Motivated Sellers and Predation in the Housing Market

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ABSTRACT: We develop an equilibrium search model of the housing market where sellers may become distressed as they are unable to sell. A unique steady state equilibrium exists where distressed sellers attempt liquidation sales by accepting prices that are substantially below fundamental values. During periods where a large number of sellers are forced to liquidate customers exhibit 'predation': they hold off purchasing and strategically slow down the speed of trade, which in turn causes more sellers to become distressed. The model naturally suggests several proxies of liquidity. Interestingly, the average time on the market (TOM), one of the most frequently used statistics in the literature, does a poor job within the context of liquidation sales and predation. Specifically we show that TOM falls during periods of predatory buying, which, if interpreted on face value, indicates that the market becomes more liquid with predation. We propose an alternative proxy–the profit loss in fire sales–which appears to be a more robust measure of liquidity than TOM.
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JEL: D39, D49, D83

1 Introduction

Selling a house involves a long and non-trivial search process where the home seller faces a trade-off between the price and the time to sale. With sufficient time and no pressure to sell immediately, a seller can afford to wait to receive a price commensurate with the market value. However, due to factors such as bankruptcy, job loss, foreclosure, relocation, divorce etc. some sellers become 'distressed' and attempt to quickly sell and exit the market.

The presence of distressed sellers seems to affect buyers' purchasing behavior as well. During the recent housing crisis, for instance, where presumably a large number of sellers became distressed, buyers exhibited what can be termed as 'predation'. Despite falling prices customers were reluctant to purchase—appearing to be strategically delaying purchasing in an effort to obtain even better deals.

Based on these observations we develop an equilibrium search model of the housing market with two distinctive features. First, buyers' willingness to pay is private information and more importantly, second, sellers may become distressed, or 'motivated' in real estate parlance, as they wait to sell. We show that in equilibrium, financially distressed sellers accept prices substantially below fundamental values and consequently sell faster than regular sellers (liquidation sales). The more painful the shock, the lower the sale price and the quicker the sale.

Moreover, during periods where many sellers encounter financial distress (e.g. a crisis or recession) the following occur. First, the number of liquidation sales rises. Second, all sellers, regular and distressed, drop their prices. And most importantly, third, buyers exhibit predation: they become more selective and hold off purchasing despite the abundance of distressed sales and lower prices. By doing so, customers strategically slow down the speed of trade causing more sellers to become distressed, which in turn, exerts more pressure on sellers forcing them for further price cuts, and so on. From buyers' point of view such behavior is optimal as it allows them to acquire better houses at lower prices, but from sellers' point of view it is the worst possible outcome. Indeed, for distressed sellers liquidity disappears when it is most needed.

The model naturally suggests several proxies measuring liquidity from different angles. Curiously, though, the expected time on the market (TOM)—one of the most frequently used and referenced statistics in the literature—does a poor job in this context. We show that TOM falls during periods of predatory buying, which, if interpreted on face value, indicates that the market becomes more liquid with predation. We propose an alternative proxy, the profit loss in liquidation sales, which appears to be a more robust measure than TOM.

Finally, the model provides simple and intuitive answers to two puzzles raised by Merlo and Ortalo-Magné [15]. Based on a unique data set of individual residential property transactions in England, the authors document that about 2/3 of sellers do not change the listing price at all, while remaining sellers revise the listing price at least once (typically once). The fact that some sellers revise the listing price while others do not and that price revisions are infrequent and sizable are in stark contrast to the predictions of most existing theories in the housing market. In addition, based on the same data set the authors document a negative correlation between the sale price and the duration of the sale—the longer the time on the market, the lower the sale price. This fact, again, is inconsistent with most of the existing theoretical models.

According to our model some sellers revise the listing price while others do not, simply because some sellers become distressed while others do not. The revision occurs only once (when the shock hits) and it can be sizable if the shock is severe. The negative correlation is also easy to explain. Properties sold soon after the listing date are most likely 'regular sales'. Sellers of such properties are unlikely to become distressed within a short period of time. Sales taking place long after the listing date are most likely 'distressed', because the longer a seller waits, the more likely he is to become distressed. Since distressed sales occur at lower prices, the aforementioned negative correlation follows.

When constructing the model what we had in mind was the housing market, however the model is applicable in other settings characterized by (i) search frictions, (ii) informational asymmetry between buyers and sellers and (iii) the prospect of becoming distressed. As an example, consider the over the counter (OTC) markets; in particular markets for mortgage-backed securities, bank loans and derivatives among others. These markets share all three of the aforementioned characteristics. Indeed, search is a fundamental feature in many OTC markets, just as it is in the housing market, as it is difficult to identify a counterparty with whom there are likely gains from trade. Similarly informational asymmetry between buyers and sellers is a prevalent feature of the OTC markets as buyers' valuations are private information and it is not uncommon at all for parties to simply walk away without trading. Finally, traders may become financially distressed due to, for instance, pressing debt obligations, nearing margin calls, hedging motives or being caught in a "short squeeze". The model, therefore, is potentially applicable in this setting as well and anecdotal evidence suggests that the main results of the paper (fire sales and predation) indeed hold true in the OTC markets.[1]

This paper belongs to a literature that studies the housing market using search theory, e.g. see Yavas and Yang [21], Krainer [14], Wheaton [19] and Albrecht et al. [1], among others. The paper by Albrecht et al. is perhaps the closest to our model in terms of motivation and setup; however, it is based on complete information while ours is based on incomplete information. This difference is crucial because incomplete information is key in obtaining the predation result.

The paper proceeds as follows. In the next section we lay out the model. Section 3 presents the predation result, section 4 discusses prices, section 5 discusses liquidity and section 6 concludes.

2 Model

Time is continuous and infinite. The economy consists of a continuum of risk neutral buyers and sellers. Each seller is endowed with a house and each buyer seeks to purchase one. Buyers and sellers differ in terms of their intrinsic preferences towards ownership of a house, which creates the incentive to trade. For simplicity, we assume that the utility to the seller from keeping the house is zero. Buyers on the other hand receive periodic dividends (housing services) starting at the period after the purchase of the house and continuing forever. Following the asset pricing interpretation, we assume that the value of a house is captured by the discounted sum of the future dividends.

Sellers' personal circumstances may change for the worse if they are unable to sell for too long a period. All sellers enter the market in regular circumstances, though, eventually as they are unable to sell they might be hit by an idiosyncratic shock and become motivated or distressed. The adverse shock arrives at an exogenous Poisson rate $\mu > 0$ and may be associated with difficulties, financial or otherwise, forcing sellers into early liquidation. Regular and distressed sellers differ in terms of their time preferences. Buyers and regular sellers discount future utility by $(1+\delta)^{-1} > 0$, whereas distressed sellers are more impatient and discount the future by $\left(1+\overline{\delta}\right)^{-1} < (1+\delta)^{-1}$, which means that $\overline{\delta} > \delta$. Sellers do not exit the market until they sell and a distressed seller remains distressed. The parameters of interest are the frequency of the shock, μ, and the severity of the shock, $\overline{\delta}$.

[1] For an application of search theory in OTC markets see [10], for predation in financial markets see Attari et al. [3], Brunnermeier and Pedersen [5] or Carlin et al. [7], Ozcan et al. [17] among others.

Transactions are bilateral and involve a non-trivial search process. At any point in time buyers and sellers meet each other at a constant Poisson rate $\alpha > 0$.[2] Upon inspecting the house, a buyer realizes his own valuation of the house $v \in [0,1]$, which is a random draw from the unit interval via c.d.f. $F(v)$. Buyers are identical in the sense that their valuations are generated by the same random process, however they may differ in their valuations for any particular house. This specification captures the notion that different buyers have different tastes and preferences and, therefore, will have different reservation prices. The realization of $v \in [0,1]$ is match specific, so when buyers search they in fact search for a high v. We assume that v is time invariant; so, once a buyer finds and purchases a house with a sufficiently high v then he continues to enjoy the same v forever. We impose log-concavity on the survival function, which is a crucial technical assumption to obtain several key results in the paper.[3]

Assumption 1. *The density function $f(v)$ is strictly positive, whereas the survival function $S \doteq 1-F$ is log-concave, that is*

$$f^2(v) + f'(v)\, S(v) > 0, \ \forall v.$$

The realization of v is unobservable to the seller. The seller only knows the c.d.f. F generating v, so, he advertises a *list price* l, trading off the probability of sale with revenue. The *sale price* $p(l)$, depends on the list price but may involve a non-trivial renegotiation process (more on this later). If agents agree to trade at price p then the seller receives payoff p; the buyer receives dividends v starting at the beginning of the next period and continuing forever; both agents leave the search market and are replaced by a buyer and a regular seller. The replacement assumption is standard in the literature; it is needed to maintain stationarity. Agents who do not trade receive a period payoff of zero and continue to the next round to play the same game.

2.1 Sale Price

In the housing market, transactions rarely occur at the list price; the sale price typically involves a hard bargain between the buyer and the seller. We are not particularly interested how agents interact with each other as they negotiate, so we treat the renegotiation mechanism (be it Nash bargaining, strategic bargaining or even some esoteric price formation procedure) as a black box; however, we specify some mild properties that the resulting sale price ought to satisfy. As long as the renegotiation mechanism satisfies these properties our results go through. More formally, let $G\langle l, \alpha \rangle$ denote an extensive form game that induces some expected sale price $p(l) : [0,1] \to [0,1]$ for any given list price l and contact frequency α.

