Vendor Managed Inventory and bullwhip reduction in a two level supply chain

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Abstract
This paper compares the bullwhip properties of a Vendor Managed Inventory (VMI) supply chain with those of a traditional “serially-linked” supply chain. The emphasis of this investigation is the comparative impact the two structures have on the “Bullwhip Effect” generated. Particular attention is paid to the manufacturer’s production ordering activities as demonstrated using a simulation model based on difference equations. Each of the four important sources of the Bullwhip Effect is documented and considered in turn. The analysis shows that with VMI implementation two sources of the Bullwhip Effect may be completely eliminated, i.e. rationing and gaming or the Houlihan Effect, and the order batching effect or the Burbidge Effect. VMI is also significantly better at responding to rogue changes in demand due to the Promotion Effect or to price induced variations. However the effect of VMI on demand signal processing induced bullwhip or the Forrester Effect is less clear cut. The paper concludes that on balance VMI offers a significant opportunity to reduce the Bullwhip Effect in real-world supply chains.

Key Words
Bullwhip Effect, Vendor Managed Inventory, Supply Chain Dynamics, Production and Inventory Control.

1. Introduction
It is well established that removing an echelon in a supply chain can be of great benefit in improving dynamic performance (Wikner, Towill and Naim, 1992). This is because there is potential for a two-fold improvement. This is firstly due to elimination of delays in both information and material flow. Secondly a decision-
making activity that customarily increases distortion in the order waveform as it is flows upstream is eliminated (Towill and del Vecchio, 1994). VMI is one practical way of seeking to obtain the benefits of echelon elimination. Hence the need for a detailed investigation using the traditional supply chain as a benchmark to be bettered via a suitable design. As Maloni and Benton (1997) have indicated, there exists a large amount of literature on the concepts of supply chain partnerships projecting extremely optimistic views about their promise as win-win partnerships without any rigorous analysis to support the cause of optimism. This paper is a response to the shortfall in research that adopts a more rigorous analytical approach to examine supply chain partnership issues.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Order lead time</td>
<td>15</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>(days) (from customer’s order entry to delivery)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On Time Deliveries</td>
<td>20%</td>
<td>98%</td>
<td>99.8%</td>
</tr>
<tr>
<td>(% of orders delivered on time)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inventory Turnover Rate</td>
<td>5</td>
<td>35</td>
<td>80</td>
</tr>
<tr>
<td>Total Overhead Cost (index)</td>
<td>100</td>
<td>120</td>
<td>80</td>
</tr>
</tbody>
</table>

**Table 1. Impact of the Change Supply Chain Strategy and Implementation of NMS PipeChain Version of VMI at Ericsson Radio Systems**
(Source: Gustafsson and Norrman, 2001)

It is already known that when properly implemented, VMI healthily impacts the bottom line, for example, as shown in Table 1 (Gustafsson and Norrman, 2001). Note that there has been a two-stage programme of supply chain re-engineering supporting the introduction of VMI. This is via changed responsibility for orders (NMS phase) followed by total pipeline control (Pipechain phase). However there are both positive and negative aspects of implementing the NMS/Pipechain mode of VMI. These are listed in Table 2, and the downside is a warning to potential users falsely thinking that implementation is straightforward and trivial. It is good to see that benefits are visible
within months. But in a global BPR Programme, Towill and McCullen (1999) have observed significant improvements in supply chain performance occurring on a year-by-year basis for some time after changeover. Hence to maximise impact it is essential to ensure that adequate monitoring systems are in place. These will firstly ensure that regeneration to previous working practices is avoided, and secondly to help ensure that beneficial learning curve effects are forthcoming.

A. The Upside

- Main benefits visible shortly after an implementation (months)
- The investment pays off shortly (months)
- The customers and suppliers in the network have gained a greater knowledge and understanding of each others’ working processes and businesses
- The software tool is fast to implement (weeks-month)
- The users of the software tool rely on the system and find it logical and process oriented
- The work load for the people working with operative logistics has been less fluctuating

B. The Downside

- Although the concept is easy to understand, accepting the change of working procedures and shift of responsibility takes time
- Even though a standard interface is used to integrate the ERP systems it must be adapted to the process. This should not be underestimated, it creates work and takes time
- The software tool does not fit certain businesses (e.g. short term relationships with suppliers and seldom supply)

| Table 2. Positive and Negative Experiences of Implementing NMS/PipeChain Version of VMI at Ericsson Radio Systems | (Source: Gustafsson and Norrman, 2001) |

The particular emphasis of this paper is the relative impact these two supply chain structures have on the “Bullwhip Effect”, (Lee, Padmanabhan and Whang, 1997a and b) generated in the supply chain, which is investigated using a simulation model. Focussing on a one supplier, one customer relationship particular attention is given to the manufacturer’s production scheduling activities. To achieve this aim, an overview of a traditional supply chain and a VMI supply chain is given. The Bullwhip Effect is then outlined and the various causes are highlighted. Next, the two supply chain
structures are compared with respect to the Bullwhip Effect, with each source being investigated in turn in order to verify the research findings. This gives confidence to the potential VMI system performance benefits via time-series displays readily identified by managers as comparators with present day dynamic behaviour.

2. Using Bullwhip to Assess VMI Capability

The magnitude of the Enterprise Resource Planning (ERP) interface problem currently facing the manufacturing industry is well known in the literature (Thaler, 1999). Table 3 lists some of the snags already noted by Lee and Whang (2000). It is clear that substantial financial investment is required to move forward in this respect. But who pays for the new communication system? Whereas the UK DTI/CBI expectancy of partnering arrangements was of benefits to be equally shared (Towill and Naim, 1993), present-day customer pressures have tended to negate this aim. For example, Clark and Hammond (1997) infer that in much VMI experience to date, cost-benefit analysis is arguably that the supplier bears the cost of implementation, but the customer reaps the benefit. Similar conclusions may be drawn from the study by Lamming (2001) on the Japanese supply chain relationships in recession. He states that suppliers cannot now rely on retaining business in this new environment. Instead they must work in innovative ways so as to enable their customers to concentrate on real-time, market driven configuration of products coupled with minimum stocks in their supply chains (Lamming, 2001). When implemented properly VMI is clearly a step in the right direction.

