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The Role of Price-Volatility Cojumps in Volatility Forecasting

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ABSTRACT

This paper investigates whether simultaneous jumps in prices and volatility improve volatility forecasting. Using up-to-date high-frequency S&P 500 and VIX data, we identify price-volatility cojumps at the intraday granularity and construct upside, downside, and asymmetric measures. Embedding these into the Heterogeneous Autoregressive (HAR) model, we provide new empirical evidence that downside cojumps increase future volatility, upside cojumps reduce volatility. Out-of-sample analysis further shows that incorporating these impacts of cojumps significantly enhances HAR model forecasting performance. Moreover, our results reveal that recent price jumps become important predictors of volatility when accompanied by simultaneous volatility jumps, an effect not previously documented in the literature. Finally, we also document the economic interpretation, policy implications, and economic value of price-volatility cojumps.

JEL Classification: G10, C58

1 | Introduction

A growing strand of the literature documents strong evidence of simultaneous jumps in equity prices and volatility, known as price-volatility cojumps. Todorov and Tauchen (2011) and Bandi and Reno (2016) provide systematic evidence of such cojumps in equity markets. Bibinger et al. (2019) report a significant discontinuous leverage effect in NASDAQ firms. Andersen et al. (2015) exploit a novel "corridor implied volatility" index to show that volatility jumps are common and that their co-movement with returns is more pronounced than previously recognized. Aït-Sahalia et al. (2017) distinguish between continuous and discontinuous leverage effects, demonstrating that the latter accounts for a substantial share of the overall leverage effect.

Because prior evidence on price-volatility cojumps is based on earlier samples, we begin our analysis by reassessing whether price-volatility cojumps remain a stylized fact using stock market data extended through September 2025. Using high-frequency data for the S&P 500 and the CBOE Volatility Index (VIX), we let the data speak and estimate intraday cojumps using the nonparametric measures of Jacod and Todorov (2009). Consistent with existing research (Tauchen and Zhou 2011; Jacod et al. 2017), we document the continued presence of price-volatility cojumps throughout our sample from 2003 to 2025. Our results confirm that price-volatility cojumps continue to be observed as a stylized fact in equity markets.

While most of the literature focuses on the statistical detection of price-volatility cojumps, less attention has been paid to their economic underpinnings (Todorov and Tauchen 2011; Bandi and Reno 2016; Aït-Sahalia et al. 2017; Bibinger et al. 2019; Bollerslev and Todorov 2023). To address this gap, we investigate whether cojumps are linked to macroeconomic and financial conditions. Specifically, we relate the estimated cojump series to measures of business cycles, policy uncertainty, and credit and liquidity conditions. Our regression analysis reveals that cojumps intensify during recessions and periods of elevated policy uncertainty. Credit and liquidity conditions also matter, with the default spread, illiquidity, and TED spread all positively connected to cojumps. These results indicate that cojumps may reflect systemic shocks in fragile markets, which help reconcile the findings that not all jumps coincide with volatility jumps in prior studies (Tauchen and Zhou 2011; Jacod et al. 2017) and interpret the evidence that price-volatility cojumps command a nontrivial equity risk premium (Bollerslev and Todorov 2023). Moreover, we also show that, by capturing persistent uncertainty and market stress, price-volatility cojumps have important implications for margin regulation and systemic risk monitoring for policymakers.

The strong link between price-volatility cojumps and market stress suggests that cojumps may also capture shocks with lasting effects on future volatility. Forecasting volatility is a central task in financial econometrics, with important implications for asset pricing (Bollerslev et al. 2009; Engle et al. 2013), option valuation (Black and Scholes 1973), and market risk management (Christoffersen and Diebold 2000). According to the leverage effect of Black (1976), equity volatility reacts more strongly to negative price moves than to positive ones. We therefore construct upside and downside cojump measures, defined as cojumps associated with positive and negative price jumps, respectively. To capture whether volatility predictability is dominated by downside or upside cojumps, we propose a cojump asymmetry measure based on the difference between upside and downside cojumps.

We assess the impact of the cojump variables we constructed on volatility forecasting within the Heterogeneous Autoregressive (HAR) framework of Corsi (2009). Our results show that downside cojumps significantly increase future volatility while upside cojumps reduce volatility, with asymmetric cojumps negatively associated with volatility. These effects are robust to alleviating spurious jump detection due to price staleness and to addressing the loss of jump-detection power arising from non-zero drift in high-frequency data, as well as to alternative HAR model transformations based on variance, standard deviation, and logarithmic volatility.

Our out-of-sample evaluations confirm that models incorporating cojumps yield significantly superior predictive performance compared with the standard HAR model and its jump-augmented extensions in prior research. Importantly, we find that these volatility forecast gains translate into significantly more accurate Value-at-Risk (VaR) forecasts, a key input into regulatory capital and margin requirements, underscoring the economic value of price-volatility cojumps.

We further uncover that the predictive power of both downside and upside cojumps depends critically on the sign of volatility jumps. Cojumps characterized by rising prices and falling volatility are associated with lower future volatility, consistent with episodes of uncertainty resolution, whereas cojumps featuring falling prices and rising volatility predict higher subsequent volatility, reflecting adverse shocks that heighten market fears. By contrast, cojumps in which prices and volatility move in the same direction carry little predictive content, suggesting that offsetting economic forces limit their impact on future volatility.

Existing evidence shows that price jumps have no statistically significant impact on volatility forecasting, with studies consistently reporting a lack of predictive power (Andersen, Bollerslev, and Diebold 2007; Busch et al. 2011; Audrino and Hu 2016; Prokopczuk et al. 2016; Caporin 2023). In lieu of this, we find that price jumps can significantly predict volatility when they coincide with contemporaneous volatility jumps. We also find that such price jumps significantly improve out-of-sample forecasts. This finding aligns with the intuition that price jumps linked to volatility jumps reflect heightened perceived risk.

This paper also adds to the growing body of literature on the role of the VIX in volatility forecasting. Prior studies document the superiority of the VIX in predicting equity market volatility (Busch et al. 2011; Bekaert and Hoerova 2014; Pati et al. 2018; Kambouroudis and McMillan 2016; Kambouroudis et al. 2021; Dutta and Das 2022; Wu et al. 2023; Pan et al. 2024; Jiang et al. 2025), though most directly use the index as a predictor. The VIX itself exhibits large jumps, particularly those accompanied by jumps in the S&P 500 index (Todorov and Tauchen 2011; Bandi and Reno 2016; Bollerslev and Todorov 2023). Such cojumps generate a substantial risk premium (Bollerslev and Todorov 2023), and should therefore translate into elevated volatility. We extend this literature by moving beyond the direct use of VIX levels and demonstrating that VIX jump components concurrent with equity jumps contain incremental predictive power for equity volatility, thereby uncovering a new channel through which the VIX forecasts market volatility.

Our analysis is closely connected to Jiang et al. (2025), who study the predictability of volatility from price-volatility cojumps. Jiang et al. (2025) estimate S&P 500-VIX cojumps by first identifying daily jumps in each series and then assessing their concurrence using the co-exceedance rule. By contrast, we identify cojumps using the nonparametric estimator of Todorov and Tauchen (2011), which registers cojumps only when price and volatility jumps take place within the same intraday interval. This intraday precision is crucial for accurately capturing instantaneous jump events and may more accurately reflect their impact on volatility forecasts. Hence, relative to daily co-exceedance methods, our approach provides an improved identification of cojumps and a clearer assessment of their forecasting power.

The remainder of the paper is organized as follows. Section 2 outlines the theoretical background and the nonparametric estimation of price jumps and price-volatility cojumps. Section 3 presents descriptive statistics of the volatility and cojump estimators. Section 4 examines the impact of jumps and cojumps on volatility forecasting, both in-sample and out-of-sample. Section 5 conducts further analysis on the effects of jumps and cojumps on volatility forecasting. Section 6 reports robustness checks, and Section 7 concludes.

2 | Volatility, Jump, and Cojump Variations

2.1 | Set-Up and Volatility Estimation

Let X_t denote the logarithmic equity price at time t. Its dynamics are conventionally expressed as following components:

$$X_t = \int_{t-1}^{t} \mu_s \mathrm{d}s + \int_{t-1}^{t} \sigma_s \mathrm{d}W_s + J_t, \tag{1}$$

where μ denotes a locally bounded drift coefficient, σ is the stochastic volatility process, W is a standard Brownian motion, and J_t is the sum of all jump price movements up to time t, defined as $J_t = \sum_{s\in[t-1,t)} \Delta X_s^J$, with ΔX_s^J representing a jump at time s. The quadratic variation of returns from $t-1$ to t is given by,

$$QV_t = \int_{t-1}^{t} \sigma_s^2 \mathrm{d}s + \sum_{s\in[t-1,t)} \left(\Delta X_s^J\right)^2, \tag{2}$$

where $C_t = \int_{t-1}^{t} \sigma_s^2 \mathrm{d}s$ is the continuous component of quadratic variation, and $\sum_{s\in[t-1,t)} \left(\Delta X_s^J\right)^2$ represents the jump variation.

Let X_{t_i} be observed at n equally spaced intervals within trading day t, such that $0 = t_0 < t_1 < t_2 < \ldots < t_n = t$. For ease of notation, we normalize the daily time interval to unity, making the time gap between consecutive observations as $\Delta = t_i - t_{i-1} = 1/n$. The corresponding intraday returns are defined as $\Delta X_{t_i} = X_{t_i} - X_{t_{i-1}}$, for $i = 1, 2, \ldots, n$. As demonstrated by Andersen and Bollerslev (1998), realized variance (RV), estimated by the sum of squared intraday returns, provides an asymptotically consistent estimator of the quadratic variation of day t:

$$RV_t = \sum_{i=1}^{n} r_{t_i}^2 \to \int_{t-1}^{t} \sigma_s^2 \mathrm{d}s + \sum_{s\in[t-1,t)} \left(\Delta X_s^J\right)^2, \tag{3}$$

where $\to$ denotes convergence in probability. However, RV is unable to distinguish between upside and downside risk. To address this, Barndorff-Nielsen et al. (2010) propose realized semivariances, where the upside (downside) realized semivariance is computed from the sum of squared positive (negative) intraday returns, thereby capturing upside (downside) risk. They further show that the limiting behavior of realized semivariance consists of one-half of the continuous variation in the price process together with the jump component.

$$RS_t^+ = \sum_{i=1}^{n} \Delta X_{t_i}^2 I(\Delta X_{t_i} > 0) \to 0.5\int_{t-1}^{t} \sigma_s^2 \mathrm{d}s + \sum_{s\in[t-1,t)} \left(\Delta X_s^J\right)^2 I\left(\Delta X_s^J > 0\right),$$

$$RS_t^- = \sum_{i=1}^{n} \Delta X_{t_i}^2 I(\Delta X_{t_i} < 0) \to 0.5\int_{t-1}^{t} \sigma_s^2 \mathrm{d}s + \sum_{s\in[t-1,t)} \left(\Delta X_s^J\right)^2 I\left(\Delta X_s^J < 0\right).$$

If the objective is to measure the integrated variance only, RV and semivariances may not be ideal, as both capture jump components. To improve robustness, Barndorff-Nielsen and Shephard (2004) introduce bipower variation (BV), which converges to the continuous component of variation as the observation interval becomes small,

$$BV_t = 1.57\sum_{i=2}^{n} |\Delta X_{t_i}||\Delta X_{t_{i-1}}| \to \int_{t-1}^{t} \sigma_s^2 \mathrm{d}s. \tag{4}$$

Intuitively, BV is robust to jumps because a jump inflates only one of two consecutive return terms, so their product is far less affected than in RV or semivariance, where the jump is squared.