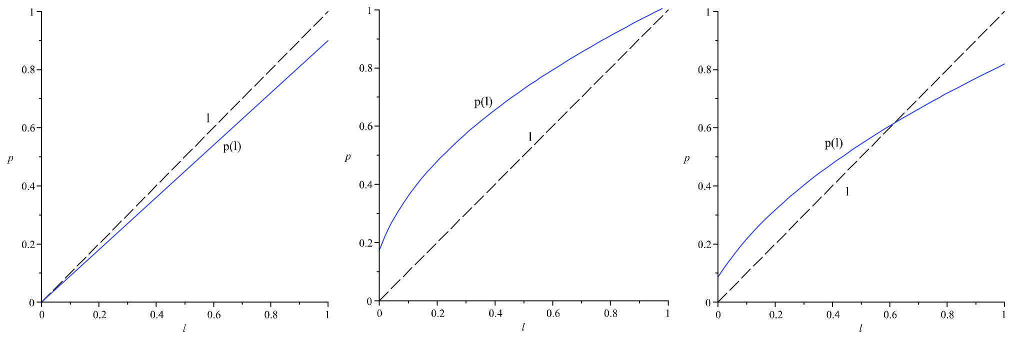
Assumption 2. *The sale price $p(l) : [0,1] \to [0,1]$ is an increasing and differentiable function of l.*

If $G\langle l, \alpha \rangle$ has multiple equilibria and, therefore, generates multiple sale prices (which, typically, is the case with bargaining models with private information, e.g. see the survey by Kennan and Wilson

[2] What we have in mind is a Mortensen-Pissarides style random matching function where arrival rates are functions of the market tightness (buyer-seller ratio). Typically, one assumes different measures of buyers and sellers so that arrival rates for buyers and sellers may vary. However, to avoid excessive parameterization, we simply assume equal measures, which means that agents meet each other with the same rate α.

[3] Log-concavity of the survival function is equivalent to the ratio of the density to the survival being monotone increasing and many well known distributions including Uniform, Normal, Exponential, χ^2 satisfy this property. See [4] for more details.

[12] and the references therein), then we assume that there is an equilibrium selection device that uniquely pins down $p(l)$. The game, the selection device and the resulting sale price function $p(l)$ are all common knowledge.



1a 1b 1c

Figure 1 – Sale Prices

Figure 1 illustrates some possible sale price functions. Panel a depicts an environment where the transaction takes place at 10% below list price. In 1b, sale price almost always exceeds list price—much as the real estate market in Santa Monica, CA. In 1c, sale price is above or below list price depending on how much sellers ask for; the house is sold above list price if list price is low and it is sold below list price, otherwise.

We admit that the shape of the sale price function should be endogenous and depend on the fundamentals of the market. This paper's purpose, however, is not to explain why a certain pricing practice emerges in this market and not in another. Instead, we want to investigate what happens to prices, volume of trade, and above all, buyers' purchasing behavior when some sellers become distressed. For this purpose, the shape of the sale price function can take any form; all we need is that it satisfies Assumption 2.

We move on to discuss buyers' and sellers' problems. We denote a seller's type by $j = r, d$ (where r refers to regular sellers and d refers to distressed sellers). We focus on a symmetric steady state equilibrium where identical agents follow identical strategies. In particular, a type j seller advertises a list price l_j, corresponding to the sale price $p_j \doteq p(l_j)$. Buyers, upon meeting a type j seller, purchase if their private valuation v (willingness to pay) of the house exceeds an endogenous threshold v_j.

2.2 Buyer's Problem

The problem of a representative buyer has a recursive formulation. We use a dynamic programming approach, letting Ω denote the value of search to a buyer. In a symmetric pure strategy equilibrium the distribution of prices $\mathbf{p}^* = (p_r^*, p_d^*)$ is degenerate. Clearly Ω is a function of $\mathbf{p}^*$, however, we omit the argument when this is understood. We have

$$\delta\Omega = \alpha\theta \int_0^1 \max\{v/\delta - p_s - \Omega, 0\}\, dF(v) + \alpha(1-\theta) \int_0^1 \max\{v/\delta - p_r - \Omega, 0\}\, dF(v).$$

A buyer's lifetime utility from owning a house that yields v per period equals v/δ. The parameter θ is the endogenous fraction of distressed sellers; so, with probability $\alpha\theta$ a buyer meets a distressed seller who sells for p_d. If the consumer surplus $v/\delta - p_d$ exceeds the value of search Ω then the buyer purchases, otherwise he walks away. Similarly, with probability $\alpha(1-\theta)$ the buyer encounters a regular seller who sells for p_r. Again, if the consumer surplus exceeds the value of search then the buyer purchases, otherwise, he keeps searching.

For any given sale price p_j we conjecture an associated reservation value

$$v_j = \delta(p_j + \Omega) \tag{1}$$

such that the customer purchases only if $v \geq v_j$. The implication is that a buyers' search process amounts to finding a house with a sufficiently high v. Obviously, not all meetings result in trade; for trade to occur the house must turn out to be a good match for the buyer, which happens with probability

$$\Pr(v \geq v_j) = 1 - F(v_j) = S(v_j).$$

A high v_j means that buyers are unlikely to purchase (they are selective). Observe that there are two types of trading frictions in the model. The first is locating a vacant house, which is captured by the meeting probability α, and the second is whether the house, once found, is a good match, which is captured by the probability $S(v_j)$.[4] Inserting the reservation values into Ω and using integration by parts we obtain

$$\Omega = \frac{\alpha\theta}{\delta^2}\int_{v_d}^{1} S(v)\,dv + \frac{\alpha(1-\theta)}{\delta^2}\int_{v_r}^{1} S(v)\,dv. \tag{2}$$

2.3 Fraction of Distressed Sellers

The steady state fraction of distressed sellers, θ, is endogenous and can be obtained by equating the inflow into the pool of distressed sellers to the outflow from the pool. The inflow equals to $(1-\theta)\mu$, whereas the outflow is $\theta\alpha S(v_d)$. Therefore,

$$\theta = \frac{\mu}{\mu + \alpha S(v_d)} \in (0,1). \tag{3}$$

Observe that θ depends on arrival rate of the adverse shock, μ, and meeting probability, α. It is easy to see that θ rises in μ and falls in α. More importantly, θ depends on the probability of trade $S(v_d)$ which is endogenous and controlled by buyers. Observe that buyers can squeeze the outflow and raise θ by becoming more selective (i.e. by raising the threshold v_d). Put differently, buyers can strategically slow down the speed of trade and, thereby, cause more sellers to become distressed. This is the basic mechanism behind the predation result in Section 3.

Lemma 1 *We have $\partial\Omega/\partial v_d < 0$ and $\partial\Omega/\partial v_r < 0$.*

The Lemma has two implications. First, buyers' value of search falls as the market becomes less liquid, i.e. Ω falls as v_r and v_d go up. Second, sellers face a trade-off between revenue and liquidity. Indeed, the indifference condition (1) implies that

$$\frac{dv_j}{dp_j} = \frac{\delta}{1 - \delta\partial\Omega/\partial v_j} > 0,$$

[4] The fact that some meetings do not result in trade is in line with the empirical observation by Merlo and Ortalo-Magné [15]. Analyzing transaction histories of residential properties sold in England between 1995 and 1998, they find that about a third of all meetings resolve with no agreement. Most of the existing theoretical models of the housing market are in clear contradiction with this empirical observation, e.g. Arnold [2], Chen and Rosenthal [8], Yavas [20], Yavas and Yang [21]. Assuming complete information, these models imply that a match necessarily results in trade.

which says that the higher the price, the higher the threshold v_j. From seller's perspective, raising sale price p_j (by advertising a higher l_j) brings in a larger revenue, but lowers the chance of a sale. The seller's task is to find a balance between these two effects, which we discuss next.

2.4 Seller's Problem

A type $j = r, d$ seller advertises a list price l_j taking as given the sale price function $p(\cdot)$ and buyers' search decisions. The value functions are given by

$$\bar{\delta}\Pi_d = \alpha S(v_d)\max\{p_d - \Pi_d, 0\} \tag{4}$$

$$\delta\Pi_r = \alpha S(v_r)\max\{p_r - \Pi_r, 0\} + \mu(\Pi_d - \Pi_r). \tag{5}$$

A distressed seller who lists l_d (and consequently sells for p_d) meets a buyer with probability α, who purchases with probability $S(v_d)$. The seller agrees to trade only if the price exceeds his continued value of search i.e. if $p_d - \Pi_d \geq 0$. The second line can be interpreted similarly.

Conjecturing that $p_j \geq \Pi_j$, a type j seller solves

$$\max_{l_j} \Pi_j \ \text{ subject to } \ v_j = \delta(p_j + \Omega)$$

taking Ω as given.

