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Impact of market demand mis-specification on a two-level supply chain

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This paper investigates the impact of mis-specifying the market demand process on a serially linked two-level supply chain. Box-Jenkins models are used to represent both the true and a mis-specified market demand processes. It is shown that the impact of mis-specification on cost is minor if the supply chain tries to minimise the market demand forecast errors. Furthermore, our analysis suggests that mis-specification does not always result in additional costs. A managerial insight is revealed; *poor forecast accuracy is not always bad for the total supply chain costs.* In other words, *employing more accurate forecasting methods may actually result in higher total supply chain costs.*

Key words: order-up-to policy; base-stock policy; mis-specification; forecasting; Box-Jenkins model

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1 Introduction

A typical and common concern for managers is to minimise demand forecast errors in order to avoid unnecessary supply chain costs. To minimise forecast errors, some knowledge of the demand process is essential. For example, if the demand process can be represented in a mathematical form, such as the Box-Jenkins Model (Box *et al.*, 1994), managers can use conditional expectation to create a Minimum Mean Square Error (MMSE) forecast. There

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are many supply chain studies that take this approach to represent a market demand process and use conditional expectation as a forecasting method (e.g. Graves, 1999; Lee *et al.*, 2000; Gilbert, 2005; Hosoda and Disney, 2006). Many of these studies assume that the Order-Up-To (OUT) policy is exploited in a supply chain, the true demand process structure and the true values of parameters of the market demand process are correctly known and an MMSE forecast is employed to determine the OUT level. However, in real supply chains due to a lack of familiarity of the Box-Jenkins technique and the high level of skill required to use it, simple forecasting methods, such as Exponentially Weighted Moving Average (EWMA), are still quite popular (see, McCarthy *et al.*, 2006, for example), even though the importance and the usefulness of correctly identifying the true demand process might be realised among managers. Thus there are many opportunities to suffer from demand process mis-specification in real supply chains.

Demand process mis-specification is not a new research topic, but it does appear to be relatively under researched. To the best of our knowledge, Badinelli (1990) may be the first to research this topic. More recent papers were produced by Kim and Ryan (2003) and Zhang (2004). All of these studies have used the Box-Jenkins model to represent the market demand process and have quantified the cost of using the EWMA forecasting technique. This research differs from these prior works in two ways.

First, we will consider the situation that the forecasting method used by a supply chain is conditional expectation. However, the market demand process is mis-specified. The forecast actually used by a supply chain, therefore, is not the "correct" MMSE forecast. The motivation of this setting came from the fact that generating "correct" MMSE forecasts may be difficult due to a lack of skills and knowledge of the Box-Jenkins approach. Furthermore, since the true market demand process is unknown to everybody, assuming that the demand process is mis-specified may be a better representation of reality.

Secondly, we will consider a two-level supply chain and the reason of this is that one of our interests is to quantify the impact of mis-specification on a multi-level supply chain. From prior work, such as Badinelli (1990), we can reasonably conjecture that a forecast based on a mis-specified demand process would have a negative impact on the first level player in a supply chain, since the forecast accuracy directly increases the inventory cost. Through the first level player's ordering policy, this impact is then transfered to the second level player. This is because the first level's ordering decision is, in part, based on the mis-specified demand process. The second level then generates a forecast of the demand from the first level, again using conditional expectation. This generated forecast is, however, also affected by the shared, but incorrect, knowledge given by the first level on the demand process structure. Therefore, the second level has also suffered from mis-specification and its performance has also been affected.

It is shown that even if the market demand process is mis-specified, as long as the supply chain tries to minimise forecast errors the impact from mis-specification on the supply chain cost is minor. Furthermore, we highlight a counter-intuitive conclusion; market demand process mis-specification does not always increase the supply chain costs. Indeed, by exploiting mis-specification, the supply chain can reduce both supply chain inventory costs and the production costs simultaneously.

This paper is organised as follows. After the literature review shown in Section 2, the analytical model is described in Section 3. Via numerical analysis some interesting managerial insights are presented in Section 4. We conclude in Section 5.

2 Literature Review

It has been long recognised that the forecasting method has an impact on the performance of an ordering policy, and there are many studies that analyse the impact of forecasting methods assuming that the true market demand process is unknown or ignored (see, Badinelli, 1990; Chen *et al.*, 2000; Xu *et al.*, 2001; Kim and Ryan, 2003; Zhang, 2004, for example). Badinelli (1990) analyses the impact of the market demand mis-specification using a single level supply chain model with the OUT policy. Demand is assumed to be a general ARMA process. Badinelli provides an analytical model for the no mis-specification case. Using exponential smoothing, the impact of mis-specification is quantified via numerical analysis. Badinelli concludes that the market demand mis-specification results in a higher inventory cost.

Assuming an AR(1) demand process and a demand process with a linear trend, Chen *et al.* (2000) study the impact of using EWMA and the Moving Average (MA) forecasting methods on the bullwhip effect. They conclude that the smoother the market demand forecast, the lower the bullwhip effect. Xu *et al.* (2001) and Kim and Ryan (2003) both study a two-level supply chain model where each player adopts EWMA forecasts even though an AR(1) market demand process is assumed. After quantifying the cost of mis-specification of the forecast model, they investigate the benefit of an information sharing strategy by measuring the bullwhip (Xu *et al.*, 2001) and forecast errors generated (Kim and Ryan, 2003). In terms of information sharing, Xu *et al.* (2001) assume the smoothing constant in EWMA used at the retailer is shared with the supplier, and Kim and Ryan (2003) assume the market demand information is exchanged. Kim and Ryan (2003) report that the cost impact of mis-specification on the first level player sometimes shows "strange and unexpected" patterns. Using a single level supply chain model, Zhang (2004) analyses the impact of forecasting methods on the bullwhip effect using MA in addition to EWMA and MMSE forecast methods when the market demand follows an AR(1) process. It is concluded that an MMSE forecast yields the minimum inventory cost. EWMA and/or MA forecasting may, however, outperform the MMSE when the demand process is not well specified.

After showing the benefit of sharing the market demand information in a two-level supply chain using a real set of data, Hosoda *et al.* (2008) discuss a potential impact of the market demand mis-specification on the benefit of information sharing. In their model, an MMSE forecasting method is used. They suggest that the market demand mis-specification does affect the magnitude of the benefit coming from the information sharing strategy, and in an extreme case, mis-specification can eliminate this benefit. Watson and Zheng (2007) consider the situation where the true market demand process is known but the forecasts of the demand are affected by a manager's subjective bias. They quantify its impact on the bullwhip effect and the inventory cost in a single-level supply chain. It is shown that under-reaction to the change in the demand can mitigate the bullwhip effect. In Hosoda and Disney (2009), from the point of the total supply chain cost, two different two-level supply chain settings are investigated. In both settings, the OUT policy is used at each level of the supply chain. In the first setting, it is assumed that the OUT level is updated every time period by using an MMSE forecast. In the second setting, the OUT level is determined based on a mean of the demand that is time invariant. An AR(1) process is used to represent the demand. It is shown that the second setting can yield lower supply chain cost, expecially when the market demand is positively correlated over time.

3 The Model

A serially linked two-level supply chain system is analysed. We assume that up-to-date market demand information and the mis-specified knowledge of the demand process captured by the first level player is shared and is common knowledge in the supply chain. This assumption enables us to negate the benefit coming from sharing the market demand information.

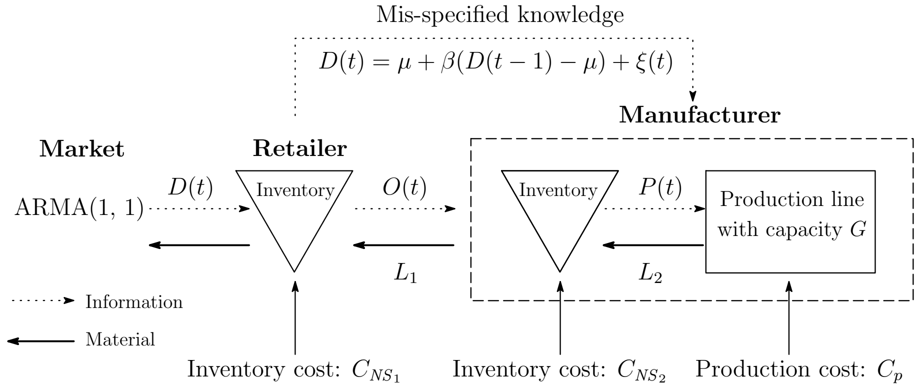

Figure 1: Two-level supply chain model

Therefore, the supply chain under investigation is a vertically integrated two-level supply chain. It is assumed that each player cooperates to minimise the total supply chain cost and the benefit generated by the cooperation will be reallocated to ensure that each player is better off. Thus, there is no conflict in the supply chain. For convenience, we call the first level player, the retailer and the second level player, the manufacturer. Both players exploit a periodic review system, and the replenishment lead-time is constant and known. Fig. 1 shows a schematic of our two-level model. In what follows, the details of our model will be described.

The ordering policy used herein is the OUT policy. The OUT level is adjusted in each time period according to the latest demand forecast. The market information sharing scheme is assumed. The knowledge about the market demand process captured by the retailer is shared with the manufacturer. In addition, the manufacturer has access to the most recent instance of the external market demand. The cost parameters and the ordering policies in the supply chain are common knowledge. We assume that the true market demand process is unknown; however, the mean level is known[2]. The market demand is unintentionally mis-specified by the retailer, and this mis-specified knowledge of the market demand process is shared with the manufacturer and exploited by both players.