In a recent seminal paper, Buxey (2001) has argued that production strategy drives the planning process. By having a clear view of what that strategy should be, management decisions regarding order fulfilment, capacity requirements, workforce-manning levels (and skills) become simpler and more transparent. The whole business is then much more readily aligned with the production strategy. As Buxey points out, the strategy is decided in the knowledge of customer requirements taking both short term and medium term horizons into account. A careful review of the Case Studies reported in Buxey (2001) suggests that generally the production strategy selected is the simplest and most robust capable of satisfying requirements. It is clear that VMI has much to offer in this scenario. Working closely with the end customer reduces uncertainty that in turn enables simplicity and reliability of operations.
<table>
<thead>
<tr>
<th>POTENTIAL SNAGS</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>MULTIPLE STANDARDS</td>
<td>There are multiple industry-specific standards. So a company with multiple business interests has to face dealing with multiple standards.</td>
</tr>
<tr>
<td>INFLEXIBILITY</td>
<td>EDI is designed on a one-size-fits-all basis. It may not meet the exact needs of any particular supply chain.</td>
</tr>
<tr>
<td>LIMITED FUNCTION</td>
<td>EDI is primarily designed around transaction processing. It may not cope with other kinds of information sharing such as databases, bar codes, images etc.</td>
</tr>
<tr>
<td>FIXED OPERATING MODE</td>
<td>EDI is batch operated. It works only in operational windows.</td>
</tr>
<tr>
<td>COST</td>
<td>There is a high financial cost and high resource cost to installing EDI. This discourages small and medium size companies.</td>
</tr>
</tbody>
</table>

Table 3. Potential EDI Implementation Snags or Why VMI May Not Happen Overnight
(Source: Authors Based on Lee & Whang, 2000)

We have selected Bullwhip as a measure of performance because it is a transparent and readily identifiable metric that can be used to establish if a course of action has been beneficial to the system. In that sense it is analogous to the use of elapsed time as an independent and unambiguous metric used for assessing process re-engineering programmes (Thomas, 1990; Stalk and Hout, 1990). Recent advances in costing the bullwhip effect include predictions from an OR model developed by Metters (1997). He concludes that avoidable on-costs range from 10% to 30% (depending on bullwhip source) calculated at the manufacturing stage alone. As Fisher, Hammond, Obermeyer and Raman (1997) point out, the on-costs throughout the chain can be very substantial, especially where an artificially high load is placed on system capacity. So in that sense the Metters (1997) figures can be regarded as underestimates. However, in our search herein for generic solutions we concentrate on bullwhip reduction alone. We believe it is a valid metric for VMI insight and exploitation in customer/vendor negotiations and in subsequent system re-design. It is simple enough to satisfy the Buxey (2001) need for basing production strategy around rules-of-thumb. At the same time it is a meaningful driver towards cost reduction (Metters, 1997).
3. Overview of a Traditional Supply Chain

A supply chain is a system consisting of material suppliers, production facilities, distribution services, and customers who are all linked together via the downstream feed-forward flow of materials (deliveries) and the upstream feedback flow of information (orders), (Stevens, 1989). As shown in Figure 1, in a traditional supply chain each “player” is responsible for his own inventory control and production or distribution ordering activities. One fundamental characteristic and problem that all players in a traditional supply chain (such as retailers, distributors, manufacturers, raw material suppliers) must solve is “just how much to order the production system to make (or the suppliers to supply) so as to enable a supply chain echelon to satisfy its customers’ demands”. This is the production/inventory control problem.

![Diagram of a traditional supply chain showing order fluctuations and bullwhip effect](Image)

Figure 1. Sequential Information Flow Causing Bullwhip in a “Traditional” Clothing Supply Chain
(Source: Towill and McCullen, 1999, based on the description by Stalk and Hout, 1990)

According to Axsäter (1985), “the purpose of a production/inventory control system (the method used to control inventory levels and production rates) is to transform incomplete information about the marketplace into co-ordinated plans for production and replenishment of raw materials”. The production/inventory control problem is...
tackled by practitioners inspecting data relating to demands, inventory levels and orders in the pipeline. Then, either in a structured, mathematical way (for example, by using a decision support system with properly engineered, well designed replenishment rules), or in a less formal way (by using their own experience and judgement), they place orders up the supply chain. The structure of the traditional supply chain shown in Figure 1 has developed partly as a result of the need for a company to be in control of its own assets and partly because, until recently, it has been uneconomic to pass vast amounts of information around the system. The traditional supply chain is characterised by each player in the supply chain basing his production orders or delivery orders solely on his sales to his customer, on his own inventory levels and, sometimes, on WIP (pipeline) targets. Each echelon in the supply chain only has information about what their customers want and not on which products the end customer is actually buying today. The clothing supply chain shown in Figure 1 typifies this state of affairs. It especially does not allow suppliers to gain any insight into what their customers are ordering to cover their own Customer Service Level (CSL) and cost requirements and what the customers are ordering to satisfy immediate customer demand (Kaipia, Holmström and Tanskanen, 2002).

This lack of visibility of real demand leads to a double-guessing culture. It can and does cause a number of problems in a supply chain if it is not properly designed and even then fluctuations in the supply chain cannot be completely eliminated. Such a state of affairs certainly causes the Forrester Effect, as a particular player over-orders in response to genuine changes in demand to account for his inventory deviations that result from the production/distribution lead-time. This over-ordering is then amplified up the supply chain, creating wide (and wild) fluctuations in the demand signal as it passes through the supply chain. Those shown in Fig. 1 are typical of real-world supply chains (Olsmats, Edghill and Towill, 1988). As we shall see later, an amplification of demand of 2:1 is typical as orders pass through a single supply chain echelon. Our purpose herein is to look further into the causes of this phenomenon, and to see how VMI helps to reduce this amplification on a source-by-source basis.

4. The Bullwhip Effect
The “Bullwhip Effect” is a new term (but not a new phenomenon since it has been debated in the literature for over four decades) coined by Lee, Padmanabhan and
Whang (1997a, b). It refers to the scenario where the orders to the supplier tend to have larger fluctuations than sales to the buyer. This distortion subsequently propagates upstream in an amplified form. Generally speaking, the further upstream the echelon, the more distorted and amplified is the waveform. Lee et al (1997a and b) state that there are five fundamental causes of Bullwhip; non-zero lead-times, demand signalling processing, price variations, rationing and gaming, and order batching. In any practical supply chain these may all be present and interact as shown in Fig. 2. Note that we consider both zero lead-time and demand signal processing to be the essence of the well-known Forrester effect (Forrester, 1961). It is our intention in this paper to show how each of these bullwhip sources is affected by the introduction of VMI. This will be done using a dynamic model of a particular VMI system capable of representing current industrial practice.

