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**SUMMARY**

<table>
<thead>
<tr>
<th>Purpose</th>
<th>The aim is to provide a concise methodology for the design of a widely used class of Decision Supply Systems (DSS) which will enable precise control of bullwhip variance and inventory variance induced within a supply chain echelon.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methodology/Approach</td>
<td>It exploits recent research which derived analytical formulae for calculating these performance metrics germane to the delivery process when the demand is randomly varying about a constant mean value. These formulae have been verified via extensive simulation based cross-checks.</td>
</tr>
<tr>
<td>Findings</td>
<td>The design methodology focuses on the specification of bullwhip variance as an input. The output is to identify combinations of parameter settings to meet this target. Hence these parameters may be mapped to provide a visual display of competing designs with their associated inventory variance.</td>
</tr>
<tr>
<td>Limitations/Implications</td>
<td>Although the analytical solutions apply only to the case where the pipeline error and inventory error correction terms are equal, this is not a severe limitation. Both theoretical studies of dynamic response and industrial experience support this feedback gain equality as enabling good practice.</td>
</tr>
<tr>
<td>Practical Implications</td>
<td>Design of this particular DSS to control bullwhip is now greatly simplified, and guaranteed via extensive verification tests. The formulae are equally sound as a means of establishing system robustness.</td>
</tr>
<tr>
<td>Originality/Value of Paper</td>
<td>The methodology is unique in enabling transparency of both bullwhip variance and inventory variance computation. Not only is system design time saved and normal performance guaranteed, but considerable management insight is generated thereby.</td>
</tr>
<tr>
<td>Type of Paper</td>
<td>Technical paper.</td>
</tr>
<tr>
<td>Key Words</td>
<td>Bullwhip avoidance: replenishment rules: DE-APIOBPCS: system damping: bullwhip benchmarking: robustness</td>
</tr>
</tbody>
</table>
“A METHODOLOGY FOR BENCHMARKING REPLENISHMENT INDUCED BULLWHIP”

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"A METHODOLOGY FOR BENCHMARKING REPLENISHMENT INDUCED BULLWHIP"

Abstract
Demand amplification (or “bullwhip” as it is now popularly called) is not a new phenomenon. Evidence of its existence plaguing supply chain operatives has been recorded at least as far back as the start of the twentieth century and is particularly well known to economists. Industry typically has to cope with bullwhip measured not just in terms of the 2:1 amplification in orders which is frequently encountered across a single echelon, but sometimes it is as high as 20:1 across the extended enterprise. This can be very costly in terms of capacity on-costs and stock-out costs on the upswing and stockholding and obsolescence costs on the downswing. In this paper we consider how bullwhip due to various “Forrester effects” may be avoided. This leads to the exploitation of a particular Replenishment Rule already widely used in industry and for which analytical formulae for bullwhip generation and inventory variance have been recently published. Hence given the replenishment lead time we show how an appropriate algorithm is readily designed which follows genuine trend changes yet damps down random fluctuations. Succinct user guidelines are included, including benchmarking via bullwhip contours.

Key Words: Bullwhip avoidance: replenishment rules: DE-APIOBPCS: system damping: trend detection following: bullwhip benchmarking

1. Introduction

Jay Forrester (1958) has been rightly viewed in many quarters as a pioneer of modern day supply chain management. His seminal work on demand amplification as studied via Systems Dynamics simulation demonstrated phenomena that many practising managers had experienced in real-world value streams. This included such events as demand waveforms being amplified as they were propagated upstream in the supply chain, the induction of “rogue seasonality” into the order patterns and the consequent wrong-footing of decision makers on both accounts. Nor was such demand amplification as studied in 1958 a relatively recent phenomenon, since evidence of its existence has been recorded at least as far back as the start of the twentieth century.

