

# SIZE EFFECT AND MULTISCALE FRACTURE

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## ABSTRACT

We study fracture of notched samples made of quasi-brittle, polyphase materials like rock, concrete or ceramics. The fracture demonstrates the size effect during loading. This means that a full-size sample made of such a material exhibits different fracture behaviour than a laboratory-size sample. The effect is explained by the existence of an extended zone of distributed defects and cracks (process zone) that surrounds the tip of the propagating fracture. The growth mechanisms of the process zone is scale-dependent: in an unbounded sample or a full-size structure, the zone develops until its maximum width and then it remains of the same width, while in a bounded sample that is less than some critical size, the process zone cannot be fully developed. Various similarity and scaling approaches to mechanics of multiple fracture are discussed. The growing process zone is modelled as a pattern of fractures having fractal properties on the intermediate stage of the development of the pattern. A formula is derived for the critical tensile stress that depends on both the sample size and the size of the process zone.

## 1 INTRODUCTION

During loading of notched samples made of quasi-brittle, polyphase materials like rock, concrete or ceramics several stages of fracture may be observed. In particular, micro-defects grow and become microcracks, in turn these coalesce and isolated mesocracks are formed. Then a macrocrack grows slowly by breaking bridges between the macrocrack tip and mesocracks of the process zone (Botvina [1], Huang [2]). The process zone consists of a cascade of interacting defects of various length scales (Barenblatt [3]). Hence the process of multiple fracture in these materials should be considered on several scales: micro, meso and macro. The concept of quasi-brittle materials supposed that there is a narrow layer of non-elastic deformations near the fracture surfaces. During the crack propagation, both this layer and the new fracture surface absorb energy. One can also observe the fracture size effect, namely a sample made of polyphase material exhibits different behaviour when it is of laboratory-size and when the sample size increases. Such a behaviour can be explained by the screening effect of the process zone and scale-dependent growth mechanisms of the process zone. To study the process within the intermediate scale, new postulates which are outside the hypotheses of classical continuum mechanics, are employed. Various similarity and scaling approaches to mechanics of multiple fracture including geometric similarity, statistical self-similar scaling of patterns, parametric-homogeneous scaling, fractal scaling, will be discussed in order to describe the cascade of developing defects and cracks. To study multiple fracture, the following postulates will be employed (Borodich [4]): (i) the width  $h$  of the layer of inelastic deformation near any fracture surface is constant and it is the same for mesocracks and the macrocrack; (ii) material of every cube of size  $h$  centred in a point of fracture surface absorbs the same quantity of energy  $g_f$ ; (iii) no new microcracks arise in the compressed region; (iv) the process zone develops during loading, the size of process zone (its width and length) depends on both the fine structure of material and the stress field; (v) the jump of the main crack tip leads to a relaxation of the stress field in some domain behind the tip, and the micro- and mesocracks situated in the domain will not grow further, therefore, the process zone can be

separated on an active  $S_A$  and a passive part  $S_P$ , the active part only is essential for propagation of the main crack; (vi) the stress concentration regions near the mesocracks of the periphery of the active domain are sources for growing of new microcracks. Let us cover all mesocracks of the active part of the process zone by cubes of the size  $h$  centred at points of fracture surfaces. Then the average amount  $\langle W \rangle$  of absorbed energy can be calculated as