The value functions are linked to each other and, therefore, it requires some algebra to solve the maximization problems. A complete analysis is provided in Appendix I; here, we simply record some key steps. The FOC of seller $j = r, d$ is given by

$$p_j - \Pi_j = \frac{S(v_j)}{\delta f(v_j)}.$$

Using the FOC and manipulating the value functions with straightforward algebra one can obtain profit maximizing sale prices for regular and distressed sellers

$$P_r = \frac{S(v_r)}{\delta f(v_r)} + \frac{\alpha S(v_r)^2}{\delta(\mu+\delta) f(v_r)} + \frac{\alpha\mu S(v_d)^2}{\delta\bar{\delta}(\mu+\delta) f(v_d)}, \tag{6}$$

$$P_d = \frac{S(v_d)}{\delta f(v_d)} + \frac{\alpha S(v_d)^2}{\bar{\delta}\delta f(v_d)}. \tag{7}$$

Notice that the upper case P_r and P_d denote the profit maximizing prices and the lower case p_r and p_d denote generic prices.

Lemma 2 *We have $\partial P_d/\partial v_d < \partial P_r/\partial v_d < 0$ and $\partial P_r/\partial v_r < \partial P_d/\partial v_r = 0$.*

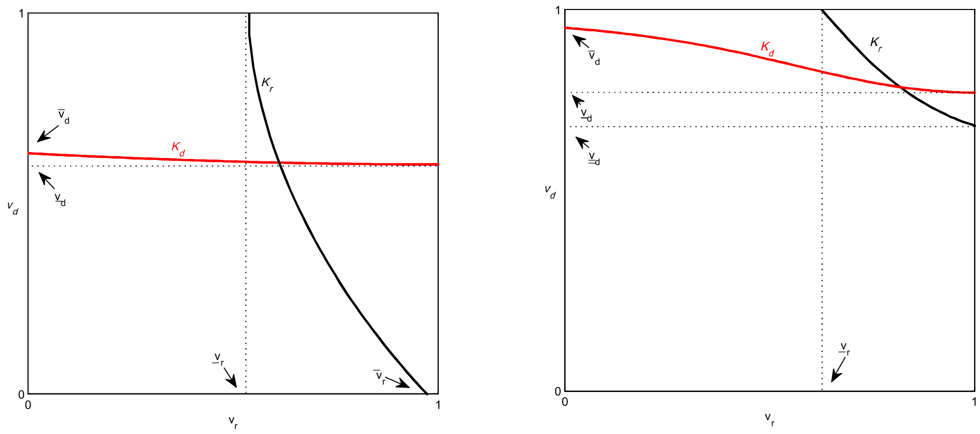
The Lemma has two implications. First, the negative relationship between prices and reservation values reflect the aforementioned trade-off between revenue and liquidity. For low values of v_j the probability of a sale is high, so sellers can afford to charge high prices; however, as v_j rises, liquidity concerns are initiated and prices fall. Second, a type j seller is more sensitive to his probability of sale than the other type is, which is why $\partial P_d/\partial v_d < \partial P_r/\partial v_d$ and $\partial P_r/\partial v_r < \partial P_d/\partial v_r$. Now, we are ready to close down the model.

Definition 3 *A steady-state symmetric equilibrium is characterized by the pair* $\mathbf{v}^* = (v_r^*, v_d^*)$ *satisfying*

$$\Delta_r := P_r + \Omega - v_r/\delta = 0 \text{ and } \Delta_d := P_d + \Omega - v_d/\delta. \tag{8}$$

Once the thresholds v_r^* and v_d^* are pinned down it is easy to obtain the equilibrium sale prices, list prices and the steady state fraction of distressed sellers. Specifically sale prices p_r^* and p_d^* are obtained by substituting v_r^* and v_d^* into (6) and (7). List prices l_r^* and l_d^* can be recovered from the sale price function $p(l)$. Finally the steady state fraction θ is obtained using (3).

Existence (and uniqueness) of the equilibrium amounts to showing that there exists a unique $\mathbf{v}^* \in [0,1]^2$ satisfying (8). To do so one needs to demonstrate that the locuses of Δ_r and Δ_d intersect once in the $v_r - v_d$ space. Lemma 8 in Appendix II establishes that the locuses look as they do in Figure 2. The fact that κ_r is steeper than κ_d, and the specific locations of the boundaries ($\underline{v}_d, \overline{v}_r$ etc.), together guarantee a unique intersection.



2a 2b

Figure 2 – Locuses

3 Distressed Sales and Predation

Proposition 4 *A steady state symmetric equilibrium exists and it is unique. In equilibrium distressed sellers accept lower prices and sell faster than regular sellers; more specifically* $p_d^* < p_r^*$ *and* $S(v_d^*) > S(v_r^*)$. *If the shock becomes more severe then prices fall even further and trade speeds up (i.e.* $dp_j^*/d\overline{\delta} < 0$ *and* $dS(v_j^*)/d\overline{\delta} > 0$*).*

A distressed home owner is impatient to sell, which is why he undercuts his competitors. The price cut produces the desired outcome. Indeed $S(v_d^*) > S(v_r^*)$ implies that distressed home owners are more likely to sell than regular home owners. The signs of the derivatives suggest that the more painful the shock, the lower the price and the quicker the trade. The simulation in Figure 6c provides further insight on this, where we plot a distressed seller's percentage wise profit loss against the severity of the shock $\overline{\delta}$. The profit loss is measured by

$$z = \frac{p_r^* - p_d^*}{p_r^*} \in (0,1).$$

Had the seller not become desperate, he would have been able to sell at p_r^*, but in a distressed sale he can only get p_d^*, so the difference $p_r^* - p_d^*$ equals his forgone profits. A high value of z means that distressed sellers need to offer substantial price cuts in order to sell quickly. Fig 6c shows that if the shock is mild ($\overline{\delta} \approx \delta$) then there is not much difference between what regular and distressed sellers charge, however, as the shock starts to bite ($\overline{\delta} \gg \delta$), then distressed sellers face considerable losses. We will come back to this point later in Section 5 when we discuss liquidity.

There is a particular study by Glower et al. [11] that we would like to mention here. The paper's goal is to determine the effects of seller motivation on prices, the time on the market, the speed of trade, etc. To do so, the authors survey sellers in Columbus, OH area to obtain information on sellers' motivations by asking whether or not they have a planned date to move out or they accepted a job offer elsewhere or they bought another house. The conclusion is that motivated sellers accept lower prices and sell more quickly. This seems to be consistent with the preceding discussion.

Proposition 5 *We have $dp_j^*/d\mu < 0$ and $dS(v_j^*)/d\mu < 0$ for $j = d, r$. If the adverse shock starts to arrive more often then prices fall, yet buyers hold off purchasing and strategically slow down the speed of trade, which in turn rises the percentage of desperate sellers in the market and reduces prices even further—an outcome which we term as 'predation'.*

When μ rises regular sellers face a higher likelihood of becoming distressed in the future. They accept lower prices to sell quickly before being hit by the shock, which is why $dp_r^*/d\mu < 0$ (one can call this the 'spill-over effect' of distressed sales on regular sales). Desperate sellers, on the other hand, face stiffer competition. Indeed the percentage of desperate sellers θ rises with the arrival rate of the adverse shock μ, so, realizing that there are many other sellers in the same dire situation, desperate sellers are forced to cut their already low prices. This is why $dp_d^*/d\mu < 0$.

Customers, on the other hand, exhibit what we call predatory buying; they delay purchasing despite falling prices. The reason is that, unlike sellers, buyers benefit from the growing θ.[5] Realizing that there are plenty of good deals in the market (higher θ) buyers find it optimal to search longer, which means that they become more selective and increase thresholds v_r^* and v_d^*. This response has a spiral effect. By raising the thresholds, buyers strategically slow down the speed of trade, causing more sellers to become distressed. The growing θ puts additional downward pressure on prices and the speed of trade, and so on.

The argument can be better understood by decomposing the effect of μ as follows.

$$\frac{dp_j^*}{d\mu} = \underbrace{\frac{\partial p_j^*}{\partial \mu}}_{\text{direct effect}} + \underbrace{\frac{\partial p_j^*}{\partial v_r^*}\frac{dv_r^*}{d\mu} + \frac{\partial p_j^*}{\partial v_d^*}\frac{dv_d^*}{d\mu}}_{\text{indirect effect due to buyers' behavior}}.$$

The first expression–the "direct effect"–captures the partial change in price p_j^*, ignoring the change in buyer's purchasing behavior. In the proof we show that $\frac{\partial p_j^*}{\partial \mu} \leq 0$, so prices fall when the μ rises. The second expression –the "indirect effect"–captures the change in p_j^* as a result of a change in v_r^* and v_d^*, which are controlled by buyers. In the proof we establish that this indirect effect is always negative which means that buyers' becoming more selective triggers further price cuts.

Predation is well documented in financial markets, see for instance Attari et al. [3], Brunnermeier and Pedersen [5] or Carlin et al. [7], Ozcan et al. [17] among others. Casual observations suggest that

[5] Buyers' value of search Ω increases in θ. In the proof of Proposition 5 we establish that $\Omega_\mu > 0$. Since $\theta_\mu > 0$ it follows that $\Omega_\theta > 0$.

in the real estate market, too, various forms of predation take place. Newspaper stories abound about potential buyers delaying their purchase and waiting for the 'right time' to enter the market. The number of such stories seems to have escalated during the recent housing crisis where, presumably, the arrival rate of the adverse shock μ went up. These observations seem to be consistent with the implications of the model. To the best of our knowledge, predation is not empirically documented in the real estate market.

