FDI versus Exporting under Cournot Oligopoly and Trade in a Hotelling model of Differentiated Duopoly

by

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Thesis Submitted in Partial Fulfilment of the Requirements for the Degree of

Doctor of Philosophy

Economics Section
Cardiff Business School
Cardiff University

May 2013

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Abstract

This study investigates FDI versus export decisions under oligopoly in the trade liberalization, and examines how trade liberalization affects welfares in the Hotelling model of differentiated Bertrand duopoly. It uses a four-firm two-country Cournot oligopoly model to resolve the conflict between the theory, which predicts that a reduction in trade costs discourages FDI, and the empirical evidence, which is that trade liberalisation has led to an increase in FDI, and shows that both FDI and exporting can co-exist in the same market, in line with recent trends. In the static game, a reduction in trade costs causes a decrease in FDI, and the outcomes are often a prisoners' dilemma where the firms are worse off when they all undertake FDI than when all firms from both markets choose to export. To avoid it, firms can tacitly collude over their FDI versus export decisions when the game is infinitely-repeated. Then a reduction in trade costs may lead firms to switch from exporting to undertaking FDI when trade costs are sufficiently high. The robustness of the analysis is checked by using the constant elasticity demand function.

A two-country Hotelling model of spartial duopoly is developed to consider the welfare effects of trade liberalisation. It is shown that gains from trade occur when products are highly differentiated, and losses from trade occur when products are close substitutes, as the positive effect of more product choices overweighs the negative effect of the decreased home sales caused by trade liberalization when products are highly differentiated. This contrasts to Fujiwara (2009) who prove that there are always losses from trade in the Hotelling model. The reason behind is that the kinked demand market structure is often ignored, and by considering the full features of the Hotelling model, welfare effects depend on product differentiation and trade costs.

Acknowledgement

It is a great honour for me to have this opportunity to extend my appreciation and acknowledge the contribution of many people who have helped me during this doctorate research.

First, I would like to express my deeply-felt gratitude to my supervisor, Professor David R. Collie for providing me with inspiration, guidance, encouragement and many more. I have benefited incredibly from those regular and irregular meetings I had with him. The customary acknowledgement seems far from adequate. Without his continuous advice, and constructive comments, I would not reach where I am today with this work in its present form. I would also like to thank Dr. Helmuts Azacis who was kind enough to give comments and share his knowledge with me.

Special appreciation and sincere thanks to Professor Patrick Minford for providing the scholarship and teaching opportunities to support my study in the UK. I gained not only valuable experience but also financial support from the self-organized teaching during my research abroad.

I would like to thank the members of the departmental staffs, Ms. Elsie Phillips, Ms. Laine Clayton, and Ms. Karen Jones who supported generously in many ways and helped in seeing this thesis to fruition. I would also like to express many thanks to my friends and course-mates in Cardiff and in China whose names were not acknowledged here due to volume capacity. All these wonderful friends change my life differently and make me realize that they are and will always be there for me. Their invaluable concern, warm friendship, constant personal advice will always be remembered.

Lastly, I am very much indebted to my beloved father, Mr. Sijiu Hu, mother Mrs. Yuzhen Huang for their patience, continuous support and encouragement.

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Chapter 1: Introduction

1.1 Background and the Motivations for this Research

Prior to the 1980s, trade theory was dominanted by perfectly competitive market, which relied on the assumptions of constant returns to scale and perfect competition in production. Then the so-called new trade theory adds the elements of increasing returns to scale, imperfect competition and product differentiation to the traditional trade model. In the new theory, trade and gains from trade can arise independently of any pattern of comparative advantage because of the product differentiation and the economies of scale.

Oligopoly is the competition among a small number of large firms in the market, and plays a small role in the trade theory. However, many industries are dominated by a small number of firms empirically, and an increasing number of papers indicates that large firms account for the major share of exports and FDI as well as research and development[1]. In fact, the assumptions of perfect and monopolistic competition do not fit in the trade theory. For example, the assumption of infinitely elastic supply of atomistic firms, which do not engage in strategic interaction[2], is not appropriate to the global market. Consequently, the study of oligopoly is suited to the special feasure of concentrated industries, such as the strategic behaviour and the persistence of profits by firms.

This research joints the new industrial organization models, oligopoly in particular, with theories of trade and multinational enterprise, or foreign direct investment (FDI). It examines two aspects of trade under oligopoly: firstly, to look at FDI vs exporting decisions in Cournot oligopoly model, and secondly, to investigate the welfare effects in a Hotelling model of differentiated duopoly.

According to IMF/OECD IMF (1993),OECD (1996), FDI is an investment in a foreign company where the foreign investor owns at least 10% of the ordinary shares,

[1] See Bernard, A. B., J. B. Jensen, et al. (2007).
[2] See Leahy, D. and J. P. Neary (2010).

undertaken with the objective of establishing a 'lasting interest' in the country, a long-term relationship and significant influence on the management of the firm. FDI grew dramatically over the last few decades, far outpacing the growth of trade and income[3]. The period 1986-2000 saw an explosive growth of activity by multinational enterprises, as measured by flows of foreign direct investment. In that period, worldwide real GDP increases by 2.5% per year, and exports by 5.6% whereas worldwide real inflows of FDI increased by 17.7%[4]. Compare this with the pre-1985 data, real world GDP, exports and FDI grew at closer trends.

Other important facts of FDI include: the predominant source of supply of FDI is the advanced countries[5]. The US is the world's largest foreign investor, followed by EU as a whole, which accounted for 71.2% of all outward stocks. However, the share of developing countries has been rising, and the increase of FDI flows to developing countries reflects the growing importance of FDI as a source of financing of these economies. A foreign subsidiary may take place in one of the two forms: either as a 'greenfield investment', where a new plant is set up from scratch, or as a merger with or acquisition of an existing firm, known as M&A. The majority of FDI activities take place through M&A rather than through greenfield investment, especially to high-income countries. In addition, most FDI is concentrated in skill- and technology-intensive industries, and multinational enterprises (MNEs) are large companies compared with national firms, both in home and host countries. They are sometimes more productive than national firms. Finally, multinational firms are increasingly engaged in international production networks. This is to do with vertical specialisation, in which different stages of the production of a good takes place in different countries. There are generally two types of FDI: vertical and horizontal FDI.

Vertical foreign direct investment refers to those that geographically fragment the production process by stages of production[6]. These stages could be the production of components or stages of the manufacturing process or service activities in a separate

[3] Stylized facts were discussed in Barba Navaretti, Giorgio, et al. (2004), Markusen, J. R. (2002), and Caves, R. E. (2007).
[4] See UNCTAD (2000), chapter 1 of Markusen, J. R. (2002), and chapter 1of Barba Navaretti, Giorgio, et al. (2004)
[5] According to Barba Navaretti, Giorgio, et al. (2004): 15 countries of the EU are classified as advanced countries in 2003.
[6] See Markusen, J. R. (2002)

foreign plant. This is referred to as 'vertical' investment, as it breaks of the value-added chain. The main drive of this type of FDI is that it enables them to benefit from lower production costs by moving different stages of the production process to countries with lower costs. Vertical FDI tend to be drawn to markets with lower factors costs. Trade costs are important for this type of FDI as products at different stages of the production may cross the board for quite a few times. Therefore, Vertical FDI favours low wage locations with good transport and trade links.

Horizontal foreign direct investment refers to the foreign production of products and services roughly similarly to those the firm produces for its home market. For example, setting up a foreign plant to serve the foreign market, this is referred to as 'horizontal' investment, as the same stage of the production process is duplicated. Firms undertake investment in order to gain some advantages in supplying local or regional markets, even though it may incur some plant-level costs. Horizontal FDI tend to locate in markets where it can improve its market access, but sales will be large enough to cover fixed costs of the plant. Market access may be good as the country itself has a large high-income population, or as the country is well located to access such markets[7].

FDI is primarily horizontal rather than vertical, so instead of breaking up the production process, and producing in different countries to lower the factor costs as in vertical FDI, the bulk of horizontal FDI aims to replicate production facilities abroad to benefit from good market access. Empirical evidence generally confirms this result: they show that the location of foreign subsidiaries is mostly driven by factors consistent with horizontal FDI, for example, Markusen (2001) shows that bilateral flows of FDI depend on the similarity of the market size as well as the ratio of skilled and unskilled labour between markets, and uses this evidence to oppose the influence of the vertical FDI. Brainard (1997) finds that FDI is increasing in transport costs, but decreasing in production scale economies.

Multinationals have grown fast over the last three decades, far outpacing the growth of the trade. The experience of the 1990s shows that Foreign direct investment (FDI)

[7] See Barba Navaretti, Giorgio, et al. (2004)

has grown rapidly and trade costs have been reduced by trade liberalisation. An intriguing question is why has FDI grown so fast in an era when trade costs have been reduced by trade liberalisation. Intuitively, trade costs are associated with export, so a reduction in trade costs would increase the profitability of exporting relative to the profitability of undertaking FDI. However, the theory is in contradiction to the empirical evidence in 1990s. This is one of the key problems this study is trying to solve.

1.1.1 Trade versus FDI under Oligopoly

Having the conventional view that FDI should be horizontal, it is expected a fall in trade costs should discourage FDI as predicted in a proximity-concentration trade-off. However, the experience of the 1990s shows that FDI has grown rapidly when trade costs have been reduced by trade liberalisation. A standard theoretical framework proximity-concentration trade-off[8]suggests that firms invest in a foreign market when the benefits of avoiding trade costs outweigh the loss of economies of scale from producing exclusively in the home market. So it predicts that the horizontal FDI is discouraged when trade costs fall as the benefits of concentrated production outweigh the gains from improved market access. This concept, however, is in contradiction with the trend when trade costs[9] fell dramatically during 1990s both technically and politically.

The foundation of the proximity-concentrated trade-off has been analysed by Neary (2009). He shows that higher fixed costs are associated with more exports relative to FDI, and higher trade costs is associated with more FDI relative to exports[10]. This could be interpreted across time, across space, and across sectors. In terms of time, his model implies that a reduction in trade costs should encourage FDI relative to export. In terms of sectors, it implies that when trade costs are low, exports are preferred to FDI. In terms of space, a closer market favours exports and a further one favours FDI.

[8] See Horstmann, I. J. and J. R. Markusen (1992)
[9] Trade costs include both tariffs and transport costs.
[10] One firm can not engage in both trade and FDI as in Helpman, E., M. J. Melitz, et al. (2004)

There are some econometric evidence and case-study evidence to support the proximity-concentration hypothesis. Brainard (1993) considers the U.S. data, and shows that the level of outward FDI falls as trade costs increase, but the share of FDI in affiliate sales plus U.S. exports rises. Hence he confirms the prediction of the theory at least in relative terms that lower trade costs are associated with more exports than FDI, leading a substitution away from FDI towards exports, and Carr, Markusen et al. (2001) find the similar results. Brainard (1997) has provided empirical evidence to support the hypothesis, and he finds that the share of affiliate sales in industry-country is increasing in transport costs, trade barriers, and corporate scale economies, and decreasing in production scale economies.

Case study of Ireland in the 1930's[11] has proven the proximity-concentration trade off hypothesis: in 1937, a change in the Irish government had transformed this country from an open economy to a highly protected economy. Although Irish market is small, still the theory would expect a large inflow of FDI following the reduction of the trade costs and tariffs. However this did not happen immediately, but waited until six years later. The reason behind was to do with the political context: protection was introduced by the new government as part of campaign to cut down the influence of Briton at that time. So when British firms try to set up manufactories in Ireland, the new legislation forbid them doing so. Only when this law was relaxed in 1938, FDI increased significantly in Ireland.

All these evidence are consistent with the implication of the proximity-concentration trade-off. However, this theory is contradicted with the huge increase in FDI in the era of trade liberalisation in 1990s. The hypothesis explained the Irish example in 1930s, when Ireland benefited from a huge increase in the FDI inflow, but it could not explain the experience in the 1990s, while FDI rose much faster than exports when trade costs fell dramatically. Neary (2009)suggests that this comes from an old literature initiated by Mundell (1957), who showed that exports and FDI are perfect substitutes rather than perfect complements in two-sector two-country Heckscher-Ohlin model, and trade barriers encourage international capital flows. Then his model was extended by Markusen (1983), Jones and Peter Neary (1984), and they showed

[11] See Neary, J. P. (2009)

that if the induced capital flows enter export sector, falls in trade costs can encourage FDI, as countries are different either in technology or in endowments of sector-specific factors.

Leahy and Pavelin (2003)used an infinitely-repeated game to demonstrate the follow-my-leader character in FDI observed by Knickerbocker (1973). It implies the positive interdependence between firms' FDI decisions, so domestic firms may be motivated to set up foreign production in the same country and to tacitly collude over outputs, which implies foreign investment by one firm bring incentive for others to follow suit. Neary (2009) explores two resolutions to the paradox: Firstly, intra-bloc trade liberalisation encourage horizontal FDI in trading blocs, since foreign firms establish plants in one country as export platforms to serve the bloc as a whole, and secondly, falling trade costs encourage cross-border mergers and acquisitions (M&As), where a foreign firm purchase an existing firm in the host country. This form of FDI (M&As) are quantitatively more important than greenfield FDI. In his paper, he resolves the paradox built on export-platform FDI in Neary (2002).Mrazova and Neary (2011) derives a general result that characterizes how firms select themselves into exporting or FDI to serve a group of foreign countries, and how many plants they plan to establish. They show that only if firms' maximum profits are supermodular in tariffs and production costs, then the most efficient firms establish one plant in each country, firms of intermediate efficiency establish only one plant and use it as an export platform, and the least efficient firms choose to export.

Collie (2009) resolves the problem by using an infinitely-repeated game with both Cournot duopoly and Bertrand duopoly models, and he only considers collusion over the choices of undertaking FDI or exporting. This study adopts Collie (2009)'s framework, extending his model into a four firms oligopoly, that located in two countries, and solved the conflicts between the theory and the empirical experience. Additionally, it adds two contributions, which will be mentioned in section 1.3.

While the first part of the study focuses on the strategic choices between FDI and trade under oligopoly in the trade liberalization, the second part of this study examines how trade liberalization affects the welfare gains, profits, and the volume of trade in the Hotelling model of differentiated Bertrand duopoly.

1.1.2 Trade under Oligopoly

Welfare analysis has been studied in many literature, and under different types of markets. In a simple framework of the trade model under oligopoly, in order to look at each market in isolation, it is more convenient to make the assumption that markets are segmented. This enables firms to decide their outputs or prices for each market endogenously. Another common assumption to consider one market in isolation is to assume that firms produce at constant marginal costs. This is to make sure that outputs or prices decision in one market have no implications for the costs at which other markets can be served.

Brander (1981) first presents a reciprocal-markets model, he considers a Cournot duopoly model, where products are identical and outputs are shared between home and foreign countries, and there is only one firm from each country that compete in this industry. This model is symmetric and both home and foreign firms have the same marginal cost and the same trade costs. Brander and Krugman (1983) also uses the reciprocal-markets model to consider free trade under Cournot oligopoly with identical products. Both of them have identified that intra-industry trade can be expected even in the identical goods. Meanwhile, the welfare under the multilateral free trade is in a U-shape of the trade costs, and the reason for which will be illustrated next when considering a Cournot competition.

Output Competition

Considering the output competition of symmetric multilateral free trade between two identical countries first, if product differentiation is allowed, the Cournot-Nash equilibrium can occur. Leahy and Neary (2010) presents a general setup and concludes that oligopoly competition is an independent determinant of trade. Brander (1981) analyses two-way trade in identical products and shows that it is true even when the products are identical. When goods are more differentiated, the volume of trade increases further, because consumers can enjoy a variety of goods. Secondly, when the trade cost is closer to its prohibitive level, each firm is selling more in its home market than its exports to the foreign market, as there is a penalty on the foreign sales. Then there is a 'reciprocal dumping' by Brander and Krugman (1983): the price of each firm in equilibrium yields a lower mark-up over cost on its exports than on its sales in the home country.

Then the effect of trade costs on the profits of the firm will be examined. By focusing on the home firm, its total profits are the sum of its home sales and its export to the foreign market. Leahy and Neary (2010) derives the profits that are decreasing in trade costs at free trade, but increasing in them at neighbourhood of autarky in a linear demand function. So the profits are in a U-shape in trade costs. There are two reasons for the shape: Firstly, at free trade, an increase in the trade costs has a negative effect on the exports, but has a positive effect on a firm's home sales, and the negative effect dominants, so total profits and sales decrease in trade costs. Secondly, at autarky, there is no export initially, so a change in trade costs affects profits on exports rarely, but a fall in the trade costs of the foreign firm will reduce the sales and the profits of the home sales, as they were at the monopoly level initially. Consequently, a reduction in trade costs reduces the home firm's total profits.

Finally, the effects of changes of trade costs on welfare will be looked at. Since the model is symmetric, if focusing on home firm only (symmetric model), its total welfare is the sum of the home consumer surplus and its total sales in both markets. Consumer surplus will rise when trade costs fall, as a reduction in trade costs lowers the prices to the home consumers. Home firm's total profits are illustrated earlier that it is a U-shape in relation to the trade costs. Adding up the profits of the home firm and consumer surplus, the welfare can be analysed as follows:

- Starting from autarky, if trade costs fall, consumer surplus increases as the competition intensify the market, leading to lower prices. On the other hand, home profits and sales fall for the reduced prices. Hence these two effects in the home market cancel out, the fall in total profits overweight the rise in consumer surplus, and the welfare fall when trade costs falls starting from autarky. Alternatively speaking, opening up to trade will always lead to a welfare gain as stated in Brander and Krugman (1983).

- Starting from free trade, if trade costs rise, consumer surplus falls as the prices of both firms to the home consumers will increase. Home firm's total sales and profits will fall as well as illustrated earlier. Hence the overall welfare of home firm fall when trade costs rise starting from free trade.

Consequently, home firm's welfare is also in a U-shape with the trade costs. An alternative explanation of the U-shape of the welfare in trade costs was provided by Brander and Krugman (1983), where they think the welfare effects are interesting. When trade costs fall if they are closer to a prohibitive level, the welfare will decline because a competitive effect, which is positive in sales, is dominated by the increased waste due to the trade costs. When the trade costs are low, the competitive effect dominants the increased waste.

Price Competition

This subsection will consider the price competition of symmetric multilateral free trade between two identical countries, and analyse how the effects of trade liberalisation affects trade and welfare in this case. The welfare effects under the Bertrand competition are first derived by Clarke and Collie (2003). They add the effect of product differentiation in the free trade under Bertrand duopoly, and prove that there are always gains from trade. They present a two country Bertrand duopoly model with linear demands and constant marginal costs, and allow for differences between the two countries in terms of demand and cost functions. Their conclusion is that the level of welfare never falls below the autarky level under both unilateral and multilateral free trade.

According to Clarke and Collie (2003), profits are also U-shaped under Bertrand competition. Together with the fact that consumer surplus is monotonically decreasing in the trade costs[12], the welfare under free trade in the Bertrand model is also U-shaped as in the Cournot case. However, the competition effect under Bertrand duopoly is stronger than under Cournot duopoly, which means that even when trade may not take place in the home market, it can still affect home firm's behaviour as there is a potential threat of export to the home country. Leahy and Neary (2010)

[12] A rise in trade costs will reduce the prices of both firms in the home country, which will reduce the welfare of home consumers.

derives a general result showing that when the trade costs reach its prohibitive level[13] under Bertrand competition, home firm's outputs are still higher than the unconstrained monopoly outputs. Yet the home firm would not raise its price, as the foreign competitor would have earned positive profits by exporting, and lower home firm's profits. Only when the trade costs reach a prohibitive level under Cournot competition, home firm can be an unconstrained monopolist facing no threat or potential competition. This pro-competitive effect where home firm is constrained by the threat only occurs in the Bertrand competition, whereas free trade only has an effect if trade actually happens in the Cournot competition.

The welfare effects under Bertrand duopoly are slightly different from the ones under Cournot duopoly due to this pro-competitive effect where trade or competition does not actually occur. Start from autarky where trade costs are at a Cournot prohibitive level under Bertrand competition, trade liberalisation will increase the welfare of the home country initially, because this is the region where home firm faces the potential competition from foreign firm, but trade does not actually happen. Thus there is no waste on the transport cost, and the prices of the products are lowered by the trade liberalisation, leading to welfare and profit gains. As trade costs decrease further to below the threshold level of Bertrand competition, trade occurs, and the welfare in terms of trade costs will be the usual U-shape as in the Cournot competition[14]. Nevertheless, Clarke and Collie (2003) finds that the welfare never fall below the autarky level under Bertrand oligopoly, while it could do under Cournot oligopoly, and induce losses from trade.

Finally, to conclude the difference between the Bertrand and Cournot results: the fact that trade promotes a competition effect applies to both cases, but it is stronger under the Bertrand competition, in which home firm behaves as a constrained monopoly even when trade may not take place. In this case, welfare increases as a result of a reduced price and this pro-competitive effect under the trade liberalisation. When trade costs fall further, however, the relationship between the welfare and trade costs

[13] This prohibitive trade costs are at some intermediate level, which is lower than the prohibitive cost under Cournot competition. For details see Leahy, D. and J. P. Neary (2010) and Clarke, R. and D. R. Collie (2003)

[14] See Leahy, D. and J. P. Neary (2010): figure 1: welfare and trade costs under Cournot and Bertrand competition, both are U-shaped in trade costs, but welfare falls after certain point under Bertrand competition.

are in a similar U-shape fashion. In addition, the welfare under Bertrand completion is always above the autarky level, but it is not always the case under Cournot competition. The Hotelling model this study has adopted exhibits the features of Bertrand duopoly with product differentiation, and without market expansion effect.

1.2 Objectives of this study

The context of this study is embedded in a large volume of literature analysing trade versus FDI in a Cournot Oligopoly, as well as the welfare effects of free trade and gains from trade under imperfect competition by using a Hotelling model. The specific objectives of this study are as follows:

I. To examine the FDI versus exporting decision under Cournot oligopoly by using linear-demand function in a two-country four-firm model;
II. To re-examine the FDI versus exporting decision under Cournot oligopoly by using constant elasticity function and two-country four-firm model;
III. To analyse the welfare effects of free trade, gains from trade and the volume of trade by constructing a product space in a two-country Hotelling model of a spatial duopoly.

1.3 Outline and Contributions of this study

This study includes three analyses, which are to be found in chapter two to four. The organisation of this study, together with a brief description of the main contribution of each chapter is as follows:

Chapter 2 analysed the export versus FDI decisions in a two-country four-firm model with identical products under Cournot oligopoly. In the static game, a reduction in the trade cost will lead the firms switch from undertaking FDI to exporting. The outcomes are that two firms in the same country choose to undertake FDI if the fixed cost is relatively low, or one firm chooses to export while its competitor from the same country chooses to undertake FDI if the fixed cost is relatively high. Thus, this model shows that both export and FDI can exist as an equilibrium outcome in the world when the trade cost is sufficiently high. However, prisoners' dilemmas might exist. If based on one market, both firms in the same country might make lower profits when they both undertake FDI than when they both export. If based on both markets, if the fixed cost is relatively low, all firms might make lower profits when they all undertake FDI than when they export. If the fixed cost is relatively high, the equilibrium profits when one firm in each country undertakes FDI while its competitor in the same country export might be higher than the profits when all firms export. This is due to the intensified competition caused by FDI. The prisoners' dilemma can be avoided in an infinitely-repeated game when all firms tacitly collude over their FDI versus export decisions, as collusion over FDI can be sustained by the threat of Nash-reversion strategies if the trade cost is sufficiently high. Then a reduction in trade costs may lead firms to switch from exporting to undertaking FDI when the trade cost is sufficiently high, as in the infinitely-repeated game , a reduction in a sufficiently high trade costs lessen the profitability of collusion, and that explains the experience of the increasing FDI in 1990s. Also it is shown that a reduction in the fixed cost may lead firms to switch from undertaking FDI to exporting when the fixed cost is relatively high.

Chapter 3 studies uses constant elasticity demand function to check the robustness of the results from chapter 2, and it has been confirmed that all the results are quite general. In the static game, a reduction in the trade cost will lead the firms switch from undertaking FDI to exporting. The same outcomes are achieved that two firms in the same country choose to undertake FDI if the fixed cost is relatively low, or one firm chooses to export while its competitor from the same country chooses to undertake FDI if the fixed cost is relatively high. Again it shows that both export and FDI can exist as an equilibrium outcome in the world when the trade cost is sufficiently high. The prisoners' dilemmas still exist. If the fixed cost is relatively low, all firms might make lower profits when they all undertake FDI than when they export. If the fixed cost is relatively high, the equilibrium profits when one firm in each country undertakes FDI while its competitor in the same country export might be higher than the profits when all firms export, due to the intensified competition caused by FDI. In an infinitely-repeated game, the prisoners' dilemma can be avoided, as collusion over FDI can be sustained by the threat of Nash-reversion strategies if the trade cost is sufficiently high. Then a reduction in trade costs may lead firms to switch from exporting to undertaking FDI.

The main contribution of chapter 2 and chapter 3 to the current literature are: firstly, a reduction in the fixed cost may increase the incentive to collude and therefore lead firms to switch from undertaking FDI to exporting when the fixed cost is relative high in an infinitely repeated game. Secondly, there exist multiply equilibia in both static game and infinitely-repeated game, so both export and FDI can co-exist in the same market when the trade cost is relatively high, which is in line with the current trend in the globalised world.

Chapter 4 aims to examine how trade liberalisation affects the welfare gains from trade, and volume of trade in the Hotelling model of differentiated Bertrand duopoly by constructing a product space between the trade costs and marginal disutility, which associated with product differentiation. The results turn out to be different from Fujiwara (2009), who proves losses-from-trade proposition at any trade costs. By considering the kinked-demand structure in the Hotelling model which Fujiwara (2009)ignores, it shows that there are gains from trade when products are highly differentiated and losses when products are close substitutes. When there is pro-competitive effect in the competitive market, there are neither gains nor losses. The volume of trade is increasing in the degree of product differentiation when products are close substitutes, and decreasing in the degree of product differentiation when products are sufficiently differentiated.

The last chapter is the conclusion. It provides a discussion of the overall findings and implications of this research.

Chapter 2: FDI versus Exporting under Cournot Oligopoly

2.1 Introduction

Multinationals have grown fast over the last three decades, far outpacing the growth of the trade. Especially the period 1986 to 2000 saw a dramatic growth in foreign direct investment (FDI) in real terms, yet FDI flows remain much smaller than trade flows in the same period[15] . The experience of the 1990s shows that FDI has grown rapidly and trade costs have been reduced by trade liberalisation.

A standard theoretical framework proximity-concentration trade-off [16] has been discussed in a lot of literature and it predicts that the horizontal FDI is discouraged when trade costs fall. This concept, however, is in contradiction with the trend. The proximity-concentration trade-off suggests that firms invest in a foreign market when the benefits of avoiding trade costs outweigh the loss of economies of scale from producing exclusively in the home market. Brainard (1997) provides empirical evidence to support this hypothesis, but does not explain the fast growth of FDI in an era of trade liberalisation. Neary (2009) explores two resolutions to the paradox: First, intra-bloc trade liberalisation encourage horizontal FDI in trading blocs, as foreign firms build plants in one country as platforms for the whole bloc, and second, cross-border mergers, which are quantitatively more important than greenfield FDI, are encouraged by falling trade costs. Leahy and Pavelin (2003) used an infinitely-repeated game to demonstrate the follow-my-leader character in FDI observed by Knickerbocker (1973). It implies the interdependence between firms' FDI decisions, so domestic firms may be motivated to tacitly collude over outputs.

Collie (2009) resolves the problem by using an infinitely-repeated game with both Cournot duopoly and Bertrand duopoly models, and he considers collusion over the choices of undertaking FDI or exporting. His theory started with the static game in a symmetric Cournot duopoly model. Two firms located in two countries may export to their competitor's market or undertake FDI. The decision is based on the trade cost incurred with exporting and the fixed cost incurred with undertaking FDI. Firms are

[15] Stylised facts on FDI are presented in chapter one of Markusen, J. R. (2002), and chapter one of Barba Navaretti, Giorgio, et al. (2004)
[16] See Horstmann, I. J. and J. R. Markusen (1992)

more likely to undertake FDI if the trade cost is high or the fixed cost is low. In a static game, one firm's decision of undertaking FDI will lead to an intensified competition in its competitor's market, and reduce its competitor's profits in its home market. Hence, when both firms undertake FDI, both of them make lower profits in each market than when they both export. This is often a prisoners' dilemma. He then solves this problem by looking at an infinitely-repeated game when the firms tacitly collude over their choice of undertaking FDI or exporting, and a reduction in trade costs may lead firms to switch from exporting to undertaking FDI when the trade cost is high. However, his model is based on a two firm two country world, where his results might not cover all the equilibrium cases. For example, there is only one Nash equilibrium that both firms would undertake FDI, and this approache does not really address that both FDI and exports can co-exist in the same market, even with identical firms, in line with recent trends in the globalized world.

The key innovation in this model is to adopt Collie (2009)'s framework, extending his model into a four firms oligopoly, that located in two countries. It then allows Cournot oligopolistic competition between the firms producing identical products. A four-firm two-country world has added three more contributions: firstly, there exist multiply equilibria: the new equilibrium is that one firm in each country will choose to undertake FDI while its competitor in the same country exports when the fixed cost of undertaking FDI is relatively high. Hence firms choose different internationalisation strategies such that both export and FDI can co-exist in the same market. Secondly, the asymmetric equilibria in which ex-ante identical firms choose differenet strategies can emerge in an infinitely-repeated game. Finally, a reduction in the fixed cost may increase the incentive to collude and therefore lead firms to switch from undertaking FDI to exporting when the fixed cost is relative high in an infinitely repeated game. The model has checked the robustness of Collies (2009)'s work, and conclude the similar results: a reduction in trade costs may lead firms to switch from exporting to undertaking FDI when the trade cost is relatively high, and collusion over FDI can be sustained as Nash equilibrium using Nash-reversion trigger strategies.

Brander (1981) and Brander and Krugman (1983) develop a simple Cournot duopoly model with trade, where two firms produce commodity in two identical countries. Horstmann and Markusen (1987) and Smith (1987) view FDI as a strategic investment in models of intra-industry trade under oligopoly.[17] They consider the case of a horizontal FDI and assume that the technology of production involves a firm-specific cost such as R&D and a plant-specific cost. In this research, there is only a plant-specific cost, and it is considered as a sunk cost when a firm builds a factory in its own country. Each firm needs to decide whether to export to their competitor's market or to undertake FDI. Undertaking FDI incurs the fixed cost while exporting incurs the trade costs, and firms are more likely to undertake FDI if the trade costs (the fixed cost) are low (high) as it will increase the profitability of exporting relative to the profitability of undertaking FDI.

Influenced by the investment behaviour of Horstmann and Markusen (1987) and Smith (1987), Horstmann and Markusen (1992)and Rowthorn (1992) developed symmetric two-country trade models in which market structure is determined endogenously as a result of plant location choices by firms. The result in these models is the existence of multiple equilibria, given by the production technology of both firm specific and plant specific fixed costs, due to the endogenized multinational firms. The market structure in the present model is in the similar fashion, but by assumption firms have already built their manufactories in the market, so that their decision is only whether to export or to undertake FDI in the foreign market. This assumption is made to avoid the problem of multiple equilibria arising from the complex market. Nevertheless, there exist two equilibria in this model: (1) all firms undertake FDI when the fixed cost is relatively low as in Collie (2009) and (2) one firm in each country undertakes FDI while other firms export to the foreign country when the fixed cost is relatively high. This equilibrium is an important result that shows the possibility of the existence of both FDI and exports in one market, which reflect the exact trading activities in a globalised world.

Motta (1992) analysed the impact of a tariff by a foreign country. In his model, a multinational competes with a local firm in the foreign country. The tariff may induce the multinational to shift away from the investment as the local firm may enter the industry as a result of the tariff, and this result runs contrary to the traditional tariff argument. Motta (1996), Norman and Motta (1993) and Neary (2002) have

[17] See Caves, R. E. (2007) for a broader knowledge about empirical literature on multinational enterprise and FDI.

considered the effects of the internal trade liberalisation on the pattern of FDI into a two-country model and three-country model, that a reduction in trade cost within a trade bloc may encourage FDI. While most papers assume that all firms are equally likely to engage in FDI, Neary (2002) assumes that the potential multinational has a first-mover advantage. The present model also considers the first-mover advantage in equilibrium (2) in an infinitely-repeated game as mentioned above. The concept fits well the case when one firm in each country possesses more technological or organisational advantage (for example, more information) which may make it more likely than its competitor from the same country to become a multinational in the first place.

Most literature on FDI and export under oligopolistic competition has used static game theory models, except for Leahy and Pavelin (2003), who used an infinitely-repeated game to demonstrate the follow-my-leader character in FDI observed by Knickerbocker (1973). It implies the interdependence between firms' FDI decisions, so domestic firms may be motivated to tacitly collude over outputs. The present chapter does not consider collusion over outputs but the collusion over the decision of undertaking FDI or exporting.

This chapter is organised in the following way. Section two presents a theoretical framework of FDI under Cournot oligopoly. Section three presents the static game theory model, followed by the infinitely-repeated game in section four. The conclusions are in section five.

2.2 The model

In the symmetric model, the world consists of two countries with two firms in each country, and they produce homogeneous products and compete as Cournot oligopolists in the two markets.

These two countries are labelled A and B, and the firms in country A are labelled one and two, while the firms in country B are labelled three and four. Firm one and firm two are owned by shareholders who are resident in country A, referred as the home market, and firm three and firm four are owned by shareholders who are resident in country B, referred as the foreign market. It is assumed that firm one and firm two have incurred sunk costs to design their products and to build factories in country A, and by symmetry, firm three and four have incurred the same sunk costs in country B.

The firms play a two-stage static game. In the first stage, the firms independently decide whether to export to supply the other country or to undertake FDI by building a factory in the other country. Export incurs a trade cost (transport cost or import tariff) of k per unit, and undertaking FDI incurs a fixed cost of G per period. In the second stage, they compete in a Cournot outputs game in the two markets, which are assumed to be segmented. All firms incur a constant marginal cost of c regardless the location of their production. In each country, there is a representative agent who has an identical, quadratic, quasi-linear utility function, which yields linear demand functions.

The utility function in country A is:

$$U_A = \alpha\left(x_{1A} + x_{2A} + x_{3A} + x_{4A}\right) - \frac{\beta}{2}\left(x_{1A} + x_{2A} + x_{3A} + x_{4A}\right)^2 + z_A \qquad (1)$$

where x_{iA} is the consumption of the product of firm i, and z_A is the consumption of the numeraire good in country A. Parameter α is the consumers' maximum willingness to pay for the product, and β is inversely related to the size of the market. The representative consumer in country A then solves the utility maximisation problem by the first order condition, which yields the inverse demand functions in country A, and it is linear in x_{iA}. The products are assumed to be perfect substitutes, so the inverse demand functions are the same for four firms in the home market (country A):

$$p_A = \alpha - \beta\left(x_{1A} + x_{2A} + x_{3A} + x_{4A}\right) \qquad (2)$$

Since the model is symmetric, the inverse demand functions are the same for country B. In order to analyse the profits and the outputs of the firms under all strategic combinations in the static game, it is easier to look at the profits of the home firms in country A first, in response to the strategies of the foreign firms in country B. Then the profits of the foreign firms in country B are symmetric. Consequently, the total profits of all firms in both markets (the world) will be the summation of the above two. There are three strategic combinations in each market: (1) both firms in the same market choose to export, (2) one firm undertakes FDI while its competitor in the same market chooses to export, or (3) both firms undertake FDI.

2.2.1 Both firms choose to export

Consider the home market (country A), when firm three and firm four in the foreign country choose to export, the marginal cost of firm one and firm two will be c and the marginal cost of firm three and firm four will be $c+k$, assuming an interior solution where all firms have positive profits in both countries. Thus the operating profits of the firms in country A will be:

$$\pi_{1A} = (p_A - c)x_{1A} \qquad \pi_{2A} = (p_A - c)x_{2A}$$
$$\pi_{3A} = (p_A - c - k)x_{3A} \qquad \pi_{4A} = (p_A - c - k)x_{4A} \tag{3}$$

In Cournot competition, each firm makes the best response to its competitors' outputs, so maximises the profits in response to its outputs while holding the other firms' outputs fixed. Taking the derivatives of Cournot oligopoly ($\partial\pi_{iA}/\partial x_{iA} = 0$), as shown in the appendix, the outputs, prices and profits of the four firms are solved. For country A variables, the superscript EE denotes that both firm three and firm four are exporting to country A:

$$x_{1A}^{EE} = x_{2A}^{EE} = \frac{\alpha - c + 2k}{5\beta} \qquad x_{3A}^{EE} = x_{4A}^{EE} = \frac{\alpha - c - 3k}{5\beta}$$
$$p_A^{EE} = \frac{\alpha + 4c + 2k}{5} \tag{4}$$
$$\pi_{1A}^{EE} = \pi_{2A}^{EE} = \frac{(\alpha - c + 2k)^2}{25\beta} \qquad \pi_{3A}^{EE} = \pi_{4A}^{EE} = \frac{(\alpha - c - 3k)^2}{25\beta}$$

Regardless what strategies firm three and firm four would choose, firm one's outputs and profits are always equal to those of firm two, as the above equations simply show how foreign firms' strategies affect home firms' decisions. If the trade cost is prohibitive, i.e. $x_{3A}^{EE} = x_{4A}^{EE} = 0$, when $k \geq \bar{k} \equiv (\alpha - c)/3$, the exports from firm three and firm four to country A and the profits of both firms from exports will be zero. When the trade cost reaches its prohibitive level $\bar{k}$, firms will stop trading between the two markets, then firm one and firm two share the duopoly profits in country A, while firm three and firm four produce outputs and sell them solely in country B. Symmetry of the model implies that: $x_{1A}^{EE} = x_{3B}^{EE}$, $x_{2A}^{EE} = x_{4B}^{EE}$, $x_{3A}^{EE} = x_{1B}^{EE}$, $x_{4A}^{EE} = x_{2B}^{EE}$, $p_A^{EE} = p_B^{EE}$, $\pi_{1A}^{EE} = \pi_{3B}^{EE}$, $\pi_{2A}^{EE} = \pi_{4B}^{EE}$ $\pi_{3A}^{EE} = \pi_{1B}^{EE}$, and $\pi_{4A}^{EE} = \pi_{2B}^{EE}$,where the superscript EE of country B variables denotes that firm one and firm two are exporting to country B.

2.2.2 One firm exports, the other firm undertakes FDI

When firm three chooses to export to country A, and firm four chooses to undertake FDI in country A, the marginal cost of firm one, firm two and firm four will be c, but the marginal cost of firm three will be $c+k$. Assuming the outcome is an interior solution where all firms have positive sales, the operating profits (before the fixed cost) of the firms in the home market (country A) will be:

$$\pi_{1A} = (p_A - c)x_{1A} \qquad \pi_{2A} = (p_A - c)x_{2A}$$
$$\pi_{3A} = (p_A - c - k)x_{3A} \qquad \pi_{4A} = (p_A - c)x_{4A} \tag{5}$$

If the superscript EF of country A variables denotes that firm three is exporting to country A, and firm four is undertaking FDI to supply country A, the usual derivations for a Cournot oligopoly yield the outputs, prices and profits of the four firms as shown in the appendix:

$$x_{1A}^{EF} = x_{2A}^{EF} = x_{4A}^{EF} = \frac{\alpha - c + k}{5\beta} \qquad x_{3A}^{EF} = \frac{\alpha - c - 4k}{5\beta}$$
$$p_A^{EF} = \frac{\alpha + 4c + k}{5} \tag{6}$$
$$\pi_{1A}^{EF} = \pi_{2A}^{EF} = \pi_{4A}^{EF} = \frac{(\alpha - c + k)^2}{25\beta} \qquad \pi_{3A}^{EF} = \frac{(\alpha - c - 4k)^2}{25\beta}$$

By comparison, the prices in (6) are lower than the prices in (4). This is because FDI intensifies competition. When firm four undertakes FDI, the prices set by firms are lower than the prices when it exports to country A, which reduces the profits of firm one and firm two in country A. Moreover, the outputs and profits (before the fixed cost) of firm four has increased from exporting to undertaking FDI in country A, i.e. $x_{4A}^{EF} > x_{4A}^{EE}$, $\pi_{4A}^{EF} > \pi_{4A}^{EE}$ while the outputs and profits of firm three, which does not change its exporting strategy, are lower when firm four switches to undertaking FDI in country A, i.e. $x_{3A}^{EF} < x_{3A}^{EE}$, $\pi_{3A}^{EF} < \pi_{3A}^{EE}$. Symmetry of the model implies that $x_{3B}^{EF} = x_{4B}^{EF} = x_{2B}^{EF} = x_{1A}^{EF}$, $x_{1B}^{EF} = x_{3A}^{EF}$, $p_B^{EF} = p_A^{EF}$, $\pi_{3B}^{EF} = \pi_{4B}^{EF} = \pi_{2B}^{EF} = \pi_{1A}^{EF}$, $\pi_{1B}^{EF} = \pi_{3A}^{EF}$, where the superscript EF of country B variables denotes that firm one is exporting to country B, and firm two is undertaking FDI in country B.

The exports of firm three to country A and its profits from exports will be zero if the trade cost is prohibitive, i.e. x_{3A}^{EF} =0, so $k \geq \tilde{k} \equiv (\alpha - c)/4$. When the trade cost reaches its prohibitive level $\tilde{k}$, firm three will stop exporting to country A, yet it still produces and sells the products in country B. In this case, only firm four will supply country A by undertaking FDI. The profits and the sales of exporting from firm three to country A becomes zero, hence there is a corner solution equilibrium, where firm three sets its price equals to the marginal cost $c + k$. The profits, outputs and prices of the four firms under the Cournot oligopoly when $k > \tilde{k}$ are:

$$\pi_{1A} = (p_A - c)x_{1A} \qquad \pi_{2A} = (p_A - c)x_{2A} \tag{7}$$
$$\pi_{4A} = (p_A - c)x_{4A} \qquad \pi_{3A} = (p_A - c - k)x_{3A}$$

Which yields:

$$x_{1A}^{*F} = x_{2A}^{*F} = x_{4A}^{*F} = \frac{\alpha - c}{4\beta} \qquad x_{3A}^{*F} = 0$$
$$p_A^{*F} = \frac{\alpha + 3c}{4} \tag{8}$$
$$\pi_{1A}^{*F} = \pi_{2A}^{*F} = \pi_{4A}^{*F} = \frac{(\alpha - c)^2}{16\beta} \qquad \pi_{3A}^{*F} = 0$$

where the superscript $*F$ of country A variables denotes that firm three is not exporting to country A, but firm four is undertaking FDI in country A. Symmetry of the model implies that if $k > \tilde{k}$: $x_{2B}^{*F} = x_{3B}^{*F} = x_{4B}^{*F} = x_{1A}^{*F}$, $p_B^{*F} = p_A^{*F}$, $\pi_{2B}^{*F} = \pi_{3B}^{*F} = \pi_{4B}^{*F} = \pi_{1A}^{*F}$, $\pi_{1B}^{*F} = \pi_{3A}^{*F} = 0$, $x_{1B}^{*F} = x_{3A}^{*F} = 0$ where the superscript $*F$ of country B variables denotes that firm one stops trading with country B, but firm two is undertaking FDI in country B.

When firm three chooses to undertake FDI, and firm four chooses to export, the marginal cost of firm one, firm two and firm three will be c, and the marginal cost of firm four will be $c+k$. The results are reciprocal to the above ones when firm three chooses to export and firm four undertakes FDI. The interior outcome is as follows:

$$
\begin{aligned}
& x_{1A}^{FE} = x_{2A}^{FE} = x_{3A}^{FE} = \frac{\alpha - c + k}{5\beta} && x_{4A}^{FE} = \frac{\alpha - c - 4k}{5\beta} \\
& p_A^{FE} = \frac{\alpha + 4c + k}{5} && \\
& \pi_{1A}^{FE} = \pi_{2A}^{FE} = \pi_{3A}^{FE} = \frac{(\alpha - c + k)^2}{25\beta} && \pi_{4A}^{FE} = \frac{(\alpha - c - 4k)^2}{25\beta}
\end{aligned}
\tag{9}
$$

Symmetry of the model implies that $x_{3B}^{FE} = x_{4B}^{FE} = x_{1B}^{FE} = x_{1A}^{FE}$, $x_{2B}^{FE} = x_{4A}^{FE}$, $p_B^{FE} = p_A^{FE}$, $\pi_{3B}^{FE} = \pi_{4B}^{FE} = \pi_{1B}^{FE} = \pi_{1A}^{FE}$, $\pi_{2B}^{FE} = \pi_{4A}^{FE}$, where the superscript FE of country B variables denotes that firm one is undertaking FDI in country B, and firm two is exporting to country B.

Similarly, the corner solution occurs when firm four stops exporting to the country A, only firm three is trading with country A. That is when $x_{4A}^{FE} = 0$, $k \geq \tilde{k} \equiv (\alpha - c)/4$. Hence firm four sets its price equal to marginal cost $c+k$ and its sales in country A are zero. Recalculate the profits, outputs and prices of the three firms under the Cournot oligopoly: $x_{1A}^{F*} = x_{2A}^{F*} = x_{3A}^{F*} = (\alpha - c)/4\beta$, $p_A^{F*} = (\alpha + 3c)/4$, $\pi_{1A}^{F*} = \pi_{2A}^{F*} = \pi_{3A}^{F*} = (\alpha - c)^2/16\beta$, $\pi_{4A}^{F*} = 0$. Symmetrically, It implies that $x_{1B}^{F*} = x_{3B}^{F*} = x_{4B}^{F*} = x_{1A}^{F*}$, $\pi_{1B}^{F*} = \pi_{3B}^{F*} = \pi_{4B}^{F*} = \pi_{1A}^{F*}$, and $p_B^{F*} = p_A^{F*}$, $\pi_{2B}^{F*} = \pi_{4A}^{F*} = 0$ where the superscript $F*$ of country B variables denotes that firm one is undertaking FDI in country B, but firm two stops trading with country B when trade cost reaches $\tilde{k}$.

2.2.3 Both firms undertake FDI

When both firm three and firm four undertake FDI, the marginal cost of all firms will be c, so the outcome is an interior solution where all firms generate positive sales. The operating profits (before the fixed cost) of the firms will be:

$$\pi_{1A} = \left(p_A - c\right)x_{1A} \qquad \pi_{2A} = \left(p_A - c\right)x_{2A}$$
$$\pi_{3A} = \left(p_A - c\right)x_{3A} \qquad \pi_{4A} = \left(p_A - c\right)x_{4A} \tag{10}$$

If the superscript FF of country A variables denotes that both firm three and firm four are undertaking FDI to supply country A, the derivations for a Cournot oligopoly yield the outputs, prices and profits of the four firms, as in the appendix, are:

$$x_{1A}^{FF} = x_{2A}^{FF} = x_{3A}^{FF} = x_{4A}^{FF} = \frac{\alpha - c}{5\beta} \qquad p_A^{FF} = \frac{\alpha + 4c}{5}$$
$$\pi_{1A}^{FF} = \pi_{2A}^{FF} = \pi_{3A}^{FF} = \pi_{4A}^{FF} = \frac{\left(\alpha - c\right)^2}{25\beta} \tag{11}$$

Compare prices in (11) with (4) and (6), the prices set by all firms are the lowest when both firm three and firm four undertake FDI in country A than when one of them exports or both of them export to country A, i.e. $p_A^{FF} < p_A^{EF} = p_A^{FE} < p_A^{EE}$, which further proved that FDI intensifies competition. In addition, it is worth noting that the outputs and profits (before the fixed cost) of firm three and firm four in country A are higher when both firms undertake FDI than when both firms export to country A. i.e. $\pi_{3A}^{FF} > \pi_{3A}^{EE}$, $\pi_{4A}^{FF} > \pi_{4A}^{EE}$, which is not consistent with what has been discussed in literature survey.[18] This is because the fixed cost of undertaking FDI is not taken into account here yet. Next section will present a static game theory model of FDI under

[18] In a two-country two-firm model, when both firms undertake FDI, the outcome is a prisoners'dilemma where both firms have lower profits when they both undertake FDI than when they both export.

Cournot oligopoly, where the fixed cost of undertaking FDI enters the decisions of the game.

2.3 Static Game

Since each firm can choose to export or to undertake FDI, and there are four firms, there are sixteen possibilities to consider for interior outcomes. The model is symmetric so the profits of the firms are also symmetric. Denote the operating profits (before the fixed cost) of a firm from sales in the two countries as: Π_{EEEE} when all firms choose to export; Π_{EEFF} when home firms choose to export and foreign firms choose to undertake FDI; Π_{EFEF} when firm one and firm three choose to export and firm two and four choose to undertake FDI; Π_{FFFF} when all firms choose to undertake FDI etc. Meanwhile, there is a possibility of a corner solution when one firm undertakes FDI, and its competitor from the same market exports to the other country. It happens when the trade cost arrives at $\tilde{k}$ ($k \geq \tilde{k}$), the firm that has chosen to export will stop trading and its sales become zero, denoted by subscript $*$. Hence there are Π_{F*EE} when firm one is undertaking FDI, firms two stops trading while firm three and firm four are exporting to country A, so firm two's profit in foreign country becomes zero. Similar rule applies to Π_{F*FE}, Π_{F*EF}, Π_{F*FF}, Π_{*FEE}, Π_{*FEF}, Π_{*FFE}, Π_{*FFF} and Π_{F*F*} etc. Therefore, using(4), (6), (8), (9) and (11), the operating profits (before the fixed cost) of firm one or firm two from sales in the two countries are as follows:

<u>Interior solutions:</u> (when $k < \tilde{k}$)

$$\Pi_{EEEE} = \pi_{1A}^{EE} + \pi_{1B}^{EE} = \frac{(\alpha - c + 2k)^2}{25\beta} + \frac{(\alpha - c - 3k)^2}{25\beta} = \pi_{2A}^{EE} + \pi_{2B}^{EE}$$

$$\Pi_{EEFF} = \pi_{1A}^{FF} + \pi_{1B}^{EE} = \frac{(\alpha - c)^2}{25\beta} + \frac{(\alpha - c - 3k)^2}{25\beta} = \pi_{2A}^{FF} + \pi_{2B}^{EE}$$

$$\Pi_{EEEF} = \Pi_{EEFE} = \pi_{1A}^{EF} + \pi_{1B}^{EE} = \frac{(\alpha - c + k)^2}{25\beta} + \frac{(\alpha - c - 3k)^2}{25\beta} = \pi_{2A}^{EF} + \pi_{2B}^{EE}$$

$$\Pi_{EFEE} = \pi_{1A}^{EE} + \pi_{1B}^{EF} = \frac{(\alpha - c + 2k)^2}{25\beta} + \frac{(\alpha - c - 4k)^2}{25\beta} = \pi_{2A}^{EE} + \pi_{2B}^{FE}$$

$$\Pi_{EFEF} = \Pi_{EFFE} = \pi_{1A}^{EF} + \pi_{1B}^{EF} = \frac{\left(\alpha - c + k\right)^2}{25\beta} + \frac{\left(\alpha - c - 4k\right)^2}{25\beta} = \pi_{2A}^{EF} + \pi_{2B}^{FE}$$

$$\Pi_{EFFF} = \pi_{1A}^{FF} + \pi_{1B}^{EF} = \frac{\left(\alpha - c\right)^2}{25\beta} + \frac{\left(\alpha - c - 4k\right)^2}{25\beta} = \pi_{2A}^{FF} + \pi_{2B}^{FE}$$

$$\Pi_{FEEE} = \pi_{1A}^{EE} + \pi_{1B}^{FE} = \frac{\left(\alpha - c + 2k\right)^2}{25\beta} + \frac{\left(\alpha - c + k\right)^2}{25\beta} = \pi_{2A}^{EE} + \pi_{2B}^{EF} \tag{12}$$

$$\Pi_{FEEF} = \Pi_{FEFE} = \pi_{1A}^{EF} + \pi_{1B}^{FE} = \frac{2\left(\alpha - c + k\right)^2}{25\beta} = \pi_{2A}^{EF} + \pi_{2B}^{EF}$$

$$\Pi_{FEFF} = \pi_{1A}^{FF} + \pi_{1B}^{FE} = \frac{\left(\alpha - c\right)^2}{25\beta} + \frac{\left(\alpha - c + k\right)^2}{25\beta} = \pi_{2A}^{FF} + \pi_{2B}^{EF}$$

$$\Pi_{FFEE} = \pi_{1A}^{EE} + \pi_{1B}^{FF} = \frac{\left(\alpha - c + 2k\right)^2}{25\beta} + \frac{\left(\alpha - c\right)^2}{25\beta} = \pi_{2A}^{EE} + \pi_{2B}^{FF}$$

$$\Pi_{FFEF} = \Pi_{FFFE} = \pi_{1A}^{EF} + \pi_{1B}^{FF} = \frac{\left(\alpha - c + k\right)^2}{25\beta} + \frac{\left(\alpha - c\right)^2}{25\beta} = \pi_{2A}^{EF} + \pi_{2B}^{FF}$$

$$\Pi_{FFFF} = \pi_{1A}^{FF} + \pi_{1B}^{FF} = \frac{2\left(\alpha - c\right)^2}{25\beta} = \pi_{2A}^{FF} + \pi_{2B}^{FF}$$

<u>Corner solutions</u> (when $\tilde{k} \le k \le \bar{k}$)

$$\Pi_{F*EE} = \pi_{1A}^{EE} + \pi_{1B}^{F*} = \frac{\left(\alpha - c + 2k\right)^2}{25\beta} + \frac{\left(\alpha - c\right)^2}{16\beta}$$

$$\Pi_{F*FE} = \Pi_{F*EF} = \pi_{1A}^{EF} + \pi_{1B}^{F*} = \frac{\left(\alpha - c + k\right)^2}{25\beta} + \frac{\left(\alpha - c\right)^2}{16\beta}$$

$$\Pi_{F*FF} = \pi_{1A}^{FF} + \pi_{1B}^{F*} = \frac{\left(\alpha - c\right)^2}{25\beta} + \frac{\left(\alpha - c\right)^2}{16\beta} \tag{13}$$

$$\Pi_{*FEE} = \pi_{1A}^{EE} + \pi_{1B}^{*F} = \frac{\left(\alpha - c + 2k\right)^2}{25\beta} + 0$$

$$\Pi_{*FFE} = \Pi_{*FEF} = \pi_{1A}^{EF} + \pi_{1B}^{*F} = \frac{\left(\alpha - c + k\right)^2}{25\beta} + 0$$

$$\Pi_{*FFF} = \pi_{1A}^{FF} + \pi_{1B}^{*F} = \frac{\left(\alpha - c\right)^2}{25\beta} + 0$$

$$\Pi_{F*F*} = \Pi_{F**F} = \pi_{1A}^{F*} + \pi_{1B}^{F*} = \frac{(\alpha - c)^2}{8\beta}$$

$$\Pi_{*FF*} = \Pi_{*F*F} = \pi_{1A}^{F*} + \pi_{1B}^{*F} = \frac{(\alpha - c)^2}{16\beta} + 0$$

The above profits are the total profits of firm one (firm two) in both countries (the world). for example, Π_{EEEF} is the profits of firm one in country A when firm three exports to country A and firm four undertakes FDI to supply country A, plus the profits of firm one in country B when both firm one and firm two export to the foreign market, that is, $\pi_{1A}^{EF} + \pi_{1B}^{EE}$. There are twelve equations in (12), not sixteen as mentioned previously, because when considering firm one's profits, it does not matter whether firm three exports and firm four undertakes FDI, or the other way around, both have the same effect on the profits of firm one and firm two in the home market. Hence, $\Pi_{EEEF} = \Pi_{EEFE}$, $\Pi_{EFEF} = \Pi_{EFFE}$, $\Pi_{FEEF} = \Pi_{FEFE}$ and $\Pi_{FFEF} = \Pi_{FFFE}$ for firm one and firm two.

