# Does the DSGE Model Fit the Chinese Economy? A Bayesian and Indirect Inference Approach

by

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### Abstract

This thesis makes three main contributions to the literature on Dynamic Stochastic General Equilibrium (DSGE) models in Macroeconomics. As no previous studies have studied the Chinese economy from the perspective of DSGE, the first contribution of this thesis is estimating a DSGE model for China through a Bayesian approach using the Chinese quarterly post-economic reform data representing the main macro-economic time series 1978.Q1-2007.Q4. Second, this thesis adopts a new method of evaluating macro-economic models in its evaluation of the estimated DSGE model for China. Rather than the classical methods used to evaluate a macro-economic model such as the Maximum Likelihood method, the method of Indirect Inference is used to test the DSGE model. This method differs from other methods in its adoption of a VAR as the auxiliary model that mimics reality. A hybrid model is adopted to improve the ability of the DSGE model to replicate real world results and compared to the original New Keynesian version of the DSGE model developed by Smets and Wouters. Third, considering the restrictions that the prior distribution imposed on the estimated parameters of the model in the Bayesian estimation, the estimation method of Indirect Inference is used in the last chapter of this thesis and compared with the Bayesian estimation.

The results of the Bayesian estimation are in agreement with most of the existing literature on DSGE models. However, the results of Indirect Inference testing suggest that the adopted DSGE model does not closely resemble the real data, with a Hybrid model with 50% weight on the NK part performing significantly better. Indirect Inference estimation produces the same results and provides a better estimation of the model.

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### **General Introduction**

#### **Previous studies on DSGE models**

Chapter 1 estimates a dynamic stochastic general equilibrium model for the Chinese economy through a Bayesian likelihood approach using seven macro-economic time series. This project will involve two academic fields, the study of DSGE models and Bayesian econometrics.

The past decade has seen an increase in the popularity of a new generation of small-scaled monetary micro-founded business cycle models with sticky prices and wages in monetary policy analysis, which are usually named New Keynesian or New Neoclassical Synthesis (NNS) models. Smets and Wouters (2003) presented and estimated a dynamic stochastic general equilibrium model for the Euro area and then modelled the US economy (2007) in the same way as they modelled the EU economy. This study follows the work of Smets and Wouters and adopts extended versions of these models that are largely based on Christiano, Eichenbaum and Evans (2005). The model features a number of frictions that are believed to be necessary to capture the empirical persistence in the Chinese macro-economic data. Many of these frictions are becoming quite standard in the DSGE literature. The model exhibits both sticky nominal prices and wages that adjust in a Calvo mechanism following Kollmann (1997) and Erceg, Henderson and Levin (2000). This allows for backward inflation indexation, but the introduction of partial indexation of the prices and wages, which cannot be re-optimised, will result in a more general dynamic inflation and wage specification that will also depend on past inflation. Following Greenwood, Hercowitz and Huffmann (1988) and King and Rebelo (2000), this model contains a variable capital utilisation rate, which tends to smooth the adjustment of the rental rate of capital in response to changes in output. The cost of adjusting the utilisation rate is expressed in terms of consumed goods as explained in Christiano, Eichenbaum and Evans (CEE, 2001). Rather than following the common approach of expressing the cost of adjusting the capital stock as a function of the level of investment, we follow CEE (2001) by modelling the cost of adjusting the capital stock as a function of the change in investment. As expressed in Fuhrer (2000) and McCallum and Nelson (1999), external habit formation in consumption is used to introduce the necessary empirical persistence in the consumption process. While the models used by

Smets and Wouters (2003) for the EU and the US (2007) have many elements in common with that used in CEE (2001), the main differences between these analyses are the number of structural shocks that have been introduced into the models and the methodology used to estimate the DSGE models.

A considerable number of structural shocks are introduced in our model. The stochastic dynamics are driven by seven orthogonal structural shocks. In addition to total factor productivity shocks, two shocks that affect the intertemporal margin (risk premium shocks and investment-specific technology shocks) are included in the model, along with two shocks that affect the intratemporal margin (wage and price mark-up shocks) and two policy shocks (government spending and monetary policy shocks). The productivity shock is classified as a "supply" shock, while the government spending shock is a "demand" shock, and shocks to the mark-ups in the goods and labour markets and the shock to the required risk premium on capital are "cost-push" shocks. The model used here can be distinguished from the model used in Smets and Wouters (2003) in three ways. First, the number of structural shocks in the estimation has been reduced to seven. For example, we exclude the labour supply shock and time-varying inflation target. Second, the model features a deterministic growth rate that is driven by labour-augmenting technological progress. Third, in the intermediate goods and labour markets, the Dixit-Stiglitz aggregator is replaced by a more general aggregator developed by Kimball (1995). This new aggregator implies that the demand elasticity of differentiated goods and labour depends on their relative prices. We are able to estimate a more reasonable degree of price and wage stickiness due to the introduction of this real rigidity.

We are very interested in whether the NNS models can explain the main features of the Chinese macro-economic data, as NNS models have become the standard workhorses for monetary policy analysis. The introduction of a large number of frictions in these models raises the question of whether each of those frictions are really necessary to describe the seven main macro-economic data sources. CEE (2005) showed that allowing for nominal wage rigidity eliminates the need for additional price rigidity to capture the impulse responses to a monetary policy shock. The single-equation results presented by Gali and Gertler (1999) showed that price and wage stickiness are equally important, while on the

other hand, indexation was found to be relatively unimportant in both the goods and labour markets. The real frictions help to reduce the prediction errors of the NNS model. It is very important to determine the main driving forces for the main macro-economic variables in the Chinese economy. Introducing and estimating the set of orthogonal structural shocks provides a way to examine the relative contributions of the various shocks to the empirical dynamics of the macro-economic time series.

#### The Bayesian approach to estimating the DSGE model

The parameters of the model and the stochastic processes governing the structural shocks are estimated using seven key macro-economic time series in China: real GDP, consumption, investment, inflation, real wage, employment and the nominal short-term interest rate. We follow the Bayesian estimation techniques developed recently by Geweke (1999), Schorfheide (2002) and An and Schorfheide (2007) to estimate the model by minimising the posterior distribution of the model parameters based on the linearised state-space representation of the DSGE model.

In contrast to the various ways of estimating or calibrating the parameters of DSGE models, Geweke (1999) distinguished between weak and strong econometric interpretations. The original RBC model developed by Kydland and Prescott (1982) suggested that the model economy is intended to "mimic the world along a carefully specified set of dimensions," close in spirit to the weak interpretation. According to this interpretation, the aim of choosing the parameters of a DSGE model is to make the selected theoretical moments match as closely as possible to those observed in the data. Minimising some distance function between the theoretical and empirical moments of interest is a way to achieve this aim. A significant advantage of this approach is that moment estimators are often more robust than full information estimators. Using these estimation methods, researchers are able to focus on the characteristics in the data for which the DSGE model, which is necessarily an abstraction of reality, is most relevant. On the other hand, providing a full characterisation of the observed data series distinguishes the strong econometric interpretation from the weak econometric interpretation. A number of authors, for example Sargent (1989), have estimated the structural parameters of DSGE models through classical maximum likelihood methods.

Comparing the maximum likelihood and Bayesian estimation of DSGE models, we find that most works are similar except for the last few steps. As an alternative within the strong econometric interpretation, the Bayesian approach can be followed by combining the likelihood function with prior distributions of the parameters of the model to form the posterior density function. Then this posterior can be optimised with respect to the model parameters either directly or through Monte-Carlo Markov-Chain sampling methods. Many researchers are attracted by the strong econometric interpretation because it can provide a full characterisation of the data generating process and allows for proper specification testing and forecasting when successful. In our study, we adopt the strong econometric interpretation of the DSGE model.

Bayesian techniques are used for two basic reasons, as described in recent studies by Geweke (1998), Fernandez-Villaverde and Rubio-Ramirez (2001), Schorfheide (2002), An and Schorfheide (2007) and Landon-Lane (2000). The use of prior information that comes either from micro-econometric studies or previous macro-econometric studies can be formalised in this approach, which therefore explicitly links this study with the previous calibration-based literature. Additionally, the Bayesian approach can evaluate fundamentally misspecified models on the basis of the marginal likelihood of the model or the Bayes factor, although we do not apply this evaluation method here. One example is provided by Geweke (1998), in which the marginal likelihood of a model is directly linked to the predictive density function. A natural criterion that can be easily accepted for validating models for forecasting and policy analysis is prediction performance.

Our DSGE model is estimated using seven key macro-economic time series of data from China from 1978:1-2007:4. The estimation procedure produces a set of estimates for the structural parameters of the New Keynesian DSGE model that seems to be plausible. We find that there is a considerable degree of price stickiness in China, which is in line with the results of CEE (2001). This is an important feature that accounts for the empirical persistence of inflation in spite of the presence of sticky wages and variable capacity utilisation. The effects of various structural shocks on the Chinese economy are analysed. From a general point of view, those effects are quantitatively in line with the existing evidence. In particular, a temporary monetary policy shock is accompanied by a temporary rise in nominal and real interest rates and a decrease in both output and inflation as shown by Peersman and Smets (2000). A positive productivity shock is followed by a gradual increase in output, consumption, investment and real wages but a decrease in employment as expressed by Gali (1999) for the US economy. The importance of each shock to the development of the main macro-economic variables is explained. The results indicate that the price and wage mark-up shocks explain the most variation in inflation and interest rates, that the risk premium shock accounts for a large proportion of the development of output, and that other shocks with the exception of the price mark-up shock also contribute to output to some extent.

#### **Indirect Inference as a method of testing the DSGE model**

Smets and Wouters (2003 and 2007) presented a method of evaluating the estimated DSGE model called the marginal likelihood criterion. This criterion captures the out-of-sample prediction performance and therefore can be used to test the NNS model against standard and Bayesian Vector Autoregression (BVAR) models. As a result, they found that the fit provided by the NNS model is comparable to that of BVAR models. Because this is a weak method of testing DSGE models, we decided to apply another more powerful method to evaluate the DSGE model estimated on Chinese macro-economic data, the Indirect Inference testing method.

A key feature of the New Keynesian model is that it has rigid prices and wages, although the extent of nominal rigidity is a major area of disagreement between economists. Therefore, to mimic reality to the greatest possible extent, we consider the possibility that the economy consists of a mixture of both New Keynesian and New Classical features, with the result that some parts of the economy display nominal rigidity and other parts do not.

A long-standing problem has been the ability to test a calibrated or even partially Bayesianestimated DSGE model such as the model of Smets and Wouters. Particular features of data simulated from the calibrated or estimated models correspond with the real data in early DSGE models. The current study attempts to formalise this testing approach based on the methods of indirect inference. The methods of indirect inference exploit the fact that the solution of a log-linearised DSGE model can be represented by a restricted Vector-Autoregressive-Moving-Average (VARMA) model in levels, or when there are permanent shocks in first differences; and all of these can be closely represented by a Vector Autoregression (VAR). In principal, the DSGE model is tested by comparing unrestricted VAR estimates derived using data simulated from the DSGE model with unrestricted VAR estimates obtained from actual data. In some cases, we can only compare functions of the unrestricted VAR estimates derived from simulated data such as the value of the log likelihood function or the impulse response function with the unrestricted VAR estimates from real data. When carrying out the test, we apply a Wald test on the VAR estimates. The testing procedure involves several steps: first, choose an auxiliary model (usually the auxiliary model is a VAR model of order one or higher, i.e., VAR (1), VAR (2), and VAR (3), etc.) and estimate it on the actual data to produce benchmark descriptors of reality; second, assign initial values of structural parameters to be estimated and use these to generate a number of pseudo samples of simulated data with the theoretical model; third, estimate the selected auxiliary model on the simulated data to produce the joint distribution of the chosen descriptors and the mean of this distribution; finally, calculate the Wald statistic and the square of the "Mahalanobis distance" to formally measure the distance between the data descriptors and the means obtained by estimating the auxiliary model on simulated data. If the DSGE model is correctly specified then the simulated data and the VAR estimates based on these data will be close to the real data.

The significant advantage of this indirect inference testing procedure over the classical likelihood ratio test is that there is no need to specify a different DSGE model as the alternative hypothesis because the unrestricted VAR model based on actual data will automatically generate an alternative hypothesis suitable for testing of the specification of the model. The only requirement in this procedure is that the DSGE model generates an identified VAR. Instead of using the asymptotic distribution of the test statistic, in our case, a Wald statistic, an empirical estimate of its small sample distribution obtained by bootstrap methods, is used.

As a result, we find that for the whole post economic reform period, the data are matched most closely by a Hybrid model in which half of the economy enjoys price and wage flexibility but the other half of the economy is subject to nominal contracts, whereas the NK model is seriously at odds with the real data.

The Indirect Inference testing of the DSGE model conducted here makes use of a large and rapidly expanding body of literature, for example, Minford et al. (2009), Theodoridis (2006) and Le, Minford, Meenagh and Wickens (2011). Some issues stand out in this literature, such as the ability to measure the closeness of DSGE models to the data (see Watson (1993), Canova (1994, 1995, 2005), Del Negro and Schorfheide (2004, 2006), Corradi and Swanson (2007) and Del Negro et al. (2007a)) and also how well the simulations of the model perform when compared with various descriptions of the data such as impulse response coefficients, moments and cross-moments. The exercise here makes two contributions: first, it provides a formal statistical basis for comparing simulations of a previously estimated DSGE model with key features of the real data; second, detailed information on which features of the data the model is able or unable to capture are provided, supplementing currently available closeness measures.

#### Indirect Inference as a method of estimating the DSGE model

Indirect inference testing on structural models can be distinguished from indirect inference estimation of structural models. The method of indirect inference was originally designed for structural model estimation before it was recently developed for model evaluation. Indirect Inference Estimation has been widely applied for a long time; see Smith (1993), Gregory and Smith (1991, 1993), Gourieroux et al. (1993), Gourieroux and Monfort (1995) and Canova (2005). The parameters of the structural model are chosen in the estimation so that simulating the model generates the estimates of the auxiliary model, which will be similar to the estimates obtained from the actual data. We know that when indirect inference is used for model evaluation, the parameters of the structural model are taken as givens. However, fixing the model parameters is a very strong assumption to make when testing and comparing DSGE models. The reason is that the parameter values of the model could in principle be calibrated or estimated anywhere within a range permitted by the theoretical structure of the model; therefore, the rejection of a model with one set of assumed parameters does not imply its rejection with another set. Researchers have to be careful of whether their models are

saved by a set of "good" structural parameters, so the modes' parameters should not always be assumed to be fixed at some particular values. Generally speaking, the models should be fully estimated before they are tested and evaluated against each other. The procedure of Indirect Inference Estimation can be summarised as the following steps:

- 1. Choose an auxiliary model and estimate the model on the real data to obtain the benchmark estimates;
- 2. Assign initial values to the structural parameters that have to be estimated and use them to generate a number of pseudo samples of simulated data with the theoretical model;
- 3. Estimate the chosen auxiliary model on the simulated data obtained from step 2 to obtain the joint distribution of the chosen estimates (in step 1) and their mean;
- 4. Calculate the Wald statistics and the square of the "Mahalanobis distance" to measure the distance between the benchmark estimates obtained in step 1 and the mean of the estimates obtained in step 3.
- 5. Repeat the steps 2, 3 and 4 above until the Wald statistic is minimised.

It is clear that the procedure of Indirect Inference Estimation is rather similar to that of Indirect Inference Testing except for the final step. The method of indirect inference is used differently here as the aim is no longer to measure the distance between the theoretical model and the data but rather to find a set of parameters that minimise this distance when the theoretical model is taken as true. The common feature of test and estimation is to calculate the Wald statistic based on the estimates of the auxiliary model.

After re-estimating the DSGE model on the same Chinese data through the method of indirect inference, we find that there is a set of best fitting parameters that significantly improve the model performance, as shown by a full Wald value in the non-rejection area. The results obtained with this method are significantly different when compared with the Bayesian estimated values. However, the Hybrid model remains superior to the NK model as proven in the Indirect Inference testing exercise in Chapter 2, so that even under estimation, the results suggest that the Hybrid model can replicate the Chinese economy in a more sensible way.

# Chapter 1

# Estimating the DSGE model for China through a Bayesian approach

## **1.1 Introduction**

The Chinese economy has achieved huge growth during the past decades (since the economic reforms in 1978) and continues to grow rapidly. The Chinese economy is now the largest in the world. The rapid development of this "giant" has led many researchers to use existing and newly developed techniques to explain the world's most significant economic development in recent years. This chapter attempts to model the Chinese economy in a popular Dynamic Stochastic General Equilibrium pattern and estimates the model using Bayesian econometric methods. Smets and Wouters have created DSGE models for the EU area (2003) and the US economy (2007). The Chinese economy is quite similar to the EU and US economies as it is also a large closed continental economy, suggesting that we can use the same methods for the Chinese economy that Smets and Wouters used for the EU and US economies.

The DSGE model contains many frictions that affect both nominal and real decisions made by households and firms. The model of this paper is based on Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003). Smets and Wouters (2005) extend the model to be consistent with a balanced steady state growth path driven by deterministic labouraugmenting technological progress. In the extended model, households maximise a nonseparable utility function with goods and labour effort over an infinite life horizon. A timevarying external habitual variable consumption appears in the utility function. The existence of a union leads to some monopoly power over wages (Calvo, 1983) and so labour is differentiated by the union. Households face capital adjustment costs when they decide how much capital to accumulate and rent capital services to firms. The utilisation of capital stock can be adjusted at increasing cost due to changes in the rental price of capital. Firms input labour and capital, produce differentiated goods and set prices according to the Calvo model.

There are a few amendments in the model of Smets and Wouters (2005). First, to match the number of observables that are used in estimation the number of structural shocks is reduced to seven. Second, an aggregator that allows for a time-varying demand elasticity that depends on the relative price as in Kimball (1995) has taken the place of the Dixit-Stiglitz aggregator in both goods and labour markets. Eichenbaum and Fischer (2007) proved that the

introduction of this real rigidity allows for the estimation of a more reasonable degree of price and wage stickiness.

### **1.2 Model description**

Dynamic stochastic general equilibrium (DSGE) models are micro-founded optimisationbased models that have become very popular in macroeconomics in recent decades. However, for a long time, the quantitative evaluation of DSGE models was conducted without formal statistical methods. DSGE models provide a complete representation of the data using multivariate stochastic processes, but simple models impose very strong restrictions on actual time series and are in many cases rejected against less restrictive specifications such as vector autoregressions (VAR). Model misspecifications are an argument in favour of informal calibration approaches.

A prototypical DSGE model should consist of a firm producing final goods, a continuum of firms producing intermediate goods, households, and a monetary as well as fiscal authority. The final good  $Y_t$  is a composite made up of a continuum of intermediate goods  $Y_t(i)$ . The final goods producers buy the intermediate goods on the market and package  $Y_t$  for resale to consumers, investors and the government in a perfectly competitive market. The decision problem for the final goods producers is:

 $\max_{Y_{t},Y_{t}(i)} P_{t}Y_{t} - \int_{0}^{1} P_{t}(i)Y_{t}(i)di$  $s.t.\left[\int_{0}^{1} G\left(\frac{Y_{t}(i)}{Y_{t}};\lambda_{p,t}\right)di\right] = 1 \left(\mu_{f,t}\right)$ 

where  $P_t$  and  $P_t(i)$  are the prices of the final good and intermediate goods, respectively, and *G* is a strictly concave and increasing function characterised by G(1) = 1. There is also an exogenous process  $\varepsilon_t^p$  that reflects shocks to the aggregator function that result in changes in

the elasticity of demand and therefore in the markup. We will constrain  $\mathcal{E}_t^p \in (0, \infty)$ , and this process follows the exogenous ARMA process:

$$\ln \varepsilon_t^p = (1 - \rho_p) \ln \varepsilon^p + \rho_p \ln \varepsilon_{t-1}^p - \theta_p \eta_{t-1}^p + \eta_t^p, \ \eta_t^p \sim N(0, \sigma_p)$$

Expressing the first order conditions with respect to  $Y_t$  and  $Y_t(i)$  yields:

$$Y_t(i) = Y_t G^{-1} \left[ \frac{P_t(i)}{P_t} \int_0^1 G' \left( \frac{Y_t(i)}{Y_t} \right) \frac{Y_t(i)}{Y_t} di \right]$$

The assumptions made regarding G() imply that the demand for input  $Y_t(i)$  is decreasing with its relative price, while the elasticity of demand is a positive function of the relative price (or a negative function of the relative output).

The intermediate goods producers use the following technology:

$$Y_t(i) = \varepsilon_t^a K_t^s(i)^\alpha \left[ \gamma^t L_t(i) \right]^{1-\alpha} - \gamma^t \Phi$$

where  $K_t^s(i)$  is capital services used in production,  $L_t(i)$  is aggregate labour input,  $\Phi$  is a fixed cost and  $\gamma^t$  is the labour-augmenting deterministic growth rate of the economy.  $\varepsilon_t^a$  represents the total factor productivity and follows the process:

$$\ln \varepsilon_t^a = (1 - \rho_z) \ln \varepsilon^a + \rho_z \ln \varepsilon_{t-1}^a + \eta_t^a, \ \eta_t^a \sim N(0, \sigma_a)$$

The profit for the intermediate producers is given by:

$$P_t(i)Y_t(i) - W_tL_t(i) - R_t^kK_t(i)$$

where  $W_t$  is the aggregate nominal wage rate and  $R_t^k$  is the rental rate on capital. We take the first order conditions with respect to capital and labour to obtain the cost minimisation conditions, and combining these with the first order conditions yields:

$$K_t^s = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k} L_t$$

Note that the capital-labour ratio is constant across firms. The marginal cost is the same for all firms and is given by:

$$MC_{t} = \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} W_{t}^{1-\alpha} R_{t}^{k\alpha} \gamma^{-(1-\alpha)t} (\varepsilon_{t}^{a})^{-1}$$

Under Calvo pricing with partial indexation, the optimal price set by the firm that is allowed to re-optimise results from the following optimisation problem:

$$\max_{\tilde{P}_{t}(i)} E_{t} \sum_{s=0}^{\infty} \xi_{p}^{s} \frac{\beta^{s} \Xi_{t+1} P_{t}}{\Xi_{t} P_{t+s}} \Big[ \tilde{P}_{t}(i) (\Pi_{l=1}^{s} \pi_{t+l-1}^{lp} \pi_{*}^{1-lp}) - MC_{t+s} \Big] Y_{t+s}(i)$$

$$s.t.Y_{t+s}(i) = Y_{t+s} G^{'-l} \left( \frac{P_{t}(i) X_{t}^{s}}{P_{t+s}} \tau_{t+s} \right)$$

where  $\tilde{P}_{t}(i)$  is the newly set price,  $\xi_{p}$  is the Calvo probability of being allowed to optimise price,  $\pi_{t}$  is inflation defined as  $\pi_{t} = P_{t}/P_{t-1}$ , and  $\left[\frac{\beta^{s}\Xi_{t+s}P_{t}}{\Xi_{t}P_{t+s}}\right]$  is the nominal discount factor for firms that equals the discount factor for the households that are firm owners.  $\tau_{t} = \int_{0}^{1} G^{t-1}\left(\frac{Y_{t}(i)}{Y_{t}}\right) \frac{Y_{t}(i)}{Y_{t}} di$  and  $X_{t,s} = \begin{cases} 1 \text{fors} = 0\\ (\prod_{l=1}^{s} \pi_{l+l-l}^{l-p} \pi_{s}^{1-lp}) \text{fors} = 1, \dots, \infty \end{cases}$ 

The first order condition is given by:

$$E_{t} \sum_{s=0}^{\infty} \xi_{p}^{s} \frac{\beta^{s} \Xi_{t+1} P_{t}}{\Xi_{t} P_{t+s}} Y_{t+s}(i) \left[ X_{t,s} \widetilde{P}_{t}(i) + \left( \widetilde{P}_{t}(i) X_{t,s} - MC_{t+s} \right) \frac{1}{G^{-1}(z_{t+s})} \frac{G'(x_{t+s})}{G''(x_{t+s})} \right] = 0$$
  
where  $x_{t} = G^{-1}(z_{t})$  and  $z_{t} = \frac{P_{t}(i)}{P_{t}} \tau_{t}$ .

