"Controlling Replenishment Rule Induced Bullwhip via Good Systems Design"

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Abstract

Demand amplification (or "bullwhip" as it is now popularly called) is not a new phenomenon. Industry typically has to cope with bullwhip measured not just in terms of the 2:1 amplification in orders which is frequently encountered across a single echelon, but sometimes it is as high as 20:1 across the extended enterprise. In this Chapter we consider how bullwhip due to various "Forrester effects" may be avoided. This leads to our exploitation of a particular Replenishment Rule already widely used in industry and for which analytical formulae for bullwhip generation and inventory variance have been recently derived.

Key Words: Bullwhip avoidance: replenishment rules: system damping: bullwhip benchmarking

1. Introduction

We know that there are many causes of bullwhip (Geary et al, 2002). In this Chapter we are concerned only with those causes leading to the "Forrester Effects" (Jay Forrester, 1958). These particular causes include coping with practical lead times inevitably met within supply chains, the structure of information flow within the system, and how this is used within feedback paths and feed forward paths in inventory replenishment rules. The latter aspects include incorporating forecasting mechanisms and both inventory and pipeline controls therein. So the purpose of this paper is to ensure that our replenishment rule detects and follows genuine changes in customer trends. At the same time random fluctuations are to be adequately damped (Dejonckheere et al. 2002). In particular we ensure that feedback settings are selected which avoid the bullwhip effect due to this source. As this decision is within our remit, there is no excuse for inducing avoidable bullwhip. A bad example of this is clearly observable in the data for a real-world retail supply chain as shown in figure 1.

The replenishment rule used herein is frequently met in industry. It may be embedded within commercial software, although often it is developed and implemented on an ad-hoc basis. Either *modus operandi* can result in rather arbitrary settings of system parameters (Coyle, 1982 and Edghill et al, 1987). Suitable theoretical models already exist, with much supporting simulation evidence (van Aken, 1978, John et al, 1994 and van Ackere et al 1993) including optimisation procedures. However Disney and Towill (2003) have now provided an analytic solution for calculating bullwhip and inventory variance in response to random customer demand. We now exploit this solution herein by providing bandwidth contours for various values of replenishment lead time. Hence given the expected lead time, the user may select replenishment rule settings so as to avoid bullwhip. The analytic solutions additionally enable the system user to evaluate their particular performance trade-offs, including robustness.

**Figure 1. Bullwhip in the Real World ~ Substantial Demand Amplification
Observed in the UK Retail Sector**
(Source: Jones, Hines and Rich, 1997)

2. Choosing a Replenishment Rule

Thus we seek to avoid bullwhip by designing it out of the system via the appropriate choice of replenishment rule **and** parameter settings which deliver the performance expected by the user. Our rule uses exponential smoothing of demand as one component of a replenishment order. To this is added a further two inputs. These are a fraction of the inventory error, and a fraction of the "goods-in-the-pipeline" error. It

is an important feature of the paper that these fractions are always equal, for reasons which will be detailed later. This idea is originally due to Deziel and Eilon (1967). We show this is very powerful as an adjunct in bullwhip avoidance since it results in a very conservative design. What follows is a condensed version of the paper which details our approach (Disney and Towill, 2005).

The particular replenishment rule exploited in this paper is a special case of the **A**utomatic **P**ipeline **I**nventory and **O**rder **B**ased **P**roduction **S**ystem (APIOBPCS). This expressed in words is "Let the replenishment orders be equal to the sum of an exponentially smoothed demand (averaged over Ta time periods), plus a fraction (1/Ti) of the inventory difference between target stock and actual stock, plus a fraction (1/Tw) of the difference between target orders-in-the-pipeline and actual orders placed but not yet received" (John et al, 1994). APIOBPCS encapsulates the general principles for replenishment rules as advocated by Popplewell and Bonney (1987). In particular it gives due prominence to the importance of including pipeline feedback (OPL), a factor further emphasised by Bonney (1990).