2.2 | Jump Variation Estimation

2.2.1 | *Jump Variation Estimation by BV*

Jumps are essential in financial modeling because they capture sudden price movements that continuous diffusions cannot. They are key drivers of risk premia (Bollerslev and Todorov 2011), and play an important role in asset pricing (Eraker et al. 2003) and derivative valuation (Hu and Liu 2022). Recent limit theory in financial econometrics facilitates statistical inference on jump components by contrasting estimators that are robust to jumps with those that are not. In particular, Barndorff-Nielsen and Shephard (2004) show that the difference between RV and robust BV provides an intuitive measure of jump variation,

$$JV_t = RV_t - BV_t \to \sum_{s\in[t-1,t)} \left(\Delta X_s^J\right)^2. \tag{5}$$

The limit results of Barndorff-Nielsen et al. (2010) further show that the discrepancy between realized semivariance and one-half of BV can be used to identify upside and downside jumps,

$$JV_t^+ = RS_t^+ - 0.5BV_t \to \sum_{s\in[t-1,t)} \left(\Delta X_s^J\right)^2 I\left(\Delta X_s^J > 0\right), \tag{6}$$

$$JV_t^- = RS_t^- - 0.5BV_t \to \sum_{s\in[t-1,t)} \left(\Delta X_s^J\right)^2 I\left(\Delta X_s^J < 0\right). \tag{7}$$

In volatility forecasting experiments, Patton and Sheppard (2015) construct an estimator of asymmetric jump variation (AJV) to capture the difference between upside and downside jump components,

$$AJV_t = JV_t^+ - JV_t^- \to \sum_{s\in[t-1,t)} \left(\Delta X_s^J\right)^2 I\left(\Delta X_s^J > 0\right) - \sum_{s\in[t-1,t)} \left(\Delta X_s^J\right)^2 I\left(\Delta X_s^J < 0\right). \tag{8}$$

2.2.2 | *Jump Variation Estimation by High-Order Power Variation*

In the previous section, we used BV to assess the presence of jumps. A key difficulty with this approach is that BV is not the only integrated variance estimator, and the resulting jump estimates may vary across alternative estimators (Andersen, Bollerslev, and Dobrev 2007; Mancini 2009; Corsi et al. 2010; Andersen et al. 2012). Recent theoretical developments suggest that power variations can estimate jumps without relying on an integrated variance estimator. In particular, high-order realized power variation (RP), defined using powers greater than two of absolute intraday log returns, provides an measure of jump power variation,

$$RP_t^p = \sum_{i=1}^{n} |\Delta X_{t_i}|^p \to \sum_{s\in[t-1,t)} |\Delta X_s^J|^p,$$

where $p > 2$ denotes the power of log returns. Following Duong and Swanson (2015), we adopt $p = 4$ to balance the trade-off between enhancing sensitivity to jumps and avoiding the slow convergence and high variance associated with very high powers.[1] In line with the high-frequency econometrics literature, we assume that at most one large jump can occur within a trading day.[2] This assumption implies that $\left(\sum_{s\in[t-1,t)}|\Delta X_s^J|^4\right)^{1/2} \approx \sum_{s\in[t-1,t)}|\Delta X_s^J|^2$, which allows the jump variation to be measured by taking the square root of the fourth-order realized power estimator:

$$JP_t = \left(\sum_{i=1}^{n}|\Delta X_{t_i}|^4\right)^{1/2} \approx \sum_{s\in[t-1,t)}|\Delta X_s^J|^2. \tag{9}$$

According to Duong and Swanson (2015), upside and downside jump variations can be identified by decomposing realized power variation based on the sign of intraday log returns,

$$JP_t^+ = \left(\sum_{i=1}^{n}|\Delta X_{t_i}|^4 I\left(\Delta X_{t_i} > 0\right)\right)^{1/2} \approx \sum_{s\in[t-1,t)}|\Delta X_s^J|^2 I\left(\Delta X_s^J > 0\right), \tag{10}$$

and

$$JP_t^- = \left(\sum_{i=1}^{n}|\Delta X_{t_i}|^4 I\left(\Delta X_{t_i} < 0\right)\right)^{1/2} \approx \sum_{s\in[t-1,t)}|\Delta X_s^J|^2 I\left(\Delta X_s^J < 0\right). \tag{11}$$

To measure jump asymmetry, Duong and Swanson (2015) propose the difference between upside and downside realized power variations, defined as

$$AJP_t = JP_t^+ - JP_t^-. \tag{12}$$

2.3 | Price-Volatility Cojump Variation

Cojumps between prices and volatility are a well-established stylized fact in financial econometrics. They may reflect the leverage effect (Black 1976), whereby falling equity values raise financial leverage and risk. Empirically, negative return shocks are strongly associated with volatility spikes (Christie 1982; Todorov and Tauchen 2011; Caporin et al. 2017; Bollerslev and Todorov 2023). Todorov and Tauchen (2011) propose estimating cojumps between asset prices and option-implied volatility using the limit results of Jacod and Todorov (2009).

$$coJ_t = \sum_{i=1}^{n}|\Delta_n X_{t_i}|^2|\Delta_n Y_{t_i}|^2 \rightarrow \sum_{s\in[t-1,t)}|\Delta X_s^J|^2|\Delta Y_s^J|^2,$$

where Y_{t_i} denotes the option-implied volatility level at intraday time t_i, and ΔY_s^J represents the implied volatility jump. The cojump estimator is based on the product of squared volatility returns and squared price returns, corresponding to a fourth-order power variation. The cojump variation (JC_t) is then defined as the square root of this estimator,

$$JC_t = \left(\sum_{i=1}^{n}|\Delta X_{t_i}|^2|\Delta Y_{t_i}|^2\right)^{1/2} \approx \sum_{s\in[t-1,t)}|\Delta X_s^J||\Delta Y_s^J|. \tag{13}$$

Todorov and Tauchen (2011) suggest that cojump variation can be decomposed into upside and downside components, depending on whether the associated price return is positive or negative,

$$JC_t^+ = \left(\sum_{i=1}^{n}|\Delta X_{t_i} I\left(\Delta X_{t_i} > 0\right)|^2|\Delta Y_{t_i}|^2\right)^{1/2} \approx \sum_{s\in[t-1,t)}|\Delta X_s^J||\Delta Y_s^J|I\left(\Delta X_s^J > 0\right), \tag{14}$$

and

$$JC_t^- = \left(\sum_{i=1}^{n}|\Delta X_{t_i} I\left(\Delta X_{t_i} < 0\right)|^2|\Delta Y_{t_i}|^2\right)^{1/2} \approx \sum_{s\in[t-1,t)}|\Delta X_s^J||\Delta Y_s^J|I\left(\Delta X_s^J < 0\right). \tag{15}$$

Building on the estimated upside and downside cojumps, we construct a realized measure of cojump variation asymmetry, termed asymmetric price-volatility cojump variation (AJC_t), defined as:

$$AJC_t = JC_t^+ - JC_t^-. \tag{16}$$

A positive AJC_t indicates that upside cojumps dominate, whereas a negative AJC_t reflects the dominance of downside cojumps. In our forecasting experiments, we further evaluate the predictive power of this new cojump asymmetry measure.

2.4 | Significant Jumps and Cojumps

Jump estimation is defined within continuous-time models, whereas in practice, only discrete samples are observed, introducing measurement error. Such error typically induces attenuation bias, leading to downward-biased estimates of the impact of jumps on volatility forecasting. To help distinguish true discontinuities in asset prices from measurement error, we implement the jump test methodology of Huang and Tauchen (2005) and Barndorff-Nielsen and Shephard (2006). The essence of this approach is that, in the absence of jumps, estimators of continuous variation and total quadratic variation should coincide, so that their difference approximately zero. Following Duong and Swanson (2015), we apply the adjusted jump ratio statistics developed by Huang and Tauchen (2005):

$$Z_t^J = \frac{1 - TBV_t/RV_t}{\sqrt{0.61M^{-1}\max\left(1, TPQ_T/TBV_t^2\right)}}, \tag{17}$$

where TBV_t denotes the tripower variation, $TBV_t = 1.93\sum_{i=3}^{n}|\Delta_n X_{t_{i-2}}|^{2/3}|\Delta_n X_{t_{i-1}}|^{2/3}|\Delta_n X_{t_i}|^{2/3}$, and TPQ_t represents the realized tripower quarticity, which consistently estimates the integrated quarticity $\int_{t-1}^{t}\sigma_s^4 ds$, defined as $TPQ_t = 1.74 \sum_{i=3}^{n}|\Delta_n X_{t_{i-2}}|^{4/3}|\Delta_n X_{t_{i-1}}|^{4/3}|\Delta_n X_{t_i}|^{4/3}$.

Significant jumps are identified on a given day if the test statistic exceeds the universal threshold (UT) proposed by Bajgrowicz et al. (2016), $\sqrt{2\log N}$, where N is the number of sample days. The UT provides a robust, data-driven procedure for jump detection that substantially reduces spurious identifications[3] caused by multiple testing and adapts naturally with sample size, making it particularly

TABLE 1 | Descriptive statistics for realized variance, continuous variation, and jump/cojump measures.

Variable	Mean	Standard deviation	Min	Max	Median
RV	1.164	4.035	0.000	162.6	0.413
C	1.159	4.035	0.013	162.6	0.410
Equity jumps estimated via bipower variation					
JV	0.225	0.358	0.000	1.899	0.098
JV^+	0.158	0.304	0.004	2.107	0.058
JV^-	0.141	0.301	0.000	2.540	0.058
AJV	0.021	0.386	−2.540	2.107	0.006
Equity jumps estimated via high-order power variation					
JP	0.217	0.387	0.000	2.660	0.092
JP^+	0.138	0.274	0.005	2.183	0.053
JP^-	0.135	0.291	0.000	2.658	0.057
AJP	0.002	0.349	−2.547	2.118	0.000
Equity-VIX cojumps					
JC	1.496	1.771	0.301	8.871	0.912
JC^+	0.871	1.251	0.069	5.969	0.497
JC^-	1.007	1.431	0.106	8.867	0.659
AJC	−0.136	1.678	−8.582	4.690	−0.141

Note: This table reports the descriptive statistics of realized volatility (RV), continuous variation (C), and various jump and cojump measures. JV, JV^+, JV^-, and AJV denote price jump variation, upside jump variation, downside jump variation, and asymmetric jump variation, respectively, estimated using bipower variation. JP, JP^+, JP^-, and AJP are the corresponding measures estimated using high-order realized power variation. JC, JC^+, JC^-, and AJC denote price-volatility cojump variation, upside cojump variation, downside cojump variation, and asymmetric cojump variation, respectively. The data sample consists of high-frequency observations of the S&P 500 E-mini and VIX index from September 22, 2003, to September 12, 2025.

suitable for long high-frequency panels. Thus, we model the significant component of jumps using the indicator $I_{jump} = I(Z_t > \sqrt{2\log N})$. For bipower-based measures, $JV_t = (RV_t - BV_t) I_{jump}$, with upside, downside, and asymmetric components $JV_t^+ = \left(RS_t^+ - \frac{1}{2}BV_t\right)^{1/2} I_{jump}, JV_t^- = \left(RS_t^- - \frac{1}{2}BV_t\right)^{1/2} I_{jump}$, and $AJV_t = \left(JV_t^+ - JV_t^-\right) I_{jump}$. For power-variation-based measures, $JP_t = (RP_t)^{1/2} I_{jump}$, with upside, downside, and asymmetric components defined analogously. Continuous variation is estimated using BV when jumps are present and RV otherwise, $C_t = RV_t(1 - I_{jump}) - BV_t I_{jump}$.

To mitigate the risk that our cojump analysis is affected by measurement error, we focus on "significant" price-volatility cojumps identified using the realized correlation statistic of Jacod and Todorov (2009),

$$\text{corr}_t = \frac{\sum_{i=1}^n |\Delta_n X_{t_i}|^2 |\Delta_n Y_{t_i}|^2}{\sqrt{\sum_{i=1}^n |\Delta_n X_{t_i}|^4 \sum_{i=1}^n |\Delta_n Y_{t_i}|^4}}. \tag{18}$$

A value of zero for this statistic indicates disjoint arrivals of jumps, whereas a value close to one implies strong dependence between jumps in the two series. When jumps are present, the realized correlation converges to the true correlation between price and volatility jumps as the sampling interval shrinks, $\text{corr}_t \to \frac{\sum_{s\in[t-1,t)} |\Delta X_s^J|^2 |\Delta Y_s^J|^2}{\sqrt{\sum_{s\in[t-1,t)} |\Delta X_s^J|^4 \sum_{s\in[t-1,t)} |\Delta Y_s^J|^4}}$. Motivated by this property, we designate cojumps as instances where the realized correlation exceeds 0.85, a threshold commonly regarded as very strong dependence. To reduce the possibility that high correlation is driven by small continuous comovements rather than genuine cojumps, we further require that the BNS statistic for price jumps exceeds the UT. Accordingly, we define cojumps $JC_t = \left(\sum_{i=1}^n |\Delta X_{t_i}|^2 |\Delta Y_{t_i}|^2\right)^{1/2} I_{cojump}$, with upside, downside, and asymmetric counterparts JC_t^+, JC_t^-, and AJC_t defined analogously. Here, $I_{cojump} = I\left(Z_t^J > \sqrt{2\log N} \cap \text{corr}_t > 0.85\right)$ indicates the presence of significant cojumps.

3 | Data Description

Our sample consists of more than two decades of high-frequency data on the S&P 500 E-mini futures and the VIX index obtained from Pi-trading Inc. The data provide 5-min observations during regular trading hours, 9:30 a.m. to 4:00 p.m. EST. The sample spans 22 years, from September 22, 2003, to September 12, 2025, ensuring that the analysis reflects the most up-to-date market dynamics. Table 1 reports descriptive statistics for volatility estimators, price jumps, and price-volatility cojumps. Jumps estimated by BV, JV, and high-order power variation, JP are comparable in magnitude, indicating that both estimators consistently measure jumps. We also find that price-volatility cojumps, indicated by JC, are on average much larger than JV or JP. One explanation is that VIX movements are typically larger than those of the S&P 500. Another interpretation, following Bollerslev and Todorov (2011), is that option-implied volatility reacts more strongly to S&P 500 jumps than realized volatility, as investors demand compensation for bearing tail risk.