The aggregate price index in this case is given by:

$$P_{t} = (1 - \xi_{p})P_{t}(i)G^{-1}\left[\frac{P_{t}(i)\tau_{t}}{P_{t}}\right] + \xi_{p}\pi_{t-1}^{lp}\pi_{*}^{1-lp}P_{t-1}G^{-1}\left[\frac{\pi_{t-1}^{lp}\pi_{*}^{1-lp}P_{t-1}\tau_{t}}{P_{t}}\right]$$

Each household chooses consumption  $C_t(j)$ , hours worked  $L_t(j)$ , bonds  $B_t(j)$ , investment  $I_t(j)$  and capital utilisation  $Z_t(j)$  to maximise the following objective function:

$$E_{t}\sum_{s=0}^{\infty}\beta^{s}\left[\frac{1}{1-\sigma_{c}}(C_{t+s}(j)-\lambda C_{t+s-1})^{1-\sigma_{c}}\right]\exp\left(\frac{\sigma_{c}-1}{1+\sigma_{l}}L_{t+s}(j)^{1+\sigma_{l}}\right)$$

subject to the budget constraint:

$$C_{t+s}(j) + I_{t+s}(j) + \frac{B_{t+s}(j)}{\varepsilon_{t}^{b}R_{t+s}P_{t+s}} - T_{t+s}$$

$$\leq \frac{B_{t+s}(j)}{P_{t+s}} + \frac{W_{t+s}^{h}(j)L_{t+s}(j)}{P_{t+s}} + \frac{R_{t+s}^{k}Z_{t+s}(j)K_{t+s-1(j)}}{P_{t+s}} - a(Z_{t+s}(j))K_{t+s-1(j)} + \frac{Div_{t+s}}{P_{t+s}}$$

and the capital accumulation equation:

$$K_{t}(j) = (1 - \delta)_{t-1}(j) + \varepsilon_{t}^{i} \left[ 1 - S \left( \frac{I_{t}(j)}{I_{t-1}(j)} \right) \right] I_{t}(j)$$

External habit formation is captured by the parameter h. The one-period bond is expressed on a discount basis.  $\varepsilon_t^b$  is an exogenous premium in the return to bonds, which might reflect inefficiencies in the financial sector leading to some premium on the deposit rate versus the risk free rate set by the central bank or could represent a risk premium that households require to hold the one period bond.  $\varepsilon_t^b$  follows the stochastic process:

$$\ln \varepsilon_t^b = \rho_b \ln \varepsilon_{t-1}^b + \eta_t^b, \ \eta_t^b \sim N(0, \sigma_b)$$

 $\delta$  is the depreciation rate,  $S(\cdot)$  is the adjustment cost function, with  $S(\gamma) = 0$ ,  $S'(\gamma) = 0$ , and  $S''(\cdot) > 0$ ,  $\varepsilon_t^i$  is a stochastic shock to the price of investment relative to consumption goods and follows an exogenous process:

$$\ln \varepsilon_t^i = \rho_i \ln \varepsilon_{t-1}^i + \eta_t^i, \ \eta_t^i \sim N(0, \sigma_i)$$

In the budget constraint equation,  $T_{t+s}$  represents the lump sum taxes or subsidies and  $Div_t$  captures the dividends distributed by the labour unions.

Households choose the utilisation rate of capital. The effective amount of capital that households can rent to firms is:

$$K_t^s(j) = Z_t(j)K_{t-1}(j)$$

The income from capital renting services is:

$$R_t^k Z_t(j) K_{t-1}(j)$$

and the capital utilisation is:

$$P_t a(Z_t(j)) K_{t-1}(j)$$

At equilibrium, every household will make the same choices for consumption, hours worked, bonds, investment and capital utilisation. The first order conditions for consumption, hours worked, bond holdings, investment, capital and capital utilisation can be written as:

$$\begin{split} \partial C_{t} &: \ \Xi_{t} = \exp\left(\frac{\sigma_{c}-1}{1+\sigma_{l}}L_{t}(j)^{1+\sigma_{l}}\right) (C_{t}-\lambda C_{t-1})^{-\sigma_{c}} \\ \partial L_{t} &: \left[\frac{1}{1-\sigma_{c}}(C_{t}-hC_{t-1})^{1-\sigma_{c}}\right] \exp\left(\frac{\sigma_{c}-1}{1+\sigma_{l}}L_{t}^{1+\sigma_{l}}\right) (\sigma_{c}-1)L_{t}^{\sigma_{l}} = -\Xi_{t}\frac{W_{t}^{h}}{P_{t}} \\ \partial B_{t} &: \ \Xi_{t} = \beta\varepsilon_{t}^{b}R_{t}E_{t}\left[\frac{\Xi_{t+1}}{\pi_{t+1}}\right] \\ \partial I_{t} &: \ \Xi_{t} = \Xi_{t}^{k}\varepsilon_{t}^{i}\left(1-S(\frac{I_{t}}{I_{t-1}})-S'(\frac{I_{t}}{I_{t-1}})\frac{I_{t}}{I_{t-1}}\right) + \beta E_{t}\left[\Xi_{t+1}^{k}\varepsilon_{t+1}^{i}S'(\frac{I_{t+1}}{I_{t}})(\frac{I_{t+1}}{I_{t}})^{2}\right] \\ \partial \overline{K}_{t} &: \ \Xi_{t}^{k} = \beta E_{t}\left[\Xi_{t+1}\left(\frac{R_{t+1}^{k}}{P_{t+1}}Z_{t+1}-a(Z_{t+1})\right) + \Xi_{t+1}^{k}(1-\delta)\right] \\ \partial u_{t} &: \ \frac{R_{t}^{k}}{P_{t}} = a'(Z_{t}) \end{split}$$

where  $\Xi_t$  and  $\Xi_t^k$  are the Lagrange multipliers associated with the budget and capital accumulation constraints, respectively. Tobin's is  $Q_t = \Xi_t^k / \Xi_t$  and equals one the in the absence of adjustment costs.

In the intermediate labour union sector, households supply their homogenous labour to an intermediate labour union that differentiates the labour services and sets wages subject to the Calvo wage schema with intermediate labour packers. In the labour market, labour used by the intermediate goods producers  $L_t$  is a composite:

$$L_{t} = \left[\int_{0}^{1} L_{t}\left(l\right)^{\frac{1}{1+\lambda_{w,t}}} dl\right]^{\lambda_{w,t}}$$

Labour packers buy the labour from the unions, package  $L_t$ , and resell it to the intermediate goods producers. Labour packers maximise profits in a perfectly competitive environment. Taking the first order conditions of the maximisation problem of the labour packers and rearranging yields:

$$L_{t}(l) = \left(\frac{W_{t}(l)}{W_{t}}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}}L_{t}$$

Combining this condition with the zero profit condition produces an expression for the wage cost for the intermediate goods producers:

$$W_{t} = \left[\int_{0}^{1} W_{t}(l)^{\frac{1}{\lambda_{w,t}}} dl\right]^{\lambda_{w,t}}$$

 $\lambda_{w,t}$  is assumed to follow the exogenous ARMA process:

$$\ln \lambda_{w,t} = (1 - \rho_w) \ln \lambda_w + \rho_w \ln \lambda_{w,t-1} - \theta_w \varepsilon_{w,t-1} + \varepsilon_{w,t}$$

Labour packers buy labour from the labour unions. The unions are an intermediary between households and labour packers. The labour unions allocate and differentiate the labour services provided by households and have sufficient market power to choose the wage subject to the labour demand equation. The households' budget constraints now contain the union dividends distributed to the households:

$$C_{t+s}(j) + I_{t+s}(j) + \frac{B_{t+s}(j)}{b_t^2 R_{t+s} P_{t+s}} + A_{t+s}(j) - T_{t+s}$$

$$\leq \frac{B_{t+s-1(j)}}{P_{t+s}} + \frac{W_{t+s}^h(j)L_{t+s}(j)}{P_{t+s}} + \frac{Div_{t+s}}{P_{t+s}} + \frac{R_{t+s}^k u_{t+s}(j)\overline{K}_{t+s-1}(j)}{P_{t+s}} - a(u_{t+s}(j))\overline{K}_{t+s-1}(j)$$

The household labour supply decision is expressed by the following first order condition and is the same for all households:

$$\frac{W_{t}^{h}}{P_{t}} = -\frac{\left[\frac{1}{1-\sigma_{c}}(C_{t}-hC_{t-1})^{1-\sigma_{c}}\right]\exp\left(\frac{\sigma_{c}-1}{1+v_{l}}L_{t}^{1+v_{l}}\right)(\sigma_{c}-1)L_{t}^{v_{l}}}{\Xi_{t}}$$

The real wage desired by the households here reflects the marginal rate of substitution between leisure and consumption. Labour unions take this rate of substitution as the cost of the labour services in their negotiations with the labour packers. The union is also subjected to the Calvo nominal rigidity. Unions can readjust wages with probability  $1-\zeta_w$  in each period. For those that cannot adjust wages,  $W_t(l)$  will increase at the deterministic growth rate  $\gamma$  and weighted average of the steady state inflation  $\pi_*$  and the last period's inflation  $(\pi_{t-1})$ . For those that can adjust, the objective is to choose a wage  $\tilde{W}_t(l)$  that maximises the wage income in all states where the union is bound to that wage in the future:

$$\max_{\widetilde{W}_{t}(l)} E_{t} \sum_{s=0}^{\infty} \zeta_{w}^{s} \left[ \frac{\beta^{s} \Xi_{t+s} P_{t}}{\Xi_{t} P_{t+s}} \right] \left[ W_{t+s}(l) - W_{t+s}^{h} \right] L_{t+s}(l)$$
  
and  $L_{t+s}(l) = \left( \frac{W_{t+s}(l)}{W_{t+s}} \right)^{-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}} L_{t+s}$   
with  $W_{t+s}(l) = \widetilde{W}_{t}(l) \left( \prod_{l=1}^{s} \gamma \pi_{t+l-1}^{l_{w}} \pi_{s}^{1-l_{w}} \right)$  for  $s = 1, \dots, \infty$ 

The first order condition with respect to the wage then becomes:

$$(\partial W_{t}) \quad 0 = E_{t} \sum_{s=0}^{\infty} \zeta_{w}^{s} \frac{\beta^{s} \Xi_{t+s} P_{t}}{\Xi_{t} P_{t+s}} \left[ (W_{t+s}(l) - W_{t+s}^{h}) \left( \frac{X_{t,s} \widetilde{W}_{t}(l)}{W_{t+s}} \right)^{-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}-1} \right] \\ \left( -\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}} \right) \left( \frac{X_{t,s}}{W_{t+s}} \right) L_{t+s} - X_{t,s} L_{t+s}(l) \right]$$

where

$$X_{t,s} = \begin{cases} 1 \text{ for } s = 0\\ (\prod_{l=1}^{s} \gamma \pi_{t+l-1}^{l_w} \pi_*^{1-l_w}) \text{ for } s = 1, \dots, \infty \end{cases}$$

Substituting for individual labour and multiplying by the optimal wage yields:

$$E_{t}\sum_{s=0}^{\infty}\zeta_{w}^{s}\frac{\beta^{s}\Xi_{t+s}P_{t}}{\Xi_{t}P_{t+s}}\left[(X_{t,s}\widetilde{W}_{t}(l)-W_{t+s}^{h})(-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}})L_{t+s}(l)-X_{t,s}\widetilde{W}_{t}(l)L_{t+s}(l)\right]=0$$

or

$$E_{t}\sum_{s=0}^{\infty}\zeta_{w}^{s}\frac{\beta^{s}\Xi_{t+s}P_{t}}{\Xi_{t}P_{t+s}}L_{t+s}(l)\frac{1}{\lambda_{w,t+s}}\left[(1+\lambda_{w,t+s})W_{t+s}^{h}-X_{t+s}\widetilde{W}_{t}(l)\right]=0$$

The aggregate wage expression is:

$$W_{t} = \left[ (1 - \zeta_{w}) \widetilde{W}_{t}^{\frac{1}{\lambda_{w,t}}} + \zeta_{w} (\gamma \pi_{t-1}^{l_{w}} \pi_{*}^{1-l_{w}} W_{t-1})^{\frac{1}{\lambda_{w,t}}} \right]^{\lambda_{w,t}}$$

In this model, the government, which is the central bank, follows a nominal interest rate rule by adjusting its instrument in response to deviations of inflation and output from their respective target levels:

$$\frac{R_{t}}{R^{*}} = \left(\frac{R_{t-1}}{R^{*}}\right)^{\rho_{R}} \left[ \left(\frac{\pi_{t}}{\pi_{*}}\right)^{\psi_{1}} \left(\frac{Y_{t}}{Y_{t}^{*}}\right)^{\psi_{2}} \right]^{1-\rho_{R}} \left(\frac{Y_{t}/Y_{t-1}}{Y_{t}^{*}/Y_{t-1}^{*}}\right)^{\psi_{3}} r_{t}$$

where  $R^*$  is the gross nominal rate in the steady state and  $Y_t^*$  is the natural output. The parameter  $\rho_R$  determines the degree of interest rate smoothing. The monetary policy shock  $r_t$  is determined as follows:

$$\ln r_t = \rho_r \ln r_{t-1} + \varepsilon_{r,t}$$

The central bank supplies the money demanded by the household to support the desired interest rate.

The government budget constraint is:

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t}$$

where  $T_t$  represents nominal lump-sum taxes (or subsidies) that also appear in the household budget constraint. Government spending expressed relative to the steady state output path  $g_t = G_t / (Y\gamma^t)$  follows the process:

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \rho_{ga} \ln Z_t - \rho_{ga} \ln Z_{t-1} + \varepsilon_{g,t}, \ \varepsilon_{g,t} \sim \text{White noise}$$

We allow for government spending to respond to the productivity process:

$$\ln g_t - \rho_{ga} \ln Z_t = (1 - \rho_g) \ln g + \rho_g (\ln g_{t-1} - \rho_{ga} \ln Z_{t-1}) + \varepsilon_{g,t}, \ \varepsilon_{g,t} \sim \text{White noise}$$

In this model, the natural output level is defined as the output of the flexible price and wage economy. Here, the question is which shocks should be taken into account when calculating the natural output level, particularly, whether the markup shocks in prices and wages should be taken into account. If the markup shocks are not taken into account, there will be a trade-off problem between output-gap stabilisation and inflation stabilisation. Persistent markup shocks could result in persistent conflicts between the two objectives and therefore in persistent deviations of inflation from the inflation target.

The resource constraints in this model can be derived as follows.

To obtain the market clearing condition for the final goods market, we first integrate the household's budget constraint across households and combine it with the government budget constraint:

$$P_tC_t + P_tI_t + P_tG_t \le \prod_t + \int W_t(j)L_t(j)dj + R_t^k \int K_t(j)dj - P_ta(u_t) \int \overline{K}_{t-1}(j)dj$$

In the case of the labour unions:

$$P_{t}C_{t} + P_{t}I_{t} + P_{t}G_{t} \leq \Pi_{t} + \int W_{t}^{h}(j)L_{t}(j)dj + Div_{t} + R_{t}^{k}\int K_{t}(j)dj - P_{t}a(u_{t})\int \overline{K}_{t-1}(j)dj$$

then we realise that:

$$\Pi_t = \int \Pi_t(i) di = \int P_t(i) Y_t(i) di - W_t L_t - R_t^k K_t$$

where  $L_t = \int L(i)_t di$  is total labour supplied by the labour packers that is demanded by the firms and  $K_t = \int K_t(i) di = \int K_t(j) dj$ . Substituting the definition of  $\Pi_t$  into the household budget constraint and realising that by the labour and goods packers' zero profit conditions  $W_t L_t = \int W_t(j) L_t(j) dj = \int W_t^h(j) L_t(j) dj + Div_t$  and  $P_t Y_t = \int P(i)_t Y(i)_t di$ , we can obtain:

$$P_tC_t + P_tI_t + P_tG_t + P_ta(u_t)\overline{K}_{t-1} = P_tY_t$$

or

$$C_t + I_t + G_t + a(u_t)\overline{K}_{t-1} = Y_t$$

The data represent  $\overline{Y}_t$  and  $\overline{L}_t$  instead of  $Y_t$  and  $L_t$ , where

$$\overline{Y}_t = \int Y_t(i) di$$

Then we look at the output, aggregate inputs, labour and capital:

$$\overline{Y}_{t} = \int \left(\frac{P_{t}(i)}{P_{t}}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_{t} di$$
$$= Y_{t}(P_{t})^{\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} \overline{P}_{t}^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}}$$

where

$$\overline{P}_t\left[\int P_t(i)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}}di\right]^{-\frac{\lambda_{p,t}}{1+\lambda_{p,t}}}$$

and

$$\begin{split} \overline{L}_t &= \int L_t(j) dj \\ &= \int \left( \frac{W_t(j)}{W_t} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t dj \\ &= L_t(W_t)^{\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} \overline{W}_t^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} \end{split}$$

where

$$\overline{W}_{t} = \left[\int W_{t}(j)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} dj\right]^{-\frac{\lambda_{w,t}}{1+\lambda_{w,t}}}$$

As described by the above model, there are seven exogenous processes:

The first exogenous process is the technology process, which is also known as a productivity shock:

$$\ln Z_t = (1 - \rho_z) \ln Z + \rho_z \ln Z_{t-1} + \varepsilon_{z,t}$$

The investment relative price process is also known as the investment shock:

$$\ln \mu_t = (1 - \rho_\mu) \ln \mu + \rho_z \ln Z_{t-1} + \varepsilon_{\mu,t}$$

The intertemporal preference shifter is also known as the financial risk premium shock:

$$\ln b_t = (1 - \rho_b) \ln b + \rho_b \ln b_{t-1} + \varepsilon_{b,t}$$

The government spending shock:

$$\ln g_{t} = (1 - \rho_{g}) \ln g + \rho_{g} \ln g_{t-1} + \rho_{ga} \ln Z_{t} - \rho_{ga} \ln Z_{t-1} + \varepsilon$$
$$\ln g_{t} - \rho_{ga} \ln Z_{t} = (1 - \rho_{g}) \ln g + \rho_{g} (\ln g_{t-1} - \rho_{ga} \ln Z_{t-1}) + \varepsilon_{g,t}$$

Monetary policy shock:

$$\ln r_t = \rho_r \ln r_{t-1} + \varepsilon_{r,t}$$

Price mark-up shock:

$$\ln \lambda_{p,t} = (1 - \rho_p) \ln \lambda_p + \rho_p \ln \lambda_{p,t-1} - \theta_p \varepsilon_{p,t-1} + \varepsilon_{p,t}$$

Wage mark-up shock:

$$\ln \lambda_{w,t} = (1 - \rho_w) \ln \lambda_w + \rho_w \ln \lambda_{w,t-1} - \theta_w \varepsilon_{w,t-1} + \varepsilon_{w,t}$$

The innovations  $\varepsilon$  are distributed as i.i.d. Normal innovations:

$$\varepsilon_{i,t} \sim N(0,\sigma_i)$$

After solving the model, we can derive its steady state. The model can be detrended with the deterministic trend  $\gamma$ , and nominal variables can be replaced by their real counterparts. Combining the steady state expressions, we obtain:

The equation for labour and capital:

$$k_* = \frac{\alpha}{1-\alpha} \frac{w_*}{r_*^k} L_*$$

The aggregate price index equation:

$$\widetilde{p}_* = 1 = (1 + \lambda_p) \alpha^{-\alpha} (1 - \alpha)^{-(1 - \alpha)} w_*^{1 - \alpha} (r_*^k)^{\alpha} Z_*^{-1}$$

The first order conditions for the household problem:

$$R_* = \overline{\beta}^{-1} \pi_*$$

$$\Xi_* = b_*^1 \exp\left(\frac{\sigma_c - 1}{1 + \nu_l} L_*^{1 + \nu_l}\right) c_*^{-\sigma_c} \left(1 - (h/\gamma)\right)^{-\sigma_c}$$

$$r_*^k = \beta^{-1} \gamma^{\sigma_c} - (1 - \delta)$$

$$r_*^k = a'(u_*)$$

The equation for the effective capital that households rent to the firms:

$$\overline{k}_* = \lambda k_*$$

The capital accumulation equation for households:

$$i_* = (1 - (1 - \delta/\gamma)\overline{k}_*)$$

The equation for the aggregate wage:

$$w_* = \overline{w}_* = (1 + \lambda_w) w_*^h$$

where

$$w_*^h = -\frac{U'_{l,*}}{\Xi_*} = \left[ (c_* - (h/\gamma)c_*) \right] L_*^{\nu_l} = (1 - h/\gamma)c_* L_*^{\nu_l}$$

The aggregate production function:

$$y_* = Z_* k_*^{\alpha} L_*^{1-\alpha} - \Phi$$

The resource constraint:

$$\frac{c_*}{y_*} + \frac{i_*}{y_*} + g_* = 1$$

The ratio:

$$\frac{w_*^h L_*}{c_*} = \frac{1}{1 + \lambda_w} \frac{w_* L_*}{c_*}$$
$$= \frac{1}{1 + \lambda_w} \frac{1 - \alpha}{\alpha} \frac{r_*^k k_*}{c_*}$$
$$= \frac{1}{1 + \lambda_w} \frac{1 - \alpha}{\alpha} r_*^k \frac{k_*}{y_*} \frac{y_*}{c_*}$$

where

$$\frac{k_{*}}{y_{*}} = \frac{y_{*} + \Phi}{y_{*}} \left(\frac{L_{*}}{k_{*}}\right)^{\alpha - 1}$$
$$\frac{c_{*}}{y_{*}} = \left(1 - g - \frac{i_{*}}{y_{*}}\right)$$
$$= \left(1 - g - \frac{i_{*}}{k_{*}}\frac{k_{*}}{y_{*}}\right)$$

Then we log-linearise the model, collect the equations that are estimated and rescale the residuals.

