategy as f_{i,t,j_new} and keeping the fitness of the original trading strategy as $f_{i,t,j}$, then, $|f_{i,t,j\_new}|$ and $|f_{i,t,j}|$ are compared. Investor i will keep the strategy with a lower absolute value of fitness in his or her trading strategy pool, which means that if the performance, measured by fitness, of the updated trading strategy is worse than the original one, the original trading strategy will be kept, while if the performance of the updated trading strategy is better, the original trading strategy will be dropped from the trading strategy pool. Therefore, at the end of this step, there are still two trading strategies in the trading strategy pool.

Third, the trading strategy in the trading strategy pool with the lowest absolute value of fitness $|f_{i,t,j}|$ is selected as the optimal trading strategy for the coming trading time, and the $f_{i,t,j}$ of the optimal trading strategy is selected as the optimal fitness for investor i at trading time t.

In summary, the evolutionary scheme of the trading strategy based on GP is a renewal process where the original tree is updated to a new tree through genetic operations. The optimal trading strategy of investor i at trading time t is determined by evaluating the fitness of trees in investor i's trading strategy pool. Once the optimal trading strategy at time t of each investor is determined, investors can generate their quoted prices and demands, as explained earlier.

3.3. Designs of the SPRs

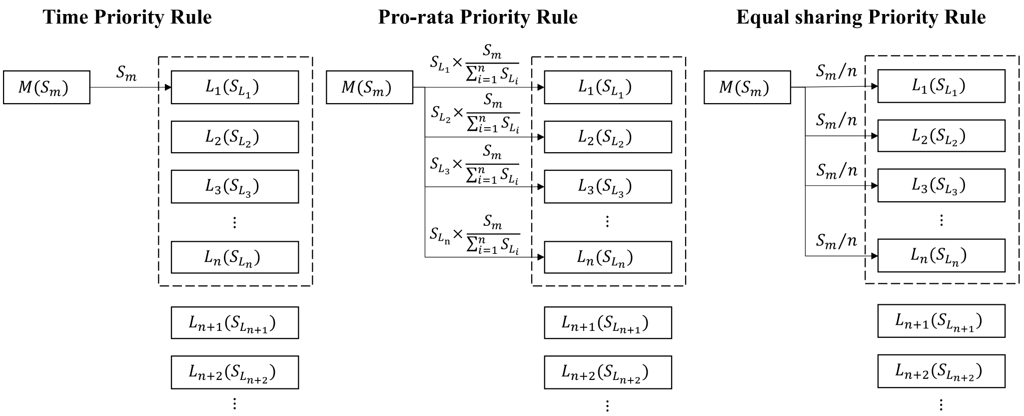
In this model, we set the price priority rule as the first-priority rule, i.e., the highest bid price and the lowest ask price are given precedence in transaction matching. If there is more than one order with the highest bid price and the lowest ask price, orders will be matched according to the SPRs. We implement time, pro-rata, and equal-sharing SPRs within the HAM framework and investigate their impacts on market quality. Our model expands the designs of order precedence rules by programming rich order matching algorithms, which are essential for well-functioning financial markets.

Assuming there are n limit orders $L_1, L_2, \dots, L_n$ with size $S_{L_1}, S_{L_2}, \dots, S_{L_n}$ respectively at the best price, under the time priority rule, a new market order M with size S_m is matched with the earliest submitted limit order L_1 at the best price. Under the pro-rata priority rule, the new market order M is matched with all limit orders at the best price $L_1, L_2, \dots, L_n$ on a pro-rata basis, with the specific match quantity of each limit order QP_{L_i} determined by equation (8). The equal-sharing priority rule also involves matching the new market order with every limit order at the best price, but allocations are equal for each order, and the matching quantity QE_L, which is the same for all limit orders at the best price, is determined by equation (9). Figure 4 illustrates how three SPRs work when a market order enters the market.

$$QP_{L_i} = \frac{S_m}{\sum_{i=1}^{n} S_{L_i}} \times S_{L_i}, \quad (8)$$

$$QE_L = \frac{S_m}{n}, \quad (9)$$

Figure 4. Work process of three SPRs



Notes. This figure depicts the working process of three SPRs, namely the time priority rule, the pro-rata priority rule, and the equal-sharing priority rule. *M* (*L*) respectively represent the market (limit) order. S_x denotes the order size of order x. The limit orders within the dashed boxes signify the limit orders with the best price.

SPRs not only govern order matching mechanics on trading platforms but also influence market quality through various strategies adopted by traders (Aspris, Foley, Harris, and O'Neill, 2015). The core of these strategies revolves around gaining higher priority in trading. To ensure the credibility of simulated results, it is crucial to mimic the trading strategies under each SPR. Based on the discussion in previous studies, the trading strategies involve accelerating quote speed, oversizing orders, and splitting orders under time, pro-rata, and equal-sharing priority rules, respectively. The design of a random quoting sequence could mimic the competition on speed under the time priority rule. As the demand determined in the basic model represents the optimal trading quantity, we further design trading strategies involving order oversizing and splitting under the pro-rata and equal-sharing priority rules, respectively. Under the pro-rata rule, traders increase their order sizes in proportion to the transaction ratio in the current market, subject to their available wealth constraints. For the equal-sharing rule, after determining the total demand at each time step, traders randomly divide their orders into 1 to 10 smaller orders in the simulation.

3.4. Model calibration

The parameter values used in this model are determined based on commonly used methods in previous studies on stock market-related HAMs (Yang, Zhang, and Ye, 2020). Initially, the parameter values are determined based on previous models. Then, the model is tested to see if it can reproduce uncontested phenomena observed in real stock markets, such as skewness and kurtosis of return. If the model fails to replicate these statistical properties, the parameter values are fine-tuned until the properties of the model are consistent with the desired statistical properties observed in real stock markets. However, previous studies have focused solely on macro-level stylized facts without considering trader behavior at the micro level. As a specific trading mechanism in stock markets, SPR has been empirically demonstrated to significantly influence the traders' trading strategies. Thus, the heterogeneous behaviors of traders under different SPRs constitute another important stylized fact that can enhance the reliability of our basic model. Thus, in addition to the macro-level stylized facts examined in prior research, we incorporate micro-level stylized facts, i.e., traders' behavior, to further strengthen the credibility of the model. The parameter values are determined such that the model successfully reproduces both macro-level market characteristics and micro-level trading behaviors observed in real stock markets.