The situation facing much of industry worldwide is exacerbated because “bullwhip”, as it is now termed following Lee et al (1997), tends to be either misunderstood, or ignored (McCullen and Towill (2002). Familiar arguments include that such “whiplash” behaviour (Hayes and Wheelright, 1984) is someone else’s problem, or it does not (apparently) cost anything to this particular “player”, or it is an unavoidable fact of life. But industry, in the meantime, has to cope with real-world bullwhip measured not just in terms of 2:1 amplification which is bad enough, but 20:1 and has been observed to be even higher (Holmström, 1997). This behaviour can be very costly in terms of capacity on-costs and in stock-out costs (Metters, 1997). Equally, because in such a boom-and-bust scenario there are consequential downturns in demand stock-holding and obsolescence costs will also increase (Fisher, 1997).
Figure 1 shows a simple supply chain in which materials flow downstream and orders flow upstream from echelon to echelon (McCullen and Towill 2002). Each echelon (retailer: garment maker: fabric maker: and yarn maker) is subject to delays and needs to make a decision on passing orders to “their” vendor in the light of “orders” received from “their” customer, work-in-progress and stock levels (Stalk and Hout, 1990). It can take many months for a complete cycle of orders to pass up the chain and the corresponding products to flow down to the marketplace. By this time the customer wants something quite different to what was originally forecast. Bullwhip is a consequence of such a long and protracted chain, with every “player” double-guessing on what is really required. We have arbitrarily but realistically shown the bullwhip increasing by a factor of 2:1 as orders pass across each interface. Since bullwhip tends to be multiplicative this means the demand amplification is likely to be 8:1 across the complete supply chain. As Holmström (1997) has shown this behaviour is regrettably typical of real-world supply chains.

![Figure 1. The Classical Generation of the Bullwhip Effect in a Clothing Supply Chain](source: McCullen and Towill, 2002)

2. An Early Perspective of Bullwhip

However despite the foregoing interest in the theory and occurrence of bullwhip evident over the last four decades bullwhip is not a new phenomenon. The following excellent early description of bullwhip occurring in the retail sector testifies (Mitchell, 1923).

“Retailers find that there is a shortage of merchandise at their sources of supply. Manufacturers inform them that it is with regret that they are able to fill their orders only to the extent of 80 per cent: there has been an unaccountable
shortage of materials that has prevented them from producing to their full capacity. They hope to be able to give full service next season, by which time, no doubt, these unexplainable conditions will have been remedied. However, retailers, having been disappointed in deliveries and lost 20 per cent or more of their possible profits thereby, are not going to be caught that way again. During the season they have tried with little success to obtain supplies from other sources. But next season, if they want 90 units of an article, they order 100, so as to be sure, each, of getting the 90 in the pro rata share delivered. Probably they are disappointed a second time. Hence they increase the margins of their orders over what they desire, in order that their pro rata shares shall be for each the full 100 per cent that he really wants. Furthermore, to make doubly sure, each merchant spreads his orders over more sources of supply.”

It may come as some surprise to many present day researchers that this powerful quotation is attributable and more recently cited by Sterman (1986). But not only was bullwhip historically known (but not defined), so were some little publicised and very interesting proposed solutions. For example, Zymelman (1965) predicted a control law for damping down the cotton industry supply chain cycle. It was based on proportional feedback of inventory error and WIP error. His verification was via an analogue computer simulation since at that time digital simulation was still in its infancy. In comparison to those labours the bullwhip avoidance replenishment rule proposed herein is trivial to demonstrate via spreadsheets and has also been incorporated into commercial software. What the prospective user needs to know is “What does it do? How is it adapted for my purpose? What is the trade-off? How robust is the system?” It is the purpose of this paper to specifically address these issues and thus help avoid some of the circumstances described so succinctly by Mitchell (1923). By using the recommended pipeline ordering system, the value stream controls will minimise the disruption caused by such chaotic behaviour.

3. Contribution of the Present Paper
There are many causes of bullwhip (Geary et al, 2002). In this paper we are concerned only with those causes leading to the “Forrester Effect” (Jay Forrester, 1958). These compliment the Houlihan (1986) flywheel effects highlighted in Figure 2. The Forrester effect, in particular, is caused by the physical lead times inevitably met within supply chains, the structure of information flow within the system, and how this is used within feedback paths and feed forward paths in inventory replenishment rules. The latter aspects include incorporating forecasting mechanisms and both inventory and pipeline controls therein. So the purpose of this paper is to ensure that our replenishment rule detects and follows genuine changes in customer trends. At the same time random fluctuations are to be adequately damped (Dejonckheere et al. 2002). In particular the methodology enables feedback settings to be selected which avoid the bullwhip effect due to this source. As this decision is within our remit, there is no excuse for inducing avoidable bullwhip.