These profit levels in the second-stage outputs game can be put into a two-by-two payoff matrix, where the rows denote the strategies of firm one and the columns denote the strategies of firm two. As markets are segmented, firm one and firm two's decisions in the foreign country will not be affected by the choices of their competitors from the foreign country[19]. Therefore the game will only focus on firm one and firm two's payoffs in the foreign market (country B). The payoff matrix is as follows with the first number in a cell the profits of firm one in country B and the second number is the profits of firm two in country B. The matrices in table 2-1 and table 2-2 show the interior payoffs where firms take different strategies when the trade cost is smaller than the prohibitive trade cost $\tilde{k} = (\alpha - c)/4$.

[19] Notice that firm one and firm two's outputs in the home country will be affected by foreign firms' decision, as different strategy yields different outputs that affect home products. However home firms' decision in the foreign market will not be affected by foreign firms's decisions in the home market. We will focus on one home firm's decision (FDI vs Export) in the foreign country later on, and its profits at home will be cancelled out regardless what foreign firms' decision. Only its profits in the foreign market is taken into account.

Payoff Matrix		Firm 2	
		Export	FDI
Firm 1	Export	π_{1B}^{EE} , π_{2B}^{EE}	π_{1B}^{EF} , $\pi_{2B}^{EF}-G$
	FDI	$\pi_{1B}^{FE}-G$, π_{2B}^{FE}	$\pi_{1B}^{FF}-G$, $\pi_{2B}^{FF}-G$

Table 2-1: interior payoff matrix when $k<\tilde{k}$

The first cell on the left hand side shows a pair of payoffs in country B when both firm one and firm two decide to export, and the second cell on the left presents a pair of payoffs in country B when firm one undertakes FDI while firm two exports, so the fixed cost of undertaking FDI G is taken off from firm one's profits. Cells on the right hand side show the pairs of payoffs in country B when firm two undertakes FDI while firm one chooses to export (top cell) or to undertake FDI (bottom cell). Table 2-2 is the substitution of the payoffs in Table 2-1 from (12), and all the payoffs contain the trade cost k and the fixed cost G.

Payoff Matrix		Firm 2	
		Export	FDI
Firm 1	Export	$\frac{(\alpha-c-3k)^2}{25\beta}$, $\frac{(\alpha-c-3k)^2}{25\beta}$	$\frac{(\alpha-c-4k)^2}{25\beta}$, $\frac{(\alpha-c+k)^2}{25\beta}-G$
	FDI	$\frac{(\alpha-c+k)^2}{25\beta}-G$, $\frac{(\alpha-c-4k)^2}{25\beta}$	$\frac{(\alpha-c)^2}{25\beta}-G$, $\frac{(\alpha-c)^2}{25\beta}-G$

Table 2-2: substitution of the interior payoff matrix when $k<\tilde{k}$

The matrices in table 2-3 and table 2-4 show the corner payoffs where firm one or firm two stops exporting and its sales become zero in country B, $\pi_{1B}^{*F}=\pi_{2B}^{F*}=0$, when the trade cost is between $\tilde{k}=(\alpha-c)/4$ and $\bar{k}=(\alpha-c)/3$.

Payoff Matrix		Firm 2	
		Export	FDI
Firm 1	Export	π_{1B}^{EE} , π_{2B}^{EE}	π_{1B}^{*F} , $\pi_{2B}^{*F}-G$
	FDI	$\pi_{1B}^{F*}-G$, π_{2B}^{F*}	$\pi_{1B}^{FF}-G$, $\pi_{2B}^{FF}-G$

Table 2-3: corner payoff matrix when $\tilde{k}\le k<\bar{k}$

Table 2-4 is the substitution of the payoffs in Table 2-3 from (13). When firm one (firm two) undertakes FDI, firm two (firm one) will stop trading if the trade cost reaches $\tilde{k}$, and its sales and profits from exporting become zero as shown in Table 2-4.

<table>
<tr><td colspan="2" rowspan="2">Payoff Matrix</td><td colspan="2">Firm 2</td></tr>
<tr><td>Export</td><td>FDI</td></tr>
<tr><td rowspan="2">Firm 1</td><td>Export</td><td>$\frac{(\alpha-c-3k)^2}{25\beta}$, $\frac{(\alpha-c-3k)^2}{25\beta}$</td><td>0 , $\frac{(\alpha-c)^2}{16\beta}-G$</td></tr>
<tr><td>FDI</td><td>$\frac{(\alpha-c)^2}{16\beta}-G$, 0</td><td>$\frac{(\alpha-c)^2}{25\beta}-G$, $\frac{(\alpha-c)^2}{25\beta}-G$</td></tr>
</table>

Table 2-4: substitution of the corner payoff matrix when $\tilde{k} \le k < \bar{k}$

They are all symmetric matrices that follow from equations (12) and (13). Notice that when a firm chooses to undertake FDI, the payoff is the total profits minus the fixed cost G of building a factory in the other market. That is to say, in the static game, undertaking FDI is profitable for a firm if the operating profits (before the fixed cost) of undertaking FDI minus the fixed cost are greater than the profits of exporting.

2.3.1 Equilibria for the game

Solving the game for equilibria, there are two cases: a firm's competitor in the same market chooses to export or a firm's competitor in the same market chooses to undertake FDI to supply foreign country. Considering *firstly* that when a firm's competitor in the same country chooses to export, the firm will only choose to undertake FDI if it gives higher payoffs than it chooses to export. Refer back to the left hand side of table 2-1, where firm one and firm two compete in country B, and firm two chooses to export to country B. When firms in country B (firm three and firm four) choose to export, undertaking FDI is profitable for firm one if $\Pi_{FEEE} - G > \Pi_{EEEE}$. When firm three (firm four) chooses to export, firm four (firm three) chooses to undertake FDI, undertaking FDI is profitable for firm one if $\Pi_{FEEF} - G > \Pi_{EEEF}$ ($\Pi_{FEFE} - G > \Pi_{EEFE}$). When both foreign firms choose to undertake FDI, undertaking FDI is optimal for firm one if $\Pi_{FEFF} - G > \Pi_{EEFF}$. As markets are segmented, the decisions of the firms in the foreign market are independent of the choices of their competitors in the home country, thus only the profits of firm one and firm two in the foreign market (country B) are considered, and their profits in the home market are not affected[20].

When the trade cost is smaller than the prohibitive trade cost $\tilde{k}$ (the interior solution), if the fixed cost of FDI of firm one is less than the critical value: $\bar{G} = \Pi_{FEEE} - \Pi_{EEEE} = \Pi_{FEEF} - \Pi_{EEEF} = \Pi_{FEFE} - \Pi_{EEFE} = \Pi_{FEFF} - \Pi_{EEFF}$, by using (12), it would choose to undertake FDI, when its competitor in the same country chooses to export up to $\tilde{k}$.

$$\bar{G} \equiv \pi_{1B}^{FE} - \pi_{1B}^{EE} \qquad if \qquad k < \tilde{k} \tag{14}$$

When firm one undertakes FDI, there is a corner solution where firm two will stop exporting to the foreign market at the prohibitive level $\tilde{k}$. Once the trade cost arrives at $\tilde{k}$, only Π_{F*EE}, Π_{F*EF}, Π_{F*FE} and Π_{F*FF} are taken into account, referring to the left hand side of table 2-3. Regardless what strategies firm three and firm four choose, it is profitable for firm one to undertake FDI if $\Pi_{F*EE} - G > \Pi_{EEEE}$, $\Pi_{F*EF} - G > \Pi_{EEEF}$, $\Pi_{F*FE} - G > \Pi_{EFEE}$ or $\Pi_{F*FF} - G > \Pi_{EEFF}$. Hence, for $\tilde{k} \le k < \bar{k}$, if the fixed cost of FDI is less than the critical value: $\bar{G} = \Pi_{F*EE} - \Pi_{EEEE} = \Pi_{F*EF} - \Pi_{EEEF} = \Pi_{F*FE} - \Pi_{EEFE} = \Pi_{F*FF} - \Pi_{EEFF}$, firm one will choose to undertake FDI than to export, by using (13) and (12):

$$\bar{G} \equiv \pi_{1B}^{F*} - \pi_{1B}^{EE} \qquad if \qquad \tilde{k} \le k < \bar{k} \tag{15}$$

When firm two exports, firm one will export up to the prohibitive cost $\bar{k} = (\alpha - c)/3$, where both firms in country A stop exporting as it is not profitable to do so. Therefore, undertaking FDI is a preferred strategy for a firm if the fixed cost of FDI is less than

[20] Regard to the strategic choice, home firms only need to consider their profits in the foreign market. For example, if foreign firm three and four decide to export (undertake FDI) to the home market, firm one's profit in the home market is π_1^{EE} (π_1^{FF}), which will be cancelled out when firm one decides whether to undertake FDI or export to the foreign market. Thus only the profits if home firms in the foreign market matter, and the strategic choice by foreign firms do not affect them.

the critical value $\bar{G}$ when the firm's competitor in the same market chooses to export, using (14), (15), (12) and (13), it can be shown that:

$$\bar{G} \equiv \begin{cases} \dfrac{8k(\alpha - c - k)}{25\beta} > 0 & \text{if } k < \tilde{k} \\ \dfrac{3(3\alpha - 3c - 4k)(\alpha - c + 12k)}{400\beta} > 0 & \text{if } \tilde{k} \le k < \bar{k} \end{cases} \tag{16}$$

The critical value of the fixed cost of FDI $\bar{G}$ above is a two-segmented concave quadratic curves in the trade cost up to the prohibitive $\bar{k}$, where $\bar{G}$ reaches its peak. It is shown in figure 2-1 with the parameter values: $\alpha = 40$, $\beta = 1$ and $c = 14$. Undertaking FDI is preferred to exporting for a firm in the region where $G < \bar{G}$ when its competitor in the same market chooses to export before the trade cost reaches $\tilde{k} = (\alpha - c)/4$ and stop trading thereafter, whereas exporting is preferred for both firms in the region $G > \bar{G}$. Thus a reduction in trade costs will only shift firms from undertaking FDI to exporting as suggested in most theoretical literature.

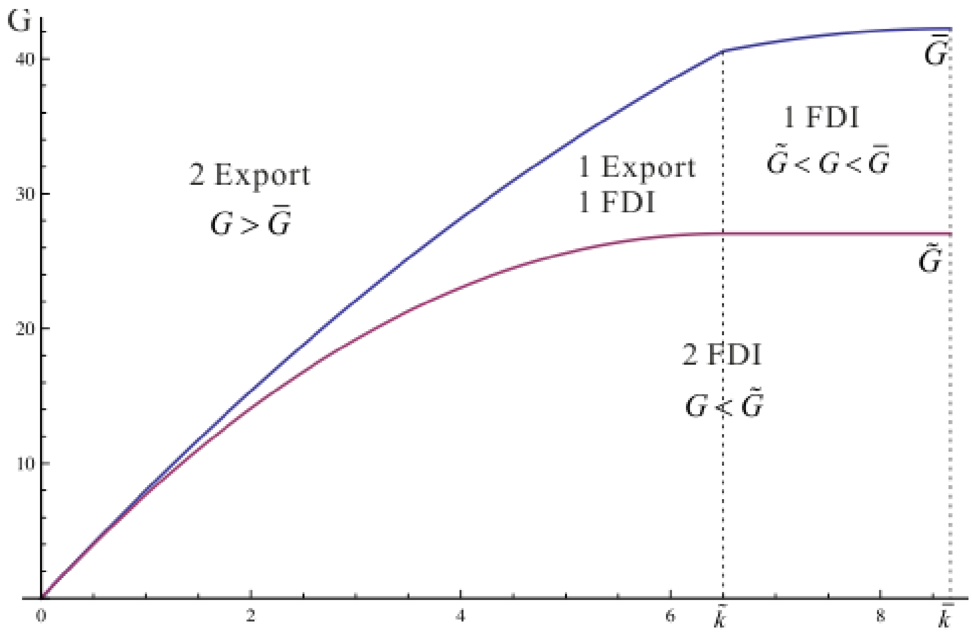

Figure 2-1: Static game under Cournot Oligopoly

Considering *secondly* that when a firm's competitor in the same country chooses to undertake FDI, a firm will only choose to undertake FDI if it brings higher payoffs than it chooses to export. By looking at the right hand side of table 2-1, regardless what the foreign firms' strategies are, when firm two chooses to undertake FDI, undertaking FDI is more profitable for firm one if $\Pi_{FFEE} - G > \Pi_{EFEE}$, where firm three and firm four are exporting. The same principle applies to the states where firm three exports and firm four undertakes FDI or the other way around or both undertake FDI, so firm one will choose to undertake FDI if $\Pi_{FFEF} - G > \Pi_{EFEF}$, or $\Pi_{FFFE} - G > \Pi_{EFFE}$ or $\Pi_{FFFF} - G > \Pi_{EFFF}$. Hence undertaking FDI is preferred to exporting for a firm when its competitor in the same market has chosen to undertake FDI, if the fixed cost of FDI is less than the critical value $\tilde{G} \equiv \Pi_{FFEE} - \Pi_{EFEE} = \Pi_{FFEF} - \Pi_{EFEF} = \Pi_{FFFE} - \Pi_{EFFE} = \Pi_{FFFF} - \Pi_{EFFF}$:

$$\tilde{G} \equiv \pi_{1B}^{FF} - \pi_{1B}^{EF} \qquad if \qquad k < \tilde{k} \tag{17}$$

With a corner solution, where firm one would stop exporting if the trade cost reaches $\tilde{k}$, the profits and the outputs of firm one become zero (refer back to the right hand side of table 2-4). For $\tilde{k} \le k < \bar{k}$, if the fixed cost of FDI is less than the critical value $\tilde{G}$, Firm one will choose to undertake FDI:

$$\tilde{G} \equiv \pi_{1B}^{FF} - \pi_{1B}^{*F} = \frac{(\alpha - c)^2}{25\beta} - 0 = \frac{(\alpha - c)^2}{25\beta} \qquad if \quad \tilde{k} \le k < \bar{k} \tag{18}$$

To sum up, if the fixed cost of FDI is less than the critical value $\tilde{G}$, undertaking FDI is preferred to exporting for a firm when its competitor in the same country undertakes FDI, using (18), (17) (12) and (13), it can be shown that:

$$\tilde{G} \equiv \begin{cases} \dfrac{8k(\alpha - c - 2k)}{25\beta} & if \quad k < \tilde{k} \\ \dfrac{(\alpha - c)^2}{25\beta} & if \quad \tilde{k} \le k < \bar{k} \end{cases} \tag{19}$$

The critical value of the fixed cost of FDI $\tilde{G}$ is shown in figure 2-1. For $k < \tilde{k}$, it is a concave quadratic curve that is increasing in the trade cost k. For $\tilde{k} \le k < \bar{k}$, it is horizontal and is independent of the trade cost, as the firm's profits from exporting to the other country become zero. By comparing (19) and (16), it can be shown that $\tilde{G} < \bar{G}$.

$$\bar{G} - \tilde{G} = \begin{cases} \dfrac{8k^2}{25\beta} > 0 & \text{if } k < \tilde{k} \\ \dfrac{(7\alpha - 7c - 12k)(c - \alpha + 12k)}{400\beta} > 0 & \text{if } \tilde{k} \le k < \bar{k} \end{cases}$$

In the region under $\tilde{G}$, a firm will choose to undertake FDI when its competitor in the same country undertakes FDI, and the firm will also choose FDI when that competitor chooses to export ($\tilde{G} < \bar{G}$). Therefore undertaking FDI is a dominant strategy for both firms in the same market when $G < \tilde{G}$, whereas only one firm in each market chooses FDI in the region where $\tilde{G} < G < \bar{G}$. Again, the figures show that a reduction in trade costs will only lead firms to shift from undertaking FDI to exporting according to $\tilde{G}$. This leads to the following proposition:

Proposition 1*: under a static game in Cournot oligopoly, undertaking FDI is a dominant strategy for both firms in a market in the region where $G < \tilde{G}$, while only one firm in each market chooses to undertake FDI in the region $\tilde{G} < G < \bar{G}$.*

In addition, there is a possibility that prisoners' dilemma exists in the region $G < \tilde{G}$, as undertaking FDI is the dominant strategy for both firms in the same country, yet the profits (before the fixed cost) when both firms undertake FDI are lower than the profits when both firms export. This is due to the more intense competition brought by the FDI in the market.

2.3.2 Prisoners' dilemma

The Prisoners' dilemma is the most fundamental game that involves two suspects for a crime might not cooperate even if it is in both of their best interests to do so. In a classic form of a static game, cooperating is strictly dominated by defecting, thus the Nash equilibrium of the game is to defect for both players, as it gives both higher payoffs. In this model of four-firm two-country Cournot oligopoly, two firms compete with each other in the same country, and each firm chooses whether to export or to undertake FDI to supply the other country. Undertaking FDI will intensify competition in the other country and therefore reduce the profits of the competitor in the home market. Thus if both firms decide to undertake FDI, both will end up with lower profits than when both export. This outcome of the game is therefore prisoners' dilemma.

When the fixed cost is between $\tilde{G}$ and $\bar{G}$, the Nash equilibrium is that one firm undertakes FDI while its competitor in the same market exports up to $\tilde{k}$. The profits of the equilibrium when undertaking FDI while its competitor in the same market exports up to $\tilde{k}$ might be lower than the profits when all firms export. Therefore there is a possibility of Prisoners' dilemma in this region.

When the fixed cost is below $\tilde{G}$, the Nash equilibrium is that both firms in the same market undertake FDI, but the profits when both firms undertake FDI could be lower than when both firms export. Hence there is a possibility of Prisoners' dilemma for this equilibrium when $G < \tilde{G}$ as well. To examine the prisoners' dilemma, this section considers the equilibria of all firms in both countries.

Prisoners' dilemma between four firms in two countries

Since two markets are symmetric, the equilibria for both countries are also symmetric. When the fixed cost is between $\tilde{G}$ and $\bar{G}$, the Nash equilibrium is that one firm in each market chooses to undertake FDI, while its competitor in the same country chooses to export up to the prohibitive trade cost $\tilde{k}$ (Π_{FEFE} or Π_{F*F*}). However, there is a possibility of Prisoners' dilemma, as the profits of the Nash equilibrium might be lower than the profits when all firms choose to export.

When one firm in each country undertakes FDI, the firm will have a higher profits than when all firms choose to export if $\Pi_{FEFE} - G > \Pi_{EEEE}$ ($\Pi_{F*F*} - G > \Pi_{EEEE}$) when $k < \tilde{k}$ ($\tilde{k} \leq k < \bar{k}$). This occurs if the fixed cost of FDI is less than the critical value:

$$G_n \equiv \begin{cases} \Pi_{FEFE} - \Pi_{EEEE} \\ \Pi_{F*F*} - \Pi_{EEEE} \end{cases} = \begin{cases} \left(\Pi_{FEEE} - \Pi_{EEEE}\right) + \left(\Pi_{FEFE} - \Pi_{FEEE}\right) & if\ k < \tilde{k} \\ \left(\Pi_{F*EE} - \Pi_{EEEE}\right) + \left(\Pi_{F*F*} - \Pi_{F*EE}\right) & if\ \tilde{k} \le k < \bar{k} \end{cases}$$

Where the first bracket shows the profits gain of a firm from undertaking FDI while its competitors from both home and foreign market choose to export up to the prohibitive level, and the second bracket shows the effect on a firm's profits when one of its competitors from the foreign market also chooses to undertake FDI. Using the definition of $\bar{G}$ in (14) and (15) , the critical value G_n can be described as:

$$G_n \equiv \bar{G} + \begin{cases} \Pi_{FEFE} - \Pi_{FEEE} & if\ \ k < \tilde{k} \\ \Pi_{F*F*} - \Pi_{F*EE} & if\ \ \tilde{k} \le k < \bar{k} \end{cases}$$

By using equations (12) and (13),

$$\begin{cases} \Pi_{FEFE} - \Pi_{FEEE} = \dfrac{-k\left(2\alpha - 2c + 3k\right)}{25\beta} < 0 & if\ \ k < \tilde{k} \\ \Pi_{F*F*} - \Pi_{F*EE} = \dfrac{\left(9\alpha - 9c + 8k\right)\left(\alpha - c - 8k\right)}{400\beta} < 0 & if\ \ \tilde{k} \le k < \bar{k} \end{cases} \tag{20}$$

A firm that undertakes FDI makes lower profits when one of its competitors from foreign market also chooses to undertake FDI than when all of its competitors from both markets choose to export, $\Pi_{FEFE} < \Pi_{FEEE}$ ($\Pi_{F*F*} < \Pi_{F*EE}$) when $k < \tilde{k}$ ($\tilde{k} \le k < \bar{k}$), so that G_n is always smaller than $\bar{G}$, as FDI intensifies competition. Substitute (20) into G_n:

$$G_n = \begin{cases} \dfrac{k\left(6\alpha - 6c - 11k\right)}{25\beta} > 0 & if\ \ k < \tilde{k} \\ \dfrac{9\left(\alpha - c\right)^2 + 8k\left(2\alpha - 2c - 13k\right)}{200\beta} > 0 & if\ \ \tilde{k} \le k < \bar{k} \end{cases} \tag{21}$$

To compare G_n and $\tilde{G}$, subtract G_n from $\tilde{G}$ using (21) and (19):

$$\tilde{G}-G_n=\begin{cases}\dfrac{k(2\alpha-2c-5k)}{25\beta}>0 & \text{if } \ k<\tilde{k}\\[2ex] \dfrac{8k(13k-2\alpha+2c)-(\alpha-c)^2}{200\beta}>0 & \text{if } \ \tilde{k}\le k<\bar{k}\end{cases}$$

The critical value G_n is a two-segmented concave quadratic that is increasing in the trade cost up to the prohibitive trade cost $\bar{k}$ as shown in figure 2-2, and it is below $\bar{G}$ and $\tilde{G}$. In the region under G_n, there is no prisoners' dilemma, as the profits of the Nash equilibrium where one firm in each country undertakes FDI while their competitors export, is greater than the profits when all firms export. In the region above $\tilde{G}$, on the other hand, indicating a prisoners' dilemma, where the equilibrium profits are smaller than the profits when all firms export. Since this Nash equilibrium only occurs when the fixed cost is between $\bar{G}$ and $\tilde{G}$, and G_n is below this region (G_n is below $\tilde{G}$), the shaded area under G_n in figure 2-2 is not relevant. Thus there is always a prisoners' dilemma when $\tilde{G}<G<\bar{G}$.

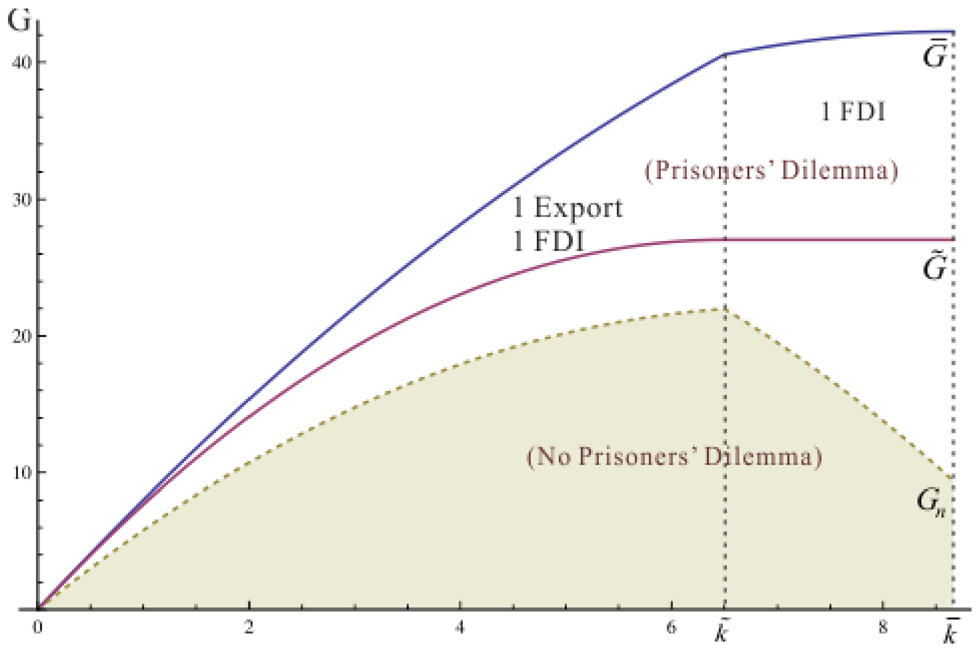

Figure 2-2: Prisoners' dilemma in the world in the static game when $\tilde{G}<G<\bar{G}$

When the fixed cost is below $\tilde{G}$, the Nash equilibrium of both countries is that all firms choose to undertake FDI. When all firms in both countries undertake FDI, they will have higher profits than when they all export if $\Pi_{FFFF} - G > \Pi_{EEEE}$, and this happens if the fixed cost of FDI is less than the critical value:

$$\hat{G} \equiv \Pi_{FFFF} - \Pi_{EEEE} = \begin{cases} \left(\Pi_{FEEE} - \Pi_{EEEE}\right) - \left(\Pi_{FFFF} - \Pi_{FEEE}\right) & if \;\; k < \tilde{k} \\ \left(\Pi_{F*EE} - \Pi_{EEEE}\right) - \left(\Pi_{FFFF} - \Pi_{F*EE}\right) & if \;\; \tilde{k} \le k < \bar{k} \end{cases}$$

Where the first bracket indicates the profits gain of a firm from undertaking FDI while its competitors from both countries export up to $\tilde{k}$, and the second bracket shows the effect on a firm's profits if its competitors from both home and foreign markets switch from exporting to undertaking FDI. Using the description of $\bar{G}$, the critical value $\hat{G}$ can be described as $\hat{G} = \bar{G} + \begin{cases} \Pi_{FFFF} - \Pi_{FEEE} & if \;\; k < \tilde{k} \\ \Pi_{FFFF} - \Pi_{F*EE} & if \;\; \tilde{k} \le k < \bar{k} \end{cases}$, by using (12) and (13):

$$\begin{cases} \Pi_{FFFF} - \Pi_{FEEE} = \dfrac{-k\left(6\alpha - 6c + 5k\right)}{25\beta} < 0 & if \;\; k < \tilde{k} \\ \Pi_{FFFF} - \Pi_{F*EE} = \dfrac{7\left(\alpha - c\right)^2 - 16\left(\alpha - c + 2k\right)^2}{400\beta} < 0 & if \;\; \tilde{k} \le k < \bar{k} \end{cases} \tag{22}$$

Once again it shows that FDI intensifies competition, a firm that undertakes FDI makes lower profits when its competitors from both countries undertake FDI than when they export, $\Pi_{FFFF} < \Pi_{FEEE}$ ($\Pi_{FFFF} < \Pi_{F*EE}$) when $k < \tilde{k}$ ($\tilde{k} \le k < \bar{k}$), so $\hat{G}$ is always smaller than $\bar{G}$. To look at the relationship between $\hat{G}$ and $\tilde{G}$, $\hat{G}$ can be decomposed into the following:

$$\hat{G} \equiv \Pi_{FFFF} - \Pi_{EEEE} = \begin{cases} \left(\Pi_{FFFF} - \Pi_{EFFF}\right) + \left(\Pi_{EFFF} - \Pi_{EEEE}\right) & if \;\; k < \tilde{k} \\ \left(\Pi_{FFFF} - \Pi_{*FFF}\right) + \left(\Pi_{*FFF} - \Pi_{EEEE}\right) & if \;\; \tilde{k} \le k < \bar{k} \end{cases}$$

As $\tilde{G}$ can be defined as $\Pi_{FFFF} - \Pi_{EFFF}$ or $\Pi_{FFFF} - \Pi_{*FFF}$, according to (18) and (17):

$$\hat{G} = \tilde{G} + \begin{cases} \Pi_{EFFF} - \Pi_{EEEE} & if \ \ k < \tilde{k} \\ \Pi_{*FFF} - \Pi_{EEEE} & if \ \ \tilde{k} \le k < \bar{k} \end{cases}$$

By using (12) and (13):

$$\begin{cases} \Pi_{EFFF} - \Pi_{EEEE} = \dfrac{-3k\left(2\alpha - 2c - k\right)}{25\beta} < 0 & if \ \ k < \tilde{k} \\ 0 - \Pi_{EEEE} = -\dfrac{\left(\alpha - c - 3k\right)^2 + \left(\alpha - c + 2k\right)^2}{25} < 0 & if \ \ \tilde{k} \le k < \bar{k} \end{cases} \tag{23}$$

A firm that exports up to $\tilde{k}$ makes lower profits when its competitors from both markets undertake FDI than when they export, $\Pi_{EFFF} < \Pi_{EEEE}$ ($\Pi_{*FFF} < \Pi_{EEEE}$). Therefore $\hat{G}$ is always smaller than $\tilde{G}$ as shown in figure 2-4. Substitute (22) into $\hat{G}$:

$$\hat{G} = \frac{k\left(2\alpha - 2c - 13k\right)}{25\beta} \tag{24}$$

In the region under $\hat{G}$ in figure 2-3, all firms prefer undertaking FDI to exporting, and all firms prefer undertaking FDI to the case that one firm in each country undertake FDI while its competitor in the same country exports ($\hat{G}$ is in the region under $\tilde{G}$). Therefore undertaking FDI is a dominant strategy for all firms under $\hat{G}$ ($G < \hat{G}$), and the profits when all firms undertake FDI are higher than the profits when all firms export, there is no prisoners' dilemma when $G < \hat{G}$ for all firms.

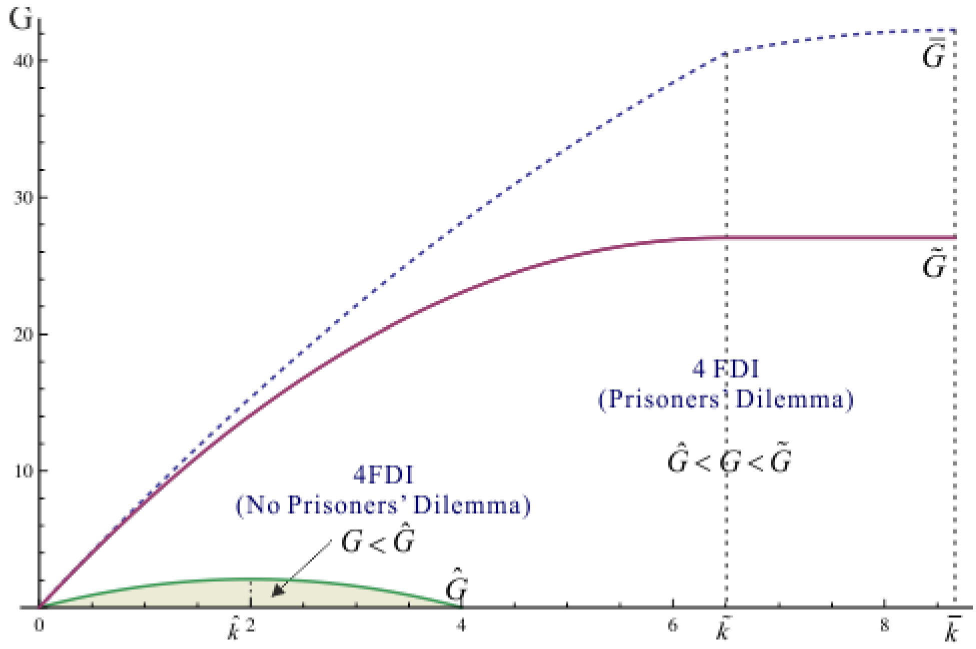

Figure 2-3: Prisoners' dilemma in the static game when $G < \tilde{G}$

Proposition 2*: Under Cournot oligopoly, for firms in the same country, undertaking FDI is a dominant strategy when $G < G_s$, and there is no prisoners' dilemma. For all firms in both countries, undertaking FDI is a dominant strategy when $G < \hat{G}$, and there is no prisoners' dilemma.*

The critical value $\hat{G}$ in (24) is a concave quadratic that reaches its maximum point when trade cost is $\hat{k} = (\alpha - c)/13$ where it is smaller than $\tilde{k}$ and $2\hat{k} < \tilde{k}$ as shown in figure 2-3. The critical value $\hat{G}$ is increasing for $k < \hat{k}$ and decreasing for $k > \hat{k}$. The explanation of the quadratic concave is that when $k < \hat{k}$, the profits of a firm when all country export (Π_{EEEE}) is decreasing and it is increasing when $k > \hat{k}$. This is because an increase in the trade cost will increase the marginal cost of the firm while it will also increase the marginal costs of its competitors. Hence it is a situation of a negative effect on export from the firm versus a positive effect on domestic sales from its competitors. Thus when the trade cost is relatively low, the price-cost margins are similar for the firms in both markets, but the effect on export is direct while the effect on domestic sales is indirect. Then the absolute size of the negative effect on the export is bigger than that of the positive effect on domestic sales, and the profits of exporting is decreasing. On the other hand, when the trade cost is relatively high, the price-cost margins affect domestic market more than on exports than the absolute size of the positive effect on domestic sales is lager than that of the negative effect on export, and the profits of exporting is increasing.

To sum up, comparing figure 2-2 with figure 2-1, undertaking FDI is a dominant strategy for all firms in both countries in the region $G < \hat{G}$ and there is no prisoners' dilemma. In the region $\hat{G} < G < \tilde{G}$, there is a prisoners' dilemma when all firms will choose to undertake FDI, but profits are lower than when they all export due to the same reason (more intensive competition and the fixed cost of undertaking FDI). In the region $\tilde{G} < G < \bar{G}$ of figure 2-2, there is a Prisoners' dilemma where one firm in each country undertakes FDI while their competitors from both markets choose to export up to the trade cost $\tilde{k}$, but the profits are lower than when all firms export.

Consequently, in the region $\tilde{G} < G < \bar{G}$, the Nash equilibrium is that only one firm undertakes FDI while its competitor in the same market chooses to export up to the prohibitive level $\tilde{k}$ in figure 2-3. In the region $G < \tilde{G}$, the Nash equilibrium is that both firms from the same country undertake FDI. Since the model is symmetric, the equilibira are that one firm from each market undertakes FDI when $\tilde{G} < G < \bar{G}$ or all firms undertake FDI when $G < \tilde{G}$. When the fixed cost is above $\bar{G}$, both firms in the same country will choose to export due to the high fixed cost of FDI. Next section will extend the static game to an infinitely-repeated game theory of FDI to avoid the prisoners' dilemma.

2.4 The Infinitely-Repeated Cournot Oligopoly Game

In the repeated prisoner's dilemma, each player has an opportunity to punish the other player for previous non-cooperative play. If the number of steps is known by both players in advance, economic theory says that the two players should defect again and again, no matter how many times the game is played. However, this analysis fails to predict the behaviour of human players in a real iterated prisoners' dilemma situation. Only when the players play an indefinite or random number of times can cooperation be an equilibrium. Technically a subgame perfect equilibrium means that both players defecting always remain an equilibrium and there are many other equilibrium outcomes. In this case, the incentive to defect can be overcome by the threat of punishment.

In the infinitely-repeated Cournot oligopoly game, firms can overcome the prisoners' dilemma problem by tacitly colluding. Friedman (1971) proposed that Nash reversion trigger strategies can sustain a subgame perfect Nash equilibrium in an infinitely repeated game. A Nash reversion trigger strategy is a grim trigger strategy, which applies to repeated prisoner's dilemmas: a player begins by cooperating in the first period, and continues to cooperate until a single defection by her opponent, following which, the player defects forever. In this infinitely-repeated game, firms cooperate until one firm deviates, and any deviation triggers a permanent retaliation in which all firms revert to the static game equilibrium at every period thereafter. So it is a punishment with a static equilibrium forever. A subgame perfect Nash equilibrium is an equilibrium such that firms' strategies constitute a Nash equilibrium in every subgame, even in the continuation after a deviation. Hence the equilibrium in the infinitely-repeated game is a subgame perfect Nash equilibrium since Nash equilibrium strategies are used in all subgames.

The collusive outcome where all firms choose to export could be sustained as a subgame perfect Nash equilibrium by the threat of Nash reversion trigger strategies in the infinitely-repeated game. If the discount factor is sufficiently high, the collusive outcome (all firms choose to export) is sustained by the threat of reversion to the Nash equilibrium outcomes in the infinitely-repeated game. Therefore, the static game Nash equilibria outcomes where all firms undertake FDI or one firm in each country undertakes FDI while their competitors export are not preferred.

In an infinitely repeated game, in each period, all firms simultaneously decide whether to export or to undertake FDI to supply the other market in the first stage and choose their outputs as Cournot oligopolies in the second stage of the game. It is assumed that firms only collude over the undertaking FDI versus exporting decision. The grim trigger strategy is the collusion that all firms choose to export, and the profits are Π_{EEEE}. However, if one firm deviates in one period, it would be follwed by the static game Nash equilibria profits: (1) Π_{FFFF} in the region below $\tilde{G}$, where all firms in both markets choose to undertake FDI, or (2) $\Pi_{FEFE}\left(\Pi_{F*F*}\right)$ in the region between $\bar{G}$ and $\tilde{G}$, where one firm in each country chooses to undertake FDI and their competitors from each country choose to export up to the trade cost $\tilde{k}$. This is repeated in every period and all firms know all previously chosen outputs. Oligopolies can use the threat of the static Nash equilibrium (deviation) to sustain the collusive outcomes if firms care enough about the future.

Since there are two Nash equilibrium profits in the infinitely-repeated game as mentioned previously, they should be taken into account separately according to the size of the fixed cost G of FDI. The Next two sub-sections will discuss the grim trigger strategies.

2.4.1 Collusion when $G < \tilde{G}$

Following the "grim trigger strategy" for firm one when the fixed cost G of FDI is in the region under $\tilde{G}$, there are two options:

- All firms have played the collusive profits Π_{EEEE} in all previous periods, and they would play in this period as well.
- One firm did not play the collusive profits hence deviate with profits $\Pi_{FEEE}\left(\Pi_{F*EE}\right)-G$ in one period, followed by Nash equilibrium profits $\Pi_{FFFF}-G$.

Then firm one's present value of all its future profits at time t if all firms play the grim trigger strategy (export) is:

$$\begin{aligned} V^c &= \Pi_{EEEE} + \delta\, \Pi_{EEEE} + \delta^2 \Pi_{EEEE} + \ldots \\ &= \sum_{t=1}^{\infty} \delta^{t-1} \Pi_{EEEE} = \frac{1}{1-\delta} \Pi_{EEEE} \end{aligned}$$

Where δ is the discount factor and is between 0 and 1. The firms' discount rate δ depends on interest rate and other factors such as the time needed for cheating to be detected and the probability that the product may be outdated. All firms' objective is to maximise the present value of profits, and that depends on the discount factor which is largely affected by the interest rate. Firm one's discounted value of profits if deviating from the equilibrium path at $t=1$ is:

$$
\begin{aligned}
V^d &= \left(\Pi_{FEEE} - G\right) + \sum_{t=2}^{\infty} \delta^{t-1}\left(\Pi_{FFFF} - G\right) \\
&= \left(\Pi_{FEEE} - G\right) + \left(\Pi_{FFFF} - G\right)\left(\delta + \delta^2 + \delta^3 + \ldots\right) \\
&= \left(\Pi_{FEEE} - G\right) + \frac{\delta}{1-\delta}\left(\Pi_{FFFF} - G\right)
\end{aligned}
$$

That is to say, firm one has no incentive to deviate if $V^c > V^d$. Hence all firms choosing to export is a Nash equilibrium if the present discounted profits from collusion (all firms export) exceed the present discounted value of profits from cheating (choosing to undertake FDI when its competitors from the home country and the foreign country have chosen to export) for one period, and thereafter followed by the Nash equilibrium profits of all firms (all firms choose to undertake FDI):

$$
\begin{cases}
\frac{1}{1-\delta}\Pi_{EEEE} > \left(\Pi_{FEEE} - G\right) + \frac{\delta}{1-\delta}\left(\Pi_{FFFF} - G\right) & if\ k < \tilde{k} \\
\frac{1}{1-\delta}\Pi_{EEEE} > \left(\Pi_{F*EE} - G\right) + \frac{\delta}{1-\delta}\left(\Pi_{FFFF} - G\right) & if\ \tilde{k} \le k < \bar{k}
\end{cases}
\qquad (25)
$$

When the fixed cost G of FDI is in the region under $\tilde{G}$, a subgame Nash equilibrium could be the collusive outcome where all firms choose to export, and firms are earning jointly maximised profits. However, if a firm deviates (undertaking FDI) to increase its single period (discounted) profits, the other firms are, in response, likely to revert to the safe position (static Nash equilibrium) where all firms undertake FDI for the rest of the periods. So by deviating, a firm makes a short-term gain but get a lower profit in all future periods. That means if the discount factor is sufficiently high or the firms are patient enough, then they can resist the temptation to deviate, and the collusive outcome where all firms choose to export can be sustained by the threat of Nash reversion strategy where all firms choose to undertake FDI in an infinitely repeated game.

Since this equilibrium involves one firm exports, and the other firm in the same country undertakes FDI for the first period, there will be an interior solution and a corner solution. Firm two will stop exporting when the trade cost reaches its prohibitive level $\tilde{k}$. So the first equation shows that when $k < \tilde{k}$, the present value of the profits of a firm when all firms export is greater than the present value of the profits when cheating (undertake FDI) for one period, and followed by all firms choosing to undertake FDI. This agrees with the first-mover advantage in section 2.1, that the firm which undertakes FDI first gains the profits while its competitors who have chosen to export suffer, resulting in the retaliation by its competitors who will also choose to undertake FDI for the following periods, so that all firms earn lower profits than the subgame Nash equilibrium position.

The second equation shows that when $\tilde{k} \le k < \bar{k}$, once firm one decides to deviate by undertakeing FDI, firm two will stop exporting to the foreign market due to the high trade cost, while firm three and firm four have chosen to export for the fist period, followed by all firms will revert to the static game equilibrium where all firms choose to undertake FDI forever. Thus only when (25) is satisfied, the collusive outcome can be sustained as a subgame Nash equilibrium and all firms can maximise their joint profits. In addition, as the profits of the firm when all firms export are greater than the profits when all firms undertake FDI in (25), the prisoners' dilemma is avoided in this infinitely repeated game.

Rearrange(25), the collusive outcome can be sustained as a Nash equilibrium if the fixed cost of FDI is greater than the critical value:

$$G^*(\delta) = \begin{cases} (1-\delta)(\Pi_{FEEE} - \Pi_{EEEE}) + \delta(\Pi_{FFFF} - \Pi_{EEEE}) & if\ k < \tilde{k} \\ (1-\delta)(\Pi_{F*EE} - \Pi_{EEEE}) + \delta(\Pi_{FFFF} - \Pi_{EEEE}) & if\ \tilde{k} \le k < \bar{k} \end{cases} \tag{26}$$

Since $\bar{G}$ is defined as $\Pi_{FEEE} - \Pi_{EEEE}$ when $k < \tilde{k}$, or $\Pi_{F*EE} - \Pi_{EEEE}$ when $\tilde{k} \le k < \bar{k}$, and $\hat{G}$ is defined as $\Pi_{FFFF} - \Pi_{EEEE}$, the critical value $G^*(\delta)$ can be described as:

$$G^*(\delta) = (1-\delta)\bar{G} + \delta\hat{G} \tag{27}$$

$G^*(\delta)$ is a convex combination of $\bar{G}$ and $\hat{G}$, weighted by the discount factor δ. When the discount factor becomes zero, the critical value G^* equals $\bar{G}$ (i.e. $\delta = 0$ implies $G^* = \bar{G}$), and when the discount factor becomes one, the critical value G^* equals $\hat{G}$ (i.e. $\delta = 1$ implies $G^* = \hat{G}$). Meanwhile, the critical value of the fixed cost is always lower in the infinitely repeated game than in the static game if the discount factor is above zero, this is because $\bar{G}$ is above $\hat{G}$, and $G^*(\delta)$ is decreasing in the discount factor δ.

If substitute (16)and (24) into (27), the critical value of the fixed cost of FDI that determines whether the collusion outcome is a Nash equilibrium over all firms in both markets is:

$$G^*(\delta) \equiv \begin{cases} \dfrac{k\left[8(\alpha - c - k) - \delta(6\alpha - 6c + 5k)\right]}{25\beta} & \text{if} \quad k < \tilde{k} \\ \dfrac{3(3\alpha - 3c - 4k)(\alpha - c + 12k) - \delta\Omega}{400\beta} & \text{if} \quad \tilde{k} \le k < \bar{k} \end{cases} \tag{28}$$

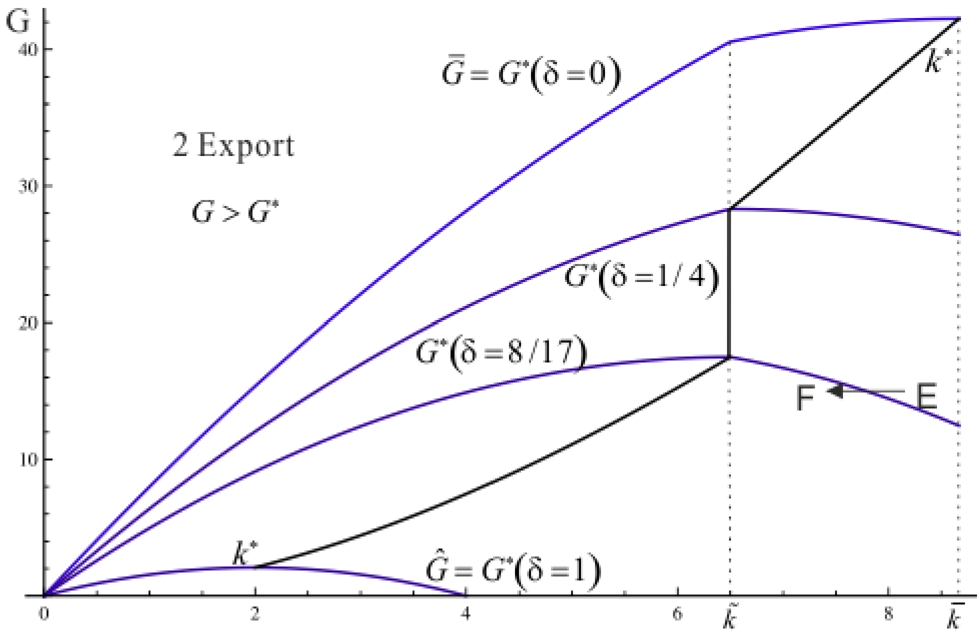

Figure 2-4: Infinitely-Repeated Game under Cournot Oligopoly when $\tilde{G} < G < \bar{G}$

Where $\Omega = 9(\alpha - c)^2 + 64k(\alpha - c + k) > 0$. The critical value of the fixed cost $G^*(\delta)$ is shown in figure 2-4 by using the same parameters as in figure 2-1, and it is a two-segmented concave quadratic of the trade cost for a number of discount factors $\delta \in \{0, 1/4, 8/17, 1\}$ up to the prohibitive trade cost $\bar{k}$ [21]. However, if combining figure 2-3 and figure 2-4 (see figure 2-5), only the area under $\tilde{G}$ is relevant to this game, as this is the area where the Nash equilibrium is that both firms in the same market undertake FDI in the static game. In the region between $\hat{G}$ and $\tilde{G}$ in figure 2-5, the collusive outcome where all firms export can be sustained as Nash equilibrium in the infinitely repeated game whereas all firms would undertake FDI in the static game, and the prisoners' dilemma as discussed in the static game is avoided here. Hence the collusive equilibrium can be sustained whenever the static game is a prisoners' dilemma in the region $\tilde{G} > G^* \geq \hat{G}$.

It is natural to assume that the discount factor changes over time. Clearly, the collusive outcome is easier to be sustained when the discount factor is larger. The higher the discount factor in a given period, the higher the collusive profits could be supported as a subgame Nash equilibrium in that period. The reason behind it is: a realisation of a higher discount factor leads to a stronger threat of future punishment, and higher profits without firms deviating.

[21] The reason of the chosen discount factors will be explained later.