$$\langle W \rangle = g_f N(h) + \text{const} \quad (1)$$

where  $N(h)$  is the minimal number of the cubes in the cover. The idea of covering the cracks of the process zone by cubes in order to calculate the amount of absorbed energy is very close to the methods of applied fractal geometry. Evidently, this approach can be used if the width of the plastic layer is small with respect to the average distance between mesocracks. The author has modelled the discrete propagation of fracture using parametric-homogeneous scaling (Borodich [4]). However, here a physical fractal model of size effect in a sample subjected to stretching stresses is presented. The growing process zone is modelled as a pattern of fractures having fractal properties on the intermediate stage of development of the pattern. The main part of the developing process zone is assumed to be wedge-shaped, while its head (the active process zone) is bounded by an arc of a circle with the centre at the main crack tip. Fractal models are quite popular in mechanics of fracture (see, e.g. review by Borodich [5]). It was found (Chelidze [6], Zhao [7]) that patterns of multiple fracture in marble have fractal features. The author defines fractals as sets with non-integer fractal dimension, and emphasizes that it is necessary to split the term in two: mathematical and physical fractals. Confusion of these two kinds of fractals led often to various erroneous or at least unjustified conclusions. Mathematical fractals or fractal geometry studies various fractal and multifractal measures and fractal dimensions, in particular Hausdorff dimension and box-counting dimension (this can be obtained by consideration the limit behaviour of covers of a set by boxes of size at most  $\delta$  when  $\delta \rightarrow 0$ ), while physical (natural, empirical) fractals which are real world or numerically simulated objects exhibiting a kind of self-similarity (this is the so-called fractal behaviour) in a bounded region between upper ( $\Delta^*$ ) and lower ( $\delta^*$ ) cut-offs. The main distinction between physical and mathematical fractals is that the power law of natural objects (empirical fractals) is observed on a bounded region of scales only, while mathematical fractals consider limits when the scale of consideration goes to zero. Hence, the scaling approach to these two kinds of fractals is not the same. In 1992 a model of a fractal single crack was presented by the author (Borodich [8]). Both mathematical and physical fractal approaches were employed. Then it was shown (Borodich [9]) that the same scaling arguments are valid for fractal fracture patterns. Let us use the fractional part  $D^*$  of the fractal dimension,  $0 < D^* < 1$ . Hence, the fractal dimension of a fractal curve is  $D = 1 + D^*$ , while the fractal dimension of a fractal surface is  $D = 2 + D^*$ . We suppose that the fractal dimension of the mesocrack pattern is equal to some value  $C$  and it is constant during the cloud growth. It is known that the fractional parts of the fractal dimensions of profiles of fracture surfaces ( $D^*$ ) and patterns of fracture ( $C^*$ ) for quasi-brittle materials belong mainly to the following intervals:  $0.04 < D^* < 0.33$  and  $0.47 < C^* < 0.79$ . It was shown that fractal dimension of a fracture surface can correlate with fracture energy only in quasi-brittle materials with very narrow plastic zones. If fracture surface exhibits fractal features, however the width of the plastic zone  $h$  is about the upper cut-off of the fractal law, i.e.  $h \approx \Delta^*$ , then fracture energy is mainly related to the work done within the non-fractal zone of inelastic deformation of the crack and there is no correlation (Borodich [6]). Analysis of experimental data showed that for metals  $\Delta^* \approx 0.1$  mm (Bouchaud [11]), while  $h \approx 0.4$  mm (Botvina [12]), i.e.,  $\Delta^* \leq h$ . Hence, the fractal properties of fracture surface are not essential for fracture energy of ductile materials. This conclusion is in agreement with experimental data. On the other hand, the experimental studies show (Chelidze [6], Zhao [7]) that for polyphase materials the upper cut-off for fractal law for fracture patterns is  $\Delta^* > 1$  cm, i.e.,  $\Delta^* \gg h$ . Therefore, the

fractal properties of the pattern of mesocracks could characterize the fracture energy of the materials when the fractal properties of the main crack surface are not essential. Thus, the experimental evaluation of the dimension  $C$  of the fracture pattern is more important for fracture mechanics than the evaluation of the dimension  $D$  of the fracture surface. It was found by the Jerusalem group (Avnir [10]) that the overwhelming majority of reported physical fractals span about 1.5 orders of magnitude. In other words, the average ratio of the fractal law cut-offs ( $\Delta^*/\delta^*$ ) is about 31.6.

## 2 MULTIPLE FRACTURE: FRACTURE ENERGY AND SIZE EFFECT

It is assumed in our model that (i) the growth of the process zone results in a continuous growth of the main crack; (ii) the fractal scaling is applicable to describe the beginning of the self-similar growth of the process zone; (iii) while a fracture pattern is already fully developed, i.e., when the width of the process zone reaches some critical size  $w_*$ , the pattern picture is repeated. This means that the active part of the process zone is invariant with respect to continuous shifting. Let us use two systems of coordinates, namely  $(X, y)$  and  $(x, y)$  where  $x \equiv X - l$ . If the width of the sample is  $L$  and the initial notch length is  $l$  then the sample points are situated in the range  $0 \leq X \leq L$ . The process zone is assumed to consist of two parts, namely a wedge-shaped part with some angle  $2\alpha$  at the wedge vertex that is at  $x=0$ , and the second part of the zone is described as a segment of a circle. Hence, if the main crack has the tip at a point  $x = x_0$  then for  $0 \leq x \leq x_0$ , it is wedge-shaped, while it is described by a segment of a circle of radius  $w_c(x_0) = x_0 \sin \alpha$  for  $x \geq x_0$ . It is assumed that the active part of the process zone is situated within the segment that is a physical fractal of dimension  $C$  for scales  $\delta_* = h < R < w_c(x_0)$ . Substituting the fractal number-radius relation  $N(\delta_*) \approx (R/\delta_*)^C$  into eqn (1), one can calculate the average amount of energy  $\langle W \rangle$  absorbed by mesocracks of the process zone

$$\langle W \rangle \propto g_f (w_c/h)^C + const \quad (2)$$

where  $w_c(x)$  is the current width of the zone. The fracture energy  $G_F = dW/dx$ . Hence, differentiating eqn (2), one obtains

$$G_F = \begin{cases} G_f (x/\Delta_*)^{C^*}, & x \leq \Delta_* \\ G_f, & \Delta_* < x \end{cases}, \quad G_f = g_f (1 + C^*) (w_*/h)^{C^*} \sin \alpha / h.$$