The replenishment rule used herein is not new and is frequently met in industry. It may be embedded within commercial software, although often it is developed and implemented on an ad-hoc basis. Either modus operandi can result in rather arbitrary settings of system parameters by operations managers and production schedulers (Coyle, 1982 and Edghill et al, 1987). Suitable theoretical models already exist, with
much supporting simulation evidence (John et al, 1994, van Ackere et al 1993 and Riddalls and Bennett, 2002) including optimisation procedures. However Disney and Towill (2003) have now provided an analytic solution for calculating the bullwhip variance and inventory variance in response to random customer demand. We now exploit this solution herein by providing bullwhip variance contours for various values of replenishment lead time. These may be used as benchmarks for system design. Hence given the expected lead time, the user may select replenishment rule settings so as to reduce or avoid bullwhip induced by the ordering process.

![Figure 2. Bullwhip Explained via the Houlihan Production Flywheel Effect](source: Houlihan 1986, adapted by authors)

The recommended policy is to seek to avoid bullwhip by designing it out of the system via the appropriate choice of replenishment rule and parameter settings which deliver the performance expected by the user. Our rule uses exponential smoothing of demand as one component of a replenishment order. To this is added a further two inputs. These are a fraction of the inventory error, and a fraction of the “goods-in-the-pipeline” error. It is an important feature of the paper that these fractions are always equal, for reasons which will be detailed later. This idea is originally due to Deziel and Eilon (1967). It is a very powerful principle as an adjunct in bullwhip avoidance since it results in a very conservative design. The analytic solutions additionally enable the system user to evaluate particular performance trade-offs, including investigating robustness. As Parnaby (1991) has emphasised, coping with change (i.e. in this case delivery lead time) is a very important factor in system design.

4. What is Bullwhip?
Broady speaking bullwhip is the phenomenon of demand (variance) amplification of orders as they are passed up the supply chain. This is readily and very enlighteningly portrayed as a propagation curve (van Aken, 1978). It is important to understand the three dimensions of bullwhip as they affect the extended enterprise (McCullen and Towill, 2002). Firstly there is the dimension of amplitude i.e. things may get much worse. Secondly there is the time dimension since orders may take a significant time
to move up the chain. Thirdly, there is the distance dimension in which the outsourced pipeline may be thousands of miles from the marketplace. So the opportunities of masking what is really happening to customer demand are numerous. Hence information systems need to be implemented which ensure that data is both uncorrupted and readily available to all “players” in the chain. “Double guessing” between the various participants simply makes things worse (Mitchell, 1923 and Lee et al, 1997).

Frequently used bullwhip measures included peak overshoot to “shock” demands (van Ackere et al, 1993, and Wikner et al, 1991) thus following in the footsteps of Jay Forrester (1958); frequency response (Dejonckheere et al, 2001) which is of particular interest in coping with rogue seasonality; and finally order variance (Adelson, 1966, Fransoo and Wouters, 2000; Lee et al, 1997, and Disney and Towill, 2003). It is the latter measure which is of prime interest herein. Fig. 3 is a typical example of bullwhip occurring in practice (Jones et al, 1997). Retail sales therein appear by eye to be reasonably constant over the time period observed. However there is clearly some variation with the extreme swings being approximately ± 25% of average sales. But the orders placed on the suppliers are much amplified. Now the extreme swings are roughly ± 50% of average sales. In other words, based on deviations about the average, the bullwhip evident in this simple scenario is already 2:1 across just one business interface. However the realities for the transportation process are much more extreme. For example, if the supplier delivers exactly what is ordered by the retailer, his lorry requirements will vary not by 2:1, but by 3:1, that is the capacity needed could drop as low as that provided by (say) 2 lorries/day but can rise as high as 6 lorries/day. In contrast, if the supplier was required to replenish sales as they actually occur, four lorries would be adequate for most deliveries, a conclusion readily exploited in industrial practice via Vendor Management Inventory (VMI) Systems (Disney et al, 2003).

![Figure 3. An Example of Bullwhip in the Real-World ~ Demand Amplification Observed in the UK Retail Sector](Source: Jones, Hines and Rich, 1997)
5. The Real-World Bullwhip Environment

There is much anecdotal evidence available on what happens in present day supply chain operations. The attitude with respect to bullwhip varies widely according to the observer’s perception, their position within the supply chain, and the power associated with their particular function. For example, McCullen and Towill (2002) have developed a list of clichés commonly met in the real-world discussions with business managers. These are given in Table I, and it is emphasised that they represent commonly held views concerning bullwhip. They give an indication of the cultural barriers to change when endeavouring to engineer smooth material flow within a supply chain. This is a topic discussed in depth by Childerhouse et al. (2003a: 2003b).