The reliability of our model is confirmed by testing whether the statistical properties of the simulation at the macro level are consistent with those of real stock markets. We test skewness, kurtosis, range (represented by maximum return and minimum return), and the average of the absolute value of return ($|Return|$). The five indicators in Panel A of Table 3 are calculated based on the stock price across 5,000 trading times, which present the statistical properties of a typical simulation under the three SPRs, respectively.[4] The prevalent statistical properties of realistic stock markets are that returns present negative skewness and a high peak. For example, according to the summary of Yeh and Yang (2014), the skewness of the Dow Jones Industrial Average Index, the Nasdaq Composite Index, and the S&P 500 are −2.64, −0.53, and −2.02, respectively, while kurtosis is 77.03, 13.53, and 53.96, respectively. The maximum, minimum, and average of the absolute value of return for the seven well-known stock indexes range from 7.27% to 15.84%, -49.99% to -10.37%, and 0.70% to 1.07%, respectively. It is obvious that the statistical properties from the simulation are consistent with real-world stock markets. All results show negative skewness and a high peak. In addition, the maximal return, minimal return, and average return all fall within a reasonable range.

Table 3. Simulation statistical properties

Priority Rules / Indicator	Time	Pro-rata	Equal-sharing
Panel A: Macro-level			
Skewness	-1.52	-2.51	-2.01
Kurtosis	7.47	12.73	9.03
Maximum return (%)	10.02	13.72	8.42
Minimum return (%)	-11.30	-13.27	-13.36
|Return| (%)	0.87	0.87	0.94
Panel B: Micro-level			
Order size	2.75	9.31	1.49
Order number	77.36	66.28	310.34

Notes. This table reports the statistical properties from both macro-level (skewness, kurtosis, maximum return, minimum return, the average of the absolute value of return |Return|) and micro-level (order size and order number) of a typical simulation under the three SPRs. The results are based on a collection of data from 5,000 periods in simulated experiments.

We also examine micro-level stylized facts. As discussed above, trading strategies differ across different SPRs. Prior research suggests that investors have an incentive to submit larger orders under the pro-rata priority rule than under the time priority rule, while the number of quoted orders tends to increase under the equal-sharing priority rule. To assess whether trader behavior at the micro level in real stock markets can be closely replicated, we measure two indicators, namely order size and order number. Order size is defined as the average volume of orders quoted by each trader per period, while order number is calculated as the average count of unfilled bid and ask orders at the end of each period. Panel B of Table 3 shows that both order size and order number in the HAM simulations align with findings in the literature and with the expectations outlined above. Specifically, in the simulated experiments, traders quote larger orders under the pro-rata priority rule compared with the time priority rule, and under the equal-sharing priority rule, they quote more numerous orders. These results are consistent with trader behavior in real stock markets under different SPRs, thereby supporting the validity of the model.

[4] We check the statistical properties under other seeds, which are consistent with the statistical properties of real stock markets.

4. Results and Discussion

Consistent with previous studies, we treat market quality as a multi-dimensional concept encompassing market liquidity, price efficiency, and market volatility (Dai, Zhang, and Chang, 2025). Among these, trading activity, bid-ask spreads, and market depth are primary indicators of liquidity, while price efficiency reflects how closely transaction prices align with fundamental values. Market volatility is also relevant, though it can be considered as a secondary dimension. The potentially positive relationship between trading volume and market volatility suggests that increased volatility may, to some extent, accompany higher trading activity. Moreover, as noted by the NYSE, a key objective of reforming priority rules is to enhance market liquidity. Therefore, a moderate rise in volatility may be viewed as a trade-off for achieving a more active market. Accordingly, we consider an SPR to improve overall market quality if it promotes market liquidity and enhances price efficiency, even if this entails somewhat slightly higher volatility. Beyond the direct effects of SPRs on market quality, we also explore the underlying mechanisms through which SPRs influence market outcomes. Specifically, we examine these mechanisms from a micro-level perspective by analyzing the trading behavior of investors.

All baseline experiments have been conducted using the same 10 random seeds to ensure the reproducibility of the results. The experimental setup utilized the following hardware and software configurations: the Processor is a 1.8 GHz Dual-Core Intel Core i5, Memory is 8 GB 1600 MHz DDR3, and the Graphics are Intel HD Graphics 6000 1536 MB. The software environment employed was macOS Monterey, and the experiments were run using NetLogo (version 5.3.1).[5] The results in Tables 4 and 5 are based on a collection of data from 5,000 periods in each experiment, and the reported values represent the average results of 10 random seeds.[6] In order to compare and analyze market quality under each SPR, the statistical results obtained under the time priority rule are used as the benchmark. This choice is based on the fact that the time priority rule is widely adopted as the SPR in most stock markets.

To investigate the impacts of SPRs on trading behavior, we adopt order size, order number, and the aggressive degree as the main indicators. Order size and order number are calculated using the same methods as described for Table 3. However, the results presented in Table 4 reflect the average order size and order number across 10 random seeds, rather than a single typical simulation as shown in Table 3. The aggressive degree is measured as the average distance of the best bid and best ask prices from the stock price at the end of the trading period. Therefore, a smaller value shows that quotes are closer to the stock price, which corresponds to more aggressive pricing behavior.

[5] The experiment is suitable for most hardware environments and software environments with NetLogo (version 5.3.1).

[6] Each experiment runs 6,000 periods, while the first 1,000 periods are used to stabilize the market, and the corresponding results during that period are excluded from the analysis. For robustness, we also conduct additional simulations with different random seeds, and the results are quantitatively similar to those presented in Tables 4 and 5.