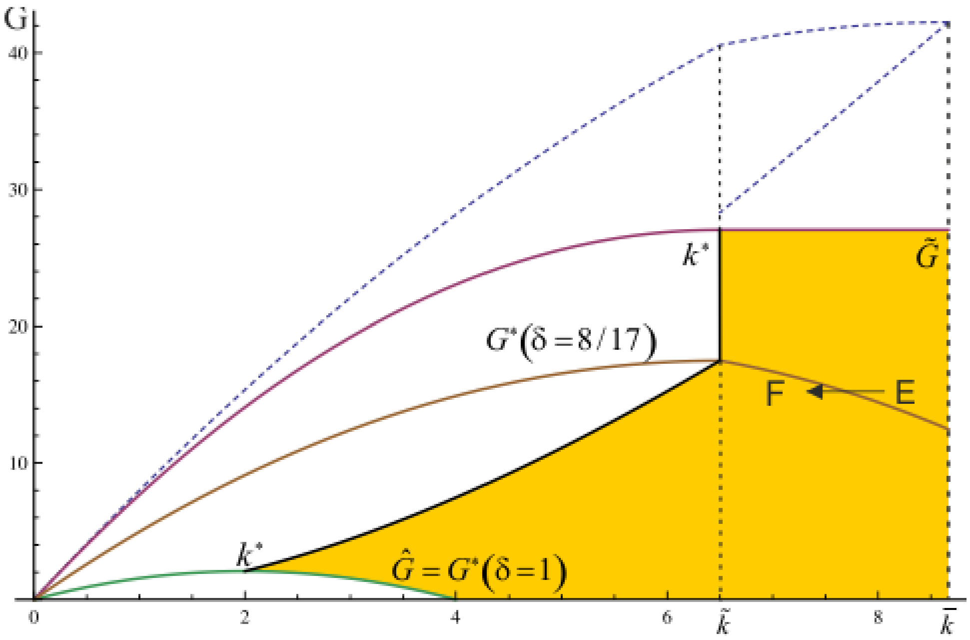

Figure 2-5: Infinitely-Repeated Game under Cournot Oligopoly when $G < \tilde{G}$

2.4.2 Collusion when $G > \tilde{G}$

When the fixed cost G of FDI is in the region between $\tilde{G}$ and $\bar{G}$, a subgame Nash equilibrium is the collusive outcome where all firms choose to export, and firms are earning jointly maximised profits. However, like what happens in section 2.4.1, if a firm deviates (undertaking FDI) to increase its single period (discounted) profits, the other firms are, in response, likely to revert to the safe position (static Nash equilibrium) where one firm in each country chooses to undertake FDI for the rest of the periods, while their competitors sustain their exporting strategy up to the trade cost $\tilde{k}$. Nevertheless, if the discount factor is sufficiently high, the collusive outcome where all firms choose to export can be sustained by the threat of Nash reversion strategy in an infinitely repeated game, and the prisoners' dilemma in the static game can be avoided.

When the fixed cost of FDI G is in the region above $\tilde{G}$, all firms choosing to export is a Nash Equilibrium if the present discounted profits from collusion (all firms export) exceed the present discounted value of profits from cheating (choosing to undertake FDI when its competitors from both countries have chosen to export) for one period, and thereafter followed by the Nash equilibrium profits above $\tilde{G}$ (one firm in each country chooses to undertake FDI when their competitors in the same country have chosen to export), for both interior and corner solutions:

$$\begin{cases} \frac{1}{1-\delta}\Pi_{EEEE} > \left(\Pi_{FEEE} - G\right) + \frac{\delta}{1-\delta}\left(\Pi_{FEFE} - G\right) & if\ k < \tilde{k} \\ \frac{1}{1-\delta}\Pi_{EEEE} > \left(\Pi_{F*EE} - G\right) + \frac{\delta}{1-\delta}\left(\Pi_{F*F*} - G\right) & if\ \tilde{k} \le k < \bar{k} \end{cases} \tag{29}$$

The first equation shows that when $k < \tilde{k}$, the present value of the profits of a firm when all firms export is greater than the present value of the profits when cheating (undertake FDI) for one period, and followed by the static Nash equilibrium where one firm in each country undertakes FDI, while the other firms have chosen to export thereafter. The Nash reversion strategy happens because when the firm cheats for one period, firms in the other market notice the advantage of cheating, and then one of them will make the move of switching strategy first, resulting the Nash equilibrium under the static game for the following periods. It means that the first firm in each market switching strategy by choosing FDI (cheating) has the advantage of gaining more profits, while its competitor in the same market loses the profits from sustaining its export strategy.

The second equation shows the corner solution when $\tilde{k} \le k < \bar{k}$, once firm one decides to deviate by undertaking FDI, firm two which has chosen to export will stop trading with the other country and the sales from exporting to the other country becomes zero, while firm three and firm four have chosen to export for the fist period, followed by all firms will revert to the static game equilibrium where only one firm in each country chooses to undertake FDI and their competitors will stop trading forever, due to the high trade cost ($k \ge \tilde{k}$). It all comes to the problem of the timing, only the first firm in each market switching to the strategy of undertaking FDI wins, whereas its competitor in the same market will be forced out of the trading activity.

Only when (29) is satisfied that the profits of the firm when all firms export are greater than the profits when one firm in each country undertakes FDI, leaving their competitors exporting up to $\tilde{k}$, the collusive outcome can be sustained as a subgame Nash equilibrium and there is no prisoners' dilemma in this infinitely repeated game.

By rearranging (29), the collusive outcome can be sustained as a Nash equilibrium if the fixed cost of FDI is greater than the critical value when $k < \tilde{k}$, that is:

$$G^{**}(\delta) = \begin{cases} (1-\delta)\,(\Pi_{FEEE} - \Pi_{EEEE}) + \delta(\Pi_{FEFE} - \Pi_{EEEE}) & if\ k < \tilde{k} \\ (1-\delta)\,(\Pi_{F*EE} - \Pi_{EEEE}) + \delta(\Pi_{F*F*} - \Pi_{EEEE}) & if\ \tilde{k} \le k < \bar{k} \end{cases} \tag{30}$$

Since G_n is defined as $\Pi_{FEFE} - \Pi_{EEEE}$ when $k < \tilde{k}$, or $\Pi_{F*F*} - \Pi_{EEEE}$ when $\tilde{k} \le k < \bar{k}$ and the definition of $\bar{G}$, the critical value $G^{**}(\delta)$ can be described as:

$$G^{**}(\delta) = (1-\delta)\bar{G} + \delta G_n \tag{31}$$

$G^{**}(\delta)$ is a convex combination of $\bar{G}$ and G_n, weighted by the discount factor δ. When the discount factor becomes zero, the critical value G^{**} equals $\bar{G}$, and when the discount factor becomes one, the critical value G^{**} equals G_n. Similar to G^{*}, the critical value of fixed cost G^{**} is decreasing in the discount factor δ, and it is always lower in the infinitely repeated game than the critical value of the fixed costs in the static game.

If substitute (21) and (16) into (31), the critical value of the fixed cost in the infinitely repeated game when there is only one firm in each market undertakes FDI is:

$$G^{**}(\delta) \equiv \begin{cases} \dfrac{k\left[\,8(\alpha - c - k) - \delta(2\alpha - 2c + 3k)\right]}{25\beta} & if\quad k < \tilde{k} \\ \dfrac{3(3\alpha - 3c - 4k)(\alpha - c + 12k) - \delta\,\phi}{400\beta} & if\quad \tilde{k} \le k < \bar{k} \end{cases} \tag{32}$$

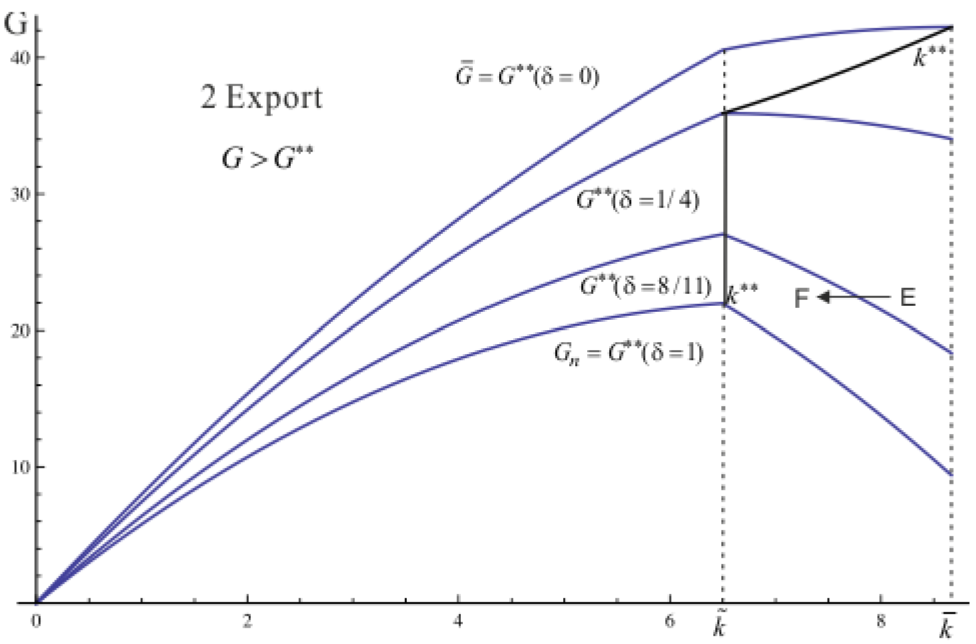

Figure 2-6: Infinitely-Repeated Game under Cournot Oligopoly when $G > \tilde{G}$

Where $\phi = (9\alpha - 9c + 8k)(-\alpha + c + 8k) > 0$. $G^{**}(\delta)$ is shown in figure 2-6 by using the same parameters as the previous figures, and it is a two-segmented concave quadratic of the trade cost for a number of discount factors $\delta \in \{0, 1/4, 8/11, 1\}$ up to the prohibitive trade cost $\bar{k}$. However, only the area between $\tilde{G}$ and $\bar{G}$ in figure 2-2 is relevant to this game, as this is the area where the Nash equilibrium is that one firm in each country undertakes FDI and their competitors export up to the trade cost $\tilde{k}$ in the static game. Combining figure 2-2 and figure 2-6, yields figure 2-7, which shows that in the region between $G^{**}(\delta)$ and $\bar{G}$, the collusive outcome where all firms export can be sustained as a Nash equilibrium in the infinitely repeated game whereas only one firm in each market would undertake FDI in the static game and there is no prisoners' dilemma. Clearly, the firm that switches to undertaking FDI firstly will gain more profits at the expenses of its competitor's profits in both countries, and the collusive outcome is easier to be sustained when the discount factor is larger ($\delta \to 1$). That is, if firms care about the future enough, they would look at the discount factor (interest rate) fluctuations. The higher the discount factor, the higher the threat of future punishment if firms deviate, which allows a higher collusive profits to be sustained.

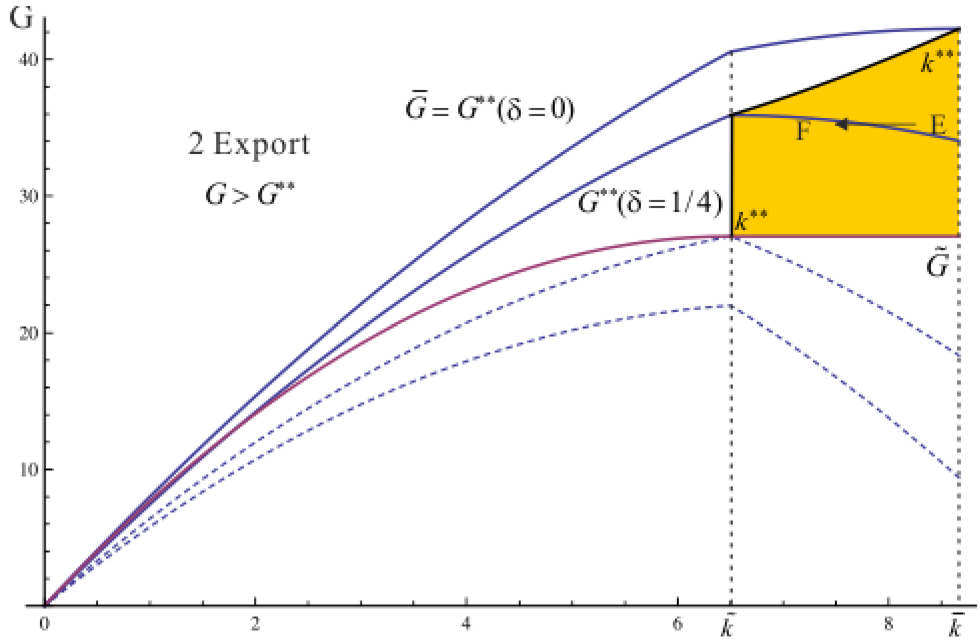

Figure 2-7: Infinitely-Repeated Game under Cournot Oligopoly when $G > \tilde{G}$

Proposition 3: *Under Cournot oligopoly, the collusive outcome where all firms export can be sustained as a subgame perfect Nash equilibrium in the infinitely repeated game if* $G > G^*$ *in the region* $\tilde{G} > G > \hat{G}$, *or* $G > G^{**}$ *in the region under* $\bar{G} > G > \tilde{G}$, *and the critical value is decreasing in the discount factor,* $\partial G^* / \partial \delta < 0$, $\partial G^{**} / \partial \delta < 0$.

2.4.3 Effect of *k* and *G* on collusion

To understand that how the trade cost affects the sustainability of collusion, it is worthwhile to look at how the critical value of the fixed cost of FDI depends on the trade cost. Considering the area under $\tilde{G}$ first (see figure 2-5), differentiate (28) with respect to k, it gives:

$$\frac{\partial G^*}{\partial k} \equiv \begin{cases} \dfrac{2\left[(\alpha - c)(4 - 3\delta) - k(5\delta + 8)\right]}{25\beta} & if \quad k < \tilde{k} \\ \dfrac{2\left[(\alpha - c)(3 - 2\delta) - k(4\delta + 9)\right]}{25\beta} & if \quad \tilde{k} \le k < \bar{k} \end{cases} \tag{33}$$

For a sufficient lower trade cost where $k < k^*$ the derivative is positive, but for higher trade costs, the derivatives are negative. The derivatives are negative if the trade cost is greater than critical value of k when $\partial G^* / \partial k = 0$:

$$k^*(\delta) \equiv \begin{cases} \dfrac{(\alpha - c)(4 - 3\delta)}{8 + 5\delta} & if \quad \delta \in [\,0,\ 1/4\,) \\ \tilde{k} & if \quad \delta \in [\,1/4,\ \ 8/17\,) \\ \dfrac{(\alpha - c)(3 - 2\delta)}{9 + 4\delta} & if \quad \delta \in [\,8/17,\ 1\,] \end{cases} \tag{34}$$

The critical value of the trade cost k^* is equal to the prohibitive trade cost, $k^* = \bar{k}$, when the discount factor $\delta = 0$, and it decreases to $k^* = \tilde{k}$ when $\delta = 1/4$, then it becomes a vertical line where $k^* = \tilde{k}$ when $\delta \in (1/4, 8/17)$. As the discount factor increases, i.e. $\delta > 8/17$, k^* decreases to the maximum of $\hat{G}$ when $\delta = 1$. In figure 2-5, the curve labelled k^*k^* represents k^* in (34) which joins the maximum point of the critical value of G^* so that G^* is decreasing to the right of k^*k^*, and this is the region where a decrease in the trade cost will shift firms from exporting to undertaking FDI as shown in the shaded area. For example, in figure 2-5, when $\delta = 8/17$ in the shaded region, a reduction in the trade cost might shift the equilibrium from E (where all firms export) to F (where all firms undertake FDI).

When the trade cost is high, above $\tilde{k}$, a reduction in the trade cost will reduce the collusive outcome of the firm, as the negative domestic effect dominates the positive export effect abroad, and it will increase the profit of deviation. Hence the future punishment is smaller that it is hard to sustain the grim trigger strategy, and then firms would switch to the cheating mode.

When the trade cost is lower, below $\tilde{k}$, it is less likely that firms would choose to undertake FDI when the trade cost is reduced, as a reduction in the trade cost will increase the collusive profits when the trade cost is low. The positive export effect will dominate the negative home sales effect. Then both future profits of collusion and future profits of cheating are increasing that the absolute value of the amount depends on the trade cost and the fixed costs. So it is more likely that firms would sustain the collusive outcome in the region $k < \tilde{k}$, and the shaded area where firms would like to switch the strategy is smaller.

To look at the relationship between the trade cost and sustainability of collusion in the area above $\tilde{G}$, differentiate the critical value of the fixed cost (32) with respect to k, yields:

$$\frac{\partial G^{**}}{\partial k} \equiv \begin{cases} \dfrac{2\left[(\alpha - c)(4-\delta) - k(3\delta + 8)\right]}{25\beta} & if \quad k < \tilde{k} \\ \dfrac{2\left[(\alpha - c)(3-2\delta) - k(4\delta + 9)\right]}{25\beta} & if \quad \tilde{k} \le k < \bar{k} \end{cases} \tag{35}$$

For low trade costs when $k < k^{**}$ the derivative is positive, but for higher trade costs in the region $k^{**} \le k < \bar{k}$, the derivative is negative. The derivatives are negative if the trade cost is greater than k when $\partial G^{**} / \partial k = 0$:

$$k^{**}(\delta) \equiv \begin{cases} \dfrac{(\alpha - c)(\delta + 4)}{3\delta + 8} & if \quad \delta \in [0,\ 1/4) \\ \tilde{k} & if \quad \delta \in [1/4,\ 1] \end{cases} \tag{36}$$

The critical value of the trade cost k^{**} is equal to the prohibitive trade cost, $k^{**} = \bar{k}$, when the discount factor $\delta = 0$, and it decreases to $k^{**} = \tilde{k}$ when the discount factor increases to $\delta = 1/4$, then it becomes a vertical line where $k^{**} = \tilde{k}$ until $\delta = 1$. In figure 2-7, the curve labelled $k^{**}k^{**}$ represents k^{**} in (36), joining the maximum point of the critical value of G^{**}, and G^{**} is increasing to the left of $k^{**}k^{**}$ but decreasing to the right of $k^{**}k^{**}$. The shaded area in figure 2-7 represents the regime that firms switch from exporting to undertaking FDI as the trade cost is reduced. For example, when $\delta = 1/4$ in the shaded region, a reduction in the trade cost might shift the equilibrium from E (where all firms export) to F (where one firm in each country undertakes FDI while the other firm stops trading). Notice that it occurs in the region where one firm in each market stop trading, $\tilde{k} < k < \bar{k}$. These results lead to the following proposition:

Proposition 4: *Under Cournot oligopoly, if the trade cost* $k > k^*$ *(* $k > k^{**}$ *), the critical value of the fixed cost* G^* *(* G^{**} *) is decreasing in the trade cost,* $\partial G^* / \partial k < 0$ *(* $\partial G^{**} / \partial k < 0$ *), and a reduction in trade costs may lead firms to switch from exporting to undertaking FDI.*

The combination of figure 2-5 and figure 2-7 lead to figure 2-8, where the shaded area is to the right of $k^{**}k^{**}$ (k^*k^*) if $G > \tilde{G}$ ($G < \tilde{G}$). The reduction in the trade cost will lead the firms shift from exporting to undertaking FDI in the shaded area. The reason is that when the trade cost is sufficiently high, even there is a reduction, the profits of collusion Π_{EEEE} will be reduced, and the profits of the firm if cheating Π_{FEEE} (Π_{F*EE}) (undertakes FDI while its competitors have chosen to export up to k^*) will increase, and the profits of the Nash equilibria where all firms undertake FDI, Π_{FFFF} , or one firm in each market undertakes FDI , Π_{FEFE} (Π_{F*F*}), will not be affected. As a result, a reduction in trade costs makes it harder to sustain the collusive outcome, and may lead to a switch to the static game Nash equilibrium where one firm undertakes FDI while its competitor exports, or to the Nash equilibrium where both firms undertake FDI.

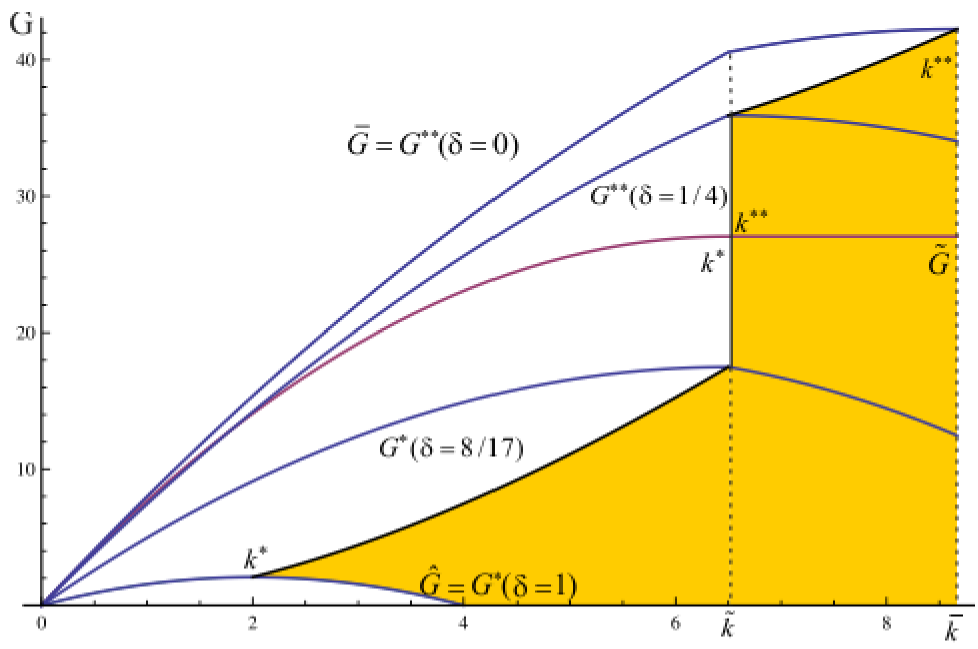

Figure 2-8: Combination of Figure 2-5 and Figure 2-7

Interestingly, the switch of the strategies could happen outside the shaded region in figure 2-8 as well. To illustrate the relationship between the trade cost and FDI further, a certain discount rate (e.g. $\delta = 1/3$) will be used for both G^{*} and G^{**}, so that $G^{*}(\delta = 1/3)$ is below $\tilde{G}$ and $G^{**}(\delta = 1/3)$ is above $\tilde{G}$ as shown in figure 2-9. For simplicity, figure 2-9 ignores the boundary critical value of the fixed cost $k^{*}k^{*}$ and $k^{**}k^{**}$ from figure 2-8, leaving $\bar{G} = G^{**}(\delta = 0)$, $G^{**}(\delta = 1/3)$, $\tilde{G}$, $G^{*}(\delta = 1/3)$ and $\hat{G} = G^{*}(\delta = 1)$ from top to bottom. In the region of $G^{**}(\delta = 1/3) < G < \bar{G}$, the subgame equilibrium is that all firms export (collusive outcome 2E[22]): in the region $\tilde{G} < G < G^{**}(\delta = 1/3)$, the equilibrium is that one firm in each country undertakes FDI and their competitor in the same country exports (1E1F) up to $\tilde{k}$ when their competitors stop trading (1F); in the region $G^{*}(\delta = 1/3) < G < \tilde{G}$, the equilibrium is the collusive outcome (all firms export 2E), and in the region under $G^{*}(\delta = 1/3)$, all firms choose to undertake FDI (2F), as shown in figure 2-9.

[22] Notice that it only shows one country's equilibrium. 2E means both firms in one country will export. As the model is symmetric, so firms in the other country will also export. When one firm in each country undertakes FDI. We use 1E1F instead on 2E2F to avoid the confusion.

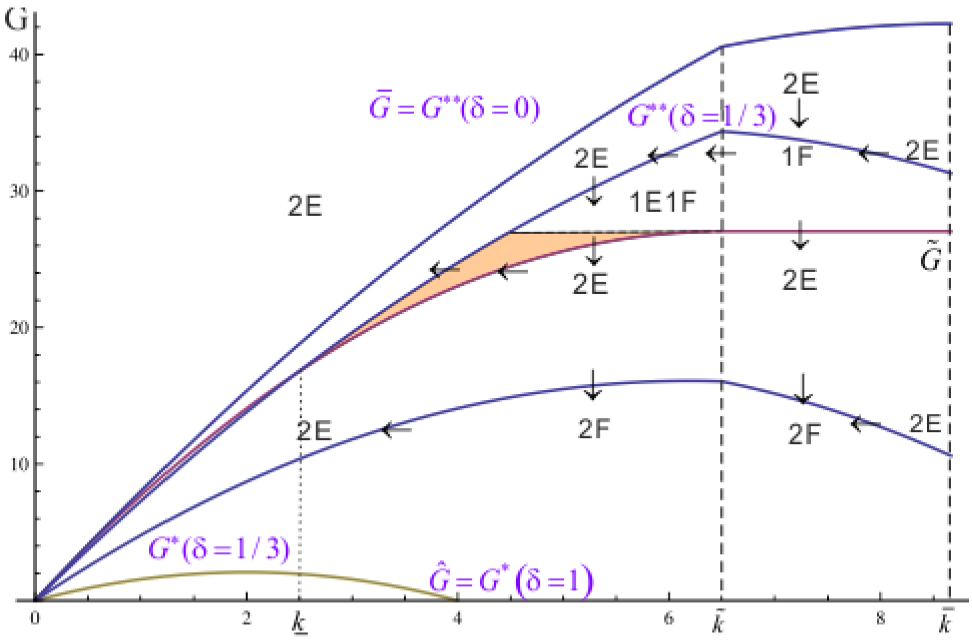

Figure 2-9: Infinitely-Repeated Game under Cournot Oligopoly ($\delta = 1/3$)

Suppose $\tilde{G}$ intersect $G^{**}(\delta = 1/3)$ at the point $k = k^{**}$, if the trade cost falls in the region $\underline{k} < k < \tilde{k}$ and within the region between $G^{*}(\delta = 1/3) < G < G^{**}(\delta = 1/3)$, firms might switch from 2E (both firms in the same county choose to export) to 1E1F (one firm in each country chooses to undertake FDI) just across $\tilde{G}$ as shown in the shaded area in figure 2-9. If the trade cost is reduced further to the outside of the shaded area, both firms from the same country will choose to export. Hence, a reduction in the trade cost may also lead firms to switch from exporting to undertaking FDI outside the shaded area in figure 2-8. Note that when the discount factor δ is decreasing, the critical value of the fixed cost G^{**} is increasing, therefore the area where the Nash equilibrium outcome (one firm exports while its competitor in the same country undertakes FDI) sustains is bigger, and so is the shaded area in figure 2-8. It leads to the following proposition:

Proposition 5: *Under Cournot oligopoly, a reduction in trade costs in the region $\underline{k} < k < \tilde{k}$ may also lead firms to switch from exporting to undertaking FDI. The larger the discount factor δ, the better chance that firms will switch the strategy.*

This is due to the same reason: when the trade cost is relatively high, a reduction in trade costs reduces the profitability of collusion (all firms export) but increase the profitability of cheating (undertakes FDI while its competitors in the world have chosen to export). This result confirms that there is an increase in the number of firms which undertake FDI when the trade cost is reduced by multilateral trade liberalisations in an era.

It is also interesting to examine the relationship between the fixed cost and FDI. By looking at figure 2-9 vertically, the level of the fixed cost G will affect firms' decision about their strategies. In the area of $G^{*}\left(\delta = 1/3\right) < G < G^{**}\left(\delta = 1/3\right)$, a reduction in the fixed costs across $\tilde{G}$ might shift the equilibrium from 1E1F (undertake FDI, while its competitor in the same country export if $k < \tilde{k}$) or 1F(the firm stops trading if $k > \tilde{k}$) to the collusive outcome (both firms chose to export), indicating that firms might switch from undertaking FDI to exporting as the fixed cost G decreases. It leads to the following proposition:

Proposition 6: *Under Cournot oligopoly, if the fixed cost* G *is reduced from the region* $\tilde{G} < G < G^{**}$ *to the region* $G^{*} < G < \tilde{G}$ *, it may lead firms to switch from undertaking FDI to exporting.*

When G is greater than $\hat{G}$, a reduction in the fixed cost may lead firms to switch from the collusive outcome $2E$ to the static equilibrium $1F1E$ ($1F1*$). This is because that if one firm deviates by undertaking FDI, it would be too expensive for its competitor in the same market to deviate. However, when the fixed cost is relatively lower (between $G^{**}\left(\delta = 1/3\right)$ and $G^{*}\left(\delta = 1/3\right)$), a reduction in fixed costs makes it easier for its competitor to undertake FDI. Yet if all firms choose to undertake FDI, the punishment could be so bad that firms might as well collude instead of cheating, and the worse the punishment is, the easier it is to sustain the collusive outcomes (δ is closer to one). Hence a reduction in the fixed cost could make it hard for the firms to deviate and sustain the Nash equilibrium (undertaking FDI while its competitor in the same country exports up to $\tilde{k}$), and it may lead to a switch to the collusive outcome (both firms export). When the fixed costs decrease further to a even lower level, the punishment of deviating is not so bad and the discount value of δ is closer to zero, the long term profits of the Nash equilibrium would be greater than the long term profits of the grim trigger strategy, then all firms would undertake FDI. This finding is one of the points which Collie (2009) did not address.

2.5 Conclusion

The export versus FDI decisions have been analysed in a two-country four-firm model with identical products under Cournot oligopoly. In the static game, a reduction in the trade cost will lead the firms switch from undertaking FDI to exporting as shown in the literature. The outcomes are that two firms in the same country choose to undertake FDI if the fixed cost is relatively low, or one firm chooses to export while its competitor from the same country chooses to undertake FDI if the fixed cost is relatively high. Thus, this model shows that both export and FDI can exist as an equilibrium outcome in the world when the fixed cost is sufficiently high.

However, there might be prisoners' dilemmas. If based on one market, both firms in the same country might make lower profits when they both undertake FDI than when they both export. When based on the two markets, if the fixed cost is relatively high, all firms might make lower profits when they all undertake FDI than when they export. If the fixed cost is sufficiently high, the equilibrium profits when one firm in each country undertakes FDI while its competitor in the same country export might be lower than the profits when all firms export. This is a prisoner' dilemma caused by the intensified competition.

The prisoners' dilemma can be avoided in an infinitely-repeated game when all firms tacitly collude over their FDI versus export decisions, as collusion over FDI can be sustained by the threat of Nash-reversion strategies if the trade cost is sufficiently high. Then a reduction in trade costs may lead firms to switch from exporting to undertaking FDI when the trade cost is sufficiently high, as in the infinitely-repeated game , a reduction in a sufficiently high trade costs lessen the profitability of collusion, and that explains the experience of the increasing FDI in 1990s. Also it is shown that a reduction in the fixed cost may lead firms to switch from undertaking FDI to exporting when the fixed cost is relatively high.

This chapter uses linear demand function as in most of the literature, the constant elasticity demand function will be adopted to test the results in the subsequent chapter.

Appendix

Operating profits and outputs

Firms employ a Cournot strategy. That is, each firm maximises its profit assuming the outputs of other firms in each market remain the same. Firms stay in business as long as they make non-zero positive profits in each market. To begin with, we will look at the case when both firms in country B export to supply country A. Suppose that X_A is the total outputs supplied to country A:

$$X_A = x_{1A} + x_{2A} + x_{3A} + x_{4A}$$
$$P_A = \alpha - \beta\left(x_{1A} + x_{2A} + x_{3A} + x_{4A}\right) = \alpha - \beta X_A$$

1. When both firms choose to export

Suppose both firms in country B choose to export, substitute (2) into (3), the operating profits (before the fixed costs) of the firms in country A are:

$$\begin{aligned}
\pi_{1A}^{EE} &= \left[\alpha - \beta X_A - c\right] x_{1A} \\
\pi_{2A}^{EE} &= \left[\alpha - \beta X_A - c\right] x_{2A} \\
\pi_{3A}^{EE} &= \left[\alpha - \beta X_A - c - k\right] x_{3A} \\
\pi_{4A}^{EE} &= \left[\alpha - \beta X_A - c - k\right] x_{4A}
\end{aligned} \tag{37}$$

The first-order conditions are:

$$\frac{\partial \pi_{1A}^{EE}}{\partial x_{1A}} = \alpha - \beta x_{1A} - \beta X_A - c = 0 \tag{38}$$

$$\frac{\partial \pi_{2A}^{EE}}{\partial x_{2A}} = \alpha - \beta x_{2A} - \beta X_A - c = 0 \tag{39}$$

$$\frac{\partial \pi_{3A}^{EE}}{\partial x_{3A}} = \alpha - \beta x_{3A} - \beta X_A - c - k = 0 \tag{40}$$

$$\frac{\partial \pi_{4A}^{EE}}{\partial x_{4A}} = \alpha - \beta x_{4A} - \beta X_A - c - k = 0 \tag{41}$$

The four first-order conditions are the reaction functions and constitute four equations with four unknowns x_{1A}, x_{2A}, x_{3A} and x_{4A}. Adding up equations from (38) to (41), we get:

$$4\alpha - 5\beta X_A - 4c - 2k = 0 \tag{42}$$

Solve for X_A, we get:

$$X_A = \frac{4\alpha - 4c - 2k}{5\beta} \tag{43}$$

Now substitute (43) into equation (38), we get Cournot equilibrium $x_{1A} = \frac{\alpha - c + 2k}{5\beta}$.

Then substitute X_A into the equation (39), (40), and (41), we can solve for the Cournot equilibria x_{2A}, x_{3A} and x_{4A}, which yields equation (4) in section 3. There fore the operating outputs, prices and profits of the firms in country A are:

$$x_{1A}^{EE} = x_{2A}^{EE} = \frac{\alpha - c + 2k}{5\beta} \qquad x_{3A}^{EE} = x_{4A}^{EE} = \frac{\alpha - c - 3k}{5\beta}$$

$$p^{EE} = \alpha - \beta\left(x_{1A} + x_{2A} + x_{3A} + x_{4A}\right) = \frac{\alpha + 4c + 2k}{5}$$

$$\pi_{1A}^{EE} = \pi_{2A}^{EE} = \left(p^{EE} - c\right)x_{1A}^{EE} = \frac{\left(\alpha - c + 2k\right)^2}{25\beta}$$

$$\pi_{3A}^{EE} = \pi_{4A}^{EE} = \left(p^{EE} - c - k\right)x_{3A}^{EE} = \frac{\left(\alpha - c - 3k\right)^2}{25\beta}$$

2. When one firm exports and the other firm in the same country undertakes FDI

Suppose firm three chooses to export to country A, and firm four chooses to undertake FDI in country A. For the interior solution, substitute (2) into(5), the operating profits (before the fixed costs) of the firms in country A are:

$$\pi_{1A}^{EF} = \left[\alpha - \beta X_A - c\right]x_{1A}$$

$$\pi_{2A}^{EF} = \left[\alpha - \beta X_A - c\right]x_{2A}$$

$$\pi_{3A}^{EF} = \left[\alpha - \beta X_A - c - k\right] x_{3A} \tag{44}$$

$$\pi_{4A}^{EF} = \left[\alpha - \beta X_A - c\right] x_{4A}$$

The first-order conditions are:

$$\frac{\partial \pi_{1A}^{EF}}{\partial x_{1A}} = \alpha - \beta x_{1A} - \beta X_A - c = 0$$

$$\frac{\partial \pi_{2A}^{EF}}{\partial x_{2A}} = \alpha - \beta x_{2A} - \beta X_A - c = 0$$

$$\frac{\partial \pi_{3A}^{EF}}{\partial x_{3A}} = \alpha - \beta x_{3A} - \beta X_A - c - k = 0 \tag{45}$$

$$\frac{\partial \pi_{4A}^{EF}}{\partial x_{4A}} = \alpha - \beta x_{4A} - \beta X_A - c = 0$$

Solving for the Cournot equilibria using the same method as above, adding up all the first order conditions from (45), yields:

$$4\alpha - 5\beta X_A - 4c - k = 0$$

then solve for x_{1A} by combining the above equation and the first equation of (45), solve for x_{2A} by substitute the above equation to (45) and so on and so forth, which yields (6) in section 3. The operating outputs, prices and profits of the firms in country A are:

$$x_{1A}^{EF} = x_{2A}^{EF} = x_{4A}^{EF} = \frac{\alpha - c + k}{5\beta} \qquad x_{3A}^{EF} = \frac{\alpha - c - 4k}{5\beta}$$

$$p^{EF} = \alpha - \beta X_A = \frac{\alpha + 4c + k}{5}$$

$$\pi_{1A}^{EF} = \pi_{2A}^{EF} = \pi_{4A}^{EF} = \left(p^{EF} - c\right) x_{1A}^{EF} = \frac{\left(\alpha - c + k\right)^2}{25\beta}$$

$$\pi_{3A}^{EF} = \left(p^{EF} - c - k\right) x_{3A}^{EF} = \frac{\left(\alpha - c - 4k\right)^2}{25\beta}$$

For the corner solution, firm three will stop exporting up to a prohibited trade cost, thus from (7), the operating profits (before the fixed costs) of the firms in country A become:

$$
\begin{aligned}
&X_{A*} = x_{1A} + x_{2A} + x_{4A} \\
&\pi_{1A}^{*F} = \left[\alpha - \beta X_{A*} - c\right] x_{1A} \\
&\pi_{2A}^{*F} = \left[\alpha - \beta X_{A*} - c\right] x_{2A} \\
&\pi_{4A}^{*F} = \left[\alpha - \beta X_{a*} - c\right] x_{4A}
\end{aligned} \tag{46}
$$

Where $*$ means firm three stops trading with country A, so its profits and outputs I country A are zero, i.e. $x_{3A}^{*F} = 0$ and $\pi_{3A}^{*F} = 0$. The price of firm three will just cover its marginal cost $p_{3A}^{*F} = c + k$ .The first-order conditions of (46)are:

$$
\begin{aligned}
&\frac{\partial \pi_{1A}^{*F}}{\partial x_{1A}} = \alpha - \beta x_{1A} - \beta X_{A*} - c = 0 \\
&\frac{\partial \pi_{2A}^{*F}}{\partial x_{2A}} = \alpha - \beta x_{2A} - \beta X_{A*} - c = 0 \\
&\frac{\partial \pi_{4A}^{*F}}{\partial x_{4A}} = \alpha - \beta x_{4A} - \beta X_{A*} - c = 0 \\
&3\alpha - 4\beta X_{A*} - 3c = 0 \\
&X_{A*} = \frac{3\alpha - 3c}{4\beta}
\end{aligned} \tag{47}
$$

Solving for the Cournot equilibria yields(8), where the operating outputs, prices and profits of the firms in country A are:

$$x_{1A}^{*F} = x_{2A}^{*F} = x_{4A}^{*F} = \frac{\alpha - c}{4\beta}$$

$$p^{*F} = \alpha - \beta X_{A*} = \frac{\alpha + 3c}{4}$$

$$\pi_{1A}^{*F} = \pi_{2A}^{*F} = \pi_{4A}^{*F} = \left(p^{*F} - c\right) x_{1A}^{*F} = \frac{\left(\alpha - c\right)^2}{16\beta}$$

Similarly, when firm three chooses to undertake FDI and firm four chooses to export, the results are reciprocal.

3. When both firms choose to undertake FDI

Suppose both firms in country B choose to undertake FDI, substitute (2) into (10), the operating profits (before the fixed costs) of the firms in country A are:

$$\begin{aligned} \pi_{1A}^{FF} &= \left[\alpha - \beta X_A - c\right] x_{1A} \\ \pi_{2A}^{FF} &= \left[\alpha - \beta X_A - c\right] x_{2A} \\ \pi_{3A}^{FF} &= \left[\alpha - \beta X_A - c\right] x_{3A} \\ \pi_{4A}^{FF} &= \left[\alpha - \beta X_A - c\right] x_{4A} \end{aligned} \tag{48}$$

The first-order conditions are:

$$\begin{aligned} \frac{\partial \pi_{1A}^{FF}}{\partial x_{1A}} &= \alpha - \beta x_{1A} - \beta X_A - c = 0 \\ \frac{\partial \pi_{2A}^{FF}}{\partial x_{2A}} &= \alpha - \beta x_{2A} - \beta X_A - c = 0 \\ \frac{\partial \pi_{3A}^{FF}}{\partial x_{3A}} &= \alpha - \beta x_{3A} - \beta X_A - c = 0 \\ \frac{\partial \pi_{4A}^{FF}}{\partial x_{4A}} &= \alpha - \beta x_{4A} - \beta X_A - c = 0 \\ 4\alpha - 5\beta X_A - 4c &= 0 \\ X_A &= \frac{4\alpha - 4c}{5\beta} \end{aligned} \tag{49}$$

Solving for the Cournot equilibria yields (11), where the operating outputs, prices and profits of the firms in country A are:

$$x_{1A}^{FF} = x_{2A}^{FF} = x_{3A}^{FF} = x_{4A}^{FF} = \frac{\alpha - c}{5\beta}$$

$$p^{FF} = \alpha - \beta X_A = \frac{\alpha + 4c}{5}$$

$$\pi_{1A}^{FF} = \pi_{2A}^{FF} = \pi_{3A}^{FF} = \pi_{4A}^{FF} = \left(p^{FF} - c \right) x_{1A}^{FF} = \frac{\left(\alpha - c \right)^2}{25\beta}$$

Chapter 3: FDI versus Exporting under Cournot Oligopoly-The constant elasticity demand function case

3.1 Introduction

The analysis of the sustainability of collusion in an infinitely-repeated Cournot game have been focused on either proving that collusion can be sustained for a sufficiently large discount factor, or assuming a linear demand function so that the discount factor can be solved explicitly by relating to the parameters of the model. For instance, many authors have used linear functions to analyse how different mode of competition (Bertrand or Cournot competition), cost asymmetries, the number of the firms or product differentiation affect the sustainability of collusion. These factors do affect the sustainability of collusion to some extent, but there is a limitation that the results are restricted by this particular functional form: linear demand. One of the important factors that might affect the result is the elasticity of demand, and the effect of which on the sustainability of collusion could not be captured by linear demand functions, but can be addressed by constant elasticity demand function. This form of a function, however, has a disadvantage, that it could not give an explicit analytical result when a firm tries to deviate from collusion. Hence it does not allow an explicit solution for the discount factor to sustain the collusion in an infinitely repeated game, as it does by using linear demand in chapter two. The way to deal with this situation is to use numerical solutions, and the results are general and are surprisingly similar to those in chapter two, which confirms the robustness of the analysis.

Chapter two assumes linear demand functions and uses a Cournot oligopoly to resolve the conflict between the theory, which predicts that trade liberalisation discourages foreign direct investment (FDI), and the empirical evidence, which is that a reduction in trade costs has led to an increase in FDI. When firms tacitly collude over FDI versus export decisions in an infinitely repeated game, the puzzle can be explained and collusion can be sustained as a subgame perfect equilibrium for a sufficiently large discount factor. Therefore by assuming linear demand functions, the discount factor can be solved explicitly and relate to the parameters of the model. For instance, the previous analysis has indicated that how the trade costs k (incurred with export) and the fixed costs G (incurred with FDI) affect the sustainability of collusion by using linear demand functions. However, such form of a function limits the analysis of how the parameters of the model affect the sustainability of collusion and thus the solution to the argument. By adapting constant elasticity demand function, the effect of the elasticity of demand on sustainability of collusion is analysed and it is proved that the outcomes are similar to those by linear demand function.

3.2 The model

The model in this chapter is similar to the one in chapter two, except that the demand function exhibits constant elasticity. The model is symmetric, and again consists of two countries with two firms in each country, labelled firm one and firm two in country A, along with firm three and firm four in country B. These firms produce homogeneous products and compete as Cournot oligopolists in the two markets (country A and country B).

The firms play a two-stage game. At stage one, firms independently choose whether to export to the other country or to undertake FDI in the other country. At stage two, these firms compete in a Cournot oligopoly game in the two segmented markets. The products are assumed to be perfect substitutes, so each firm is facing the same constant elasticity demand function in country A:

$$p_A\left(Q_A\right) = Q_A^{-1/\eta} \tag{50}$$

$$Q_A = q_{1A} + q_{2A} + q_{3A} + q_{4A} \tag{51}$$

Where p_A is the market price in country A, q_{iA} is the output of each firm in country A, Q_A is the total industry output in country A, and η is the constant elasticity of demand, which is assumed to be larger than one. All firms are identical and incur a constant marginal cost of c.

3.2.1 Both firms choose to export:

Using the example of country A, the profits of both firms in country A are $\pi_{iA} = \left(p_A - c \right) q_{iA}$. When both firm three and firm four export, their marginal cost will be $c + k$, assuming an interior solution, so the operational profits of all firms are:

$$
\begin{aligned}
&\pi_{1A} = \left(p_A - c \right) q_{1A} && \pi_{2A} = \left(p_A - c \right) q_{2A} \\
&\pi_{3A} = \left(p_A - c - k \right) q_{3A} && \pi_{4A} = \left(p_A - c - k \right) q_{4A}
\end{aligned} \tag{52}
$$

The outputs, prices and profits of the firms are then solved by taking the derivatives of the above:

$$
\begin{aligned}
&p_A = \left(4c + 2k \right) / \left(4 - 1/\eta \right) \\
&Q_A = p_A^{-\eta} = \left[\left(4c + 2k \right) / \left(4 - 1/\eta \right) \right]^{-\eta}
\end{aligned}
$$

$$
\begin{aligned}
&q_{1A}^{EE} = q_{2A}^{EE} = \eta \left(1 - c/p_A \right) Q_A = \left(4c + 2k \right)^{-1-\eta} \left(4 - 1/\eta \right)^{\eta} \left(c + 2k\eta \right) \\
&q_{3A}^{EE} = q_{4A}^{EE} = \eta \left[1 - \left(c + k \right) / p_A \right] Q_A = \left(4c + 2k \right)^{-1-\eta} \left(4 - 1/\eta \right)^{\eta} \left(c + k - 2k\eta \right)
\end{aligned} \tag{53}
$$

$$
\begin{aligned}
&\pi_{1A}^{EE} = \pi_{2A}^{EE} = \left(4c + 2k \right)^{-1-\eta} \eta^{-\eta} \left(-1 + 4\eta \right)^{-1+\eta} \left(c + 2k\eta \right)^2 \\
&\pi_{3A}^{EE} = \pi_{3A}^{EE} = \left(4c + 2k \right)^{-1-\eta} \eta^{-\eta} \left(-1 + 4\eta \right)^{-1+\eta} \left(c + k - 2k\eta \right)^2
\end{aligned}
$$

The outputs of firms are shown in figure 3-1 in the appendix, where the sales of firm one and firm two are upward sloping, and those of firm three and firm four are downward sloping. This is because firm three and firm four export to country A, so an increase in the trade cost will reduce the motivation of exporting, and then reduce the supply to the home country, while home firms' outputs are increasing with the trade cost. Since the sales from export are more sensitive to the trade cost, the outputs of firm three and firm four in the home market are steeper than the ones of home firms as shown in figure 3-1.

Firms stop trading when the trade cost is prohibitive, i.e. $q_{3A}^{EE} = q_{4A}^{EE} = 0$, meaning $k \geq \bar{k} \equiv c/(2\eta - 1)$. Then the exports from firm three and firm four in country B to supply country A will be zero.

3.2.2 One firm exports and the other firm undertakes FDI

Using the example of country A again, if firm three chooses to export to country A, and firm four chooses to undertake FDI in country A, the marginal cost of country three will be $c+k$, while the marginal costs of the other three firms are c. Assuming the outcome is an interior solution where all firms have positive sales, the operating profits (before the fixed cost) of the firms in country A will be:

$$\begin{aligned} \pi_{1A} &= (p_A - c) q_{1A} & \pi_{2A} &= (p_A - c) q_{2A} \\ \pi_{3A} &= (p_A - c - k) q_{3A} & \pi_{4A} &= (p_A - c) q_{4A} \end{aligned} \tag{54}$$

The usual derivations for a Cournot oligopoly yield the outputs, prices and profits of the four firms as shown in the appendix:

$$\begin{aligned} p_A^{EF} &= (4c+k)/(4-1/\eta) \\ Q_A &= p_A^{-\eta} = \left[(4c+k)/(4-1/\eta)\right]^{-\eta} \end{aligned}$$

$$\begin{aligned} q_{1A}^{EF} = q_{2A}^{EF} = q_{4A}^{EF} &= \eta\left[1-(c/p_A)\right]Q_A = (4c+k)^{-1-\eta}\left[4-(1/\eta)\right]^{\eta}(c+k\eta) \\ q_{3A}^{EF} &= \eta\left[1-(c+k)/p_A\right]Q_A = (4c+k)^{-1-\eta}(4-1/\eta)^{\eta}(c+k-3k\eta) \end{aligned} \tag{55}$$

$$\begin{aligned} \pi_{1A}^{EF} = \pi_{2A}^{EF} = \pi_{4A}^{EF} &= (4c+k)^{-1-\eta}\eta^{-\eta}(4\eta-1)^{\eta-1}(c+k\eta)^2 \\ \pi_{3A}^{EF} &= (4c+k)^{-1-\eta}\eta^{-\eta}(4\eta-1)^{\eta-1}(c+k-3k\eta)^2 \end{aligned}$$

Symmetry of the model implies that $q_{3B}^{EF} = q_{4B}^{EF} = q_{2B}^{EF} = q_{1A}^{EF}$, $q_{1B}^{EF} = q_{3A}^{EF}$, $p_B^{EF} = p_A^{EF}$, $\pi_{3B}^{EF} = \pi_{4B}^{EF} = \pi_{2B}^{EF} = \pi_{1A}^{EF}$, $\pi_{1B}^{EF} = \pi_{3A}^{EF}$.

When firm three stops trading with country A, there is a corner solution, where the exports of firm three to country A and its profits from exports will be zero. It happens if the trade cost reaches its prohibitive level, i.e. q_{3A}^{EF} =0, when $k \geq \tilde{k} \equiv c/(3\eta - 1)$. Therefore, the profits, outputs and prices of the four firms under the Cournot oligopoly when $k > \tilde{k}$ are:

$$\pi_{1A} = (p_A - c)q_{1A} \qquad \pi_{2A} = (p_A - c)q_{2A}$$

$$\pi_{4A} = (p_A - c)q_{4A} \qquad \pi_{3A} = 0$$

$$\begin{aligned}
&p_A^{*F} = 3c/(3 - 1/\eta) \\
&Q_A^{*F} = \left(p_A^{*F}\right)^{-\eta} = \left[3c/(3 - 1/\eta)\right]^{-\eta} \\
&q_{1A}^{*F} = q_{2A}^{*F} = q_{4A}^{*F} = \eta\left(1 - \frac{c}{p_A^{*F}}\right)Q_A^{*F} = \frac{1}{3}\left(\frac{3\eta - 1}{3c\eta}\right)^{\eta} \qquad q_{3A}^{*F} = 0 \\
&\pi_{1A}^{*F} = \pi_{2A}^{*F} = \pi_{4A}^{*F} = 3^{-1-\eta}c^{1-\eta}\eta^{-\eta}(3\eta - 1)^{\eta - 1} \qquad \pi_{3A}^{*F} = 0
\end{aligned} \tag{56}$$

The outputs and sales in this scenario are shown in figure 3-2, where the demands of firm one, firm two and firm four are upward sloping, but the outputs for firm four is downward sloping for the same reason: the export from firm three is decreasing in the trade cost until k reaches its prohibitive level $\tilde{k}$.

When firm three chooses to undertake FDI, and firm four chooses to export, the results are reciprocal to the above ones, so the interior outcome is as follows: $q_{1A}^{FE} = q_{2A}^{FE} = q_{3A}^{FE} = q_{1A}^{EF}$, $q_{4A}^{FE} = q_{3A}^{EF}$, $p_A^{FE} = p_A^{EF}$, $\pi_{1A}^{FE} = \pi_{2A}^{FE} = \pi_{4A}^{FE} = \pi_{1A}^{EF}$, $\pi_{4A}^{FE} = \pi_{3A}^{EF}$. Symmetry of the model implies that $q_{3B}^{FE} = q_{4B}^{FE} = q_{1B}^{FE} = q_{1A}^{FE}$, $q_{2B}^{FE} = q_{4A}^{FE}$, as shown in figure 3-3, $p_B^{FE} = p_A^{FE}$, $\pi_{3B}^{FE} = \pi_{4B}^{FE} = \pi_{1B}^{FE} = \pi_{1A}^{FE}$, $\pi_{2B}^{FE} = \pi_{4A}^{FE}$. The corner solution occurs when firm four stops exporting to the country A. That is, when q_{4A}^{FE} =0, when $k \geq \tilde{k} \equiv c/(3\eta - 1)$. The profits, outputs and prices of the three firms under the Cournot oligopoly are: $q_{1A}^{F*} = q_{2A}^{F*} = q_{3A}^{F*} = q_{1A}^{*F}$, $q_{4A}^{F*} = q_{3A}^{*F} = 0$, $p_A^{*F} = p_A^{F*}$, $\pi_{1A}^{F*} = \pi_{2A}^{F*} = \pi_{3A}^{F*} = \pi_{1A}^{*F}$, $\pi_{4A}^{F*} = \pi_{3A}^{*F} = 0$.