[2]This assumption is set according to the fact that the estimation of unbiased value of the mean is less problematic than that of the whole Box-Jenkins model.

3.1 Sequence of Events and Costs

The sequence of the events at period t in the model is as follows: at the beginning of the period t, the retailer receives its order placed L_1 periods ago, $O(t - L_1)$, from the manufacturer, where $L_1 = 1, 2, \ldots$ includes both the review period and the transport delay between the manufacturer and the retailer. The retailer then observes and fills the market demand, $D(t)$. Unmet demand is backlogged. At the end of the period, the retailer updates its OUT level, $S_1(t)$, with incorrect knowledge of the market demand process. Then the retailer places an order to the manufacturer, $O(t)$. The manufacturer receives this order without information delay. From its on-hand inventory, the manufacturer ships the order to the retailer immediately after observing $O(t)$. Using knowledge of the mis-specified market demand process and the shared value of $D(t)$, the manufacturer updates the forecast and then adjusts its OUT level, $S_2(t)$. Then to maintain its inventory position, $S_2(t)$, the manufacturer makes a production request, $P(t)$. After $L_2(= 1, 2, \ldots)$ time periods, $P(t)$ will be completed and ready for filling orders from the retailer. At the end of each period, the retailer incurs a holding cost (h_1) per unit of on-hand inventory plus a backlog cost (b_1) per unit of backlogged demand. The manufacturer is charged a holding cost (h_2) per unit of on-hand inventory at the end of each period. In this paper, the expediting strategy is assumed as in Lee *et al.* (2000) and Gavirneni (2006). Therefore, if the manufacturer cannot meet all the demand from the retailer, unmet demand is filled by an external source. Only the manufacturer incurs this expediting cost, b_2. The manufacturer's production line has a time invariant standard capacity, G. When the volume of the production request, $P(t)$, exceeds G, the manufacturer will run its production line in over-time to meet the full demand and be charged an extra overwork cost (w) per period for each product over the capacity G. In the case that $P(t)$ is lower than G, the production line is not fully utilised, and the manufacturer incurs an opportunity cost (u) per period for each unit of production below the capacity G. It is also assumed that infinite raw material is available for production.

3.2 Market Demand

A first-order auto-regressive and moving average process, ARMA(1, 1), will be used to represent the unknown true external demand,

$$D(t+1) = \mu + \rho\,(D(t) - \mu) + \varepsilon(t+1) - \theta\,\varepsilon(t), \tag{1}$$

where μ is the known mean of the demand, ρ and θ are the unknown auto-regressive and the moving average parameters respectively. $\varepsilon(t)$ is an unknown normally distributed "white noise" element in period t with the mean of zero and the standard deviation of σ_ε. It is assumed that $\mu \gg 4\sigma_\varepsilon$ so that negative demand is extremely rare (Johnson and Thompson, 1975). Details of an ARMA(1, 1) process are described in Box *et al.* (1994). For stability, the values of ρ and θ should be $|\rho| < 1$ and $|\theta| < 1$, respectively. Note that by setting $\theta = 0$, an ARMA(1, 1) process becomes an AR(1) process. Literature supports our ARMA(1, 1) and AR(1) demand process assumption. For instance, Lee *et al.* (2000) present evidence that market demand processes for a real supermarket are positively correlated over time. Using a real set of demand data from a retail supply chain, Hosoda *et al.* (2008) provide examples of AR(1) consumer demand processes in a retailer and ARMA(1, 1) demand processes in a supplier. Disney *et al.* (2006) show that the demand processes for Procter & Gamble products can be modelled as an ARMA(1, 1) process. These papers suggest that, the most of the time, the value of ρ is positive. Therefore we mainly focus on the case of $0 < \rho < 1$.

It is assumed herein that due to a lack of skill and knowledge, a time series data analyst mis-specified the true market demand process and believes that $D(t)$ follows an AR(1) process,

$$D(t+1) = \mu + \beta\,(D(t) - \mu) + \xi(t+1), \tag{2}$$

where β and $\xi(t)$ are the autoregressive parameter and the error term in period t respectively. It is assumed that $|\beta| < 1$. In this research, we have assumed that by analysing the historical market demand data, the retailer believes that (2) is the true process of the market demand, and this knowledge and the estimated value of β are shared with the manufacturer. Therefore, the manufacturer also believes that the market demand process can be described as (2) and exploits this knowledge to determine its own OUT level.

Generally, when the value of $|\theta|$ is much smaller than the value of $|\rho|$ (i.e. $|\theta| \ll |\rho|$), an ARMA(1, 1) process becomes quite similar to an AR(1) process. Therefore, assuming that the market demand process tends to be mis-specified in a practical setting where $|\theta| \ll |\rho|$, this research will focus on the situation that the value of θ is quite small. From (1) and (2), $\xi(t)$ can be written as

$$\xi(t) = (\rho - \beta)(D(t-1) - \mu) + \varepsilon(t) - \theta\,\varepsilon(t-1).$$

We can see that $E[\,\xi(t)\,] = 0$, irrespective of the value of β. In addition, since $\xi(t)$ contains both $\varepsilon(t)$ and $\varepsilon(t-1)$, $\xi(t)$ is not an i.i.d. process anymore. If the value of $|\theta|$ is small, however, $\xi(t)$ might be mis-specified as an i.i.d. process. This tricks the supply chain into believing that its forecast is a correct MMSE forecast. This also supports our assumption that when the value of θ is small, mis-specification tends to occur.

3.3 Ordering Policy and Performance Measures

The ordering policy for the retailer is

$$\begin{aligned} O(t) &= D(t) + (S_1(t) - S_1(t-1)), \\ S_1(t) &= \hat{D}_{L_1}(t) + z_1 \cdot \sigma_{NS_1}, \end{aligned}$$

where $\hat{D}_{L_1}(t)$ is an MMSE forecast of the market demand over the lead-time plus review period, L_1, made at period t. σ_{NS_1} is the long-run standard deviation of the end of period net stock levels at the retailer. z_1 is a predetermined constant that is selected to minimise the retailer's inventory costs. The inventory balance equation for the retailer is

$$NS_1(t) = NS_1(t-1) + O(t - L_1) - D(t),$$

where $NS_1(t)$ is the retailer's net stock level at the end of period t. The value of $NS_1(t)$ can be negative and in such a case $NS_1(t)$ represents the retailer's backorder level. The retailer's expected inventory cost at the end of each period can be represented as

$$C_{NS_1} = h_1 E\left[\Big(NS_1(t)\Big)^+\right] + b_1 E\left[\Big(-NS_1(t)\Big)^+\right]. \tag{3}$$

Here $(\,x\,)^+$ is the maximum operator, that is, $\max(\,0, x\,)$. The minimised value of C_{NS_1}, $C^*_{NS_1}$, can be obtained from the classical newsvendor problem and is $C^*_{NS_1} = \Big(h_1 \cdot z_1 + (h_1 + b_1) \cdot L(z_1)\Big)\sigma_{NS_1}$, where $z_1 = \Phi^{-1}\Big(b_1/(b_1 + h_1)\Big)$ for the standard normal distribution Φ and $L(\cdot)$ is the standard loss function.

Similarly, the production request policy used by the manufacturer is

$$\begin{aligned} P(t) &= O(t) + (S_2(t) - S_2(t-1)), \\ S_2(t) &= \hat{O}_{L_2}(t) + z_2 \cdot \sigma_{NS_2}, \end{aligned}$$

where $\hat{O}_{L_2}(t)$ is an MMSE forecast of the retailer's order over the lead-time plus review period (L_2), made at period t. σ_{NS_2} is the stable standard deviation of the end period net stock levels at the manufacturer. z_2 is a predetermined constant selected to minimise the manufacturer's inventory costs. The inventory balance equation for the manufacturer is

$$NS_2(t) = NS_2(t-1) + P(t - L_2) - O(t),$$

where $NS_2(t)$ is the manufacturer's net stock level at the end of period t. The manufacturer's expected inventory cost at the end of each period can be written as

$$C_{NS_2} \;=\; h_2 E\left[\Big(NS_2(t)\Big)^+\right] + b_2 E\left[\Big(-NS_2(t)\Big)^+\right], \tag{4}$$

and the minimised value of C_{NS_2} is $C^*_{NS_2} = \Big(h_2 \cdot z_2 + (h_2+b_2)\cdot L(z_2)\Big)\sigma_{NS_2}$, where $z_2 = \Phi^{-1}\Big(b_2/(b_2+h_2)\Big)$. We can express C_P, the manufacturer's expected production cost, as

$$C_P \;=\; uE\left[\Big(G-P(t)\Big)^+\right] + wE\left[\Big(P(t)-G\Big)^+\right]. \tag{5}$$

The classical newsvendor procedure can also be used to obtain C^*_P, the optimal value of C_P that minimises the sum of overwork and opportunity costs. It provides $C^*_P = \Big(u\cdot z_P + (u+w)\cdot L(z_P)\Big)\sigma_P$, where $z_P = \Phi^{-1}\Big(w/(w+u)\Big)$ and σ_P is the long-run standard deviation of $P(t)$.

