Essays on Price Rigidity in the UK: Evidence from Micro Data and Implications for Macro Models

by

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ABSTRACT

This study consists of three individual essays which all shed light on assessing the price rigidity by using price micro data in the UK. The relevant implications for macro models are also discussed in each essay respectively.

The first essay gives a unified framework *a la* Dixon (2012) to gauge the price rigidity from three perspectives: frequency, hazard function and distribution across firms. On average, the monthly frequency of consumer price change is 19% between 1996 and 2007. Sales and substitutions will significantly affect the frequency of consumer price change. The frequency of consumer price change varies considerable across sectors. The fraction of price changes which are decreasing is about 40%. The hazard function is downward sloping with 12-month spike. The censoring and sampling issues in the estimation of hazard function are discussed thoroughly. The distribution across firms is derived from estimated hazard function, which is consistent with the frequency of price changes. Two benchmark sticky price models are calibrated and simulated. Furthermore, a multiple Calvo and multiple menu costs model are also simulated, based on the empirical finding in micro data. The simulation results suggest that introducing heterogeneity into sticky price models can improve models' fitness in respect to matching micro evidence.

The second essay mainly focus on "the monthly frequency of price changes", which is a prominent feature of many studies of the CPI micro-data. In this essay, we see how much the frequency ties down the behavior of price-setters ("firms") in steady-state in terms of the average length of price-spells across firms. We are able to divide an upper and lower bound for the mean duration of price-spells averaged across firms. We use the UK CPI data at the aggregate and sectoral level and find that the actual mean is about twice the theoretical minimum consistent with the observed frequency. We estimate the distribution using the hazard function and find that although the estimated hazard differs significantly from the Calvo distribution, the means and medians are similar. However, despite the micro differences, we find that the artificial Calvo distributions generated using the sectoral frequencies result in very similar impulse responses to the estimated hazards when used in the Smets-Wouters (2003) model.

The third essay examines the behavior of individual producer prices in the UK. A number of stylized facts about price setting behavior are uncovered. A time-varying Ss model is set up in a way that is consistent with the stylized facts obtained from the UK PPI data. A duration model (semiparametric survival analysis model) is built in line with the time-varying Ss model. This duration model is estimated by controlling for observed and unobserved heterogeneity across firms. The estimation results suggest that the increase in the inflation rate will significantly increase the hazard rate of price change. The other factors considered in the model will also affect the hazard rate of price change, while in different magnitude.

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[1]We provide empirical evidence about frequency of price changes, hazard functions, and DAF. We do not provide detailed empirical findings about the magnitude of price changes since this chapter is not focus on this issue. However, we do find that the size of price change on average is 15%. And there is significant heterogeneity in the magnitude of price change among different COICOP sectors.

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Chapter 1

General Introduction

In this thesis, I intend to analyze nominal price rigidities under a unified framework proposed by Dixon (2012), employing micro price data underlying CPI and PPI in the United Kingdom. I also attempt to reconcile micro and macro evidence, adopting a common general framework that allows for an explicit modeling of the distribution of contract lengths and for different types of price setting. I also evaluate how far the theories are consistent with the micro evidence on price rigidity. I investigate the firm's decision to change its price by developing a time-varying Ss band model, which allows evaluation of differences across firms and economic sectors in the hazard rate of price changes. In this chapter, I will provide a brief overview of the area of study explored in this thesis. The motivation behind the study will be outlined and introductions to the areas of research, the research questions and an insight into the structure of the thesis will be explored.

1.1 Research Background and Current Situation

The study of nominal price rigidities is "one of the hot topics of research in macro today". Blanchard (2009) highlighted the role of nominal rigidities in his paper of "The State of Macro":

> The new-Keynesians...accepted the need for better foundations for the various imperfections underlying that approach. The research program became one of examining, theoretically and empirically, the nature and the reality of various imperfections, from nominal rigidities, to efficiency wages, to credit market constraints.

There is a considerable amount of theoretical and empirical literature on nominal price rigidities.

1.1.1 Models of pricing

There are several theoretical approaches in the literature modelling nominal rigidities at the individual level. They are based on various assumptions for price non-adjustment: Calvo/Taylor type contract; menu costs, sticky information; customer anger. I will review the assumptions and implication of these pricing models, so as to check them against micro evidence and macro evidence.

Taylor/Calvo type model   Nominal prices, according to Taylor (1980), are fixed by assumption for a certain number periods. If price changes were perfectly staggered over

time, duration of nominal prices remains constant for all .rms. In the Taylor model, prices are fixed for N periods and the hazard rate is zero for all duration except N; when the period is N, the hazard rate is one. Hence, the hazard rate is the measurement of likelihood of a price change depending on the elapsed duration of a price spell.

In the Calvo (1983) model, the probability of a price change is constant. Each period, a fixed proportion of firms are able to change prices; the remaining firms keep their nominal prices fixed. The probability of being able to change price is the same for all firms, regardless of when they changed price last. This means that the hazard rate is constant.

The simple Taylor and Calvo type price setting models are inadequate in generating enough persistence of output and inflation to monetary policy shocks (e.g., Fuhrer and Moore, 1995, Chari *et al.,* 2000, Christian *et al.*, 2005).

One popular theoretical justification is to add indexation to the Calvo model (e.g., Smets and Wouters 2003, Woodford 2003, Christiano *et al.* 2005). Price is set at the beginning of the contract, and for the contract duration this is updated by the previous period's inflation. Though the Calvo with indexation can model inflation and output persistence well, it is at the cost of having prices changing every period.

Generalized Taylor (GT)[1] model and Generalized Calvo model[2] are introduced to explain inflation and output persistence while being consistent with the microevidence on nominal rigidity. In GT model, there are many sectors with different price-spell

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[1] See Taylor (1993), Coenen *et al* (2007), and Dixon and Kara (2010,2011), Dixon (2012) for details.
[2] See Wolman (1999), Mash (2003), Sheedy (2007), Dixon (2012).

lengths, and within each sector there is a simple Taylor process. In GC model, the reset probability is duration-dependent. Additionally, we can model the price setting strategy as a multiple sector Calvo model (MC), which is special case of GC[3]. A key feature of these generalized models is that they reflect the substantial heterogeneity observed in micro data. Price rigidity varies across sectors. In the presence of pricing complementarities the slow adjusting sectors have a disproportionally large effect on overall price adjustment, slowing the price response, and increasing the output response to shocks. The intuition is as follows. When a heterogeneous economy is hit by a shock, the initial adjustment takes place by firms mostly in the fast adjusting sectors. As time passes, a large proportion of firms that still have to adjust are firms in the slow-adjusting sector. In other words, the adjustment process is dominated initially by high frequency adjusters and later by low frequency adjusters. Furthermore, the presence of slow adjusting sectors and strategic complementarities slows down price adjustment in the fast adjusting sectors.

As supposed by Dixon (2012), we can link the GT with micro data by looking at the cross-sectional distribution of duration across firms, and link the GC to the micro data through the hazard function. Moreover, MC can be linked to micro data by looking at the frequency of price changes at sectoral level.

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[3]See Carvalho (2006), and Dixon (2012).

Menu costs

The menu costs model assumes that the price change is costly and these costs prevent firms from changing prices in a continuous manner. Sheshinski and Weiss (1977) show that in presence of price changing costs, the optimal pricing policy is of the (S,s) type. The S and s indicate the upper and lower bound for the real price, respectively. Once the real price lies within the bounds, the nominal price will be kept constant. Over the pricing period, the optimal policy is a kind of state-contingent policy.

Under state-contingent policy, firms adjust prices within a relative large range due to the fact that the current prices had been deviated from the optimal price. That the aggregate price level will be adjusted more rapidly to nominal changes is resulted from the selection effect, in particular, when firms are constrained by state-contingent instead of time-contingent pricing. Monetary shocks have been evidenced to have longer lasting real output effect in time-dependent (i.e. Calvo) models than in menu costs model[4]. In addition, monetary impulses make prices elicit more rapid response in menu costs models than in time-dependent models.

The menu costs models are usually solved using numerical methods, so no analytical expression is available for the hazard rate. In most calibrations made by previous researchers, the hazard is increasing with the duration.

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[4]See Caplin and Spulber (1987), Dotsey et al. (1999), Gertler and Leahy (2006), and Golosov and Lucas (2007).

Sticky information

It is assumed that it is costly for firms to gather information about the current economic conditions (Mankiw and Reis, 2002). Fischer (1977) pointed out those opportunities to adopt new price paths had been evidenced to arise stochastically. New information about the state of the economy has been adopted and a new path of optimal prices has been updated each period. Outdated information is used to make pricing decisions by the rest of firms. Inflation, therefore, depends on previous expectations of current inflation and output.

Because substantially larger and persistent real effects are resulted from monetary shocks, the sticky information model fit macroeconomic facts better. However, in absence of other frictions, some form of indexation no matter it is to the general, sectoral or price level is involved in firms' optimal price plans in absence of other frictions. Therefore, all firms change price all the time in sticky information model. This argument, however, is contradictory to empirical evidence based on micro economic data. Prior studies make attempts to solve this problem by combining sticky information with menu costs (Klenow and Willis, 2006; Knotek, 2006). Using both assumptions, namely, sticky information and menu costs, causes two consequences: prices are not constantly changed; and old aggregate level inflation innovations will be reflected by the new prices when both assumptions change. This is due to the fact that price changes are not in accordance with getting informed about the state of the economy. Knoteck (2006) suggested that the micro features of the economy can be inferred by adjusting parameters to match general

macroeconomic behaviour. He found that information is updated once every 7 quarters typically; price is changed once every 2 quarters; and 10% of prices remain unchanged for more than 2 years. An economy in which firm face both menu costs and a cost of knowing macroeconomic conditions needs to be considered (Gorodnichenko, 2008). Two methods that firms can adopt to get information either paying the costs or by learning the actions of other .rms. This results in an information externality and encourages firms delay their price adjustment. Following a shock, price adjustment is postponed because firms expect to catch the chance to observe other firm's actions in order to achieve better pricing choice. As a result, inflation responds to nominal shocks slowly and the response is hump-shaped, and inflation is persistent.

Customer anger

Rotemberg (2005) develops a model to explain the price rigidity. This model indicates that consumers analyse firms' pricing decision depending on the perception of fairness. If consumers are convinced that price are unfair, they will have adverse reaction towards relevant products or services. Hence, firms may reluctant to change prices to avoid potential anger. Chances are that firms keep prices the same even if they desire to change prices due to the fact that price changing will elicit consumers' evaluation of fairness of prices and cause potential negative reactions. Overall, firms respond to macroeconomics conditions and change prices. For example, consumers are unlikely to have negative reaction and accept prices change under the circumstances of rapid inflation. In addition,

customers who are heterogeneous in information holding will have various responses to price changing. Firms will change the prices within different time schedule pertinent to the development of consumers beliefs.

1.1.2 Empirical studies

Micro price studies have been conducted using different sources of data, such as the micro data underlying national CPIs and PPIs, sub-national store scanner data, "scraping" prices from internet, and the survey data. A set of stylized facts have been found in empirical analyses. They are summarized as following[5]:

Prices change infrequently

Dhyne *et al.* (2006) find that the monthly frequency of consumer price change is about 15% in the euro area. Excluding the countries with the highest and the lowest frequency, the resulting frequency of price changes is 16.9%. The implied average duration, which equals to the inverse of frequency, is about a year. Vermeulen *et al.* (2006) show that the average frequency of producer price change is 21% in the euro area. Bils and Klenow (2004) report that the average frequency of consumer price change in the US for 1995-1997 period is 26.1%; While Klenow and Kryvtsov (2008) find that the monthly frequency of the US consumer price change is 29.3% between 1998 and 2003.

Sales are common in the US consumer price data, the share of sales prices in the US

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[5]This study mainly reviews the empirical finding from using CPI and PPI micro data in the US and the euro area, respectively. For a good broaden review, see Klenow and Malin (2009).

CPI data is about 20%. Specifically, Nakamura and Steinsson (2008) define sale as a situation in which a price falls temporarily and then returns to the price in effect just before the decrease, and in such situation no regular price change is recorded. There are some other methods to identify sales. For example, Kehoe and Midrigan (2007) indicates a sale if "price decrease is followed by any price increase thereafter". Nakamura and Steinsson (2008) find that after excluding the sale prices, the median frequency of the US consumer prices is 11.1% in 1988-1997, and 8.7% in 1998-2005.

Substitutions are observed in many sectors. Nakamura and Steinsson (2008) report that the monthly rate of forced item substitution is about 10 percent in "Apparel" and "Transportation Goods", and about 6 percent in "Recreation Groups". Klenow and Willis (2007) estimate that price changes associated with item substitutions are sensitive to inflation. However, the findings of Klenow and Willis (2007) depend to some degree on their modeling assumptions.

The frequency of price changes varies across sectors

Dhyne *et al.* (2006) document the heterogeneity with respect to frequency of consumer price changes across sectors in the euro area. The frequency of price changes ranges from 5.6% in "Services" to 28.3% in "Unprocessed Food" and 78% in "Oil Products". Nakamura and Steinsson (2008) report the distribution of the frequency of price changes for about 300 Entry Level Items (ELI). The ELI with the highest frequency of price changes, 100%, is "Used Cars". The ELI with the lowest frequency of price changes,

1.6%, is "Legal Services".

The micro data show that, in the real world, there are many heterogeneous firms. In general, a macro model with a "representative firm" does not behave like a model with many heterogeneous firms. One can construct and calibrate a model with a representative firm and compare selected predictions of this model to a version of the same model with many heterogeneous firms. If the predictions from the representative firm model come close to those obtained from heterogeneous firm model, then the predictions of the representative firm model are reliable.

Price decreases are common

Dhyne *et al.* (2006) report that 42% of price changes are decreases in the euro area. While Nakamura and Steinsson (2008) find this fraction to be roughly one-third in both consumer prices excluding sales and finished-goods producer prices. Food and energy price increases and decreases are almost equally likely. For industrial goods 43% price changes are decreases. Price decreases are less common for services, for which they constitute only 20% of price changes. However, micro data suggest that there is no stronger downward nominal price stickiness.

The finding that price decreases are common has important implications for the calibration of models of price rigidity. As shown in Nakamura and Steinsson (2008), the fraction of price changes that are decreases helps to pin down the key parameters in a benchmark menu costs model along the lines of Golosov and Lucas (2007). It also

provides strong evidence for the hypothesis that idiosyncratic shocks are an important driving force for price changes.

Downward sloping hazard function

A "dynamic" feature that has been documented in many studies is the shape of the hazard function of price change. The general finding in the literature is that hazard function is decreasing. Alvarez (2008) reports that the frequency of price changes conditional on reaching a given age is downward sloping when all goods are considered together, but note that this could be the consequence of heterogeneity in the probability of price adjustment. For a price changed recently there is a high likelihood that the good is a flexible price good and so the probability of the occurrence of a price change is high; for a price that has been unchanged for a long time it is likely that the good is a sticky price good and the probability of price change is low. This is so called "selection effect". Hence the empirical hazard rate is downward sloping.

Nakamura and Steinsson (2008) estimate the hazard function of price change for consumer and producer prices, controlling for heterogeneity across products. The hazard functions are downward sloping for the first few months, then mostly flat except for a large twelve-month spike in all major groups. Accounting for heterogeneity leads to a substantial flattening of the hazard function. But they do not find evidence to support for upward sloping hazard function. Furthermore, they suggest that menu costs model can generate a variety of shapes of hazard functions, depending on the relative importance

of transitory and permanent shocks to marginal costs. Firms may be more attentive to getting prices right when revenue is temporarily high for a product due to idiosyncratic supply or demand considerations. Large idiosyncratic shocks tend to produce a downward sloping hazard.

1.1.3 Summary

Previous research has demonstrated the importance of assessing the nominal price rigidity using micro data. The stylized facts obtained from micro data can help us to examine pricing behaviour at the firm level, where pricing decisions are actually made. Individual information on price setting allows determining to which extent the assumptions used in deriving theoretical models are actually realistic, which helps refine modeling strategies.

1.2 Research objectives

My thesis consists of three individual essays which all shed light on assessing the price rigidity by using price micro data in the UK. The relevant implications for macro models are also discussed in each essay respectively. My thesis mainly focuses on a few research questions shown as follows:

1. How often do consumer prices change?

2. How can we deal with the censoring and sampling issue when the hazard function is estimated from the UK CPI micro data?

3. Would the probability of price change vary along the duration of price spells?

4. How can we derive the distribution across firms which is consistent with a given mean frequency of price changes?

5. Whether the workhorse pricing models can fit the empirical evidence in micro data, if not, what can be done to refine modeling strategies?

6. What is the minimum (maximum) mean duration of price-spells averaged across firms consistent with a given frequency of price changes?

7. Can we just use frequencies of price change to generate corresponding hypothetical distributions which can match the "true" distribution across firms?

8. Do the DSGE models under the Calvo distribution hypothesis behave in a way similar to models calibrated with the microdata ("true" DAF)?

9. How can we build a time-varying Ss model which has implication consistent with the micro evidence found in the UK PPI micro data?

10. How can we evaluate the effects of covariates on the hazard rate?

11. How can we control for the unobservable heterogeneity when hazard function is estimated?

1.3   Research outline

The rest of this thesis can be briefly described as following:

In Chapter 2, I assess the price rigidity in a unified framework *a la* Dixon (2012), using CPI micro data in the UK. I estimate the frequency of price change, given the information about sales and substitutions. I give a detailed discussion about the effect of censoring and sampling on the estimation of hazard function. I derive the distribution across firms which are consistent with a given mean frequency of price changes in terms of the corresponding proportion of firms resetting prices. I exam pricing behaviour of two benchmark pricing models: menu costs and Calvo. And I build models with heterogeneous structure in price setting to improve models' fitness in respect to matching microevidence.

In Chapter 3, I seek to analyze how much the frequency could tie down the behaviour of firms in steady-state in terms of the average length of price spells across firms. I derive an upper and lower bound for the mean duration of price-spells averaged across firms. Then the UK CPI micro data at the aggregate and sectoral level are used to find that the actual mean is about twice the theoretical minimum consistent with the observed frequency. I estimate the distribution using the hazard function and find that although the estimated hazard differs significantly from the Calvo distribution, the means and medians are similar. However, despite the micro differences, I find that the artificial Calvo distributions generated using the sectoral frequencies result in very similar impulse responses to the estimated hazard when used in the Smets-Wouters (2003) model.

In Chapter 4, I examine the behaviour of individual producer prices in the UK. A number of stylized facts about price setting behaviour are uncovered. The weighted average frequency of price change is 25%, and there are about 44% of price adjustments

are price decreases. The frequency of price changes varies substantially across industry groups and product sectors. The unconditional hazard function displays a downward sloping pattern with annual spikes. Then I set up a time-varying Ss model which is consistent with the micro evidence. Then a duration model is specified according to the Ss model. I estimate the duration model which controls for observable and unobservable heterogeneity across firms in assessing the effect of changes in inflation, interest rate, oil price, industrial output, and exchange rate on the hazard rate of price changes.

In Chapter 5, I discuss the major findings of this research. I also discuss the limitations of this research and suggest areas for future research.

Chapter 2

Frequency, Hazard Function, and Distribution across Firms

2.1   Introduction

In monetary macroeconomics, the price rigidity plays a key role in many models' setup. Not only it affects the "real effect" of monetary policy and the dynamic of inflation, but it also has a deep influence on optimal monetary policy. There are quite a few important questions which need to be addressed in the micro price data set, such as: how often do the consumer price and/or producer price change, would the probability of price change vary along the duration of price spell, and what does the distribution of price durations look like? Furthermore, we would like to investigate whether the workhorse pricing models can fit the empirical evidence in micro price data. In this chapter, we intend to

indicate the stylized facts of the price setting mechanism in the UK economy, and take important steps at discriminating among models by assessing the ability of "two of the most popular models of price rigidity"– the Calvo and menu costs models – to match key empirical features of firm's price-setting behaviour we found in the UK CPI micro data.

Price stickiness has been studied at the level of individual firms since 1920s. Mills (1927) and Means (1935) distinguish two types of products: the one with flexible price, and the other one with "administered price", and they find that many prices change infrequently and the frequency of price changes varies widely across goods. Their work has spurred a voluminous literature. Early studies mainly focus on price adjustment for particular products. Mussa (1981) and Weiss (1993) describe the behavior of newspaper prices during the 1920s German hyperinflation. Carlton (1986) finds evidence on the dynamics of industrial prices. Cecchetti (1986) investigates the magazine cover prices. Lach and Tsiddon (1992) study the behavior of the prices of 26 foodstuffs at grocery store in Israel. Kashyap (1995) checks the catalogue product.

Recently, many researchers broaden the source of micro price data. Blinder et al (1998) make surveys of firm managers. Eichenbaum, Jaimovich and Rebelo (2011) use scanner data from supermarkets, drugstores, and mass merchandisers in the U.S. Cavallo (2009) collects the online-shopping price data in Brazil.

Date sets underlying official CPIs have become available to researchers since new millennium. For the U.S., the leading research was conducted by Bils and Klenow (2004). In their influential work, they focus on the micro-data of CPIs between 1995 and 1997

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in U.S. They find that the price rigidity is not serious. On average, the price can remain the same for 3 to 4 months, which is much shorter than the estimate from macro model. Klenow and Kryvtsov (2008), Nakamura and Steinsson (2008) both extend the work of Bils and Klenow (2004) in different directions. Nakamura and Steinsson (2008) not only investigate the CPI micro data, but also do the thorough research on PPI micro data. For the euro area, Dhyne et al. (2006) have a good summary of all the related work done by the members of Inflation Persistence Network[1]. For the United Kingdom, Bunn and Ellis (2012a) (BE hereafter) is the first trial. Our study differs from BE from three aspects. First, we assess the price rigidity in a unified framework, which includes frequency, hazard function, and distribution across firms. Second, we estimate the hazard function using all the normal (uncensored) and right-censored price spells. Whereas BE only choose one complete price spell for each item to estimate the hazard function. We point out that not only this method suffers the loss of important information from discarding large amount of spells, but also faces the problem of selection bias. Third, we derive out the distribution of durations across firms. And we argue that the cross-sectional feature of price setting behaviour is important to make macro-model fit the micro-evidence, as suggested by Dixon and Kara(2010).

Individual firms do not continuously adjust their prices in response to shocks that hit

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[1]For the other countries: Baharad and Eden (2004) point out that the popular method adopted by most researchers suffer the over-sampling bias. They announce a method to correct the downward bias in estimation of price rigidity. Gagnon (2007) investigates the price rigidity for Mexico during the hyperinflation period. Kovanen (2006) gets the result for Sierra Leone; Goueva (2007) has the estimation of price rigidity for Brazil; Masahiro and Saita (2007) report the conclusion for Japan; Hofstetter (2010) provides evidence on how often prices change in Colombia.

the economy. To model this fact, the economic literature considers mainly two types of pricing behaviour: time dependent and state dependent pricing rules. According to the former, firms are assumed to change their prices periodically using either a deterministic (Taylor, 1980) or a stochastic (Calvo, 1983) process of price adjustment. More specifically, in the Calvo model, firms face a probability of optimizing their price, which is exogenous and constant. Therefore, the timing of the price changes is exogenous and does not depend either on the timing of the shocks or on the state of the economy. Due to its tractability, the Calvo model enjoys popularity in the macroeconomic literature.

Firms following state-dependent pricing rules are usually assumed to review their prices whenever relevant shocks hit the economy but, due to the existence of fixed costs of changing prices (e.g., the costs of printing and distributing new price lists), they change their prices only when the difference between the actual and target prices is large enough (see, for example, Sheshinski and Weiss, 1977, Caplin and Spulber, 1987, Caballero and Engel, 1993, Dotsey *et al.*, 1999). Thus, a company facing these menu costs will change its price less frequently than an otherwise identical firm without such costs.

There are debates on how to embed price stickiness inside macroeconomic models, and whether one can construct a price-setting model consistent with the micro evidence that has plausible macroeconomic implications.

In this chapter we examine the frequency of price changes underlying the UK CPI micro data, with the consideration on the effect of sales and substitutions. The major aim is to analyze the degree of the nominal rigidity present in United Kingdom consumer

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prices and trying to applying a unified framework (proposed by Dixon, 2012) to study the characteristics in price setting behaviour.

We find that the monthly average frequency of price changes is 19 percent for identical items in CPI micro data. Moreover, the frequency of price changes varies considerably across sectors and products. For various fuel types and seasonal food products the average frequency can be as high as 90 percent per month. However, for several services and administered prices, such as automatic car wash, and digital photo printing, the frequency can be as low as 3 percent per month. To provide a measure of distribution of price durations across firms, we estimate the aggregate hazard function for all price spells. Similar to previous studies, we find that the aggregate hazard function is decreasing with most marked annual spike. We derive the distribution of durations across firms from the hazard function. In line with previous studies (Baharad and Eden 2004, Dixon 2009), the distribution of duration across firms has fatter long tail than the distribution of duration across contract, indicating that the price is stickier than we thought. Moreover, the "stickier" sectors dominate the behaviour of inflation in the longer term.

The potential contributions of this chapter are three folds: we firstly take empirical study of price rigidity under the framework of distribution across firms. A unified framework for using micro-data *a la* Dixon (2012) combines frequency method with hazard function, and then derives the distribution across firms (hereafter DAF). Secondly, we investigate the censoring issue in detail, and discuss the different effect on the estimation of hazard function resulted from different method on dealing with right-censoring. Thirdly,

we calibrate a benchmark menu costs model and Calvo model to match the evidence we obtained from micro-data, and we check the implication of different pricing models with heterogeneous structure.

The structure of the chapter is allocated as following: Section 2 provides the data description and some specific data issues; Section 3 discusses the methodology used in assessing price rigidity, pointing out the connection among frequency, hazard function and DAF; Section 4 provides the empirical results we get from CPI micro data; Section 5 maps micro-evidence with macro model; Section 6 provides possible extension to the benchmark pricing model; and in Section 7 we conclude.

2.2 Data

2.2.1 Date description

The micro data used in this paper are produced by the Office of National Statistics (ONS hereafter), Because of the confidentiality issues relating to information collected about individual firms, it is not possible to makes this type of data widely available. The micro data that underlie the consumer and producer price indices used in this research were made accessible via the VML[2].

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2.2.2 CPI data

The ONS collect a longitudinal micro data set of monthly price quotes from over ten thousands of outlets to compute the national index of consumer prices. There are two basic price collection methods: local and central. Local collection is used for most items. There are about 150 locations around country, and around 120,000 quotations are obtained each month by local collection. For some items, collection in individual shops across the 150 areas is not required- for example, for larger chain stores who have a national pricing policy or where the price is the same for all UK residents or the regional variation in prices can be collected centrally. Central collected data cover about 33% of CPI, but they are not available to our research[3]. Therefore, our CPI research data mainly are local collected, covering about two thirds of total CPI. The sample spans over the time period from January 1996 to December 2007 and contains between 112,676 (1996) and 99,524 (2007) elementary price quotations per month. And our data sample includes over 14 million observations. The coverage and classification of the CPI indices are based on the international classification system for household consumption expenditures known as COICOP (classification of individual consumption by purpose). This is a hierarchical classification system comprising: *divisions* e.g. 01 Food and non-alcoholic beverages, *groups* e.g. 01.1 Food, and *classes* (the lowest published level) e.g. 01.1.1 Bread and

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[3]The sample excludes 33% of CPI items which are central collected. The central collected data set include price quotes for education, some of the energy goods, and some of the communication services. However, we don't have the detail of the description. We do recognize the potential sample selection bias. But the data availability is the most common issue in this kind of micro price data research. And we argue that we have the most representative data set so far to fulfil our research objectives.

COICOP division	Freq.	Percent	Cum.
Food and Non-Alcoholic Beverages	25,191.51	17.62	17.62
Alcoholic Beverages and Tobacco	10,083.28	7.05	24.67
Clothing and Footwear	13,323.33	9.32	33.98
Housing and Utilties	9,350.23	6.54	40.52
Furniture and Home Maintenance	16,211.75	11.34	51.86
Health	2,705.55	1.89	53.75
Transport	14,800.15	10.35	64.1
Communications	237.2797	0.17	64.27
Recreation and Culture	14,085.51	9.85	74.12
Education	6.340364	0	74.12
Restaurants and Hotels	25,087.06	17.54	91.67
Miscellaneous Goods and Services	11,918.02	8.33	100
Total	143,000	100	

Table 2.1: CPI share in COICOP sectors

cereals. As table 2.1 shows[4], the division Food and non-alcoholic beverages accounts for about 17% of the CPI weight in the subsample available in the dataset. The education division is excluded from our research due to the lack of observation.

In our CPI research data set, each individual price quote consists of information on the item code, the outlet, the region, the date and etc. And we define the product category at the elementary level, for example, an item could be indicated as large loaf, white, unsliced (800g). There are a total of 675 products categories in our raw dataset, and the product varies by its specific variety and brand with each product category. However, the data set has been anonymized with respect to the variety and brand of the product.

---

[4]Similar to the situation illustrated in Bunn and Ellis (2012a), only the locally collected data that account for about two thirds of the CPI are available. While centrally collected data are not available for the research. Therefore, the average weights in this study do not equal to the average weights underlying official CPI. This is in line with the finding in Bunn and Ellis (2012a, Figure 1). Specifically, the average weight of Food and Non-Alcoholic Beverages in this study is about 17%, which is higher than the average published CPI weight (about 12%) ; the average weight of Communication is this study is less than 1%, while the average published CPI weight is about 2%.

With the information on the item i, the shop j, the location k, and the date t, we can construct a price trajectory $P_{ijk,t}$, which is sequence of price quotes for a specific item belonging to a product category in a specific shop over time. Specifically, we take two sequential price quotes belong to the same price trajectory if they have the same product identity, location and shop code. There are about $614,000$ price trajectories. And the average length of each price trajectory is about 24 months.

As Table 2.2 described, the average length of price trajectory differs quite a lot among different COICOP sectors. The sector with longest price trajectory in average is health sector, indicating that the item belonging to health sector appear in CPI research data as long as two and a half year. On contrary, an item in the communication sector usually disappears from our CPI basket after one and a half year. It shows that the communication sector is updated more frequently in CPI basket.

Furthermore, we can define a price spell as the sequence of price quotes with the same price for a specific item in a specific shop. The length of price spell (duration) is a key factor when we assess the price rigidity. Moreover, the distribution of duration will provide us a possible perspective to model the price rigidity. We will try to measure the average length (duration) of price spell and get the distribution of duration in following sector.

COICOP Sector	Mean length of price trajectory	Median length of price trajectory
Food and Non-Alcoholic Beverages	26.0	21
Alcoholic Beverages and Tobacco	28.6	22
Clothing and Footwear	19.6	13
Housing and Utilties	25.2	21
Furniture and Home Maintenance	24.5	19
Health	30.1	23
Transport	26.7	23
Communications	18.7	12
Recreation and Culture	23.3	19
Education	na	na
Restaurants and Hotels	25.3	23
Miscellaneous Goods and Services	26.1	22

Table 2.2: Average length of price trajectory by COICOP sector

2.2.3 Specific data issues

Censoring

Censoring is a word often used in survival analysis literature, which is defined as a situation when the failure event occurs and the subject is not under observation. In this paper, a failure event is specified as a price change. Suppose that the surveys sample individual price quote between the date B and E (as in Figure2-1. The date B is the beginning of the survey period, while date E is the ending of it. We define a normal price spell as the period between two price changes. Censoring occurs when you are prevented from seeing the exact time of the failure event (price change, in our context).Therefore, censoring entails a downward bias in the estimation of the duration of price spells, as

longer price spells are more likely to be censored. In general, we can classify the price spells into four groups with different censoring type as shown in Figure2-1:

1. Non-censored price spell, or normal price spell: we can observe the beginning and ending of the price spell explicitly, which can be seen as S1 in Figure2-1

2. Left-censored price spell: if we do not know the true starting date of the price spell. This usually happens when new item or product is introduced to CPI basket. S2 in Figure2-1 gives an example. Spells such as S4 may be recorded as a duration beginning at B and lasting until the completion of the spell, in which case the actual duration is unknown since the time from the inception of the spell to the beginning of survey (B) is unknown.

3. Right-censored price spell: if we do not know the ending date of the price spell. E.g. a product withdraws prematurely; a product no longer be available in an outlet, or the outlet shut down; or at the end of sample period. We can illustrate this type of spell as S3 in Figure2-1.

4. Double-censored price spell: as shown as S4 in Figure2-1,if both the start and the end of the spell is not observed, i.e. the price spell of the product actually begins before the statistical agency starts to observe the product, and ends after the statistical agency stops to observe the product.