![Figure 2. Four Major Causes of the Bullwhip Effect (Source: Disney and Towill, 2001)](image)

Demand signal processing has in the past been called the “Demand Amplification” or the “Forrester Effect” after Jay Forrester (1961) who encountered the problem and subsequently demonstrated it via DYNAMO simulation. The Forrester Effect is also encompassed by Sterman’s bounded rationality, (Sterman, 1989), terminology that is common in the field of psychology as used to describe players sub-optimal but
seemingly rational decision making behaviour. This particular source of bullwhip was fully understood and the phenomenon well described and publicised by Stalk and Hout (1990). It is thus clear that the Boston Consultancy Group were fully conversant with the existence of bullwhip problems, which they then studied further and proposed solutions specific via a dynamic simulation.

Order batching is also known as the **Burbidge Effect** (Burbidge, 1991). It refers to the practise of placing orders up the supply chain (or on the various manufacturing processes) in batches. The philosophy behind this action is to gain economies of scale in set-up activities (such as setting up a specific machine or placing and receiving an order). It is often the result of the application of an Economic Order Quantity calculation or similar technique. Burbidge discusses the problems this causes on the shop floor in considerable detail. To deal with these problems Towill (1997) outlined the contributions of Forrester and Burbidge for avoiding the Bullwhip Effect brought together in an integrated approach termed “Forridge”. The Input-Output diagram in Figure 3 highlights the root causes of demand amplification that can be attributed to either the Forrester Effect or the Burbidge Effect and in some cases both.

![Figure 3. “FORRIDGE” Input-output diagram of demand amplification resulting from evidence provided by Jay Forrester and Jack Burbidge (Source: Towill, 1997)](image_url)

Within the production context, rationing and gaming, or the **Houlihan Effect** was highlighted by Houlihan (1987) who recognised that as shortages or missed deliveries occur in traditional supply chains, customers overload their schedules or orders. This in turn places more demands on the production system that inevitably leads to more unreliable deliveries. Customers then increase their safety stock target in a vicious
circle that further distorts the demand signal, giving rise to the Bullwhip Effect. Houlihan has summarised this phenomenon as the Flywheel effect as shown in Fig. 4. This simple diagram conveys, in terms readily recognised by top management, the dilemma facing production schedulers in “traditional” supply chains, such as previously reported in an automotive sector Case Study (Edghill, Olsmats and Towill, 1988). It deserves to be much more widely known and used.

![Diagram of Houlihan Flywheel]

Fig. 4. The Houlihan Flywheel Describing One Aspect of Bullwhip
(Houlihan, 1988)

Price variations or the Promotion Effect refers to the practise of offering products at reduced prices so as to stimulate demand. Assuming an elastic demand, this creates temporary increases in orders where customers take advantage of this opportunity and forward buy or “stock up”. However this has serious impacts on the dynamics of the supply chain, as when the price is released from the discounted level, demand slumps, creating a perceived need for further discounting in order to stimulate demand. A famous real-world example is due to Fisher et al (1997), with the resulting time-series being shown in Fig. 5.

As can be seen, the enticement of a discount offered by Campbells Soups to the retailer caused an unpredictable change in behaviour to which all suppliers have to
respond. This produces a typical bullwhip profile with demand being amplified as it is passed upstream. As can be seen from Fig. 5, this self-induced bullwhip requires a peak capacity well over twice the average demand. The resultant on-costs are considerable for all ‘players’ in the chain, including overtime, shift premiums, quality variances, and additional distribution, handling and storage charges. Furthermore, actual point-of-sales data suggests that adaptive level scheduling would be sufficient to meet real demand.

![Diagram](image-url)

Fig. 5. Example of a Price Discount Induced Bullwhip Recorded in Campbell’s Soups Supply Chain (Source: Fisher, Hammond, Obermeyer and Raman, 1997)

5. Measuring The Bullwhip Effect in Real Supply Chains

Many authors have recently supported using statistical measures of the Bullwhip Effect, for example, Chen, Ryan and Simchi-Levi (2000). Herein ORATE refers to the orders placed on our supplier and CONS represents sales or consumption by our customer. The bullwhip effect metric of choice is then:
\[
\text{Bullwhip} = \frac{\sigma^2_{\text{orate}} / \mu_{\text{orate}}}{\sigma^2_{\text{cons}} / \mu_{\text{cons}}} = \frac{\sigma^2_{\text{orate}}}{\sigma^2_{\text{cons}}} \quad \text{Eq 1}
\]

Where;

- \( \sigma^2 \) is the unconditional variance of the orders (subscript ORATE) and consumption (CONS).
- \( \mu \) is the unconditional mean of the orders (subscript ORATE) and consumption (CONS).

We may expect, as we are considering a single customer and supplier that the unconditional means are identical and thus they cancel. There is already a considerable amount of evidence in the literature that bullwhip exists in real-world supply chains (as distinct from simulation model results). Fransoo and Wouters (2000) used statistical techniques to measure the Bullwhip Effect experienced in a grocery supply chain. They considered the practical aspects of using the standard deviation ratios (rather then the variance) as a bullwhip measure and concluded that four Bullwhip metrics should be used. These focus on:

- a specific product for a specific outlet;
- a specific product demand aggregated across all outlets;
- aggregated products for individual outlets;
- aggregated outlets against aggregate products.

Fransoo and Wouters (2000) highlight the fact that each bullwhip measure is useful for investigating somewhat different circumstances. For example, Table 4 summarises the Bullwhip metrics estimated in their particular grocery supply chain. The four methods of calculation clearly enables bullwhip to be associated in turn with specific products and/or specific outlets as required by the systems designer.
<table>
<thead>
<tr>
<th>Coefficient of Variation Estimated for</th>
<th>Product</th>
<th>Outlet</th>
<th>Bullwhip</th>
<th>Weighted Average Bullwhip</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual Products and Aggregated Outlets</td>
<td>1</td>
<td>A</td>
<td>2.449</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>A</td>
<td>4.796</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>B</td>
<td>4.796</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>B</td>
<td>4.796</td>
<td></td>
</tr>
<tr>
<td>Aggregate Products and Aggregate Outlets</td>
<td>1</td>
<td>(A+B)</td>
<td>2.796</td>
<td>3.6205</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>(A+B)</td>
<td>4.472</td>
<td></td>
</tr>
<tr>
<td>Aggregate products and Individual Outlets</td>
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<td>A</td>
<td>4.583</td>
<td>4.619</td>
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<tr>
<td></td>
<td>(1+2)</td>
<td>(B)</td>
<td>4.712</td>
<td></td>
</tr>
<tr>
<td>Aggregate Products and Aggregate Orders</td>
<td>(1+2)</td>
<td>(A+B)</td>
<td>4.712</td>
<td>4.712</td>
</tr>
</tbody>
</table>

