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*Inflation and Balanced-Path Growth with Alternative
Payment Mechanisms*

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Inflation and Balanced-Path Growth with Alternative Payment Mechanisms[*]

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Abstract

The paper shows that contrary to conventional wisdom an endogenous growth economy with human capital and alternative payment mechanisms can robustly explain major facets of the long run inflation experience. A negative inflation-growth relation is explained, including a striking non-linearity found repeatedly in empirical studies. A set of Tobin (1965) effects are also explained and, further, linked in magnitude to the growth effects through the interest elasticity of money demand. Undisclosed previously, this link helps fill out the intuition of how the inflation experience can be plausibly explained in a robust fashion with a model extended to include credit as a payment mechanism.

Keywords: Human capital, cash-in-advance, interest-elasticity, credit production

JEL: O42, E31, E22

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1   Introduction

The evidence on the effect of inflation on growth has continued to show a strong negative relation. Recent panel studies report strong inflation effects, both for developed and developing country samples. Further in the evidence has emerged a striking nonlinearity of this effect. Here there is a stronger negative effect of inflation at lower rates of inflation, and this becomes weaker as the inflation rate rises. This still makes for a rising cumulative effect of inflation rate increases, but it makes for a significantly weaker, negative, marginal effect on growth as the rate of inflation becomes higher.[1]

The achievement of the theoretical literature in replicating such results has been more mixed. It has been unclear whether a monetary general equilibrium economy with a payments technology can explain the evidence of how inflation affects economic growth and other related activity. One emphasis has been on calibrating the marginal effect on growth of an increase in the inflation rate, from a level typically of 10%, and then matching that to the average estimates in the empirical literature. A variety of endogenous growth models have been offered in this regard, with widely varying results. For example, both Chari, Jones, and Manuelli (1996), using human capital, and Dotsey and Sarte (2000), using an AK model with uncertainty, present endogenous growth models with cash-in-advance technologies in which inflation has an insignificant effect on growth. In contrast, for example, both Gomme (1993), in a human capital model with a cash-in-advance constraint, and Haslag (1998),

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[1]A debate has arisen on the effects of inflation below certain "threshold" rates of inflation, with some findings of insignificant inflation effects at inflation rates below the threshold. But this rate has been found to be close to 0 for developed country samples. In developing country samples, the threshold tends to be higher, near 10%, but a strong negative effect is typically re-established at all rates of inflation in all samples when instrumental variables are used, as in Ghosh and Phillips (1998) and in Gillman, Harris, and Matyas (2004). These studies also find the marked nonlinearity, as do Khan and Senhadji (2000) and Judson and Orphanides (1996). Bruno and Easterly (1998) provide statistical averages of high inflation episodes whereby high inflation is correlated with lower growth rates than both before and after the episode; Gylfason and Herbertsson (2001) and Chari, Jones, and Manuelli (1996) provide reviews of earlier evidence of a negative inflation effect; Barro (2001) finds a significant negative effect while emphasizing human capital.

in an AK model with money used for bank reserves, find a significant effect of inflation on growth.[2] Thus these models have been ambivalent. And in focusing on just one level of the inflation rate, this literature has begged the question of how inflation affects growth over a wide range of inflation rates, and on whether the models can replicate the nonlinear profile of the inflation-growth effect. Also, after a strong appearance in the older exogenous growth literature, the recent growth literature has largely ignored the issue of whether the models generate empirically consistent Tobin (1965) effects.[3]

The main contribution of the paper here is that it presents a model in which a reasonable calibration can account for the empirical evidence, across the range of inflation rates, on inflation and growth. It does this in a robust fashion, and with an extension of a standard model using human capital and cash-in-advance. The paper also shows that the inflation-growth explanation is fully consistent with evidence on the existence of the Tobin (1965)-like effects, including a rise in output per effective labor, even as the balanced-path growth rate declines as a result of an inflation rate increase.[4] Further it presents a novel, systemic, link between the strength of the growth effect and the strength of the Tobin (1965) evidence. This fills another gap in the theoretical literature and opens up a new line of model predictions that have yet to

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[2] Dotsey and Sarte (2000) also present a deterministic AK version of the Stockman (1981) model in which there is a significant negative effect. And in an more robust reformulation of the Haslag (1998) model, using instead a cash-in-advance approach, Gillman and Kejak (2004b) also find this strong negative effect. For a comparison of such models, see Gillman and Kejak (2004a)

[3] For example, neither Dotsey and Ireland (1996), Aiyagari, Braun, and Eckstein (1998), or Gomme (1993) indicate Tobin type results, although Gomme (1993) is clearly consistent with them. The original Tobin (1965) effect is within an exogenous growth model in which an increase in the inflation rate causes an increase in the capital to labor ratio and in per capita output; see Walsh (1998) for a review. Ahmed and Rogers (2000) compare the Tobin (1965) effect across various exogenous growth models. Gillman and Kejak (2004a) compare Tobin-like effects across endogenous growth models.

[4] Ahmed and Rogers (2000) report long run US evidence showing that inflation has had a negative effect on the real interest rate historically, which would be expected if inflation causes the capital to effective labor ratio to rise as in the Tobin (1965) effect. Gillman and Nakov (2003) report long run US and UK evidence of an increase in the capital to effective labor ratio as a result of inflation.

be empirically examined: that the magnitude of the Tobin (1965) effect is roughly proportional to the magnitude of the growth effect, and that these magnitudes vary monotonically from higher to lower as the inflation rate increases.

The key mechanism that gives our model the added flexibility to explain the evidence is the ability of the representative consumer to choose between competing payment mechanisms, money and credit, so that in equilibrium the marginal cost of each is equal. With such credit available to purchase the good, the nonlinearity is greatly magnified. When inflation rises up, the exchange cost of goods rises, but with credit available it rises by less than otherwise. So the consumer substitutes from goods to leisure, but uses credit to decrease the amount of substitution towards leisure. And this credit is relied upon increasingly more as the inflation rate goes up, and leisure is relied upon increasingly less as a substitution channel. This is because the marginal utility of goods gets increasingly high as less goods are consumed, while the marginal utility of leisure becomes increasingly lower as more leisure is consumed. This inflation-induced distortion in the marginal rate of substitution between goods and leisure is alleviated by the consumer's use of credit, and so accordingly the credit gets used more as the distortion gets bigger. And this results despite the increasing marginal cost of credit use, and in a way that is robust to the nature of the marginal cost specification. Because credit gets used increasingly more, and leisure is used increasingly less as a substitution channel, the inflation-growth nonlinearity results. Leisure plays a key role in determining the growth rate: increased leisure use causes a lower return on human capital and a lower growth rate. So the use of increasingly less leisure makes for the decrease in the growth rate to be of increasingly lower magnitude, as the inflation rate rises. The resulting inflation-growth profile is shown to be very nonlinear compared to the model without credit and it qualitatively matches the profile in the evidence, unlike in the previous literature.

The use of credit has a residual implication for the use of money. And the nature of the model's money demand function is an alternative way to explain the basis for the inflation-growth nonlinearity. The

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money demand can be described as being similar to a general equilibrium version of the Cagan (1956) function, in that it has an approximately constant semi-interest elasticity. This means that as the inflation rate rises, the interest elasticity rises substantially in magnitude. And this results because of the decreasing use of real money as credit is instead used to ameliorate the rising goods–to-leisure inflation-induced distortion, as the inflation rate rises. As part of this rising magnitude of the interest elasticity, in the model with credit, the use of money is much more interest elastic at all levels of the inflation rate relative to the same model without credit available.[5] And the approximate semi-interest elasticity is a testable model implication that has substantial support, such as in recent international panel evidence by Mark and Sul (2002). It thereby provides a parallel dimension to the nonlinear inflation-growth evidence.[6]

In particular, the rising interest elasticity and its correspondence to the nonlinearity of the inflation-growth profile involves a previously unreported systemic link between the strength of the growth and of the Tobin (1965) effects: when the inflation rate is low and the money demand function is in the relatively inelastic range, the growth and Tobin (1965) effects are both marginally stronger, that is, of greater magnitude. When the inflation rate is relatively high and the money demand is in a relatively elastic range, these effects are weak, of small magnitude. Credit takes most of the substitution burden, instead of leisure, of an increase in the inflation rate when the level of the inflation rate is already high. This results in less growth and capital reallocation effects in re-equilibrating the return on human and physical capital at a lower rate of return.