The marginal cost equation:

$$\hat{m}c_t = (1 - \alpha)\hat{w}_t + \alpha\hat{r}_t^k - \hat{Z}_t$$

The equation of the first order condition for the Calvo wage with indexation:

$$(1+\overline{\beta}\mathcal{A}_p)\hat{\pi}_t = l_p\hat{\pi}_{t-1} + \overline{\beta}\mathcal{Y}E_t[\hat{\pi}_{t+1}] + A\frac{(1-\zeta_p\overline{\beta}\mathcal{Y})(1-\zeta_p)}{\zeta_p}(\hat{m}c_t) + \hat{\lambda}_{p,t}$$

The equation of wage setting option 2:

$$(1 + \overline{\beta}\gamma)\hat{w}_{t} - \hat{w}_{t-1} - \overline{\beta}\gamma E_{t}[\hat{w}_{t+1}]$$

$$= \frac{(1 - \zeta_{w}\overline{\beta}\gamma)(1 - \zeta_{w})}{\zeta_{w}} \left[\frac{1}{1 - h/\gamma}\hat{c}_{t} - \frac{h/\gamma}{1 - h/\gamma}\hat{c}_{t-1} + v_{t}\hat{L}_{t} - \hat{w}_{t}\right]$$

$$- (1 + \overline{\beta}\gamma I_{w})\hat{\pi}_{t} + l_{w}\hat{\pi}_{t-1} + \overline{\beta}\gamma E_{t}[\hat{\pi}_{t+1}] + \hat{\lambda}_{w,t}$$

The equation of the first order condition with respect to bonds in the household problem:

$$\hat{c}_{t} = \frac{1}{(1 + (h/\gamma))} E_{t} [\hat{c}_{t+1}] + \frac{(h/\gamma)}{(1 + (h/\gamma))} \hat{c}_{t-1}$$

$$- \frac{(1 - h/\gamma)}{\sigma_{c} (1 + (h/\gamma))} (\hat{b}_{t}^{2} + \hat{R}_{t} - E_{t} [\hat{\pi}_{t+1}])$$

$$- \frac{(\sigma_{c} - 1)(w_{*}^{h} L/c_{*})}{\sigma_{c} (1 + (h/\gamma))} (E_{t} [\hat{L}_{t+1}] - \hat{L}_{t})$$

The equation of capital and labour:

$$\hat{k}_t = \hat{w}_t - \hat{r}_t^k + \hat{L}_t$$

The equation of the first order condition with respect to capital utilisation for the household problem:

$$r_*^k \hat{r}_t^k = a'' \hat{u}_t$$

The capital accumulation equation for the household problem:

$$\hat{\bar{k}}_{t} = (1 - \frac{i_{*}}{\bar{k}_{*}})\hat{\bar{k}}_{t-1} + \frac{i_{*}}{\bar{k}_{*}}\mu_{t} + \frac{i_{*}}{\bar{k}_{*}}\hat{i}_{t}$$

The equation of the first order condition with respect to investment in the household problem:

$$\hat{i}_{t} = \frac{1}{(1+\overline{\beta}\gamma)}(\hat{i}_{t-1} + (\overline{\beta}\gamma)E_{t}[\hat{i}_{t+1}] + \frac{1}{\gamma^{2}S^{\prime\prime}}\hat{Q}_{t}^{k} + \frac{1}{\gamma^{2}S^{\prime\prime}}\hat{\mu}_{t}$$

The equation of the first order condition with respect to capital in the household problem:

$$\hat{Q}_{t}^{k} = -\hat{b}_{t}^{2} - (\hat{R}_{t} - E_{t}[\hat{\pi}_{t+1}]) + \frac{r_{*}^{k}}{r_{*}^{k} + (1-\delta)} E_{t}[r_{t+1}^{k}] + \frac{(1-\delta)}{r_{*}^{k} + (1-\delta)} E_{t}[Q_{t+1}^{k}]$$

The equation of the Taylor rule:

$$\begin{split} \hat{R}_{t} &= \rho_{R} \hat{R}_{t-1} + (1 - \rho_{R}) (\psi_{1} \hat{\pi}_{t} + \psi_{2} (\hat{y}_{t} - \hat{y}_{t}^{flex})) \\ &+ \psi_{3} (\hat{y}_{t} - \hat{y}_{t-1} - (\hat{y}_{t}^{flex} - \hat{y}_{t-1}^{flex})) + r_{t} \end{split}$$

The equation of the aggregate production function:

$$\hat{y}_{t} = \alpha \frac{y_{*} + \Phi}{y_{*}} \hat{k}_{t} + (1 - \alpha) \frac{y_{*} + \Phi}{y_{*}} \hat{L}_{t} + \frac{y_{*} + \Phi}{y_{*}} \hat{Z}_{t}$$

The equation of the resource constraint:

$$\hat{y}_{t} = \hat{g}_{t} + \frac{c_{*}}{y_{*}}\hat{c}_{t} + \frac{i_{*}}{y_{*}}\hat{i}_{t} + \frac{r_{*}^{k}k_{*}}{y_{*}}\hat{u}_{t}$$

### **1.3 Estimation Methodology**

Dynamic stochastic general equilibrium models are micro-founded optimisation based models that have become very popular in macroeconomics in recent decades. The quantitative evaluation of DSGE models was once conducted without formal statistical methods. A complete representation of the data using a multivariate stochastic process can be provided by DSGE models, while simple models impose very strong restrictions on actual time series and, in many cases, reject less restrictive specifications. Due to improvements in the structural models and the relaxation of many misspecified restrictions of the first generation of DSGE models, more traditional econometric techniques have become applicable. In recent years, DSGE models have grown beyond simple theoretical attractiveness to emerge as useful tools for forecasting and quantitative policy analysis in macroeconomics. These models are gaining credibility in policy-making institutions such as central banks.

There are classical methods of estimation for the DSGE models, including calibration, Kydland and Prescott (1982), the generalised method of moment, Christiano and Eichenbaum (1992), minimum distance estimation based on the discrepancy among VAR and DSGE model impulse response functions, Rotemberg and Woodford (1997) and Christiano, Eichenbaum and Evans (2005), and full-information likelihood-based estimation, Altug (1989), McGranttan (1994), Leeper and Sims (1994), and Kim (2000). This work applies a new method that has been widely used in recent years for empirical work with DSGE models, that is, Bayesian estimation and evaluation techniques. The Bayesian framework should be able to address several challenges including potential model misspecification, identification problems, and the computational burden. Due to the computational burden associated with the construction of the likelihood function for nonlinear solutions of the DSGE model, we estimated a linearised DSGE model. Random Walk Metropolis Algorithm and Importance Sampler are used to evaluate the posterior moments of DSGE model parameters and transformations. Model evaluation will be described in the next chapter of this thesis.

# **1.3.1** The basic rules of Bayesian econometrics

The fundamental rule of Bayesian econometrics is based on the fundamental probability rules that can be used to estimate the parameters of a model, compare different models, and obtain predictions. The most important rule is Bayes' rule:

$$p(A,B) = p(A|B)p(B) = p(B|A)p(A) \Longrightarrow p(B|A) = \frac{p(A|B)p(B)}{p(A)}$$

### **1.3.1.1** Application of the fundamental rule

Estimation

Different from Frequentist econometrics, Bayesian econometrics says that the parameter of the model  $\theta$  is also a random variable. Bayes' rule is applied by changing the corresponding components:

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)} \Longrightarrow \underbrace{p(y|\theta)}_{p(\theta|y)} \propto \underbrace{p(y|\theta)}_{likelihood function} prior density} prior density$$

The prior  $p(\theta)$  contains any non-data information available about  $\theta$ . This information could take any form. In particular, a *Conjugate Prior* is a distribution that, when combined with the likelihood, yields a posterior that falls in the same class of distributions. A *Natural Conjugate Prior* has the same functional form as the likelihood function, which means that the prior arises from a *fictitious data* set from the same process that generated the actual data.

The *likelihood*  $p(y|\theta)$  is the density of the data conditional on  $\theta$ .

The posterior  $p(\theta|y)$  summarises all that is known about  $\theta$  after seeing the data.

The expected value of the posterior  $\theta | y$  can be determined to provide a point estimate. Meanwhile, the *Highest Posterior Density Interval* (HPDI) provides the smallest *credible interval* [a,b]:  $p(a \le \theta_i \le b | y) = \int_a^b p(\theta_i | y) d\theta_i = 95\%$ , i.e., an interval estimate.

### **Posterior Model Probability**

The first method of comparing models is to compare the *posterior model probabilities*  $p(M_i|y)$  to assess the degree of support for  $M_i$  with different  $\theta^i$ , which can be compared by applying Bayes' rule:

$$p(M_i | y) = \frac{p(y | M_i) p(M_i)}{p(y)} \Longrightarrow \underbrace{p(M_i | y)}_{p(M_i | y)} \propto \underbrace{p(y | M_i)}_{marginal \ likelihood} \underbrace{p(y | M_i)}_{p(M_i)} \underbrace{p(M_i)}_{p(M_i)}$$

The marginal likelihood can be calculated by applying Bayes' rule in another way and making use of the fact that  $\int p(\theta^i | y, M_i) d\theta^i = 1$ :

$$p(\theta^{i} | y, M_{i}) = \frac{p(y | \theta^{i}, M_{i}) p(\theta | M_{i})}{p(y | M_{i})}$$
$$\Rightarrow p(y | M_{i}) \int p(\theta^{i} | y, M_{i}) d\theta^{i} = \int p(y | \theta^{i}, M_{i}) p(\theta^{i} | M_{i}) d\theta^{i}$$
$$\Rightarrow p(y | M_{i}) = \int p(y | \theta^{i}, M_{i}) p(\theta^{i} | M_{i}) d\theta^{i}$$

### Prediction

Some future unobserved data  $y^*$  can be predicted based on the observed data y using the *predictive density*  $p(y^*|y)$ . This can be rewritten using the basic rules for joint and marginal probability density functions as well as Bayes' rule:

$$p(y^*|y) = \int p(y^*, \theta|y) d\theta = \int p(y^*|y, \theta) p(\theta|y) d\theta$$

### **Prior Postulation**

Prior distributions play a very important role in the estimation of DSGE models. When the parameter space is at odds with observations that are not contained in the estimation sample, prior distributions may weigh down regions of the parameter space. The priors can strongly influence the shape of the posterior distribution by adding curvature to a likelihood function that is nearly flat in some dimensions of the parameter space. In practice, priors can be chosen based on some observations together with personal introspection, reflecting strongly held beliefs about the validity of economic theories.

Parameters ranging from 0 to 1 are usually assigned beta distributions. Parameters defined over the whole real axis are assigned with a normal distribution, while positive parameters are assigned with a gamma distribution. However, the standard deviation is usually assigned with an inverted gamma distribution.

#### **Posterior Simulation**

Most problems related to *parameter uncertainty* (cases where we do not know exactly what the parameters of the model are) in Bayesian econometrics can be summarised by the *function of interest*  $E[g(\theta)|y] = \int g(\theta)p(\theta|y)d\theta$ , which involves calculating the integral. However, few posterior densities for  $g(\theta)$  could be analytically obtained. The most common approach is to use *posterior simulation*, which generates random draws  $\theta^{(s)}$ . If the posterior  $p(\theta|y)$  has a common form such as a Normal or Chi Squared distribution, then *Monte Carlo Integration* can be directly implemented to estimate the mean and variance of the posterior density as well as the *Numerical Standard Error*. If, however, the posterior  $p(\theta|y)$  does not have a common form, then we cannot directly obtain random draws from the posterior. In this case, we can either use independent draws from a certain *source density* that approximates the posterior density well or else use dependent draws based on a *Markov chain*. Independent posterior simulations include *Acceptance Sampling* and *Importance Sampling*, while the dependent posterior simulation includes *Markov Chain Monte Carlo* (MCMC) *Algorithms* such as *Gibbs Sampler* and the *Metropolis-Hastings* (MH) *Algorithm*.

In the scenario of *model uncertainty* (cases where we do not know exactly which model is correct), we have to use *Bayesian Model Averaging* (BMA) to obtain the function of interest:

$$E[g(\phi)|y] = \sum_{r=1}^{R} E[g(\phi)|y, M_r] p(M_r|y).$$

There are *R* possible models  $M_r$  with a vector of parameters  $\theta_r$  termed prior  $p(\theta_r | M_r)$ , likelihood  $p(y | \theta_r, M_r)$  and posterior parameters  $p(\theta_r | y, M_r)$ . Thus, the *posterior model probability*  $p(M_r | y)$  can also be obtained.  $\phi$  is a vector of parameters that has a common interpretation in all models, so it will be the focus of interest in the empirical study. However, most cases feature myriad potential explanatory variables and thus numerous candidate models. For example, if *K* is the number of potential explanatory variables for the linear regression model, then there are  $2^K$  possible models defined by the inclusion or exclusion of each explanatory variable. To surmount this problem, the *Markov Chain Monte Carlo Model Composition* (MC<sup>3</sup>) *Algorithm* is developed.

#### Metropolis-Hastings (MH) Algorithm

The MH algorithm is an MCMC algorithm because the current draw always depends on the previous draw. These draws are made to mimic draws from the posterior by taking many draws from regions of the parameter space where the posterior probability is higher and few draws from regions where the posterior probability is low.

Step 0: Choose a starting value  $\theta^{(0)}$ .

Step 1: Take a candidate draw  $\theta^*$  from the *candidate generating density*  $q(\theta^{(s-1)};\theta)$ , which denotes that the density of  $\theta$  depends on  $\theta^{(s-1)}$ .

Step 2: Calculate the *acceptance probability*  $\alpha(\theta^{(s-1)}, \theta^*)$ , which gives the probability of accepting  $\theta^*$  as a draw from the posterior. The acceptance probability tends to move the chain away from areas of low posterior probability towards areas of higher probability.

$$\alpha(\theta^{(s-1)},\theta^*) = \min\left[\frac{p(\theta=\theta^*|y)q(\theta^*;\theta=\theta^{(s-1)})}{p(\theta=\theta^{(s-1)}|y)q(\theta^{(s-1)};\theta=\theta^*)},1\right]$$
Step 3: Set  $\theta^{(s)} = \theta^*$  with probability  $\alpha(\theta^{(s-1)}, \theta^*)$  and set  $\theta^{(s)} = \theta^{(s-1)}$  with probability  $1 - \alpha(\theta^{(s-1)}, \theta^*)$ .

Step 4: Repeat Steps 1, 2 and 3 S times.

Step 5: Take the average of the *S* draws  $g(\theta^{(1)}), \dots, g(\theta^{(S)})$ .

$$\hat{g}_{s} = \frac{1}{S} \sum_{s=1}^{S} g(\theta^{(s)}) \xrightarrow{p} E[g(\theta)|y]$$

To ensure that the effect of the starting value has vanished and draws converge to the posterior, the first  $S_0$  draws are discarded and the remaining  $S_1$  draws are retained as draws from the posterior for the estimate of  $E[g(\theta|y)]$ .

#### **1.3.1.2** Application in Estimating the DSGE Model

#### Estimation

Bayesian estimation of DSGE models has three features. First, it is system-based and fits the entire solved DSGE model, in contrast to GMM estimation, which is based on some equilibrium conditions such as Euler equations. Second, it is based on the likelihood function generated by the DSGE model rather than the discrepancy in the impulse response functions between DSGE and VAR. Third, prior distributions can be used to incorporate additional information into the estimation. Bayesian estimation can resolve both potential model misspecification problems and identification problems.

Two components are needed to apply Bayesian estimation to obtain the posterior distribution: the prior distribution and the likelihood function.

The posterior distribution of the parameters (or functions of the parameters) can be obtained by two posterior simulation approaches. Both approaches provide converging stable posterior distributions as the simulated sample size tends towards infinity.

- (i) Independent Draw Approach: draws from the posterior distribution are independent of each other, e.g., importance sampling and acceptance sampling.
- (ii) Dependent Draw Approach: draws from the posterior distribution are dependent on the previous draws (called "Markov Chain Monte Carlo (MCMC)"), e.g., the Metropolis-Hastings and Gibbs sampler methods.

A direct comparison of priors and posteriors can often provide valuable insights about the extent to which data provide information about the parameters of interest. If the posteriors do not differ much from the priors, then the data fail to provide any information. If, however, the posteriors are more concentrated around a certain point compared to the priors, then the data are informative.

It is helpful to begin the estimation from different points in the parameter space to increase the possibility that the global, rather than local, optimal estimators are found. This is important because the posterior might have multiple modals.

The priors are specified in such a way that all parameters are a priori independent. Then the posterior can be constructed as  $p(\theta|z) \propto p(z|\theta)p(\theta)$ . Posterior simulations have to be employed because the posterior does not have a standard density. Random Walk Chain MH Algorithm (RWM) and Importance Sampling (IS) are used to generate draws from the posterior distribution of  $\theta$ .

#### Random Walk Chain MH Algorithm (RWM)

Step 1: Use a numerical optimisation routine to maximise  $\ln p(y|\theta) + \ln p(\theta)$ . Denote the posterior mode by  $\tilde{\theta}$ .

Step 2: Let  $\tilde{\Sigma}$  be the inverse of the Hessian computed at the posterior mode  $\tilde{\theta}$ .

Step 3: Draw  $\theta^{(0)}$  from  $N(\tilde{\theta}, c_0^2 \tilde{\Sigma})$  or directly specify a starting value.

Step 4: For  $s = 1, ..., n_{sim}$ , draw  $\mathcal{G}$  from the proposed distribution  $N(\theta^{(s-1)}, c^2 \tilde{\Sigma})$ . The jump from  $\theta^{(s-1)}$  is accepted ( $\theta^{(s)} = \mathcal{G}$ ) with probability min $\{1, r(\theta^{(s-1)}, \mathcal{G}|Y)\}$  and rejected ( $\theta^{(s)} = \theta^{(s-1)}$ ) otherwise. The acceptance probability is expressed as:

$$r(\theta^{(s-1)}, \mathcal{G}|Y) = \frac{p(Y|\mathcal{G})p(\mathcal{G})}{p(\theta^{(s-1)}|Y)p(\theta^{(s-1)})}.$$

Step 5: Approximate the posterior expected value of a function by  $\frac{1}{n_{sim}} \sum_{s=1}^{n_{sim}} h(\theta^{(s-1)})$ .

#### **Model Comparison**

To evaluate the performance of the estimated DSGE model, we can either use an absolute model fit such as a posterior predictive model check or use the relative model comparison such as the posterior odds comparison.

#### Posterior Predictive Check (absolute model fit)

The predicted distribution of an observed sample  $Y^{rep}$  can be derived as:

$$p(Y^{rep}|Y) = \int p(Y^{rep}|\theta) p(\theta|Y) d\theta$$

A model is considered as discredited by the data if the observed sample is very unlikely according to the model. A quantitative model check can be based on the Bayesian p-value:

$$p-value = \int I(h(Y^{rep}) \ge h(Y)) p(Y^{rep}|Y) dY^{rep}$$

#### Posterior Odds Comparisons (relative model fit)

The PO ratios can also be used to compare two different models. This approach requires the marginal likelihood to be calculated, which can be done using the Gelfand-Dey method.

For example, to compare DSGE with the VAR model, we can compare the posterior distributions of interesting population characteristics such as impulse-response functions obtained from the DSGE model and from a VAR representation that does not impose restrictions. This benchmark model is termed the DSGE-VAR model:

$$y_t = \Phi_0 + \Phi_1 y_{t-1} + \ldots + \Phi_p y_{t-p} + u_t$$
, where  $E[u_t u_t'] = \Sigma$ .

We assume that:

$$\Sigma | \theta \sim IW(\lambda T \Sigma^*(\theta), \lambda T - k, n) \text{ and } \Phi | \Sigma, \theta \sim N(\Phi^*(\theta), \frac{1}{\lambda T} [\Sigma^{-1} \otimes \Gamma_{XX}(\theta)]^{-1}).$$

 $\lambda$  is a hyperparameter that scales the prior covariance matrix. The prior matrix is diffuse for small values of  $\lambda$  and shifts its mass closer to the DSGE model restrictions as  $\lambda$  goes towards infinity. At the limit, the VAR model is estimated subject to the restrictions. The prior is proper provided that  $\lambda T \ge n+k$ .

The joint posterior density of VAR and DSGE model parameters can be factorised as:

$$p_{\lambda}(\Phi, \Sigma, \theta | Y) = p_{\lambda}(\Phi, \Sigma | Y, \theta) p_{\lambda}(\theta | Y).$$

The posterior can be expressed as  $\overline{\Sigma}|Y, \theta \sim IW$  and  $\overline{\Phi}|Y, \Sigma, \theta \sim N$ . Hence, the larger the weight  $\lambda$  of the prior, the closer the posterior mean of the VAR parameters is to the values that respect the cross-equation restrictions of the DSGE model. On the other hand, if  $\lambda$  is equal to (n+k)/T, then the posterior mean is close to the OLS estimate.

# 1.3.2 Potential problems associated with Bayesian econometrics

Subsequently, we will elaborate on the challenges that any estimation and evaluation method is confronted with and discuss how a Bayesian approach can be used to cope with these challenges.

#### **1.3.2.1** Potential Model Misspecification

In a normal DSGE model, we would predict a vector of time series, such as a vector composed of output growth, inflation, and nominal interest rates, by a function of past values of the several variables in the vector. The resulting forecast error covariance matrix is non-singular. For this reason, any DSGE model that generates a rank-deficient covariance matrix for the vector is at odds with the data and suffers from an obvious form of misspecification. Likelihood estimation cannot be conducted in this case because of the singularity problem. To remove the singularity problem we can simply modify the model specification by adding so-called measurement errors. Smets and Wouters (2003) included additional structural shocks in the DSGE model.

A second source of misspecification is potentially invalid cross-coefficient restrictions on the time series representation of the vector of variables generated by the DSGE model. Compared to more densely parameterised reference models such as VARs, invalid restrictions in the DSGE models will be revealed. The primary goal of the study of DSGE models is to overcome discrepancies between the models and the real data; thus, it is important to find empirical methods to cope with potential model misspecifications. If the DSGE model is acknowledged as only providing an approximation to the law of motion of the vector of time series, then there is no such parameter vector that can deliver the true intertemporal substitution elasticity or price adjustment costs and the most precise impulse responses to a technology or monetary policy shock. In reality, each estimation method is associated with a particular level of discrepancy between the model and the real data.

Maximum likelihood estimation of the DSGE model is not chosen as the estimation method due to the "dilemma of absurd parameter estimates." The estimates of structural parameters generated with maximum likelihood procedures based on a set of observations of the vector of variables are often at odds with out-of-sample information. Due to the stylised nature and the resulting misspecifications of most DSGE models, extraneous information often leads the likelihood function to peak in regions of the parameter space that appear to be inconsistent with the out-of-sample information. This problem can be properly addressed within the framework of Bayesian estimation by re-weighting the likelihood function by a prior density. The prior contains the information that is not present in the estimation of the vector of variables. The prior density allows for the weighting of information about different parameters according to their reliability. A shift in the prior distribution to the posterior distribution indicates tension between different information sources. The marginal data density will be low if the likelihood function peaks at a value that is at odds with the out-ofsample information that has been used to construct the prior distribution. Hence, if there are odds in the posterior comparison, then the DSGE model will automatically be penalised for the inability to reconcile the two sources of information with a single set of parameters.

#### **1.3.2.2** Identification problem

A probability model that implies that different values of structural parameters lead to the same joint distribution for the vector of observable variables can lead to a problem of identification due to a lack of informative observations. Beyer and Farmer (2004) provided an algorithm to construct families of observationally equivalent linear rational expectations models and Canova and Sala (2005) compared the informativeness of different estimators with respect to key structural parameters in a variety of DSGE models. To illustrate the identification problem, we can specify two simple models:

Model 1:

$$y_t = \frac{1}{\alpha} E_t y_{t+1} + u_t, \quad u_t = \rho u_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim iid(0, (1 - \rho \alpha)^2).$$

In model 1, y is the observable endogenous variable and  $u_t$  is an unobserved shock process and is serially correlated.

Model 2:

$$y_{t} = \frac{1}{\alpha} E_{t} y_{t+1} + \phi y_{t-1} + u_{t}, \quad u_{t} = \varepsilon_{t}, \quad \varepsilon_{t} \sim iid \left( 0, \left[ \frac{\alpha + \sqrt{\alpha^{2} - 4\phi\alpha}}{2\alpha} \right]^{2} \right)$$

Model 2 contains the backward-looking term y, and the shocks are serially uncorrelated. In both model specifications, y always follows the law of motion:

$$y_t = \psi y_{t-1} + \eta_t, \ \eta_t \sim iid \ (0,1).$$

We can obtain the following relationship by imposing restrictions that guarantee uniqueness of a stable rational expectations solution:

In model 1:

 $\psi = \rho$ 

In model 2:

$$\psi = \frac{1}{2} \left( \alpha - \sqrt{\alpha^2 - 4\phi \alpha} \right)$$

In model 1, parameter  $\alpha$  is not identifiable, and in model 2, parameters  $\alpha$  and  $\phi$  are not separately identifiable. Models 1 and 2 are observationally equivalent. In this case, the likelihood functions of these two models have ridges that can cause serious problems for numerical optimisation. It is difficult to trace the parameter subspace along which the likelihood function is constant, making the calculation of the valid confidence set challenging. For the likelihood function in the Bayesian inference, even a weakly informative prior can introduce curvature into the posterior density surface that facilitates numerical maximisation and the use of MCMC methods. In manipulations based on Bayesian theorem, the prior distribution is flat. In large DSGE models, the mapping from the vector of structural parameters into the state-space representation that determines the joint probability distribution of the vector of variables is nonlinear and can only be evaluated numerically, making it difficult to directly detect identification problems. If the joint prior distribution of the parameters is proper then the posterior distribution is also well-defined. One of the advantages of the Bayesian framework is that it allows for the direct comparison of the prior and posterior, providing valuable insight about the extent to which data provide information about the parameters of interest. When using Bayesian methods researchers could suffer from the problem that posteriors are quite different from their priors due to some identification problems. Koop, Pesaran and Smith (2011) showed that priors can differ from posteriors even for unidentified parameters. There are reasons for that. First, although the approximate solution of DSGE models is taken to be linear, the structural parameters are complicated nonlinear functions of the parameters of the linearized model and as a result the likelihood function for the structural parameters may be very badly behaved; when the model involves unobserved variables the solution is of a VARMA form rather than a VAR, thus some of the associated reduced form parameters may not be identified; also the requirement for a determine solution puts restrictions on the joint parameter space, which may create dependency between identified and unidentified parameters.