4 Prices

4.1 Price Trajectories

According to the model, for any given property the trajectory of the list price is either flat or looks like a step function with a sizeable jump-down. Some sellers manage to sell without becoming distressed; so, for those properties the trajectory remains flat, throughout. Others, however, are hit by the adverse shock, so they have trajectories that look like a step function.

Interestingly, this is exactly what Merlo and Ortalo-Magné [15] observe empirically. Based on home sale transaction data from England, they find that 2/3 of sellers do not change their list price, 1/4 reduce only once, and the rest reduce twice or more. The individual list price trajectories are either flat or piecewise flat with, typically, one discontinuous jump-down (see Figure 2.1 in Merlo et al. [16]). Sellers wait, on average, 11 weeks to modify prices and reductions can be as high as 10%. These sizable and infrequent price revisions are inconsistent with most of the theoretical literature. Indeed existing models imply that, in equilibrium, either sellers never revise the price (e.g., Arnold [2], Chen and Rosenthal [8], Yavas and Yang [21]), or they gradually lower it in a continuous fashion (Coles [9]).

4.2 The Negative Relationship between Duration and Expected Sale Price

Merlo and Ortalo-Magné [15] document a negative correlation between sale price and duration of the sale (the longer the time on the market, the lower the sale price). This fact, again, is inconsistent with most of the existing theoretical models. Our setup provides a simple explanation: If a property is sold shortly after the listing date then it is most likely a regular sale. Indeed, given the Poisson arrival process, the owner is unlikely to become distressed within a short period of time. On the other hand, if the sale occurs long after the listing date then most likely it is a distressed sale. The longer the wait, the more likely is a seller to be hit by the shock. Since distressed sales occur at lower prices the aforementioned negative correlation follows. Below, we make these arguments more precise.

Consider a seller who enters the market at time 0 (wlog). The probability that he remains non-distressed without a sale until time t is given by

$$r(t) = e^{-(\mu+\alpha_r)t}. \tag{9}$$

The probability that he becomes distressed at some time $y \leq t$ while he is still unable to sell at t equals to

$$\varrho(t) = \int_0^t \mu e^{-\mu y} e^{-\alpha_r y} e^{-\alpha_d (t-y)} dy, \tag{10}$$

where $\mu e^{-\mu y}$ is the density of transition time y (exponential pdf). Now, consider all sales completed with duration t. The fraction of distressed sales equals

$$g(t) = \frac{\varrho(t)}{r(t) + \varrho(t)}.$$

One can easily verify that g rises in t (see the proof of Proposition 6), i.e., the longer the duration, the more likely sellers are to be distressed.

An immediate corollary is that the expected sale price falls with the duration. To see this more precisely, define the expected sale price

$$\overline{p}(t) = g p_d^* + (1-g) p_r^*$$

and the variance

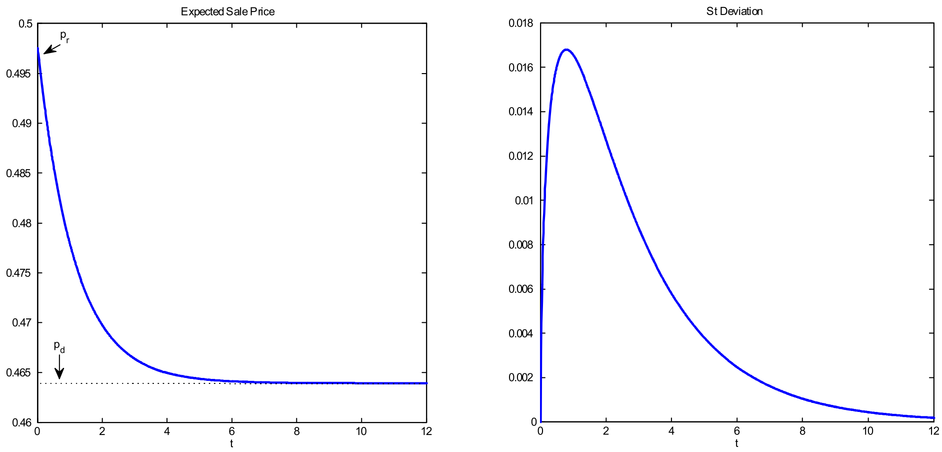
$$\sigma^2(t) = g\left(p_d^* - \overline{p}(t)\right)^2 + (1-g)\left(p_r^* - \overline{p}(t)\right)^2.$$

Proposition 6 *The expected sale price $\overline{p}(t)$ is monotone decreasing and the variance $\sigma^2(t)$ is hump-shaped in t.*

Figures 3a and 3b simulate $\overline{p}$ and σ. But before we proceed, let us outline which parameter values are used in these and subsequent simulations. The justification for these values comes from Merlo and Ortalo-Magné [15].

- $p(l) = 0.96l$: The sale price $p(l)$ equals to the 96% of the list price l. This follows from the observation that properties in [15]'s sample sell at about 96% of their listing price.
- $\alpha = 0.11$: In the sample in [15] the time it takes to meet a buyer is about 9 weeks, so we set $\alpha = 1/9$, which is about 0.11.
- $\mu = 0.09$: Before a price change sellers wait 11 weeks on average. We interpret the price change as a result of becoming distressed, so the the frequency of the shock $\mu = 1/11$, which roughly equals to 0.09.

The continuously downward slope in $\overline{p}$ may be somewhat misleading and may create an illusion that the transaction price continuously falls with respect to duration. We emphasize that an individual transaction price trajectory is piecewise flat with a discontinuous drop from p_r^* to p_d^* at the time the seller is hit by the shock. It is the *expected* price that falls monotonically; the transaction price is either p_r^* or p_d^*.



3a 3b

Figure 3 – Exp. Sale Price and the St. Dev.

The shape of the standard deviation is also intuitive. For very short or very long durations the sale is either non-distressed or distressed. Only for intermediate values of t there is ambiguity; hence, the hump shape.

5 Liquidity

The working definition of *liquidity* in this paper is the capacity of how fast one can sell a property without any 'loss in value'. There are two aspects of liquidity that we are interested in: speed of trade and profit loss in liquidation sales. The former can be measured either by probability of sale

$$\alpha_j \doteq \alpha S(v_j^*)$$

or expected time on the market, TOM. The probability of sale measures the speed of trade from an individual seller's point of view, whereas TOM is a market-wide weighted average taking into account all sellers, regular and distressed.

The second aspect of liquidity is the loss of value in liquidation sales. To measure it, we use the index

$$z = \frac{p_r^* - p_d^*}{p_r^*} \in (0, 1)\,, \tag{11}$$

which is distressed seller's percentage-wise profit loss compared to regular seller. A high value of z means that distressed sales occur far below regular sales, which indicates illiquidity. Below, we discuss the performance of these proxies within the context of liquidation sales and predation.

5.1 Time on the Market: TOM

We know that during periods when μ goes up customers exhibit predation and the probability of trade falls. So, one naturally expects TOM to go up in such times because sellers are less likely to trade but things are more subtle than that. Because of the rising μ more sellers become distressed and distressed sellers trade faster than regular sellers. This *transition effect* puts a downward pressure on TOM and blurs the picture. Below, we make these arguments more precise.

Proposition 7 *Density of time on the market is given by*

$$\gamma = \frac{\mu\alpha_d e^{-\alpha_d t} - (\alpha_d - \alpha_r)(\mu + \alpha_r) e^{-(\mu+\alpha_r)t}}{\mu - \alpha_d + \alpha_r}. \tag{12}$$

The pdf is hump-shaped if $\frac{\mu}{\alpha} > \frac{S(v_r^*)^2}{F(v_r^*) - F(v_d^*)}$ *and monotone decreasing, otherwise. The time on the market is given by*

$$TOM = \frac{\mu + \alpha S(v_d^*)}{\alpha S(v_d^*) \times \{\mu + \alpha S(v_r^*)\}}.$$

Now, we can analyze how TOM responds to a change in μ. We have

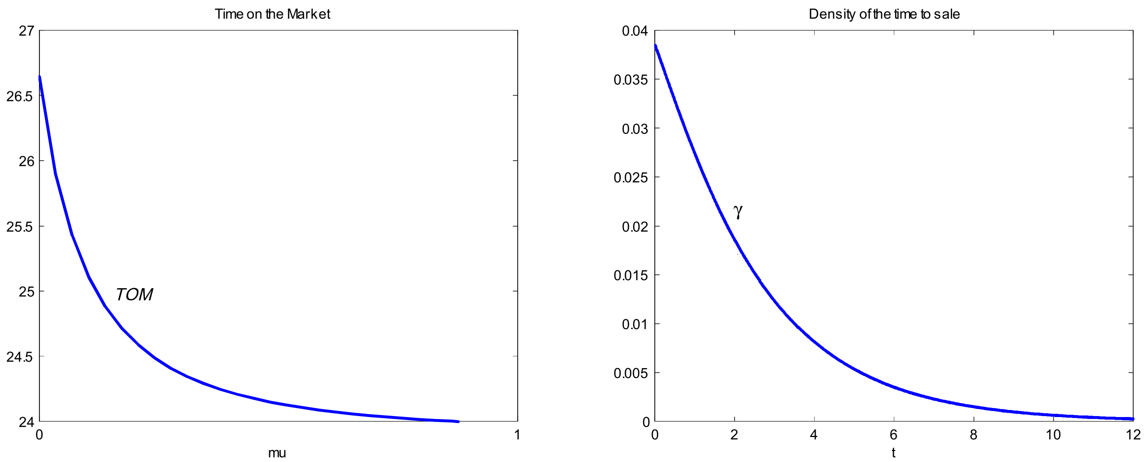
$$\frac{dTOM}{d\mu} = \frac{\partial TOM}{\partial v_r^*}\frac{dv_r^*}{d\mu} + \frac{\partial TOM}{\partial v_d^*}\frac{dv_d^*}{d\mu} + \frac{\partial TOM}{\partial \mu}.$$

