RESTRICTION MAPS IN EQUIVARIANT KK-THEORY.

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ABSTRACT. We extend McClure's results on the restriction maps in equivariant K-theory to bivariant K-theory:

Let G be a compact Lie group and A and B be G-C^*-algebras. Suppose that $KK_n^H(A,B)$ is a finitely generated $R(G)$-module for every $H \leq G$ closed and $n \in \mathbb{Z}$. Then, if $KK_*^F(A,B) = 0$ for all $F \leq G$ *finite cyclic*, then $KK_*^G(A,B) = 0$.

0. INTRODUCTION

One of the basic facts about the representation theory of a *compact Lie* group is that any virtual representation which restricts trivially to every *finite cyclic* subgroup is itself trivial.

McClure studied how far this generalizes to equivariant K-theory and proved the following. Recall that a finite group is called *elementary* if it is a direct product of a cyclic group and a p-group.

Theorem 0.1 (McClure [McC86]). *Let G be a compact Lie group and let X be a finite G-CW-complex.*

(a) If $K_F^*(X) = 0$ for all $F \leq G$ finite cyclic*, then $K_G^*(X) = 0$.*
(b) If $x \in K_G(X)$ restricts to zero in $K_H(X)$ for every finite elementary *subgroup H of G, then $x = 0$.*

Remark 0.2. (i) Theorem 0.1(a) was proved by Jackowski for G finite (cf. [Jac77, Corollary 4.3]) and McClure proved the general case by reducing to the finite case using Theorem 0.1(b).
(ii) Theorem 0.1(b) cannot be strengthened by replacing "finite elementary" by "finite cyclic" (cf. [Jac77, page 89] and [McC86, page 404]).

We extend these to bivariant K-theory as follows. Let $R(G) = K_G(*)$ denote the representation ring of G.

Theorem 0.3. *Let G be a compact Lie group and A and B be G-C^*-algebras.*[1] *Suppose that $KK_n^H(A,B)$ is a finitely generated $R(G)$-module for every $H \leq G$ closed and $n \in \mathbb{Z}$.*

(a) If $KK_*^F(A,B) = 0$ for all $F \leq G$ finite cyclic*, then $KK_*^G(A,B) = 0$.*

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[1]We only consider separable C^*-algebras.

(b) Suppose, in addition, that $KK_n^F(A,B)$ is a finitely generated group for all $F \leq G$ finite and $n \in \mathbb{Z}$. Then, if $x \in KK^G(A,B)$ restricts to zero in $KK^H(A,B)$ for all $H \leq G$ finite elementary*, then $x = 0$.*

Remark 0.4. If A and B are both KK^G-equivalent to the algebra of continuous functions on a G-CW-complex, then the finite generation assumptions in Theorem 0.3 are automatic. Hence Theorem 0.3 extends Theorem 0.1.

In fact, we prove the following. This is done mainly for clarity, but as an added bonus, we see that Theorem 0.3 holds for equivariant E-theory as well.

Theorem 0.5. *Let G be a compact Lie group and let $\widetilde{E}_G^*$ be an $RO(G)$-gradable module theory over $\widetilde{K}_G^*$. Suppose that $\widetilde{E}_H^n(S^0)$ is a finitely generated $R(G)$-module for every $H \leq G$ closed and $n \in \mathbb{Z}$. Let X be a finite based G-CW-complex.*

(a) If $\widetilde{E}_F^*(X) = 0$ for all $F \leq G$ finite cyclic*, then $\widetilde{E}_G^*(X) = 0$.*
(b) Suppose, in addition, that $\widetilde{E}_F^n(S^0)$ is a finitely generated group for all $F \leq G$ finite and $n \in \mathbb{Z}$. Then, if $x \in \widetilde{E}_G^*(X)$ restricts to zero in $\widetilde{E}_H^*(X)$ for all $H \leq G$ finite elementary*, then $x = 0$.*

The proof follows [McC86] rather closely. In Section 1, we show that Theorem 0.5 implies Theorem 0.3. In Section 2, we extend the generalized Atiyah-Segal completion theorem of [AHJM88a] to modules over K-theory. Using the completion theorem, we prove Theorem 0.5 in Section 3. However, unlike [McC86], we prove part (a) of Theorem 0.5 directly in order to avoid the additional finite generation assumptions of part (b). In the final section, we apply Theorem 0.3 to prove a variation of [MN06, Theorem 9.3].