3. Causal Loop Representation

Of course, APIOBPCS is not new: it is well established in industry and has the additional advantage of describing data from some 2000 Beer Game "plays" as modelled by John Sterman (1989). What we are doing herein is to show how the control parameters Ta, Ti and Tw depicted in the Causal Loop of Figure 2 can be set to avoid bullwhip. The causal loop is a pictorial description of APIOBPCS which corresponds to the preceding verbal rule. Note that there is a replenishment time delay of Tp units. The practical interpretation of Tp (for mathematical reasons) is that if we update replenishment orders daily, deliveries become available (Tp + 1) days later. So if supplies arrive on day 4, then Tp = 3 days. Note also that to avoid inventory drift the OPL target is also set at a multiple of exponentially smoothed demand. The OPL target multiplier is $\overline{Tp}$, our best estimate of current replenishment lead time.
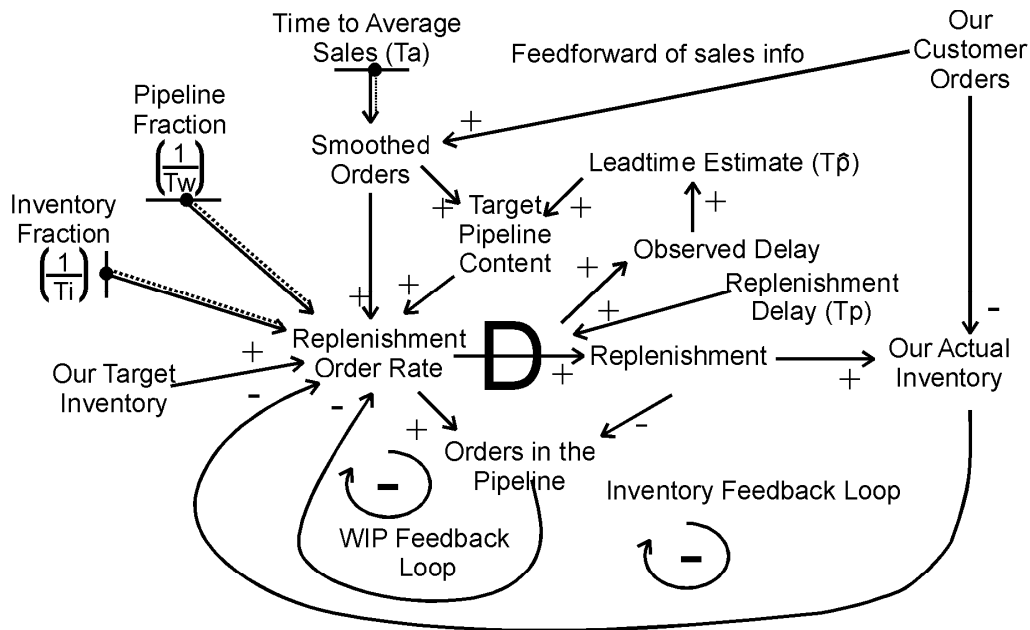
Figure 2. Causal Loop Diagram of the APIOBPCS Replenishment Rule
(Source: John et al. 1994)

