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Genetic Algorithm Optimisation of a Class of Inventory Control Systems.

By

S. M. Disney, M. M. Naim and D.R. Towill.

Logistics Systems Dynamics Group, Department of Maritime Studies & International Transport, University of Wales, Cardiff, P.O. Box 907, Cardiff, CF1 3YP. UK.

Abstract.

The paper describes a procedure for optimising the performance of an industrially designed inventory control system. This has the three classic control policies utilising sales, inventory and pipeline information to set the order rate so as to achieve a desired balance between capacity, demand and minimum associated stock level. A first step in optimisation is the selection of appropriate "benchmark" performance characteristics. Five are considered herein and include inventory recovery to "shock" demands; in built filtering capability; robustness to production leadtime variations; robustness to pipeline level information fidelity; and systems selectivity. A genetic algorithm for optimising system performance, via these five vectors is described. The optimum design parameters are presented for various vector weightings. This leads to a Decision Support System for the correct setting of the system controls under various operating scenarios. The paper focuses on a single supply chain interface, however the methodology is also applicable to complete supply chains.

Nomenclature.

$\overline{Tp}$	Production WIP Gain.
ω	Frequency (rads/time period)
λ_a	Normalised Parameter = (*Ta/Tp*)
λ_i	Normalised Parameter = (*Ti/Tp*)
ω_N	Noise Bandwidth
Δt	Simulation time increment.
AINV	Actual Inventory Holding
APIOBPCS	Automatic Pipeline, Inventory and Order Based Production Control System.
AVCON	Average Consumption.
COMRATE	Completion Rate
CONS	Consumption or Market Demand
CSL	Customer Service Levels
DINV	Desired Inventory Holding
DSS	Decision Support System
DWIP	Desired Work In Progress
E	Error
EINV	Error in Inventory Holding
EWIP	Error in Work In Progress
ITAE	Integral of Time*Absolute Error.
ORATE	Order Rate
PR	Robustness to production leadtime variations
s	Laplace Operator, and (S) Normalised Laplace Operator.
SV	Systems selectivity
t	Time
Ta	Consumption Averaging Time Constant.
Ti	Inverse of Inventory Based Production Control Law Gain.
Tp	Production Lag Time Constant.
Tw	Inverse of WIP Based Production Control Law Gain.
WIP	Work In Progress
WIPR	Robustness to pipeline level information fidelity

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Introduction.

Burbidge's Law of Industrial Dynamics states that "If demand is transmitted along a series of inventories using stock control ordering, then the amplitude of demand variation will increase with each transfer", (Burbidge, 1984). This results in excessive inventory, production, labour, capacity and learning curve costs, due to unnecessary fluctuations in perceived demand, (Disney et al, 1997a). One major cause is the time lag between a decision to make or order a unit of inventory and the realisation of that order, Sterman (1989). It has long been understood (from knowledge of controller design), that the way to minimise this effect is to design the decision appropriately using control theory techniques to customise the response of the production/ distribution decision (Forrester, 1961). This implies that the decision has a known structure and there are a number of such generic structures in use. A generic production/ distribution control algorithm, termed the Automatic Pipeline Inventory and Order Based Production Control System (APIOBPCS) can be expressed in words as follows; "Production targets are equal to demand averaged over *Ta* time units (the demand policy), plus a fraction, *Ti*, of the inventory deficit in stores (the inventory policy), plus a fraction, *Tw*, of the WIP deficit (the WIP policy)." We usually express the model in continuous control form. However the discrete version is also available, if preferred (Popplewell and Bonney, 1987).
The model can also be expressed in block diagram form as shown in Figure 1 (John et al 1994). The block diagram describes, in a form suitable for building a simulation model, the controllers that are used to place production orders. It is also representative of Sterman's work (1989), (where a simplified model of a beer production/ distribution system was used to convince senior executives that they did not fully understand the concept of the supply chain, especially the effect of system structure on system behaviour). His heuristics can be directly related to the control parameters *Ta*, *Tw* and *Ti,* via mathematical manipulation, (Naim & Towill 1995). It should also be noted that by appropriate selection of system parameters the model can cover a wide spectrum of supply philosophies ranging from make-to-stock to make-to-order.

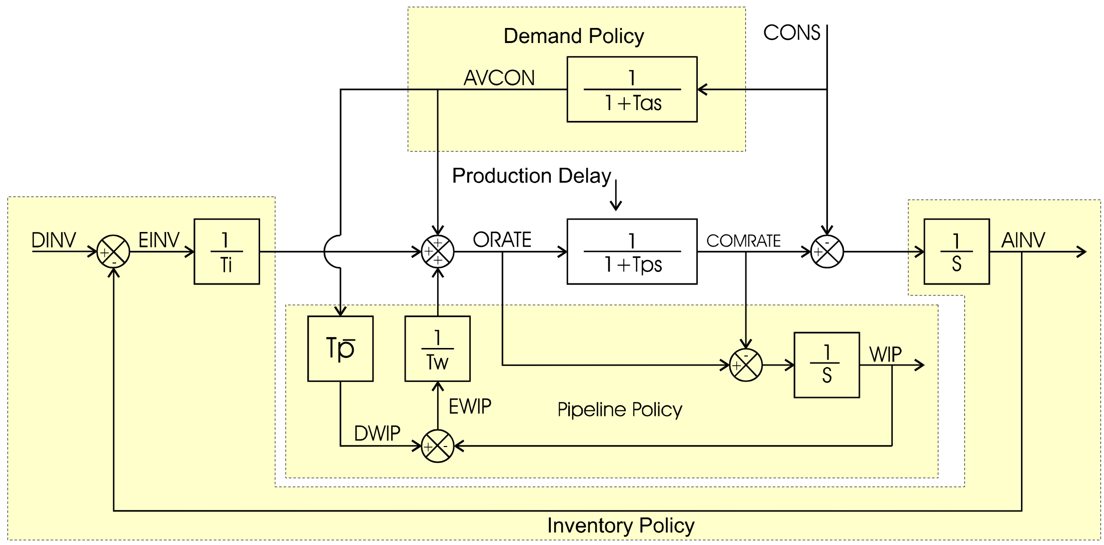

Figure 1. Block Diagram Laplace Representation of APIOBPCS.

Within the block diagram we have approximated the production delay in the Laplace domain by using $\frac{1}{\left(1+\frac{Tps}{n}\right)^n}$, where n is either 1, for a first order delay, or 3, for a third order delay. Proof of this approximation is shown in Chen (1957); and it is also the *n*th order delay used in the DYNAMO/ STELLA/ iTHINK simulation packages (Forrester, 1961). It has been shown by various authors that these are good representation of real world production lead-time distributions, for example Wolstenholme (1990).

The focus of this paper will be to show how the control parameters, Ti, Ta and Tw change as the balance between a situation's inventory carrying costs and production on-costs (capacity related costs) alters. We will present a table of control parameters which reflect various weightings on inventory and on-costs. To do this we will make extensive use of some control theory techniques coupled with an optimisation procedure termed, Genetic Algorithms. As an aid to the structure and usefulness of the DSS and to the content of this paper, Figure 2, shows the inputs, outputs, assumptions and tools and techniques that are used to develop the DSS.

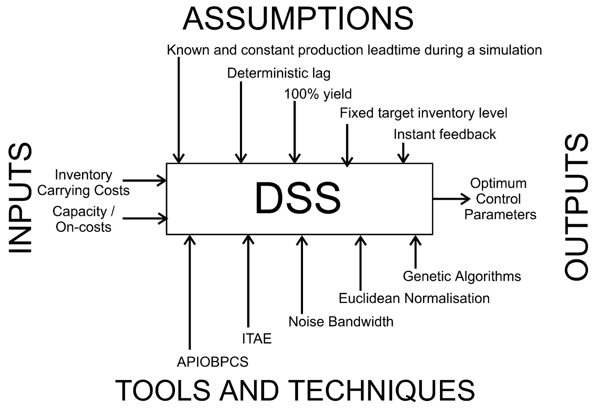

Figure 2. Outline Description of the DSS.