Table 4. Bullwhip Found in a Grocery Supply Chain (Fransoo and Wouters 2000)

Note that Bullwhip Factors yield important insights into the real-world behaviour of the different ‘players’ in the chain. This is shown in Table 5, based on a European retail supply chain (Holmström, 1997). He analysed the orders flowing upstream from the retail outlets right through the various echelons and ultimately back to the factory. Using the bullwhip measure (Eq 1) Holmström studied in depth a traffic building (high volume, low margin) product and a low traffic (low volume, high margin) product. This established that the downstream players (shops and wholesalers) are the biggest culprits in the particular sense of bullwhip generation. Furthermore the decision-makers exhibit little difference in their attitude to ordering policies for either the low margin or the high margin products, with Bullwhip Factors at around 3 to 1 at each stage. Not so the factory scheduler who clearly matches the ordering policy to SKU. He visibly treats the two products differently, and significantly dampens down the demand volatility in the factory orders placed for the high volume product. This is most likely to have been achieved via some version of level scheduling (Suzaki, 1987). In contrast, the same scheduler is quite prepared to induce further substantial bullwhip into the system when considering the low volume product. Finally, deliveries from the factory also exhibit some bullwhip but it is of a smaller order of magnitude than that generated by the downstream ‘players’
### Table 5. Actual Demand Amplification Recorded within a Real-World Supply Chain
(McCullen and Towill, 2001, based on results by Holmström, 1997)

<table>
<thead>
<tr>
<th>Supply Chain Echelon</th>
<th>High Volume Low Margin Product</th>
<th>Low Volume High Margin Product</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient of Variation</td>
<td>Comments on Waveforms</td>
</tr>
<tr>
<td>Retailer</td>
<td>2.60</td>
<td>Primarily Forrester Effect</td>
</tr>
<tr>
<td>Wholesaler</td>
<td>2.88</td>
<td>Forrester and Burbidge Effects</td>
</tr>
<tr>
<td>Factory Planner</td>
<td>0.72</td>
<td>Levelled Scheduling</td>
</tr>
<tr>
<td>Factory Production/Distribution</td>
<td>1.67</td>
<td>Forrester and Burbidge Effects</td>
</tr>
</tbody>
</table>

The composite Bullwhip Factor over the entire retail chain is obtained here by multiplying together the bullwhip at each stage. The result is 9:1 for the high volume product, but nearly 29:1 for the low volume product. These results show that “demonstrator” bullwhip values of 2 or 3 to 1 per stage as recorded by Sterman (1989) during the playing of the MIT Beer Game are realistic benchmarks. This is good to verify, as critics of the game have doubted its real-world relevance. In terms of generation of insight the retail supply chain results puts the value of the game into a new and enhanced perspective. Inspection of the time series presented by Holmström (1997) also enables some comment to be made on likely causes of the bullwhip in this retail supply chain. Those in Table 5 follow from the observations by McCullen and Towill (2001). They argue that Forrester Effects appear to dominate downstream ordering, with Burbidge Effects becoming much more evident as the waveform propagates upstream.

6. Overview of a VMI Supply Chain

In recent years, many companies have been compelled to improve their supply chain operations by sharing demand and inventory information with their suppliers and customers. Different industries have coined different terms for VMI, but all are based essentially on the same idea. VMI is a supply chain strategy whereby the vendor or supplier is given the responsibility of managing the customer’s stock. For clarity the term “distributor” for the customer in the VMI relationship and “manufacturer” for
the supplier or vendor is the VMI relationship will be used. VMI has become more popular in the grocery sector in the last 15 years due to the success of retailers such as Wal-Mart. Additionally, it is only relatively recently that the necessary information and communication technology has become economically available to enable the strategy, although Holmström (1998) has shown that it can be readily enabled via fax or emails and spreadsheets. As proof, Disney, Holmström, Kaipia and Towill (2001) have implemented VMI in a real-world supply chain using data available from a popular ERP system and a spreadsheet based decision support system.

Moreover, VMI is not a new philosophy. It was initially discussed by Magee (1958, pp 298) in a presentation of a conceptual framework for designing a production control system. Quoting directly from the text (as it prophetically and very concisely portrays what we believe VMI actually is):

“Frequently there is argument as to who should control inventories. For example, should it be the sales organisation or (some) other unit that draws on the stocks and wants to be sure they are there, or the operation that supplies the stock point and wants to feed it economically? There is probably no resolution to this question as stated; the difficulty is that both have a legitimate interest. It is possible to restate the question slightly and reach a solution. The user has to be sure the material he needs is there. He has corresponding responsibility to state what his maximum and minimum requirements will be. Once these limits are accepted as reasonable, the supplier has the responsibility of meeting demand within these limits, making whatever use he can of the flexibility the inventory provides. Thus both have a share in the responsibility for and control over a stock unit. One specifies what the maximum and minimum demands on the stock unit will be; the other has the responsibility of keeping the stock unit replenished but not overloaded as long as demand stays within the specified limits”, Magee (1958, pp298).

VMI comes in many different forms. Familiar names are Quick Response (QR), (Lee, So and Tang, 2000), Synchronised Consumer Response (SCR), Continuous Replenishment (CR), Efficient Consumer Response (ECR), (Cachon and Fisher, 1997), Rapid Replenishment (RR), Collaborative Planning, Forecasting and
Replenishment (CPFR), Holmström et al, (2000) and Centralised Inventory Management (Lee, Padmanabhan and Whang, 1997a), the terminology depending on sector application, ownership issues and scope of implementation. However, in essence, they are all specific applications of VMI that is summarised conceptually in Figure 6. This is the system to be used to benchmark bullwhip reduction.

![Figure 6. Overview of the VMI Scenario](image)

Note that we do not consider those supply chain scenarios that exploit only the data about end consumer demand in the ordering decisions to be true VMI. We term this kind of supply chain as possessing “information sharing” and it is a distinct (but equally valid) strategy. However, the lack of customer inventory information in the suppliers ordering decision makes it a fundamentally different system. Examples of information sharing can be found in Yu, Yan and Cheng (2001), Chen, Drezner, Ryan and Simchi-Levi (2000), Lee, So and Tang (2000) and Mason-Jones and Towill (1997).

