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General Equilibrium with Monopolistic Firms and Occasionally Binding Cash-in-Advance Constraints

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Abstract

We show a simple way to introduce monopolistic competition in a general equilibrium model where prices are fully flexible, the velocity of money is variable and cash-in-advance (CIA) constraints occasionally bind.We establish the conditions under which money has real effects and demonstrate that an equilibrium that occurs at a binding CIA constraint is welfare inferior to any equilibrium that occurs at a non-binding CIA constraint with the same level of technology. We argue that even though the probability of a binding CIA constraint can be increasing with money supply, under certain conditions, expansionary money supply is welfare improving.

JEL Classification Codes: D43; E31; E41; E51

Keywords: cash-in-advance; general equilibrium; monopolistic competition

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1 Introduction

In this paper, we introduce and analyze an analytically tractable cash-in-advance (CIA) general equilibrium model populated by risk-neutral monopolists and risk averse workers. We show that the CIA constraint is occasionally binding, depending on both the current state of the economy and the expectation about the future state of the economy. We demonstrate the real effects of money without requiring the presence of other physical assets.[1] We argue that even when money is the only asset in the economy, monetary policy is non-trivial.

Cooley and Hansen (1989), introduce a cash-in-advance (CIA) constraint in a stochastic optimal growth model with capital, endogenous indivisible labor and perfectly competitive markets.[2] Assuming that the CIA constraint always binds, they find that the impact of money on real quantities is small at business cycle frequencies whereas the impact is significantly large in the long-run.[3] While unanticipated inflation has no role, anticipated inflation induces a considerable effect on real variables at the steady state.[4] Note that whether the

[1] As noted by Chamley and Polemarchakis (1984), a standard argument for money non-neutrality in a general equilibrium framework lies on the existence of other real assets. Changes in money supply affect the price level which in turn affects the return of money as an asset relative to the other physical assets. As a result, individuals realign their portfolios and the equilibrium holdings of physical assets change.

[2] Svensson (1985), intoduced money via a CIA constraint in a general equilibrium model where other financial assets are also traded. Due to the absence of physical capital, the equilibrium consumption always equals output which is specified as a stochastic endowment process. In such setting, it is unclear whether output is dependent or independent of monetary expansion. His model is differentiated from that of Lucas (1982) in that consumers decide on their cash balances before they know the current state of nature and hence before they know their consumption. This feature leads to potential variation in the velocity of money as the CIA constraint is sometimes non-binding.

[3] The impact of money on real variables results from the inflation tax. That is, increases in the growth rate of money lead agents to expect higher inflation. The positive inflation tax on the consumption good induces agents to lower work effort and as a result, output and consumption. In other words, the agents substitute away from activities that involve the use of cash (consumption good) in favor of activities that do not require cash (leisure). Among others, Cooley and Hansen (1995, 1997) adopt a similar framework.

[4] In other words, the agents substitute away from activities that involve the use of cash (consumption good) in favor of activities that do not require cash (leisure). Among others, Cooley and Hansen (1995, 1997) adopt a similar framework.

CIA constraint binds or not is independent of the presence of capital. Another strand of the literature focusses on nominal rigidities of one kind or another which result in real effects of monetary policy in the short-run.[5]

In this paper we offer an alternative general equilibrium framework of a CIA economy where output is produced by monopolistic firms. Specifically, risk averse workers supply labor and risk-neutral monopolists set prices and produce output (the labor market is competitive). To keep our analysis simple and tractable and since our objective is to examine the qualitative aspects of money rather than to match features of the data, we abstact from the presence of physical assets such as capital. We allow for a very general set of possibilities about how the velocity of money is determined. We show that velocity has an upper bounded which is decreasing with the elasticity of demand of the consumption good (increasing in the markup of marginal productivity over the real wage). There are two main sources of uncertainty, one associated with random technology innovations and the other with random money transfers. Money tranfers take place at the beginning of the period whereas technology innovations are revealed at the end of the period. The state vector consists of a technology innovation, money balances and a velocity specific-shock. As Cooley and Hansen (1989), suggest in their conclusion (p 746), "... the most important influence of money on short-run fluctuations are likely to stem from the influence of the money supply process on expectations of relative prices". Here, we establish this argument analytically. When particular state vectors occur, the CIA constraint binds because the agents expect that the value of money will decrease next period (i.e the expected discounted relative price will rise). As a result, they rush to

[5]This is the case in the neoclassical synthesis framework (e.g. Don Patinkin 1956) and also the *new* neoclassical synthesis (e.g. Woodford 2003).

spend all their money holdings the current period which leads to an increase in the velocity of money to the extent that it hits its upper bound. We demonstrate that in this case, there is a unique equilibrium where money induces real effects: equilibrium output, consumption, work effort and real profits are functions of money balances as well as the expectations for future money transfers, technology innovations and velocity-specific shocks. This result does not require any sort of nominal rigidity or the presence of other physical assets.

When the expected value of money equals its current value (i.e the expected discounted relative price remains unchanged), the CIA constraint does not bind because the consumers are indifferent between spending a unit of money today and holding it for one period. We demonstrate that in this case there is a unique equilibrium where money does not affect real quantities. In other words, real variables are driven only by current technology innovations, whereas money transfers and velocity-specific shocks only affect the price level. Unlike the case of a binding CIA constraint, the price level does not depend on expectations about future technology innovations and money transfers. We also show that the equilibrium at a binding CIA constraint is inferior (in terms of current welfare) to any equilibrium with the same level of technology that occurs at a non-binding CIA constraint.