Description of APIOBPCS.

A description of the relevance of each policy element within APIOBPCS is now given (Disney et al., 1997a).

Demand Policy.

Current demand is an important factor because if it is omitted from the scheduling algorithm, it can be easily shown mathematically and experimentally that there is a continuing freefall following a ramp demand and a permanent inventory deficit following a step increase in demand. This is typical of many real world stock replenishment systems, (Disney et al., 1997a). It is also accepted that allowing demand to be used for scheduling without some form of averaging will result in excessive fluctuations in production rates, which in turn leads to increased production on-costs, (Shalk and Haut, 1990). Therefore we need to utilise an average measure of the current market demand in the proposed scheduling algorithm. The most suitable method of doing this is by exponential smoothing, as it requires little data storage, is relatively accurate for short term forecasts, is a close approximation to the first order lag used in control theory and is readily understood. ***The question to be answered here is, how much weighting do we give to the recent demand figures to attenuate the fluctuations in demand but at the same time respond to genuine changes?***

Inventory Policy.

The inventory policy is to be considered because the rate at which we recover deviations in inventory will have a profound effect on production fluctuations, (Disney et al., 1997a). It is often a misguided practice in industry that production targets are set to recover all the inventory deficit in a single time period, even though it may take many more time periods for the product to be manufactured and appear in inventory holding (Berry & Towill, 1995). This is continued for the whole of the production lead time. By the time the products begin to appear in the inventory there is significant excess WIP on the shop floor, which will inevitably increase the stock holding beyond the desired level. This will have to be reduced by producing less than the average market demand until the latter has reduced the inventory levels to the desired level. However the same target overshoots happen here and the production rates are continuously fluctuating. As stock levels are related to our customer service levels and inventory offsets result from decision making without explicit inventory policies, it is desirable to correct these discrepancies. ***The question to be answered here is, how much of the inventory discrepancy do we correct each time we set production/ distribution requirements to avoid excessive overshoots and undershoots around the target level?***

Pipeline Policy.

The pipeline policy is concerned with how much WIP is present on the shopfloor, (Disney et al., 1997a). The desired WIP level is a function of the average demand and the time it takes to produce the product, i.e. the lead time. Throughout this paper we are going to assume when setting targets that the production lead time is known via shop floor feedback. However, it is not proposed to update the system controller settings in real-time during the robustness experiments since this accords with known industrial practice, (Cheema, 1994). During periods when there is insufficient WIP, for example, just after a genuine step increase in demand, then it would be beneficial for the pipeline policy to increase the demands on the shop floor to account for the shortfall in WIP. However there will be periods when there is excessive WIP on the shop floor due to the inventory and demand policies not considering the effects of the time delays in the system. It would then be beneficial for the pipeline policy to reduce the production targets. ***The question to be answered here is, how quickly do we correct the pipeline deviation each time we set the production/ distribution requirements?***

The Effect of Controller Settings on System Response.

On the Production Orders.

The effect of each controller in making up the production target (ORATE) is clearly shown in Figure 3, (Disney et al., 1997a), (a simulation model of APIOBPCS can be built in a spreadsheet environment using the difference equations shown in Appendix 1). The smoothed sales signal (AVCON) is responsible for feeding forward the change from one level of sales to the other. The inventory feedback signal (EINV/Ti) is the main contributor to ORATE overshoot and oscillatory behaviour. The WIP signal (EWIP/Tw) is self adjusting, i.e. during times when WIP is too small it bolsters ORATE, and during times when WIP is too great it reduces ORATE. The effect of the WIP signal reduces the rise time of ORATE, and decreases the percentage overshoot; both favourable traits. However the downside costs must appear somewhere, which in this case is in the extended settling time. The more emphasis that is given to WIP feedback, the longer it takes to reach steady state. This is due to the negative WIP signal cancelling out the inventory signal, thus giving more responsibility to the contribution of AVCON in reaching the steady state.

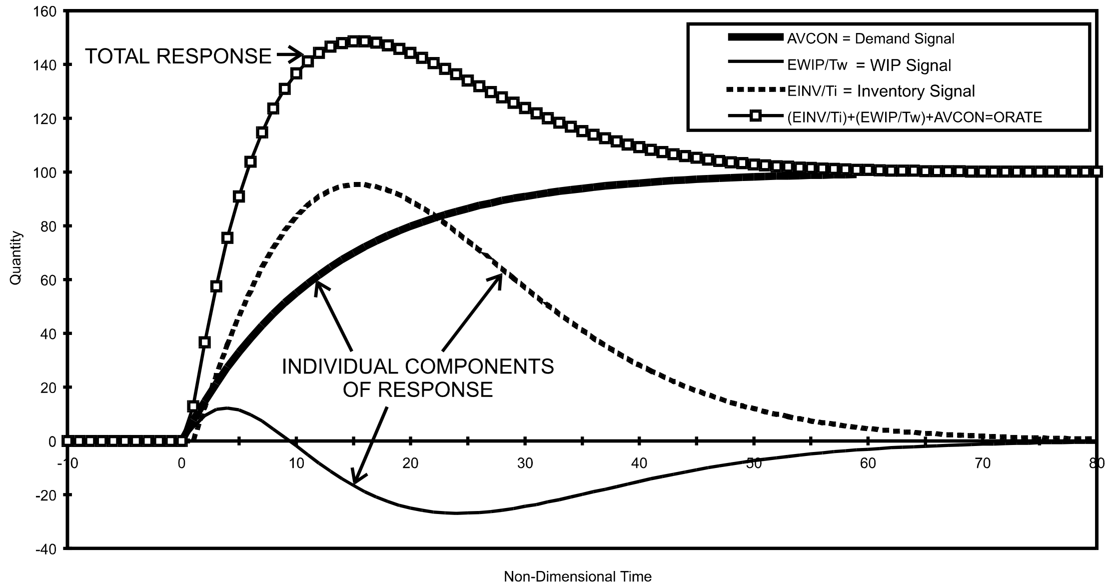

Figure 3. The Effect of the Individual Controllers on Production Order Rates.

The problem can now be fully stated. We need to determine the optimum setting of the design parameters Ta, Ti and Tw. An optimum setting of the parameters will;

- respond to genuine changes in demand quickly,
- filter out random noise in the sales pattern,
- be robust to unknown changes in production lead-time and to changes in production lead-time distributions. Although it is an objective of lean manufacturing to achieve short, consistent lead-times, this is more difficult in a multi product environment. The competition for resources means that achievement in practice is still difficult (Harrison, Holloway, and Patell, 1990). It would therefore be beneficial to consider the consequences of production variations on the dynamic response in our optimisation procedure, and thus ensure that the detrimental effects are minimised,
- be robust to delays in the WIP feedback loop as this information is not always easy to collect, (Cheema, 1994).
- be selective, in the sense that minor changes to the control parameters by real world users will not result in significant degradation of performance.

On the Inventory Levels.

Figure 4, shows the effect of the controller parameters on the actual inventory levels (AINV), following a step increase in sales from 100 to 200 widgets/time unit at time t = 0. Each controller setting has been reduced by 25% of its nominal value. Decreasing Tw will slightly reduce the maximum inventory deficit, but it will take longer for the inventory levels to fully recover. Decreasing Ta has a similar effect to Ti, but is less pronounced. Ti will reduce the inventory freefall considerably more than reducing Tw and the recovery is much quicker. Ti also has the advantage over Ta in that after the freefall it is quicker in the recovery. However, too small a value of Ti results in very poor filtering properties in the presence of a random signal.