Table 4 presents the trading behavior of investors under different SPRs. Compared with the average order size of 2.602 under the time priority rule, traders in markets adopting the pro-rata priority rule tend to quote larger orders, averaging about 7.859. This result aligns with existing empirical evidence that traders oversize their quoted orders under the pro-rata priority rule to secure a higher execution proportion. In contrast, the equal-sharing priority rule primarily affects the number of orders quoted. As shown in the third row of Table 4, the order number under the equal-sharing priority rule is approximately five times that under the time and pro-rata priority rules. Beyond examining the impact of SPRs on trading behavior from the perspective of order characteristics, we also analyze quoted prices. The results regarding the aggressive degree in Table 4 suggest that both the pro-rata and equal-sharing priority rules encourage traders to quote more aggressively, with prices much closer to the stock price.

Table 4. Traders' behavior under different SPRs

Indicators	Secondary Priority Rules (Benchmark: Time priority rule)		
	Time	Pro-rata	Equal-sharing
Order size	2.602	7.859†††	1.371†††
	(0.031)	(0.119)	(0.018)
	Time	Pro-rata	Equal-sharing
Order number	73.12	63.90†††	325.6†††
	(0.126)	(0.141)	(0.535)
	Time	Pro-rata	Equal-sharing
Aggressive degree	0.051	0.024†††	0.041†††
	(0.002)	(0.001)	(0.001)

Notes. Entries report the mean across random seeds over 5,000 periods. Standard errors of the period-level means are reported in parentheses. ††† indicates that the mean under the reported SPR differs from the benchmark (time priority) at the 1% level, based on two-sided t-tests of period-level mean differences.

In addition, to highlight the connection between micro- and macro-level behavior under different trading rules, we conduct additional experiments that allow for the micro-level variables to vary to mimic different market scenarios, including order book oversizing and normal-sizing under the pro-rata priority rule, and orders are split and not split under the equal-sharing priority rule. The results shown in Appendix A demonstrate how the micro-level changes feed into the market level, thereby driving changes in market quality.

Table 5 presents the market qualities under the three SPRs. Two indicators are used to measure trading activity, which are trading volume and price volume. Trading volume is calculated as the average number of stocks traded at each time, while price volume is obtained by averaging the product of trading volume and stock price at each time. The equations for these two indicators are shown as follows:

$$Trading\ volume = \frac{1}{N_t}\sum_{t=1}^{N_t} TV_t, \tag{10}$$

$$Price\ volume = \frac{1}{N_t}\sum_{t=1}^{N_t}(TV_t \times P_t), \tag{11}$$

where N_t indicates the total number of trading times in each experiment, TV_t denotes the trading volume in trading time t, and P_t represents the stock price in trading time t. The higher values of trading volume and price volume indicate more active trading activities, reflecting higher market liquidity and better market quality.

Panel A of Table 5 presents the performance of trading activity under three SPRs. The analysis of trading volume and price volume discloses consistent patterns across the three SPRs. The pro-rata priority rule exhibits higher trading activity, which reflects greater market liquidity, compared to the time priority rule, as evidenced by increased trading volume and price volume. In detail, the adoption of the pro-rata priority rule leads to a substantial increase of 108.6% in trading volume and a significant improvement of 157.2% in price volume when compared to the time priority rule. On the other hand, the equal-sharing priority rule results in decreased trading activity, with trading volume and price volume reducing to 4.615 and 42.16, respectively, in contrast to 5.748 and 54.86 observed under the time priority rule. These findings collectively suggest that the pro-rata priority rule promotes the highest level of trading activity. The trading volume under diverse SPRs is associated with the quoted order size, which is determined by the trading strategies employed by traders. As supported by results in Table 4, since the execution size of limit orders under the pro-rata priority rule is uncertain, investors are incentivized to inflate the order size (Aldridge, 2014). In this context, the pro-rata priority rule could attract a greater volume (Jurich, 2020). In contrast, the equal-sharing priority rule undermines the advantages of large traders and further leads to smaller order sizes, thereby leading to a decrease in trading volume.

Table 5. Market quality under different SPRs

Panel A: Trading activity under different SPRs			
Indicators	Secondary Priority Rules (Benchmark: Time priority rule)		
	Pro-rata	Time	Equal-sharing
Trading volume	11.99†††	5.748	4.615†††
	(0.077)	(0.041)	(0.029)
	Pro-rata	Time	Equal-sharing
Price volume	141.1†††	54.86	42.16†††
	(1.002)	(0.461)	(0.288)
Panel B: Bid-ask spread under different SPRs			
	Pro-rata	Equal-sharing	Time
Quoted spread	0.042†††	0.075†††	0.093
	(0.001)	(0.002)	(0.003)
	Pro-rata	Equal-sharing	Time
Effective spread	0.043†††	0.079†††	0.102
	(0.001)	(0.002)	(0.003)
	Pro-rata	Equal-sharing	Time
%Quoted spread	0.453†††	0.793†††	0.899
	(0.012)	(0.018)	(0.021)
	Pro-rata	Equal-sharing	Time
%Effective spread	0.462†††	0.838†††	0.993
	(0.012)	(0.019)	(0.022)
Panel C: Market depth under different SPRs			
Market depth	Pro-rata	Time	Equal-sharing

	$17.87^{\dagger\dagger\dagger}$	3.460	$2.211^{\dagger\dagger\dagger}$
	(0.308)	(0.039)	(0.025)
Panel D: Price deviation under different SPRs			
	Pro-rata	Time	Equal-sharing
Price deviation	$44.91^{\dagger\dagger\dagger}$	54.13	$55.20^{\dagger\dagger\dagger}$
	(0.091)	(0.101)	(0.074)
Panel E: Market volatility under different SPRs			
	Time	Pro-rata	Equal-sharing
Rolling volatility	0.0128	$0.0140^{\dagger\dagger\dagger}$	$0.0143^{\dagger\dagger\dagger}$
	(0.000)	(0.000)	(0.000)

Notes. Entries report the mean across random seeds over 5,000 periods. Standard errors of the period-level means are reported in parentheses. ††† indicates that the mean under the reported SPR differs from the benchmark (time priority) at the 1% level, based on two-sided t-tests of period-level mean differences.

