

Resolving Super Massive Black Holes with LISA

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We study the angular resolution of the gravitational wave detector LISA and show that numerical relativity can drastically improve the accuracy of position location for coalescing Super Massive Black Hole (SMBH) binaries. For systems with total redshifted mass above $10^7 M_\odot$, LISA will mainly see the merger and ring-down of the gravitational wave (GW) signal, which can now be computed numerically using the full Einstein equations. Using numerical waveforms that also include about ten GW cycles of inspiral, we improve inspiral-only position estimates by an order of magnitude. We show that LISA localizes half of all such systems at $z = 1$ to better than 3 arcminutes and the best 20% to within one arcminute. This will give excellent prospects for identifying the host galaxy.

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Introduction.— Astronomical observation provides very strong evidence for the existence of a “dark” compact massive ($10^6 - 10^9 M_\odot$) object in the core of every galaxy for which the central parsec region can be resolved [1]. These objects are believed to be supermassive black holes (SMBH), and are of great interest to researchers in fundamental physics, astrophysics and cosmology. Their formation and the observed correlation between SMBH mass and galaxy morphology (see [2] for an overview) are still open questions. SMBHs probably arise at least in part through the mergers of smaller-mass BHs [3]. These mergers and the mergers of SMBH binaries following collisions of galaxies constitute some of the most powerful sources of gravitational waves (GW) predicted by current models. We will be able to detect such events throughout the Universe with LISA [4], a proposed space-borne gravitational-wave observatory, scheduled for launch in 2018+ and designed to be sensitive to GW signals in the range $10^{-4} - 0.1$ Hz.

Realizing the full scientific benefit of the LISA mission will require accurate estimates of the binary’s parameters. Precise measurements of the masses, spins and distance will allow us to probe models of SMBH formation. Accurate localization of the source in the sky is crucial to relate gravitational-wave and electromagnetic observations of the coalescence event, and hopefully will allow identification of the host galaxy. Optical observations are required to measure the redshift to the object, while gravitational-wave observations yield precise calibration-free estimates of the distance. Such LISA events with optical counterparts will determine the redshift-distance relationship, which in turn will allow us to map the geometry of the Universe and measure the amount of dark energy.

Recently several groups [5, 6, 7, 8] have evaluated the accuracy of parameter estimation using the inspiral part of the GW signal. It was shown that the spin and higher orbital harmonics are necessary to de-correlate the parameters and therefore improve the parameter estimation.

tion.

In this Letter we assess the angular resolution of LISA for SMBH binaries with (red-shifted) masses above $10^7 M_\odot$. We expect several such merger signals per year at relatively close distance [3]. For those heavy systems the inspiral signal may be smaller or at least not much larger than the instrumental noise, and the signal will be dominated by the merger and ring-down. We use numerical relativity to compute a waveform which contains about ten GW cycles, plus merger and ring-down. We fix the *redshifted* masses of two non-spinning BHs to $4.44 \times 10^6 M_\odot$ and $8.88 \times 10^6 M_\odot$ and vary the “extrinsic” parameters: sky ecliptic coordinates θ_S, ϕ_S , inclination ι of the orbital angular momentum to the line of sight, polarization angle ψ , fiducial arrival time T_0 which we fix to be the time when the binary separation equals $10 M$, orbital phase ϕ_0 at T_0 , and luminosity distance D_L .

Data analysis for LISA is based on time-delay interferometry (TDI, see [9] for an overview). We convert the strain polarizations h_+ and h_\times given in the source frame to first generation unequal-arm Michelson streams X, Y, Z [9, 10] and use two combinations $A = (2X - Y - Z)/3$, $E = (Z - Y)/\sqrt{3}$ with uncorrelated noise. Due to technical difficulties explained below, we do not take into account the third independent combination, which has poor sensitivity to GW at low frequencies, and which adds only a few percent to the combined signal-to-noise ratio (SNR). By fixing masses we underestimate the errors, but at the same time not including the third TDI combination overestimates the error boxes. We mainly concentrate here on the estimation of the localization of the source and usually [8] the directional angles very weakly correlate with masses. Since we use the merger for localizing the hosting galaxy, we cannot produce an early warning for the merger itself; however, we can identify the hosting galaxy by the afterglow [11].

