

Use of a Lateral Shearing Plate Interferometer for Verifying Diffraction-Limited Performance of a Near-Infrared Spectrometer

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ABSTRACT

Creating a new observatory or optical system includes the development of an *alignment plan*, the procedure for assembling parts and subsystems into a fully functioning observatory. Part of developing the alignment plan is choosing which metrology method is the best fit for each step, ideally with an emphasis on reducing cost and complexity. In this paper, we show how lateral shearing interferometry can be used for quick, low-cost verification of diffraction-limited performance of an optical system using a so-called *horizontal line test*. This horizontal line test does not require numerical methods and is relatively simple to implement, making it a low-complexity and flexible metrological method to include as part of an alignment plan. We discuss the optical theory behind this method, and provide specific examples of aberration testing with both simulated and laboratory demonstrations. We also discuss how linear shearing interferometry can be applied to observatory alignment plans, using the NASA EXoplanet Climate Infrared TElescope (EXCITE) spectrograph as an example of real-life implementation. Overall, the lateral shearing plate horizontal line test is particularly useful in situations where logistics constrain available resources, such as deployment in a remote location like Antarctica.

Keywords: Exoplanets, hot Jupiters, High altitude balloons, Astronomical instrumentation

1. INTRODUCTION

Deploying astronomical instruments requires a comprehensive alignment plan, one that brings a collection of parts to a fully functioning observatory. One example step of an alignment plan is verifying the performance of individual optics, before assembly. Another, related step is alignment verification: checking that the assembled optics produce a beam quality that meets the spec-

ifications required by the science goals of the project. Part of developing the alignment plan is choosing which metrology method is the best fit for each step, ideally with an emphasis on reducing cost and complexity (P. Coulter et al. 2016). In this paper, we show how *lateral shearing interferometry* can be used for quick, low-cost verification of diffraction-limited performance of an optic, both individually and as part of an instrument.

Optical interferometry is a technique for estimating the quality of an optical beam by measuring the shape of a wavefront of light, or by measuring the slope (a.k.a. gradient) of the wavefront, then using that gradient to

infer the shape. This wavefront shape can be used to infer the shape of an optical surface, or to estimate the alignment of an optic in use. Shearing interferometry, first described in M. V. R. K. Murty (1964), is an interferometry technique that splits a wavefront in two, displaces the split wavefronts with respect to each other, then interferes them at some observation plane. While there are radial and azimuthal approaches to this (K.-N. Joo & H. M. Park 2023), we focus on *lateral shearing*, which works by using parallel, or nearly parallel, partially reflective surfaces (the so-called *shearing plate*) to split the wavefront and displace it in a lateral direction in a single optical step. This saves the need for generating a reference beam and allowing wavefront shape measurements in a compact footprint, at the cost of requiring a monochromatic light source. Lateral shearing plate interferometry is sensitive to optical wavefront error at the fraction of a wavelength scale.

The interference pattern produced by lateral shearing provides information on the wavefront gradient, which is commonly used to infer the wavefront shape and quantify any errors. One approach is to estimate the wavefront slope at discrete points across the pattern, then using the difference between those points to reconstruct the wavefront (W. Southwell 1980). This approach is similar to the one used with Shack-Hartmann sensors, which have similar advantages in compactness and no need for a reference wavefront. Like Shack-Hartmann sensors, shearing interferometry can also be used in a modal fashion (albeit without the inherent limit of a Shack-Hartmann’s spatial sampling frequency), using the entire interference pattern to estimate coefficients of a polynomial aperture function, such as Zernike polynomials. Either approach allows shearing interferometry to map a nearly arbitrary wavefront using numerical methods, which has led to their use in a broad range of applications, from adaptive optics to characterizing lens shape (K.-N. Joo & H. M. Park 2023).

However, we use shearing plate interferometry to estimate wavefront error without numerical techniques. In the limit of a nearly flat wavefront, such as produced by a nearly aligned optic, one can estimate whether or not an optic is producing diffraction-limited performance by simply comparing the interference pattern to a reference line parallel with the direction of shear, which we refer to as the horizontal direction. This allows us to verify diffraction-limited performance at a glance, without the need for a detector, complex light source, or computational analysis, greatly simplifying the logistics of verifying instrument performance.

In this paper we show how this *horizontal line test* relates to Strehl performance. In **Section 2** we describe

the geometry and interference pattern of linear shearing plates. In **Section 3** we describe qualitative diagnosis of aberrations. In **Section 4**, we describe using shearing plates patterns to quantify optic alignment, first for the well-known case of defocus, then for off-axis astigmatism. In **Section 5**, we show how we use these techniques to verify performance of optics, using the alignment plan for the NASA EXoplanet Climate Infrared Telescope (EXCITE) spectrograph as an example.

1.1. Summary of the EXCITE mission

EXCITE is a NASA high-altitude balloon mission that will use infrared spectroscopy to study hot Jupiter exoplanets. Hot Jupiters are close-orbiting extra-solar planets that have no Solar System analogue, presenting a unique opportunity to provide context for our own Solar System. The atmosphere of these large planets contain signatures of the conditions under which they formed, making hot Jupiters records of the formation history of their planetary system and providing an opportunity to test models of planetary system formation in extreme scenarios. And their short orbital period (< 10 days) and high temperatures makes hot Jupiters optimal targets for *phase resolved spectroscopy*. For a more detailed description of hot Jupiter science and spectroscopic phase curve observations, see N. Madhusudhan et al. (2014), (V. Parmentier & I. J. M. Crossfield 2018), N. Madhusudhan (2019), and E. M.-R. Kempton & H. A. Knutson (2024). The goal of the EXCITE mission is enabling phase resolved spectroscopy of hot Jupiter exoplanet atmospheres.

EXCITE is a novel instrument that uses a 0.5 meter telescope operating from a high-altitude balloon platform, with a visible-light camera providing fine guidance and a two-channel NIR spectrograph (short channel 0.8-2.5 μm , long channel 2.5-3.5 μm , $R \geq 50$) providing the science data. A dedicated mission, EXCITE will use a long-duration, high-altitude balloon flight to observe targets nearly uninterrupted for tens of hours, using near-infrared (NIR) spectroscopy to provide phase curve measurements of transiting hot Jupiters. EXCITE is expected to produce a first-of-kind survey of full spectroscopic phase curves of transiting hot Jupiters, with about 10 phase curve measurements expected per science flight. A full description of the mission is found in P. C. Nagler et al. (2019) and P. C. Nagler et al. (2022). A description of the spectrograph design can be found in L. Bernard et al. (2022), and the cryogenic receiver design in T. Rehm et al. (2022). A description of the spectrograph assembly and integration can be found in L. Bernard et al. (2024), with a description of the integration and testing of the cryogenic receiver in A. Kleyheeg