We now propose to make a very simple, but operationally profound, modification to APIOBPCS by making Ti ≡ Tw. This simplified model we describe as DE-APIOBPCS, since this modification was first advocated by **D**eziel and **E**ilon (1967) in an OR context. The importance of their contribution was first emphasised to the present authors by Bertrand (2001). Our simple demonstration of the importance of the DE settings of (Ti ≡ Tw) is given in Figure 3, (Disney and Towill, 2002). Here the stability boundary for the replenishment rule for the particular case where Tp = 3 days is shown. The critical stability contour separates the stable regime (controlled behaviour) from the unstable regime (uncontrolled behaviour). It can be seen by inspection that the DE contour lies well within the stable regime with extremely well behaved dynamic response.
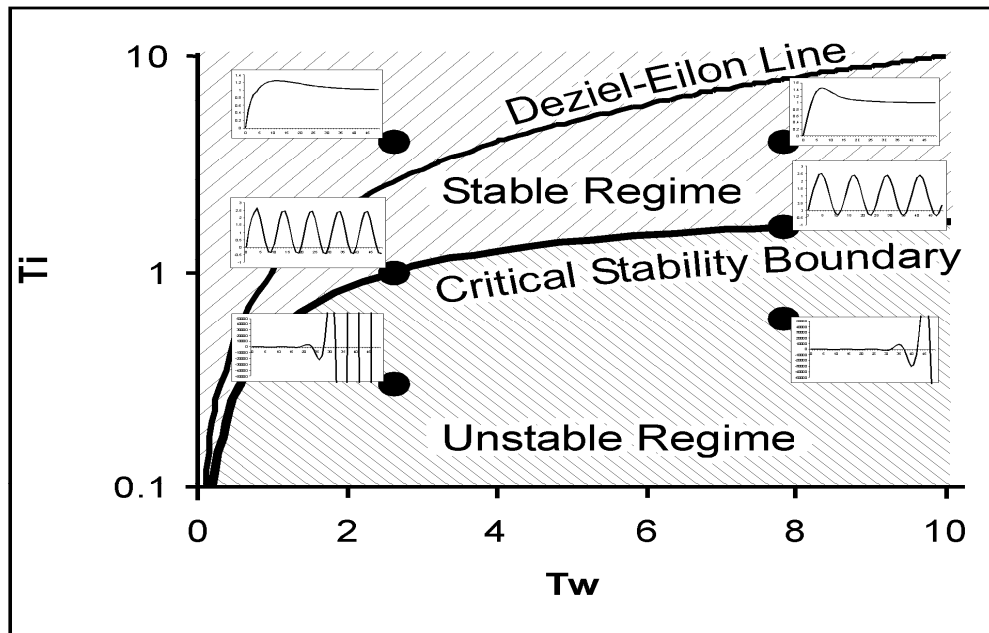
Figure 3. Relevance of the Deziel-Elion Line to System Design for Bullwhip Avoidance
(Source: Authors based on Disney and Towill, 2002)

This realisation of the significance of the DE relationship has led to further research and the analytic determination of one particular bullwhip measure (order variance/demand variance) and associated (inventory variance/demand variance) on the assumption of random demand (Disney and Towill, 2003). These formulae are shown in Table I. Hence any potential user of this replenishment rule is now able to estimate the bullwhip associated with any particular settings proposed for the DE-APIOBPCS model. As we shall see in the next section, by setting the (order variance/demand variation) equation equal to unity, we can produce the bullwhip boundary. It is then extremely straightforward to propose suitable parameter settings to guarantee bullwhip avoidance.

Performance Metric	Mathematical Expression
VAR $_{ORATE}$ (Bullwhip)	$$\dfrac{2Ta^2 + 3Ti + 2Tp + 2(Ti+Tp)^2 + Ta(1+6Ti+4Tp)}{(1+2Ta)(Ta+Ti)(-1+2Ti)}$$
VAR $_{INV}$ (Inventory Variation)	$$1+Tp+\dfrac{2Ta^2(-1+Ti)^2 + Ti(1+Tp)^2 + Ta(1+Tp)(1+(-1+2Ti)Tp)}{(1+2Ta)(Ta+Ti)(-1+2Ti)}$$

Table I. Formulae for the Bullwhip and Inventory Performance Metrics Corresponding to the Recommended Replenishment Rule
(Source: Disney and Towill, 2003)