It should be noted that in our model the costless return assumption is needed (see, Lee *et al.*, 1997; Dong and Lee, 2003, for example). This allows $O(t)$ and $P(t)$ to be negative. Negative values of $O(t)$ or $P(t)$ would occur when the sum of the on-hand inventory and the work-in-progress is higher than the target OUT level. In such a case, the excess inventory does not move out of a current stock point but is considered as the upstream player's inventory and will stay there until being used as part of a future replenishment. However, if we assume that $\mu_O \gg 4\sigma_O$ and $\mu_P \gg 4\sigma_P$ (where μ_O and μ_P are the means of $O(t)$ and $P(t)$ respectively, and σ_O is the standard deviation of $O(t)$), we may reasonably expect that the possibility of negative values of $O(t)$ or $P(t)$ is quite rare.

From the model, the transmission of the impact of mis-specification over the supply chain can be described as follows. Firstly, the market demand mis-specification directly affects the values of the forecasts, $\hat{D}_{L_1}(t)$ and $\hat{O}_{L_2}(t)$. Then, these forecast values affect the volume of the order, $O(t)$ and $P(t)$ via $S_1(t)$ and $S_2(t)$, respectively. $O(t)$ and $P(t)$ affect the value of σ_P, which in turn affects C^*_P in a linear fashion. Through the inventory balance equations, $O(t)$ and $P(t)$ also influence the values of σ_{NS_1} and σ_{NS_2}. These two standard deviations affect respectively $C^*_{NS_1}$ and $C^*_{NS_2}$, in a linear manner as well.

To quantify the impact of mis-specification, analytical expressions for σ_{NS_1}, σ_{NS_2} and σ_P will be provided in the next section. In what follows, since means do not affect the values of the standard deviations, we set $\mu = 0$ without loss of generality to ease the mathematical exposition.

3.4 Variances and Standard Deviations

From our definition, σ_{NS_1} and σ_{NS_2} represent the stable long-run standard deviations of the net stock levels at the end of each period when the market demand is mis-specified; σ_{NS_1} refers to the retailer and σ_{NS_2} to the manufacturer, respectively. As shown in Vassian (1955) for a single level and Hosoda and Disney (2006) for a multi-level supply chain, when the OUT policy is exploited, the net stock level at the end of period t is identical to the forecast error over the lead-time made at $t-L$. $L(=1,2,\ldots)$ is the lead-time including the review period. This characteristic is valid irrespective of the forecasting method used. This knowledge allows us to employ the forecast errors over the lead-time to obtain analytical expressions of σ_{NS_1} and σ_{NS_2}, instead of considering $NS_1(t)$ and $NS_2(t)$ directly. In our experience, this method easily yields analytical expressions of the standard deviations of net stock levels which are often quite complicated. By using the forecast errors, the analytical expressions for $\sigma^2_{NS_1}$ and $\sigma^2_{NS_2}$ are obtained as

$$\begin{aligned}\sigma^2_{NS_1} &= \frac{(\theta-\rho)^2\left(\rho-\beta(1+\beta^{L_1}(\rho-1))+(\beta-1)\rho^{L_1+1}\right)^2}{(\beta-1)^2(1-\rho)^3(1+\rho)}\sigma^2_\varepsilon + \\ &\left(\frac{\beta(\beta^{L_1}-1)}{\beta-1}+\frac{(\theta-\rho)(\rho^{L_1}-1)}{\rho-1}\right)^2\sigma^2_\varepsilon+\bar{\sigma}^2_{NS_1}, \end{aligned} \tag{6}$$

$$\begin{aligned}\sigma^2_{NS_2} &= \frac{\left(\begin{array}{l} 2(\theta-\rho)(\theta\rho-1)\times \\ \left(\begin{array}{l} 1+\beta^2+\beta^{L_1+2}(\rho-1)(1+\beta^{L_2-1})+ \\ \rho^{L_2-1}\left(\beta+\beta^{L_1+1}(\rho-1)-\rho\right)\left(1+\beta^{1+L_1+L_2}(\rho-1)-\beta\rho\right)\end{array}\right)+ \\ \beta^{2(1+L_1)}(\rho-1)^2\left(2\theta\rho-1-\theta^2\right)\left(1+\beta^{2L_2}\right)+ \\ 2\beta\left(2\theta\rho(1+\rho^2)-(\theta-1)^2-(1+\theta)^2\rho^2\right)- \\ 2\beta^{1+L_1}(\rho-1)((1-\theta)^2+\theta(1-\rho)^2)(1+\beta^{1+L_2})\end{array}\right)}{(\beta-1)^2(\rho-1)^3(1+\rho)}\sigma^2_\varepsilon + \\ &\frac{2(\theta-1)\rho^{L_1}(\rho-\theta)(\rho^{L_2}-1)+\frac{(\theta-\rho)^2\rho^{2L_1}(\rho^{2L_2}-1)}{1+\rho}}{(1-\rho)^3}\sigma^2_\varepsilon+\bar{\sigma}^2_{NS_2}, \end{aligned} \tag{7}$$

where $\bar{\sigma}^2_{NS_1}$ and $\bar{\sigma}^2_{NS_2}$ are the values of $\sigma^2_{NS_1}$ and $\sigma^2_{NS_2}$ when there is no demand process mis-specification[3]. The detailed process to obtain these expressions is shown in Appendix 1. Note that since an unbiased estimator for σ_{NS_1} is obtainable from the historical data of the net stock levels, $\{NS_i(t), NS_i(t-1), \ldots | i \in 1, 2\}$, it is assumed herein that even though they do not know the RHS's of (6) and (7), both the retailer and the manufacturer do know the

[3] In (7), when $\rho = 0$ and $L_2 = 1$, the value of ρ^{L_2-1} should be set to unity.

values of σ_{NS_1} and σ_{NS_2}. They can then exploit these values to establish safety stock levels via the newsvendor approach. From (6), we have the following property.

Property 1 *When $|\rho| < 1$, $|\theta| < 1$ and $|\beta| < 1$, we have $\sigma_{NS_1} \geq \bar{\sigma}_{NS_1}$.*

Proof 1 *The first two terms in (6) are never negative.*

From Property 1, we can conclude that if the market demand is mis-specified, the inventory cost for the retailer is never lower than that in the no mis-specification case, since the retailer's inventory cost is linear in σ_{NS_1}. Therefore, demand process mis-specification never reduces the retailer's inventory cost. Notice from (6) that only when $\rho = \beta$ and $\theta = 0$ or $\rho = \theta$ and $\beta = 0$, does $\sigma_{NS_1} = \bar{\sigma}_{NS_1}$ (that is, the retailer's inventory cost will not increase). The first case is obviously, as it is the case when there is no mis-specification. The second case is also a no mis-specification case because an ARMA(1, 1) process with $\rho = \theta$ is identical to an i.i.d. white noise process (Box *et al.*, 1994), and the MMSE forecast for an i.i.d. white noise process can be achieved by setting $\beta = 0$. Therefore, when $\rho = \beta$ and $\theta = 0$ or $\rho = \theta$ and $\beta = 0$, the market demand is correctly specified, and the forecast given by the supply chain is the MMSE forecast.

On the other hand, σ_{NS_2} can be either smaller or greater than $\bar{\sigma}_{NS_2}$. For example, consider the case: $\rho = 0.7$, $\theta = 0.1$, $\sigma_\varepsilon = 10$, $L_1 = 3$, and $L_2 = 4$. Here, the value of $\bar{\sigma}_{NS_2}$ is 51.4. If the supply chain chooses $\beta = 0.6$, $\sigma_{NS_2} = 48.8$. If, however, $\beta = 0.7$ is chosen, then the value of σ_{NS_2} increases to 52.5. This will be explored further in the next section.

Recall σ_P is used to represent the stable standard deviation of the production request, $P(t)$. If the process of $P(t)$ can be expressed as a Box-Jenkins model, σ_P can be obtained easily. As shown in Appendix 1, $P(t)$ can is an ARMA(1, 2) process, and σ_P^2 is

$$\sigma_P^2 = \frac{\begin{pmatrix} 1+\theta^2-2\theta\rho + \\ 2\beta(\theta-\rho)(1-\theta\rho)+\beta^2(1+\theta^2-2\theta\rho)- \\ 2\beta^{L_1+L_2+1}(\beta^{L_1+L_2+1}-1-\beta)(\rho-1)(1+\theta+\theta^2-\theta\rho) \end{pmatrix}}{(\beta-1)^2(1-\rho^2)}\sigma_\varepsilon^2. \tag{8}$$

Note that it is assumed that the manufacturer does not know the RHS of (8). However, the manufacturer does have easy access to an unbiased estimator of σ_P^2 from historical data of $P(t)$, $\{P(t), P(t-1), \ldots\}$.

Property 2 *When ρ, θ, σ_ε and β are given, σ_P is a function of L_1+L_2 and the impact of L_1 on σ_P is identical to that of L_2, irrespective of demand mis-specification.*

Proof 2 *The proof is omitted since it is straightforward from (8) and (10) in Appendix 1.*

The production cost is linear in σ_P when $C_P = C_P^*$. Thus we can conclude that not only does shorter L_2, but also shorter L_1 (which is usually not under control of the manufacturer but is under control of the retailer) reduces the production cost, irrespective of market demand mis-specification.