<table>
<thead>
<tr>
<th>Cliché</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ignorance cliché</td>
<td>Bullwhip? Doesn’t exist in the real world</td>
</tr>
<tr>
<td>Arrogance cliché</td>
<td>Bullwhip? Is just an academic invention</td>
</tr>
<tr>
<td>Negligence cliché</td>
<td>Bullwhip? Doesn’t cost me any money</td>
</tr>
<tr>
<td>Indifference cliché</td>
<td>Bullwhip? The customer can wait</td>
</tr>
<tr>
<td>Transference cliché</td>
<td>Bullwhip? So what? – the suppliers can cope. That’s what service level agreements are for!</td>
</tr>
<tr>
<td>Acceptance cliché</td>
<td>Bullwhip? It’s like tax – always with us</td>
</tr>
<tr>
<td>Despondence cliché</td>
<td>Bullwhip? It’s a systems problem – nothing I can do about it</td>
</tr>
<tr>
<td>Decadence cliché</td>
<td>Bullwhip? It’s old hat – surely it’s been eradicated by now?</td>
</tr>
<tr>
<td>Intolerance cliché</td>
<td>Bullwhip? Japanese solutions don’t work here</td>
</tr>
<tr>
<td>Avoidance Cliché</td>
<td>Bullwhip? Those solutions are all very well – but not in my industry</td>
</tr>
</tbody>
</table>

Table 1. Cultural Barriers to Change ~ Bullwhip Clichés Frequently Encountered in Supply Chain Practice
(Source: McCullen and Towill, 2002)

These bullwhip clichés of Table I clearly cover a wide range of sentiment. This is not unexpected given the comment by Andraski (1994) that supply chain problems are “80% people/20% technology” in origin which is certainly one experience. So the “actors” making the foregoing remarks are just going through normal routines of “turf protection” and “buck passing”. In the meantime, as the Fisher’s (1997) classic Campbell’s Soup example, coupled with the Metters (1997) OR based cost model, show, the ramp-up / ramp down production on-costs, and stock-holding / stock-out costs add significantly to the price paid by the customer. The DSS to be discussed in the next section gives the operations manager the opportunity to control the impact of the boom-and-bust cycle on his business.

So reducing bullwhip really is important, as otherwise the supply chain players are in a classic lose-lose scenario. Nor are these added costs likely to be equitably spread throughout the chain. Since the seminal Marshall Fisher paper first appeared in 1997, the power and hence the influence of the retailer has increased yet further.
Consequently their “mark-ups” tend to have risen significantly with the bullwhip on-costs being disproportionately borne upstream so these players are most at risk of bankruptcy. As Levy (1995) has shown, this situation may be worsened, not bettered, when adopting third world outsourcing. His analysis demonstrated that in practice lowest apparent price is often far from minimum real cost.

6. The DSS Replenishment Rule

The particular replenishment rule advocated and exploited in this paper is a special case of the Automatic Pipeline Inventory and Order Based Production System (APIOBPCS). This is readily expressed in words is “Let the replenishment orders be equal to the sum of an exponentially smoothed demand (averaged over Ta time periods), plus a fraction (1/Ti) of the inventory difference between target stock and actual stock, plus a fraction (1/Tw) of the difference between target orders-in-the-pipeline and actual orders placed but not yet received” (John et al, 1994). APIOBPCS encapsulates the general principles for replenishment rules as advocated by Popplewell and Bonney (1987). In particular it gives due prominence to the importance of including pipeline feedback (OPL) in replenishment decision making, a factor further emphasised by Bonney (1990). Of course APIOBPCS is not a new concept. It is well established in industry and has the additional advantage of reasonably describing data from 2000 Beer Game “plays” as elegantly modelling by John Sterman (1989). Furthermore the variant known as the “to-make model” has successfully controlled 6000 multi-product pipelines in the orthopaedic industry via empirically derived parameter settings (Cheema et al, 1989).

For the present statistical method of analysis the demand pattern is assumed to be random with constant mean value. This statistical approach to bullwhip is OR based (Adelson, 1966). Under these conditions the performance criteria usually adopted are the bullwhip variance and the inventory variance. A bullwhip variance of unity means that the variance of the replenishment orders placed is exactly equal to the demand variance. For this critical value of 1.0, there will be zero bullwhip, since the demand signal is not being amplified according to this particular statistical definition.