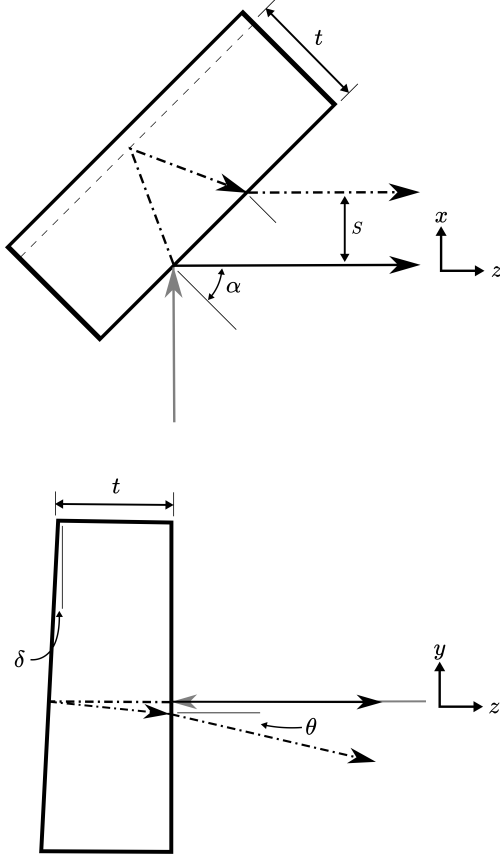


Figure 1. Diagram representing a shearing plate interferometer (not to scale). **Top:** Top-down view. The z axis is defined as the direction of propagation of the first reflected beam, and the x axis is defined as the direction of the shear. **Bottom:** Side view. Note the wedge is perpendicular to the direction of shear. The wedge angle δ and divergent beam angle θ are exaggerated for clarity.

et al. (2024). The telescope gondola pointing and stabilization qualification can be found in L. J. Romualdez et al. (2024). A full description of the current status of EXCITE can be found in T. D. Rehm et al. (2026).

2. BASIC PHYSICS OF LATERAL SHEARING PLATES

Here we provide a brief description of a lateral shearing plate interferometer. A detailed description of lateral shearing plate interferometers can be found in M. E. Riley & M. A. Gusinow (1977); in this section we update and expand upon Riley and Gusinow's notation.

2.1. Shearing Plate Geometry

A shearing plate interferometer is an optical glass plane plate with parallel or nearly parallel surfaces, which operates by reflecting overlapping beams from

both surfaces of the plate. (See **Figure 1.**) When the plate is placed in a monochromatic beam at some angle α relative to the beam (typically 45°), the plate produces two reflected beams. The thickness of the plate t introduces both an offset (the so-called *shear distance*) and an optical path length (OPL) difference between the two beams. Choosing a plate thickness such that the shear distance S is less than the beam diameter D causes the reflected beams to overlap, resulting in interference; a slight wedge angle δ in the plate introduces a linear modulation in the phase difference of the beam wavefronts, producing sinusoidal fringes in the intensity pattern. This fringe pattern is a function of the local radius of curvature of the wavefront, enabling a quick and convenient direct estimate of wavefront error.

Assuming δ is very small, such that $D\delta \ll t$, the shear distance S is given by

$$S = t \frac{\sin 2\alpha}{\sqrt{n^2 - \sin^2 \alpha}}. \quad (1)$$

The wedge angle δ produces an angle θ between the two reflected wavefronts

$$\theta = 2\delta \sqrt{n^2 - \sin^2 \alpha}. \quad (2)$$

And the optical path difference ΔZ between the two beams is given by

$$\Delta Z = 2t \sqrt{n^2 - \sin^2 \alpha}, \quad (3)$$

where t is the shearing plate thickness and α is the angle between the incident beam and the normal of the shearing plate.

Using the geometry of shear plate defined above, we can describe the pair of reflected beam in terms of a unified coordinate system. As in Riley and Gusinow, we consider two optical axis, where z is along the axis of the first reflected beam and z' is along the axis of the second reflected beam, with (x, y) and (x', y') defining the plane perpendicular to their respective directions of propagation. The transformation from the first reflection to the second reflection is then given by

$$x' = x - S, \quad (4)$$

$$\begin{bmatrix} y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} \simeq \begin{bmatrix} 1 & \theta \\ -\theta & 1 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix}. \quad (5)$$

2.2. Linear Shearing Interference

A common use of shearing plate interferometers is to measure the collimation of a beam of monochromatic light, which we use here as an example. We model collimated light as a beam emitted by an ideal spherical

source. As the radius of curvature (R_c) of the beam increases, the beam collimation improves, with $R_c = \infty$ constituting an ideally collimated source (defined as a perfect plane wave). This allows us to conveniently quantify the collimation of a source in terms of R_c , which we derive here.

The shearing plate produces an intensity pattern when the reflected beams are projected onto an observation plane. This pattern is due to the difference in phase of the overlapping reflected beams, which causes interference that results in an intensity variation that can be described by a sinusoidal function (M. E. Riley & M. A. Gusinow 1977). The phase of the wavefront, Φ , is the wave-front's optical path length (OPL) as it propagates along the z -axis multiplied by the wavenumber $k = \frac{2\pi}{\lambda}$. For the sheared beam, the wavefront phase is

$$\Phi(x, y, z) = k(z + W) - k(z' + \Delta Z + W'), \quad (6)$$

with W defined as the deviation in OPL from a perfect plane wave. In the case of an ideal spherical wavefront, W is given by

$$W = \frac{x^2 + y^2}{2R_c}, \quad (7)$$

where x and y are the axii perpendicular to the beam propagation direction z , and R_c is the radius of curvature of the incident beam at the shear plate. The radius of curvature, R_c , corresponds to the focal length of an ideal optic that would perfectly collimate the light (a plane wave). E.g., for a collimated incident beam $R_c = \infty$.

With x, y, z, x', y', z' and ΔZ as defined in **Section 2.1**, placing an observation plane at a distance L produces an effective optical path length of

$$\text{OPL} = \frac{S}{R_c}x + \theta(1 - \frac{L}{R_c})y + C, \quad (8)$$

where $C = \frac{1}{2R_c}(\theta^2 L^2 + s^2) - \Delta Z$ is a constant. Note that multiplying the effective optical path length by the wavenumber produces the phase of the wavefront. Also note how in the limit of $R_c \rightarrow \infty$ (a flat wave-front, *i.e.*, collimated light), the phase difference collapses to $\Phi = k\theta y$, meaning in the case of collimated light the shearing pattern produces parallel fringes exactly perpendicular to the wedge.

With a spherical wavefront, the phase equation is linear in the $x - y$ plane. This means that as the light deviates from a perfect plane wave and gains a radius of curvature, the fringes remain parallel but gain a slope, changing angle with respect to the direction of the shear. Consequently, when dealing with a wavefront that is approximate to the ideal spherical case (*e.g.*, defocus), the

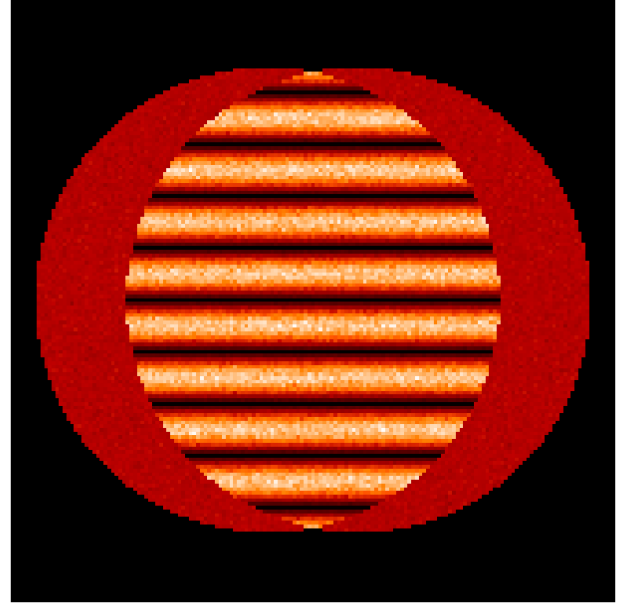


Figure 2. Top: Fringe pattern of an ideal collimated beam, as produced by numerical simulation (see Appendix A). **Bottom:** An actual fringe pattern, produced in our lab during parts acceptance testing. This provides a direct measurement of wavefront error, which in this case is approximately $\lambda/3$.

collimation of a beam of light can be directly measured with high accuracy by simply measuring the angle of the fringe pattern.