Property 3 *1) When $|\rho| < 1$, $|\theta| < 1$ and $0 \leq \beta < 1$, σ_P is increasing in β, and 2) when $|\rho| < 1$, $|\theta| < 1$ and $|\beta| < 1$, the value of β which minimises σ_P is negative.*

Proof 3 *The proof is given in Appendix 2.*

The first part of Property 3 suggests that if the current value of β used in the supply chain is positive, setting $\beta = 0$, the value of σ_P will be lower, which results in a lower production cost. Note that when $\beta = 0$, the ordering/production request policies used by the supply chain become the classical base stock policy which has a time invariant OUT level. The proof is shown in Appendix 3. One of the interesting characteristics of the classical base stock policy is that the order process given by this policy is identical to the demand process (see, Chen, 1999, for example). Therefore, if the supply chain sets $\beta = 0$, both $O(t)$ and $P(t)$ will have the same ARMA(1, 1) structure as $D(t)$, and there is no bullwhip effect. A similar discussion can be seen in Lee *et al.* (2000). The second part in Property 3 means that if minimisation of σ_P is the first priority for the supply chain, a negative value of β should be considered, even though the true value of ρ is positive. However, in a practical setting, since the supply chain does not know (8), and thus does not know values of ρ, θ and σ_ε, it may not be easy to find the exact value of β that minimises σ_P.

From the properties shown above, the following points have grabbed our attention. Property 1 suggests that if the market demand is mis-specified, σ_{NS_1} will always increase. An interesting point is that how much the impact of the market demand mis-specification is on the cost when the supply chain chooses the value of β which minimises forecast errors. If the amount of the cost increase is minor, we may conclude that the impact of the mis-specification is negligible. If not, avoiding the demand process mis-specification would be a critical issue for managers, and to achieve this, a certain amount of investment in forecasting systems might be necessary. Another interesting observation is that by exploiting the market demand mis-specification *intentionally*, the supply chain may be able to reduce its

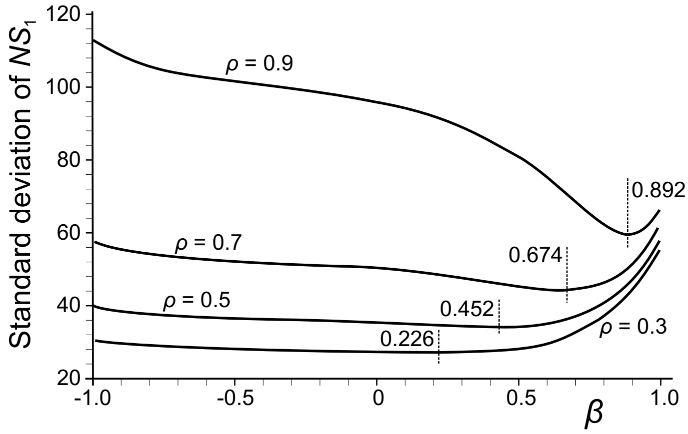

Figure 2: Values of σ_{NS_1} when $|\beta| < 1$, $\theta = 0.1$ and $L_1 = 5$

total costs. For example, since a value of β can decrease the value of σ_P, we may reasonably expect that the supply chain can reduce its production cost, even though the retailer's inventory cost will increase under such a condition. Furthermore, it might be possible that the benefit coming from the production cost is large enough to compensate for the increased inventory cost. To explore further details of the impact of mis-specification, we will now resort to numerical analysis due to the complexity of equations (6), (7) and (8).

4 Numerical Analysis

In this section, two scenarios are investigated to verify properties and to illustrate the impact of market demand mis-specification. The first scenario will consider the case when β is chosen to minimise σ_{NS_1}. This is identical to the standard deviation of the market demand forecast error over L_1 time periods. We consider this scenario a representative of a typical attitude of a real world retailer. In the second scenario we will assume that the supply chain will manipulate β to control the dynamics of the supply chain. In other words, β is considered to be a controllable variable that is exploited to achieve a lower total supply chain cost. In both scenarios, the following cost parameters are assumed; $h_1 = 2$, $b_1 = 50$, $h_2 = 1$, $b_2 = 25$, $u = 2$ and $w = 50$. Lead-times are $L_1 = 5$ and $L_2 = 5$. We focus on the case in which $0 < \rho < 1$, $|\theta| \ll \rho$ and $|\beta| < 1$. For the true market demand, we assume that $\sigma_\varepsilon = 10$. In what follows, we will use the parameter settings shown above unless otherwise noted.

4.1 Scenario 1: Minimising the retailer's forecast error

Fig. 2 shows the values of σ_{NS_1} for each value of ρ when $|\beta| < 1$, $\theta = 0.1$ and $L_1 = 5$. Observe that there is a unique value of β that minimises σ_{NS_1} for each value of ρ and we call that value $\beta^*_{NS_1}$. Table 1 shows $\beta^*_{NS_1}$ and each cost ($C^*_{NS_1}$, $C^*_{NS_2}$ and C^*_P), together with corresponding costs of the no mis-specification case, when $0.3 \leq \rho \leq 0.9$ and $\theta = -0.1$ and 0.1. As we expect, $C^*_{NS_1}$ in the case of the demand mis-specification is higher than that in the case of no mis-specification for all settings. However, the other two costs show a different pattern. In some cases mis-specification yields higher costs and in other cases they can lower. In terms of the total cost, when $\theta = -0.1$, the total costs for the mis-specified case are always lower than those for the no mis-specification case. On the other hand, when $\theta = 0.1$, the total cost for the mis-specification case is higher than that for no mis-specification case. All in all, however, the difference of each cost between the mis-specification case and the no mis-specification case is rather small. From Table 1, we can also conclude that if the supply chain employs $\beta^*_{NS_1}$, the impact of the market demand mis-specification on total cost is minor. Demand mis-specification always increases $C^*_{NS_1}$; however, its impact on the total cost is quite small. Furthermore, Table 1 suggests that there is a possibility that demand mis-specification may actually reduce the total cost. This leads to our next interesting question: Can the supply chain reduce its total cost by manipulating β ? Scenario 2 will consider this question.

4.2 Scenario 2: Manipulating the value of β

In this scenario, we vary the value of β to analyse its impact on the supply chain costs. Fig. 3 shows the impact of β on the total cost, $C^*_{NS_1} + C^*_{NS_2} + C^*_P$. In Fig. 3, the cost when the market demand is correctly specified is also indicated (shown as "No mis-specification" in the graph). Clearly, the total cost is affected by the value of β. The values of β that minimise the total cost, β^*_{total}, are always $-1 < \beta^*_{\text{total}} < \rho$ and in many cases the values are negative. Only when the value of ρ is large (i.e. $\rho = 0.9$) does β^*_{total} have a large positive value.

Table 2 shows the results under the same parameter setting as in Fig. 3 with β^*_{total} used for the mis-specification case, and the results of no mis-specification case for comparison. It shows that the total cost of mis-specification case (B) is always lower than that of the no mis-specification case (A). When $\theta = 0$, since the market demand process becomes an AR(1) process, setting $\beta = \rho$ yields the no mis-specification case. Table 2 shows, however, that

Table 1: Impact of $\beta^*_{NS_1}$ on the total supply chain cost

ρ	0.3	0.5	0.7	0.9	0.3	0.5	0.7	0.9
	$\theta = -0.1$				$\theta = 0.1$			
Mis								
$\beta^*_{NS_1}$	0.356	0.535	0.719	0.906	0.226	0.452	0.674	0.892
$C^*_{NS_1}$	138	173	224	298	117	147	191	256
	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)
$C^*_{NS_2}$	77	107	167	301	62	86	135	251
	(1.01)	(1.01)	(1.01)	(1.00)	(0.99)	(0.99)	(0.99)	(1.00)
C^*_P	67	93	151	310	56	79	132	289
	(0.99)	(0.98)	(0.97)	(0.94)	(1.00)	(1.01)	(1.03)	(1.07)
Total cost	282	373	542	909	235	312	458	796
	(1.00)	(0.999)	(0.992)	(0.98)	(1.00)	(1.00)	(1.01)	(1.02)
No mis								
$C^*_{NS_1}$	138	173	224	297	117	147	191	256
$C^*_{NS_2}$	76	106	166	301	62	87	137	251
C^*_P	68	95	156	328	56	78	128	271
Total cost	282	374	546	926	235	312	456	778

Mis: Mis-specification case
No mis: No mis-specification case
($\cdot$): Ratio to the corresponding value of no mis-specification case

even if $\theta = 0$, the value of β^*_{total} is far from ρ. Furthermore, together with the results of no mis-specification case shown in Table 2, Fig. 3 suggests that even if β is not exactly equal to β^*_{total}, by exploiting demand mis-specification, the supply chain can achieve lower total cost, especially when $0.3 \leq \rho \leq 0.7$ (although more care has to be taken when $\rho = 0.9$). Table 2 also suggests that the value of β^*_{total} is affected by the value of θ. As the value of θ increases, the value of β^*_{total} decreases.