# **1.3.3** Computational burden

The computational burden related to solving the model depends on the model size and the desired accuracy. The DSGE models can be solved using linear and quadratic approximations. The linear approximations are based on generalised eigenvalue decompositions of the system matrices associated with the rational expectation system, while the quadratic approximations can be obtained by constructing a linear approximation and then solving a discrete Lyapunov equation to obtain the second-order terms for the model solution. Usually, after solving the model by the linear approximation method, the likelihood function has to be solved with the Kalman filter or with a nonlinear particle filter if the model is solved with a nonlinear method. The computational burden is significant in both cases.

The Smets and Wouters DSGE model is large compared to others. As the model increases in dimension and sample size, more time will be required to generate a single draw for the Markov chain of the posterior simulator. The Markov chain will converge to its ergodic

distribution, and the speed at which it converges is a function of the dimensionality of the parameter space. Because of the large parameter space of the Smets and Wouters DSGE model, it is likely that the MCMC algorithm explores the posterior distribution only locally in the neighbourhood of the mode.

# **1.4 Estimation**

The model is estimated with Bayesian estimation techniques using the following macroeconomic quarterly Chinese time series as observable variables: the log difference of real GDP, real consumption, real investment, real wages, log hours worked, the log difference of the GDP deflator and the central bank's funds rate.

Prior distributions of the parameters are set following the parameters for the American economy determined by Smets and Wouters (2007).

# **1.4.1 Prior distributions**

The priors on the standard errors of the innovations are assumed to follow an inverse-gamma distribution with a mean of 0.10 and two degrees of freedom, corresponding to a rather loose prior. The persistence of the AR(1) processes follows the beta distribution with a mean of 0.5 and a standard deviation of 0.2. Similarly, the MA parameter in the process for the price and wage mark-up is assumed to follow the beta distribution. The quarterly trend growth rate is assumed to follow a Normal distribution with a mean of 0.4 and a standard deviation of 0.1. A gamma distribution with a mean of 2.5% and 1% on the annual basis is set for the steady-state inflation rate and the discount rate.

There are five fixed parameters in the estimation procedure: the depreciation rate  $\delta$  fixed at 0.025, the exogenous spending GDP ratio  $g_y$  set at 0.18, the steady-state mark-up in the labour market  $\lambda_w$  set at 1.5, and the curvature parameters of the Kimball aggregators in the goods and labour market  $\varepsilon_p$  and  $\varepsilon_w$ , both set at 10.

The Taylor rule is the basis for setting the parameters that describe the monetary policy rule. The long-run reaction to the inflation and output gaps are set to follow a Normal distribution, with means of 1.5 and 0.125 and standard errors of 0.125 and 0.05, respectively. The coefficient on the lagged interest rate is set to follow a Normal distribution with a mean of 0.75 and a standard error of 0.1, and this coefficient also describes the persistence of the policy rule. The coefficient of the short-run reaction to the change in the output gap is set to 0.125.

The parameters of the utility function are distributed as follows. There are three quite standard calibrations: the intertemporal elasticity of substitution is set at 1.5 with a standard error of 0.375; the habit parameter is assumed to be 0.7 with a standard error of 0.1; and the elasticity of the labour supply is set at 2 with a standard error of 0.75. The prior for the adjustment cost parameter for investment is 4 with a standard error of 1.5 (CEE, 2005), and the capacity utilisation elasticity is set at 0.5 with a standard error of 0.15. The share of fixed costs in the production function is set at 0.25. The priors of the parameters describing the price and wage setting are as follows. For both prices and wages, the Calvo probabilities are assumed to be around 0.5, indicating an average length of price and wage contracts of half a year. The prior mean of the degree of indexation to past inflation is set at 0.5 in both goods and labour markets.

# **1.4.2** Posterior estimates of the parameters

The information of priors (distribution type, mean and standard deviation) and the estimated posterior means of the structural parameters and shock processes are shown in Table 1 and Table 2, respectively.

| -                                       |              | Posterior |           |         |  |
|---|--------------|-----------|-----------|---------|--|
| Parameter                               | Distribution | Mean      | Std. Dev. | Mean    |  |
| <sup>𝖗</sup> Adj. Cost                  | Normal       | 4         | 1.5       | 10.6638 |  |
| $\sigma_c$ Inter. Sub.                  | Normal       | 1.5       | 0.37      | 1.2432  |  |
| <sup>h</sup> Habit Form.                | Beta         | 0.7       | 0.1       | 0.8769  |  |
| $\xi_w$ Calvo Wage                      | Beta         | 0.5       | 0.1       | 0.7876  |  |
| $\sigma_{LElas.}$ L Suply               | Normal       | 2         | 0.75      | 2.7213  |  |
| $\xi_p$ Calvo Price                     | Beta         | 0.5       | 0.1       | 0.8776  |  |
| <i>l</i> <sup><i>w</i></sup> Wage Index | Beta         | 0.5       | 0.15      | 0.3623  |  |
| $l_p$ Price Index                       | Beta         | 0.5       | 0.15      | 0.1705  |  |
| ΦFixed Cost                             | Normal       | 1.25      | 0.12      | 1.2318  |  |
| $r_{\pi}$ Inf. Policy                   | Normal       | 1.5       | 0.25      | 2.1304  |  |
| $ ho_{\text{Coeff. Lag r}}$             | Beta         | 0.75      | 0.1       | 0.9676  |  |
| <sup>r</sup> <sub>y</sub> Output Policy | Normal       | 0.12      | 0.05      | 0.1079  |  |
| $r_{\Delta y} \Delta Y$ Policy          | Normal       | 0.12      | 0.05      | 0.0023  |  |
| $100(\beta^{-1}-1)$                     | Gamma        | 0.25      | 0.1       | 0.05    |  |
| $\overline{\gamma}$ Trend Growth        | Normal       | 0.4       | 0.1       | 0.5848  |  |
| $\alpha$ Capital Share                  | Normal       | 0.3       | 0.05      | 0.199   |  |

 Table 1. Prior and posterior distribution of structural parameters.

|                                 |              | Posterior |           |        |
|---------------------------------|--------------|-----------|-----------|--------|
| Parameter                       | Distribution | Mean      | Std. Dev. | Mean   |
| $\sigma_{a}$ Std. Dev.          | Invgamma     | 0.1       | 2         | 2.49   |
| $\sigma_{\scriptscriptstyle b}$ | Invgamma     | 0.1       | 2         | 0.1817 |
| $\sigma_{_g}$                   | Invgamma     | 0.1       | 2         | 0.7257 |
| $\sigma_{_{I}}$                 | Invgamma     | 0.1       | 2         | 0.3112 |
| $\sigma_r$                      | Invgamma     | 0.1       | 2         | 0.057  |
| $\sigma_{_p}$                   | Invgamma     | 0.1       | 2         | 0.0535 |
| $\sigma_{_{w}}$                 | Invgamma     | 0.1       | 2         | 0.1048 |
| $\rho_{a}$ Shock AR             | Beta         | 0.5       | 0.2       | 0.987  |
| $ ho_b$                         | Beta         | 0.5       | 0.2       | 0.6225 |
| $ ho_{g}$                       | Beta         | 0.5       | 0.2       | 0.9938 |
| $ ho_{I}$                       | Beta         | 0.5       | 0.2       | 0.8631 |
| $ ho_r$                         | Beta         | 0.5       | 0.2       | 0.7368 |
| ${oldsymbol{ ho}_p}$            | Beta         | 0.5       | 0.2       | 0.9863 |
| $ ho_{\scriptscriptstyle w}$    | Beta         | 0.5       | 0.2       | 0.9763 |
| $\mu_p$ Price Mk                | Beta         | 0.5       | 0.2       | 0.645  |
| $\mu_{_{w}}$ Wage Mk            | Beta         | 0.5       | 0.2       | 0.2302 |
| $\rho_{ga}$ Cross Coef.         | Beta         | 0.5       | 0.2       | 0.5523 |

Table 2. Prior and Posterior distributions of shock processes.

The tables above give the mean of the posterior distribution of the parameters obtained by the Metropolis-Hastings algorithm. The estimation shows that the trend of the growth rate is around 0.58, which is smaller than the average growth rate of output per capita over the sample. The mean of the discount rate is estimated to be 0.99% on an annual basis.

Among the main behavioural parameters, some means of the posterior distribution are relatively close to the means of the prior assumptions; however, in our study, most of the estimated posterior means are not especially close to their pre-assumed priors. The degrees of price and wage stickiness are estimated to be quite a bit higher than the prior assumption of 0.5, which suggests an average length of price and wage contracts of half a year. The degree of price stickiness is 0.8776 and the stickiness of wages is 0.7876; the average duration of price contracts is less than a year, whereas the average duration of wage contracts is about three quarters. The elasticity of the cost of changing investments is estimated to be 10.6638, quite a bit higher than the assumed prior 4.0, suggesting an even slower response of investment to changes in the value of capital. The mean degrees of price and wage indexation are estimated to be 0.1705 and 0.3623, respectively, which are less than the pre-assumed prior 0.5. The posterior mean of the fixed cost parameter is estimated to be 1.2318, slightly higher than the assumed prior 1.2. The mean of the discount rate is estimated to be 0.99, higher than the prior 0.5, and the mean of the elasticity of the labour supply is estimated to be 2.7213, higher than the prior 2.0. The posterior mean of the share of capital in production is 0.1990, much lower than its prior as a share of capital.

Next, we examine the parameters of the monetary policy reaction. The mean of the long-run reaction coefficient to inflation is estimated to be 2.1304, which is relatively higher than the assumed prior 1.5. The mean of the coefficient on the lagged interest rate is estimated to be 0.9676, indicating a considerable degree of interest rate smoothing. The mean of the parameters of policy reactions to an output gap and changes in the output gap are estimated to be 0.1079 and 0.0023, respectively, indicating that the policy reaction to an output gap is stronger than to changes in the output gap; however, in general, policy does not appear to react that strongly to either parameter.

Some observations can be made regarding the estimated processes for the exogenous shock variables. The data appear to be highly informative on the stochastic processes for the exogenous disturbances. The productivity, government spending, price mark-up and wage mark-up processes are estimated to be the most persistent with AR(1) coefficients of 0.9870, 0.9938, 0.9863 and 0.9763, respectively. According to the beta distribution whose mean is away from one in the priors of the shock persistence parameters, the estimated shock persistence should not be so high; however sometimes the posterior could be different from the prior (as discussed in section 1.3.2.2) and here of course it shows that some of the shocks to the productivity process is 2.4900, which indicates that China has experienced and is experiencing an intense technological revolution. The high persistence of the productivity, price mark-up and wage mark-up processes implies that at long horizons most of the forecast error variance of the real variables will be explained by those three shocks. The persistence of the risk premium and monetary policy shocks are 0.6225 and 0.7368, respectively, lower than those of the other shocks.

# **1.5 Application**

We also investigate a number of key macroeconomic variables in terms of the main driving forces of those variables, whether the model can replicate the cross correlation between output and inflation, the effect of a productivity shock on hours worked, and whether output and inflation have become less volatile.

The following table presents the forecast variance decomposition of output, inflation and the PBC (People's Bank of China) funds rate, consumption and investment.

| Shocks        | Prodty | Risk Pre | Gov.<br>Spend | Investmt | Monetary | Price<br>Mark-up | Wage<br>Mark-up | Total |
|---------------|--------|----------|---------------|----------|----------|------------------|-----------------|-------|
| Output        | 12.62  | 27.93    | 10.37         | 13.96    | 13.66    | 4.39             | 17.06           | 100   |
| Interest rate | 7.68   | 1.55     | 0.65          | 4.85     | 1.72     | 35.21            | 48.33           | 100   |
| Inflation     | 2.13   | 0.44     | 0.12          | 2.44     | 5.63     | 36.89            | 52.35           | 100   |
| Consumption   | 13.67  | 37.86    | 0.5           | 3.34     | 13.66    | 5.52             | 25.44           | 100   |
| Investment    | 16.58  | 14.23    | 0.13          | 35.07    | 12.58    | 9.32             | 12.09           | 100   |

Table 3. Forecast error variance decomposition.

The above table clearly indicates the main driving forces of each of the three key macroeconomic variables. Output is mainly driven by wage mark-up shock and risk premium shock, which together account for 44.99% of the variation in output. Other shocks' contributions to output are comparably smaller than these two shocks. According to the results of Shapiro and Watson (1989), the two supply shocks of the wage mark-up shock and the productivity shock together contribute most of the output variation in the medium to long run, however productivity seems to contribute less in the long run to GDP in the Chinese economy. The risk premium shock affects both the consumption and investment Euler equations and the investment shock affects the investment Euler equation, and these shocks together account for more than nearly 42 percent of the forecast error variance of output. Smets and Wouters (2003) state that those two shocks can be categorised as demand shocks due to their positive effects on output, hours worked, inflation and the nominal interest rate under the estimated policy rule and that they have strong effects on the output in the short run (up to one year) but not in the long run. They also argued that in the long run, the wage mark-up shock dominates the productivity shock, which is consistent with the case of the Chinese economy. The monetary policy shock is an important factor controlling the movement of the Chinese GDP, accounting for 13.66% of the forecast error variance of output in the long run. Inflation is mainly driven by the price mark-up and wage mark-up shocks, which together account for 89.24% of the forecast error variance of inflation. The movement of the nominal interest rate is most affected by the price mark-up and wage mark-up shocks, which together account for 83.54% of the forecast error variance of the nominal interest rate. The price mark-up shock dominates the wage mark-up shock in the long run movement of the nominal interest rate.

The case for the variation in consumption seems very similar to that for output, as the risk premium shock and the wage mark-up shock together account for 63.3% of the forecast error variance of consumption. The investment shock is the main driving force for the development of investment; other shocks are almost equally important to investment except for the government spending shock, which does not play a significant role.

Figure 1 shows the estimated impulse response functions to the "demand" shocks of the exogenous spending shock, the risk premium shock and the investment shock.



Panel 1



Panel 2



Panel 3

Figure 1. The estimated impulse responses to "demand" shocks (dy=output, pinfobs=inflation, robs=interest rate, dc=consumption, dinve=investment, dw=real wage, labobs=labour). Panel 1: exogenous spending shock; Panel 2: risk premium shock; Panel 3: investment shock.

The first to the third panels (from up to down) of Figure 1 show the impulse responses to the exogenous spending shock, risk premium shock and investment shock. These three shocks have positive effects on output, inflation, nominal interest rate and hours worked, consistent with the estimated policy rule. In the short run (within a year), the three "demand" shocks mainly drive movements in real GDP. This is shown in the upper left diagram (dy) of each panel in Figure 1. These three shocks have long term effects on the movements in the nominal interest rate (robs), with the exogenous spending shock and the risk premium shock exhibiting especially strong effects on this outcome. The risk premium shock and the investment shock have a positive effect on inflation (pinfobs), but these effects become negative after a few quarters. The exogenous spending shock has a strong effect on the long run variation of hours worked, increasing hours worked by 0.7 percent with a slow return to its normal level (after over 20 quarters), while the risk premium shock and the investment shock have significant positive effects on hours worked in the short run only. Shapiro and Watson (1989) argued that the movements in output in the medium to long run are mainly driven by two "supply" shocks, the productivity shock and the wage mark-up shock. Figure 2 shows the estimated impulse responses to the "supply" shocks.



Panel 1



Panel 2

Figure 2. Estimated impulse responses to "supply" shocks (dy=output, pinfobs=inflation, robs=interest rate, dc=consumption, dinve=investment, dw=real wage, labobs=labour). Panel 1: productivity shock; Panel 2: wage mark-up shock.

Figure 2 shows the impulse responses to the supply shocks. Panel 1 of Figure 2 shows the impulse responses to the productivity shock and Panel 2 presents the impulse responses to the wage mark-up shock. Table 3 shows that the productivity shock is not a main force driving the movement of output in the Chinese economy, although the top left diagram in Panel 1 of Figure 2 shows that the productivity shock has a long run effect on output. The top left diagram in Panel 2 of Figure 1 shows that the risk premium shock increases output by nearly 1 percent followed by a rapid decrease within a year, which is consistent with Table 3 showing that the risk premium shock is a main driving force of output in the short run. The top left diagram in Panel 2 of Figure 2 shows that the wage mark-up shock has a strong effect on output and that this effect lasts for a long time. First, the wage mark-up shock increases output by almost 0.5 percent, and then after 6 quarters, the effect becomes negative, with output reduced by roughly 0.25 percent. The diagrams in the middle left of both panels show that the "supply" shocks become dominant forces in the long-run development of consumption (dc). The diagrams in the centre of both panels show that the "supply" shocks do not have long run effects on investment.

Christiano, Eichenbaum and Evans (2000) confirmed that the monetary policy shock contributes only a small fraction of the forecast variance of output. Smets and Wouters (2003) also estimated that monetary policy shocks only played a dominant role in the recession of the early 1980s in the US economy when the Federal Reserve under the chairmanship of Paul Volker started the deflation process. Table 3 shows that the monetary policy shock contributes 13.66% of the forecast error variance of Chinese output, which is not in line with the literature, indicating that this shock is important in the movement of the Chinese GDP but is not the dominant force.



Figure 3. Estimated impulse responses to a monetary policy shock (dy=output, pinfobs=inflation, robs=interest rate, dc=consumption, dinve=investment, dw=real wage, labobs=labour).

Figure 3 shows that the monetary policy shock has negative effects on output and inflation and a positive effect on nominal interest rate, consistent with the literature. However, the effects of the monetary policy shock on the key variables are not highly significant, showing that the monetary policy shock is not the main driving force of the movements of the economy. Additionally, the impulse response functions show that the effects of the monetary policy shock cannot last for a long time.



Figure 4. Estimated impulse responses of inflation to all shocks (from top left: productivity, risk premium, exogenous spending, monetary policy, price mark-up, investment, and wage mark-up shocks).

Figure 4 presents the estimated impulse responses of inflation to each of the seven shocks. From the top left, the shocks are the productivity shock, risk premium shock, exogenous spending shock, monetary policy shock, price mark-up shock, investment shock, and wage mark-up shock. The productivity and monetary policy shocks have negative effects on inflation and other shocks have positive effects on inflation. Note that the investment shock increases inflation slightly at first by about 0.005 percent at first and later decreases inflation for the rest of the time period studied. Except for the price mark-up and wage mark-up shocks, all shocks have short run influences on inflation. The price and wage mark-up shocks dominate other shocks in explaining the total variation in inflation, and between these two shocks the wage mark-up shock is comparably more important (which is also shown in Table 3). The magnitudes of the effects of the "demand" and productivity shocks on inflation can be explained by at least two facts, the estimated slope of the New Keynesian Phillips curve and the way that the central bank reacts to inflation. Only large and persistent changes in the marginal cost can have an impact on inflation due to the small estimated slope of the New Keynesian Phillips curve. The People's Bank of China responds aggressively to emerging output gaps and their impact on inflation. These two facts led the various "demand" shocks and productivity shock to have only limited impacts on inflation.

| Variables | Labour  | Int. Rate | Infl.   | GDP     | Cons.   | Invst   | Wage    |
|-----------|---------|-----------|---------|---------|---------|---------|---------|
| Labour    | 1       | -0.5634   | -0.3593 | 0.0397  | 0.2422  | -0.2104 | 0.2492  |
| Int. rate | -0.5634 | 1         | 0.7572  | -0.2472 | -0.3085 | -0.118  | -0.2201 |
| Infl.     | -0.3593 | 0.7572    | 1       | -0.0346 | 0.0034  | -0.099  | 0.1613  |
| GDP       | 0.0397  | -0.2472   | -0.0346 | 1       | 0.8111  | 0.7826  | 0.675   |
| Cons.     | 0.2422  | -0.3085   | 0.0034  | 0.8111  | 1       | 0.4195  | 0.6629  |
| Invst     | -0.2104 | -0.118    | -0.099  | 0.7826  | 0.4195  | 1       | 0.4738  |
| Wage      | 0.2492  | -0.2201   | 0.1613  | 0.675   | 0.6629  | 0.4738  | 1       |

Table 4. Correlations of key macro-economic variables.

Table 4 shows the cross correlations of all the variables in the structural model. As highlighted in Gali and Gertler (1999), it is interesting to determine whether the structural model can replicate the empirical correlation functions between different variables. Output has a negative relationship with inflation, with a cross correlation coefficient of -0.0346. The output and inflation data are further analysed below. The data source is the National Bureau of Statistics of China.



Figure 5. Plots of output and inflation in the sample period. Source: National Bureau of Statistics of China

The left panel of Figure 5 shows a moderate increase of Chinese real GDP from the start of the economic reform until the early 1990s, after which the Chinese economy began a period of rapid growth that has lasted until now. The right panel of Figure 5 shows the process of inflation. From 1978 to 1990 the inflation rate was kept in an acceptable range; however, in the first half of the 1990s, there was a significant increase in inflation. Then inflation was cut down fiercely and kept at a low level during the period of Prime Minister Zhu Rongji. Inflation started to increase again in 2004. Combining the two diagrams in Figure 5 demonstrates that output and inflation are positively correlated from 1978 to 1995, and significantly negatively correlated after 1995. After all, the structural model can replicate the empirical correlation between output and inflation. Moreover, the correlations generated by the DSGE model are significantly different from zero. Note that the monetary policy shocks only account for a small fraction of the changes in output and inflation, and as shown in the top

and middle diagrams of Figure 3, the peak effect of a policy shock on output occurs before its peak effect on inflation.

Table 4 also shows that investment and output exhibit a strong positive relationship with a correlation coefficient of 0.7826, indicating that investment makes a very important contribution to Chinese GDP, in agreement with the fact that investment accounts for a large fraction of Chinese GDP. Inflation and nominal interest rate also have a strong positive correlation with a coefficient of 0.7572. The nominal interest rate is negatively correlated with many other variables, in agreement with the earlier investigation of the analysis of impulse response functions. Consumption is negatively correlated with nominal interest rate, reflecting the reality in China of traditional and historical thinking in which people always deposit their money in the bank and have a low propensity to consume. Both hours worked and wage levels are negatively correlated with the nominal interest rate. Consumption is slightly positively correlated with inflation with a cross correlation coefficient of 0.0034.

We have analysed the basic properties of output and inflation of China, however except for output and inflation it is also important to look into the basic properties of other main macroeconomic variables. The other 5 main variables, which are consumption, employment, interest rate, investment and wage, are plotted against time in Figure 6.





First we can see that the real variables such as consumption and investment increased tremendously. Both consumption and investment started to rise from 1990, while consumption kept a high growth since the middle of 1990s and investment rose very rapidly since the beginning of this century. The growth of consumption is very much in line with the growth of output, on the other hand investment growth is mainly due to the government's

policy which tended to stimulate the real estate industry and expands the export intensively, these policies need the planning of new factories, new towns, infrastructure, etc. all these demand lots of investments. There is no doubt that employment increased significantly in respond of the rapid growth of output and investment. Wage started to rise in a high speed since the mid-1990s, which is in line with consumption and employment, when people have more money consumption will increase. The data plots are in line with the statistical analysis in Table 4. The diagram of interest rate shows that there was a large interest rate cut due to the financial crisis in 1998. Chinese government adopted a loose monetary policy in order to stimulated consumption and keep the high growth rate of output.

#### The effect of a productivity shock on hours worked

The study of US business cycles has included a debate about the effects of productivity shocks on hours worked and about the implications of this finding for the role of those shocks in the business cycles.