3.2.3 Both firms undertake FDI

When both firm three and firm four undertake FDI, the marginal cost of all firms will be c, so the outcome is an interior solution where all firms generate positive sales. The operating profits (before the fixed cost) of the firms will be:

$$\pi_{1A} = (p_A - c)q_{1A} \qquad \pi_{2A} = (p_A - c)q_{2A}$$
$$\pi_{3A} = (p_A - c)q_{3A} \qquad \pi_{4A} = (p_A - c)q_{4A}$$

The derivations for a Cournot oligopoly yield the outputs, prices and profits of the four firms, as in the appendix, are:

$$\begin{aligned}
p_A &= 4c/(4-1/\eta) \\
Q_A &= p_A^{-\eta} = \left[4c/(4-1/\eta)\right]^{-\eta} \\
q_{iA}^{FF} &= \eta\left[1-(c/p_A)\right]Q_A = 1/4\left[(4\eta-1)/4c\eta\right]^{\eta} \\
\pi_{iA}^{FF} &= c/4(4c\eta)^{-\eta}(4\eta-1)^{\eta-1}
\end{aligned} \tag{57}$$

The outputs for all firms are the same horizontal line. Because the trade costs do not affect FDI, and do not play any role here, the outputs of the firms are not affected by the trade cost. Next section will present a static game theory model of FDI under Cournot oligopoly, where the fixed cost of undertaking FDI enters the decisions of the game.

3.3 Static Game

In the first stage, each firm chooses whether to undertake FDI or to export to supply the other country. There are four firms, so there will be sixteen combinations among them. The model is symmetric so the profits of the firms are also symmetric. All the interior combinations and the corner combinations are similar to the ones in chapter two section 2.3. Although the numerical results are different, this step can be skipped, as the markets are assumed to be segmented.

The profits in the second-stage outputs game can be put into a two-by-two payoff matrix, where the rows denote the strategies of firm one and the columns denote the strategies of firm two. The payoff matrix is as follows with the first number in a cell the profits of firm one in country B and the second number the profits of firm two in country B. The matrix in table 3-1 shows the interior payoffs where firms take different strategies when the trade cost is smaller than $\tilde{k}$.

<table>
<tr><td colspan="2" rowspan="2">Payoff Matrix</td><td colspan="2">Firm 2</td></tr>
<tr><td>Export</td><td>FDI</td></tr>
<tr><td rowspan="2">Firm 1</td><td>Export</td><td>π_{1B}^{EE} , π_{2B}^{EE}</td><td>π_{1B}^{EF} , $\pi_{2B}^{EF} - G$</td></tr>
<tr><td>FDI</td><td>$\pi_{1B}^{FE} - G$, π_{2B}^{FE}</td><td>$\pi_{1B}^{FF} - G$, $\pi_{2B}^{FF} - G$</td></tr>
</table>

Table 3-5: Interior payoff matrix when $k < \tilde{k}$

The first cell on the left hand side shows a pair of payoffs in country B when both firm one and firm two decide to export, and the second cell on the left presents a pair of payoffs in country B when firm one undertakes FDI (after the fixed cost) while firm two exports. Cells on the right hand side show the pairs of payoffs in country B when firm two undertakes FDI while firm one chooses to export (top cell) or to undertake FDI (bottom cell).

The matrix in table 3-2 shows the corner payoffs when the trade cost is between $\tilde{k}$ and $\bar{k}$. Then firm one or firm two stops exporting and its sales become zero in country B, $\pi_{1B}^{*F} = \pi_{2B}^{F*} = 0$,

<table>
<tr><td colspan="2" rowspan="2">Payoff Matrix</td><td colspan="2">Firm 2</td></tr>
<tr><td>Export</td><td>FDI</td></tr>
<tr><td rowspan="2">Firm 1</td><td>Export</td><td>π_{1B}^{EE} , π_{2B}^{EE}</td><td>π_{1B}^{*F} , $\pi_{2B}^{*F} - G$</td></tr>
<tr><td>FDI</td><td>$\pi_{1B}^{F*} - G$, π_{2B}^{F*}</td><td>$\pi_{1B}^{FF} - G$, $\pi_{2B}^{FF} - G$</td></tr>
</table>

Table 3-6: Corner payoff matrix when $\tilde{k} \le k < \bar{k}$

They are all symmetric matrices that follow from equations (53) to (55). Notice that when a firm chooses to undertake FDI, the payoff is the total profits minus the fixed cost G of building a factory in the other market.

3.3.1 Equilibria for the game

Solving the game for equilibria, there are two cases: a firm's competitor in the same country chooses to export or to undertake FDI. In terms of the first case, the firm will choose to undertake FDI if it gives higher payoffs than it chooses to export. Refer back to the left hand side of table 3-1, where firm one and firm two compete in country B, and firm two chooses to export to country B. Then undertaking FDI is optimal for firm one if $\pi_{1B}^{FE} - G > \pi_{1B}^{EE}$ when $k < \tilde{k}$, or $\pi_{1B}^{F*} - G > \pi_{1B}^{EE}$ when $\tilde{k} \le k < \bar{k}$

Therefore if the fixed cost of FDI of firm one is less than the critical value: $\bar{G} \equiv \pi_{1B}^{FE} - \pi_{1B}^{EE}$ or $\bar{G} \equiv \pi_{1B}^{F*} - \pi_{1B}^{EE}$, given different value of the trade cost k, it will choose to undertake FDI when its competitor in the same country chooses to export, using (53) (56) and (55):

$$\bar{G} \equiv \begin{cases} \frac{1}{2}\eta^{-\eta}(4\eta-1)^{\eta-1}\left[2(4c+k)^{-\eta-1}(c+k\eta)^2 - 2^{-\eta}(2c+k)^{-\eta-1}(c+k-2k\eta)^2\right] & \text{if } k < \tilde{k} \\ \frac{1}{6}\eta^{-\eta}\left[2\cdot 3^{-\eta}c^{1-\eta}(3\eta-1)^{\eta-1} - 3\cdot 2^{-\eta}(2c+k)^{-\eta-1}(4\eta-1)^{\eta-1}(c+k-2k\eta)^2\right] & \text{if } \tilde{k} \le k < \bar{k} \end{cases} \tag{58}$$

The critical value of the fixed cost of FDI $\bar{G}$ is a two-segmented concave quadratic curves in the trade cost up to the prohibitive trade cost $\bar{k}$, similar to $\bar{G}$ in chapter two. It is shown in figure 3-5 with the parameter values: $c=1$, and $\eta=10$. Undertaking FDI is preferred to exporting for a firm in the region where $G<\bar{G}$ when its competitor in the same market chooses to export before the trade cost reaches $\tilde{k} = c/(3\eta-1)$ and stop exporting thereafter, whereas exporting is preferred for both firms in the region $G>\bar{G}$. Thus a reduction in trade costs will only shift firms from undertaking FDI to exporting as suggested in most theoretical literature.

Considering the second case when a firm's competitor in the same country chooses to undertake FDI, a firm will only choose to undertake FDI if it brings higher payoffs. Then undertaking FDI is more profitable for firm one if $\pi_{1B}^{FF} - G > \pi_{1B}^{EF}$ when $k < \tilde{k}$, or $\pi_{1B}^{FF} - G > \pi_{1B}^{*F}$ when $\tilde{k} \le k < \bar{k}$. Hence undertaking FDI is a better strategy for a firm when its competitor in the same market has chosen to do it if the fixed cost of FDI is less than the critical value $\tilde{G} \equiv \pi_{1B}^{FF} - \pi_{1B}^{EF}$ or $\tilde{G} \equiv \pi_{1B}^{FF} - \pi_{1B}^{*F}$, when the trade cost is prohibitive for firm two. Using (53), (56)and (55)

$$\tilde{G} \equiv \begin{cases} \frac{1}{4}\eta^{-\eta}\left(4\eta-1\right)^{\eta-1}\left[4^{-\eta}c^{1-\eta} - 4\left(4c+k\right)^{-\eta-1}\left(c+k-3k\eta\right)^2\right] & if\ k < \tilde{k} \\ 4^{-1-\eta}c^{1-\eta}\eta^{-\eta}\left(4\eta-1\right)^{\eta-1} & if\ \tilde{k} \le k < \bar{k} \end{cases} \tag{59}$$

The critical value of $\tilde{G}$ is shown in figure 3-5. For $k < \tilde{k}$, it is a concave quadratic curve that is increasing in the trade cost k. For $\tilde{k} \le k < \bar{k}$, it is horizontal and is independent of the trade cost, as the firm's profits from exporting to the other country become zero. By comparing (58) and (59), it can be shown that $\tilde{G} < \bar{G}$.

$$\bar{G} - \tilde{G} = \begin{cases} \frac{1}{4}\eta^{-\eta}(-1+4\eta)^{\eta-1}\left[-4^{-\eta}c^{1-\eta} + 4(4c+k)^{-1-\eta}(c+k-3k\eta)^2 + 2(-2^{-\eta}(2c+k)^{-1-\eta}(c+k-2k\eta)^2 + 2(4c+k)^{-1-\eta}(c+k\eta)^2)\right] > 0 & if\ k < \tilde{k} \\ \frac{1}{12}\eta^{-\eta}(-34^{-\eta}c^{1-\eta}(-1+4\eta)^{-1+\eta} + 2(23^{-\eta}c^{1-\eta}(-1+3\eta)^{-1+\eta} - 32^{-\eta}(2c+k)^{-1-\eta}(-1+4\eta)^{-1+\eta}(c+k-2k\eta)^2)) > 0 & if\ \tilde{k} \le k < \bar{k} \end{cases}$$

In the region under $\tilde{G}$, a firm will choose to undertake FDI when its competitor in the same country undertakes FDI, and the firm will also choose FDI when that competitor chooses to export ($\tilde{G} < \bar{G}$). Therefore undertaking FDI is a dominant strategy for both firms in the same market when $G < \tilde{G}$, whereas only one firm in each market chooses FDI in the region where $\tilde{G} < G < \bar{G}$.

Result 1*: under a static game in Cournot oligopoly, undertaking FDI is a dominant strategy for both firms in a market in the region where $G < \tilde{G}$, while only one firm in each market chooses to undertake FDI in the region $\tilde{G} < G < \bar{G}$.*

3.3.2 Prisoners' dilemma

There is a possibility of prisoners' dilemma in the region $G < \tilde{G}$ and $\tilde{G} < G < \bar{G}$, as the dominant strategy of undertaking FDI might give lower payoffs than the strategy of export, due to the more intense competition by FDI.

Since two markets are symmetric, the equilibria for all firms between two markets are also symmetric. When the fixed cost is **between** $\tilde{G}$ and $\bar{G}$, the equilibrium is that one firm in each country undertakes FDI. They will have a higher profits than when they all choose to export if $\Pi_{FEFE} - G > \Pi_{EEEE}$ ($\Pi_{F*F*} - G > \Pi_{EEEE}$) when $k < \tilde{k}$ ($\tilde{k} \le k < \bar{k}$). This occurs if the fixed cost of FDI is less than the critical value:

$$G_n \equiv \begin{cases} \Pi_{FEFE} - \Pi_{EEEE} & if\, k < \tilde{k} \\ \Pi_{F*F*} - \Pi_{EEEE} & if\, \tilde{k} \le k < \bar{k} \end{cases}$$

By using equations from chapter two:

$$G_n = \begin{cases} \dfrac{1}{2}\eta^{-\eta}(-1+4\eta)^{-1+\eta}\left[4(4c+k)^{-1-\eta}(c+k\eta)^2 + 2^{-\eta}(2c+k)^{-1-\eta}\phi\right] & if\, k < \tilde{k} \\ \dfrac{1}{6}\eta^{-\eta}\left[43^{-\eta}c^{1-\eta}(-1+3\eta)^{-1+\eta} + 32^{-\eta}(2c+k)^{-1-\eta}(-1+4\eta)^{-1+\eta}\phi\right] & if\, \tilde{k} \le k < \bar{k} \end{cases} \tag{60}$$

where $\phi = -2c^2 - 2ck + k^2(-1+4\eta-8\eta^2)$. The critical value G_n is a two-segmented concave quadratic that is increasing in the trade cost up to the prohibitive trade cost $\bar{k}$ as shown in figure 3-6. It is below $\bar{G}$ and $\tilde{G}$ [23]. In the region under G_n, there is no prisoners' dilemma. In the region above $\tilde{G}$, there is a prisoners' dilemma, where the equilibrium profits are smaller than the profits when all firms export. However, this Nash equilibrium only occurs between $\bar{G}$ and $\tilde{G}$, and G_n is below this region, so the shaded area in figure 3-6 which represents no prisoners' dilemma is irrelevant. Therefore prisoners' dilemma always exists when one firm undertakes FDI while its competitor from the same market exports in the region $\tilde{G} < G < \bar{G}$.

[23] The calculation of the fact that the critical value G_n is below $\bar{G}$ and $\tilde{G}$ is not mentioned here, because it is similar to what happened in chapter two.

When the fixed cost is **below** $\tilde{G}$, the Nash equilibrium is that all firms undertake FDI. They will have higher profits than when they all export if $\Pi_{FFFF} - G > \Pi_{EEEE}$, and this happens if the fixed cost of FDI is less than the critical value:

$$\hat{G} \equiv \Pi_{FFFF} - \Pi_{EEEE}$$

By using equations:

$$\hat{G} = 2^{-1-2\eta} c^{-\eta} (2c+k)^{-1-\eta} \eta^{-\eta} (-1+4\eta)^{-1+\eta} \left[c(2c+k)^{1+\eta} - 2^{\eta} c^{\eta} \left(2c^2 + 2ck + k^2 (1 - 4\eta + 8\eta^2) \right) \right]$$

In the region under $\hat{G}$ in figure 3-7, all firms prefer undertaking FDI to exporting, and all firms prefer undertaking FDI to the case that one firm in each country undertake FDI while its competitor in the same country exports ($\hat{G}$ is in the region under $\tilde{G}$ [24]). Therefore undertaking FDI is a dominant strategy for all firms under $\hat{G}$ ($G < \hat{G}$), and the profits (before the fixed cost) when all firms undertake FDI is higher than the profits when all firms export, there is no prisoners' dilemma when $G < \hat{G}$.

Result 2: *Under Cournot oligopoly, for all firms in both markets, undertaking FDI is a dominant strategy when* $G < \hat{G}$*, and there is no prisoners' dilemma. Prisoners' dilemma exists between the region* $\tilde{G} < G < \overline{G}$ *and the region* $\hat{G} < G < \tilde{G}$.

To sum up, in the region $\tilde{G} < G < \overline{G}$, the Nash equilibrium is that only one firm undertakes FDI while its competitor in the same market chooses to export up to the prohibitive level $\tilde{k}$. In the region $G < \tilde{G}$, the Nash equilibrium is that both firms from the same country undertake FDI.

The prisoner's dilemma effect is decomposed into figure 3-6 and 3-7. In the region $\tilde{G} < G < \overline{G}$ of figure 3-6, there is a prisoners' dilemma where one firm in each country undertakes FDI while their competitors from both markets choose to export up to the trade cost $\tilde{k}$, but the profits are lower than when all firms export. In the

[24] The calculation of which is not mentioned as it is similar to the one in chapter two.

region $\hat{G} < G < \tilde{G}$ of figure 3-7, there is a prisoners' dilemma when all firms will choose to undertake FDI, but profits are lower than when they all export due to the same reason (more intensive competition and the fixed cost of undertaking FDI). Next section will extend the static game to an infinitely-repeated game theory of FDI to avoid the prisoners' dilemma.

3.4 The infinitely repeated game

In the infinitely-repeated Cournot oligopoly game, firms can overcome the prisoners' dilemma problem by tacitly colluding, and Nash reversion trigger strategies (Grim trigger strategy) can sustain a subgame perfect Nash equilibrium in an infinitely repeated game. If the discount factor is sufficiently high, the collusive outcome (all firms choose to export) is sustained as a subgame perfect Nash equilibrium by the threat of reversion to the Nash equilibrium outcomes in the infinitely-repeated game. It is assumed that firms only collude over the undertaking FDI versus exporting decision, and choose outputs as Cournot oligopolies in the second stage of the game.

In the static game, there are two Nash Equilibria, thus when considering the Nash equilibrium profits from the second period onwards in the infinitely-repeated game, these two static equilibria should be taken into account separately according to the size of the fixed cost G.

3.4.1 Collision when $G < \tilde{G}$

When the fixed cost in the region under $\tilde{G}$, a subgame Nash equilibrium is the collusive outcome where all firms choose to export, and firms are earning jointly maximised profits. However, if one firm deviates (undertaking FDI) to increase its single period (discounted) profits, the other firms are, in response, likely to revert to the safe position (static Nash equilibrium) where all firms undertake FDI for the rest of the periods.

All firms choosing to export is a Nash equilibrium if the present discounted profits from collusion (all firms export) exceed the present discounted value of profits from cheating (choosing to undertake FDI when its competitors from the home country and the foreign country have chosen to export) for one period, and thereafter followed by the Nash equilibrium profits of all firms (all firms choose to undertake FDI).

$$\begin{cases} \dfrac{1}{1-\delta}\Pi_{EEEE} > \left(\Pi_{FEEE} - G\right) + \dfrac{\delta}{1-\delta}\left(\Pi_{FFFF} - G\right) & if\, k < \tilde{k} \\ \dfrac{1}{1-\delta}\Pi_{FEEE} > \left(\Pi_{F*EE} - G\right) + \dfrac{\delta}{1-\delta}\left(\Pi_{FFFF} - G\right) & if\, \tilde{k} \le k < \bar{k} \end{cases}$$

There will be an interior solution and a corner solution. Firm two will stop exporting when the trade cost reaches its prohibitive level $\tilde{k}$. Rearrange the above, as shown in chapter two, the collusive outcome can be sustained as a Nash equilibrium if the fixed cost of FDI is greater than the critical value:

$$G^*\left(\delta\right) = \left(1-\delta\right)\bar{G} + \delta\hat{G} \tag{61}$$

$G^*\left(\delta\right)$ is a convex combination of $\bar{G}$ and $\hat{G}$, weighted by the discount factor δ. When the discount factor becomes zero, the critical value G^* equals $\bar{G}$, and when the discount factor becomes one, the critical value G^* equals $\hat{G}$. Meanwhile, as $\bar{G} > \hat{G}$ and the critical value of fixed cost is decreasing in the discount factor δ, it is always lower in the infinitely repeated game than in the static game if $\delta > 0$.

$$G^*\left(\delta\right) \equiv \begin{cases} \frac{1}{2}\eta^{-\eta}(-1+4\eta)^{-1+\eta}((1-\delta)(-2^{-\eta}(2c+k)^{-1-\eta}(c+k-2k\eta)^2 + 2(4c+k)^{-1-\eta}(c+k\eta)^2) \\ +4^{-\eta}\delta(c^{1-\eta} + 2^{\eta}(2c+k)^{-1-\eta}\phi)) & if \quad k < \tilde{k} \\ \frac{1}{6}\eta^{-\eta}((1-\delta)(23^{-\eta}c^{1-\eta}(-1+3\eta)^{-1+\eta} - 32^{-\eta}(2c+k)^{-1-\eta}(-1+4\eta)^{-1+\eta}(c+k-2k\eta)^2) \\ +34^{-\eta}c^{-\eta}(2c+k)^{-1-\eta}\delta(-1+4\eta)^{-1+\eta}(c(2c+k)^{1+\eta} + 2^{\eta}c^{\eta}\phi)) & if \quad \tilde{k} \le k < \bar{k} \end{cases} \tag{62}$$

Where $\phi = -2c^2 - 2ck + k^2(-1+4\eta-8\eta^2)$. The critical value of the fixed cost $G^*\left(\delta\right)$ is shown in figure 3-8 and figure 3-9 by using the same parameters, and it is a two-segmented concave quadratic of the trade cost for a number of discount factors $\delta \in \{0, 0.264, 0.497, 1\}$ up to the prohibitive trade cost $\bar{k}$. However, only the area under $\tilde{G}$ is relevant to this game. The collusive outcome is easier to be sustained when the discount factor is larger, and the collusive equilibrium can be sustained whenever the static game is a prisoners' dilemma, $\tilde{G} > G^* \geq \hat{G}$.

3.4.2 Collusion when $G > \tilde{G}$

When the fixed cost of FDI G is in the region above $\tilde{G}$, all firms choosing to export is a Nash equilibrium if the present discounted profits from collusion (all firms export) exceed the present discounted value of profits from cheating (choosing to undertake FDI when its competitors from both countries have chosen to export) for one period, and thereafter followed by the Nash equilibrium profits above $\tilde{G}$ (one firm in each country chooses to undertake FDI when their competitors in the same country have chosen to export), for both interior and corner solutions:

$$\begin{cases} \dfrac{1}{1-\delta}\Pi_{EEEE} > \left(\Pi_{FEEE} - G\right) + \dfrac{\delta}{1-\delta}\left(\Pi_{FEFE} - G\right) & if\, k < \tilde{k} \\ \dfrac{1}{1-\delta}\Pi_{EEEE} > \left(\Pi_{F*EE} - G\right) + \dfrac{\delta}{1-\delta}\left(\Pi_{F*F*} - G\right) & if\, \tilde{k} \leq k < \bar{k} \end{cases} \tag{63}$$

Only when (29) is satisfied, the collusive outcome can be sustain as a subgame Nash equilibrium and there is no prisoners' dilemma in this infinitely repeated game.

By rearranging (29), the collusive outcome can be sustained as a Nash equilibrium if the fixed cost of FDI is greater than the critical value when $k < \tilde{k}$, by using the definition of G_n,

$$G^{**}\left(\delta\right) = \left(1-\delta\right)\bar{G} + \delta G_n \tag{64}$$

$G^{**}\left(\delta\right)$ is a convex combination of $\bar{G}$ and G_n, weighted by the discount factor δ. When the discount factor becomes zero, the critical value G^{**} equals $\bar{G}$, and when the discount factor becomes one, the critical value G^{**} equals G_n. Similar to G^*, the critical value of fixed cost G^{**} is decreasing in the discount factor δ, and it is always lower in the infinitely repeated game than in the static game.

If substitute (59) and (60) into(64), the critical value of the fixed cost in the infinitely repeated game when there is only one firm in each market undertakes FDI is:

$$G^{**}(\delta) \equiv \begin{cases} \frac{1}{2}\eta^{-\eta}(-1+4\eta)^{-1+\eta}(2(4c+k)^{-1-\eta}(1+\delta)(c+k\eta)^2+2^{-\eta}(2c+k)^{-1-\eta}\omega) & if \quad k<\tilde{k} \\ \frac{1}{6}\eta^{-\eta}(23^{-\eta}c^{1-\eta}(1+\delta)(-1+3\eta)^{-1+\eta}+2^{-\eta}(2c+k)^{-1-\eta}(-1+4\eta)^{-1+\eta}3\omega) & if \quad \tilde{k}\le k<\bar{k} \end{cases} \tag{65}$$

Where $\omega=-c^2(1+\delta)-2ck(1+2(-1+\delta)\eta)-k^2(1+4\eta(-1+\eta+\delta\eta))$. $G^{**}(\delta)$ is shown in figure 3-10 by using the same parameters as the previous figures, and it is a two-segmented concave quadratic of the trade cost for a number of discount factors $\delta\in\{0,0.27,0.76,1\}$ up to the prohibitive trade cost $\bar{k}$. However, only the area between $\tilde{G}$ and $\bar{G}$ is relevant to this game. Combining figure 3-6 and figure 3-10, yields figure 3-11. It shows that in the region between $G^{**}(\delta)$ and $\bar{G}$, the collusive outcome where all firms export can be sustained as a Nash equilibrium in the infinitely repeated game whereas only one firm in each market would undertake FDI in the static game and there is no prisoners' dilemma. Clearly, the firm that switches to undertaking FDI faster will gain more profits at the expenses of its competitor's profits in both countries, and the collusive outcome is easier to be sustained when the discount factor is larger.

Result 3: *Under Cournot oligopoly, the collusive outcome where all firms export can be sustained as a subgame perfect Nash equilibrium in the infinitely repeated game if* $\tilde{G}>G>G^{*}$, *or* $\bar{G}>G>G^{**}$ *and the critical value is decreasing in the discount factor,* $\partial G^{*}/\partial\delta<0$, $\partial G^{**}/\partial\delta<0$

3.4.3 Effect of k and G on collusion

To understand how the trade cost affects the sustainability of collusion, it is worthwhile to look at how the critical value of the fixed cost of FDI depends on the trade cost. Considering the area under $\tilde{G}$, take the first order condition of $G^{*}(\delta)$ in (62) with respect to k ($\partial G^{*}/\partial k=0$). The solution to this gives the trade cost

$k = k^*(\delta)$, which is a function of discount factor and the maximum point of the critical value G^*. Unfortunately, this function could not be solved analytically, but it can be solved by a given value for elasticity of demand. The solution then can be solved for the maximum of $\hat{G}$ as a function of δ. By substituting the discount factor from zero to one in $k^*(\delta)$, then a line of k^*k^* can be plotted by Mathematica as a function of the trade cost[25], jointing the maximum points of the critical value of G^*. Hence, for low trade costs when $k < k^*k^*$ the derivatives of $\partial G^{**}/\partial k$ are positive, but for higher trade costs in the region $k^*k^* \le k < \bar{k}$, the derivatives are negative.

A light blue line in figure 3-9 shows k^*k^* for given value of the elasticity ($\eta = 10$) in the area under $\tilde{G}$, where G^* decreases on the left hand side of k^*k^*, and it increases on the right hand side of k^*k^*, i.e. the shaded area, and this area represents that a reduction in the trade cost cause firms switch from exporting to undertaking FDI, as shown by the arrow in figure 3-13.

Using the same method to test the relationship between the trade cost and sustainability of collusion in the area above $\tilde{G}$, take the first order condition of $G^{**}(\delta)$ in (65) with respect to k ($\partial G^{**}/\partial k = 0$). The solution of the critical value of $k = k^{**}$ shows the boundary of how the derivatives of $\partial G^{**}/\partial k$ change when trade costs increase, i.e. they are negative if $k > k^{**}$. Again, by using a given value for elasticity of demand ($\eta = 10$), and substituting the numerical discount factor δ from 0 to 1, a line of $k^{**}k^{**}$ can be plotted by Mathematica as a function of the trade cost, jointing the maximum points of the critical value of G^{**}. On the right hand side of $k^{**}k^{**}$, the critical value of G^{**} is decreasing with trade costs, indicating that a reduction in trade costs will cause firms switch from exporting to undertaking FDI, as shown in the shaded area in figure 3-11.

Result 4: *Under Cournot oligopoly, if the trade cost $k > k^*$ ($k > k^{**}$), the critical value of the fixed cost G^* (G^{**}) is decreasing in the trade cost, $\partial G^*/\partial k < 0$ ($\partial G^{**}/\partial k < 0$),*

[25] The code for calculating k^*k^* is in the Appendix.

and a reduction in trade costs may lead firms to switch from exporting to undertaking FDI.

Figure 3-12 is the combination of figure 3-9 and figure 3-11, so the shaded area is to the right of $k^{**}k^{**}$ above $\tilde{G}$ and $k^{*}k^{*}$ below $\tilde{G}$. There is a possibility that a reduction in the trade cost will lead the firms shift from exporting to undertaking FDI in the shaded area as shown in proposition four. Interestingly, the switch of the strategies could happen outside the shaded area in figure 3-12 as well. To illustrate the relationship between the trade cost and FDI further, a certain discount rate (e.g. $\delta = 0.38$) will be used for both G^{*} and G^{**}, so that $G^{*}(\delta = 0.38)$ is below $\tilde{G}$ and $G^{**}(\delta = 0.38)$ is above $\tilde{G}$ as shown in figure 3-13. For simplicity, figure 3-13 ignores the boundary critical value of the fixed cost $k^{*}k^{*}$ and $k^{**}k^{**}$, leaving $\bar{G} = G^{**}(\delta = 0)$, $G^{**}(\delta = 0.38)$, $\tilde{G}$, $G^{*}(\delta = 0.38)$ and $\hat{G} = G^{*}(\delta = 1)$.

The relationship between the trade cost and FDI can be checked in more detail by looking at figure 3-13 horizontally. Clearly, the right hand side of $k^{**}k^{**}$ and $k^{*}k^{*}$ from figure 3-12 indicate that a reduction in trade cost may lead firms switch from exporting to undertaking FDI. The question is if this is the only area that would happen. In the region of $G^{**}(\delta = 0.38) < G < \bar{G}$, the subgame equilibrium is that all firms export (collusive outcome 2E): in the region $\tilde{G} < G < G^{**}(\delta = 0.38)$, the equilibrium is that one firm in each country undertakes FDI and their competitor in the same country exports (1E1F)up to $\tilde{k}$ (1F); in the region $G^{*}(\delta = 0.38) < G < \tilde{G}$, the equilibrium is the collusive outcome (all firms export 2E), and in the region under $G^{*}(\delta = 0.38)$, the equilibrium becomes all firms choose to undertake FDI (2F), as shown in 3-14.

Suppose $\tilde{G}$ intersect $G^{**}(\delta = 0.38)$ at the point $k = k^{**}$, if the trade cost falls in the region $k^{**} < k < k^{*}$ and within the region between $G^{*}(\delta = 0.38) < G < G^{**}(\delta = 0.38)$, the firms might switch from 2E (both firms in the same county choose to export) to 1E1F (one firm in each country chooses to undertake FDI) just across $\tilde{G}$ as shown in the shaded area in figure 3-13. If the trade cost is reduced further to the outside of the shaded area, both firms from the same country will choose to export. Hence, a reduction in the trade cost may lead firms switch from exporting to undertaking FDI even outside the area as shown in figure 3-12. Note that when the discount factor δ is decreasing, the critical value of the fixed cost G^{**} is increasing, therefore the area where the Nash equilibrium outcome (one firm exports while its competitor in the same country undertakes FDI) sustains is bigger, and so is the shaded area in figure 3-12. It leads to the following proposition:

Result 5: *Under Cournot oligopoly, a reduction in trade costs in the region $k^{**} < k < \tilde{k}$ may also lead firms to switch from exporting to undertaking FDI. The larger the discount factor δ, the better chance that firms will switch the strategy.*

By looking at figure 3-13 vertically, the relationship between the fixed cost and FDI can be examined. This is because the level of the fixed cost G will affect the firms' decision about their strategies. In the area of $G^{*}(\delta = 0.38) < G < G^{**}(\delta = 0.38)$, a reduction in fixed costs across $\tilde{G}$ might shift the equilibrium from 1E1F (undertake FDI, while its competitor in the same country export) or 1F(the firm stops trading when $k > \tilde{k}$) to the collusive outcome (both firms chose to export), indicating that firms might switch from undertaking FDI to exporting as the fixed cost G decreases. which leads to the proposition six in chapter two:

Result 6: *Under Cournot oligopoly, if the fixed cost G is reduced from the region $\tilde{G} < G < G^{**}$ to the region $G^{*} < G < \tilde{G}$, it may lead firms to switch from undertaking FDI to exporting.*

The same reason applies that when the fixed cost is relatively high (between $G^{**}(\delta = 0.38)$ and $G^{*}(\delta = 0.38)$), a reduction in fixed costs makes it hard for firms to sustain the Nash equilibrium (undertaking FDI while its competitor in the same country exports up to $\tilde{k}$), and may lead to a switch to the collusive outcome. This is because that if one firm deviates by undertaking FDI, it would be too expensive for its competitor in the same market to deviate. Thus when the fixed cost is relatively high, a reduction in fixed costs will reduce the profits from cheating, and reduce the profits from Nash equilibria where all firms undertake FDI, or one firm in each market undertakes FDI, but increase the profits from the collusion outcome where all firms export.

3.5 Conclusion

Chapter two has analysed the export versus FDI decisions in a two-country four-firm model under Cournot oligopoly by using linear demand function. This chapter uses constant elasticity demand function to check the robustness of the results from the last chapter, and it has been confirmed that all the results are quite general. In the static game, a reduction in the trade cost will lead the firms switch from undertaking FDI to exporting. The same outcomes are achieved that two firms in the same country choose to undertake FDI if the fixed cost is relatively low, or one firm chooses to export while its competitor from the same country chooses to undertake FDI if the fixed cost is relatively high. Again it shows that both export and FDI can exist as an equilibrium outcome in the world when the trade cost is sufficiently high.

The prisoners' dilemmas still exist. If the fixed cost is relatively low, all firms might make lower profits when they all undertake FDI than when they export. If the fixed cost is relatively high, the equilibrium profits when one firm in each country undertakes FDI while its competitor in the same country export might be higher than the profits when all firms export, due to the intensified competition caused by FDI.

The prisoners' dilemma can be avoided in an infinitely-repeated game when all firms tacitly collude over their FDI versus export decisions, as collusion over FDI can be sustained by the threat of Nash-reversion strategies if the trade cost is sufficiently high. Then a reduction in trade costs may lead firms to switch from exporting to undertaking FDI when the trade cost is sufficiently high, as in the infinitely-repeated game ,a reduction in a sufficiently high trade costs lessens the profitability of collusion.

Appendix

Operating profits and outputs

Firms employ a Cournot strategy. That is, each firm maximises its profit assuming the outputs of other firms in each market remain the same. Firms stay in business as long as they make non-zero positive profits in each market. The demand function exhibits constant elasticity by using the function, Q_A is total outputs supplies to country A.

$$p_A\left(Q_A\right) = Q_A^{-1/\eta} \tag{66}$$

$$Q_A = q_{1A} + q_{2A} + q_{3A} + q_{4A}$$

1. When both firms choose to export

Suppose both firm three and firm four in country B choose to export to country A, substitute (66) into **Error! Reference source not found.**, the operating profits (before the fixed costs) of the firms in country A are:

$$\pi_{1A}^{EE} = \left(Q_A^{-1/\eta} - c\right) q_{1A} \qquad \pi_{2A}^{EE} = \left(Q_A^{-1/\eta} - c\right) q_{2A}$$

$$\pi_{3A}^{EE} = \left(Q_A^{-1/\eta} - c - k\right) q_{3A} \qquad \pi_{4A}^{EE} = \left(Q_A^{-1/\eta} - c - k\right) q_{4A}$$

The first-order conditions are:

$$\frac{\partial \pi_{1A}^{EE}}{\partial q_{1A}} = p_A\left(1 - \frac{1}{\eta}\frac{q_{1A}}{Q_A}\right) - c = 0 \tag{67}$$

$$\frac{\partial \pi_{2A}^{EE}}{\partial q_{2A}} = p_A\left(1 - \frac{1}{\eta}\frac{q_{2A}}{Q_A}\right) - c = 0 \tag{68}$$

$$\frac{\partial \pi_{3A}^{EE}}{\partial q_{3A}} = p_A\left(1 - \frac{1}{\eta}\frac{q_{3A}}{Q_A}\right) - c - k = 0 \tag{69}$$

$$\frac{\partial \pi_{4A}^{EE}}{\partial q_{4A}} = p_A\left(1 - \frac{1}{\eta}\frac{q_{4A}}{Q_A}\right) - c - k = 0 \tag{70}$$

The four first-order conditions are the reaction functions and constitute four equations with four unknowns q_{1A}, q_{2A}, q_{3A} and q_{4A}. Adding up equations from (67) to (70), we get:

$$p_A\left(4-\frac{1}{\eta}\right)=4c+2k \tag{71}$$

Solve for p_A, we get:

$$p_A=\left(4c+2k\right)/\left(4-1/\eta\right) \tag{72}$$

Now substitute (72) into equation (66), we get Cournot equilibrium $Q_A=p_A^{-\eta}=\left[\left(4c+2k\right)/\left(4-1/\eta\right)\right]^{-\eta}$. Then substitute Q_A and p_A into the equations (67), (68), (69), and (70), we can solve for the Cournot equilibria q_{1A}, q_{2A}, q_{3A} and q_{4A}, which yields equation (53) in section 3.2.1.

2. When one firm exports and the other firm in the same country undertakes FDI

Suppose firm three chooses to export to country A, and firm four chooses to undertake FDI in country A. For the interior solution, substitute (66) into (54), the operating profits (before the fixed costs) of the firms in country A are:

$$\pi_{1A}^{EF}=\left(Q_A^{-1/\eta}-c\right)q_{1A} \qquad \pi_{2A}^{EF}=\left(Q_A^{-1/\eta}-c\right)q_{2A}$$

$$\pi_{3A}^{EF}=\left(Q_A^{-1/\eta}-c-k\right)q_{3A} \qquad \pi_{4A}^{EF}=\left(Q_A^{-1/\eta}-c\right)q_{4A}$$

The first-order conditions are:

$$\frac{\partial\pi_{1A}^{EF}}{\partial q_{1A}}=p_A\left(1-\frac{1}{\eta}\frac{q_{1A}}{Q_A}\right)-c=0 \tag{73}$$

$$\frac{\partial \pi_{2A}^{EF}}{\partial q_{2A}} = p_A \left(1 - \frac{1}{\eta} \frac{q_{2A}}{Q_A}\right) - c = 0 \tag{74}$$

$$\frac{\partial \pi_{3A}^{EF}}{\partial q_{3A}} = p_A \left(1 - \frac{1}{\eta} \frac{q_{3A}}{Q_A}\right) - c - k = 0 \tag{75}$$

$$\frac{\partial \pi_{4A}^{EF}}{\partial q_{4A}} = p_A \left(1 - \frac{1}{\eta} \frac{q_{4A}}{Q_A}\right) - c = 0 \tag{76}$$

Solving for the Cournot equilibria using the same method as above, adding up all the first order conditions from(73) to (76), yields:

$$p_A \left(4 - \frac{1}{\eta}\right) = 4c + k$$

then solve for p_A:

$$p_A = (4c + k) / (4 - 1/\eta)$$

Now substitute p_A into (66), we get Cournot equilibrium $Q_A = p_A^{-\eta} = \left[(4c+k)/(4-1/\eta)\right]^{-\eta}$. Then substitute Q_A and p_A into the equations (73), (74), (75), and (76), we can solve for the Cournot equilibria q_{1A}, q_{2A}, q_{3A} and q_{4A}, which yields equation (55) in section 3.2.2.

For the corner solution, firm three will stop exporting up to a prohibited trade cost, the operating profits (before the fixed costs) of the firms in country A become:

$$Q_{A*} = q_{1A} + q_{2A} + q_{4A}$$

$$\pi_{1A}^{*F} = \left(Q_A^{-1/\eta} - c\right) q_{1A} \qquad \pi_{2A}^{*F} = \left(Q_A^{-1/\eta} - c\right) q_{2A}$$

$$\pi_{4A}^{*F} = \left(Q_A^{-1/\eta} - c\right) q_{4A}$$

Where $*$ means firm three stops trading with country A, so its profits and outputs in country A are zero, i.e. $q_{3A}^{*F} = 0$ and $q_{3A}^{*F} = 0$. The price of firm three will just cover its marginal cost $p_{3A}^{*F} = c + k$.The first-order conditions of are:

$$\frac{\partial \pi_{1A}^{*F}}{\partial q_{1A}} = p_A \left(1 - \frac{1}{\eta}\frac{q_{1A}}{Q_A}\right) - c = 0 \tag{77}$$

$$\frac{\partial \pi_{2A}^{*F}}{\partial q_{2A}} = p_A \left(1 - \frac{1}{\eta}\frac{q_{2A}}{Q_A}\right) - c = 0 \tag{78}$$

$$\frac{\partial \pi_{4A}^{*F}}{\partial q_{4A}} = p_A \left(1 - \frac{1}{\eta}\frac{q_{4A}}{Q_A}\right) - c = 0 \tag{79}$$

Solving for the Cournot equilibria by adding up the above equations, the price is $p_A^* = 3c/(3 - 1/\eta)$, substitutes it into (66), the total outputs supply in country A is $Q_{A*} = \left(p_A^*\right)^{-\eta} = \left[3c/(3-1/\eta)\right]^{-\eta}$. Then the outputs and profits are shown in (56).

Similarly, when firm three chooses to undertake FDI and firm four chooses to export, the results are reciprocal.

3. When both firms choose to undertake FDI

Suppose both firms in country B choose to undertake FDI, the operating profits (before the fixed costs) of the firms in country A are:

$$\pi_{1A}^{FF} = \left(Q_A^{-1/\eta} - c\right) q_{1A} \qquad \pi_{2A}^{FF} = \left(Q_A^{-1/\eta} - c\right) q_{2A}$$

$$\pi_{3A}^{FF} = \left(Q_A^{-1/\eta} - c\right) q_{3A} \qquad \pi_{4A}^{FF} = \left(Q_A^{-1/\eta} - c\right) q_{4A}$$

The first-order conditions are:

$$\frac{\partial \pi_{1A}^{FF}}{\partial q_{1A}} = p_A \left(1 - \frac{1}{\eta}\frac{q_{1A}}{Q_A}\right) - c = 0 \tag{80}$$

$$\frac{\partial \pi_{2A}^{FF}}{\partial q_{2A}} = p_A\left(1-\frac{1}{\eta}\frac{q_{2A}}{Q_A}\right)-c=0 \tag{81}$$

$$\frac{\partial \pi_{3A}^{EE}}{\partial q_{3A}} = p_A\left(1-\frac{1}{\eta}\frac{q_{3A}}{Q_A}\right)-c=0 \tag{82}$$

$$\frac{\partial \pi_{4A}^{EE}}{\partial q_{4A}} = p_A\left(1-\frac{1}{\eta}\frac{q_{4A}}{Q_A}\right)-c=0 \tag{83}$$

Solving for the Cournot equilibria yields (57) by using the same method, where the operating outputs, prices and profits of the firms in country A are:

$$p_A\left(4-\frac{1}{\eta}\right)=4c$$

$$p_A = 4c/\left(4-1/\eta\right)$$

$$Q_A = p_A^{-\eta} = \left[4c/\left(4-1/\eta\right)\right]^{-\eta}$$

$$q_{iA}^{FF} = \eta\left(1-\frac{c}{p_A}\right)Q_A = \frac{1}{4}\left(\frac{4\eta-1}{4c\eta}\right)^{\eta}$$

$$\pi_{iA}^{FF} = \frac{c}{4}\left(4c\eta\right)^{-\eta}\left(4\eta-1\right)^{\eta-1}$$

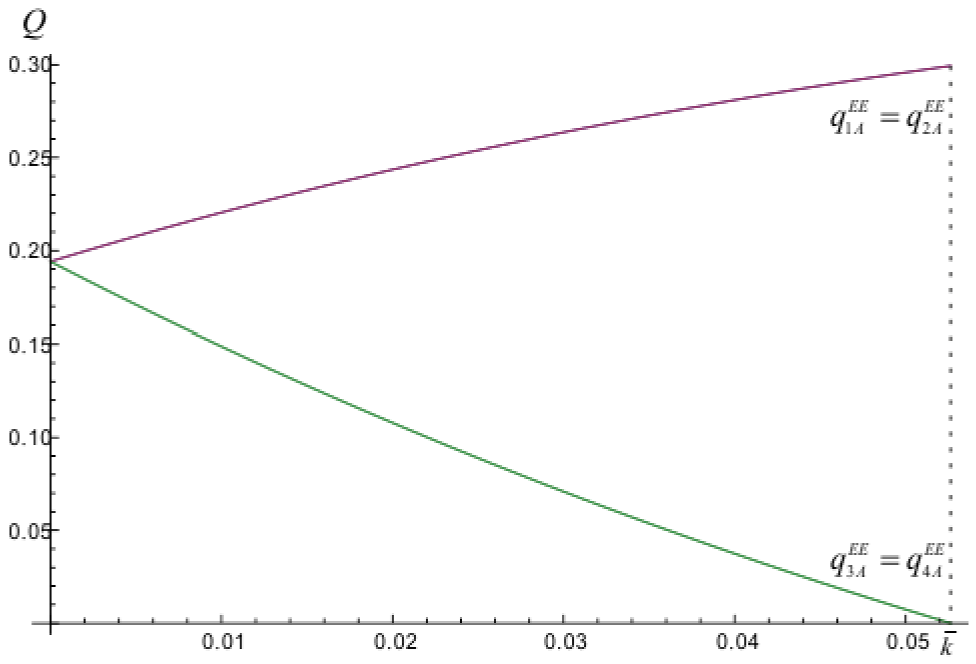

Figure 3-1: Outputs of firms when firm 3 and firm 4 export

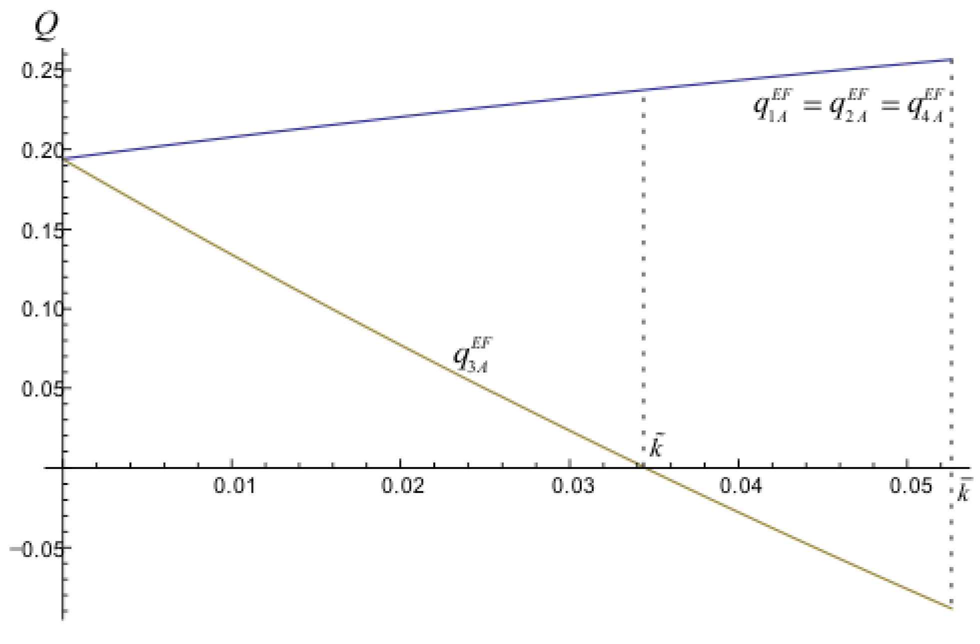

Figure 3-2: Outputs of firms when firm 3 exports and firm 4 undertakes FDI

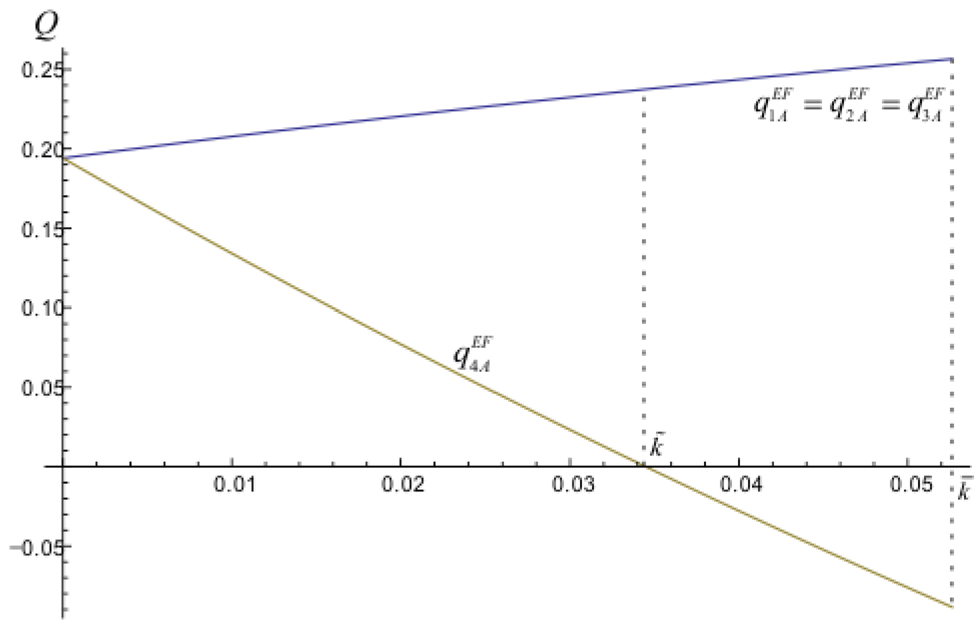

Figure 3-3: Outputs when firm 3 undertakes FDI, and firm 4 exports

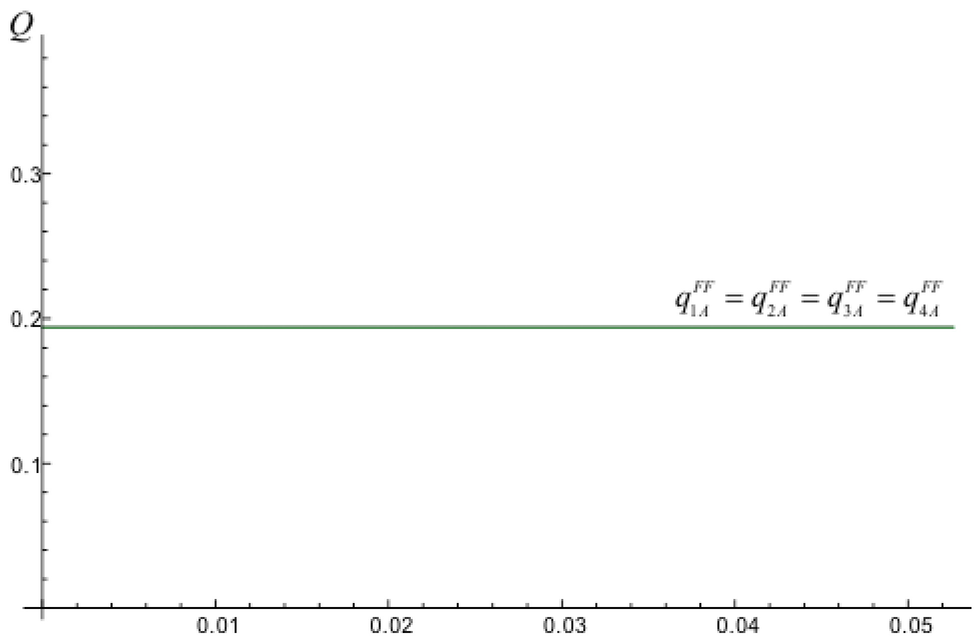

Figure 3-4: Outputs when both firm 3 and firm 4 undertake FDI

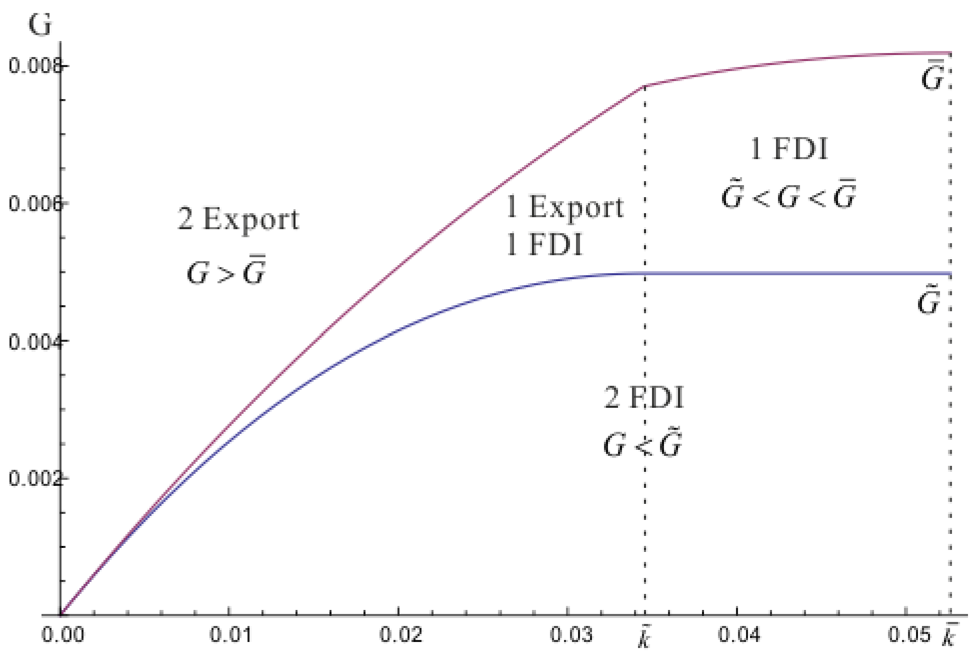

Figure 3-5: Static Game under Cournot duopoly

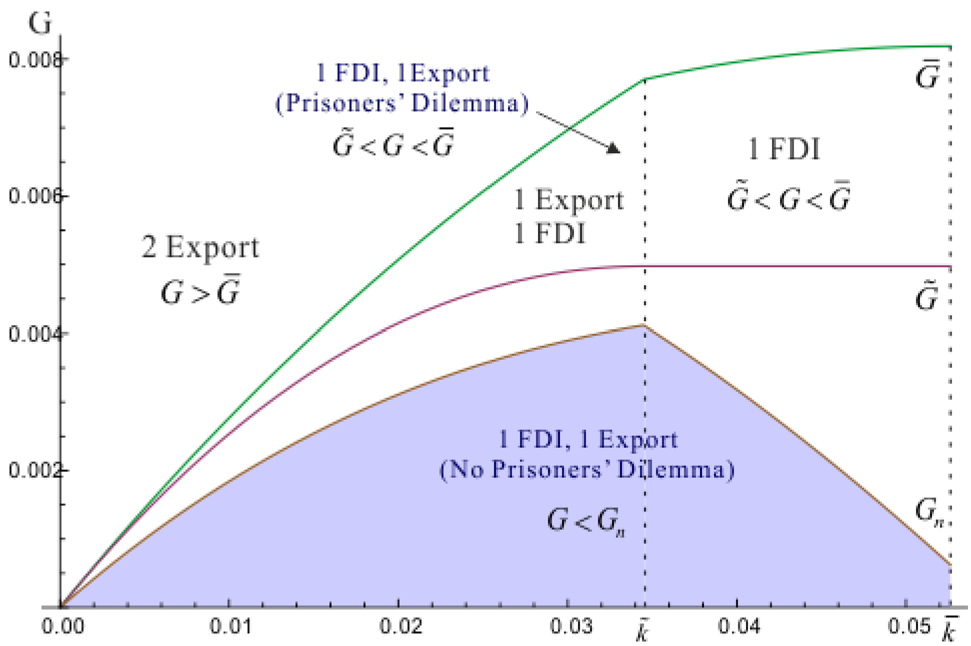

Figure 3-6: Prisoners' dilemma in both countries in the static game when $\tilde{G} \leq G < \bar{G}$