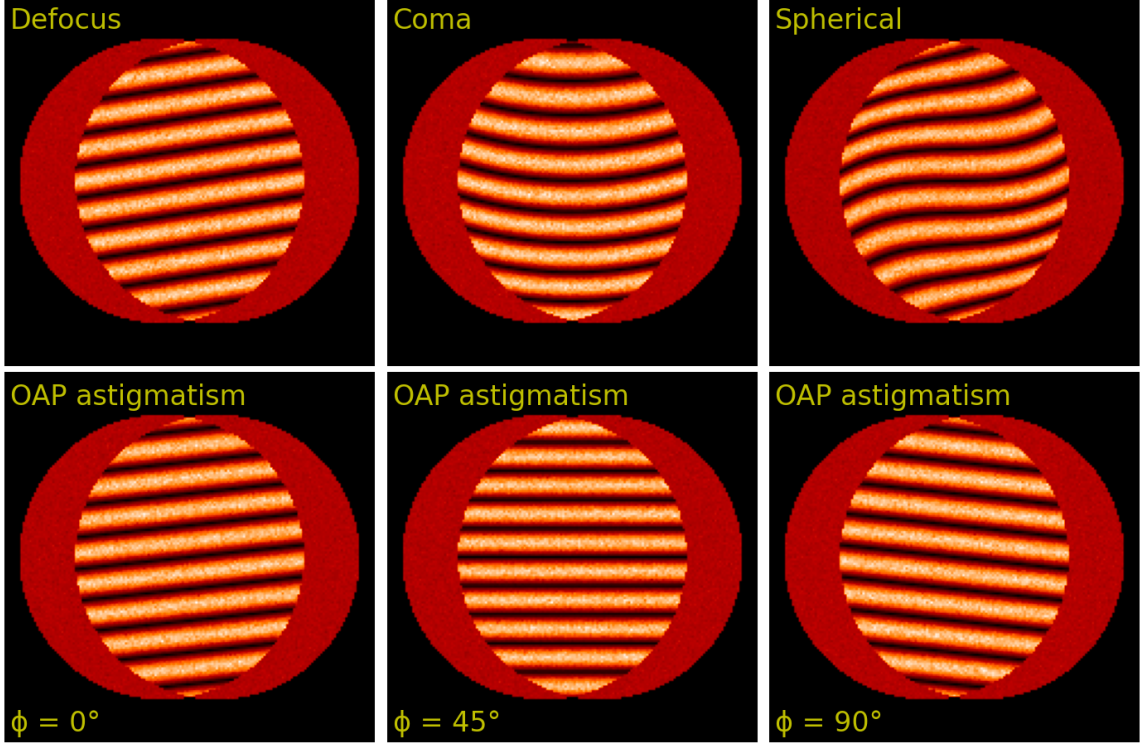


Figure 3. Simulated interference patterns, with the top row showing fringing produced by defocus, coma, and spherical aberration. The bottom row shows fringing produced by OAP astigmatism, with the shearing plate interferometer rotated at 0° , 45° , and 90° relative to the axis of the astigmatism. All six patterns were generated with the same wavefront error of $\lambda/4$ rms. Note that when the shearing plate is rotated to 45° (**Center**), the interference pattern appears to indicate no aberration.

3. RELATING LATERAL SHEARING INTERFEROMETRY PATTERNS TO OPTICAL ABERRATIONS

A benefit of defining the shear pattern in terms of effective optical path length is it simplifies including aberrations. Aberrations can be accounted for by simply adding the OPL difference due to the aberration, ΔW , to the ideal spherical wavefront produced by defocus. Here we describe how to use the shearing interference pattern to qualitatively identify common aberrations.

In general, the OPL produces a intensity pattern at the observation plane

$$I = \sin^2 \frac{1}{2} [\Phi(x, y, z)], \quad (9)$$

$$\Phi(x, y, L) = k \left(\frac{S}{R_c} x + \theta \left(1 - \frac{L}{R_c} \right) y \right) + k \left(\Delta W(x, y) - \Delta W(x - S, y + \theta L) \right). \quad (10)$$

Note how the aberration component of the fringe pattern depends on the shearing action of the plate.

Optical aberrations produce deviations in the wavefront, which appear as non-linear curves in the horizontal fringe pattern produced by ideally collimated light.

This curved pattern makes the presence of non-linear aberrations (*e.g.*, Seidel aberrations) conveniently apparent when testing assembled optical systems. Combined with the inherently small footprint of the interferometer, this enables quick assessment if an aberration is due to misalignment in an optic, making it practical for us to iteratively assemble and align optical instruments that make use of collimating optics (such as a spectrograph). In particular, using a shear plate in this fashion makes it convenient to align off-axis parabolic mirrors, which due to their asymmetrical nature can otherwise be challenging and time-consuming to align.

In the case of small aberrations, *i.e.*, when the optic is nearly aligned, the fringe pattern is an approximately linear projection of the two wavefront's interference pattern, with the peak-to-peak of the pattern corresponding to one wavelength λ . The error in the wavefront can be estimated by estimating how much of a fringe crosses a horizontal reference line. This makes the shearing plate interferometer convenient for estimating surface quality of an optic as a fraction of λ . As we describe in **Section 4**, this also allows us to estimate the Strehl performance of an optical system.

Larger aberrations reveal themselves as complex patterns in the interferometry projection. For our purposes, we do not use shearing interferometry to estimate the magnitude of these aberrations. However, the standard Seidel aberrations have characteristic fringe patterns, making shearing plates convenient for detecting if those aberrations are present. See **Figure 3** for simulated examples of how select Seidel aberration interference patterns appear. These simulated images were made in Python, using the Numpy package (C. R. Harris et al. 2020). See Appendix A for the source code.

4. QUANTIFYING STREHL PERFORMANCE USING A LATERAL SHEARING INTERFEROMETRY PATTERN

As we show in **Section 2**, the intensity pattern of the shearing plate interferometry relates to the wavefront error. This implies one can use the interference pattern to estimate Strehl performance. We derive the relation between Strehl and the interference pattern here.