We can summarize the reasons which can explain the censored price spells in our observations. First, the observation period is restricted by the database availability.
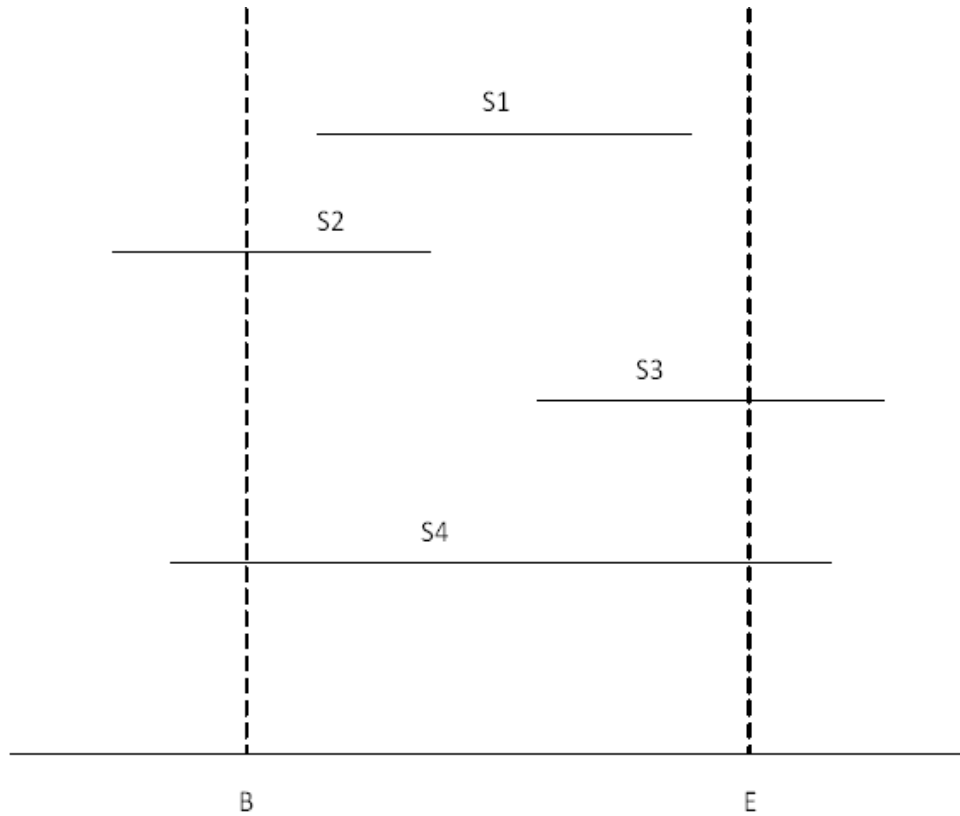
Figure 2-1: Type of price spells

This makes it very likely to observe product prices in a given outlet after the current price of the product was actually set and/or before that price ceases to exist. Indeed, the probability that the first spell in a price trajectory is left-censored is high, as is one of the last spell being right-censored. Second, the sampling of products and outlets by the statistical agency is also likely to generate some censoring. Indeed, the statistical institute may decide to discard a specific product from the "representative" CPI basket due to changes in technology or consumer behavior that shrink the expenditure share on these products (e.g. black and white TV sets) although they may still be sold in outlets. Then,

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the last price spell of such a product will be right-censored. Conversely, products may be included in the CPI basket and their price observed after they were actually available for consumers. This will generate left-censoring of the price spell. Third, outlets and firms may decide to stop selling a product which price was followed by the statistical agency. Then, the procedure which is often adopted by statistical agencies in charge of computing the CPI consists in replacing the "old product" by another one, either a close substitute in the same outlet or the same product but in another outlet. It is then very likely that the price of the "replacing product" was set before the first price observation for this product. We then have left-censoring of the price spell for this new product.

In our CPI data set, there are over 3 million price spells. The majority of price spells are uncensored, covering 42% of whole sample; the share of left-censored price spells is 20%; the share of right-censored price spells is 30%; and the share of double-censored price spells is 8%[5].

Sales

Nakamura and Steinsson (2008) suggest that temporary sales have "strikingly different macroeconomic implications". Guimaraes and Sheedy (2011) point out that sales have important implication on monetary policy. The ONS gathers consumer price data on whether a product was "on sale" or "recovering from sale" when its price was sampled in a particular month. Sales prices are recorded if they are temporary reductions on

---

[5]Double censored price spells are 8% of the sample. However, it does not mean that 8% of prices were unchanged for 12 years. It is because that sometimes a shop is shut down or an item is out of stock.

goods likely to be available again at normal prices or end of season reductions. Prices in closing down sales and for special purchase of end of range, damaged, shop soiled or defective goods are not recorded as they are deemed not to be the same quality as, or comparable with, goods previously priced or those likely to be available in future. We identify temporary "sales" with the flag provided by ONS[6]. However, alternative "sales" filters are proposed by other researchers. There are three kinds of price filters used by previous studies:

1. The AC Neilsen filter, which is used by Kehoe and Midrigan (2007) (KM hereafter), indicates a sale if "price decrease is followed by *any* price increase thereafter".

2. Nakamura and Steinsson (2008) (NK hereafter)suggest a sale filter that flag a sale only when a price decrease is followed by a return to the price in effect just before the decrease.

3. Eichenbaum, Jaimovich and Rebelo (2010) (EJR hereafter) identify the most frequently observed price in a given quarter as "reference price", which means that it excludes an even larger portion of price changes than sale filters, yielding "more persistent series and suggesting a stronger role for nominal rigidities."[7]

The EJR filter restricts regular prices to change only on certain dates, and therefore greatly increases estimates of price persistence. The KM filter is much more likely to

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[6] All the discounting available for all customers are recorded by ONS, labelled as sale. While discounting only available for loyalty card members are not recorded by ONS.

[7] Chahrour (2010) proposes a new price filter similar to the EJR (2010) and show that implications for price duration depend on the choice of filter.

records a sale even if it is a reversion in regular price, and therefore it may identify spurious sales. The NS filter is more strict, which will typically identify fewer sales and more frequent price changes. We choose NS filter to identify the "sales", however, the empirical finding suggests that there is trivial difference between NS filter and ONS' sale flag. Furthermore, we find that sales have some kind of "seasonal pattern", which can be shown in Figure 2-2.
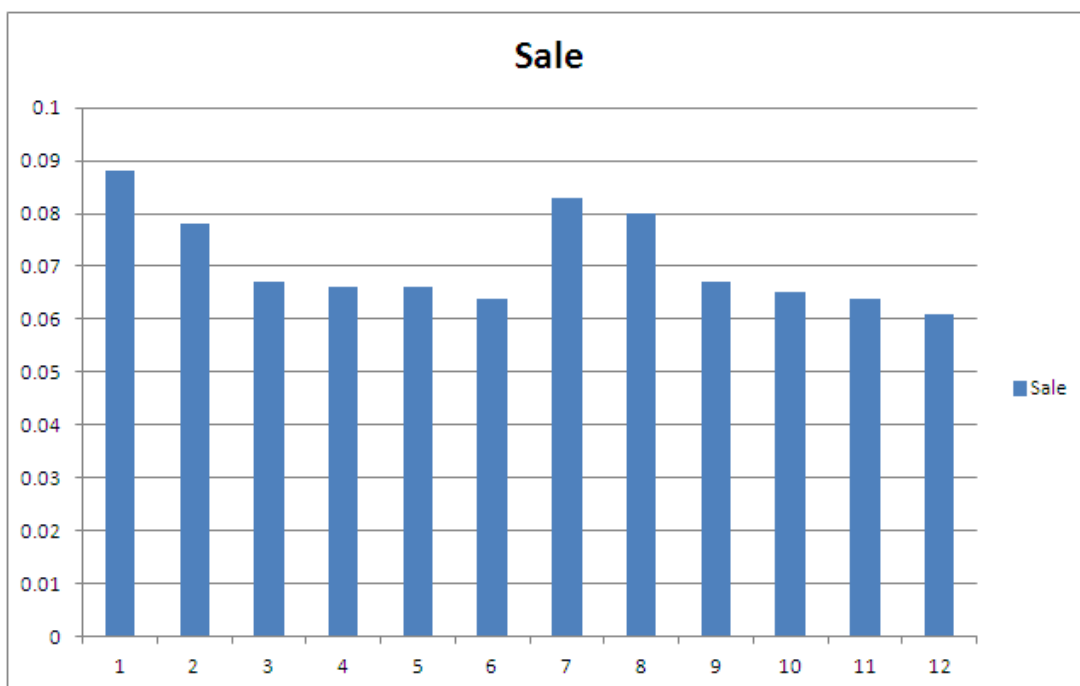
Figure 2-2: Sales as percentage of total price quotes in each calendar month

Concerning the price changes associated to sales we decided to follow a dual approach: In the baseline version of the results we treat sales as regular price changes which terminate a price spell. However, it can be argued that these price changes merely reflect noise in the price setting process and are not due to changes in fundamental price deter-

mining factors (as e.g. monetary policy and business cycle developments) and therefore they should be ignored from the viewpoint of monetary policy analysis[8]. Therefore, we also provide an alternative set of results without taking into account the price changes induced by sales. In order to exclude price changes induced by flagged sales from our analysis, we replace all flagged sales prices with the last regular price[9], i.e. the price before the sale started.

The sales price quotes account for about 7% of whole sample[10]. It is lower than the share of sales prices in the US CPI data, which account for about 20% (Nakamura and Steinsson,2008). There is significant difference in our estimation of price rigidity if we choose to include or exclude sales data.

Substitutions

Nakamura and Steinsson (2008) claim that the importance of substitution draws "attention to the question of whether the relative frequency of different types of price changes is an important determinant of the macroeconomic implications of price rigidity".As a measure of price change alone, the CPI should reflect the cost of buying a fixed basket of goods and services of constant quality. However, products often disappear or are re-

---

[8]However, Guimaraes and Sheedy (2007) argue that even if firms can adjust sales without cost, monetary policy has large real effects owing to sales being strategic substitution. Thus the flexibility seen in individual prices due to sales does not translate into flexibility of the aggregate price level.

[9]However, Eichenbaum et al. (2009) replace sale prices with reference prices which defined as the most common prices within a given quarter.

[10]Some authors, e.g. Baumgartner et al 2005, argue that the reporting of sales is generally up to the interviewer and therefore cannot be expected to be complete and consistent across all products. They additionally define a "v-shaped" sale indicator. However, this filter generates substantial fraction of sales when prices are perfectly flexible.

placed with new versions of a different quality or specification, and brand new products also become available. When such a situation arises, direct comparison is adopted. If there is another product which is directly comparable (that is, it is so similar to the old one that it can be assumed to have the same base price), for example a garment identical except that it is a different colour, then the new one directly replaces the old one and its base price remains the same. This is described as "obtaining a replacement which may be treated as essentially identical" (CPI Technical Manual,2007), and is equivalent to saying that any difference in price level between the new and the old product is entirely due to price change and not quality differences. In CPI data, such "comparable" substitution is not uncommon. It accounts for a little more than 4 percent of our total CPI research dataset. The substitution happens more likely in the January, August, and September. This partially reflects the fact that ONS adjust the basket of CPI in the beginning of the year. Beside, the clothing and footwear are more likely to change the style when summer ends. We can show the substitutions as percentage in whole price quotes in each calendar month as Figure 2-3.

Outliers

We remove some price quotes from the data set mainly because they display unrealistic price movements. We set a very large pre-defined threshold value. Any individual price changes exceed this value will be detected as outliers and excluded from our study. We don't want to apply a harsh filter here. Therefore, only if individual price changes with
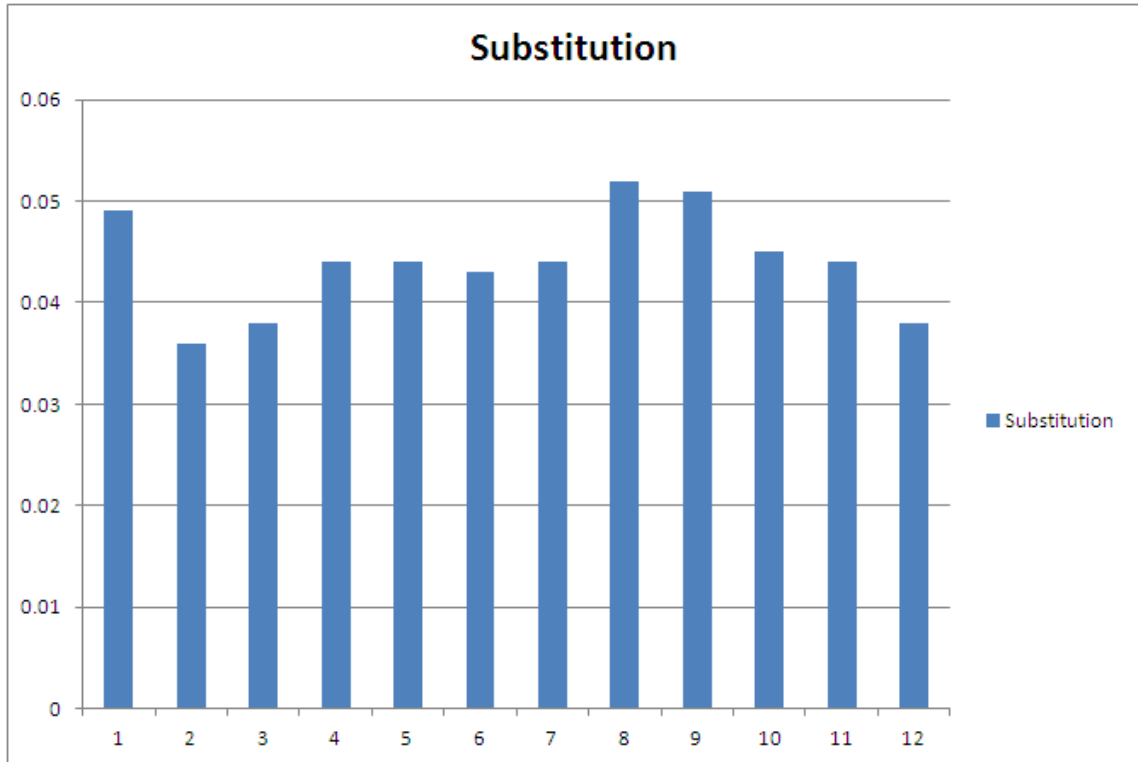
Figure 2-3: Substitution as percentage of total price quotes in each calendar month

$(P_{ijk,t} - P_{ijk,t-1})/P_{ijk,t-1} > 100$ will be defined as outliers. This is such a conservative rule that only a few observations have to be discarded. The percentage of the exclusion is very trivial, less than 0.01%. Robustness checks have done on frequency of price changes. And it does not alter our result. However, it will affect the mean size of price changes in some specific time periods for a few specific items. Though we do not focus on the magnitude of price changes. We only exclude the zero price quotes (prices dropping to zero) as outlier. Once again, the data treatment does not alter our results on frequencies of price changes.

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Data Gaps

After removing the outliers from our data set, we faces the gaps in our data set. Specifically, the price quotes for some items are missing during some periods. This situation would be even worse when we exclude the sales and substitutions from our data set. We have to fill the gaps when we estimate the duration of price spells. Therefore, we adopt a "carry forward" strategy. In another word, we will fill the gaps with the last observable price. This strategy is consistent with Klenow and Kryvtsov (2008). But it will generate upward bias in estimating the duration of price spells. In order to make this potential bias as small as possible, we add another restriction when we adopt the "carry forward" strategy. That is, we only fill the gaps not longer than 3 months. If the gaps are longer than 3 months, we consider the later one be a new price spell. This 3-month window is also used by Nakamura and Steinsson (2008) when they construct their own sale filter. Alternative treatment would be treating the missing data as "loss". Then this would effectively increase the right censoring and left censoring observations, but to a very slightly higher level (less than 1%). However, our results are not significantly affected.

Weights

In CPI dataset, each individual price quote is attached with weight, which is given by ONS. The CPI weights cover monetary expenditure within the UK on goods and services. The weights are based on expenditure within the domestic territory by all

private households, foreign visitors to the UK and residents of institutions. The individual weights[11] which initially do not sum to one as not 100 percent of the CPI is covered in our sample, are then rescaled such that the sum of the weights equals one and the relative weights among the goods are preserved.

2.3   A unified framework in assessing price rigidity

In this section, we provide a unified framework in assessing price rigidity from three perspectives: the frequency, the hazard function method, and the distribution of duration across firms.

2.3.1   Frequency

The frequency of price changes is given for each product as the number of price changes in each period over the total number of price quotes for that product in that period. We assume that the proportion of firms resetting price corresponds to the proportion of prices changing. It imply that each price trajectory corresponds to a different firm. We

---

[11]The weights are used at the "item" level, which is the most disaggregated level. More specifically, the weights are assigned to a specific item sold in a specific shop belonging to a specific location.

can use dummies $I_{j,t}$ to define the price change[12]:

$$
\begin{cases}
I_{j,t} = 0 \text{ if } P_{j,t} = P_{j,t-1} \\
\\
I_{j,t} = 1 \text{ if } P_{j,t} \neq P_{j,t-1}
\end{cases}
$$

where $P_{j,t}$ and $P_{j,t-1}$ belong to the same trajectory. Similarly, we can define the price increase/decrease with dummies:

$$
\begin{cases}
U_{j,t} = 1 \text{ if } I_{j,t} = 1 \& P_{j,t} > P_{j,t-1} \\
\\
 U_{j,t} = 0 \text{ all else}
\end{cases}
$$

$$
\begin{cases}
D_{j,t} = 1 \text{ if } I_{j,t} = 1 \& P_{j,t} < P_{j,t-1} \\
\\
 D_{j,t} = 0 \text{ all else}
\end{cases}
$$

And the weighted average frequency of price changes is defined as

$$
f = \frac{\sum_{t=1}^{T} \sum_{j=1}^{J} \omega_{j,t} I_{j,t}}{\sum_{t=1}^{T} \sum_{j=1}^{J} \omega_{j,t}},
$$

The frequency approach now is standard in the empirical sticky-price literature[13]. It has following advantages:

---

[12]This is a kind of backward comparing scheme. This method is used by a lot of researchers, such as Fougere et al. (2007), Nakamura and Steinsson (2008), Bunn and Ellis (2012a). The price change can also be defined in a forward comparing way. However, we argue that when sample is big enough. The estimates from two methods are consistent.

[13]We estimate the frequency of price changes by using STATA, which include standard command to calculate the weighted mean (frequency of price changes). The incorrect formula (a typo in previous version) was not used.

1. not require a long span of data.

2. allows to use the maximum amount of information from the data.

3. does not require an explicit treatment of the censoring of price spell.

The measure f is an average incorporating price changes of all firms and over all periods of time. It can also be interpreted as a flow of new contracts in each period.

In discrete time, the frequency of price change f implies an average of price duration:

$$d = \frac{1}{f},$$

while in continuous time[14] framework (as Bils and Klenow 2004, Nakamura and Steinsson 2008), the implied average duration equals:

$$d = \frac{-1}{\ln(1 - f)}$$

Whther we choose a discrete or continuous setup, there are a number of difficulties with the concept of implied duration:

1. The method above makes the simplifying assumption of constant hazards, where the probability of a price change is independent from the amount of time elapsed since the previous adjustment.

---

[14]The prices can change within a month.

2. The estimator to be consistent, homogeneity of observations in the cross-sectional dimension is required.

3. The estimator suffers downward aggregation bias due to the Jensen' inequality. Specifically, under the situation where heterogeneity exists, we have:$1/E(x) < E(1/x)$ (Baharad and Eden,2004).

2.3.2 Hazard function

We start to think about a price spell j, which begins at time $t_{j,start}$ and remains until some time point $t_{j,end}$ where the precise date $t_{j,end}$ is not known but is observed to be somewhere between dates $t-1$ and t,due to the discrete nature of sampling. However, we can be sure that the price spell has kept on more than $t-1$ months and at most t months. Hence, we can define the probability for a price change to occur after some time has elapsed since the previous price change. First,we have the probability for a spell to last at least $t-1$ months (the survivor function) as:

$$\Pr(T \geq t-1) = \Omega(t-1)$$
$$= 1 - F(t-1)$$

where $\Omega(\cdot)$ and $F(\cdot)$ are the survivor function and the cumulative density function of T correspondingly. After that, we can define the probability that price spell keeps on at

least $t-1$ months but less than t months as:

$$\Pr\left(t-1 < T \le t\right) \;=\; F\left(t\right) - F\left(t-1\right)$$

$$= \; \Omega\left(t-1\right) - \Omega\left(t\right).$$

We introduce the concept of hazard function ω_t , which is the instantaneous probability of price change at month t, conditional on the price not changing until that point in time. It can be shown as a function of cumulative density function $F\left(\cdot\right)$ or the survivor function $\Omega\left(\cdot\right)$:

$$
\begin{aligned}
\omega_t \;&=\; \Pr\left(t-1 < T \le t \mid T > t-1\right)\\
&= \; \frac{\Pr\left(t-1 < T \le t\right)}{\Pr\left(T > t-1\right)}\\
&= \; \frac{F\left(t\right) - F\left(t-1\right)}{1 - F\left(t-1\right)}\\
&= \; \frac{\Omega\left(t-1\right) - \Omega\left(t\right)}{\Omega\left(t-1\right)}\\
&= \; 1 - \frac{\Omega\left(t\right)}{\Omega\left(t-1\right)}.
\end{aligned}
$$

If we define the distribution of ages in steady-state as $\boldsymbol{\alpha}^A \in \Delta_M^{F-1}$,[15] the corresponding hazard rate is given by

$$
\begin{aligned}
\omega_i &= \frac{\alpha_i^A - \alpha_{i+1}^A}{\alpha_i^A}, \quad i = 1, \cdots, F-1 \\
\omega_F &= 1
\end{aligned}
$$

Here the α_i^A are monotonic decreasing[16], $\alpha_i^A \leq 1$ with $\alpha_{F+1}^A = 0$. Accordingly, we can derive the survival probability as the probability at birth that the price survives for at least i periods, with $\Omega_1 = 1$[17] and for $i = 2, \ldots, T$

$$
\Omega_i = \prod_{k=1}^{i-1} (1 - \omega_k)
$$

and the sum of survival rates is

$$
\sum_{\Omega} = \sum_{i=1}^{T} \Omega_i
$$

Proposition 1 *If we define $\varpi = \frac{1}{\sum_{\Omega}}$, then at steady state, the frequency of price change f equals ϖ.*

    Proof. Think of the age distribution: in steady state, the flow of new contracts is f. The share of each age is $f * \Omega_i$, $i = 1, \ldots, T$. The sum of all shares of age distribution is

---

[15] The age of firm's price-spell is the period of time that has elapsed since the price spell started. Subscript M refers to monotonicity; supscript A refers to age.

[16] We cannot have more older price spells than younger price spells, since to become old must first be young.

[17] Here we assume the price can last at least a month. This is in line with the frequency of sampling by ONS in constructing the CPI.

one. Therefore,

$$\sum_{i=1}^{T} f \cdot \Omega_i \;\; = \;\; 1$$

$$\Rightarrow \;\; f \cdot \sum_{i=1}^{T} \Omega_i = 1$$

$$\Rightarrow \;\; f = \frac{1}{\sum_{i=1}^{T} \Omega_i}$$

$$\Rightarrow \;\; f = \varpi$$

∎

We estimate the hazard function using Survival Analysis, widely used in the life science to study the time elapsed from the "onset of risk" until the occurrence of a "failure" event. Economists mostly use survival analysis to study unemployment spells. In our practice, we estimate hazard function based on Kaplan-Meier nonparametric estimator, since it does not need for the assumption of the distribution and is purely data driven. However, some adjustments are needed:

1. in discrete time macroeconomic framework, the firm believes that it has a probability of 1 that its price lasts for at least one period, so $\Omega_1 = 1$;

2. we reconcile the estimated hazard function with the data on the proportion of firms changing price per month, as the lemma above tells, $\varpi = f$

Next, we define the set of price spells for the estimation of the hazard function. There is no agreement in literature about the selection of price spells in estimating the hazard

function. Some use uncensored price spells, arguing that these price spells are properly defined, while the censored price spells are much vague in this sense, such as Klenow and Kryvtsov (2008). Some just pick one complete price spell in each price trajectory to make the estimation (Bunn and Ellis 2012a). These treatments are quite ad hoc. Therefore, we will quite a few options in estimating hazard function and get some preliminary results.

First, we categorize the price spells into four groups, and estimate hazard function within each groups.

1. Uncensored price spells only group. For each price spells, a price change happens at the end. Therefore, all censored spells are excluded. The figure 2-4 reports that during the first a few months, the hazard function is downward sloping. Then it becomes relatively flat, with significant 12 month spike. This finding is consistent with the previous findings in other countries, such as Alvarez et al.(2005) and Nakamura and Steinsson (2008). However, the reciprocal of sum of survival rates $\varpi = 0.29$, which is too much higher than the frequency of price change we directly calculated from whole sample. This indicates that the uncensored price spells scheme suffer the loss of long price spells.

2. Left censored price spells only group. The reciprocal of sum of survivor rate $\varpi = 0.18$. As Figure 2-5 shows, the shape of hazard function is similar to what we get from the normal price spell, though with smaller annual spikes. The biggest problem when we included the left censored price spell is that we do not really know when the price spells start. The statistical treatment of such spells induces more
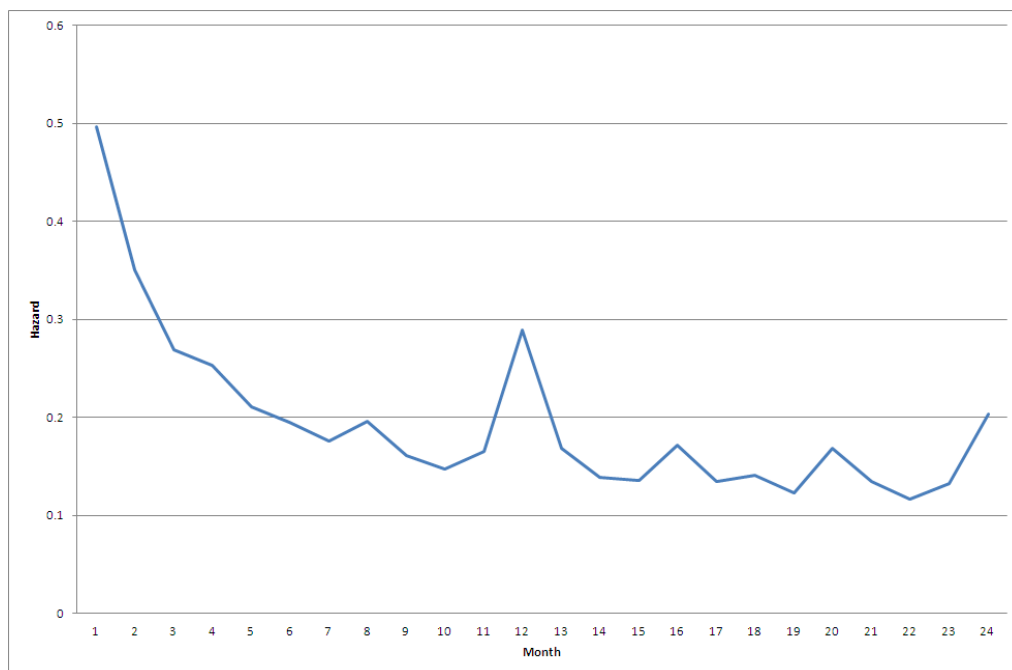
Figure 2-4: Hazard function for normal price spells

difficulties than accounting for the sole right-censoring. However, the sample we have is made of thousands of spells for similar products in quite similar outlets -i.e. we have many spells that concern a given particular product sold in a given type of outlet (e.g. in small corner grocery shops, or in supermarkets). Excluding the left censored spells will not result in a substantial loss of information nor produce a selection bias as the left-censoring is independent of the duration of price spells. Therefore, following most literature, we will not include the left censored price spells.

3. Right censored price spells only group. As shown in the Figure 2-6[18], the hazard

[18]For the right censored price spells, there are no defined end of price spells, since no price changes
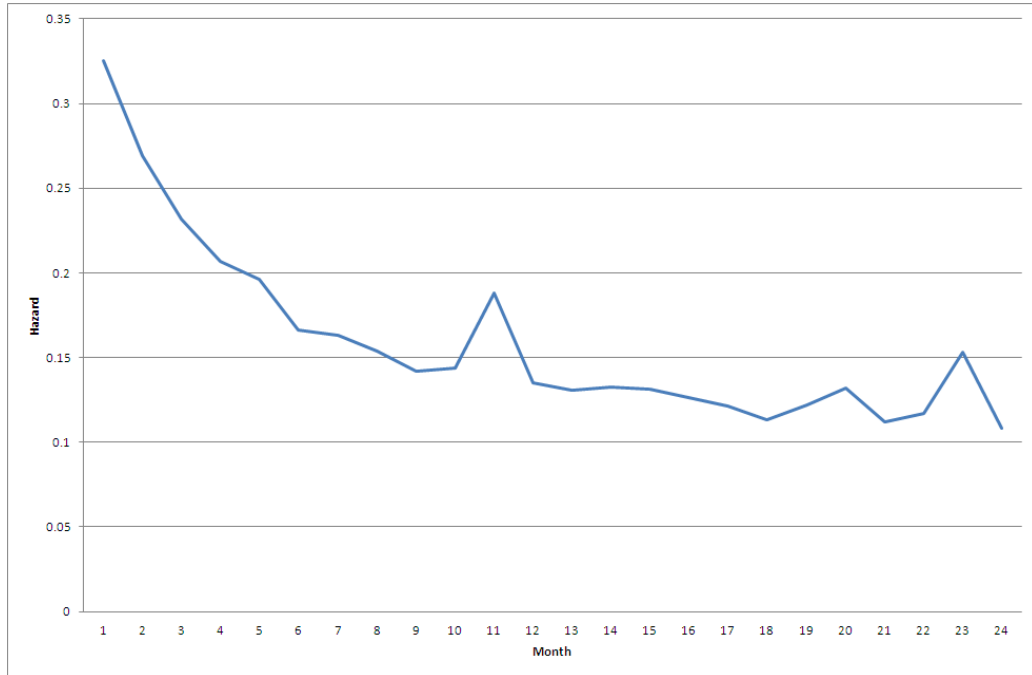
Figure 2-5: Hazard function for left-censored price spells

function is quite smooth comparing to the last two situations. It is also downward sloping for first a few months, and then becomes relative flat. No annual spike is found here. The implied frequency of price change $\varpi = 0.16$, indicating that right-censored spells tend to belong to those long lasting spells.

4. Double censored price spells only group. The double censored price spells are most likely to be very long price spells. Therefore, this group features extremely sticky prices. And the result proves our hypothesis. The implied frequency of price change $\varpi = 0.09$. As Figure 2-7 shows, the hazard function shows about yearly large spikes,

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are observed. We assume price spells end at the last observation. The same assumption is also made when we estimate the hazard function for the double-censored price spells.
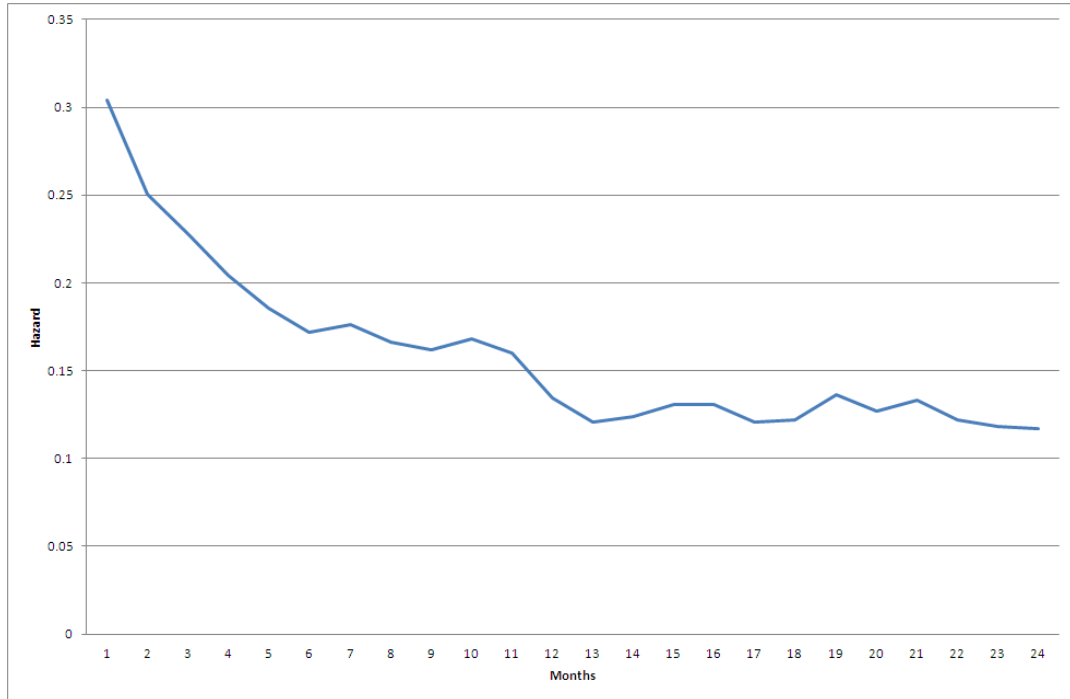
Figure 2-6: Hazard function for right-censored price spells

indicating that the statistical agency usually updates the basket annually.