The problem of the monetary authority is not modeled explicitly and money transfers are treated as random variables (with a known distribution) by firm owners and consumers. For illustrative purposes we assume that the velocity of circulation is an increasing function of technology and money transfers. Then, an increase of money supply increases the probability of a binding CIA constraint. We argue that the monetary authority would not necessarily avoid expansionary money supply because, as we show, there are cases where it might be welfare improving. When the monetary authority decides the transfer of money, neither the technology innovation nor the velocity-specific shock are known. Therefore, the transfer may be optimal ex-ante based on current information and expectations but not optimal ex post, after technology and velocity shocks are revealed.

The rest of the paper is organized as follows. In section 2, we describe the economic environment which includes the problem of the firms, the problem of the workers and the analysis of the equilibrium conditions. In section 3 we provide an example of welfare improving expansionary money supply while conclusions are presented in section 4.

2 Model Economy

The economy is populated by risk averse workers and monopolistic firms which are owned by risk-neutral entrepreuneurs. There are incomplete financial markets which mean that there is no source of insurance for workers. There is a perfectly competitive labor market and a goods market where the workers and the firms trade labor services and the final good. The agents exchange goods and labor services using cash which is the only medium of exchange. As the quantity theory of money indicates, at the aggregate level, nominal output varies with the nominal money balances times its velocity that is,

$$P_t y_t \equiv \overline{M}_t q_t \tag{1}$$

where P_t is the aggregate price level, y_t is aggregate real output, $\overline{M}_t$ is the total quantity of money and q_t is the velocity of money. This is an identity which we can think of as defining the velocity q_t. The velocity of money is not a choice variable of a single agent but it is rather determined at the aggregate level.

2.1 Firms

There is a number $n > 1$ of firms, each producing a good $x_i \geq 0$, $i = 1, ..., n$. The firms are managed by risk-neutral entrepreneurs that consume the firms' profits. The price for good i is denoted by $p(x_i)$, where $p'(\cdot) \leq 0$ and $p''(\cdot) \leq 0$, while the n-vector of prices is denoted by $\mathbf{p} \in \Re_+^n$ and general price level P: $\Re_+^n \rightarrow \Re_+$. The firm produces output by employing a fixed number $m \geq 1$ of workers, each providing h_i hours of work, via technology $x_i(h_i; m, \theta_i)$, where $x_i'(\cdot) > 0$ and $x_i''(\cdot) \leq 0$. $\theta_i > 0$ is an exogenous productivity shock. The latter is distributed according to the conditional p.d.f. $\vartheta\left(\widetilde{\theta}; \theta'\right)$ for $\widetilde{\theta} \in \Theta \subset \Re_+$ where the θ' denotes the previous period realized value of θ. The objective function of firm i can be written as

$$\Pi_i = p(x_i)\, x_i - Pwmh_i \tag{2}$$

where Π_i are profits and w is the real hourly wage rate. The problem of the firm is to maximize its profits by choosing hours, taking as given the aggregate price level and the real hourly wage rate. The necessary and sufficient condition for profit maximization is

$$x_i'\left[p'(x_i)\, x_i + p(x_i)\right] = Pwm \tag{3}$$

In our analysis, we assume that firms employ the same technology and thereby, $\theta_i = \theta$. To obtain an analytical solution we assume that the technology is described by the linear production function $x_i(h_i; m, \theta) = \theta m h_i$. The inverse demand for good i is also linear and of the form

$$p_i = A - Bx_i$$

where $A > 0$ and $B > 0$. The demand function is a special case generated from a class of linear- homothetic (LH) preferences[6]. The aggregate price level is defined as

$$P(\mathbf{p}) = \mu + \gamma(\mu - s) \tag{4}$$

$$\mu = \frac{\sum_{i=1}^{n} p_i}{n} \ , \ s = \left(\frac{\sum_{i=1}^{n} p_i^2}{n} \right)^{\frac{1}{2}}$$

where $\gamma > 1$ (γ is the absolute value of the elasticity of demand when prices are equal). Notice that (4) implies that when all prices are equal then, $p_i = s = \mu = P$. Coefficients A and B correspond to

$$A = \frac{1+\gamma}{\gamma} s \ \ , \ \ B = \frac{snP}{\gamma Y}$$

where Y denotes the economy's total nominal expenditure. Hence we can solve (3) for labor demand function, nominal price and profits:

$$h = \frac{1}{2Bm\theta}\left[A - P\frac{w}{\theta}\right] \tag{5}$$

$$p_i = \frac{1}{2}\left[A + \frac{w}{\theta}P\right] \tag{6}$$

$$\Pi_i = \frac{1}{4B}\left[A - P\frac{w}{\theta}\right]^2 \tag{7}$$

[6] See Datta and Dixon (2000, 2001).

Since firms face the same technology shock θ and there are no frictions in the economy, the equilibrium will be symmetric. In other words, in equilibrium all firms will set their price equal to P. Thus, the individual product demand reduces to an expression of aggregate real output, $y = nx$, where $y = Y/P$. Then, condition (6) reduces to

$$w = \frac{\gamma - 1}{\gamma}\theta \tag{8}$$

(8) is the simple condition that the real wage is the marginal product of labor times the inverse of the markup. Condition (5) reduces to

$$h^d = \frac{y}{nm\theta} \tag{9}$$

where h^d denotes labor demand. Finally, real profits reduce to

$$\pi = \frac{y}{\gamma n} \tag{10}$$

where $\pi = \Pi/P$.