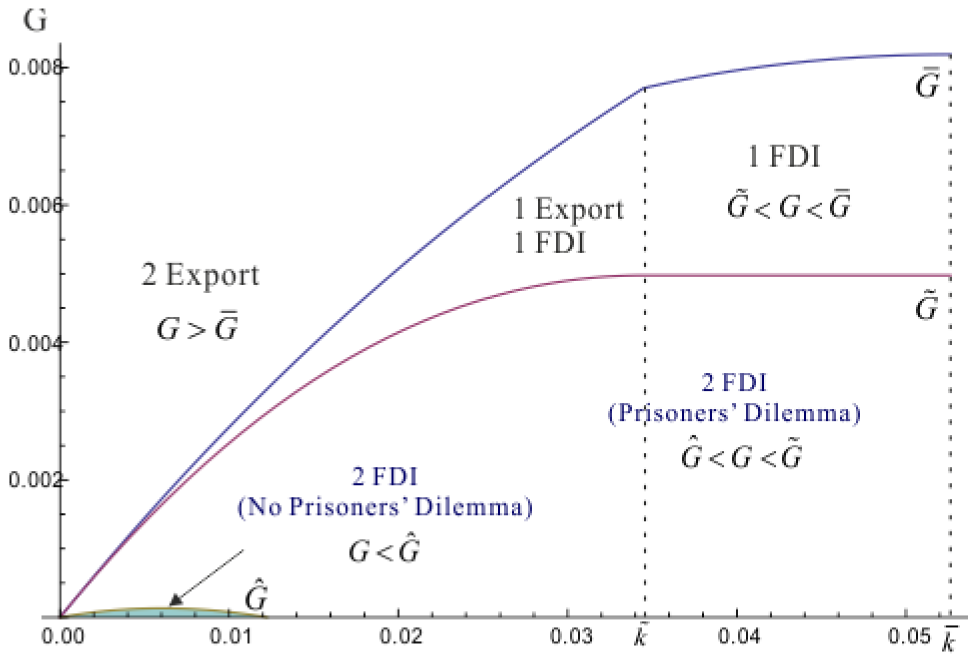

Figure 3-7: Prisoners' dilemma in the static game when $G < \tilde{G}$

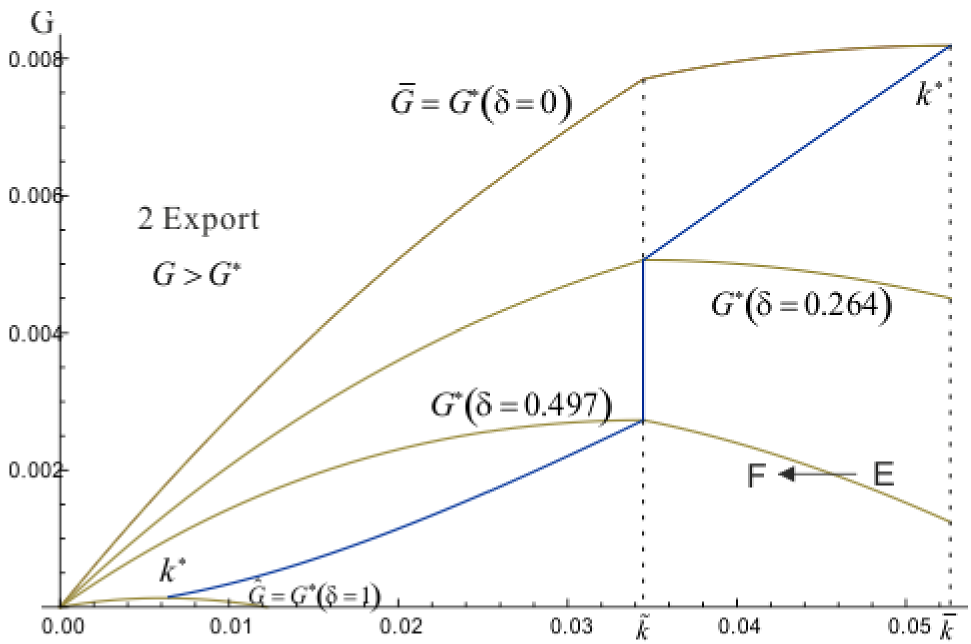

Figure 3-8: Infinitely-repeated game under Cournot oligopoly when $G < \tilde{G}$

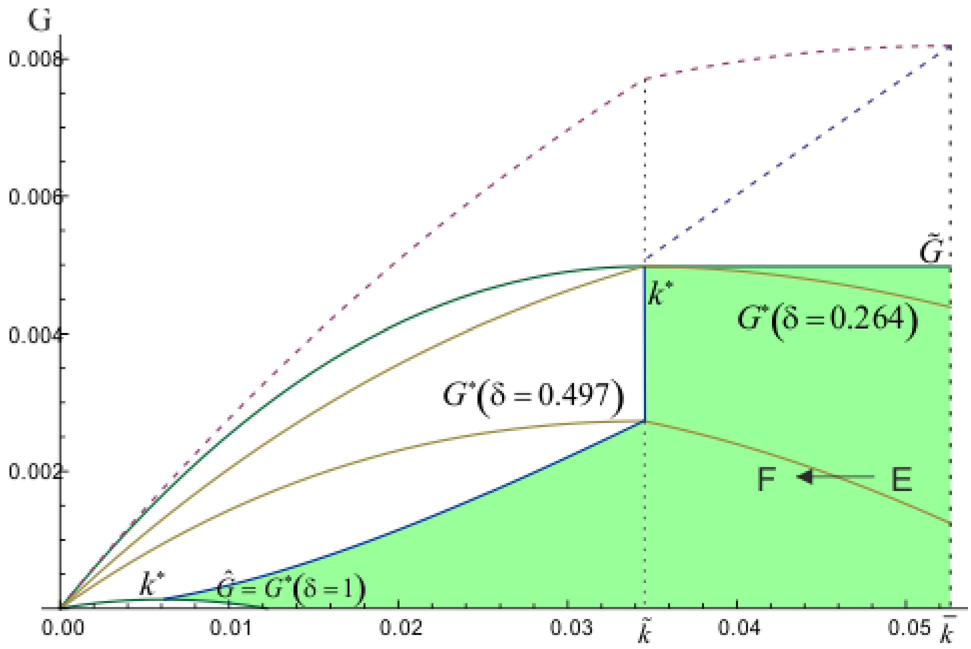

Figure 3-9: Infinitely-repeated game under Cournot oligopoly when $G < \tilde{G}$

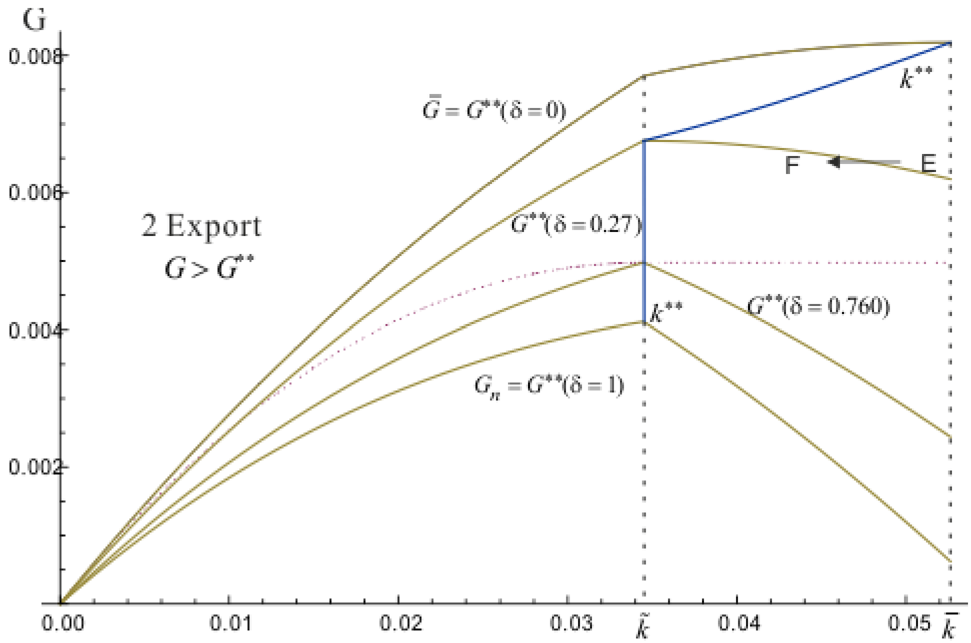

Figure 3-10: Infinitely-repeated game under Cournot oligopoly when $G > \tilde{G}$

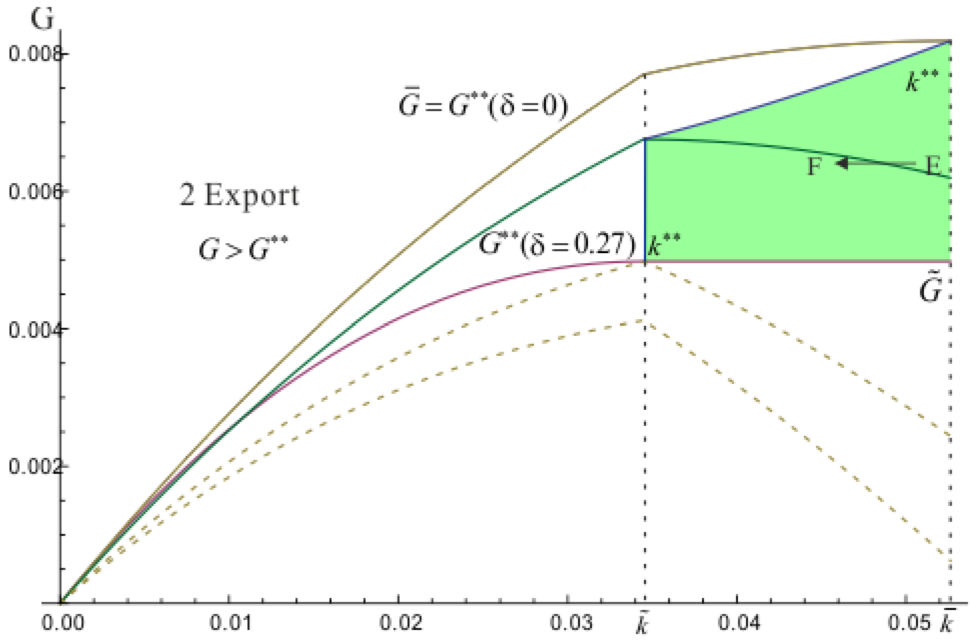

Figure 3-11: Infinitely-repeated game under Cournot oligopoly when $G > \tilde{G}$

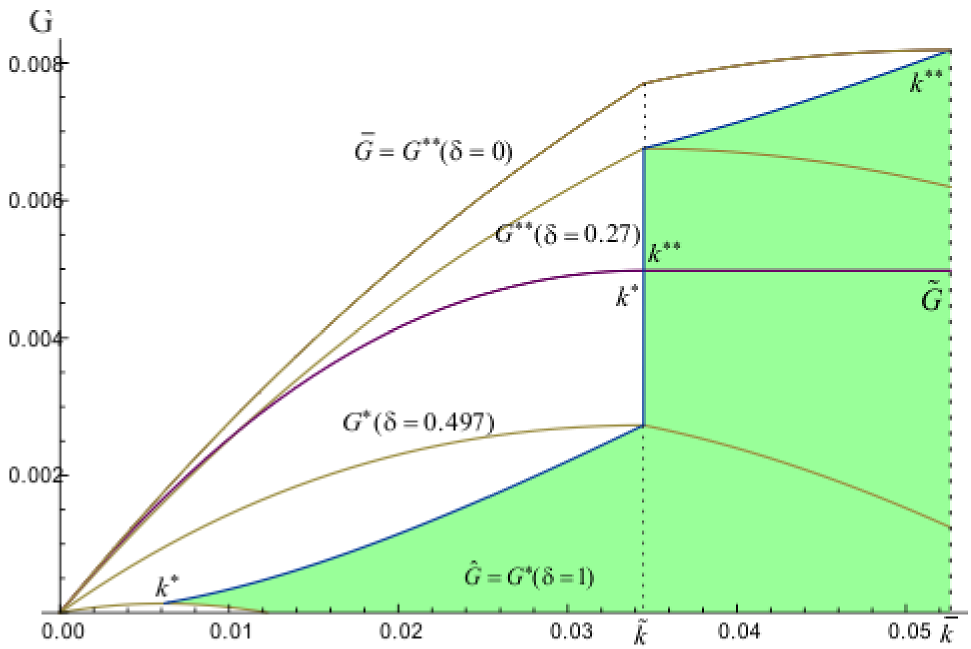

Figure 3-12: Combination of figure 3-9 and figure 3-11

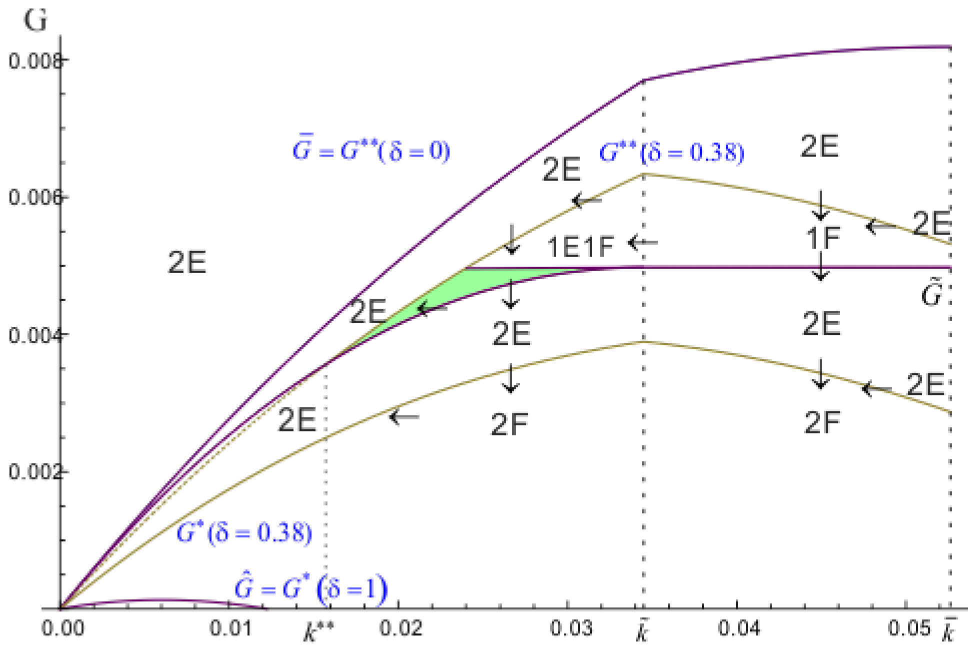

Figure 3-13: Infinitely-repeated game under Cournot oligopoly if $\delta = 0.38$

Code for k^*k^*

```
{c = 1, η = 10}
```

```
{1, 10}
```

```
FullSimplify[D[Gstar1, k]]
```

$$\frac{208\,728\,361\,158\,759 \left(\left(\frac{11\,(1-19\,k)^2}{1024\,(2+k)^{12}}+\frac{19\,(1-19\,k)}{512\,(2+k)^{11}}-\frac{22\,(1+10\,k)^2}{(4+k)^{12}}+\frac{40+400\,k}{(4+k)^{11}}\right)(1-\delta)+\frac{9\,(2+k\,(-336+761\,k))\,\delta}{1024\,(2+k)^{12}}\right)}{20\,000\,000\,000}$$

```
FullSimplify[Solve[% == 0, k]]
```

```
{{k → Root[
    679 477 248 - 578 813 952 δ + (-5 905 580 032 - 10 703 863 808 δ) #1 + (308 281 344 - 12 324 962 304 δ)
      #1^2 + (31 993 102 336 + 13 500 416 000 δ) #1^3 + (51 409 256 448 + 48 649 666 560 δ) #1^4 +
      (30 584 733 696 + 68 457 725 952 δ) #1^5 + (-6 194 479 104 + 67 198 500 864 δ) #1^6 +
      (-25 093 939 200 + 50 908 323 840 δ) #1^7 + (-21 969 775 872 + 29 793 519 360 δ) #1^8 +
      (-11 442 734 336 + 13 166 616 320 δ) #1^9 + (-4 004 008 800 + 4 279 124 256 δ) #1^10 +
      (-957 471 120 + 988 551 600 δ) #1^11 + (-151 209 603 + 153 572 073 δ) #1^12 +
      (-14 284 624 + 14 393 200 δ) #1^13 + (-613 317 + 615 600 δ) #1^14 &, 1]},
 {k → Root[679 477 248 - 578 813 952 δ + (-5 905 580 032 - 10 703 863 808 δ) #1 +
      (308 281 344 - 12 324 962 304 δ) #1^2 + (31 993 102 336 + 13 500 416 000 δ) #1^3 +
      (51 409 256 448 + 48 649 666 560 δ) #1^4 + (30 584 733 696 + 68 457 725 952 δ) #1^5 +
```

```
      (-6 194 479 104 + 67 198 500 864 δ) #1^6 + (-25 093 939 200 + 50 908 323 840 δ) #1^7 +
      (-21 969 775 872 + 29 793 519 360 δ) #1^8 +
      (-11 442 734 336 + 13 166 616 320 δ) #1^9 + (-4 004 008 800 + 4 279 124 256 δ) #1^10 +
      (-957 471 120 + 988 551 600 δ) #1^11 + (-151 209 603 + 153 572 073 δ) #1^12 +
      (-14 284 624 + 14 393 200 δ) #1^13 + (-613 317 + 615 600 δ) #1^14 &, 2]},
{k → Root[679 477 248 - 578 813 952 δ + (-5 905 580 032 - 10 703 863 808 δ) #1 +
      (308 281 344 - 12 324 962 304 δ) #1^2 + (31 993 102 336 + 13 500 416 000 δ) #1^3 +
      (51 409 256 448 + 48 649 666 560 δ) #1^4 + (30 584 733 696 + 68 457 725 952 δ) #1^5 +
      (-6 194 479 104 + 67 198 500 864 δ) #1^6 + (-25 093 939 200 + 50 908 323 840 δ) #1^7 +
      (-21 969 775 872 + 29 793 519 360 δ) #1^8 +
      (-11 442 734 336 + 13 166 616 320 δ) #1^9 + (-4 004 008 800 + 4 279 124 256 δ) #1^10 +
      (-957 471 120 + 988 551 600 δ) #1^11 + (-151 209 603 + 153 572 073 δ) #1^12 +
      (-14 284 624 + 14 393 200 δ) #1^13 + (-613 317 + 615 600 δ) #1^14 &, 3]},
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      (51 409 256 448 + 48 649 666 560 δ) #1^4 + (30 584 733 696 + 68 457 725 952 δ) #1^5 +
      (-6 194 479 104 + 67 198 500 864 δ) #1^6 + (-25 093 939 200 + 50 908 323 840 δ) #1^7 +
      (-21 969 775 872 + 29 793 519 360 δ) #1^8 +
      (-11 442 734 336 + 13 166 616 320 δ) #1^9 + (-4 004 008 800 + 4 279 124 256 δ) #1^10 +
      (-957 471 120 + 988 551 600 δ) #1^11 + (-151 209 603 + 153 572 073 δ) #1^12 +
      (-14 284 624 + 14 393 200 δ) #1^13 + (-613 317 + 615 600 δ) #1^14 &, 4]},
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      (51 409 256 448 + 48 649 666 560 δ) #1^4 + (30 584 733 696 + 68 457 725 952 δ) #1^5 +
      (-6 194 479 104 + 67 198 500 864 δ) #1^6 + (-25 093 939 200 + 50 908 323 840 δ) #1^7 +
      (-21 969 775 872 + 29 793 519 360 δ) #1^8 +
      (-11 442 734 336 + 13 166 616 320 δ) #1^9 + (-4 004 008 800 + 4 279 124 256 δ) #1^10 +
      (-957 471 120 + 988 551 600 δ) #1^11 + (-151 209 603 + 153 572 073 δ) #1^12 +
      (-14 284 624 + 14 393 200 δ) #1^13 + (-613 317 + 615 600 δ) #1^14 &, 5]},
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      (51 409 256 448 + 48 649 666 560 δ) #1^4 + (30 584 733 696 + 68 457 725 952 δ) #1^5 +
      (-6 194 479 104 + 67 198 500 864 δ) #1^6 + (-25 093 939 200 + 50 908 323 840 δ) #1^7 +
      (-21 969 775 872 + 29 793 519 360 δ) #1^8 +
      (-11 442 734 336 + 13 166 616 320 δ) #1^9 + (-4 004 008 800 + 4 279 124 256 δ) #1^10 +
      (-957 471 120 + 988 551 600 δ) #1^11 + (-151 209 603 + 153 572 073 δ) #1^12 +
      (-14 284 624 + 14 393 200 δ) #1^13 + (-613 317 + 615 600 δ) #1^14 &, 6]},
{k → Root[679 477 248 - 578 813 952 δ + (-5 905 580 032 - 10 703 863 808 δ) #1 +
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      (51 409 256 448 + 48 649 666 560 δ) #1^4 + (30 584 733 696 + 68 457 725 952 δ) #1^5 +
```

$(-6\,194\,479\,104 + 67\,198\,500\,864\,\delta)\ \#1^{6} + (-25\,093\,939\,200 + 50\,908\,323\,840\,\delta)\ \#1^{7} +$
$(-21\,969\,775\,872 + 29\,793\,519\,360\,\delta)\ \#1^{8} +$
$(-11\,442\,734\,336 + 13\,166\,616\,320\,\delta)\ \#1^{9} + (-4\,004\,008\,800 + 4\,279\,124\,256\,\delta)\ \#1^{10} +$
$(-957\,471\,120 + 988\,551\,600\,\delta)\ \#1^{11} + (-151\,209\,603 + 153\,572\,073\,\delta)\ \#1^{12} +$
$(-14\,284\,624 + 14\,393\,200\,\delta)\ \#1^{13} + (-613\,317 + 615\,600\,\delta)\ \#1^{14}\ \&,\ 7]\},$

$\{k \to \mathrm{Root}[679\,477\,248 - 578\,813\,952\,\delta + (-5\,905\,580\,032 - 10\,703\,863\,808\,\delta)\ \#1 +$
$(308\,281\,344 - 12\,324\,962\,304\,\delta)\ \#1^{2} + (31\,993\,102\,336 + 13\,500\,416\,000\,\delta)\ \#1^{3} +$
$(51\,409\,256\,448 + 48\,649\,666\,560\,\delta)\ \#1^{4} + (30\,584\,733\,696 + 68\,457\,725\,952\,\delta)\ \#1^{5} +$
$(-6\,194\,479\,104 + 67\,198\,500\,864\,\delta)\ \#1^{6} + (-25\,093\,939\,200 + 50\,908\,323\,840\,\delta)\ \#1^{7} +$
$(-21\,969\,775\,872 + 29\,793\,519\,360\,\delta)\ \#1^{8} +$
$(-11\,442\,734\,336 + 13\,166\,616\,320\,\delta)\ \#1^{9} + (-4\,004\,008\,800 + 4\,279\,124\,256\,\delta)\ \#1^{10} +$
$(-957\,471\,120 + 988\,551\,600\,\delta)\ \#1^{11} + (-151\,209\,603 + 153\,572\,073\,\delta)\ \#1^{12} +$
$(-14\,284\,624 + 14\,393\,200\,\delta)\ \#1^{13} + (-613\,317 + 615\,600\,\delta)\ \#1^{14}\ \&,\ 8]\},$

$\{k \to \mathrm{Root}[679\,477\,248 - 578\,813\,952\,\delta + (-5\,905\,580\,032 - 10\,703\,863\,808\,\delta)\ \#1 +$
$(308\,281\,344 - 12\,324\,962\,304\,\delta)\ \#1^{2} + (31\,993\,102\,336 + 13\,500\,416\,000\,\delta)\ \#1^{3} +$
$(51\,409\,256\,448 + 48\,649\,666\,560\,\delta)\ \#1^{4} + (30\,584\,733\,696 + 68\,457\,725\,952\,\delta)\ \#1^{5} +$
$(-6\,194\,479\,104 + 67\,198\,500\,864\,\delta)\ \#1^{6} + (-25\,093\,939\,200 + 50\,908\,323\,840\,\delta)\ \#1^{7} +$
$(-21\,969\,775\,872 + 29\,793\,519\,360\,\delta)\ \#1^{8} +$
$(-11\,442\,734\,336 + 13\,166\,616\,320\,\delta)\ \#1^{9} + (-4\,004\,008\,800 + 4\,279\,124\,256\,\delta)\ \#1^{10} +$
$(-957\,471\,120 + 988\,551\,600\,\delta)\ \#1^{11} + (-151\,209\,603 + 153\,572\,073\,\delta)\ \#1^{12} +$
$(-14\,284\,624 + 14\,393\,200\,\delta)\ \#1^{13} + (-613\,317 + 615\,600\,\delta)\ \#1^{14}\ \&,\ 9]\},$

$\{k \to \mathrm{Root}[679\,477\,248 - 578\,813\,952\,\delta + (-5\,905\,580\,032 - 10\,703\,863\,808\,\delta)\ \#1 +$
$(308\,281\,344 - 12\,324\,962\,304\,\delta)\ \#1^{2} + (31\,993\,102\,336 + 13\,500\,416\,000\,\delta)\ \#1^{3} +$
$(51\,409\,256\,448 + 48\,649\,666\,560\,\delta)\ \#1^{4} + (30\,584\,733\,696 + 68\,457\,725\,952\,\delta)\ \#1^{5} +$
$(-6\,194\,479\,104 + 67\,198\,500\,864\,\delta)\ \#1^{6} + (-25\,093\,939\,200 + 50\,908\,323\,840\,\delta)\ \#1^{7} +$
$(-21\,969\,775\,872 + 29\,793\,519\,360\,\delta)\ \#1^{8} +$
$(-11\,442\,734\,336 + 13\,166\,616\,320\,\delta)\ \#1^{9} + (-4\,004\,008\,800 + 4\,279\,124\,256\,\delta)\ \#1^{10} +$
$(-957\,471\,120 + 988\,551\,600\,\delta)\ \#1^{11} + (-151\,209\,603 + 153\,572\,073\,\delta)\ \#1^{12} +$
$(-14\,284\,624 + 14\,393\,200\,\delta)\ \#1^{13} + (-613\,317 + 615\,600\,\delta)\ \#1^{14}\ \&,\ 10]\},$

$\{k \to \mathrm{Root}[679\,477\,248 - 578\,813\,952\,\delta + (-5\,905\,580\,032 - 10\,703\,863\,808\,\delta)\ \#1 +$
$(308\,281\,344 - 12\,324\,962\,304\,\delta)\ \#1^{2} + (31\,993\,102\,336 + 13\,500\,416\,000\,\delta)\ \#1^{3} +$
$(51\,409\,256\,448 + 48\,649\,666\,560\,\delta)\ \#1^{4} + (30\,584\,733\,696 + 68\,457\,725\,952\,\delta)\ \#1^{5} +$
$(-6\,194\,479\,104 + 67\,198\,500\,864\,\delta)\ \#1^{6} + (-25\,093\,939\,200 + 50\,908\,323\,840\,\delta)\ \#1^{7} +$
$(-21\,969\,775\,872 + 29\,793\,519\,360\,\delta)\ \#1^{8} +$
$(-11\,442\,734\,336 + 13\,166\,616\,320\,\delta)\ \#1^{9} + (-4\,004\,008\,800 + 4\,279\,124\,256\,\delta)\ \#1^{10} +$
$(-957\,471\,120 + 988\,551\,600\,\delta)\ \#1^{11} + (-151\,209\,603 + 153\,572\,073\,\delta)\ \#1^{12} +$
$(-14\,284\,624 + 14\,393\,200\,\delta)\ \#1^{13} + (-613\,317 + 615\,600\,\delta)\ \#1^{14}\ \&,\ 11]\},$

$\{k \to \mathrm{Root}[679\,477\,248 - 578\,813\,952\,\delta + (-5\,905\,580\,032 - 10\,703\,863\,808\,\delta)\ \#1 +$
$(308\,281\,344 - 12\,324\,962\,304\,\delta)\ \#1^{2} + (31\,993\,102\,336 + 13\,500\,416\,000\,\delta)\ \#1^{3} +$
$(51\,409\,256\,448 + 48\,649\,666\,560\,\delta)\ \#1^{4} + (30\,584\,733\,696 + 68\,457\,725\,952\,\delta)\ \#1^{5} +$