3.1 | Time Series Pattern of Cojumps

Figure 1 depicts daily S&P 500 realized volatility, $\sqrt{RV}$, over the sample period. The series from 2005 to 2025 shows strong

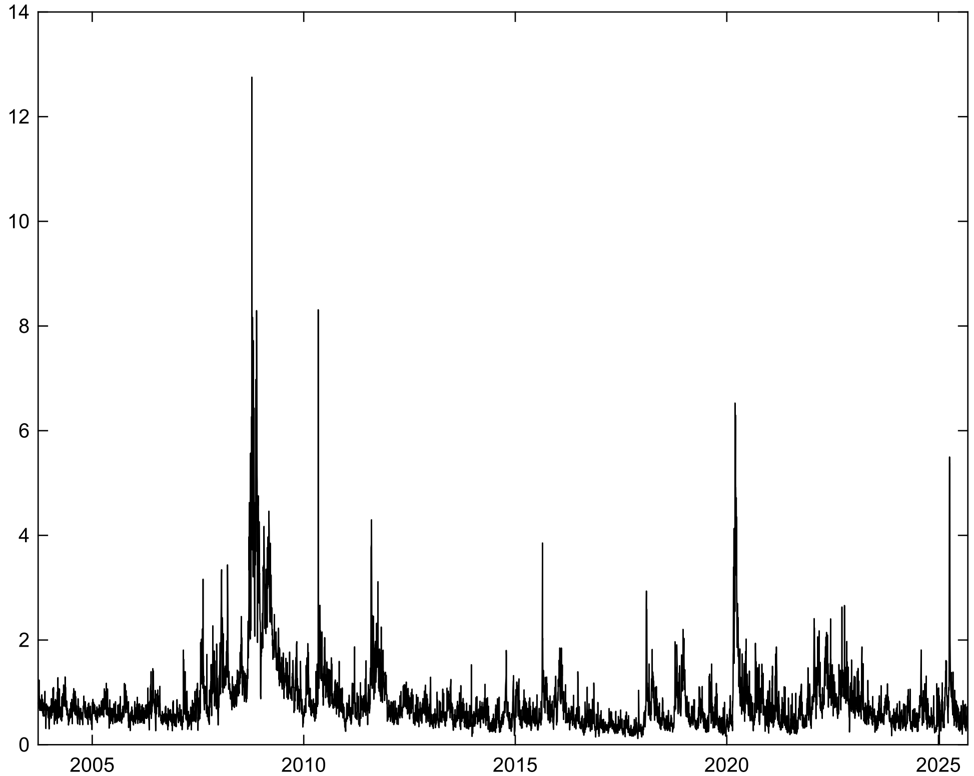

FIGURE 1 | Daily realized volatility of the S&P 500 E-mini futures from 2005 to 2025, estimated from 5-min returns.

clustering, persistence, and sharp spikes during major crises such as the 2008 global financial crisis and the 2020 COVID-19 shock, consistent with well-documented volatility dynamics in financial markets. Figure 2 plots jumps estimated using BV, high-order power variation, and price-volatility cojumps. All series are shown in square-root form. As the upper two panels illustrate, the two jump estimators yield similar estimates of jump magnitudes. Jumps are rare and occur on 2.26% of sample days, consistent with the recent literature (Bajgrowicz et al. 2016; Caporin 2023; Christensen et al. 2014).

The bottom panel shows that price-volatility cojumps appear less frequent than price jumps. We find that the occurrence rate of cojumps is 0.78%, indicating that not all price jumps coincide with volatility jumps. This finding is consistent with Tauchen and Zhou (2011) and Jacod et al. (2017) and may be because not all price jumps reflect changes in risk perceptions: some arise from the rapid incorporation of well-understood information without materially changing perceived risk, while others are associated with heightened uncertainty and increased risk aversion. Based on this intuition, price-volatility cojumps should tend to occur during extreme market episodes, when markets experience elevated uncertainty and increased risk aversion. Indeed, as illustrated in Figure 2, cojumps often occur around periods of market stress, including the 2008 global financial crisis, the 2020–2021 COVID-19 pandemic, and the more recent U.S. tariff shock in April 2025.

3.2 | Economic Interpretation and Policy Implications

The preceding section provides visual evidence that periods of heightened financial stress, elevated uncertainty, and liquidity frictions may be associated with a higher likelihood of simultaneous jumps in prices and volatility. In this section, we formally test whether these factors help explain the occurrence of price-volatility cojumps.

We retrieve the daily U.S. Economic Policy Uncertainty (EPU) index of Baker et al. (2016), available at www.policyuncertainty.com, which quantifies policy uncertainty based on the frequency of related terms in leading newspapers.[4] We also include a daily measure of credit risk, the default spread (DEF), defined as the yield difference between Moody's seasoned BAA and AAA corporate bonds from the Federal Reserve Bank of St. Louis (FRED). In addition, we use the daily TED spread from FRED, defined as the difference between the 3-month USD LIBOR and the 3-month U.S. Treasury bill.[5] Finally, we construct an illiquidity measure (ILLIQ) following Amihud (2002), based on daily returns, closing prices, and trading volumes of S&P 500 E-mini futures. We then regress the cojump measures on these variables to investigate their economic determinants:

$$JC_t = \beta_0 + \beta_{REC} REC_t + \beta_{EPU} EPU_t + \beta_{DEF} DEF_t + \beta_{ILLIQ} ILLIQ_t + \beta_{TED} TED_t + \epsilon_t,$$

where we further control for a recession dummy variable (REC), equal to one during NBER-dated recessions and zero otherwise. The estimation results are reported in Table 2. We find that the recession dummy (REC) and the EPU index enter with positive and significant coefficients across all specifications, indicating that cojumps are amplified during recessions and periods of elevated policy uncertainty. Credit and liquidity variables also play a role: the default spread (DEF), the illiquidity measure (ILLIQ), and the TED spread

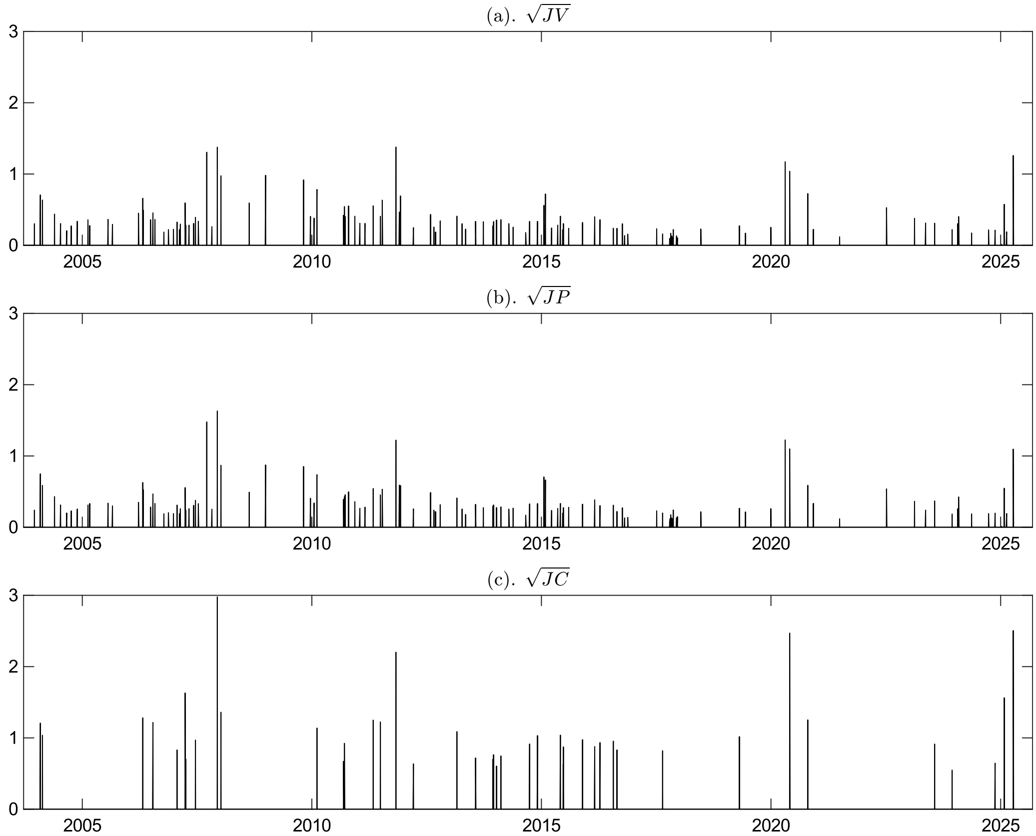

FIGURE 2 | Daily S&P 500 jumps and S&P 500-VIX cojumps from 2003 to 2025, with all series shown in square root form. (a) and (b) Jump variation estimated via bipower and high-order power variation, respectively. (c) Plots cojumps.

TABLE 2 | Explaining price-volatility cojumps with financial indicators.

	(1)	(2)	(3)	(4)	(5)
β_0	0.906***	0.929***	0.908***	0.952***	1.001***
	(32.2)	(31.5)	(31.6)	(32.6)	(34.1)
β_{REC}	1.610***	1.339***	1.586***	1.064***	0.462***
	(16.2)	(9.93)	(14.8)	(8.29)	(3.07)
β_{EPU}	0.358***				0.389***
	(13.1)				(14.2)
β_{DEF}		0.194***			0.068*
		(5.24)			(1.76)
β_{ILLIQ}			0.167***		0.113***
			(5.67)		(3.62)
β_{TED}				0.326***	0.345***
				(8.92)	(9.71)
R^2	0.0860	0.0621	0.0629	0.0716	0.1088

Note: The table reports regression results linking daily cojumps to macro-financial indicators. Explanatory variables include the recession dummy (REC) defined by NBER, the U.S. Economic Policy Uncertainty index (Baker et al. 2016) (EPU), the default spread (DEF), the Amihud illiquidity measure (Amihud 2002) (ILLIQ), and the TED spread (TED). Columns (1)–(5) present results for specifications including these factors separately and jointly. Coefficient estimates are reported with t-statistics in parentheses. Statistical significance is denoted by *, **, and *** at the 10%, 5%, and 1% levels, respectively.

are all significantly and positively related to cojumps. Viewed through a disaster risk lens, cojumps may reflect rare, adverse macro-financial shocks that occur in states of heightened uncertainty and stress, helping explain why exposure to cojump risk earns a sizable equity risk premium (Bollerslev and Todorov 2023).

Because price-volatility cojumps are linked to persistent uncertainty and market stress, they may also have important regulatory implications for preserving financial stability. For example, elevated cojumps during periods of widening credit spreads and illiquidity suggest that the margin requirements of central counterparties (CCPs) may warrant a more responsive calibration to cojump signals. Moreover, since the TED spread is a standard indicator of funding conditions, its strong association with cojumps implies that cojumps can serve as timely early-warning signals of systemic funding stress. Finally, because cojumps are closely related to EPU, their occurrence may reflect the market impact of policy actions, providing feedback to regulators on the credibility and clarity of policy communication.

4 | The Impact of Cojumps on Volatility Forecasting

The previous section shows that cojumps are closely tied to EPU, market stress, and financial frictions. Because these conditions generate persistent uncertainty and risk aversion, the occurrence of price-volatility cojumps may serve as an early indicator of elevated future volatility. This section, therefore, further explores the role of cojumps in volatility forecasting.

4.1 | Model Setup

The HAR RV model of Corsi (2009) is the workhorse for RV forecasting, capturing volatility persistence through lagged daily, weekly, and monthly components. The HAR model was later augmented by Patton and Sheppard (2015) and Caporin (2023), who decomposed the daily lag into separate continuous and jump variation components, yielding the HAR-C-JV specification. We adopt this framework for forecasting RV and extend it further by incorporating price-volatility cojumps. Formally, Corsi (2009) defines the HAR model as:

$$RV_t = \beta_0 + \beta_d RV_t^{(d)} + \beta_w RV_t^{(w)} + \beta_m RV_t^{(m)} + \epsilon_t. \tag{19}$$

Here, the terms $RV_t^{(d)} = RV_{t-1}, RV_t^{(w)} = \frac{1}{5}\sum_{i=1}^{5} RV_{t-i}$, and $RV_t^{(m)} = \frac{1}{22}\sum_{i=1}^{22} RV_{t-i}$ correspond to the daily, weekly, and monthly lagged average RV, respectively. This framework captures the heterogeneous components of volatility across short-, medium-, and long-term horizons. Following Corsi (2009), the HAR model is estimated via ordinary least squares. The variability of RV evolves over time, with volatility clustering that alternates between periods of high and low uncertainty, which indicates pronounced heteroskedasticity. This feature may result in inconsistent inference based on conventional standard errors and in inadequately calibrated out-of-sample prediction intervals. To mitigate these issues, a logarithmic transformation of RV is employed, as this renders the distribution more closely Gaussian and thereby better fits linear time-series modeling (Andersen et al. 2003). The logarithmic HAR also ensures strictly positive predictions, in contrast to the HAR estimated in levels, which may yield negative variance forecasts. We adopt this transformation throughout all subsequent augmented HAR specifications.