2.2 Worker-Consumers

Time is discrete and infinite, $t \in \mathbb{Z}_+ = \{1, 2...\infty\}$. There are ($n$ x m) identical worker-consumers with preferences over leisure, l, and consumption, c. The utility function is given by $u(c_t, l_t) = \ln c_t + \phi \ln l_t$ where $\phi > 0$. Each worker-consumer is endowed with one unit of time which is split between work and leisure that is, $l + h = 1$.

The consumer's wealth constraint is given by

$$M_{t+1}^c + P_t c_t = M_t^c + \nu_t + P_t w_t h_t \tag{11}$$

where M^c are the consumer's nominal money holdings and ν is a money increase or decrease such that $M^c > |\nu|$. The transfer ν_t is made at the end of period $t-1$ and before θ_t is realized. It takes a while for the transfer to be completed but the timing is such that the money is availabe at the beginning of the period. Consumers treat ν as a random variable that is distributed according to $\xi\left(\widetilde{\nu};\nu'\right)$ for $\widetilde{\nu} \in N$ where ν' denotes the previous period transfer and $N = \{\widetilde{\nu} \in \Re : \nu + M^c > 0\}$. The consumer receives her labor earnings at the end of the period but purchases consumption at the beginning of the period. As a result, she faces a cash-in-advance constraint:

$$P_t c_t \leq M_t^c + \nu_t \tag{12}$$

The problem of the consumer is to choose consumption, labor supply and money balances to maximize utility subject to the budget contraint and the CIA constraint. We will say that the CIA is binding whenever $P_t c_t = M_t^c + \nu_t$. It is *weakly binding* when the household does not wish to consume more; it is *strictly binding* when the household is constrained to consume less than it would like to in the absence of the CIA.

The Lagrangian function, $\mathcal{L}$, associated with the consumer's problem is the following:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t E_t \left\{ u\left(c_t, l_t\right) - \lambda_{1t} \left[M_{t+1}^c + P_t c_t - M_t^c - \nu_t - P_t w_t h_t \right] \right.$$
$$\left. -\lambda_{2t} \left[P_t c_t - M_t^c - \nu_t \right] \right\}$$

where β is the discount factor, λ_{it} is the shadow price of the standard budget constraint and λ_{2t} is the shadow price of the CIA constraint.

This yields the following necessary and sufficient first-order conditions:

$$u_c\left(c_t, l_t\right) = \lambda_{1t} P_t + \lambda_{2t} P_t \tag{13}$$

$$u_l\left(c_t, l_t\right) = \lambda_{1t} P_t w_t \tag{14}$$

$$\lambda_{1t} = \beta E_t \left\{ \lambda_{1t+1} + \lambda_{2t+1} \right\} \tag{15}$$

Notice that in equilibrium, $M_t^c = M_t$. Combining (13), (14) and (15) yields

$$\frac{u_l\left(c_t, l_t\right)}{w_t} = \beta E_t \left\{ \frac{u_c\left(c_{t+1}, l_{t+1}\right)}{1 + \inf_{t+1}} \right\}$$

where $\inf_t$ denotes the inflation rate in period t. If the CIA constraint does not bind or is only weakly binding in period t ($\lambda_{2t} = 0$), the left-hand side of the above condition is also equal to the marginal utility of consumption, which implies that the marginal benefit of work will equal the marginal cost of work, i.e $u_c\left(c_t, l_t\right)\, w_t = u_l\left(c_t, l_t\right)$. On the other hand, if the CIA constraint is strictly binding ($\lambda_{2t} > 0$) then the marginal benefit of work will be greater than the marginal cost of work, i.e $u_c\left(c_t, l_t\right)\, w_t > u_l\left(c_t, l_t\right)$. Using the fact that utility is separable in consumption and leisure, it is straightforward to show that

$$E_t\left[\left(\frac{\beta u_c\left(c_{t+1}\right)}{u_c\left(c_t\right)}\right)\left(\frac{1}{1+\inf_{t+1}}\right)\right]\begin{cases} <1, \text{ binding CIA constraint} \\ =1, \text{ nonbinding CIA constraint}\end{cases} \tag{16}$$

The term $\left[1/\left(1+\inf_{t+1}\right)\right]$ is the gross return of money, $R_{t+1}^M \equiv 1+r_{t+1}^M$.[7] The left hand side of the above condition can also be written as $E_t\left[\psi_{t+1}R_{t+1}^M\right]$, where ψ_{t+1} is the stochastic discount factor or pricing kernel which is equal to the intertemporal rate of substiturion (IRS) between next period consumption and current consumption. The term on the left hand side of (16) is the expected return of money measured in next period's utility per unit of current utility. When the expected return of a unit of money measured in next period's utility units is the same as the value of a unit of money measured in current utility units (i.e $E_t\left[\psi_{t+1}R_{t+1}^M\right]=1$), the CIA contraint does not bind because the agents are indifferent between spending a unit of money today and holding it for one period. On the other hand, when the expected return of a unit of money, measured in next period's utility units, is smaller than the current utility value of a unit of money (i.e $E_t\left[\psi_{t+1}R_{t+1}^M\right]<1$), the CIA constraint binds because agents are not indifferent between spending a unit of money today and holding it for one period; they strongly prefer to spend it today.