Let's consider the relationship between the total inventory cost, $C^*_{NS_1} + C^*_{NS_2}$, and the production cost, C^*_P. Fig. 4 illustrates the relationship between those two costs. It is suggested that the supply chain can reduce both the total inventory cost and the production cost simultaneously by changing a value of β, which should be smaller than the value of ρ. In most cases, setting $\beta = 0$ can reduce both the total inventory cost and the production cost at the same time. Fig. 4 also suggests that when ρ is a large value (e.g. $\rho = 0.9$), the value of β should be about 0.8 to avoid higher inventory costs.

All these findings suggest that 1) the market demand mis-specification does not always increase the total supply chain cost, and 2) even though the true values of ρ and θ are

Hosoda, T. and Disney, S.M., (2009), “Impact of market demand mis-specification on a two-level supply chain”,
International Journal of Production Economics, Vol. 121, No. 2, pp739–751. DOI: 10.1016/j.ijpe.2009.04.024.

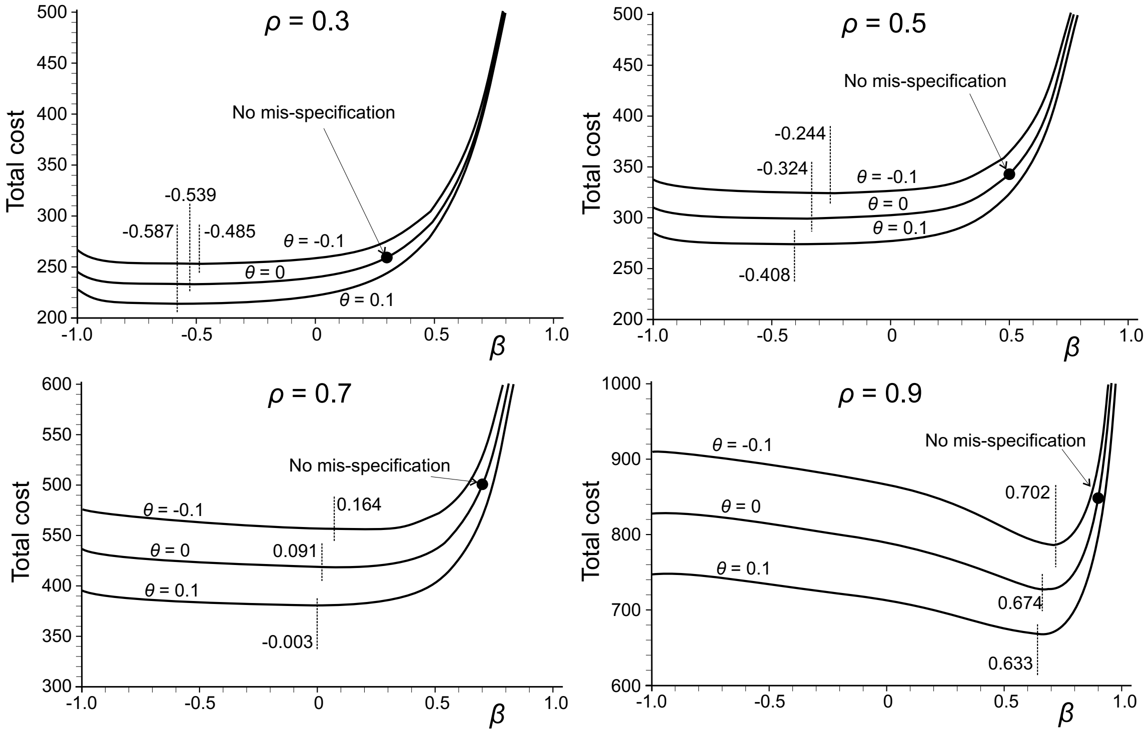

Figure 3: Impact of β on the total cost when $0.3 \leq \rho \leq 0.9$ and $-0.1 \leq \theta \leq 0.1$.

Table 2: Costs for no mis-specification and mis-specification cases

		No mis-spec. case				Mis-spec. case					
ρ	θ	$C^*_{NS_1}$	$C^*_{NS_2}$	C^*_P	TC (A)	β^*_{total}	$C^*_{NS_1}$	$C^*_{NS_2}$	C^*_P	TC (B)	B/A
0.3	−0.1	138	76	68	282	−0.485	144	68	40	253	0.895
	0.0	128	69	62	259	−0.539	133	63	38	234	0.899
	0.1	117	62	56	235	−0.587	121	57	35	213	0.905
0.5	−0.1	173	106	95	374	−0.244	186	90	49	325	0.869
	0.0	160	96	87	343	−0.324	171	82	45	298	0.870
	0.1	147	87	78	312	−0.408	156	74	42	272	0.873
0.7	−0.1	224	166	156	546	0.164	255	132	69	456	0.836
	0.0	207	151	142	500	0.091	235	120	63	418	0.836
	0.1	191	137	128	456	−0.003	216	108	57	381	0.836
0.9	−0.1	297	301	328	926	0.702	356	268	164	788	0.850
	0.0	276	276	299	851	0.673	335	244	148	727	0.854
	0.1	256	251	271	778	0.633	316	219	133	668	0.859

TC: Total Cost

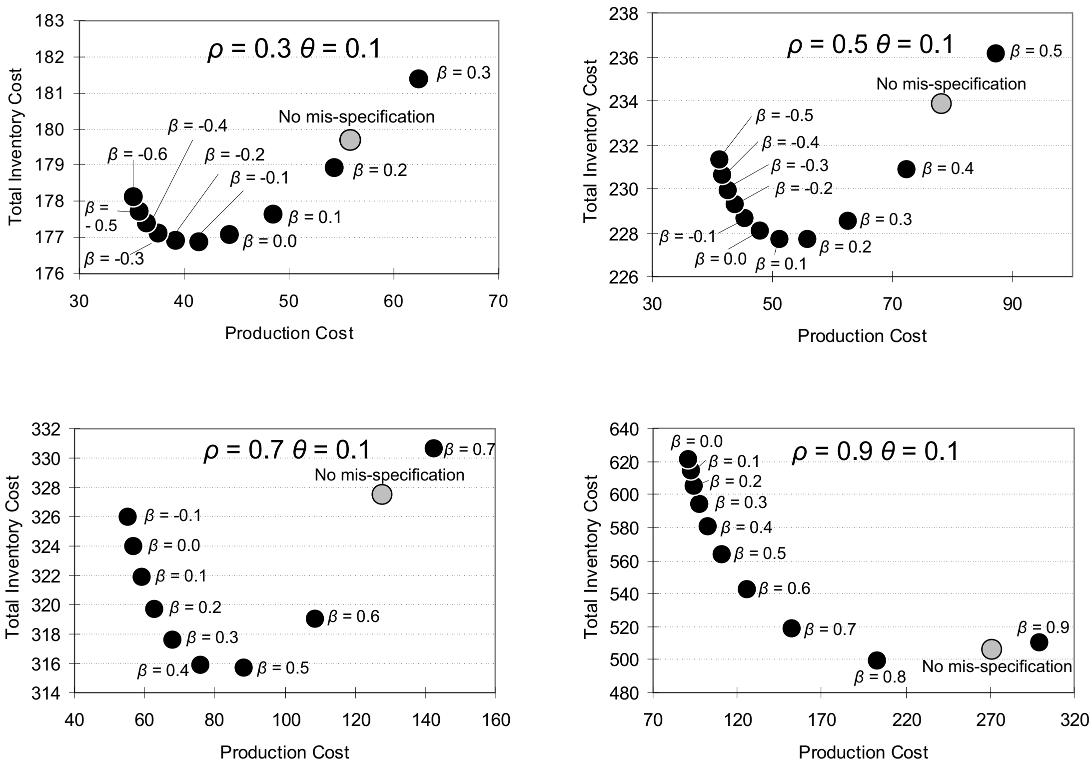

Figure 4: Impact of β when $0.3 \leq \rho \leq 0.9$ and $\theta = 0.1$

unknown, by exploiting a small value of β, ($\beta = 0$, for example), the total supply chain cost can be reduced. Setting $\beta = 0$ means sacrificing MMSE forecasting and employing instead the classical base stock policy with a time invariant OUT level, as shown in Appendix 3.

It should be noted that when ρ is negative, even if β^*_{total} is used in the supply chain, the total supply chain cost in the case of mis-specification could be higher than that in the case of no mis-specification. Some results are shown in Appendix 4. For example, when $\rho = -0.7$ and $\theta = 0.1$, the value of β^*_{total} is -0.752 and the total cost ratio (= B/A) becomes 1.026. This fact can be explained as follows. It is well known that when ρ is negative and the market demand is specified correctly, there is no bullwhip effect in a supply chain (see, Kahn, 1987, for example). Therefore, the total cost in the case of no mis-specification is quite low and the power of β to control the supply chain cost is not strong enough to outperform the no mis-specification case.