Let us assume a beam of light is emitted from an ideal spherical source, and is collimated by an ideal optic with focal length f . If the shearing plate interferometer is placed in collimated output of the optic, we know that the OPL at the pattern viewing plate is given by Equation 8. The variation in intensity (the fringe pattern) is produced by variation in the OPL, with multiples of the light's wavelength corresponding to the periodicity of the fringes. Thus, the variation in intensity relates to the variation in optical path length difference by

$$dW(x, y) = \frac{S}{R_c} dx + \theta(1 - \frac{L}{R_c}) dy = n\lambda, \quad (11)$$

where n is a number of fringe peaks. This implies the angle (ϑ) of the tilt pattern is

$$\tan \vartheta = \frac{dy}{dx} = \frac{-S}{\theta R_c (1 - L/R_c)}. \quad (12)$$

Now consider the limit where R_c is very large (e.g, when the optic is nearly aligned), such that $R_c \gg S$. In this limit, the spacing of a single fringe corresponds to the y-direction:

$$dy \approx \frac{\lambda}{\theta}, \quad n = 1 \quad (13)$$

$$\rightarrow \tan \vartheta \approx \frac{-S}{\theta R_c}. \quad (14)$$

Across the width of the shear pattern ($\Delta x = D - S$), the corresponding Δy will be some fraction f of the

fringe spacing ($n \Rightarrow f$),

$$\frac{\Delta y}{\Delta x} = \frac{f dy}{(D - S)} = \tan(\vartheta) \quad (15)$$

$$\rightarrow f = \frac{-S(D - S)}{R_c \lambda}, \quad R_c \gg L. \quad (16)$$

For a focus error ε_z , the amplitude of the error A relates to the OPL difference (ΔW) by

$$\Delta W = A(x^2 + y^2), \quad A = \frac{\varepsilon_z}{2f^2}, \quad (17)$$

where f is the focal length of the optic. If the optic is being used to collimate a beam, the focus error will produce a defocus aberration in the collimation with a magnitude of $A = \frac{1}{2R_c}$, where R_c is the radius of curvature of the collimated beam, the same R_c the shearing equation is sensitive to. This allows us to infer a relation between R_c and the focus error ε_z , $R_c = \frac{f^2}{\varepsilon_z}$, where f is the focal length of the collimating optic. From this, we can relate ε_z to how much the shearing pattern deviates from horizontal, such that

$$f = \frac{-S(D - S)\varepsilon_z}{f^2 \lambda} \quad (18)$$

$$\rightarrow \varepsilon_z = f \lambda \frac{F^2}{\frac{S}{D}(1 - \frac{S}{D})}, \quad (19)$$

where F is the f-number ($f/\#$) of the optic. Taking the root-mean-square (rms) of the wavefront error over a circular aperture with a diameter of D and a central obscuration fraction ϵ provides

$$W_{\text{rms}} = A(D/2)^2 \frac{(1 - \epsilon^2)}{\sqrt{12}} \quad (20)$$

$$= \varepsilon_z \frac{(1 - \epsilon^2)}{8F^2 \sqrt{12}} \quad (21)$$

$$= D^2 \frac{(1 - \epsilon^2)}{8R_c \sqrt{12}}. \quad (22)$$

The well-known Marechal formula relates the wavefront error root-mean-square (W_{rms}) to the Strehl ratio, with diffraction limited performance defined as ≥ 0.8 Strehl at some criteria wavelength λ_0

$$\text{Strehl} \approx e^{-(\frac{2\pi}{\lambda} W_{\text{rms}})^2}, \quad (23)$$

$$W_{\text{rms}} \approx 0.075 \lambda_0, \quad \text{Strehl} = 0.8. \quad (24)$$

Relating Equation 20 to Equation 24, the fraction f of fringe spacing that deviates from horizontal corresponding to diffraction limited performance is

$$f \approx \frac{\lambda_0}{\lambda} \frac{2}{(1 - \epsilon^2)} \frac{S}{D} (1 - \frac{S}{D}). \quad (25)$$

Aberration	ΔW	ΔW_{rms}
Defocus	$A(x^2 + y^2)$	$A(D/2)^2 \frac{\sqrt{1-\epsilon^2}}{\sqrt{12}}$
Coma	$Ax(x^2 + y^2)$	$A(D/2)^2 \frac{\sqrt{1+\epsilon^2+\epsilon^4+\epsilon^6}}{\sqrt{8}}$
Spherical Aberration	$A(x^2 + y^2)^2$	$A(D/2)^2 \frac{2(1-\epsilon^2)\sqrt{1+\frac{7}{4}\epsilon^2+\epsilon^4}}{3\sqrt{5}}$
Astigmatism	Ax^2	$A(D/2)^2 \frac{\sqrt{1+\epsilon^4}}{4}$
OAP Astigmatism	$A(x^2 - y^2)$	$A(D/2)^2 \frac{\sqrt{1+\epsilon^2+\epsilon^4}}{\sqrt{6}}$

Table 1. Typical aberrations, the functional form of their wave-front error, and associated root-mean-square of the wave-front error.

From Equation 25 one can infer the size of f depends on the ratio of the shearing distance S and the beam diameter D . The optimal case is when $\frac{S}{D} = \frac{1}{2}$, which maximizes the apparent size of f . In this case, Equation 25 becomes

$$f = \frac{1}{2} \frac{\lambda_0}{\lambda} \frac{1}{(1 - \epsilon^2)}. \quad (26)$$

This implies that when we observe the deviation of the shearing pattern from horizontal is no more than 1/2 of a fringe across the shearing distance, the collimating optic is focused to within the diffraction limit (for $\epsilon = 0$).

Some things to note. While $f = \frac{1}{2}$ is not difficult to confirm, it's possible to make life even easier by using a shorter wavelength light to generate the shearing interference pattern. For example, we could be using a shearing plate interferometer to align the optics for a near-infrared system, with a criteria wavelength of 1.2 μm . If we generate the interference pattern using a HeNe laser ($\lambda = 633 \text{ nm}$), then a fringe deviation of 1 f corresponds approximately to diffraction limited performance at $\lambda_0 = 1.2 \mu\text{m}$. Also note that for pupils with a central obscuration ($\epsilon \neq 0$), such as the beam from a telescope, the requirement of $f < 0.5$ relaxes.

4.1. Special case: Quantifying Strehl for Off-Axis Parabolic Mirror Astigmatism

EXCITE's spectrograph makes use of OAP mirrors. OAPs are light-weight, high-throughput optics, making them well-suited to a balloon-born, on-axis instrument like EXCITE. However, their asymmetrical nature makes it possible to misinterpret the interference pattern when using shearing interferometry check an OAP's optical quality. Here, we discuss how an OAP's asymmetrical nature impacts the shearing interference pattern, and show how to make use of a shearing plate interferometer to verify diffraction-limited performance of an OAP.

For an OAP, angular misalignment produces on-axis asymmetric astigmatism (referred to here-after as *OAP*

astigmatism), which takes the form of

$$\Delta W = \frac{x^2 - y^2}{2R_c}. \quad (27)$$

Since OAP astigmatism is caused by the tangential and sagittal planes having different effective focal lengths, the associated wavefront error is effectively a defocus. This means the method given above for quantifying Strehl performance also applies, with some caveats.