Dividing (13) over (14) and using (3) results to:

$$\frac{u_l\left(c_t,l_t\right)}{u_c\left(c_t,l_t\right)}=\frac{\lambda_{1t}}{\lambda_{1t}+\lambda_{2t}}\frac{x_{it}'\left[p'\left(x_{it}\right)x_{it}+p\left(x_{it}\right)\right]}{P_t m} \tag{17}$$

[7] Note that $r^M=-\inf/\left(1+\inf\right)$ is non-positive as long as inflation is strictly non-negative.

or

$$\phi\frac{c_t}{l_t} = \frac{\lambda_{1t}}{\lambda_{1t}+\lambda_{2t}}\frac{\gamma-1}{\gamma}\theta_t \tag{18}$$

When $\lambda_{2t} = 0$ and the CIA constraint for that perod is not binding, this is the usual intra-temporal condition which states that the marginal rate substitution (MRS) between leisure and consumption equals marginal productivity times the inverse of the markup. However, when the CIA constraint is binding and $\lambda_{2t} > 0$, the MRS is lower in the case of a nonbinding CIA constraint, which implies that consumption is less and/or the labor supply larger.

2.3 Equilibrium

There is a sequence of productivity levels and money supplies $\{\theta_t, M_t\}_{t=1}^{\infty}$ that evolve according to ϑ and ξ and the initial conditions $\{\theta_1, M_1\}$. Whilst we have treated q_t as given at the household level, we now need to define the aggrgegate relationship which determines the velocity of circulation:

Assumption *Let us define a velocity shock φ_t which has an initial condition φ_1 and the conditional p.d.f. $\bar{\Phi}(\widetilde{\varphi};\varphi')$ for $\widetilde{\varphi} \in \Phi \subset \Re_+$ where φ' denotes the previous period realized value of φ. The velocity of circulation is determined by the mapping: $q_t \in Q_t$: $\mathbb{Z}_+$ x Φ x N x $\Theta \rightarrow (0, q^b]$* which we can write as $q_t = q(t, \theta_t, \varphi_t, \nu_t)$.

Thus we allow for a very general set of possibilities about how the velocity is determined: there is a general function (which may be time specific) which relates the velocity q_t to the two shocks determining θ_t, M_t as well as a possible velocity shock. The assumption allows for the velocity to be constant, or to be a decreasing or increasing in its arguments and there is no requirement for smoothness or differentiability. An equilibrium consists of a sequence pairs of $\{w_t, P_t\}_{t=1}^{\infty}$ that clear the labor and the goods market (notice that w is the *real wage* and P is the *nominal price* of output) given $\{\varphi_t, \theta_t, M_t\}_{t=1}^{\infty}$. Associated with $\{w_t, P_t, \theta_t, \varphi_t, \upsilon_t\}_{t=1}^{\infty}$ are the sequences $\{q_t, \lambda_{1t}, \lambda_{2t}, y_t, c_t, h_t, \pi_t\}_{t=1}^{\infty}$.

We can characterize the equilibrium sequence by dividing it into two possible states: one where the CIA is binding, and one where it is not. Of course, how this divides up will depend on the sequence of productivity and monetary shocks. The two extremes are that the CIA constraint is always binding (as in Cooley and Hansen 1989), or never binding. The following propositions allow us to determine how the economy behaves in the case of an intermittently binding CIA.

For all t, the real wage is related to the current productivity level by the markup equation (8), $w_t = \frac{\gamma-1}{\gamma}\theta_t$. The nominal price P_t thus becomes the key variable for establishing equilibrium in each period. A useful way to sort the sequence into binding and non-binding is to note that there is an upper bound to the velocity of circulation: the CIA constraint binds only when this upper bound is reached.

Proposition 1 *For all t there is an upper bound $q^b = \gamma/(\gamma-1)$ on the equilibrium q_t. The CIA contraint binds at time t when $q_t = q^b$ and it does not bind at time t when $q_t < q^b$.*[8]

[8] Recall that whether the CIA constraint binds or not depends on the expectation about next period's relative value of money (condition (16)). This expectation is conditional on the current state of the economy.

All proofs are in the appendix. This enables us to partition time into two sets: times when the CIA is binding, and times when it is not binding:

$$\mathcal{B} = \left\{t \in \mathbb{Z}_+ : q_t = q^b\right\};\ \mathcal{NB} = \left\{t \in \mathbb{Z}_+ : q_t < q^b\right\}$$

Note that whenever $t \in \mathcal{NB}$, $\lambda_{2t} = 0$. However, for $t \in \mathcal{B}$, $\lambda_{2t} \geq 0$. Clearly from a macroeconomic point of view, we are mostly interested in the case where $t \in \mathcal{B}$ and $\lambda_{2t} > 0$. Hence, we can define the subset of $\mathcal{B}$

$$\mathcal{SB} = \left\{t \in \mathbb{Z}_+ : q_t = q^b \text{ and } \lambda_{2t} > 0\right\}$$

Notice that the upper bound of q_t is decreasing with the elasticity of demand of the consumption good (i.e $dq^b/d\gamma = -1/\left(\gamma - 1\right)^2 < 0$); equivalently, it is increasing with the markup of marginal productivity over the real wage. Now, we can define the *proportion* of periods in which the CIA constraint is binding. If we define

$$\mathcal{B}(T) = \left\{t \in \{1, 2...T\} : q_t = q^b\right\}$$

and likewise $\mathcal{NB}(T)$ and $\mathcal{SB}(T)$. For any $T \geq 1$ we can define the proportion of times the CIA constraint binds:

$$\mathcal{P}(\mathcal{B}, T) = \frac{\#\mathcal{B}(T)}{T}$$

The stationarity of the conditional distributions of θ, ν and φ is sufficient to ensure that $\lim_{T\to\infty} \mathcal{P}(\mathcal{B}, T) = \varkappa$, where $\varkappa \in [0, 1]$.