From Table 2, it can be seen that C^*_P is the major contributor to the lower total cost in the case of mis-specification. Since C^*_P is affected by not only σ_P but also both u and w, we have used different values of u and w to analyse the impact of these cost parameters on

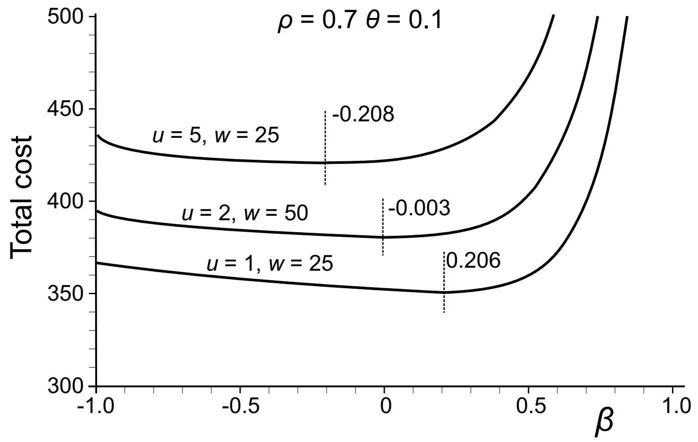

Figure 5: Impact of u and w on the total cost

Table 3: Impact of u and w on on each cost when $\rho = 0.7$ and $\theta = 0.1$

	$u = 2$ $w = 50$	$u = 1$ $w = 25$	$u = 5$ $w = 25$
Mis-specification case			
β^*_{total}	-0.003	0.206	-0.208
$C^*_{NS_1}$	216	209	221
$C^*_{NS_2}$	108	110	107
C^*_P	57	31	93
Total cost	381	351	420
No mis-specification case			
$C^*_{NS_1}$	191	191	191
$C^*_{NS_2}$	137	137	137
C^*_P	128	64	221
Total cost	456	392	549

the total cost. Fig. 5 and Table 3 provide the results for when $\rho = 0.7$ and $\theta = 0.1$. Fig. 5 shows that the value of β^*_{total} tends to be smaller (or more negative) as u is larger. This may be because when u is large, σ_P should be smaller to minimise the impact of this large value of u on the total cost. A smaller value of σ_P can be achieved by using a small value of β, as shown in Property 3. With the aid of Fig. 5 and Table 3, it can be seen that setting $\beta = 0$ can bring the benefit for the total supply chain, irrespective of the values of u and w.

Fig. 6 shows the impact of the lead-time on the total cost and illustrates that the smaller $L_1 + L_2$ leads to the lower total costs. The difference between two cases where $L_1 + L_2 = 15$ comes from the total inventory cost, since from Property 2, the production cost, C^*_P, is the

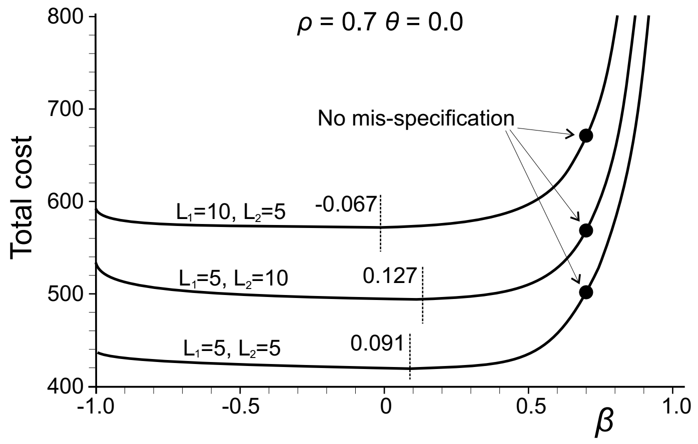

Figure 6: Impact of L_1 and L_2 on the total cost

same for both cases. Fig. 6 suggests that when $L_1 + L_2$ is constant, smaller L_1 (or larger L_2) can result in lower total costs. Again, we can see that $\beta = 0$ is not a bad alternative.

The numerical analysis shown herein highlights the following operational insights. Although true values of ρ and θ are not known, by using a small value of β, the supply chain can reduce its total cost in most cases. A guideline we suggest is as follows: if the value of ρ is considered $0.3 \leq \rho \leq 0.7$ and the value of θ is quite small, set $\beta = 0$. That is, by using the classical base stock policy the supply chain can reduce its cost (see, Fig. 3 − Fig. 6). If the supply chain can reasonably assume that ρ is about 0.9, using a large value of β ($\beta = 0.6$ or 0.7) can achieve lower total cost (see, Fig. 3 and Fig. 4). If it is known that the value of ρ is a small value (i.e. $0.3 \leq \rho \leq 0.5$), negative value of β should be considered to minimise the total cost (see, Table 2). In the case of negative ρ, mis-specification may not bring benefit to the supply chain (see, Table 4 in Appendix 4).

5 Conclusion

Assuming the true stochastic nature of the market demand process is unknown, we have analysed the impact of market demand process mis-specification on the costs in a serially linked two-level supply chain. We have shown that the impact of the demand process mis-specification on the costs is minor if a supply chain tries to minimise market demand forecast errors. Furthermore, by exploiting demand process mis-specification, a supply chain can reduce both the inventory cost and the production cost simultaneously. A "rule of thumb" for the selection of the value of β is presented. The managerial insight derived from our study is that *poor forecasts are not always bad from a total supply chain cost viewpoint.* Poor forecasts always increase the first-level player's inventory cost. However, the cost reduction generated at the second-level player can be large enough to compensate for the increase at the first-level. This managerial insight can be restated; *employing more accurate forecasting methods may actually result in higher total supply chain costs.* It is possible that a supply chain who has mis-specified the market demand process may be enjoying a lower cost at the second-level unintentionally. And it is also possible that if such a supply chain incorporates a more accurate forecasting method, only the inventory cost at the first-level is guaranteed to decrease. The cost at the second-level, however, may increase. This may result in a higher total supply chain cost. Therefore, when a supply chain considers changing its forecasting method to a more accurate one, it should be very careful. The result of improving forecast accuracy may mean that the supply chain loses hidden benefits and incurs new additional costs.

Acknowledgements

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Appendix 1: The variances

No Mis-Specification Case

In the no mis-specification case, the supply chain knows the true structure and the values of parameters of the market demand process. Under this condition, the MMSE forecast is

$$\begin{aligned}
\hat{D}_{L_1}(t) &= E\left[\sum_{i=1}^{L_1} D(t+i)\middle| D(t), \rho, \theta, \varepsilon(t)\right] \\
&= \rho(1+\rho+\cdots+\rho^{L_1-1})D(t) - \theta(1+\rho+\cdots+\rho^{L_1-1})\varepsilon(t) \\
&= \frac{1-\rho^{L_1}}{1-\rho}\left(\rho D(t) - \theta\varepsilon_t\right) \\
&= \Lambda_{L_1}\left(\rho D(t) - \theta\varepsilon_t\right),
\end{aligned}$$

where $\Lambda_{L_1} = (1-\rho^{L_1})/(1-\rho)$. Vassian (1955) and Hosoda and Disney (2006) have shown that the value of the net stock variance is identical to the variance of the forecast error over the lead-time. By exploiting this, we can obtain the long-run variance of the net stock levels at the retailer when the demand process is specified correctly, $\bar{\sigma}^2_{NS_1}$, as follows.

$$\bar{\sigma}^2_{NS_1} = E\left[Var\left[\sum_{i=1}^{L_1} D(t+i) - \hat{D}_{L_1}(t)\right]\right]$$

$$
\begin{aligned}
&= \sum_{i=1}^{L_1}\left[\left(1+\sum_{j=2}^{i}(\rho-\theta)\rho^{j-2}\right)^2\right]\sigma_\varepsilon^2 \\
&= \frac{\begin{pmatrix} L_1(\theta-1)^2(\rho^2-1)+ \\ (\rho-\theta)(\rho^{L_1}-1)(\theta(1+2\rho-\rho^{L_1})+\rho(\rho^{L_1}-1)-2) \end{pmatrix}}{(\rho-1)^3(1+\rho)}\sigma_\varepsilon^2.
\end{aligned}
$$

As shown in Hosoda and Disney (2006), the OUT policy transforms an ARMA(1, 1) demand process into another ARMA(1, 1) order process, which is

$$
\begin{aligned}
O(t+1) &= \rho O(t) + (1+\rho\Lambda_{L_1}-\theta\Lambda_{L_1})\,\varepsilon(t+1) - (\theta+\rho\Lambda_{L_1}-\theta\Lambda_{L_1})\,\varepsilon(t) \\
&= \rho O(t) + \varepsilon_o(t+1) - \theta_o\,\varepsilon_o(t),
\end{aligned}
$$

where $\varepsilon_o(t) = (1+\rho\Lambda_{L_1}-\theta\Lambda_{L_1})\varepsilon(t)$ and $\theta_o = (\theta+\rho\Lambda_{L_1}-\theta\Lambda_{L_1})/(1+\rho\Lambda_{L_1}-\theta\Lambda_{L_1})$. Therefore, since the demand for the manufacturer is also an ARMA(1, 1) process (as it was for the retailer), the same approach used to obtain $\bar{\sigma}^2_{NS_1}$ can be used to obtain $\bar{\sigma}^2_{NS_2}$. The result is

$$
\begin{aligned}
\bar{\sigma}^2_{NS_2} &= \frac{\begin{pmatrix} L_2(\theta_o-1)^2(\rho^2-1)+ \\ (\rho-\theta_o)(\rho^{L_2}-1)(\theta_o(1+2\rho-\rho^{L_2})+\rho(\rho^{L_2}-1)-2) \end{pmatrix}}{(\rho-1)^3(1+\rho)}\sigma^2_{\varepsilon_o} \\
&= \frac{\begin{pmatrix} L_2(\theta-1)^2(\rho-1)+ \\ 2(\theta-1)\rho^{L_1}(\rho-\theta)(\rho^{L_2}-1)+\frac{(\theta-\rho)^2\rho^{2L_1}(\rho^{2L_2}-1)}{1+\rho} \end{pmatrix}}{(\rho-1)^3}\sigma^2_\varepsilon, \qquad (9)
\end{aligned}
$$

where $\sigma^2_{\varepsilon_o}$ is the variance of $\varepsilon_o(t)$. Since $O(t)$ is an ARMA(1, 1) process and the manufacturer uses the OUT policy for its production request, $P(t)$ is also an ARMA(1, 1) process and is described as

$$
\begin{aligned}
P(t+1) &= \rho P(t) + (1+\rho\Lambda_{L_2}-\theta_o\Lambda_{L_2})\,\varepsilon_o(t+1) - (\theta_o+\rho\Lambda_{L_2}-\theta_o\Lambda_{L_2})\,\varepsilon_o(t) \\
&= \rho P(t) + \varepsilon_p(t+1) - \theta_p\,\varepsilon_p(t).
\end{aligned}
$$