Specification 1: To examine the impact of jumps on volatility forecasting, Andersen, Bollerslev, and Dobrev (2007) propose the HAR-C-JV model, which augments the HAR by decomposing RV into continuous and jump components:

$$RV_t = \beta_0 + \beta_d C_t^{(d)} + \beta_w RV_t^{(w)} + \beta_m RV_t^{(m)} + \beta_J JV_{t-1} + \epsilon_t, \tag{20}$$

where $C_t^{(d)} = C_{t-1}$ is the daily lag of continuous variation, and JV_t denotes jump variation as defined in Equation (5).[6]

Specification 2: The HAR-C-UDJV model of Patton and Sheppard (2015) distinguishes between upside and downside jumps:

$$\begin{aligned}RV_t = \beta_0 + \beta_d C_t^{(d)} + \beta_w RV_t^{(w)} + \beta_m RV_t^{(m)} + \beta_{J^+} JV_{t-1}^+ \\ + \beta_{J^-} JV_{t-1}^- + \epsilon_t.\end{aligned} \tag{21}$$

Specification 3: The HAR-C-AJV model of Patton and Sheppard (2015) incorporates AJV:

$$\begin{aligned}RV_t = \beta_0 + \beta_d C_t^{(d)} + \beta_w RV_t^{(w)} + \beta_m RV_t^{(m)} + \beta_{AJ} \\ AJV_{t-1} + \epsilon_t.\end{aligned} \tag{22}$$

Specification 4: The HAR-C-JP model of Duong and Swanson (2015) extends the HAR by incorporating jumps estimated via high-order realized power variation:

$$RV_t = \beta_0 + \beta_d C_t^{(d)} + \beta_w RV_t^{(w)} + \beta_m RV_t^{(m)} + \beta_J JP_{t-1} + \epsilon_t, \tag{23}$$

where JP_t denotes the high-order power jump estimator (Equation 9).

Specification 5: The HAR-C-UDJP model of Duong and Swanson (2015) decomposes jump power variation into upside and downside components:

$$\begin{aligned}RV_t = \beta_0 + \beta_d C_t^{(d)} + \beta_w RV_t^{(w)} + \beta_m RV_t^{(m)} + \beta_{J^+} JP_{t-1}^+ \\ + \beta_{J^-} JP_{t-1}^- + \epsilon_t.\end{aligned} \tag{24}$$

Specification 6: The HAR-C-AJP model of Duong and Swanson (2015) incorporates asymmetric jump power variation:

$$RV_t = \beta_0 + \beta_d C_t^{(d)} + \beta_w RV_t^{(w)} + \beta_m RV_t^{(m)} + \beta_{AJ} AJP_{t-1} + \epsilon_t. \tag{25}$$

Specification 7: Our HAR-C-JC model includes price-volatility cojumps as an explanatory variable:

$$RV_t = \beta_0 + \beta_d C_t^{(d)} + \beta_w RV_t^{(w)} + \beta_m RV_t^{(m)} + \beta_{JC} JC_{t-1} + \epsilon_t, \tag{26}$$

where JC_t denotes the cojump estimate (Equation 13).

Specification 8: Our HAR-C-UDJC model distinguishes between upside and downside cojumps:

$$\begin{aligned}RV_t = \beta_0 + \beta_d C_t^{(d)} + \beta_w RV_t^{(w)} + \beta_m RV_t^{(m)} + \beta_{JC^+} \\ JC_{t-1}^+ + \beta_{JC^-} JC_{t-1}^- + \epsilon_t,\end{aligned} \tag{27}$$

where JC_t^+ and JC_t^- denote upside and downside cojumps (Equations 14 and 15).

Specification 9: Our HAR-C-AJC model incorporates asymmetric cojump variation:

$$\begin{aligned}RV_t = \beta_0 + \beta_d C_t^{(d)} + \beta_w RV_t^{(w)} + \beta_m RV_t^{(m)} + \beta_{AJC} \\ AJC_{t-1} + \epsilon_t,\end{aligned} \tag{28}$$

where AJC_t is the cojump asymmetry measure (Equation 16). Table 3 summarizes the HAR model specifications considered in this study, including baseline versions from the literature and our extensions incorporating price-volatility cojumps.

4.2 | Price Jumps and RV Forecasting

Table 4 reports the in-sample estimation results and goodness-of-fit measures for all HAR specifications. The first column presents the baseline HAR model, while the second and third panels report models incorporating jump variables measured by BV and high-order power variation, respectively. The results for the HAR model indicate that daily, weekly, and monthly RV components all significantly contribute to explaining future RV. This confirms the persistence of volatility across different time horizons, consistent with the findings of Corsi (2009).

The second panel reports results for the three RV forecasting models that incorporate jump variables based on BV. The HAR-C-JV model of Andersen, Bollerslev, and Dobrev (2007) evaluates the impact of unsigned jumps, while the HAR-C-UDJV and HAR-C-AJV models of Patton and Sheppard (2015) examine the effects of upside and downside jumps and jump asymmetry. The results show that the coefficients on unsigned, positive, negative, and asymmetric jumps (β_J, β_J^+, β_J^-, and β_{AJ}) are not statistically significant at conventional levels. The coefficient on daily continuous variation (β_d) in these models is nearly identical to that on daily RV in the baseline HAR model. These findings suggest that the persistence of RV arises almost entirely from its continuous component, with jumps contributing little explanatory power. Our results, therefore, reinforce prior evidence from Andersen, Bollerslev, and Dobrev (2007), Busch et al. (2011), Prokopczuk et al. (2016), and Caporin (2023), showing that jump-based predictors identified from realized (semi)variance and BV offer limited forecasting value.

The third panel reports results for the three jump models of Duong and Swanson (2015), namely HAR-C-JP, HAR-C-UDJP, and HAR-C-AJP. These specifications parallel those in the second panel but employ alternative high-order realized power estimators for the jump measures. The results indicate that jumps continue to lack predictive power for RV, as the coefficients on all jump components (β_J, β_J^+, β_J^-, and β_{AJ}) are not statistically significant at the 5% level. Consistent with our findings, Duong and Swanson (2015) also report that jumps estimated from high-order power variation exert no significant effect on future RV.

4.3 | Price-Volatility Cojumps and RV Forecasting

Table 5 reports the in-sample estimation results for the newly proposed HAR-C-JC, HAR-C-UDJC, and HAR-C-AJC specifications. These models extend the HAR framework by incorporating overall cojumps, upside and downside cojumps, and cojump asymmetry, respectively, as predictors of RV. The three panels of the table report results for realized correlation thresholds of 0.85, 0.95, and 0.99, which measure the dependence between price and volatility jumps. Across all correlations, the coefficient on overall cojumps (β_{JC}) is small and statistically insignificant. In contrast, the coefficients on upside and downside cojumps (β_{JC^+} and β_{JC^-}) are positive and negative, respectively, and both are highly significant, as is the negative coefficient on cojump asymmetry (β_{AJC}). These results indicate that downside cojumps are associated with higher future volatility, while upside cojumps predict lower volatility; when averaged, their opposing effects imply little impact of overall cojumps on volatility. The significance of price-volatility cojumps stands in sharp contrast to the weak evidence for price jumps reported in Table 4, suggesting that cojumps embed information related to market risk aversion and thus provide predictive power beyond that of price jumps. Finally, Table 5 shows that the t-statistics for downside cojumps increase with the correlation threshold, consistent with the interpretation that stronger jump dependence reduces measurement error and enhances the reliability of cojumps in forecasting future volatility.

4.4 | Out-of-Sample Forecast

We now examine whether the significant in-sample effects of price-volatility cojumps on RV forecasting also translate into superior out-of-sample performance. The forecasting exercise considers all $k = 10$ models summarized in Table 3, including

TABLE 3 | Summary of HAR model specifications.

Model	Description	References	Equations
HAR	Baseline heterogeneous autoregressive model	Corsi (2009)	(19)
HAR-C-JV	Jump variation	Andersen, Bollerslev, and Dobrev (2007)	(20)
HAR-C-UDJV	Upside and downside jump variations	Patton and Sheppard (2015), Caporin (2023)	(21)
HAR-C-AJV	Asymmetric jump variation	Patton and Sheppard (2015), Caporin (2023)	(22)
HAR-C-JP	Jumps power variation	Duong and Swanson (2015), Caporin (2023)	(23)
HAR-C-UDJP	Upside and downside jump power variation	Duong and Swanson (2015), Caporin (2023)	(24)
HAR-C-AJP	Asymmetric jump power variation	Duong and Swanson (2015), Caporin (2023)	(25)
HAR-C-JC	Price-volatility cojumps variation	This paper	(26)
HAR-C-UDJC	Upside and downside cojump variations	This paper	(27)
HAR-C-AJC	Asymmetric cojump variation	This paper	(28)

Note: This table summarizes the HAR model of Corsi (2009) and its extensions. Prior models incorporate jump variation (Andersen, Bollerslev, and Dobrev 2007; Patton and Sheppard 2015; Duong and Swanson 2015; Caporin 2023), while our new specifications include price-volatility cojumps, their upside and downside decompositions, and cojump asymmetry. The last column reports the corresponding equation numbers in this paper.

TABLE 4 | In-sample estimation results for HAR jump models.

		Jumps estimated via bipower variation			Jumps estimated via high-order power variation		
	HAR	**HAR-C-JV**	**HAR-C-UDJV**	**HAR-C-AJV**	**HAR-C-JP**	**HAR-C-UDJP**	**HAR-C-AJP**
β_0	−0.103***	−0.072***	−0.072***	−0.072***	−0.072***	−0.072***	−0.072***
	(−11.0)	(−7.45)	(−7.45)	(−7.45)	(−7.43)	(−7.42)	(−7.46)
β_d	0.485***	0.465***	0.465***	0.464***	0.465***	0.465***	0.465***
	(28.0)	(27.8)	(27.8)	(27.8)	(27.8)	(27.8)	(27.8)
β_w	0.293***	0.307***	0.307***	0.307***	0.307***	0.307***	0.307***
	(11.7)	(12.4)	(12.4)	(12.4)	(12.4)	(12.4)	(12.4)
β_m	0.168***	0.170***	0.170***	0.170***	0.170***	0.170***	0.170***
	(9.05)	(9.13)	(9.11)	(9.11)	(9.14)	(9.12)	(9.12)
β_J		0.020			−0.027		
		(0.09)			(−0.12)		
β_{J^+}			−0.336			−0.494*	
			(−1.26)			(−1.92)	
β_{J^-}			0.445			0.451	
			(1.36)			(1.39)	
β_{AJ}				−0.367			−0.387
				(−1.61)			(−1.54)
R^2	0.750	0.749	0.749	0.749	0.749	0.749	0.749

Note: This table reports in-sample estimation results for the HAR model and its jump-augmented extensions. Results for the jump models are grouped by jumps estimated using bipower variation and high-order power variation. Reported are coefficient estimates with t-statistics in parentheses. Statistical significance is denoted by *, **, and *** at the 10%, 5%, and 1% levels, respectively.

TABLE 5 | In-sample estimation results for HAR cojump models.