Consider the case in which the shearing plate is rotated by an angle ϕ relative to the OAP. The aberration then becomes

$$\Delta W = \frac{1}{2R_c} (\cos 2\phi x^2 - \cos 2\phi y^2 + 2 \sin 2\phi \cos 2\phi xy). \quad (28)$$

The shearing plate will then produce an optical path length difference of

$$OPL = \left(\frac{S}{R_c} \cos 2\phi - \frac{\theta L}{R_c} \sin 2\phi \right) x + \left(\theta + \frac{\theta L}{R_c} \cos 2\phi + \frac{S}{R_c} \sin 2\phi \right) y. \quad (29)$$

This implies a linear fringe pattern, with a tilt angle dependent on both the magnitude of the aberration and the rotation of the shearing plate relative to the beam. The tilt angle of this fringe pattern ϑ is

$$\tan(\vartheta) = \frac{dy}{dx} = \frac{S \cos 2\phi - \theta L \sin 2\phi}{R_c \theta + \theta L \cos 2\phi + S \sin 2\phi}. \quad (30)$$

In the case where the shearing plate axis and the aberration axis are aligned or perpendicular, the shearing angle simplifies to

$$\tan(\vartheta) = \frac{S}{R_c \theta (1 + L/R_c)} \approx \frac{S}{R_c \theta}, \quad \phi = 0 \quad (31)$$

or

$$\tan(\vartheta) = \frac{-S}{R_c \theta (1 - L/R_c)} \approx \frac{-S}{R_c \theta}, \quad \phi = \frac{\pi}{2}, \quad (32)$$

which is similar to the defocus case but with the sign of the slope changing as the shearing plate rotates relative to the OAP. In the case where $\phi = \pi/4$,

$$\tan(\vartheta) = \frac{-\theta L}{R_c \theta + S \sin 2\phi} \quad (33)$$

$$\approx 0, R_c \gg L, \theta \ll S. \quad (34)$$

So as the shearing plate rotates with respect to an OAP aberration, the fringe pattern becomes horizontal, then begins to slope in the opposite direction. (See **Figure 3**). Not accounting for the angle of rotation between the astigmatism and the shearing plate could lead one to falsely conclude that the OAP is aligned within tolerance.

We mitigate this by placing the shearing plate on a rotation stage. As we freely rotate the interferometer relative to the collimated beam, we can observe the change of the fringe pattern angle in real time. When the fringe pattern no longer changes angle with rotation of the shearing plate, we know the OAP is aligned in angular space.

Furthermore, this behavior is useful as an intermediary step in alignment. One of the issues with aligning OAPs is the degeneracy between roll and pitch alignments: since both affect the pitch of the collimated beam, beam direction alone is not enough to determine which axis is out of alignment. By rotating the shearing plate, we can identify the axial alignment of the OAP astigmatism relative to an optical table, and from this infer the alignment of roll angle of the OAP.

In the case where the OAP astigmatism and the shearing plate are well-aligned ($\phi \approx 0$), we can estimate the Strehl performance from the interference fringes in a manner similar to the defocus case. The functional form of OAP astigmatism (Equation 27) has the coefficient

$$A = \frac{\varepsilon_z}{2f^2} = \frac{1}{2R_c} = \frac{\varepsilon_\alpha \tan(\alpha/2)}{2f}. \quad (35)$$

In the case where the aberration and shearing plate axis are aligned ($\phi = 0$), the fraction of the fringe spacing for OAP aberration is

$$f = \frac{1}{\lambda} \frac{(D - S)S}{R_c}, R_c \gg L, \quad (36)$$

which is the same as defocus aberration save for a sign change. However, the relation between the fringe spacing and Strehl is slightly different in the OAP case, due to the difference in the root-mean-square of the wavefront error (see **Table 1**). Using Equation 35 and a Strehl criteria of 0.8, the maximum angular alignment error is

$$\varepsilon_\alpha \approx \frac{1.5\lambda_0}{\sqrt{1 + \epsilon^2 + \epsilon^4}} \frac{F_0}{D_0 \tan(\alpha/2)}, \quad (37)$$

where λ_0 , D_0 , and F_0 are the wavelength, diameter, and F-number respectively used to define diffraction-limited operation of the OAP. Using $\frac{S}{D} = \frac{1}{2}$ from the defocus case, the fringe fraction that corresponds to diffraction-limited performance is

$$f = \frac{3}{8} \frac{\lambda_0}{\lambda} \left(\frac{D}{D_0} \right)^2 \frac{1}{\sqrt{1 + \epsilon^2 + \epsilon^4}}. \quad (38)$$

Note this fringe fraction is slightly more restrictive than defocus. Also note that contrary to estimating defocus Strehl, the presence of a central obscuration also increases the restriction. This motivates using some combination of a larger diameter beam and/or a shorter wavelength when aligning with the shearing plate than will be used in instrument operations.

5. PRACTICAL APPLICATION: ALIGNING THE EXCITE SPECTROGRAPH

The techniques described in this paper were developed for building and aligning the primary science instrument of the EXCITE mission, the spectrograph. When operating on-axis, which is the case with EXCITE's spectrograph, OAP misalignment primarily produces defocus and astigmatism aberrations (L. Bernard et al. 2022). These specific aberrations are particularly well-suited to identification with the shear plate, due to the linear pattern they produce. With this linear pattern, we can quickly estimate the performance of an optic in terms of Strehl by simply counting the number of lines that cross a center reference mark on the pattern viewing plate. Here, we show how we made use of this to verify the performance of EXCITE's OAPs.

5.1. Parts Acceptance Testing

We tested the performance of our optical components using a combination of a HeNe laser source ($\lambda = 633$ nm) and optics to produce a precision light source, with measurements provided by a 25 mm diameter shearing plate interferometer. The test light source has two configurations, one producing collimated light and the other producing focused light. Diffraction-limited performance of the light source is verified with the shearing plate. We used the collimated mode to test flat optics, such as the fold mirror and the dichroics, by measuring the shearing plate interference pattern before and after the flat and comparing the differences (if any exist). For testing the OAPs we used focused mode, using the OAP to collimate the focused source and then measuring how well-collimated the output with the shearing plate.

While our design criteria wavelength is 1.0 μm , the 633 nm light of the laser is not only easier to use in the laboratory but is more sensitive to error than infrared

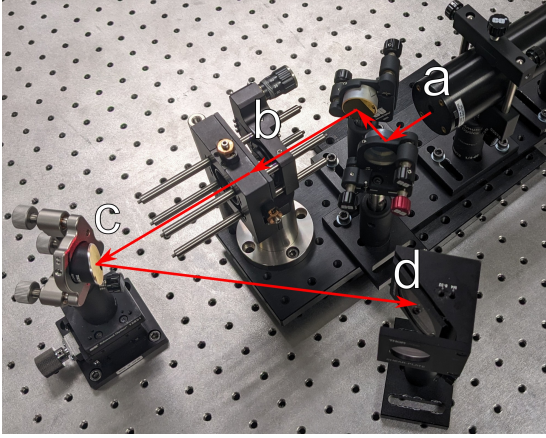


Figure 4. Isometric view of an example of the setup used to verify the performance of optical components, in this case the 45° OAP. The test setup is mounted to a 1-inch grid optical table. We direct light from a He-Ne laser (a) into a microscope objective lens (b) to direct light onto the focal plane of the OAP (c), which the OAP then collimates and directs into a shearing plate (d). Note how compact the measurement system is.

light. We choose an optical surface quality tolerance of $\lambda/2$, which translates to an accuracy of $> \lambda/4$ for the Nyquist-sampled spectrograph wavelengths. Additionally, we test the optics with a 25 mm diameter beam, instead of the actual instrument’s 8 mm, further increasing our test’s sensitivity to error over real operations.