Here $\Lambda_{L_2} = (1-\rho^{L_2})/(1-\rho)$, $\varepsilon_p(t) = (1+\rho\Lambda_{L_2}-\theta_o\Lambda_{L_2})\varepsilon_o(t)$ and $\theta_p = (\theta_o+\rho\Lambda_{L_2}-\theta_o\Lambda_{L_2})/(1+\rho\Lambda_{L_2}-\theta_o\Lambda_{L_2})$. Generally, once an ARMA(1, 1) process is described as (1), its variance can be obtained as $(1+\theta^2-2\theta\rho)\sigma^2_\varepsilon/(1-\rho^2)$ (Box *et al.*, 1994). From this knowledge, we can easily obtain the variance of $P(t)$, $\bar{\sigma}^2_P$, and it is

$$
\bar{\sigma}^2_P = \frac{2\rho^{2(L_1+L_2)}(\theta-\rho)^2+(\theta-1)^2(1+\rho)+2\rho^{L_1+L_2}(\theta-1)(1+\rho)(\rho-\theta)}{(\rho-1)^2(1+\rho)}\sigma^2_\varepsilon. \qquad (10)
$$

Mis-Specification Case

Since the retailer believes that the demand process is the AR(1) structure as shown in (2), the MMSE forecast of the lead-time demand made by the retailer at period t, $\hat{D}_{L_1}(t)$ can be written as

$$\begin{aligned}
\hat{D}_{L_1}(t) &= E\left[\sum_{i=1}^{L_1} D(t+i) \,\Big|\, D(t), \beta\right] \\
&= E\left[\beta(1+\beta+\cdots+\beta^{L_1-1})D(t) + \sum_{i=1}^{L_1}\sum_{j=0}^{L_1-i}\beta^j \xi(t+i)\right] \\
&= \beta(1+\beta+\cdots+\beta^{L_1-1})D(t) \\
&= \beta\,\Lambda'_{L_1} D(t), \qquad (11)
\end{aligned}$$

where $\Lambda'_{L_1} = (1-\beta^{L_1})/(1-\beta)$. Similar to the no mis-specification case, we will exploit the forecast errors over the lead-time to obtain $\sigma^2_{NS_1}$.

$$\begin{aligned}
\sigma^2_{NS_1} &= E\left[Var\left[\sum_{i=1}^{L_1} D(t+i) - \hat{D}_{L_1}(t)\right]\right] \\
&= \sum_{i=1}^{L_1}\left[\left(1+\sum_{j=2}^{i}(\rho-\theta)\rho^{j-2}\right)^2\right]\sigma_\varepsilon^2 + \\
&\quad \left((\rho-\theta)(1+\rho+\cdots+\rho^{L_1-1}) - \beta\frac{1-\beta^{L_1}}{1-\beta}\right)^2\sigma_\varepsilon^2 + \\
&\quad \frac{(\rho-\theta)^2}{1-\rho^2}\left(\rho(1+\rho+\cdots+\rho^{L_1-1}) - \beta\frac{1-\beta^{L_1}}{1-\beta}\right)^2\sigma_\varepsilon^2 \\
&= \frac{(\theta-\rho)^2\left(\rho-\beta(1+\beta^{L_1}(\rho-1))+(\beta-1)\rho^{L_1+1}\right)^2}{(\beta-1)^2(1-\rho)^3(1+\rho)}\sigma_\varepsilon^2 + \\
&\quad \left(\frac{\beta(\beta^{L_1}-1)}{\beta-1} + \frac{(\theta-\rho)(\rho^{L_1}-1)}{\rho-1}\right)^2\sigma_\varepsilon^2 + \bar{\sigma}^2_{NS_1}.
\end{aligned}$$

In the setting that both the retailer and the manufacturer believe that the market demand process is an AR(1) process, the retailer's ordering process that the manufacturer expects can be written as

$$\begin{aligned}
O(t) &= D(t) + \beta\,\Lambda'_{L_1}\Big(D(t) - D(t-1)\Big) \\
&= \beta\,D(t-1) + \xi(t) + \beta\,\Lambda'_{L_1}\Big(\beta\,D(t-1) + \xi(t) - D(t-1)\Big) \\
&= \beta^{L_1+1}\,D(t-1) + (1+\beta\Lambda'_{L_1})\xi(t)
\end{aligned}$$

$$
\begin{aligned}
&= \beta^{L_1+1} D(t-1) + (1-\beta^{L_1+1})/(1-\beta)\xi(t) \\
&= \beta^{L_1+1} D(t-1) + \Lambda'_{L_1+1}\xi(t), \qquad (12)
\end{aligned}
$$

where $\Lambda'_{L_1+1} = (1-\beta^{L_1+1})/(1-\beta)$. Therefore, the forecast used by the manufacturer, the conditional expectation of (12) over the L_2 time periods is given by

$$
\begin{aligned}
\hat{O}_{L_2}(t) &= E\left[\sum_{i=1}^{L_2} O(t+i) \;\middle|\; D(t), \beta\right] \\
&= E\left[\beta^{L_1+1}(1+\beta+\cdots+\beta^{L_2-1})D(t) + \sum_{i=1}^{L_2}\left(\left(\Lambda'_{L_1+1} + \sum_{j=0}^{L_2-1-i}\beta^{L_1+1+j}\right)\xi(t+i)\right)\right] \\
&= \beta^{L_1+1}(1+\beta+\cdots+\beta^{L_2-1})D(t) \\
&= \beta^{L_1+1}\Lambda'_{L_2}D(t), \qquad (13)
\end{aligned}
$$

where $\Lambda'_{L_2} = (1-\beta^{L_2})/(1-\beta)$. In our model, it is assumed that the manufacturer exploits (13) believing that it provides MMSE forecasts. However, the actual process of $O(t)$ is different from (12). Using (1), the actual process of $O(t)$ when the retailer mis-specified the market demand process can be written as

$$
\begin{aligned}
O(t) &= D(t) + \beta\Lambda'_{L_1}\left(D(t) - D(t-1)\right) \\
&= D(t) + \beta\Lambda'_{L_1}D(t) - \beta\Lambda'_{L_1}D(t-1) \qquad (14) \\
&= \rho\left(D(t-1) + \beta\Lambda'_{L_1}(D(t-1) - D(t-2))\right) + \\
&\quad \left(1+\beta\Lambda'_{L_1}\right)\varepsilon(t) - \left(\theta + \beta\Lambda'_{L_1}\theta + \beta\Lambda'_{L_1}\right)\varepsilon(t-1) + \beta\Lambda'_{L_1}\theta\varepsilon(t-2) \\
&= \rho O(t-1) + \left(1+\beta\Lambda'_{L_1}\right)\varepsilon(t) - \\
&\quad \left(\theta + \beta\Lambda'_{L_1}\theta + \beta\Lambda'_{L_1}\right)\varepsilon(t-1) + \beta\Lambda'_{L_1}\theta\varepsilon(t-2), \qquad (15)
\end{aligned}
$$

which is an ARMA(1, 2) process. From (1), (13) and (14), we can find $\sigma^2_{NS_2}$. This is identical to the variance of the forecast error over the L_2 time periods.

$$
\begin{aligned}
\sigma^2_{NS_2} &= E\left[Var\left[\sum_{i=1}^{L_2} O(t+i) - \hat{O}_{L_2}(t)\right]\right] \\
&= \left(1+\beta\Lambda'_{L_1}\right)^2\sigma^2_\varepsilon + \\
&\quad \sum_{i=2}^{L_2}\left(1 + \rho^{i-2}(1+\beta\Lambda'_{L_1})(\rho-\theta) + (\rho-\theta)\sum_{j=3}^{i}\rho^{j-3}\right)^2\sigma^2_\varepsilon + \\
&\quad \left(\left(\rho\Lambda_{L_2} + \beta\Lambda'_{L_1}(\rho^{L_2}-1) - \beta^{L_1+1}\Lambda'_{L_2}\right) - \theta\left(\rho^{L_2-1}(1+\beta\Lambda'_{L_1}) + \sum_{i=2}^{L_2}\rho^{i-2}\right)\right)^2\sigma^2_\varepsilon +
\end{aligned}
$$

$$\frac{\left(\rho\Lambda_{L_2} + \beta\Lambda'_{L_1}(\rho^{L_2} - 1) - \beta^{L_1+1}\Lambda'_{L_2}\right)^2 (\rho - \theta)^2}{1 - \rho^2}\sigma_\varepsilon^2. \tag{16}$$