Proposition 2 *When the CIA constraint binds* ($t \in \mathcal{SB}$) *there is a unique equilibrium where*

$$P_t = (1+\chi_t)\, q^b \left[\frac{M_t+\nu_t}{\theta_t}\right] \textit{ with } \chi_t = \frac{\phi}{Z_t(M_t+\nu_t)} > \phi > 0 \textit{ and } Z_t = \beta E_t \left\{\lambda_{1t+1} + \lambda_{2t+1}\right\}.$$

Proposition 2 implies that equilibrium output, consumption, work effort and real profits are functions of technology innovations, the level of money supply, money tranfers and expectations for next period's technology and velocity-specific innovations and money transfers:

$$y_t = \frac{nm}{1+\chi_t}\theta_t,\ c_t = \frac{(\gamma-1)}{\gamma(1+\chi_t)}\theta_t, h_t = \frac{1}{1+\chi_t},\quad \pi_t = \frac{m}{\gamma(1+\chi_t)}\theta_t$$

Corollary 1 *The CIA constraint strictly binds at time* t *when* $Z_t(M_t+\nu_t) = \phi/\chi_t < 1$ *and does not bind when* $Z_t(M_t+\nu_t) = q^b/q_t > 1$.

Corollary 1 also indicates that the velocity of circulation is related to the expectations about the future state of the economy via Z since $q_t = q^b/Z_t(M_t+\nu_t)$.

Proposition 3 For a given $\{\theta_t, M_t, \varphi_t\}_{t=1}^{\infty}$, consider γ_1 and γ_2 with corresponding sequences of equilibria and $\mathcal{B}_1(T)$ and $\mathcal{B}_2(T)$ for $\infty > T \geq 1$. If $\gamma_1 > \gamma_2$ then $\mathcal{B}_2(T) \subseteq \mathcal{B}_1(T)$.

As the market becomes more competitive (as $\lim_{\gamma\longrightarrow\infty} \frac{\gamma}{\gamma-1} = 1$), it is "more likely" that the CIA constraint will bind (or certainly no less likely). It needs to be stressed that Proposition 3 does not imply that in a perfectly competitive market the CIA constraint will always bind. Whilst it is possible that the CIA constraint will be binding all the time[9] and $\mathcal{NB} = \varnothing$, it is perfectly possible that in the competitive case the CIA constraint never

[9]Cooley and Hansen (1989), assume that the consumption good is traded in a perfectly competitive market. They establish the condition under which the CIA constraint binds and they assume that this condition is met at all times. This condition is a version of the condition established in corollary 1.

(strictly) binds (for example, if θ_t and M_t are constant) and hence $\mathcal{SB} = \varnothing$. However, what is clear from the proof of proposition 3 is that for some $\{\theta_t,\ M_t,\ \varphi_t\}_{t=1}^{\infty}$ and for some pairs (γ_1, γ_2), and some T, $\mathcal{B}_2(T) \subset \mathcal{B}_1(T)$.

Proposition 4 *When the CIA constraint does not bind ($t \in \mathcal{NB}$) there is a unique equilibrium where $P_t = (1+\phi)\, q_t \left[\frac{M_t+\nu_t}{\theta_t}\right]$ with $q_t < q^b$ and $\phi > \chi_t$.*

Corollary 2 *When the CIA constraint is non-binding money does not affect real quantities.*

Corrolary 2 implies that equilibrium real output, real consumption, work effort and real profits are functions of only technology innovations:

$$y_t = \frac{nm}{1+\phi}\theta_t,\ \ c_t = \frac{\gamma - 1}{\gamma\,(1+\phi)}\theta_t,\ \ h_t = \frac{1}{(1+\phi)},\ \ \pi_t = \frac{m}{\gamma\,(1+\phi)}\theta_t$$

Note that when the CIA constraint does not bind, changes in money supply affect only the price level and as proposition 4 and corrolary 2 indicate, expectations have no effect on either real quantities or the price level.

Proposition 5 *For any $t_1 \in \mathcal{SB}$ and any $t_2 \in \mathcal{NB}$* such that $\theta_{t_1} = \theta_{t_2}$, *$u(\theta_{t_2}) > u(\theta_{t_1})$ and $\pi(\theta_{t_2}) > \pi(\theta_{t_1})$.*

The above proposition states that an equilibrium that occurs at a binding CIA constraint is inferior, in terms of the current welfare, to any equilibrium that occurs at a non-binding CIA constraint with the same level of technology. Notice that changes in the monopoly power, do not directly affect aggregate production as output is not a function of γ. Nevertheless, changes in monopoly power, change the distribution of aggregate consumption; as the monopoly decreases (i.e γ increases), the consumption of the workers increases while the real profits decrease. Then, for time-invariant velocity function and probability distributions for θ, ν and φ, it becomes more likely that the CIA constraint will bind (proposition 3) which implies that it becomes more likely that both workers and entrepreneurs will be worse-off (proposition 5).