Alternative solutions to the problem, of explaining the inflation experience, that rely on popular existing payment mechanisms all face inadequacies. The Lucas (1988) model with a standard payment mechanism potentially can produce both significant calibrated effects of the

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[5] As shown in a related model in Gillman (1993).

[6] Another testable hypothesis here is the models ability to explain velocity; in a closely related model, Gillman and Kejak (2004b) are able to explain velocity trends for an array of monetary aggregates.

inflation-growth effect as well as the Tobin (1965) effects, but it yields a weakly non-linear inflation-growth profile that is strained to match the evidence. Models with Lucas and Stokey (1983) cash goods and credit goods, but without a payments mechanism specified for credit, can only explain the effects of inflation through the agent's preference for credit goods versus cash goods. The lack of microeconomic evidence for this dichotomy makes the model difficult to calibrate in a non-arbitrary way. And while it has been common to interpret leisure as the credit good, making leisure the credit good in the endogenous growth models simply reduces the model back to the cash-only model with goods and leisure in the utility function.[7] Shopping time economies, a now commonly used alternative approach, in one sense improve on other standard payments mechanisms by allowing time to be used as a substitute to using money. But it is unclear what this shopping time is meant to represent as it has no obvious market analogy. With little to guide the specification, the fashion has been to use a constant interest elasticity to set the shopping time parameters, similar to how the preference-for-money parameters have been set in the money-in-the-utility function approach.[8] Some have interpreted shopping time as banking time, but have not taken the approach of modeling any part of banking. This is precisely what we do with our credit sector. And the result is a Cagan (1956)-like strongly rising interest elasticity, not a constant one, that is robust to a range of credit production function parameters, and is key to explaining the nonlinear nature of the evidence.

2 The Economy with Goods, Human Capital, and Exchange Production

2.1 The Consumer Problem

The representative consumer's utility at time t depends on goods consumption, c_t, and leisure, x_t, in the constant elasticity form. Lifetime utility is

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[7]Hodrick, Kocherlakota, and Lucas (1991) found a Lucas and Stokey (1983)-type economy unable to explain velocity movements.

[8]See Goodfriend (1997), Lucas (2000), and Gavin and Kydland (1999).

$$U_0 = \int_0^\infty e^{-\rho t} \frac{c_t^{1-\theta} x_t^{\alpha(1-\theta)}}{1-\theta} dt. \tag{1}$$

Output of goods, denoted by y_t, can be turned costlessly into physical capital. Both goods output and human capital are produced with physical capital and human capital -indexed labor in constant-returns-to-scale functions. Let k_t and h_t denote the stocks of physical capital and human capital, with the fixed depreciation rate of the capital stocks denoted by δ_k and δ_h. Let s_{Gt}, and s_{Ht} denote the fraction of capital that the agent uses in the goods production and human capital production, whereby

$$s_{Gt} + s_{Ht} = 1, \tag{2}$$

and $s_{Gt}k_t$, and $s_{Ht}k_t$ are the amounts of capital used in each sector. Similarly, let l_{Gt}, l_{Ht}, and l_{Ft} denote the fraction of time the agent uses in the goods, human capital, and credit sectors. This makes the allocation of time constraint

$$l_{Gt} + l_{Ht} + l_{Ft} = 1 - x_t, \tag{3}$$

and making $l_{Gt}h_t$, $l_{Ht}h_t$, and $l_{Ft}h_t$ the effective labor in each sector.

With $\beta, \varepsilon \in [0,1]$ and A_G and A_H being positive shift parameters, the goods production function is

$$y_t = A_G \left(s_{Gt}k_t\right)^{1-\beta} \left(l_{Gt}h_t\right)^\beta. \tag{4}$$

The marginal product of capital $s_{Gt}k_t$, denoted by r_t, and the marginal product of effective labor $l_{Gt}h_t$, denoted by w_t, are

$$r_t = (1-\beta)A_G \left(s_{Gt}k_t\right)^{-\beta} \left(l_{Gt}h_t\right)^\beta, \tag{5}$$

$$w_t = \beta A_G \left(s_{Gt}k_t\right)^{1-\beta} \left(l_{Gt}h_t\right)^{\beta-1}. \tag{6}$$

The human capital equation of motion, given $h_0 > 0$, is

$$\dot{h}_t = A_H[(1 - s_{Gt})\, k_t]^{1-\varepsilon}[(1 - l_{Gt} - l_{Ft} - x_t)\, h]^\varepsilon - \delta_h h_t. \tag{7}$$

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Note that this human capital investment equation is the same as in Lucas (1988) except that there is also physical capital used as an input along with the effective labor. This follows the King and Rebelo (1990) extension of the Lucas (1988) model which makes it more suitable for calibration purposes. While in the Lucas (1988) model the growth rate of human capital is proportional to the labor time devoted to human capital accumulation, or to "learning", here the growth rate is a combination of the fraction of time and the fraction of capital devoted to human capital accumulation. In both the Lucas (1988) model and this extension, the balanced-path growth rate equals the human capital stock growth rate, and both are reduced when leisure time increases.

The goods output forms an input into the Becker (1965) household production of the consumption good c_t. The goods used as an input for producing the consumption are denoted by y_{ct}. The other input is exchange, denoted by y_{et}, which enters the production function $f_c(\cdot)$:

$$c_t = f_c(y_{ct}, y_{et}). \tag{8}$$

The production function for the consumption good is assumed to be Leontieff, with the isoquant ray from the origin having a slope of one:

$$c_t = y_{ct}, \tag{9}$$

$$c_t = y_{et}. \tag{10}$$

This technology ensures that the amount of consumption goods equals the amount of physical goods, and that the value of the physical goods is equal to the value of the amount that is paid (or exchanged) for the goods. This one-to-one relation is the most intuitively appealing; other specifications are possible but would require some extended justification.

The exchange in turn is produced using two inputs: real money balances, denoted by m_t, and real credit, denoted by d_t. These inputs are perfect substitutes, implying that

$$y_{et} = m_t + d_t. \tag{11}$$

Real money balances are defined as the nominal money stock, denoted by M_t, divided by the nominal price of goods output, denoted by

P_t ; $m_t \equiv M_t/P_t$. The initial nominal money stock M_0 is given to the consumer. Additional money stock is transferred to the consumer exogenously in a lump sum fashion by an amount V_t. The consumer uses the money to buy some fraction of the output goods with money, and the rest with credit. Let $a_t \in (0,1]$ denote the fraction of output goods bought with money.[9] Then the agents demand for money is constrained to be this fraction of goods purchased. In real terms,

$$m_t = a_t y_{ct}. \tag{12}$$

Substitution from equation (9) gives a Clower (1967) constraint:

$$m_t = a_t c_t; \tag{13}$$

$$M_t = P_t a_t c_t. \tag{14}$$

Credit demand is the residual fraction of output goods purchases,

$$d_t = (1 - a_t) y_{ct}, \tag{15}$$

or substituting in from equation (9),

$$d_t = (1 - a_t) c_t. \tag{16}$$

With $\gamma \in (0,1)$, and A_F a shift parameter, the credit production function is specified as

$$d_t = A_F (l_{Ft} h_t)^\gamma c_t^{1-\gamma}. \tag{17}$$

This function can be interpreted using duality. Because the total cost of production in the credit sector is the wage bill of the effective labor, $w_t l_{Ft} h_t$, equation (17) implies the marginal cost (MC_t) function of

$$MC_t = (w_t/\gamma) A_F^{-1/\gamma} (d_t/c_t)^{(1-\gamma)/\gamma}. \tag{18}$$

With $\gamma < 0.5$, this gives a marginal cost of credit output, per unit of consumption, that rises at an increasing rate as in a traditional U-shaped cost curve. Figure 1 graphs the three cases of $\gamma = 0.3$ (thicker line), $\gamma = 0.5$ (middle, straight, line) and $\gamma = 0.7$ (and with $w_t = A_F = 0.2$).