The first two terms are positive because of the earlier predation result. The last term, however, is negative because

$$\frac{\partial TOM}{\partial \mu} \propto F(v_d^*) - F(v_r^*) < 0,$$

which simply reflects the fact that distressed sellers trade faster than regular sellers—confirming our intuition about the aforementioned transition effect. Analytically, it is difficult to sign $dTOM/d\mu$ but numerical simulations suggest that the transition effect is, in fact, more dominant; see Figure 4a.

Why is this important? TOM is one of the most frequently used and referred statistics in the housing literature. Low values of TOM are interpreted as an indication of high liquidity e.g. Krainer [14], Knight [13], Taylor [18]. Going with this interpretation the fact that $dTOM/d\mu < 0$ indicates that the market becomes *more* liquid during times where many sellers become distressed and attempt fire sales. It appears that in this particular setting the probability of trade for individual sellers α_j is a better proxy of liquidity than TOM. In data α_j is the percentage of meetings resulting in a sale and it clearly falls with μ.[6] This, in turn, means that the market becomes less liquid if μ rises.



4a 4b

Figure 4 – TOM and the density function

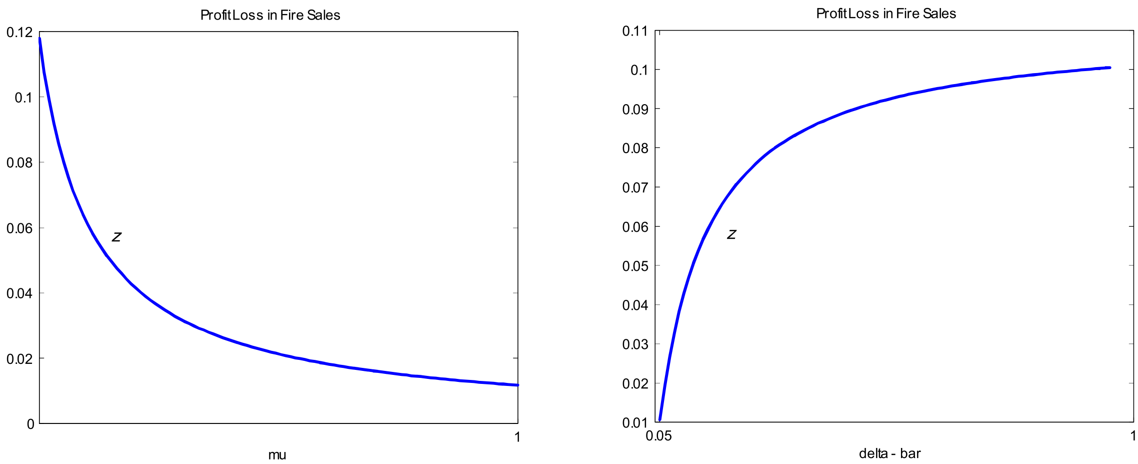
Finally, note that the density function γ is endogenous and it is derived from maximization behavior of buyers and sellers. Note that γ is skewed to the right because of the Poisson arrivals and it may

[6] Observe that $\frac{d\alpha_j}{d\mu} = -\alpha F(v_j^*) \frac{dv_j^*}{d\mu} < 0$.

be hump-shaped if the ratio μ/α is sufficiently large, i.e. if buyers are scarce and the adverse shock is frequent (Figure 4b). The shape of γ is indeed realistic. Merlo et al. [16], based on transaction data from England, obtain the empirical distribution of times to sale, which is right skewed and hump-shaped; see Figure 2.3, therein.

5.2 Profit Loss: z

Figures 5a and 5b below illustrate z against the frequency and the severity of the adverse shock.[7]



5a 5b

Figure 5 – Profit Loss in Fire Sales

The simulation in 5a suggests that attempting a liquidation sale is, in fact, *less* costly when μ is high. The reason is simple. During such times regular sellers, afraid of becoming distressed, substantially lower their prices to sell quickly and exit the market. This is the aforementioned 'spillover effect' of liquidation sales onto the regular sales. Regular sellers are more sensitive to a rise in μ than distressed sellers. Distressed sellers do not worry about being hit by the shock because they are already distressed. So, although both prices fall, the drop in p_r^* is sharper than the one in p_d^*, which is why z declines in μ.

Again, one has to be careful when interpreting this rather positive-looking result. In absolute terms, all sellers are worse off (all prices fall in μ). Only in relative terms, distressed sellers appear to be better off.

The relationship between z and $\overline{\delta}$ is more straightforward. The simulation in Figure 5b suggests that if the shock is mild ($\overline{\delta} \approx \delta$) then a liquidation sale is not too costly; however, as the shock starts to bite ($\overline{\delta} \gg \delta$), then distressed sellers face considerable losses. The reason is that desperate sellers are directly affected by a rise in $\overline{\delta}$, whereas, regular sellers worry about $\overline{\delta}$ only because they may become desperate in the future. The fall in p_d^* is sharper than the one in p_r^*, which is why z goes up.

[7] In panel a we fix $\delta = 0.05$ and $\overline{\delta} = 0.2$ which means that regular sellers' discount factor is about 95% and distressed sellers' discount factor is about 83%. In panel b the parameter $\overline{\delta}$, which by definition must exceed δ, ranges from 0.05 to 1. Recall that the higher the value of $\overline{\delta}$ the more severe the shock.

6 Conclusion

We have presented an equilibrium search model with three distinctive characteristics: (i) trade is decentralized; agents search for a counterparty to trade, (ii) a buyer's willingness to pay is private information and (iii) sellers may become financially distressed as they are unable to sell. We have found that, once distressed, sellers attempt liquidation sales by accepting prices that are substantially below fundamental values. In addition, during periods where a large number of sellers are forced to liquidate customers strategically hold off purchasing and slow down the speed of trade in an effort to obtain better deals—an outcome which we call predatory buying. The model suggests several proxies measuring liquidity, which we discuss in detail. Interestingly, the expected time on the market (TOM) appears to be doing a poor job in measuring liquidity. Indeed, we show that TOM falls during periods of predatory buying, which simply says that the market becomes more liquid with predation. We argue that, in this context, the percentage of profit loss in liquidation sales is a better proxy of liquidity than TOM.

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Appendix: Omitted Proofs

Proof of Lemma 1. Observe that θ does not depend on v_r. Hence

$$\frac{\partial\Omega}{\partial v_r} = -\frac{\alpha(1-\theta)}{\delta} S(v_r),$$

which clearly is negative. Now consider

$$\frac{\partial\Omega}{\partial v_d} = \frac{\alpha\theta'}{\delta}\int_{v_s}^{v_r} S(v)\,dv - \frac{\alpha\theta}{\delta} S(v_d), \tag{13}$$

where

$$\theta' = \frac{\partial\theta}{\partial v_d} = \frac{\theta\alpha f(v_d)}{\mu + \alpha S(v_d)} > 0.$$

To show that $\partial\Omega/\partial\underline{v}_d < 0$ it suffices to demonstrate that

$$\eta(v_d) := \int_{v_d}^{1} S(v)\,dv - \frac{\mu}{\alpha}\frac{S(v_d)}{f(v_d)} - \frac{S(v_d)^2}{f(v_d)} < 0.$$

Omitting the argument and differentiating with respect to $\underline{v}_d$ we have

$$\eta' = \frac{f^2 + f'S}{f'^2}\left[\frac{\mu}{\alpha} + S\right]$$

which is positive under Assumption 1. Since η increases in v_d and $\eta(1) = 0$, it follows that $\eta(v_d) < 0$, $\forall v_d \in [0,1)$. ■

Maximization Problems of Distressed and Regular Sellers

Distressed Sellers. We start with the distressed sellers' problem. Note the followings:

- The sale price p is a function of l; p_j simply stands for $p(l_j)$. Because of Assumption 2 we have $p_j' > 0$.
- The indifference constraint $v_d = \delta(p_d + \Omega)$ implies that $v_d' = \delta p_d' > 0$.
- Sellers take Ω as given, thus $\Omega' = 0$.