Figure 7 Estimated impulse responses to productivity shocks

Gali (1999), Francis and Ramey (2005) and Gali and Rabanal (2004) argue that positive productivity shocks lead to an immediate decrease in hours worked due to the existence of

nominal price rigidities, habit formation and adjustment costs to investment. The diagram in the third row and first column of Figure 6 makes it obvious that there is a significant decrease in hours worked after positive productivity shocks of more than 2%. The effect of productivity shocks fades away in three years. Table 4 shows that there is a positive correlation between output and hours worked over the business cycle which implies that productivity shocks cannot play an important role in the business cycle. However, Christiano et al. (2004), Dedola and Neri (2004) and Peersman and Straub (2005) have argued that the effect of a productivity shock on hours worked is not very robust and could be consistent with a positive impact on hours worked through their empirical evidence using alternative VAR specifications and identification strategies. Actually, the effect of a productivity shock on hours worked turns out to be positive after four years in our model. The upper left diagram of Figure 6 demonstrates that productivity shocks raise output dramatically beyond the one year horizon in our estimated model, but Table 3 shows that productivity shocks are not the main driving force of the movements of output, so productivity shocks play an important but not a dominant role in driving changes in output. A positive productivity shock leads to an expansion of output, aggregate demand and real wages. The diagrams with titles dc, dinve, and dw show that both consumption and real wages increase and reach a maximum (0.25%)around the tenth quarter. Investment rises by 1% immediately, and this effect will fade away in about 15 quarters. Nominal and real interest rates fall under the estimated monetary policy reaction function as shown in the top right diagram of Figure 6, but not enough to prevent the fall in inflation that can be seen from the diagram in the middle of the first row. Confirming the analysis of Francis and Ramey (2004), the estimation results of our model show that the estimated degree of habit persistence and the importance of capital adjustment costs are the main explanations of the negative impact of productivity shocks on hours worked. It turns out that the effects on employment are not governed by monetary policy. The temporality of the productivity shock, although persistent, can be one reason for the relatively low medium-run positive effects on hours worked. The result is that when the effects on hours worked start materialising, output already starts returning to its baseline. Rotemberg (2003) argued that the appearance of major productivity advances in output growth could be very slow, which implies that a higher total productivity could increase the effect on hours worked. Another potential reason for the negative impact of productivity on hours worked is that the fixed cost per unit of production is reduced by a higher productivity. Therefore, less labour is required to achieve a given level of output.

#### **1.6 Conclusion**

In the last decade, a new generation of small-scale monetary business cycle models have been developed that are generally referred to as New Keynesian or New Neoclassical Synthesis models (Goodfriend and King (1997), Rotemberg and Woodford (1997), Gali and Gertler (1999)). Some of the new findings, ideas and features of these models relative to the traditional Keynesian literature were highlighted by Gali (2000). In this chapter, we first investigated the possibility of modelling the Chinese economy according to a DSGE pattern. Although no such modelling has yet been reported, we carefully searched through the existing literature and found some very useful previous reports on DSGE modelling of the US economy, particularly the work of Smets and Wouters (2007). The monetary DSGE model used in this chapter shares the essential features of the class of models mentioned above, particularly the sticky price setting. There are some additional frictions in our model following Christiano, Eichenbaum and Evans (2001) that are necessary to capture the empirical persistence and covariances in the main macro-economic data of China; these frictions are namely sticky nominal wage setting, variable capital utilisation, adjustment cost in capital accumulation and habit formation in consumption. Finally, this model contains a full set of structural shocks: a supply shock (productivity shock), demand shocks (investment and government spending shocks), mark-up shocks (price and wage mark-up shocks), a risk premium shock, and a monetary policy shock; these structural shocks account for the stochastics in the empirical data. Compared to the canonical two equation models these extensions allow us to perform more thorough analyses. We can estimate the model parameters with Bayesian techniques using the main Chinese macroeconomic data on output, inflation, real wages, investment, consumption, the short-term interest rate and employment; we can examine the sources of business cycle dynamics in the Chinese economy; and we can also analyse some new features of this class of models in an empirically plausible set-up.

Among various methods that can be used to estimate or calibrate the parameters of a linearised DSGE model, we chose the Bayesian estimation method. Previous studies have estimated the parameters in monetary DSGE models by minimising the difference between an empirical and the theoretical impulse response to a monetary shock (Rotemberg and Woodford 1998, and Christiano, Eichenbaum and Evans 2001). A number of other authors have estimated the structural parameters of DSGE models using classical maximum

likelihood methods (Sargent, 1989). As an alternative within this group of strong interpretations, a Bayesian approach can be followed by combining the likelihood function with prior distributions for the parameters of the model to form the posterior density function. Then the posterior density functions can be optimised with respect to the model parameters either directly or through Monte Carlo Markov Chain (MCMC) sampling methods. The attractions of the strong econometric interpretations include their ability to provide a full characterisation of the data generation process and to allow for proper specification testing and forecasting only if these econometric interpretations are successful. This chapter has followed the strong econometric interpretation approach to DSGE models. There are two main reasons for our application of Bayesian techniques: this approach allows us to formalise the use of prior information that comes from either micro-econometric studies or all the previous macro-econometric studies, therefore allowing for an explicit link with the previous calibration-based literature. Based on the marginal likelihood of the model or Bayes' factor, the Bayesian approach can provide a framework for evaluating fundamentally misspecified models.

The exercises carried out in this chapter as described above have led to the following results. The parameter estimates of the DSGE model of Chinese macroeconomic data suggest that there is a considerable degree of price and wage stickiness in the Chinese economy. In the results, prices respond slowly to changes in expected marginal costs and wages adjust slowly to deviations from their efficient levels. Both price and wage inflation depend on past inflation, which introduces a backward-looking component; however, the forward-looking component makes the major contribution to the price setting equation. As mentioned above, our estimated model features a full set of structural shocks arising from technologies, preferences and inefficient mark-ups, clarifying that the appropriate output gap does not exist solely from a monetary policy perspective. From a monetary policy point of view, the appropriate estimate of the potential output level should only take into account that part of the natural level of output is driven by shocks arising from preferences and technologies. Two comparably important driving forces of output are the risk premium shock and wage mark-up shock, which are both demand shocks. For variations of inflation and interest, the price and wage mark-up shocks make the greatest contributions. Consumption is mainly driven by the risk premium and wage mark-up shocks, which is in line with the case of output. Investment is most affected by the investment shock, while other shocks affect investment at almost the same magnitude except for the government spending shock. The estimates of the effects of a temporary monetary policy shock to the main macroeconomic variables are largely in line with the existing literature. The nominal and real interest rates are raised by the monetary policy shock, leading to a hump-shaped fall in output, consumption and investment, with the investment exhibiting a significantly stronger response, and with a gradual fall in prices. However, a persistent monetary shock exhibits some different effects. According to the arguments made by Gali (2000), as the decrease in the nominal component outweighs the increase in the real component of the short-term interest rate, there is no liquidity effect. Also because the shock in policy is credible and implemented gradually, expectations have time to adjust and therefore the effects on output are much smaller than those on other components. As emphasised by Gali (1999), in models with sticky prices, the employment is likely to drop in the short run in response to a productivity shock unless the monetary shock is sufficiently accommodating. This chapter shows that there is a significant negative effect on employment under the estimated policy reaction function to a positive productivity shock, confirming previous results. Because of the high labour supply elasticity, it is worth noting that even in a flexible price and wage economy, i.e., a New Classical economy, the productivity shock would have a negative effect on employment. Gali (2000) seriously doubted the quantitative significance of productivity shocks as a source of aggregate fluctuation because of the empirical procyclicality of employment. In practice, our results suggest that in contrast to many other studies, the productivity shocks only account for 12.62% of the output variance. Indeed, productivity shocks, risk premium shocks, government spending shocks and monetary shocks are the most important sources of variation in output, inflation and interest rates.

Overall, the results presented in this chapter show that an estimated version of the Chinese DSGE model with sticky prices and wages can be used for monetary analysis in an empirically plausible set-up; these results are to some extent representative of the Chinese economy and suitable for consultation by policy analysts in China.

# Chapter 2

**Testing the Chinese DSGE model using indirect inference** 

## **2.1 Introduction**

This Chapter proposes a method to evaluate DSGE models that is based on indirect inference and applies this method to the Chinese DSGE model developed in Chapter 1. The Chinese DSGE model features sticky prices and wages, making the model a New Keynesian (NK) model. Because of the major disagreement between economists regarding the degree of nominal rigidity, it is worth to compare the Chinese DSGE model with a New Classical (NC) version that contains flexible prices and wages but lagged information for households with a one quarter delay in the receipt of macroeconomic information. Although these analyses represent the two major fields of economics, in the real world, economies may not behave like either model alone but rather like a mixture of the two in which some parts of the economy exhibit nominal rigidity and other parts do not. Therefore, we also consider the possibility of a mixed model which we call a hybrid model of NK and NC.

The problem of how to test a calibrated and estimated DSGE model has existed for a long time. Economists previously simply compared some particular features of the data that were simulated from the calibrated or estimated model with the true data. The method of indirect inference proposed in this Chapter will formalise the approach of evaluating DSGE models. We note that the solution of a log-linearised DSGE model can be closely represented by a VAR model due to the fact that the solutions can be represented as a restricted Vector Autoregressive Moving Average (VARMA) model. These can be settled by the approach of indirect inference. We can impose restrictions on the VAR according to the a priori structural restrictions of the DSGE model. To test the DSGE model we have to compare the unrestricted VAR estimates that are derived from the data simulated from the DSGE model with the unrestricted VAR estimates obtained using the true data. We conduct a Wald test based on the VAR estimates to determine whether the complicated DSGE model is correctly specified. If the conclusion is that the model is correct then the actual data and the simulated data together with the VAR estimates from these data will be very close to each other. As described above, the obvious advantage of the indirect inference testing procedure is that it is not necessary to specify a different DSGE model as the alternative hypothesis. In our procedure, the alternative hypothesis that is suitable for testing the specification of the model can be generated automatically by an unrestricted VAR model based on the true data. Of course, the auxiliary VAR that is generated by the DSGE model has to be identified.

The work in this chapter is builds on a large body of literature on the evaluation of DSGE models. Starting from measuring the closeness of DSGE models to the actual data, particularly relevant studies include Watson (1993), Canova (1994, 1995), Del Negro and Schorfheide (2004, 2006), Canova (2005), Corradi and Swanson (2007), and Del Negro et al. (2007a). Other authors have compared simulations of the model with various aspects of the actual data such as moments, cross-moments, and impulse response functions, see for example Theodoridis (2006), Minford et al. (2009) and Le et al. (2010). Also, Le et al. (2011) pointed out a direction of further model development for resolving the "puzzles methodology" due to the poor agreement between the model and the actual data.

# 2.2 Methodology

After the estimation of the Chinese DSGE model we wish to determine whether the structural model fits the real data. The Chinese DSGE model is adopted from Smets and Wouter's model, so just like in the Smets and Wouter model the Chinese DSGE model incorporates a full range of structural shocks with nominal and real frictions and is estimated through a Bayesian approach. This model is considered to be satisfactory in a variety of ways. In this chapter, we focus on the model's dynamic performance with regard to the data. We adopt the evaluation procedure based on indirect inference, an approach that has been widely used in the estimation of structural models in the literature. Here, we make a different use of the indirect inference aim to evaluate an already estimated or calibrated model. This new procedure will exploit the properties of the model's error processes through bootstrap simulations, and both the simulations and the actual data are represented by the dynamic behaviour of the auxiliary model. We test whether the simulated data from the structural model can explain the actual data. The test is a form of Wald statistic that focuses on the overall capacity of the model to fit the dynamic performance of the data.

In the indirect inference, we use an auxiliary time series model. In the previous chapter, the parameters of the structural model are chosen in the Bayesian estimation so that when this model is simulated it generates estimates of the auxiliary model similar to those obtained from actual data. The parameters should be chosen to minimise the distance between a given function of the two sets of estimated coefficients of the auxiliary model. These functions are actual coefficients and the impulse response functions. The aim of the evaluation is to

compare the performance of the auxiliary model estimated on simulated data derived from the given estimates of the structural model (in the previous chapter) with the performance of the auxiliary model when estimated from actual data. The comparison is based on the distributions of the two sets of parameter estimates of the auxiliary model; if the structural model is correct then its predictions about the impulse responses, moments and time series properties of the simulated data should match the actual data. To carry out the test we first have to construct the errors derived from the previously estimated structural model and the actual data. Then these errors are bootstrapped and used to generate new data based on the bootstrap model for each bootstrap. Then an auxiliary time series model is fit to each set of data, both actual and simulated, and the sampling distribution of the coefficients of the auxiliary time series model is obtained from these estimates of the auxiliary model. Whether the functions of the parameters of the time series model estimated on the actual data lie in the confidence interval implied by the sampling distribution of the coefficients of the auxiliary time series model is determined by a Wald statistic.

There is another issue that is important to consider regarding the probability distribution of our Wald test statistic. This chapter will use an empirical estimate of the test statistic's small sample distribution obtained by bootstrap methods rather than the asymptotic distribution of the Wald statistics.

# 2.2.1 The procedures of indirect inference testing

The aim of this chapter is to evaluate the already estimated Chinese DSGE macroeconomic model by the method of indirect inference; the evaluation procedure involves carrying out classical statistical inference on the previously estimated macro model. The common feature between estimation and evaluation of a macroeconomic model by indirect inference is that both exercises use an auxiliary time series model in addition to the structural macroeconomic model. The estimation of a macroeconomic model by indirect inference involves several steps.

First, suppose that  $y_t$  is an  $m \times 1$  vector of actual observed data and  $x_t(\theta)$  is an  $m \times 1$  vector of simulated time series generated from the structural macroeconomic model, both of which have *T* time periods, and that  $\theta$  is a  $k \times 1$  vector of the parameters of the macroeconomic

model and both the actual and simulated data are assumed to be stationary and ergodic. The auxiliary model is  $f[y_t, \alpha]$ , where  $\alpha$  is a vector of parameters of the auxiliary model. The estimation of indirect inference is that a particular value of  $\theta$  exists given by  $\theta_0$  such that

$$f[x_t(\theta_0), a] = f[y_t, \alpha]$$

The maximum likelihood estimators of the parameters of the model based on actual and simulated data are

$$a_T = \arg \max_{a} \zeta_T(y_t; \alpha)$$
 and  $a_S = \arg \max_{a} \zeta_S(y_t; \alpha)$  correspondingly.

Then, the simulated quasi-maximum likelihood estimator of  $\theta$  is

$$\theta_{T,S} = \arg \max \zeta_T[y_t; \alpha_S(\theta)]$$

This is the value of  $\theta$  that produces parameter values of the auxiliary model that maximise the likelihood function using the actual data. We can use the extended method of simulated moments estimator (EMSME) as an alternative to the simulated quasi-maximum likelihood estimator; this can be obtained as follows. Consider the continuous  $p \times 1$  vector of functions  $g(a_T)$  and  $g(\alpha_s(\theta))$  which could be moments or scores, for examples g(.) could be impulse response functions, then let  $G_T(a_T) = \frac{1}{T} \sum_{t=1}^T g(a_T)$  and  $G_s(\alpha_s(\theta)) = \frac{1}{s} \sum_{s=1}^S g(\alpha_s(\theta))$ . And there is a requirement that  $a_T \to \alpha_s$  in probability and that  $G_T(a_T) \to G_s(\alpha_s(\theta))$  in probability for each  $\theta$ . Then the EMSME is:

$$\theta_{T,S} = \arg\min_{\theta} [G_T(a_T) - G_S(\alpha_S(\theta))]' W(\theta) [G(a_T) - G_S(\alpha_S(\theta))]$$

We now have the structural macroeconomic model and the estimation of its parameters by a Bayesian method based on the actual data. We have to bootstrap the structural disturbances of the macroeconomic model to obtain its simulations and then compare the performance of the auxiliary model based on actual data with its performance based on the simulations. We obtain the joint distribution of the auxiliary model from these simulations and use the joint distribution to perform a Wald test that can test whether the estimates of the auxiliary model based on actual data could have come from the particular realisation of the structural model. The choice of the auxiliary model will be a VAR and the test will be based on a function of the VAR coefficients. Usually a VAR(1) on a limited number of key variables will be used. If the null hypothesis is not rejected then the estimates of the auxiliary model based on actual data come from the particular realisation of the dynamic

behaviour of the macroeconomic model is not significantly different from that of the actual data. The rejection of the null hypothesis implies that the macroeconomic model is incorrectly specified and cannot reproduce the economy's behaviour. The impulse response function of the observed and simulated data can determine in what respects the macroeconomic model fails to capture the auxiliary model.

To obtain the Wald test statistics we again assume that a particular value of  $\theta$  exists given by  $\theta_0$  such that the actual data and the simulated time series share the same distribution. If  $\hat{\theta}$  is the estimated value of  $\theta$ , then the null hypothesis can be expressed as  $H_0: \hat{\theta} \to \theta_0$ . Given an auxiliary model and a function of its parameters, the Wald test statistic for evaluating the structural macroeconomic model is based on the distribution of  $G_T(a_T) - G_S(\alpha_S(\hat{\theta}))$  and can be written as

$$[G_T(a_T) - G_S(\alpha_S(\hat{\theta}))]' W[G_T(a_T) - G_S(\alpha_S(\hat{\theta}))]$$

where the estimate of the optimal weighting matrix is

$$W(\hat{\theta}) = \left\{ \left[ \frac{\partial G(\alpha(\hat{\theta}))}{\partial \alpha} \right] (\hat{\theta}) \left[ \frac{\partial G(\alpha(\hat{\theta}))}{\partial \alpha} \right] \right\}^{-1}$$

We obtain the distribution of  $G_T(a_T) - G_S(\alpha_S(\hat{\theta}))$  and the Wald statistic through bootstrapping.

The procedure of performing the Wald test by bootstrapping can be summarised as follows. First, estimate the errors of the economic model conditional on the observed data and  $\hat{\theta}$ . The number of independent structural errors of the macroeconomic model has to be less than or equal to the number of endogenous variables. Then estimate the empirical distribution of the structural errors. The structural errors give the empirical distribution of the  $\{\varepsilon_t\}_{t=1}^T$  errors that are omitted in the null hypothesis. The simulated disturbances are drawn from these structural errors. We draw these disturbances from a time vector to preserve the simultaneity between them. Finally, we compute the Wald statistic.

In addition to the basic Wald statistic, a number of related Wald statistics are considered. We refer to the Wald test based on the full joint distribution of the VAR coefficients as implied by their full covariance matrix as the full Wald test. This Wald test checks whether the

coefficients based on the VAR data lie within the DSGE model's implied joint distribution and is a test of the DSGE model's specification. The Mahalanobis distance based on the same joint distribution is used to measure the overall closeness between the model and the data and is normalised as a t-statistic.

We also want to check on the specific features of the macroeconomic model, for example, how well the model can reproduce the behaviour of Chinese GDP and inflation. This can be done using a Wald statistic based on the VAR equation for these two variables alone. This type of Wald test is referred to as a Directed Wald statistic that can be used to evaluate how well a particular variable or limited set of variables is modelled. The Directed Wald test can also be used to determine how well the structural model captures the effects of a particular set of shocks. This requires creating a joint distribution of the impulse response functions for the particular set of shocks and calculating a Wald statistic for this distribution. Even if a macroeconomic model is rejected by the indirect inference test, the Directed Wald test can still evaluate whether the model is well-specified enough to deal with specific aspects of economic behaviour.

Traditional statistics are also useful in this test, such as the ability to match data variances, cross-correlations, and VAR-based Impulse Response Functions. The ability to match data variance is included in the full Wald statistic, and the failure of this test is essentially terminal. The cross-correlations and the Impulse Response Functions are all derived from the VAR coefficients, so we focus on themes coefficients.

# 2.2.2 Testing the Chinese DSGE model using the method of indirect inference

The proposed procedure described above will be used to test the Chinese DSGE model. The actual data start from quarter 1 of 1978 (when the economic reform started) and extend to quarter 4 of 2008. The data need to be stationarised by a filtering method. The filters we use here are the same as used by Smets and Wouter in which all variables are log differenced except for inflation, log of hours worked and interest rates which are left in levels. Here we difference all variables by log differencing as in Smets and Wouter's model but simply
difference the remaining three variables (inflation, labour and interest rate), and we also use an HP filter and linear detrending.

We use a VAR(1) as the auxiliary model. The estimation of the VAR is performed with five main observable variables: output, investment, consumption, the quarterly interest rate and the quarterly inflation rate.

First, we test the Chinese DSGE macroeconomic model, which was designed according to the Smets and Wouter model, using the error properties assumed in the model. The posterior means form the Bayesian estimation of the Chinese DSGE model and are used for the error variances and autoregressive coefficients.

Then, we evaluate the Chinese DSGE model based on the actual errors derived from estimation on the observed data. This requires an estimation of the model's structural errors, which are the residuals in each structural equation given by the actual data and the expected variables in that equation. To estimate the structural errors we follow a procedure suggested by McCallum (1976) and Wickens (1982) under which the expectations on the right hand side of each equation are generated by an instrumental variable regression that is implied by the model. The instruments are the lagged values of the endogenous variables. Seven behavioural residuals are estimated: consumption, investment, productivity, monetary policy, wage and price setting, and government spending. Five of these residuals follow an AR (1) processes but the price and wage residuals follow ARMA (1, 1) processes.

Next we evaluate the New Classical version of the Chinese DSGE model in which we adopted the New Keynesian rule (Taylor Rule) except the potential output was set to a constant. The aim of evaluating the New Classical version of the model is to compare the performance of these two versions of the DSGE model and determine which provides a better fit to the actual data.

After evaluating the Chinese DSGE model adopted from Smets and Wouters and the New Classical version of the Chinese DSGE model, we can assess the mechanisms of the two rather different macroeconomic models. The Smets and Wouter DSGE model is a New Keynesian model that is highly rigid with Calvo price and wage settings, while the New Classical model is flexible in prices and wages with only a simple one-period information lag

for labour suppliers. The two different versions of the DSGE model must have different performances, they may both fit the actual data very well or they may both fit the data poorly, and it is possible that one fits the data while the other does not. Hence, it is necessary to propose a third version of model, that is, a weighted combination of New Keynesian and New Classical models that is called the hybrid model.

Le, Minford and Wickens (2008) proposed a hybrid model that adopted the Smets and Wouter DSGE model. In their model, they assume that the wage and price setters find themselves supplying labour and intermediate output, partly in a competitive market with flexible wages and prices and partly in a market with imperfect competition. The size of the competitive and imperfectly competitive sectors depends on the facts of competition and do not vary in sample size but do vary in the degree of imperfect competition between labour and goods markets. The monetary authority pursues a Taylor Rule that reflects the properties of the hybrid model. The price and wage setting equations are the weighted averages of the corresponding New Keynesian and New Classical equations. The weighting process attempts to find the combination of weights and Taylor coefficients that make the combined model perform the best when compared with the auxiliary model. Le, Minford and Wickens (2008) determine an optimal weight for the New Keynesian wage setting of 0.1 and an optimal weight for New Keynesian price setting of 0.2, which indicates that only 10% of labour markets and only 20% of product markets are imperfectly competitive. Therefore, the model requires only a small amount of nominal rigidity to match the data. However, the case of the Chinese economy is different. Considering the large number of state owned enterprises in China with wages set by the government that seldom change and react very slowly to economic information, we set the New Keynesian share for wages to 0.5 in the Chinese DSGE model. In the Chinese economy, some portion of prices in product markets are competitive such as the prices for retail goods, but another portion is under imperfect competition such as the prices of oil, steel, and rice, so we set the New Keynesian share of prices to 0.5. That is, 50% of the labour market and 50% of the product market are imperfectly competitive according to this model.

As described before, we use the Directed Wald test to check the models' performance for particular aspects of the data. In the case of a single variable, we estimate the best ARMA(i, j) and estimate a VAR(1) for a group of variables. To assess the individual shocks

we take the Impulse Response Functions of the shock for the variables on which the shock exerts a major impact, generate the model-implied joint distribution of these Impulse Response Functions, and then compute the Wald statistic for the joint values in the data. To confirm our judgments on individual variances we examine the joint distribution of the variances.