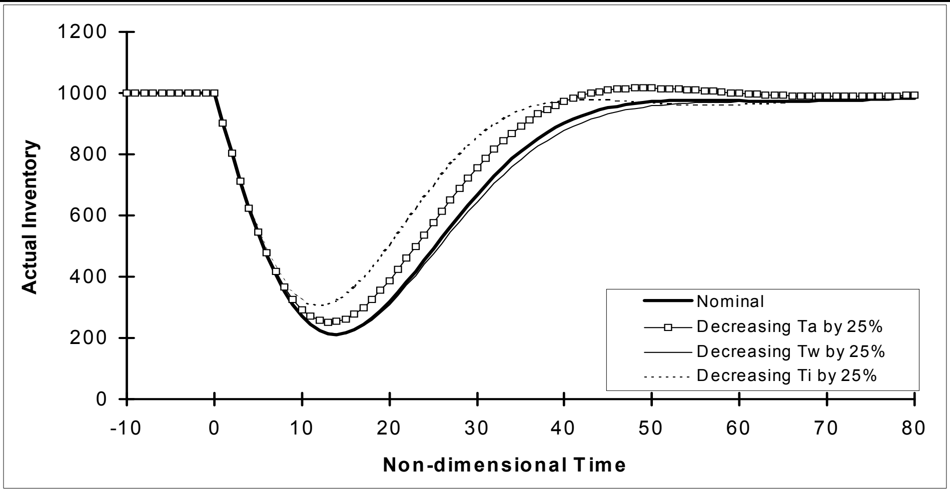

Figure 4. The Effect of Ta, Tw and Ti on Inventory Recovery Following a Step Input.

Performance Characteristics of a Scheduling Algorithm.

It is the aim of this section to highlight a method to determine how to judge the fitness of the values of the control parameters (Ti, Ta and Tw), for the five characteristics above. Summarising, the optimisation routine will assess the trade off between;

- inventory recovery
- noise attenuation
- production robustness,
- robustness to WIP information lags
- selectivity.

Quantification of Characteristics.

Each of the above five criteria for assessing the scheduling algorithms performance will now be quantified to allow a GA to determine the fitness of a design, in order to optimise the scheduling algorithm. However, first we will introduce the transfer functions of the systems as these play an important role in the subsequent analysis. Then after the first two characteristics have been quantified (inventory recovery and noise bandwidth) we will show some analysis to describe the nature of the basic trade-off and the importance of the cost structure of real situations on the optimisation procedure. Finally, the remaining desirable characteristics will be described.

Transfer Function Analysis.

By manipulating the block diagram in Figure 1, a set of system transfer functions can be derived. Of particular interest is the ORATE/CONS and the AINV/CONS transfer functions, shown in Eq. 1 and 2 respectively. The ORATE transfer function is useful as it yields information about the dynamic performance of the production targets and it plays an important role in defining the filtering ability of the system. The addition of the WIP controller into the production control system has allowed us to de-couple the damping ratio from the natural frequency. This was not possible in the Inventory and Order Based Production Control System (IOBPCS), Towill (1982). It also allows the filtering characteristics of the system to be improved as the WIP controller forces the ORATE signal to be closer to the AVCON signal, at the expense of the inventory signal. The more the system is driven by the inventory signal the faster the system will become and the more reliant it is on the WIP controller to check its performance. Therefore in these situations it is important that the WIP data acquisition system is operating, hence the WIP robustness performance vector.

$$\frac{ORATE}{CON} = \frac{1 + \left(Ta + Ti + Tp + \frac{T\overline{p}Ti}{Tw}\right)s + \left(TpTa + TiTp + \frac{TpT\overline{p}Ti}{Tw}\right)s^2}{(1 + Tas)\left[1 + \left(Ti + \frac{TpTi}{Tw}\right)s + TpTis^2\right]} \quad \text{.......Eq 1.}$$

$$\frac{AINV}{CONS} = \frac{Ti\{(T\overline{p} + Tw) - (1 + Tas)[Tp + Tw(1 + Tps)]\}}{(1 + Tas)[Tw(1 + Tis) + TiTps(1 + Tws)]} \quad \text{........Eq 2.}$$

The AINV/CONS transfer function is useful because it gives information on the dynamic response of the inventory performance. The final value theorem can be applied to the AINV transfer function as shown in Equations 3 to 4. It shows us the importance of being able to accurately determine the production lead-time. If an inaccurate production lead-time has been used to set the desired WIP targets, then there is a permanent error in the steady state inventory levels (John et al. 1995). This is clearly jeopardising CSL's. Therefore if a WIP controller is used to set production targets the lead-time must be accurately known. It will be assumed in this paper that the lead-time is known and constant throughout a particular simulation for the purpose of setting WIP targets only. This is representative of the real world scenario where companies monitor their lead-time for setting WIP targets, but do not use it to update their optimum controller settings. It is all too often forgotten that the controller settings are a function of the production lead-time (Cheema 1994).

The Final Value Theorem is given by: $\lim_{t\to\infty}\{f(t)\} = \lim_{s\to 0}\{s * f(s)\}$ ....Eq 3.

where $f(s) = \frac{AINV}{CONS}$, therefore in the limit the final value = $Ti\left[\frac{T\overline{p} - Tp}{Tw}\right]$ ....Eq 4.

Inventory Recovery.

The Integral of Time * Absolute Error (ITAE) is generally agreed to be the most intuitive criterion following a step, for assessing transient deviations from a target. It is inevitable that a large error is present shortly after the step and it penalises more heavily, errors that are present later, by a suitable weighting in the time domain, (Towill 1970). The ITAE also penalises positive and negative errors equally, and is the simplest measure that is reliable, applicable and selective, (Graham et al 1953).
The ITAE is defined in Equation 5. Throughout this paper the ITAE was calculated following a step input in CONS that increased from 100 to 200 widgets per time period at time = zero.

$itae = \int_{\infty}^{0} t|E|dt$ ....Eq 5, where t= time period and |E|= absolute error in inventory

Noise Attenuation.

The ORATE noise bandwidth is important because it is a measure of the ability of the Sales Averaging Time (*Ta*), Time to Adjust Inventory (*Ti*), and Time to Adjust WIP (*Tw*), to filter out the random higher frequency content of the demand, when setting production targets. The noise bandwidth is defined as the area under the system amplitude ratio squared curve, (Towill 1982). For a linear system it is also proportional to the output variance. The noise bandwidth is a useful method of condensing frequency domain information into one criterion, and can be estimated from the frequency response plot. Alternatively it may be evaluated algebraically via Parsevals Theorem (Garnell and East 1977 and Newton et al 1957). In the case of APIOBPCS for an assumed first order production lag the noise bandwidth equation is shown in Eq 6, below, with the derivation later in Appendix 2.

$$\omega_N = \frac{\frac{\left(TpTa + TpTi + \frac{Tp\overline{T}pTi}{Tw}\right)^2\left(Ti + \frac{TpTi}{Tw} + Ta\right)}{TpTiTa} + \left(Ta + Ti + Tp + \frac{\overline{T}pTi}{Tw}\right)^2 - 2\left(TpTa + TpTi + \frac{Tp\overline{T}pTi}{Tw}\right) + \left(TpTi + TaTi + \frac{TpTiTa}{Tw}\right)}{\left(TpTi + TaTi + \frac{TpTiTa}{Tw}\right)\left(Ta + Ti + \frac{TpTi}{Tw}\right) - TaTiTp} \quad \text{....Eq 6.}$$

The Nature of the Trade-Off and the Importance of the Cost Structure.

Figure 5, below shows the nature of the trade-off between the inventory recovery and noise bandwidth for a given set of control parameters (where Ta and Tw were kept constant at 16 and 27 respectively, and Ti is changing). The inventory recovery has been scaled (divided by 1,600,000, a value determined via experimentation) so that the its magnitude does not swamp out the noise bandwidth signal. With both vectors the smaller the better, the optimum value of Ti is around 7. However, it can be envisioned that if the scaling factor on the ITAE value is reduced then the nearest point of Ti to zero increases.