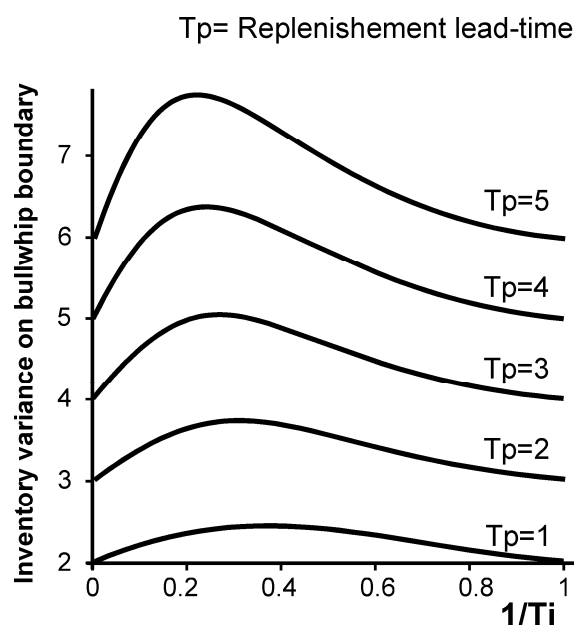## 4. Bullwhip and Inventory Variance Performance Metrics

We have solved the bullwhip and inventory equations previously presented in Table III for the DE-APIOBPCS replenishment rules to determine "zero bullwhip" contours. The bullwhip results are shown in Figure 4. If the system design parameters are located above the contour indicated for a particular value of replenishment lead time, then bullwhip exists. On the boundary the replenishment order variance is equal to the customer demand variance, hence in this respect the rule is "neutral". For parameter settings below the contour, the replenishment order variance falls below the demand variance. In this case the ordering system "smoothes" demand, usually by filtering out the high frequency "noise" (Dejonckheere et al, 2002).

a) Bullwhip boundary for various lead-times
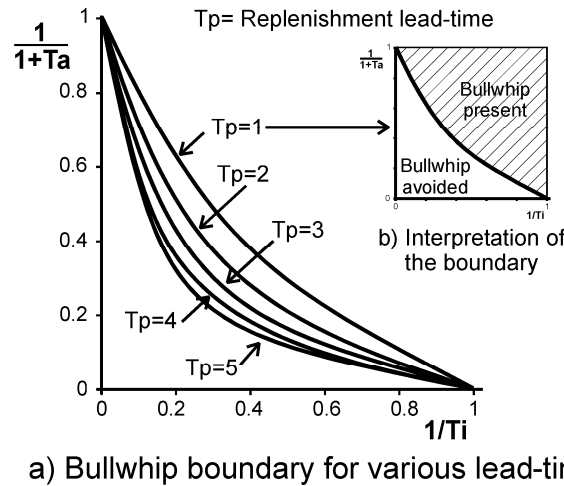
Figure 4. Bullwhip Contours Corresponding to Recommended Replenishment Algorithms
(Source: Authors)

The corresponding inventory variance in response to random customer demand is shown in Figure 5. At the extreme values of inventory control $(1/Ti)$, this variance is $(1 + Tp)$, as can be determined by inspection of Table I. This explicit dependence on lead time is yet further proof of the benefit of implementation of the Time Compression Paradigm (Towill, 1996). It emphasises the need to reduce lead times throughout the supply chain if better material flow control is to be enabled. The inventory variance does peak, but as Figure 5 shows, even for Tp=5 days the worst value is about eight i.e. only 46% greater than the minimum possible value for this replenishment lead time. So provided we incorporate the Dezeil-Eilon parameter settings of for $(Ti/Tw) = 1$ and the associated values of Ta as shown in Figure 4 we can avoid bullwhip. At the same time we have constrained the inventory variance so

that it does not exceed that determined by the replenishment lead time and the review period.

Tp= Replenishement lead-time

Figure 5. Associated Inventory Variance at the Bullwhip Boundaries
(Source: Authors)

5. Exploring Performance Trade-Offs and System Robustness

To further examine and understand the performance of the recommended replenishment rule the solutions obtained have been cross-checked by sampling systems along the line of symmetry (45º slope through the origin) in Figure 4. The results for Tp = 1; 2; 3; 4, and 5 days are shown in Disney and Towill (2005). In each case the theoretical bullwhip and inventory variance predictions (calculated from formulae in Table 1) have been compared with the results obtained by extensive simulation of the system response to a random demand signal. To ensure convergence these tests were conducted on 30 trials each with a time span of 10,000 days. The worst bullwhip difference observed between theory and simulation is only 0.4%! Finally, the worst observed inventory variance is 0.8%, hence considerable confidence may be attached to this replenishment rule theory as summarised in Table 1.