The intuition behind the real effect of money is that sometimes the CIA constraint binds. When this happens, the consumer faces a trade-off between consuming less now and holding more cash in order to benefit in the future. This decision is inherently inter-temporal: it derives from (15) in which equates the current shadow price of money to its expected value next period. To make matters concrete, for illustrative purposes, let us assume that the velocity of circulation is an increasing function of θ and ν[10]. For a massive monetary expansion or a substantial technology improvement or a combination of the two, the CIA constraint will then bind because the agents expect that the value of money next period will be smaller than the value of money the current period (see condition (16)). As a result, they rush to spend all their money holdings the current period which increases the velocity of money to the extent that it hits its upper bound. It follows that the equilibrium output, consumption, work effort and profits, all depend on the current money supply as well as expectations for future money transfers and technology innovations.

In general, a higher level of technology would imply a higher welfare. In addition, for any given technology level, a binding CIA constraint implies a lower welfare than a nonbinding CIA constraint (proposition 5). A higher level of technology would also imply a higher

[10] This is a special case of a velocity function where $q_t = q(\underset{+}{\theta_t}, \underset{+}{\nu_t})$.

probability of a binding CIA contraint (under our illustrative assumption). If the CIA constraint binds, larger money transfers will, in general, increase the welfare. The monetary effect on real quantities comes through variable χ. The smaller χ is the higher the welfare of both consumers and firm owners. There are two channels through which money transfers can affect χ, a direct channel in which there is a negative relationship between ν and χ, and an 'indirect' channer (through Z) in which the direction of the relationship is not obvious. The latter depends on the conditional probability distributions of ν, θ and φ. Assuming that the direct effect of ν on χ dominates the indirect effect, an increase in the supply of money decreases χ and thereby, increases welfare along a binding CIA constraint.

Note that when the monetary authority decides the transfer ν_t, the values of θ_t and φ_t are not known. For a given technology innovation and velocity-specific shock the monetary authority can increase the likelihood of a binding CIA constraint by transfering a large amount of money to the agents. A binding CIA constraint can occur even with moderate levels of technology. If such a case occurs then, according to proposition 5, the welfare for both firm owners and consumers will deteriorate.[11] The monetary authority cannot entirely prevent the CIA constraint from binding because the condition that determines a binding CIA constraint does not depend only on ν but also on θ and φ, which are not under the control of the monetrary authority. One may argue that the monetary authority should keep money supply constant, making zero transfers, in order to decrease the likelihood of a binding CIA constraint. Variation in the supply of money however does not necessarily make the consumers worse off. As mentioned above, there might be values of ν (within

[11] If the CIA constraint did not bind utility and real profits would have been higher at the same level of technology.

the set of equilibria with binding CIA constraints) that make the agents better off. In the absence of velocity shocks, if there was no time lag between the decision of the transfer and the realization of technology innovation then the monetary authority could have made appropriate transfers so that the agents achieve the highest level of welfare for any realization of θ.

3 Welfare Improving Expansionary Money Supply: Example

For simplicity, we abstract from velocity-specific shocks and assume that velocity is time invariant (i.e $q_t \in Q_t$: N x $\Theta \rightarrow (0, q^b])$. Let $\boldsymbol{\theta} = [\theta_1,\, \theta_2,\, \theta_3]' \in \Re^3_+$ and $\boldsymbol{\nu} = [\nu_1,\, \upsilon_2,\, \nu_3] \in N^3$ be vectors containing the possible values of θ and ν, respectively. Specifically, $\theta_1 < \theta_2 < \theta_3$ and $\nu_1 < \upsilon_2 < \nu_3$. The 3 x 3 transition matrices of $\boldsymbol{\theta}$ and $\boldsymbol{\nu}$ are denoted by $\boldsymbol{\vartheta}$ and ξ, respectively. Consider the following case:[12]

state	$\theta_3,\, \nu_1$	$\theta_3,\, \nu_2$	$\theta_3,\, \nu_3$	$\theta_2,\, \nu_1$	$\theta_2,\, \nu_2$	$\theta_2,\, \nu_3$	$\theta_1,\, \nu_1$	$\theta_1,\, \nu_2$	$\theta_1,\, \nu_3$
CIA const. binds	yes	yes	yes	no	no	yes	no	no	no

Notice that having a high ν increases the likelihood of a binding CIA constraint which is a welfare inferior outcome as regards to the current welfare of the agents for any given level of technology. Nevertheless, the monetary authority will not necessarily choose a low

[12]Suppose the economy is at state (θ_k, ν_f) then, $Z_t\,(\theta_k, \nu_f) = \beta \sum_{j=1}^{3} \sum_{i=1}^{3} \vartheta_{kj} \xi_{fj} \frac{q^b}{q(\theta_i, \nu_j)} \frac{1}{M_t + \upsilon_f + \nu_j}$, where ϑ_{ij} and ξ_{ij} are the ith, jth elements of matrices $\boldsymbol{\vartheta}$ and ξ, respectively. Note also that q_t is such that $q_t = q^b / Z_t(M_t + \nu_t) < 1$ when the CIA constraint binds and $q_t = q^b > 1$ when the CIA constraint does not bind.

value of ν in order to decrease the probability of a binding CIA constraint. If θ_3 occurs the CIA constraint will bind no matter what ν is. Then, it could be the case that among the binding CIA-contraint equilibria, $\chi(\theta_3, \nu_3) < \chi(\theta_3, \nu_1)$ where $\nu_1 < \nu_3$. The latter implies that both current profits and current utility are higher under ν_3 than under ν_1; as shown in the proof of proposition 5, $du/d\chi < 0$ and $d\pi/d\chi < 0$. Whether $\chi(\theta_3, \nu_3)$ is smaller than $\chi(\theta_3, \nu_1)$ depends on the expectations about the future state of the economy. Consequently, there might be a senario where there is a trade off between choosing a low value of ν that reduces the probability of a binding CIA constraint and a high value of ν that increases current welfare among binding CIA-contraint equilibria.