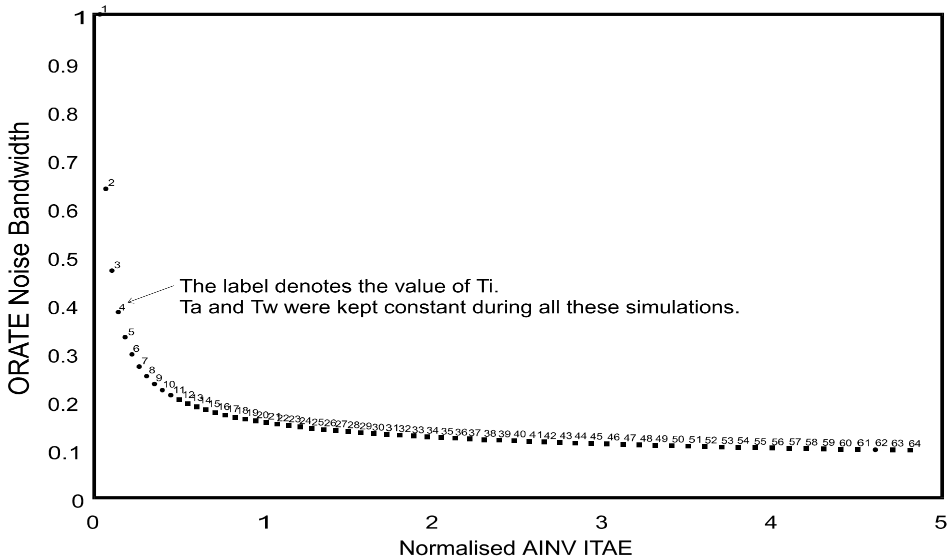

Figure 5. Ti Trade-Off Curve Between Noise Bandwidth and ITAE.

Clearly, the scaling factor that should be used is dependant on the cost structure of the business involved. To determine the exact scaling factor an analysis of the inventory carrying costs and the production on-costs needs to be conducted. Therefore, for each situation there is a unique set of optimum control parameters, dependant on the scaling factor. Hence, the optimisation presented later determines the values of the control parameters based on weighting the inventory recovery and noise bandwidth criteria to reflect the various cost structures that could be present.

Production Robustness.

The production robustness vector is a measure of the robustness of the design parameters, with respect to changes in the average production lead-time and to its distribution. The method used is based on the two vectors outlined above. The robustness vector is a measure of how much the performance alters with respect to ITAE and noise bandwidth for all combinations of production lead-time at 50%, 100%, and 150% of the nominal value (Tp = 8), and a production distribution of first or third order. It is assumed that $\overline{Tp}$ =Tp at all times. It can be visualised as shown by Figure 6. The quantification of this vector is based on the distance of the performance points from the centre of gravity. The centre of gravity is calculated by finding the average ORATE Noise Bandwidth and the average ITAE. The spread is calculated using Euclidean Normalisation, from Equation 7. Appendix 3 lists the additional transfer functions used in the production robustness vectors.

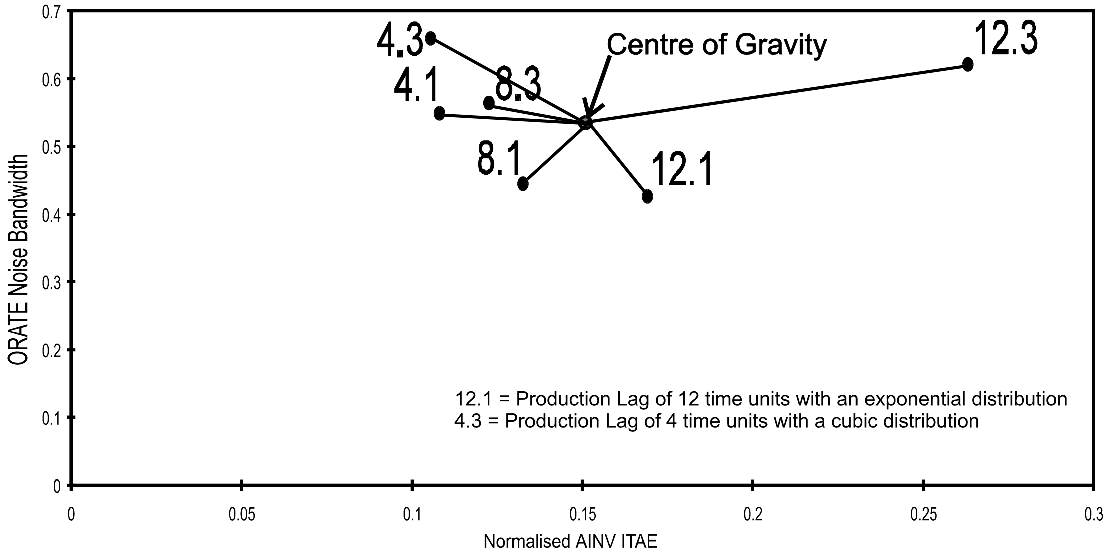

Figure 6. Visualisation of the Production Robustness Vector.

$$PR = \frac{\sum_{i=1}^{p}\sum_{j=1}^{q}\sqrt{\left[ITAE_{ij} - \frac{\sum_{i=1}^{p}\sum_{j=1}^{q} ITAE_{ij}}{pq}\right]^2 + \left[\omega_{Nij} - \frac{\sum_{i=1}^{p}\sum_{j=1}^{q}\omega_{Nij}}{pq}\right]^2}}{pq}, \text{ .... Eq 7.}$$

where, PR= Production Robustness Vector,
$ITAE_{ij}$ is the ITAE for AINV under conditions i and j, (1<=i<=p), (1<=j<=q), where p=q=3.
ω_{Nij} = Noise Bandwidth of ORATE under conditions i and j;

Conditions i;
@ i= 1, Production Lag = 4 time units,
@ i= 2, Production Lag = 8 time units,
@ i= 3, Production Lag = 12 time units,
@ j= 1, Production Leadtime Distribution = Exponential Lag,
@ j= 2, Production Leadtime Distribution = Cubic Lag.

WIP Robustness.

To reduce the order of the transfer function of an APIOBPCS model, a first order lag is initially used to approximate a pure time delay in the WIP feedback loop. This is to simplify analysis. The purpose of this criterion is to establish the robustness of the design parameters to possible delays in the WIP feedback loop. Such delays as these may be present due to inaccuracies in the recording of WIP on the real world shop floor. Like the production robustness vector and the selectivity vector the WIP robustness vector is a measure of how much the performance alters in the ITAE and Noise Bandwidth plane for all of the following conditions; No time delay; a first order lag of 4 time units; and 8 time units. See Appendix 2 for the additional transfer functions required. It is defined in Equation 8.

$$WIPR = \frac{\sum_{i=1}^{p}\sqrt{\left[ITAE_{i} - \frac{\sum_{i=1}^{p} ITAE_{i}}{p}\right]^2 + \left[\omega_{Ni} - \frac{\sum_{i=1}^{p}\omega_{Ni}}{p}\right]^2}}{p}, \text{ ....Eq 8.}$$

where; WIPR = WIP Feedback Delays Robustness,

$ITAE_i$ = the ITAE for AINV under conditions I, (1<=i<=p) where p =3,
ω_{Ni} = Noise Bandwidth of ORATE under condition i,

conditions i;

@ i= 1, no WIP feedback delay,
@ i= 2, there is an exponential lag in the WIP feedback loop of 4 time units,
@ i= 3, there is an exponential lag in the WIP feedback loop of 8 time units.

Selectivity.