The above relations have been derived for unbounded samples. They are in qualitative agreement with experimental results concerning the behaviour of fracture energy of large concrete samples (Bramshuber [13]). However, we assume that the main cause of the size effect is that the process zone cannot be fully developed in a bounded model of a real size construction. Hence, when mesocracks ahead of the tip of the main crack reach the sample boundary, the value of the fracture energy decrease from  $G_F$  to the fracture energy of the ultimate link  $g_f$ . Thus, a large sample of the size  $L^{(F)}$  with a notch of the size  $l^{(F)}$  has fully developed process zone, while the zone width cannot reach the value  $w_*$  in a small model of the size  $L^{(M)}$  with a notch of the size  $l^{(M)}$ ,  $L^{(M)} - l^{(M)} < \Delta_* + w_* = \Delta_* (1 + \sin \alpha)$ . Hence, the growth of the process zone stops at some

size  $w_c < w_*$ , when cracks ahead of the tip of the main crack are reaching the boundary of  $L^{(M)} - l^{(M)} = \Delta_c + w_c = \Delta_c (1 + \sin \alpha)$ .

In the case of nonlinear fracture of an elastic body with a cut that is under a uniaxial tensile stress  $\sigma$  perpendicular to the cut, the fracture energy criterion can be formulated that the elastic energy release rate  $G(\sigma, l) = R(l)$ . Here  $R(l)$  is the  $R$ -curve (the resistance curve) of the sample that is equal to its fracture energy  $G_F$ . The criterion says that if  $G < R$  then the crack does not propagate and if  $G > R$  then the crack is unstable and its propagation is catastrophic. In an elastic solid under a uniaxial tensile stress  $\sigma$ , the energy released after creating a straight-through planar crack perpendicular to the stress direction can be estimated using the following idea: there exists a domain where the stress field relaxes, and employing the principle of independence of released energy of crack trajectory: the total released energy in an elastic solid does not depend on crack trajectories if the crack shape is fixed in some small regions at the crack tips and the trajectory variations are within the domain of the stress field relaxation. It follows from the principle that the total released energy does not change if there are some micro- and mesocracks within the domain of the stress field relaxation. Thus,  $G(\sigma, l)$  is a linear function of the crack length  $l$ . If the line  $G$  is tangential to  $R$  at some length  $l = l_0 + x_c$  then  $G(\sigma, l_0 + x) > R$  for  $x > x_c$  and the crack starts to propagate catastrophically. However, if  $x_c > \Delta_*$  in the full-scale structure then the catastrophic propagation starts at  $x = \Delta_*$ . The catastrophic propagation starts at  $x^{(M)} = \Delta_c$  in a model. Therefore, we can write the following formulae for the fracture stresses  $k(\sigma_f^{(F)})^2 (l^{(F)} + \Delta_*) = G_f$  and  $k(\sigma_f^{(M)})^2 (l^{(M)} + \Delta_c) = G_f (\Delta_c / \Delta_*)^{C^*}$ , where  $k$  is a coefficient and  $\sigma_f^{(F)}$  and  $\sigma_f^{(M)}$  are the fracture stress in the full-size sample and in a small model respectively. Thus, if  $L^{(M)} - l^{(M)} < \Delta_* + w_*$  then we obtain the size effect law of fracture of polyphase samples under tension

$$\sigma_f^{(F)} = \sigma_f^{(M)} \sqrt{\frac{l^{(M)} + \Delta_c}{l^{(F)} + \Delta_*}} \left(\frac{\Delta_*}{\Delta_c}\right)^{C^*/2} = \sigma_f^{(M)} \lambda^{-1/2} \sqrt{\frac{1 + \Delta_c / l^{(M)}}{1 + \Delta_* / l^{(F)}}} \left(\frac{\Delta_*}{\Delta_c}\right)^{C^*/2} \quad (3)$$

where  $\lambda = L^{(F)} / L^{(M)}$ . One can see that eqn (3) depends on both the sample size and the size of the process zone. Eqn (3) allows us to analyse the case when there is no initial notch, i.e. then the process zone starts to develop from an initial flaw.

It is known that scaling methods may be very effective in description of fracture processes (Barenblatt [3], Borodich [4]). In the above considered case, fractal scaling allowed us to bridge scales of multiple fracture. Assuming that the process zone cannot be fully developed in a bounded model, the size effect eqn (3) was obtained.

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