Conjecturing that $p_d \geq \Pi_d$ rewrite (4) as $\bar{\delta}\Pi_d = \alpha S(v_d)(p_d - \Pi_d)$. Keeping the preceding points in mind differentiate Π_d with respect to l_d to obtain

$$\bar{\delta}\Pi_d' = -\alpha\delta f(v_d)\,p_d'(p_d - \Pi_d) + \alpha S(v_d)\left(p_d' - \Pi_d'\right).$$

The FOC is given by

$$\Pi_d' = 0 \Rightarrow p_d - \Pi_d = \frac{S(v_d)}{\delta f(v_d)}. \tag{14}$$

Substitute the FOC into the expression for Π_d to obtain

$$\Pi_d = \frac{\alpha S(v_d)^2}{\bar{\delta}\delta f(v_d)}. \tag{15}$$

Expressions (14) and (15) together imply that the profit maximizing price P_d equals to the expression on display at (7). To verify the second order condition, differentiate the expression Π_d' above and use the FOC (14) along with the fact that $\Pi_d' = 0$ to obtain (we omit the argument v_d where understood):

$$\Pi_d'' = -\frac{\delta p_d'^2}{\bar{\delta}/\alpha + S} \times \left[\frac{f'S + f^2}{f} + f\right].$$

The first multiplicative term is negative, whereas the second term is positive because of log-concavity. It follows that $\Pi'' < 0$; thus the solution to the first order condition yields a maximum.

Regular Sellers. The problem of a regular seller is similar. We have

$$\delta \Pi_r = \alpha S(v_r)(v_r - \Omega - \Pi_r) + \mu(\Pi_d - \Pi_r). \tag{16}$$

Differentiate Π_r with respect to l_r to obtain the first-order condition (imposing $\Omega' = \Pi_d' = 0$ since they are taken as given):

$$\Pi_r' = 0 \Leftrightarrow p_r - \Pi_r = \frac{S(v_r)}{\delta f(v_r)}. \tag{17}$$

The second order condition can be verified similarly. Use (15), (16) and (17) to get

$$\Pi_r = \frac{\alpha S(v_r)^2}{\delta(\mu+\delta) f(v_r)} + \frac{\alpha \mu S(v_d)^2}{\bar{\delta}\delta(\mu+\delta) f(v_d)}.$$

Substitute Π_r into (17) to obtain the profit maximizing price of a regular seller, given by (6). ■

Proof of Lemma 2.

Differentiate P_r and P_d, given by (6) and (7), with respect to v_r and v_d to obtain:

$$\frac{\partial P_r}{\partial v_r} = -\frac{f_r^2 + f_r' S_r}{\delta f_r^2} - \frac{\alpha S_r}{\delta(\mu+\delta)}\left[\frac{2f_r^2 + f_r' S_r}{f_r^2}\right] < 0, \tag{18}$$

$$\frac{\partial P_r}{\partial v_d} = -\frac{\alpha \mu S_d}{\delta \bar{\delta}(\mu+\delta)}\left[\frac{2f_d^2 + f_d' S_d}{f_d^2}\right] < 0, \quad \frac{\partial P_d}{\partial v_r} = 0, \tag{19}$$

$$\frac{\partial P_d}{\partial v_d} = -\frac{f_d^2 + f_d' S_d}{\delta f_d^2} - \frac{\alpha S_d}{\delta \bar{\delta}}\left[\frac{2f_d^2 + f_d' S_d}{f_d^2}\right] < 0, \tag{20}$$

where $S_j := S(v_j)$. The expression $\frac{\partial P_r}{\partial v_r}$, $\frac{\partial P_r}{\partial v_d}$ and $\frac{\partial P_d}{\partial v_d}$ are negative because of log-concavity. The fact that $\frac{\partial P_d}{\partial v_d} < \frac{\partial P_r}{\partial v_d}$ is immediate after comparing them term by term. ■

Proof of Proposition 4. The proof of existence and uniqueness is in Appendix II. In what follows we show that $S(v_d^*) > S(v_r^*)$ and $p_r^* > p_d^*$ (the "fire sales result"). By contradiction, suppose that $v_r^* = v_d^* = v$ and notice that

$$\Delta_r(v,v) - \Delta_d(v,v) = \frac{\alpha\left(\bar{\delta} - \delta\right)}{\bar{\delta}\delta(\mu+\delta)} \times \frac{S(v)^2}{f(v)} > 0,$$

which contradicts the equilibrium condition $\Delta_r(v_r^*, v_d^*) - \Delta_d(v_r^*, v_d^*) = 0$. Observe that

$$\frac{\partial(\Delta_r - \Delta_d)}{\partial v_r} = \frac{\partial P_r}{\partial v_r} - \frac{1}{\delta} - \frac{\partial P_d}{\partial v_r} < 0$$

because $\frac{\partial P_r}{\partial v_r} < 0$ and $\frac{\partial P_d}{\partial v_r} = 0$ (Lemma 2). It follows that $\Delta_r(v_r^*, v_d^*) = \Delta_d(v_r^*, v_d^*)$ is satisfied only when $v_r^* > v_d^*$, which in turn means that $S(v_d^*) > S(v_r^*)$. The inequality $p_r^* > p_d^*$ is immediate since $p_r^* - p_d^* = (v_r^* - v_d^*)/\delta > 0$. For future reference we note that

$$p_r^* - p_d^* = \frac{S_r}{\delta f_r} - \frac{S_d}{\delta f_d} + \frac{\alpha}{(\mu+\delta)}\left[\frac{S_r^2}{\delta f_r} - \frac{S_d^2}{\overline{\delta} f_d}\right] > 0. \tag{21}$$

The rest of the proposition, namely the claims $dp_j^*/d\overline{\delta} < 0$ and $dS(v_j^*)/d\overline{\delta} > 0$, are proved below. ■

Proof of Proposition 5. The equilibrium values of v_r^* and v_d^* simultaneously satisfy

$$\Delta_r(v_r^*, v_d^*) = 0 \text{ and } \Delta_d(v_r^*, v_d^*) = 0.$$

Omit the superscript * and note that (General Implicit Function Theorem)

$$\frac{dv_j}{du} = \frac{\det B_j(u)}{\det A}, \text{ for any } u = \overline{\delta}, \mu \text{ and } j = r, d,$$

where

$$B_r(u) = \begin{bmatrix} -\frac{\partial \Delta_r}{\partial u} & \frac{\partial \Delta_r}{\partial v_d} \\ -\frac{\partial \Delta_d}{\partial u} & \frac{\partial \Delta_d}{\partial v_d} \end{bmatrix},\ B_d(u) = \begin{bmatrix} \frac{\partial \Delta_r}{\partial v_r} & -\frac{\partial \Delta_r}{\partial u} \\ \frac{\partial \Delta_d}{\partial v_r} & -\frac{\partial \Delta_d}{\partial u} \end{bmatrix},\ A = \begin{bmatrix} \frac{\partial \Delta_r}{\partial v_r} & \frac{\partial \Delta_r}{\partial v_d} \\ \frac{\partial \Delta_d}{\partial v_r} & \frac{\partial \Delta_d}{\partial v_d} \end{bmatrix}.$$

Note that

$$\det A = \frac{\partial \Delta_r}{\partial v_r}\frac{\partial \Delta_d}{\partial v_d} - \frac{\partial \Delta_d}{\partial v_r}\frac{\partial \Delta_r}{\partial v_d} > 0$$

since

$$\frac{\partial \Delta_r}{\partial v_r} < \frac{\partial \Delta_d}{\partial v_r} < 0 \text{ and } \frac{\partial \Delta_d}{\partial v_d} < \frac{\partial \Delta_r}{\partial v_d} < 0 \text{ (see (22) and (23))}.$$

It follows that

$$sign\left(\frac{\partial v_j}{\partial u}\right) = sign\left(\det B_j(u)\right).$$

Below we investigate the signs of the determinants. To do so we need the following partial derivatives.

Partial Derivatives. Here we obtain the partial derivatives of Ω, P_d and P_r with respect to $\overline{\delta}$ and μ. To start, differentiate Ω, given by (2), to obtain

$$\frac{\partial \Omega}{\partial \overline{\delta}} = 0 \quad \text{and} \quad \frac{\partial \Omega}{\partial \mu} = \frac{\theta^2 \alpha S_d}{\delta \mu^2}\int_{v_d}^{v_r} S(v)\,dv > 0.$$

Notice that $\frac{\partial \Omega}{\partial \mu}$ is positive since $v_r^* > v_d^*$. Now differentiate P_r and P_d, given by (6) and (7), to obtain

$$\frac{\partial P_d}{\partial \overline{\delta}} = -\frac{\alpha S_d^2}{\overline{\delta}^2 \delta f_d} < 0, \qquad \frac{\partial P_r}{\partial \overline{\delta}} = \frac{\mu}{(\mu+\delta)}\frac{\partial P_d}{\partial \overline{\delta}} < 0$$

$$\frac{\partial P_d}{\partial \mu} = 0, \qquad \frac{\partial P_r}{\partial \mu} = -\frac{\alpha}{(\mu+\delta)^2}\left[\frac{S_r^2}{\delta f_r} - \frac{S_d^2}{\overline{\delta} f_d}\right] < 0.$$

Note that f_j and F_j stand for $f(v_j^*)$ and $F(v_j^*)$. The signs of the first three expressions are obvious. To see why $\frac{\partial P_r}{\partial \mu} < 0$ focus on the inequality above in (21) and notice that $\frac{\partial P_r}{\partial \mu}$ is negative if in (21) the expression in square brackets is positive. The term

$$\frac{S_r}{f_r} - \frac{S_d}{f_d}$$

in (21) is negative because S/f decreases in v (log-concavity) and $v_r^* > v_d^*$. Therefore the expression in square brackets in (21) must be positive.