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      (-6 194 479 104 + 67 198 500 864 δ) #1^6 + (-25 093 939 200 + 50 908 323 840 δ) #1^7 +
      (-21 969 775 872 + 29 793 519 360 δ) #1^8 +
      (-11 442 734 336 + 13 166 616 320 δ) #1^9 + (-4 004 008 800 + 4 279 124 256 δ) #1^10 +
      (-957 471 120 + 988 551 600 δ) #1^11 + (-151 209 603 + 153 572 073 δ) #1^12 +
      (-14 284 624 + 14 393 200 δ) #1^13 + (-613 317 + 615 600 δ) #1^14 &, 12]},
 {k → Root[679 477 248 - 578 813 952 δ + (-5 905 580 032 - 10 703 863 808 δ) #1 +
      (308 281 344 - 12 324 962 304 δ) #1^2 + (31 993 102 336 + 13 500 416 000 δ) #1^3 +
      (51 409 256 448 + 48 649 666 560 δ) #1^4 + (30 584 733 696 + 68 457 725 952 δ) #1^5 +
      (-6 194 479 104 + 67 198 500 864 δ) #1^6 + (-25 093 939 200 + 50 908 323 840 δ) #1^7 +
      (-21 969 775 872 + 29 793 519 360 δ) #1^8 +
      (-11 442 734 336 + 13 166 616 320 δ) #1^9 + (-4 004 008 800 + 4 279 124 256 δ) #1^10 +
      (-957 471 120 + 988 551 600 δ) #1^11 + (-151 209 603 + 153 572 073 δ) #1^12 +
      (-14 284 624 + 14 393 200 δ) #1^13 + (-613 317 + 615 600 δ) #1^14 &, 13]},
 {k → Root[679 477 248 - 578 813 952 δ + (-5 905 580 032 - 10 703 863 808 δ) #1 +
      (308 281 344 - 12 324 962 304 δ) #1^2 + (31 993 102 336 + 13 500 416 000 δ) #1^3 +
      (51 409 256 448 + 48 649 666 560 δ) #1^4 + (30 584 733 696 + 68 457 725 952 δ) #1^5 +
      (-6 194 479 104 + 67 198 500 864 δ) #1^6 + (-25 093 939 200 + 50 908 323 840 δ) #1^7 +
      (-21 969 775 872 + 29 793 519 360 δ) #1^8 +
      (-11 442 734 336 + 13 166 616 320 δ) #1^9 + (-4 004 008 800 + 4 279 124 256 δ) #1^10 +
      (-957 471 120 + 988 551 600 δ) #1^11 + (-151 209 603 + 153 572 073 δ) #1^12 +
      (-14 284 624 + 14 393 200 δ) #1^13 + (-613 317 + 615 600 δ) #1^14 &, 14]}}
```

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kstar = Part[k /. %, 2]
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Root[
 679 477 248 - 578 813 952 δ + (-5 905 580 032 - 10 703 863 808 δ) #1 + (308 281 344 - 12 324 962 304 δ) #1^2 +
   (31 993 102 336 + 13 500 416 000 δ) #1^3 + (51 409 256 448 + 48 649 666 560 δ) #1^4 +
   (30 584 733 696 + 68 457 725 952 δ) #1^5 + (-6 194 479 104 + 67 198 500 864 δ) #1^6 +
   (-25 093 939 200 + 50 908 323 840 δ) #1^7 + (-21 969 775 872 + 29 793 519 360 δ) #1^8 +
   (-11 442 734 336 + 13 166 616 320 δ) #1^9 + (-4 004 008 800 + 4 279 124 256 δ) #1^10 +
   (-957 471 120 + 988 551 600 δ) #1^11 + (-151 209 603 + 153 572 073 δ) #1^12 +
   (-14 284 624 + 14 393 200 δ) #1^13 + (-613 317 + 615 600 δ) #1^14 &, 2]
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Chapter 4: International trade in a Hotelling model of differentiated duopoly

4.1 Introduction

One of the central topics in international trade is the welfare effects of free trade under imperfect competition. The welfare effects of free trade under Cournot duopoly have been analysed by many literature, and they proved that losses from trade could happen. However, the analysis of welfare effect on another important case, Bertrand duopoly with product differentiation, has been rarely mentioned in the past, as it is expected to be similar to the Cournot duopoly case. Clarke and Collie (2003) analyse the welfare effects in the Bertrand duopoly model with differentiated products, and they prove that there are always gains from trade under both unilateral free trade and multilateral free trade. Nevertheless their results are sharply contradicted by Fujiwara (2009), who considers a Hotelling model of differentiated Bertrand duopoly, and proves that welfare under free trade is less than welfare under autarky for any transport cost, and there are always losses from trade. Then it would seem interesting to consider what exactly happens to the welfare in the Bertrand duopoly model with product differentiation, so this chapter re-examines the welfare effect of the Hotelling model of the spatial duopoly by constructing a product space between the trade costs and marginal disutility, which associated with product differentiation and concludes that there is a possibility of losses from trade.

Brander (1981) and Brander and Krugman (1983) are the first to analyse the welfare effects of trade in a Cournot duopoly with homogeneous products. Assuming segmented markets, they show that there might be losses from multilateral free trade when the initial transport cost is too high (close to the prohibitive level). This is due to the fact that the negative effect of waste in transport outweighs the positive effect of competition promoted by trade. When the transport cost is sufficiently low, on the other hand, there are always gains from trade. Brander and Krugman (1983) also show that there will be gains from trade provided there is free entry of firms. They conclude that the relationship between welfare of trade and the trade costs is in a U-shape under Cournot duopoly. Markusen (1981) assumes integrated market, and proves that there are gains from trade even no trade will actually occur. In his two-country model, outputs of both firms increase with a pro-competitive effect. In our model, pro-competitive effect exists in the competitive market, but the outputs could not change due to the structure of the Hotelling model, whereas the price of the home firm is reduced so that foreign firm could not enter the home market.

Clarke and Collie (2003) introduce Bertrand duopoly with differentiated product to re-analyse the welfare gains/losses by using a two country Bertrand duopoly model with linear demands and constant marginal costs to demonstrate that welfare in trade is always larger than welfare in autarky for any transport cost. The reason is that the increase of the consumer surplus caused by a lower price of the home firm, and the positive effect of a wider variety of differentiated products overweigh a fall in the profits of the home firm, resulting gains from trade in the unilateral free trade for any transport cost. Since welfare under multilateral free trade is the welfare under unilateral free trade plus the profits of the export to the foreign country by the home firm, there must also be gains from trade.

Fujiwara (2009) reconsiders the result of Clarke and Collie (2003) in a Hotelling model of differentiated Bertrand duopoly, and his conclusion sharply contrasts to theirs. He proves that welfare under free trade is less than welfare under autarky for any transport cost, and there are always losses from trade. There might be a few reasons behind the difference between these two papers, one is that he ignores the kinked-demand structure in the spatial model[26]. In fact, this structure is often overlooked in many literature, due to the complexity arises when calculating the range of the prices in equilibria. Another reason is that he has made an unrealistic assumption: $a-c>4\tau$[27], so that the prohibitive trade costs of pro-competitive effect is greater than the prohibitive trade costs of autarkic equilibrium. These two points might lead to the incorrect result of the welfare analysis: losses from trade are everywhere.

This chapter builds upon the welfare analysis of Clarke and Collie (2003) and Fujiwara (2009), and adopts the Hotelling's model of spatial duopoly to analyse the welfare under both unilateral free trade and multilateral free trade as well as gains

[26] The Kinked demand structure is not an assumption, and exists in the Hotelling model

[27] Refer to Fujiwara K.(2009), where τ is marginal disutility

from trade. The innovation of the current chapter is: 1) It discusses the kinked demand market structure which is ignored in Fujiwara (2009), 2) it considers the pro-competitive effect without making any assumptions of the values of the factors so the results could be more comprehensive, and 3) It analyses the volume of the trade and how product differentiation affects the welfare and the trade volume. The results are losses are possible in the Bertrand duopoly model, which is similar to those in the Cournot duopoly model. So gains from trade occur when products are highly differentiated, and losses from trade occur when products are closer substitutes. In another word, welfare under free trade is less than welfare under autarky when the market is competitive; welfare under free trade is higher than welfare under autarky when the market is uncovered; welfare under free trade is the same as welfare under autarky when there is a pro-competitive effect; and finally welfare gain/loss is ambiguous when there is kink in demand.

To our knowledge, a spatial competition model of Hotelling (1929) type can provide a different implication of trade liberalisation and welfare analysis and captures very realistic consumer's and producer's features. The modified version of the linear model of Hotelling (1929) is used in this chapter instead of the circular road model of Salop (1979). There are three types of Hotelling models: 1) those both prices and locations are chosen, 2) those locations are fixed and price is chosen[28], and 3) those prices are fixed and locations are chosen. The second type of price competition will be focused on[29], so it is a Bertrand duopoly. MÃrel and Sexton (2010) uses the canonical Hotelling duopoly model to investigate the existence and characteristics of asymmetric kinked-demand equilibria, and characterized the continuum of asymmetric kinked-demand equilibria. The current paper has adopted the similar method to derive the kinked-demand equilibria, but assuming firms incur asymmetric costs: foreign firm bear the trade costs. Troncoso-Valverde (November 2004) allow firms to choose the degree of purposeness of the product, i.e. the transportation cost coefficient, before compete in prices.

Another important aspect of international trade is to look at the relationship between the volume of trade and the degree of product differentiation. Bernhofen (2001)

[28] See Tirole, J. (1988)
[29] The assumption of exogenous location can be very restrictive, as in Fujiwara, K. (2009)

integrates the product expansion and import competition aspects of trade into an oligopoly model, and shows that the volume of trade is increasing in the degree of differentiation under Bertrand oligopoly and Cournot oligopoly by using Bowley demand function. Collie and Le (2010) uses Shubik-levitan demand function to avoid market expansion effect from product differentiation, and they find that the volume of trade is decreasing in the degree of product differentiation when the trade cost is low, and increasing in the degree of product differentiation when the trade cost is high for both Cournot and Bertrand oligopoly. Collie and Le (2011) analyses product differentiation, the volume of trade and profits under both Cournot and Bertrand Duopoly. The Hotelling model of Bertrand oligopoly developed in this chapter has the same advantage that an increase in product differentiation does not affect the size of the market, thus no market expansion effect[30]. The results are the volume of trade is increasing in the degree of differentiation when products are close substitutes, but decreasing in the degree of differentiation when products are sufficiently differentiated.

The model in this chapter assumes that trade is costly due to the transport costs of trade costs as it is referred, there is no free entry, marginal cost is constant, and markets are segmented. The contributions of this chapter are: firstly, it develops the Hotelling model of differentiated Bertrand duopoly in the international study, so there is no market expansion effect, meaning increasing the degree of product differentiation will not affect the market size. Secondly, this study completes the unanswered part of the Hotelling model: the kinked demand market structure in the trade, where the welfare effect is umbiguous. Thirdly, the best response functions of the Bertrand trade model are not symmetric as the traditional Bertrand Hotelling model stated, and this asymmetry is caused by the trade cost associated with the foreign export.

[30] Intuitively, market size and trade affect the toughness of competition in the market, which then influence the producer and exporter in that market. Thus average markups and productivety respond to market size and its intergration through trade. Larger markets exhibit higher level of product differentiation. Thus an increase in product differentiation would increase the size of the market and that would influence the volume of trade too. We try to discuss the relationship between product differentiation and trade volume, so it is better not to consider the effect of market size on trade volume. Then it is important to analyse trade volume in a model in which the degree of product differentiation does not affect the size of the market.

This chapter is organised as follows. Section 4.2 presents a basic Hotelling model, characterizes autarkic equilibria, derives reaction functions and then trade equilibria, Section 4.3 considers the welfare effects of free trade, section 4.4 illustrates the gains from trade proposition, section 4.5 and section 4.6 demonstrate the volume of trade and how trade liberalisation affect profits, and section 4.7 concludes this chapter.

4.2 The model

Consider a Hotelling's linear city model where goods are horizontally differentiated. Suppose there are two firms (one and two) allocated in two countries (A and B). Country A is the home country where firm one is based, and country B is the foreign country where firm two is based. In each country, there is a continuum of consumers uniformly distributed along a one-dimensional space of product characteristics, defined by the unit length interval in $[0,1]$, and each of them wanted to buy one unit of the product or none. The disutility from consuming a variety other than one's ideal variety is in the distance alone this interval.

In product space, the two firms have the same constant marginal cost of production. They are located at the two ends of the unit segment, where home firm one is at point 0, and foreign firm two is at point 1, and these locations are exogenously determined. Thus x is the distance between consumer and firm one in the home market, whereas $(1-x)$ is the distance to firm two in the home market. Consequently, the consumer's utility is $a-p_1-\sigma x$ if buying from firm one, or $a-p_2-\sigma(1-x)$ if buying from firm two, where a is the reservation price that is large enough for consumer to buy a unit of the product[31], $p_1\left(p_2\right)$ is the price of the good produced by firm one (firm two), and σ denotes the marginal disutility per unit of distance. Then σx is disutility of the distance between the chosen product and consumer x's ideal product.[32] Notice that the distance in this model is measured in a product space rather than in a physical distance. This model is symmetric and the markets are assumed to be segmented, thus it is efficient enough by only looking at home market's economy. In addition, the

[31] assuming $a \geq c$

[32] Here we do not interpret σ as the usual transportation costs as it does in large Literature, because it will cause confusion between consumer's disutility from consuming different good and the trade costs associated with export.

products are differentiated more for consumers when σ is higher. the degree of product differentiation is inversely related to the intensity of competition among firms. When σ increases, competition between the two firms falls, and when it becomes zero, all consumers can go to either firm for the same cost zero in the absence of product differentiation.

4.2.1 An Autarkic equilibrium

Under autarky, each imperfectly competitive firm acts as a monopoly in its own country and faces no competition. There are two cases for autarky in the Hotelling model: 1) the firm covers the market partially, i.e. $x<1$, and 2) the firm covers the entire market i.e. $x_1=1$. For the first case, any consumer at location x is willing to buy the product from firm one if $a-p_1-\sigma x\geq 0$, so the aggregate demand is determined under $a-p_1-\sigma x=0$, leading to $x=(a-p_1)/\sigma$. Firm one's profit under autarky is $(p_1-c)x=(p_1-c)(a-p_1)/\sigma$. The usual profit maximisation yields $x^{A1}=(a-c)/2\sigma$, $p^{A1}=\dfrac{a+c}{2}$, and $\pi^{A1}=\dfrac{(a-c)^2}{4\sigma}$. However, the market is not completely covered by firm one, so $x^{A1}=(a-c)/2\sigma\leq 1$, meaning $\Sigma\equiv\dfrac{\sigma}{a-c}\geq 0.5$. The subscript $A1$ denotes autarky case one when the market is covered partially.

$$x^{A1}=(a-c)/2\sigma, \quad p^{A1}=\frac{a+c}{2}, \quad \pi^{A1}=\frac{(a-c)^2}{4\sigma} \quad \text{if} \quad \Sigma\geq 0.5 \tag{84}$$

In addition, consumer surplus when $\Sigma\geq 0.5$ can be expressed as:

$$CS^{A1}=\int_0^{x^{A1}}\left(a-p^{A1}-\sigma x\right)dx=\frac{(a-c)^2}{8\sigma}$$

The welfare of the home market under autarky is the sum of consumer surplus and the profits of the home firm,

$$W^{A1} = CS^{A1} + \pi^{A1} = \frac{(a-c)^2}{8\sigma} + \frac{(a-c)^2}{4\sigma} = \frac{3(a-c)^2}{8\sigma} \tag{85}$$

The second case occurs when $\Sigma \equiv \frac{\sigma}{a-c} < 0.5$, firm one is supplying the whole market, i.e. $x^{A2} = 1$, and the price is $p^{A2} = a - \sigma$. Firm one's profits under autarky are therefore $\pi^{A2} = \left(p^{A2} - c\right)x^{A2} = a - c - \sigma$. The superscript $A2$ denotes autarky case two when the market is entirely covered. In both cases, firms two's profits, market share and outputs in the home market are zero as shown in Table 4-1 in the appendix.

$$x^{A2} = 1, \quad p^{A2} = a - \sigma, \quad \pi^{A2} = a - c - \sigma \quad \text{if} \quad \Sigma < 0.5 \tag{86}$$

The consumer surplus of the home country under autarky when $\Sigma < 0.5$ is:

$$CS^{A2} = \int_0^1 \left(a - p^{A2} - \sigma x\right) dx = \frac{\sigma}{2}$$

The welfare of the home market is equal to consumer surplus CS^{A2} plus the profits of the home firm under autarky case two, i.e. $\Sigma < 0.5$, and the market is entirely covered.

$$W^{A2} = CS^{A2} + \pi^{A2} = \frac{\sigma}{2} + \left(a - c - \sigma\right) = a - c - \frac{\sigma}{2} \tag{87}$$

This welfare analysis under autarky provides the benchmark for gains from trade in section 4.4. The Equilibria under trade will be analysed in the subsequent section.

4.2.2 Trade Equilibria

Since consumers only purchase the good if the utility generated from the consumption is greater or equal to zero, the utility function is given by:

$$U = \begin{cases} a - p_1 - \sigma x_1 & \text{if buying from firm one;} \\ a - p_2 - \sigma x_2 & \text{if buying from firm two;} \\ 0 & \text{if not buying at all} \end{cases} \tag{88}$$

Where x_1 is the consumer's distance to firm one, and x_2 is his/her distance to firm two. These distances, that are equal to the number of consumers in the market, could be treated as the demand for both goods. If prices and disutility of the distance are not too large, everyone buys from the firm that offers cheaper goods, so consumers from the x_1 segment buy from firm one, and consumers from x_2 segment buy from firm two.

Competitive market:

When there is a consumer who is indifferent between buying from firm one and firm two, the market is completely covered, and this post trade home market is a duopoly by home firm one and foreign firm two, referred as the competitive market in this chapter. If the market is fully covered, then $x_2 = 1 - x_1$, consumer who is indifferent from buying at either firm is determined by $a - p_1 - \sigma x_1 = a - p_2 - \sigma(1 - x_1)$. Solving for x_1, the demands for each good are:

$$\begin{cases} x_1 = \dfrac{p_2 - p_1 + \sigma}{2\sigma} \\ x_2 = \dfrac{p_1 - p_2 + \sigma}{2\sigma} \end{cases} \tag{89}$$

Assuming firm two chooses to export to country A (home market), recall that demand for foreign goods is represented by the number of consumers located on the x_2 segment. Export will incur a per-unit trade cost $k > 0$ under free trade. So each firm's profit is defined as follows:

$$\begin{gathered} \pi_{1A}^E = (p_1 - c)x_1 = \frac{(p_1 - c)(p_2 - p_1 + \sigma)}{2\sigma} \\ \pi_{2A}^E = (p_2 - c - k)x_2 = \frac{(p_2 - c - k)(p_1 - p_2 + \sigma)}{2\sigma} \end{gathered} \tag{90}$$

The home market is fully covered, the Bertrand Nash equilibrium profits, prices and outputs are solved by $\partial\pi_{1A}^E / \partial p_1 = 0$, $\partial\pi_{2A}^E / \partial p_2 = 0$,

$$p_{1A}^{E} = \frac{3c + 3\sigma + k}{3},\ p_{2A}^{E} = \frac{3c + 3\sigma + 2k}{3}$$

$$x_{1A}^{E} = \frac{3\sigma + k}{6\sigma},\ x_{2A}^{E} = \frac{3\sigma - k}{6\sigma} \tag{91}$$

$$\pi_{1A}^{E} = \frac{\left(3\sigma + k\right)^2}{18\sigma},\ \pi_{2A}^{E} = \frac{\left(3\sigma - k\right)^2}{18\sigma}$$

For country A variables, the superscript E denotes that firm two is exporting to country A. The prohibitive trade cost happens when $x_{2A}^{E} = 0$, that is $x_{2A}^{E} = \frac{3\sigma - k}{6\sigma} = 0$, so $\bar{k} = 3\sigma$. The prohibitive trade cost is increasing in the degree of product differentiation, so trade is more likely to happen when products are more differentiated[33]. After this point, the export from firm two to country A will be zero, and firm one will act as a monopolist in the home market, yet still facing the competition from foreign firm two, which only happens in Bertrand competition as mentioned in Chapter 1, known as the pro-competitive effect, and it will be explained in details in the next section. Furthermore, the consumer at point x_1 must obtain a positive utility:

$$U = a - p_1 - \sigma x_1 = \frac{2a - 2c - 3\sigma - k}{2} > 0 \tag{92}$$

That is: $k < 2\left(a - c\right) - 3\sigma$ must hold.

Local Monopoly case (uncovered market):

If prices and disutility of the distance are too high, some consumers on the unit segment might choose not to buy any goods at all. In this case, home market is not completely covered, and firms do not compete with each other in this market, as each firm is a monopolist that has its own market share, and it is not affected by the other firm's strategy. This is referred as Local monopoly.

[33] Because consumers can enjoy a wider varieties of goods.

A consumer who is indifferent between buying from firm one and firm two does not exist, so $x_1 + x_2 < 1$. Any customers between 0 and x_1 are willing to buy the good from the monopolist firm one, so the demand function that firm one faces is given by equating $U = a - p_1 - \sigma x_1 = 0$, hence $x_1 = \frac{a - p_1}{\sigma}$, and the same argument applies for $x_2 = \frac{a - p_2}{\sigma}$. If foreign firm two chooses to export to home country A, the profit for each firm is:

$$\pi_{1A}^{E} = (p_1 - c)x_1 = \frac{(p_1 - c)(a - p_1)}{\sigma}$$
$$\pi_{2A}^{E} = (p_2 - c - k)x_2 = \frac{(p_2 - c - k)(a - p_2)}{\sigma} \tag{93}$$

Where x_2 is not equal to $(1 - x_1)$, as the market is not completely covered. The equilibrium prices, profits and outputs are solved by $\partial\pi_{1A}^{E} / \partial p_1 = 0$, $\partial\pi_{2A}^{E} / \partial p_2 = 0$, and denoting $x_{2A}^{E} = x_2$:

$$p_{1A}^{E} = \frac{a + c}{2} \quad , \quad p_{2A}^{E} = \frac{a + c + k}{2}$$
$$x_{1A}^{E} = \frac{a - c}{2\sigma} \quad , \quad x_{2A}^{E} = \frac{a - c - k}{2\sigma} > 0 \tag{94}$$
$$\pi_{1A}^{E} = \frac{(a - c)^2}{4\sigma} \quad , \quad \pi_{2A}^{E} = \frac{(a - c - k)^2}{4\sigma}$$

By looking at x_{1A} and x_{2A}, only if $k \le a - c$, and $x_{1A} + x_{2A} < 1$, both firms supply the home market, so the prohibitive trade cost is $k = \tilde{k} = a - c$. Nevertheless, the market is not completely covered, to satisfy this condition, the total demand in the home market is less than one unit. That is,

$$x_{1A} + x_{2A} = \frac{2a - 2c - k}{2\sigma} < 1$$

That is, $k > 2(a-c) - 2\sigma$ must hold. The question arises here: what happens in the home market when the trade cost is between $\hat{k}$ and $\tilde{k}$? There is a kink in the demand structure in this range, so market is still completely covered, but unlike the competitive market, the gains from purchasing the goods from both firms become zero. The details of kinked demand will be discussed in the next section.

If trade costs and the marginal disutility are normalized by $\mathrm{K} \equiv \dfrac{k}{a-c}$, and $\Sigma \equiv \dfrac{\sigma}{a-c}$, these three types of market structure in the home country are separated as follows:

$$\begin{cases} \mathrm{K} \le 2 - 3\Sigma & \text{Competitive market} \\ 2 - 3\Sigma < \mathrm{K} \le 2 - 2\Sigma & \text{Kink in demand} \\ \mathrm{K} > 2 - 2\Sigma & \text{Local monopoly} \end{cases} \tag{95}$$

The equilibrium prices, outputs and profits under the competitive market and the local monopoly have been derived in (91) and (94) from the trade point of view. However, the equilibria under the kink in demand are unknown, and have not been analysed in many literature. This could be discovered from Hotelling model in the following sections. As complication arises when considering the best response functions in each case, and the best response function for each firm is made up by two or three segments, equilibria is therefore, determined by the intersection of the two correspondences (p_1 crosses p_2). The following sections will go through the demand functions, the best reaction functions and the equilibria in the competitive market, kink in demand and local monopoly.

4.2.3 The Nash equilibria

In general, there are two cases of the demand functions of the Hotelling model: firstly, when the prices are low, the market is completely covered, so there exists a consumer who is indifferent between purchasing from firm one or firm two. Suppose such a consumer is allocated at x_1 as shown previously, i.e. $a - p_1 - \sigma x_1 = a - p_2 - \sigma(1 - x_1)$, then $x_1 = \dfrac{p_2 - p_1 + \sigma}{2\sigma}$. This was competitive market mentioned above, thus this consumer must also obtain positive utility from purchasing from either firm, which means that[34] $p_1 \le 2a - \sigma - p_2$. In addition, the constraint that $0 \le x_1 \le 1$ implies that[35] $p_1 \le p_2 + \sigma$ and $p_1 \ge p_2 - \sigma$. The demand for firm one's product is then x_1 shown as in the first equation in (96).

Secondly, when the prices are high, the market is not completely covered and an indifferent consumer does not exist, referred to as the local monopoly[36]. As an indifferent customer does not exist in such a market structure, the consumer located at the market boundary of firm one, which happens at the point when $U = a - p_1 - \sigma x_1 = 0$ (i.e. $x_1 = \dfrac{a - p_1}{\sigma}$), would not purchase from firm two, i.e. $p_1 > 2a - \sigma - p_2$. The demand for firm one is summarized as:

$$x_1(p_1, p_2, a, \sigma) = \begin{cases} \dfrac{p_2 - p_1 + \sigma}{2\sigma} & if \quad p_2 - \sigma \le p_1 \le \min\{p_2 + \sigma, 2a - p_2 - \sigma\} \\ \dfrac{a - p_1}{\sigma} & if \quad 2a - p_2 - \sigma < p_1 \le \min\{a, p_2 + \sigma\} \end{cases} \tag{96}$$

Where the first equation represents the competitive market, and the second equation represents a monopoly. There are two possibilities with the ranges of price p_1. One is: $p_2 + \sigma \ge 2a - p_2 - \sigma$, i.e. $p_2 + \sigma \ge a$, then the demand function x_1 has a kink structure at $p_1 = 2a - \sigma - p_2$,

$$x_1(p_1, p_2, a, \sigma) = \begin{cases} \dfrac{p_2 - p_1 + \sigma}{2\sigma} & if \quad p_2 - \sigma \le p_1 \le 2a - p_2 - \sigma \\ \dfrac{a - p_1}{\sigma} & if \quad 2a - p_2 - \sigma < p_1 \le a \end{cases} \tag{97}$$

And the other one is: if $p_2 + \sigma < 2a - p_2 - \sigma$, i.e. $p_2 + \sigma < a$ and then $x_1 = \dfrac{p_2 - p_1 + \sigma}{2\sigma}$ is on the relevant range, i.e.

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[34] Substitute the location of the indifferent consumer x_1 into the utility function $U = a - p_1 - \sigma x_1 \ge 0$

[35] So the indifferent customer is between the unit segment end points: $0 \le \dfrac{p_2 - p_1 + \sigma}{2\sigma} \le 1$

[36] Assume the case that one firm captures the whole market is ruled out, each firm must obtain some of the market share.

$$x_1(p_1, p_2, a, \sigma) = \frac{p_2 - p_1 + \sigma}{2\sigma} \quad if \quad p_2 - \sigma \le p_1 \le p_2 + \sigma \tag{98}$$

and the monopoly case does not exist. In all cases, x_1 is continuous in p_1. The demand function is kinked for values of p_2 such that $p_2 + \sigma \ge a$. Firms' best response functions are derived in Appendix A, and that resulting the different expressions of the reaction curves listed in the end of Appendix A. As the Nash Equilibria (p_1, p_2) are determined by the intersection of the reaction curves p_1^* and p_2^*, we have the following situations[37]:

1. $\Sigma \le \frac{1}{4} - \frac{1}{4}\mathrm{K}$: In this case, $c < c + 3\sigma < c + k + 3\sigma < a - \sigma < a$, the value of $\frac{3}{2}a$ denote $\frac{3}{2}a - \frac{c}{2} - \sigma$ or $\frac{3}{2}a - \frac{c+k}{2} - \sigma$ and, $\frac{4}{3}a$ denote $\frac{4}{3}a - \frac{c}{3} - \sigma$ or $\frac{4}{3}a - \frac{c+k}{3} - \sigma$, are above a. The reaction function $p_1^*(p_2^*)$ consists of two upward sloping segments on $[c, a]$, it is increasing with slope $1/2$ on $[c, c+3\sigma]$ or $[c, c+k+3\sigma]$ for p_2^*, while it is increasing with slope 1 on $[c+3\sigma, a]$ or $[c+k+3\sigma, a]$ for p_2^*. The break point is at $p_2 = c + 3\sigma$ for p_1^* and $p_1 = c + k + 3\sigma$ for p_2^*. There is a unique equilibrium located on the segment $[c, c+3\sigma]$ at the intersection point $(p_1^*, p_2^*) = \left(c + \sigma + \frac{k}{3}, c + \sigma + \frac{2}{3}k\right)$ (see figure A1).

[37] All cases are distinguished by the term $a - c$ and k.

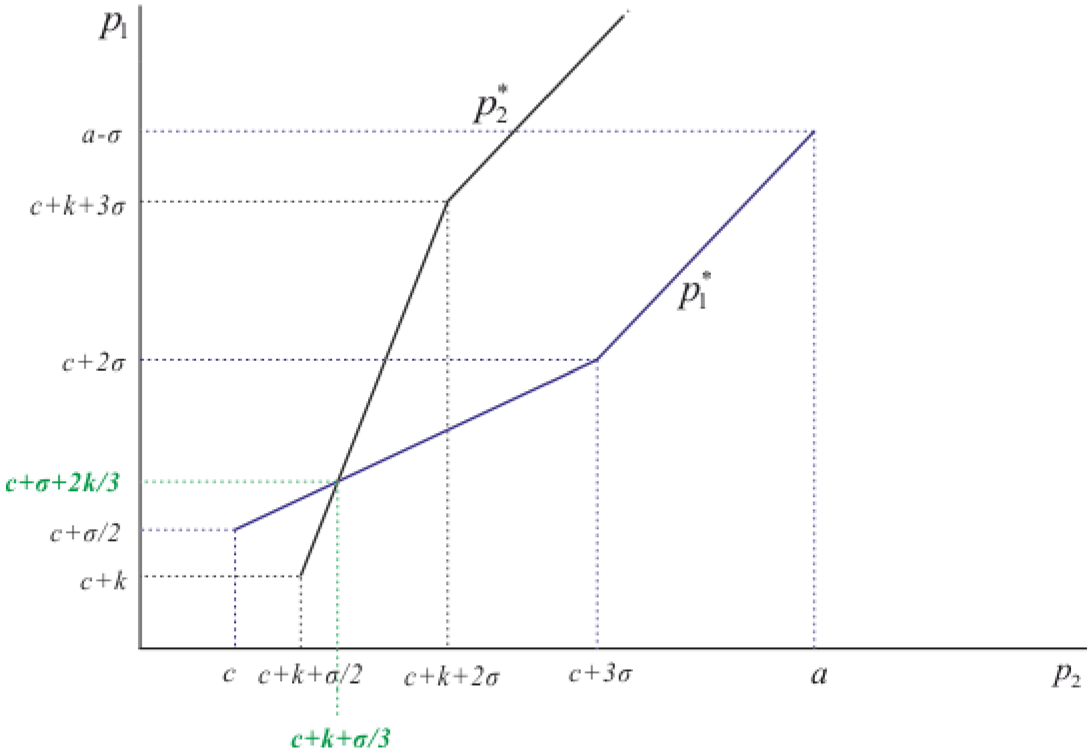

Figure A 1: The best response correspondence when $\Sigma \leq \frac{1}{3} - \frac{1}{3}K$

The first part of the best response function represents the competitive market portion with a slope of $1/2$, so the demand and the price depend on the price charged by the other firm. In this range, a decrease in price p_1 by Δ will lead to a rise in sales by $\Delta/2\sigma$. A decrease in price p_2 by Δ will lead to a reduction in price p_1 by $\Delta/2$ and a reduction in sales by $\Delta/2\sigma$. The second part of the best response function is upward sloping with a slope of 1, which corresponds to the competitive market with pro-competitive effect. So the firm dominates the market but facing the competition from the other firm. A reduction in price p_2 by Δ will reduce the price of p_1 by Δ.

2. $\frac{1}{4} - \frac{1}{4}K < \Sigma \leq \frac{1}{4}$: This case is similar to the above case in the sense that $a - \sigma$ is between $c + 3\sigma$ and $c + k + 3\sigma$. The reaction functions and the unique equilibrium are the same (see figure A1).

3. $\frac{1}{4} < \Sigma \le \frac{1}{3} - \frac{1}{3}\mathrm{K}$: This case is similar to the above case in the sense that $a-\sigma$ is between $c+k$ and $c+3\sigma$. The reaction functions and the unique equilibrium are the same (see figure A1).

4. $\frac{1}{3} - \frac{1}{3}\mathrm{K} < \Sigma \le \frac{1}{3}$: Now a is between $c+3\sigma$ and $c+k+3\sigma$, so $c+k+3\sigma$ is no longer on the segment $[c,a]$, but $\frac{4}{3}a - \frac{c+k}{3} - \sigma$ is. The reaction function for p_1^* has not changed, but the one for p_2^* has, the relevant range becomes $c < c+k < c+3\sigma < \frac{4}{3}a - \frac{c+k}{3} - \sigma < a$ (with $a-\sigma$ on the segment $[c,a]$). The reaction function p_2^* consists of two segments on $[c,a]$, it is increasing with slope $1/2$ on $\left[c+k, \frac{4}{3}a - \frac{c+k}{3} - \sigma\right]$, while it is decreasing with slope –1 on $\left[\frac{4}{3}a - \frac{c+k}{3} - \sigma, a\right]$. Two reaction functions will intersect at on the increasing part with slope of $1/2$. Therefore there is a unique equilibrium $\left(p_1^*, p_2^*\right) = \left(c+\sigma+\frac{k}{3}, c+\sigma+\frac{2}{3}k\right)$ on this range (see figure A2).

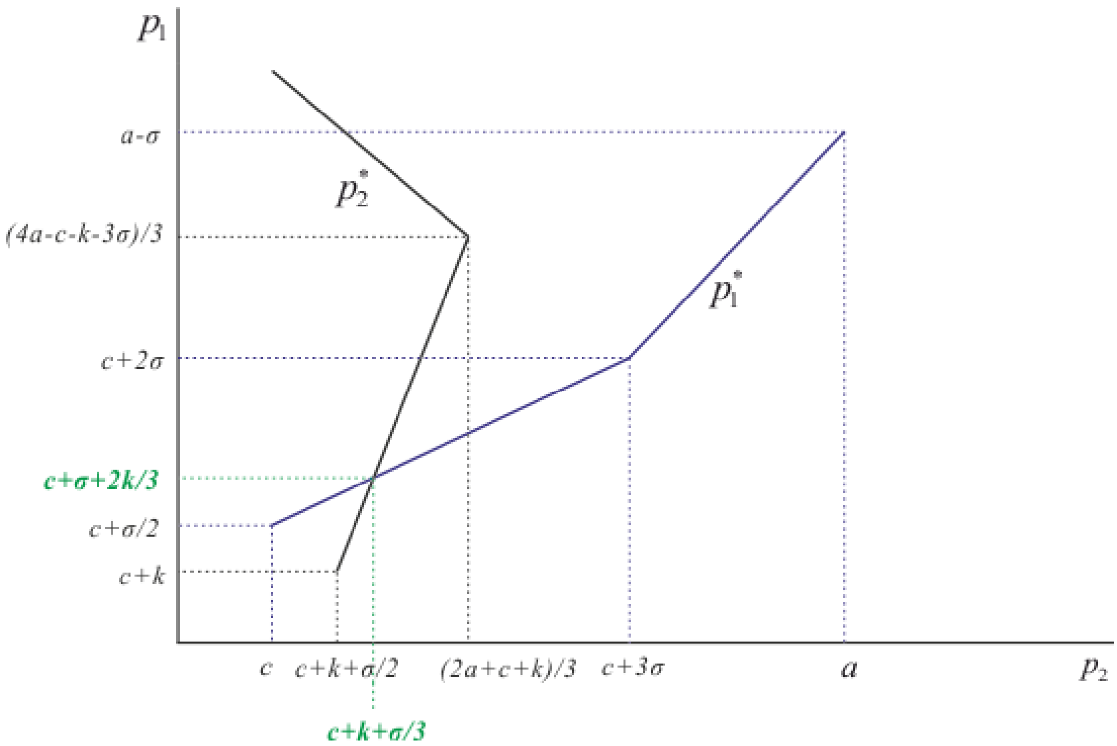

Figure A 2: The best response correspondence when $\frac{1}{3}-\frac{1}{3}K<\Sigma\leq\frac{1}{3}$

Firm one's best response curve is the same, but the second part of the best response curve for firm two is downward sloping with the slope of -1. This range represents the kinked demand portion of the market, where the marginal revenue of the firm is discontinued, so the price of firm two, given a range of values of p_1, is set at the kink of the demand function. That is to say: If firm one increases its price p_1, firm two would decrease its price p_2 just enough to cover the market. This is the reason for the hump-shaped best response curve.

5. $\frac{1}{3}<\Sigma\leq\frac{1}{2}-\frac{1}{2}\mathrm{K}$: Now a is greater than $\frac{4}{3}a-\frac{c}{3}-\sigma$, and $c+3\sigma$ is no longer on the segment $[c,a]$, and the relevant range is $c<c+k<\frac{4}{3}a-\frac{c+k}{3}-\sigma<\frac{4}{3}a-\frac{c}{3}-\sigma<a$ (with $a-\sigma$ on the segment $[c,a]$). Both reaction functions will increase with slope of $1/2$ on $\left[c,\frac{4}{3}a-\frac{c}{3}-\sigma\right]$ for p_1^* and $\left[c+k,\frac{4}{3}a-\frac{c+k}{3}-\sigma\right]$ for p_2^*. They will intersect on the increasing part

if and only if the kink of the function p_1^* lies below the 45 degree line, that is, $p_1^*\left(\frac{4}{3}a-\frac{c}{3}-\sigma\right)<\frac{4}{3}a-\frac{c+k}{3}-\sigma$. This last condition is satisfied if $\sigma\le\frac{2}{3}(a-c)-\frac{k}{3}$, which holds on this range of parameter values. Therefore there is a unique equilibrium on this range as well, $\left(p_1^*,p_2^*\right)=\left(c+\sigma+\frac{k}{3},c+\sigma+\frac{2}{3}k\right)$ (see figure A3).

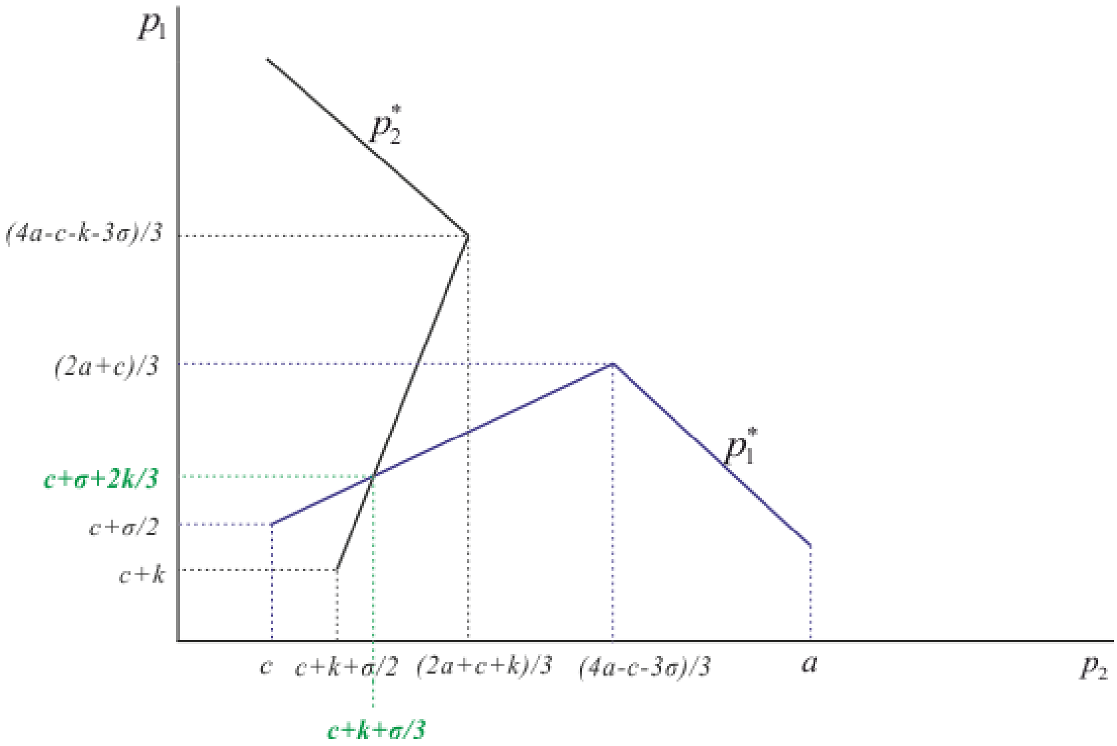

Figure A 3: The best response correspondence when $\frac{1}{3}<\Sigma\le\frac{1}{2}-\frac{1}{2}K$

6. $\frac{1}{2}-\frac{1}{2}\mathrm{K}<\Sigma\le\frac{1}{2}$: Now $\frac{3}{2}a-\frac{c+k}{2}-\sigma$ is on the segment $[c,a]$, and the relevant range is $c<c+k<\frac{4}{3}a-\frac{c+k}{3}-\sigma<\frac{4}{3}a-\frac{c}{3}-\sigma<\frac{3}{2}a-\frac{c+k}{2}-\sigma<a$ (with $a-\sigma$ on the segment $[c,a]$). The graph is similar to the above one, but the reaction function of p_2^* has three segments, it has slope $1/2$ on $\left[c+k,\frac{4}{3}a-\frac{c+k}{3}-\sigma\right]$,

slope -1 on $\left[\frac{4}{3}a-\frac{c+k}{3}-\sigma,\frac{3}{2}a-\frac{c+k}{2}-\sigma\right]$, and slope 0 on $\left[\frac{3}{2}a-\frac{c+k}{2}-\sigma,a\right]$. As $\sigma\leq\frac{2}{3}(a-c)-\frac{k}{3}$ is satisfied within this range, two reaction functions will intersect at on the increasing part with slope of $1/2$. Again there is a unique equilibrium $\left(p_1^*,p_2^*\right)=\left(c+\sigma+\frac{k}{3},c+\sigma+\frac{2}{3}k\right)$ on this range (see figure A4).

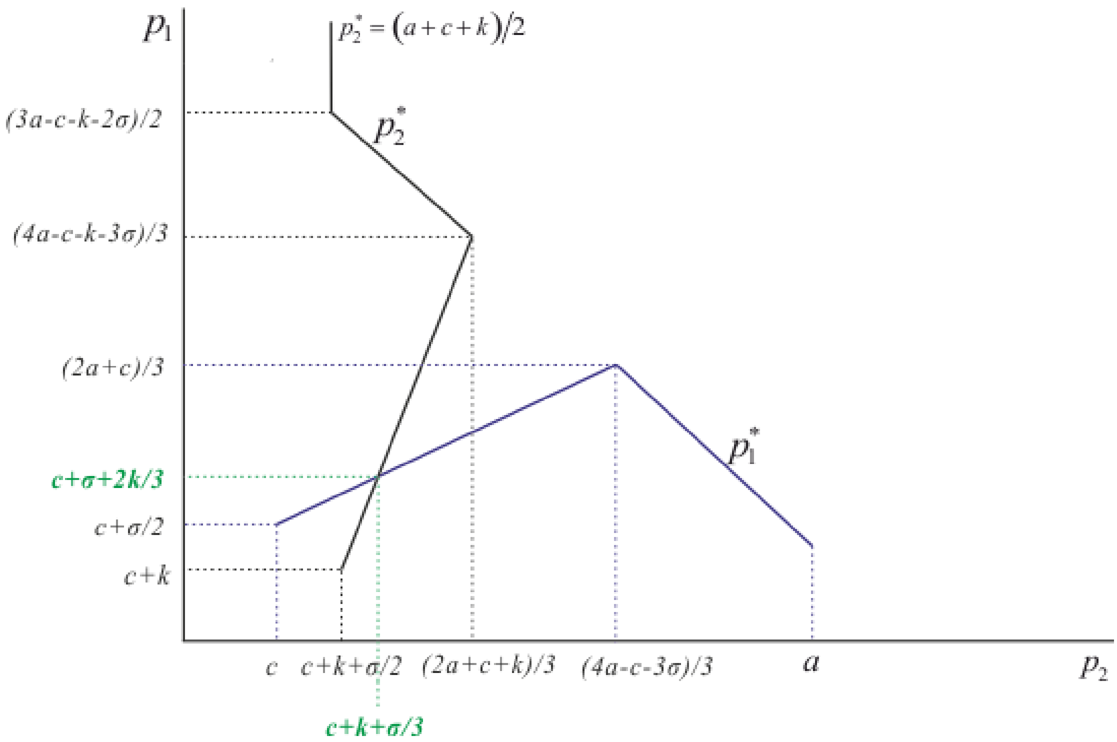

Figure A 4: The best response correspondence when $\frac{1}{2}-\frac{1}{2}K<\Sigma\leq\frac{1}{2}$

The third part of the reaction curve for firm two is a vertical line, corresponding to the local monopoly portion of the market. So the demand for good two only depends on the price p_2, and it is a monopoly demand. In this range, a decrease in price p_2 by Δ will lead to a rise in sales by Δ/σ, which is twice as much as that in the competitive market portion. The price of firm one has no effect on the demand or price of firm two.

7. $\frac{1}{2} < \Sigma \le 1$: Now $\frac{3}{2}a - \frac{c}{2} - \sigma$ is on the segment $\left[c, a\right]$, and the relevant range is $c < c + k < \frac{4}{3}a - \frac{c+k}{3} - \sigma < \frac{4}{3}a - \frac{c}{3} - \sigma < \frac{3}{2}a - \frac{c+k}{2} - \sigma < \frac{3}{2}a - \frac{c}{2} - \sigma < a$ (with $a - \sigma$ on the segment $\left[c, a\right]$). Compared with the previous case, the reaction function of p_1^* also has three segments, it has slope $1/2$ on $\left[c, \frac{4}{3}a - \frac{c}{3} - \sigma\right]$, slope -1 on $\left[\frac{4}{3}a - \frac{c}{3} - \sigma, \frac{3}{2}a - \frac{c}{2} - \sigma\right]$, and a flat line on $\left[\frac{3}{2}a - \frac{c}{2} - \sigma, a\right]$. It is known that when $\sigma \le \frac{2}{3}(a-c) - \frac{k}{3}$, the intersection of the two reaction functions will occur on the increasing part, with unique equilibrium $\left(p_1^*, p_2^*\right) = \left(c + \sigma + \frac{k}{3}, c + \sigma + \frac{2}{3}k\right)$. However, for values $\sigma > \frac{2}{3}(a-c) - \frac{k}{3}$, the reaction functions will intersect on their decreasing portion as long as the break point $\frac{3}{2}a - \frac{c+k}{2} - \sigma < \frac{a+c}{2}$, that is, $\sigma < \left(a-c\right) - \frac{k}{2}$. There are three subcases according to whether the segment with slope of -1 of the foreign reaction curve is above, within or below the segment of -1 of the home reaction curve (see figure A5 in the appendix). The critical value of the three cases depend on whether the value of $\frac{4}{3}a - \frac{c+k}{3} - \sigma$ exceeds the monopoly price $\frac{a+c}{2}$ for home firm and whether the value of $\frac{4}{3}a - \frac{c}{3} - \sigma$ exceeds the monopoly price $\frac{a+c+k}{2}$ for foreign firm [38]. Notice $\sigma < \left(a-c\right) - \frac{k}{2}$ is within this range, thus if $\sigma > \left(a-c\right) - \frac{k}{2}$, there is a unique equilibrium where two flat lines intersect. This leads to the following Equilibria:

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[38] If $\frac{4}{3}a - \frac{c+k}{3} - \sigma > \frac{a+c}{2}$, then $\sigma < \frac{5}{6}(a-c) - \frac{k}{3}$, and if $\frac{4}{3}a - \frac{c}{3} - \sigma > \frac{a+c+k}{2}$, then $\sigma < \frac{5}{6}(a-c) - \frac{k}{2}$. Therefore when the negative sloped segment of foreign reaction curve lies within the negative sloped segment of home reaction curve, it is case where $\frac{5}{6}(a-c) - \frac{k}{2} < \sigma \le \frac{5}{6}(a-c) - \frac{k}{3}$.

- If $\frac{1}{2} < \Sigma \le \frac{2}{3} - \frac{1}{3}\mathrm{K}$, there is a unique equilibrium $\left(p_1^*, p_2^*\right) = \left(c + \sigma + \frac{k}{3}, c + \sigma + \frac{2}{3}k\right)$ (see figure A5 below).
- If $\frac{2}{3} - \frac{1}{3}\mathrm{K} < \Sigma \le \frac{5}{6} - \frac{1}{2}\mathrm{K}$, there is an infinity of Nash Equilibria characterized by $p_1^* \in \left(\frac{2a+c}{3}, \frac{4}{3}a - \frac{c+k}{3} - \sigma\right)$ and $p_2^* \in \left(\frac{4}{3}a - \frac{c}{3} - \sigma, \frac{2a+c+k}{3}\right)$, and $p_1^* + p_2^* = 2a - \sigma$ (see figure A6 below).
- If $\frac{5}{6} - \frac{1}{2}\mathrm{K} < \Sigma \le \frac{5}{6} - \frac{1}{3}\mathrm{K}$, there is an infinity of Nash Equilibria characterized by $p_1^* \in \left(\frac{4}{3}a - \frac{c+k}{3} - \sigma, \frac{3}{2}a - \frac{c+k}{2} - \sigma\right)$ and $p_2^* \in \left(\frac{2a+c+k}{3}, \frac{a+c+k}{2}\right)$, and $p_1^* + p_2^* = 2a - \sigma$ (see figure A7 below).
- If $\frac{5}{6} - \frac{1}{3}\mathrm{K} < \Sigma \le 1 - \frac{1}{2}\mathrm{K}$, there is an infinity of Nash Equilibria characterized by $p_1^* \in \left(\frac{3}{2}a - \frac{c+k}{2} - \sigma, \frac{a+c}{2}\right)$ and $p_2^* \in \left(\frac{a+c+k}{2}, \frac{3}{2}a - \frac{c}{2} - \sigma\right)$, and $p_1^* + p_2^* = 2a - \sigma$ (see figure A8 below).
- If $1 - \frac{1}{2}\mathrm{K} < \Sigma \le 1$, there is a unique Nash Equilibria $\left(p_1^*, p_2^*\right) = \left(\frac{a+c}{2}, \frac{a+c+k}{2}\right)$ (see figure A9 below).

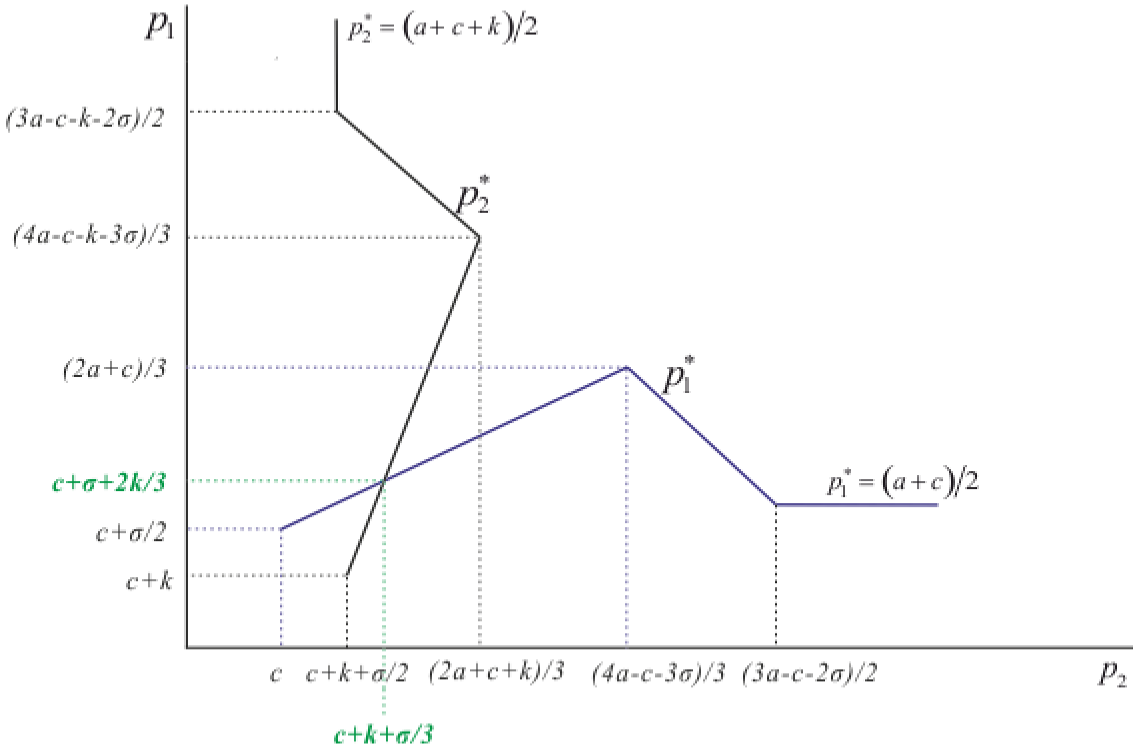

Figure A 5: The best response correspondence when $\frac{1}{2} < \Sigma \leq \frac{2}{3} - \frac{1}{3}K$

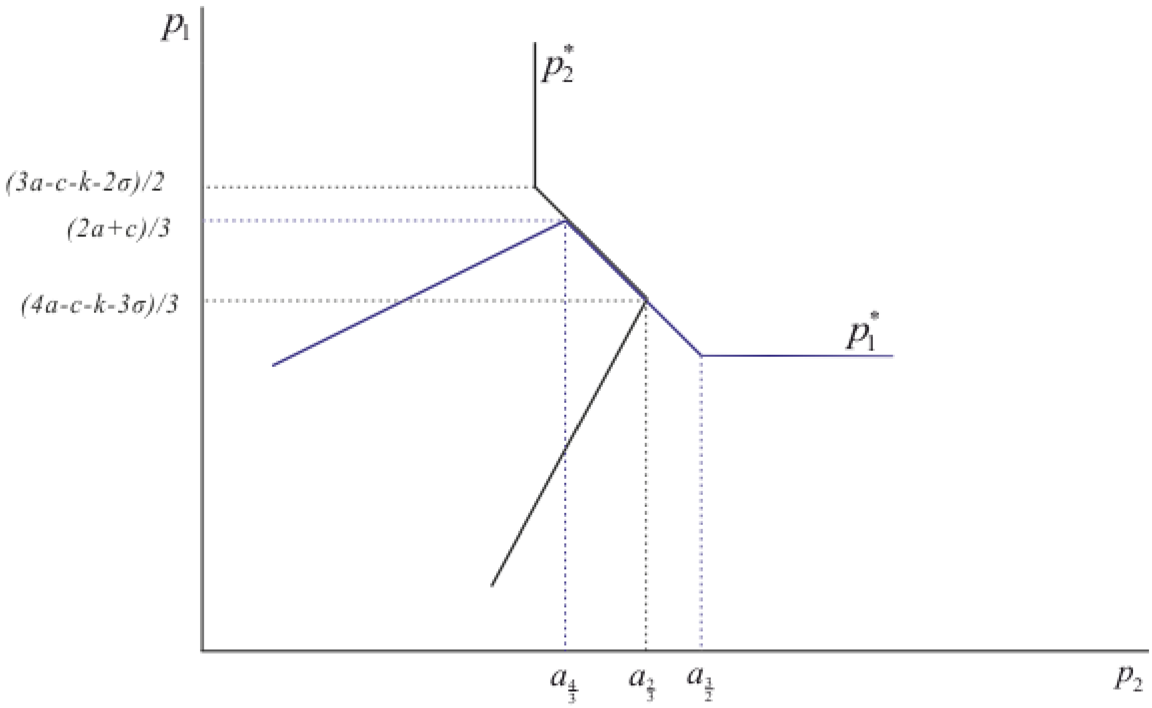

Figure A 6: The best response correspondence when $\frac{2}{3} - \frac{1}{3}K < \Sigma \leq \frac{5}{6} - \frac{1}{2}K$

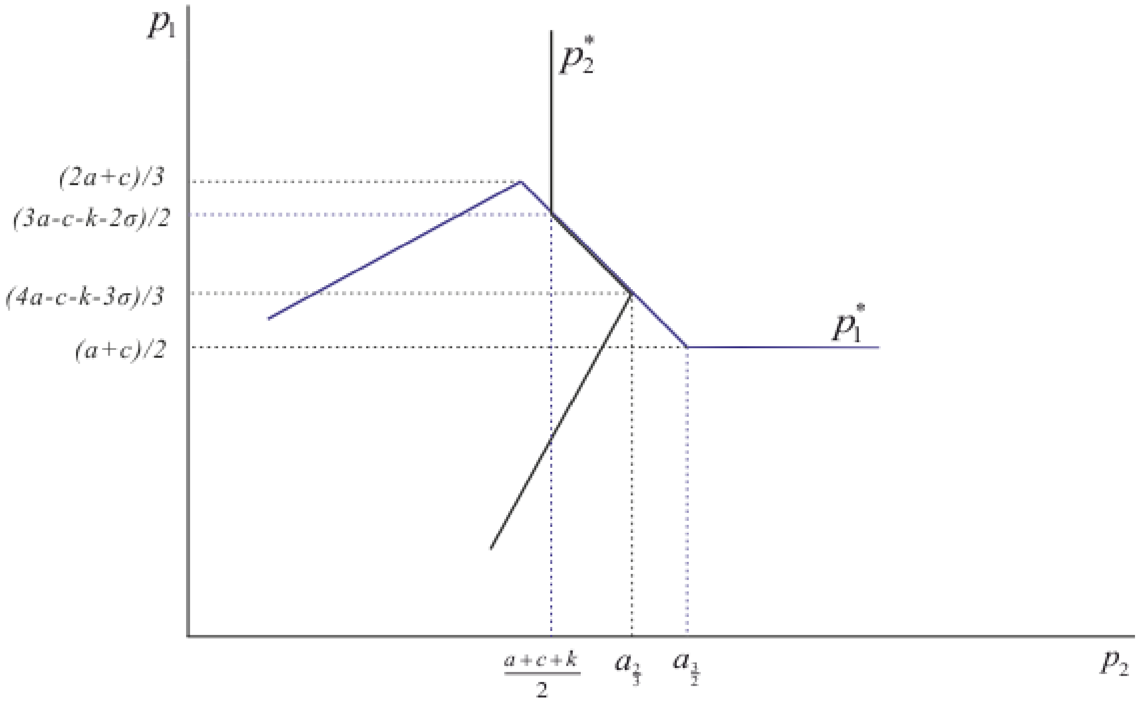

Figure A 7: The best response correspondence when $\frac{5}{6}-\frac{1}{2}K<\Sigma\le\frac{5}{6}-\frac{1}{3}K$

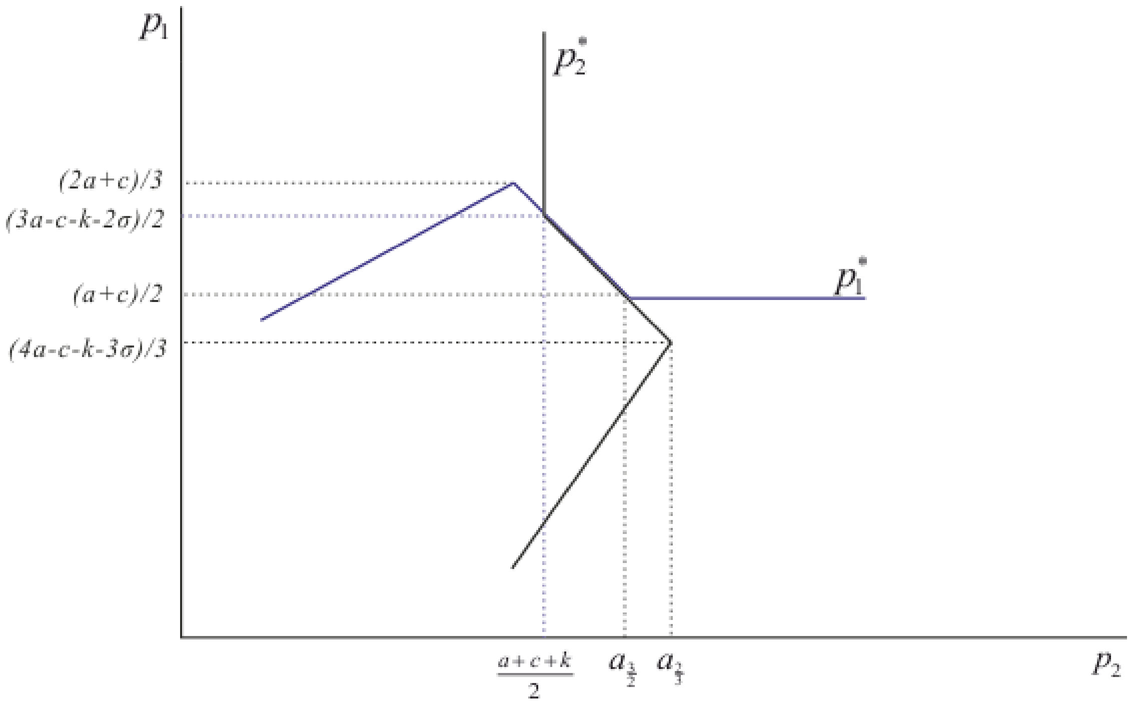

Figure A 8: The best response correspondence when $\frac{5}{6}-\frac{1}{3}K<\Sigma\le 1-\frac{1}{2}K$

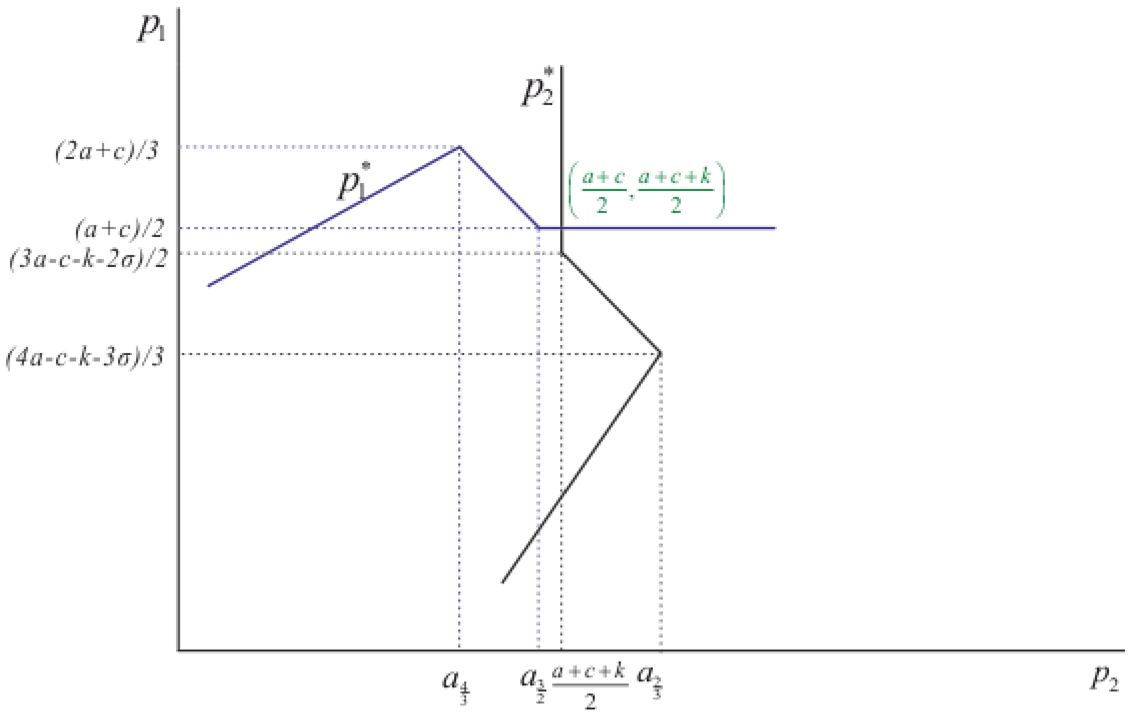

Figure A 9: The best response correspondence when $1-\frac{1}{2}K<\Sigma\le\frac{4}{3}-\frac{4}{3}K$

8. $1<\Sigma\le\frac{4}{3}-\frac{4}{3}\mathrm{K}$: on this range, the overall shape of the reaction function is the same as the previous case, but now the two reaction functions intersect on their flat range. There is a unique Nash Equilibria $\left(p_1^*,p_2^*\right)=\left(\frac{a+c}{2},\frac{a+c+k}{2}\right)$ (see figure A9 above).

9. $\frac{4}{3}-\frac{4}{3}\mathrm{K}<\Sigma\le\frac{4}{3}-\mathrm{K}$: on this range, $c+k>\frac{4}{3}a-\frac{c+k}{3}-\sigma$, and the relevant range is $c<\frac{4}{3}a-\frac{c+k}{3}-\sigma<c+k<\frac{4}{3}a-\frac{c}{3}-\sigma<\frac{3}{2}a-\frac{c+k}{2}-\sigma<\frac{3}{2}a-\frac{c}{2}-\sigma<a$, the reaction function of p_2^* is composed of only two segment, it has slope –1 on $\left[c+k,\frac{3}{2}a-\frac{c+k}{2}-\sigma\right]$, and a flat line on $\left[\frac{3}{2}a-\frac{c+k}{2}-\sigma,a\right]$. There is a unique equilibria $\left(p_1^*,p_2^*\right)=\left(\frac{a+c}{2},\frac{a+c+k}{2}\right)$ (see figure A10).

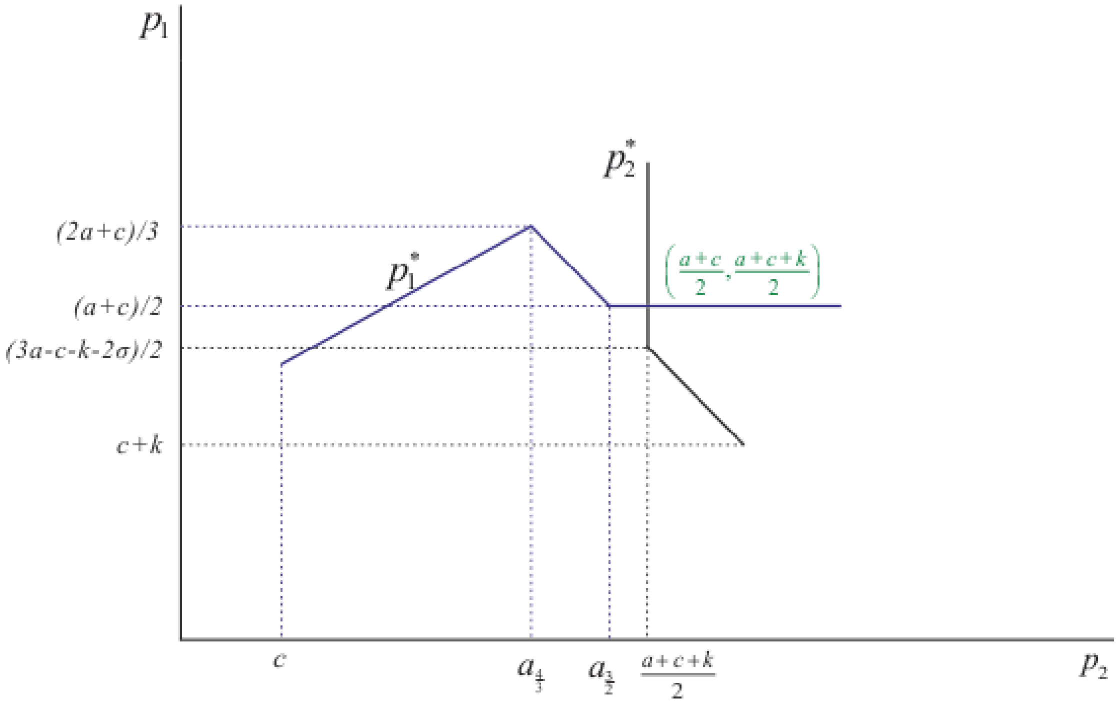

Figure A 10: The best response correspondence when $\frac{4}{3}-\frac{4}{3}K<\Sigma\leq\frac{4}{3}$

10. $\frac{4}{3}-\mathrm{K}<\Sigma\leq\frac{4}{3}-\frac{1}{3}\mathrm{K}$: On this range, the case differs from previous in the sense that $c+k>\frac{4}{3}a-\frac{c}{3}-\sigma$, the reaction functions have the same shapes, and the unique equilibrium is at $\left(p_1^*,p_2^*\right)=\left(\frac{a+c}{2},\frac{a+c+k}{2}\right)$ (see figure A10).

11. $\frac{4}{3}-\frac{1}{3}\mathrm{K}<\Sigma\leq\frac{4}{3}$: This case differs from previous in the sense that $c>\frac{4}{3}a-\frac{c+k}{3}-\sigma$ which does not affect the result, the reaction functions have the same shapes, and the unique equilibrium is at $\left(p_1^*,p_2^*\right)=\left(\frac{a+c}{2},\frac{a+c+k}{2}\right)$ (see figure A10).

12. $\frac{4}{3}<\Sigma\leq\frac{3}{2}-\frac{3}{2}\mathrm{K}$: On this range, $c>\frac{4}{3}a-\frac{c}{3}-\sigma$, and the relevant range is $c<c+k<\frac{3}{2}a-\frac{c+k}{2}-\sigma<\frac{3}{2}a-\frac{c}{2}-\sigma<a$, the reaction function of p_1^* is

