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Development of a Fitness Measure for an Inventory and Production Control System

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Abstract

This paper outlines a method of developing a fitness measure for use in a Genetic Algorithm for assessing the performance of a generic production control system. The performance criteria is based on the ability to recover from inventory variations, the ability to filter out noise, robustness to production delays, robustness to WIP information delays and selectivity.

Nomenclature

APIOBPCS	Automatic Pipeline Inventory and Order Based Production Control System.
Tp	Production Lag Time Constant.
$\overline{Tp}$	Production WIP Gain.
Ti	Inverse of Inventory Based Production Control Law Gain.
Tw	Inverse of WIP Based Production Control Law Gain.
Ta	Consumption Averaging Time Constant.
λ_i	Normalised Parameter = (*Ti/Tp*)
λ_a	Normalised Parameter = (*Ta/Tp*)
s	Laplace Operator, and (S) Normalised Laplace Operator.
ω_N	Noise Bandwidth
ω	Frequency (rads/time period)
CONS	Consumption or Market Demand
AVCON	Average Consumption.
DINV	Desired Inventory Holding
EINV	Error in Inventory Holding
ORATE	Order Rate
COMRATE	Completion Rate
WIP	Work In Progress
DWIP	Desired Work In Progress
EWIP	Error in Work In Progress
AINV	Actual Inventory Holding
ITAE	Integral of Time*Absolute Error.
E	Error
t	Time

Introduction

Production scheduling algorithms are required to ensure high customer service levels whilst at the same time reducing total costs. To enable good customer service levels a minimum reasonable inventory should be maintained but stock holding costs and production on-costs have to be minimised. Therefore the scheduling algorithm needs to be responsive to genuine changes in demand to minimise stock holding costs. Also it is important to attenuate fluctuations in consumption so as to keep a smooth production level, and hence reduce production on-costs.

A common production control algorithm, called the Automatic Pipeline Inventory and Order Based Production Control System (APIOBPCS) can be expressed in words as follows; "Production targets are equal to demand averaged over *Ta* time units, plus a fraction, *Ti*, of the inventory deficit in stores, plus a fraction, *Tw*, of the WIP deficit." It can also be expressed in block diagram form as shown in Figure 1 (John et al 1994).

It can be appreciated that the response of the algorithm to different inputs will be depend on the values of the parameters (*Ta, Ti, Tw*) chosen in the system. It is the aim of this paper to highlight a method to determine how to judge the fitness of the values of these parameters. It is a relatively trivial task, to optimise the algorithm when it is an exact representation of the real world, but this is seldom the case. The robustness of the algorithm to incorrect estimations of production lag and the production lag distribution, will be considered via a production robustness vector. The robustness of the design parameters to the information lags in the WIP feedback loop will also be considered. Finally a selectivity vector will be developed to help identify optimum "ballpark" figures that they are robust to "fine tuning" by real world users. Therefore the optimisation routine will assess the trade off between;

- speed of response

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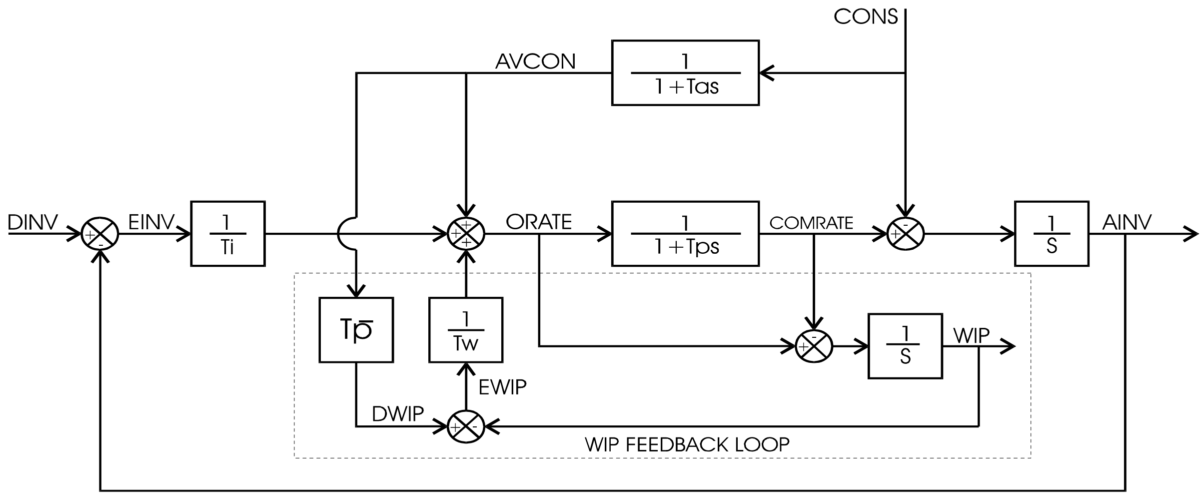