As Parnaby (1991) has emphasised, the design of any delivery pipeline must be checked for **robustness**. What exactly does this mean? To date our discussion has been on the design and performance of the recommended replenishment rule under **normal** or **steady state** conditions. That is, the analysis is conducted on the basis that the replenishment lead time is fixed at the value chosen at the start of the investigation. So how well does our pipeline perform when things change from

nominal? One obvious robustness check is therefore to estimate the impact on performance of the lead time actually being at some other (almost certainly greater) value.

Replenishment Rule Settings		System Performance for Expected Lead Time		System Performance for Substantially Increased Lead Time	
$1/Ti$	$1/(1+Ta)$	Bullwhip (Tp=1)	Inventory Variance (Tp=1)	Bullwhip (Tp=3)	Inventory Variance (Tp=3)
0.1	0.803	1	2.205	1.352	4.781
0.2	0.626	1	2.392	1.586	5.432
0.4	0.371	1	2.541	1.727	5.915
0.6	0.216	1	2.417	1.612	5.515
0.8	0.102	1	2.209	1.356	4.794

Table II. Checking the Robustness of the Recommended Replenishment Algorithm to Lead Time Changes
(Source: Authors)

To demonstrate the robustness of the recommended replenishment rule, we have calculated the corresponding change in performance when our system has been designed under an assumption that Tp = 1 day, but the actual in-service value is 3 days. The results of our analysis are shown in Table II, for a range of combinations of the parameters Ta and Ti (=Tw) as we traverse the bullwhip = 1.0 contour. Table II shows that under these adverse conditions bullwhip can increase by between 35% and 73% depending on the design selected. (This is still much less than the real-world bullwhip observed in the retail supply chain documented in Figure 1). The corresponding increase in inventory variance is between 117% and 133% of the original value for that particular design. Considering the excellent performance of he system achieved when Tp = 1, and acceptable behaviour of lead time inadvertently drops off to 3 days, this range of parameters might well be argued as providing a suitably robust (and certainly well controlled) system. But this is a trade-off debate to be considered further in the user guideline section.

6. Some Recommended User Guidelines
Having shown that bullwhip, as defined by the variance of the replenishment orders as a ratio of the variance of our customers' orders is readily avoided by suitable system

design, how is this knowledge best exploited by the user? Obviously, as shown in Tables IV and V, many combinations of replenishment rule parameters may be suitable. Which one might be best for our purpose? Unless the user is in a position to undertake a comprehensive investigation for every SKU in the catalogue, including simulation cross checks, a rough-but-reasonably-ready guideline needs to be used. Preferably such a guideline should be backed up by already existing and comprehensive knowledge and a range of proven practical applications.