Fig 4 shows in much simplified diagrammatic form the various control parameters Ta, Ti, and Tw which will be set in such a way as to reduce bullwhip. This is a pictorial description of APIOBPCS which implements the preceding verbal rule (Disney and Towill, 2003). Note that there is a replenishment time delay of Tp units. The practical interpretation of Tp (for mathematical reasons) is that if replenishment orders are updated daily, deliveries become available (Tp + 1) days later. So if supplies arrive on day 4, then Tp = 3 days. Note also that to avoid inventory drift the OPL target is also set at a multiple of exponentially smoothed demand. The OPL target multiplier is $\overline{Tp}$ and this is the best available estimate of the current replenishment lead time.
7. Equalising the Pipeline and Inventory Recovery Times

A very simple, but operationally profound, modification to APIOBPCS is undertaken by making $T_i = T_w$. This simplified model is designated as DE-APIOBPCS, since this modification was first advocated by Deziel and Eilon (1967) in an OR context. A simple but powerful demonstration of the importance of the DE settings of $(T_i = T_w)$ is given in Figure 5, (Disney and Towill, 2002). Here the stability boundary for the replenishment rule for the particular case where $T_p = 3$ days. The critical stability contour separates the stable regime (controlled behaviour) from the unstable regime (uncontrolled behaviour). It can be seen by inspection that the DE contour lies well within the stable regime with extremely well behaved dynamic response.

This understanding of the fundamental significance of the DE relationship has led to further research. This includes the analytic determination of one particular bullwhip measure (order variance / demand variance) and associated (inventory variance/demand variance) on the assumption of random demand (Disney and Towill, 2003). These formulae are summarised in Table 2. Hence any potential user of this replenishment rule is now able to estimate the bullwhip variance associated with any particular settings proposed for the DE-APIOBPCS model. As we shall see in the next section, by setting the (order variance / demand variation) equation equal to unity, the bullwhip boundary can be readily produced. It is then extremely straightforward to propose suitable parameter settings to guarantee bullwhip avoidance by appropriate damping of demand.
Figure 5. Charting Dynamic Response – the Relevance of the Deziel-Eilon Line to System Design for Bullwhip Avoidance
(Source: Authors based on Disney and Towill, 2002)

<table>
<thead>
<tr>
<th>Performance Metric</th>
<th>Mathematical Expression</th>
</tr>
</thead>
</table>
| \( \text{VAR ORATE} \) (Bullwhip) | \[
\frac{2Ta^2 + 3Ti + 2Tp + 2(Ti + Tp)^2 + Ta(1 + 6Ti + 4Tp)}{(1 + 2Ta)(Ta + Ti)(-1 + 2Ti)}
\] |
| \( \text{VAR INV} \) (Inventory Variation) | \[
1 + Tp + \frac{2Ta^2(-1 + Ti)^2 + Ti(1 + Tp)^2 + Ta(1 + Tp)(1 + (-1 + 2Ti)Tp)}{(1 + 2Ta)(Ta + Ti)(-1 + 2Ti)}
\] |

Table 2. Theoretical Formulae Derived for Calculating Bullwhip and Inventory Performance Metrics Corresponding to the Recommended Replenishment Rule
(Source: Disney and Towill, 2003)

8. Performance Contour Benchmarks
The bullwhip and inventory equations previously presented in Table 2 for the DE-APIOBPCS replenishment rule has been solved thus determining the “zero bullwhip” contours. The bullwhip results are shown in Figure 6. If the system design parameters are located above the contour indicated for a particular value of replenishment lead time, then bullwhip exists. On the boundary the replenishment order variance is exactly equal to the customer demand variance, hence in this respect the rule is “neutral”. For parameter settings below the contour, the replenishment
order variance falls below the demand variance. In this case the ordering system “smoothes” demand, usually by filtering out the high frequency “noise” (Dejonckheere et al, 2002). The bullwhip variance = unity contour is thus especially suitable for benchmarking DSS performance.