Remark 0.6. (i) Chris Phillips extended the Atiyah-Segal completion theorem to C^*-algebras in [Phi89]. See also the comments at the end of Section 2 of loc.cit.
(ii) Michel Matthey and Guido Mislin obtained results dual to McClure's theorem, for restriction maps in K-homology of spaces with proper actions of discrete groups (cf. [MM04]).
(iii) Heath Emerson studied C^*-algebras with a circle action and showed that there are many C^*-algebras that are not *equivariantly* KK-equivalent to a commutative C^*-algebra, even though they and their crossed products are KK-equivalent to commutative C^*-algebras (cf. [Eme10]). Hence Theorem 0.3 covers many more examples than just the commutative ones.

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1. $RO(G)$-GRADED COHOMOLOGY THEORIES

Let G be a compact Lie group. A based G-space is a G-space with a G-fixed base point. In the rest of the paper, we assume that all G-spaces are G-CW-complexes and all cohomology theories are equivariant and reduced cohomology theories.

For a finite-dimensional representation V of G, we write S^V for the one-point compactification of V, considered a based G-space with base point the point at infinity.

1.1. $RO(G;U)$-gradable theories.

We fix a complete universe U. (cf. [May96, Definition IX.2.1]).

Definition 1.1. An $RO(G)$-*graded* cohomology theory is an $RO(G;U)$-graded cohomology theory in the sense of [May96, Definition XIII.1.1]. A $\mathbb{Z}$-*graded* cohomology theory is an $RO(G;U^G)$-graded cohomology theory (any trivial universe would work). We say that a $\mathbb{Z}$-graded cohomology theory is $RO(G)$-*gradable* if it is the $\mathbb{Z}$-graded part of an $RO(G)$-graded theory.

Let $\widetilde{E}^*_G$ be a $\mathbb{Z}$-graded cohomology theory. For a closed subgroup $H \leq G$ and a based H-CW-complex X, we define

$$\widetilde{E}^*_H(X) \coloneqq \widetilde{E}^*_G(G_+ \wedge_H X). \tag{1.1}$$

Then $\widetilde{E}^*_H$ is a $\mathbb{Z}$-graded cohomology theory on based H-spaces. If X is actually a based G-CW-complex, then we have a natural G-equivariant identification

$$G_+ \wedge_H X \cong G/H_+ \wedge X \tag{1.2}$$

and the collapse map $G/H \to *$ gives rise to a natural transformation

$$\mathrm{res}^G_H : \widetilde{E}^*_G \to \widetilde{E}^*_H \tag{1.3}$$

called the *restriction* map.

1.2. Bivariant K-theory.

The following is the main example we have in mind. First note that $\widetilde{K}^*_G$ is an $RO(G)$-graded commutative ring theory with $\widetilde{K}^V_G(X) = KK^G(C_0(S^V), C_0(X))$ and $R(G) \cong \widetilde{K}_G(S^0)$.

Proposition 1.2. *Let G be a compact Lie group and let A and B be G-C^*-algebras. For a finite based G-CW-complex X and finite-dimensional real representation V of G, we define*

$$\widetilde{E}^V_G(X) \coloneqq KK^G(A \otimes C_0(S^V), B \otimes C_0(X)). \tag{1.4}$$

Then the following holds.

(i) $\widetilde{E}^*_G$ defines an $RO(G)$-graded cohomology theory on the category of finite based G-CW-complexes.
(ii) $\widetilde{E}^*_G$ extends to an $RO(G)$-graded cohomology theory on the category of based G-CW-complexes.

(iii) $\widetilde{E}^*_G$ *is a module theory over* $\widetilde{K}^*_G$.

Proof. (i) See [Kas88].
(ii) By Adams' representation theorem [May96, Theorem XIII.3.4], $\widetilde{E}^*_G$ is represented by an Ω-G-prespectrum, hence extends to an $RO(G)$-graded cohomology theory on the category of G-CW-complexes. See [Sch92].
(iii) The module structure

$$\widetilde{E}^V_G(X) \times \widetilde{K}^W_G(Y) \to \widetilde{E}^{V+W}_G(X \wedge Y). \tag{1.5}$$

is given by the Kasparov product

$$KK^G(A(S^V), B(X)) \times KK^G(C_0(S^W), C_0(Y)) \tag{1.6}$$

$$\to KK^G(A(S^{V+W}), B(X \wedge Y)). \tag{1.7}$$

□

It is well-known that for $H \leq G$,

$$KK^G(A, B \otimes C_0(G/H_+)) \cong KK^H(A, B) \tag{1.8}$$

and the restriction map is induced by $G/H_+ \to S^0$. Hence we obtain the following corollary.