By simplifying (16), we can have (7). Using (13) as its forecasting method together with the observed demand (15), the manufacturer places a production request, $P(t)$, which is

$$\begin{aligned} P(t) &= O(t) + \beta^{L_1+1}\Lambda'_{L_2}\Big(D(t) - D(t-1)\Big) \\ &= \rho\Big(O(t-1) + \beta^{L_1+1}\Lambda'_{L_2}(D(t-1) - D(t-2))\Big) + \Big(1 + \beta\Lambda'_{L_1} + \beta^{L_1+1}\Lambda'_{L_2}\Big)\varepsilon(t) - \\ &\quad \Big(\theta + \beta\Lambda'_{L_1}\theta + \beta\Lambda'_{L_1} + \beta^{L_1+1}\Lambda'_{L_2} + \beta^{L_1+1}\Lambda'_{L_2}\theta\Big)\varepsilon(t-1) + \\ &\quad \Big(\beta\Lambda'_{L_1}\theta + \beta^{L_1+1}\Lambda'_{L_2}\theta\Big)\varepsilon(t-2) \\ &= \rho P(t-1) + \Big(1 + \beta\Lambda'_{L_1} + \beta^{L_1+1}\Lambda'_{L_2}\Big)\varepsilon(t) - \\ &\quad \Big(\theta + \beta\Lambda'_{L_1}\theta + \beta\Lambda'_{L_1} + \beta^{L_1+1}\Lambda'_{L_2} + \beta^{L_1+1}\Lambda'_{L_2}\theta\Big)\varepsilon(t-1) + \\ &\quad \Big(\beta\Lambda'_{L_1}\theta + \beta^{L_1+1}\Lambda'_{L_2}\theta\Big)\varepsilon(t-2). \end{aligned} \tag{17}$$

(17) shows that $P(t)$ is an ARMA(1, 2) process as is $O(t)$. Generally, when an ARMA(1, 2) process defined as

$$P(t) = \rho P(t-1) - \theta_0\varepsilon(t) - \theta_1\varepsilon(t-1) - \theta_2\varepsilon(t-2),$$

where θ_0, θ_1 and θ_2 are moving average parameters, its variance is given by

$$\frac{\sigma_\varepsilon^2}{1-\rho^2}\left((\theta_0^2 + \theta_1^2 + \theta_2^2) + 2\rho\left(\theta_0\theta_1 + \theta_2(\rho\theta_0 + \theta_1)\right)\right). \tag{18}$$

From (17) and (18) and after algebraic simplification, we obtain (8).

Appendix 2: Property 3

Property 3 can be proven by considering the derivative of σ_P^2 with respect to β and showing that $(\partial\sigma_P^2)/(\partial\beta)$ is positive for $|\rho| < 1$, $|\theta| < 1$ and $0 \le \beta < 1$.

$(\partial\sigma_P^2)/(\partial\beta)$ can be written as

$$\frac{\partial\sigma_P^2}{\partial\beta} = \frac{\begin{pmatrix} 2\left(1 + \mathcal{L}_{12}\beta^{\mathcal{L}_{12}+1} - (\mathcal{L}_{12}+1)\beta^{\mathcal{L}_{12}}\right) \times \\ \left(2\beta^{\mathcal{L}_{12}+1} - \beta - 1\right)(1 + \theta^2 + \theta(1-\rho)) \end{pmatrix}}{(\beta-1)^3(1+\rho)}\sigma_\varepsilon^2,$$

where $\mathcal{L}_{12} = L_1 + L_2$. For $|\rho| < 1$ and $0 \le \beta < 1$, it is easy to check

$$\begin{aligned} 2\beta^{\mathcal{L}_{12}+1} - \beta - 1 \le 2\beta - \beta - 1 = \beta - 1 &< 0, \\ (\beta-1)^3 &< 0, \\ 1 + \rho &> 0. \end{aligned}$$

Since $0 < 1-\rho < 2$, when $0 \leq \theta < 1$, we have

$$1 + \theta^2 + \theta(1-\rho) > 0,$$

and when $-1 < \theta < 0$, we have

$$1 + \theta^2 + \theta(1-\rho) > 1 + \theta^2 + 2\theta = (1+\theta)^2 > 0.$$

Therefore, we can conclude that when $|\rho| < 1$ and $|\theta| < 1$, $1 + \theta^2 + \theta(1-\rho)$ is positive.

For convenience, we set

$$f(\beta) = 1 + \mathcal{L}_{12}\beta^{\mathcal{L}_{12}+1} - (\mathcal{L}_{12}+1)\,\beta^{\mathcal{L}_{12}},$$

and will show that $f(\beta)$ in non-negative. Differentiating $f(\beta)$ with respect to β yields

$$\frac{\partial f(\beta)}{\partial \beta} = \mathcal{L}_{12}\,(\mathcal{L}_{12}+1)\,\beta^{\mathcal{L}_{12}} - \mathcal{L}_{12}\,(\mathcal{L}_{12}+1)\,\beta^{\mathcal{L}_{12}-1} \leq 0.$$

Therefore, since $0 \leq \beta < 1$, the minimum value of $f(\beta)$ is achieved when $\beta \to 1$:

$$\min f(\beta) = \lim_{\beta \to 1} f(\beta) = 0.$$

This shows $f(\beta) > 0$ when $0 \leq \beta < 1$, and thus $(\partial\sigma_P^2)/(\partial\beta) > 0$. Therefore, since $(\partial\sigma_P^2)/(\partial\beta)$ is still positive when $\beta = 0$, we can conclude that the value of β which minimises the value of σ_P^2 should be negative. □

Appendix 3: Classical Base Stock Policy, $\beta = 0$

By using the recursive characteristic of $D(t)$ given in (2), the total demand over the L_1 time periods can be written as

$$\begin{aligned}\sum_{i=1}^{L_1} D(t+i) &= \mu \cdot L_1 + \beta\frac{1-\beta^{L_1}}{1-\beta}\,(D(t)-\mu) + \\ &\quad \varepsilon(t+L_1) + (1+\beta)\varepsilon(t+L_1-1) + \cdots + \\ &\quad (1+\beta+\cdots+\beta^{L_1-1})\varepsilon(t+1). \qquad (19)\end{aligned}$$

Taking the expected value of (19) yields the forecast model used by the retailer. When the retailer sets $\beta = 0$, the expected value of (19) becomes $\mu \cdot L_1$. Therefore, when $\beta = 0$, $S_1(t)$ is given by

$$S_1(t) = \mu \cdot L_1 + z_1 \cdot \sigma_{NS_1},$$

which is the time invariant base stock level used in the classical base stock policy. Using this method it can be shown that the ordering policy used by the manufacturer is also the classical base stock policy when $\beta = 0$. □

Appendix 4: The Case where ρ is not Positive

Table 4 shows the results of cases where the value of ρ is not positive. The values of other parameters used herein, such as cost parameters and lead-times, are the same as in Table 2. Note that when $\rho = 0$ and $\theta \neq 0$, the true market demand process becomes an MA(1) process. If $\rho = \theta = 0$, it becomes an i.i.d. white noise process. Even though an i.i.d. case, mis-specification can bring some benefits to the supply chain, since the value of B/A is less than unity (see Table 4). When $-0.9 \leq \rho \leq -0.3$, most of the time β^*_{total} does bring benefit to the supply chain, but in some settings, it does not. All in all, however, the benefit or the loss for the total supply chain cost due to the mis-specification is quite minor. Therefore, when ρ is non-positive, intentional exploitation of the mis-specification is not strongly recommended. Since true values of ρ and θ are not known, finding β^*_{total} may not be easy. Even if the exact value of β^*_{total} is obtainable, the benefit might be quite small, or there may be no benefit.

Table 4: Costs when ρ is non-positive

		No mis-spec. case				Mis-spec. case					
ρ	θ	$C^*_{NS_1}$	$C^*_{NS_2}$	C^*_P	TC (A)	β^*_{total}	$C^*_{NS_1}$	$C^*_{NS_2}$	C^*_P	TC (B)	B/A
0	−0.1	105	53	48	206	−0.637	107	50	33	190	0.924
	0.0	97	49	43	189	−0.663	99	46	31	176	0.932
	0.1	89	44	39	172	−0.683	91	42	30	163	0.945
−0.3	−0.1	85	41	37	163	−0.696	86	39	29	154	0.947
	0.0	79	37	33	149	−0.708	80	36	28	144	0.960
	0.1	73	34	30	137	−0.717	74	33	27	134	0.979
−0.5	−0.1	76	36	32	144	−0.719	77	34	27	138	0.962
	0.0	71	32	29	132	−0.726	71	32	26	129	0.979
	0.1	66	29	26	121	−0.731	67	29	26	122	1.003
−0.7	−0.1	71	31	29	131	−0.744	71	30	26	127	0.976
	0.0	67	28	26	121	−0.749	67	28	26	121	0.997
	0.1	64	25	24	113	−0.752	64	26	26	116	1.026
−0.9	−0.1	71	28	42	141	−0.789	73	27	29	129	0.910
	0.0	70	26	43	139	−0.792	72	25	30	127	0.912
	0.1	68	24	45	137	−0.794	71	23	31	125	0.914

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