![Graph](image)

**Figure 6. Bullwhip Boundaries for Benchmarking Recommended Replenishment Algorithms**
(Source: Authors)

The corresponding inventory variance in response to random customer demand is shown in Figure 7. At the extreme values of inventory control (1/Ti), this variance is (1 + Tp), as can be determined by inspection of Table 2. This explicit dependence on lead time is yet further proof of the benefit of implementation of the Time Compression Paradigm (Towill, 1996 and de Treville et al, 2004). It emphasises the need to reduce lead times throughout the supply chain if better material flow control is to be enabled. The inventory variance does peak, but as Figure 7 shows, even for Tp=5 days the worst value is about eight units i.e. only 46% greater than the minimum possible value for this replenishment lead time. So provided the Deziel-Eilon parameter settings of (Ti/Tw) = 1 are incorporated together with the associated values of Ta as shown in Figure 5 bullwhip can be avoided. At the same time we have reasonably constrained the inventory variance.
Figure 7. Prediction of Inventory Variance at the Bullwhip Benchmark Boundaries
(Source: Authors)

9. Checking the Bullwhip Predictions
To further examine and understand the performance of the recommended replenishment rule the solutions obtained are cross-checked via simulation. This is undertaken by sampling systems along the line of symmetry (45° slope through the origin) in Figure 5. The results obtained for $T_p = 1; 2; 3; 4,$ and 5 days are shown in Table 3. The corresponding replenishment rule parameter settings to achieve this performance are also shown. In each case the theoretical bullwhip and inventory variance predictions have been compared with the results obtained by simulation of the system response to a random demand signal.

To ensure convergence these tests were conducted 30 times each simulating a period of 10,000 days. The worst bullwhip difference observed between theory and simulation is 0.4%. Finally, the worst observed inventory variance is 0.8%, hence considerable confidence may be attached to this replenishment rule theory. To summarise, it is clear that the formulae given in Table 3 accurately predict bullwhip behaviour for this particular ordering system. These formulae lie at the heart of bullwhip reduction via the DSS.

10. System Robustness
As Parnaby (1991) has emphasised, the design of any delivery pipeline must be checked for robustness. What exactly does this mean? So far, our discussion in this paper has been on the design and performance of the recommended replenishment rule under normal or steady state conditions. That is, the analysis is conducted on the basis that the replenishment lead time is fixed at the value chosen at the start of the investigation. One obvious robustness check is therefore to estimate the impact on performance of the lead time actually being at some other (almost certainly greater)
value. In other words, can the system cope when delivery lead times are different from those assumed by the system designer?

<table>
<thead>
<tr>
<th>Lead Time</th>
<th>Replenishment Rule Settings</th>
<th>Bullwhip</th>
<th>Inventory Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td><strong>Theory</strong></td>
<td><strong>Simulation</strong>*</td>
</tr>
<tr>
<td>Tp</td>
<td>1/Ti, 1/(1+Ta)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.385</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>0.333</td>
<td>1.000</td>
<td>0.998</td>
</tr>
<tr>
<td>3</td>
<td>0.298</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>4</td>
<td>0.272</td>
<td>1.000</td>
<td>1.004</td>
</tr>
<tr>
<td>5</td>
<td>0.252</td>
<td>1.000</td>
<td>1.002</td>
</tr>
</tbody>
</table>

* Simulation results are the average for 30 experiments each of 10,000 time periods duration.

Table 3. Sample Bullwhip and Inventory Variance Estimates Used to Demonstrate the Correctness of the Theoretical Analyses
(Source: Authors)

<table>
<thead>
<tr>
<th>Replenishment Rule Settings (based on expected lead time)</th>
<th>Predicted Performance for Expected Lead Time</th>
<th>Predicted Performance for Substantially Increased Lead Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/Ti, 1/(1+Ta)</td>
<td>Bullwhip (Tp=1)</td>
<td>Inventory Variance (Tp=1)</td>
</tr>
<tr>
<td>0.1</td>
<td>0.803</td>
<td>1</td>
</tr>
<tr>
<td>0.2</td>
<td>0.626</td>
<td>1</td>
</tr>
<tr>
<td>0.4</td>
<td>0.371</td>
<td>1</td>
</tr>
<tr>
<td>0.6</td>
<td>0.216</td>
<td>1</td>
</tr>
<tr>
<td>0.8</td>
<td>0.102</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4. Checking the Performance Robustness of the Recommended Replenishment Algorithm to Changes in Delivery Lead Time
(Source: Authors)