The selectivity vector is a measure of the robustness of a design to arbitrary changes to the values of the control parameters by users of the ordering algorithm. It is particularly useful for determining the terrain in the solution space so that we can recommend an optimum that is robust, in the sense that minor deviations around it will not degrade performance greatly. i.e. it is not at the top of a sharp peak in the solution space. It is also representative of inaccurate estimations of the systems state, such as inventory levels and WIP levels. Like the production robustness vector it is based on the ITAE and Noise Bandwidth plane. It is a measure of how much the performance alters, when each parameter is set at 75%, 100% and 125% of the nominal value and is calculated in Equation 9 below.

$$SV = \frac{\sum_{i=1}^{p} \sqrt{\left[ITAE_i - \frac{\sum_{i=1}^{p} ITAE_i}{p} \right]^2 + \left[\omega_N - \frac{\sum_{i=1}^{p} \omega_N}{p} \right]^2}}{p} \quad \text{............Eq 9.}$$

where; SV = Selectivity Vector,
$ITAE_i$ is the ITAE for AINV under conditions i (1<=i<=p), where p=9,
and ω_{Ni} = Noise Bandwidth of ORATE under condition i,

Conditions i;

@ i= 1, $Ta=Ta_{nom}$*75%, $Ti=Ti_{nom}$, $Tw=Tw_{nom}$.
@ i= 2, $Ta=Ta_{nom}$*100%, $Ti=Ti_{nom}$, $Tw=Tw_{nom}$.
@ i= 3, $Ta=Ta_{nom}$*125%, $Ti=Ti_{nom}$, $Tw=Tw_{nom}$.
@ i= 4, $Tw=Tw_{nom}$*75%, $Ti=Ti_{nom}$, $Ta=Ta_{nom}$.
@ i= 5, $Tw=Tw_{nom}$*100%, $Ti=Ti_{nom}$, $Ta=Ta_{nom}$
@ i= 6, $Tw=Tw_{nom}$*125%, $Ti=Ti_{nom}$, $Ta=Ta_{nom}$
@ i= 7, $Ti=Ti_{nom}$*125%, $Tw=Tw_{nom}$, $Ta=Ta_{nom}$
@ i= 8, $Ti=Ti_{nom}$*125%, $Tw=Tw_{nom}$, $Ta=Ta_{nom}$
@ i= 9, $Ti=Ti_{nom}$*125%, $Tw=Tw_{nom}$, $Ta=Ta_{nom}$

Aggregation of the Characteristics.

The overall score assigned to a set of design parameters (Ta, Ti and Tw) is now given by Equation 10.

$$SCORE = \frac{1}{\sqrt{ITAE^2 + \omega_N^{\ 2} + PR^2 + WIPR^2 + SV^2}} \quad \text{..............Eq 10.}$$

The reciprocal has been introduced so that the higher the score, the better the dynamic performance of the ordering algorithm.

Optimisation of APIOBPCS via a Genetic Algorithm.

The optimum values of the control parameters in APIOBPCS to maximise the SCORE as described earlier was determined by a Genetic Algorithm (GA). Genetic Algorithms (GA) are an attempt to simulate Darwin's Theory of Evolution (Darwin 1859). Darwin stipulated that more favourable characteristics in an individual would increase the individual's chance of passing those favoured characteristics to the next generation via reproduction. The important struggle for life filtered out the weaker individuals and fitter individuals survived to pass on their genes to the next generation. The fitness of the population increased over the generations as individuals inherited the favoured designs of the their ancestors. Holland (1975) recognised that this could be a very useful technique for searching the solution space of problems that have many local minima, and adopted the idea for computer based directed random searches.
GA's have two main areas of application (Everett 1995), the first is the optimisation of the performance of a system, such as traffic lights or a gas distribution pipeline system. They typically depend on the

selection of parameters, perhaps within certain constraints, whose interaction restricts a more analytical approach. The second area of application for GA's is in the field of testing or fitting of quantitative models. In this case the aim of the GA is the minimisation of the error between the model and the data. The controller order reduction problem by Caponetto et al (1996) fits this type of application. This paper is concerned with the first, i.e. an optimisation type application (Disney et al 1997b). The GA approach was specifically used in this case due to their robustness to the possible shape of the solution space, the ease of implementation and acceptance as a good non-greedy searching mechanism.

The Operation of the GA.

Let $\mathbf{X}^{M}=\{\mathbf{x}|x_j^{(L)} <=x<= x_j^{(U)}\}$, (1,<=j<=M) be the search space where M is the dimension of **x** and $x_j^{(L)}$ and $x_j^{(U)}$ is the upper and lower limit of the jth component x_j of vector **x**, respectively.
Let **P**(k)=N binary chromosome structures $\mathbf{s}_i(k)$, (1<=i<=N) in generation k.
Let $f_i(k)$ be the fitness of the ith structure in generation k as defined by *SCORE* described earlier.
Let $\mathbf{x}_b(k)$ be the best parameter vector with the largest fitness $f_b(k)$.

The GA works in the following way;
Set k=0
Create initial random population **P**(k)
Decode $\mathbf{s}_i(k)$, (1<=i<=N) into $\mathbf{x}_i(k)$
Evaluate fitness $f_i(k)$, (1<=i<=N)
Determine the best $\mathbf{x}_b(k)$ and copy into **P**(k+1)
DO WHILE <termination conditions are not met>
    Crossover and mutate **P**(k) to form N-1 chromosome structures and copy into population **P**(k+1)
    Decode $\mathbf{s}_i(k+1)$, (1<=i<=N) into $\mathbf{x}_i(k+1)$
    Evaluate fitness $f_i(k+1)$, (1<=i<=N)
    Determine the best $\mathbf{x}_b(K)$ and copy into **P**(k+1)
    Set k=k+1
END WHILE

After convergence the GA was restarted several times to check for the true optimum. In the description of the GA above Tp and $\overline{Tp}$ have been set at 8 time units, the dimension of **x** (M) is 3 (for Ti, Ta and Tw), and the upper and lower limit of each x is 255 and 0 respectively. Thus the binary structures (s_i) are 24 bits long and there were N= 60 binary chromosome structures.

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Implementation of the Optimisation Procedure.

The optimisation procedure was carried out within a spreadsheet environment. A continuous simulation model of APIOBPCS was developed using the difference equations shown in Appendix 1. This model was then used to determine the inventory response of the system and to calculate its ITAE. The noise bandwidth was calculated by applying Parsvels Relation to the systems transfer functions. The search mechanism (GA's) was also implemented in the spreadsheet making extensive use of the Macro functions (described in the pseudo code earlier) for its implementation. See Figure 7 for a visual description of this procedure.

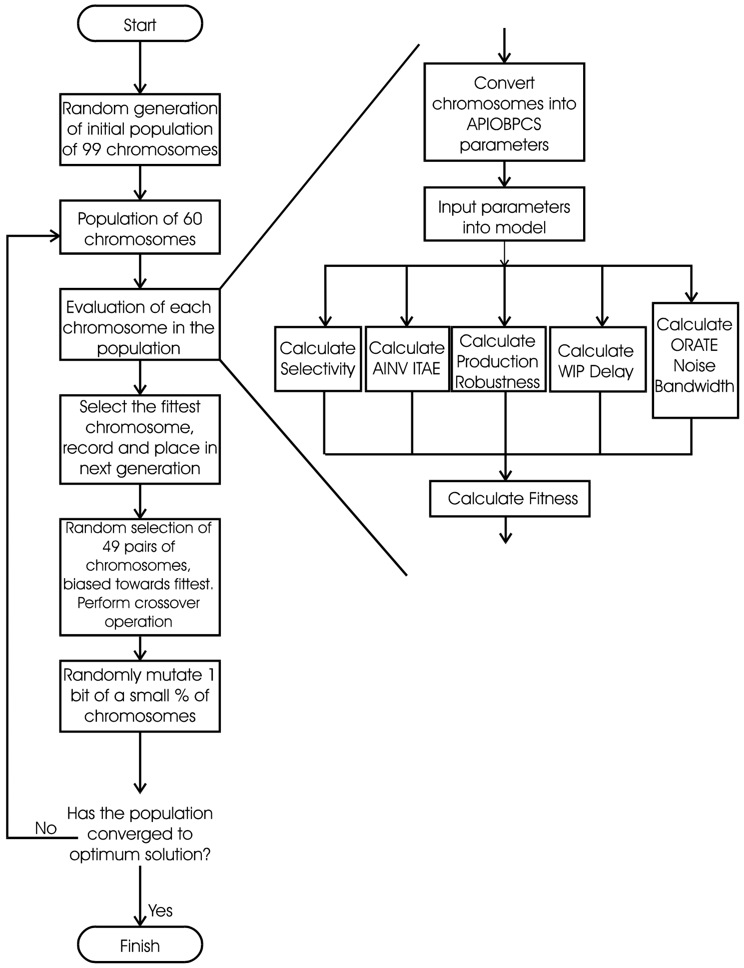

Figure 7. Flow Diagram of the Simulation and Optimisation Procedure.