Reserve Values v_r^* and v_d^*. Now we can investigate the signs of $dv_j^*/d\overline{\delta}$ and $dv_j^*/d\mu$. To do so we need to determine the signs of $\det B_j\left(\overline{\delta}\right)$ and $\det B_j\left(\mu\right)$.

- Since $\partial\Omega/\partial\overline{\delta} = 0$ we have
$$\det B_d\left(\overline{\delta}\right) = \frac{\partial\Delta_d}{\partial v_r}\frac{\partial P_r}{\partial\overline{\delta}} - \frac{\partial\Delta_r}{\partial v_r}\frac{\partial P_d}{\partial\overline{\delta}}.$$
Furthermore, since
$$\frac{\partial P_d}{\partial\overline{\delta}} < \frac{\partial P_r}{\partial\overline{\delta}} < 0 \text{ and } \frac{\partial\Delta_r}{\partial v_r} < \frac{\partial\Delta_d}{\partial v_r} < 0$$
it follows that $\det B_d\left(\overline{\delta}\right) < 0 \Longrightarrow dv_d^*/d\overline{\delta} < 0$. Hence $dS(v_d^*)/d\overline{\delta} > 0$.

- Because $\partial P_d/\partial\mu = 0$ we have
$$\det B_d\left(\mu\right) = \frac{\partial\Delta_d}{\partial v_r}\frac{\partial P_r}{\partial\mu} + \frac{\partial\Omega}{\partial\mu}\left[\frac{\partial\Delta_d}{\partial v_r} - \frac{\partial\Delta_r}{\partial v_r}\right].$$
Since
$$\frac{\partial P_r}{\partial\mu} < 0,\ \frac{\partial\Omega}{\partial\mu} > 0 \text{ and } \frac{\partial\Delta_r}{\partial v_r} < \frac{\partial\Delta_d}{\partial v_r} < 0$$
it follows that $\det B_d\left(\mu\right) > 0 \Longrightarrow dv_d^*/d\mu > 0$. Hence $dS(v_d^*)/d\mu < 0$.

- Since
$$\frac{\partial P_r}{\partial\overline{\delta}} = \frac{\mu}{\mu+\delta}\frac{\partial P_d}{\partial\overline{\delta}} < 0 \text{ and } \frac{\partial\Omega}{\partial\overline{\delta}} = 0$$
it is easy to verify that
$$\det B_r\left(\overline{\delta}\right) = \frac{\partial P_d}{\partial\overline{\delta}}\left[\frac{\partial P_r}{\partial v_d} - \frac{\mu}{\mu+\delta}\frac{\partial P_d}{\partial v_d} + \frac{\delta}{\mu+\delta}\frac{\partial\Omega}{\partial v_d} + \frac{\mu}{\mu+\delta}\right].$$
The expressions for $\frac{\partial\Omega}{\partial v_d}$, $\frac{\partial P_r}{\partial v_d}$ and $\frac{\partial P_d}{\partial v_d}$ are given by (13), (18) and (20). Using these, one can show that the expression inside the square brackets equals to
$$\frac{\mu}{\mu+\delta}\frac{f_d^2 + f_d' S_d}{f_d^2} + \frac{\alpha\theta'}{\mu+\delta}\int_{v_d}^{v_r} S\left(v\right)dv + \frac{\mu\theta}{\mu+\delta} > 0.$$
The first term is positive because of log-concavity and the second term is positive since $\theta' > 0$ and $v_r^* > v_d^*$. It follows that $\det B_r\left(\overline{\delta}\right) < 0 \Longrightarrow dv_r^*/d\overline{\delta} < 0$. Thus $dS(v_r^*)/d\overline{\delta} > 0$.

- Recalling $\partial P_d/\partial\mu = 0$ we obtain
$$\det B_r\left(\mu\right) = \frac{\partial\Omega}{\partial\mu}\left[\frac{\partial\Delta_r}{\partial v_d} - \frac{\partial\Delta_d}{\partial v_d}\right] + \frac{\partial\Delta_d}{\partial v_d}\frac{\partial p_r}{\partial\mu}.$$
The first term is positive since
$$\frac{\partial\Delta_d}{\partial v_d} < \frac{\partial\Delta_r}{\partial v_d} < 0 \text{ and } \frac{\partial\Omega}{\partial\mu} > 0.$$
The second term is also positive since $\partial\Delta_d/\partial v_d < 0$ and $\partial P_r/\partial\mu < 0$. It follows that $\det B_r\left(\mu\right) > 0 \Longrightarrow dv_r^*/d\mu > 0$. Hence $dS(v_r^*)/d\mu < 0$.

Finally, we investigate the signs of $dp_j^*/d\overline{\delta}$ and $dp_j^*/d\mu$.

Prices. Totally differentiating p_j^* with respect to μ one obtains

$$\frac{dp_j^*}{d\mu} = \frac{\partial p_j^*}{\partial \mu} + \frac{\partial p_j^*}{\partial v_r^*}\frac{dv_r^*}{d\mu} + \frac{\partial p_j^*}{\partial v_d^*}\frac{dv_d^*}{d\mu}.$$

Recall that

$$\frac{\partial p_r^*}{\partial \mu} < \frac{\partial p_d^*}{\partial \mu} = 0,\ \frac{\partial p_j^*}{\partial v_r} \leq 0,\ \frac{\partial p_j^*}{\partial v_d} < 0 \text{ and } \frac{dv_j^*}{d\mu} > 0.$$

Hence $dp_j^*/d\mu < 0$. To show $dp_j^*/d\overline{\delta} < 0$, recall that $p_j^* = v_j^*/\delta - \Omega$ in equilibrium. Differentiation with respect to $\overline{\delta}$ yields

$$\frac{dp_j^*}{d\overline{\delta}} = \frac{dv_j^*}{\delta d\overline{\delta}} - \frac{\partial \Omega}{\partial v_r}\frac{dv_r^*}{d\overline{\delta}} - \frac{\partial \Omega}{\partial v_d^*}\frac{dv_d^*}{d\overline{\delta}},$$

which is negative since $dv_j^*/d\overline{\delta} < 0$ and $\partial\Omega/\partial v_j^* < 0$. ■

Proof of Proposition 6. Notice that

$$\frac{d\overline{p}\,(t)}{dt} = -\frac{dg\,(t)}{dt}\,(p_r^* - p_d^*)$$

One can verify that

$$\frac{dg\,(t)}{dt} \propto \mu e^{-(\alpha_d+\alpha_r+\mu)} > 0.$$

It follows that $\overline{p}' < 0$ since $p_r^* > p_d^*$. Finally note that

$$\frac{d\sigma^2}{dt} = (p_r^* - p_d^*)\left[g'\,(2\overline{p} - p_r^* - p_d^*) + 2g\overline{p}'\right].$$

Clearly $d\sigma^2/dt$ shares the sign of the expression in the square brackets, since $p_r^* > p_d^*$. One can verify that $\lim_{t\to 0} g\,(t) = 0$ and $\lim_{t\to\infty} g\,(t) = 1$ so that $\lim_{t\to 0}\overline{p}\,(t) = p_r^*$ and $\lim_{t\to\infty}\overline{p}\,(t) = p_d^*$. It follows that $d\sigma^2/dt$ is positive for t small and negative for t large because $g' > 0$ and $\overline{p}' < 0$. In other words σ^2 first rises and subsequently falls with t. ■

Proof of Proposition 7. Using (9) and (10) one can obtain density of time to sale γ and expected time to sale TOM. We have

$$TOM = \int_0^\infty [r\,(t) + \varrho\,(t)]\,dt \quad \text{and} \quad \gamma = -\left[dr\,(t) + d\varrho\,(t)\right]/dt.$$

Basic algebra reveals that TOM and γ are given by the expressions on display in Proposition 7. It is easy to verify that γ is positive and that

$$\int_0^\infty \gamma dt = -\left[r\,(t) + \varrho\,(t)\right]|_0^\infty = 1.$$

To analyze the shape of γ note that

$$\gamma' = \frac{-\mu\alpha_d^2 e^{-\alpha_d t} + (\alpha_d - \alpha_r)\,(\mu + \alpha_r)^2\,e^{-(\mu+\alpha_r)t}}{\mu - \alpha_d + \alpha_r},$$

where

$$\alpha_d - \alpha_r = \alpha\left[F\,(v_r^*) - F\,(v_d^*)\right] > 0 \text{ since } v_r^* > v_d^*.$$