4 Conclusion

The paper lays out a simple framework in a general equilibrium model with money where the consumption good is produced by monopolistic firms via labor services provided by risk-averse workers. As in Lucas (1982) and Svensson (1985), money is introduced by means of a cash-in-advance constraint. Within this framework, we demonstrate that money is a liquidity vehicle which can have real effects on the economy without requiring the presence of other real assets or any sort of price rigidity. We allow for a very general function for the velocity of money which depends on the current state of the economy. When consumers expect that the value of money will decrease, they rush to spend all their money holdings. Then velocity reaches its maximum value and the cash-in-advance constraint binds. In this case, both the current variation of money supply as well as the expected variation of money supply, expected technology innovations and velocity-specific shocks affect real variables. When consumers expect that the value of money will remain unchanged, the cash-in-advance constraint does not bind as consumers do not spend all their money holdings. In this case, real variables are driven only by the current technology innovation and money supply variation affects only the price level. We show that a binding cash-in-advance constraint is a welfare inferior outcome for both the workers and firm owners as it delivers lower current utility and lower current real profits for any given level of technology. We also argue that even though the monetary authority can increase the probability of a binding CIA constraint by increasing money supply, expansionary monetary policy can be welfare improving.

From a more methodological point of view, we have shown a simple way to introduce monopolistic competition in a general equilibrium monetary model with divisible labor. The model can be easily extented in various ways to incorporate other real and nominal assets.

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Appendix: Proofs

Proof of proposition 1. Suppose the CIA constraint binds. Then, the resource constraint becomes

$$y_t = \underbrace{nm\left[\frac{M_t}{P_t} + \frac{\nu_t}{P_t}\right]}_{C_{WORKERS}} + \underbrace{\frac{1}{\gamma} y_t}_{C_{ENTREPREUNEURS}}$$

which is equivalent to the quantity theory of money equation, $P_t y_t = q^b \overline{M}_t$, where $q^b \equiv \gamma / (\gamma - 1)$.

Next, suppose the CIA constraint does not bind; then, $\lambda_{2t} = 0$. Substituting out $P_t c_t$ from (13) using (11), λ_{1t} from (14) using (13), w_t from (14) using (8) and imposing the equilibrium condition $h_t^s = h_t^d$ we obtain

$$M_{t+1} = \frac{(1+\phi)(\gamma - 1)}{\phi \gamma n m} Y_t - \frac{(\gamma - 1) P_t}{\gamma \phi} \theta_t + [M_t + \nu_t]$$

Using the worker's budget constraint the equilibrium consumption can be written as a linear combination of productivity and real expenditures:

$$c_t = \frac{\gamma - 1}{\gamma \phi} \theta_t - \frac{\gamma - 1}{n m \phi \gamma} y_t$$

It follows that the resource constraint becomes

$$y_t = \underbrace{\frac{nm(\gamma - 1)}{\gamma \phi} \theta_t - \frac{(\gamma - 1)}{\phi \gamma} y_t}_{C_{WORKERS}} + \underbrace{\frac{1}{\gamma} y_t}_{C_{ENTREPRENEURS}}$$

The latter and the quantity theory of money equation imply

$$y_t = \frac{nm}{1+\phi}\theta_t \quad , \quad c_t = \frac{\gamma - 1}{\gamma (1+\phi)}\theta_t \ \text{ and } \ P_t = (1+\phi)\, q_t^{nb} \left[\frac{M_t + \nu_t}{\theta_t} \right]$$

Then, since $0 < P_t c_t < [M_t + \nu_t]$, it must be the case that

$$0 < \frac{\gamma - 1}{\gamma} [M_t + \nu_t]\, q_t^{nb} < [M_t + \nu_t]$$

which holds only if $0 < q_t^{nb} < \gamma / (\gamma - 1) \equiv q^b$. ■

Proof of proposition 2. When the CIA contraint binds, equations (8), (9), (13) and (14) imply

$$\lambda_{1t} = \frac{\phi \gamma nm}{(\gamma - 1)\, P_t \left[nm\theta_t - y_t \right]} > 0$$

$$\lambda_{2t} = \frac{(\gamma - 1)\left[nm\theta_t - y_t \right] - nm\phi\gamma \left[\frac{M_t}{P_t} + \frac{\nu_t}{P_t} \right]}{(\gamma - 1)\left[M_t + \nu_t \right]\left[nm\theta_t - y_t \right]} > 0$$

Recall that

$$y_t = \frac{\gamma}{\gamma - 1} nm \left[\frac{M_t}{P_t} + \frac{\nu_t}{P_t} \right] \qquad \text{(A.1)}$$

which can be rewritten as

$$P_t y_t = q^b \overline{M}_t$$

Since $\lambda_{1t} > 0$ and given $(A.1)$, it follows that

$$P_t > q^b \left[\frac{M_t + \nu_t}{\theta_t} \right] \qquad \text{(A.2)}$$

Likewise, since $\lambda_{2t} > 0$ it follows that

$$P_t > (1+\phi)\, q^b \left[\frac{M_t + \nu_t}{\theta_t}\right] \tag{A.3}$$

Then, $(A.2)$ and $(A.3)$ imply that

$$P_t = (1+\chi_t)\, q^b \left[\frac{M_t + \nu_t}{\theta_t}\right] \text{ where } \chi_t > \phi > 0$$