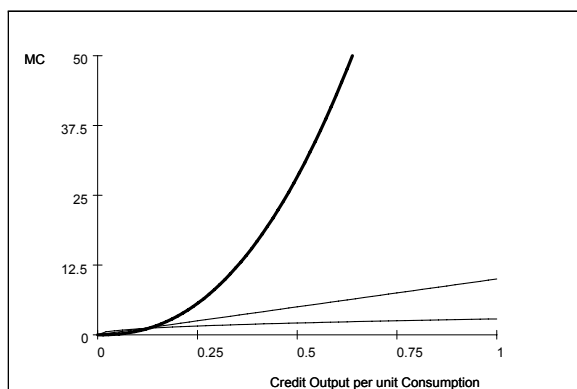[9] An equilibrium with $a = 0$ does not have well-defined nominal prices.

Figure 1. Marginal Cost of Credit

A rising marginal cost function per unit of consumption is the same devise used in Gillman (1993). The difference is that in that model there was a continuum of goods and of stores each with a different time cost of supplying credit to buy their good. In aggregate the stores present an upward sloping marginal cost curve, so that a unique equilibrium with the nominal interest exists at each nominal interest rate. However here there is only one consumption good and one credit production function, with γ being the diminishing returns parameter that determines the shape of the curve; the unique equilibrium results as long as $\gamma < 1$, although $\gamma > 0.5$ seems unlikely in that they indicate a marginal cost that rises at a decreasing rate in contrast to typical industrial organization evidence.

The upward sloping cost curve, for example, with $\gamma = 0.3$ as in Figure 1, can also be interpreted in terms of the value-added of the credit sector. This requires an explicit price for the credit service through a decentralization of the sector.[10] Given the decentralization, it is found that the price of the credit service is the nominal interest rate. In market clearing equilibrium, this price equals the marginal cost given above. And indeed the equality of the nominal interest rate and the marginal cost of credit is one of the below key equilibrium conditions (equation (32)). This "price" can also be used to define the value-added, or total revenues as in national accounts, of the credit sector; this equals

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[10]See Gillman and Kejak (2004b).

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the nominal interest rate factored by the quantity of the credit supplied. Given the assumed production function, in equilibrium it can be shown that this value-added is proportional to the cost of production ($(Rc(1-a))/(wl_Fh) = \gamma$). This gives another way to interpret the assumed production specification. Even more simply the specification implies that the per unit marginal cost is higher than average cost by a fixed proportion for all levels of credit output, resulting in a constant profit rate. Thus the assumption is the same as assuming an upward sloping marginal cost curve, proportional to average cost, with a constant profit rate, which has intuition based firmly in standard price theory.

Note that the output of such a service sector is necessarily proportional to aggregate consumption. Factoring out this proportionality factor to determine what is being produced gives the share of the output for which the service is provided. If it is also assumed that the production function has diminishing returns, then the production of the share necessarily includes an "externality" effect from the aggregate consumption. Were constant returns to scale specified for the service, while at the same time there is a substitute price that exhibits a constant marginal cost, which is what the nominal interest rate presents for the marginal cost of real money, then there is no unique equilibrium between the two alternatives. Thus the production function for credit must be specified with diminishing returns in order to have a unique equilibrium, and as a service proportional to aggregate consumption, it must include the externality effect. However consider an illustration of what this really means in the model economy. A credit card company such as American Express, in a decentralized setting, would maximize profit while taking as given how much is spent on goods for consumption. American Express would not try to change this goods expenditure but must consider it in making its optimal credit supply available to the consumer. By making its inputs grow as the consumption of goods grows, it can maintain its share of supplying credit. This simply means that if the aggregate consumption increases, and the credit sector does not increase its effective labor proportionally, then it will lose its share of output for which it

provides the service.

Setting credit demand equal to credit supply, in equations (16) and (17),

$$(1 - a_t) = A_F(l_{Ft}h_t/c_t)^\gamma. \tag{19}$$

Substituting into equation (14) for a_t from equation (19), the money and credit constraints can be written as

$$M_t = \left(1 - A_F\left(\frac{l_{Ft}h_t}{c_t}\right)^\gamma\right)P_tc_t. \tag{20}$$

2.2 Government Money Supply

The initial money stock M_0 is given to the representative agent, and the only role of the government is to change the money supply from its initial value. To do this, the government transfers to the consumer each period an exogenous lump sum money supply of V_t at a constant rate of σ;

$$\dot{M}_t = V_t = \sigma M_t. \tag{21}$$

The stock V_t is the inflation "proceeds" that result when the government buys output/capital (they are costlessly interchangeable) with freshly printed fiat and then gives this (thereby producing real money) to the consumer as an income transfer. Net government spending equals zero and is omitted for notational simplification. The only effect of such "production" is a relative price distortion if the inflation rate ends up non-optimal.

In real terms, dividing equation (21) by P_t implies that the government's investment rate in real money is the supply growth rate minus the inflation-based depreciation of $\pi \equiv \dot{P}_t/P_t$:

$$\dot{m}_t = (\sigma - \pi)m_t. \tag{22}$$

2.3 Definition of Equilibrium

The consumer's total nominal financial wealth, denoted by Q_t, is the sum of the money stock M_t and the nominal value of the physical capital stock P_tk_t:

$$Q_t = M_t + P_t k_t; \tag{23}$$

$$\dot{Q}_t = \dot{M}_t + P_t \dot{k}_t + \dot{P}_t k_t. \tag{24}$$

The consumer's change in the financial wealth over time, $\dot{Q}_t$, is equal to the sum of V_t by equation (21), plus the nominal value of the change in physical capital $P_t \dot{k}_t$, and plus the nominal price appreciation factor $\dot{P}_t k_t$. The $P_t \dot{k}_t$ term is the output of goods, which can be written in terms of marginal products using equation (5) and (6), minus the output of goods that are purchased for consumption, which by equation (9) equals $P_t c_t$, and minus capital depreciation $P_t \delta_k k_t$. This gives

$$\dot{Q}_t = P_t r_t s_{Gt} k_t + P_t w_t l_{Gt} h_t + V_t - P_t c_t - P_t \delta_k k_t + \dot{P}_t k_t. \tag{25}$$

Equations (4), (5), (6), (25) and (21) imply the social resource constraint

$$y_t = c_t + \dot{k}_t + \delta_k k_t. \tag{26}$$

Given M_0, k_0, h_0, and the normalization of $P_0 = 1$, equilibrium consists of the values of the prices $\{r_t, w_t, P_t\}_{t=0}^{\infty}$ and the allocations $\{c_t, x_t, s_{Gt}, l_{Gt}, l_{Ft}, M_t, Q_t, k_t\}_{t=0}^{\infty}$ that satisfy i) the representative consumer's maximization of the lifetime utility (1) subject to the constraints in equations (7), (20), (23), and (25), taking as given the prices and the transfer V_t, ii) the firm's maximization problem taking prices as given, iii) the government supply of money in equation (21), and iv) the clearing of all markets in the economy, with equation (26) for the goods market.

2.4 Balanced Growth Path

On the balanced-growth path, c_t, k_t, h_t, m_t and y_t grow at the same rate, denoted by g. The variables x_t, l_{Gt}, l_{Ft}, l_{Ht}, s_{Gt}, s_{Ht}, w_t, r_t are stationary.