Optimum Control Parameters.

Using the optimisation procedure outlined above the optimum control parameters where noise bandwidth and ITAE were given equal importance i.e., the scaling factor on ITAE was 1600000, is Ta= 2*Tp, Tw= 5.125*Tp and Ti=0.875*Tp. Clearly in some situations the value of the inventory costs will be a greater proportion of the sum of inventory costs and capacity costs than others, due to differing ratios between inventory and capacity costs in situ. Therefore it would be beneficial to increase the speed of response in inventory recovery in these cases and vice versa. To this aim, the weighting between inventory recovery and the noise bandwidth has been altered in the optimisation procedure to determine how the parameters change with the differing ratios between inventory holding and capacity costs. This was implemented in such a way that the relative importance of inventory recovery and noise bandwidth was reflected in all five vectors of the overall performance vector. The results of this exercise are shown in Table 1 and graphically in Figures 8 and 9.

Weightings⇒ / Vector being weighted⇓		Nominal	*2	*4	*8	*16
ITAE	Tw	5.125*Tp,	4.375*Tp,	3.375*Tp,	2.5*Tp,	1.5*Tp,
	Ta	2.125*Tp,	1.75*Tp,	1.625*Tp,	1.625*Tp,	1*Tp,
	Ti	0.875*Tp.	0.75*Tp.	0.625*Tp.	0.5*Tp.	0.5*Tp.
Noise Bandwidth	Tw	5.125*Tp,	5.625*Tp,	5.875*Tp,	9.125*Tp,	11.375*Tp,
	Ta	2.125*Tp,	2.125*Tp,	1.875*Tp,	2*Tp,	2*Tp,
	Ti	0.875*Tp.	0.875*Tp.	1*Tp	1*Tp	1*Tp.

Table 1. The Optimum Control Parameters (Tw, Ta and Ti) for Various Weightings of the Inventory Recovery and Noise Bandwidth.

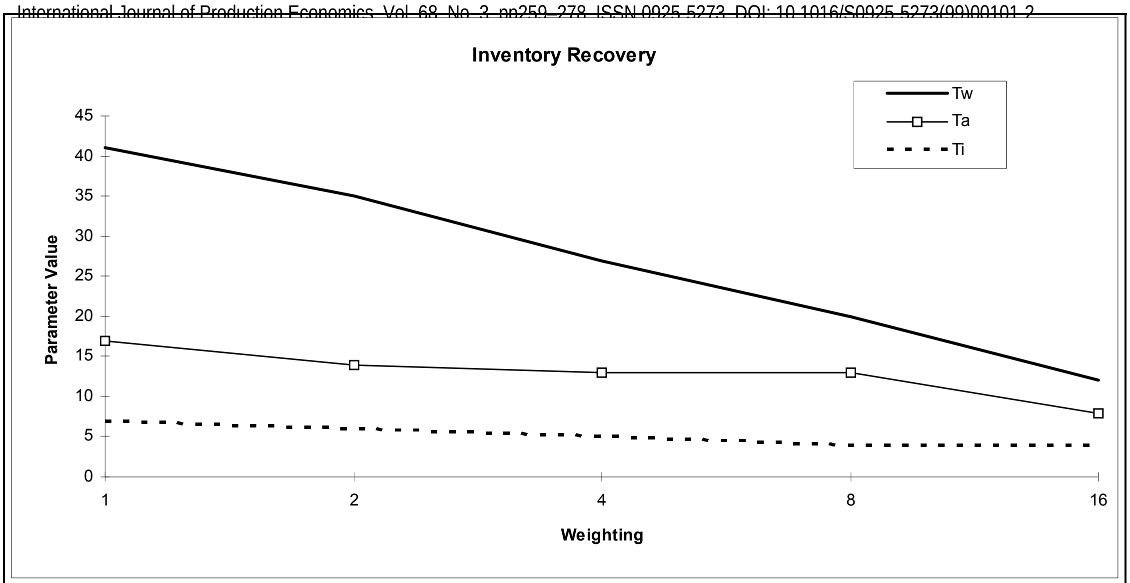

Figure 8. The Optimum Control Parameters for Various Weightings in Inventory Recovery.

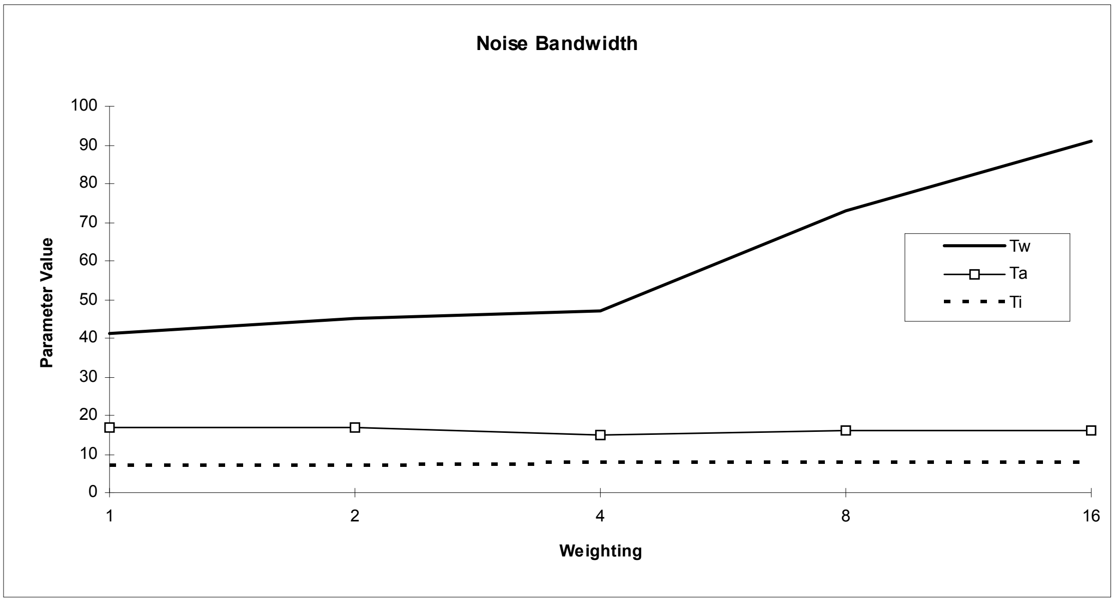

Figure 9. The Optimum Control Parameters for Various Weightings in Noise Bandwidth.

The dynamic performance of these optimal parameters when subjected to step input can be seen in Figures 10 and 11 for the inventory recovery and the noise bandwidth weightings respectively, and similarly their response to a random demand is shown in Figures 12 and 13.

Disney, S.M., Naim, M.M. and Towill, D.R., (2000), "Genetic algorithm optimisation of a class of inventory control systems", International Journal of Production Economics, Vol. 68, No. 3, pp259–278. ISSN 0925-5273. DOI: 10.1016/S0925-5273(99)00101-2.