Notice that the denominator could be either positive or negative. It follows that:

$$\text{If } \mu > \alpha_d - \alpha_r \text{ then } \gamma'(t) > 0 \Leftrightarrow \frac{(\alpha_d - \alpha_r)(\mu + \alpha_r)^2}{\mu \alpha_d^2} > e^{(\mu + \alpha_r - \alpha_d)t}.$$
$$\text{If } \mu < \alpha_d - \alpha_r \text{ then } \gamma'(t) > 0 \Leftrightarrow \frac{(\alpha_d - \alpha_r)(\mu + \alpha_r)^2}{\mu \alpha_d^2} < e^{(\mu + \alpha_r - \alpha_d)t}.$$

First note that $\lim_{t\to\infty} \gamma' < 0$, i.e., γ is monotone decreasing for t large. Now evaluate $\lim_{t\to 0} \gamma$. Note that in the first line the exponential term is minimum when $t = 0$ whereas in the second line it is maximum when $t = 0$. Hence

$$\gamma'(0) > 0 \text{ if } \frac{\mu}{\alpha} > \frac{S(v_r^*)^2}{F(v_r^*) - F(v_d^*)}.$$

Clearly if $\gamma'(0) > 0$ then γ first rises and then falls (hump-shape). Otherwise if $\gamma'(0) < 0$ it falls monotonically. ■

Appendix II: Existence and Uniqueness of Equilibrium

Let

$$\kappa_j(v_r) := \{v_d \in [0,1] \mid \Delta_j(v_r, v_d) = 0\}$$

be the locus of $\Delta_j(v_r, v_d)$. The following Lemma guarantees that the κ_d and κ_r intersect once in the $v_r - v_d$ space. Then using standard arguments we complete the proof of existence.

Lemma 8 *The simultaneous equations*

$$\Delta_r(v_r, v_d) = p_r - \frac{v_r}{\delta} + \Omega \quad \text{and} \quad \Delta_d(v_r, v_d) = p_d - \frac{v_d}{\delta} + \Omega$$

define κ_r and κ_d as implicit and strictly decreasing functions of v_r with $\frac{d\kappa_r}{dv_r} < \frac{d\kappa_d}{dv_r} < 0$. Furthermore there exists some $0 < \underline{v}_d < \overline{v}_d < 1$ and $\underline{v}_r \in (0,1)$ such that $\kappa_d(0) = \overline{v}_d$, $\kappa_d(1) = \underline{v}_d$ and $\kappa_r(\underline{v}_r) = 1$. Last either there exists some $\overline{v}_r \in (\underline{v}_r, 1)$ such that $\kappa_r(\overline{v}_r) = 0$ as in Figure 2a or there exists some $\underline{\underline{v}}_d \in (0, \underline{v}_d)$ such that $\kappa_r(1) = \underline{\underline{v}}_d$ as in Figure 2b.

Proof. We will first demonstrate that $\frac{d\kappa_r}{dv_r} < \frac{d\kappa_d}{dv_r}$ and then we will focus on the existence of boundaries $\underline{v}_j$, $\overline{v}_j$. Notice that

$$\frac{\partial \Delta_r}{\partial v_r} = \frac{\partial p_r}{\partial v_r} - \frac{1}{\delta} + \frac{\partial \Omega}{\partial v_r} < \frac{\partial \Delta_d}{\partial v_r} = \frac{\partial p_d}{\partial v_r} + \frac{\partial \Omega}{\partial v_r} < 0, \tag{22}$$

$$\frac{\partial \Delta_d}{\partial v_d} = \frac{\partial p_d}{\partial v_d} - \frac{1}{\delta} + \frac{\partial \Omega}{\partial v_d} < \frac{\partial \Delta_r}{\partial v_d} = \frac{\partial p_r}{\partial v_d} + \frac{\partial \Omega}{\partial v_d} < 0. \tag{23}$$

These inequalities follow from the facts that $\frac{\partial \Omega}{\partial v_j} < 0$ (Lemma 1) and $\frac{\partial p_j}{\partial v_j} < \frac{\partial p_{\bar{j}}}{\partial v_j} < 0$ (Lemma 2). Therefore $\Delta_j(v_r, v_d) = 0$ defines $v_d = \kappa_j(v_r)$ as an implicit function of v_r (Implicit Function Theorem) with

$$\frac{d\kappa_j}{dv_r} = -\frac{\partial \Delta_j / \partial v_r}{\partial \Delta_j / \partial v_d} < 0.$$

Since $\frac{\partial \Delta_r}{\partial v_r} < \frac{\partial \Delta_d}{\partial v_r} < 0$ and $\frac{\partial \Delta_d}{\partial v_d} < \frac{\partial \Delta_r}{\partial v_d} < 0$ it is obvious that $\frac{d\kappa_r}{dv_r} < \frac{d\kappa_d}{dv_r} < 0$.

Boundaries. Start by evaluating $\Delta_d(v_r, v_d)$ at end points. Observe that

$$\Delta_d(0,0) > \Delta_d(1,0) = p_d(1,0) + \Omega(1,0) > 0.$$

In addition

$$\Delta_d(0,1) = \Delta_d(1,1) = -1/\delta < 0,$$

because $\theta(1) = 1$. Since $\Delta_d(1,0) > 0$ and $\Delta_d(1,1) < 0$ and Δ_d decreases in v_d the Intermediate Value Theorem guarantees existence of some $\underline{v}_d \in (0,1)$ such that $\Delta_d(1, \underline{v}_d) = 0$, i.e., $\kappa_d(1) = \underline{v}_d$. Similarly $\Delta_d(0,0) > 0$ and $\Delta_d(0,1) < 0$ implies existence of some $\overline{v}_d \in (0,1)$ such that $\Delta_d(0, \overline{v}_d) = 0$, i.e., $\kappa_d(0) = \overline{v}_d$. Note that $\kappa_d(1) < \kappa_d(0)$ and since κ_d decreases in v_r we have $\underline{v}_d < \overline{v}_d$.

Now evaluate $\Delta_r(v_r, v_d)$ at end points. Similar to above, one can show that $\Delta_r(0,0) > \Delta_r(0,1) > 0$ and $\Delta_r(1,1) = -1/\delta < 0$. However $\Delta_r(1,0)$ can be positive or negative.

The existence of $\underline{v}_r \in (0,1)$ follows from the facts that $\Delta_r(0,1) > 0$, $\Delta_r(1,1) < 0$ and that Δ_r decreases in v_r. The Intermediate Value Theorem guarantees that there is some $\underline{v}_r \in (0,1)$ such that $\Delta_r(\underline{v}_r, 1) = 0$ which is equivalent to $\kappa_r(\underline{v}_r) = 1$. Existence of $\overline{v}_r$ or $\underline{\underline{v}}_d$ hinges on the sign of $\Delta_r(1,0)$, as we show below.

- Suppose $\Delta_r(1,0) < 0$: Since $\Delta_r(0,0) > 0$ there exists some $\overline{v}_r \in (0,1)$ such that $\Delta_r(\overline{v}_r, 0) = 0$ or equivalently $\kappa_r(\overline{v}_r) = 0$, and since κ_r is a decreasing function of v_r we have $\underline{v}_r < \overline{v}_r$.

- Suppose $\Delta_r(1,0) > 0$: First we will show that $\Delta_r(1, \underline{v}_d) < 0$. Notice that

$$\Delta_d(1, \underline{v}_d) - \Delta_r(1, \underline{v}_d) = \frac{S(\underline{v}_d)}{\delta f(\underline{v}_d)} + \frac{\alpha}{\overline{\delta}(\mu+\delta)} \frac{S(\underline{v}_d)^2}{f(\underline{v}_d)} + \frac{1-\underline{v}_d}{\delta} > 0.$$

Since $\Delta_d(1, \underline{v}_d) = 0$ it is must be that $\Delta_r(1, \underline{v}_d) < 0$. Now, since $\Delta_r(1,0) > 0$ there exists some $\underline{\underline{v}}_d \in (0, \underline{v}_d)$ such that $\Delta_r\left(1, \underline{\underline{v}}_d\right) = 0$ or equivalently $\kappa_r(1) = \underline{\underline{v}}_d$. ■

Existence and Uniqueness. Below we argue that there exists a unique interior v_r^* satisfying $\kappa_r(v_r^*) = \kappa_d(v_r^*)$. Define $\varkappa(v_r) := \kappa_r(v_r) - \kappa_d(v_r)$ and notice that it decreases in v_r since

$$\frac{d\varkappa}{dv_r} = \frac{d\kappa_r}{dv_r} - \frac{d\kappa_d}{dv_r} < 0.$$

Now we will verify that $\varkappa(\underline{v}_r) > 0$ and $\varkappa(1) < 0$. Indeed $\varkappa(\underline{v}_r) = \kappa_r(\underline{v}_r) - \kappa_d(\underline{v}_r) = 1 - \kappa_d(\underline{v}_r) > 0$ since $\kappa_d(\underline{v}_r) < \kappa_d(0) = \overline{v}_d < 1$. Similarly $\varkappa(1) = \kappa_r(1) - \kappa_d(1) = \kappa_r(1) - \underline{v}_d < 0$ since $\kappa_r(1)$ is either negative or equals to $\underline{\underline{v}}_d$ both of which are smaller than $\underline{v}_d$. Consequently the Intermediate Value Theorem guarantees existence of a unique $v_r^* \in (\underline{v}_r, 1)$ such that $\kappa_r(v_r^*) = \kappa_d(v_r^*) = v_d^*$.