We believe that for the DE-APIOBPCS replenishment rule a useful and comprehensive guideline is already available (Mason-Jones et al 1997). This was based on an exhaustive comparison of a number of "recommended" settings tested against a range of competing performance criteria. For the purpose of this paper these published "user guideline" settings may be interpreted as follows: $Ti = Tw = (Tp+1)$, and $Ta = 2(Tp+1)$. Substituting these values into the bullwhip equation in Table I, for a demonstration value of $Tp = 3$ days, gives an estimate of 0.378. In other words the replenishment order variance is about 38% of our customer order variance. In other words the *range* of the replenishment orders has been *reduced* (or "smoothed") by about 40%.
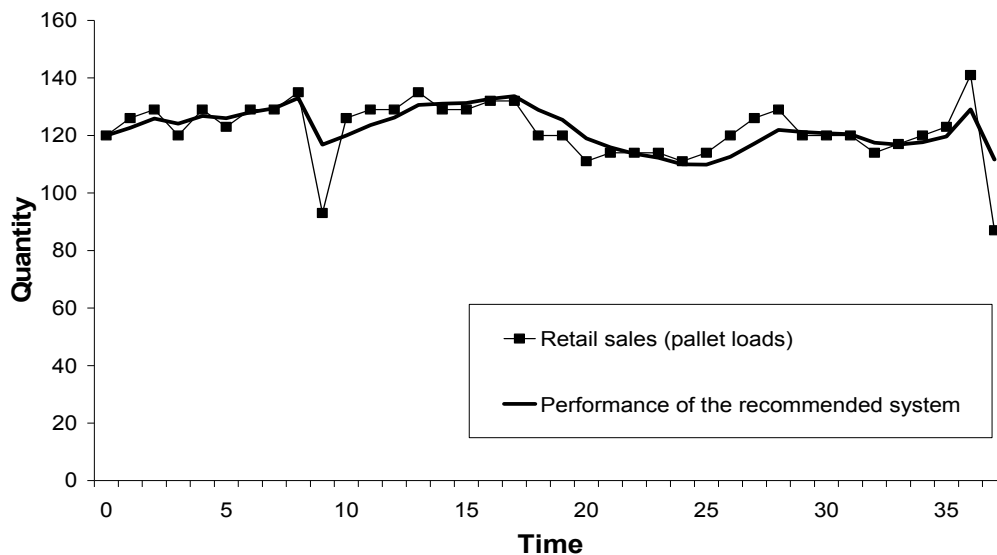
Figure 6. Response of "User Guideline" Setting Replenishment Rule to UK Retail Sector Demand Pattern Demonstrating Bullwhip Avoidance
(Source: Authors)

This is confirmed in the simulation study shown in Figure 6 on the same retail data. In contrast to the original real-world supply chain order patterns shown earlier herein

in Figure 1, where bullwhip undoubtedly exists, since the *range* of the replenishment orders is *amplified* by 100% (i.e. 2:1). So the improvement may be viewed as resulting in an order of magnitude in bullwhip reduction, and not just a fractional change in performance. It is obvious by comparing the replenishment orders actually placed in Figure 1 with those proposed in Figure 6 that the new rule is much better. Furthermore, this improvement is so blatant that detailed cost-benefit analysis to support the change is unnecessary. This is fortuitous since as Buxey (2001) has demonstrated, such cost data is often noted for its absence from much of the real-world industry. Hence the bigger the expected improvement, the more likely is the business process to be re-engineered.

7. Conclusions

We know that bullwhip is costly to all players in the supply chain. Consequential alternating "boom-and-bust" scenarios incur additional acquisition costs and additional stock-out costs. The ideal solution is to design and manage such on-costs out of the chain in such a way that the only uncertainty left is due to the marketplace (Childerhouse et al, 2003a, 2003b). One major source of internally generated bullwhip is due to poorly selected ordering policies in use at all levels within the chain. Our European retail supply chain provides ample evidence that this is indeed the case in the real-world. Hence this paper is concerned with selecting a replenishment ordering rule which can be structured to avoid bullwhip as observed in order variance behaviour. At least the various "players" can now adopt a policy to avoid self-induced bullwhip.

The DE-APIOBPCS replenishment rule exploited herein is well established in the literature. What we have demonstrated in the paper is that having the right decision making structure is insufficient. Additionally we need to select the re-ordering rule parameters to suit the replenishment lead time appropriate to the SKU (and thence, by appropriate scaling, for each and every SKU under consideration). We have therefore provided a formula for calculating both bullwhip and inventory variance when responding to our random customer demand. The formula then permits us to project a set of contours identifying bullwhip regions thus enabling "boom-and-bust" operating scenarios to be avoided. Finally we have interpreted these results via recommended and extremely robust replenishment rule settings which guarantee to avoid bullwhip

under these stated conditions. If this is tackled automatically then executive efforts can be concentrated on interface management so as to reduce bullwhip at this more difficult level.

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