Figure 1. Block Diagram of an Automatic Pipeline, Inventory and Order Based Production Control System

- noise attenuation
- production robustness,
- WIP information lags
- selectivity.

The Transfer Functions

Manipulating the block diagram in Figure 1 for ORATE/SALES and AINV/SALES will produce the following transfer functions (Equation 1 is written in normalised standard notation);

$$\frac{ORATE}{CONS}=\frac{1+\left(1+\lambda_a+\lambda_i+\frac{\overline{\lambda}_p\lambda_i}{\lambda_w}\right)s+\left(\lambda_a+\lambda_i+\frac{\overline{\lambda}_p\lambda_i}{\lambda_w}\right)s^2}{\left(1+\lambda_a s\right)\left[1+\left(\lambda_i+\frac{\overline{\lambda}_p\lambda_i}{\lambda_w}\right)s+\lambda_i s^2\right]} \text{ ....1.}$$

$$\frac{AINV}{CONS}=\frac{Ti\left\{(\overline{T}p+Tw)-(1+Tas)[Tp+Tw(1+Tps)]\right\}}{(1+Tas)[Tw(1+Tis)+TiTps(1+Tws)]} \text{ ....2}$$

The Fitness Measure

Any fitness measure needs to be directly related to the objectives of the algorithm. These are the minimisation of the stock levels and hence stock costs and the attenuation of demand changes and inventory recovery. Considering stock levels, a direct measure of the relative improvement in inventory recovery is the Integral of Time x Absolute Error (ITAE) after a step increase in consumption.

The measure that will be assigned to the relative improvements in the attenuation of random inputs is the noise bandwidth. The noise bandwidth is directly related to the variance of the output when subjected to an input of pure white noise. Therefore the system which is a balanced compromise of minimum ITAE in AINV following a step and minimum noise bandwidth in ORATE, will be a compromise between the two criteria that make up customer service levels.

The Inventory Recovery Vector

The ITAE is generally agreed to be the most intuitive criterion following a step, for assessing inventory recovery, as it is inevitable that a large error is present shortly after the step and it penalises more heavily, errors that are present later, by a suitable weighting in the time domain, (Towill 1970). The ITAE also penalises positive and negative errors equally, and is then the simplest measure that is reliable, applicable and selective, (Graham et al 1953).

The ITAE is defined in Equation 3. Throughout this paper the ITAE was calculated following a step input in CONS that increased from 100 to 200 widgets per time period at time = zero.

$$itae=\int_{\infty}^{0} t\left|E\right|dt \text{ ....3.}$$

The Noise Filtering Vector

The noise bandwidth is defined as the area under the system amplitude ratio squared curve, (Towill 1982). The noise bandwidth is a useful method of condensing frequency domain information into one criteria. The ORATE noise bandwidth is important because it is a measure of the ability of the Sales Averaging (*Ta*), Time to Adjust Inventory (*Ti*), and Time to Adjust WIP (*Tw*), to filter out the higher frequency content of the demand, when setting production targets. The noise bandwidth equation (Garnell and East 1977 and Newton et al 1957) for APIOBPCS for a first order production lag, is shown by Equation 4.

@ j= 1, Production Distribution = 1st Order Lag,
@ j= 2, Production Distribution = 3rd Order Lag.
(See appendix for additional transfer functions used in the production robustness vector.)