Second, we focus on the mix group with normal price spells and right-censoring price spells. We distinguish the estimations of hazard function by treating right-censoring as "failure" (the same as price change) or as "loss" (out of scope of observation, not necessarily end at that point of time):

1. The first assumption about right censoring is adopted by Kaplan-Meier estimator for the survival probability, which treats the right censoring as *loss*. This method assumes that the right censoring price spell keep the same price on to the end of estimated period. Especially, we define the number of price spells that have lasted
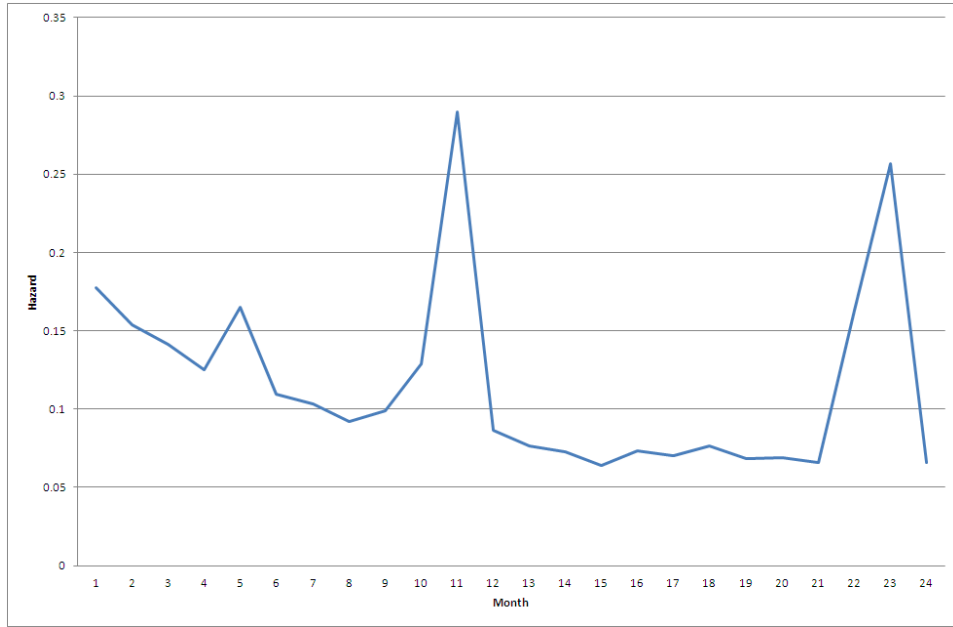
Figure 2-7: Hazard function for double-censored price spells

up to the ith period as n_i, of these, f_i fail (price-change), l_i are lost due to the right censoring, which is defined in the previous sub-section. The basic KM estimator for the survival probability up to period i is:

$$\widehat{\Omega}\left(i\right) = \prod_{j=1}^{i}\left(1 - \frac{f_j}{n_j}\right)$$

The key assumption here is that failure and loss are mutually exclusive, too strong assumption in our case.

2. The other assumption on right censoring is to treat it as failure. Therefore, the right-censoring is the same as price change. Both end a price spell definitely. Again, this is kind of strong and ad hoc assumption. Under this assumption, the survival

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probability can be calculated as:

$$\widehat{\Omega}\left(i\right) = \prod_{j=1}^{i} \left(1 - \frac{f_j + l_j}{n_j}\right)$$

We provide a kind of sensitivity analysis to check how robust our results from these two assumptions among different data samples in CPI micro data. The detail of the results will be reported in next section.

2.3.3 Distribution across Firms (DAF)

The distribution across firms in our context corresponds to the cross-section of completed price-spells, which implies the average completed price-spell across firms. This concept shed light on the fact that firms set price. In order to measure nominal rigidity in meaningful way, we must focus on the behavior of firms. "*The degree of nominal rigidity is the average over time prices remain unchanged for a typical firm in the economy*" (Dixon, 2012). This is in contrast with the concept of the average length of a price-spell across contracts, a measure that is frequently used (e.g. Bils and Klenow 2004, Nakamura and Steinsson 2008). The empirical evidence shows that short price spells are dominant, because they more likely to be counted. In the conventional method (distribution across contracts), such assumptions are made: each price spell is unique and there is no link between different price spells. However, taking the average over price-spells (distribution across contracts) gives an excessive weight to short durations, moreover, it is in effect ignoring the panel structure and the fact that it is firms which are generating the price-

spells. The conventional method also generates an underestimation of the actual degree of nominal rigidity in an economy as argued by Baharad and Eden (2004), Hoffman and Kurz-Kim (2006), and Dixon (2012). Furthermore, we can justify the reason that we should look at the cross-sectional measure from firms and/or households' point of view. When firms set price, they maximize the discounted sum of future profits up to some time T (which may be infinite). Thus the weight put on a particular price spell is, in a sense, "proportionate" to its duration, notwithstanding the effects of discounting. Since the objective function is additive across time, a longer duration adds more items into the summation than a shorter one. Hence firms pay attention to the flow of profits earned during price-spells roughly proportionate to their duration, given discounting. The cross-sectional approach (DAF) is a form of length-biased sampling, which weights price-spells in proportion to their duration. This enables us to focus on the behavior of the firms which are generating the price-spells. Moreover, the information on sequence of price-spells generated by the same firm over time is fully used. This is obviously an advantage over some so-called one spell per firm sampling scheme. We will check this result later.

The cross-sectional distribution of completed price-spell across firms is just another different way to look at the same object: a panel of price events. Each row of the panel is a cross-section of all of the prices set by firms at a point in time. Each column is a trajectory of prices corresponding to a particular firm. And we can find a 1-1 mapping from hazard rates to completed price-spells across firms originally proposed by Dixon (2012).

Proposition 2 *Given the hazard rate $\omega \in [0,1]^{F-1}$, there exists a unique $\alpha \in \Delta_M^{F-1}$ corresponding to ω*

$$\alpha_i = i \cdot \overline{\omega} \cdot \Omega_i \cdot \omega_i \; , \;\; i = 1, \cdots, F \tag{2.1}$$

Proof. The proportion of firms that have a contract that last for exactly 1 period are those are born (age1) and do not go on the age 2. The proportion of firms that last for exactly i periods in any one cohort (born at the same time) is given by those who attain the age i but who do not make it to $i+1$: this is $\overline{\omega} \cdot \Omega_i \cdot \omega_i$. Given the flow of new contracts is $f = \overline{\omega}$ each period. To survive for exactly i periods, you have to survive to period i which happens with probability Ω_i, and then start a new contract which happens with probability ω_i. Hence from a single cohort $\overline{\omega} \cdot \Omega_i \cdot \omega_i$ will have contracts that last for exactly i periods. Clearly, We can sum over the i cohorts (to include all of the contracts which are in the various stages moving towards their final period i) to get the expression.

$$
\begin{aligned}
\sum_{i=1}^{F} \alpha_i &= \sum_{i=1}^{F} i \left(\overline{\omega} \cdot \Omega_i \cdot \omega_i \right) \\
&= \sum_{i=1}^{F} i \left(\alpha_i^A \cdot \omega_i \right) \\
&= \sum_{i=1}^{F} i \left(\alpha_i^A - \alpha_{i+1}^A \right) \\
&= \left(\alpha_1^A - \alpha_2^A \right) + 2 \left(\alpha_2^A - \alpha_3^A \right) + 3 \left(\alpha_3^A - \alpha_4^A \right) + \cdots + F \cdot \alpha_F^A \\
&= \sum_{i=1}^{F} \alpha_i^A \\
&= 1
\end{aligned}
$$

Hence $\boldsymbol{\alpha} \in \boldsymbol{\Delta}^{F-1}$. The derivation of second equation can be referred to Observation 1 by Dixon (2012). Rearranging the $F-1$ equations (2.1) we have:

$$\frac{\alpha_1}{\overline{\omega}} = \omega_1; \frac{\alpha_2}{2\overline{\omega}} = \omega_2 \left(1 - \omega_1\right); \cdots ; \frac{\alpha_i}{i \cdot \overline{\omega}} = \omega_i \Omega_i; \cdots ; \frac{\alpha_F}{F\overline{\omega}} = \Omega_F.$$

∎

In terms of cross-section, we are in effect selecting over all price events in the cross-section. The probability of a price-spell being observed is proportional to its length: a 8-period spell is 8 times more likely to be observed than a 1-period spell. Now, for a given mean frequency of prices change f, there are many distributions of duration across firms $\{\alpha_i\}$ consistent with that mean frequency. Since we know from Proposition 1 that the mean frequency of price change equals the flow of new contracts in each period $f = \overline{\omega}$, we can define the mapping $H\left(\overline{\omega}\right) : [0,1] \rightarrow \Delta^{F-1}$

$$H\left(\overline{\omega}\right) = \left\{ \alpha \in \Delta^{F-1} : \sum_{i=1}^{F} \frac{\alpha_i}{i} = \overline{\omega} \right\}$$

$H\left(\overline{\omega}\right)$ is the set of all DAFs which are consistent with a given mean frequency of price changes expressed in terms of the corresponding proportion of firms resetting prices $\overline{\omega}$.

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2.4 Empirical findings from micro data[19]

2.4.1 Frequency of price change

As described in previous sector, the frequency of price changes is computed as the ratio of observed price changes to all valid price records. Thus, this measure is an average incorporating price changes of all firms where the product has been recorded and over all periods of time. In Tables 2.3 to 2.5, the results aggregated on the COICOP division and CPI sectors are presented. On average, 21 percent of all prices are changed every month. Therefore, there is overwhelming evidence that most products exhibit a significant degree of price stickiness. Our finding is somehow between those in the US and in the Euro area. Bils and Klenow (2004) report that the average monthly frequency of price adjustment is about 25 percent in the US, while Dhyne et al. (2005) report the share of price adjustment is around 15 percent each month in Euro area. If we exclude the price quotes which are indicated by "Sales", the frequency of price changes is 17 percent, showing that during the sales period the prices change more frequently. If we exclude the price quotes which are indicated by "Substitution", the result of frequency drops to 19 percent. If, say, we only look at the regular prices, or the prices excluding both sales and substitutions, the result of frequency is 15 percent only. Moreover, the frequency of price changes varies considerably (see Table 5). Energy goods display a rather high frequency of price

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[19]**We provide empirical evidence about frequency of price changes, hazard functions, and DAF. We do not provide detailed empirical findings about the magnitude of price changes since this chapter is not focus on this issue. However, we do find that the size of price change on average is 15%. And there is significant heterogeneity in the magnitude of price change among different COICOP sectors.**

changes (65 percent) and thus a short implied duration. Within these categories fuels of different types are most flexible. This is in line with the other's find with weekly data (Eichenbaum, Jaimovich and Rebelo 2011), since fuel prices are indeed changed with a very high frequency - sometimes even on a daily basis. In contrast, some service items as well as products with administered prices display a very low frequency of price changes and, on average, a duration which is almost three times as long as for unprocessed food. For example, automatic car wash, parking and digital photo printing show an estimated average duration of 24 months or longer.

Table 2.3 report the mean frequency of prices change (including sales and substitutions) for each COICOP Division. The division "Transport" is the one with highest frequency of 36 percent per month, which means that more than one third of items in this division will reset the prices each month. The sales and substitutions play a trivial role in this division. The division "Clothing and Footwear" also change their prices frequently, as high as 27 percent per month, but largely due to sales and product substitution. After excluding the effect from sales and substitutions, the "Regular Price" in this division, actually, is quite sticky (frequency drops to only 8.9 percent per month). Our finding is consistent with what reported in Nakamura and Steinsson (2008). In the Table II of NS's paper, they report that the major group "Transportation goods", on average, about 31 percent of prices/ regular prices change each month. While the frequency varies dramatically between posted prices and regular prices in the major group "Apparel". This is because many clothes undergo sales and substitution at the beginning of the spring

and fall seasons. In addition, the "Recreation and Culture" division also has high rate of substitution, close to 5 percent. As argued in NS (2008), the timing of product substitution is primarily motivated by factors such as product cycles, fashion, and seasonal demand variation rather than a firm's desire to change its price. Price changes often occur as introduction of new products. However, the introduction of new products is not due to the misprice of the old products. Therefore, the "selection effect associated with price changes for identical items" may be stronger than for price changes because of product substitution (Nakamura and Steinsson 2008). Since the effect associated with substitution is quite different from the price changes for identical items, we calibrate our benchmark models in section 5 in accordance with the findings in the micro price dataset without substitutions.

COICOP Division	Weight	Prices		Prices ex. Sales		Prices ex. Sub.		Regular Prices	
		Mean Freq	Frac. Up	Mean Freq.	Frac. Up	Mean Freq.	Frac. Up	Mean Freq.	Frac. Up
Food and Non-Alcoholic Beverages	17.6	26.0	60.4	21.9	63.5	24.5	60.7	20.3	64.5
Alcoholic Beverages and Tobacco	7.1	27.6	72.1	23.3	78.4	27.1	72.3	22.8	79.0
Clothing and Footwear	9.3	27.2	54.5	17.3	62.2	19.6	46.3	8.7	70.4
Housing and Utilities	6.5	12.7	65.4	12.0	66.7	11.2	66.7	10.5	68.3
Furniture and Home Maintenance	11.3	22.7	56.4	12.3	66.3	20.9	55.6	10.2	69.4
Health	1.9	10.4	77.1	9.5	80.4	9.8	78.3	8.9	82.0
Transport	10.4	36.0	60.9	35.9	61.1	35.8	60.8	35.7	61.1
Communications	0.2	22.5	45.8	16.6	52.3	19.6	42.0	13.6	50.8
Recreation and Culture	9.9	20.0	57.6	16.7	62.1	15.5	57.5	11.9	66.3
Education	0	NA	NA	NA	NA	NA	NA	NA	NA
Restaurants and Hotels	17.5	10.5	81.9	10.2	82.7	9.8	83.3	9.6	84.1
Miscellaneous Goods and Services	8.3	13.7	69.6	11.4	74.7	12.5	70.1	10.3	76.3
Total	100	21.1	-	17.1	-	19.0	-	15.2	-

Table 2.3: Frequency of Price Change by COICOP Division in 1996-2007

All Frequencies are reported in percent per month. Fractions are reported as percentages. "Weight" denotes the CPI expenditure weight of the CPI sectors. "Mean Freq." denotes the expenditure weighted mean frequency of price change. "Frac. Up" denotes the expenditure weighted mean fraction of price changes that are price increases. "Price ex. Sales" denotes prices excluding sales. "Price ex. Sub" denotes prices excluding substitutions. "Regular Prices" denote prices excluding sales and substitutions.

		Prices		Prices ex. Sales		Prices ex. Sub.		Regular Prices	
CPI Sectors	Weight	Mean Freq	Frac. Up	Mean Freq	Frac. Up	Mean Freq	Frac. Up	Mean Freq	Frac. Up
Food and Non-Alcoholic Beverages	17.6	26.0	60.4	21.9	63.5	24.5	60.7	20.3	64.5
Alcoholic Beverages and Tobacco	7.1	27.6	72.1	23.3	78.4	27.1	72.3	22.8	79.0
Energy Goods	5.2	64.8	58.5	64.8	58.5	64.8	58.4	64.8	58.5
Non-Energy Industrial Goods	35.7	22.0	56.6	14.6	63.7	18.3	54.0	10.5	67.8
Housing Services	5.7	9.8	71.1	9.8	71.3	8.3	74.3	8.2	74.5
Transport and Travel Services	4.4	9.1	78.7	9.0	79.4	9.0	78.9	8.9	79.7
Communication	0.2	22.5	45.8	16.6	52.3	19.6	42.0	13.6	50.8
Recreational and Personal Services	20.8	10.9	79.8	10.7	80.5	9.7	82.9	9.5	83.7
Miscellaneous Services	3.5	10.0	85.8	10.0	86.3	10.0	85.9	9.9	86.4

Table 2.4: Frequency of Price Change by CPI Sectors in 1996-2007

All Frequencies are reported in percent per month. Fractions are reported as percentages. "Weight" denotes the CPI expenditure weight of the COICOP Division. "Mean Freq." denotes the expenditure weighted mean frequency of price change. "Frac. Up" denotes the expenditure weighted mean fraction of price changes that are price increases. "Price ex. Sales" denotes prices excluding sales. "Price ex. Sub" denotes prices excluding substitutions. "Regular Prices" denote prices excluding sales and substitutions. The sum of weights differ from unity due to rounding.

		Prices		Prices ex. Sales		Prices ex. Sub.		Regular Prices	
Major Group	Weight	Mean Freq	Frac. Up	Mean Freq	Frac. Up	Mean Freq	Frac. Up	Mean Freq	Frac. Up
Processed Food	12.4	20.1	60.4	16.8	63.5	18.3	60.7	14.5	64.5
Non-Processed Food	12.2	30.3	51.4	25.4	55.6	20.1	72.3	18.7	79.0
Energy Goods	5.2	64.8	58.5	64.8	58.5	64.8	58.4	64.8	58.5
Non-Energy Industrial Goods	35.7	22.0	56.6	14.6	63.7	18.3	54.0	10.5	67.8
Services	34.5	10.5	70.1	9.2	66.2	8.3	61.8	8.2	74.5

Table 2.5: Frequency of Price Change by major CPI Groups in 1996-2007

All Frequencies are reported in percent per month. Fractions are reported as percentages. "Weight" denotes the CPI expenditure weight of the Major Group. "Mean Freq." denotes the expenditure weighted mean frequency of price change. "Frac. Up" denotes the expenditure weighted mean fraction of price changes that are price increases. "Price ex. Sales" denotes prices excluding sales. "Price ex. Sub" denotes prices excluding substitutions. "Regular Prices" denote prices excluding sales and substitutions. The sum of weights differ from unity due to rounding.

If we analyze price increases and decreases separately and excluding the effect of substitution, we realize that price increase more often than they decrease: the frequency of price increase is 11 percent compared to 8 percent for price decreases. Exceptions from this pattern can be found in the categories, e.g. "clothing &foot wear" and "communication", where price decreases appear more frequent than price increases. Moreover, if we look at the share of price increase/decrease in the total price changes, we can find that the price increases account for roughly 60 percent of price changes, whereas the price decreases account for the remaining 40 percent. Therefore, from the evidence of the frequency of price changes, we do not find significant downward price change rigidity, which is in contrast with some theories.

It is true that this study only covers a part of "Great Moderation" period. During this period, inflation was relatively low, and the price decreases are not uncommon. This finding is consistent with one of "five facts about prices in the U.S" established by Nakamura and Steinsson (2008). In their paper, they claimed that "one-third of nonsale price changes are price decreases". In Bunn and Ellis (2012a), they also find that "across the CPI microdata as whole, 45% of the changes in goods price changes are falls". However, as pointed out by Gagnon (2008), price decreases are key to the dramatically different behaviours of low- and high- inflation economies. Gagnon (2008) find that the frequency of price decreases "diminishes rapidly as inflation rises from 0 to 10-15%". According to Nakamura and Steinsson (2008), we should interpret these results with caution "given the small amount of inflation variability over the period we

consider".

As has been mentioned before, the results on the frequencies of price changes and implied duration of price spells are also computed without sales. For all product groups the frequency of price changes have to be smaller (or equal) compared to the figures in the "Prices" column. As expected, these effects are most pronounced for food and alcoholic beverages where temporary promotions are a common practice to attract new customers, as well as for cloth and footwear where end of season sales are a common practice to clear inventories.

We can also calculate the average frequency of price changes for each calendar month. When looking at the frequency of price changes over each month we can see that there is a clear seasonal pattern visible in Figure 2-8 [20]: The spikes in January indicate that most prices are changed in January. More specifically, price decreases are more likely to be observed in January. This is partly due to the post-Christmas sales. Price increases are more common during April, since many utility bills tend to rise at the beginning of a financial year. A modest jump in the frequency of prices decreases in July may reflect the summer sales in clothing and footwear. We also plot annually frequency of price changes. As Figure 2-9[21] shows, there is no trend in the annual frequency of price changes visible over the period considered. Furthermore, price increases and decreases show a similar pattern over years.

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[20] Ch, Ch Up and Ch Dn represent frequency of overall price changes, frequency of price increase, and frequency of price decrease respectively.

[21] Ch, Ch.Up and Ch.Dn represent frequency of overall price changes, frequency of price increase, and frequency of price decrease.
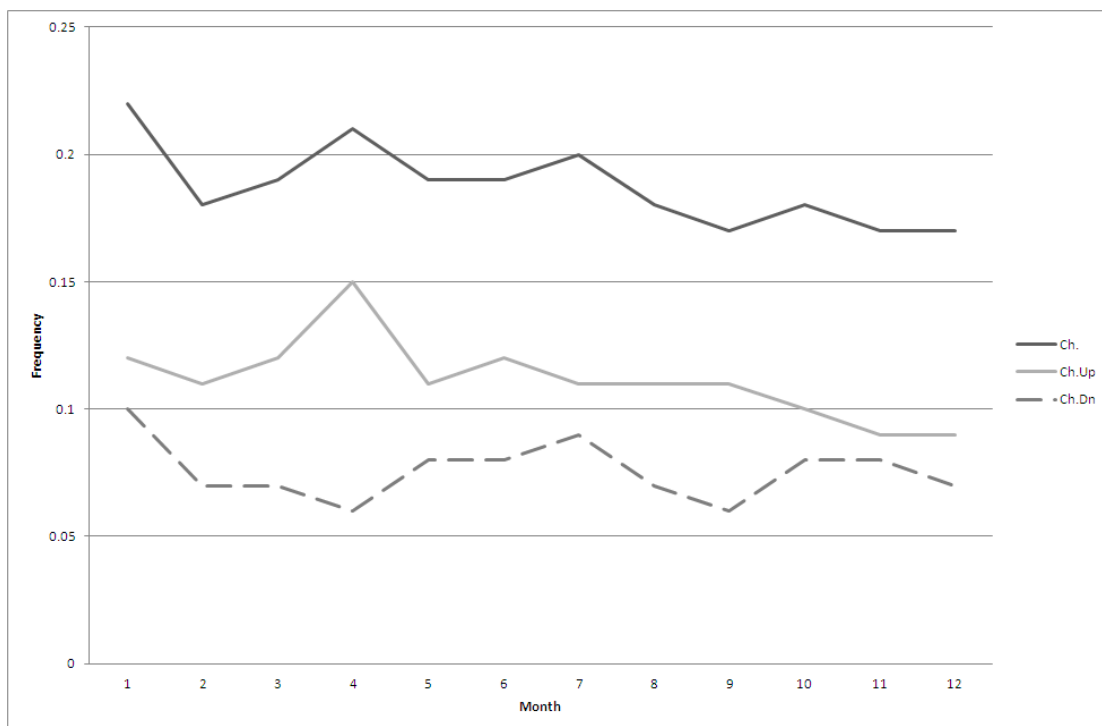
Figure 2-8: Frequency of price changes in each calendar month

2.4.2 Hazard function

Aggregate hazard function

We remove all the left-censored and double-censored spells from the original dataset prior to estimation. Indeed, spells that are double-censored are very likely to be very long, typically longer than 11 months. However, this exclusion, as suggested by Heckman and Singer (1984), helps to avoid making non-testable assumptions on the price setting behaviour before the start of the observation period. As discussed in previous subsection, we make two different assumptions on the last observation of right-censored price spells. Since we cannot observe a price change at the end of right-censored price spells, there
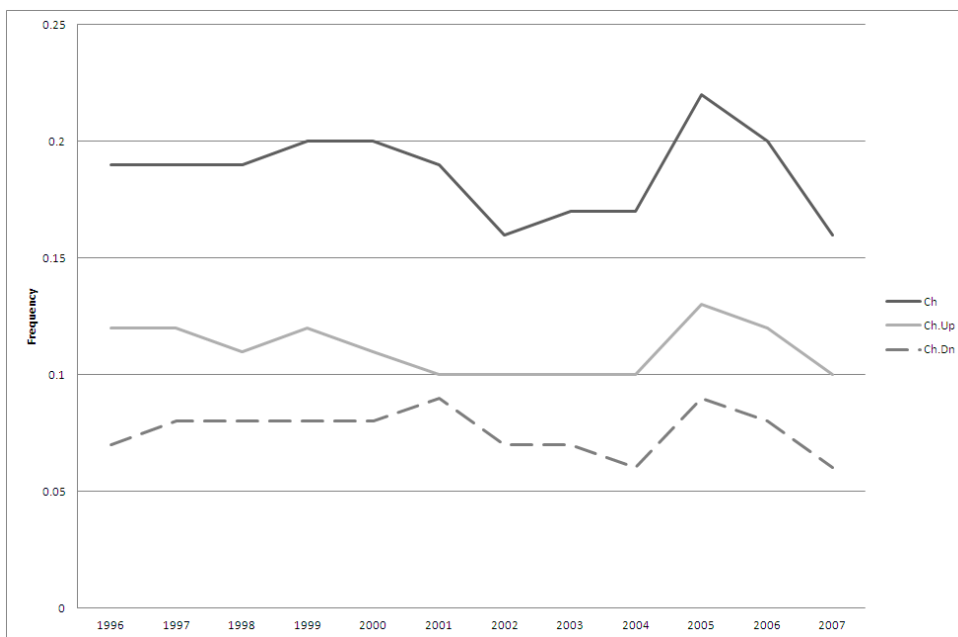
Figure 2-9: Frequency of price changes in each year

is no cut clear definition on hazard rate at that point of time. We could either assume an artificial price change happens at the end of right-censored price spell, or we could assume that the right-censored price spell is keeping on going which can last forever and beyond our observation. Therefore, we can plot the Kaplan-Meier Hazard functions based on two assumptions in the following Figure 2-10. As can be seen from Figure 2-10, those two hazard function have similar shape, both are downward sloping for first few months and then becoming relatively flat with large annual spike. In the first few months, the two curves stay quite close. However, as period becomes longer, the gap between two curves becomes more significant. This is due to the fact that as time goes by, the right-censoring is more likely to appear. The hazard rate would be lower if we assume the price spell lasts even out of our observation.
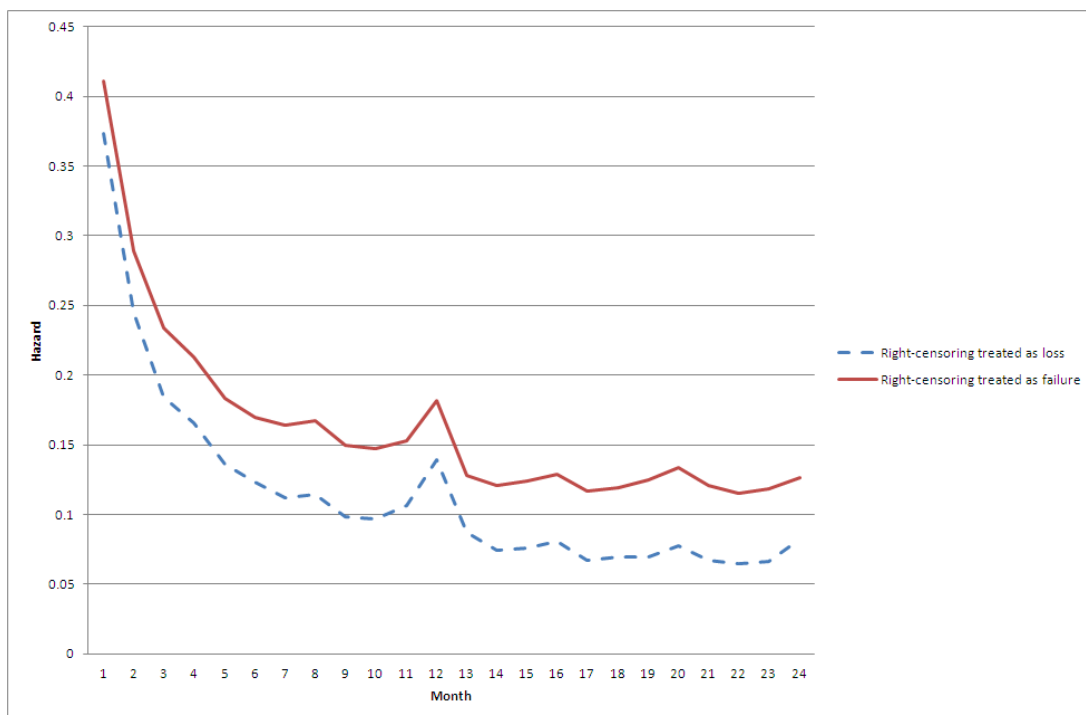
Figure 2-10: Aggregate hazard function for consumer prices

The downward sloping hazard function, taken at face value, means that the probability that a firm will change its price may be lower as it has kept it unchanged longer. It may seem count-intuitive at first glance. However, we can look at this in an alternative way. As we are estimating the aggregate hazard function, which includes the firms with sticky pricing strategies and those with flexible pricing strategies. The firms with flexible pricing strategies are more likely to be in the "young age" zone. As firms become older, the share of price changes by firms with flexible pricing strategy will decrease. As argued in Alvarez (2008), only price changes which belong to sticky firms can be observed at high ages. The downward slope of hazard function reflects the "aggregation of heterogeneous

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price setters".

In light of state dependent pricing models, the declining hazard may be explained by the variation in the "Ss band", or the width of the inaction region (Klenow and Kryvtsov 2008). Let's consider the following scenerio: When a firm faces persistent idiosyncratic shock with high level, it tends to sell a large quantity under a low price. Therefore, the profit of the firm is mainly decided by choosing the right price. This will lead to a narrow Ss band. However, when the idiosyncratic shock is at low level, the firm's inaction region becomes wider. Furthermore, when Ss band is narrow and hazard rate is high, the young prices are more common; while the old prices are more common when Ss band is wider and hazard rate is lower.

The existence of annual spikes is a reflection of the seasonality that is present in most quantitative study using micro price data. The annual spikes we find in aggregate hazard function is consistent with Taylor type price setters or the annual Calvo model of Alvarez *et al.*(2005). These spikes could also arise in models with information cost. Moreover, these spikes may reflect the seasonality of demand or cost shock.