To analyze bid-ask spread, we employ four measurements: quoted spread, effective spread, percentage quoted spread (% quoted spread), and percentage effective spread (% effective spread). Let's define $m_t = (A_t + B_t)/2$, where A_t and B_t represent the best ask and bid prices at the end of trading time t, respectively. The bid-ask spread fundamentally measures the difference between the best ask price and the best bid price. Therefore, a smaller difference indicates lower transaction costs and higher market liquidity. These four measurements can be calculated as follows:

$$Quoted\ Spread = \frac{1}{N_t}\sum_{t=1}^{N_t}(A_t - B_t), \tag{12}$$

$$Effective\ Spread = \frac{1}{N_t}\sum_{t=1}^{N_t}(2 \times |P_t - m_t|), \tag{13}$$

$$Percentage\ Quoted\ Spread = \frac{1}{N_t}\sum_{t=1}^{N_t}\left(100 \times \left[\frac{A_t - B_t}{m_t}\right]\right), \tag{14}$$

$$Percentage\ Effective\ Spread = \frac{1}{N_t}\sum_{t=1}^{N_t}\left(200 \times \left[\frac{|P_t - m_t|}{m_t}\right]\right), \tag{15}$$

In terms of bid-ask spread performance, as shown in Panel B of Table 5, the pro-rata priority rule stands out as the most effective among the three SPRs. It achieves the smallest spread, reducing the quoted spread by 54.8% compared to time rules. Under the equal-sharing priority rule, the quoted spread decreases by 19.4% relative to the time priority rule. Similarly, the three other types of spread also experience reductions compared to the time priority rule. The observed differences in bid-ask spread can be attributed to the incentive for updating the best quoted price under different priority rules. Hersch (2023) points out that the time priority rule is not fair in practice. One of the most salient issues is that traders with time advantages, such as larger traders, typically possess more resources to ensure early arrival, which implies that these traders enjoy higher priority under the time priority rule (Perry and Zarsky, 2014). Nevertheless, the pro-rata priority rule reduces the priority of traders with time advantages by allocating market orders based on the limit order size. In this case, they have to share the market orders with other traders. Therefore, traders with time advantages would have a more vigorous motivation to update the best quoted price under the pro-rata priority rule than under the time priority rule. Analogously, the motivation for updating the best quoted price is also more intense under the equal-sharing priority rule than under the time priority rule, as traders are also required to share the market order under the equal-sharing priority rule. Thus, in comparison with the time priority rule, the pro-rata and equal-sharing priority rules can lead to a narrower bid-ask spread, which suggests superior market liquidity. This mechanism is also supported by the traders' behavior observed in our study, as shown in Table 4, where traders exhibit a higher level of aggressiveness under both the pro-rata priority rule and the equal-sharing priority rule.

Market depth is defined as the aggregate quantity of order size on the best ask price and best bid price at the end of each trading time. It is calculated as follows:

$$Market\ Depth = \frac{1}{N_t}\sum_{t=1}^{N_t}\left(\frac{AO_t+BO_t}{2}\right), \tag{16}$$

where AO_t and BO_t represent the aggregate quantity of order size on the best ask price and best bid price at the end of trading time t, respectively. A deep market depth suggests a higher level of stability, as it suggests that the market can accommodate relatively large market orders without causing significant price changes in security (Foucault, Pagano, and Röell, 2023). Therefore, a deeper market depth is associated with higher market liquidity and better market quality.

Panel C of Table 5 shows that the adoption of the pro-rata priority rule has a positive impact on market depth. The market depth increases significantly from 3.460 under the time priority rule to 17.87 under the pro-rata priority rule. In contrast, the equal-sharing priority rule reduces market depth compared with the time priority rule. Similar to the mechanism through which SPRs affect trading volume, market depth, defined as the aggregate quantity of unfilled order size at the best prices, is notably influenced by the quoted order size of traders. As shown in Table 4, order size increases under the pro-rata rule and decreases under the equal-sharing rule, leading to a corresponding rise and fall in market depth under these respective rules.

Price efficiency is assessed using price deviation P_D, which measures the difference between the stock price and the fundamental price. The fundamental price P_f is determined based on an average dividend $\overline{Div}$ of 0.2 and an interest rate r_f of 0.01, resulting in $P_f = \overline{Div}/r_f = 20$. The price deviation represents the average deviation of the stock price from the fundamental price. A lower price deviation is associated with higher price efficiency and better market quality. By defining t as the trading period and denoting N_t as the total trading periods in each experiment, which is 5000, the price deviation is calculated as follows:

$$P_D = \frac{100}{N_t}\sum_{t=1}^{N_t}\left|\frac{P_t-P_f}{P_f}\right|, \tag{17}$$

The results in Panel D of Table 5 reveal that the pro-rata priority rule achieves the smallest price deviation of 44.91, indicating a higher level of price efficiency compared to the time priority rule. The price deviation under the pro-rata priority rule improves by approximately 17.0% relative to the time priority rule. On the other hand, the equal-sharing priority rule shows a slight decline in price efficiency of around 2.0%. According to Chung and Chuwonganant (2023), continuous trading can drive a market to approach the fundamental price. The pro-rata priority rule, with its larger trading volume, leads to a lower price deviation, bringing prices closer to the fundamental value. In contrast, the equal-sharing priority rule, which has a smaller trading volume, results in prices that deviate further from the fundamental price.

Regarding market volatility, the average rolling standard deviation of return every 20 times is used as a measure. This indicator provides insights into the stability of the stock market, with lower rolling volatility values indicating a more stable market. The results presented in Panel E of Table 5 show that both the pro-rata priority rule and the equal-sharing priority rule show an increase in rolling volatility compared to the time priority rule. For the pro-rata priority rule, the rolling volatility changes from 0.0128 to 0.0140, while for the equal-sharing priority rule, from 0.0128 to 0.0143, compared with the time priority rule. These results suggest that neither the pro-rata priority rule nor the equal-sharing priority rule successfully stabilizes the market compared to the traditional time priority rule.