We conduct Monte Carlo simulations by randomly choosing 600 points in the extrinsic parameter space (with fixed masses, and choosing an example distance

of $z = 1$ or $D_L \approx 6.4$ Gpc), and estimate the errors in the parameter error box with two different methods: the first is based on computing the variance-covariance matrix, the second is a Bayesian method based on the evaluation of the marginalized posterior distribution function using a Metropolis-Hastings Markov chain (MHMC) [12, 13, 14]. We have shown that both methods give comparable results. For 50% of the randomly chosen parameters we can localize the source down to a box 3 arcminutes on a side, and the best 20% of the events are localized to better than one arcminute. The relative error in the distance is less than 1%, the largest error in the distance being due to weak lensing [15]. We also estimate robustness of our results with respect to errors in the numerical waveform. By conducting another Monte Carlo simulation, we have found that the errors in the sky locations are good to within factor two at worst (but usually much better than that).

Numerical relativity waveforms.— We use numerical-relativity waveforms from simulations of non-spinning black holes at mass ratio 1:2, which have initially been presented in [16], and are discussed in more detail in [17], where the quadrupole spherical harmonic mode has been compared with effective-one-body waveforms, and excellent agreement in the phase evolution has been found. Here we now include results for higher spherical-harmonic contributions. Our simulations follow the now standard moving-puncture approach [18, 19], using the BAM code [20, 21] to numerically solve the Einstein equations. Black hole initial data are modeled as conformally flat puncture data [22, 23]. The initial data parameters then reduce to the black-hole masses, momenta and separation. We choose a separation of $10 M$ (we refer to the total initial black-hole mass as M). The momenta are specified to give quasicircular inspiral with minimal eccentricity of $e \approx 0.003$ [24].

The gravitational-wave signal is extracted at five surfaces of constant radial coordinate, $R_{ex} = 40, 50, 60, 80, 90M$ by means of the Newman-Penrose Weyl tensor component Ψ_4 , as described in [20]. At every extraction radius the gravitational wave strain is obtained from Ψ_4 by double time integration as described in [16]. The analysis carried out in this paper will use, as approximate asymptotic amplitude, the curvature perturbation extracted at radius $90M$, at our highest numerical resolution, as has been done in [17]. For the modes with $l > 2$ we filter out low frequencies and frequencies higher than the Nyquist frequency to avoid the pathologies discussed in Sec. II.A. of [25]. For these simulations, we find that the finite extraction radii dominate the error, and the amplitude error is below 5% prior to merger, and the accumulated phase error is below 0.25 radians for the $700M$ up to $M\omega = 0.1$. The fall-off in the amplitude error with respect to radiation extraction radius is not clean around merger time, preventing us from performing an accurate extrapolation to infinity. As such, we would

conservatively give an uncertainty estimate of 10% of the amplitude at merger and later. We estimate the *relative* error in the amplitude between different modes (which is what dominates the results presented here) as below 4%.

Parameter estimation.— We have used two methods to estimate accuracy in measuring the parameters of the GW signal. The first method is based on computing the variance-covariance matrix. This method is widely used and well described in numerous publications (see for example [15, 26, 27, 28]). It is based on inverting the Fisher matrix Γ , which is the matrix of the inner products between derivatives of the signal with respect to the parameters $h_{,i} = \partial h / \partial \lambda_i$:

$$\Gamma_{ij} = 4\Re \int_0^\infty df \frac{\tilde{h}_{,i}(f)\tilde{h}_{,j}^*(f)}{S_n(f)}, \quad (0.1)$$

where $S_n(f)$ is the one-sided noise power spectral density. We use two TDI measurements and the combined Fisher matrix is a sum of Fisher matrices for A and E : $\Gamma_{ij} = \Gamma_{ij}^A + \Gamma_{ij}^E$. The presence of the noise might cause a deviation of the recovered parameters of the GW signal from the true values. The diagonal elements of the variance-covariance matrix are maximum likelihood estimators of the variance of parameters around the true value in the case of a large SNR (which is always the case here). We have computed derivatives numerically using forward differencing and checked the robustness with respect to the step using several randomly chosen points in the parameter space. We have tried to choose a parametrization of the signal which would reduce the dynamical range of the eigenvalues of the Fisher matrix, however there are still six or seven orders of magnitude between the smallest and largest eigenvalues.