Hazard Function by Each Major Group

As shown in the previous subsection, the frequency of price adjustment varies a lot between different CPI major groups. There is heterogeneity in price adjustment across firms selling different products. As shown in Figure2-11, the hazard function for non-energy goods is downward sloping in first three months, and becomes relatively flat

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afterwards. The highest hazard rate appears in the first month, showing that there are
a lot of short price spells in this product category. Moreover, there are annual spikes
in the hazard function, indicating sort of Taylor-type contract scheme. The Figure 2-12
reports the hazard function for household service. It is contrast to the hazard function
for non-energy goods. There are quarter spikes in this hazard function, reflecting the
facts that the Taylor-type contracts are more common in the service market than in the
goods market.
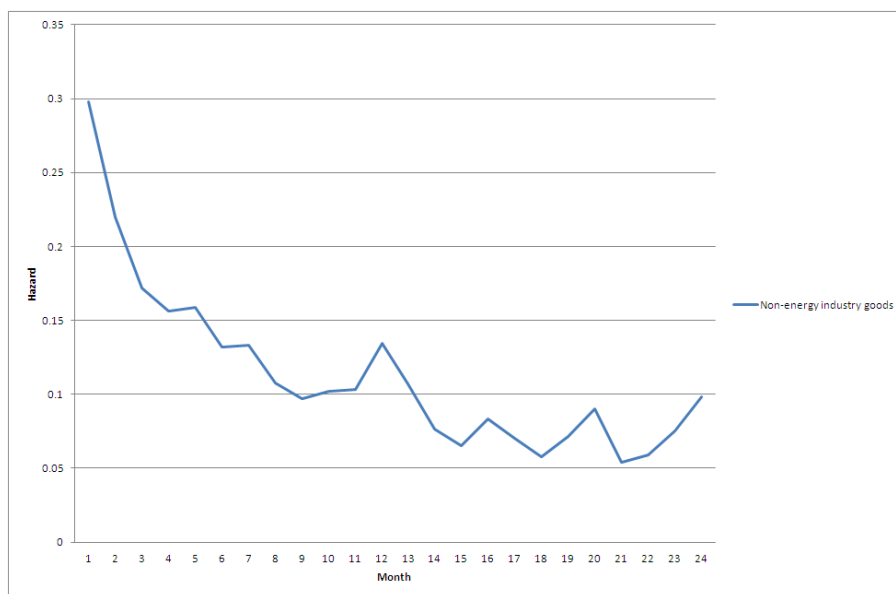
Figure 2-11: Hazard function for non-energy goods

Bunn and Ellis' Method Revisited

In Bunn and Ellis (2012a) work, they estimate the hazard function by choosing one
price spell only in each price trajectory. We argue that this kind of treatment suffers

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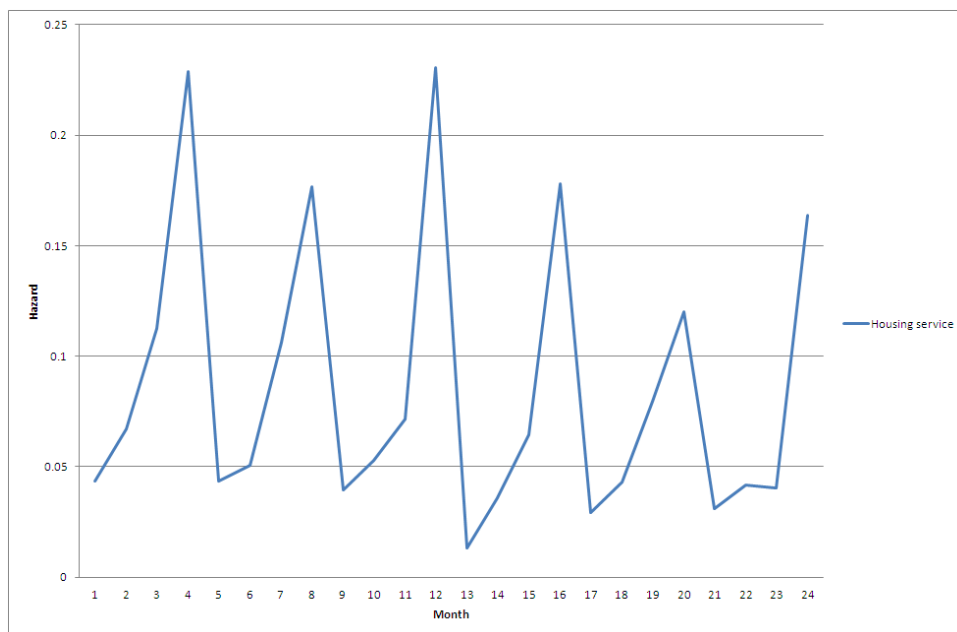
Figure 2-12: Hazard function for household service.

information loss. Moreover, the sampling scheme in BE (2012a) is not random. Therefore, it may suffer the sampling bias. If we estimate the frequency of price change in BE's subsample, we can find that the frequency of price change drops to 12%, which can not reflect the aggregate flexibility in price-setting behaviour. A large amount of short spells are excluded from their subsample. If we draw our hazard function, which is estimated by using all the non-left censored price spells, we can find our estimation differs from BE's in the first 6 months. Especially for the first 3 months, our estimation of hazard rate lies high above the BE's, indicating that the BE's estimation ignores the most flexible part in the price-setting. However, after 8 months, two estimations become relatively closer.
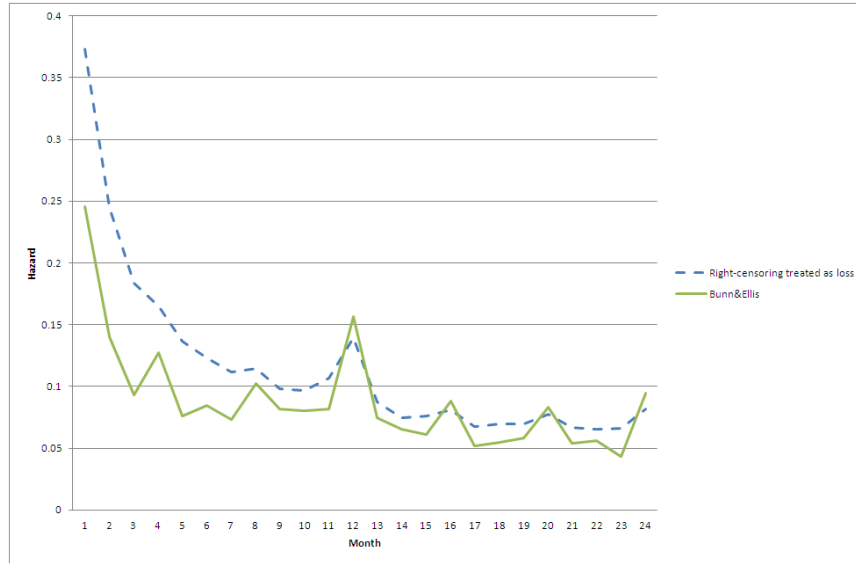
Figure 2-13: Comparison of ours and BE's estimation of hazard function.

2.4.3 Distribution of Durations across Firms

Dixon (2012) argues that "price spells across time are linked by the fact that they are set by the same firm", and "focusing on the distribution of durations (across contracts) is in effect ignoring the panel structure and the fact that it is firms which are generating the price spells". Since we have obtained the hazard function from the estimation on all non-left-censoring spells (right-censorings are treated as failure), the distribution across firms (DAF) can be estimated as Equation (2.1). Hence, we can depict the DAF as Figure 2-14. The first feature observed here is that the share of one-month price spells is the largest. This reflects that there are many flexible price-setters in the economy, even after we correct the oversampling issue. The second feature is that the DAF is decreasing with a spike around 12 months. The third feature is that there is fat 'tail' sticking up at the

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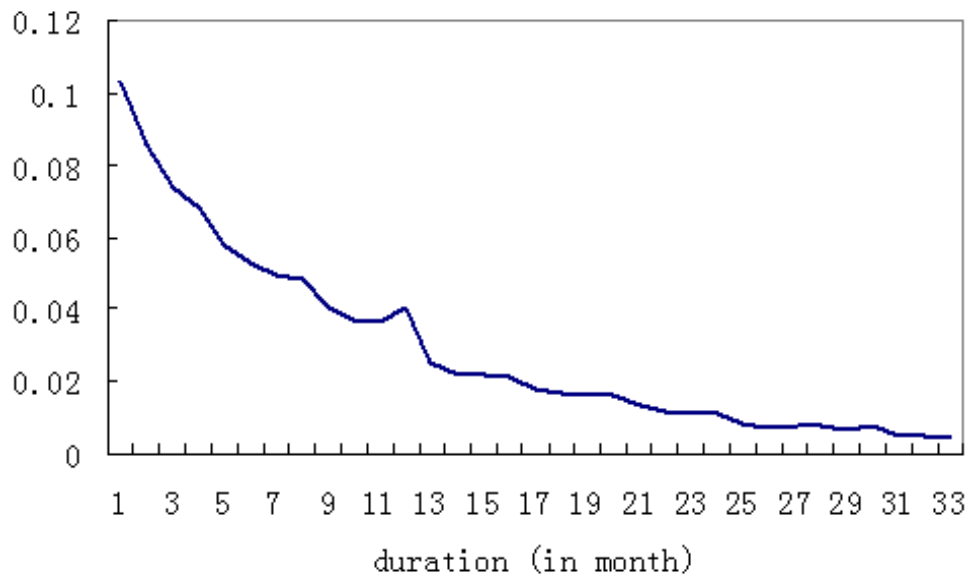
Figure 2-14: Distribution of durations across firms.

end. This feature becomes more clearly when we compare the DAF with the conventional distribution of durations across contracts.

According to Dixon (2012), it is important to distinguish the distribution of duration across firms (DAF) with conventional distribution of duration across contracts. This is because the conventional distribution has a problem of the oversampling of short price spells. Since the conventional distribution of durations ignores the role of firms in setting prices and the panel structure of the economy, it does not directly relate to firms pricing behaviour. As suggested by Dixon and Kara (2011), we can also generate a distribution using aggregate Calvo probability if no estimated hazard function is available. These three distributions (DAF, conventional, Calvo) are shown in the Figure 2-15. It is clear that the conventional distribution of duration has too large share of 1-month spells,
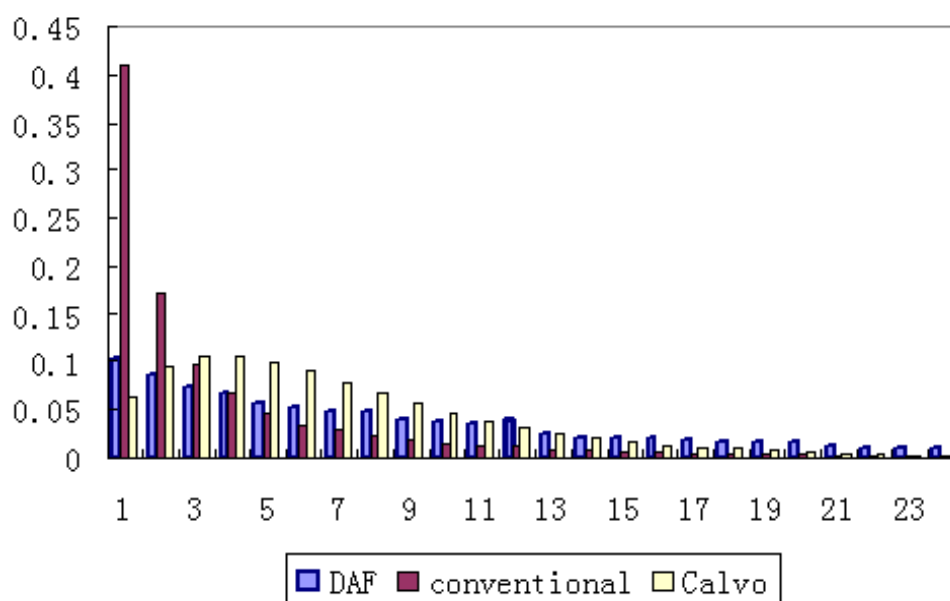
Figure 2-15: Comparision of distributions

while the Calvo distribution does not have enough 1-month spells. There is no 12-month

spike in Calvo distribution or the conventional distribution. Furthermore, the Calvo

distribution and conventional distribution both have thin tails in the long term, while

the DAF has a relatively fat tail in the long term.

2.5   Two benchmark pricing models

Here we try to distinguish between different models of price-setting behaviour by using

the empirical evidence we have found in previous section. This work is inspired by

Eichenbaum, Jaimovich, and Rebelo (2011). In their work, they use their empirical

findings, which come from the weekly data in frame work of "reference price", to discuss

the implications for menu costs models and Calvo models[22]. Similarly, we focus on these two kinds of model: Calvo (1983) and menu costs model developed by Barro (1972), and Sheshinski and Weiss (1977). These two models are most popularly embedded in macroeconomics models, especially in monetary economy.

2.5.1 A common model setup

In line with Wieland et al. (2009), we start by presenting our approach to pricing models comparison. A general class of dynamic stochastic macroeconomic models is augmented with a space of common comparable variables, parameters and shocks. Augmenting models in this manner is a necessary pre-condition for a systematic comparison of particular model characteristics. We derive comparable objects that may be produced as model output, such as average frequency of price change, hazard function[23], and the DAF.

We assume that a firm uses a linear technology to produce a differentiated good. And following most literature in this area[24], we assume an economy without capital, and leave the labor as the only input.

$$y_t \left(q \right) = A_t \left(q \right) L_t \left(q \right) \tag{2.2}$$

From this equation (2.2) , we define the following variables. The firm q produce $y_t \left(q \right)$

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[22]Nakamura and Steinsson (2008) calibrate a benchmark menu cost model to match the main five facts they find about prices.

[23]The slope and shape of the hazard function are important questions, because they are related to such important features of price setting: whether the recently changed prices are more likely to change again, or is it the case that "prices become more likely to change the longer they have remained unchanged"? Nakamura and Steinsson (2008), and Klenow and Kryvtsov (2008) are two among others which highlighted the importance of the slope and shape of the hazard function.

[24]See Golosov and Lucas (2007), Nakamura and Steinsson (2008), Klenow and Kryvtsov (2008).

output in period t. In order to produce this amount of output in period t, the firm need to employ a quantity of labour as $L_t(q)$. A labour combined technology in period t can be defined as $A_t(q)$.

Differentiated goods $y_t(q)$ can be used to produce a final consumption good Y_t. We assume the production function exhibit a CES love of variety over a continuum of differentiated goods y that are indexed by $q \in [0, 1]$:

$$Y_t = \left[\int_0^1 y_t(q)^{\frac{\eta-1}{\eta}} \, dq \right]^{\frac{\eta}{\eta-1}}.$$

And we assume the corresponding unit cost function P_t is:

$$P_t = \left[\int_0^1 p_t(q)^{1-\eta} \, dq \right]^{\frac{1}{1-\eta}}.$$

where $p_t(q)$ denotes the nominal price the firm charges in period t. As is standard in this setup, the demand for the output of firm q is given by

$$y_t(q) = \left(\frac{p_t(q)}{P_t} \right)^{-\eta} Y_t \tag{2.3}$$

where $y_t(q)$ denotes the quantity demanded of the firm's good.

Given aggregate output level Y_t, aggregate nominal price index P_t, and the wage rate for each firm as $W_t(f)$, the firm chooses $\{p_t(q), y_t(q), L_t(q)\}$ to maximize profits subject to equation (2.2,2.3).

The households' preferences are defined over a final composite consumption good C_t, and leisure $1 - L_t$, where L_t denotes the time devoted to market employment. Households maximize their expected lifetime utility of consumption, a discounted consumption stream at time t, $E_t \sum_{t=0}^{\infty} \beta^t U(C^t, 1 - L_t)$, where they exhibit a CES love of variety $C_t = \left[\int_0^1 c_t(h)^{\frac{\eta-1}{\eta}} dz \right]^{\frac{\eta}{\eta-1}}$ over a continuum of final goods c that are indexed by $h \in [0,1]$. For simplicity, we let aggregate consumption is a constant C that determines the size of the market for the firm's good. As is standard in this setup, consumer demand for good is

$$c_t(h) = C \left(\frac{p_t(h)}{P_t} \right)^{-\eta}, \tag{2.4}$$

where $c_t(h)$ denotes the quantity demanded of the firm's good in period t, $p_t(h)$ denotes the nominal price the firm charges in period t. P_t denotes the nominal price index the firm charges in period t, which can be defined as: $P_t = \left[\int_0^1 p_t(h)^{1-\eta} dz \right]^{\frac{1}{1-\eta}}$. The parameter η governs the price elasticity of demand for the individual goods. And for symmetry, we have $q = h$.

For simplicity, we assume that a constant-elasticity demand in the economy, which implies a constant markup pricing strategy:

$$P_t = \frac{\eta}{\eta - 1} W_t$$

Therefore, the real wage in our economy is a constant:

$$\frac{W_t}{P_t} = \frac{\eta - 1}{\eta},$$

(2.5)

where W_t denotes nominal wage rate in the economy at time t. As Nakamura and Steinsson (2008) argues that, more generally, if the degree of the monetary nonneutrality is small, variation in C^t will be small and the real wage will be approximately constant.

Assume that the logarithm of technology of the firm's labour force follows an AR(1) process:

$$\log\left(A_t\left(q\right)\right) = \rho \log\left(A_{t-1}\left(q\right)\right) + \varepsilon_t\left(q\right)$$

(2.6)

where $\varepsilon_t\left(f\right) \sim \mathbf{N}\left(0, \sigma_\varepsilon^2\right)$ is an idiosyncratic technology shock.

Assume that the logarithm of the price level fluctuates around a trend:

$$\log P_t = \mu + \log P_{t-1} + \theta_t$$

(2.7)

where $\theta_t \sim \mathbf{N}\left(0, \sigma_\theta^2\right)$.

2.5.2 Calvo price-setting

We assume that prices vary across firms. In every period t some firms can adjust the price level others cannot. Hence, there is price stickiness. The possibility of changing the price level is determined at random, i.e. price adjustment is random, and this probability

does not change over time.

In particular, we denote by α the probability that the price level is unchanged in a generic period t, while $1 - \alpha$ is the probability that the price level changes.

Using equations (2.2),(2.4),and (2.5) and the fact that markets clear $C = Y$, we can write real profits as

$$\Pi\left(p_t\left(q\right); P_t, A_t\left(q\right)\right) = C\left(\frac{p_t\left(q\right)}{P_t}\right)^{-\eta}\left(\frac{p_t\left(q\right)}{P_t} - \frac{\eta - 1}{\eta}\frac{1}{A_t\left(q\right)}\right) \tag{2.8}$$

When a firm adjusts its price, it maximizes the present discounted value of profits. Firms are assumed to know the current values of both the current exogenous and endogenous state variables, when making their decision about their current price, and assumed to satisfy all demand on this price. The state variables of the system are denoted by Υ, where $\Upsilon_t = \left(p_t\left(q\right), P_t, A_t\left(q\right)\right)$. Let the value function of the firms be

$$V_t\left(\Upsilon_t\right) = \max_{\{V^{NC}, V^C\}}\left(\alpha V_t^{NC}\left(\Upsilon_t\right) + \left(1 - \alpha\right) V_t^C\left(\Upsilon_t\right)\right) \tag{2.9}$$

where the value function in case of no price change NC is given by

$$V_t^{NC} = \max_{p_{t-1}(q)} E_t\left\{\sum_{j=0}^{\infty}\beta^j\Pi\left(p_{t-1}(q); P_{t+j}, A_{t+j}(q)\right)\right\}$$

Because each firm adjusts its price with constant probability $1 - \alpha$ in any period, the

value of a firm upon adjustment is give by

$$V_t^C = \max_{p_{t-1}(q)} E_t \left\{ \sum_{j=0}^{\infty} \beta^j \Pi\left(p_t\left(q\right); P_{t+j}, A_{t+j}\left(q\right)\right) \right\} \tag{2.10}$$

where E_t denotes the expectations operator conditional on information known at time t.

2.5.3 Menu costs

Instead of assuming that the probability of changing the price level is determined at random, we assume that the firm must hire an extra M units of labour in order to change its price, which generate price rigidity[25]. Again, using equations (2.2),(2.4),and (2.5) and the fact that markets clear, we can write real profits as

$$\Pi_t\left(q\right) = C\left(\frac{p_t\left(q\right)}{P_t}\right)^{-\eta}\left(\frac{p_t\left(q\right)}{P_t} - \frac{\eta-1}{\eta}\frac{1}{A_t\left(q\right)}\right) - \frac{\eta-1}{\eta}MI_t\left(q\right) \tag{2.11}$$

where $I_t\left(f\right)$ is a dummy variable indicates whether the price changes or not.

The firm maximizes profits discounted at a constant rate β. The value function of the firm is given by solution to

$$V\left(p_{t-1}\left(q\right); P_t, A_t\left(f\right)\right) = \max_{p_t(q)} \left[\Pi_t\left(q\right) + \beta E_t V\left(p_t\left(q\right); P_{t+1}, A_{t+1}\left(q\right)\right)\right],$$

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[25] Klenow and Kryvstov (2008), Nakamura and Steinsson (2008) suggest the same simple assumption on menu cost.

2.5.4 Solution method

We follow the solution method suggested by Nakamura and Steinsson (2010). Basically, it is a standard iterative procedure on finite grid of points which is proposed by Tauchen (1986). We solve for the firm's policy function by value function iteration on the grid. We choose a relative erro tolerance level, ε. We obtain the numerical estimates of the value and policy function when value function has converged, or $d < \varepsilon$. Here, $d = ||V - V'||.V'$ is the update of initial value function V.

As pointed out by Nakamura and Steinsson (2010), the value function algorithm has a draw back that "it is difficult to prove uniqueness". In the menu cost model, there is a non-convexity because the firm has the option of not changing the price if the cost of doing so is too high. However, the large idiosyncratic shocks assumed in the model significantly reduce the scope for multiplicity, which is in line with Nakamura and Steinsson (2010). It is important that the large idiosyncratic shocks prevent "sufficient synchronization across firms".

2.5.5 Calibration and simulation

Following Hansen (1985) and Rogerson (1988), we assume linear disutility of labour and log-utility in consumption. We set annual discount factor as 0.97 as Klenow and Kryvtsov (2008), consistent with steady state annual real interest rate of 3 per cent, which also leads to the monthly discount factor as $\beta = 0.97^{1/12}$. The value of η defines the markups of prices over marginal cost, or equivalently, the elasticity of demand. Studies

by Eichenbaum and Fisher (2004) and Kimball (1995) suggest a value of $\eta = 11$. Chari, Kehoe, and McGrattan (2000) use a value of $\eta = 10$. Midrigan (2006) uses $\eta = 3$ while Nakamura and Steinsson (2008) use $\eta = 4$, which they think it imply a markup similar to the mean markup estimated by Berry, Levinsohn, and Pakes (1995). Golosov and Lucas (2007) use $\eta = 7$. Dixon and Kara (2010) set $\eta = 12$. In light of these studies, we set $\eta = 5$. We estimate $\mu = 0.0014$ and $\sigma_\theta = 0.00029$ from data on the UK CPI from the period January 1996 to December 2007, implying the average annual inflation rate is about 1.7 percent.[26] We calibrate the rest parameters of the model to match statistics for price changes, which include the frequency of price change and the fraction of price changes that are price increases in CPI data from 1996 to 2007. We find that the implied first order autocorrelation in technology is $\rho = 0.7$, the idiosyncratic technology shock $\sigma_\varepsilon = 0.0425$[27]. For the menu costs mode, we let the menu costs comparing with whole economy to be $M/C = 0.0119$. For the Calvo model, we let the the probability of price changes be consistent with our empirical finding $\omega = 0.1855$.

We simulate model given the grid for idiosyncratic technology shock $\varepsilon_t(q)$, the grid for the real price $(p_t(q)/P_t)$, autoregressive coefficient in process for technology ρ, standard deviation of idiosyncratic shocks σ_ε, average monthly inflation rate μ, standard deviation of shocks to price level σ_θ. The number of time periods to be simulated is set to $300,000$. And we drop the first 200 time periods when we start the simulation. Furthermore, we

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[26] Annual inflation rate can be calculated as $(1 + \mu)^{12} - 1$.

[27] These calibrations are quite similar to those in Nakamura and Steinsson (2008). In their paper, $\rho = 0.66$, and $\sigma_\varepsilon = 0.0428$.

create duration data from the simulated price data, and chop off left censored spells to calculate a Kaplin-Meier estimate of the hazard function of price changes. Afterwards, we generate the distribution of durations across firms from hazard function of price changes.

Simulation result part1: Calvo model

In this part, we report the result of simulation on Calvo model. We have around $300,000$ nominal price quotes which are generate from our Calvo model. We estimate the frequency of price changes, the fraction of price changes that are increases from the simulation dataset by using the same formula we used for the empirical analysis, and we manage to make our estimation close to our previous empirical findings. The frequency of price changes in our Calvo-dataset equals 18.17 percent. The fraction of price changes which are increases is 58.52 percent. The figure2-16 shows the hazard function which is estimated from a dataset simulated from Calvo model. It is clear that the hazard function is almost a horizontal line[28], which means that the probability of price change is constant in Calvo model.

The Figure 2-17 describe the distribution of duration across firms implied by Calvo model. This is consistent with the shape of exponential distribution, just as Dixon and Kara (2005) point out.

We also generate the graph which shows the evolution of price trajectory, the path

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[28]The main reason for the hazard function for the Calvo case deviates from a completely horizontal line is that the right censorings will affect the calculation of the empirical hazard function. Theoretically, some firms with Calvo pricing strategy may keep their price unchanged forever. However, in the simulation, we must treat the unchanged price spells as right censorings, and this will affect the simulated hazard function.
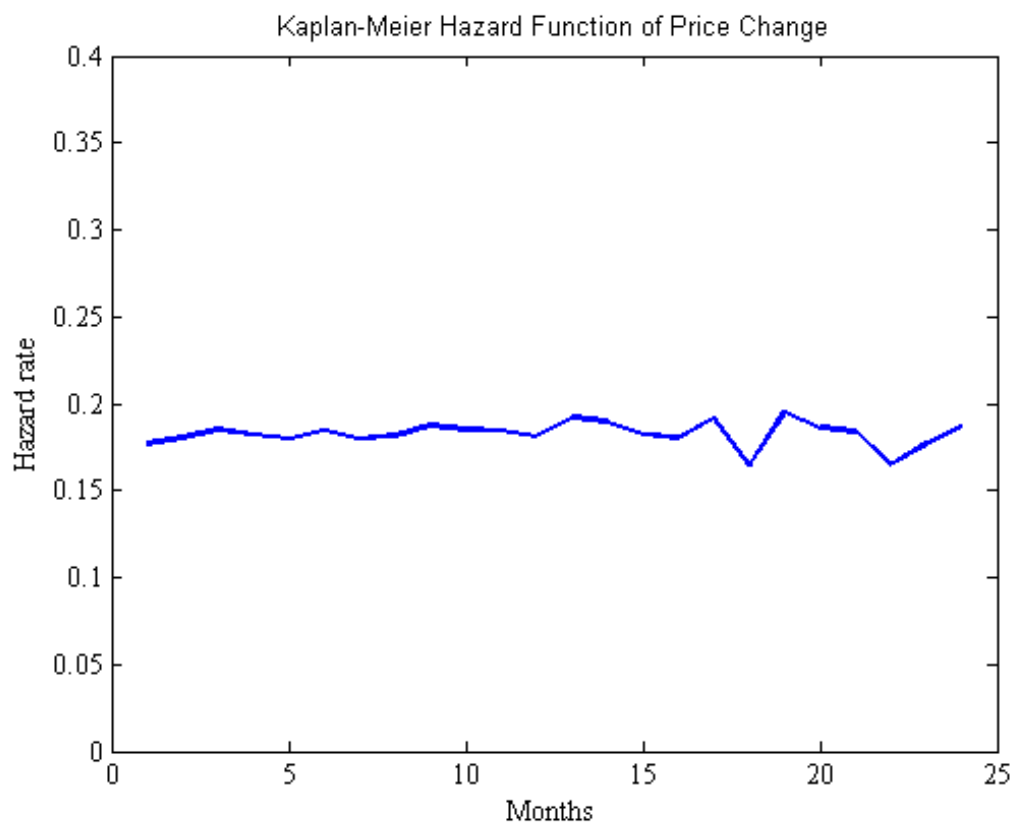
Figure 2-16: Hazard function of price change in Calvo pricing.
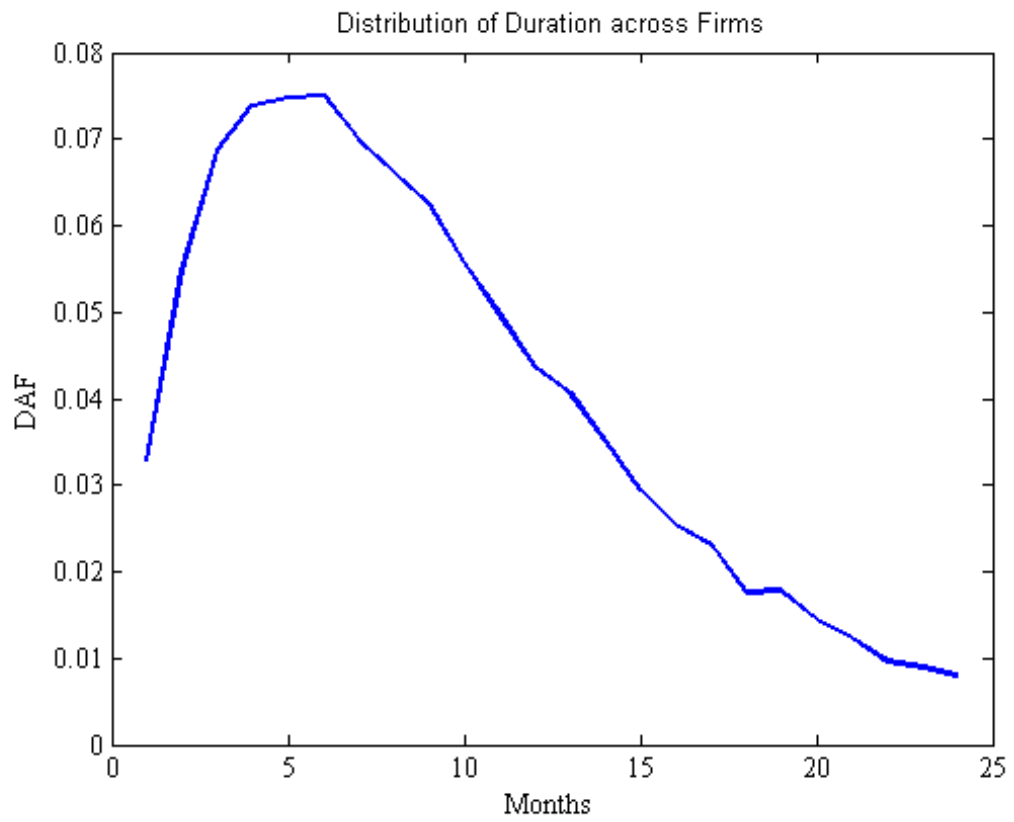
Figure 2-17: Distribution of duration across firms implied by Calvo model
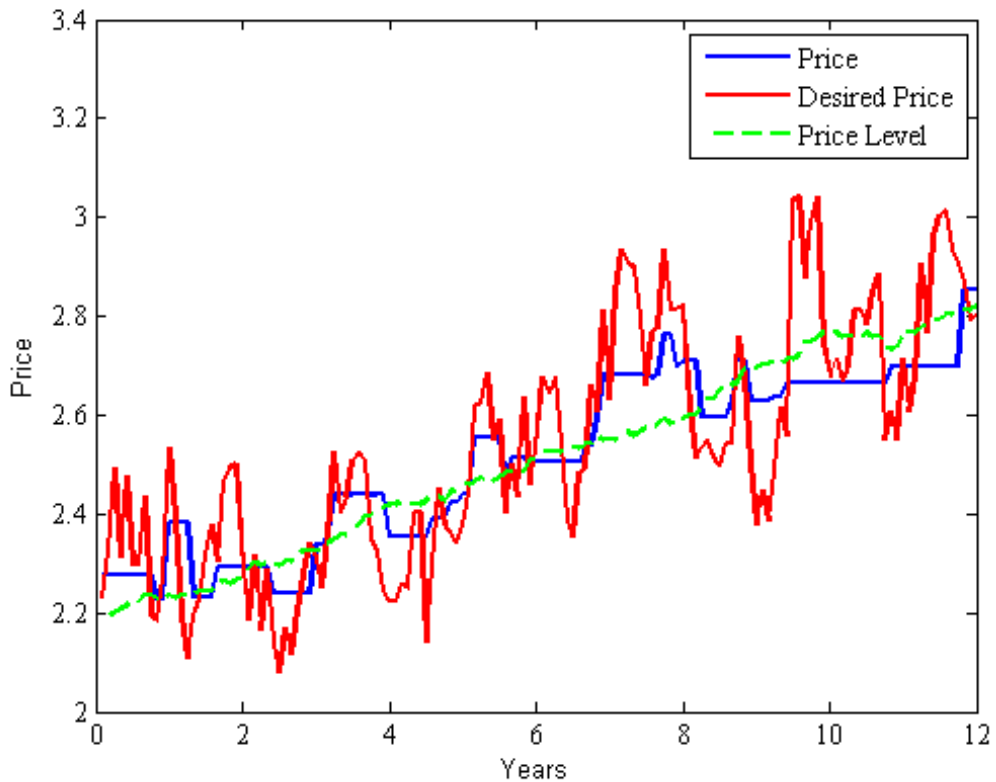
Figure 2-18: Simulation of price trajectory from Calvo model.

of desired price, and general price level.The Figure 2-18 indicate that the desired price ,which can be measured by the product between general price P_t and inverse of $A(f)$, is the most volatile one. This is to say, due to the idiosyncratic technology shock, the desired price level are quite flexible and volatile. But according to our Calvo model setup, the fraction ω of firms are not able to change the price. Therefore, the blue line which shows the actual price is sticky. This phenomenon shows that pass-through from marginal cost to prices is incomplete. This finding from our simulation is consistent with those findings from aggregate time-series data, such as Bils (1987), Rotemberg and
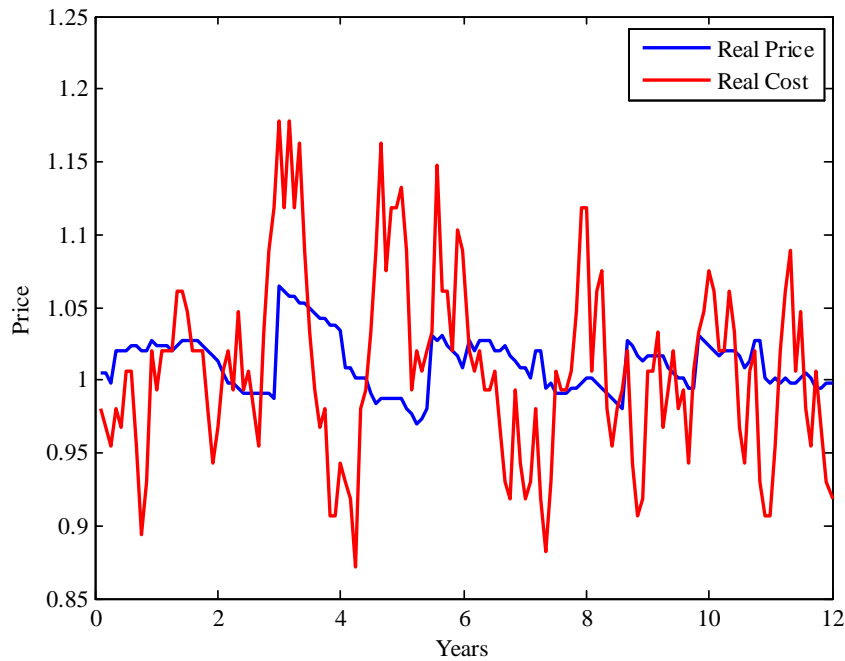
Figure 2-19: Real price and real cost simulated by Calvo model.