By comparing the effectiveness of the three SPRs, namely time priority, pro-rata priority, and equal-sharing priority, we have investigated market quality in terms of market liquidity (including trading activity, bid-ask spread, and market depth), price efficiency, and market volatility. Our results suggest that the pro-rata priority rule improves overall market quality relative to time priority as it could consistently increase trading activity, narrow spreads, deepen market depth, and enhance price efficiency. The observed moderate rise in volatility can be understood in the context of this heightened trading activity, considering the well-known positive relationship between trading volume and market volatility. The improved market quality under the pro-rata priority rule can be attributed to the specific trading strategies it induces. As shown in Table 4, this rule motivates traders to submit larger orders at more aggressive prices, which not only enhances market liquidity but also helps drive market prices closer to the fundamental value. In summary, compared to the traditional time priority rule commonly used in stock markets, our findings suggest that the pro-rata priority rule appears to enhance market performance in terms of both market liquidity and price efficiency, which suggests that pro-rata priority can be a promising alternative to time priority in equity limit-order-book settings, subject to market design objectives and implementation constraints.

On the other hand, the equal-sharing priority rule, although adopted by some financial markets as an SPR, shows significantly worse performance compared to both the time and pro-rata priority rules. The poorer market quality observed under the equal-sharing priority rule may stem from both its impact on trading strategies, i.e., order splitting and reduced order size, and its fundamental design flaw. The equal-sharing priority rule disregards order size when allocating execution at the best price. As a result, an investor submitting a one-share order receives the same expected execution probability as another posting a much larger order at the same price. This distortion incentivizes "pepper-noodle" strategies, defined as order-fragmentation (order-splitting) strategies that exploit equal-sharing allocation, under which traders fragment a desired quantity into many tiny orders not to improve price discovery, but simply to game the allocation mechanism. At the same time, large institutional investors face a disproportionately low execution probability relative to their order size, which diminishes their incentive to provide market depth. In summary, the equal-sharing rule may not be a suitable secondary priority rule for stock markets.

5. Sensitivity Analyses

Our basic results suggest that the pro-rata priority rule can enhance market liquidity in contrast to the traditional time priority rule. In this section, we further explore whether the pro-rata priority rule remains effective under different financial environments and, if so, which market conditions are better suited for the pro-rata priority rule. Three parameters, stock price, interest rate, and the wealth of each investor, are selected in this section, and there are two primary reasons for choosing these three parameters for sensitivity analyses. On the one hand, the parameter values used in the basic model have been calibrated to replicate the stylized facts in real stock markets, signifying that modifying the values of basic parameters might result in the invalidation of the model. These three parameters are associated with the external environment of stock markets and have no relation to the fundamental structure of the models. Hence, adjusting their values would not affect the reliability of the model. On the other hand, these parameters vary among different stock markets. The results could offer more customized advice for real stock markets with distinct features. In the basic model, settings for stock price, interest rate, and investor wealth are 20, 0.01, and 200, respectively. A higher initial stock price (30), a lower interest rate (0.005), and a greater initial wealth for each investor (400) are used for the sensitivity analyses. Five seeds are randomly chosen, and the average values across seeds are regarded as the final results[7].

Panel (a) of Figure 5 provides a visual comparison of the effectiveness of the three SPRs in a market featuring a higher stock price[8]. Taking the market quality under the time priority rule as the benchmark, i.e., on the x-axis, the positive and negative values respectively signify the superior and inferior performance of each indicator. Evidently, the performance of both the pro-rata and equal-sharing rules in the market with a higher stock price, represented by solid lines, is highly consistent with their performance in the basic experiment, represented by dotted lines. The results suggest the robustness of our basic findings. To further explore the sensitivity of the effectiveness of the pro-rata rule to stock prices, we compare the performance of the pro-rata rule in markets with a broader range of stock prices, namely 20, 40, 60, 80, and 100. The figure given in Appendix C depicts the performance trends of the main indicators along with the variations in stock prices. The results show that the performance of the pro-rata rule is consistent with the findings presented in Panel A of Figure 5 across a broader range of stock prices. However, as the stock price increases, the effectiveness of the pro-rata priority rule tends to decline gradually. This finding is rational as the higher trading volume under the pro-rata rule is strongly correlated with the trading strategy of oversizing orders. When the stock price is sufficiently high, the wealth of investors would constrain their oversized range and further restrain the efficacy of the pro-rata priority rule.

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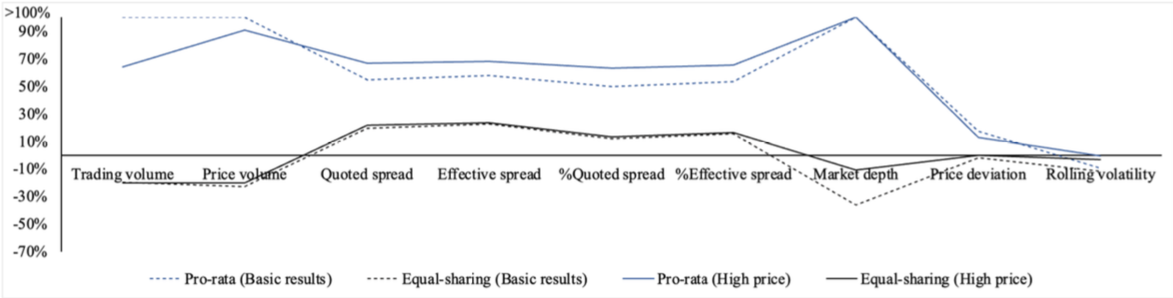
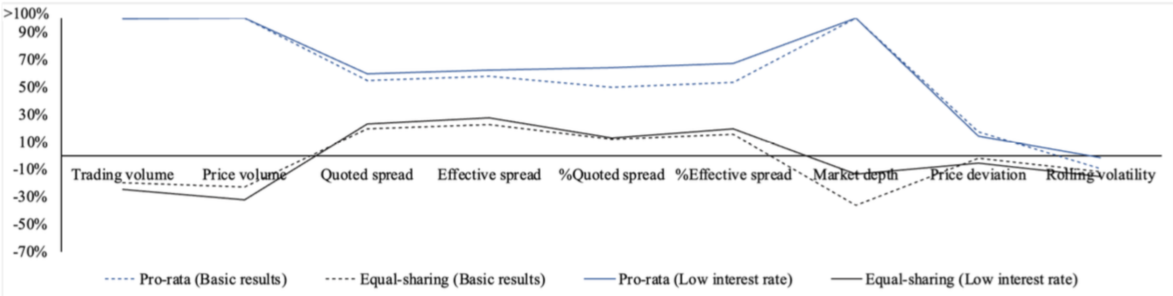
[7] Sensitivity analyses are averaged over 5 seeds due to computational cost; results are qualitatively unchanged when using 10 seeds (untabulated).