A balanced growth path reduced set of equilibrium conditions are set out below, with time subscripts dropped and assuming $\delta_k = \delta_h$:

$$\frac{u_c(c,x)}{u_x(c,x)} = \frac{x}{\alpha c} = \frac{1 + aR + wl_F h/c}{wh}, \tag{27}$$

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$$\frac{w}{r} = \frac{\beta}{1-\beta}\frac{s_G k}{l_G h} = \frac{\varepsilon}{1-\varepsilon}\frac{s_H k}{l_H h}, \tag{28}$$

$$g \equiv \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{h}}{h} = \frac{\dot{m}}{m} = \frac{r - \delta_k - \rho}{\theta} \tag{29}$$

$$= \frac{\varepsilon(1-x)A_H[(s_{Ht}k_t)/(l_{Ht}h_t)]^{1-\varepsilon} - \delta_h - \rho}{\theta}, \tag{30}$$

$$r - \delta_k + \frac{\dot{P}}{P} \equiv R, \tag{31}$$

$$R = w / \left(\gamma A_F \left(\frac{l_{Ft}h_t}{c_t}\right)^{\gamma-1}\right). \tag{32}$$

Because of the novel nature of the credit sector, a focus on this last equation (32) helps describe the model. In the Baumol (1952) model, the consumer chooses between two payment mechanisms: the use of money and the use of banking in which interest is earned on the income. The banking of these models is similar to the credit in the model here. Also similar is that the consumer optimally chooses between the two according to the cost of each relative to the other. This choice yields the only equilibrium condition in Baumol (1952). There is no such margin in the standard cash-only Lucas (1980) or Lucas and Stokey (1983) economies. The model here follows Baumol (1952) and adds this as an additional margin relative to the standard cash-in-advance economy with the following equilibrium condition. The cost of money, R, equals the marginal cost of credit, which is the marginal factor cost of effective labor in the credit sector, w_t, divided by the marginal product of labor in the credit sector. This is a standard microeconomic pricing condition for factor market equilibrium. The existence of this condition, not found in Baumol (1952), takes the important margin that Baumol (1952) develops, and places it securely within microeconomic theory, while using the single-good standard neoclassical growth framework.[11] This makes standard monetary theory tractable back to the production structure of credit, unlike in Baumol (1952).

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[11] One comparison in the literature to equation (32) can be found in an innovative paper by Canzoneri and Diba (2003); it follows more of the Tobin (1956) approach by specifying bonds that back up a non-money exchange service (not dissimilar to credit), and it uses this to solve the price indeterminacy problem.

The marginal rate of substitution of goods relative to leisure is given by equation (27), and can be understood as the ratio of the shadow price of the consumption good to leisure. The shadow price of consumption goods is one, the goods cost, plus the exchange cost of $aR + wl_F h/c$ per unit. If only money is used in exchange, this is just the nominal interest R. But with credit also used this exchange cost is less than R and can be expressed as a weighted average of money and credit use, or $1 + aR + (1 - a)\gamma R$. Or with a focus on a, this writes as $1 + \gamma R + aR(1 - \gamma)$. When the inflation rate goes up the cost of exchange rises. But because of substitution towards credit, the cash share a falls, the shadow exchange price rises by less than proportionately to R, and so it rises by less than in the cash-only model. Thus there is substitution towards leisure as in the cash-only model, but less such substitution.

Other balanced-growth path equilibrium conditions here show that the growth rate equals the return on capital minus the time preference rate, in the log-utility case, and that the returns of human and physical capital are equal; with equal depreciation rates, $r = \varepsilon(1 - x)A_H[(s_{Ht}k_t)/(l_{Ht}h_t)]^{1-\varepsilon}$. This last expression highlights how the increased leisure can act to decrease the growth rate, while the Tobin (1965) effect towards greater capital intensity in both goods and human capital sectors, as w/r increases because of an inflation increase, can partially offset the decrease in the growth rate.

2.5 Effect of Inflation on Balanced-Growth Path

Technically, the effect of a change in the inflation rate on the balanced-growth path equilibrium can be solved analytically for certain parameter specifications by solving all equations in terms of leisure and then solving for the change in leisure from one implicit equation in terms of only leisure. Then the main results follow and can be summarized in the following two lemmas. For analytic tractability, log-utility is assumed and in addition no physical capital is assumed for the second lemma and its two corollaries. These assumptions are relaxed in the calibration.

Note that the results state what happens when there is an increase in the money supply growth rate. The inflation rate, as in all such

models, increases because the exogenous rate of money supply growth is assumed to increase. The inflation rate goes up a bit more than the money supply growth rate increase, because the balanced-path growth rate falls somewhat, while the sum of the inflation rate and the balanced-path growth rate are constrained to equal the money supply growth rate; from equation (22), $\pi = \sigma - g$. So while this is generally thought of as the effect of inflation on growth in such models, and this is the usage made in this paper, the inflation-growth relation is more precisely a result of the money supply changes.

Lemma 1 *An increase in the money supply growth rate σ causes an increase in leisure time, a decrease in the real interest rate, an increase in the capital to effective labor ratio in the goods and human capital production sectors, an increase in the goods capital to output ratio, and a decrease in the balanced-growth path growth rate. It is assumed that $\theta = 1$, $\beta = \varepsilon = \gamma = 0.5$, $A_G = A_H$, and that the change in the money supply growth rate is evaluated at the Friedman optimum of $R = 0$.*

   Proof. Please see Appendix A.1. ∎

The increase in the exchange cost of goods causes a relative decrease in the opportunity cost of leisure, thereby inducing a shift back in the supply of labor for goods production, while there is a shift of labor into credit production. The real wage rises (by less than does the exchange cost of goods) in order to clear the labor market, inducing firms to realign inputs towards capital and away from labor. The increase in the capital to effective labor ratios, across both goods and human capital production sectors, lowers the marginal product of capital and the real interest rate.[12] Here the rising capital to effective labor effect marks the Tobin (1965) effect in the human capital model, rather than the rising capital per worker as in the Solow exogenous growth model without leisure. Output per effective labor also goes up in a way similar to Tobin (1965). And a lower real interest rate from an inflation increase can be viewed as part of this Tobin (1965) effect. But unlike in Tobin (1965), here the growth rate goes down.

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[12]We thank an anonymous referee for a suggested description here.

Note that in the Lucas (1988) model, only effective labor is used in human capital accumulation and there is no leisure in the utility function; in this case the rate of return on human capital in equilibrium is just proportional to the time spent accumulating human capital, or $A_H l_H$. When the time spent in human capital production goes down, the growth rate goes down. In the monetary extension of the human capital growth model, leisure plays a critical role with respect to inflation. For example, with no physical capital and log-utility (as assumed in the next Lemma), the rate of return on human capital is proportional to the time spent working in all sectors, or $A_H (1 - x)$. And in this case the change in the total time spent working $(1 - x)$ (in all three sectors) is exactly equal to the change in the time spent in human capital accumulation l_H; here the Lucas (1988) explanation of the growth rate, as being proportional to the time spent in human capital accumulation, is perfectly interchangeable with the time spent working. With physical capital the growth rate more generally depends on the rate of return to human capital, in which a falling amount of leisure time because of inflation is the primary effect, while an increase in the capital to effective labor ratio is of secondary magnitude, moderating the decrease in the growth rate.

Lemma 2 *The magnitude of the change in the balanced-path growth rate, from a change in the money supply growth rate, is determined inversely by the magnitude of the interest elasticity of money demand, given that $\beta = \varepsilon = \theta = 1$, and given that the interest elasticity is less than one in magnitude. Further with a cash-only restriction ($a \equiv 1$), the inflation-growth profile is exactly linear.*

Proof. Please see Appendix A.2. ∎

This is the log-utility and no physical capital case. At the Friedman (1969) optimum of $R = 0$, the marginal rate of substitution between goods and leisure is undistorted and leisure is a close substitute for goods because there is no tax wedge to force their marginal utilities to diverge. As the inflation rate rises from the optimal rate, leisure tends to be used readily to avoid the inflation tax, while credit use is relegated to a secondary role in avoiding inflation, despite the fact that the marginal cost

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of credit is relatively low at low inflation rates since there is a rising marginal cost curve. However at higher rates of inflation, the inflation tax wedge makes the use of more leisure increasingly less attractive relative to the use of more credit because leisure's diminishing marginal utility, and goods increasing marginal utility, in effect dominate the rising cost of the credit. Credit is used increasingly more and therefore the interest elasticity of money demand is increasingly high. Because the growth rate effect is dependent directly on how much leisure is used when inflation rises, this effect is strongest when the inflation rate is rising up from the optimum and the wedge in the goods-leisure rate of substitution is at its smallest. The growth rate falls by increasingly less as the inflation rate rises, and the interest elasticity of money demand rises in magnitude.