Using the horizontal line test described in **Section 4**, we found wavefront error to be $< \lambda/4$ for all optics, with the exception of the 45° OAP where we found a wavefront error of $\sim \lambda/3$ (see **Figure 4**). We note that the 45° OAP was tested with an F/4 beam, so while this larger error could be due to optic surface quality, it could also be the limitations of our laser source optics; limitations that would not affect the other, slower-beam tests. We also note that our 25 mm beam will detect defects in the edge of the mirror, defects which would not affect the 8 mm beam of the operating spectrograph. Regardless, all optics exceed our criteria of $\lambda/2$

We used a straight-edge and a precision square to check the accuracy of the machined optical mounts and the optical bench, making use of a feeler gauge to measure any inaccuracy. All mounts were within 0.0015 inches (38 μm) of design on critical surfaces, with the exception of some slight warping on the base of the OAP mount which we corrected with a 0.002 in shim. We measured the optical bench as flat within 0.007 in (178 μm) over the 11 inch (280 mm) diameter.

As a final test, we made a mock assembly of the spectrograph parts on the optical bench, and verified system throughput with HeNe laser. (See **Figure 5**.)

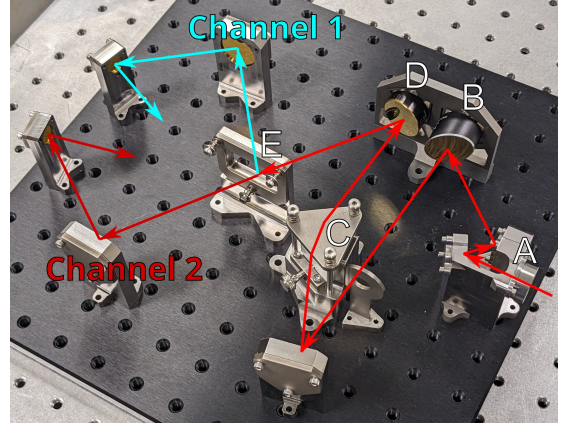


Figure 5. Mock assembly of the spectrograph, during the last stage of acceptance testing before coating. The major components are the 100 μm by 3 mm input field stop (A), the 45° collimating OAP 1, the calcium-fluoride prism (C), the 30° camera OAP 2, and the dichroic (E) that reflects Channel 1 (0.8 μm -2.5 μm) and transmits Channel 2 (2.5 μm -3.5 μm). All non-labeled optics are flat fold mirrors.

5.2. Alignment Criteria

The alignment criteria of EXCITE’s spectrograph can be found in [L. Bernard et al. \(2022\)](#). That work identifies the collimating off-axis parabolic mirror (OAP 1) as the most sensitive optic in the assembly, and describes how it drives the tolerances of EXCITE’s spectrograph. We summarize the tolerances of OAP 1 here.

Our criteria is maintaining diffraction-limited performance, which for the purposes of alignment we define as a Strehl ratio of 0.8 at a wavelength (λ) of 1.0 μm . In our case, the secondary mirror obscures 38% of the telescope aperture. With this $\epsilon = .38$ obscuration ratio, a Strehl ratio of 0.8 corresponds to a defocus error ϵ_z of

$$\epsilon_z = 2.41 \lambda F^2 \quad (39)$$

where F is the telescope focal ratio ([V. N. Mahajan 1982](#)). With EXCITE’s $F = 12$ telescope beam at 1.0 μm , we can tolerate up to 347 μm of focus error.

As shown in [L. Bernard et al. \(2022\)](#), OAP misalignment appears as an effective defocus, which can be translated into an angular misalignment after accounting for the working angle of the OAP. The angular misalignment (ϵ_α) for an OAP with focal length f and working angle α is given by

$$\epsilon_\alpha = \frac{\epsilon_z}{f \tan \frac{\alpha}{2}} = \frac{2.41 \lambda F^2}{f \tan \frac{\alpha}{2}}. \quad (40)$$

For OAP 1 this produces a maximum alignment tolerance of ± 28 arcminutes.

For our system ($\epsilon = 0.38$, $\lambda_0 = 1.0 \mu\text{m}$, $\lambda = 0.633 \mu\text{m}$), the fringing fraction that corresponds to this alignment is $f = 0.55$ (or approximately 1/2).

5.3. Optics Alignment

OAPs asymmetric nature makes them challenging to align. They must be aligned in 6 degrees of freedom: x , y , z , rotation in the x - y plane (*roll*), rotation in the x - z plane (*yaw*), and rotation in the y - z plane (*pitch*), where z is the axis along the beam path, x is the horizontal perpendicular axis, and y is the vertical perpendicular axis. These axes are degenerate, meaning any particular aberration could be due to misalignment in one of two axis, or a combination of misalignment in both. To break this degeneracy, we use the mechanical design of the spectrograph to eliminate degrees of freedom wherever possible.

We use the same HeNe laser and shearing plate used for acceptance testing to align the spectrograph. For alignment, we secured the spectrograph to an optical bench, with the entrance aperture aligned to the optical grid. We mounted the laser source to an optical island, allowing us to move the light source while maintaining alignment relative to the bench. We made use of two source modes, a collimated light mode and an simulated single ray mode (described below). We developed a third mode, the focused input configuration we made use of in acceptance testing, but found it unnecessary for our 28 arcminute alignment criteria.

For rough alignment, we use the 100 μm entrance slit to filter the 633 nm laser beam to an approximation of a single chief ray (the *simulated chief ray*). The diffraction pattern produced by the slit tells us the orientation of the slit, which we adjust to be horizontal (ensuring a vertical slit). We then center the core of the diffraction pattern beam on each optic in turn, ensuring the beam is level throughout the system and centered on each optic.

Once the spectrograph is roughly aligned, we align the critical optic, the 45° OAP that collimates the telescope beam (OAP 1). To align OAP 1, we make use of the rotational symmetry of the collimation side of the optic. When collimated light is directed into OAP 1, operating the optic in reverse (see **Figure 6**), the x - y - z axes are eliminated as variables (as collimated light is translation-independent). Any *roll* misalignment will not affect the focus, but will direct the focus light at some angle; when the focused beam projection is horizontal, the *roll* axis is aligned to 0° . We control the *pitch* angle with the OAP mount, which was measured as 90° with a precision square during acceptance testing and verified with the simulated chief ray. To control the *yaw* angle, we use the tri-mount's beam folding as a reference. The tri-mount is a monolithic mount holding the field stop entrance to the spectrograph and two fold mirrors that combine to redirect field stop input by 45° into the OAP. If the reverse operation emits focused

light through the field stop at exactly 90° relative to the collimated beam, we know the 45° working angle of OAP 1 is aligned to field stop.

There is a degeneracy between the *yaw* alignment and the x -axis alignment; any misalignment in the *yaw* can be compensated for by translation in the x direction, at the cost of producing astigmatism and changing the angle of the output beam. We break this degeneracy by comparing the angle of the reverse operation output beam to the angle of the simulated chief ray. The optical island enabled us to switch between chief ray mode and reverse operation mode while maintaining alignment, using precision stops mounted to the optical bench to ensure a mechanical $90^\circ \pm 1.6'$ between the modes. This allowed us to iterate between modes until we achieved alignment. Using this procedure, we achieved an alignment of 12 arcminutes or better, well under our budget of 28 arcminutes, with the primary limiting factor being the accuracy of the measurement of the beam angle between the two modes.