[8] See Appendix B for the specific values of each market indicator under different SPRs.

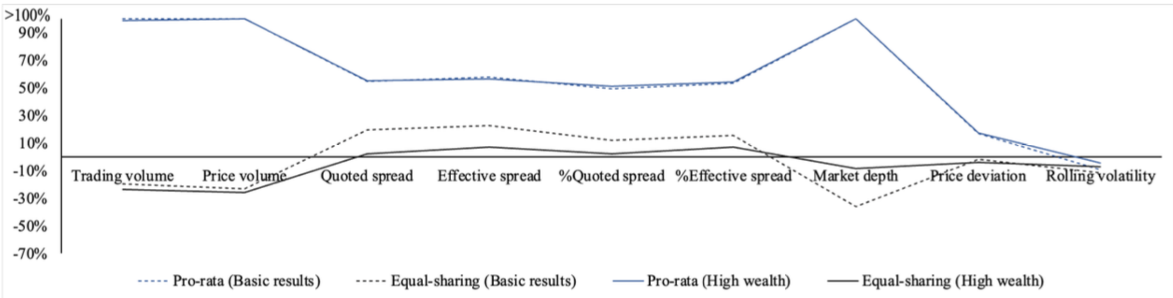
Figure 5. Market quality of sensitivity analyses



(a)



(b)



(c)

Notes. This figure illustrates the market quality under three SPRs across three distinct market scenarios. Panel (a) depicts the market quality under three SPRs in both the basic market and the market with a higher stock price. Panel (b) depicts the market quality under three SPRs in both the basic market and the market confronted with a low interest rate. Panel (c) depicts the market quality under three SPRs in both the basic market and the market where each investor has higher wealth. The *x*-axis represents the performance of the time priority rule, while the lines signify the performance of the pro-rata and equal-sharing priority rules. The *y*-axis indicates the degree of improvement or deterioration of each indicator. Specifically, positive (negative) values imply higher (lower) trading volume, higher (lower) price volume, narrower (wider) quoted spread, narrower (wider) effective spread, narrower (wider) percentage quoted spread, narrower (wider) percentage effective spread, deeper (shallower) market depth, smaller (larger) price deviation, and smaller (larger) rolling volatility.

Panel (b) of Figure 5 presents the market quality under three SPRs when the interest rate is low.[9] The outcomes suggest that the pro-rata priority rule can still increase trading volume, narrow bid-ask spread, and reduce price deviation, in contrast to the time priority rule. Both

[9] See Appendix D for the specific values of each market indicator under different SPRs.

the dotted lines and solid lines exhibit highly similar trends, thereby verifying the reliability of the basic results. We also explore the efficacy of the pro-rata priority rule under various interest rates. The figure given in Appendix E shows the performance of the pro-rata priority rule under interest rates of 0.01, 0.015, 0.02, and 0.025. The outcomes in this figure once more manifest that the pro-rata priority rule remains more effective than the time priority rule under different interest rates. However, when the interest rate rises from 0.025 to 0.03, the trading in the stock market comes to a standstill. This is because when the interest rate is sufficiently high, the returns from risk-free assets would definitely be higher than those from stocks. In this situation, all investors would opt to invest in risk-free assets rather than stocks.

In the market where traders have higher wealth, the results presented in Panel (c) of Figure 5 show that the performance of the pro-rata priority rule still surpasses the time priority rule and the equal-sharing priority rule.[10] It is manifest that the pro-rata priority rule can still enhance trading activity, narrow the bid-ask spread, and reduce price deviation, which is in line with our fundamental findings. We have also compared the efficacy of the pro-rata rule in markets where investors have different initial wealth levels ranging from 200 to 1000, with an increment of 200. Nevertheless, when the wealth exceeds 400, there is no alteration in the market performance regardless of the wealth value. This phenomenon is rational as investors' demands for stock are determined by the stock performance rather than the investors' financial conditions. The results suggest that a wealth of 400 is sufficient to meet the trading demand for stocks for all investors in this experiment. In this situation, even if investors possess more wealth, they will not invest in the stock.

Overall, the sensitivity analyses depicted in Figure 5 validate the qualitative outcomes presented in Section 4. Generally, the pro-rata priority rule outperforms the time priority rule in terms of market liquidity and price efficiency, while the equal-sharing priority rule has a detrimental impact on market quality. Additionally, we conduct a further investigation into the application conditions of pro-rata priority rules. Our discoveries imply that the efficacy of the pro-rata priority rule appears to be sensitive to stock price, rather than to the interest rate and the wealth of investors. Specifically, the effectiveness of the pro-rata priority rule would be undermined when the stock price is high.

6. Conclusion

The sophisticated system of pro-rata and equal-sharing priority rules restricts their large-scale applications in financial markets, particularly in stock markets, despite the fact that they have been theoretically demonstrated to be superior to the time priority rule in certain aspects. Currently, the enhancement of computational power undermines the advantage of simplicity of the time priority rule and makes the use of the pro-rata and equal-sharing priority rule in stock markets possible. For instance, the NYSE reforms the SPR into parity, which is a combination of time and pro-rata priority rules. Nevertheless, the reforms of SPRs in stock markets are in their infancy, and the reforms mainly focus on the combination of time and pro-rata priority rules, making it challenging to separately identify the effectiveness of pure time

[10] See Appendix F for the specific values of each market indicator under different SPRs.

and pure pro-rata priority rules. Therefore, in this paper, we compare the effectiveness of time, pro-rata, and equal-sharing priority rules in stock markets based on a HAM.