7. Description of the VMI Supply Chain Simulation Model
The difference equations required to model our version of the VMI scenario are shown in Appendix 1. These difference equations can quickly be turned into a mathematical model of the VMI supply chain by using z-transforms. The formulation and exploitation of such a mathematical model is not presented in this contribution due to space restrictions but can be found in Disney (2001) and Disney and Towill
(2001 and 2002). Herein, the difference equation representation will be exploited. The difference equations may be quickly realised by interested readers in “spreadsheet” applications such as Microsoft Excel. Difference equations can also be implemented in standard computer languages with relative ease, as shown in Disney and Towill, (2001). The specific “flavour” of VMI that the difference equations represent in Appendix 1 is termed VMI-APIOBPCS, or **Vendor Managed Inventory, Automatic Pipeline, Inventory and Order Based Production Control System.**

The VMI term in VMI-APIOBPCS reflects the most significant fact about a VMI supply chain, i.e. that the distributor (the customer in the VMI relationship) passes inventory information and Point of Sales (POS) data to their suppliers rather than orders, (Kaipia et al 2002, Cottrill (1997)). The actual inventory at the customer is then compared to a re-order point that has been agreed on by both parties. This re-order point is set to ensure adequate availability without building up excessive stocks. It triggers a replenishment order that is delivered to the customer if the actual inventory is below the re-order point in each planning period. Each party also agrees the order-up-to point, O. The dispatches between the two echelons are equal to the order-up to level, O, minus the re-order point, R, and the dispatches can be of a constant or varying size within this framework.

The re-order point is set dynamically so as to reflect perceived changes in demand. This is done by exponentially smoothing (over Tq time units) the sales signal and multiplying it by a constant (G) that ensures appropriate customer service levels at the distributor, taking into account the transportation lead-time between the two parties in the supply chain. Exponential smoothing was chosen as the forecasting mechanism because it is; simple to implement in computer systems (requiring less data storage), readily understood and the most favoured technique by both industrialists and academics. It should be noted that the net change in the re-order point from one time period to another is added to the sales signal and the vendor treats this a demand. So, when demand is increasing and the distributors re-order point grows, the supplier or vendor treats the stock (re-order point) requirements at the distributor as demand and incorporates that into his forecasts and stock levels, as he clearly should do. Obviously, the negative argument also applies, i.e. when the re-order point is reducing in size over time, demand signals to the manufacture reflect this.
The APIOBPCS term reflects the components of the structure of the ordering decision at the VMI supplier. In words it is “let our production orders be equal to the sum of three components; the forecasted demand, (exponential smoothed over $T_a$ time units), a fraction $(1/T_i)$ of the difference between target stock and actual stock and a fraction $(1/T_w)$ of the difference between target WIP and actual WIP.

8. Description of the Traditional Supply Chain Simulation Model

The APIOBPCS model, John, Naim and Towill (1994), was chosen to represent a traditional supply chain. This was due to a number of reasons. Firstly it was felt important that it is desirable that like (APIOBPCS) is compared to like as much as possible (VMI-APIOBPCS) in order to gain as much understanding as possible on the fundamental structure of VMI. Secondly APIOBPCS was chosen for VMI and the traditional supply chain, as it is recognised as good practice, incorporates all commonly available forms of information, represents human behaviour (Sterman, 1989 and Naim and Towill, 1995) and is a well-understood member of the IOBPCS (Towill, 1982) family. The APIOBPCS model can be expressed in words as outlined in the previous section. It incorporates three variables:

- $T_a$, a parameter that describes how quickly demand is tracked in the forecasting mechanism,
- $T_i$, a parameter that describes of much of the discrepancy between actual inventory and target inventory levels should be added to the production/distribution order rate and
- $T_w$, a parameter that describes how much of the discrepancy between actual WIP and target WIP levels should be added to the production/distribution order rate.

Individual echelons, or APIOBPCS models, can be linked together to form a supply chain, by coupling the ORATE signal of the consuming echelon to the CONS signal of the supplying echelon, as recognised by Burns and Sivazlian (1978) and further exploited by Towill and del Vecchio (1994). The difference equations required to model a two-level APIOBPCS supply chain (for example in a spreadsheet) are shown
in Appendix 2. Like the VMI model the production and distribution delays are arbitrarily assumed to be of four time units.

9. Impact of VMI on the Promotions Induced Bullwhip

To investigate the impact of VMI on the promotions induced Bullwhip Effect, the Factory Order Rate response of the two supply chain structures to a step input will be used. This produces a very “rich picture” of the associated system dynamics. Understanding the dynamic response to a step input will thereby yield insight into how the system will be affected by various promotions. As there are an infinite number of designs for VMI and traditional supply chains that might be compared, previous best practise designs will be used to compare the two supply chains via the step response. The following designs were chosen to represent good designs of a traditional supply chain with a production lead-time of 4 time periods;

- John et al (1994) recommended settings (Ta=8, Ti=4, Tw=8). This was derived using classical control theory and simulation and may be considered to a fairly conservative design.
- Disney et al (1997) recommended settings (Ta=8, Ti=4, Tw=15). This was based on a Genetic Algorithms search, using Laplace transforms, simulation with the aim of minimising the Forrester Effect, inventory holding, selectivity, whilst maximising robustness to errors in estimation of WIP levels and production lead-times.
- Naim and Towill (1995) values of (Ta=8, Ti=4, Tw=4). These were derived from inspecting Sterman’s (1989) Beer Game derived optimum settings. This may be considered to a reactive version of the John et al (1994) settings.
- Disney (2001) recommended settings (Ta=4, Ti=7, Tw=28). This was based on the full solution based search using z-transforms and simulation aimed at balancing the Forrester Effect and inventory holding requirements.
Operational Setting | Parameters of “Optimum” VMI System
--- | ---
| G | W# | Ta | Ti | Tq | Tw |
| 1 | 0.01 | 1 | 3 | 1 | 3 |
| 1 | 1 | 6 | 7 | 6 | 42 |
| 1 | 100 | 18 | 23 | 6 | 63 |
| 4 | 0.01 | 1 | 14 | 1 | 14 |
| 4 | 1 | 4 | 14 | 4 | 63 |
| 4 | 100 | 22 | 27 | 6 | 63 |

- G = Re-order point level
- W# = Weighting function used in optimisation routine to trade-off production capacity requirements against stock requirements