At this point OAP 1 is ready for fine alignment using the shearing plate. During laboratory demonstrations, we prepared a focused light source and directed it into OAP 1. Using the rotation technique described in **Section 4.1**, we were able to demonstrate OAP alignment to within the diffraction limit of a F/4 beam. However, with the relatively relaxed tolerances of the spectrograph, we found this fine alignment step unnecessary for the final assembly of the flight version of the spectrograph. Note that we could further improve the alignment of OAP 1 using the shearing plate, without disassembling the spectrograph, should future engineering tests show any unexpected sources of error.

Finally, we adjust focus (z -axis alignment) by maximizing the output through the field stop. This places the focus of OAP 1 within the field stop, and because this instrument is effectively a slit-less spectrometer, we can use the telescope focus to perform spectrograph fine focus alignment during flight. Alignment of OAP 1 in the y -translation (cross-dispersion, a.k.a., slit direction) is non-critical, with the entire field stop being usable.

OAP 2 is co-mounted with OAP 1, with their co-alignment ensured by the machined surface and precision pins. This co-mounting ensures OAP 2 shares alignment with OAP 1 in all axes, except for *roll*. As mentioned earlier, a misalignment in *roll* does not affect the Strehl ratio but merely tilts the beam path. Additionally, OAP 2 has a large depth of focus ($\sim \pm 600 \mu\text{m}$) which prevents any aberration due to tilt of the focal plane. This makes OAP 2 simple to align by leveling its output beam, with alignment in all other degrees of freedom constrained by the alignment of OAP 1. (See

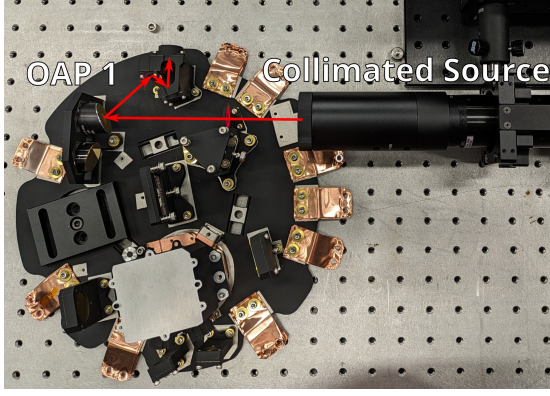


Figure 6. We aligned OAP 1 by removing Fold Mirror 3 and replacing it with a collimated source, operating the OAP in reverse. In this configuration, we were able to achieve the target alignment of 28 arcminutes. As the two OAPs are co-mounted, this also aligns the lighter tolerance OAP 2.

L. Bernard et al. (2022) for a more detailed description of the relaxed alignment tolerance of OAP 2.)

All fold mirrors and the prism are aligned by centering and leveling the simulated chief ray, with appropriate calculated offsets to account for the prism dispersion.

6. CONCLUSIONS

In this paper, we have discussed how shearing plates are useful for verifying the alignment of the optics of a optical/NIR spectrograph, using EXCITE as an example. We discussed a formalism for shearing plate geometry, and used that formalism to show how shearing plates can provide a quantitative estimate of Strehl performance. We show this approach to estimating Strehl is particularly well-suited to aligning OAPs, with OAP misalignment showing as a clear, linear variation in the interferometry fringe pattern.

The main advantages of this approach are the small footprint of a shearing plate, both spatially and logistically. The compact size of a shearing plate enables us to measure collimation in-situ, without significant disassembly of the spectrograph, allowing one to rigorously verify the alignment of a collimating OAP independently from the camera optic. With the alignment of the collimating optic confirmed, we can know the camera optic is receiving a well-collimated beam, allowing us to align the camera optic to the focal plane independently from the collimating optic.

The fact that we can use the interference pattern to estimate Strehl performance without digitization or numerical analysis, combined with the small size of the interferometer, makes performance verification possible with only the spectrograph, a light source, and an optical bench to hold them, with no need for a large beam path or detectors. This simple setup makes for fewer single points of failure, decreasing the risk of equipment issues. Crucially, the compact size requires a relatively small optical bench, which makes this setup easy to transport and deploy for field operations in a remote location such as Antarctica.

Another advantage of lateral shearing interferometers is they can be used in double-pass mode. Since the interferometer can be a simple glass plate, it acts as an effective beam-splitter, with a portion of the incident beam passing through. This transmitted beam can be reflected back into the shearing plate, producing a second interference pattern that is projected in the opposite direction from the incident beam pattern. This makes for convenient A-B comparison between the incident and reflected beam, which allows one to account for and subtract off any error in the source beam. This is useful for high-sensitivity measurements of optical surfaces.

This approach is generally applicable to any optical train involving a collimated beam. One can imagine a use case where we place a source at the focal plane, operating the instrument in reverse and allowing us to use the shearing plate to align the camera optic. Overall, the ability of shearing plates to directly measure beam collimation in a compact space allows for iterative assembly of an optical/NIR instrument.

ACKNOWLEDGMENTS

This work is supported by NASA award 18-APRA18-0075 selected under NASA Research Announcement NNH18ZDA001N, Research Opportunities in Space Science – 2018 (ROSES-2018). Work by Kyle Helson is supported by NASA under award number 80GSFC24M0006.

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APPENDIX

A. LATERAL SHEARING PLATE SIMULATION CODE

```

837
838 """
839 Module for simulating the interference pattern of a linear shearing
840 interferometer. This code is based upon the Thorlabs shearing plate
841 interferometer. This plate comes in several sizes, with the model number
842 and specs given below.
843
844 SI254P: 10-25.4 mm beam diameter (R=21.5), ang=18, T=6.35 (plate=0)
845 SI100P: 5-10 mm beam diameter (R=11.), ang=40, T=2.6 (plate=1)
846 SI500P: 25.4-50 mm beam diameter (R=30.), ang=10, T=13 (plate=2)
847         Note, R is more like 70-80.
848
849 This version is intended for release to the public domain as open source
850 software, under MIT License 2026.
851
852 """
853
854 from numpy import arange, zeros, ones
855 from numpy import sqrt, sin, cos, pi, arctan2, sign, ceil, mean, diff
856 from numpy.random import randn
857
858
859 def shear_plate(Rc=1.e5, D=25.4, Dmax=36.0, eps=0, Rs=50, alpha=45, plate=-1,
860               T=6.35, wedge_ang=18., n_idx=1.46, lam=632.8e-6, atype='coma',
861               wfe=0, wfe_phi=0, acenter=[0., 0.], N=203, visual=True):
862     """
863
864     Parameters
865     -----
866     Rc: radius of curvature of the wavefront
867     D: Diameter of the beam (in the crossplane direction)
868     Dmax: Diameter of the shearing plate. This also sets the image size
869     eps: fraction of central obscuration, assuming a centered, circular
870         obscuration.
871     Rs: Distance from the shearing plate to the observation plane
872     alpha: angle of incidence of the beam upon the shearing plate
873     plate: option for a preset shearing plate geometry
874     T: thickness of the shearing plate
875     wedge_ang: wedge angle of the shearing plate
876     n_idx: index of refraction of the shearing plate glass
877     lam: wavelength of the beam of light
878     atype: option for choosing which wavefront aberration to display
879     wfe: magnitude of the wavefront error (rms, in units of wavelength)
880         due to aberration
881     wfe_phi: angle of the wavefront error in the cross-plane
882     acenter: offset of the wavefront aberration center from the beam center
883     N: Parameter controlling resolution of the sample. Number of rays
884         generated is NxN