# 2.3 Testing the DSGE model of China using the method of indirect inference

We apply the proposed testing procedure to the SW model for the period after the Economic reform of China (1978) until 2007.

As mentioned above, before proceeding to the testing we have to filter the data to make the data stationary. The unit root tests for the macro data show that some of the data are not stationary. We apply the Hodrick-Presscot (HP) filter to stationarise the data. The auxiliary VAR model has five main observable variables: output, consumption, investment, quarterly interest rate and quarterly inflation rate, and the VAR is of order one.

# 2.3.1 Evaluating the Chinese DSGE model using its own assumed error properties

Chapter 1 estimated the model through a Bayesian approach and have obtained the posterior means for the error variances and their autoregressive coefficients. Then we can test the original Chinese DSGE model using the Bayesian estimate posterior means for the error variances and the autoregressive coefficients. We performed these tests for 3 (output, interest rate, and inflation) and 5 (output, interest rate, consumption, investment and inflation) variables to determine the effect of adding more variables to the test on the model behaviour.

|                                   | Actual Est          | Lower Bound | Upper Bound       | IN/OUT  |
|-----------------------------------|---------------------|-------------|-------------------|---------|
| $A_Y^Y$                           | 4.02625             | 3.95142     | 4.08392           | IN      |
| $A_Y^R$                           | 0.27160             | 0.20850     | 0.32016           | IN      |
| $A_Y^{\pi}$                       | -0.14892            | -0.15351    | -0.14293          | IN      |
| $A_R^Y$                           | 0.91987             | 0.93066     | 0.94107           | OUT     |
| $A_R^R$                           | -0.05707            | -0.04834    | -0.03925          | OUT     |
| $A_R^{\pi}$                       | 0.00540             | 0.00371     | 0.00458           | OUT     |
| $A^{ Y}_{\pi}$                    | 62.70774            | 61.2500     | 63.56152          | IN      |
| $A_{\pi}^{R}$                     | 4.86894             | 3.63281     | 5.58887           | IN      |
| $A^{\pi}_{\pi}$                   | -2.00703            | -2.07513    | -1.89014          | IN      |
| $\sigma_{\scriptscriptstyle Y}^2$ | 2.6427              | 2.4325      | 7.0141            | IN      |
| $\sigma_{\scriptscriptstyle R}^2$ | 2.7256              | 2.3567      | 6.4021            | IN      |
| $\sigma^2_{\pi}$                  | 2.0206              | 1.2397      | 3.4422            | IN      |
| Overall                           | Wald percentile 100 | )           | Overall M-distand | ce 2.39 |

Table 5. VAR Parameters, Data Variances and Model Bootstrap Bounds of the Chinese

 DSGE Model with its Own Error Properties (3 variables).

The overall Wald percentile is 100, indicating that the model is rejected; the normalised overall Mahalanobis Distance is 2.39, indicating that the dynamic properties of the data are not very close to those implied by the model. Out of 9 unrestricted VAR parameters, only 3 lie outside the 95% bounds for the VAR based on the model simulation. All 3 data variances lie inside the 95% bounds. Table 5 shows the parameter and variance estimates and also their 95% bounds. Particularly for the partial autocorrelation of interest rate and output and the autocorrelation of interest rate, the model's bounds lie above the unrestricted VAR estimates. We can interpret these results as indicating excessive interest rate and output persistence, especially indicating excessive persistence of interest rates in the model. Further, the bottom 3 entries in Table 5 show that for the variances of output, interest rate and inflation lie inside the model bounds. However, the variances of output and interest rate are low compared with the data. Overall, therefore, based on the assumed error properties of the Chinese DSGE model, the model does not accurately fit the data.

| VAR coeffs   | Actual Est          | Lower Bound | <b>Upper Bound</b>  | IN/OUT  |
|--|---------------------|-------------|---------------------|---------|
| $A_Y^Y$  | 1.28021             | 0.87134     | 1.49128             | IN      |
| $A_{Y}^{R}$  | 1.23168             | 1.16902     | 1.37533             | IN      |
| $A_{\scriptscriptstyle Y}^{\scriptscriptstyle C}$      | -0.04922            | -0.04935    | -0.04906            | IN      |
| $A_{Y}^{I}$  | 0.01744             | 0.01281     | 0.01779             | IN      |
| $A_{\scriptscriptstyle Y}^{\pi}$                       | -0.01896            | -0.03462    | 0.00333             | IN      |
| $A_R^Y$  | 0.82890             | 0.65552     | 0.93551             | IN      |
| $A_R^R$  | -0.05278            | -0.08665    | -0.02454            | IN      |
| $A_R^C$  | -0.00215            | 0.00135     | 0.00400             | OUT     |
| $A_R^I$  | -0.00135            | -0.00443    | -0.00128            | IN      |
| $A_R^\pi$  | 0.00968             | 0.00485     | 0.01704             | IN      |
| $A_C^Y$  | 6.87409             | 5.83594     | 7.52692             | IN      |
| $A_C^R$  | -15.56754           | -12.01990   | -11.01242           | OUT     |
| $A_C^C$  | 0.72206             | 0.63020     | 0.65997             | OUT     |
| $A_C^I$  | 0.15347             | 0.17040     | 0.22212             | OUT     |
| $A_C^{\pi}$  | 0.82433             | 0.68402     | 0.78467             | OUT     |
| $A_I^Y$  | -4.58005            | -7.25293    | -1.72516            | IN      |
| $A_I^R$  | 6.25826             | 5.97684     | 6.56737             | IN      |
| $A_I^C$  | 0.51108             | 0.51012     | 0.53453             | IN      |
| $A_I^I$  | -0.00182            | -0.03414    | -0.00093            | IN      |
| $A^{\pi}_{I}$  | -0.16172            | -0.32712    | -0.02070            | IN      |
| $A^{\scriptscriptstyle Y}_{\pi}$                       | 27.84046            | 20.86719    | 30.98505            | IN      |
| $A_\pi^{\scriptscriptstyle R}$                         | 21.11750            | 19.66614    | 23.82722            | IN      |
| $A^{C}_{\pi}$  | -0.85979            | -0.86986    | -0.86432            | OUT     |
| $A^{I}_{\pi}$  | 0.50278             | 0.43376     | 0.51681             | IN      |
| $A^{\pi}_{\pi}$  | -0.35478            | -0.60110    | 0.04053             | IN      |
| $\sigma_{\scriptscriptstyle Y}^{\scriptscriptstyle 2}$ | 1.9043              | 1.0183      | 1.2572              | OUT     |
| $\sigma_{\scriptscriptstyle R}^{\scriptscriptstyle 2}$ | 3.5278              | 3.6303      | 5.3280              | IN      |
| $\sigma_c^2$   | 1.1557              | 2.1299      | 3.7248              | OUT     |
| $\sigma_{\scriptscriptstyle I}^{\scriptscriptstyle 2}$ | 1.8009              | 1.1252      | 2.3354              | IN      |
| $\sigma^2_{\pi}$                                       | 6.0517              | 5.1503      | 5.3335              | OUT     |
| Over all W   | Vald percentile 100 | )           | Over all M-distance | e 7.468 |

After testing 3 variables we repeated the test for 5 variables, and these results are reported in Table 6.

Table 6. VAR Parameters, Data Variances and Model Bootstrap Bounds of the ChineseDSGE Model with its own error properties (5 variables).

The overall Wald percentile is 100, indicating that the model is again rejected. The normalised overall Mahalanobis distance is 7.468, a poor result that indicates that the model is far away from the dynamic properties it implies. This result is worse than the result from the test of the 3 variables, indicating that adding more nominal variables to the test decreased the ability of the model to pass the test. Out of 25 unrestricted VAR parameters, 6 lie outside the 95% bounds for the VAR based on the model simulation, and 3 out of 5 data variances lie outside their 95% bounds. The actual estimates and the upper and lower bounds of the parameters and variances are shown in the Table. Examining the actual estimates that lie outside their bounds shows that the model's bounds lie above the unrestricted VAR estimates for the partial autocorrelation of interest rate and consumption and the partial autocorrelation of consumption and investment. This can be interpreted as indicating excessive persistence in interest rates, consumption and investment. This is consistent with the results of the 3 variables test. The bottom 5 entries of Table 6 show the relationship of the data variances and their model bounds for each nominal variable. For output and inflation, the data variances lie above the 95% bound, however for consumption the model's bounds lie above the unrestricted VAR estimation of its variance. This could be interpreted as indicating excessive variation of consumption in the Chinese economy.

Overall, the results obtained by testing both 3 and 5 variables with the assumed error properties of the Chinese DSGE model indicate that the model is out of line with the data. If we compare the rejection here with Del Negro et al.'s (2007a) estimate of the  $\lambda$  weight for model when combined with an unrestricted VAR, it is clear that their estimate is consistent with our rejection of the model's overall specification.

# 2.3.2 Evaluating a hybrid model: a weighted combination of New Keynesian and New Classical models

In the field of macroeconomics, there are two rather different models with particular differences in the degrees of price and wage rigidity. The New Classical model has flexible prices and wages with only a simple one-period information lag for labour suppliers, while the New Keynesian model is highly rigid with Calvo price and wage setting. The Chinese DSGE model we have analysed thus far is adopted from Smets and Wouters' New Keynesian model.

In Smets and Wouters' New Keynesian model, the capacity utilisation is fairly flexible, so output is strongly affected by demand shocks, which in turn affects inflation via the Phillips Curve and then moves interest rates through the Taylor Rule. Demand can be affected by supply shocks directly, for example the return on capital can be changed by productivity shocks, thus affecting investment, and supply shocks play the role of "cost-push" inflation shocks such as price or wage mark-up shocks. Persistent shocks to demand will raise "Q" and this effect will be persistent, eventually creating an "investment boom", which will finally make the demand shocks more persistent via demand effects. This procedure makes the model act as an accelerator of shocks both on the demand and supply sides.

The New Classical model is significantly different. Fluctuations in output that are dominated by supply shocks such as productivity and labour supply shocks can be caused by an inelastic labour supply. However, investment and consumption respond to output in a standard Real Business Cycle manner. These reactions with the demand shocks can lead to market-clearing movements in real interest rates, and then movements of real interest rates will affect inflation via the Taylor rule. In the New Classical model, the main drivers of all variable movements are supply shocks, while demand shocks only contribute to the variability of nominal variables. For this reason, suitably sized and persistent supply shocks are needed to replicate the real variability and persistence in the NC model. On the other hand, a limited variance in demand shocks would be sufficient if we only want to replicate the limited variability in inflation and interest rates, and the supply shocks must be sufficiently autocorrelated to replicate their persistence.

The analysis above shows that the New Keynesian version of the Smets and Wouters model could not match the Chinese macro data. Essentially, the model generates too little nominal variation. Given this fact, we propose a hybrid model that merges the properties of both the New Keynesian and New Classical models. We assume that the wage and price setters supply labour and intermediate goods partly in a competitive market in which prices and wages are flexible and partly in a market with imperfect competition. We also assume that the size of each sector is determined by the facts of competition and will not be changed in our sample. However, the degree of imperfect competition is allowed to differ between labour and product markets. The general idea behind these assumptions is that there are some product sectors of economies where rigidity prevails and also other sectors in which prices are flexible; essentially this reflects the degree of competition in these sectors. Similarly, we can

apply these assumptions to labour markets to make some markets much more competitive than others. An economy could contain more or less flexibility in prices and wages due to the level of competition within the economy. A final assumption is that a Taylor Rule that reflects the properties of the hybrid model must be pursued by the monetary authority.

Formally, to model the price and wage setting for the hybrid model we assume that firms that produce intermediate goods have a production function that combines a fixed proportion of labour in imperfect competition with labour from competitive markets, so the labour used by intermediate producers is:

$$n_{t} = n_{1t} + n_{2t} = \left\{ \left[ \int_{0}^{1} (n_{1it})^{\frac{1}{1+\lambda_{w,t}}} di \right]^{1+\lambda_{w,t}} + \left[ \int_{0}^{1} (n_{2it}) di \right] \right\}$$

where  $n_{1it}$  is the imperfect competitive labour and  $n_{2it}$  is the competitive labour provided by the *i* th household at time *t*. To make things more clear, we can imagine that  $n_t$  represents the activities of an intermediary "labour bundler".

Note that  $n_{1t} = v_w n_t$ , where  $v_w$  is the share of total labour that is in the imperfectly competitive market, so  $n_{2t} = (1 - v_w)n_t$  and then  $W_t = v_w W_{1t} + (1 - v_w)W_{2t}$ . Every household utility contains the two sorts of labour in the same way, that is:

$$U_{it} = \dots - \frac{n_{1it}^{1+\sigma_n} \varepsilon_{1nt}}{1+\sigma_n} - \frac{n_{2it}^{1+\sigma_n} \varepsilon_{2nt}}{1+\sigma_n} \dots$$

Now  $W_{1t}$  is set according to the Calvo wage-setting equation and  $W_{2t}$  is set equal to the current expected marginal disutility of work. In the latter case, there is a one quarter information lag for current inflation, but this is ignored as usual for convenience as it is unimportant in the Calvo setting over the whole future horizon. The labour bundler has these wages in hand and offers a labour unit as above this weighted average wage, and then firms buy these labour units for use in production.

By the same logic, retail output is made up of a fixed proportion of intermediate goods in an imperfectly competitive market and intermediate goods bought from an imperfectly competitive market. The retail output is:

$$y_{t} = y_{1t} + y_{2t} = \left\{ \left[ \int_{0}^{1} (y_{j1t})^{\frac{1}{1+\lambda_{p,t}}} dj \right]^{1+\lambda_{p,t}} + \left[ \int_{0}^{1} y_{j2t} dj \right] \right\}$$

The intermediate producers set the prices for  $y_{1t}$  according to the Calvo mark-up equation on marginal costs and set the prices for  $y_{2t}$  at marginal costs. Note that  $y_{1t} = v_p y_t$ , in which  $v_p$  is the share of the imperfectly competitive goods market; so  $y_{2t} = (1 - v_p)y_t$  and  $P_t = v_p P_{1t} + (1 - v_p)P_{2t}$ . The retailers in the economy then combine these goods as above in a bundle that they sell at this weighted average price.

Apart from these equations mentioned above the first order conditions of households and firms will not be changed no matter what markets they operate in.

According to the assumptions above, the price and wage setting equations in the hybrid model will be weighted averages of the corresponding New Keynesian and New Classical equations. To achieve the best performance of the combined model when compared with the auxiliary model, the weighting process used to find the combination of the weights and Taylor coefficients is already an informal use of indirect inference. We carried out a grid search over the two weights, one for the product market and one for the labour market, to find the optimal set of weights using the criterion of minimising the normalised Mahalanobis distance.

Based on the facts of the Chinese economy, we find that the optimal weights are  $v_w = 0.5$  (the New Keynesian share for wages) and  $v_p = 0.5$  (the New Keynesian share for prices). That is, 50% of the labour market and 50% of product market are imperfectly competitive. The hybrid model requires a large amount of nominal rigidity to match the data. This reflects the reality in China in which even after the economic reforms of 1978 there are still large parts of the product and labour markets with imperfect competition.

The VAR results for the hybrid model with 3 variables (output, interest rate, and inflation) are reported in Table 7.

|  | Actual Est         | Lower Bound | Upper Bound      | IN/OUT   |
|--|--------------------|-------------|------------------|----------|
| $A_{Y}^{Y}$  | 1.36615            | 1.18176     | 2.42883          | IN       |
| $A_{\scriptscriptstyle Y}^{\scriptscriptstyle R}$      | -1.96991           | -2.12537    | -1.07443         | IN       |
| $A_Y^{\pi}$  | 0.06304            | -0.02164    | 0.07774          | IN       |
| $A_R^Y$  | 0.37517            | 0.19879     | 0.28845          | OUT      |
| $A_R^R$  | -0.51606           | -0.66469    | -0.58936         | OUT      |
| $A_R^\pi$  | 0.04880            | 0.05571     | 0.06285          | OUT      |
| $A^{\scriptscriptstyle Y}_{\pi}$                       | 22.24055           | 19.26953    | 34.86621         | IN       |
| $A^{\scriptscriptstyle R}_{\pi}$                       | -29.23034          | -31.73633   | -18.59131        | IN       |
| $A^{\pi}_{\pi}$  | 1.21738            | 0.21136     | 1.45435          | IN       |
| $\sigma_{\scriptscriptstyle Y}^{\scriptscriptstyle 2}$ | 1.8478             | 1.2336      | 1.8871           | IN       |
| $\sigma_{\scriptscriptstyle R}^2$                      | 1.5007             | 1.2669      | 1.6096           | IN       |
| $\sigma_{\pi}^{2}$                                     | 8.7810             | 1.1256      | 9.2801           | IN       |
| Overall V  | Vald percentile 10 | 00          | Overall M-distan | ace 2.09 |

 Table 7. VAR Parameters, Data Variances and Model Bootstrap Bounds of the

 Weighted Model with Estimated Coefficients (3 variables).

The full Wald percentile is 100, indicating that the hybrid model is rejected, as was the case for the original NK model. The overall normalised Mahalanobis distance of 2.09 implies that the hybrid model is substantially closer to the data than the NK model and that the hybrid model performs better when compared to the 3-variables test for the NK model which had an overall M-distance of 2.39. The difference between the hybrid model and the original NK model is the hybrid model's ability to reproduce the variances of the data. We used the structural errors from the model and the observed data and find that all of the data variances lie within the model's 95% confidence intervals. The first 9 entries of Table 7 show that only 3 out of 9 unrestricted VAR parameters lie outside the 95% bounds for the VAR based on the model simulation. There is no evidence that there is any excessive persistence in any variables. Overall, therefore, the assumed error properties of the Chinese DSGE model indicate that the model is out of line with the data, however the hybrid model performs better than the NK model in mimicking the Chinese economy and is substantially closer to the data.

| VAR coeffs   | Actual Est          | Lower Bound | <b>Upper Bound</b> | IN/OUT  |
|--|---------------------|-------------|--------------------|---------|
| $A_Y^Y$  | 0.36267             | 0.24133     | 0.63374            | IN      |
| $A_Y^R$  | 0.51018             | 0.55078     | 1.01527            | IN      |
| $A_{Y}^{C}$  | -0.03638            | -0.05173    | -0.03481           | IN      |
| $A_Y^I$  | -0.00261            | -0.00740    | 0.01558            | IN      |
| $A_Y^\pi$  | 0.04397             | 0.02208     | 0.04737            | IN      |
| $A_R^Y$  | 0.18050             | 0.07085     | 0.40857            | IN      |
| $A_R^R$  | -0.08336            | -0.40561    | -0.09790           | OUT     |
| $A_R^C$  | -0.00104            | 0.00087     | 0.01080            | OUT     |
| $A_R^I$  | -0.00867            | -0.02240    | -0.00797           | IN      |
| $A_R^\pi$  | 0.04210             | 0.03122     | 0.05257            | IN      |
| $A_C^Y$  | 5.42137             | 3.70117     | 9.12891            | IN      |
| $A_C^R$  | -1.37819            | -4.66968    | 2.15625            | IN      |
| $A_C^C$  | 0.74361             | 0.67842     | 0.79700            | IN      |
| $A_C^I$  | -0.14262            | -0.19080    | -0.05227           | IN      |
| $A_C^\pi$  | 0.35841             | 0.09033     | 0.52887            | IN      |
| $A_I^Y$  | 2.18995             | 1.45105     | 2.77344            | IN      |
| $A_I^R$  | 2.48024             | 2.09778     | 3.06348            | IN      |
| $A_I^C$  | 0.57725             | 0.54253     | 0.56778            | OUT     |
| $A_{I}^{I}$  | 0.06175             | 0.07178     | 0.11194            | IN      |
| $A_{I}^{\pi}$  | -0.39457            | -0.42188    | -0.33844           | IN      |
| $A^{ Y}_{\pi}$   | 7.50172             | 7.36914     | 9.45923            | IN      |
| $A_\pi^{\scriptscriptstyle R}$                         | 8.85850             | 9.58594     | 11.89307           | OUT     |
| $A_{\pi}^{C}$  | -0.63879            | -0.70946    | -0.63920           | OUT     |
| $A^{I}_{\pi}$  | 0.08919             | 0.07548     | 0.17102            | IN      |
| $A^{\pi}_{\pi}$  | 0.94345             | 0.78201     | 0.92383            | OUT     |
| $\sigma_{\scriptscriptstyle Y}^{\scriptscriptstyle 2}$ | 1.9218              | 1.2678      | 1.8059             | OUT     |
| $\sigma_{\scriptscriptstyle R}^2$                      | 1.5455              | 1.0861      | 1.7501             | IN      |
| $\sigma_c^2$   | 1.2479              | 1.0398      | 1.6350             | IN      |
| $\sigma_{\scriptscriptstyle I}^2$                      | 2.2040              | 1.5573      | 2.8125             | IN      |
| $\sigma^2_{\pi}$                                       | 9.0918              | 1.1488      | 9.2962             | IN      |
| Over all V   | Vald percentile 100 | )           | Over all M-distand | ce 4.43 |

 Table 8. VAR Parameters, Data Variances and Model Bootstrap Bounds of the

 Weighted Model with Estimated Coefficients (5 variables).

After testing 3 variables we repeated the test for the 5 main macroeconomic variables (output, interest rate, inflation, consumption, and investment) to see whether the hybrid model still performs better than the NK model. The results are presented in Table 8.

The overall Wald percentile of 100 tells shows that the hybrid model is rejected when we test for 5 variables, and the overall normalised Mahalanobis distance of 4.43 indicates that the dynamic properties of the data are not close to those implied by the model. This result is worse than the result from the test of 3 variables for the hybrid model, demonstrating that adding more nominal variables to the test reduces the possibility of passing the test. Compared to the M-distance of the test of 5 variables for the original NK model the hybrid model is closer to the data. Out of 25 unrestricted VAR parameters, 6 lie outside the 95% bounds for the VAR based on the model simulation. The model's bounds lie above the unrestricted VAR estimates for the partial autocorrelation of interest rate and consumption and the partial autocorrelation of inflation and interest rate. This can be interpreted as indicating the presence of excessive persistence in interest rates, consumption and inflation. The bottom 5 entries of Table 8 show the relationship of the data variances and their model bounds for each nominal variable. Only 1 out of 5 data variances lie outside their 95% bounds which represent the estimated variance of output. Comparing this result with the fact that 3 out of 5 data variances lie outside their 95% confidence intervals for the test of the NK model, again the testing results demonstrate the ability of the hybrid model to reproduce the variances of the data; this is the main difference between the hybrid model and the NK model. There is no evidence of any excessive variations in any variables. In summary, according to the assumed error properties of the Chinese DSGE model it is considered to be out of line with the data, however both the tests of 3 and 5 main macro-economic variables of the hybrid model prove that it performs better than the NK model in mimicking the Chinese economy and is substantially closer to the data.

Because the optimal combination indicates that half of the market participants behave in a competitive manner, it is not surprising to see that the variance decomposition of the hybrid model (shown in Table 9) shows that the supply shocks, in particular the productivity shock, explain most of the movements of the real variables except for investment, the movement of which is largely explained by the investment shock as well. Moreover, the productivity shock also explains a significant portion of the variations in the nominal variables. The demand shocks also make significant contributions to movements in interest rates and inflation.

| Shocks        | Prodty | Risk Pre | Gov.<br>Spend | Investmt | Monetary | Price<br>Mark-up | Wage<br>Mark-up | Total |
|---------------|--------|----------|---------------|----------|----------|------------------|-----------------|-------|
| Output        | 79.91  | 7.42     | 8.26          | 1.42     | 0.21     | 1.2              | 1.58            | 100   |
| Interest rate | 64.94  | 16.94    | 11.28         | 2.22     | 0.2      | 3.03             | 1.4             | 100   |
| Inflation     | 67.58  | 17.36    | 10.61         | 1.37     | 0.56     | 1.17             | 1.34            | 100   |
| Consumption   | 73.62  | 11.1     | 10.99         | 1.84     | 0.33     | 0.69             | 1.43            | 100   |
| Investment    | 47.75  | 2.88     | 9.49          | 38.97    | 0.1      | 0.4              | 0.41            | 100   |

Table 9. Variance Decompositions of the Weighted Model.