	Corr = 0.85			Corr = 0.95			Corr = 0.99		
	HAR-C-JC	**HAR-C-UDJC**	**HAR-C-AJC**	**HAR-C-JC**	**HAR-C-UDJC**	**HAR-C-AJC**	**HAR-C-JC**	**HAR-C-UDJC**	**HAR-C-AJC**
β_0	−0.073***	−0.073***	−0.072***	−0.072***	−0.072***	−0.072***	−0.072***	−0.072***	−0.072***
	(−7.51)	(−7.49)	(−7.48)	(−7.47)	(−7.47)	(−7.51)	(−7.46)	(−7.46)	(−7.49)
β_d	0.465***	0.464***	0.464***	0.465***	0.464***	0.464***	0.465***	0.464***	0.464***
	(27.8)	(27.7)	(27.8)	(27.8)	(27.7)	(27.7)	(27.8)	(27.7)	(27.7)
β_w	0.306***	0.308***	0.308***	0.307***	0.307***	0.307***	0.306***	0.307***	0.307***
	(12.4)	(12.4)	(12.4)	(12.4)	(12.4)	(12.4)	(12.4)	(12.4)	(12.4)
β_m	0.170***	0.170***	0.170***	0.170***	0.170***	0.170***	0.170***	0.170***	0.170***
	(9.14)	(9.11)	(9.11)	(9.13)	(9.11)	(9.12)	(9.14)	(9.12)	(9.11)
β_{JC}	0.081			−0.001			−0.053		
	(0.64)			(−0.00)			(−0.22)		
β_{JC^+}		−0.507***			−0.649***			−0.637***	
		(−2.95)			(−4.35)			(−4.34)	
β_{JC^-}		0.528***			0.450***			0.392***	
		(3.86)			(5.90)			(6.54)	
β_{AJC}			−0.411***			−0.436***			−0.440***
			(−3.99)			(−5.11)			(−4.76)
R^2	0.749	0.749	0.749	0.749	0.749	0.749	0.749	0.749	0.749

Note: This table reports in-sample estimation results for the HAR model augmented with price-volatility cojump measures. Corr = 0.85, 0.95, and 0.99 indicate the realized correlation coefficients that gauge the level of jump dependence between price and volatility. Reported are coefficient estimates with t-statistics in parentheses. Statistical significance is denoted by *, **, and *** at the 10%, 5%, and 1% levels, respectively.

the standard HAR, jump-augmented models commonly used in the literature, and our newly proposed cojump specifications. We generate forecasts for RV over the most recent 5 years using both rolling and recursive estimation windows, with parameters updated daily. Out-of-sample forecast accuracy is evaluated using the heteroskedasticity-adjusted mean squared error and the Gaussian quasi-likelihood:

$$\text{HMSE}=\left(\frac{RV_t - F_t}{F_t}\right)^2,$$

$$\text{QLIKE}=\frac{RV_t}{F_t} - \ln\frac{RV_t}{F_t} - 1,$$

where F_t indicates the out-of-sample forecast from the models. The QLIKE loss function has been shown to be robust to noise in volatility proxies (Patton 2011). Given the relatively large number of models under comparison, differences in forecasting precision are assessed using the model confidence set (MCS) procedure of Hansen et al. (2011). The MCS applies a sequence of tests to construct a set of "superior" models for which the null hypothesis of equal predictive ability cannot be rejected at a chosen confidence level. Models included in the superior set significantly outperform those excluded from it. The following provides additional details on the implementation of the MCS.

Let $M = \{1, ..., k\}$ denote a set of competing forecasting models and let $L_{i,t}$ be the loss of model i at time t. Define the loss differential between models i and j as

$$d_{ij,t} = L_{i,t} - L_{j,t}.\ d_{i,t} = (k-1)^{-1}\sum_{j\in M} d_{ij,t},\ i = 1, ..., k,$$

where d_{ij} is the loss difference between models i and j, and $d_{i\cdot}$ is the loss difference between model i and the averages losses across models in the set.

The equal predictive ability hypothesis for the set of models M may be defined in two ways

$$H_0 : a_{ij} = 0, \forall\ i, j = 1, 2, ..., k,$$
$$H_A : a_{ij} \neq 0, \exists\ i, j = 1, 2 ..., k,$$

or

$$H_0 : a_{i\cdot} = 0, \forall\ i = 1,\ 2, ...,\ k,$$
$$H_A : a_{i\cdot} \neq 0, \exists\ i = 1,\ 2, ...,\ k,$$

where $a_{ij} = \mathbb{E}(a_{ij})$ and $c_{i\cdot} = \mathbb{E}(d_{i\cdot})$ indicate the expected loss differentials. Correspondingly, Hansen et al. (2011) proposed the following two test statistics,

$$T_R = \max_{i,j\in M}\frac{\bar{d}_{ij}}{\sqrt{\hat{\Omega}(\bar{d}_{ij})}},\ T_{SQ} = \max_{i\in M}\frac{\bar{d}_{i\cdot}}{\sqrt{\hat{\Omega}(\bar{d}_{i\cdot})}},$$

where $\bar{d}_{ij}$ and $\bar{d}_{i\cdot}$ denote sample means, with $\bar{d}_{ij} = \frac{1}{k}\sum_{t=1}^{k} d_{ij,t}$ and $\bar{d}_{i\cdot} = \frac{1}{k-1}\sum_{j\in M}\bar{d}_{ij}$. The quantities $\hat{\Omega}(\bar{d}_{ij})$ and $\hat{\Omega}(\bar{d}_{i\cdot})$ are bootstrapped variance estimators for $\bar{d}_{ij}$ and $\bar{d}_{i\cdot}$, respectively. We employ a stationary bootstrap with 1,000 resamples and a block length of 10. For presentation purposes, we focus on results based on the T_{SQ} statistic in the MCS framework; results are robust to the use of the T_R statistic and are available upon request.

If H_0 is rejected, the model with the worst predictive performance is removed from M. This procedure is iterated until the null hypothesis can no longer be rejected. The remaining models constitute the MCS, $\hat{M}_{1-\alpha}$, which contains the models with superior predictive ability under confidence level $1-\alpha$. We set $\alpha = 0.10$ as the significance level.

Table 6 reports the ratios of forecast losses for the jump- and cojump-augmented models relative to those of the standard HAR benchmark. Results are shown for three levels of realized correlation thresholds. The lowest ratio in each row is highlighted in bold, while the superscript "c" indicates models included in the MCS. From the top panel (correlation 0.85), we find that most jump and cojump models struggle to outperform the baseline HAR, with the exception of the HAR-C-AJV and HAR-C-AJC models, which consistently achieve lower losses under both HMSE and QLIKE across rolling and expanding windows. Both models are significantly superior to the HAR in terms of HMSE, while the HAR-C-AJC specification, which incorporates cojump asymmetry, achieves the lowest HMSE and QLIKE values overall, ranking best across all scenarios considered.

The superiority of cojumps in out-of-sample forecasting is further evidenced as the correlation threshold increases to 0.95, as reported in the middle panel of Table 6. At this correlation level, the HAR-C-UDJC model, which incorporates upside and downside cojump information, consistently delivers the smallest prediction losses under both rolling and expanding windows, although it is not always included in the MCS. The bottom panel, corresponding to the highest correlation threshold (0.99), reveals clearer results: the HAR-C-UDJC specification significantly outperforms almost all other models across evaluation criteria. This stronger evidence likely reflects the reduction in measurement error when cojumps are filtered using higher jump dependence between price and volatility. Overall, the results highlight the important role of cojumps in improving RV forecasts out-of-sample.

In our earlier descriptive analysis in Section 3, we find that cojumps cluster around periods of market turbulence. This behavior underscores the potential value of cojumps for volatility forecasting under these extreme environments. Accordingly, we conduct a subsample analysis to evaluate the model's forecasting performance during periods of extreme market conditions. For our out-of-sample evaluation, these conditions include the COVID-19 pandemic in 2020–2021 and the U.S. tariff shock in April 2025.

Table 7 replicates the out-of-sample results in Table 6, conditioning on the pandemic and the tariff shock. A comparison of the two tables reveals a substantial reduction in the number of HAR and jump-based models included in the MCS during stress periods, whereas the cojump models such as HAR-C-UDJC and HAR-C-AJC are able to consistently remain in the confidence set across the forecast scenarios considered. This evidence

TABLE 6 | Out-of-sample realized variance forecast performance.

			Jumps estimated via bipower variation			Jumps estimated via high-order power variation			Volatility price cojumps		
		HAR	HAR-C-JV	HAR-C-UDJV	HAR-C-AJV	HAR-C-JP	HAR-C-UDJP	HAR-C-AJP	HAR-C-JC	HAR-C-UDJC	HAR-C-AJC
Corr = 0.85											
HMSE	RW	1.0000	0.9972	0.9976	0.9963[c]	0.9968	0.9969	0.9962[c]	0.9973	0.9962[c]	**0.9948**[c]
	IW	1.0000	0.9965[c]	0.9970	0.9956[c]	0.9961[c]	0.9963[c]	0.9956[c]	0.9967	0.9954[c]	**0.9945**[c]
QLIKE	RW	1.0000[c]	1.0002[c]	1.0004[c]	0.9999[c]	1.0001[c]	1.0002[c]	1.0000[c]	1.0003[c]	1.0004[c]	**0.9993**[c]
	IW	1.0000[c]	1.0001[c]	1.0002[c]	0.9997[c]	1.0000[c]	1.0002[c]	0.9999[c]	1.0002[c]	0.9998[c]	**0.9992**[c]
Corr = 0.95											
HMSE	RW	1.0000	0.9972	0.9976	0.9963[c]	0.9968	0.9969	0.9962[c]	0.9963[c]	**0.9960**[c]	0.9965
	IW	1.0000	0.9965	0.9970	0.9956[c]	0.9961	0.9963	0.9956[c]	0.9957[c]	**0.9954**[c]	0.9961
QLIKE	RW	1.0000[c]	1.0002[c]	1.0004[c]	0.9999[c]	1.0001[c]	1.0002[c]	1.0000[c]	1.0000[c]	**0.9999**[c]	1.0001[c]
	IW	1.0000[c]	1.0001[c]	1.0002[c]	0.9997[c]	1.0000[c]	1.0002[c]	0.9999[c]	0.9998[c]	**0.9997**[c]	0.9999[c]
Corr = 0.99											
HMSE	RW	1.0000	0.9972	0.9976	0.9963	0.9968	0.9969	0.9962	0.9960	**0.9956**[c]	0.9960
	IW	1.0000	0.9965	0.9970	0.9956	0.9961	0.9963	0.9956	0.9954	**0.9951**[c]	0.9956
QLIKE	RW	1.0000	1.0002	1.0004	0.9999	1.0001	1.0002	1.0000	0.9999	**0.9998**[c]	0.9999
	IW	1.0000	1.0001	1.0002	0.9997[c]	1.0000	1.0002	0.9999	0.9997	**0.9997**[c]	0.9998

Note: Entries are forecast loss ratios of models relative to the standard HAR benchmark. HMSE denotes heteroskedasticity-adjusted mean squared error and QLIKE is the Gaussian quasi-likelihood loss function. Corr = 0.85, 0.95, and 0.99 indicate the realized correlation coefficients that gauge the level of jump dependence between price and volatility. In each row, bold entries mark the lowest loss, while the superscript "c" identifies models retained in the 90% model confidence set of Hansen et al. (2011).

TABLE 7 | Out-of-sample realized variance forecast performance conditional on market stress episodes.

			Jumps estimated via bipower variation			Jumps estimated via high-order power variation			Volatility price cojumps		
		HAR	HAR-C-JV	HAR-C-UDJV	HAR-C-AJV	HAR-C-JP	HAR-C-UDJP	HAR-C-AJP	HAR-C-JC	HAR-C-UDJC	HAR-C-AJC
Corr = 0.85											
HMSE	RW	1.0000	0.9909	0.9910	0.9890[a]	0.9904	0.9901	0.9889[a]	0.9902	0.9902	**0.9887**[a]
	IW	1.0000	0.9915	0.9916	0.9893[a]	0.9912	0.9908	0.9893[a]	0.9907	0.9904	**0.9893**[a]
QLIKE	RW	1.0000	0.9943	0.9950	0.9933	0.9939	0.9942	0.9932[a]	0.9941[a]	0.9943[a]	**0.9920**[a]
	IW	1.0000	0.9954	0.9961	0.9937[a]	0.9950	0.9949	0.9935[a]	0.9947	0.9936[a]	**0.9923**[a]
Corr = 0.95											
HMSE	RW	1.0000	0.9909	0.9910	0.9890	0.9904	0.9901	0.9889	0.9887[a]	**0.9883**[a]	0.9890
	IW	1.0000	0.9915	0.9916	0.9893	0.9912	0.9908	0.9893	0.9891[a]	**0.9890**[a]	0.9897
QLIKE	RW	1.0000	0.9943	0.9950	0.9933	0.9939	0.9942	0.9932	0.9930[a]	**0.9929**[a]	0.9931
	IW	1.0000	0.9954	0.9961	0.9937	0.9950	0.9949	0.9935[a]	0.9932[a]	**0.9932**[a]	0.9935
Corr = 0.99											
HMSE	RW	1.0000	0.9909	0.9910	0.9890	0.9904	0.9901	0.9889	0.9885	**0.9881**[a]	0.9884
	IW	1.0000	0.9915	0.9916	0.9893	0.9912	0.9908	0.9893	0.9889[a]	**0.9887**[a]	0.9892
QLIKE	RW	1.0000	0.9943	0.9950	0.9933	0.9939	0.9942	0.9932	0.9929	**0.9927**[a]	0.9929
	IW	1.0000	0.9954	0.9961	0.9937	0.9950	0.9949	0.9935	0.9931[a]	**0.9930**[a]	0.9932

Note: This table presents the realized variance out-of-sample forecasts conditional on the COVID-19 pandemic in 2020–2021 and the U.S. tariff shock in April 2025. In each row, bold entries mark the lowest loss. All remaining specifications and definitions follow Table 5.
[a]Identifies models retained in the 90% model confidence set of Hansen et al. (2011).

points to an even stronger relative performance of cojump-based models during periods of market stress, indicating that price-volatility cojumps are particularly effective to predict volatility during turbulent market conditions.