composed of only two segment, it has slope –1 on $\left[c, \frac{3}{2}a - \frac{c}{2} - \sigma\right]$, and a flat line on $\left[\frac{3}{2}a - \frac{c}{2} - \sigma, a\right]$, similar to p_2^*. there is a unique equilibrium $\left(p_1^*, p_2^*\right) = \left(\frac{a+c}{2}, \frac{a+c+k}{2}\right)$ (see figure A11).

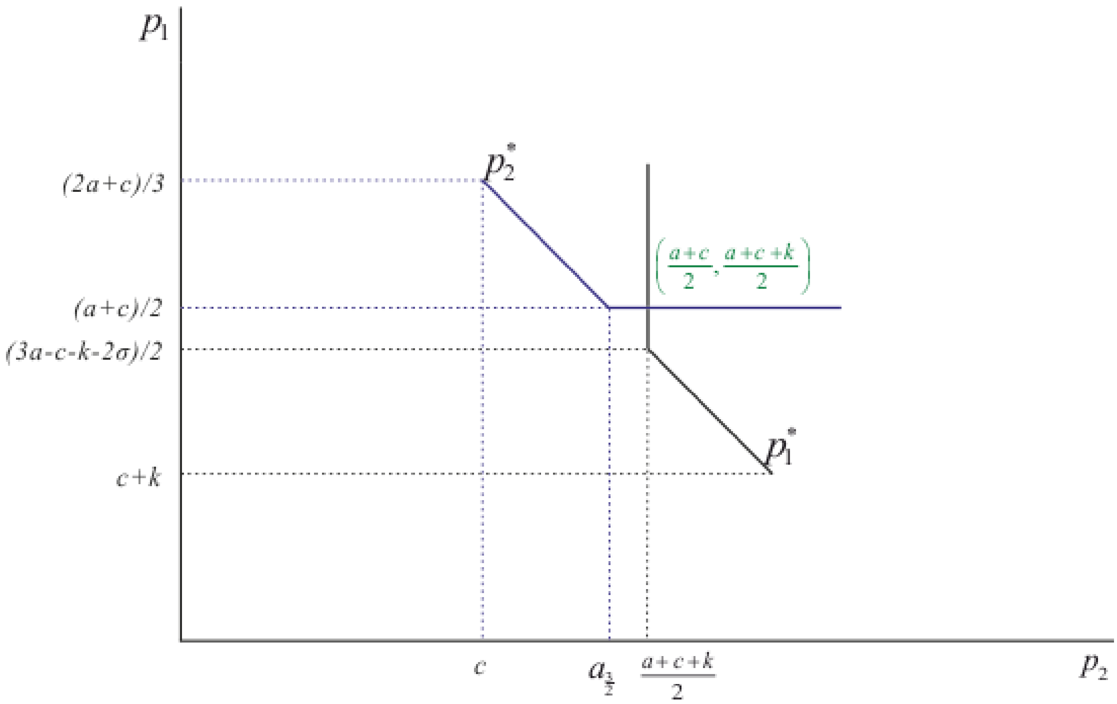

Figure A 11: The best response correspondence when $\frac{4}{3} < \Sigma \leq \frac{3}{2} - \frac{3}{2}K$

13. $\frac{3}{2} - \frac{3}{2}\mathrm{K} < \Sigma \leq \frac{3}{2} - \mathrm{K}$: On this range, $c + k > \frac{3}{2}a - \frac{c+k}{2} - \sigma$, this means the shape of the reaction function for p_2^* is a vertical line (monopoly) segmented in the range $[c+k, a]$. There is a unique equilibrium $\left(p_1^*, p_2^*\right) = \left(\frac{a+c}{2}, \frac{a+c+k}{2}\right)$ (see figure A12).

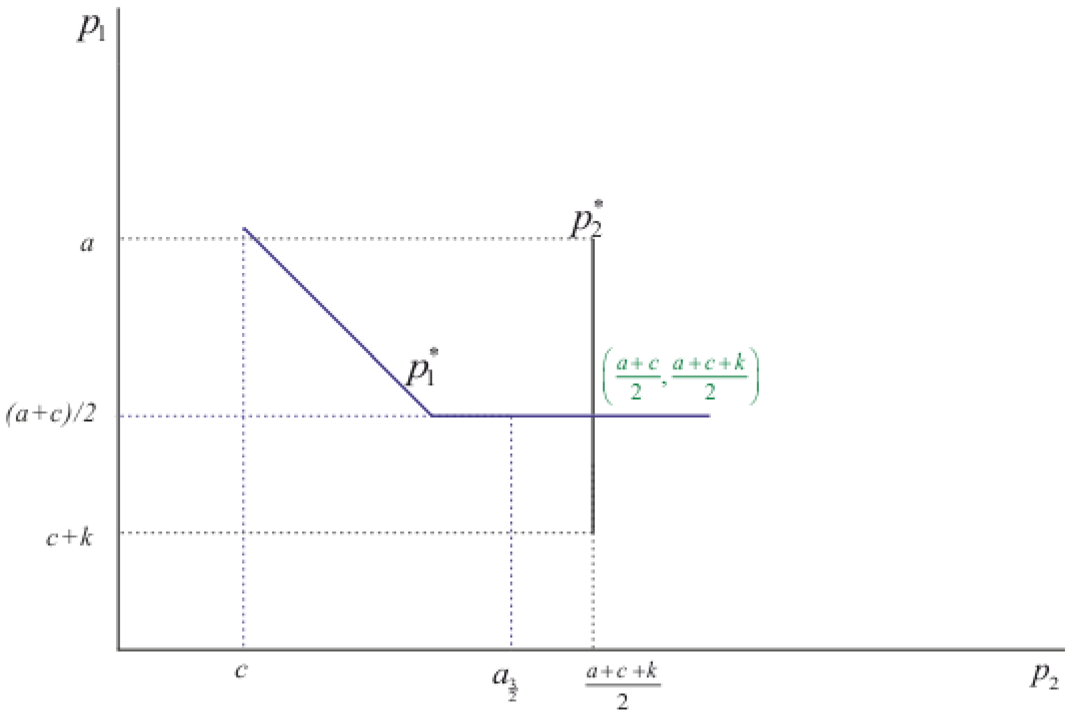

Figure A 12: The best response correspondence when $\frac{3}{2}-\frac{3}{2}K<\Sigma\leq\frac{3}{2}$

14. $\frac{3}{2}-\mathrm{K}<\Sigma\leq\frac{3}{2}-\frac{1}{2}\mathrm{K}$: On this range, $c+k>\frac{3}{2}a-\frac{c}{2}-\sigma$, again the reaction function of p_2^* is a vertical line (monopoly) segmented in the range $[c+k,a]$. There is a unique equilibrium $\left(p_1^*,p_2^*\right)=\left(\frac{a+c}{2},\frac{a+c+k}{2}\right)$ (see figure A12).

15. $\frac{3}{2}-\frac{1}{2}\mathrm{K}<\Sigma\leq\frac{3}{2}$: On this range, $\frac{3}{2}a-\frac{c}{2}-\sigma>c>\frac{3}{2}a-\frac{c+k}{2}-\sigma$, the reaction function of p_1^* is still composed of two segments, so the graph is the same as the above ones (see figure A12).

16. $\Sigma>\frac{3}{2}$: On this range, the only relevant segment of the reaction curve p_1^* is the flat one, same as p_2^* , and therefore there is a unique equilibrium $\left(p_1^*,p_2^*\right)=\left(\frac{a+c}{2},\frac{a+c+k}{2}\right)$ (see figure A13).

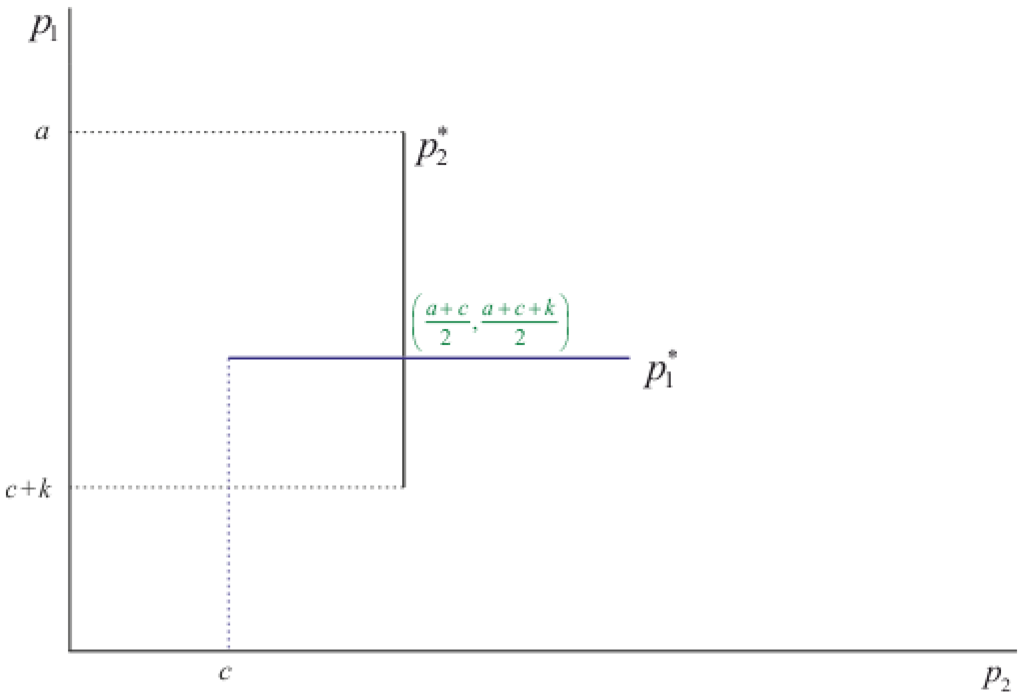

Figure A 13: The best response correspondence when $\Sigma > \frac{3}{2}$

Proposition 1: *In Hotelling model of differentiated Bertrand duopoly, there are three competition region:*

1. *For* $\mathrm{K} \in (0\ ,\ 2-3\Sigma)$, *it is a competitive regime. The Nash equilibrium is unique and asymmetric, characterized by* $(p_1^*, p_2^*) = \left(c+\sigma+\frac{k}{3}, c+\sigma+\frac{2}{3}k\right)$. *The market is covered and the indifferent customer is located at*[39] $x_1 = \frac{3\sigma + k}{6\sigma}$.

2. *For* $\mathrm{K} \in (2-3\Sigma\ ,\ 2-2\Sigma)$, *there are multiple kinked demand Equilibria. Each firm prices at the kink of its demand function, in the segment* $x_1 \in \left[\frac{a-c}{3\sigma}, \frac{a-c}{2\sigma}\right]$ *and there are three subcases:*

[39] According to the formula for location $x_1 = \frac{p_2 - p_1 + \sigma}{2\sigma}$

a) *When* $\mathrm{K} \in \left(2-3\Sigma\ ,\ \frac{5}{3}-2\Sigma\right)$, *equilibrium prices are characterized by* $p_1^* \in \left(\frac{2a+c}{3},\ \frac{4}{3}a-\frac{c+k}{3}-\sigma\right)$ *and* $p_2^* \in \left(\frac{4}{3}a-\frac{c}{3}-\sigma,\ \frac{2a+c+k}{3}\right)$, *and* $p_1^*+p_2^*=2a-\sigma$. *The market is covered, and the indifferent customer is located at* $x_1=\frac{a-p_1}{\sigma}$, *within the segment* $\left[\frac{a-c}{3\sigma},\ 1-\frac{a-c-k}{3\sigma}\right]$.

b) *When* $\mathrm{K} \in \left(\frac{5}{3}-2\Sigma\ ,\ \frac{5}{2}-3\Sigma\right)$, *equilibrium prices are characterized by* $p_1^* \in \left(\frac{4}{3}a-\frac{c+k}{3}-\sigma, \frac{3}{2}a-\frac{c+k}{2}-\sigma\right)$ *and* $p_2^* \in \left(\frac{2a+c+k}{3},\ \frac{a+c+k}{2}\right)$, *and* $p_1^*+p_2^*=2a-\sigma$. *The market is covered, and the indifferent customer is located at* $x_1=\frac{a-p_1}{\sigma}$, *within the segment* $\left[1-\frac{a-c-k}{3\sigma},\ 1-\frac{a-c-k}{2\sigma}\right]$.

c) *When* $\mathrm{K} \in \left(\frac{5}{2}-3\Sigma\ ,\ 2-2\Sigma\right)$, *equilibrium prices are characterized by* $p_1^* \in \left(\frac{3}{2}a-\frac{c+k}{2}-\sigma,\ \frac{a+c}{2}\right)$ *and* $p_2^* \in \left(\frac{a+c+k}{2},\ \frac{3}{2}a-\frac{c}{2}-\sigma\right)$, *and* $p_1^*+p_2^*=2a-\sigma$. *The market is covered, and the indifferent customer is located at* $x_1=\frac{a-p_1}{\sigma}$, *within the segment* $\left[1-\frac{a-c-k}{2\sigma},\ \frac{a-c}{2\sigma}\right]$.

3. *When* $\mathrm{K} > 2-2\Sigma$, *each firm is a monopolist: each firm sets price independently and the equilibrium price is* $\left(p_1^*, p_2^*\right)=\left(\frac{a+c}{2},\ \frac{a+c+k}{2}\right)$.

Figure A1 to A13 show the best response functions that are made up by two or three segments, depending on the normalised marginal disutility of Σ and normalised transportation cost K. The first part of the best response function is a upward slopping segment (with slope of $1/2$). When this part of the best response segments cross, the equilibrium corresponds to the competitive market equilibrium in summary

1: a typical Hotelling model equilibrium where the market is completely covered and the consumer who is indifferent from buying at either firm earns positive rents. This equilibrium occurs when the transportation cost K is small, i.e. it is within the range $\left(0,\ 2-3\Sigma\right)$. Define the boundary trade costs $K_c = 2-3\Sigma$.

In the second part of the best response function, it is either upward sloping (with the slope of 1) or downward slopping (with the slope of -1). In the former case, the home market is solely supplied by one firm, facing the competition from the other firm, yet the equilibrium does not exist here, see figure A1 in the appendix. In the latter case, the best response function is downward slopping (with slope of -1) as shown in figure 3A to 10A. When the reaction functions of both firms overlap, there exists a continuum of equilibria. The reason is that the optimal price of firm one (two), given a range of values of $p_2\left(p_1\right)$, is set at the kink of the demand function, which is a result of a discontinuity in the marginal revenue of firm one (two). If firm two (one) increases its price, firm one (two) would decrease its price just enough to cover the market. Hence, by setting the price at the kink of the demand function, the market is completely covered and the indifferent consumer earns zero surplus at the point $x_1 = \dfrac{p_2 - p_1 + \sigma}{2\sigma}$, the optimal price of p_1^* is $p_1^* + p_2 = 2a - \sigma$ in this range of p_2. This equilibrium (summary 2) arises when the two reaction functions overlap for the intermediate values of the normalised transportation cost $\mathrm{K} \in \left(2-3\Sigma\ ,\ 2-2\Sigma\right)$. Define the boundary trade costs as $K_m = 2-2\Sigma$.

Finally, the third part of the reaction function is a constant $p_1^* = \dfrac{a+c}{2}$ for firm one, and $p_2^* = \dfrac{a+c+k}{2}$ for firm two. When these parts of the two best response curves cross, both firms charge the monopoly price and part of the home market is not covered. Thus potentially there is no competition between these two firms in the home market, the best response of the firm is to charge the monopoly price, facing the high price charged by the other firm. This case (proposition 1 point 3) takes place when the normalised transportation cost is sufficiently high, i.e. $\mathrm{K} > K_c$. Notice that unlike the classical analysis of the kinked demand equilibria in the Hotelling model, the marginal costs are different for firm one and firm two in the home market, the reaction curve of p_2^* and the reaction curve of p_1^* are not symmetric with respect to $p_1 = p_2$. However, the overlapping portions of the reaction curves still exist, implying that the indifferent consumer who earns zero rent is located close to the mid-point of the $[0,1]$ segment, then prices and market shares of both firms are bounded.

4.3 Profits and Welfare

Welfare in a market is the sum of the consumer surplus and producer surplus. Consumer surplus in a Hotelling model is the area below the line $U = a - p - \sigma x$, and producer surplus is the profits earned my domestic firm in the home country and foreign country if it exports.

The autarky case has been analysed in section 4.2.1, where firm one can either cover the whole market, or the partial market. Under free trade, when $K < K_c$, it is the competitive market region, where it comes across a prohibitive trade cost $\bar{K} \equiv \dfrac{\bar{k}}{a-c}$. Hence within this region, if $K < \bar{K}$, the normal Hotelling competitive equilibrium applies, if $K \geq \bar{K}$, there is a pro-competitive effect, in which case firm one covers the entire home market, facing the competition from firm two. When $K_c \leq K < K_m$, there is a kinked demand region, where a upper bound and a lower bound exist due to the nature of the marginal revenue at the kink. When $K \geq K_m$ [40], it is a local monopoly, where both firms supply the market, yet part of the market is not covered, so each firm acts as a local monopoly. Bear in mind that, if the trade cost $K \geq \tilde{K}$, firm two stops exporting to the home market, then it will be an autarky. Thus trade only occurs when $K < \tilde{K}$, which will be a necessary condition for all the above cases. There are two types of autarky, firm one supplies the entire home market, or supplies only partial market (refer back to section 4.2.1). Consequently, a trade cost and a marginal disutility space could be established in figure 4-1, showing all the relevant areas above as follows[41]:

[40] If normalise the prohibitive trade cost $\bar{K} \equiv \dfrac{\bar{k}}{a-c}$, and the same as $\tilde{K} \equiv \dfrac{\tilde{k}}{a-c}$ later on.

[41] Assuming that $a - c = 1$ for the normalised trade costs K and marginal disutility Σ

- Area A: a competitive region when $K_c \geq K > \bar{K}$ and $K < \tilde{K}$ with a pro-competitive effect
- Area B: a competitive region when $K < \bar{K}$, $K \leq K_c$, and $K < \tilde{K}$
- Area C: kinked demand region with a upper and a lower bound of equilibrium prices when $K_c < K \leq K_m$ and $K < \tilde{K}$
- Area D: a local monopoly region (uncovered market) when $\tilde{K} > K > K_m$
- Area E: an autarky when $K \geq \tilde{K}$

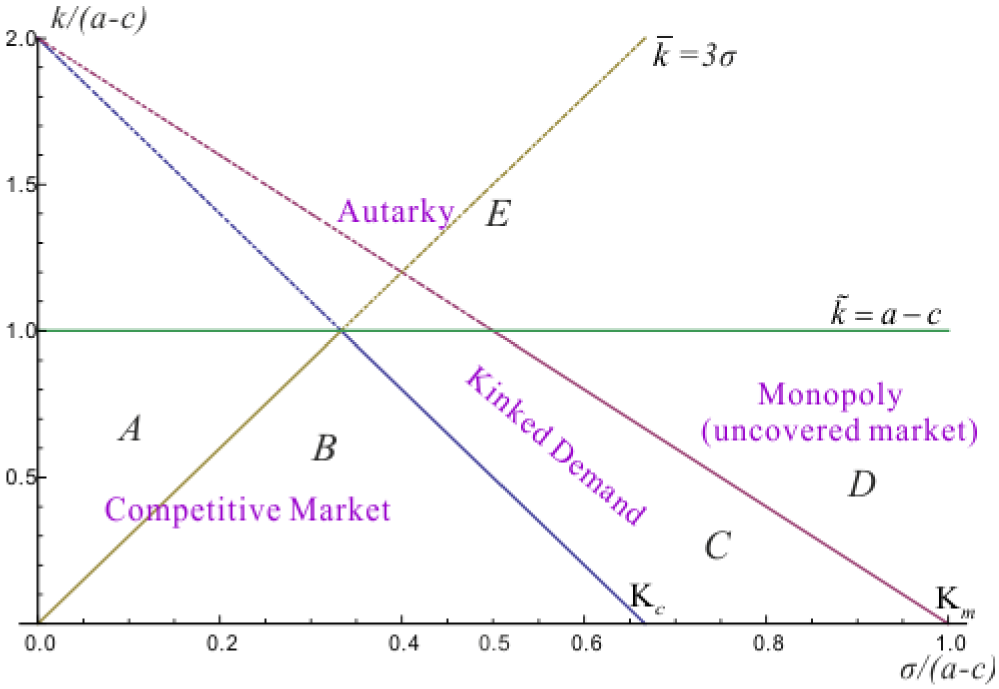

Figure 4-1: Equilibria in a normalised trade cost – marginal disutility space

In this chapter, markets are assumed to be segmented, and two forms of free trade will be considered: unilateral free trade and multilateral free trade. Unilateral free trade is a one way trade, implying two firms compete as a Bertrand duopoly in the home country, while foreign firm faces no competition and acts as a monopolist in the foreign country. Multilateral free trade, on the other hand, is a two way trade. Firm one and firm two compete as a Bertrand duopoly in the home country as well as in the foreign country. Subsequently, the profits of the home firm under the multilateral free trade include the additional profits from exporting to the foreign market, and then the welfare of home country under multilateral free trade is greater than that under unilateral free trade. In the following section, the profits and the welfare of these two types of trade will be analysed according to the areas in figure 4-1:

Area A: competitive region when $\bar{K} < K \leq K_c$ and $K \leq \tilde{K}$ with a pro-competitive effect

In this region, firm two does not supply the home market, but it is a potential competitor for firm one. There is a pro-competitive effect: the price of the home firm must be lower than the price of the foreign firm, so that foreign firm does not sell in the home market[42]. The demand for good one in the home market is therefore one ($x_1 = 1$). Using equation (131) and the price of good one $p_1 = p_2 - \sigma$, its price can be expressed as $p_1 = c + 2\sigma$, while $p_2 = c + 3\sigma$. The profits of firm one in the home market is

$$\pi_{1A}^E = (p_1 - c)\, x_1 = 2\sigma \tag{99}$$

and the welfare in area A is:

$$\begin{aligned} W_1 &= CS + \pi_1 \\ &= \int_0^1 (a - p_1 - x)\, dx + 2\sigma \\ &= a - c - \frac{5}{2}\sigma + 2\sigma \\ &= a - c - \frac{1}{2}\sigma \end{aligned} \tag{100}$$

Notice that the boundary lines K_c, $\bar{K}$ and $\tilde{K}$ intersect at one point where $\Sigma = 1/3$ and $K = 1$ in the figure, so area A is a triangle. In addition, this area is on the left hand side of the line $\Sigma = 0.5$, which is particularly useful when looking at gains from trade later on, as this is the area where firm one would have supplied the home market under autarky case two (i.e. the market is entirely covered).

---

[42] The critical trade cost is $\bar{k} = 3\sigma$, $x_1 = 1$, so the home price is smaller than the foreign price $p_1 + \sigma < c + \bar{k}$, $p_1 < c + 3\sigma - \sigma = c + 2\sigma$.

Area B: a competitive region ($K \le K_c$ ***,*** $K < \bar{K}$ ***and*** $K < \tilde{K}$ ***)***

The profits for area B (a competitive market when $K < \bar{K}$) has been discussed in equation (91), where the demand for good one in the home country is $x_1 = \dfrac{3\sigma + k}{6\sigma}$.

The consumer surplus is therefore:

$$
\begin{aligned}
CS &= \int_0^{x_1} \left(a - p_1 - \sigma x_1\right)\, dx_1 + \int_{x_1}^{1} \left[a - p_2 - \sigma\left(1 - x_1\right)\right] dx_1 \\
&= a - c - \frac{3}{2}\sigma - \frac{2}{3}k + \frac{\left(3\sigma + k\right)^2}{36\sigma}
\end{aligned}
$$

Using the Bertrand equilibrium prices, outputs, profits from equations (91) and consumer surplus above, welfare of the home firm one under ***unilateral free trade*** is:

$$
\begin{aligned}
W_1^u &= CS + \pi_{1A}^E \\
&= a - c - \frac{3}{2}\sigma - \frac{2}{3}k + \frac{\left(3\sigma + k\right)^2}{36\sigma} + \frac{\left(3\sigma + k\right)^2}{18\sigma} \\
&= \frac{3k^2 - 6k\sigma + 9\sigma\left[4\left(a - c\right) - 3\sigma\right]}{36\sigma}
\end{aligned}
\tag{101}
$$

If firm one decides to export as well, substituting the profits of firm one in the foreign market from equation (91), the welfare of the home firm one under ***multilateral free trade*** is:

$$
\begin{aligned}
W_1^m &= CS + \pi_{1A}^E + \pi_{2A}^E \\
&= a - c - \frac{3}{2}\sigma - \frac{2}{3}k + \frac{\left(3\sigma + k\right)^2}{36\sigma} + \frac{\left(3\sigma + k\right)^2}{18\sigma} + \frac{\left(3\sigma - k\right)^2}{18\sigma} \\
&= \frac{5k^2 - 18k\sigma + 9\sigma\left[4\left(a - c\right) - \sigma\right]}{36\sigma}
\end{aligned}
\tag{102}
$$

Where W_1^u represents welfare under unilateral free trade, and W_1^m represents welfare under multilateral free trade. As it is mentioned above, area B is right under $\tilde{K}$, so it is also a triangle, but $\Sigma = 0.5$ line go through this area. Thus there will be two cases in terms of gains from trade.

Area C: kinked demand region with a upper and a lower bound of equilibrium prices ($K_c < K \le K_m$ ***and*** $K \le \tilde{K}$***)***

In this region, the normalised trade cost is between K_c and K_m, and it is smaller than one. In the home country, both firms sell to the home consumers and cover the entire market. However, the kink of the demand function determines that the optimal price of good one is such that $p_1^* + p_2^* = 2a - \sigma$ in this range of the price of good two. Hence, there is a upper bound and a lower bound of the equilibrium prices for both p_1 and p_2. Refer back to Proposition 1 point 2, there are three cases in this region, as shown in Appendix A6, A7 and A8, but only the boundaries of all cases are of the interest. The price of good one is in a range of $p_1 \in \left[\frac{2a+c}{3},\ \frac{a+c}{2}\right]$, the location of the indifferent customer is located within the range of $x_1 \in \left[\frac{a-c}{3\sigma},\ \frac{a-c}{2\sigma}\right]$.

With a lower bound of the equilibrium price, $p_1 = \frac{2a+c}{3}$, $p_2 = \frac{4}{3}a - \frac{c}{3} - \sigma$, and $x_1 = \frac{a-c}{3\sigma}$. Consumer surplus is expressed as:

$$CS = \int_0^{x_1} \left(a - p_1 - \sigma x_1\right)\, dx_1 + \int_{x_1}^{1} \left[a - p_2 - \sigma\left(1 - x_1\right)\right] dx_1$$

Substitute the prices and the location of x_1 into the above formula, consumer surplus is :

$$CS_{2l} = \frac{\left(a-c\right)^2}{9\sigma} + \frac{3\sigma - 2a + 2c}{6}$$

Where the subscript $2l$ represents kinked demand lower bound equilibrium. Using the Bertrand equilibrium prices, outputs, profits from equations (91) and consumer surplus above, welfare of the home firm one under ***unilateral free trade*** is:

$$
\begin{aligned}
W_{2l}^{U} &= CS + \pi_{1A}^{E} \\
&= \frac{(a-c)^2}{9\sigma} + \frac{3\sigma - 2a + 2c}{6} + \frac{(3\sigma + k)^2}{18\sigma} \\
&= \frac{4(a-c)^2 + 2k^2 + 12\sigma k - 12\sigma\left[(a-c) - 3\sigma\right]}{36\sigma}
\end{aligned} \tag{103}
$$

The welfare of the home firm one under ***multilateral free trade*** is:

$$
\begin{aligned}
W_{2l}^{M} &= CS + \pi_{1A}^{E} + \pi_{2A}^{E} \\
&= \frac{(a-c)^2}{9\sigma} + \frac{3\sigma - 2a + 2c}{6} + \frac{(3\sigma + k)^2}{18\sigma} + \frac{(3\sigma - k)^2}{18\sigma} \\
&= \frac{2(a-c)^2 + 2k^2 + 3\sigma\left[9\sigma - 2(a-c)\right]}{18\sigma}
\end{aligned} \tag{104}
$$

With a upper bound of the equilibrium price, $p_1 = \frac{a+c}{2}$, $p_2 = \frac{3}{2}a - \frac{c}{2} - \sigma$, and $x_1 = \frac{a-c}{2\sigma}$. Using the expression for consumer surplus above, and substitute the prices and the location of indifferent consumer:

$$
CS_{2h} = \frac{(a-c)^2}{4\sigma} + \frac{\sigma - a + c}{2}
$$

Where the subscript $2h$ represents kinked demand upper bound equilibrium The welfare of the home firm one under ***unilateral free trade*** is

$$
\begin{aligned}
W_{2h}^{U} &= CS + \pi_{1A}^{E} \\
&= \frac{(a-c)^2}{4\sigma} + \frac{\sigma - a + c}{2} + \frac{(3\sigma + k)^2}{18\sigma} \\
&= \frac{9(a-c)^2 + 2k^2 + 12\sigma k - 18\sigma\left[(a-c) - 2\sigma\right]}{36\sigma}
\end{aligned} \tag{105}
$$

The welfare of the home firm one under ***multilateral free trade*** is:

$$
\begin{aligned}
W_{1h}^{M} &= CS + \pi_{1A}^{E} + \pi_{2A}^{E} \\
&= \frac{(a-c)^2}{4\sigma} + \frac{\sigma - a + c}{2} + \frac{(3\sigma + k)^2}{18\sigma} + \frac{(3\sigma - k)^2}{18\sigma} \\
&= \frac{9(a-c)^2 + 4k^2 + 9\sigma\left[6\sigma - 2(a-c)\right]}{36\sigma}
\end{aligned}
\tag{106}
$$

Overall, the welfare for firm one in the kinked demand region with lower and upper bound of the equilibrium prices can be summarised in the following way:

For the ***Unilateral case***: $W^{U} \in \left[W_{2l}^{U},\ W_{2h}^{U}\right]$, where

$$W_{2l}^{U} = \frac{4(a-c)^2 + 2k^2 + 12\sigma k - 12\sigma\left[(a-c) - 3\sigma\right]}{36\sigma}, \text{ and}$$

$$W_{2h}^{U} = \frac{9(a-c)^2 + 2k^2 + 12\sigma k - 18\sigma\left[(a-c) - 2\sigma\right]}{36\sigma}$$

For the ***Multilateral case***: $W^{M} \in \left[W_{2l}^{M},\ W_{2h}^{M}\right]$, where

$$W_{2l}^{M} = \frac{2(a-c)^2 + 2k^2 + 3\sigma\left[9\sigma - 2(a-c)\right]}{18\sigma}, \text{ and}$$

$$W_{2h}^{M} = \frac{9(a-c)^2 + 4k^2 + 9\sigma\left[6\sigma - 2(a-c)\right]}{36\sigma}$$

Area D: a local monopoly region (uncovered market) when $\tilde{K} \geq K > K_m$

In this region, both firms supply the home country, but the market is not covered completely, so each firm acts as a monopoly and does not compete with each other. The prices, outputs and profits have been discussed in equations (94): $x_1 = \dfrac{a-c}{2\sigma}$, $1 - x_2 = \dfrac{2\sigma - a + c + k}{2\sigma}$ and $\pi_{1A}^{E} = \dfrac{(a-c)^2}{4\sigma}$, the consumer surplus in the home country is:

$$
\begin{aligned}
CS_3 &= \int_0^{x_1} \left(a - p_1 - \sigma x_1\right)\, dx_1 + \int_{x_2}^{1} \left[a - p_2 - \sigma\left(1 - x_2\right)\right] dx_2 \\
&= \frac{\left(a-c\right)^2 + \left(2\sigma - a + c + k\right)^2}{8\sigma} + \frac{a - c - k - \sigma}{2} \\
&= \frac{2\left(a-c\right)^2 + k^2 - 2k\left(a-c\right)}{8\sigma}
\end{aligned}
$$

Welfare in the trade equilibrium under ***Unilateral free trade*** *is:*

$$
\begin{aligned}
W_3^U &= CS + \pi_{1A}^E \\
&= \frac{\left(a-c\right)^2 + \left(2\sigma - a + c + k\right)^2}{8\sigma} + \frac{a - c - k - \sigma}{2} + \frac{\left(a-c\right)^2}{4\sigma} \\
&= \frac{4\left(a-c\right)^2 + k^2 - 2k\left(a-c\right)}{8\sigma}
\end{aligned} \tag{107}
$$

Welfare in the trade equilibrium under ***Multilateral free trade*** *is :*

$$
\begin{aligned}
W_D^M &= CS + \pi_{1A}^E + \pi_{2A}^E \\
&= \frac{\left(a-c\right)^2 + \left(2\sigma - a + c + k\right)^2}{8\sigma} + \frac{a - c - k - \sigma}{2} + \frac{\left(a-c\right)^2}{4\sigma} + \frac{\left(a-c-k\right)^2}{4\sigma} \\
&= \frac{6\left(a-c\right)^2 + 3k^2 - 6k\left(a-c\right)}{8\sigma}
\end{aligned} \tag{108}
$$

Area E: an autarky when $K \geq \tilde{K}$

This area is where the trade costs are too high for firm two to supply the home country. Home firm one is the only seller at home, facing no potential threat from foreign firm. Thus this is an autarky, and there are two cases as analysed in section 4.2.1. When the normalised marginal disutility is smaller than 0.5, firm one covers the whole market, otherwise it only covers part of the home market. The results of the welfares under these two cases are detailed in equations (85) and (87). The welfare of trade under all kinds of market structures in the Hotelling model are summarized in table 4-2 in the appendix.

4.4 Gains from Trade

Gains from unilateral free trade

The welfare of area A, B, C and D have been analysed under free trade, the gains from trade are given by subtracting welfare under autarky (85) and (87) from welfare under unilateral free trade or welfare under multilateral free trade. The gains from unilateral free trade are shown in figure 4-2 and the gains from multilateral free trade are shown in figure 4-3. For the case of the competitive market area A, there is a pro-competitive effect so the Bertrand equilibrium is a boundary solution where there is no export from the foreign firm and the home firm acts as a monopoly. As this area is on the left hand side of $\Sigma = 0.5$, hence the home market is covered entirely, and welfare under autarky case two, equation (87) will be applied. The gains from trade in area A is:

$$\begin{aligned} G_{A} &= W_1 - W^{A2} \\ &= a - c - \frac{1}{2}\sigma - \left(a - c - \frac{\sigma}{2} \right) = 0 \end{aligned}$$

There are no gains or losses in the region A from unilateral free trade or multilateral free trade, this is due to the fact that home firm one sets the monopoly price and sells the monopoly output in the home market, which is the same as under autarky case two. Trade liberalisation start from autarky would have increased welfare in the competitive market with pro-competitive effect, but without actual trade taking place, there is no wasteful trade costs occurred. Thus there are no gains or losses from trade.

Secondly, consider the usual Hotelling competitive market in area B, so that both home firm and foreign firm compete as a Bertrand duopolists to supply the home market, and all consumers earn positive surplus. In the trade cost-marginal disutility space, the $\Sigma = 0.5$ line goes through area B, splitting this area into B_1 and B_2 as shown in figure 4-2, here both autarky cases 1 and 2 exist when calculating gains from trade. Using equations (101), (87) and (85), when $\Sigma < 0.5$, using (101) and (87), the gains from unilateral trade are:

$$G^u_{B1} = W^u_1 - W^{A2} = \frac{3k^2 - 6k\sigma + 9\sigma\left[4(a-c) - 3\sigma\right]}{36\sigma} - \left(a - c - \frac{\sigma}{2}\right)$$
$$= \frac{3k^2 - 6k\sigma - 9\sigma^2}{36\sigma} \tag{109}$$

which is referred to as area B_1 in figure 4-2. To see whether this is a gain or a loss, a critical value of the normalised trade cost in terms of the marginal disutility will be examined, i.e. $G^u_{B1} = 0$, assuming $a - c = 1$,

$$K^u_{B1} \equiv \frac{k^u_{B1}}{a-c} = 3\sigma \quad \text{or} \quad -\sigma$$

Welfare is higher under unilateral free trade if the relative trade cost is outside the range of the critical values ($k > 3\sigma$ or $k < -\sigma$). Nevertheless, area B_1 is above zero and below $\bar{K} = 3\sigma$, therefore G^u_{B1} is strictly negative and there are no gains from trade. When $\Sigma \geq 0.5$, by using (101) and (85), the gains from unilateral trade are:

$$G^u_{B2} = W^u_1 - W^{A1} = \frac{3k^2 - 6k\sigma + 9\sigma\left[4(a-c) - 3\sigma\right]}{36\sigma} - \frac{3(a-c)^2}{8\sigma}$$
$$= \frac{6k^2 - 12k\sigma + 18\sigma\left[4(a-c) - 3\sigma\right] - 27(a-c)^2}{72\sigma} \tag{110}$$

This is referred to as area B_2 in figure 4-2. Comparing total welfare under unilateral free trade with total welfare under autarky as in (110), welfare is higher under unilateral free trade if the relative trade cost is outside the range of the critical values below, i.e. K^u_{B2}. However, the area B_2 is inside this range, therefore G^u_{B2} is strictly negative and there are no gains from trade.

$$K^u_{B2} \equiv \frac{k^u_{B2}}{a-c} = \sigma \pm \frac{1}{2}\sqrt{18 - 48\sigma + 40\sigma^2}$$

To summarise, there are always losses from trade in area B, shown as follows:

$$G_B^u \equiv \begin{cases} G_{B1}^u = \dfrac{3k^2 - 6k\sigma - 9\sigma^2}{36\sigma} < 0 & if \ \Sigma < 0.5 \\ G_{B2}^u = \dfrac{6k^2 - 12k\sigma + 18\sigma\left[4(a-c) - 3\sigma\right] - 27(a-c)^2}{72\sigma} < 0 & if \ \Sigma \geq 0.5 \end{cases}$$

In figure 4-2, start from Area A, when trade costs fall, and marginal disutility increases to area B, the products are more differentiated, leading to welfare gains. Meanwhile, the consumer surplus increases as the falling trade costs intensify the competition, leading to lower prices. However, home sales and profits will decrease for the reduced prices, the fall in the home profits overweight the rise in the consumer surplus as well as gains from enjoying various products.

Thirdly, consider the kinked demand Bertrand equilibrium in area C, where the kink occurs at profit maximising price. If the firm reduces its price then it faces monopoly demand, but if the firm increases its price then it faces competitive demand. Since home firm sets its price lower than that under autarky, more sales are obtained and gains from trade should be positive. In figure 4-2, the $\Sigma = 0.5$ line also goes through area C, crossing the point where K_m intersects $k = a - c$. On the left hand side when $\Sigma < 0.5$, using (103), (105) and (87), gains from unilateral free trade are expressed as:

$$\begin{aligned} G_{2l}^U = W_{2l}^U - W^{A2} &= \frac{4(a-c)^2 + 2k^2 + 12\sigma k - 12\sigma\left[(a-c) - 3\sigma\right]}{36\sigma} - \left(a - c - \frac{\sigma}{2}\right) \\ &= \frac{4(a-c)^2 + 2k^2 + 12k\sigma - 6\sigma\left[8(a-c) - 9\sigma\right]}{36\sigma} \end{aligned} \tag{111}$$

$$\begin{aligned} G_{2h}^U = W_{2h}^U - W^{A2} &= \frac{9(a-c)^2 + 2k^2 + 12\sigma k - 18\sigma\left[(a-c) - 2\sigma\right]}{36\sigma} - \left(a - c - \frac{\sigma}{2}\right) \\ &= \frac{9(a-c)^2 + 2k^2 + 12\sigma k - 54\sigma\left[a - c - \sigma\right]}{36\sigma} \end{aligned} \tag{112}$$

This is shown in area $C_1 + C_2$, the critical values of the gains above can be calculated by setting both G_{2l}^u and G_{2h}^u to zero. Then the total welfare is higher under unilateral free trade if the relative trade cost is outside the range of the two roots of each equation. When $\Sigma < 0.5$,

$$K_{2l}^{u} \equiv \frac{k_{2l}^{u}}{a-c} = -3\sigma \pm \sqrt{-2+24\sigma-18\sigma^2}$$

$$K_{2h}^{u} \equiv \frac{k_{2h}^{u}}{a-c} = -3\sigma \pm \frac{3}{2}\sqrt{-2+12\sigma-8\sigma^2}$$

These critical values are shown in the region $C_1 + C_2$, since K_{2h}^{u} is smaller than K_{2l}^{u} for each marginal disutility, only K_{2l}^{u} will be taken into account. The area below shows a loss from unilateral free trade, represented as area C_1, and the area above K_{2l}^{u} shows a gain from trade, represented as area C_2. On the right hand side when $\Sigma \geq 0.5$, using (103), (105) and (85), gains from unilateral free trade are expressed as:

$$\begin{aligned} G_{2l}^{U} = W_{2l}^{U} - W^{A1} &= \frac{4(a-c)^2 + 2k^2 + 12\sigma k - 12\sigma\left[(a-c)-3\sigma\right]}{36\sigma} - \frac{3(a-c)^2}{8\sigma} \\ &= \frac{-19(a-c)^2 + 4k^2 + 24k\sigma - 24\sigma\left[(a-c)-3\sigma\right]}{72\sigma} \end{aligned} \tag{113}$$

$$\begin{aligned} G_{2h}^{U} = W_{2h}^{U} - W^{A1} &= \frac{9(a-c)^2 + 2k^2 + 12\sigma k - 18\sigma\left[(a-c)-2\sigma\right]}{36\sigma} - \frac{3(a-c)^2}{8\sigma} \\ &= \frac{-9(a-c)^2 + 4k^2 + 24\sigma k - 36\sigma\left[(a-c)-2\sigma\right]}{72\sigma} \end{aligned} \tag{114}$$

which is shown in area $C_3 + C_4$, the critical values of the gains are found by setting both $G_{2l}^{u} = 0$ and $G_{2h}^{u} = 0$. Then again the total welfare is higher under unilateral free trade if the relative trade cost is outside the range of the two roots of each equation. when $\Sigma \geq 0.5$,

$$K_{2l}^{u} \equiv \frac{k_{2l}^{u}}{a-c} = -3\sigma \pm \frac{1}{2}\sqrt{19+24\sigma-36\sigma^2}$$

$$K_{2h}^{u} \equiv \frac{k_{2h}^{u}}{a-c} = -3\sigma \pm \frac{1}{2}\sqrt{9+36\sigma-40\sigma^2}$$

Once again, K_{2h}^{u} is smaller than K_{2l}^{u} for each marginal disutility in area C_3+C_4, so only K_{2l}^{u} will be considered. The area below K_{2l}^{u} shows a loss from unilateral free trade, represented as area C_4, and the area above K_{2l}^{u} shows a gain from trade, represented as area C_3. Consequently, in the kinked demand region, the area labelled C_1+C_4 shows losses from unilateral free trade, and area C_2+C_3 shows gains from trade. In figure 4-2, from area B to area C, marginal disutility increases further, implying higher differentiated products, which is clearly a welfare gain as home consumers face more product choices. Trade liberalisation increases consumer surplus as prices are reduced by an intensified competition. Again, the home sales and profits will decrease for the reduced prices, but the overall effect is ambiguous: the fall in the home profits overweight the rise in the consumer surplus plus gains from consuming more variety of products in area C_1+C_4, and the opposite happens in area C_2+C_3, which is to the right of area C_1+C_4, representing more differentiated products and gains from trade.

Finally, consider the local monopoly Bertrand equilibrium in area D of figure 4-1, so both home and foreign firm have a non-empty sale in the home country, yet part of the market is not covered by any firms. Here, both firm one and firm two set monopoly price and output. The gains from unilateral free trade can be calculated by using (107) and (87):

$$\begin{aligned} G_D^U = W_3^U - W^{A1} &= \frac{4(a-c)^2 + k^2 - 2k(a-c)}{8\sigma} - \frac{3(a-c)^2}{8\sigma} \\ &= \frac{(a-c)^2 + k^2 - 2k(a-c)}{8\sigma} \end{aligned} \tag{115}$$

As the trade cost is smaller than $a-c$ in this region, G_D^u must be a positive. Hence there are always gains from unilateral free trade in the local monopoly region. This is due to the fact that the products are highly differentiated in this area when the home market is uncovered, so home consumers gain enormous welfare from a wider variety of goods such that the welfare losses from the reduced home sales/profits are dominated by the gains.

To summarise, there is no gains or losses in the competitive market region A with pro-competitive effect, and there are always losses from unilateral free trade in the shaded area B_1, B_2 (competitive market region), C_1 and C_4 (part of the kinked demand region). On the other hand, part of the kinked demand region C_3, C_2 and the local monopoly region D always have gains from unilateral free trade.

Proposition 2: *In a Hotelling model of differentiated Bertrand duopoly, there are losses from unilateral free trade in the region* $B_1 + B_2 + C_1 + C_4$, *there is no gains or losses in the region* A, *and there are always gains from unilateral free trade for the rest of the region under* $k = a - c$.

Gains from unilateral free trade are summarized as follows:

$$G^u \equiv \begin{cases} G_A = 0 \\ G_B = \begin{cases} \dfrac{6k^2 - 12k\sigma + 18\sigma\left[4(a-c) - 3\sigma\right] - 27(a-c)^2}{72\sigma} & if\ \ \Sigma \geq 0.5 \\ \dfrac{3k^2 - 6k\sigma - 9\sigma^2}{36\sigma} & if\ \ \Sigma < 0.5 \end{cases} \\ G_C \in \begin{cases} G_{2l} = \begin{cases} \dfrac{-19(a-c)^2 + 4k^2 + 24k\sigma - 24\sigma\left[(a-c) - 3\sigma\right]}{72\sigma} & if\ \ \Sigma \geq 0.5 \\ \dfrac{4(a-c)^2 + 2k^2 + 12k\sigma - 6\sigma\left[8(a-c) - 9\sigma\right]}{36\sigma} & if\ \ \Sigma < 0.5 \end{cases} \\ G_{2h} = \begin{cases} \dfrac{-9(a-c)^2 + 4k^2 + 24\sigma k - 36\sigma\left[(a-c) - 2\sigma\right]}{72\sigma} & if\ \ \Sigma \geq 0.5 \\ \dfrac{9(a-c)^2 + 2k^2 + 12\sigma k - 54\sigma\left[a - c - \sigma\right]}{36\sigma} & if\ \ \Sigma < 0.5 \end{cases} \end{cases} \\ G_D = \dfrac{(a-c)^2 + k^2 - 2k(a-c)}{8\sigma} \end{cases}$$

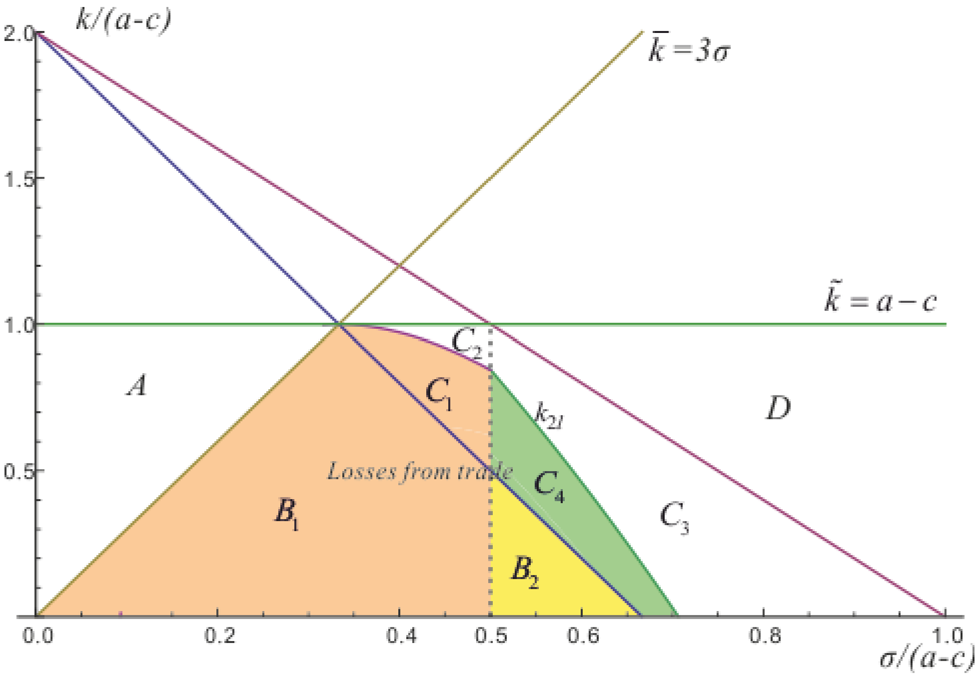

Figure 4-2: Gains from unilateral free trade

Gains from Multilateral free trade

The above result can be extended to the multilateral free trade case. Theoretically, the case of multilateral free trade should be similar to the one under unilateral free trade due to the segmented markets. Yet, despite home firm one' profits in the home market, it would also earn profits from the exports to the foreign country under multilateral free trade, adding extra positive profits itself. Thus it increases the welfare of the home country under free trade. If there are gains from unilateral free trade, there would definitely be gains from trade from multilateral free trade. If there are losses from unilateral free trade, there would be uncertain about the gains from multilateral free trade. The gains from multilateral free trade are illustrated in figure 4-3.

Firstly, for the case of the competitive market region A with a pro-competitive effect, the Bertrand equilibrium is a boundary solution where there is no export from the foreign firm and the home firm acts as a monopoly. So the gains from multilateral free trade in this region are the same as the one under unilateral free trade, i.e. $G_A = 0$

Secondly, consider the usual Hotelling competitive market in area B. Again, in the trade cost-marginal disutility space, the $\Sigma = 0.5$ line goes through area B, splitting this area into two regions. Using equations (102), and (87), when $\Sigma < 0.5$, the gains from multilateral free trade are:

$$\begin{aligned} G_{Bl}^{m} &= W_1^{m} - W^{A2} = \frac{5k^2 - 18k\sigma + 9\sigma\left[4(a-c)-\sigma\right]}{36\sigma} - \left(a-c-\frac{\sigma}{2}\right) \\ &= \frac{5k^2 - 18k\sigma + 9\sigma^2}{36\sigma} \end{aligned} \tag{116}$$

which is referred to as area $B_1 + B_2$ in figure 4-3. To see whether this is a gain or a loss, a critical value of the normalised trade cost in terms of the marginal disutility will be examined, i.e. $G_{Bl}^{m} = 0$, and assuming $a-c=1$,

$$K_{Bl}^{u} \equiv \frac{k_{Bl}^{u}}{a-c} = 3\sigma \quad \text{or} \quad 0.6\ \sigma$$

Welfare is higher under multilateral free trade if the relative trade cost is outside the range of the critical values ($k > 3\sigma$ or $k < 0.6\sigma$). Region B_1 in figure 4-3 is between 3σ and 0.6σ, therefore G_{Bl}^{m} is strictly negative and there are no gains from trade. Region B_2, on the other hand, shows strictly positive gains from multilateral free trade. When $\Sigma \geq 0.5$, the relevant region is $B_3 + B_4$, by using (102) and (85), the gains from multilateral free trade are:

$$\begin{aligned} G_{Br}^{m} &= W_1^{m} - W^{A1} = \frac{5k^2 - 18k\sigma + 9\sigma\left[4(a-c)-\sigma\right]}{36\sigma} - \frac{3(a-c)^2}{8\sigma} \\ &= \frac{10k^2 - 36k\sigma + 18\sigma\left[4(a-c)-\sigma\right] - 27(a-c)^2}{72\sigma} \end{aligned} \tag{117}$$

Comparing total welfare under multilateral free trade with total welfare under autarky as in (117), it is higher under multilateral free trade if the relative trade cost is outside the range of the critical values below.

$$K_{Br}^{m} \equiv \frac{k_{Br}^{m}}{a-c} = \frac{9\sigma}{5} \pm \frac{3\sqrt{30 - 80\sigma + 60\sigma^2}}{10}$$

Only region B_4 is within this range, and region B_3 is outside the range, leading G_{Br}^{m} to be strictly negative in region B_4 and strictly positive in B_3. Therefore, in the normal Hotelling competitive region, there are always gains from multilateral free trade in region B_2+B_3, and always losses from unilateral free trade in region B_1+B_4. Compare area B (competitive market region) in figure 4-3 to the same area in figure 4-2, under multilateral free trade, home firm benefits from foreign sales in addition to the home sales under unilateral free trade. It is expected the area of losses from trade is smaller in this case. Figure 4-3 shows that there are gains from trade in the competitive market (B_2+B_3).

Thirdly, consider the kinked demand Bertrand equilibrium in area C, where the kink occurs at profit maximising price. In figure 4-3, the $\Sigma=0.5$ line also goes through area C. On the left hand side when $\Sigma<0.5$, using (104), (106) and (87), gains from multilateral free trade are expressed as:

$$\begin{aligned} G_{2l}^{M} &= W_{2l}^{M}-W^{A2}=\frac{2(a-c)^2+2k^2+3\sigma\left[9\sigma-2(a-c)\right]}{18\sigma}-\left(a-c-\frac{\sigma}{2}\right) \\ &= \frac{2(a-c)^2+2k^2-12\sigma\left[2(a-c)-3\sigma\right]}{18\sigma} \end{aligned} \tag{118}$$

$$\begin{aligned} G_{2h}^{M} &= W_{2h}^{M}-W^{A2}=\frac{9(a-c)^2+4k^2+9\sigma\left[6\sigma-2(a-c)\right]}{36\sigma}-\left(a-c-\frac{\sigma}{2}\right) \\ &= \frac{9(a-c)^2+4k^2-18\sigma\left[3(a-c)-4\sigma\right]}{36\sigma} \end{aligned} \tag{119}$$

Which is shown in area C_1+C_2, the critical values of the gains above are calculated by setting both G_{2l}^{M} and G_{2h}^{M} to zero. Then the total welfare is higher under unilateral free trade if the relative trade cost is outside the range of the two roots of each equation. When $\Sigma<0.5$,

$$K_{2l}^{m}\equiv\frac{k_{2l}^{m}}{a-c}=\pm\sqrt{-1+6\sigma(2-3\sigma)}$$

$$K_{2h}^{m} \equiv \frac{k_{2h}^{m}}{a-c} = \pm\frac{3}{2}\sqrt{-1+6\sigma-8\sigma^{2}}$$

These critical values are shown in the region $C_1 + C_2$, since K_{2h}^{m} is below this region, only K_{2l}^{u} will be taken into account. The area below K_{2l}^{u} shows a loss from unilateral free trade, represented as area C_1, and the area above it shows a gain from trade, represented as area C_2. On the right hand side when $\Sigma \geq 0.5$, using (104), (106) and (85), gains from unilateral free trade are expressed as:

$$\begin{aligned} G_{2l}^{M} &= W_{2l}^{M} - W^{A1} = \frac{2(a-c)^2 + 2k^2 + 3\sigma\left[9\sigma - 2(a-c)\right]}{18\sigma} - \frac{3(a-c)^2}{8\sigma} \\ &= \frac{-19(a-c)^2 + 8k^2 + 12\sigma\left[9\sigma - 2(a-c)\right]}{72\sigma} \end{aligned} \tag{120}$$

$$\begin{aligned} G_{2h}^{M} &= W_{2h}^{M} - W^{A1} = \frac{9(a-c)^2 + 4k^2 + 9\sigma\left[6\sigma - 2(a-c)\right]}{36\sigma} - \frac{3(a-c)^2}{8\sigma} \\ &= \frac{-9(a-c)^2 + 8k^2 + 18\sigma\left[6\sigma - 2(a-c)\right]}{72\sigma} \end{aligned} \tag{121}$$

which is shown in area $C_3 + C_4$, the critical values of the gains are found by setting both $G_{2l}^{m} = 0$ and $G_{2h}^{m} = 0$. The total welfare is higher under unilateral free trade if the relative trade cost is outside the range of the two roots of each. when $\Sigma \geq 0.5$,

$$K_{2l}^{m} \equiv \frac{k_{2l}^{m}}{a-c} = \pm\frac{1}{2}\sqrt{\frac{19}{2} + 6\sigma(2-9\sigma)}$$

$$K_{2h}^{m} \equiv \frac{k_{2h}^{m}}{a-c} = \pm\frac{3}{2}\sqrt{\frac{1}{2} + 2\sigma - 6\sigma^{2}}$$

Once again, K_{2h}^{m} is below the region $C_1 + C_2$, so only K_{2l}^{m} will be considered. The area below K_{2l}^{m} shows a loss from unilateral free trade, represented as area C_4, and the area above K_{2l}^{u} shows gains from trade, represented as area C_3. Consequently, in the kinked demand region, a small area labelled $C_1 + C_4$ shows losses from multilateral free trade, and the rest area $C_2 + C_3$ shows gains from trade. Again, the area of gains from trade ($C_2 + C_3$) under multilateral free trade is larger than the one under unilateral free trade as shown in figure 4-2.

Finally, consider the local monopoly Bertrand equilibrium in area D of figure 4-1, so both home and foreign firm have a non-empty sale in the home country. Here, both firm one and firm two set monopoly price and output. The gains from unilateral free trade can be calculated by using (108) and (87):

$$\begin{aligned} G_D^M &= W_3^M - W^{A1} = \frac{6\left(a-c\right)^2 + 3k^2 - 6k\left(a-c\right)}{8\sigma} - \frac{3\left(a-c\right)^2}{8\sigma} \\ &= \frac{3\left(a-c\right)^2 + 3k^2 - 6k\left(a-c\right)}{8\sigma} \end{aligned} \tag{122}$$

As G_D^u is always positive from unilateral free trade, G_D^m must be always positive from multilateral free trade in the local monopoly region. To summarise, there is no gains or losses in the competitive market region A with pro-competitive effect, and there are always losses from multilateral free trade in the shaded area B_1, B_4 (part of the competitive market region), C_1 and C_4 (part of the kinked demand region). On the other hand, there are always gains from multilateral free trade in part of the competitive region B_2, B_3, part of the kinked demand region C_3, C_2 and the local monopoly region D.

Proposition 3*: In a Hotelling model of differentiated Bertrand duopoly, there are losses from multilateral free trade in the region $B_1 + B_4 + C_1 + C_4$, there is no gains or losses in the region A, and there are always gains from multilateral free trade for the rest of the region under $k = a - c$.*

Gains from multilateral free trade are summarized as follows:

$$
G \equiv \begin{cases}
G_1 = \begin{cases} \dfrac{10k^2 - 36k\sigma + 18\sigma\left[4(a-c)-\sigma\right] - 27(a-c)^2}{72\sigma} & if \;\; \sigma \geq 0.5 \\[2ex] \dfrac{5k^2 - 18k\sigma + 9\sigma^2}{36\sigma} & if \;\; \sigma < 0.5 \end{cases} & \text{if} \;\; k < k_c \\[6ex]