Table 6. Sample Optimum Parameter Values for VMI System Simulation

As outlined earlier, the VMI strategy has 5 key parameters:
- Tq - the forecasting parameter used to generate the re-order point,
- G - the gain on the forecast generated by Tq used to calculate the re-order point,
- Ta - the forecasting parameter used to forecast demand by the manufacturer,
- Ti – the fraction of inventory error accounted for in a single order and
- Tw – the fraction of the WIP error accounted for in a single order

These parameters determine the dynamic response of the system. The terms Ta, Ti, Tq and Tw depend on the parameter G that is independently set to reflect the desired CSL given the transportation lead-time between the manufacturer and the distributor, via the re-order point equation. A full-scale optimisation procedure (Disney (2001) and Disney and Towill (2002)) has been applied to these parameters for a range of ratios of production adaptation costs (due to the Forrester Effect) to the associated inventory holding costs and for different values of the re-order point G. The resulting optimal parameter settings for Ta, Ti, Tq and Tw for the case when G = 1 and 4 are shown in Table 6. There is a complex relationship between these parameters for example; higher values of G generally induce more bullwhip into the manufacturer’s orders. Furthermore, higher values of Tq help to reduce the bullwhip experienced by the manufacturer but at the expense of longer inventory settling time. It is not our intention to explore this here. In this Section it is sufficient to illustrate the VMI system step response for the case where production adaptation and inventory holding costs were given equal importance, for the two designs chosen to represent good...
solutions for a VMI supply chain. Hence the “best practice” settings for the VMI supply chain used were;

- The optimum parameter setting when the distributor has a re-order point level set at 1 planning periods average demand, (i.e. G=1, Ta=6, Ti=7, Tq=6, Tw=42)
- The optimum parameter setting when the distributor has a re-order point level set at 4 planning periods average demand, (i.e. G=4, Ta=4, Ti =14, Tq=4, Tw=63)

It can be seen from inspection of Figure 7 that the VMI design outperforms the traditional supply chain, with less peak overshoot, faster settling time and a generally quicker response.

![Figure 7. Impact of VMI on the Promotions Bullwhip Effect](image)

10. Impact of VMI on System Induced Bullwhip Effect

We now estimate the impact of VMI on Forrester source induced bullwhip. In Table 7 we have compared VMI and traditional supply chains across a range of performance metrics. The peak ORATE overshoot is the simple measure of bullwhip already met in Fig. 7. Note that for completeness Table 7 includes three optimal solutions for each of the two values of $G$ (1 and 4). These are for ratios of production
adaptation/inventory holding costs $W = 0.01$; $W = 1.0$; and $W = 100$. The reason for this is that $W = 0.01$ approximates an agile system; $W = 100$ approximates a lean (level scheduling) system; whilst $W = 1.0$ is a compromise solution. As noted by Christopher and Towill (2000) there are occasions where “agile” is the best business solution, and where “lean” is the best business solution, and where some “mix” is required.

For the optimal VMI supply chains, the bullwhip is reasonably unaffected by varying $W$ for a given value of $G$. This is because the optimisation programme (Disney, 2001) drives the VMI parameters to yield the best possible response. (As we have seen in Table 6, the parameter settings to achieve this goal are substantially different.) If the peak ORATE overshoot is 2.5, then $X$ is a bullwhip effect of 150% and so on. So comparing the optimal VMI system with the nearest equivalent traditional supply chain i.e. $G = 1$, $W = 1.0$, and with VMI optimal parameter setting, we see VMI reduces the bullwhip effect from 144% to 69%. Some authors (for example Chen, Ryan and Simchi-Levi [2000]) use the ratio of order and sales variance as a bullwhip measure, others (for example Fransoo and Wouters (2000) have been using ratios involving the standard deviation. Whilst both conceptually similar, the variance ratio is preferred as this can be calculated directly from a system’s transfer function, Disney and Towill (2001) or efficiently enumerated with difference equations. Hence in Table 6 we have included an estimate of variance obtained via evaluation of system noise bandwidth (Towill, 1982). This bullwhip measure has been reduced from 0.93 (Traditional supply chain) to 0.46 (VMI system), a factor of 2 to 1. So on both bullwhip measures using VMI is a great improvement in coping with Forrester sourced bullwhip.

11. The Impact of VMI on the Houlihan Effect

In the VMI supply chain the responsibility for managing the stock at the customer’s premises clearly lies with the manufacturer. Therefore, the Houlihan Effect is completely eliminated as the manufacturer is generating the despatches in the supply chain rather than the distributor. With this configuration it is not possible to “game” against yourself. VMI has the advantage that on-time delivery does not need to be monitored, because for as long as there is stock availability at the distributor, no one cares (including the end customer) if a delivery is missed. In fact, it is unlikely that
<table>
<thead>
<tr>
<th>System Performance</th>
<th>Optimal VMI supply chain</th>
<th>Traditional supply chain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W=1</td>
<td>W=0.01</td>
</tr>
<tr>
<td>Bullwhip Measures</td>
<td>Peak ORATE overshoot</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>Noise Bandwidth/(\pi)</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>(\sigma^2) (calculated from real time series)</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 7. Impact of VMI on Forrester Sourced Bullwhip Effect
the distributor would even know if a delivery is on time, as he does not even generate orders to compare against shipments.

VMI has another unique advantage over the traditional supply chain; it aligns the necessary measures of performance required in the VMI supply chain to the customer expectations, which has also been noticed by Kaipia, Holmström and Tanskanen (2002). This comes from the fact that the only two measures that are important in the VMI supply chain (at least in a logistical sense), are whether there is a lost sale due to a stock-out at the distributor and how much inventory there is in the supply chain, as this influences the costs to the end consumer. So clearly VMI eliminates one very common source of bullwhip. It is also arguably the most tenuous and irritating source of bullwhip. More often than not it is enflamed by secrecy, lack of trust, and the general adversarial nature of “traditional” supply chains.

12. The Impact of VMI on the Burbidge Effect

The Burbidge Effect in a traditional supply chain can be avoided by despatching every time period only the requirements for that time period. However, it is often the case that under such conditions the transportation (or receiving facilities) cost is hugely inflated. Thus, companies often resort to a batching mentality, thereby introducing a huge source of Bullwhip Effect into the supply chain. If only the current time period’s requirements are despatched then, as shown in Fig. 8, the amount transported will need to change every time period. So there is an apparent conflict between reducing bullwhip and obtaining economies of scale on transportation costs.