As a second way to evaluate the parameter estimation error we use a Metropolis-Hastings Markov chain (MHMC) [12] to estimate the posterior probability function

$$p(\vec{\lambda}|s) \propto \pi(\vec{\lambda})L(s|\vec{\lambda}), \quad (0.2)$$

where $\pi(\vec{\lambda})$ is the prior distribution of parameters and $L(s|\vec{\lambda})$ is the likelihood function. The MHMC uses a proposal distribution $q(\vec{\lambda}_{(i)}|\vec{\lambda}_{(j)})$ and Metropolis-Hastings ratio

$$H = \frac{p(\vec{\lambda}_{(j)}|s)q(\vec{\lambda}_{(j)}|\vec{\lambda}_{(i)})}{p(\vec{\lambda}_{(i)}|s)q(\vec{\lambda}_{(i)}|\vec{\lambda}_{(j)})}, \quad (0.3)$$

which gives a probability $\alpha(\vec{\lambda}_{(i)}|\vec{\lambda}_{(j)}) = \min(1, H)$ of accepting the jump from the point $\vec{\lambda}_{(i)}$ to $\vec{\lambda}_{(j)}$. MHMC gives the best result (better convergence) if the proposal distribution matches the shape of the target distribution. For the proposal distribution we take normal jumps in the eigen-directions of the Fisher matrix [14], yielding acceptance rates $> 30\%$. We have

generated simulated data using the *lisatools* software [http://code.google.com/p/lisatools/], consisting of instrumental noise and a simulated reduced Galaxy confusion noise with more than 5×10^7 chirping binaries [29, 30]. We have generated signals for 600 randomly chosen parameters (600 data sets), and performed 600 mappings of posterior distribution functions using 3×10^5 long chains. We have also used mild simulated annealing for the first two thousands steps, which helps the chain to migrate from the point with true parameters to the point with better likelihood. Note that due to the presence of the Galaxy the noise is strictly speaking not Gaussian. Because the jumps are rather small, we have made the assumption that the Fisher matrix does not change notably within the jumping region and therefore $q(\vec{\lambda}_{(i)}|\vec{\lambda}_{(j)}) = q(\vec{\lambda}_{(j)}|\vec{\lambda}_{(i)})$ and the Metropolis-Hastings ratio is reduced to Metropolis. This assumption results in a small bias in the estimation of the variance, but it significantly speeds up the computations.

The results of both methods are presented in the top two plots of Figure 1, as cumulative histograms for the 600 realizations. Both methods give comparable estimation of the error box for the sky location (θ_S, ϕ_S) . One can see that 50% of the trials give an error box smaller than 3 arcminutes for a source located at $z = 1$.

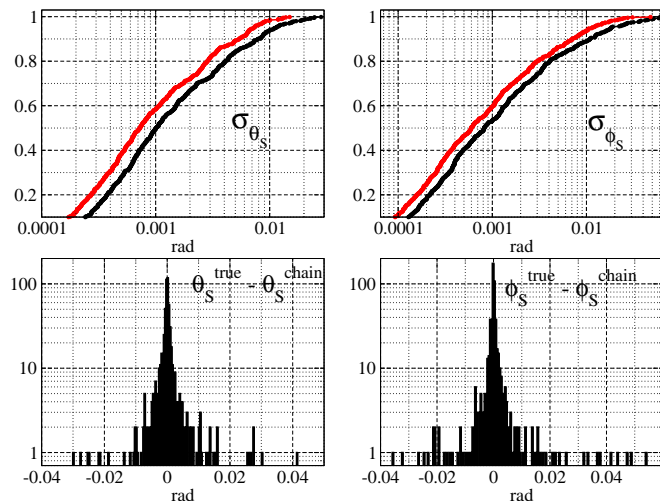


FIG. 1: Top plots: cumulative histograms for the standard deviation estimators from 600 randomly chosen points in the parameter space. The black (lower) curve corresponds to the variance σ evaluated by computing the variance-covariance matrix and the red (upper) curve is the Bayesian estimator obtained using the MHMC. Lower plots: deviation of the mean value of the chain from the true parameter of the signal. The plotted results correspond to a distance of 6.4 Gpc ($z \approx 1$ and total mass of $\approx 0.7 \times 10^7 M_\odot$),