Woodford (1999) and Altig, Christiano, Eichenbaum and Linde (2005), all of which argue that prices are less volatile than marginal cost. The green dash line shows the general price level, is quite smooth and upward sloping.

Additionally, we can look at the behaviour of real price $\frac{p_t(f)}{P_t}$ and real cost $\frac{1}{A_t(f)}$ implied by Calvo model in Figure 2-19. The real cost changes are far more volatile than the real price changes.

Simulation result part2: menu costs model

In line with the operation we did in part 1, we also simulate a large micro price dataset from our benchmark menu costs model. We estimate the frequency of price changes from
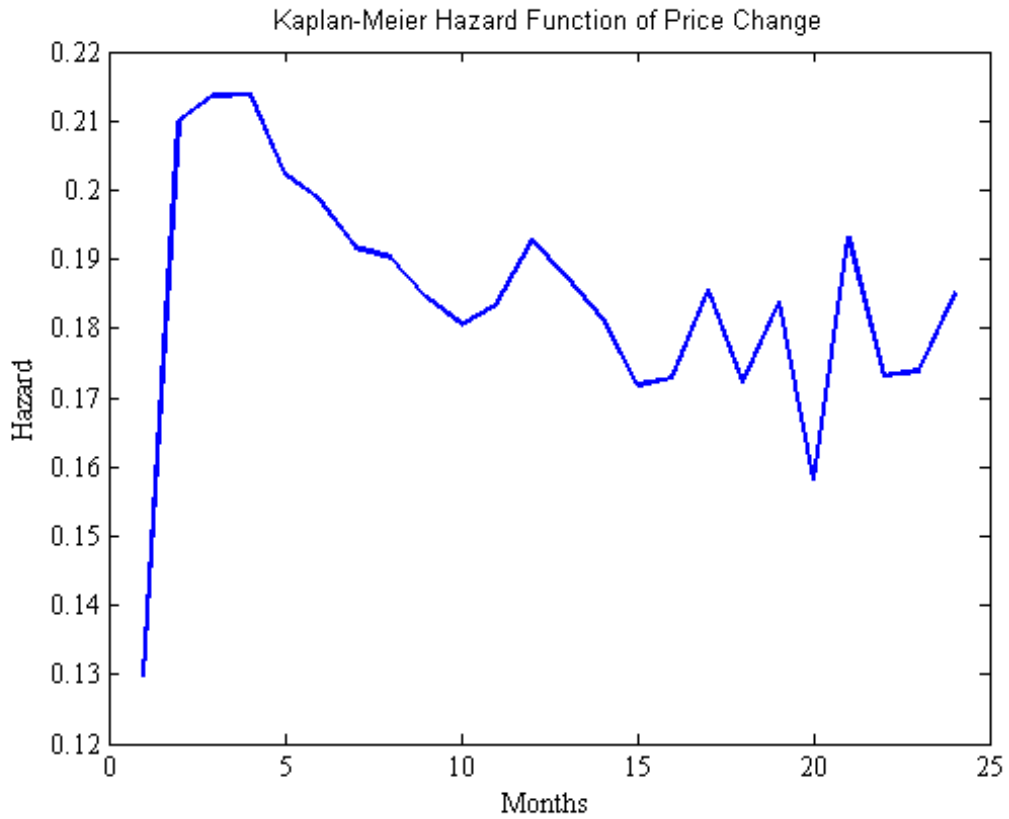
Figure 2-20: Hazard function of price change from menu cost model.

this dataset. The statistics on frequency of price change equal 18.77 percent. And the fraction of price changes which are increases accounts for 55.5 percent. The implied hazard function from our benchmark menu costs model is shown in Figure 2-20. The hazard function from menu costs model is quite different from Calvo model. It is increasing in first few periods, and then it is becoming relatively flat (after downward sloping by a few months) with a few spikes. The hazard rate are moving around 0.18 which close to the frequency of price change in this dataset. This finding is consistent with Nakamura and Steinsson (2008).
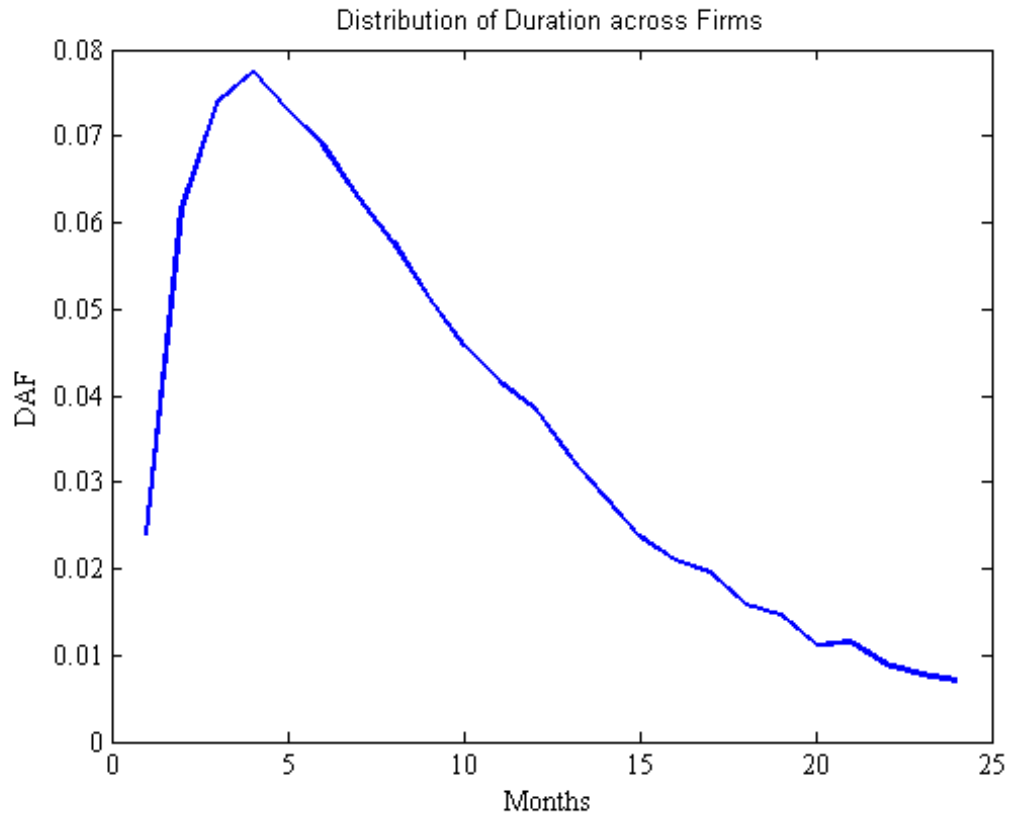
Figure 2-21: Distribution of duration across firms implied by menu cost model.

Furthermore, we can generate the distribution of duration across firms from the hazard function implied by menu costs. The most striking result is that the Figure 2-21 is similar to what we find in Calvo model. In this micro dataset, the most common price duration is about 4 or 5 months in length. However, we should notice that, the DAF generated by menu costs model differs from the DAF generated by Calvo model in skewness and kurtosis.

Moreover, we generate the paths for desirable price, actual price and price level from the price dataset simulated from menu costs model. We can identify a key feature of

Figure 2-22, which is that the blue line is quite likely to change with the red line when large jumps appear in red line. That is to say, when idiosyncratic technology shock is large enough, the price setters intend to change their actual price to make it in line with the desired price. This can also be shown in Figure2-23, which plots the real price and real cost[29] simulated by menu costs model. However, we cannot observe this feature from Figure 2-18. This is because the Calvo model does not have the channel to connect the idiosyncratic shock with the timing of price change. However, in menu costs model, the nominal price is still less volatile than the marginal cost.

2.6 Extension: Multiple-sector model

2.6.1 Multiple Calvo model

As we find in previous sector, the Calvo model and Menu costs model both fail to replicate the micro evidences we find on hazard function and the distribution across firms. The reason is that both models assume homogeneous structure in price setting. However, our empirical finding suggests that there is significant heterogeneity in price setting behaviours. As pointed out by Carvalho (2006) and Dixon (2012), the multiple Calvo model can mimic the heterogeneity in price settings. *A la* Dixon(2009), a multiple Calvo model can be defined as $MC(\omega, \lambda)$ where $\omega_k \in (0,1)$which is the frequency of price change for

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[29]As a we have defined, the real cost equals to the inverse of $A_t(f)$.
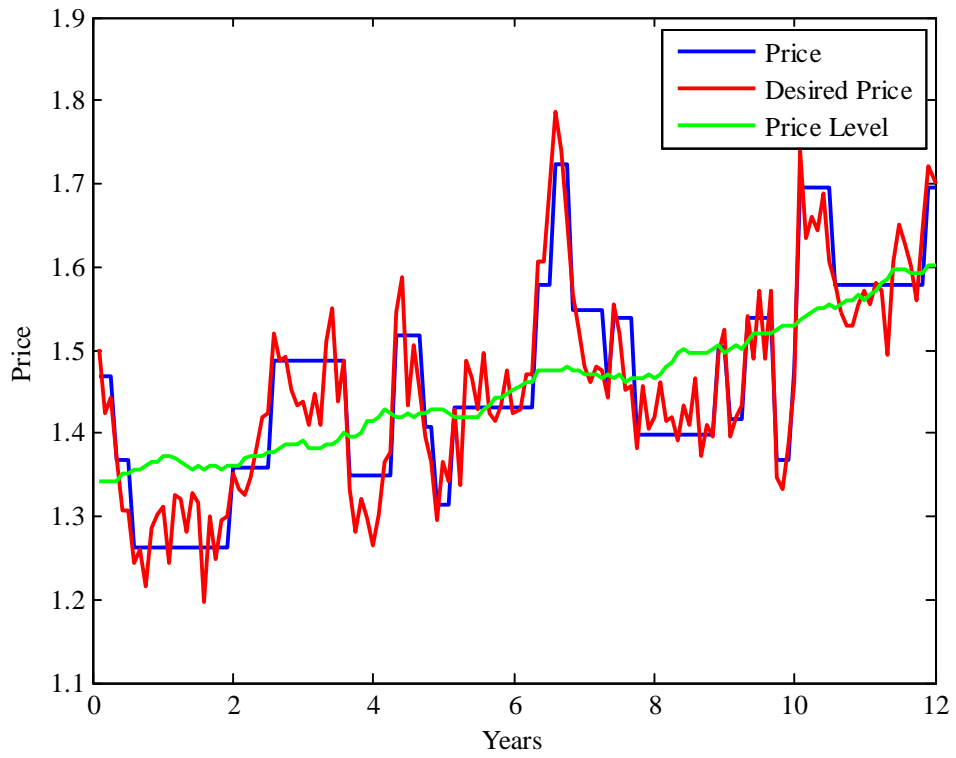
Figure 2-22: Actual price trajectory, desired nominal price, and general price level simulated from menu cost model.
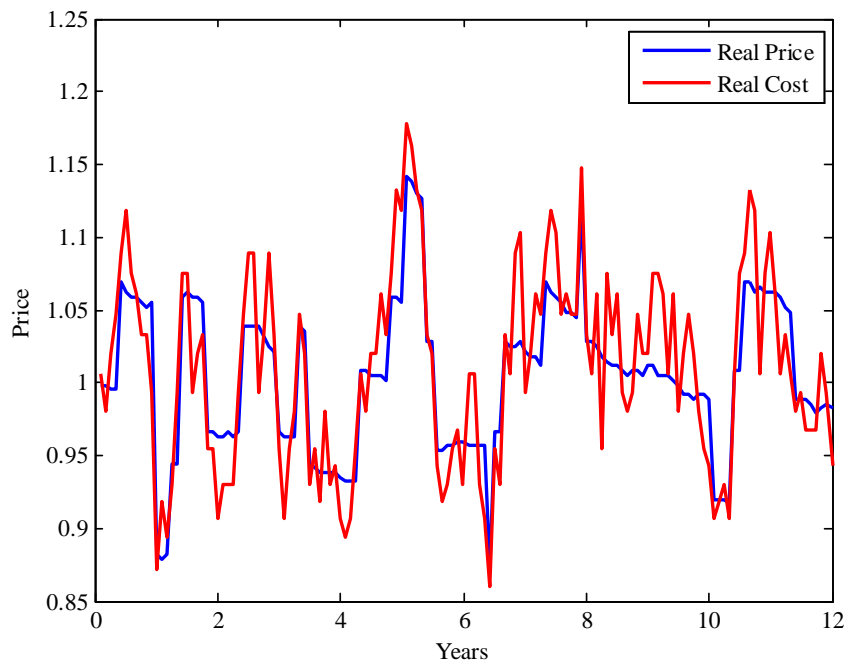
Figure 2-23: Real price and real cost simulated by menu cost model.

sector k, and $k = 1, \ldots, n$ and $\lambda \in \triangle^{n-1}$ is the vector of CPI weights. We can get the reset price for each sector as :

$$x_{kt} = \frac{1}{\sum_{j=1}^{F} (1 - \omega_k)^{j-1} \beta^{j-1}} \sum_{j=1}^{F} (1 - \omega_k)^{j-1} \beta^{j-i} p_{t+j-1}^*$$

The average price in each sector k is :

$$p_{kt} = \sum_{j=1}^{F} (1 - \omega_k)^{j-1} \beta^{j-1} x_{kt-j+1}$$

And the aggregate price level is:

$$p_t = \sum_{k=1}^{n} \lambda_k p_{kt}$$

We solve the multiple Calvo model by using the value function iteration method, and simulate the model given the price frequency and CPI weight in each of 570 item-level sectors in our micro data. Actually, we get a downward sloping hazard function, albeit no annual spikes are found in our model simulation (see Figure 2-24). And we also get a hump shaped DAF in Figure 2-25, which is consistent with the finding in Dixon and Tian (2011). The simulation result from multiple Calvo model suggests that adding in heterogeneity in our pricing model can improve the fitness of micro evidence.
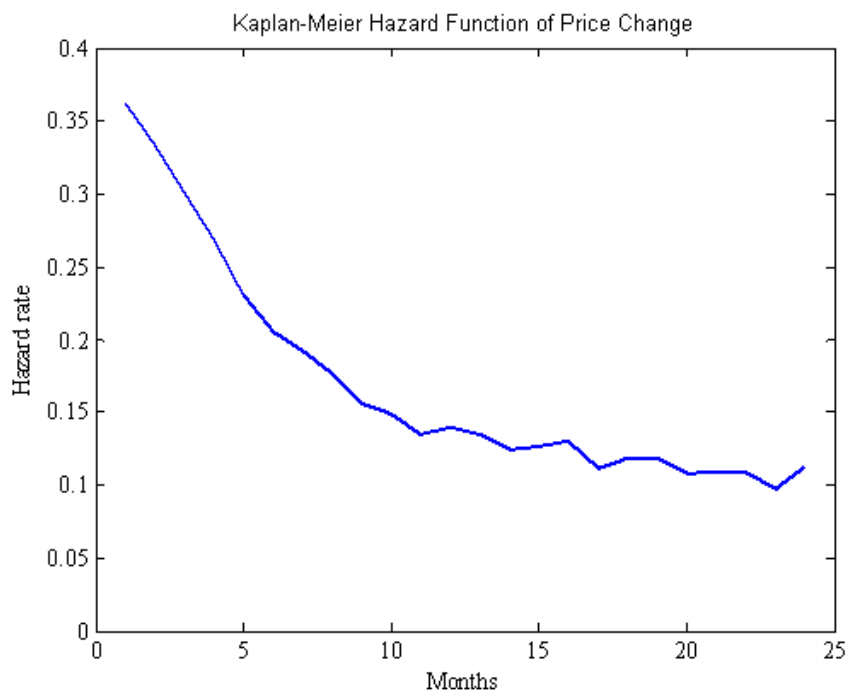
Figure 2-24: Hazard function from Multiple Calvo model
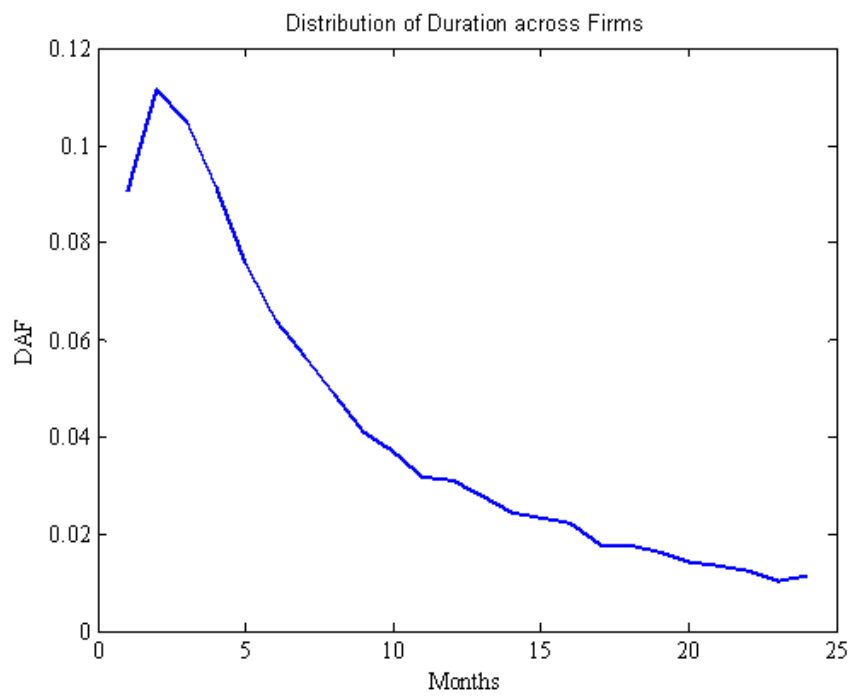
Figure 2-25: DAF from Multiple Calvo model

2.6.2 Multiple-sector menu costs model

If we assume that there are many sectors in the economy, and in each sector a specific product is produced, facing a specific menu cost. *A la* multiple Calvo model in previous subsection, the multiple menu costs model is able to generate a downward sloping hazard function, which is shown in Figure 2-26. In order for a downward sloping aggregate hazard function to occur, there must be many firms with a low hazard, and the fraction of firms with a low hazard must be increasing. The reasoning is analogous to that which explains a downward sloping aggregate hazard in a model with many types of sectors with different Calvo probabilities of adjustment. As the age of a price progresses, the fraction of firms who have not adjusted is increasingly dominated by those with the lower hazard–implying a downward sloping aggregate hazard rate. Furthermore, adding firm-level productivity shocks creates the possibility that unexpected increases in productivity lead to situations where firms wish to decrease their price, and where the probability of price adjustment decreases over time even though a firm's relative price depreciates with inflation. The DAF for multiple menu costs model can be seen as Figure $2 - 27$. The DAF generated from multiple menu costs model becomes more close to the empirical DAF estimated from micro data. Though there is a small hump in the first 2 months in our simulation result.
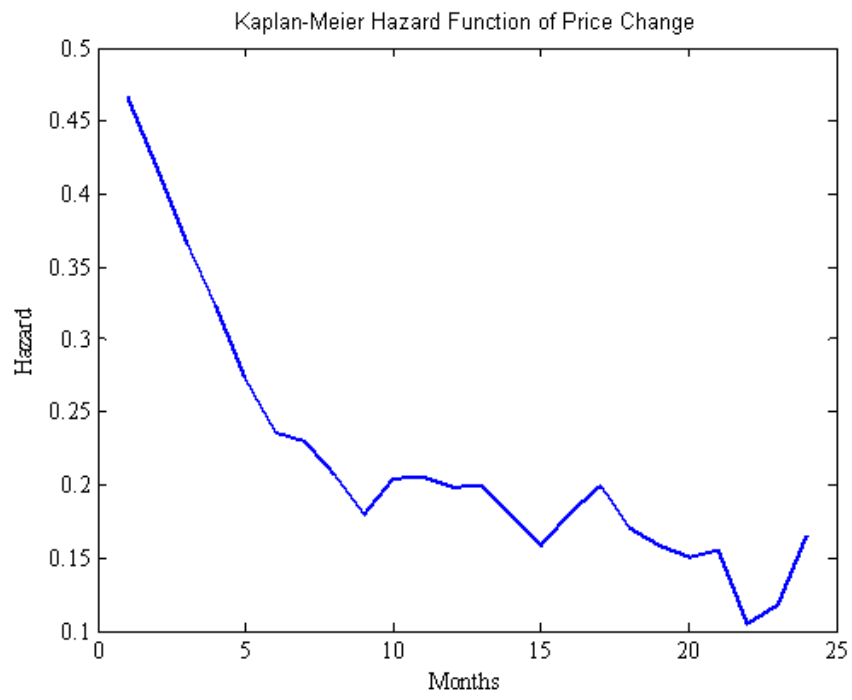
Figure 2-26: Hazard function generated from Multiple Menu Costs model
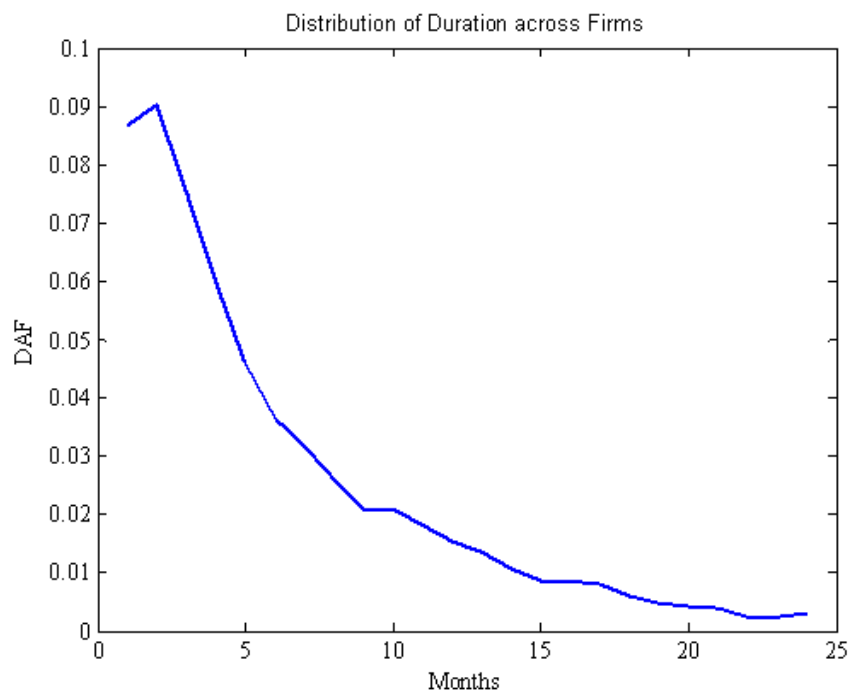
Figure 2-27: DAF generated from Multiple Menu Cost model

2.7 Conclusion

We propose a unified framework to assess the price rigidity. The frequency, hazard function, and the distribution across firms are three different perspectives to look at the same thing. We prove that these three methods can give the consistent estimates if we choose the proper treatment on censoring price spells. We also compare the different assumption on the right-censoring, and how robust our results are.

Employing the CPI data collected by the ONS, we find some empirical facts about price setting from late 1990s to 2007. In CPI micro- data set, price changes are frequent (about 21% on average) and the frequency of price changes varies a lot between different sectors. When we estimate the frequency of price changes excluding the sales from CPI data, the frequency of price changes becomes much lower.

The aggregate hazard function is downward sloping with large annual spikes. The downward sloping hazard function cannot be explained by standard time-dependent or state-dependent model. However, it can be rationalized by a customer relation model (Vincent, 2012). It is assumed that firms understand that their pricing decisions will affect their customer base and hence future profits. As the firm maintains prices constant, it attracts new custmoers and retains its loyal clients. This mechanism may rationalize the decreasing hazard functions observed empirically. Alternatively, we see the downward sloping aggregate hazard function as a result from the mixture of heterogeneous price-setters.

We generate the distribution across firms from the hazard function we estimated

using non-parametric survival analysis. Comparing with the common used distribution of duration across contracts, we find that the DAF (the desired distribution) is much flatter than the distribution across contracts. As argued in Dixon (2012), the DAF is the proper distribution we should infer to when we assess the price rigidity in the second order moment, and the Calvo probability itself is not enough to generate the whole picture of price rigidity.

We solve and simulate two benchmark models, the basic Calvo model and menu costs model. We let both models to match the empirical findings in frequency of prices changes and the proportion of prices increases. However, these two benchmark models cannot meet the empirical findings in hazard functions and the implied DAF are different from empirical one. Moreover, the two benchmark models have different implication on the response of actual price level to the idiosyncratic shocks. All of these results suggest that we need to be careful about the pricing model set up. And a next generation of pricing models is needed if we want the macro models to meet the micro evidences. We solve and simulate a multiple-sector Calvo model and a multiple-sector menu costs model. The results suggest that adding heterogeneity to price-setting model can improve its fitness on micro evidence. As shown in Dixon and Kara (2010), allowing for a distribution of durations can take us a long way to solving the puzzle of inflation persistence. In other words, an explicit modeling of the distribution of durations can help the DSGE model to match macro data.

Chapter 3

What we can learn from the average monthly frequency of price-changes in CPI data: an application to the UK CPI micro data.

3.1 Introduction

In recent years, there have been many studies using comprehensive micro-data on pricing. In the Euro area, there has been the inflation persistence network (*IPN*) consisting of national studies of the CPI and PPI micro data[1], which are summarized in Dhyne et al

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[1]See Beaudry et al (2007) and Alvarez and Hernando (2006) for France and Spain *inter alia.*

(2006). In the US there have been similar studies: Bils and Klenow (2004), Klenow and Krytsov (2008), Nakamura and Steinsson (2008)[2]. One common focus of these studies has been the statistic of the proportion of prices changing per month (this can either be an average over several months, or a monthly statistic). This statistic can be presented in several ways, depending on the level of disaggregation and the treatment of temporary sales and so on. In this chapter, we seek to analyze what this statistic implies for the behavior of firms (or more accurately price-setters) in the economy. Each period firms set prices: they may either choose to leave the price unchanged or to change it. The proportion of firms resetting price corresponds to the proportion of prices changing (for simplicity we take a 1-1 correspondence between firms and prices). The prices of some product types change frequently (e.g. gasoline, tomatoes) while some very infrequently.

We can think of the CPI dataset as a panel of observations, each cross-section corresponding to the prices set by "firms" at that date. The cross-sectional mean completed price spell can be seen as capturing the mean behavior of the price-setters, which represents the "structure" of the economy in this respect (i.e. the average behavior of the firms in the economy). For a given frequency of price change, what can we infer about the behavior of the firms? In this paper we are able to derive a lower bound (and an upper bound) for the mean length of price-spells across firms, interpreted as the cross-sectional mean completed price-spell. The cross-sectional distribution is needed if we are to model price-setting as a Taylor process. We then use the UK CPI data for the period 1996-

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[2]See Bunn and Ellis (2009) for the UK and Baharad and Eden (2004) for Israel.

2007 and consider frequency data at three different levels of disaggregation: the 11 sector COICOP, the 67 sector COICOP and the highest possible level of disaggregation at 570 items, to see how the actual data on price-spell durations compares to the theoretical minimum. We find that the actual mean estimated from the CPI is 10.9 months, which is 62-90% higher than the theoretical minima generated from the frequency data. This is not surprising, in Proposition 1 we find that the theoretical minimum consistent with a given frequency is only attained if all price-spells have the same or almost the same duration, whilst the actual distribution contains a lot of heterogeneity in durations which implies a longer cross-sectional mean.

We also interpret the frequency data under the hypothesis that within the sector the frequency is generated by a Calvo distribution, as has been assumed in applied work by Dixon and Kara (2010, 2011). We look at this in two ways. First, we aggregate over all sectors to derive the aggregate distribution under this assumption: thus we have the distribution of durations in each sector and for each duration we aggregate over sectors using the sectoral CPI weights. We can then compare this to the "true" distribution derived from the estimated hazard function.

- The aggregate distribution derived under the Calvo hypothesis at the sectoral level has a similar mean and median to the true distribution, with the mean increasing with the level of disaggregation. For example, the 570 model yields a mean of 10.8 and median of 7.8 months, whilst the true values are 10.9 and 7.8 respectively.

- However, the implied Calvo distributions differ from the true distribution in signif-

icant ways: (i) there is a 12 month spike in the true distribution absent from the Calvo distributions, (ii) the proportion of short price-spells (1-3 months) is less in the Calvo than in the true distribution.

We also examine whether the Calvo is a good fit in each of the 11 COICOP sectors. Since the data set is large, even small deviations of the actual distribution from the hypothetical Calvo distribution cause the Calvo null to be rejected under the Kolmogorov-Smirnov test, which is indeed the case. However, whilst in some sectors the hypothesized Calvo distribution looks completely different (for example in Health which has a very large 12 month spike), in others the Calvo looks more similar (Transportation). There is clearly a variety of patterns across sectors[3] when we compare the true distributions within the 11 COICOP sectors.

Whilst the Calvo distributional hypothesis might not provide a good statistical fit in terms of the aggregate or sectoral distributions, does this matter in terms of how the economy behaves? Since the Calvo hypothesis yields a mean and median close to the true distribution, perhaps the differences will not result in different behavior of the economy in terms of impulse response functions. We explore this in the context of two macro models: a simple Quantity Theory model and the Smets and Wouters (2003) Euro area model. The pricing models used are the Generalized Taylor and Calvo as in Dixon and Le Bihan (2012), which are both consistent with any micro distributions of durations and can be calibrated to the true data and the data under the Calvo distributional hypothesis. What

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[3]This heterogeneity across sectors was also found in the French data by Fougere et al (2007).

we find is that the impulse response functions for output and inflation are very similar when we use the true distribution and hypothetical Calvo distribution at the different levels of disaggregation for both the generalized Taylor and Generalized Calvo. In the Smets-Wouters model, the IRFs are indeed almost identical. This indicates that if we are interested in modelling macroeconomic properties of an economy, using the sectoral frequency data under the Calvo distributional hypothesis might be a useful shortcut and alternative to estimating the distribution using the hazard function. Indeed, where the actual hazard is not available or not estimated reliably, we can be confident that the use of sectoral frequencies with the Calvo distributional hypothesis can be a good working approximation.

The structure of the chapter is follows. In section 2, we give a theoretical description of the steady state distributions of durations. In section 3, we derive an upper and lower bound for the mean duration of price-spells averaged across firms. We show an application to the UK CPI data in section 4. In section 5, we simulate a quantitative theory model and a Smets-Wouters model, both with Generalized Taylor and Generalized Calvo price-setting respectively. We conclude in sector 6.

3.2   Steady state distributions of durations.

The statistical framework for understanding the CPI microdata is outlined in detail by Dixon (2012), so in this paper we just summarize in a less technical manner the key properties needed for this paper.

There is a continuum of price-setting firms $f \in [0,1]$, time is discrete[4] and infinite $t \in Z_+ = \{0,1,2...\infty\}$. The price set by firm f at time t is p_{ft}. A price spell is a duration, a sequence of consecutive periods that have the same price. For every $\{t,f\} \subseteq [0,1] \times Z_+$ we can assign an integer $d(t,f)$ which is the duration of the price-spell to which p_{ft} belongs[5]. The distribution of price-spell durations is simply the proportions of all durations having length $i = 1...F$: $\boldsymbol{\alpha}^d = \left\{\alpha_i^d\right\}_{i=1}^F \in \Delta^{F-1}$. We assume a steady-state, so that the distribution of durations of new price-spells is the same for each new cohort of price-spells. This means that the distribution of price-spells is exactly the same as the distribution of new price spells at any period.