Corollary 1.3. *Suppose that Theorem 0.5 holds. Then Theorem 0.3 holds.* □

2. Atiyah-Segal Completion

First we abstract the main finiteness condition from Theorem 0.5.

Definition 2.1. Let R be a unital commutative ring and let $\widetilde{E}^*_G$ be a $\mathbb{Z}$-graded cohomology theory with values in R-modules. We say that $\widetilde{E}^*_G$ is *finite* over R if $\widetilde{E}^n_G(X)$ is a finitely generated R-module for every *finite* based G-CW-complex X and $n \in \mathbb{Z}$.

Clearly, this is equivalent to asking that $\widetilde{E}^{k-n}_H(S^0) \cong \widetilde{E}^k_G(G/H_+ \wedge S^n)$ is a finitely generated R-module for $H \leq G$.

Lemma 2.2. *Let G be a compact Lie group and let R be a unital commutative ring. Let $\widetilde{E}^*_G$ be a $\mathbb{Z}$-graded cohomology theory with values in R-modules. Suppose that R is Noetherian and $\widetilde{E}^*_G$ is finite over R. Then for any family $\mathcal{I}$ of ideals in R, the following defines a $\mathbb{Z}$-graded cohomology theory with values in pro-R-modules:*

$$\widetilde{E}^*_G(X)^\wedge_{\mathcal{I}} := \{\widetilde{E}^*_G(Y)/J \cdot \widetilde{E}^*_G(Y)\}. \tag{2.1}$$

where $Y \subseteq X$ runs over the finite based G-CW-subcomplexes of X and J runs over the finite products of ideals in $\mathcal{I}$.

Note that in this lemma, it is enough to have $\widetilde{E}^*_G$ to be a cohomology theory on *finite* based G-CW-complexes (only finite wedges are considered in the additivity axiom).

Proof. Exactness follows from the Artin-Rees lemma. See the proof of [AHJM88b, Lemma 2.1]. □

2.1. **Bott Periodicity.** Let V be a complex G-representation. By Bott periodicity [Ati68, Theorem 4.3], $\widetilde{K}^0_G(S^V)$ is a free $\widetilde{K}^0_G(S^0)$-module generated by the Bott element $\lambda_V \in \widetilde{K}^0_G(S^V)$. The *Euler class* of V is defined to be $\chi_V \coloneqq e^*(\lambda_V) \in \widetilde{K}^0_G(S^0)$, where $e : S^0 \to S^V$ is the obvious map.

Lemma 2.3. *Let $\widetilde{E}^*_G$ be an $RO(G)$-graded module theory over $\widetilde{K}^*_G$. Then for any complex representation V, multiplication by the Bott element $\lambda_V \in \widetilde{K}^0_G(S^V)$ gives an isomorphism*

$$\widetilde{E}^0_G(S^0) \cong \widetilde{E}^0_G(S^V). \tag{2.2}$$

If $V \subseteq W$ are complex representations and $i : S^V \to S^W$ is the inclusion, then the following diagram commutes

$$\begin{tikzcd} \widetilde{E}^0_G(S^0) \arrow[r] \arrow[d, "\cdot\chi_{W-V}"] & \widetilde{E}^0_G(S^W) \arrow[d, "i^*"] \\ \widetilde{E}^0_G(S^0) \arrow[r] & \widetilde{E}^0_G(S^V) \end{tikzcd} \tag{2.3}$$

Proof. Let $\lambda_V^{-1} \in \widetilde{K}^V_G(S^0)$ denote the inverse Bott element: it has the property that

$$\lambda_V \cdot \lambda_V^{-1} = \lambda_V^{-1} \cdot \lambda_V = 1 \in \widetilde{K}^V_G(S^V) \cong \widetilde{K}^0_G(S^0). \tag{2.4}$$

Then multiplication by λ_V^{-1} gives the inverse map

$$\widetilde{E}^0_G(S^V) \to \widetilde{E}^V_G(S^V) \cong \widetilde{E}^0_G(S^0). \tag{2.5}$$

The second statement is shown for $\widetilde{E}^*_G = \widetilde{K}^*_G$ in [AHJM88a, page 4]. The general case follows by functoriality. □

2.2. **Completion.** A class of subgroups of G closed under subconjugacy is called a *family*. A family $\mathcal{C}$ of subgroups of G determines a class, again denoted $\mathcal{C}$, of ideals of $R(G)$ by the kernels of the restriction maps:

$$\ker(\mathrm{res}^G_H : R(G) \to R(H)), \quad H \in \mathcal{C}, \tag{2.6}$$

hence a topology on any $R(G)$-module.