The duration of the signal from a separation of $10M$ to the merger is 26 hours, so the Doppler modulation is small, however for our choice of parameters in the Monte Carlo simulation the SNR varies between 900 and 9000,

and these large values drastically improve parameter estimation. The other crucial ingredient for the excellent angular resolution is the presence of higher orbital harmonics coming from the higher multipoles of the source. We find that the use of only the dominant ($l = |m| = 2$) mode worsens our results by up to a factor of ten. Different harmonics have different angular emission patterns and different dependencies on the inclination. These help to de-correlate the parameters, and together with the high SNR turn a seemingly small effect (higher modes are much lower in amplitude) into a crucial contribution for parameter estimation. For our analysis we have used $l = 2, 3, 4$ and $m = -l, \dots, l$ (except $m = 0$). We also find that the arrival time could be measured with an accuracy of less than a second and the luminosity distance with an accuracy less than 1.5%. However, the dominant error in estimating the luminosity distance is due to weak lensing [15]. Weak lensing gets stronger with distance [31] and we expect to have SMBH binaries at distances $z < 5$.

As we have mentioned, the Markov chain is usually migrating to the point with higher likelihood and this shift could be as large as the variance itself. We have found that the secondary maxima in the likelihood could be very close to the primary maximum both in amplitude and distance in parameter space. In our simulations we observed that due to the presence of the instrumental and Galactic confusion noise the secondary maxima could become stronger than the primary (by less than 0.01%) and the secondary could be located about a σ away, where σ is the standard deviation of the parameter assuming no secondary maxima. In the lower two plots of Figure 1 we show the histogram of the deviation mean value of the chain from the true sky location. Further work is needed to explore ways of reducing these ambiguity errors.

We have used only two TDI streams in the above analysis, because we could not compute the third independent data set constructed out of X, Y, Z , which is $T \propto X + Y + Z$, with the required accuracy: the signal cancellation at low frequencies was not perfect due to inaccuracy (less than a few percent) in the evaluation of X, Y, Z . In principle we could generate A, E, T out of the six-pulse TDI combinations as in [32] but we did not have simulated a Galactic background for those combinations, so we have decided to drop the third effective detector for the present considerations. We however checked that using T (as in [32]) would improve our SNR by a few percent, and, more importantly, could also improve parameter estimation: having three independent measurements should help to triangulate the source.

Finally, we have checked the robustness of the variance estimation with respect to possible errors in the numerical waveform. Since the results depend on the relative strengths of the different harmonics, we varied the $l = m = 2$ mode by $\pm 5\%$ with respect to the higher harmonics. We have estimated the variance by computing the variance-covariance matrix and comparing it to the

original one, and find that σ changes by at most a factor of two (much less in the majority of the cases $\lesssim 15\%$). We have also noticed that changes in the waveform resulting from enhancing the higher orbital modes improve results (makes the variance smaller) which is in agreement with our explanation of the angular resolution.

Summary.— We have presented a first study of how results from numerical relativity can improve parameter estimation for SMBH binary observations with LISA. We have chosen to first examine the issue of angular resolution of LISA – which is crucial to identify electromagnetic counterparts to gravitational wave observations. Looking at the merger signal for non-spinning SMBH binaries with redshifted total mass $1.4 \times 10^7 M_\odot$ and a mass-ratio of 2, Placing our source at a luminosity distance of 6.4 Gpc ($z \approx 1$, corresponding to a total mass of $\approx 0.7 \times 10^7 M_\odot$), we have found an excellent sky resolution: for 50% of our randomly chosen events we can localize the source down to 3 arcminutes, which roughly corresponds to an order of magnitude improvement over estimates using only the inspiral phase of the GW signal. We have also shown that we obtain consistent error estimates when comparing two independent methods based on the variance-covariance matrix and a Metropolis-Hastings Markov chain.

Our result calls for further work along several directions. First, it is necessary to cover the parameter space of numerical simulations: larger mass ratios will show two effects: a decrease in SNR, but an increase of the contributions of higher modes, which have proven crucial for parameter estimation. The inclusion of spins will lead to a much larger parameter space, but also to much more interesting waveform structures. For lower masses, it will be important to accurately match numerical waveforms to post-Newtonian results in order to cover the whole LISA band, and to understand the systematic errors in the matching procedure. The extension of existing phenomenological waveforms (e.g. along the lines of [16, 33]) to spinning binaries and non-dominant modes will allow for systematic parameter estimation studies in the regime where solving the full Einstein equations is necessary to obtain accurate gravitational wave signals, and will extend the useful sensitivity range of LISA to substantially higher masses, so that LISA can begin to explore the process whereby black holes grow from the size we see in our galaxy to the sizes that are needed in quasars.

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