Comparing market quality under the time priority rule, we discover that the pro-rata priority rule, in general, can further enhance market quality by achieving higher liquidity and more efficient asset pricing. In contrast, the equal-sharing priority rule has a detrimental impact on market quality. Furthermore, we explore the degree of effectiveness of the pro-rata rule under diverse market conditions. Specifically, we find that the pro-rata priority rule remains more effective than the time priority rule regardless of the values of investors' wealth and the interest rate. However, the effectiveness of the pro-rata priority rule diminishes with the increase in stock price. These findings suggest that the pro-rata priority rule is not invariably superior to the time priority rule in stock markets but is dependent on the specific market circumstances. Thus, it is necessary to adjust the parameter values in the HAM in accordance with the specific stock market to reevaluate the performance of the pro-rata priority rule if a certain stock market intends to reform its SPR. Our results provide new insights into the relationship between SPRs and market quality and enrich the market microstructure and HAM literature.

Appendix

Appendix A. Market quality under different trading behavior

SPRs and trading behavior / Indicators	Time	Pro-rata (without oversize)	Pro-rata (oversize)	Equal-sharing (without split)	Equal-sharing (split)
Trading volume	5.748	6.147	11.99	4.965	4.615
Price volume	54.86	77.47	141.1	56.14	42.16
Quoted spread	0.093	0.042	0.042	0.081	0.075
Effective spread	0.102	0.043	0.043	0.085	0.079
% Quoted spread	0.899	0.435	0.453	0.804	0.793
% Effective spread	0.993	0.448	0.462	0.848	0.838
Market depth	3.460	2.135	17.87	1.416	2.211
Price deviation	54.13	48.01	44.91	54.20	55.20
Rolling volatility	0.0128	0.0137	0.0140	0.0139	0.0143

Notes. Entries report the mean across random seeds.

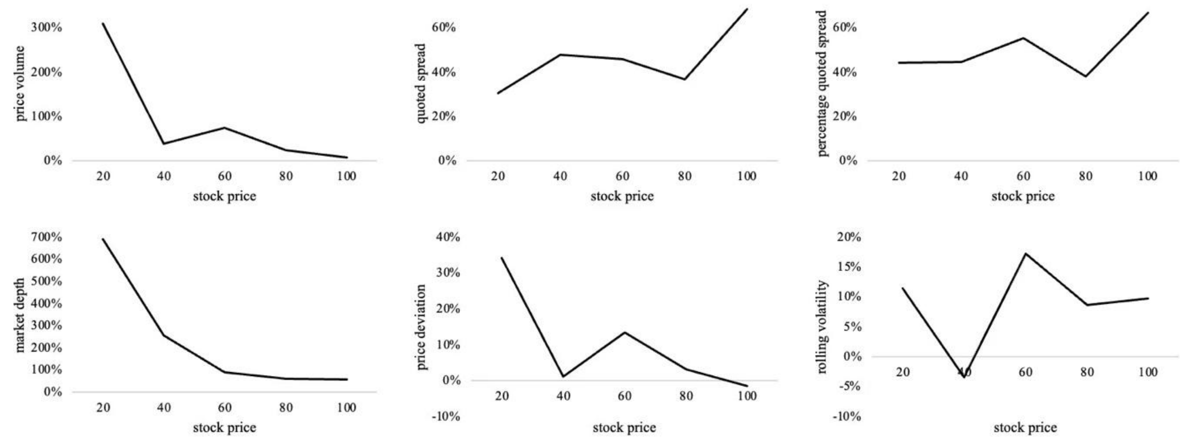
Appendix B. Sensitivity analyses of high stock price

Indicators	Secondary Priority Rules (Benchmark: Time priority rule)		
Trading volume	Pro-rata 11.79††† (0.097)	Time 7.188 (0.067)	Equal-sharing 5.729††† (0.049)
Price volume	Pro-rata 199.7†† (1.841)	Time 104.6 (1.161)	Equal-sharing 83.29††† (0.877)
Quoted spread	Pro-rata 0.059†††	Equal-sharing 0.139†††	Time 0.177

	(0.002)	(0.005)	(0.007)
	Pro-rata	Equal-sharing	Time
Effective spread	0.061†††	0.147†††	0.192
	(0.002)	(0.005)	(0.007)
	Pro-rata	Equal-sharing	Time
%Quoted spread	0.418†††	0.986†††	1.133
	(0.011)	(0.029)	(0.036)
	Pro-rata	Equal-sharing	Time
%Effective spread	0.430†††	1.040†††	1.241
	(0.012)	(0.031)	(0.036)
	Pro-rata	Time	Equal-sharing
Market depth	4.474†††	1.205	1.075†††
	(0.139)	(0.013)	(0.011)
	Pro-rata	Time	Equal-sharing
Price deviation	46.98†††	53.74	53.83
	(0.119)	(0.145)	(0.139)
	Time	Pro-rata	Equal-sharing
Rolling volatility	0.0149	0.0150	0.0154†††
	(0.000)	(0.000)	(0.000)

Notes. Entries report the mean across random seeds over 5,000 periods. Standard errors of the period-level means are reported in parentheses. ††, ††† indicate that the mean under the reported SPR differs from the benchmark (time priority) at the 5% and 1% levels, respectively, based on two-sided t-tests of period-level mean differences.

Appendix C. The effectiveness of the pro-rata rule under markets with different stock prices



Notes. This figure depicts the effectiveness of the pro-rata rule under markets with stock prices of 20, 40, 60, 80, and 100. The *x*-axis represents the performance of the time priority rule, while the lines signify the performance of the pro-rata priority rule. The *y*-axis indicates the degree of improvement or deterioration of each indicator. Specifically, positive (negative) values imply higher (lower) price volume, narrower (wider) quoted spread, narrower (wider) percentage quoted spread, deeper (shallower) market depth, smaller (larger) price deviation, and smaller (larger) rolling volatility.