However, the way a VMI supply chain copes with the Burbidge Effect in an innovative manner, is also shown in Figure 8. This is because VMI allows batching to occur in the transportation activity between the manufacturer and the distributor, without introducing the order batching effect into the production order rate. This is enabled by VMI because of the way the information flow is structured. Recall that in a VMI supply chain the stock position at the distributor is compared to a re-order point and if the stock position is below the re-order point then a despatch quantity is transported to the distributor. This is one side of an IF…. THEN rule. Capturing the other side of the IF…. THEN rule is done by adding to the distributor’s stock the
goods in transit between the two parties and the manufacturer’s stock position. When these three stock positions are summed up together the batching disappears from the supply chain dynamics. This can be easily verified by implementation of the difference equations in Appendix 1 and 2. It should noted that to account for different demand rates the frequency of deliveries changes (rather than the size of those deliveries), in a VMI supply chain, thus permitting much better scope for gaining economies of scale in transportation and packaging without introducing the Bullwhip Effect.

13. Discussion of Results
Our simulation model suggests that VMI offers significant opportunities for reducing the Bullwhip Effect in supply chains. Table 8 summarises the findings in terms of the four reported sources of bullwhip.

Two sources (The Houlihan and Burbidge Effect) of the Bullwhip Effect may be completely eliminated by the adoption of VMI in a supply chain. The Houlihan Effect is sidestepped because of the change in responsibilities in the relationships and it is unlikely that rationing and gaming effects will be introduced by the manufacturer on himself. The Burbridge or order batching effect is eliminated by VMI because of
the balancing effect of the information flows in the supply chain. The influence of price variations or the Promotion Effect on the dynamics of the supply chain is also greatly reduced by the use of VMI.

Approximately 50% less overshoot is generated in a VMI supply chain when demand shifts to a new level due to a step change in demand rates. Finally the Forrester Effect in the VMI supply chain exhibits much less variation than a traditional supply chain, although a traditional supply chain can be designed to reduce the Forrester Effect at the expense of other criteria, for example stockholding. Importantly however, VMI requires typically only approximately 50% of the inventory holding in the supply chain (Disney and Towill, 2001). Thus this paper argues that VMI can significantly improve the dynamics of supply chains and it simultaneously offers an effective mechanism for solving the Bullwhip problem.

<table>
<thead>
<tr>
<th>Source of the Bullwhip Effect</th>
<th>Traditional Supply Chain</th>
<th>VMI Supply Chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price variations (Promotion Effect)</td>
<td>Requires 50% increase in capacity to provide desired Customer Service Levels</td>
<td>Step responses show that VMI produces approximately 50% less overshoot when responding to step inputs</td>
</tr>
<tr>
<td>Rationing and gaming (Houlihan Effect)</td>
<td>Can make a significant contribution to Bullwhip in a traditional supply chain</td>
<td>Completely avoided by VMI supply chains because of the change in the nature of the relationships in the supply chain</td>
</tr>
<tr>
<td>Demand signal processing (Forrester Effect)</td>
<td>The Forrester Effect can be reduced in a traditional supply chain but it comes at the cost of twice as much system inventory holding</td>
<td>In a well designed system it is easy to substantially reduce bullwhip to about the level of a single echelon supply chain</td>
</tr>
<tr>
<td>Order batching (Burbidge Effect)</td>
<td>Can make a significant contribution to Bullwhip in a traditional supply chain. However, it can be avoided if deliveries occur every time period and variable batch sizes are used</td>
<td>Completely avoided by VMI supply chains due to the structure of the information flows</td>
</tr>
</tbody>
</table>

Table 8. The Impact of VMI on the Bullwhip Effect in Supply Chains
Our analysis herein has concentrated on the case of a single VMI customer and a single supplier. We have not considered the case of multiple (VMI and non-VMI) customers and interacting values streams in the manufacturer. This is a different problem altogether, but we note that Waller, Johnson and Davies (1999) have considered such a case. Furthermore, as Burbidge (1991) was at pains to point out, interacting value streams should be avoided if at all possible and Towill and McCullen (1999) have shown that BPR principles emerging from a simple generic model can indeed be exploited in a real world supply chain scenario. We have also not considered here the impact on tardy or inaccurate information flows on VMI performance.

14. Conclusions

Our analysis has shown that by adopting VMI can have positive impacts on the bullwhip problem in supply chains. We have investigated each of the potential sources of bullwhip identified by Lee et al (1997a and b) and shown that it is possible to completely avoid two causes of bullwhip altogether. It is also possible to reduce the impact of other sources of bullwhip. It is clear that VMI can be of great benefit to the vendor or supplier in a VMI relationship if they correctly use inventory and sales information in the production and inventory control decision-making process. However there is relatively little discussion of this in the literature, which has often focussed on benefits for the customer in the VMI relationship. In our approach has highlighted that VMI offers benefits for low volume products, which typically suffer from Burbidge effects, and high volume products that typically suffer from the Forrester effect.
References


### Appendix 1. The Difference Equations Required to Simulate the VMI-APIOBPCS Model When Inventory is Treated Separately and Transportation Despatches are Modelled Explicitly