G_2 \in \begin{cases} G_{2l} = \begin{cases} \dfrac{-19(a-c)^2 + 8k^2 + 12\sigma\left[9\sigma - 2(a-c)\right]}{72\sigma} & if \;\; \sigma \geq 0.5 \\[2ex] \dfrac{2(a-c)^2 + 2k^2 - 12\sigma\left[2(a-c) - 3\sigma\right]}{18\sigma} & if \;\; \sigma < 0.5 \end{cases} \\[6ex] G_{2h} = \begin{cases} \dfrac{-9(a-c)^2 + 8k^2 + 18\sigma\left[6\sigma - 2(a-c)\right]}{72\sigma} & if \;\; \sigma \geq 0.5 \\[2ex] \dfrac{9(a-c)^2 + 4k^2 - 18\sigma\left[3(a-c) - 4\sigma\right]}{36\sigma} & if \;\; \sigma < 0.5 \end{cases} \end{cases} & \text{if} \;\; k_c \leq k < k_m \\[6ex]
G_3 = \dfrac{3(a-c)^2 + 3k^2 - 6k(a-c)}{8\sigma} & \text{if} \;\; k \geq k_m
\end{cases}
$$

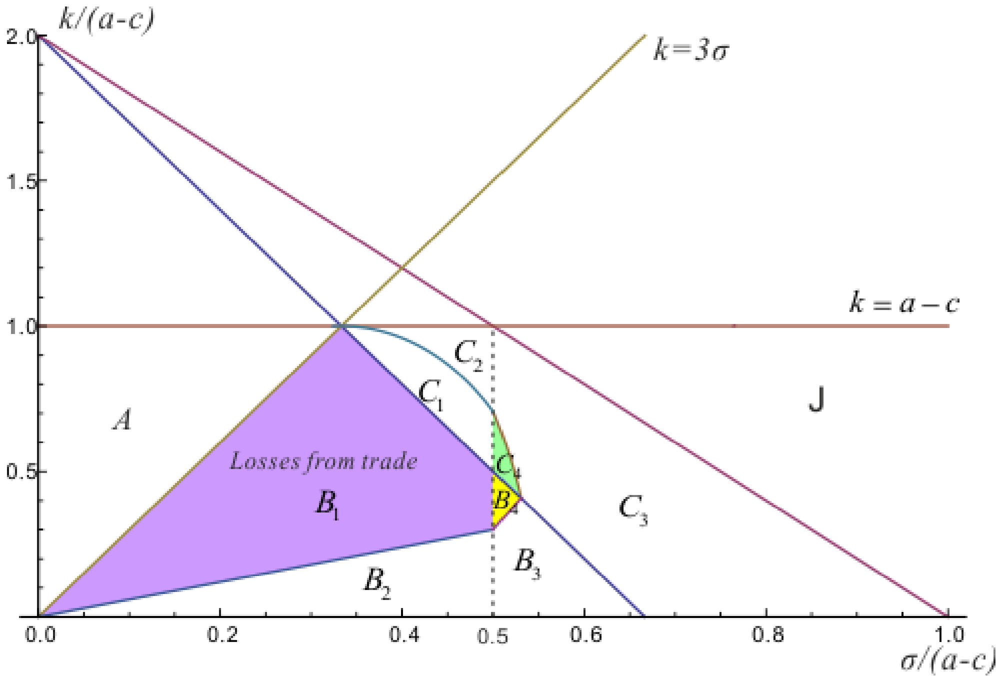

Figure 4-3: Gains from multilateral free trade

This result is different from Fujiwara (2009)'s losses-from-trade proposition. The welfare effect of trade liberalisation can be decomposed as follows: 1) trade promotes competition, so consumer surplus increases; 2) foreign import shifted home profits away, reducing home firm's welfare; 3) trade liberalisation increases wasteful resources because of transport cost, and 4) the positive variety-expanding effect from consuming different products. While Fujiwara (2009) concludes that the negative effect dominants the positive effect, so there are always losses, the result here shows that the last effect could be strong depending on product differentiation. If the products are highly differentiated, trade liberalisation leads to gains from trade, and if products are close substitutes, trade liberalisation leads to losses from trade. The product space in figure 4-2 and figure 4-3 gives clear insights of how welfare related to different parameters at the same time.

4.5 Volume of World trade

Using the same two-country, two-firm model, where a home and a foreign monopolist are producing a home and a foreign good symmetrically and exporting to the other market (multilateral free trade), it is sufficiently to analyse the home country economy only. Using the same Hotelling model, each consumer alone the unit length interval has a taste for the home and foreign firm, each firm has an incentive to supply home and foreign consumers under free trade, and these firms are engaged in Bertrand competition. The advantage of using this model is that there is no market expansion effect from product differentiation. As it mentioned before, σ represents marginal disutility in this model, when it increases, the goods are less substitutable for each other, competition between the firms fall, and products are more differentiated. Hence the higher the value of σ, the higher the degree of product differentiation, and σ measures the degree of product differentiation.

Under multilateral free trade, the volume of trade can be analysed in the normal Hotelling competitive market, the kink in demand case, and the local monopoly case (uncovered market). The volume of trade of the competitive market with a pro-competitive effect is not considered here (Area A in figure 4-1), as there is no actual trade in this case. The total volume of trade between home and foreign market is measured in terms of physical quantities of exports: $V = x_{2A}^{E} + x_{1B}^{E}$, while x_{2A}^{E} and x_{1B}^{E} are the same in the symmetric model. By using the Bertrand-Nash Equilibria prices and quantities that have been solved previously, we obtain:

Area B (Competitive market):

Assuming that the trade cost is below the prohibitive level, $k < \bar{k} = 3\sigma$, both firms export to the other market, and both markets are fully covered. By using (91), the volume of trade in terms of quantities is:

$$\begin{aligned} V_B &= x_{2A}^E + x_{1B}^E \\ &= \frac{2(3\sigma - k)}{6\sigma} = 1 - \frac{k}{3\sigma} \end{aligned} \tag{123}$$

The effect of a change of product differentiation with respect to the volume of trade can be evaluated by differentiating V_B with respect to σ, which is $\partial V_B / \partial\sigma = k/3\sigma^2 > 0$. Hence the volume of trade is increasing in the degree of product differentiation when products are close substitutes.

If σ increases, there is a higher degree of product differentiation, which lessens the competition, and the volume of world trade increases. This is due to the fact that the product differentiation is linked to import competition (competition of both firms in the home market in this case) in a negative way, implying that higher product differentiation will induce firms to supply more in both home and foreign market, as in Bernhofen (2001). In a competitive market, where market is fully covered and products are closer substitutes, the intensity of competition between two firms would be reduced by more differentiated products, therefore exporting more outputs. In addition, when goods are more differentiated, the volume of trade increase further, as consumers can enjoy a variety of goods. Hence the volume of trade increases in the degree of product differentiation when the market is fully covered.

Area C (Kinked demand Equilibria):

Assuming the trade cost is below the prohibitive trade cost $k < \bar{k}$, by using $x_{2A}^E \in \left[1 - \frac{a-c}{2\sigma},\ 1 - \frac{a-c}{3\sigma}\right]$ from summary 3, with the lower bound and the upper bound of the outputs, the volume of trade can be expressed in the range of:

$$V_C = x_{2A}^E + x_{1B}^E \in \left[2 - \frac{a-c}{\sigma},\ \ 2 - \frac{2(a-c)}{3\sigma} \right] \tag{124}$$

By differentiating V_C with respect to σ, which is from $\partial V_C / \partial\sigma = (a-c)/\sigma^2 > 0$ to $\partial V_C / \partial\sigma = 2(a-c)/3\sigma^2 > 0$, it also shows that the volume of trade increases in the degree of product differentiation when the market is completely covered and the indifferent consumer earns zero rent. Again, the explanation is that the increase of the product differentiation will increase the outputs of foreign firm, therefore increase the volume of trade.

Area D (local Monopoly)

Assuming the trade cost is below the prohibitive trade cost $k < \tilde{k} = a - c$, using equations (94), the volume of trade is expressed as:

$$V_D = x_{2A}^E + x_{1B}^E = \frac{a-c-k}{\sigma} \tag{125}$$

Differentiating V_D with respect to σ, the first derivative is $\partial V_D / \partial\sigma = -(a-c-k)/\sigma^2 < 0$, implying that the volume of world trade will decrease in the product differentiation when the home market is not fully covered. This is due to the fact that two firms do not compete with each other in either market and they act as individual monopolies in the home market, so an increase in product differentiation will not affect import competition in the home market, then export is solely caused by various consumers' tastes, which has a positive effect on trade. When the market is at local monopoly, trade costs are high and closer to its prohibitive level (refers back to equation (95)), each firm is selling more in its home market than its export to the foreign market, as the penalty on the foreign sales is higher, leading to a negative effect on the volume of trade. The negative effect of waste on trade costs dominants the positive effect of increased various consumers' tastes. Thus when products are sufficiently differentiated, the volume of trade/the market share for imports is decreasing in the degree of product differentiation.

***Proposition 4**: In a Hotelling model, the volume of trade in terms of quantities is increasing in the degree of product differentiation if the trade cost $k < \bar{k}$, $k < k_m$ and $k < \tilde{k}$, and it is decreasing in the degree of product differentiation if the trade cost $k > k_m$ and $k < \tilde{k}$.*

4.6 The profit change

This section considers how trade costs and product differentiation affects the profitability of trade liberalisation, and compares the profits under free trade and under autarky. Here only the multilateral free trade case is considered. The total profits of the firms are the sum of the profits from the home firm and the profits from the foreign firm. Thus the total profits for firm one are $\Pi_1 = \pi_{1A} + \pi_{1B}$, and they are the same for the symmetric firm two, i.e. $\Pi_1 = \Pi_2$. The profits changes across all the markets in the trade cost - marginal disutility space are shown in figure 4-4.

Area A (competitive market with pro-competitive effect):

In area A, when $\bar{K} < K \le K_c$ and $K \le \tilde{K}$ there is no trade between the domestic firm and the foreign firm, so the home firm only makes profits in the home market, but the presence of pro-competitive effect would reduce the price of the home firm. by using (99), the profits of the home firm under multilateral free trade are $\Pi_1 = \Pi_2 = \pi_{1A} + \pi_{2A} = 2\sigma$, and the profits under autarky case two are $\pi^{A2} = a - c - \sigma$. As $\Sigma \le 1/3$ and $a - c \le 1$ in this area, the profits change from trade when there is a pro-competitive effect is:

$$\Delta\Pi_A = \Pi_1 - \pi^{A2} = 3\sigma - (a - c) < 0 \tag{126}$$

Clearly, this is negative, implying that the profits of the home firm under the competitive market with a pro-competitive effect are lower than under the autarky case two, when firm one covers the entire home market. This is because of the reduced price in the home country caused by the potential competition from the foreign firm. Therefore when products are more substitutable (lower σ), import competition induces firms to reduce their supplies and trade liberalisation has a negative effect on firms' profitability.

Area B

In area B, when $K < K_c$, $K < \bar{K}$ and $K < \tilde{K}$, trade occurs between the home firm and the foreign firm. Both firms compete in the same market and the entire market is covered. Price must fall by the competition after the opening of trade in the home country, so are the profits of the home firm. By using (91), the total profits of the home firm under the free trade are $\Pi_1 = \Pi_2 = \pi_{1A}^E + \pi_{2A}^E = \frac{(3\sigma + k)^2}{18\sigma} + \frac{(3\sigma - k)^2}{18\sigma}$. As the line $\Sigma = 0.5$ goes through this area, both autarky cases one and two are to be considered. If $\Sigma < 0.5$, the profit gains from trade in the competitive market are:

$$\Delta\Pi_{B1} = \Pi_1 - \pi^{A2} = 2\sigma - (a-c) + \frac{k^2}{9\sigma}$$

It can be shown that profits are higher under the free trade if the relative trade cost is between the critical values:

$$\kappa_{\Pi 1} \equiv \frac{k_{\Pi 1}}{a-c} \equiv \pm 3\sqrt{(a-c-2\sigma)\sigma} \qquad (127)$$

The critical values are shown in figure 4-4. To the left hand side of $k_{\Pi 1}$, the profits change is negative, so the profits are lower under free trade than under autarky, whereas to the right hand side of $k_{\Pi 1}$, the profits change is positive (within the range of the critical values), so the profits are higher under free trade as shown in figure 4-4. If $\Sigma \geq 0.5$, referred as area B_2, the profits under autarky are $\pi^{A1} = \frac{(a-c)^2}{4\sigma}$, so the profits change from the trade are:

$$\Delta\Pi_{B2} = \Pi_{B2} - \pi^{A1} = \frac{(3\sigma + k)^2}{18\sigma} + \frac{(3\sigma - k)^2}{18\sigma} - \frac{(a-c)^2}{4\sigma} \qquad (128)$$

It can be shown that profits are higher under the free trade if the relative trade cost is between the critical values of trade costs:

$$\kappa_{\Pi 2} \equiv \frac{k_{\Pi 2}}{a-c} \equiv \frac{3}{2}\sqrt{\left(a-c\right)^{2} - 4\sigma^{2}}$$

This turns out that area B_2 is within the range of the above critical values, so the profits change is always positive in this area, implying that the profits are higher under free trade than under autarky. Consequently, in the competitive market, trade liberalisation is unprofitable if the products are less differentiated and close substitutes.

Area C (Kinked Demand)

In area C where a kinked demand occurs, when $K_c < K \leq K_m$, and $K < \tilde{K}$, trade also happens between the home firm and the foreign firm, and there is no overlapping of consumers who will be indifferent from buying the products, yet the entire market is covered. As it mentioned before, the price of the good is in a range, and so are the profits in this area. Again, the price is reduced by the competition in the home country, which will decrease the profits of the home firm, but the products are more differentiated in this region, which might increase the profits of the home firm by volume. This leads to the result that the profits gains from trade are uncertain when the demand is at the kink. When the demand function is at the kink, the profits of the home firm are the same as the ones in the competitive market, so the total profits of the home firm under the free trade are $\Pi_1 = \pi_{1A}^{E} + \pi_{2A}^{E} = \frac{\left(3\sigma + k\right)^{2}}{18\sigma} + \frac{\left(3\sigma - k\right)^{2}}{18\sigma}$. If $\Sigma < 0.5$, the profit gains from trade in the competitive market are: $\Delta\Pi_{C1} = \Delta\Pi_{B1} = 2\sigma - \left(a-c\right) + \frac{k^{2}}{9\sigma}$. Again, it can be shown that profits are higher under the free trade if the relative trade cost is between the critical values:

$$\kappa_{\Pi 1} \equiv \frac{k_{\Pi 1}}{a-c} \equiv \pm 3\sqrt{\left(a-c-2\sigma\right)\sigma}$$

To the left hand side of $\kappa_{\Pi 1}$ in area C of figure 4-4 (the shaded area in area C), the profits change is negative, so the profits are lower under free trade than under autarky, whereas to the right hand side of $\kappa_{\Pi 1}$, the profits change is positive, so the profits are higher under free trade. If $\Sigma \geq 0.5$ in region C, the profits change from the trade are:

$$\Delta\Pi_{C2} = \Pi_{C2} - \pi^{A1} = \frac{(3\sigma + k)^2}{18\sigma} + \frac{(3\sigma - k)^2}{18\sigma} - \frac{(a-c)^2}{4\sigma} \tag{129}$$

It can be shown that profits are higher under the free trade if the relative trade cost is between the critical values of trade costs: $\kappa_{\Pi 2} \equiv \frac{k_{\Pi 2}}{a-c} \equiv \frac{3}{2}\sqrt{(a-c)^2 - 4\sigma^2}$, which is to the left hand side of the line $\Sigma = 0.5$, so the profits change is always positive in this region, implying that the profits are higher under free trade than under autarky. Consequently, when the market is at the kinked demand $C_1 + C_2$, the free trade almost always has a positive effect on firms' profitability when the products are sufficiently differentiated.

Area D (uncovered market)

In area D, $\tilde{K} \geq K > K_m$, trade occurs but the home market is not completely covered, so both firms are monopolies in the same country. The price of the home firm under the local monopoly is the same as the one under autarky[43], i.e. $p_1 = \frac{a+c}{2}$, and the price of the foreign firm is higher with trade cost k, i.e. $p_2 = \frac{a+c+k}{2}$. Both firms sell in the home market, and the products are differentiated the most in this region, so the total sales/profits are higher than the autarky case, where only the home firm supplies the home market. Using (94), the total profits of the home firm under free trade are: $\Pi_1 = \Pi_2 = \pi_{1A} + \pi_{2A} = \frac{(a-c)^2}{4\sigma} + \frac{(a-c-k)^2}{4\sigma} = \frac{2(a-c)^2 + k^2 - 2k(a-c)}{4\sigma}$. As this is the region where the marginal disutility is greater than 0.5, i.e. $\Sigma \geq 0.5$, the

[43] Compare the price of firm one in equation (94) under local monopoly and the price in (84) under autarky, the prices of firm one are the same.

autarky is where the home firm covers the market partially. Therefore the profit gains from autarky are:

$$\Delta\Pi_D = \Pi_1 - \pi^{A1} = \frac{\left[k-(a-c)\right]^2}{4\sigma} \tag{130}$$

This is clearly positive, implying that the profits under free trade are higher than under autarky. The home firm's sales are the same in the home country, if it exports to the foreign market and still remains as a monopoly there, it will obviously gain the foreign market share and yet keep its home sale non-decreased.

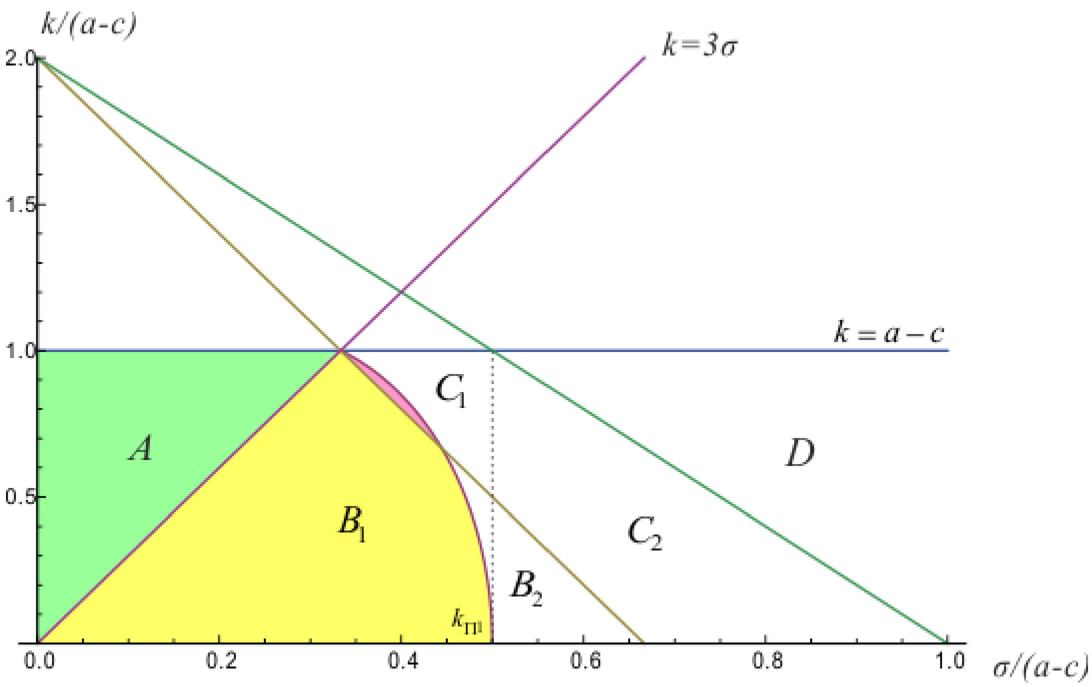

Figure 4-4: Profits change from free trade

Consequently, the shaded area in figure 4-4 represents the negative profits change. As it shows that when the products are more differentiated, the competition between firms is less intense, then firms are able to supply more goods to the market. Trade liberalisation is more profitable if the trade cost is relatively low, and products are highly differentiated. On the other hand, when products are close substitutes and not so much differentiated, trade liberalisation is unprofitable, as shown in the shaded area. These results are summarised in the following proposition:

Proposition 5*: Total profits of the firm under free trade are higher than under autarky if the trade cost* $k > k_{\Pi 1}$ *and* $k < a - c$.

4.7 Conclusion

This chapter has analysed the welfare effects of trade, gains from trade, and volume of trade in the Hotelling model of differentiated Bertrand duopoly. One of the advantages of using this model is that there is no market expansion effect, so increasing the degree of product differentiation will not affect the market size. Another distinc effect in the Hotelling model is that trade liberalisation enables those who can not buy differentiated product under autarky to consume it, so sonsumers gain from trade. The current research considers the kinked-demand market structure in the trade, which was rarely mentioned in other literature. Furthermore, the best reply functions are not symmetric as conventionally stated, this is due to the fact that foreign firm faces trade costs per unit when exporting to the home market, while the home firm does not have this addition marginal cost, causing asymmetric cost/demand in the home market.

This chapter comments on Fujiwara (2009)'s result, where he shows that there are always losses from trade in a Hotelling model of differentiated duopoly, and proves that it is an incorrect statement. While he ignores the kinked-demand structure in the spatial model, this chapter has considered the full features of the Hotelling model, and uses a trade costs-marginal disutility space to demonstrate how product differentiation and trade costs affect firms' welfare. Our research shows 1) there are gains from trade when products are highly differentiated and losses from trade when products are close substitutes. 2) welfare under free trade is less than welfare under autarky when the market is competitive; welfare under free trade is higher than welfare under autarky when the market is local monopoly; welfare under free trade is the same as welfare under autarky when there is a pro-competitive effect; and lastly welfare gain/loss is ambiguous when the market structure is kink in demand.

In addition, the volume of trade are also analysed, that it is increasing in the degree of product differentiation when products are close substitutes, and decreasing in the degree of product differentiation when products are sufficiently differentiated. When the products are more differentiated, the competition between firms is less intense, and then firms are able to supply more goods to the market. Trade liberalisation is more profitable if the trade cost is relatively low, and products are highly differentiated.

Appendix

Appendix A: The best response functions

Firm one maximises its profit by solving: $\pi_{1A}^{E}=\left(p_1-c\right)x_1$ as shown in (90) and (93). Given the expression for x_1, two cases need to be distinguished according to whether the demand function of firm one has a kink as in (97) or not as in (98).

1. If $p_2+\sigma\geq a$, there is a kink as shown in (97). The left-hand side of π_{1A}^{E} evaluated at the boundary $p_1=2a-\sigma-p_2$ is $\pi_1=\dfrac{\left(p_1-c\right)\left(p_2-p_1+\sigma\right)}{2\sigma}$, then maximise it with respect to p_1, this derivative must be smaller than the boundary: $p_1=\dfrac{p_2+\sigma+c}{2}\leq 2a-\sigma-p_2$. Solving for the relative range of p_2, $p_2\leq\dfrac{4}{3}a-\dfrac{c}{3}-\sigma$. The right-hand side of π_{1A}^{E} evaluated at $p_1=2a-\sigma-p_2$ is: $\pi_1=\dfrac{\left(p_1-c\right)\left(a-p_1\right)}{\sigma}$, then maximise it with respect to p_1, we get $p_1=\dfrac{a+c}{2}\geq 2a-\sigma-p_2$. Solving for p_2, $p_2\geq\dfrac{3}{2}a-\dfrac{c}{2}-\sigma$. It is assumed that $a>c$, so $\dfrac{3}{2}a-\dfrac{c}{2}-\sigma>\dfrac{4}{3}a-\dfrac{c}{3}-\sigma$. Hence, there are three cases when $p_2+\sigma\geq a$, and there is a kinked structure in between:

$$p_1^{*}=\begin{cases}\max\left\{p_2-\sigma,\ \dfrac{p_2+\sigma+c}{2}\right\} & \mathit{if}\quad p_2\leq\dfrac{4}{3}a-\dfrac{c}{3}-\sigma\\ 2a-\sigma-p_2 & \mathit{if}\quad \dfrac{4}{3}a-\dfrac{c}{2}-\sigma<p_2\leq\dfrac{3}{2}a-\dfrac{c}{3}-\sigma\\ \max\left\{\dfrac{a+c}{2},\ a-\sigma\right\} & \mathit{if}\quad p_2>\dfrac{3}{2}a-\dfrac{c}{2}-\sigma\end{cases}$$

- ***Strict duopoly***: when $p_2\leq\dfrac{4}{3}a-\dfrac{c}{3}-\sigma$: the peak occurs on the range $p_1\in\left[p_2-\sigma,2a-p_2-\sigma\right]$. Firm one competes with firm two to attract the buyer and marginal surplus is positive. Refer back to the first equation of (90), differentiate the profits π_{1A}^{E} with respect to p_1, the reaction function is

$$p_1 = \frac{p_2 + \sigma + c}{2} \tag{131}$$

and an indifferent consumer who earns positive surplus exists. This occurs when the trade cost k is smaller than the prohibitive trade cost $\bar{k}$. When the trade cost is greater than $\bar{k}$, firm one has an incentive to set its price such that the output supply of firm two to the home market is zero ($x_2 = 0$). By setting such a limiting price, firm one can supply the whole market yet still facing a competition from firm two in the home market, i.e. $x_2 = \frac{p_1 - p_2 + \sigma}{2\sigma} = 0$, then p_1 is $p_2 - \sigma$. This is referred to as pro-competitive effect in the Bertrand competition. Therefore the optimal price is given by $p_1 = \max\left(p_2 - \sigma, \frac{p_2 + \sigma + c}{2}\right)$ in the competitive market case.

- ***Local Monopoly***: when $p_2 \geq \frac{3}{2}a - \frac{c}{2} - \sigma$, the peak occurs when $p_1 > 2a - p_2 - \sigma$. The indifferent consumer earns negative utility, so he /she is better off not to purchase from either firm, and only the nearest customers would make a purchase. The monopoly demand is $x_1 = (a - p_1)/\sigma$. Firm one's profit is $(p_1 - c)x_1 = (p_1 - c)(a - p_1)/\sigma$, maximising it with respect to p_1 yields $x_1 = (a - c)/2\sigma$, so the optimal price in this case is: $p_1^* = \frac{a + c}{2}$. However, this only happens with the condition that $x_1 = (a - c)/2\sigma \leq 1$, which means $\Sigma \geq 1/2$. When $\Sigma < 1/2$, firm one is supplying the whole market facing no competition from firm two, i.e. $x_1 = 1 = \frac{a - p_1}{\sigma}$, then the optimal price is $p_1^* = a - \sigma$, which is the same as autarky case two. Hence, no equilibrium exists when one firm covers the entire market. This price is greater or equal to $2a-\sigma-p_2$, thus $p_2 \geq a$. These two optimal prices are mutually exclusive[44], depending on the value of Σ:

$$p_1^* = \begin{cases} a-\sigma & if \quad \Sigma < \dfrac{1}{2} \quad and \quad p_2 \geq a \\ \dfrac{a+c}{2} & if \quad \Sigma \geq \dfrac{1}{2} \quad and \quad p_2 \geq \dfrac{3}{2}a - \dfrac{c}{2} - \sigma \end{cases}$$

- ***Kinked Demand***: when $\frac{4}{3}a - \frac{c}{2} - \sigma < p_2 \leq \frac{3}{2}a - \frac{c}{3} - \sigma$. The peak occurs at $p_1 = 2a - \sigma - p_2$. On this range, the optimal strategy is to price at the kinked structure of the demand function, so the indifferent consumer earns zero surplus when purchasing from either firm. The optimal price is[45] $p_1^* = 2a - \sigma - p_2$.

2. If $p_2 + \sigma < a$, only the competitive market demand exists, as shown in (98). In this case, $x_1 = \dfrac{p_2 - p_1 + \sigma}{2\sigma}$ is on the range $p_1 \in [p_2 - \sigma, p_2 + \sigma]$. The optimal price is either $p_1 = p_2 - \sigma$ or $p_1 = \dfrac{p_2 + \sigma + c}{2}$, depending on the magnitude of the two. In addition, $p_1 = \dfrac{p_2 + c + \sigma}{2}$ must be smaller than $p_2 + \sigma$, implying $p_1 \geq c - \sigma$ .therefore:

$$p_1^* = \begin{cases} \dfrac{p_2 + \sigma + c}{2} & if \quad p_2 < c + 3\sigma \\ p_2 - \sigma & \text{if} \quad p_2 \geq c + 3\sigma \end{cases}$$

[44] Notice that when $\Sigma \geq 1/2$, $a \geq \frac{3}{2}a - \frac{c}{2} - \sigma$, and vice versa.

[45] The utility or the surplus of the indifferent consumer is zero at the kink. i.e. $U = a - p_1 - \sigma x_1 = 0$ where $x_1 = \dfrac{p_2 - p_1 + \sigma}{2\sigma}$, then $p_1 = 2a - \sigma - p_2$ is the reaction function.

If $p_2 \geq c+3\sigma$, all consumers are willing to purchase the good from firm one, as firm one sets a limiting price, such that the consumers located closer to firm two would obtain greater utility from buying at firm one than the surplus he would get from buying at firm two. Thus firm one covers the whole market, and the demand of good from firm one is equal to one ($x_1 = 1$). If $p_2 < c+3\sigma$, the usual competitive market indifferent consumer exists between zero and one segment, then consumers closer to firm two would obtain strictly smaller utility from buying at firm one than buying at firm two, which is the nearest firm for him/her.

To summarize the above, the reaction functions for firm one can be expressed as:
For $p_2 + \sigma \geq a$

- If $p_2 \leq \frac{4}{3}a - \frac{c}{3} - \sigma$ and $p_2 < c+3\sigma$: $p_1^* = \frac{p_2 + \sigma + c}{2}$ (competitive Portion of Demand)
- If $p_2 \leq \frac{4}{3}a - \frac{c}{3} - \sigma$ and $p_2 \geq c+3\sigma$: $p_1^* = p_2 - \sigma$ (competitive portion of demand)
- If $p_2 \geq \frac{3}{2}a - \frac{c}{2} - \sigma$, and $\Sigma \geq \frac{1}{2}$: $p_1^* = \frac{a+c}{2}$ (monopoly portion of demand)
- If $p_2 \geq a$, and $\Sigma < \frac{1}{2}$: $p_1^* = a - \sigma$ (monopoly portion of demand)
- If $\frac{4}{3}a - \frac{c}{2} - \sigma < p_2 < \frac{3}{2}a - \frac{c}{3} - \sigma$: $p_1^* = 2a - \sigma - p_2$ (kink in demand)

For $p_2 + \sigma < a$

- If $p_2 \geq c+3\sigma$, $p_1^* = p_2 - \sigma$ (competitive portion of demand)
- If $p_2 < c+3\sigma$, $p_1^* = \frac{p_2 + \sigma + c}{2}$ (competitive portion of demand)

Unlike MÃrel and Sexton (2010), firm two and firm one in the home market are not symmetric, a trade cost of k incurs when firm two supplies to the home market. Firm two maximises its profit, given price p_2, by solving: $\pi_{2A}^E = \left(p_2 - c - k\right)x_2$, as shown in (90) and (93). By using the same steps as for firm one in the home market, its reaction function is obtained as:

For $p_1 + \sigma \geq a$

- If $p_1 \leq \frac{4}{3}a - \frac{c+k}{3} - \sigma$ and $p_1 \leq c+k+3\sigma$: $p_2^* = \frac{p_1 + \sigma + c + k}{2}$ (competitive portion of demand)
- If $p_1 \leq \frac{4}{3}a - \frac{c+k}{3} - \sigma$ and $p_1 > c+k+3\sigma$: $p_2^* = p_1 - \sigma$ (competitive portion of demand)
- If $p_1 \geq \frac{3}{2}a - \frac{c+k}{2} - \sigma$, and $\Sigma \geq \frac{1}{2}$: $p_2^* = \frac{a+c+k}{2}$ (monopoly portion of demand)
- If $p_1 \geq a$, and $\Sigma < \frac{1}{2}$: $p_2^* = a - \sigma$ (monopoly portion of demand)
- If $\frac{4}{3}a - \frac{c+k}{2} - \sigma < p_1 < \frac{3}{2}a - \frac{c+k}{3} - \sigma$: $p_2^* = 2a - \sigma - p_1$ (kinked demand)

For $p_1 + \sigma < a$

- If $p_1 \geq c+k+3\sigma$, then $p_2^* = p_1 - \sigma$ (competitive portion of demand)
- If $p_1 < c+k+3\sigma$, then $p_2^* = \frac{p_1 + \sigma + c + k}{2}$ (competitive portion of demand)

The Nash Equilibria $\left(p_1,\ p_2\right)$ are determined by the intersection of the reaction curves p_1^* and p_2^*. Although there are a few options in the reaction functions stated above, no equilibrium exists when one firm covers the entire market facing competition from the other firm (i.e. $p_1^*\left(p_2^*\right) = a - \sigma$ do not exist). For p_1^*, p_2 always lies between c and a, and $a \geq c$, $\frac{3}{2}a - \frac{c}{2} - \sigma > \frac{4}{3}a - \frac{c}{3} - \sigma$ as well as $\frac{4}{3}a - \frac{c}{3} - \sigma \geq a - \sigma$. For p_2^*, p_1 always lies between $c+k$ and a, $a \geq c+k$, as well as $\frac{4}{3}a - \frac{c+k}{3} - \sigma \geq a - \sigma$, and $\frac{3}{2}a - \frac{c}{2} - \sigma > \frac{3}{2}a - \frac{c+k}{2} - \sigma$, $\frac{4}{3}a - \frac{c}{3} - \sigma > \frac{4}{3}a - \frac{c+k}{3} - \sigma$, $c+k+3\sigma > c+3\sigma$. Thus the shape of the equilibrium p_1^* and p_2^* are determined by using the following expressions:

- $c \le a - \sigma \quad \Rightarrow \quad \Sigma \le 1$
- $c + k \le a - \sigma \quad \Rightarrow \quad \Sigma \le 1 - \mathrm{K}$
- $c \le \frac{4}{3}a - \frac{c}{3} - \sigma \quad \Rightarrow \quad \Sigma \le \frac{4}{3}$
- $c \le \frac{4}{3}a - \frac{c+k}{3} - \sigma \quad \Rightarrow \quad \Sigma \le \frac{4}{3} - \frac{1}{3}\mathrm{K}$
- $c + k \le \frac{4}{3}a - \frac{c+k}{3} - \sigma \quad \Rightarrow \quad \Sigma \le \frac{4}{3} - \frac{4}{3}\mathrm{K}$
- $c + k \le \frac{4}{3}a - \frac{c}{3} - \sigma \quad \Rightarrow \quad \Sigma \le \frac{4}{3} - \mathrm{K}$
- $c \le \frac{3}{2}a - \frac{c}{2} - \sigma \quad \Rightarrow \quad \Sigma \le \frac{3}{2}$
- $c \le \frac{3}{2}a - \frac{c+k}{2} - \sigma \quad \Rightarrow \quad \Sigma \le \frac{3}{2} - \frac{1}{2}\mathrm{K}$
- $c + k \le \frac{3}{2}a - \frac{c+k}{2} - \sigma \quad \Rightarrow \quad \Sigma \le \frac{3}{2} - \frac{3}{2}\mathrm{K}$
- $c + k \le \frac{3}{2}a - \frac{c}{2} - \sigma \quad \Rightarrow \quad \Sigma \le \frac{3}{2}(a - c) - \mathrm{K}$
- $a \ge \frac{3}{2}a - \frac{c}{2} - \sigma \quad \Rightarrow \quad \Sigma \ge \frac{1}{2}$
- $a \ge \frac{3}{2}a - \frac{c+k}{2} - \sigma \quad \Rightarrow \quad \Sigma \ge \frac{1}{2} - \frac{1}{2}\mathrm{K}$
- $a \ge \frac{4}{3}a - \frac{c}{3} - \sigma \quad \Rightarrow \quad \Sigma \ge \frac{1}{3}$
- $a \ge \frac{4}{3}a - \frac{c+k}{3} - \sigma \quad \Rightarrow \quad \Sigma \ge \frac{1}{3} - \frac{1}{3}\mathrm{K}$
- $c + 3\sigma \le a \quad \Rightarrow \quad \Sigma \le \frac{1}{3}$
- $c + k + 3\sigma \le a \quad \Rightarrow \quad \Sigma \le \frac{1}{3} - \frac{1}{3}\mathrm{K}$
- $c + 3\sigma \le a - \sigma \quad \Rightarrow \quad \Sigma \le \frac{1}{4}$

- $c+k+3\sigma \leq a-\sigma \quad \Rightarrow \quad \Sigma \leq \frac{1}{4}-\frac{1}{4}\mathrm{K}$

All cases are distinguished by the term $a-c$ and k, and the Nash equilicria are listed in the main text.

Appendix B: Tables

Table 4-1: Profits

	x_{1A}	x_{2A}	p_1	p_2	π_{1A}	π_{2A}
Autarky ($\sigma < 0.5$)	1	0	$a-\sigma$	$a-\sigma$	$a-c-\sigma$	0
Autarky ($\sigma \geq 0.5$)	$\frac{a-c}{2\sigma}$	0	$\frac{a+c}{2}$	$\frac{a+c}{2}$	$\frac{(a-c)^2}{4\sigma}$	0
Competitive Market ($k \geq \bar{k}$)	1	0	$c+2\sigma$	$c+3\sigma$	2σ	0
Competitive Market ($k < \bar{k}$)	$\frac{3\sigma+k}{6\sigma}$	$\frac{3\sigma-k}{6\sigma}$	$\frac{3c+3\sigma+k}{3}$	$\frac{3c+3\sigma+2k}{3}$	$\frac{(3\sigma+k)^2}{18\sigma}$	$\frac{(3\sigma-k)^2}{18\sigma}$
Kinked Demand Lower band	$\frac{a-c}{3\sigma}$	$1-\frac{a-c}{3\sigma}$	$\frac{2a+c}{3}$	$\frac{4a-c-3\sigma}{3}$	$\frac{2(a-c)^2}{9\sigma}$	$\left[4(a-c)-3\sigma-3k\right]\cdot\left[3\sigma-(a-c)\right]/9\sigma$
Kinked Demand upper band	$\frac{a-c}{2\sigma}$	$1-\frac{a-c}{2\sigma}$	$\frac{a+c}{2}$	$\frac{3a-c-2\sigma}{2}$	$\frac{(a-c)^2}{4\sigma}$	$\left[3(a-c)-2\sigma-2k\right]\cdot\left[2\sigma-(a-c)\right]/4\sigma$
Monopoly (uncovered market)	$\frac{a-c}{2\sigma}$	$\frac{a-c-k}{2\sigma}$	$\frac{a+c}{2}$	$\frac{a+c+k}{2}$	$\frac{(a-c)^2}{4\sigma}$	$\frac{(a-c-k)^2}{4\sigma}$

Table 4-2: Consumer surplus and Welfare

	CS	$W_1(unilateral)$	$W_1(multilateral)$
Autarky ($\sigma < 0.5$)	$\frac{\sigma}{2}$	$a-c-\frac{\sigma}{2}$	$a-c-\frac{\sigma}{2}$
Autarky ($\sigma \geq 0.5$)	$\frac{(a-c)^2}{8\sigma}$	$\frac{3(a-c)^2}{8\sigma}$	$\frac{3(a-c)^2}{8\sigma}$
Competitive Market ($k \geq \bar{k}$)	$a-c-\frac{5}{2}\sigma$	$a-c-\frac{1}{2}\sigma$	$a-c-\frac{1}{2}\sigma$
Competitive Market($k < \bar{k}$)	$a-c-\frac{3}{2}\sigma-\frac{2}{3}k+\frac{(3\sigma+k)^2}{36\sigma}$	$\frac{3k^2-6k\sigma+9\sigma\left[4(a-c)-3\sigma\right]}{36\sigma}$	$\frac{5k^2-18k\sigma+9\sigma\left[4(a-c)-\sigma\right]}{36\sigma}$
Kinked Demand Lower band	$\frac{(a-c)^2}{9\sigma}+\frac{3\sigma-2a+2c}{6}$	$\frac{4(a-c)^2+2k^2+12\sigma k-12\sigma\left[(a-c)-3\sigma\right]}{36\sigma}$	$\frac{2(a-c)^2+2k^2+3\sigma\left[9\sigma-2(a-c)\right]}{18\sigma}$
Kinked Demand upper band	$\frac{(a-c)^2}{4\sigma}+\frac{\sigma-a+c}{2}$	$\frac{9(a-c)^2+2k^2+12\sigma k-18\sigma\left[(a-c)-2\sigma\right]}{36\sigma}$	$\frac{9(a-c)^2+4k^2+9\sigma\left[6\sigma-2(a-c)\right]}{36\sigma}$
Monopoly (uncovered market)	$\frac{2(a-c)^2+k^2-2k(a-c)}{8\sigma}$	$\frac{4(a-c)^2+k^2-2k(a-c)}{8\sigma}$	$\frac{6(a-c)^2+3k^2-6k(a-c)}{8\sigma}$

Table 4-3: Gains from trade

	σ	$G_1(unilateral)$	$G_1(multilateral)$
Competitive Market ($k \geq \bar{k}$)	$\sigma < 0.5$	0	0
Competitive Market($k < \bar{k}$)	$\sigma < 0.5$	$\frac{3k^2 - 6k\sigma - 9\sigma^2}{36\sigma}$	$\frac{5k^2 - 18k\sigma + 9\sigma^2}{36\sigma}$
	$\sigma \geq 0.5$	$\frac{6k^2 - 12k\sigma + 18\sigma\left[4(a-c) - 3\sigma\right] - 27(a-c)^2}{72\sigma}$	$\frac{10k^2 - 36k\sigma + 18\sigma\left[4(a-c) - \sigma\right] - 27(a-c)^2}{72\sigma}$
Kinked Demand Lower band	$\sigma < 0.5$	$\frac{4(a-c)^2 + 2k^2 + 12k\sigma - 6\sigma\left[8(a-c) - 9\sigma\right]}{36\sigma}$	$\frac{2(a-c)^2 + 2k^2 - 12\sigma\left[2(a-c) - 3\sigma\right]}{18\sigma}$
	$\sigma \geq 0.5$	$\frac{-19(a-c)^2 + 4k^2 + 24k\sigma - 24\sigma\left[(a-c) - 3\sigma\right]}{72\sigma}$	$\frac{-19(a-c)^2 + 8k^2 + 12\sigma\left[9\sigma - 2(a-c)\right]}{72\sigma}$
Kinked Demand Upper band	$\sigma < 0.5$	$\frac{9(a-c)^2 + 2k^2 + 12\sigma k - 54\sigma\left[a-c-\sigma\right]}{36\sigma}$	$\frac{9(a-c)^2 + 4k^2 - 18\sigma\left[3(a-c) - 4\sigma\right]}{36\sigma}$
	$\sigma \geq 0.5$	$\frac{-9(a-c)^2 + 4k^2 + 24\sigma k - 36\sigma\left[(a-c) - 2\sigma\right]}{72\sigma}$	$\frac{-9(a-c)^2 + 8k^2 + 18\sigma\left[6\sigma - 2(a-c)\right]}{72\sigma}$
Monopoly (uncovered market)	$\sigma \geq 0.5$	$\frac{(a-c)^2 + k^2 - 2k(a-c)}{8\sigma}$	$\frac{3(a-c)^2 + 3k^2 - 6k(a-c)}{8\sigma}$

Table 4-4: Change in Profits

	σ	$\Delta\Pi_1$	κ_Π
Competitive Market **($k \geq \bar{k}$)**	$\sigma < 0.5$	$3\sigma - (a-c)$	–
Competitive Market($k < \bar{k}$)	$\sigma < 0.5$	$2\sigma - (a-c) + \frac{k^2}{9\sigma}$	$3\sqrt{(a-c-2\sigma)\sigma}$
	$\sigma \geq 0.5$	$\frac{(3\sigma+k)^2}{18\sigma} + \frac{(3\sigma-k)^2}{18\sigma} - \frac{(a-c)^2}{4\sigma}$	$\frac{3}{2}\sqrt{(a-c)^2 - 4\sigma^2}$
Kinked Demand Lower band	$\sigma < 0.5$	$\frac{2(a-c)^2 + [4(a-c)-3\sigma-3k][3\sigma-(a-c)] - 9\sigma(a-c-\sigma)}{9\sigma}$	$\frac{2(a-c)}{3}$
	$\sigma \geq 0.5$	$\frac{2(a-c)^2 + [4(a-c)-3\sigma-3k][3\sigma-(a-c)]}{9\sigma} - \frac{(a-c)^2}{4\sigma}$	$\frac{17(a-c)^2 - 60(a-c)\sigma - 36\sigma^2}{12(a-c-3\sigma)}$
Kinked Demand Upper band	$\sigma < 0.5$	$\frac{(a-c)^2 + [3(a-c)-2\sigma-2k][2\sigma-(a-c)] - 4\sigma(a-c-\sigma)}{4\sigma}$	$a-c$
	$\sigma \geq 0.5$	$\frac{[3(a-c)-2\sigma-2k][2\sigma-(a-c)]}{4\sigma}$	$\frac{3(a-c)}{2} - \sigma$
Monopoly (uncovered market)	$\sigma \geq 0.5$	$\frac{[k-(a-c)]^2}{4\sigma}$	$a-c$

Chapter 5: Conclusion

This study concentrates on two aspects of international trade with Oligopoly: FDI versus exporting decision under Oligopoly, and trade liberalisation in a Hotelling model of differentiated duopoly. The trade liberalisation has led to a rapid growth in FDI during the experience of the 1990s, while the conventional theory predicts that a reduction in trade costs will discourage FDI. The first part of this study has discussed about this contradiction and analysed the FDI versus exporting decision by using a two-country four-firm model with identical products under Cournot oligopoly, assuing demand is linear.

In the static game, a reduction in the trade cost will lead the firms to switch from undertaking FDI to exporting, which confirms the proximity-concentration trade-off theory, but conflicts with the empirical result in the 1990s. The equilibria outcomes show that if the fixed cost of FDI is relatively low, both firms in the same market choose to undertake FDI. If the fixed cost is relatively high, one firm chooses to export while its competitor from the same market chooses to undertake FDI. Thus, this study contributes to the current literature by showing that both export and FDI can exist as an equilibrium outcome in the world.

It also shows that the static game is often a prisoners' dilemma, where all firms might make lower profits when they all undertake FDI than when they export, and it is likely happen when the fixed cost is relatively high. If the fixed cost is sufficiently high, the equilibrium profits when one firm in each country undertakes FDI while its competitor in the same country export might be lower than the profits when all firms export, due to the intensified competition.

The prisoners' dilemma can be avoided in an infinitely-repeated game when all firms tacitly collude over their FDI versus export decisions, as collusion over FDI can be sustained by the threat of Nash-reversion strategies if the trade cost is sufficiently high. Then this study shows that a reduction in trade costs may lead firms to switch from exporting to undertaking FDI if the trade cost is sufficiently high, as a reduction in a sufficiently high trade costs lessen the profitability of collusion in the infinitely-repeated game, and that explains the experience of the increasing FDI in 1990s. Another contribution of this study is: it is shown that a reduction in the fixed cost may lead firms to switch from undertaking FDI to exporting when the fixed cost is relatively high.

Conventionally, the demand function is assumed to be linear, as shown in chapter 2. To check the robustness of the result above, this study also examines the FDI versus exporting decisions by looking at the constant elasticity of demand function. It has been confirmed that all the results are quite similar. In the static game, a reduction in the trade cost will lead the firms switch from undertaking FDI to exporting. The same outcomes are achieved that two firms in the same country choose to undertake FDI if the fixed cost is relatively low, or one firm chooses to export while its competitor from the same country chooses to undertake FDI if the fixed cost is relatively high.

The prisoners' dilemmas still exist and can be avoided in an infinitely-repeated game when all firms tacitly collude over their FDI versus export decisions. Then a reduction in trade costs may lead firms to switch from exporting to undertaking FDI when the trade cost is sufficiently high.

While the first part investigates the strategic choices between FDI and trade under oligopoly in the trade liberalisation, the second part of this study examines how trade liberalisation affects the welfare gains, profits, and the volume of trade in the Hotelling model of differentiated Bertrand duopoly.

The welfare analysis has been evaluated in Cournot duopoly and Bertrand duopoly in a large amount of literature, but the Hotelling model has rarely been used. One of the advantages of using this model is that there is no market expansion effect, so increasing the degree of product differentiation will not affect the market size. By applying the spatial model, trade liberalisation could be assessed in a different market structure, as in Fujiwara (2009), where he ignores an important part of the Hotelling model: the kinked demand market structure, and makes an unrealistic assumption about the parameters. This study has considered the full features of the Hotelling model, and uses a trade costs-marginal disutility space to demonstrate how product differentiation and trade costs affect welfares in trade liberalisation.

The purpose of the study is not to criticize Fujiwara (2009)' losses-from-trade proposition, but to verify his result by looking at a full picture of the model. It proves that his statement of losses from trade is incorrect. The result shows that gains from trade could happen when products are highly differentiated, and losses from trade happen when products are close substitutes, because the positive effect of more product choices overweighs the negative effect of the decreased home sales caused by trade liberalisation when products are more differentiated. On the other hand, when products are closer substitutes, this positive effect is small relative to the total negative effect.

Gains from trade are also investigated by looking at different market structure in the trade costs-marginal disutility product space. Under the unilateral free trade, when there is a pro-competitive effect, there are no gains or losses from trade, as trade does not actually occur. When the market is competitive, there are always losses from trade, as the fall in the home profits overweighs the rise in the consumer surplus as well as gains from enjoying more variety of products. When the market structure is kinked-demand, gains from trade are ambiguous, depending on the product differentiation and trade costs. When there is a local monopoly, there are always gains from trade as the products are highly differentiated that consumer can benefit from a much wider range of goods. On the other hand, welfare effects under the multilateral free trade are similar, and gainsfrom trade are more likely to happen.

Furthermore, the volume of trade are also analysed, and it is increasing in the degree of product differentiation when the trade cost is sufficiently low, and decreasing in the degree of product differentiation when the trade cost is sufficiently high. This is because higher trade costs cause firm selling more in its home market than its export to the foreign market, as the penalty on the foreign sales is higher, leading to a negative effect on the volume of trade. In addition, trade liberalisation is more profitable if the trade cost is relatively low, and products are highly differentiated. On the other hand, when products are close substitutes and not so much differentiated, trade liberalisation is unprofitable.

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