4.5 | Economic Significance of Cojumps: VaR Forecasting

Volatility forecasting plays a central role in finance. It predicts market risk, which is fundamental to asset pricing, portfolio

TABLE 8 | Out-of-sample VaR forecast performance.

			Jumps estimated via bipower variation			Jumps estimated via high-order power variation			Volatility price cojumps		
		HAR	HAR-C-JV	HAR-C-UDJV	HAR-C-AJV	HAR-C-JP	HAR-C-UDJP	HAR-C-AJP	HAR-C-JC	HAR-C-UDJC	HAR-C-AJC
Corr = 0.85											
Q	RW	1.0000	0.9995	0.9993$^{\mathrm{c}}$	0.9993$^{\mathrm{c}}$	0.9995	0.9993$^{\mathrm{c}}$	0.9993$^{\mathrm{c}}$	0.9993$^{\mathrm{c}}$	**0.9986$^{\mathrm{c}}$**	0.9988$^{\mathrm{c}}$
	IW	1.0000	0.9993	0.9992	0.9992	0.9994	0.9992$^{\mathrm{c}}$	0.9992$^{\mathrm{c}}$	0.9992$^{\mathrm{c}}$	**0.9988$^{\mathrm{c}}$**	0.9988$^{\mathrm{c}}$
$\tilde{Q}$	RW	1.0000	0.9990$^{\mathrm{c}}$	0.9991$^{\mathrm{c}}$	0.9988$^{\mathrm{c}}$	0.9988$^{\mathrm{c}}$	0.9989$^{\mathrm{c}}$	0.9987$^{\mathrm{c}}$	**0.9987$^{\mathrm{c}}$**	0.9991$^{\mathrm{c}}$	0.9988$^{\mathrm{c}}$
	IW	1.0000	0.9991$^{\mathrm{c}}$	0.9993$^{\mathrm{c}}$	0.9988$^{\mathrm{c}}$	0.9990$^{\mathrm{c}}$	0.9990$^{\mathrm{c}}$	**0.9987$^{\mathrm{c}}$**	0.9988$^{\mathrm{c}}$	0.9989$^{\mathrm{c}}$	0.9988$^{\mathrm{c}}$
Corr = 0.95											
Q	RW	1.0000	0.9995	0.9993	0.9993	0.9995	0.9993$^{\mathrm{c}}$	0.9993$^{\mathrm{c}}$	0.9995$^{\mathrm{c}}$	0.9992$^{\mathrm{c}}$	**0.9989$^{\mathrm{c}}$**
	IW	1.0000	0.9993	0.9992	0.9992	0.9994	0.9992	0.9992$^{\mathrm{c}}$	0.9994$^{\mathrm{c}}$	0.9992$^{\mathrm{c}}$	**0.9989$^{\mathrm{c}}$**
$\tilde{Q}$	RW	1.0000	0.9990	0.9991	0.9988	0.9988	0.9989	0.9987	**0.9986$^{\mathrm{c}}$**	0.9987$^{\mathrm{c}}$	0.9989$^{\mathrm{c}}$
	IW	1.0000	0.9991	0.9993$^{\mathrm{c}}$	0.9988$^{\mathrm{c}}$	0.9990	0.9990$^{\mathrm{c}}$	0.9987$^{\mathrm{c}}$	**0.9986$^{\mathrm{c}}$**	0.9987$^{\mathrm{c}}$	0.9990$^{\mathrm{c}}$
Corr = 0.99											
Q	RW	1.0000	0.9995	0.9993$^{\mathrm{c}}$	0.9993$^{\mathrm{c}}$	0.9995	0.9993$^{\mathrm{c}}$	0.9993	0.9994$^{\mathrm{c}}$	0.9994$^{\mathrm{c}}$	**0.9992$^{\mathrm{c}}$**
	IW	1.0000	0.9993	0.9992$^{\mathrm{c}}$	0.9992$^{\mathrm{c}}$	0.9994	0.9992$^{\mathrm{c}}$	0.9992$^{\mathrm{c}}$	0.9993$^{\mathrm{c}}$	0.9992$^{\mathrm{c}}$	**0.9992$^{\mathrm{c}}$**
$\tilde{Q}$	RW	1.0000	0.9990	0.9991	0.9988	0.9988	0.9989	0.9987	0.9985	**0.9985$^{\mathrm{c}}$**	0.9987
	IW	1.0000	0.9991	0.9993	0.9988	0.9990	0.9990	0.9987	0.9986$^{\mathrm{c}}$	**0.9986$^{\mathrm{c}}$**	0.9987$^{\mathrm{c}}$

Note: Entries are forecast loss ratios of models relative to the standard HAR benchmark. Q denotes the quantile loss function of Koenker and Bassett (1978), and $\tilde{Q}$ is a smoothed version of Q proposed by González-Rivera et al. (2004). Corr = 0.85, 0.95, and 0.99 indicate the realized correlation coefficients that estimate the level of jump dependence between price and volatility. In each row, bold entries mark the lowest loss, while the superscript "c" identifies models retained in the 90% model confidence set of Hansen et al. (2011).

choice, and financial market regulation. Volatility forecasts are widely used in practice. The higher precision of volatility forecasts achieved by incorporating price-volatility cojumps is therefore expected to have many economic implications. As an example, this section demonstrates that improved volatility forecasts lead to more accurate VaR predictions. VaR is a risk measure that quantifies the expected loss of a portfolio over a given time horizon at a specified confidence level. VaR plays a central role in CCPs risk management by determining initial margin requirements (BCBS-CPMI–IOSCO 2022, 2025).

VaR is defined as

$$\mathrm{VaR}_t^{\alpha} = \mu_t + \Phi^{-1}(\alpha)\sqrt{F_t},$$

where μ_t denotes the conditional mean of returns and $\Phi^{-1}(\cdot)$ is the inverse of the standard normal cumulative distribution function. For simplicity, μ_t is assumed to be constant over time and is estimated by the sample mean. To evaluate VaR forecast accuracy, we adopt the quantile-based loss function of Koenker and Bassett (1978),

$$Q = \left[\alpha - I\left(r_t < \mathrm{VaR}_t^{\alpha}\right)\right]\left(r_t - \mathrm{VaR}_t^{\alpha}\right).$$

Since the Q loss function above may suffer from non-differentiability issues (Fameliti and Skintzi 2020), we also consider its smoothed version proposed by González-Rivera et al. (2004),

$$\tilde{Q} = \left[\alpha - \frac{1}{1 + \exp\left\{\psi\left(r_t - \mathrm{VaR}_t^{\alpha}\right)\right\}}\right]\left(r_t - \mathrm{VaR}_t^{\alpha}\right),$$

where the parameter $\psi > 0$ governs the degree of smoothness. Smaller values of Q and $\tilde{Q}$ correspond to more accurate VaR forecasts. Following Fameliti and Skintzi (2020) and Clements and Preve (2021), we set $\psi = 25$ and $\alpha = 0.05$.

Table 8 reports the VaR out-of-sample forecast performance, measured by loss ratios relative to the standard HAR benchmark. The MCS results indicate that the cojump HAR specifications in the final panel are consistently retained in the MCS across loss functions and dependence levels, while the benchmark HAR model and specifications incorporating general jumps (by bipower or realized power variation) are largely excluded. This contrast is consistent across quantile loss functions and forecast windows and becomes even more pronounced at higher levels of jump dependence. These results indicate the superior VaR forecasting performance of price-volatility cojump information relative to models with no jumps or price jumps alone, highlighting that improved volatility forecasts driven by cojump dynamics also translate into better tail-risk forecasting.

5 | Further Analysis

5.1 | Do Price Jumps Forecast RV? Revisited

Our analysis of price jumps and RV forecasting has so far focused on significant price jumps without distinguishing whether they coincide with volatility jumps. Price jumps that occur simultaneously with volatility jumps (cojumps) are associated with elevated perceived risk and therefore may provide stronger predictive signals for future volatility than isolated price jumps. This motivates us to compare the forecasting power of these two types of jumps. Specifically, we re-estimate

TABLE 9 | In-sample results for modified HAR jump models.

	Jumps estimated via bipower variation			Jumps estimated via high-order power variation		
	Modified HAR-C-JV	**Modified HAR-C-UDJV**	**Modified HAR-C-AJV**	**Modified HAR-C-JP**	**Modified HAR-C-UDJP**	**Modified HAR-C-AJP**
Corr = 0.85						
β_J	0.119			0.132		
	(0.39)			(0.42)		
β_{J^+}		−0.556			−0.805*	
		(−1.29)			(−1.85)	
β_{J^-}		0.789***			0.821***	
		(2.91)			(3.15)	
β_{AJ}			−0.649**			−0.692***
			(−2.35)			(−2.83)
R^2	0.7488	0.7490	0.7490	0.7488	0.7489	0.7489
Corr = 0.95						
β_J	0.060			0.098		
	(0.10)			(0.20)		
β_{J^+}		−1.558***			−1.626***	
		(−9.08)			(−9.24)	
β_{J^-}		0.711***			0.834***	
		(6.72)			(7.87)	
β_{AJ}			−0.924***			−0.916***
			(−3.78)			(−3.45)
R^2	0.7487	0.7490	0.7490	0.7487	0.7490	0.7490
Corr = 0.99						
β_J	−0.012			0.042		
	(−0.02)			(0.08)		
β_{J^+}		−1.540***			−1.659***	
		(−9.98)			(−10.71)	
β_{J^-}		0.637***			0.768***	
		(9.87)			(13.45)	
β_{AJ}			−0.877***			−0.889***
			(−3.78)			(−3.42)
R^2	0.7487	0.7490	0.7490	0.7487	0.7490	0.7490

Note: The modified HAR jump models include jumps that coincide with volatility jumps. Corr = 0.85, 0.95, and 0.99 indicate the realized correlation coefficients that gauge the level of jump dependence between price and volatility. Results for the jump models are grouped by jumps estimated using bipower variation and high-order power variation. Coefficients on the constant and volatility components are not reported for presentation purposes. Reported are coefficient estimates with t-statistics in parentheses. Statistical significance is denoted by *, **, and *** at the 10%, 5%, and 1% levels, respectively.

the previously defined jump-augmented models, HAR-C-JV, HAR-C-UDJV, HAR-C-AJV, HAR-C-JP, HAR-C-UDJP, and HAR-C-AJP, by interacting their jump components with the indicator function of price-volatility cojumps, which equals one in the presence of cojumps and zero otherwise.

Table 9 presents the estimated coefficients for the modified jump models, which assess the effects on RV forecasting of price jump variables that coincide with volatility jumps. The models are evaluated under realized correlation thresholds of 0.85, 0.95, and 0.99 between price and volatility jumps. For each case, we report results for specifications where jumps are estimated using both BV and high-order power variation. Coefficients for the volatility components are omitted from the table for brevity.

Across all settings, the coefficient on positive jumps, β_{J^+}, is negative and highly significant, while β_{J^-} is positive and often significant, consistent with leverage effects in volatility forecasts. The asymmetry term, β_{AJ}, is also negative and significant, particularly at higher correlation thresholds. These results stand in contrast to the null evidence for price jumps in volatility forecasting reported in the empirical work of Andersen, Bollerslev, and Dobrev (2007), Busch et al. (2011), Prokopczuk et al. (2016), and Caporin (2023). A plausible explanation is that equity jumps occurring jointly with implied volatility jumps trigger stronger market fears than isolated jumps, thereby exerting a more pronounced influence on future volatility. Our findings thus extend prior studies by showing that jumps are

TABLE 10 | Out-of-sample comparison of original and modified HAR jump models.