<table>
<thead>
<tr>
<th>Description</th>
<th>Difference Equations</th>
<th>Eq. No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecasted Re-order point at the distributor</td>
<td>$R_t = R_{t-1} + \frac{1}{1+T_q}((G*CONS_t) - R_{t-1})$</td>
<td>(1.1)</td>
</tr>
<tr>
<td>Order-up-to point at the distributor</td>
<td>$O_t = R_t + TQ_t$</td>
<td>(1.2)</td>
</tr>
<tr>
<td>Distributor's inventory level</td>
<td>$DINV_t = DINV_{t-1} - CONS_t + DES_{t-T}$</td>
<td>(1.3)</td>
</tr>
<tr>
<td>Goods In Transit between factory and distributor</td>
<td>$GIT_t = \sum_{i=t-T}^{t} DES_i$, where $T$ is the transportation lead-time</td>
<td>(1.4)</td>
</tr>
<tr>
<td>Despatches</td>
<td>$DES_t = \begin{cases} TQ_{t-1} &amp; \text{if } DINV_{t-1} + GIT_{t-1} &lt; R_{t-1} \ 0 &amp; \text{if } DINV_{t-1} + GIT_{t-1} &gt;= R_{t-1} \end{cases}$</td>
<td>(1.5)</td>
</tr>
<tr>
<td>Transport quantity</td>
<td>$TQ_t = CONS_t$ or $ETQ_t$, nominally set to equal 4</td>
<td>(1.6)</td>
</tr>
<tr>
<td>System inventory levels</td>
<td>$SINV_t = FINV_t + GIT_t + DINV_t - R_t$</td>
<td>(1.7)</td>
</tr>
<tr>
<td>Factory inventory levels</td>
<td>$FINV_t = FINV_{t-1} + COMRATE_t - DES_t$</td>
<td>(1.8)</td>
</tr>
<tr>
<td>Virtual consumption</td>
<td>$VCON_t = CONS_t + dSS_t$</td>
<td>(1.9)</td>
</tr>
<tr>
<td>Net changes in the distributor's re-order point</td>
<td>$dSS_t = R_t - R_{t-1}$</td>
<td>(1.10)</td>
</tr>
<tr>
<td>Forecasted consumption for the factory</td>
<td>$AVCON_t = AVCON_{t-1} + \frac{1}{1+Ta}(VCON_t - AVCON_{t-1})$</td>
<td>(1.11)</td>
</tr>
<tr>
<td>Desired WIP</td>
<td>$DWIP_t = AVCON_t * Tp$</td>
<td>(1.12)</td>
</tr>
<tr>
<td>Actual WIP</td>
<td>$WIP_t = WIP_{t-1} + ORATE_t - COMRATE_t$</td>
<td>(1.13)</td>
</tr>
<tr>
<td>Error in WIP</td>
<td>$EWIP_t = DWIP_t - WIP_t$</td>
<td>(1.14)</td>
</tr>
<tr>
<td>Order rate</td>
<td>$ORATE_t = AVCON_{t-1} + \frac{EINV_{t-1}}{Ti} + \frac{EWIP_{t-1}}{Tw}$</td>
<td>(1.15)</td>
</tr>
<tr>
<td>Completion rate</td>
<td>$COMRATE_t = ORATE_{t-Tp}$</td>
<td>(1.16)</td>
</tr>
<tr>
<td>Error in system inventory levels</td>
<td>$EINV_t = TINV_t - SINV_t$</td>
<td>(1.17)</td>
</tr>
<tr>
<td>Typical Test Input</td>
<td>$CONS_t = \begin{cases} 0 &amp; \text{if } t &lt; 0 \ 10 &amp; \text{if } t &gt;= 0 \end{cases}$, for a step input</td>
<td>(1.18)</td>
</tr>
<tr>
<td>Typical Target inventory</td>
<td>$TINV_t = 0$</td>
<td>(1.19)</td>
</tr>
</tbody>
</table>
### Appendix 2. Difference Equations Required for the Two Level APIOBPCS (Traditional Supply Chain) Simulation Model

These difference equations (where the subscript 1 denoted the distributor variables and subscript 2 denotes the manufacturer variables) are for modelling a two level APIOBPCS model are:

<table>
<thead>
<tr>
<th>Description</th>
<th>Difference Equations</th>
<th>Eq. No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distributor's actual WIP</td>
<td>$WIP_i = WIP_{i-1} + ORATE_i - COMRATE_i$</td>
<td>(2.1)</td>
</tr>
<tr>
<td>Distributor's completion rate</td>
<td>$COMRATE_i = ORATE_{i-(T_{p1})}$</td>
<td>(2.2)</td>
</tr>
<tr>
<td>Distributor's desired WIP</td>
<td>$DWIP_i = AVCON_i * T_{p1}$</td>
<td>(2.3)</td>
</tr>
<tr>
<td>Distributor's error in system inventory levels</td>
<td>$EINV_i = TINV_i - SINV_i$</td>
<td>(2.4)</td>
</tr>
<tr>
<td>Distributor's error in WIP</td>
<td>$EWIP_i = DWIP_i - WIP_i$</td>
<td>(2.5)</td>
</tr>
<tr>
<td>Distributor's forecasted consumption for the factory</td>
<td>$AVCON_i = AVCON_{i-1} + \frac{1}{1+T_{a1}}(CONS_i - AVCON_{i-1})$</td>
<td>(2.6)</td>
</tr>
<tr>
<td>Distributor's inventory levels</td>
<td>$AINV_i = AINV_{i-1} + COMRATE_i - CONS_i$</td>
<td>(2.7)</td>
</tr>
<tr>
<td>Distributor's order rate</td>
<td>$ORATE_i = AVCON_{i-1} + \frac{EINV_i}{T_{i1}} + \frac{EWIP_i}{T_{w1}}$</td>
<td>(2.8)</td>
</tr>
<tr>
<td>Distributor's typical target inventory</td>
<td>$TINV_i = 0$</td>
<td>(2.9)</td>
</tr>
<tr>
<td>Manufacturer's Actual WIP</td>
<td>$MWIP_i = MWIP_{i-1} + MORATE_i - MCOMRATE_i$</td>
<td>(2.10)</td>
</tr>
<tr>
<td>Manufacturer's Completion rate</td>
<td>$MCOMRATE_i = MORATE_{i-(T_{p2})}$</td>
<td>(2.11)</td>
</tr>
<tr>
<td>Manufacturer's Desired WIP</td>
<td>$MDWIP_i = AVCON_i * T_{p2}$</td>
<td>(2.12)</td>
</tr>
<tr>
<td>Manufacturer's error in inventory levels</td>
<td>$MEINV_i = MTINV_i - MAINV_i$</td>
<td>(2.13)</td>
</tr>
<tr>
<td>Manufacturer's Error in WIP</td>
<td>$MEWIP_i = MDWIP_i - MWIP_i$</td>
<td>(2.14)</td>
</tr>
<tr>
<td>Manufacturer's forecasted consumption for the manufacturer</td>
<td>$MAVCON_i = MAVCON_{i-1} + \frac{1}{1+T_{a2}}(ORATE_i - MAVCON_{i-1})$</td>
<td>(2.15)</td>
</tr>
<tr>
<td>Manufacturer's Inventory levels</td>
<td>$MAINV_i = MAINV_{i-1} + MCOMRATE_i - ORATE_i$</td>
<td>(2.16)</td>
</tr>
<tr>
<td>Manufacturer's Order rate</td>
<td>$MORATE_i = MAVCON_{i-1} + \frac{MEINV_i}{T_{i2}} + \frac{MEWIP_i}{T_{w2}}$</td>
<td>(2.17)</td>
</tr>
<tr>
<td>Manufacturer's typical target inventory</td>
<td>$MTINV_i = 0$</td>
<td>(2.18)</td>
</tr>
<tr>
<td>Typical test input</td>
<td>$CONS_i = \begin{cases} 0 &amp; \text{if } t &lt; 0 \ 10 &amp; \text{if } t \geq 0 \end{cases}$, for a step input</td>
<td>(2.19)</td>
</tr>
</tbody>
</table>