Using the latter we can express equilibrium real output, real consumption, work effort and real profits as functions of θ_t, χ_t and parameters:

$$y_t = \frac{nm}{1+\chi_t}\theta_t,\ c_t = \frac{\gamma - 1}{\gamma(1+\chi_t)}\theta_t,\ \ h_t = \frac{1}{1+\chi_t},\ \ \pi_t = \frac{m}{\gamma(1+\chi_t)}\theta_t$$

It is straightforward to show that variable χ_t takes a unique value. Recall the euler condition $\lambda_{1t} = \beta E_t \{\lambda_{1t+1} + \lambda_{2t+1}\}$ where

$$\{\lambda_{1t+1} + \lambda_{2t+1}\} = \begin{cases} \frac{\gamma}{(\gamma-1)(M_{t+1}+\nu_{t+1})q_{t+1}} & \text{for } q_{t+1} < q^b \\ \frac{1}{M_{t+1}+\nu_{t+1}} & \text{otherwise} \end{cases}$$

Therefore, given the probability distributions for θ and ν, the expectation $E_t \{\lambda_{1t+1} + \lambda_{2t+1}\}$ is well defined. For notational convenience let $Z_t = \beta E_t \{\lambda_{1t+1} + \lambda_{2t+1}\}$. Since

$$\lambda_{1t} = \begin{cases} \frac{\gamma}{(\gamma-1)(M_t+\nu_t)q_t} & \text{for } q_t < q^b \\ \frac{\phi}{\chi_t(M_t+\nu_t)} & \text{otherwise} \end{cases}$$

then

$$\chi_t = \frac{\phi}{Z_t\left(M_t + \nu_t\right)}$$

■

Proof of corollary 1. The euler condition implies that $Z_t\left(M_t+\nu_t\right) = q^b/q_t$ when the CIA constraint does not bind and $Z_t\left(M_t+\nu_t\right) = \phi/\chi_t$ when the CIA constraint binds. As shown in the proof of proposition 1, when the CIA constraint does not bind $q^b > q_t$ and thereby $Z_t\left(M_t+\nu_t\right) > 1$. As shown in the proof of proposition 2, when the CIA constraint binds $\chi_t > \phi$ and thereby $Z_t\left(M_t+\nu_t\right) < 1$. ■

Proof of proposition 3. For γ_1 and γ_2 the corresponding upper bounds of velocity are denoted by $q^b\left(\gamma_1\right)$ and $q^b\left(\gamma_2\right)$, respectively. Proposition 1 indicates that if $\gamma_1 > \gamma_2$ then $q^b\left(\gamma_1\right) < q^b\left(\gamma_2\right)$. It follows that for a given $\{\theta_t, M_t, \varphi_t\}_{t=1}^{\infty}$, $\mathcal{B}_2(T) \subseteq \mathcal{B}_1(T)$. ■

Proof of proposition 4. As shown in the proof of proposition 1, when the CIA constraint is non-binding, $q_t < q^b$, equilibrium output and consumption are functions of only θ_t while the price level is a function of M_t, ν_t, φ_t and θ_t. Using the solution for output in (9) and (10), equilibrium work effort and real profits are expressed as functions of only θ_t. Corollary 1 indicates that when the CIA constraint does not bind $Z_t\left(M_t+\nu_t\right) > 1$ which implies that $\chi_t < \phi$. ■

Proof of corollary 2. It follows from the proof of proposition 4. ■

Proof of proposition 5. Let $u^{nb}\left(\theta_{t_2}\right) \in \mathcal{U}^{nb} = \{u\left(t\right)\colon\ t \in \mathcal{NB}\}$ and $u^b\left(\theta_{t_1}\right) \in \mathcal{U}^b = \{u\left(t\right)\colon\ t \in \mathcal{SB}\}$ correspond to $\chi^{nb}\left(\theta_{t_2}\right)$ and $\chi^b\left(\theta_{t_1}\right)$, respectively. For any θ we know that $\chi^{nb}\left(\theta\right) < \phi < \chi^b\left(\theta\right)$. Then, for a given θ, as $\chi^b\left(\theta\right)$ decreases, $\chi^b\left(\theta\right) \rightarrow \chi^{nb}\left(\theta\right)$ and $u^b\left(\theta\right) \rightarrow u^{nb}\left(\theta\right)$. If $u^b\left(\theta\right)$ increases (decreases) as $\chi^b\left(\theta\right)$ decreases (increases) then

$u^{nb}(\theta_{t_2}) > u^b(\theta_{t_1})$. To show this write

$$u^b\left(\theta;\chi^b\right) = \ln\frac{\gamma-1}{\gamma\left(1+\chi^b\right)}\theta + \phi\ln\frac{\chi^b}{1+\chi^b},\;\; \phi < \chi^b$$

Since $0 < \phi < \chi^b$, it follows that

$$\frac{du^b(\theta;\chi)}{d\chi^b} = \frac{\phi-\chi^b}{\chi^b\left(1+\chi^b\right)} < 0$$

and thereby $u^{nb}(\theta_{t_2}) > u^b(\theta_{t_1})$. In addition, since $0 < \phi < \chi^b$,

$$\pi^b(\theta_{t_1}) = \frac{m}{\gamma\left(1+\chi^b\right)}\theta_{t_1} < \frac{m}{\gamma(1+\phi)}\theta_{t_2} = \pi^{nb}(\theta_{t_2})$$

■