		Models with jumps estimated by bipower variation			Models with jumps estimated by high-order power variation		
		HAR-C-JV original – modified	HAR-C-UDJV original – modified	HAR-C-AJV original – modified	HAR-C-JP original – modified	HAR-C-UDJP original – modified	HAR-C-AJP original – modified
Corr = 0.85							
HMSE	RW	0.51	−2.04	−0.73	−0.50	−1.58	0.75
	IW	−0.30	−2.13	−2.05	−1.64	−0.67	−0.20
QLIKE	RW	0.10	−1.36	0.65	0.03	−1.17	0.24
	IW	−0.81	−1.26	0.01	−0.92	−0.20	0.84
Corr = 0.95							
HMSE	RW	2.20**	3.32***	−0.14	1.87*	1.86*	−1.36
	IW	2.67***	5.63***	−1.42	2.25**	4.00***	−1.58
QLIKE	RW	0.25	−0.04	−0.24	−0.12	−0.18	−0.01
	IW	1.00	1.95*	−0.68	0.96	1.95*	0.44
Corr = 0.99							
HMSE	RW	3.52***	5.57***	0.55	4.18***	3.78***	0.01
	IW	2.98***	5.67***	−0.86	2.94***	4.20***	−0.98
QLIKE	RW	1.33	2.09**	0.14	2.15**	2.15**	0.62
	IW	1.07	1.95*	−0.54	1.10	1.95*	0.59

Note: The modified models contain jumps that are restricted to coincide with volatility jumps while the original models use all jumps. Reported values are Diebold-Mariano statistics (Diebold 2015) under heteroskedasticity-adjusted mean squared error (HMSE) and Gaussian quasi-likelihood (QLIKE) loss functions. Statistical significance is denoted by *, **, and *** at the 10%, 5%, and 1% levels, respectively.

not irrelevant for volatility forecasting; rather, their predictive power materializes when they coincide with large shifts in market volatility.

We next examine whether the significance of jumps associated with volatility jumps also translates into improved out-of-sample volatility forecasts. To do so, we compare the predictive performance of each original jump model (Equations 20–25) with its modified counterpart in which jump variables are restricted to those coinciding with volatility jumps. To assess significant differences for the pair comparisons, we employ the Diebold-Mariano (DM) test of Diebold (2015). Let $L_t^{(\text{original})}$ and $L_t^{(\text{modified})}$ denote the forecast losses from original and modified jump models, respectively, and define the loss differential as $d_t = L_t^{(\text{original})} - L_t^{(\text{modified})}$. The null hypothesis assumes equal predictive accuracy, against the alternative of differing predictive performance. The DM test statistic is defined as,

$$\text{DM} = \frac{\bar{d}}{\sqrt{\hat{\Omega}_d / T_f}} \xrightarrow{d} N(0, 1),$$

where $\bar{d}$ is the sample mean of $\{d_t\}_{t=1,\ldots,T_f}$ and $\hat{\Omega}_d$ is a heteroskedasticity- and autocorrelation-consistent variance estimator of Newey and West (1987). Under a two-sided test, equal predictive accuracy is rejected when the absolute DM statistic exceeds the critical value, with positive (negative) values favoring the modified (original) model.

Table 10 reports the DM statistics of Diebold (2015) comparing the predictive accuracy between the original and modified jump models, where the latter incorporate only price jumps that concur with volatility jumps. The left panel corresponds to models with jumps estimated via BV, while the right panel uses high-order power variation. At a realized correlation threshold of 0.85, none of the DM statistics are significant, indicating no discernible predictive gains from incorporating volatility-jump-related jumps. At a realized correlation of 0.95, significant improvements appear primarily in the HAR-C-UDJV and HAR-C-UDJP models under the HMSE loss function, with DM values ranging from 1.86 to 5.63. At correlation 0.99, both HAR-C-UDJV and HAR-C-UDJP exhibit large and highly significant DM statistics under HMSE, and also show significant gains under QLIKE at the 5% level for rolling windows and at the 10% level for expanding windows. These results confirm that incorporating price jumps markedly enhances forecast accuracy when their correlation with volatility jumps is very high. By contrast, the HAR-C-AJV and HAR-C-AJP models show no systematic improvements across scenarios. Taken together, the evidence highlights the critical role of price jumps in improving RV forecasts when they are linked to volatility jumps, thereby reinforcing the broader importance of price-volatility cojumps in volatility forecasting.

5.2 | The Direction of Cojumps in S&P500 and VIX

Todorov and Tauchen (2011) show that common jumps in the S&P 500 and VIX series can occur in both the same and opposite directions. They examine this by decomposing cojumps into components arising from opposite-direction and same-direction price-volatility jumps, with the corresponding estimators defined as:

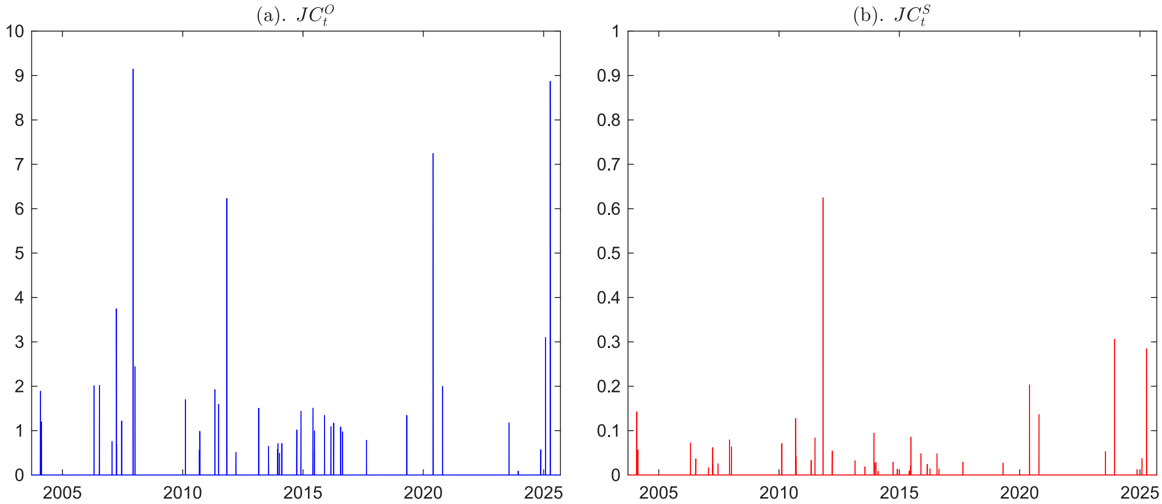

FIGURE 3 | This figure shows daily S&P 500-VIX cojumps, decomposed into (a) opposite-direction cojumps (JC_t^O, left panel) and (b) same-direction cojumps (JC_t^S, right panel). [Color figure can be viewed at wileyonlinelibrary.com]

$$JC_t^O = \left(\sum_{i=1}^{n} |\Delta_n X_{t_i}|^2 |\Delta_n Y_{t_i}|^2 I\,(\Delta_n X_{t_i}\, \Delta_n Y_{t_i} < 0) \right)^{1/2},$$

and

$$JC_t^S = \left(\sum_{i=1}^{n} |\Delta_n X_{t_i}|^2 |\Delta_n Y_{t_i}|^2 I\,(\Delta_n X_{t_i}\, \Delta_n Y_{t_i} > 0) \right)^{1/2},$$

where JC_t^O measures cojumps in opposite directions and JC_t^S measures cojumps in the same direction. Substituting this decomposition into Equation (26) yields a new specification, denoted HAR-C-JC$^{S/O}$, defined as:

$$RV_t = \beta_0 + \beta_d C_t^{(d)} + \beta_w RV_t^{(w)} + \beta_m RV_t^{(m)} + \beta_{JC^O} JC_{t-1}^O + \beta_{JC^S} JC_{t-1}^S + \epsilon_t.$$

We can also divide upside and downside cojumps based on the directions of the volatility and price jumps, resulting in price-up/volatility-down cojumps $JC^{+,-}$, price-down/volatility-up cojumps, $JC^{-,+}$, price-up/volatility-up cojumps $JC^{+,+}$, and price-down/volatility-down cojumps, $JC^{-,-}$. Using these separations, we can extend the above specification to the HAR-C-UDJC$^{S/O}$ model:

$$RV_t = \beta_0 + \beta_d C_t^{(d)} + \beta_w RV_t^{(w)} + \beta_m RV_t^{(m)} + \beta_{JC^{+,-}} JC_{t-1}^{+,-} + \beta_{JC^{-,+}} JC_{t-1}^{-,+} + \beta_{JC^{+,+}} JC_{t-1}^{+,+} + \beta_{JC^{-,-}} JC_{t-1}^{-,-} + \epsilon_t.$$

By contrasting the upside and downside cojump components in which price and volatility move in opposite directions, we construct the opposite-sign asymmetric cojump measure as $AJC^O = JC^{+,-} - JC^{-,+}$. Analogously, we define the same-sign asymmetric cojump measure as $AJC^S = JC^{+,+} - JC^{-,-}$. These two asymmetric cojump measures form the basis of the HAR-C-AJC$^{S/O}$ specification presented below.

$$RV_t = \beta_0 + \beta_d C_t^{(d)} + \beta_w RV_t^{(w)} + \beta_m RV_t^{(m)} + \beta_{AJC^O} AJC_{t-1}^O + \beta_{AJC^S} AJC_{t-1}^S + \epsilon_t.$$

Figure 3 plots the daily series of equity-VIX cojumps of opposite sign, JC_t^O, and same sign, JC_t^S, shown in the left and right panels, respectively. The results indicate that the magnitude of JC_t^O (on the order of 10^0) is greater than that of JC_t^S (order of 10^{-1}), suggesting that opposite-sign cojumps dominate overall cojump activity. This finding is consistent with Todorov and Tauchen (2011), who document that the majority of price-volatility cojumps arise from jumps in opposite directions.

Table 11 reports coefficient estimates for the models that decompose cojump measures into opposite- and same-sign components, for different realized correlation thresholds. The first column of each panel corresponds to the HAR-C-JC$^{S/O}$ model that contains the decomposed cojumps based on the directions of volatility jumps and price jumps. The results indicate that the coefficients on opposite- and same-sign cojumps (β_{JC^O} and β_{JC^S}) are insignificant for realized correlation 0.85, corr = 0.85, and only marginally significant with differing signs at corr = 0.95.

The second column of each panel reports the coefficient estimates from the HAR-C-UDJC$^{S/O}$ specification that incorporates the sign direction of upside and downside price-volatility cojumps. The coefficients on price-up/volatility-down and price-down/volatility-up cojumps, $\beta_{JC^{+,-}}$ and $\beta_{JC^{-,+}}$ are consistently negative and positive, respectively, and are generally statistically significant across all correlation thresholds. This pattern may be interpreted by noting that price-up/volatility-down cojumps ($JC^{+,-}$) reflect episodes of favorable news and are associated with lower future volatility, whereas price-down/volatility-up cojumps ($JC^{-,+}$) capture adverse shocks that heighten market fears and lead to higher subsequent volatility.

By contrast, the coefficients on the price-up/volatility-up and price-down/volatility-down cojumps, denoted by $\beta_{JC^{+,+}}$ and $\beta_{JC^{-,-}}$, are generally insignificant. The weak predictive content of price-up/volatility-up cojumps ($JC^{+,+}$) is likely because such events involve offsetting leverage and uncertainty effects, leaving little net effect on future volatility. Similarly, price-down/volatility-down cojumps ($JC^{-,-}$) may be associated with adverse but clarifying news that reduces market fears, thereby exerting only limited influence on future volatility.

TABLE 11 | Impact of opposite- and same-sign cojumps on realized variance forecasts.

	Corr = 0.85			Corr = 0.95			Corr = 0.99		
	HAR-C-JC$^{O/S}$	HAR-C-UDJC$^{O/S}$	HAR-C-AJC$^{O/S}$	HAR-C-JC$^{O/S}$	HAR-C-UDJC$^{O/S}$	HAR-C-AJC$^{O/S}$	HAR-C-JC$^{O/S}$	HAR-C-UDJC$^{O/S}$	HAR-C-AJC$^{O/S}$
β_0	−0.102***	−0.102***	−0.101***	−0.102***	−0.102***	−0.103***	−0.102***	−0.102***	−0.103***
	(−10.89)	(−10.89)	(−10.80)	(−10.92)	(−10.91)	(−10.94)	(−10.93)	(−10.92)	(−10.95)
β_d	0.484***	0.484***	0.483***	0.486***	0.485***	0.486***	0.486***	0.486***	0.485***
	(27.81)	(27.86)	(27.82)	(27.96)	(27.92)	(27.93)	(27.97)	(27.94)	(27.94)
β_w	0.293***	0.294***	0.295***	0.291***	0.293***	0.292***	0.292***	0.292***	0.292***
	(11.72)	(11.76)	(11.82)	(11.66)	(11.70)	(11.69)	(11.67)	(11.68)	(11.69)
β_m	0.169***	0.168***	0.168***	0.169***	0.168***	0.168***	0.169***	0.169***	0.169***
	(9.06)	(9.04)	(9.04)	(9.09)	(9.04)	(9.06)	(9.09)	(9.07)	(9.06)
β_{JC^O}	0.088			0.331			0.570**		
	(0.52)			(1.34)			(2.02)		
β_{JC^S}	0.089			−6.242			−11.0***		
	(0.05)			(−1.24)			(−2.87)		
$\beta_{JC^{+,-}}$		−0.577***			−0.826***			−0.473	
		(−2.66)			(−3.14)			(−0.83)	
$\beta_{JC^{-,+}}$		0.437***			0.389***			0.473**	
		(3.42)			(3.75)			(2.29)	
$\beta_{JC^{+,+}}$		−0.490			0.918			0.825	
		(−0.37)			(0.23)			(0.19)	
$\beta_{JC^{-,-}}$		5.564***			5.951			−6.673	
		(6.67)			(1.58)			(−0.44)	
β_{AJC^O}			−0.348***			−0.428***			−0.437***
			(−3.15)			(−6.18)			(−5.31)
β_{AJC^S}			−1.335			−1.662			−0.031
			(−0.72)			(−0.56)			(−0.01)
R^2	0.7489	0.7495	0.7493	0.7493	0.7494	0.7495	0.7493	0.7493	0.7494

Note: HAR-C-JC$^{S/O}$ denotes the model that includes cojumps with price moving in the opposite direction to volatility as well as cojumps with price moving in the same direction as volatility. HAR-C-UDJC$^{S/O}$ is the specification that uses price-up/volatility-down cojumps $JC^{+,-}$, price-down/volatility-up cojumps $JC^{-,+}$, price-up/volatility-up cojumps $JC^{+,+}$, and price-down/volatility-down cojumps $JC^{-,-}$. HAR-C-AJC$^{S/O}$ is the specification that depends on the corresponding asymmetry terms. Statistical significance is denoted by *, **, and *** at the 10%, 5%, and 1% levels, respectively.