To demonstrate the robustness of the recommended replenishment rule, the corresponding change in performance has been calculated when the system was originally designed under an assumption that Tp = 1 day, but the actual dilatory value is 3 days. The results of this analysis are shown in Table 4, for a range of combinations of the parameters Ta and Ti (Tw=Ti) as we traverse the bullwhip = 1 contour. Table 4 shows that under these adverse conditions bullwhip can increase by between 35% and 73% depending on the design selected. (Note: this is still much less than the real-world bullwhip observed in the retail supply chain documented in Table 1). The corresponding increase in inventory variance is between 117% and 133% of the original value for that particular design. Considering the excellent performance achieved when Tp = 1, and reasonably acceptable behaviour even when the lead time inadvertently drops off to 3 days, this range of parameters might well be
argued as providing a suitably robust (and certainly well controlled) system. But this is a trade-off debate to be considered further in the user guideline section.

11. User Guidelines

Having shown that bullwhip is readily avoided by suitable system design, how is this knowledge best exploited by the user? Obviously, as shown in Tables IV and V, many combinations of replenishment rule parameters may be suitable. Which one might be best for our purpose? Unless the user is in a position to undertake a comprehensive investigation for every SKU in the catalogue, including simulation cross checks, a rough-but-reasonably-ready guideline needs to be used. Preferably such a guideline should be backed up by already existing and comprehensive knowledge and a range of proven practical applications.

The DE-APIOBPCS replenishment rule already provides a useful and comprehensive but empirical guideline for placing orders in the pipeline (Mason-Jones et al 1997). This earlier recommendation was based on an exhaustive comparison of a number of “recommended” settings tested, via simulation only, against a range of competing performance criteria. For the purpose of this paper these published “user guideline” settings may be interpreted as follows: \( T_i = T_w = (T_p + 1) \), and \( T_a = 2 (T_p + 1) \). Substituting these values into the bullwhip equation in Table III, for a demonstration value of \( T_p = 3 \) days, gives an estimate of 0.378. In other words the replenishment order variance is about 38% of our customer order variance. In other words the range of the replenishment orders has been reduced (or “smoothed”) by about 60%.

This is confirmed in the simulation study shown in Figure 8 on the same retail data. In contrast to the original real-world supply chain order patterns shown in Figure 2, where bullwhip undoubtedly exists. The improvement may be viewed as resulting in an order of magnitude in bullwhip reduction, and not just a fractional change in performance. It is obvious by comparing the replenishment orders actually placed in

![Figure 8. Simulated Response of “User Guideline” Replenishment Rule Settings to Random Demand Pattern Observed in UK Retail Sector](Source: Authors)
Figure 3 with those proposed in Figure 8 that the new rule is better. Furthermore, it is so blatant that detailed cost-benefit analysis to support the change in ordering policy is unnecessary. This is fortuitous since as Buxey (2001) has demonstrated, such cost data is often noted for its absence from much of the real-world industry.

12. Conclusions
It is well known that bullwhip is costly to all players in the supply chain. Consequential alternating “boom-and-bust” scenarios incur additional acquisition costs and additional stock-out costs. The ideal solution is to design and manage such on-costs out of the chain in such a way that the only uncertainty left is due to the marketplace (Childerhouse et al, 2003a, 2003b). One major source of internally generated bullwhip is due to poorly selected ordering policies in use at all levels within the chain. Our European retail supply chain provides ample evidence that this is indeed the case in the real-world. Hence this paper is concerned with selecting a replenishment ordering rule which can be structured to avoid bullwhip as observed in order variance behaviour. At least the various “players” can now adopt a policy to avoid self-induced bullwhip.

The DE-APIOBPCS replenishment rule exploited herein is well established in the literature. What we have demonstrated in the paper is that having the right decision making structure is insufficient. Additionally there is a need to select the re-ordering rule parameters to suit the replenishment lead time appropriate to the SKU (and by appropriate scaling, for each and every SKU under consideration). Hence the provision herein of a novel formula for calculating both bullwhip and inventory variance when responding to our random customer demand. The formula then permits us to project a set of contours identifying bullwhip regions thus enabling “boom-and-bust” operating scenarios to be avoided. These results have been interpreted via the development of recommended and extremely robust replenishment rule settings which guarantee to avoid bullwhip under these stated conditions. If this is tackled automatically using appropriate ordering software then executive efforts can be concentrated on interface management so as to reduce bullwhip at this more difficult level.

References