The WIP Robustness Vector

To reduce the order of the transfer function of an APIOBPCS model, a first order lag is going to approximate a pure time delay in the WIP feedback loop. This is to reduce the likelihood of errors in the integration procedure used to define the noise bandwidth criteria. The purpose of this criterion is to establish the robustness of the design parameters to possible delays in the WIP feedback loop. Delays such as these may be present due to inaccuracies in the recording of WIP on the shop floor

$$\omega_N = \frac{\dfrac{\left(TpTa + TpTi + \dfrac{Tp\bar{T}pTi}{Tw}\right)^2\left(Ti + \dfrac{TpTi}{Tw} + Ta\right)}{TpTiTa} + \left(Ta + Ti + Tp + \dfrac{\bar{T}pTi}{Tw}\right)^2 - 2\left(TpTa + TpTi + \dfrac{Tp\bar{T}pTi}{Tw}\right) + \left(TpTi + TaTi + \dfrac{TpTiTa}{Tw}\right)}{\left(TpTi + TaTi + \dfrac{TpTiTa}{Tw}\right)\left(Ta + Ti + \dfrac{TpTi}{Tw}\right) - TaTiTp} \quad ....4.$$

The Production Robustness Vector

The production robustness vector is a measure of the robustness of the design parameters, with respect to changes in the production lead-time and distribution. The method used is based on the two vectors outlined above. The robustness vector is a measure of how much the performance alters with respect to ITAE and noise bandwidth for all combinations of production lead-time at 50%, 100%, and 150% of the nominal value (Tp = 8), and a production distribution of first or third order. It is assumed that $\bar{T}p$ =Tp at all times. It is defined by Equation 5.

in the real world. Like the production robustness vector and the selectivity vector the WIP robustness vector is a measure of how much the performance alters in the ITAE and Noise Bandwidth plane for all of the following conditions; No time delay, a first order lag of 4 time units, and 8 time units. It is defined below in Equation 6.
(See appendix for additional transfer function used in the WIP Robustness vector.)
where; wipr = WIP Feedback Delays Robustness,
$itae_i$ = the ITAE for AINV under conditions i,
ω_{Ni} = Noise Bandwidth of ORATE under conditions i,
conditions i;
@ i= 1, no WIP feedback delay,
@ i= 2, there is a first order lag in the WIP feedback loop

$$pr = \frac{\sum_{i=1}^{p}\sum_{j=1}^{q}\sqrt{\left[itea_{ij} - \dfrac{\sum_{i=1}^{p}\sum_{j=1}^{q} itea_{ij}}{pq}\right]^2 + \left[\omega_{Nij} - \dfrac{\sum_{i=1}^{p}\sum_{j=1}^{q}\omega_{Nij}}{pq}\right]^2}}{pq} \quad ,.......5$$

where, pr= Production Robustness Vector,
$itae_{ij}$ is the ITAE for AINV under conditions i and j,

of 4 time units,
@ i= 3, there is a first order lag in the WIP feedback loop

$$wipr = \frac{\sum_{i=1}^{p}\sqrt{\left[itae_i - \dfrac{\sum_{i=1}^{p} itae_i}{p}\right]^2 + \left[\omega_{Ni} - \dfrac{\sum_{i=1}^{p}\omega_{Ni}}{p}\right]^2}}{p} \quad ,...............6$$

ω_{Nij} = Noise Bandwidth of ORATE under conditions i and j;
Conditions i and j;
@ i= 1, Production Lag = 4 time units,
@ i= 2, Production Lag = 8 time units,
@ i= 3, Production Lag = 12 time units,

of 8 time units.
The selectivity vector is a measure of the robustness of a design to arbitrary changes to the values of a design parameters by users of the ordering algorithm. It is particularly useful for determining the terrain in the solution space so that we can recommend an optimum that

$$sv = \frac{\sum_{i=1}^{p}\sqrt{\left[itae_i - \dfrac{\sum_{i=1}^{p} itae_i}{p}\right]^2 + \left[\omega_N - \dfrac{\sum_{i=1}^{p}\omega_N}{p}\right]^2}}{p} \quad ....7$$

is robust, in the sense that minor deviations around it will not degrade performance greatly. i.e. it is not at the top of a peak in the solution space. It is also representative of inaccurate estimations of the systems state, such as inventory levels and WIP levels.