Whilst the distribution of durations $\boldsymbol{\alpha}^d \in \Delta^{F-1}$ is one way of looking at the micro-data, it ignores the *panel* structure of the data. Each row of the panel is a trajectory of prices corresponding to a particular firm (or more accurately product sold at an outlet). Each column is a cross-section of all of the prices set by firms at a point in time. The cross-sectional distribution of *completed* price-spell durations is $\boldsymbol{\alpha} \in \Delta^{F-1}$. In effect, we take a representative t, and for each firm we see the completed price-spell duration at that time $d(f,t)$.

The proportion of firms re-setting price each month is denoted as $\bar{h}$: in the UK this is equal to 21%. We define the mean duration[6] of price-spells across the Panel as a whole

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[4]Typically, CPI data are collected on a monthly basis, the price observations being obtained in the first two weeks of the calendar month.

[5]Note that in assigning an integer to a duration, we start with 1 by convention: it would be equally valid to start with 0. With our convention, a new price-spell is 1 month old, rather than becoming 1 on completion of the first month.

[6]Again, this way of defining the mean is consistent with our convention of assigning the integer 1 to the first time period. Had we instead assigned a 0 to this value, then we would have the expression

as

$$\bar{d}\left(\boldsymbol{\alpha}^d\right) = \sum_{i=1}^{F} i\alpha_i^d$$

and cross-sectional mean (across firms) as

$$\bar{T}\left(\boldsymbol{\alpha}\right) = \sum_{i=1}^{F} i\alpha_i \tag{3.1}$$

Note that the cross sectional mean in general be larger than the mean duration $\bar{T} \geq \bar{d}$: this is because in cross-section you have length-biased sampling, since the probability of a price-spell being observed in cross-section is proportional to its duration. Indeed, the two can be equal $(\bar{T} = \bar{d})$ *if and only if* $\alpha_F = \alpha_F^d = 1$, so that all price-spells are F months long and there is no heterogeneity to generate a length bias.

From Dixon (2012), we know that[7]:

$$\bar{h} = \bar{d}^{-1} \tag{3.2}$$

$$= \sum_{i=1}^{F} \frac{\alpha_i}{i} \tag{3.3}$$

That is, the proportion of firms resetting price is equal to the reciprocal of the mean duration $\bar{d}$. Furthermore, the proportion of firms resetting price is related to the cross-sectional distribution by equation (3.3). In steady-state, a proportion i^{-1} of the α_i

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$\sum_{i=1}^{F}(i-1)\alpha_i$ (as we do with human ages). An equally acceptable measure is to take the midpoint and have $\sum_{i=1}^{F}(i-0.5)\alpha_i$ We can move between these definitions simply by adding or subtracting a constant.

[7]In continuous time, we have $\bar{d} = -\frac{1}{\log(1-\bar{h})}$, which allows for the price to change more than once per period. Again, we are using a discrete time setting in which durations are integer valued.

$i-$duration firms reset their price. The aggregate proportion is simply the sum over the durations $i = 1...F$.

3.3 The average duration across firms consistent with $\bar{\text{h}}$.

Now, for a given frequency $\bar{h}$ there are many possible distributions across firms (DAF) $\boldsymbol{\alpha} \in \Delta^{F-1}$ consistent with identity (3.3) and each distribution results in a corresponding mean across firms $\bar{T}(\boldsymbol{\alpha})$. We can define the mapping $H\left(\bar{h}\right) : [0,1] \to \Delta^{F-1}$

$$H\left(\bar{h}\right) = \left\{\boldsymbol{\alpha} \in \Delta^{F-1} : \sum_{i=1}^{F} \frac{\alpha_i}{i} = \bar{h}\right\}$$

$H\left(\bar{h}\right)$ is the set of all $DAFs$ which are consistent with a given mean duration of price-spells $\bar{d}$ expressed in terms of the corresponding proportion of firms resetting prices $\bar{h}$. Clearly, since the maximum duration is F, we have $\bar{h} \geq F^{-1}$ so that H is non-empty. Since $H\left(\bar{h}\right)$ is defined by a linear restriction on the sector shares $\boldsymbol{\alpha}$, $H\left(\bar{h}\right) \subset \Delta^{F-2}$ and is closed and bounded. We can then ask what is the minimum (maximum) $\bar{T}$ consistent with a given $\bar{d}$. Since $H\left(\bar{h}\right)$ is non-empty, closed and bounded, with $\bar{T}\left(\boldsymbol{\alpha}\right)$ continuous, both a maximum and a minimum will exist. Turning to the minimization problem first: we have:

$$\min \; \bar{T}\left(\boldsymbol{\alpha}\right) \quad s.t. \; \boldsymbol{\alpha} \in H\left(\bar{h}\right) \tag{3.4}$$

Proposition 1 Let $\boldsymbol{\alpha}^{\min} \in \Delta^{F-1}$ solve (3.4) to give the shortest average contract length $\bar{T}^{\min}$.

    (a) No more than two sectors i have values greater than zero

    (b) If there are two sectors $\alpha_i > 0$, $\alpha_j > 0$ then will be consecutive integers $(|i - j| = 1)$.

    (c) There is one solution iff $\bar{h}^{-1} = k \in Z_+$. In this case, $\alpha_k = 1$.

    (d) The minimum is $\bar{T}^{\min} = \bar{h}^{-1} = \bar{d}$.

We can also ask what is the *maximum* average contract length consistent with a proportion of re-setters $\bar{h}$:

$$\max \ \bar{T}(\boldsymbol{\alpha}) \quad s.t. \ \boldsymbol{\alpha} \in H(\bar{h}) \tag{3.5}$$

Proposition 2 Let $\boldsymbol{\alpha}^{\max} \in \Delta^{F-1}$ solve (3.5). Given the longest contract duration F, the distribution of contracts that maximizes the average length of contract subject to a given proportion $\bar{h}$ of firms resetting price

$$
\begin{aligned}
\alpha_F^{\max} &= \frac{F}{F-1}\left(1 - \bar{h}\right) \\
\alpha_1^{\max} &= \frac{F}{F-1}\bar{h} - \frac{1}{F-1}
\end{aligned}
$$

with $\alpha_i^{\max} = 0$ for $i = 2...F - 1$. The maximum average contract length is

$$\bar{T}^{\max} = F\left(1 - \bar{h}\right) + 1$$

To understand Propositions 1 and 2[8], we just need to think of what is generating the mean duration $\bar{d}$ and the proportion of firms changing price each period $\bar{h}$. There is the unit interval of firms, divided into proportions with different price-spell durations $i = 1...F$. Firms with price-spell lengths i will set prices once every i^{-1} periods: the longer the price-spell, the more infrequently the firm will reset price. Hence, we can have the same proportion of firms re-setting price (and hence same mean duration) and increase the mean duration across firms by more longer price-spells. The maximum $T^{\max}$ is reached when we have as many F period contracts as possible, consistent with $\bar{h}$. In effect, this means we have a mix of 1 period and F period price-spells. The existence of a maximum relies on us assuming an upper bound F: clearly, as $F \to \infty$, $T^{\max} \to \infty$. The minimum occurs when all firms have similar price-spells: if $\bar{d}$ happens to be an integer, then all price-spells have that length and the two distributions are the same: $\boldsymbol{\alpha}^d = \boldsymbol{\alpha}$.

This section derives the lower and upper bound for the mean length of price spells. This is an important issue given many previous studies derive the distribution of price spells from sectoral or item level frequencies of price changes, using the convention method (e.g. average length of price spells=1/average frequency of price change). We argue that the convention method effectively gives the lower bound for the mean length of price spells at either sectoral or item level. We manage to derive the upper bound for the mean length of price spells, which showing the theoretical maximum average length of price spells.

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[8]The proofs of Proposition 1 and 2 are illustrated in Dixon (2012).

3.4   An application to the UK CPI micro data[9].

In this section, we take the frequency data from the UK and apply the two propositions to derive the implied upper and lower bounds for the cross-sectional mean duration. We then estimate the actual distribution and the corresponding mean and how it compares to the theoretical distributions that yield the maximum and minimum mean durations across-firms. Our research data set is the locally collected CPI price microdata covering the period from January 1996 to December 2007. The detailed description of our data can be found in the Appendix. The period covered corresponds to the Great Moderation period when the frequencies would have been stationary. We also want to see how the level of disaggregation affects the results. For the UK, we have the following levels of disaggregation available from the ONS:

- 11 COICOP categories

- 67 disaggregated COICOP categories.

- 570 items.

Each of these disaggregations represents exactly the same data. To get an idea of the level of aggregation, we can depict the broad 11 COICOP categories (excluding education which is not included in the VML dataset) in Table 3.1. For example, there

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[9]**This work contains statistical data from ONS which is Crown copyright and reproduced with the permission of the controller of HMSO and Queen's Printer for Scotland. The use of the ONS statistical data in this work does not imply the endorsement of the ONS in relation to the interpretation or analysis of the statistical data. This work uses research datasets which may not exactly reproduce National Statistics aggregates**

is the category "food and non-alcoholic beverages" which represents 17.6% of the CPI weight in the subsample available in the dataset. The second level of disaggregation subdivides these into a total of 67 COICOP subcategories. For example, within ""food and non-alcoholic beverages" there are 11 subcategories: 2 for drinks (tea, coffee and cocoa; mineral water, soft drinks and juices) and 9 for food (such as meat, fish, fruit). The lowest level of disaggregation is the item level. An item is a particular product or service on which the price observation is made. For example: canned sweet corn (198g-340g); coffee - take-away; fresh lettuce (iceberg). The 570 items we include are all of the items which were included throughout the sample period - it excludes old items which were either discontinued or new items introduced in this period. These items represent over 66.4% of the total CPI.

Firstly, we present in Table 3.1 and for each category, we have the frequency data, which gives the proportion of items changing price in a given month[10]. The items are weighted by the appropriate CPI weight. The data are represented in Table 3.1.

In the first column of Table 3.1 is the COICOP sector, in the second the CPI weight for the sector, normalized so that they add up to 100 (since Education is excluded) and the third is the frequency of price change. In the fourth we have the minimum average duration (MAD) in that sector from Proposition 1, and in the fifth the maximum from Proposition 2 based on the assumption that the longest price-spell is 44 months.

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[10]In a given month, they look at all the prices of items at an outlet and compare them, with the price the previous month. The figure excludes items for which there was no observation the month before (e.g. it is the first price observation of the item at the outlet).

COICOP Category.	CPI adj.	Freq	Min.	Max
Transport	10.4	36	2.8	29.2
Alcoholic Beverages and Tobacco	7.1	27.6	3.6	32.9
Clothing and Footwear	9.3	27.2	3.7	33
Food and Non-Alcoholic Beverages	17.6	26	3.8	33.6
Furniture and Home Maintenance	11.3	22.7	4.4	35
Communications	0.2	22.5	4.4	35.1
Recreation and Culture	9.9	20	5	36.2
Housing and Utilities	8.3	13.7	7.3	39
Miscellaneous Goods and Services	6.5	12.7	7.9	39.4
Restaurants and Hotels	17.5	10.5	9.5	40.4
Health	1.9	10.4	9.6	40.4

Table 3.1: COICOP 11 sectoral frequencies
"Freq" denotes the frequencies of prices changes, which are reported in percent per month. "CPI adj." denotes the adjusted CPI expenditure weight of the CPI sectors after excluding the Education sector. "Min." denotes the minimum average duration. "Max" denotes the maximum average duration

We next generate the cross-sectional distribution in the whole economy corresponding to the minimum average duration consistent with the observed frequencies. From Proposition 1, in each sector we will have one or two durations with a non-zero share. In recreation and culture, since 20% of prices change per month, there are just 5 month price-spells. In food and non-alcoholic beverages there will be a mixture of 20% 3 month and 80% 4 month price-spells. The shortest durations are 2 months (in transport) and the longest 10 months (in health). For each duration, we can then add up across the 11 sectors to get the weighted cross-sectional distribution. This is depicted in Figure 3-1:

We estimate the actual cross-sectional distribution using the hazard functions which are already obtained in the Chapter 2. We can see that the distributions are completely different, although derived from exactly the same data. The "minimum duration" distribution has no 1 month, 6 month, 11 month or 12 month durations; the most common
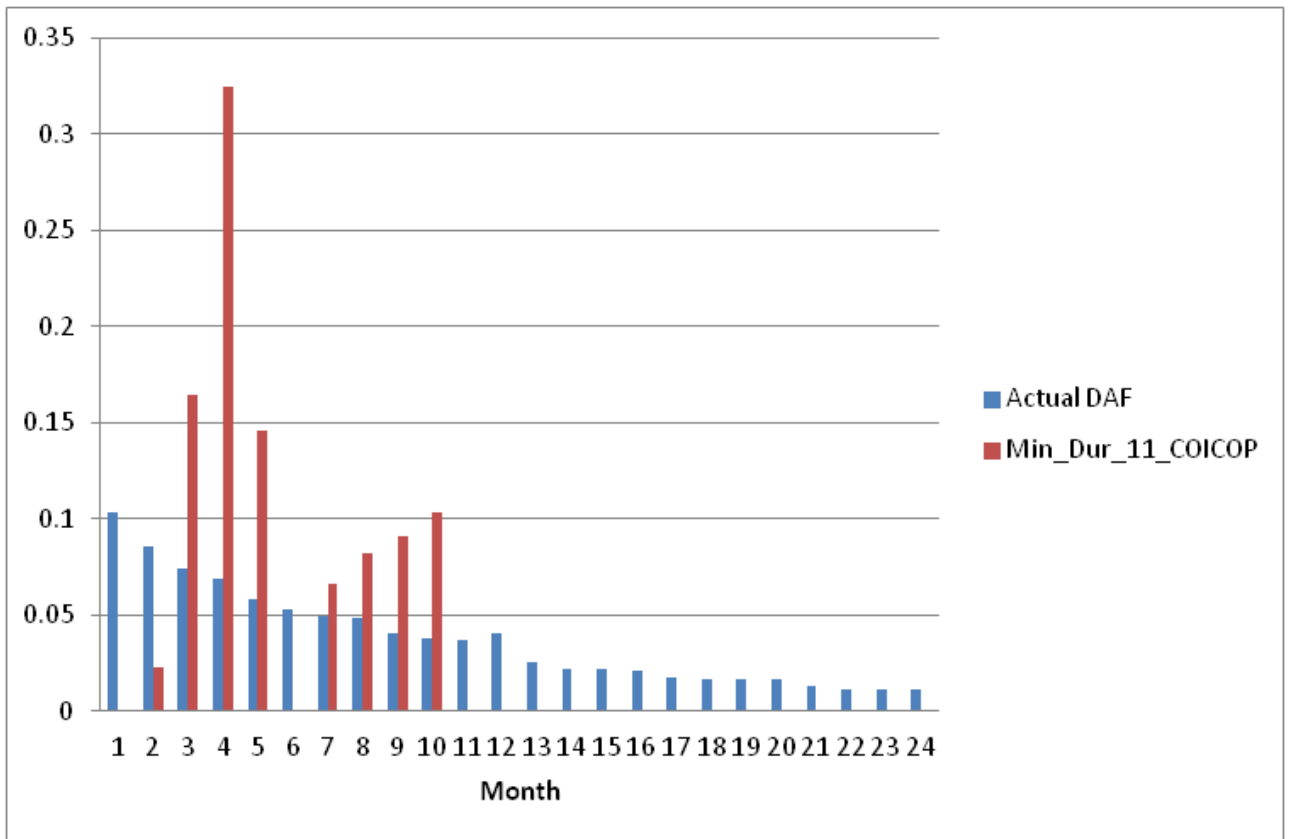
Figure 3-1: Actual DAF vs. Minimum Duration 11 COICOP

durations are 4 months (32%), then followed by the 3 months (16%) and 5 months (14%). Among the rest, we find that the share of distribution coincidently according with the length of duration, such as 10 months (10%) and 9 months (9%), 8 months (8%), 7 months (7%), 2 months (2%). In contrast, the "true" distribution is much flatter with a long tale (which we have truncated at 24 months). The most common duration is 1 month (10.3%) closely followed by 2 months (8.5%). There is an annual spike at 12 months (4%). Whilst the longer durations tend it have lower shares the cross-sectional distribution is non-monotonic. The maximum duration distribution is to have a mix of one month and the maximum duration (44 months). As a first approximation, the share of the one month durations in each sector is a little less than the frequency. This is clearly very different from the actual distribution.

We can now see what the effect of further disaggregation is: we perform the same procedure for the COICOP 67 and the 570 item level. These are all depicted in Figure 3-2, along with the COICOP 11 and the true distribution.

These are "minimum duration" distributions generated by differing levels of disaggregation. They share some common features when compared to the true distribution: they all put too little weight on month 1, month 12, and after. They all put too much weight on months 3-5 and months 9 and 10. However, they are also quite different. The level of disaggregation clearly matters when constructing a possible cross-sectional distribution. We can see this from the mean and median durations in Table 3.2:
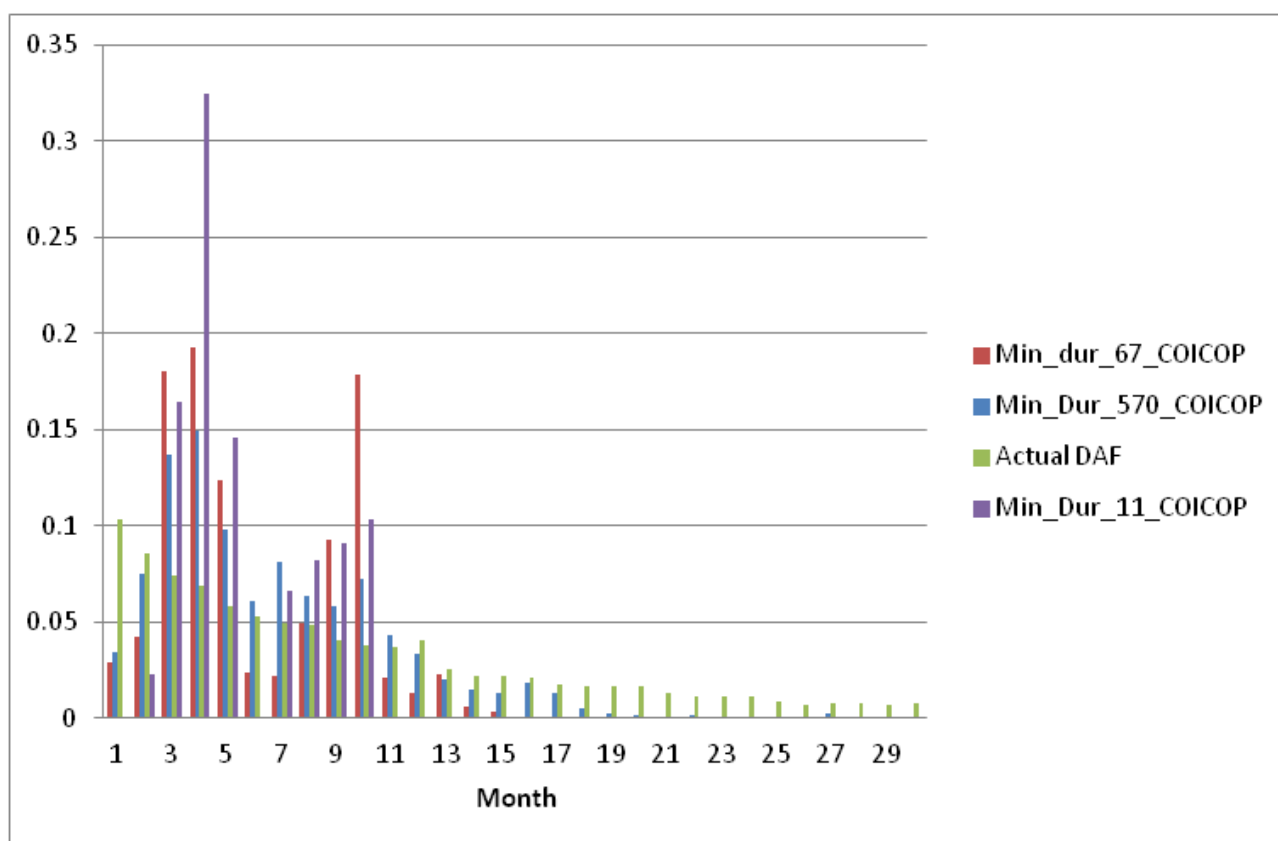
Figure 3-2: Actual DAF vs. Minimum Durations

	Mean DAF	Median DAF
True	10.9	7.8
11_min	5.5	4
67_min	6.1	4.5
570_min	6.7	5.8

Table 3.2: The minimum mean and median duration across firms comparison
"True" denotes the actual cross-sectional distribution implied by hazard function,
"11_min", "67_min", and "570_min" denote the cross-sectional distributions derived
from the "minimum method" at different disaggregation level, corresponding to 11
COICOP categories,67 disaggregated COICOP categories,and 570 items *respectively.*
"Mean DAF" and "Median DAF" denote the mean and median length of duration
across firms, and both of them are in unit of month.

It is quite clear that the mean and median length in all these "minimum duration" distributions are far too short, reflecting the fact that the minimum duration distributions put a large weight on the shorter distributions and do not have a long fat tail as in the data. In fact, the minimum durations are just over half the actual mean duration.

3.4.1 Calvo Distributions.

Clearly, the "minimum duration" distributions corresponding to Proposition 1 do not look at all like the true distribution. In this section we look at the distribution generated by the hypothesis that the sectoral frequencies are generated by a Calvo distribution. Again, as in the previous section, we are looking at exactly the same data, just at different levels of aggregation. Within each sector $k = 1...N$, we observe a frequency of h_k. As shown in Dixon and Kara (2006), this corresponds to the cross-sectional distribution for that sector $\boldsymbol{\alpha}_k = \{\alpha_{ik}\}_{i=1}^{\infty}$ where:

$$\alpha_{ki} = i.h_k^2(1 - h_k)^{i-1} \tag{3.6}$$

Each sector has a CPI weight c_k. We can then aggregate across the N sectors using the CPI weights to get the share of each duration across all sectors $\boldsymbol{\alpha} = \{\alpha_i\}_{i=1}^{\infty}$ where:

$$\alpha_i = \sum_{k=1}^{N} c_k \alpha_{ki} \tag{3.7}$$

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The mean duration of the Calvo distribution at the sectoral level is[11]:

$$\bar{T}_k^C = 2h_k^{-1} - 1. \tag{3.8}$$

The mean of the aggregate distribution is thus:

$$\bar{T} = \sum_{k=1}^{N} c_k \bar{T}_k^C$$

This is the method used in Dixon and Kara (2010, 2011) for generating the Bils-Klenow distribution based on the Bils Klenow (2004) appendix dataset of 350 sectoral frequencies for the US.

It is important to note that by assuming a Calvo *distribution*, we are not assuming a Calvo *pricing model* within each sector. We are simply describing the distribution of price-spell durations in each sector generated by a constant hazard rate that is equal to the sectoral frequency. This is purely descriptive of the distribution. It is perfectly compatible with a Taylor model, where within each sector the length of the price-spells is known ex ante. What we are doing in effect in constructing $\boldsymbol{\alpha} = \{\alpha_i\}_{i=1}^{\infty}$ using (3.7): that means we take out all of the i duration spells from each sector k and put them together into a duration sector α_i, which includes all of the price-spells of length i in the economy. The key difference between the Calvo and Taylor pricing frameworks is that under Taylor the firms know the length of the price-spell when they set the price,

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[11] Dixon and Kara (2006), Theorem 1.
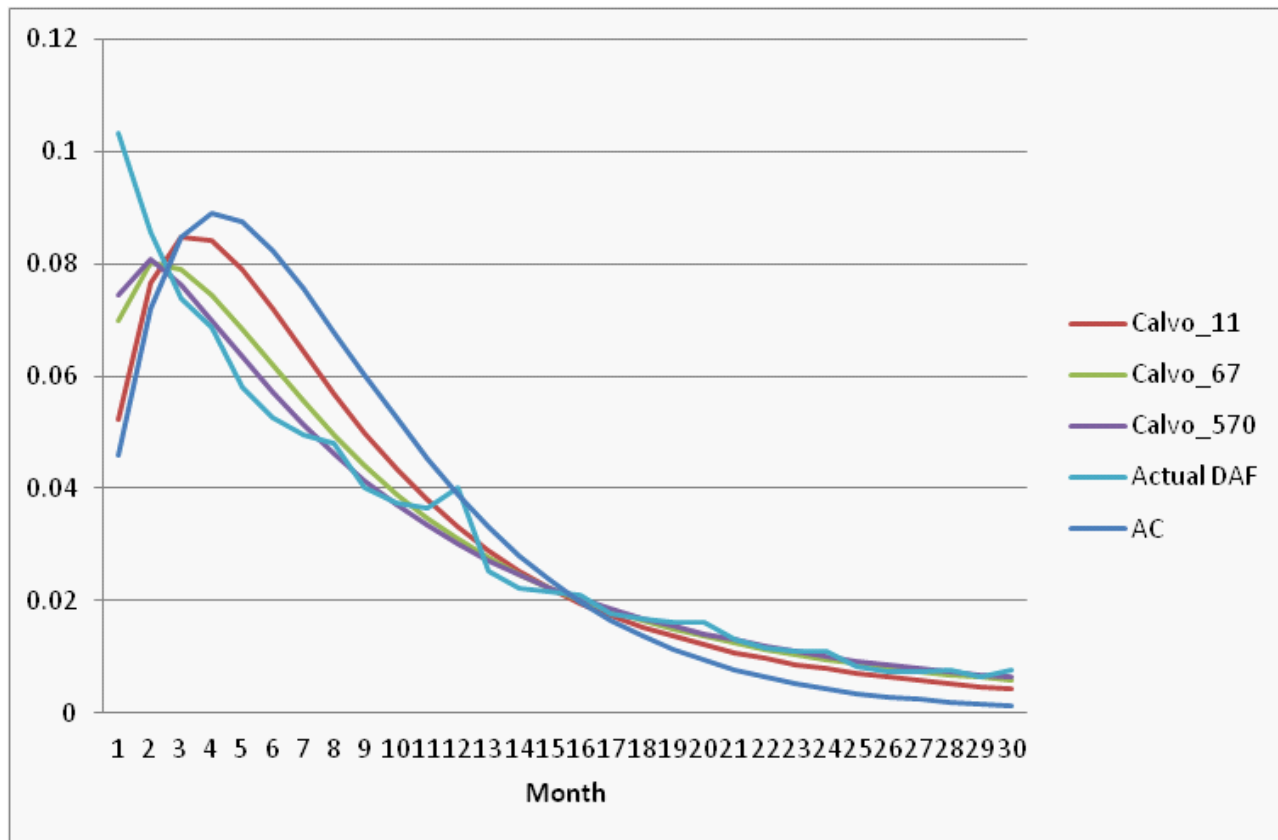
Figure 3-3: Calvo distributions at different levels of aggregation vs. Actual DAF.

whereas in the Calvo they do not.

In Figure 3-3, we represent the Calvo distributions at different levels of aggregation: the one sector "aggregate Calvo" (AC) distribution based on the mean UK frequency of 0.2140; the 11 sector COICOP, the 67 sector COICOP and the 570 item level. We have truncated the theoretical Calvo distributions at 44 months.

The first observation is that the level of aggregation influences the shape of the aggregate distribution. The distributions all have a "hump" shape[12], which peaks at 2

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[12]It worth of notice that we focus on distribution of durations across firms (DAF) instead of distribution

months (COICOP 67 and Item 570), 3 months (COICOP 11), and 4 months(AC). They all have far too few one-month shares and of course miss the 12 month spike. However, from month 8, COICOP 67 and Item 570 both track the true distribution fairly well (except for month 12). AC and COICOP 11 both overestimate the share of durations between 3 months and 16 months, and they underestimate the share of durations longer than 16 months. However, COICOP 11 is relatively closer to the true distribution than AC. The above results suggest that as sector level become more disaggregated, the implied Calvo distributions converge to the true distribution. If we look at the Item 570 Calvo distribution, the most disaggregated one we have, this peaks at month 2 and is the only Calvo distribution to be roughly close to (a little less than) the true proportion of 1 months. Furthermore, the Calvo distribution generated by Item 570 is quite similar from the months 2 onwards, only missing the 12 months spike. The UK distribution has a fatter tail, but the Calvo tails are certainly quite substantial. Essentially, the Calvo distributions put too much weight on the shorter months (1-7) and hence put less weight on the remaining durations.[13]

The means and medians of the different Calvo distributions are listed in Table 3.3. Here we can see that as the category becomes more disaggregated, the mean and median of Calvo distributions become closer to the true distribution. Indeed, the Calvo distrib-

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of durations across contracts. This is equivalent to a length biased sampling. As pointed out by Dixon (2012), the so called Calvo distribution is purely descriptive of the distribution. We find a hump shaped distribution in monthly data. This is just an empirical finding. And we do not assume a Calvo pricing model within each sector or item.

[13]We then look at the sectoral data to see what the sectoral distributions actually look like. Most of these do not fit the Calvo implied distribution. We use a Komogorov-Smirnov test to confirm this finding. See appendix 2 for detail.

	Mean DAF	Median DAF
True	10.9	7.8
11_Calvo	9.1	6.8
67_Calvo	10.5	7.2
570_Calvo	10.8	7.8
AC	8.3	6.5

Table 3.3: Mean and median durations of Calvo distributions
"True" denotes the actual cross-sectional distribution implied by hazard function, "AC","11_Calvo", "67_Calvo", and "570_Calvo" denote the cross-sectional distributions derived from the "Calvo distribution" at different aggregate level, corresponding to one aggregate sector, 11 COICOP categories,67 disaggregated COICOP categories,and 570 items respectively. "Mean DAF" and "Median DAF" denote the mean and median length of duration across firms, and both of them are in unit of month.

ution generated by Item 570 almost has the same mean and median value as what we get from the True distribution. If we compare the means of the Calvo distributions, these are linked to the distributions in Table 3.2, since the mean of the minimum duration distribution is

$$\bar{T}^{\min} = \sum_{k=1}^{N} c_k h_k^{-1}$$

which yields the theoretical relation $\bar{T}^C = 2.\bar{T}^{\min} - 1$. Hence the Calvo means are similar to the actual mean, whilst the theoretical minimum is just over half. No such exact relation holds for the medians. If we look at Tables 2 and 3, we can see that the Calvo means are less than the theoretical means. This is because we have truncated the Calvo distributions at 44 months: if we extend this then the mean will approach its theoretical value. Truncation reduces the mean quite significantly, since the long tail of the Calvo distribution will be allocated to the 44th month: whilst the shares of these longer durations are small, they are long and so affect the mean.

Whilst the Calvo assumption gives us a mean that is about right, it differs considerably from the true distribution. There are not enough one-period price-spells: in the data, there are a lot of products that have perfectly flexible prices that change almost every month (petrol, vegetables etc.) Hence the first period hazard rate needs to be higher than in the standard Calvo model. Second there are too many 3-8 month spells. This implies that the hazard needs to be lower for these months. Then of course there is a 12 month peak.

We can also look at the distributions within each of the 11 COICOP sectors. This we do in Appendix B. There is considerable heterogeneity in the sectoral distributions. Whilst most have a 12 month peak, there are several sectors which have little if any 12 month peak in the DAF or hazard: these latter include Food and non-alcoholic beverages, alcoholic beverages, clothing and footwear, communication. Also, there are sectors which have peaks other than 12 months which are important: housing and utilities has peaks every 4 months, communications at 5-6, 11 and 18 months. We compare each sectoral distribution with the corresponding Calvo distribution. Since we have such a large data set, the formal Kolmogorov-Smirnov test rejects the null hypothesis that the two distributions are the same in all of the 11 sectors. We also measure the degree of "overlap" of the two distributions for each sector. That is, the extent to which the two distributions allocate the same mass to the same values: it is the sum of the absolute deviations for each value (in this case number of months) relative to the total mass. A value of 1 means that there is no overlap at all: 0 that they are identical. In the

case of the distributions, the difference is as low as 0.18 for Alcoholic beverages, 0.20 for restaurants & hotels, and 0.23 for transportation. For the rest it is over 0.30 peaking at 0.58 for Health. Given that there must be some overlap here (all of the values for both distributions are strictly positive for months 1-44), these figures indicate a wide divergence in most sectors. In conclusion, we can say that the Calvo distribution is not a good description of the data either at the aggregate level or the COICOP 11 level.