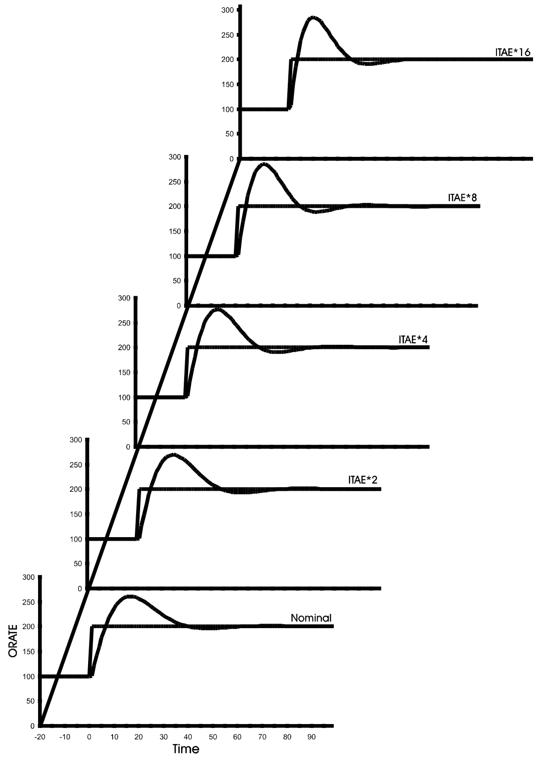

Figure 10. Dynamic Responses for the Optimal ITAE Weightings.

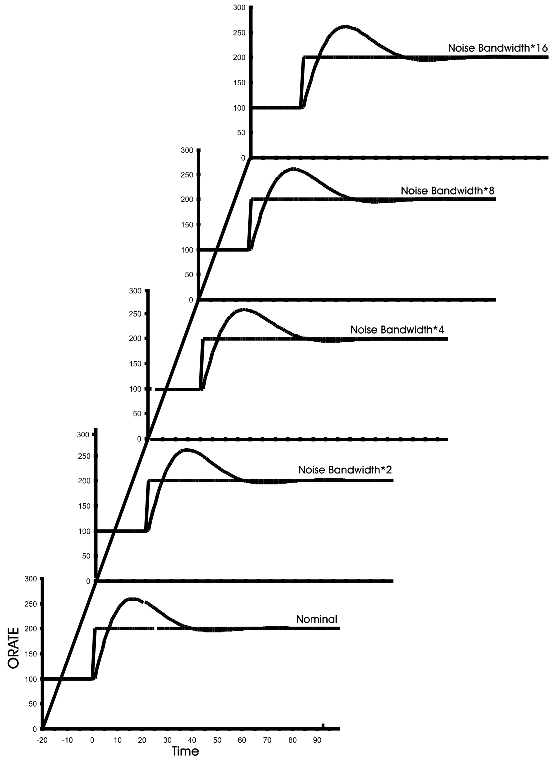

Figure 11. Dynamic Responses for the Optimal Noise Bandwidth Weightings.

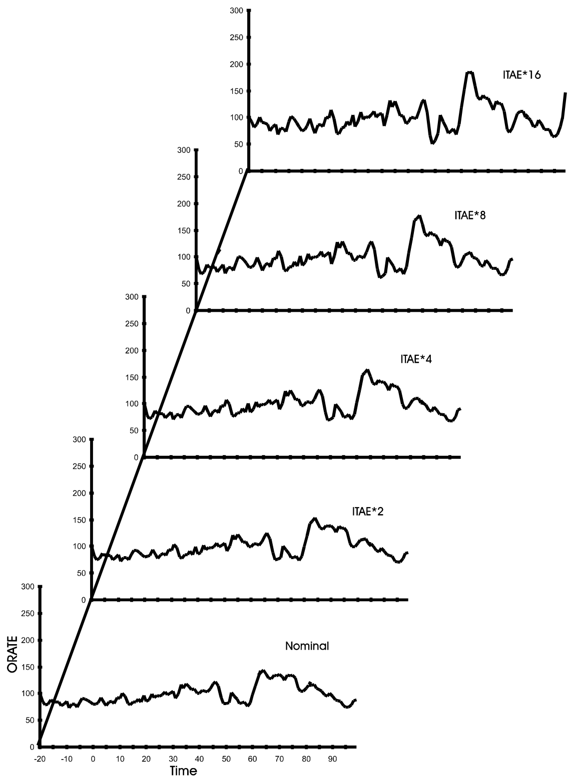

Figure 12. Dynamic Responses for Optimal ITAE Weightings to Random Demand.

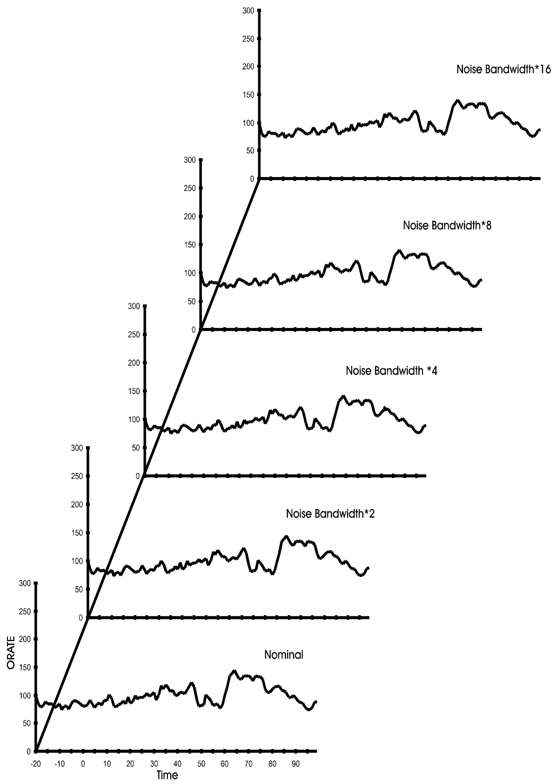

Figure 13. Dynamic Responses for Optimal Noise Bandwidth Weightings to Random Demand.

Conclusion.

A generic production and distribution control system has been outlined. The role of each of the controllers has been demonstrated via the use of simulation. Five desirable characteristics of a production distribution system have been described and quantified by drawing on control techniques developed in the 1950's and 60's for use in an optimisation procedure using a Genetic Algorithm. The contribution of this paper is to successfully combine classical control theory with modern optimisation techniques for an inventory control application. The first of the five desirable characteristics is inventory recovery which ensures that finished goods stock is kept to a minimum. The second, noise bandwidth, ensures that the production on-costs are kept to a minimum in response to random patterns of demand. Three more vectors ensure that the system is robust to uncertainties in lead-times, information fidelity and user interaction. It is reassuring to note that the optimal nominally weighted parameters are very similar to previously stated optimum controller settings derived by more intuitive techniques, John et al (1994).

The optimisation procedure was then used to determine how the controller parameters would alter with different cost structures within a production system. Considered was the weighting on inventory carrying costs and production on-costs or capacity costs reflected in each of the five performance vectors. The set of controller parameters determined for the range of cost structures available ensures that users have ***a range of solutions*** for their business situation, thus forming a DSS. The DSS shows that as capacity costs become more important to a business (i.e. in a process oriented companies), the use of WIP information becomes negligible in the ordering decision, as Tw becomes very large. However, where inventory costs are significant (i.e. in a batch/ job shop oriented business) then WIP information has a considerable effect on the dynamic performance of the ordering system. Berry et al (1998) outlined similar results from an industrial survey of how WIP information is being used in practical situations. Their main conclusions show that companies with low and constant leadtimes (generally process type industries) were not collecting WIP information in their decision making and those that did (high, variable leadtime industries, i.e. jobshop type industries) could benefit most from the inclusion of such information in their order decision making, but were failing to incorporate them formally in the feedback structure of the decision. This contribution suggests, that in industries where it is easy to collect WIP information, there is little benefit in doing so. Conversely, where it is harder to collect WIP information there is considerable benefits to be obtained.