The Selectivity Vector

Like the production robustness vector it is based on the ITAE and Noise Bandwidth plane. It is a measure of how much the performance alters, when each parameter is set at 75%, 100% and 125% the nominal value. It is defined by Equation 7.

where; sv = Selectivity Vector,

$itae_i$ is the ITAE for AINV under conditions i,

and ω_{Ni} = Noise Bandwidth of ORATE under conditions i,

Conditions i;

@ i= 1, $Ta=Ta_{nom}*75\%$, $Ti=Ti_{nom}$, $Tw=Tw_{nom}$.

@ i= 2, $Ta=Ta_{nom}*100\%$, $Ti=Ti_{nom}$, $Tw=Tw_{nom}$.

@ i= 3, $Ta=Ta_{nom}*125\%$, $Ti=Ti_{nom}$, $Tw=Tw_{nom}$.

@ i= 4, $Tw=Tw_{nom}*75\%$, $Ti=Ti_{nom}$, $Ta=Ta_{nom}$.

@ i= 5, $Tw=Tw_{nom}*100\%$, $Ti=Ti_{nom}$, $Ta=Ta_{nom}$

@ i= 6, $Tw=Tw_{nom}*125\%$, $Ti=Ti_{nom}$, $Ta=Ta_{nom}$

@ i= 7, $Ti=Ti_{nom}*125\%$, $Tw=Tw_{nom}$, $Ta=Ta_{nom}$

@ i= 8, $Ti=Ti_{nom}*125\%$, $Tw=Tw_{nom}$, $Ta=Ta_{nom}$

@ i= 9, $Ti=Ti_{nom}*125\%$, $Tw=Tw_{nom}$, $Ta=Ta_{nom}$

The Optimum APIOBPCS Design

The overall score assigned to a set of design parameters (Ta, Ti and Tw) is now given by Equation 8.

The reciprocal has been introduced so that the higher the score the better the dynamic performance of the ordering algorithm.

$$Score = \frac{1}{\sqrt{itae^2 + \omega_N{}^2 + pr^2 + wipr^2 + sv^2}} \quad ....8$$

Genetic Algorithms

The importance of GA's as tools for solving complex optimisation problems is well accepted (Pham et al 1995). They are based on the natural law of evolution of species by natural selection (Caponetto et al 1996). Originally developed by (Holland 1975) they are increasingly being used for a range of production planning type problems from scheduling to assembly line balancing.

GA's have two main areas of application (Everett 1995), the first is the optimisation of the performance of a system, such as traffic lights or a gas distribution pipeline system. They typically depend on the selection of parameters, perhaps within certain constraints, whose interaction restricts a more analytical approach.

The second area of application for GA's is in the field of testing or fitting of quantitative models. In this case the aims of the GA is the minimisation of the error between the model and the data. The controller order reduction problem by Caponetto et al (1996) fits this type of application. This paper is concerned with the first, i.e. an optimisation type application.

Operation of the GA

Let $\mathbf{X}^M=\{\mathbf{x}|x_j^{(L)} <=x<= x_j^{(U)}\}$, (1,<=j<=M) be the search space where M is the dimension of **x** and $x_j^{(L)}$ and $x_j^{(U)}$ is the upper and lower limit of the jth component x_j of vector **x**, respectively.

Let **P**(k)=N binary chromosome structures $\mathbf{s}_i(k)$, (1<=i<=N) in generation k.

Let $f_i(k)$ be the fitness of the ith structure in generation k as defined by *Score* described earlier.

Let $\mathbf{x}_b(k)$ be the best parameter vector with the largest fitness $f_b(k)$.