The following is a straightforward generalization of [AHJM88a, Theorem 3.1].

Theorem 2.4. *Let G be a compact Lie group and let $\widetilde{E}^*_G$ be an $RO(G)$-gradable module theory over $\widetilde{K}^*_G$, which is finite over $R(G)$.*

Let $\mathcal{C}$ be a family of subgroups of G. For any based G-CW-complex X, if $\widetilde{E}^*_H(X)^\wedge_{\mathcal{C}|H} = 0$ for all $H \in \mathcal{C}$, then $\widetilde{E}^*_G(X)^\wedge_{\mathcal{C}} = 0$.

Proof. By [Seg68, Corollary 3.3], $R(G) = \widetilde{K}^0_G(S^0)$ is Noetherian. Hence, by Lemma 2.2, $\widetilde{E}^*_G(X)^\wedge_{\mathcal{C}}$ is a cohomology theory.

Now the proof of [AHJM88a, Theorem 3.1] carries over ad verbatum, once we extend Bott periodicity to $\widetilde{E}^*_G$ as in Lemma 2.3. □

Corollary 2.5. *Let $E\mathcal{C}$ denote the classifying space of $\mathcal{C}$. For any finite based G-CW-complex X, the projection map $E\mathcal{C}_+ \to S^0$ gives completion*

$$\widetilde{E}^*_G(E\mathcal{C}_+ \wedge X) \cong \lim \widetilde{E}^*_G(Y \wedge X) \cong \lim \widetilde{E}^*_G(X)^\wedge_{\mathcal{C}}, \tag{2.7}$$

where Y runs over finite based subcomplexes of $E\mathcal{C}_+$.

Proof. The inverse system $\widetilde{E}^*_G(X)^\wedge_{\mathcal{C}}$ satisfies the Mittag-Leffler condition and $\widetilde{E}^*_G(Y \wedge X)$ is $\mathcal{C}$-complete for any finite based subcomplex $Y \subset E\mathcal{C}_+$ (cf. [AHJM88a, Corollary 2.1]). □

3. Proof of Theorem 0.5

3.1. **$\mathcal{F}$-spaces.** Let $\mathcal{F}$ be a family of subgroups of G. We say that a based G-CW-complex X is an $\mathcal{F}$-space if all the isotropy groups, except at the base point, are in $\mathcal{F}$. The following lemma says that in the proof of Theorem 0.5, we may assume that X is an $\mathcal{F}$-space, for any $\mathcal{F}$ containing all finite cyclic subgroups of G.

Lemma 3.1. *Let G be a compact Lie group and let $\widetilde{E}^*_G$ be an $RO(G)$-gradable module theory over $\widetilde{K}^*_G$, which is finite over $R(G)$.*

Let $\mathcal{F}$ be a family containing all finite cyclic subgroups of G. Then for any finite based G-CW-complex X, the top horizontal map in the commutative diagram

$$\begin{tikzcd} \widetilde{E}^*_G(X) \arrow[r] \arrow[d] & \lim_{Y \subset E\mathcal{F}_+} \widetilde{E}^*_G(Y \wedge X) \arrow[d] \\ \prod_{F \in \mathcal{F}} \widetilde{E}^*_F(X) \arrow[r] & \lim_{Y \subset E\mathcal{F}_+} \prod_{F \in \mathcal{F}} \widetilde{E}^*_F(Y \wedge X) \end{tikzcd} , \tag{3.1}$$

is injective. Here Y runs over the finite based subcomplexes of $E\mathcal{F}_+$, the horizontal maps are induced by the projections $Y \wedge X \to X$ and the vertical maps are restrictions.

Proof. The $\mathcal{F}$-topology on $\widetilde{E}^*_G(X)$ is Hausdorff by [McC86, Corollary 3.3]. Hence, the claim follows from Corollary 2.5. □

Let $\mathcal{C}$ denote the family of finite cyclic subgroups of G.

Proof of Theorem 0.5(a). By assumption, $\widetilde{E}^*_F(X) = 0$ for all $F \in \mathcal{C}$. Let Y be a finite based G-CW-complex, which is a $\mathcal{C}$-space. Then the zero skeleton Y^0 and the skeletal quotients Y^n/Y^{n-1} are finite wedges of G-spaces of the form $G/F_+ \wedge S^n$ with $F \in \mathcal{C}$. It follows that $\widetilde{E}^*_G(Y \wedge X) = 0$. Hence by Lemma 3.1, $\widetilde{E}^*_G(X) = 0$. □

3.2. **Induction.** We write $\mathcal{O}_G$ for the category whose objects are orbit spaces G/H, where $H \leq G$ is a closed subgroup, and whose morphisms are homotopy classes of G-maps.