3.5   The Simulation of different pricing models.

We find that we can use the sectoral frequencies to generate the corresponding hypothetical Calvo distributions. For the UK data at least, we find that at high levels of disaggregation, the resultant hypothetical aggregate distribution matches the true distribution quite well in terms of both the mean and the median. There are significant differences, most notably the hypothetical distribution has no 12 month spike and too few flexible prices. Since the mean and median are close, do the differences matter at the aggregate level? If we simulate a DSGE macro model using the hypothetical Calvo distribution, will it yield a good approximation to the simulations using the true distribution found in the UK data? If the answer is "yes", then it implies that the absence of the 12 month spike and too few flexible prices does not matter from the perspective of the macroeconomic properties of the DSGE model. This validates the approach taken in Dixon and Kara (2010,2011) and Kara (2011) which used the hypothetical Calvo distribution derived from the Bils-Klenow table of sectoral frequencies in order to calibrate

their US pricing models.

We will perform our simulations using two DSGE models: a simple Quantity theory model (QT) and the Smets and Wouters (2003) (SW) model. We will look at two pricing models in both of these cases: the Generalized Taylor (GT) and Generalized Calvo (GC) model as in Dixon and Le Bihan (2012).

3.5.1 Price setting.

There are two general time-dependent models which are capable of reflecting the underlying distribution of data found in the micro-data: the Generalized Taylor (GT) and Generalized Calvo (GC) models The key difference between the models is that in the GT the firms know how long the price spell will last when they set the price, and so each duration of price-spell will have a different reset price. In the GC, in contrast, the firms do not know how long the price spell will last and have a distribution over possible price-spells durations. All firms have the same distribution and hence there is only one reset price every period as in the simple Calvo model. In the *Generalized Taylor Economy* (*GT*) there are N sectors, $i = 1, ..., N$. In sector i there are $i-$period contracts: each period a cohort of i^{-1} of the firms in the sector sets a new price (or wage). If we think of the economy as a continuum of firms, we can describe the *GT* as a vector of sector shares: α_i is the proportion of firms that have price-spells of length i. If the longest observed price-spell is F, then we have $\sum_{i=1}^{F} \alpha_i = 1$ and $\alpha \in \Delta^{F-1}$ is the F-vector of shares $\boldsymbol{\alpha} = \{\alpha_i\}_{i=1}^{F}$. We can think of the "sectors" as "duration sectors":

we can classify the economy by the length of price-spells. The essence of the Taylor model is that when they set the price, the firm knows exactly how long its price is going to last. The simple Taylor economy is a special case where there is only one length of price-spell (e.g. $\alpha_2 = 1$ is a simple Taylor "2 quarters" economy). The GTE is based on the cross-sectional distribution of completed spell lengths: hence it can also be called the distribution across firms (DAF) in this context.

The log-linearised equation for the aggregate price p_t is a weighted average of the sectoral prices p_{it}, where the weights are α_i :

$$p_t = \sum_{i=1}^{F} \alpha_i p_{it} \tag{3.9}$$

In each sector i, a proportion i^{-1} of the α_i firms reset their price at each date. Assuming imperfect competition and standard demand curve, the optimal reset price in sector i, x_{it} is given by the first-order condition of an intertemporal profit-maximisation program under the constraint implied by price rigidity. The log-linearised equation for the reset price, as in the standard Taylor set-up, is then given by :

$$x_{it} = \left(\frac{1}{\sum_{k=0}^{i-1} \beta^k} \right) \sum_{k=0}^{i-1} \beta^k E_t p_{t+k}^* \tag{3.10}$$

where β is a discount factor, E_t is the expectation operator conditional on information available at date t , and p_{t+k}^* is the optimal flex price at time $t+k$. The reset price is thus an average over the optimal flex prices for the duration of the contract (or price-spell).

The formula for the optimal flex price will depend on the model: clearly, it is a markup on marginal cost. We will specify the exact log-linearised equation for the optimal flex-price when we specify the exact macroeconomic model we use.

The sectoral price is simply the average over the i cohorts in the sector:

$$p_{it} = \frac{1}{i} \sum_{k=0}^{i-1} x_{it-k} \tag{3.11}$$

In each period, a proportion $\bar{h}$ of firms reset their prices in this economy: proportion ι^{-1} of sector i which is of size α_i.

$$\bar{h} = \sum_{i=1}^{F} \frac{\alpha_i}{i}$$

In the GC, firms have a common set of duration-dependent reset probabilities: the probability of resetting price i periods after you last reset the price is given by h_i. This is a time-dependent model, and the profile of reset probabilities is $\mathbf{h} = \{h_i\}_{i=1}^{F}$. Clearly, if F is the longest price-spell we have $h_F = 1$ and $h_i \in [0, 1)$ for $i = 1...F - 1$. Again, the duration data can be represented by the hazard function. Estimated hazard function can then be used to calibrate $\mathbf{h}$. Since any distribution of durations can be represented by the appropriate hazard function, we can choose the GCE to exactly fit micro-data.

In economic terms, the difference between the Calvo approach and the Taylor approach is that when the firm sets its price, it does not know how long its price is going to last. Rather, it has a *survivor function* $S(i)$ which gives the probability that its price

will last at up to i periods. The survivor function in discrete time is[14]:

$$S(1) = 1 \qquad\qquad (3.12)$$

$$S(i) = \prod_{j=1}^{i-1}(1 - h_j) \quad i = 2, ..., F$$

Thus, when they set the price in period t, the firms know that they will last one period with certainty, at least 2 periods with probability $S(2)$ and so on. The Calvo model is a special case where the hazard is constant $h_i = \bar{h}$, $S(i) = (1 - \bar{h})^{i-1}$ and $F = \infty$. Of course, in any actual data set, F is finite.

In the GC model the reset price is common across all firms that reset their price. The optimal reset price, in the same monopolistic competition set-up as mentioned above, is given in log-linearised form by:

$$x_t = \frac{1}{\sum_{i=1}^{F} S(i)\beta^{i-1}} \sum_{i=1}^{F} S(i)\beta^{i-1} E_t p_{t+i-1}^* \qquad\qquad (3.13)$$

The evolution of the aggregate price-level is given by:

$$p_t = \sum_{i=1}^{F} S(i)x_{t-i+1} \qquad\qquad (3.14)$$

That is, the current price level is constituted by the surviving reset prices of the present

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[14]Note that the discrete time survivor function effectively assumes that all "failures" occur at the end of the period (or the start of the next period): this corresponds to the pricing models where the price is set for a whole period and can only change at the transition from one period to the next.

and last F periods.

3.5.2   A simple quantity theory model with price-setting.

We will first examine the GC and GT models of prices in a quantity theory model with labour as the only input of production. This model has the great advantage of being very simple, because almost all its dynamic properties are generated by the pricing models alone. DSGE models like the SW model in contrast are quite complicated with dynamic properties emerging from the interaction of pricing with many other features of the model. The model we present is in its log-linearised version (see Ascari 2003, Dixon and Kara 2005 for the derivation from macroeconomic foundation).

To model the demand side, we use the Quantity Theory:

$$y_t = m_t - p_t$$

where (p_t, y_t) are aggregate price and output and m_t the money supply. We model the monetary growth process as an autoregressive process of order one $AR(1)$:

$$m_t = m_{t-1} + \varepsilon_t$$

$$\varepsilon_t = \nu \varepsilon_{t-1} + \xi_t$$

where ξ_t is a white noise error term (effectively a monetary growth shock). Following

CEE we set $\nu = 0.5$.

The optimal flexible price p_t^* at period t in all sectors is given by:

$$p_t^* = p_t + \gamma y_t \tag{3.15}$$

The key parameter γ captures the sensitivity of the flexible price to output[15]. As discussed in Dixon and Kara (2010), there are a range of calibrated and estimated values for γ: for illustrative purposes, we use the "moderate" case of $\gamma = 0.1$ as in Mankiw and Reis (2002). As discussed in Ascari (2003) and Edge (2002), the value of γ can be interpreted as resulting from either wage or price-setting.

In Figure 3-4 and Figure 3-5, we see the IRFs for the QT model responding to a monetary 1% monetary growth shock. As we can see, the IRFs look similar in shape for both the GC and GT: for both output and inflation. There are four IRFs reflecting the distributions generated from the estimated ("true") UK distribution, and the three Calvo distributions derived from the sectoral frequencies at different levels of aggregation. The results are striking. We can summarize them in a few points:

(a) the IRFs from the four distributions are similar, with the "true" IRF lying in the middle.

(b) the 570 item Calvo has the largest and most persistent effect on output, followed by the 67 COICOP Calvo, then the "true" and the 11 COICOP Calvo showing the least

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[15]This can be due to increasing marginal cost and/or an upward sloping supply curve for labour. See for example Walsh (2003, chapter 5) and Woodford (2003, chapter 3).
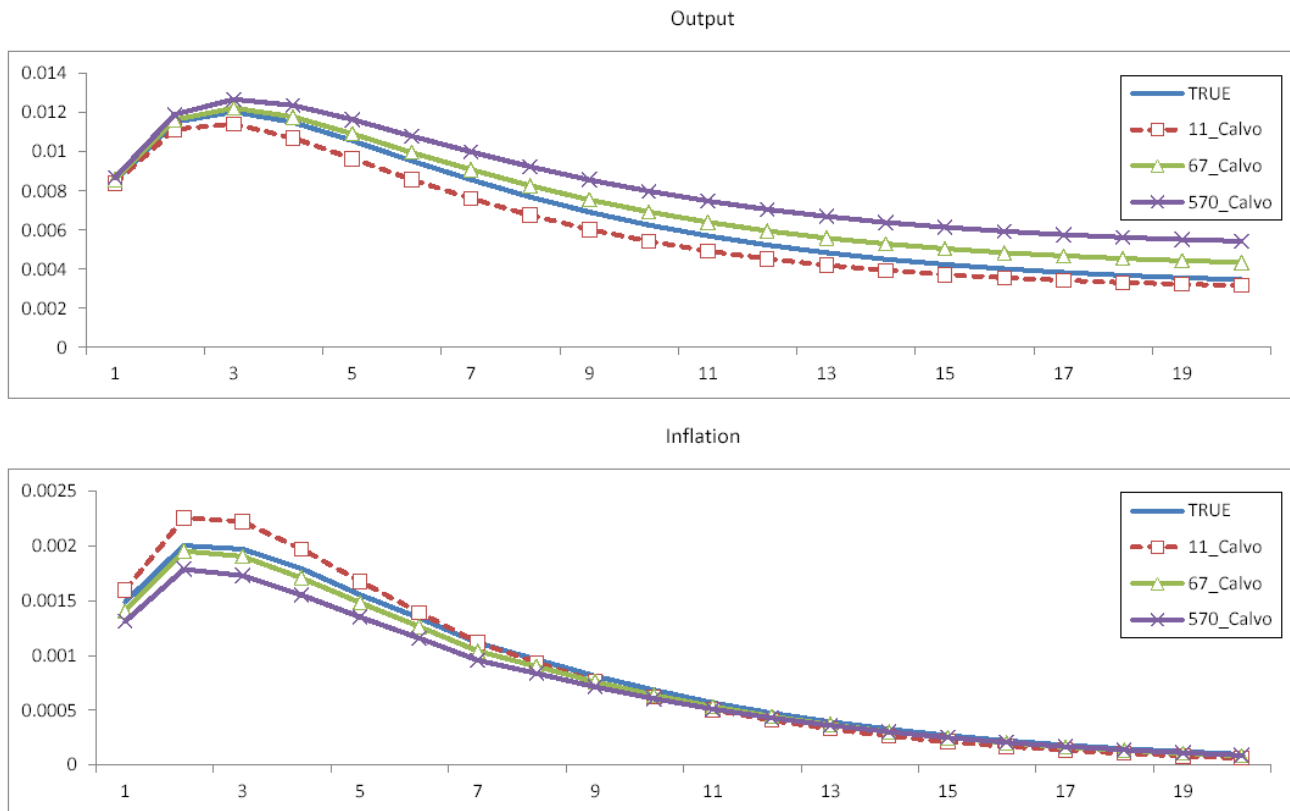
Figure 3-4: IRF of money growth shock in Quantity Theory model for GTE price-setting.
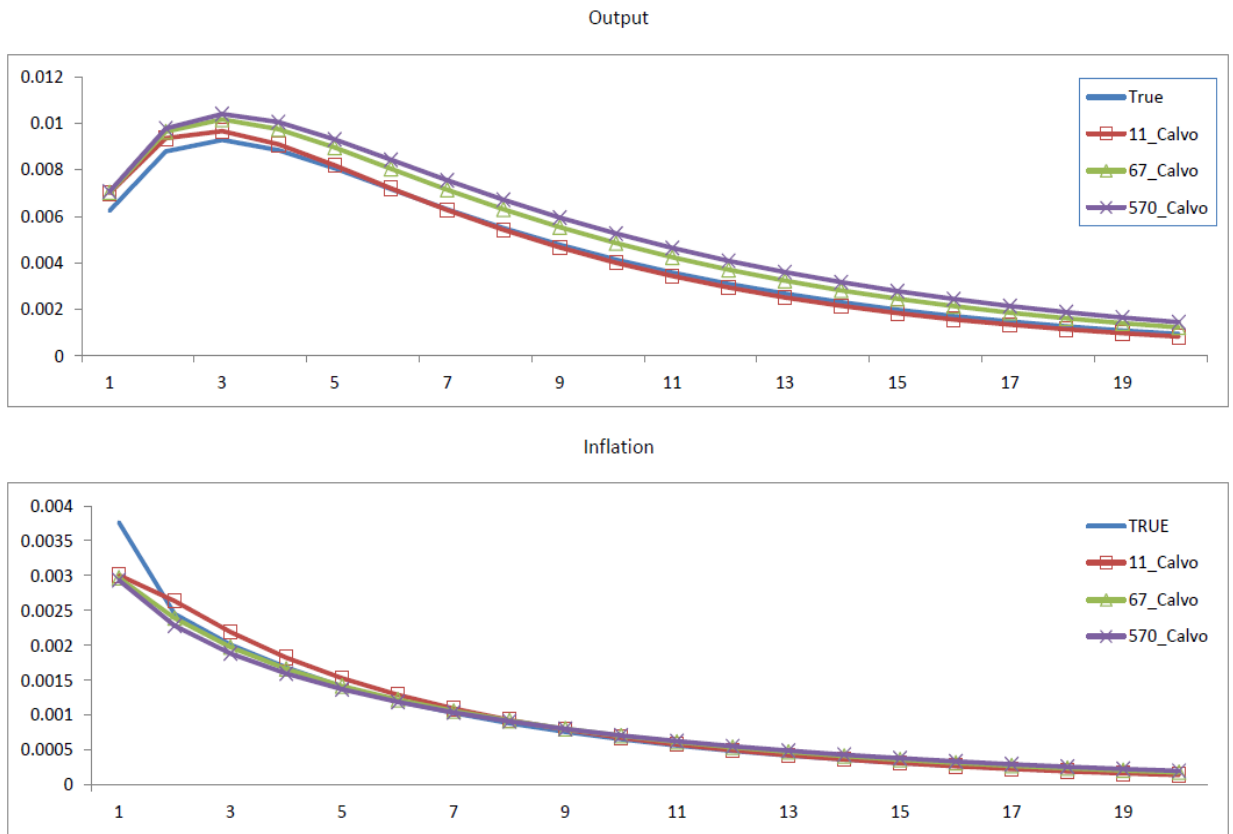
Figure 3-5: IRF of money growth shock in Quantity Theory model for GCE price-setting.

	GTE			GCE		
	True -11C	True -67C	True -570C	True -11C	True -67C	True -570C
AD in output	9.12%	8.73%	21.82%	4.34%	14.41%	21.42%
AD in inflation	9.93%	5.36%	11.87%	9.01%	7.99%	10.49%

Table 3.4: The difference in IRF from QT model

effect.

(c) for inflation, the 11COICOP has the biggest immediate effect, while it dies out relatively faster than the other cases. The 570 item Calvo has the smallest immediate effect, but it has the most persistent effect. The "true" and 67 COICOP Calvo lie in those two.

(d) the GT has a hump shaped reaction function for inflation, the GC does not. This is consistent with the stylized facts that the biggest effect of monetary policy is not on impact but after four quarters.

In order to quantify differences between the IRFs, we define the point-by-point absolute difference as a percentage of the mean "true" IRF $\delta_i = \frac{|IRF_{true} - IRF_{calvo}|}{mean(IRF_{ture})} \times 100\%$. Summing these differences over the first 20Q δ_i $(i = 1, \cdots, 20)$ and dividing by 20, we can get the average relative difference (AD), which is shown in the table 3.4.

Here we can see that with the GT model, the IRF generated by the COICOP 67 frequencies is the one most close to those generated from "true" distribution. However, in the GC model, the results are kind of mix. For the IRF in output, the COICOP 11 is the one has the smallest average difference. While for the IRF in inflation, the COICOP

67's performance is the best.

3.5.3 A DSGE model: Smets and Wouters (2003)

In this section, we use the Smets and Wouters (2003) model of the euro area commonly employed for monetary policy analysis. The *SW* model is much more complicated than the simple QT model we have just used: there are many sources of dynamics other than prices and wages, including capital adjustment, capital utilization, consumer dynamics with habit formation, and a monetary policy reaction function. The behavior of the model is the outcome of the interaction of all of these processes together as it should be in a DSGE model. Hence the effect of pricing dynamics is not isolated as in the simple QT framework of the previous section. The details of the model and calibration are outlined in Appendix as in Dixon and Le Bihan (2012) using a notation consistent with this paper.

We depict the IRFs for an interest rate shock[16], which causes output and inflation to fall initially (as shown in Figures 3-6 and 3-7. We find that again that only GT pricing can generate hump-shaped reaction function for inflation[17]. Here we see that the

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[16]The focus of this chapter is on the effect of monetary shock. However, we can find some previous studies on measuring the response to a technology shock in the framework of GT and GC. Dixon and Le Bihan point out that after a persistent but non-permanent increase in productivity, marginal cost will decline, which leads to a decrease in prices for the first 5 quarters. As the shock dies away, the price increase slowly back to its pre-shock level. See Dixon and Le Bihan (2012) for detail.

[17]According to empirical facts documented in Dixon and Kara (2010), monetary policy shocks have persistent and delayed effects on inflation. Monetary policy has "long and variable lags". Bank of England has a point of view that the impact on inflation might not peak for as long as eight quarters or even more. The European Central Bank takes the view that the maximum impact is six quarters. Smets and Wouters (2003) find that the maximum effect of monetary policy shock is about four quarters after the policy
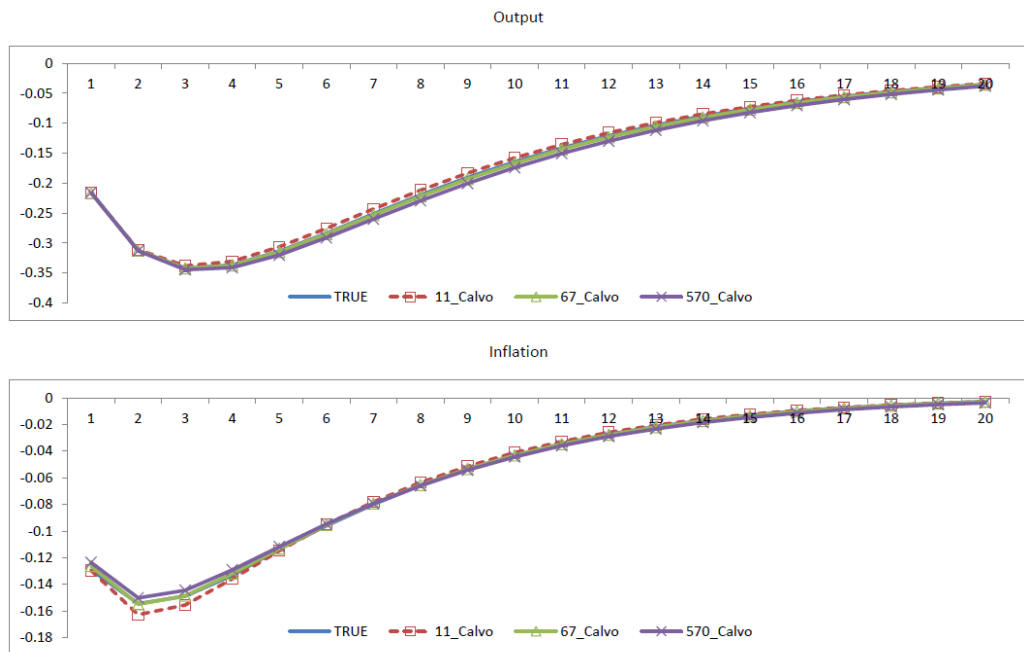
Figure 3-6: IRF for monetary shock in the Smets and Wouters model with GTE price-setting.
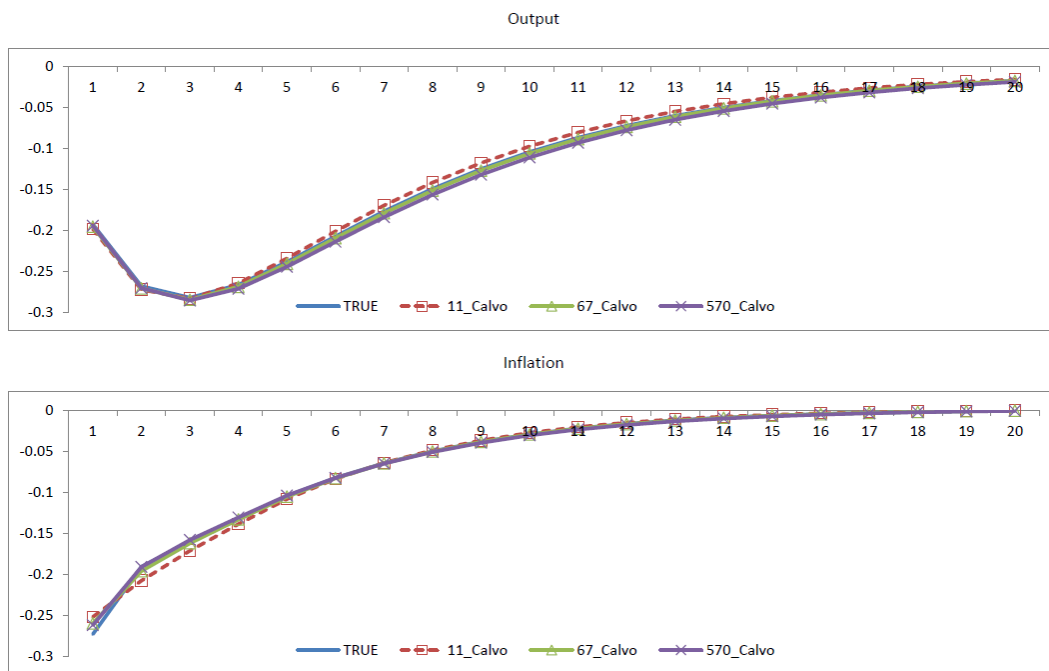
Figure 3-7: IRF for monetary policy shock in the Smets and Wouters model with GCE price-setting.

	GTE			GCE		
	True -11C	True -67C	True -570C	True -11C	True -67C	True -570C
AD in output	2.58%	0.88%	3.58%	3.71%	1.28%	3.58%
AD in inflation	3.08%	0.89%	3.27%	6.12%	2.06%	2.34%

Table 3.5: The difference in IRF from QT model

differences between the 4 IRFs are much smaller and less visible when compared to the QT model. This is probably because the structure of the dynamics is also determined by the rest of the model's complex dynamics, which leaves less room for the precise distribution of price-spell durations to matter. However, we still calculate the average relative differences between the "true" and different disaggregate level Calvo type, which are shown in the Table 3.5.

If we take the results from the simulations of both the QT model and the SW model, we can see that the microeconomic differences in durations do not matter that much. We are comparing hypothetical distributions derived from the sectoral frequencies under the assumption that within each sector there is a Calvo distribution corresponding to the frequency. As we have seen, whilst the aggregate distributions implied by this may have a similar mean and median to the true distribution, the shape differs significantly and in particular there is no 12 month spike. The results of the simulations imply that these micro differences do not matter in practice. In the SW model, the differences in distribution seem to have almost no observable effect on the IRFs.

Why is it that the hypothetical Calvo models seem to work well despite their poor

fit at the microeconomic level. We believe that there is on prime reason for this: the macroeconomic models use a *quarterly calibration*, which in effect smoothes out some of the differences we observed in the monthly data. From the perspective of GTE, for example, the 12 month spike gets smoothed out. This is shown in the Figure 3-8, in which the "true" quarterly distribution is compared to the corresponding quarterly distributions for the "11c", "67c", "570c" distributions and "ac" the distribution implied by the single aggregate Calvo frequency. The quarterly "true" distribution shows a lack of 12 month spike. And the Calvo implied distributions at different aggregate level are all quite similar to the "True" one, except for the "ac" which has a big hump in quarter 2.

If we look at the quarterly model from the perspective of GCE, we need to compare the quarterly hazards between the Calvo pricing at different aggregate level and the "true" one which is from estimated hazard function. Following Dixon (2012), given a distribution across firms by $\alpha \in \Delta^{F-1}$. The corresponding hazard profile that will generate this distribution in steady state is given by $\mathbf{h} \in [0,1]^{F-1}$ where:

$$h_i = \frac{\alpha_i}{i} \left(\sum_{j=i}^{F} \frac{\alpha_j}{j} \right)^{-1} \tag{3.16}$$

Therefore, we can calculate the monthly hazard for the Calvo pricing at different aggregate level by using the equation 3.16, and the relevant monthly survival rate will be obtained accordingly. This can then be converted into a quarterly hazard rate.

We plot the quarterly hazard functions in the Figure 3-9. The hazard functions from
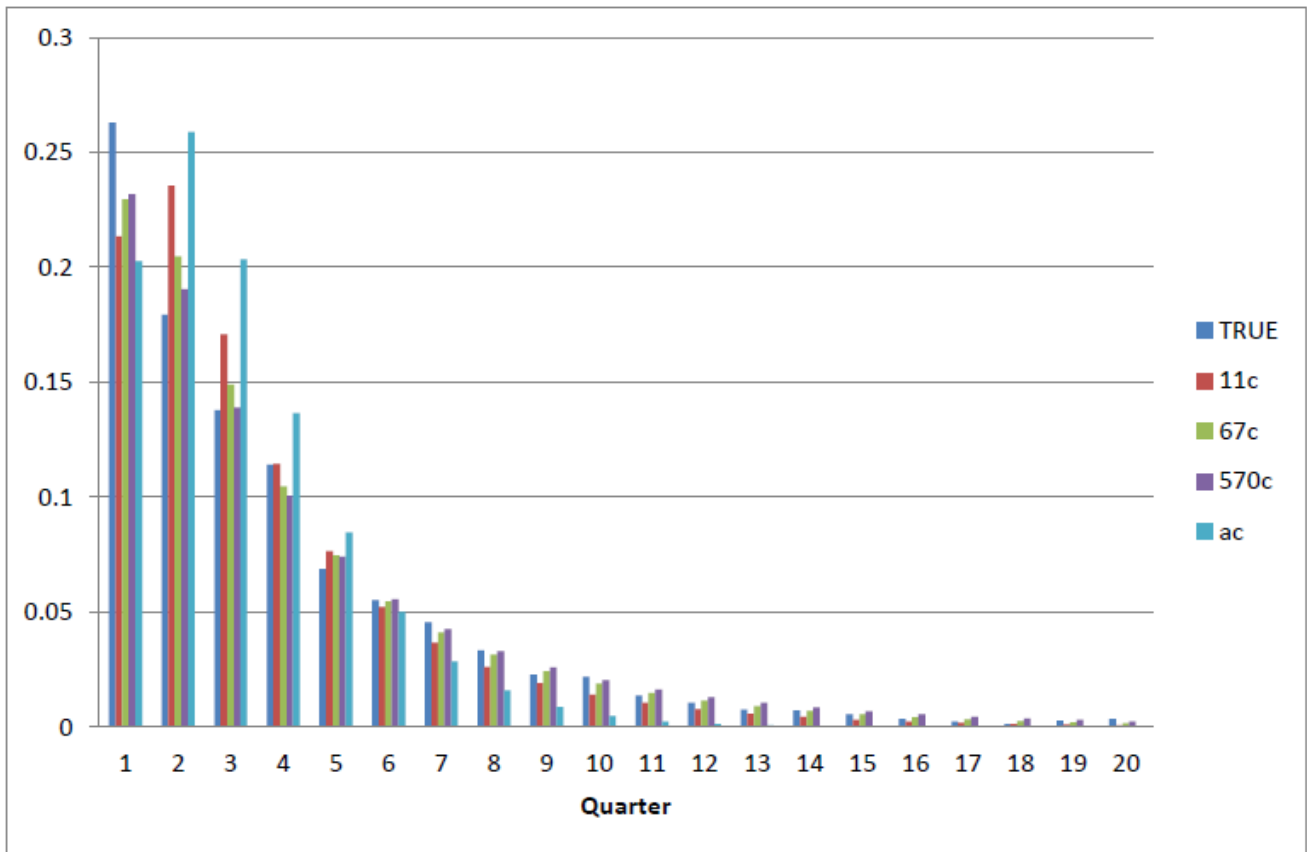
Figure 3-8: Quarterly distribution of duration across firms.

Calvo pricing at different aggregate level are downward sloping smoothly[18]resulting of aggregation of heterogeneous price setters (see Alvarez 2008). The "true" hazard function is also generally downward sloping, with several small spikes. In general, the hazards from different disaggregate level of Calvo pricing are quite similar to the "true" hazard. The aggregate Calvo on the other hand has a constant hazard and looks quite unlike the "true" hazard.

3.6   Conclusion

In this paper we asked the question what can the sectoral data on the frequency of price-change tell us? On the theoretical level, sectoral frequencies tell us what expected duration of a price-spell is. This is of some interest, but from a macroeconomic perspective we are more interested in the behavior of the economic agents setting prices - the *cross-sectional* distribution is of much more interest. Unfortunately the frequency itself says little about the cross-sectional distribution: to uncover this we need to make additional assumptions. However, we are able to say what the theoretical minimum cross-sectional mean duration is consistent with an observed frequency: it is the mean duration of a price-spell which occurs when all price-spells in the sector have the same or almost the same length. However, the cross-sectional mean can be much longer: intu-

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[18]The Calvo implied hazards are generally downward sloping. However, after 12 quarters, these hazards become upward sloping. This is due to the truncation. For Calvo pricing, truncation means that the $\sum_{j=i}^{F} \frac{\alpha_j}{j}$ is smaller than it is under the infinite sum. The reciprocal of the smaller sum is the main reason that the hazard is biased upwards after 12 quarters.
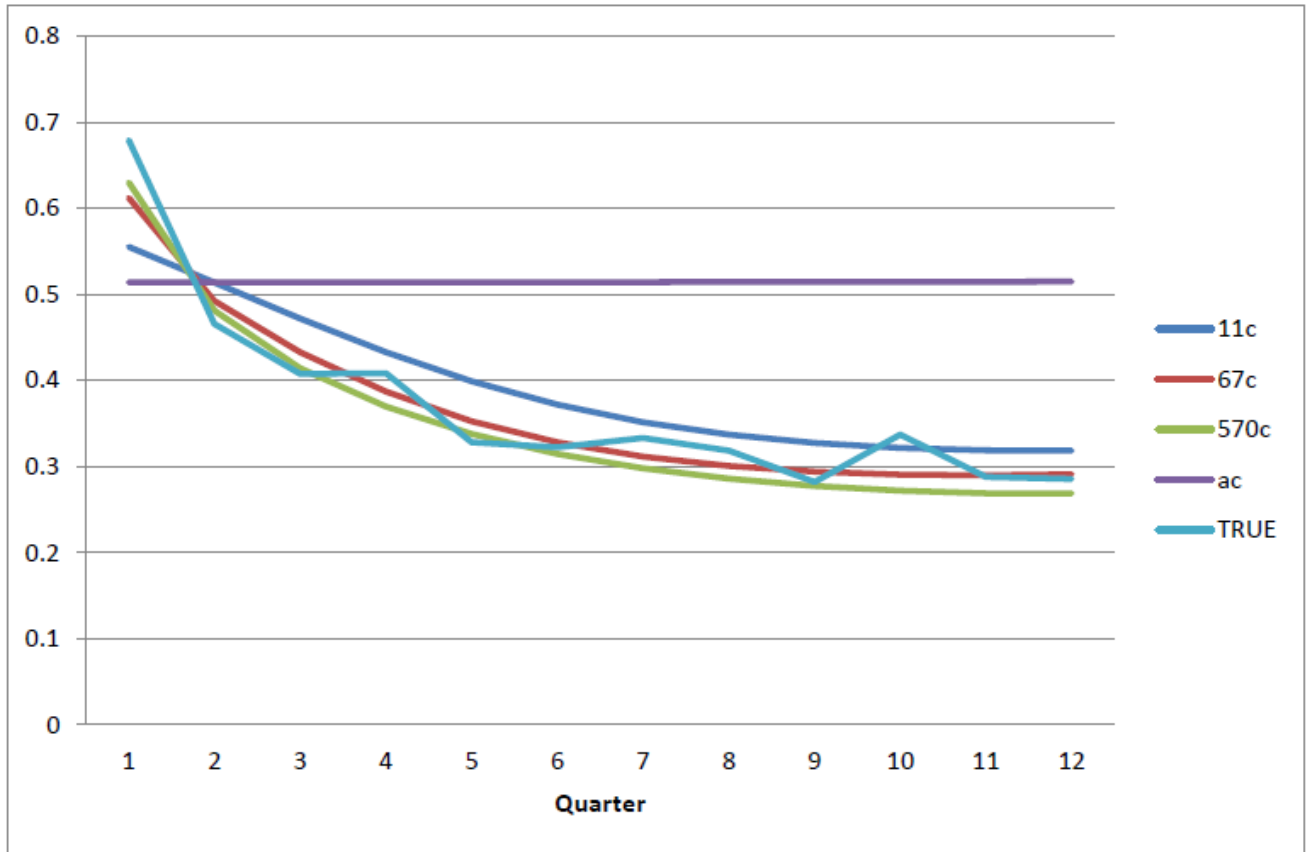
Figure 3-9: Quarterly hazard functions comparison.

itively, firms that have prices that last for a long time do not reset their prices very often. These firms contribute little to the monthly frequency but add a lot to the cross-sectional mean.