At a unitary interest elasticity, the growth rate stops falling and actually begins to rise. However the baseline calibration puts this juncture at a hyperinflation rate of inflation, above which the government makes less seigniorage anyway. This suggests that only the range of the inflation rate that induces a less than unitary elasticity is likely to be empirically relevant. Note the relation of this result to Eckstein and Leiderman (1992). They find that seigniorage in Israel rises at a steadily decreasing rate, which they model with a money demand derived from putting real money balances in the utility function. Our nonlinear inflation-growth profile, and the rising magnitude of interest elasticity, correspond directly to a seigniorage that rises at a diminishing rate. As in the Cagan (1956) model (but unlike that of Eckstein and Leiderman (1992)), the total seigniorage would begin to fall once the interest elasticity rose above one in magnitude, but we suggest that this is not an empirically relevant long run range for the elasticity.

Corollary 1 *The magnitude of the interest elasticity of the goods-normalized money demand rises with an increase in the inflation rate because the magnitude of the elasticity of substitution between money and credit, and the share of credit in purchases, each rise with an increase in the nominal interest rate.*

Proof. Please see Appendix A.3. ∎

A standard factor-price elasticity of substitution between real money and credit, as the two inputs into producing exchange, can be defined as the percentage change in inputs over the percentage change in marginal products. Then the interest elasticity of money demand can be expressed as a price elasticity of the derived input demand, in terms of the elasticity of substitution. In particular, the interest elasticity of money demand (η_m^R) equals the (negative) share of the other input credit $(1-a)$ as factored by the elasticity of substitution between money and credit (ϵ), plus a scale effect (η_c^R); or $\eta_m^R = (1-a)\epsilon + \eta_c^R$.[13] The scale effect is of secondary importance in terms of magnitude, and when normalizing the money demand by consumption, this term drops out (this is the only term in the cash-only economy). As the inflation rate rises, leisure becomes a worse substitute, even while money and credit remain perfect technical substitutes (equation (11)). This increases the two-factor elasticity of substitution; the share of credit $1 - a$ also rises unambiguously. Note that the isoquant for producing exchange is not linear because of the role of leisure.[14]

The result is insensitive to the specification of the parameters in the credit production function. Given that $\gamma \in (0,1)$ and $A_F > 0$, there is a rising marginal cost of credit, as the credit use per unit of consumption increases. The degree of diminishing returns, γ, affects shape of the marginal cost curve in an unambiguous way, but affects the normalized interest elasticity in an ambiguous fashion that depends on the calibration; the shift parameter A_F does has a clear effect on the magnitude of the normalized interest elasticity (as indicated in the next corollary). But regardless of these specifications, it is the fact of the existence of the credit (with a rising marginal cost), combined with the nature of the goods to leisure marginal rate of substitution, that

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[13]See for example Marshall (1920) or a standard microeconomic text on derived demand elasticiticies.

[14]See Gillman (2000) for another example of the input price elasticity as applied to real money, in a model using the store continuum as in Gillman (1993), Dotsey and Ireland (1996) and Aiyagari, Braun, and Eckstein (1998). Such a curved isoquant between real money and credit in general equilibrium is graphed in Gillman (1995).

produces the corollary results, of an increasing interest elasticity with inflation rate increases. This can alternatively be seen by writing the normalized elasticity as $(1-a)\epsilon = -\left[\gamma/\left(1-\gamma\right)\right]\left[\left(1-a\right)/a\right]$. All that is necessary for this elasticity to rise in magnitude is that the normalized money usage (a) falls as the inflation rate rises.

Corollary 2 *The magnitude of the interest elasticity of the goods-normalized money demand rises with an increase in productivity in the credit sector, as indicated by an increase in the total factor productivity A_F of the credit production function.*

Proof. Please see Appendix A.4. ∎

This corollary brings in one additional factor, the productivity of the credit sector. This can be important for example in analyzing changes in financial regulation. A deregulation is similar to a decrease in the implicit tax on the credit sector that has the effect of shifting up the productivity parameter A_F. Continuing the example, deregulation here has the effect on increasing the demand for credit at each nominal interest rate, making the demand for money in effect more interest elastic. The fall in the price of a substitute to money causes a shift back in the money demand function. Given the same nominal interest rate, this moves the consumer "up" the money demand function to a more interest elastic point.

3 Calibration

The analytic results of the lemmas and corollaries, on how inflation effects the balanced-growth equilibrium, are shown to apply as well in the general model through its calibration. The calibration makes clear that the model produces a significant effect of inflation on growth, within the range of empirical estimates reviewed for example by Chari, Jones, and Manuelli (1996), while showing the nonlinearity of this effect, the existence of Tobin (1965) effects, and the link between the magnitude of the growth and Tobin (1965) effects. Also the calibration shows the robustness of the results to a full range of alternative specifications of the parameters of the credit production function.

3.1 Assumed Parameter Values

Standard parameters values are assumed as in the literature. Table 1 presents the assumed values for the baseline calibration. Leisure is set as in Jones, Manuelli, and Rossi (1997); risk aversion and Cobb-Douglas parameters for goods and human capital sectors as in Gomme (1993); depreciation rates as in King and Rebelo (1990); growth rate as in Chari, Jones, and Manuelli (1996); the share of cash is similar to Dotsey and Ireland (1996); leisure preference is set within the range in the literature. For the credit sector technology, the degree of diminishing returns is set to 0.2 as based on the estimated value of this parameter that is found for the US in the money demand estimation of Gillman and Otto (2002), a companion paper. This parameter is varied below in Table 4 and a fuller set of such variations can be found in Gillman and Kejak (2002).

Table 1: Baseline Parameter and Variable Values

Parameters	ρ	δ_h	δ_k	θ	β	ε	α	γ	A_G	A_H	A_F
	0.04	0.1	0.1	1.5	0.64	0.64	4.692	0.2	1	0.581	0.801
Variables	a	x	g	π	l_G	l_H	l_F				
	0.7	0.7	0.02	0.05	0.1635	0.1355	0.00098				

3.2 The Results

Table 2 shows that the baseline calibration for the negative growth rate effect of a 10% point increase in the inflation rate is a -0.23 percentage point change in the growth rate of output, comparable to the range in Chari, Jones, and Manuelli (1996). Note that the -0.23 indicates that starting from a baseline of 0.02 percent growth (a 2% growth rate) at an inflation rate of 0.05, the growth rate falls to 0.0177 when the inflation rate rises to 0.15. Figure 2a simulates this in the solid line. The negative growth effect falls in magnitude as the inflation rate rises. This nonlinear relation, of a marginally decreasing magnitude of the negative growth effect, has been found empirically in many studies. And this occurs even while the Tobin (1965) effect is present through a higher output to effective labor ratio (Figure 2b).

Figure 2a also includes for contrast a dashed line for the cash-only

economy that is almost linear, contrary to evidence. Additionally for the economy of Lemma 2, in which there is no physical capital, Figure 2c shows that the inflation growth profile is perfectly linear for the cash-only economy (dashed line) versus the nonlinear Section 2 model with credit (solid line).

Table 2 also shows how leisure rises with inflation (Figure 3a), the real interest rate falls (Figure 3b), the real effective wage rises (Figure 3c), and the capital to effective labor ratio in the goods sector and the investment to output ratio rise (Figures 3d and 3e). The sectorial real-locations are supported empirically in Gillman and Nakov (2003), while supporting evidence for the positive investment rate effect and negative real interest rate effect are found in Ahmed and Rogers (2000). Figure 3f simulates the money demand per unit of consumption goods; this is the inverse, endogenous, consumption velocity and it contrasts for example to the assumption in Alvarez, Lucas, and Weber (2001) that velocity is exogenous. In addition, Table 2 shows the link among the magnitude of the growth and Tobin (1965) effects and the magnitude of the interest elasticity of money demand.