Recall that a compact Lie group is said to *cyclic* if it has a topological generator (an element whose powers are dense) and *hyperelementary* if it is an extension of a cyclic group by a finite p-group.

We write $\mathcal{H}$ for the class of hyperelementary subgroups of G and let $\mathcal{O}_{\mathcal{H}}$ denote the full subcategory of $\mathcal{O}_G$ of orbits G/H with H subconjugate to a subgroup in $\mathcal{H}$.

Lemma 3.2. *Let G be a compact Lie group and let $\widetilde{E}^*_G$ be an $RO(G)$-gradable module theory over $\widetilde{K}^*_G$. Then, for any based G-CW-complex, the restriction maps induce an isomorphism*

$$\widetilde{E}^*_G(X) \cong \lim_{\mathcal{O}_{\mathcal{H}}} \widetilde{E}^*_H(X). \tag{3.2}$$

Proof. Follows from Propositions 2.1 and 2.2 of [McC86]. □

For any abelian group M, let $M^\wedge_{\mathbb{Z}}$ denote its adic completion $\lim_n M/nM$.

Proof of Theorem 0.5(b). Let $\mathcal{F}$ denote the family of finite subgroups of G.

By Lemma 3.2, we may assume that G is a hyperelementary group and by Lemma 3.1, we may assume that X an $\mathcal{F}$-space.

Let G be a hyperelementary group and X an $\mathcal{F}$-space. Then the restriction map

$$\widetilde{E}^*_G(X)^\wedge_{\mathbb{Z}} \to \lim_{F \in \mathcal{O}_{\mathcal{F}}} \widetilde{E}^*_F(X)^\wedge_{\mathbb{Z}}. \tag{3.3}$$

is an isomorphism by [McC86, Theorem 1.1]. By [McC86, Corollary 3.3], the adic topologies on $\widetilde{E}^*_G(X)$ and $\widetilde{E}^*_F(X)$ are Hausdorff. This reduces the problem to the case G is finite. Now an application of [McC86, Proposition 2.1] to the class of elementary subgroups finishes the proof. See the proof of [McC86, Corollary B]. □

4. An application

The following is a variation of [MN06, Theorem 9.3].

Theorem 4.1. *Let G be a Lie group (not necessarily compact) and let A and B be G-C^*-algebras. Suppose that the following finiteness condition holds: for any closed subgroups $H \subseteq K \subseteq G$ with K compact, $K^H_n(A)$ and $K^H_n(B)$ are finitely generated $R(K)$-modules.*

Let $x \in KK^G(A,B)$ be an element with the property that for any finite cyclic subgroup $F \subseteq G$,

$$\operatorname{res}^G_F(x)_*\colon K^F_*(A) \cong K^F_*(B). \tag{4.1}$$

Then x induces an isomorphism $K^{\mathrm{top}}_*(G;A) \cong K^{\mathrm{top}}_*(G;B)$ of the topological K-groups (in the sense of [BCH94]*).*

Proof. By [CEOO04], it is enough to prove that for any *compact* subgroup $K \subseteq G$, the restriction $\mathrm{res}^G_K(x)$ induces an isomorphism $K^K_*(A) \cong K^K_*(B)$.

For this we use the triangulated category structure of equivariant KK-theory developed by Meyer-Nest (cf. [MN06]). Let C denote a mapping cone of x so that we have a distinguished triangle

$$\Sigma B \longrightarrow C \longrightarrow A \xrightarrow{x} B \ . \tag{4.2}$$

in KK^G. For any closed subgroup $H \subseteq G$, since restriction res^G_H is a triangulated functor and equivariant K-theory K^H_* is homological, we see that x induces an isomorphism on K^H_* if and only if $K^H_*(C) = 0$.

Let $K \subseteq G$ be a compact subgroup. Since G is a Lie group, so is K. Moreover, the assumptions on A and B imply that $K^H_n(C)$ is a finitely generated $R(K)$-module for any closed subgroup $H \subseteq K$ and $n \in \mathbb{Z}$. Applying Theorem 0.3(a) to K acting on $(\mathbb{C}, C)$, we see that $K^K_*(C) = 0$. This completes the proof. $\square$

Remark 4.2. (i) For G discrete, compare [MM04, Theorem 1.1].

(ii) It would be interesting to understand how much of the finiteness assumptions in Theorem 0.3 and Theorem 4.1 are really necessary.

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