```

```

885     visual: Flag. If True, the simulation adds some noise and gamma
886         correction to the simulated data
887
888     Returns
889     -----
890     A numpy array containing a simulated image of the viewing plane of
891     the lateral shearing plate interference pattern.
892     """
893     # default shearing plate geometry's. This will over-ride passed parameters
894     if (plate==0): Dmax, wedge_ang, T = 15., 10., 2.6
895     if (plate==1): Dmax, wedge_ang, T = 36., 18., 6.35
896     if (plate==2): Dmax, wedge_ang, T = 70., 40., 13.
897     print("beam radius:", D)
898     print("Radius of curvature:", Rc)
899
900     # calculate beam divergence at the edge of the beam
901     Rw = D/2
902     bdiv = 2*sign(Rc) * arctan2(Rw, sign(Rc)*Rc)
903     print(f"beam divergence: {bdiv: .2e}")
904
905     alpha *= pi/180
906     wedge_ang *= pi/180/3600
907     shear = T * sin(2*alpha) / sqrt(n_idx**2 - sin(alpha)**2)
908     theta = 2*wedge_ang * sqrt(n_idx**2 - sin(alpha)**2)
909     print (f"Ray tilt angle: {theta*180/pi*3600:.3f} arcsec")
910     print (f"Expected line spacing: {lam/theta:.3f}")
911     print (f"Shear distance: {shear:.3f}")
912     phi = arctan2(shear,theta*Rc)*180/pi
913     print (f"Angle of fringes: {phi:.2f} (degrees)")
914
915     # create some starting ray positions
916     D0 = Dmax + int(ceil(shear))
917     x0 = (arange(N) - (N-1)/2) * D0 * 1/(N-1)
918     # create the primary beam, shifting it so the center of the shearing
919     # pattern is zero
920     x = zeros((N, N), dtype='float32') + x0 + shear/2
921     y = zeros((N, N), dtype='float32')
922     yt = y.T; yt += x0
923     print(f"Image size: {x.max()-x.min():.2f}x{y.max()-y.min()} "
924         f"(pixel scale: {mean(diff(x)):.3f})")
925
926     # Generate the sheared reflected beam
927     xp = x - shear
928     yp = y + theta*Rs
929     # calculate the combined phase at the observation plane for defocus
930     phase = 2*pi/lam*((theta*(1.-Rs/Rc))*y + (shear/Rc)*x) - \
931         pi*(shear**2 + (theta*Rs)**2)/(lam*Rc)
932
933     # give the rays some aberrations
934     if wfe>0:
935         cos_fac, sin_fac = cos(wfe_phi*pi/180), sin(wfe_phi*pi/180)
936         x1 = (x - acenter[0])*cos_fac + (y - acenter[1])*sin_fac

```

```

937 y1 = -(x - acenter[0])*sin_fac + (y - acenter[1])*cos_fac
938 xp1= (xp - acenter[0])*cos_fac + (yp - acenter[1])*sin_fac
939 yp1 = -(xp - acenter[0])*sin_fac + (yp - acenter[1])*cos_fac
940 if atype=='defocus':
941     print("Calculating defocus error...")
942     eps_fac = 1-eps**2
943     w_pv = 2*sqrt(3)*wfe
944     delta_w = (x1**2 + y1**2) / Rw**2
945     delta_wp = (xp1**2 + yp1**2) / Rw**2
946     W = w_pv*(delta_w - delta_wp)/eps_fac
947 elif atype=='sa':
948     eps_fac = (1-eps**2)*sqrt( 1 + eps**2*(eps**2+7/4) )
949     # convert rms to peak to valley (same as 1/0.298)
950     w_pv = wfe*1.5*sqrt(5)
951     # calculate the path length map of beam 1
952     delta_w = (x1**2 + y1**2)**2 / Rw**4
953     # and for beam 2
954     delta_wp = (xp1**2 + yp1**2)**2 / Rw**4
955     # combine the two, and calculate the magnitude
956     W = w_pv*(delta_w - delta_wp)/eps_fac
957 elif atype=='coma':
958     eps_fac = sqrt( 1 + eps**2 + eps**4 + eps**6 )
959     w_pv = wfe*2**1.5
960     delta_w = x1*(x1**2 + y1**2) / Rw**3
961     delta_wp = xp1*(xp1**2 + yp1**2) / Rw**3
962     W = w_pv*(delta_w - delta_wp)/eps_fac
963 elif atype=='asti':
964     eps_fac = 1
965     w_pv = wfe*4
966     delta_w = x1**2/Rw**2
967     delta_wp = xp1**2/Rw**2
968     W = w_pv*(delta_w - delta_wp)/eps_fac
969 elif atype=='oap':
970     eps_fac = sqrt(1 + eps**2 + eps**4)
971     w_pv = wfe*sqrt(6)
972     delta_w = (x1**2 - y1**2)/Rw**2
973     delta_wp = (xp1**2 - yp1**2)/Rw**2
974     W = w_pv*(delta_w - delta_wp)/eps_fac
975 else:
976     W = 0
977 phase += 2*pi * W # now convert path length to a phase map
978
979 # calculate the pattern
980 # set the outer radius of the reflected beams
981 # R1 = Dmax/2
982 # R2 = Dmax/2*cos(alpha)
983 if (D<Dmax):
984     R1 = D/2
985 else:
986     R1 = Dmax/2
987
988 if (D<Dmax/2*cos(alpha)):
```

```

989         R2 = D/2
990     else:
991         R2 = Dmax/2*cos(alpha)
992     print("R1, R2:", R1, R2)
993
994     # mask the aperture
995     aperture_mask = (x/R1)**2 + (y/R2)**2 > 1
996     # mask the central obscuration, if present
997     if (eps>0):
998         aperture_mask[(x/(R1*eps))**2 + (y/(R2*eps))**2 <= 1] = True
999     norm1 = ones(x.shape)
1000     norm1[aperture_mask] = 0
1001     print("shape of norm1", norm1.shape)
1002
1003     # and again for the sheared beam
1004     aperture_mask = ((x - shear)/R1)**2 + (y/R2)**2 > 1
1005     if (eps>0):
1006         aperture_mask[((x - shear)/(R1*eps))**2 + (y/(R2*eps))**2 <= 1] = True
1007     norm2 = ones(x.shape)
1008     norm2[aperture_mask] = 0
1009
1010     # convert phase to intensity
1011     s = 0.25*(norm1**2 + norm2**2) - 0.5*norm1*norm2*cos(phase)
1012
1013     if visual:
1014         s *= 1 + randn(N, N)/10 # add some noise, to simulate laser speckle
1015         s = (s/s.max())**(1/2.2) # gamma correction
1016     return s

```