Table 2: Baseline Calibration of the Effect of Increasing the Inflation Rate

Baseline Change in Variable	Inflation Rate Change		
	$5 \to 15\%$	$15 \to 25\%$	$25 \to 35\%$
Growth Rate g	-0.00232	-0.00199	-0.00173
Leisure x	0.00878	0.00824	0.00705
Real Interest Rt r	-0.00320	-0.00304	-0.00263
Real Wage w	0.01054	0.01029	0.00914
Capit/Lab Gds $(s_G k)/(l_G h)$	0.09800	0.09753	0.08810
Capit/Lab Hum $(s_H k)/(l_H h)$	0.09800	0.09753	0.08810
Capit/Output $(s_G k)/y$	0.04086	0.04023	0.03599
Output/Eff.Labor $y/(l_G h)$	0.01647	0.01608	0.01428
Money/Consumption-Goods a	-0.04187	-0.03310	-0.02586
Point Est of Int Elast η_R^m	-0.1276	-0.1757	-0.2220

Table 3 provides a calibration with the goods sector's capital intensity increased above that of the human capital production sector, with $\beta = 0.50$, instead of $\beta = 0.64$ as in the baseline. This shows that with a greater goods sector capital intensity, the inflation-induced substitution from labor to capital is marginally greater, and the Tobin (1965)

and growth effects stronger, relative to the baseline, while the interest elasticity is of smaller magnitude. This acts to marginally shift up the inflation-growth profile; Figure 3g shows this with the solid line being the baseline and with the dashed line having $\beta = 0.50$ and all other parameters as in the baseline.

Table 3: Baseline Calibration Except for an Increase in the Capital Intensity in Goods Production

| **Baseline Except $\beta = 0.60$** | | | |
| Change in | Inflation Rate Change | | |
Variable	$5 \rightarrow 15\%$	$15 \rightarrow 25\%$	$25 \rightarrow 35\%$
Growth Rate g	-0.00232	-0.00200	-0.00174
Leisure x	0.00872	0.00820	0.00703
Real Interest Rt r	-0.00320	-0.00304	-0.00264
Real Wage w	0.01352	0.01327	0.01183
Capit/Lab Gds $(s_G k)/(l_G h)$	0.13379	0.13374	0.12133
Capit/Lab Hum $(s_H k)/(l_H h)$	0.11288	0.11284	0.10237
Capit/Output $(s_G k)/y$	0.04510	0.04452	0.03993
Output/Eff.Labor $y/(l_G h)$	0.02254	0.02211	0.01972
Money/Consumption-Goods a	-0.04063	-0.03212	-0.02507
Point Est of Int Elast η_R^m	-0.1238	-0.1699	-0.2143

Table 4 shows the effect of increasing from its baseline value the parameter that indicates the degree of diminishing returns in the credit sector. It shows that such increases cause a bigger magnitude of the growth effect and of the Tobin (1965) effects, and a smaller magnitude of the interest elasticity. This calibration is done for a neighborhood of the baseline calibration with respect to changes in γ. Simulation of the inflation-growth effect with a larger γ show that this acts to pivot down the inflation-growth profile. Figure 3h shows this with the solid line being the baseline and with the dashed line having $\gamma = 0.25$ and all other parameters as in the baseline.

While the role of financial development on the inflation-growth effect has been little studied (although there are sizeable literatures on each the inflation and growth relation, and the financial development and growth relation), Gillman, Harris, and Matyas (2004) present evidence of differences in the inflation-growth profile for APEC and OECD samples. The profiles compare closely to Figure 3h in that APEC's profile is less steep at every rate of inflation, while the profile starts at about the

same point, so that the APEC profile appears pivoted up relative to the OECD profile. The model thus suggests a comparatively greater degree of diminishing returns in credit production, and a more steeply rising marginal cost curve, in the APEC region. This offers one explanation consistent with the different inflation-growth results that cannot be provided with the standard cash-only cash-in-advance exchange technology.

Table 4: The Inflation Effects When Increasing the Degree of Diminishing Returns in Credit Production

Baseline: Inflation Rate $5 \rightarrow 15\%$ Change in Variable	Degree of Diminishing Returns in Credit Production		
	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.5$
Growth Rate g	-0.00232	-0.00273	-0.00338
Leisure x	0.00878	0.01148	0.01423
Real Interest Rt r	-0.00320	-0.00421	-0.00524
Real Wage w	0.01054	0.01398	0.01769
Capit/Lab Gds $(s_G k)/(l_G h)$	0.09800	0.13083	0.16724
Capit/Lab Hum $(s_H k)/(l_H h)$	0.09800	0.13083	0.16724
Capit/Output $(s_G k)/y$	0.04091	0.04866	0.06908
Output/Eff.Labor $y/(l_G h)$	0.01647	0.02184	0.02764
Money/Consumption-Goods a	-0.04187	-0.05434	-0.03080
Point Est of Int Elast η_R^m	-0.12757	-0.11737	-0.08745

4 Comparison to Other Payment Mechanisms

One type of comparison that can be further detailed is to use the same human capital model but with different payment mechanisms.

4.1 Cash-only Economy

The most standard is the cash-only economy of Lucas (1980). Here the consumer can use only money to buy goods. This case results from the Section 2 model when $a \equiv 1$ is imposed. Or this can be derived by having credit be prohibitively expensive (A_F close to zero). Figure 2a (dashed line) shows the resulting inflation-growth profile with the baseline calibration. The almost linear profile indicates that the growth rate becomes negative quickly as the inflation rate rises, contrary to evidence. The cash-only model overstates the inflation effect on growth at every level of the inflation rate for $R > 0$, in comparison to the Section 2 model. The reason is that when inflation increases, with cash-only the consumer can only substitute towards leisure, and so uses more leisure for

each marginal increase in the inflation rate than if credit was available. So instead of having much smaller leisure increases as the inflation rate goes higher, which is what happens when credit is available, the increases in leisure only decrease in magnitude slightly.

4.2   The Shopping Time Economy

The Lucas (2000) shopping time model focuses on the use of resources in exchange activity. Calling this activity "shopping time" after McCallum and Goodfriend (1987), and showing the sense in which it exactly equals the welfare cost of inflation in the economy (with no leisure), he specifies the shopping time exchange constraint so as to induce a constant interest elasticity. This strategy of specifying the exchange technology so as to have a constant interest elasticity is also used in Goodfriend (1997), who cites an earlier version of the Lucas (2000) paper, and in Gavin and Kydland (1999).

By assuming a constant interest elasticity, the free parameters of the shopping time function can be constrained in a non-arbitrary way. However the problem with the constant interest elasticity assumption is that it is in conflict with evidence. Lucas (2000) describes how a constant-like interest elasticity model seems to breakdown for US data during the 1980s, after which he concludes that a constant semi-interest elasticity model seems to be the preferred model. Mark and Sul (2002) find substantial cointegration panel data evidence in support of the constant semi-interest elasticity model.

If in fact a constant semi-interest elasticity is the appropriate model, then the key fact here is that the interest elasticity rises as the interest rate rises, rather than remaining constant as in the shopping time models. In this case the shopping time models are forcing an undue lack of non-linearity upon the inflation effects with respect to growth and Tobin (1965) variables. This means that the constant interest elasticity will make the effects too weak for low values of the inflation rate and too strong for higher values of the inflation rate, depending on the particulars of which constant interest elasticity is chosen.

The model of section 2 can in fact be viewed as a special case of

the shopping time economy. The special case is that the shopping time of the McCallum and Goodfriend (1987) exchange constraint becomes instead the banking time of an explicit credit production technology.[15] The credit technology parameters determine only how quickly the interest elasticity of money demand rises with the inflation rate. Corollary 1 explains why a rising interest elasticity with inflation does not depend on the exact specification of these parameters, a result confirmed with calibration. Rather through their effect on the interest elasticity they determine the degree of nonlinearity of the inflation-growth profile. Extreme values can reproduce the cash-only economy ($A_F = 0$ or $\gamma = 0$).