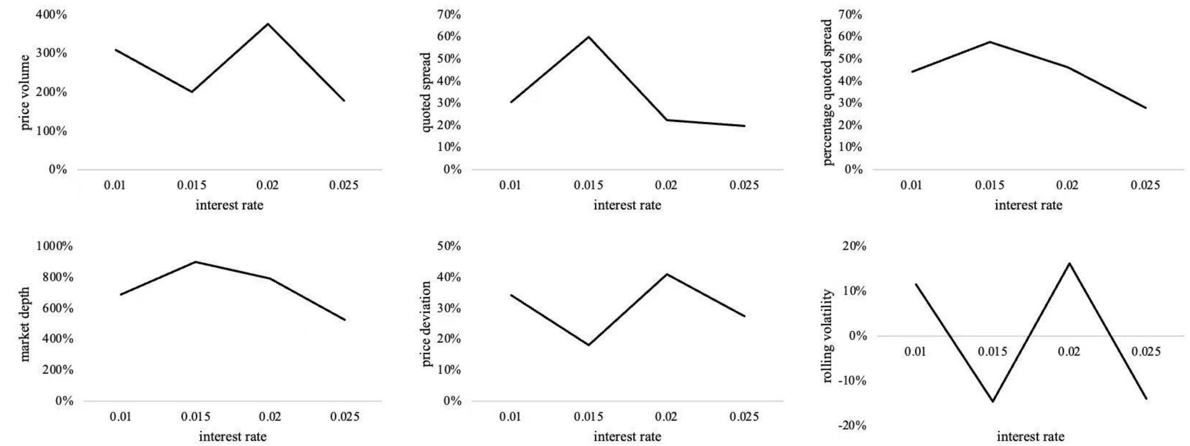
Appendix D. Sensitivity analyses of low interest rate

Indicators	Secondary Priority Rules (Benchmark: Time priority rule)		
Trading volume	Pro-rata	Time	Equal-sharing

	12.42†††	6.213	4.688†††
	(0.101)	(0.062)	(0.045)
	Pro-rata	Time	Equal-sharing
Price volume	203.0†††	78.73	53.37†††
	(1.803)	(0.923)	(0.639)
	Pro-rata	Equal-sharing	Time
Quoted spread	0.049†††	0.093†††	0.121
	(0.002)	(0.003)	(0.005)
	Pro-rata	Equal-sharing	Time
Effective spread	0.051†††	0.098†††	0.135
	(0.002)	(0.004)	(0.005)
	Pro-rata	Equal-sharing	Time
%Quoted spread	0.357†††	0.861†††	0.989
	(0.012)	(0.027)	(0.035)
	Pro-rata	Equal-sharing	Time
%Effective spread	0.370†††	0.904†††	1.122
	(0.013)	(0.029)	(0.039)
	Pro-rata	Time	Equal-sharing
Market depth	7.218†††	2.662	2.297†††
	(0.220)	(0.044)	(0.046)
	Pro-rata	Time	Equal-sharing
Price deviation	59.68†††	69.39	73.27†††
	(0.122)	(0.112)	(0.101)
	Time	Pro-rata	Equal-sharing
Rolling volatility	0.0129	0.0131†	0.0149†††
	(0.000)	(0.000)	(0.000)

Notes. Entries report the mean across random seeds over 5,000 periods. Standard errors of the period-level means are reported in parentheses. †, ††† indicate that the mean under the reported SPR differs from the benchmark (time priority) at the 10% and 1% levels, respectively, based on two-sided t-tests of period-level mean differences.

Appendix E. The effectiveness of the pro-rata rule under markets with different interest rates



Notes. This figure depicts the effectiveness of the pro-rata rule under markets with interest rates of 0.01, 0.015, 0.02, and 0.025. The *x*-axis represents the performance of the time priority rule, while the lines signify the performance of the pro-rata priority rule. The *y*-axis indicates the degree of improvement or deterioration of each indicator. Specifically, positive (negative) values imply higher (lower) price volume, narrower (wider) quoted spread, narrower (wider) percentage quoted spread, deeper (shallower) market depth, smaller (larger) price deviation, and smaller (larger) rolling volatility.

Appendix F. Sensitivity analyses of high initial wealth

Indicators	Secondary Priority Rules (Benchmark: Time priority rule)		
	Pro-rata	Time	Equal-sharing
Trading volume	12.07†††	6.066	4.623†††
	(0.102)	(0.061)	(0.043)
	Pro-rata	Time	Equal-sharing
Price volume	149.2†††	61.02	45.21†††
	(1.428)	(0.696)	(0.525)
	Pro-rata	Equal-sharing	Time
Quoted spread	0.040†††	0.087	0.089
	(0.002)	(0.004)	(0.004)
	Pro-rata	Equal-sharing	Time
Effective spread	0.042†††	0.090	0.097
	(0.002)	(0.004)	(0.004)
	Pro-rata	Equal-sharing	Time
%Quoted spread	0.418†††	0.840	0.857
	(0.016)	(0.027)	(0.032)
	Pro-rata	Equal-sharing	Time
%Effective spread	0.433†††	0.875	0.943
	(0.017)	(0.028)	(0.032)
	Pro-rata	Time	Equal-sharing
Market depth	20.54†††	3.030	2.766†††
	(0.538)	(0.042)	(0.043)
	Pro-rata	Time	Equal-sharing
Price deviation	43.31†††	52.47	54.55†††
	(0.157)	(0.133)	(0.121)
	Time	Pro-rata	Equal-sharing
Rolling volatility	0.0129	0.0135†††	0.0138†††
	(0.000)	(0.000)	(0.000)

Notes. Entries report the mean across random seeds over 5,000 periods. Standard errors of the period-level means are reported in parentheses. ††† indicates that the mean under the reported SPR differs from the benchmark (time priority) at the 1% level, based on two-sided *t*-tests of period-level mean differences.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this article.

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Data availability

Data will be made available on request.

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