The GA works in the following way;

Set k=0

Create initial random population **P**(k)

Decode $\mathbf{s}_i(k)$, (1<=i<=N) into $\mathbf{x}_i(k)$

Evaluate fitness $f_i(k)$, (1<=i<=N)

Determine the best $\mathbf{x}_b(k)$ and copy into **P**(k+1)

DO WHILE <termination conditions are not met>

Crossover and mutate **P**(k) to form N-1 chromosome structures and copy into population **P**(k+1)

Decode $\mathbf{s}_i(k+1)$, (1<=i<=N) into $\mathbf{x}_i(k+1)$

Evaluate fitness $f_i(k+1)$, (1<=i<=N)

Determine the best $\mathbf{x}_b(K)$ and copy into **P**(k+1)

Set k=k+1

END WHILE

After convergence the GA was restarted several times to check for true optimum.

In the description of the GA above Tp and $\overline{Tp}$ have been set at 8 time units, the dimension of **x** (M) is 3 (for Ti, Ta and Tw), and the upper and lower limit of each x is 255 and 0 respectively. Thus the binary structures (s_i) are 24 bits long and there were N= 60 binary chromosome structures.

Results

Where all five criteria are given equal importance the genetic algorithm produced the optimum solution such that *Ti* = slightly less then the nominal production lag, *Ta* = twice the nominal production lag and *Tw* = slightly more than thrice the nominal production lag. A more analytical approach by John et al (1994) argued that the optimum solution was to set *Ti* equal to the nominal production lag and *Ta* = *Tw* = twice the production lag.

Conclusion

This paper has outlined a procedure to evaluate the performance of a generic production control algorithm, using a GA. The use of two classical control engineering tools have been used in conjunction with simulation to determine the optimum setting of parameters in the control algorithm, so that the ordering algorithm exhibits certain characteristics, believed to be desirable in a production control system. It is interesting to note that the parameters chosen are very close to those believed to be optimal based on heuristic techniques, (John et al 1994).

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Appendix. Additional Transfer Functions Required

APIOBPCS with a First Order Production Lag, and a First Order Lag in the WIP Feedback Path, ORATE Transfer Function

$$\frac{ORATE}{CONS}=\frac{1+\left(Tq+Ta+Ti+Tp+\frac{Ti\overline{T}p}{Tw}\right)s+\left(TqTa+TiTq+TpTq+TpTa+TpTi+\frac{Ti\overline{T}pTp}{Tw}\right)s^2+\left(TpTqTa+TpTiTq\right)s^3}{1+\left(Ti+Ta+Tq\right)s+\left(TiTq+TiTa+TaTq+TaTi+\frac{TpTi}{Tw}\right)s^2+\left(TiTaTq+TaTiTq+TiTa+\frac{TiTp^2}{Tw}+\frac{TaTpTi}{Tw}\right)s^3+\left(TiTqTa^2+\frac{TaTiTp^2}{Tw}\right)s^4}$$

APIOBPCS with a Third Order Production Lag, ORATE Transfer Function

$$\frac{ORATE}{CONS}=\frac{1+\left(Tp+Ti+\frac{Ti\overline{T}p}{Tw}+Ta\right)s+\left(\frac{Tp^2}{3}+TiTp+\frac{TiTp\overline{T}p}{Tw}+TaTp\right)s^2+\left(\frac{Tp^3}{27}+\frac{TiTp^2}{3}+\frac{Ti\overline{T}pTp^2}{3Tw}+\frac{TaTp^2}{3}\right)s^3+\left(\frac{TiTp^3}{27}+\frac{Ti\overline{T}pTp^3}{27Tw}+\frac{TaTp}{27}\right)s^4}{1+\left(\frac{TpTi}{Tw}+Ti+Ta\right)s+\left(TiTp+\frac{TiTp}{3Tw}+TaTi+\frac{TaTpTi}{Tw}\right)s^2+\left(\frac{TiTp^3}{27Tw}+\frac{TiTp^2}{3}+TaTiTp+\frac{TaTiTp}{3Tw}\right)s^3+\left(\frac{TiTp^3}{27}+\frac{TiTaTp^3}{27Tw}+\frac{TiTaTp^2}{3}\right)s^4+\frac{TiTaTp^3}{27}s^5}$$