5 Conclusion

The paper shows that, contrary to what has become generally accepted, growth models with Lucas (1988) human capital, and well-defined payments mechanisms, can successfully explain major facets of how inflation affects long run economic activity. First it makes clear that point estimates, of significant magnitude, of the negative effect of inflation on the balanced-path growth rate can be found with a standard calibration that is robust to varying the parameters of the credit production function. Second the credit allows the consumer to use less leisure as inflation increases, so that the economy exhibits a significantly non-linear inflation-growth relation as has been found repeatedly in empirical studies. Third the model shows that related Tobin (1965) effects are at work in the economy, with a decrease in the real interest rate to the real wage ratio, an increase in the capital to effective labor ratios across sectors, and a rise in the output per effective labor input. This inflation-tax-induced increase in the output per effective labor hour is a result of the household trying to moderate the growth rate decrease by realigning inputs towards capital as labor becomes scarce and leisure in greater use.

The model has household production of consumption using goods and exchange. The exchange is produced interchangeably with money

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[15]In a related paper, Gillman and Yerokhin (2003) detail this connection. One implication is that shopping time function in an endogenous growth setting should include human capital in its specification, unlike in Love and Wen (1999).

or a credit sector. This offers a direction alternative to general transaction cost models such as the shopping time models. The approach is related to the cash-credit framework of Aiyagari, Braun, and Eckstein (1998), who assume a constant semi-interest elasticity of money demand. Here such a money demand is generated endogenously as the consumer equalizes the marginal cost of alternative payment mechanisms. As a result, links between the money demand function and the inflation effects are pervasive and, unlike previous work, are made explicit. The money demand's interest elasticity inversely determines the strength of the growth and Tobin (1965) effects in a way that fills out intuition of these events. This presents also an alternative research strategy towards further developing and calibrating such models: to use structural parameters of the credit production technology in addition to so-called behavioral parameters of the partial equilibrium money demand functions. This may further advance understanding of how inflation affects international growth and other aspects of the structure of the economy.

A Appendix

A.1 Proof of Lemma 1

The equilibrium conditions, including the marginal product definitions in equations (5) and (6), imply that the balanced-growth solution of all of the variables of the economy can be written in terms of $1-x$; in addition is an implicit equation in $1 - x$. The implicit equation, derived from $1-x = l_F + l_G + l_H$, is $1 - x = \frac{r^{(1-\varepsilon)/\beta}}{[A_G(1-\beta)]^{(1-\varepsilon)/\beta}} \frac{\left(A_G\left(\frac{l_G h}{s_G k}\right)^\beta (r-\rho) + \left(\frac{c}{h}\right)\left(\frac{l_H h}{s_H k}\right)^\varepsilon\right)}{\beta A_G \left(\frac{l_G h}{s_G k}\right)^\beta + \rho} + \frac{w^{\gamma/(\gamma-1)}x}{\alpha \gamma A_F R^{1/(\gamma-1)}\left[1 + aR + w\left(\frac{l_F h}{c}\right)\right]}$. With $\varepsilon = \beta = \gamma = 0.5$, and $A_G = A_H = 1$ this gives the following polynomial in $z \equiv (1-x)^{0.5}$, where $\Omega \equiv [A_F(\sigma + \rho)]^2$.

$$0 = -0.5\Omega z^3 + 2[2\alpha\rho\Omega - (1 + \rho\Omega)]z^2 - [4\alpha\rho(1 + \sigma + \rho) - 0.5\Omega]z + 1 + \rho\Omega. \tag{33}$$

Differentiating with respect to σ and z, and solving for $\partial z/\partial\sigma$, we have $\frac{\partial z}{\partial \sigma} = \frac{(\partial\Omega/\partial\sigma)\{-0.5z^3 + \rho(2\alpha-1)z^2 + 0.5z + \rho\} - 4\alpha\rho}{\Omega\{1.5z^2 - 2\rho(2\alpha-1)z + 0.5\} + 2z + 4\alpha\rho(1+\sigma+\rho)}$, where $\partial\Omega/\partial\sigma = 2A_F^2(\sigma + \rho)$. Evaluating $\frac{\partial z}{\partial\sigma}$ at the optimum of $\sigma + \rho = 0$, implies that $\frac{\partial z}{\partial\sigma} = -\frac{2\alpha\rho}{z + 2\alpha\rho}$. Since $\alpha, \rho > 0$ and $z = 1 - x \in (0,1)$, $\frac{\partial z}{\partial\sigma} = \frac{\partial(1-x)}{\partial\sigma} < 0$. Then the

equilibrium values of all variables can be examined in terms of their change with respect to $1 - x$ and σ. With the above parameter restrictions these are given by $r = 0.5(1 - x)^{0.5}$, with $\partial r / \partial (1 - x) > 0$, and $\partial r / \partial \sigma < 0$; $w = 0.5(1 - x)^{-0.5}$, $\partial w / \partial (1 - x) < 0$; $\partial w / \partial \sigma > 0$; $\frac{s_G k}{l_G h} = \frac{s_H k}{l_H h} = (1 - x)^{-1}$; $\partial (s_G k / l_G h) / \partial \sigma < 0$; $(s_G k)/y = 1/[r(1 - \beta)]$, $\partial [y/(s_G k)]/\partial \sigma > 0$; $g = r - \delta_k - \rho$, $\partial g / \partial \sigma < 0$.

Finally we derive the unique solution for x at the optimum. Evaluating equation (33) at the optimum of $\sigma + \rho = 0$, implies that $z^2 + 4\alpha\rho z + 1 = 0$. The quadratic equation has two solutions: $z_{1,2} = 2\alpha\rho(-1 \pm \sqrt{1 + 1/(4\alpha^2\rho^2)})$. One solution gives a negative x, outside of its feasible range. And it can be shown that the unique solution for leisure, $x \in [0, 1]$, is $1 - 4\alpha^2\rho^2(-1 + \sqrt{1 + 1/(4\alpha^2\rho^2)})^2$.

A.2   Proof of Lemma 2

Under the assumptions of $\beta = \varepsilon = \theta = 1$ the economy uses no physical capital and has log-utility. Here the growth rate is determined by the marginal product of human capital and is given by $g = A_H(1 - x) - \delta_h$, and $\partial g / \partial \sigma = -A_H \partial x / \partial \sigma$. The economy has a closed form solution and $x = (\rho\alpha/A_H)[(1 + aR + A_G l_F h/c)/(1 + A_G l_F h/c)$. Since $R = \sigma + \rho$, it follows that $\partial g / \partial \sigma = \partial g / \partial R$. Using this fact and the expression for x, $\partial g / \partial \sigma$ can be written as $\partial g / \partial R = -\alpha\rho[a/(1 + A_G l_F h/c)][1 + \eta_R^a - \eta_R^{l_F h/c}(A_G l_F h/c)/(1 + A_G l_F h/c)]$, where η_R^a is the elasticity of a with respect to R and is given by $\eta_R^a = -[\gamma/(1-\gamma)][(1-a)/a]$, and $\eta_R^{l_F h/c}$ is a similar elasticity given by $\eta_R^{l_F h/c} = 1/(1-\gamma)$. Further, $-\eta_R^{l_F h/c}(A_G l_F h/c)/(1 + A_G l_F h/c) = \eta_R^c$, and so $1 + \eta_R^a - \eta_R^{l_F h/c}(A_G l_F h/c)/(1 + A_G l_F h/c) = 1 + \eta_R^a + \eta_R^c = 1 + \eta_R^m$, where $\eta_R^m \leq 0$ is the interest elasticity of money demand in equation (13). Therefore $\partial g / \partial R = -\alpha\rho[a/(1 + A_G l_F h/c)][1 + \eta_R^m]$. At $R = 0$, $\eta_R^m = 0$. As R rises the elasticity becomes increasingly negative, and $1 + \eta_R^m$ gets smaller. Because it can be shown that the other term also falls unambiguously as R rises, that is $\partial [a/(1 + A_G l_F h/c)]/\partial R < 0$, the growth rate decrease that occurs for $\eta_R^m \geq -1$ becomes increasingly smaller as R increases; and its decrease is made directly less by the rising interest elasticity of money demand and the falling magnitude of the $1 + \eta_R^m$. Now if $a \equiv 1$, then from above it is clear that $\partial g / \partial R = -\alpha\rho$,

which implies a linear inflation-growth relation.