6 | Robustness

6.1 | Jump Econometric Validity

Our conclusions on the impacts of jumps and cojumps on volatility forecasting depend on the jump testing procedure, which is designed to mitigate measurement error. Recent research by Kolokolov and Renò (2024) shows that stale quotes may be erroneously classified as jumps by conventional jump tests, while Laurent and Shi (2020) demonstrate that nonzero drift (trend) in high-frequency prices can reduce the power of jump detection.

To address the impact of zero returns on jump detection, we adopt the staleness-robust version of the BNS statistic proposed by Kolokolov and Renò (2024), which down-weights price intervals with no updates to ensure that only genuine discontinuities are classified as jumps. Using this approach, we find that the occurrence rates of jumps and cojumps for our sample decline from 2.26% and 0.78% to 0.45% and 0.13%, respectively, indicating that spurious jumps induced by price staleness are substantially reduced, consistent with the findings of Kolokolov and Renò (2024).

In addition, to mitigate the effect of nonzero drift on the power of the BNS test, we follow Laurent and Shi (2020) and reconstruct the statistic using intraday log returns centered around their daily median. Intuitively, this centering procedure reduces the influence of price drift and improves jump detection power. Accordingly, the occurrence rate of jumps for our sample increases to 2.42%, in line with the empirical evidence in Laurent and Shi (2020). However, this adjustment does not lead to a higher detection rate of cojumps, as the occurrence of cojumps remains at 0.78%, likely reflecting the relatively modest gain in power for identifying joint jump events.

Table 12 reports the coefficient estimates and goodness-of-fit measures for all jump and cojump models defined in Table 2, corrected for potential biases from price staleness and nonzero drift.[7] Across both panels, we find that jumps, upside and downside jumps, and jump asymmetry remain insignificant for RV forecasts, regardless of whether jumps are measured via BV or high-order power variation.

TABLE 12 | Robustness to alternative econometric adjustments in jump detection.

	Jump variation models			Jump power variation models			Cojump variation models		
	HAR-C-JV	HAR-C-UDJV	HAR-C-AJV	HAR-C-JP	HAR-C-UDJP	HAR-C-AJP	HAR-C-JC	HAR-C-UDJC	HAR-C-AJC
Price staleness									
β_J	0.077			0.127			−0.039		
	(1.14)			(0.34)			(−0.20)		
β_{J^+}		0.045			0.048			−0.608**	
		(0.37)			(0.09)			(−2.26)	
β_{J^-}		0.042			0.163			0.710**	
		(0.36)			(0.23)			(2.23)	
β_{JC}			0.026			0.159			−0.349*
			(0.37)			(0.35)			(−1.93)
R^2	0.6344	0.6342	0.6342	0.6342	0.6342	0.6342	0.6342	0.6343	0.6343
Nonzero drift									
β_J	0.017			0.011			−0.045		
	(0.08)			(0.04)			(−0.37)		
β_{J^+}		−0.135			−0.486			−0.493***	
		(−0.39)			(−1.25)			(−2.90)	
β_{J^-}		0.147			0.378			0.338*	
		(0.44)			(1.06)			(1.92)	
β_{JC}			−0.107			−0.317			−0.275*
			(−0.39)			(−0.98)			(−1.88)
R^2	0.7513	0.7514	0.7513	0.7513	0.7515	0.7514	0.7512	0.7516	0.7516

Note: The table reports robustness checks for the estimation of HAR jump and cojump models when econometric adjustments to jump detection are applied. The upper panel shows results for the staleness-robust BNS test of Kolokolov and Renò (2024), which aims to reduce the size distortion due to stale prices. The lower panel presents results for the drift-adjusted BNS test of Laurent and Shi (2020), which is to mitigate the power loss due to nonzero drift. Coefficients on the constant and realized variance lags are suppressed for brevity. t-statistics are reported in parentheses. Statistical significance is denoted by *, **, and *** at the 10%, 5%, and 1% levels, respectively.

In contrast, upside and downside cojumps and cojump asymmetry continue to be highly significant predictors of RV under both econometric adjustments. Overall, our findings are robust to correcting for jump testing issues arising from price staleness and nonzero drift in high-frequency data.

6.2 | Variance and Standard Deviation HAR Transformations

Our empirical analysis of the HAR framework employs its logarithmic specification to improve the validity of statistical inference and to avoid negative forecasts. As a robustness check, we revisit the main in-sample estimation of the jump and cojump models reported in Table 2, using the variance and standard deviation transformations. The jump correlation coefficient is set at 0.95 when filtering cojumps. The estimation results are presented in Table 13, with coefficient estimates on RV lags omitted for brevity. Consistent with our baseline results, all price jump variables remain insignificant across both HAR forms and for both bipower and high-order power variation estimators. By contrast, upside and downside cojumps and cojump asymmetry remain highly significant: upside cojumps reduce future RV, while downside cojumps increase it. These findings confirm that our conclusions on the effects of jumps and cojumps on volatility forecasting are robust to alternative HAR model specifications.

6.3 | HAR Model With Weekly and Monthly Jumps and Cojumps

Our analysis of volatility forecasting so far has focused on the impact of daily jumps and cojumps, in order to isolate the effect of a single jump on future volatility. As a robustness exercise, we also investigate the influence of weekly and monthly jump and cojump terms. To this end, we extend all jump and cojump models in Table 2 to their "full" versions by decomposing weekly and monthly RVs into continuous variation and (co)jump components. The extended specifications and their estimation results are reported in Supporting Information S1.

Our estimation results for the models with full lags of jumps can be summarized as follows. The coefficient estimates on the weekly and monthly jump variables, including jumps, upside and downside jumps, and jump asymmetry, are generally insignificant, regardless of whether jumps are estimated using BV or high-order power variation. For the cojump components, we similarly find that weekly and monthly unsigned cojumps are insignificant, while the lags of upside and downside cojumps and cojump asymmetry also lack significance. Only the daily cojump terms remain significant. This suggests that the most recent cojumps exert a much stronger influence on volatility than more distant observations, which is consistent with economic intuition.

TABLE 13 | Robustness to HAR variance and standard deviation transformations.

	Jump variation models			Jump power variation models			Cojump variation models		
	HAR-C-JV	HAR-C-UDJV	HAR-C-AJV	HAR-C-JP	HAR-C-UDJP	HAR-C-AJP	HAR-C-JC	HAR-C-UDJC	HAR-C-AJC
Variance HAR									
β_J	−0.631			−0.422			0.064		
	(−0.97)			(−0.79)			(0.68)		
β_{J^+}		−0.845			−0.775			−0.207***	
		(−1.20)			(−1.39)			(−7.16)	
β_{J^-}		−0.088			−0.150			0.185***	
		(−0.14)			(−0.22)			(8.42)	
β_{JC}			−0.412			−0.275			−0.190***
			(−1.49)			(−1.16)			(−13.5)
R^2	0.4834	0.4833	0.4833	0.4834	0.4833	0.4833	0.4833	0.4832	0.4833
Standard deviation HAR									
β_J	0.014			0.010			0.020		
	(0.18)			(0.13)			(0.30)		
β_{J^+}		−0.107			−0.198			−0.256***	
		(−1.14)			(−1.80)			(−5.43)	
β_{J^-}		0.128			0.210			0.217***	
		(1.19)			(1.53)			(8.06)	
β_{JC}			−0.105			−0.084			−0.155***
			(−1.41)			(−0.98)			(−5.84)
R^2	0.7548	0.7548	0.7549	0.7548	0.7549	0.7548	0.7546	0.7547	0.7548

Note: This table reports robustness checks for the estimation of HAR jump and cojump models when realized variance is expressed in variance form (top panel) and standard deviation form (bottom panel). Jump variables include jumps estimated using bipower variation, high-order power variation, and cojump measures. Coefficients on the constant and realized variance lags are suppressed to save space. t-statistics are reported in parentheses. Statistical significance is denoted by *, **, and *** at the 10%, 5%, and 1% levels, respectively.

7 | Conclusion

Consistent with the literature documenting price-volatility cojumps as a stylized feature of equity markets, we find that cojumps continue to occur from 2003 to 2025 for the S&P 500. Our results indicate that price-volatility cojumps intensify during periods of elevated policy uncertainty and adverse credit and liquidity conditions, offering a potential explanation for the documented jump leverage risk premium and for the fact that price and volatility jumps do not always occur simultaneously. Moreover, the strong association between price-volatility cojumps and market stress suggests their relevance for margin regulation and systemic risk monitoring.

We further show that price-volatility cojumps have significant predictive power for future volatility, with the effects driven primarily by cojumps in which price and volatility move in opposite directions. In addition, we provide new evidence that price jumps become informative for volatility forecasting when they occur simultaneously with volatility jumps, in contrast to earlier findings that price jumps almost have no forecasting value. Importantly, models that incorporate cojump dynamics deliver superior out-of-sample forecasts of both volatility and VaR, underscoring the economic relevance of price-volatility cojumps.

Author Contributions

Kefu Liao: conceptualization, methodology, data curation, formal analysis, investigation, writing – original draft, writing – review and editing, supervision.

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Conflicts of Interest

The author declares no conflicts of interest.

Data Availability Statement

The datasets generated during the current study are available from the corresponding author on reasonable request.

Endnotes

[1] For further discussion, refer to Jacod (2008), Barndorff-Nielsen et al. (2010), Aït-Sahalia and Jacod (2012), and Duong and Swanson (2015).

[2] This convention reflects both economic intuition: major news arrivals such as macroeconomic announcements or earnings releases are typically concentrated, and established practice in the literature. See, for example, Barndorff-Nielsen and Shephard (2004, 2006), and

Tauchen and Zhou (2011), who adopt the framework of infrequent, dominant daily jumps when analyzing price discontinuities.

[3]Spurious jumps can distort the estimated jump distribution (Tauchen and Zhou 2011; Bajgrowicz et al. 2016) and undermine the reliability of derivative pricing and hedging strategies (Christensen et al. 2014; Lee and Mykland 2008).

[4]The index combines three components: (1) the frequency of articles on economic policy uncertainty in ten major U.S. newspapers, normalized to capture shifts in coverage; (2) the number of federal tax code provisions set to expire, weighted by their economic impact, as reported by the Congressional Budget Office; and (3) the dispersion among professional economic forecasters' predictions on key policy-related variables such as inflation and government spending.

[5]The TED spread is unavailable after January 31, 2022, when the 3-Month USD LIBOR was discontinued on FRED. For this subsample, we construct an alternative measure using the spread between the Secured Overnight Financing Rate (SOFR) and the 3-Month U.S. Treasury bill. SOFR has been recognized by FRED as the benchmark rate for U.S. dollar derivatives and other financial contracts.

[6]Following Corsi et al. (2010) and Patton and Sheppard (2015), we restrict attention to daily jumps to better assess the effect of a single jump on future volatility. Weekly and monthly components are considered in the robustness analysis.

[7]Realized variance and bipower variation components in these equations are also adjusted accordingly.

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Supporting Information

Additional supporting information can be found online in the Supporting Information section.
Table 1: Summary of the HAR specifications with daily, weekly, and monthly lags of jumps and cojumps. **Table 2:** In-sample estimation results for the HAR specifications with daily, weekly, and monthly lags of jumps. **Table 3:** In-sample estimation results for the HAR specifications with daily, weekly, and monthly lags of cojumps.