These results different from those of the NK model because the hybrid model acts more like a New Classical model. In the New Classical pattern, the supply shocks explain most of the variation and the demand shocks play a small part in the variability of real variables because of the one-period information lag. However, the demand shocks also contribute to the variability of nominal variables. A large portion of economic agents in the Chinese economy behave in the New Keynesian manner, with the results that aggregate supply and labour supply are more elastic and demand shocks have a greater impact on real variables. Note that the inflation variability in the model of the Chinese economy does not reflect the actual data variability very well.

We can also examine the VAR impulse response functions to two important shocks: productivity and investment shocks. Here, we identify the VAR shocks using the structural model. The impulse responses are shown in Figures 7 and 8.









We focus on the first five diagrams in each of the figures above. Comparing these impulse response functions to those of the NK model, the main difference is that the impulse responses here converge more quickly than those of the NK model. We can therefore say that the performance of the hybrid model, based on the Impulse Response Functions, is better than that of the original NK model.

## 2.3.3 The choice of Hybrid model

We have tested a Hybrid model in which the New Keynesian shares of the goods and labour markets are 50%. The choice of this weight of 0.5 should be explained. At a glance, setting the portions of the goods and labour markets with rigid prices and wages to 50% seems to be reasonable, but we must prove that this weight is an appropriate choice. In principle, the weight is to found by checking different values to see which give the best Wald statistic. This procedure is a kind of limited indirect inference estimation.

|                            | v=0.3 | $\upsilon = 0.5$ | $\upsilon = 0.8$ |
|----------------------------|-------|------------------|------------------|
| Full Wald                  | 100   | 100              | 100              |
| Overall M-distance         | 5 9/  | 2 09             | 1.96             |
| (for coeffs and variances) | 5.74  | 2.07             | 1.90             |

 Table 10. Comparison of the Goodness of Fits of Different Weights.

Table 10 reports the results of three different models with NK weights of 0.3, 0.5 and 0.8. First, not surprisingly, these three Hybrid models were all rejected despite the different weights of the NK part. This can be seen from the fact that the full Wald percentiles of these three models are 100. Therefore, we cannot judge which model is the best from the Wald percentiles. However, even though the three models were rejected totally we still must decide which is the closest to the real data. The overall Mahalanobis distances of the three Hybrid models were calculated and the reported overall M-distances suggest that increasing the NK weight in the model would make the rejection of the estimated model weaker, as the model with a weight of 0.8 is less strongly rejected than the model with a weight of 0.5, and the one with a weight 0.8; however, such a high weight for the NK part of the market is not reasonable for the Chinese economy. Looking at the M-distances of the last two columns, the

numbers are rather close, indicating that although the two models are rejected they are at almost the same distance to the data. Finally, considering both reality and the test results, we choose 0.5 as the NK weight in the Hybrid model.

# 2.4 Some issues about robustness and data stationarity2.4.1 The choice of auxiliary model

We have carried out the Indirect Inference testing for the Chinese DSGE model (NK model) and the Hybrid model. Just like any other empirical evaluation based on data, the Indirect Inference testing inevitably suffers from robustness issues. One of the issues regarding robustness in this context is the choice of auxiliary time series model that provides a benchmark description of the data against which the theoretical model is estimated and evaluated indirectly. We have chosen a VAR (1) as the auxiliary model in the Indirect Inference testing in the preceding exercise in this chapter. The descriptors chosen in the auxiliary model are the estimate of the coefficient matrix and the volatility of the data. Technically, when choosing the auxiliary model, a VAR of higher order or time series models of other types are also acceptable choices, depending on what the model must fit and to what extent. It is highly arguable why a VAR of order one could approximate the DSGE model, there are two main reasons why we have chosen VAR (1). First, as shown by the following, VAR (1) is robust when compared with higher order VARs; second, when we approximate and then test the DSGE model, especially for the first time, we would want to test the most important and basic features of the model rather than everything single bit of the whole model. We can think it in a casual way, when we compare a picture with a photograph of the same subject, obviously we could not expect the picture to be exactly the same as the photograph. This subsection examines how robust the ranking of our models is, in other words, the superiority of the hybrid model over the NK model, to the choice of differing orders of VAR.

This assessment is in principle made for the purpose of model evaluation. In a normal sense, using a VAR of higher order as the auxiliary model will make the model evaluation more demanding because the data are effectively required to fit more detailed features. Practically, this can be a way to further evaluate competing theories whose performance can be hardly

distinguished under parsimonious auxiliary models. However, this is not our purpose here since the model evaluations above showed that the Hybrid model was better than the NK model when a VAR (1) was chosen. We simply focus on whether the ranking of our models will changed when a VAR of higher orders is used as the issue of robustness is concerned.

|                            | VAR(2) |        | VAR(3) |        |  |
|----------------------------|--------|--------|--------|--------|--|
|                            | NK     | Hybrid | NK     | Hybrid |  |
| Full M-distance            |        |        |        |        |  |
| (for coeffs and variances) | 4.59   | 3.88   | 6.32   | 4.89   |  |

**Table 11. Model performance under Different Auxiliaries** 

Table 11 shows that this is not the case when a higher order VAR is chosen as the auxiliary model. The M- distances in Table 11 are in comparison with the M-distances from Table 5 (for NK model) and Table 7 (for Hybrid model). The reported overall Mahalanobis distances suggest that increasing the order of VAR would make the rejection of the estimated model even stronger in cases both for the NK model and the Hybrid model due to the increased burden placed on the models. It is also clear that in all cases the Hybrid model is always rejected less strongly than the NK model, thus the Hybrid model is more preferred compared to the NK model regardless of the order of the VAR. Expressing it in another way, the ranking of the NK and Hybrid models is robust to the choice of auxiliary models.

### 2.4.2 The use of the H-P filter

The H-P filter can be used to stationarise the data by detrending the variables. Although the H-P filter is widely used, it has a drawback in that it is an ad-hoc and arbitrary solution to the problem that over-estimates the differences and therefore induces cycles in the data. Additionally, as a two-sided filter, incorporating future outcomes in the current filtered value can distort the expectations structure of the model. If each series is filtered in this way, the structural errors would then also be H-P filtered, therefore inducing serial correlation. That is to say, the structural errors filtered are not the original errors of interest. However, if the same

H-P filter is used to filter each series and the distortion to the expectations structure is ignored, the impulse response functions of the original variables to the original errors are the same as those of the filtered data to the filtered errors. In this thesis we adopt the H-P filter to all the seven main macro-economic variables with the smooth parameter of 1600 which has been advised to be reasonable for quarterly date by Hodrick and Prescott.

A better solution that would not induce cycles or distort the expectations structure is simply to remove a linear trend from the data. However, we did not use the linear detrending method because it could not make the data stationary. Despite the deficiencies of the H-P filter mentioned above, we still used this method as it is standard practice, with the result that both the data and bootstraps were filtered in the same way.

### 2.5 Conclusion

We have applied the method of indirect inference to test two different versions of a wellknown DSGE model in terms of its dynamic performance and volatility for Chinese posteconomic reform data. We compared these two versions of models and found that neither of the models can fit the data. Additionally, choosing fewer variables in the test (output, interest rate, and inflation) could make the model closer to the data than choosing more variables (output, interest rate, inflation, consumption, investment) as more variables increased the burden on the models. Using the structural errors jointly implied by each model and the data, the NK model could not fit the data variances. The NK model generates low variation in output and interest rate but excessive variation in consumption. However, when we combine the NK and NC models as a weighted model to give a hybrid model that is a mixture of imperfectly competitive and flexible-price markets, the hybrid model provides a much better fit to the data with high flexibility in both the product and labour markets, even though it too is rejected especially with respect to interest rate behaviour. Considering the frequent rejection of the models we have undertaken an extensive examination of the robustness of our test procedure. We find that these results are not due to the uncertainty in the parameters of the original model and that the test is a consistent test with a good power. The significance of our test statistic appears to be due to the specification of the model rather than to any detectable weakness in the test procedure or to poor estimates of the original model.

Some argue that tests of DSGE models are useless as the models are misspecified. The reason that likelihood-based tests of DSGE models tend to be rejected is that they incorporate all features of a model. A test usually takes into account all aspects of a model but the theory often says more about some aspects of a model than others. For example, the field of macroeconomics is usually more interested in the long-run properties of a model rather than the short-run lag structure. Even in the situation that a model is rejected it is also important to know which aspects of the model lead to the rejection and which aspects are robust to the data. Then it may be sensible to consider particular aspects of a model such as those that would affect policymaking objectives. In our test, we found that a weighted model with high price and wage rigidities could fit the data quite closely and that it provides accurate predictions for the three key macro variables of GDP, inflation and interest rate. This finding suggests that this model can provide good advice to policymakers, who are mainly interested in these three outcomes.

## Chapter 3

## Estimating the model by Indirect Inference

#### **3.1 Introduction**

The previous chapter investigated and compared the capacities of a series of models (including the NK model and weighted model) to mimic the dynamics and volatility of the actual data, and found that the original New Keynesian DSGE model (based on Smets and Wouters' model) could not replicate the features of the Chinese economy but the weighted model with an NK weight of 0.5 did provide an accurate description. However, fixing model parameters in this way is a fairly strong assumption to make in testing and comparing DSGE models. This is because the numerical values of a model's parameters could be calibrated anywhere within a range permitted by the theoretical structure of the model, which means that a model rejected with one set of parameters may not be rejected with another. Looking back to the tests in the previous chapter, potentially the original NK model was rejected not because the model specification was incorrect but rather because the parameter set was "wrong"; in the same way the weighted model could have been saved only by a set of "better" parameters.

#### **3.2 Parameter uncertainty**

By asking whether a particular fixed set of structural parameters could have generated the data, the direct Wald test in the last chapter tried to pin down the potential sources of misspecification. However, it is possible that another set of parameters might be needed to explain how the data are generated. If no set of parameters can be found under which the model passes, then we can conclude that the model itself is rejected. In fact, this is the basis of indirect inference, a method that tries to find the parameter set that comes closest to matching the data and then decides whether this set of parameters passes the test or not. Now we propose the concept of "Indirect Inference Estimation". Generally speaking, the role of Indirect Inference Estimation in our procedure is to maximise the chance of the model passing the test. Maximising the chances that the set of parameters passes the test is in fact minimising the Wald value. The procedure begins with testing a single parameter set that provides a test of the full model with the addition of a full indirect Inference search over the whole parameter space. Consequently, we can interpret our test procedure as a way of

addressing the issue of parameter uncertainty. In a more direct way, the issue of parameter uncertainty can also be addressed by examining whether parameter sets for the model can be found that reduce the Wald statistic. A powerful algorithm based on Simulated Annealing conducts a search over a wide range around the initial values in an optimised search accompanied by random jumps around the space. We use this SA algorithm to calculate the minimum-value full Wald statistic for the Chinese DSGE model in its hybrid version over the whole sample period.

The direct Wald test has been applied in our study to examine parameter uncertainty where the original parameters are treated as exact and the test is proven to be powerful against false models. A Monte Carlo experiment can be conducted to further examine the power properties of the test. The procedure of the Monte Carlo experiment can be described as follows: first, the data have to be generated from the model and then both the Full and the Directed Wald values are computed. The rejection rate of the models (normally at 5%) has to be checked. All the even coefficients in the models are arbitrarily raised including those of the error time series; all the odd coefficients are lowered by a certain percentage x%, where x rises steadily from 1 to 7. Meanwhile, the innovations that are bootstrapped in the mis-specified models have to be kept the same for each data set just like in the true model. The consequence is that the degree of model numerical mis-specification will be steadily increased by x. This method should have considerable power against poor parameter values.

The main topic of this chapter is the alternative way of examining parameter uncertainty in which the estimation process is embedded within the testing procedure. Normally, we could estimate the parameters of a model by Bayesian, Indirect Inference or some other estimation methods and then test the unknown true model by conducting a test on the already estimated parameters. The basic idea is to test a model with an unknown parameter set, say  $\theta$ , solely using its own estimated  $\theta$ . This method can be interpreted as the estimation-augmented test. A Monte Carlo experiment is needed to determine the distribution of the test. The experiment includes a few steps of a true model with the parameter set  $\theta$ . The first step is to generate a random sample from the model and then estimate the parameters from the auxiliary equations as  $\hat{a}$ ; the second step is to estimate the parameter set  $\theta$  of the model as  $\hat{\theta}$  from this data

sample through the chosen estimation method; third, the bootstrap distribution of  $\hat{a}$  is generated from this  $\hat{\theta}$  and then the Wald value is calculated for this sample; the rest of the procedure is to repeat the work above for a large number of samples and calculate the resulting Wald distribution and its critical values. Following the procedures above would produce the distribution of a test statistic that tests whether a model is true based on its estimated parameters.

Le, Minford et al. (2011) investigated the estimation-augmented test on the Smets-Wouters model; they used both the Bayesian and Indirect Inference estimators. First they carried out a Monte Carlo experiment for the full sample and found a substantial expansion of the distribution of the Wald values. An essential feature of the Bayesian estimates from the SW model is that they have a wide joint distribution so the corresponding VARs include many model-simulated VARs that are a long way from the corresponding data-estimated VARs. Any VAR that is generated from a data sample is very likely to lie within the 95% bounds of such a wide distribution because it is not one of these very poor-fitting cases. They also performed the same Monte Carlo experiment with the Indirect Inference estimator and found a similar distribution to the Bayesian with only a limited number of replications. Thus, the estimation-augmented test has a rather low power. Le et al. tested several models that differed radically in specification ranging from New Keynesian through weighted to New classical; the test's low discrimination power is evident through the fact that each model would easily pass the test on the full VAR auxiliary model. To further investigate the test's power in a formal way these authors generated the Wald distributions for a different mis-specified model where the Wald values come from the estimation of this "wrong" model on the data generated from the "true" model, and they calculated the rejection rate of the "wrong" model at a 5% level. To make the model as close as possible to the original SW model a weighted model with New Keynesian weights of unity was chosen as the "true" model; and a weighted model with weights of 0.3 on the New Keynesian components of both labour and goods markets was chosen as the "wrong" model. The weights were fixed in estimation in both models. These results turned out to indicate that even the mis-specified "wrong" model was rejected only around 50% of the time. Comparing this low rejection rate with the 100% rejection rate of the Directed method for a model with a weight of 0.9 on NK, which is much closer to the "true" model in their paper, the estimation-augmented test has only limited

power against a highly mis-specified model. Generally speaking, although the idea of allowing for possible parameter variation occurring through estimation is attractive in that it allows for models to be tested only in terms of their estimated parameters, its power seems to be very low. Thus, further work is needed in the area of parameter uncertainty.

Thus, to find out whether the model is "true" or "wrong" we cannot assume that the model's parameters are always fixed at particular values; rather we must search over the full range of potential values that the model can take and test whether the model with the best set of parameters from a particular point of view can be accepted by the data. Rather, we can say that the models should be fully estimated before they are tested and evaluated against each other. In this chapter, we use the method of Indirect Inference to estimate the weighted model that was tested in the previous chapter. Our aim is to find the best version of parameters as suggested by the data and to test their validity.

#### **3.3 Indirect Inference as a Method of Estimation**

As mentioned in Chapter 2, the method of Indirect Inference was originally designed for the estimation of structural models and was only recently developed for model evaluation. As stated in the preceding analysis, the method of Indirect Inference is different from other methodological alternatives in its use of an auxiliary model that is totally independent from theory to generate descriptors of the data, thus indirectly estimating or evaluating the theoretical model. In the model evaluations carried out in the previous chapter, parameters were taken as given and the Wald statistics were calculated through the Indirect Inference method to see whether the real data-based estimates of the chosen auxiliary model were captured by the joint distribution suggested by the theoretical model. The purpose of this was to determine whether the proposed economic theory was close to the real data so that it could be taken as a data-generating mechanism from a statistical point of view. However, in this chapter, the method of Indirect Inference will be used in a different way. The aim here is no longer to measure how close the theoretical model is to the real data, but rather to find a set of parameters that minimises the distance between the model and the data when the theoretical

model is taken as true. The common feature of the evaluation and estimation is the calculation of the Wald statistics based on the estimates of the selected auxiliary model. The question raised here is which set of structural parameters make the real data-based estimates closest as predicted by the model simulations.

Based on the method proposed above, the Indirect Inference Estimation procedure can be summarised in five steps.

- 1. Choose an auxiliary model and estimate it on the real data to obtain the benchmark estimates;
- 2. Give initial values to the structural parameters that have to be estimated and use them to generate a number of pseudo samples of simulated data with the theoretical model;
- 3. Estimate the chosen auxiliary model on the simulated data obtained from step 2 to obtain the joint distribution of the chosen estimates (in step 1) and their mean;
- 4. Calculate the Wald statistics and the square of the "Mahalanobis distance" to measure the distance between the benchmark estimates obtained in step 1 and the mean obtained in step 3.
- 5. Repeat steps 2, 3 and 4 above until the Wald statistic is minimised.

It is not hard to see that the full estimation process of Indirect Inference would typically involve a large number of iterations based on the calculation of Wald statistics.

This chapter continues to use a VAR(1) as the auxiliary model to preserve comparability of the implications of the estimated models to those of the calibrated models, and the VAR(1) are used as descriptors of the coefficient matrix and the variances of the data. Technically, higher order VARs such as VAR(2) and VAR(3) or other alternatives such as ARIMAs are also suitable for both testing and estimation, but a VAR(1) is generally acceptable for producing a parsimonious description of the data. Just like the previous exercise, the pseudo data used to imply the joint distribution of the VAR(1) estimates are simulated by

bootstrapping the structural errors. Again, the Wald statistic is calculated when such a distribution is found and compared to the estimates based on the real data, and the method called Simulated Annealing will be used to find the set of "best" parameter values, which will help to target the minimum Wald value. To minimise the Wald value, initial values of the structural parameters are needed for numerical iterations. Here, the initial values we input are the calibrated values, and we also allow the parameters under estimation to roam around -50% to +50% of their calibrated values. Note that certain parameters such as the discount factor, the steady state debt/GDP ratio, the consumption/output ratio and other parameters in the estimation procedure, which are the depreciation rate  $\delta$  fixed at 0.025, the exogenous spending-GDP ratio  $g_y$  set at 0.18, the steady-state mark-up in the labour market  $\lambda_w$  set at 1.5, and the curvature parameters of the Kimball aggregators in the goods and labour markets  $\varepsilon_p$  and  $\varepsilon_w$ , both set at 10.

#### **3.4 Estimation Results**

In the previous chapters, the structural parameters were restricted to be the calibrated values that are commonly accepted in the literature; in this section we use the method of Indirect Inference to estimate the best fitting values of the structural parameters so that those restrictions are released. The purpose of this step is to re-evaluate the Chinese DSGE model on its best versions based on the real data. Additionally, the robustness of the earlier findings under calibration will be evaluated in the following procedures in this section.

It is reasonable for us to expect that the estimated version of the Chinese DSGE model through the method of Indirect Inference and Simulated Annealing would perform no worse than the Bayesian estimation. The reason for this is that when calibrated values are set as the "guessed" initial values for the structural parameters, the Simulated Annealing method will start searching from these initial values and replace them with "better" values according to the data whenever a smaller Wald statistic can be found. The end of this procedure will be the point where the Wald statistic can no longer decrease, indicating that we have found the best estimates of the structural parameters. Hence, if with the Bayesian estimated parameters the model is not rejected then of course we believe it would not be rejected either with the improved estimated values. Even if the model is rejected with the parameter values from Bayesian estimation, there is still a chance that the estimates from the SA estimation would help the model to pass the test. The model estimated using the SA method is expected to be more precise from the point of view of the data, and of course would be less likely to be rejected because the SA method can be thought of as an effective way of adjusting the calibrations. Table 10 compares the estimates of the Hybrid DSGE model for China under both Bayesian and Indirect Inference methods.

| Parameter  | <b>Bayesian Estimation</b> | SA Estimation |
|--|----------------------------|---------------|
| $^{\varphi}$ Adj. Cost                             | 7.6584                     | 6.8278        |
| $\sigma_{c}$ Inter. Subst.                         | 1.6667                     | 0.5851        |
| <sup>h</sup> Habit Form.                           | 0.7019                     | 0.7989        |
| $\xi_w$ Calvo Wage                                 | 0.8012                     | 0.6792        |
| $\sigma_{_L}$ Elas. L Suply                        | 2.5568                     | 2.6948        |
| $\xi_p$ Calvo Price                                | 0.8754                     | 0.8074        |
| <sup><i>t</i></sup> <sub><i>w</i></sub> Wage Index | 0.5909                     | 0.5949        |
| $l_p$ Price Index                                  | 0.683                      | 0.4395        |
| ΦFixed Cost  | 1.4474                     | 1.4912        |
| $r_{\pi}$ Inf. Policy                              | 1.855                      | 1.7552        |
| $ ho_{ m Coef.}$ Of Lag r                          | 0.7065                     | 0.9126        |
| <sup>r</sup> , Output gan Policy                   | 0.0441                     | 0.083         |
| $r_{\Delta y} \Delta Y$ Policy                     | 0.2097                     | 0.2508        |

| $100(\beta^{-1}-1)$              | 0.05   | 0.05   |
|----------------------------------|--------|--------|
| $\overline{\gamma}$ Trend Growth | 0.5225 | 0.5306 |
| $\alpha_{\rm Capital}$ Share     | 0.2832 | 0.2391 |
| $\sigma_{a}$ Shock Std. Dev.     | 2.9756 | 1.0525 |
| $\sigma_{_b}$                    | 0.7691 | 0.0266 |
| $\sigma_{_g}$                    | 1.9907 | 0.8048 |
| $\sigma_{\scriptscriptstyle l}$  | 2.4679 | 0.4619 |
| $\sigma_{r}$                     | 0.7306 | 0.0943 |
| $\sigma_{_{p}}$                  | 2.9982 | 0.0676 |
| $\sigma_{_w}$                    | 2.9968 | 0.0712 |
| $\rho_a$ Shock Persistence       | 0.8396 | 0.9805 |
| $ ho_b$                          | 0.2619 | 0.9126 |
| $ ho_{g}$                        | 0.8076 | 0.9532 |
| $ ho_l$                          | 0.3797 | 0.6066 |
| $ ho_r$                          | 0.1197 | 0.5129 |
| $ ho_p$                          | 0.9997 | 0.8865 |
| $ ho_{\scriptscriptstyle W}$     | 0.9992 | 0.7996 |
| $\mu_p$ Price Mark-un            | 0.551  | 0.7067 |
| $\mu_{w}$ Wage Mark-up           | 0.473  | 0.4104 |

 Table 12. Estimates of the Hybrid Model (Results from Bayesian Estimation and Simulated Annealing Estimation).

In Table 12, the middle column shows the posterior means of the parameters of the Hybrid model obtained through the Metropolis –Hastings algorithm and the third column presents the parameter values of the Hybrid model obtained by Simulated Annealing estimation, i.e., the Indirect Inference estimation.

We note that overall the Indirect Inference estimated parameter values of the Hybrid model are reasonably close to the Bayesian estimates; however, significant differences are observed for some parameters. The elasticity of the cost of changing investment is estimated to be 6.8278, slightly lower than that from the Bayesian estimate; this suggests that the response of investment to changes in the value of capital is comparably slow. The intertemporal elasticity of substitution is estimated to be 0.5851, much lower than the Bayesian estimate, while the elasticity of labour supply is 2.6948, only slightly higher than the Bayesian estimate. The estimate of the habit parameter is slightly higher than the Bayesian estimate, and both of these are very near to the calibrated value. The degrees of indexation to past inflation in goods and labour markets are estimated to be 0.8074 and 0.6792, respectively; they are lower than but very close to their counterparts under the Bayesian method. The Calvo probabilities for prices and wages are estimated to be 0.4395 and 0.5949, respectively, so the wage stickiness here is the same as in the Bayesian version but the price stickiness is lower than that of Bayesian estimation. In this case, the SA estimation seems to be more reasonable. The fixed cost parameter is estimated to be almost the same under the two estimation methods, and the share of capital in production is estimated to be slightly lower than the Bayesian estimation.