This paper has demonstrated that the use of a DSS coupled with a simulation facility and genetic algorithm optimisation can improve the performance of a production or distribution control system by fully understanding the trade-off between inventory levels and factory orders. Of course these benefits are even greater over many echelons in the supply chain as the effects of poor decision making are multiplicative (Towill and Del Vecchio, 1994). For example, simulation has shown that over a three level supply chain there is the potential for a 20 fold improvement in dynamic performance in a 3 level supply chain, for the nominal case, over and above the "MRP equivalent" controller settings of Tw=∞, Ti=1 and Ta=0 (Disney et al., 1997a).

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Appendices.

Appendix 1. Difference Equations Required for APIOBPCS.

$$\mathrm{CONS_t} = \begin{cases} 100 \text{ if } t <= 0 \\ 200 \text{ if } t > 0 \end{cases},$$

$$\mathrm{AVCON}_t = \mathrm{AVCON_{t\text{-}1}} + \frac{1}{1+\frac{\mathrm{Ta}}{\Delta \mathrm{t}}}(\mathrm{CONS_t} - \mathrm{AVCON_{t\text{-}1}}),$$

$$\mathrm{ORATE_t} = \mathrm{AVCON_t} + \frac{\mathrm{EINV_t}}{\mathrm{Ti}} + \frac{\mathrm{EWIP_t}}{\mathrm{Tw}},$$

$$\mathrm{COMRATE_t} = \begin{cases} \mathrm{COMRATE_{t\text{-}1}} + \frac{1}{1+\frac{\mathrm{Tp}}{\Delta \mathrm{t}}}(\mathrm{ORATE_t} - \mathrm{COMRATE_{t\text{-}1}}), \text{ if first order lag} \\ \mathrm{COMRATE_{t\text{-}1}} + \frac{1}{\left[1+\frac{\mathrm{Tp}}{3\Delta \mathrm{t}}\right]^3} + \left(\mathrm{ORATE_t} - \mathrm{COMRATE_{t\text{-}1}}\right), \text{ if third order lag} \end{cases},$$

$$\mathrm{AINV_t} = \mathrm{AINV_{t\text{-}1}} + (\mathrm{COMRATE_t} - \mathrm{CONS_t}) * \Delta t,$$

$$\mathrm{DINV_t} = 1000,$$

$$\mathrm{EINV_t} = \mathrm{DINV_t} - \mathrm{AINV_t},$$

$$\mathrm{WIP_t} = \mathrm{WIP_{t\text{-}1}} + (\mathrm{ORATE_t} - \mathrm{COMRATE_t}) * \Delta t,$$

$$\mathrm{DWIP_t} = \mathrm{AVCON_t} * \overline{\mathrm{T}}\mathrm{p},$$

$$\mathrm{EWIP_t} = \mathrm{DWIP_t} - \mathrm{WIP_t}.$$

Appendix 2. Parsvels Relation.

Integrals of the form $\int_0^\infty \frac{|b(j\omega)|^2 d\omega}{|a(j\omega)|^2}$ for a linear transfer function of the form $\frac{b_m s^{jm} + \ldots\ldots + b_1 s + b_0}{a_n s^n + \ldots\ldots + a_1 s + a_0}$

(where m<n, and a_0=1) can be determined using Parsevals Relation, which for a third order transfer

function = $\frac{\left(\frac{b_2^2 a_1}{a_3} + b_1^2 - 2b_0 b_2 + b_0^2 a_2\right)}{a_2 a_1 - a_3}$, (Garnell & East 1977).

Therefore the noise bandwidth equation for the ORATE/CONS with an exponential plant lag is;

$$\omega_N = \frac{\frac{\left(TpTa + TpTi + \frac{Tp\overline{Tp}Ti}{Tw}\right)^2 \left(Ti + \frac{TpTi}{Tw} + Ta\right)}{TpTiTa} + \left(Ta + Ti + Tp + \frac{\overline{Tp}Ti}{Tw}\right)^2 - 2\left(TpTa + TpTi + \frac{Tp\overline{Tp}Ti}{Tw}\right) + \left(TpTi + TaTi + \frac{TpTiTa}{Tw}\right)}{\left(TpTi + TaTi + \frac{TpTiTa}{Tw}\right)\left(Ta + Ti + \frac{TpTi}{Tw}\right) - TaTiTp}$$

For a fourth order transfer function, Parsevals Relation becomes;

$$= \frac{b_3^2\left(\frac{a_1 a_2}{a_3 a_4} - \frac{1}{a_4}\right) + \left(b_2^2 - 2b_1 b_3\right)\frac{a_1}{a_3} + b_1^2 - 2b_0 b_2 + b_0^2\left(a_2 - \frac{a_1 a_4}{a_3}\right)}{a_1 a_2 - \frac{a_1^2 a_4}{a_3} - a_3}$$

For a fifth order transfer function Parsevals Relation becomes;

$$= \frac{1}{\Delta_5}\left[b_4^2 m_0 + (b_3^2 - 2b_2 b_4)m_1 + (b_2^2 - 2b_1 b_3 + 2b_0 b_4)m_2 + (b_1^2 - 2b_0 b_2)m_3 + b_0^2 m_4\right]$$

, (Newton et al. 1964) where;

$$m_0 = \frac{1}{a_5}\left[a_3 m_1 - a_1 m_2\right],\ m_3 = \frac{1}{a_0}\left[a_2 m_2 - a_4 m_1\right],\ m_4 = \frac{1}{a_0}\left[a_2 m_3 - a_4 m_2\right],$$

$m_1 = -a_0 a_3 + a_1 a_4$, $m_2 = -a_0 a_5 + a_1 a_4$ and $\Delta_5 = a_0(a_1 m_4 - a_3 m_3 + a_5 m_2)$

Appendix 3. Transfer Functions Required for APIOBPCS.

A) APIOBPCS with a First Order Production Lag, and a First Order Data Collection Lag in the WIP Feedback Path, ORATE Transfer Function.

$$\frac{ORATE}{CONS} = \frac{1+\left(Tq+Ta+Ti+Tp+\frac{Ti\overline{T}p}{Tw}\right)s+\left(TqTa+TiTq+TpTq+TpTa+TpTi+\frac{Ti\overline{T}pTp}{Tw}\right)s^2+\left(TpTqTa+TpTiTq\right)s^3}{1+\left(Ti+Ta+Tq\right)s+\left(TiTq+TiTa+TaTq+TaTi+\frac{TpTi}{Tw}\right)s^2+\left(TiTaTq+TaTiTq+TiTa+\frac{TiTp^2}{Tw}+\frac{TaTpTi}{Tw}\right)s^3+\left(TiTqTa^2+\frac{TaTiTp^2}{Tw}\right)s^4}$$

Note: Tq = first order lag constant in the WIP feedback path.

b) APIOBPCS with a Third Order Production Lag, ORATE Transfer Function.

$$\frac{ORATE}{CONS} = \frac{1+\left(Tp+Ti+\frac{Ti\overline{T}p}{Tw}+Ta\right)s+\left(\frac{Tp^2}{3}+TiTp+\frac{TiTp\overline{T}p}{Tw}+TaTp\right)s^2+\left(\frac{Tp^3}{27}+\frac{TiTp^2}{3}+\frac{Ti\overline{T}pTp^2}{3Tw}+\frac{TaTp^2}{3}\right)s^3+\left(\frac{TiTp^3}{27}+\frac{Ti\overline{T}pTp^3}{27Tw}+\frac{TaTp}{27}\right)s^4}{1+\left(\frac{TpTi}{Tw}+Ti+Ta\right)s+\left(TiTp+\frac{TiTp}{3Tw}+TaTi+\frac{TaTpTi}{Tw}\right)s^2+\left(\frac{TiTp^3}{27Tw}+\frac{TiTp^2}{3}+TaTiTp+\frac{TaTiTp}{3Tw}\right)s^3+\left(\frac{TiTp^3}{27}+\frac{TiTaTp^3}{27Tw}+\frac{TiTaTp^2}{3}\right)s^4+\frac{TiTaTp^3}{27}s^5}$$