A.3   Proof of Corollary 1

Define the elasticity of substitution between cash and credit as $\epsilon \equiv \left[\partial \left(\frac{ac}{(1-a)c} \right) / \partial \left(\frac{R}{A_G/\gamma A_F^{1/\gamma}} \right) \right] \left[\left(\frac{R}{A_G/\gamma A_F^{1/\gamma}} \right) / \left(\frac{ac}{(1-a)c} \right) \right]$, which is solved as $\epsilon = -[\gamma/(1-\gamma)]/a$. In turn the interest elasticity of money is $\eta_R^m = \eta_R^a + \eta_R^c$, and this writes as $\eta_R^m = (1-a)\,\epsilon + \eta_R^c$. Normalizing the money demand m by dividing by the goods consumed, c, this gives $m/c = a$. And $\eta_R^a = (1-a)\,\epsilon$. Since $1-a = A_F^{1/(1-\gamma)}(R\gamma/A_G)^{\gamma/(1-\gamma)}$, by equations (19) and (32), then $\partial(1-a)/\partial R \geq 0$, $\partial\epsilon/\partial R \geq 0$, and so $\partial\eta_R^a/\partial R \leq 0$; for $R > 0$, $\partial\eta_R^a/\partial R < 0$.

A.4   Proof of Corollary 2

By Lemma 2 and Corollary 1, $\eta_R^a = -[\gamma/(1-\gamma)][(1-a)/a] = -[\gamma/(1-\gamma)][A_F^{1/(1-\gamma)}(R\gamma/A_G)^{\gamma/(1-\gamma)}]/[1 - A_F^{1/(1-\gamma)}(R\gamma/A_G)^{\gamma/(1-\gamma)}]$, and $\partial\eta_R^a/A_F \leq 0$ so that the magnitude of η_R^a rises as A_F rises.

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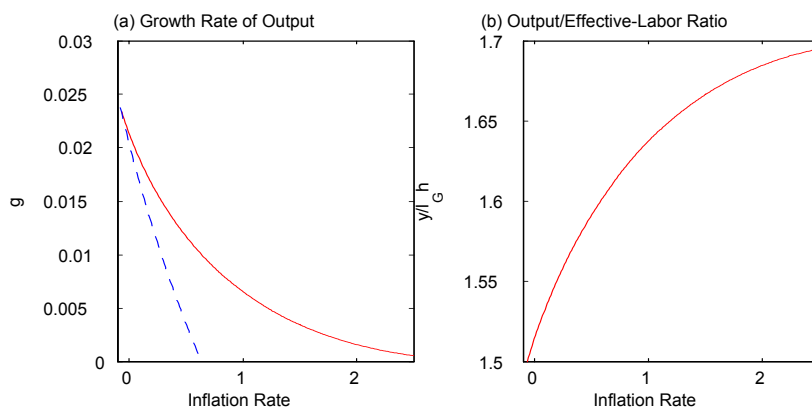
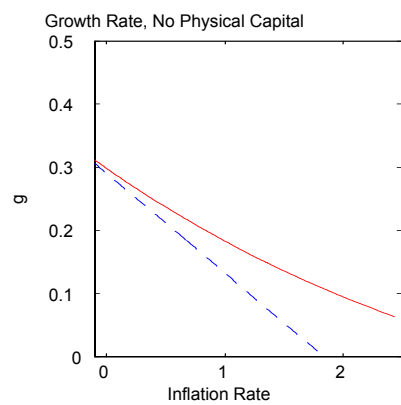Figure 2a,b. Inflation with Growth and Output per Effective Labor
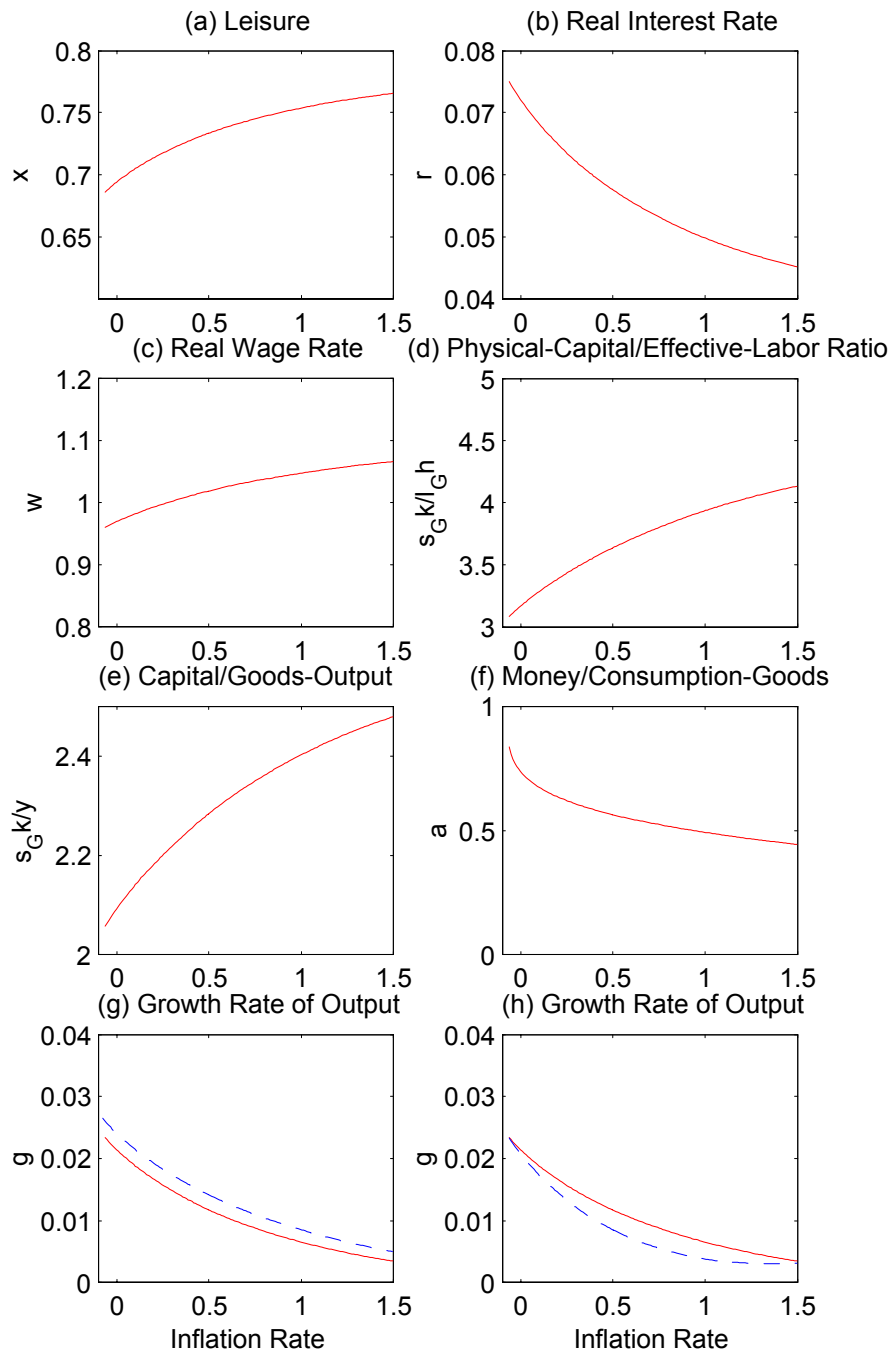
Figure 2c. Credit; Cash-Only (dashed)

Figure 3. Inflation with Other Balanced-Growth Path Variables