After checking the estimates of the main behavioural parameters we now turn to the monetary policy reaction function parameters. These parameters describing the monetary policy rule are based on a standard Taylor rule. The long-run reaction coefficient to inflation is estimated to be 1.7552, slightly lower than the Bayesian estimate, and they are both high when compared with the calibration. The long-run reaction to the output gap is estimated to be 0.0830, higher than the Bayesian estimation, indicating that policy does not appear to react very strongly to the output gap level. However, the reaction coefficient to the changes in the output gap is estimated to be 0.2508, which is higher than the Bayesian estimate and indicates that the policy responds strongly to changes in the output gap in the short run. There is a

considerable degree of interest rate smoothing as the coefficient on the lagged interest rate is estimated to be 0.9126, which is higher than both the Bayesian estimate and the calibration.

Finally, a number of observations can be made regarding the estimated processes for the exogenous shock variables. Generally, the data are highly informative on the stochastic processes for the exogenous disturbances. The productivity, government spending and risk premium processes are estimated to be rather persistent, with AR (1) coefficients of 0.9805, 0.9126 and 0.9532, respectively, much higher than their Bayesian counterparts. On the other hand, the Bayesian estimation suggests that the wage and price mark-up processes are the most persistent. The high persistence of the productivity and risk premium processes implies that at long horizons most of the forecast error variance of the real variables will be explained by these two shocks. The standard errors of all the shocks under the SA estimation are lower than those from the Bayesian estimation. In particular, the standard deviations of the risk premium shock, monetary policy shock and both price and wage mark-up shocks are relatively low. The application of the method of Indirect Inference could account for the tremendous improvements in the estimation, such as the large decrease in the standard deviations of the shocks.

|                            | Bayesian Estimation | SA Estimation |  |
|----------------------------|---------------------|---------------|--|
| Full Wald                  | 100                 | 03.2          |  |
| (for coeffs and variances) | 100                 | 93.2          |  |



Table 13 confirms that the model estimated by the method of Indirect Inference (SA estimation) fits the data better than the model developed by Bayesian estimation as expected. The reported Wald percentile has decreased significantly from the Bayesian estimation. The full Wald statistic shows that the model is not strongly rejected at 95% and could explain the data up to around 93%.

### **3.5 Applications**

### 3.5.1 Variance Decomposition

After demonstrating that the model estimated through an Indirect Inference approach can fit the Chinese macro-economic data, we now use this model to investigate some key macroeconomic issues and determine the main driving forces of the main macro-economic variables.

| Shocks        | Prodty | Risk Pre | Gov.<br>Spend | Investmt | Monetary | Price<br>Mark-up | Wage<br>Mark-up | Total |
|---------------|--------|----------|---------------|----------|----------|------------------|-----------------|-------|
| Output        | 54.94  | 0.18     | 31.61         | 12.94    | 0.31     | 0                | 0               | 100   |
| Interest rate | 13.75  | 75.33    | 0.18          | 1.62     | 9.12     | 0                | 0.01            | 100   |
| Inflation     | 13.68  | 30.76    | 0.22          | 0.2      | 55.11    | 0.01             | 0.03            | 100   |
| Consumption   | 85.33  | 0.79     | 6.58          | 6.05     | 1.25     | 0                | 0               | 100   |
| Investment    | 29.02  | 0.09     | 0.33          | 70.37    | 0.19     | 0                | 0               | 100   |

Table 14. Variance Decomposition of the Hybrid Model

Table 14 reports the forecast error variance decomposition of output, interest rate, inflation, consumption and investment. The movements in output are primarily driven by the productivity shock and government spending shock, with the investment shock only making a small contribution. The driving force of movements in consumption mainly seems to be the productivity shock. It is not surprising to see that the movements in investment are best explained by the investment shock; the same applies to movements in interest rates, which are primarily explained by the risk premium shock. The strongest driving force of movements in inflation is the monetary policy shock, and the risk premium shock also makes a considerable contribution. Overall, the supply shocks such as the productivity shock and the demand shocks such as the government spending shock, the risk premium shock and the investment shock contributed significantly to the movements in interest rates, inflation and investment.

### 3.5.2 Impulse Response Functions

To explore how the model behaves under all these shocks, we examine the effects of the shocks on the main variables. Impulse response functions are the main tools that are used to address this issue.



# Figure 10 Impulse Responses to Productivity Shocks (dy=output, pinfobs=inflation, robs=interest rate, dc=consumption, dinve=investment, dw=real wage, labobs=labour).

Figure 9 shows the impulse responses of the seven key macro-economic variables to the productivity shock. First, we can see that the productivity shock has long-run effects on output, consumption and interest rate; these three variables converge after 20 periods. Under the estimated monetary policy reaction function, nominal and real interest rates fall, but not enough to prevent the opening up of an output gap and a fall in inflation. On the other hand, the effect of an increase in productivity on investment is large but converges quickly. The

productivity shock has a negative effect on inflation and also converges quickly. There is an immediate fall in labour (hours worked) in the presence of a positive productivity shock in line with Gali (1999), Francis and Ramey (2005) and Gali and Rabanal (2004) who have argued that the immediate decrease in labour is due to the nominal price rigidities, habit formation and adjustment cost to investment.





Figure 10 presents the impulse responses of the main variables to the risk premium shock. As discussed earlier in this chapter, the risk premium shock explains most of the movements in interest rates and a considerable part of the movements in inflation. Here, we see that the risk premium shock has a negative effect on inflation and a positive effect on interest rates, both

of which last for a long time. However, the risk premium shock does not have a long-run effect on other variables.



Figure 12 Impulse Responses to an Investment Shock (dy=output, pinfobs=inflation, robs=interest rate, dc=consumption, dinve=investment, dw=real wage, labobs=labour).

Figure 11 reports the impulse responses to the investment shock. According to the estimated model, the investment shock accounts for most of the movements in investment. Here, the effect of the investment shock on investment takes place in the short-run. Except for the interest rates, the effects of the investment shock are short-lived. The investment shock also has a negative effect on inflation but positive effects on other variables.



# Figure 13 Impulse Responses to a Government Spending Shock (dy=output, pinfobs=inflation, robs=interest rate, dc=consumption, dinve=investment, dw=real wage, labobs=labour).

The government spending shock has negative effects on consumption and investment and positive effects on all other variables. Output will not be affected by this exogenous spending shock for long, but interest rate and labour did not converge quickly following a government spending shock.



Figure 14 Impulse Responses to a Monetary Policy Shock (dy=output, pinfobs=inflation, robs=interest rate, dc=consumption, dinve=investment, dw=real wage, labobs=labour).

Figure 13 shows the behaviour of the key variables to the monetary policy shock. It seems that all the variables except for interest rates are affected negatively by the monetary policy shock. The monetary policy shock is the main driving force of the movements in inflation according to our estimation results; it has comparably long effects on inflation and labour but short-lived effects for other variables.

Overall, the demand shocks such as the government spending shock, risk premium shock and investment shock have significant short-run effects on output; however, the supply shock, that is, the productivity shock is the main force that drives output in the medium to long run.
#### **3.6 Robustness check for the choice of auxiliaries**

As was done in the previous chapter, we need to check the robustness of the choice of auxiliary models. We have chosen a VAR (1) in which the chosen descriptors are the estimates of its coefficients matrix and the data variance as the auxiliary model in the Indirect Inference estimation procedure. As stated earlier, a VAR of higher order or other types of time series models are completely possible depending on what the model is required to fit and to what extent. In this section, we proceed by checking the robustness of the ranking of the NK model and the Hybrid model to determine whether the superiority of the Hybrid model over the NK model will be overturned if the choice of VAR is different. This assessment can be done for both model estimation and model evaluation, however here it is restricted to the model evaluation for simplicity based on the estimates.

|                            | VAR(2) |        | VAR(3) |        |
|----------------------------|--------|--------|--------|--------|
|                            | NK     | Hybrid | NK     | Hybrid |
| Full Wald                  |        |        |        |        |
| (for coeffs and variances) | 100    | 99.7   | 100    | 100    |

 Table 15. Performance of the Models under Different Auxiliaries.

Table 15 demonstrates that the superiority of the Hybrid model over the NK model cannot be overturned when the chosen auxiliary model is VAR (2) or VAR (3). The reported full Wald percentiles suggest that increasing the order of VAR would make the rejection of the estimated model even stronger for both the NK model and the Hybrid model due to the greater burden placed on the models. Comparing the Wald percentiles of the Hybrid model in Table 15 and that from Table 13, we can see clearly that with lower order of the auxiliary VAR, the Hybrid model is less rejected. It is also clear that the Hybrid model is always less strongly rejected than the NK model regardless of the order of the VAR. Alternatively, the ranking of the NK and Hybrid models is robust to the choice of auxiliary models. In summary, it is acceptable for us to choose a VAR (1) to mimic the theoretical model.

#### **3.7 Conclusion**

This chapter applied a new Indirect Inference method of estimation to the Hybrid DSGE model that was investigated in Chapter 2 and compared the Hybrid DSGE models based on Bayesian estimation and Indirect Inference estimation. The Hybrid model was estimated because it incorporates both the NK and NC features. The aim of this chapter was to determine whether the model's full capacity is allowed for using the method of Simulated Annealing based on indirect inference. The model contains a set of best fitting parameters, which significantly improves its performance even though there is no significant difference when compared with the Bayesian estimated values. However, the Hybrid model remains superior to the NK model as shown in the Indirect Inference testing exercise in Chapter 2, so that even under estimation the Hybrid model can better replicate the Chinese economy.

The variance decomposition shows which structural shocks playing the main roles in the development of the main macro-economic time series in the Chinese economy since the economic reform in 1978. Overall, three structural shocks explain a significant fraction of the variations of output, inflation and interest rates: the productivity shock, the risk premium shock and the monetary policy shock. The productivity shock is very important for output as it accounts more than half of the development in output. The variations in interest rates are mainly explained by the risk premium shock. Both the monetary policy shock and risk premium shock are important determinants of inflation, but the former seems to be more important than the later. The productivity shock is the main driving force of consumption and variations in investment largely depend on the investment shock.

The effects of the various structural shocks on the Chinese economy are analysed through impulse response functions. Overall, these effects are quantitatively in line with existing studies. For example, a positive productivity shock leads to an increase in output, consumption, investment and the real wage, but has a negative effect on employment as explained in many previous studies. A temporary monetary policy shock creates a temporary increase in the nominal and real interest rates and a decrease in both output and inflation. In summary, combining these findings with the preceding study in Chapter 1, although Bayesian estimation methods are effective practical tools for improving DSGE model estimates by incorporating prior information about the macro economy, this chapter shows that the method of Indirect Inference is a useful companion to the Bayesian approach.

# Chapter 4

# Summary of the thesis

#### 4.1 Some valuable summaries of the thesis

In the first chapter of the thesis we have adopted the standard Smets and Wouters (2007) DSGE model which is a New Keynesian model with a rather high price and wage rigidity (according to their estimation the rigidity is nearly 0.9). In the following chapter we first tested this model and concluded that the "pure" NK model couldn't help the Chinese model to match the real data. In order to find a model that can replicate the real economy we therefore introduced a Hybrid model in which there is a New Keynesian weight of 0.5 in both goods and labour markets, and the test results showed that this "mixed blood" model is performing better than the "pure" NK model. Actually in the real world we cannot have an economy which is "pure" Keynesian or "pure" Classical; there will always be perfect and imperfect competitions. There is a number of economists have investigated in the rigidity in DSGE models. Dixon and Lebian (2010) established a new paradigm for modelling price and wage setting behaviour, Generalised Taylor Economy (GTE) and Generalised Calvo Economy (GCE) are developed to capture the heterogeneity in price and wage setting behaviour. They presented a framework that is able to encompass all price and wage frameworks based on the idea of an economy consisting many sectors differentiated by how long a contract last; that is to say in the log-linearized equilibrium conditions of a DSGE model there can be potentially many sectors, each with a different contract length. When each sector has a Taylor-style contract then the economy is called GTE, when each sector has a Calvo-style contract then the economy is called GCE. The standard Taylor model predicts that all durations of contracts are identical and the standard Calvo model predicts that durations are distributed according to the exponential distribution. Due to these restrictive implications for the distribution of durations they established GTE and GCE set-ups which allow the distribution of durations implied by the pricing model to be exactly the same as the distribution found in micro-data. In particular, GCE assumes that firms have time-varying hazard rates, in contrast to the constant hazard rate as in simple Calvo model. At a particular point in time, prices in this economy have different ages and, thus, different hazard rates. Our Hybrid model, according to their theory, is a special case of the simple Calvo model.

Now let's look at some comparisons of NC, NK and Hybrid models.

| Parameter   | NC model | NK model | Hybrid model |
|---|----------|----------|--------------|
| ∅ Adj. Cost   | 8.1106   | 10.6638  | 6.8278       |
| $\sigma_c$ Inter. Subst.                            | 0.7547   | 1.2432   | 0.5851       |
| <i>h</i> Habit Form.                                | 0.4804   | 0.8769   | 0.7989       |
| $\xi_{w}$ Calvo Prob.                               | /        | 0.7876   | 0.6792       |
| $\sigma_{L}$ Elas. Lab. Suply                       | 0.4255   | 2.7213   | 2.6948       |
| $\xi_p$ Calvo Prob.                                 | /        | 0.8776   | 0.8074       |
| <sup><i>l</i></sup> <sub><i>w</i></sub> Wage Index. | 0.4675   | 0.3623   | 0.5949       |
| $t_p$ Price Index.                                  | 0.1165   | 0.1705   | 0.4395       |
| $\Phi$ Fixed Cost                                   | 1.4885   | 1.2318   | 1.4912       |
| $r_{\pi}$ Policy Inf.                               | 1.9522   | 2.1304   | 1.7552       |
| $\rho$ Coeff. Lag Int. Rt.                          | 0.7054   | 0.9676   | 0.9126       |
| <sup>r</sup> <sub>y</sub> Policy OutputGap          | 0.0115   | 0.1079   | 0.083        |
| $r_{\Delta y}$ Policy $\Delta Y$                    | 0.2380   | 0.0023   | 0.2508       |
| $100(\beta^{-1}-1)$                                 | 0.05     | 0.05     | 0.05         |
| $\overline{\gamma}$ Trend Growth                    | 0.4898   | 0.5848   | 0.5306       |

| $\alpha$ Capital Share          | 0.4885 | 0.1990 | 0.2391 |
|---------------------------------|--------|--------|--------|
| $\sigma_{a}$ Std. Dev. Shocks   | 1.6614 | 2.4900 | 1.0525 |
| $\sigma_{\scriptscriptstyle b}$ | 0.0457 | 0.1817 | 0.0266 |
| $\sigma_{_g}$                   | 0.7256 | 0.7257 | 0.8048 |
| $\sigma_{i}$                    | 0.3686 | 0.3112 | 0.4619 |
| $\sigma_r$                      | 0.5941 | 0.0570 | 0.0943 |
| $\sigma_{_{p}}$                 | 2.9789 | 0.0535 | 0.0676 |
| $\sigma_{_w}$                   | 2.7380 | 0.1048 | 0.0712 |
| $\rho_a$ Shock Persistences     | 0.9565 | 0.9870 | 0.9805 |
| $ ho_b$                         | 0.9736 | 0.6225 | 0.9126 |
| $ ho_{g}$                       | 0.9398 | 0.9938 | 0.9532 |
| $ ho_l$                         | 0.9431 | 0.8631 | 0.6066 |
| $ ho_r$                         | 0.3063 | 0.7368 | 0.5129 |
| ${\cal P}_p$                    | 0.9894 | 0.9863 | 0.8865 |
| $ ho_w$                         | 0.9642 | 0.9763 | 0.7996 |
| $\mu_p$ Price Mark-up           | 0.0274 | 0.6450 | 0.7067 |

| $\mu_{w}$ Wage Mark-up | 0.0190 | 0.2302 | 0.4104 |
|------------------------|--------|--------|--------|
| 8                      |        |        |        |

# Table 16 Parameter values for NC, NK and Hybrid models (the estimates of NK model are from Table 1&2 in Chapter1 and estimates of Hybrid model are from Table 12 in Chapter3)

The parameters' values of NC, NK and Hybrid models are presented in table 16 for comparison. Let's first look at the parameter of intertemporal elasticity of substitution, it is estimated to be much lower in the Hybrid model (0.5851) than those from the NK and NC models. This may tell us the situation in China that the growth rate of consumption does not response intensively to real interest rate. Habit formation is estimated to be 0.4804 in the NC model which is lower than those of the NK and Hybrid model which are 0.8769 and 0.7989 respectively. The elasticity of labour supply in NC model is estimated to be 0.4255 which is rather low when compared to those estimates from NK and Hybrid models; this indicates that in New Keynesian and Hybrid economies the change in wage rate have strong effect on labour supply, however it is not the case in the economy with perfect competition in labour market. The mean of the parameter of policy reactions to inflation is estimated to be 2.1304 in the NK model which is higher than the estimates of both NC and Hybrid models, indicating that policy react stronger to inflation under Keynesian rules than Classical rules or a combination of these two rules. The long run reaction coefficient to the output gap in the NC model is estimated to be much lower than those of the NK and Hybrid models, which tells that policy towards output gap in NK and Hybrid models are much stronger than that in NC model. And it seems that the NC model has the highest capital share in production among these three types of models.

In the NC model, the standard deviations of price and wage mark-up shocks are estimated to be 2.9879 and 2.7380 respectively which are much higher than that from the estimates of NK and Hybrid models, indicating the fact that price and wage vary a lot under perfect competition. Most of the shock processes in NC model are rather persistent except for the monetary policy shock, while for NK model there are few persistent shocks and in the Hybrid model the number of persistent shock processes become even less. One reason for NC model to have so many large AR (1) coefficients in the shock processes is that the NC model does

not have a mechanism to generate persistence as the NK and Hybrid can do thus it needs the high AR (1) coefficients to generate some necessary persistence.



Panel 1



Panel 2

#### Figure 15 Impulse Responses to Productivity shocks for NC (Panel 1) and Hybrid model (Panel 2) (dy=output, pinfobs=inflation, robs=interest rate, dc=consumption, dinve=investment, dw=real wage, labobs=labour)

Comparisons can also be drawn from the Impulse Response Functions. In figure 14 we have the impulse responses to the productivity shocks for both NC and Hybrid models. The impulse responses of the NK model look quite similar to that of Hybrid model. Panel 1 and panel 2 of figure 14 show the impulse responses to productivity shock for NC model and Hybrid model respectively. We can see that the impulse responses to productivity shock in both Hybrid and NC models are much in line with the literature; however there is an obvious difference between these IRFs in the two panels, that is, the IRFs in panel 1 converge much quicker than that of panel 2, i.e. the impulse responses to productivity shock for NC model converge much quicker than that of Hybrid model.



Panel 1



#### Panel 2

Figure 16 Impulse Responses to Monetary policy shocks for NC (Panel 1) and Hybrid model (Panel 2) (dy=output, pinfobs=inflation, robs=interest rate, dc=consumption, dinve=investment, dw=real wage, labobs=labour)

Just a glance on the two panels of figure 15 we can discover something quite similar to figure 14. The IRFs in panel 1 of figure 15 converge much quicker than that of panel 2, i.e. the impulse responses to monetary policy shock for NC model converge much quicker than that of Hybrid model. The reason is that the New Classical model cannot create enough persistence as the Hybrid model and NK model can do for us to see the hump shaped impulse responses which are presented in panel 2 of figure 14 and figure 15.

Over all, the three types of models show different performances through the estimation of parameters and impulse response functions. The behaviour of NK model and Hybrid model are to some extend close to each other, this can be shown by their estimated parameter values and the impulse response functions. However, the NC model behaves quite different from the NK and Hybrid models. In specific, the NC model does not have strong persistence like the NK and Hybrid model. Also through the testing procedure of Indirect Inference, it is clear

that neither of the "restricted" models, i.e. NC and NK model could mimic the reality; on the other hand a weighted combination of these two, i.e. the Hybrid model is more close to the real data due to its "unrestricted" properties.

### **4.2 Further research directions**

In this thesis we have estimated and tested a complicated DSGE model using Chinese macroeconomic data, the methods adopted in this thesis are Bayesian estimation and Indirect Inference testing and estimation. Some further researches which are necessary and valuable to be carried out in the future.

First, it is worth trying to build up a new DSGE model which incorporates some special features of China. DSGE models are based on the utility maximising behaviour of a forward looking representative agent and the existence of a union leads to some monopoly power over wages, for the case of China it is appropriate to consider a communist representative agent. We have treated China as a closed economy, however China exports and imports a lot and exports play a major part in the growth of Chinese economy. In the further research it is necessary to treat China as an open economy and add in exchange rate.

In the Bayesian estimation we have adopted the US priors for the parameters following Smets and Wouters (2007). Due to the differences between US and China it is worth to set some of the priors according to the reality of China. For example, some of the calibrated parameters could be different under the circumstance of Chinese economy to that of US economy; also shocks hit Chinese economy may contain different sources compared to those hit US economy thus considering different shocks would be good to try.

In the framework of Indirect Inference we have chosen a VAR (1) as the auxiliary model for both model evaluation and estimation, however just as discussed in Chapter 2 and 3 there could be some alternative choices of the auxiliary. For example, in case we have changed the model and there are unobservable features in China then a VARMA would be more appropriate as an auxiliary. Also the choice of auxiliary depends on to what extent the model must fit; if we want to test the model to a higher extent then a VAR of higher order would be a better choice.

## Appendix



#### Prior and posterior distributions for the parameters





Figure A2



Figure A3



Figure A4

Note: in each of the small figures, the grey curve is the prior distribution, the black curve is the posterior distribution and the green dashed line is the posterior mean.

#### Smoothed variables and shocks for the model



**Figure A5: Smoothed Variables** 



**Figure A6: Smoothed Shocks** 

#### The multivariate diagnostic for the Bayesian estimation



Figure A7

Supplementary Impulse Response Functions for the Classical model



#### Figure A8: IRF productivity shock



Figure A9: IRF risk premium shock



Figure A10: IRF Government spending shock



Figure A11: IRF Monetary policy shock



Figure A12: IRF Price mark-up shock



Figure A13: IRF Investment shock



Figure A14: IRF Wage mark-up shock

Cross Correlations of the SA estimated Hybrid model

| Variables | Labour  | Int. Rate | Infl.   | GDP     | Cons.   | Invst   | Wage    |
|-----------|---------|-----------|---------|---------|---------|---------|---------|
| Labour    | 1       | -0.1833   | -0.1032 | 0.1646  | 0.3061  | -0.0222 | 0.1938  |
| Int. rate | -0.1833 | 1         | 0.3917  | -0.0669 | -0.1148 | -0.0207 | -0.0075 |
| Infl.     | -0.1032 | 0.3917    | 1       | -0.2524 | 0.2164  | -0.0776 | 0.4614  |
| GDP       | 0.1646  | -0.0669   | -0.2524 | 1       | 0.5274  | 0.6173  | 0.4895  |
| Cons.     | 0.3601  | -0.1148   | 0.2164  | 0.5274  | 1       | 0.1399  | 0.5483  |
| Invst     | -0.0222 | -0.0207   | -0.0776 | 0.6173  | 0.1399  | 1       | 0.1579  |
| Wage      | 0.1938  | -0.0075   | 0.4614  | 0.4895  | 0.5483  | 0.1579  | 1       |

| Table A1: 0 | Cross corre | lations of key | variables in | the Hybrid | model (SA | estimation). |
|-------------|-------------|----------------|--------------|------------|-----------|--------------|
|             |             |                |              |            |           |              |

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