When we look at the UK data using an estimated hazard function, we find that the UK data is a long way from the distribution implied by the theoretical minimum. Looking at different levels of disaggregation, we find that whilst the minimum theoretical mean duration is around 5.5-6.7 months, the actual data reveals a mean of almost 11 months. We also find that the more disaggregated the data, the closer are both the median and mean to the true values. We also look at the aggregate data using the hypothesis that the sectoral frequencies are generated by a Calvo distribution. Under this assumption, the cross-sectional mean is much larger than the minimum, and gets closer to the actual mean as you become more disaggregated, with the most disaggregated having almost exactly the same mean and median as the data. Whilst the mean and median of the hypothetical Calvo distribution can be close to the actual values, the shape differs in two distinct ways: firstly, there is no 12 month spike, secondly there are not enough flexible prices. When we look at the COICOP 11 sectors, we find that the Calvo distribution hypothesis does not work very well: there is considerable heterogeneity across sectors in terms of the cross-sectional distribution and hazard function.

However, we do find that whilst the Calvo distribution hypothesis differs from the estimated distribution both at the aggregate and sectoral levels, it nonetheless works at the aggregate level. This is because of a combination of two factors. First, along

with getting the mean and median correct, the Calvo distribution also generates a nice long fat tail. Second, DSGE models are calibrated using quarterly periods rather than monthly: when we move to quarterly data, the differences which look significant at the monthly level get averaged out to a large extent. This means that when we look at the behavior of DSGE models under the Calvo distribution hypothesis, they behave in a very similar way to models calibrated with the microdata. This suggests that we can use the disaggregated frequency data (the more disaggregated the better) to calibrate DSGE models when we do not have reliable hazard function estimates or access to the price microdata as was done by Dixon and Kara (2010, 2011).

3.7 Appendix:

3.7.1 Data description

The data is described in some detail by Bunn and Ellis (2012a) so our description will be brief. The ONS collect a longitudinal micro data set of monthly price quotes from over ten thousand outlets to compute the national index of consumer prices. There are two basic price collection methods: local and central. Local collection is used for most items. There are about 150 areas (e.g. Cardiff,) around the UK, and around 120,000 quotations are obtained each month by local collection. For some items, collection in individual shops across the 150 areas is not required- for example, for larger chain stores

who have a national pricing policy or where the price is the same for all UK residents or the regional variation in prices can be collected centrally. The data that we were able to access for this study via the VML at Newport (Wales) consists of the locally collected data covering about two thirds of total CPI (centrally collected data covers about 33% of CPI). The sample spans over the time period from January 1996 to December 2007 and contains between 112,676 (1996) and 99,524 (2007) elementary price quotations per month, with a resulting dataset of around 14 million price observations. The coverage and classification of the CPI indices are based on the international classification system for household consumption expenditures known as COICOP (classification of individual consumption by purpose). This is a hierarchical classification system comprising: *divisions* e.g. 01 Food and non-alcoholic beverages, *groups* e.g. 01.1 Food, and *classes* (the lowest published level) e.g. 01.1.1 Bread and cereals. As table 3.6 shows, the division Food and non-alcoholic beverages accounts for about 17% of the CPI weight in the subsample available in the dataset. Education is not contained in the VML dataset, as these prices are all collected centrally: but all other CPI divisions have locally collected observations and are included in the dataset.

In our CPI research data set, each individual price quote consists of information on the item code, the outlet, the region, the date etc. The product category at the elementary level is defined as an item - for example large loaf, white, unsliced (800g). However, the data has been anonymized with respect to the variety and brand of the product. With the information on the item i, the shop j, the location k, and the date t, we can construct

COICOP division	Percent	Cum.
Food and Non-Alcoholic Beverages	17.62	17.62
Alcoholic Beverages and Tobacco	7.05	24.67
Clothing and Footwear	9.32	33.98
Housing and Utilities	6.54	40.52
Furniture and Home Maintenance	11.34	51.86
Health	1.89	53.75
Transport	10.35	64.1
Communications	0.17	64.27
Recreation and Culture	9.85	74.12
Education	0	74.12
Restaurants and Hotels	17.54	91.67
Miscellaneous Goods and Services	8.33	100
Total	100	

Table 3.6: CPI share in COICOP sectors
"Percent" shows the percentage of weighted number of observations in each COICOP division."Cum." shows the cumulative distribution.

a price trajectory $P_{ijk,t}$, which is sequence of price quotes for a specific item belonging to a product category in a specific shop over time. Specifically, we take two sequential price quotes belong to the same price trajectory if they have the same product identity, location and shop code. There are about $614,000$ price trajectories. And the average length of each price trajectory is about 24 months. Each trajectory will consist of a sequence of one or more price-spells: there are 3,174,692 price-spells in the data (i.e. on average about 5 price-spells per trajectory).

3.7.2 Comparing sectoral distribution of DAF with distribution of Calvo

This section examines the distribution of the duration of price spells at the COICOP 11 sectoral level. As in the aggregate data, we estimate the sectoral hazard functions using the KM non-parametric method and the corresponding cross-sectional DAF. We then compare these with the sectoral Calvo distributions in three ways. First, and most straightforwardly, we are comparing the distribution of DAF with the distribution of Calvo in each of 11 COICOP sectors with the "eye ball test". Even though this is not a strict statistical test, it can give us an impression how close or how far the two kinds of distributions are different. Second, we examine the distributional assumption using Kolmogorov-Smirnov test. The Kolmogorov-Smirnov statistic quantifies a distance between the empirical distribution function of the sample and the cumulative distribution function of the *reference distribution* (in our case, the reference distribution is Calvo distribution). Since we have such a large sample of price spells, this is a very strict test: standard errors are very small so that even small deviations of the reference distribution will lead to its rejection. Third, we propose a method to calculating the relative absolute difference. Instead of calculate the difference with cumulative distribution function like Kolmogorov-Smirnov test, we compute the difference of the pdf of the two distributions. Then, we divide the sum of the absolute differences by 2,since the mass of each distribution is 1. This method can provide us the deviation of the DAF from Calvo distribution in percentage. This is a descriptive statistic rather than a test, but it provides a measure

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of "overlap".

Distribution graphs: DAF vs. Calvo

Follow the derivation in Dixon (2010), we derive the cross-sectional distribution (DAF) from hazard function estimated by Kaplan-Meier method. As the distribution graphs show, there are significant differences between the distribution of DAF and the distribution of Calvo in most COICOP sectors. First, the Calvo distribution has less one-period price-spells than DAF, indicating that Calvo price setting mechanism fails to replicate the evidence of large volume of flexible price setters. Second, there is no 12-month spike in Calvo distribution. But this 12-month spike does appear in many COICOP sectors, such as health, housing, culture, etc.. Third, the distribution of DAF is non-monotonic even in the long period, and the DAF has a fatter long tail comparing to Calvo. Again,Calvo distribution miss the part of sticky price setters. Above all, the sectoral Calvo underestimates the heterogeneity within each sector.

Kolmogorov-Smirnov test

The Kolmogorov-Smirnov statistic for a given cumulative distribution function $F(x)$ is

$$D_n = \sup_x |F_n(x) - F(x)|$$

where $\sup x$ is the supremum of the set of distances. By the Glivenko-Cantelli theorem, if the sample comes from distribution $F(x)$, then D_n converges to 0 almost surely. Kol-

Figure A1: Food and Non-Alcohol

Figure A2: Alcohol

Figure A3: Cloth and Footwear

Figure A4: Housing and Utilities

Figure A5: Furniture and Home Maintance

Figure A7: Transportation

Figure A6: Health

Figure A8: Communication

Figure A9: Recreation and Culture

Figure A11: Miscellenious Goods and Services

Figure A10: Restaurant and Hotel

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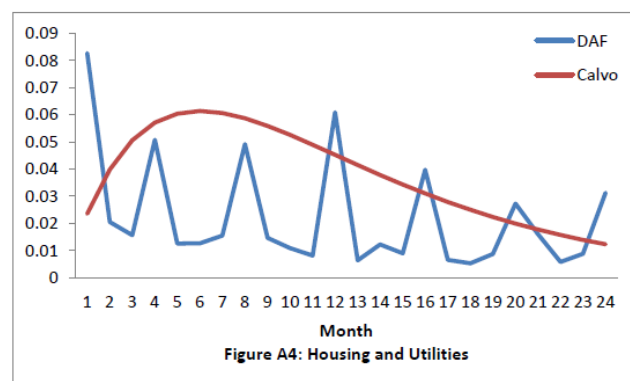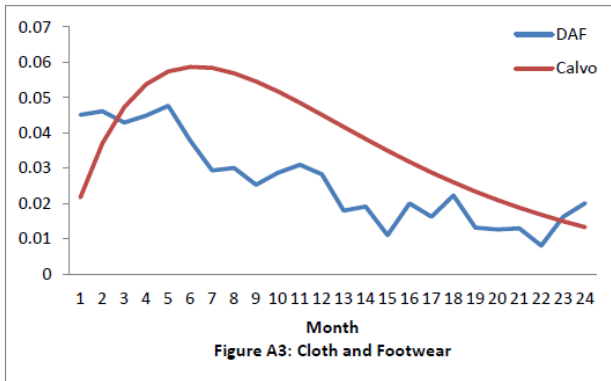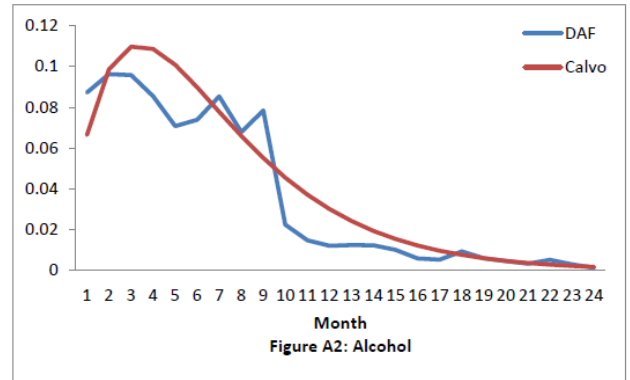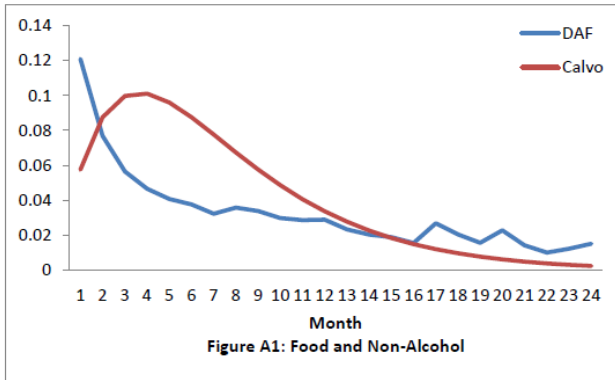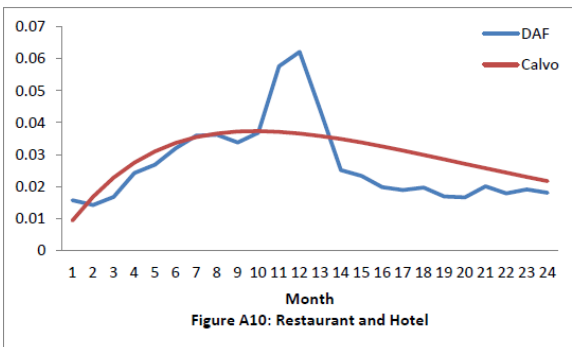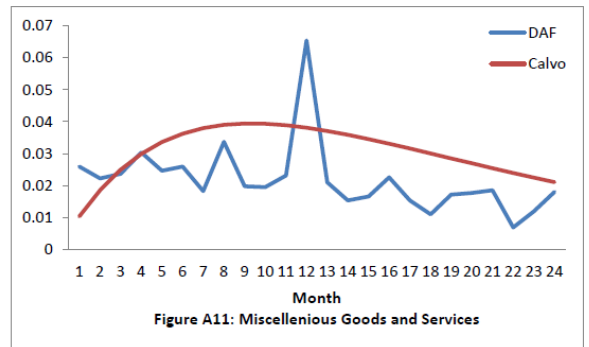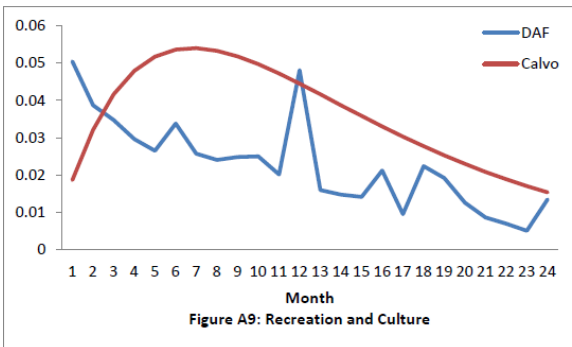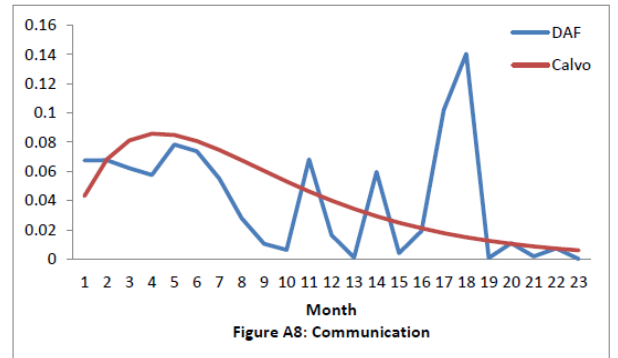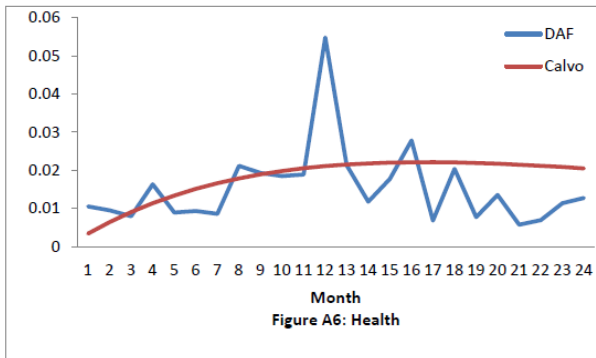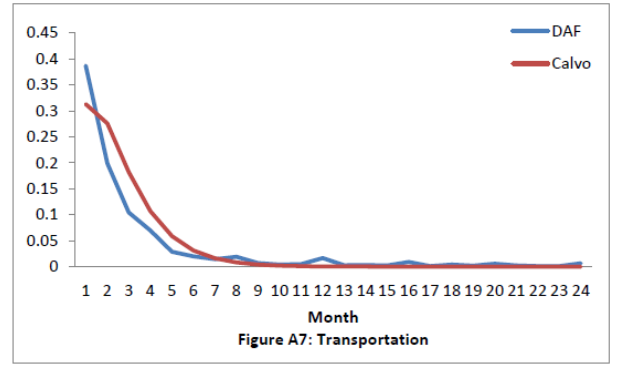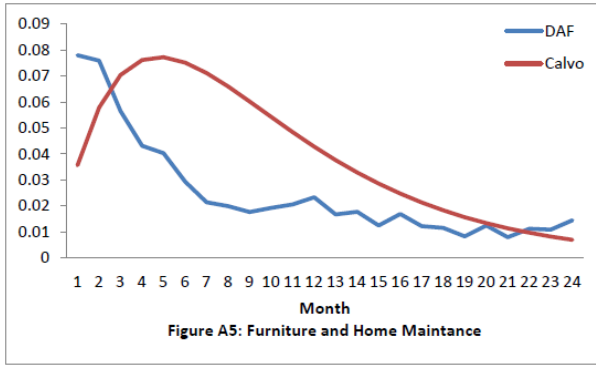
COICOP Sectors	Distance	K-alpha/sqrt(n)	Test result
Food & Beverages	0.293	0.009	Reject null
Alcoholic & Tobacco	0.130	0.060	Reject null
Clothing & Footwear	0.281	0.060	Reject null
Housing & Utilities	0.401	0.049	Reject null
Furniture & Home	0.377	0.013	Reject null
Health	0.399	0.078	Reject null
Transport	0.161	0.111	Reject null
Communications	0.227	0.136	Reject null
Recreation & Culture	0.337	0.090	Reject null
Restaurants & Hotels	0.105	0.102	Reject null
Miscellaneous	0.295	0.082	Reject null

Table 3.7: Kolmogorov-Smirnov test results.

mogorov strengthened this result, by effectively providing the rate of this convergence. In practice, the statistic requires relatively large number of data to properly reject the null hypothesis.

Under null hypothesis that the sample comes from the hypothesized distribution $F(x)$,

$$\sqrt{n}D_n \overset{n\to\infty}{\to} \sup_t |B(F(t))|$$

in distribution, where $B(t)$ is the Brownian bridge. The Kolmogorov-Smirnov test is constructed by using the critical values of the Kolmogorov distribution. The null hypothesis is rejected at level α if

$$\sqrt{n}D_n > K_\alpha,$$

where K_α is found from

$$\Pr(K \leq K_\alpha) = 1 - \alpha.$$

As described in Table3.7, the test results reject that sectoral distribution of DAF is the same as sectoral Calvo distribution. This result is consistent with the finding in Matsuoka (2009), who found that over 90 percent of the 429 tested items in the Japanese retail price data for 2000-2005 reject the hypothesis that the underlying distribution is exponential, which corresponds to the time-dependent pricing model of Calvo.

Relative difference:DAF vs. Calvo

We propose a new method to calculating the relative absolute difference between the distribution of DAF and the Calvo distribution. We calculate and absolute difference point-by-point for probability density function between two distributions:

$$d_i = |f_{DAF} - f_{Calvo}|$$

and then we add up all the d_i and let it be divided by 2. We denote the result as RD

$$RD = \frac{\sum_{i=1}^{T} d_i}{2}$$

The results of *relative difference* are shown in Table 3.8.

3.7.3 The log-linearised Smets and Wouters Model (2003)

Following Dixon and Le Bihan (2012), we present the full list of log-linearised Smets-Wouters model below.

COICOP	Relative Difference (DAF Vs. Calvo)
Food and Non-Alcoholic	0.36
Alcoholic	0.18
Cloth and footwear	0.33
Housing and utilities	0.49
Furniture and home maintanance	0.44
Health	0.58
Transportation	0.23
Communication	0.35
Recreation and culture	0.41
Restaurant and hotel	0.20
Miscellaneous goods	0.37

Table 3.8: Relative Difference: DAF vs. Calvo

First, the consumption Euler equation with habit persistence:

$$\hat{c}_t = \frac{b}{1-b}\hat{c}_{t-1} + \frac{1}{1+b}\hat{c}_{t+1} - \frac{1-b}{(1+b)\,\sigma_c}\left(r_t - E_t\pi_{t+1}\right) + \frac{1-b}{(1+b)\,\sigma_c}\varepsilon_t^b.$$

Second, the investment equation:

$$\hat{I}_t = \frac{1}{1+\beta}\hat{I}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{I}_{t+1} + \frac{\gamma}{1+\beta}q_t + \varepsilon_t^I.$$

Here $\hat{I}_t$ is investment in log-deviation, q_t is the shadow real price of capital, γ is the inverted investment adjustment cost.

Third, Tobin's q equation

$$q_t = -\left(r_t - E_t\pi_{t+1}\right) + \frac{1-\tau}{1-\tau+\bar{r}^k}E_tq_{t+1} + \frac{\bar{r}^k}{1-\tau+\bar{r}^k}E_t\hat{r}_{t+1}^k + \eta_t^Q$$

where τ is the rate of depreciation, $\bar{r}^k$ is the rental rate of capital.

Capital accumulation:

$$\hat{K}_t = (1 - \tau)\,\hat{K}_{t-1} + \tau\hat{I}_{t-1}$$

Wage equation:

$$
\begin{aligned}
\hat{W}_t - \hat{P}_t \;=\; & \frac{\beta}{1+\beta}\left(E_t\hat{W}_{t+1} - \hat{P}_{t+1}\right) + \frac{1}{1+\beta}\left(\hat{W}_{t-1} - \hat{P}_{t-1}\right) + \frac{\beta}{1+\beta}E_t\pi_{t+1} \\
& -\frac{1+\beta\gamma_w}{1+\beta}\pi_t + \frac{\gamma_w}{1+\beta}\pi_{t-1} \\
& -\frac{1}{1+\beta}\frac{(1-\beta\xi_w)(1-\xi_w)}{\left(1+\frac{1+\lambda_w}{\lambda_w}\sigma\right)\xi_w}\left[\hat{W}_t - \hat{P}_t - \sigma_L\hat{L}_t - \frac{\sigma_c}{1-h}\left(\hat{c}_t - h\hat{c}_{t-1}\right) + \varepsilon_t^L\right] + \eta_t^W
\end{aligned}
$$

Labour demand:

$$\hat{L}_t = -\hat{W}_t + (1+\psi)\,\bar{r}_t^k + \hat{K}_{t-1}$$

Goods supply

$$\hat{Y}_t = \phi\alpha\hat{K}_{t-1} + \phi\alpha\psi\hat{r}_t^K + \psi(1-\alpha)\hat{L}_t + \phi\hat{\varepsilon}_t^A$$

Goods demand

$$\hat{Y}_t = (1 - \tau k_y - g_y)\,\hat{c}_t + \tau k_y\hat{I}_t + g_y\hat{\varepsilon}_t^g$$

Monetary policy

$$
\begin{aligned}
\hat{\imath}_t \;=\; & \rho\hat{\imath}_{t-1} + (1-\rho)\left[\bar{\pi}_t + r_\pi\left(\hat{\pi}_{t-1} - \bar{\pi}_t\right) + r_Y\left(\hat{Y}_t - \hat{Y}_t^P\right)\right] \\
& + r_{\Delta\pi}\left(\hat{\pi}_t - \hat{\pi}_{t-1}\right) + r_{\Delta Y}\left[\left(\hat{Y}_t - \hat{Y}_t^P\right) - \left(\hat{Y}_{t-1} - \hat{Y}_{t-1}^P\right)\right] + \eta_t^R
\end{aligned}
$$

Parameter	Value	Interpretation
β	0.99	Discount rate
τ	0.025	Depreciation rate
α	0.30	Capital share
λ_w	0.5	Mark-up wage
γ^{-1}	6.771	Inv.adj.cost
σ_c	1.353	Consumption utility elasticity
b	0.573	Habit formation
σ_L	2.400	Labour utility elasticity
ϕ	0.169	Capital util.adj.cost
		Reaction function coefficients:
r_π	1.684	Response to inflation
$r_{\Delta\pi}$	0.140	Response to change in inflation
ρ	0.961	Response to lagged interest rate
r_y	0.099	Response to the output gap
$r_{\Delta y}$	0.159	Response to change in the output gap
ρ_a	0.823	Persistence, productivity shock

Table 3.9: Calibration Parameters

Shocks are specified below:

$$\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$$

$$\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$$

$$\varepsilon_t^I = \rho_I \varepsilon_{t-1}^I + \eta_t^I$$

$$\varepsilon_t^Q = \rho_Q \varepsilon_{t-1}^Q + \eta_t^Q$$

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g$$

The Calibration of the parameters is based on the mode of the posterior estimates, as reported in Smets and Wouters (2003).

Chapter 4

The time-varying Ss rule: an application to the UK PPI micro data

4.1   Introduction

At the heart of New Keynesian models is the assumption that nominal rigidities - most notably price stickiness - are preventing resources from being allocated efficiently. There is a large amount of theoretical research which focused on the micro foundations of sticky prices, which is a key element in explanations of the real effects of monetary policy. However, the empirical literature on price stickiness has been relatively thin. In recent years, large-scale data sets of individual prices, in particular those assembled for

the purpose of constructing price indices, have been made available to researchers. The empirical research has significantly broadened knowledge about the prevalence of price stickiness, and the characteristics of individual price changes.

One typical finding of the empirical studies using micro price data is that prices at the micro level remain unchanged for some periods. And this stylized fact was documented in, among many others, Bils and Klenow (2004), Klenow and Kryvtsov (2008), and Nakamura and Steinsson (2008), who study consumer prices in the U.S., and Dhyne *et al.* (2006) and Vermeulen *et al.* (2006), who give a synthesis of studies carried out in euro area. For example, Dhyne *et al.* (2006) find that the monthly frequency of consumer price changes is about 15% in the euro area. These results are consistent with evidence from survey data (see Fabiani *et al.* 2005).

The infrequent adjustment observed in micro price data is often described by an Ss rule. The Ss rule model indicating that there is a range of values of state variable for which it is optimal not to adjust. This range of state is called "band of inaction". Sheshinski and Weiss (1977) derived the Ss rule from optimal price setting problem in the presence of adjustment cost. The ensuing empirical studies show that Ss rules are convenient reduced forms which can be confronted to the data.

However, the standard fixed Ss band model faces some empirical difficulties. It indeed predicts that prices become more likely to change the longer they have remained unchanged. If we define the hazard of a price change at time t is the probability that price will change after t periods given that it has survived for t periods. The standard

fixed Ss band model suggests that the hazard function of price change is upward sloping. This prediction is at variance with patterns often observed in micro price data. Nakamura and Steinsson (2008) find that the hazard function of regular prices is somewhat *downward* sloping for the first few months and then mostly flat after that, and they do not find any evidence of upward-sloping hazard function. Furthermore, they find that "the hazard function including sales is much more steeply *downward* sloping than the hazard function of regular prices". Klenow and Kryvtsov (2008) confirm the finding of downward sloping hazard function and give a possible explanation that the downward sloping hazards reflect the time-varying Ss band. Gautier and Le Bihan (2011) also point out that the hazard decreases with the size of the threshold.

In this chapter, I aim to analyze the determinants of hazard rate of price changes. Firm's decision to change its price is described as a time-varying Ss model. The time-varying Ss model is set up in a way that is consistent with the stylized fact I obtained from UK PPI micro data. Then a duration model is set up which is in line with the time-varying Ss model. More specifically, the duration model is specified in form of Cox proportional hazard, which is formed by two parts: a baseline hazard function and a function with covariates of interest. The baseline hazard function can be seen as a term which captures the feature that the threshold is time-varying. I estimate the duration model which controls for observed and unobserved heterogeneity across firms in assessing the effect of changes in inflation, interest rate, oil price, industrial output, and exchange rate on the hazard rate of price changes.

The chapter is structured as follows. Section 2 provides a description of the PPI micro data set and some stylized facts about price changes. Section 3 describes a time-varying Ss band model. Section 4 gives empirical specification of the time-varying Ss band model and describes the covariates of interest. In Section 5, I illustrate the estimated results. I conclude in Section 6.

4.2 The data set and some stylized facts

4.2.1 Data description

This study uses micro-dataset on producer prices collected by the Office for National Statistics (ONS)[1]. These individual price quotes are weighted and aggregated to form domestic Producer Price Index.[2] There are two types of PPI series: output price indices and input indices. The output price indices measure the change in the price of goods *sold* by UK manufacturers, and input price indices measure the change in price of goods *bought* by manufacturers for use in the manufacturing process. Due to the data availability, this study only focuses on the output prices. Products are grouped with the Standard Industrial Classification (SIC) with weighting patterns based on overall sales by manufactures within those groupings. The PPI uses sales data taken from PRODCOM survey

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[1]This work contains statistical data from ONS which is Crown copyright and reproduced with the permission of the controller of HMSO and Queen's Printer for Scotland. The use of the ONS statistical data in this work does not imply the endorsement of the ONS in relation to the interpretation or analysis of the statistical data. This work uses research data sets which may not exactly reproduce National Statistics aggregates.

[2]The micro data that underlie the producer price index used in this research were made accessible via VML. The terms and condition of the VML is described in Richie (2008).

to update weights. Price quotes are collected from the products which are manufactured in the UK and sold to the home market, excluding VAT and after discounts. Price quotes reflect orders delivered in current month, and they reflect actual prices achieved rather than any notional list price. Excise duties (on cigarettes, tobacco, alcoholic etc.) are included to compile PPI. Above all, service sector prices are not included in the PPI.

As stated in Morris and Green (2007), the output producer price index (PPI), produced by the Office for National Statistics (ONS), is exposed to several sources of potential error. The total error consists of two elements, the sampling error and the non-sampling error. The random sampling techniques are used to minimize the sampling error. However, non-sampling errors are not easy to quantify and include errors of coverage, measurement processing and non-response. Various procedures are in place to ensure that errors are minimized. Validation checks on data, based on percentage movements from quarter to quarter, are conducted to highlight unusual price changes for items. Disparities in data are investigated by contacting respondents if not explained on the returned form. Letters are sent to respondents where no price change has been evident for eighteen months and analysts liaise with respondents to ensure that the prices they provide meet the specified criteria.

The final dataset that our analysis is based on included approximately $960,000$ individual producer price quotes, covering $24,000$ products by $12,000$ firms. Our sample covers the period between January 1998 and February 2008. The PPI basket is updated annually to incorporate new products and changes in demand patterns for existing prod-

ucts. While there are around $1,050$ products are present in our data set for all 122 months, less than 5% of total. On average, a product is included in our "raw"[3] data set for about 37 months.

The PPI computer programs impute for non-response in the most recent few months. Thus if the price £14.99 is recorded for a specific item in date t, but the price information becomes unavailable for following 9 months. Then the PPI computer programs let the prices for that 9 months remain at £14.99. Imputation can help avoid the data gaps, mitigating the problem induced by censored price spells. However,as the duration of missing price quote keeps longer, an unobserved price change becomes more and more likely. Another disadvantage of imputation is that they are not true price observations but are "pseudo observations", which would introduce an upward bias in the estimation of the duration of price spells. Therefore, we discard the price spells with imputation prices. Above all, Imputation represents about 3% of our PPI research dataset.

In our PPI dataset, we do not have weights which are attached to individual price quotes before 2003. Following Nakamura and Steinsson (2008) and Gopinath and Rigobon (2008), we obtain value weights for the PPI at the four digit SIC commodity code, then divide the value weights equally within the four-digit code, calculating the weight for each price quote within the same item group by same method. Although the calculated weights are not necessarily equal to the actual PPI weights,as the result of a robustness check shows,the effect on aggregate measures of the statistics described in next section

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[3]Here "raw" data set means the data set provided by ONS without any filtering or manipulation.

is trivial.

As described in Chapter2, censoring is an important characteristic of price data, and it needs to be taken into account. Censoring is defined as when the failure event occurs and the subject is not under observation. In our sample we have a total of 162,731 price spells. Of those, 122,462 (75.25%) are uncensored, 18,681 (11.48%) are left censored, 15,787 (9.7%) are right censored, and 5,801 (3.6%) are double censored.

4.2.2   The frequency of price changes

The frequency of price changes can be defined as the ratio of the number of non-zero price changes observations divided by the total number of observations. Following previous studies (e.g. Alvarez *et al*, 2010; Bunn and Ellis, 2012b), the observations that there is no information on the price in the previous month are dropped from our sample. It is not possible to measure whether the prices has changed for these observations. As reported in Table 4.2.2, for all items in our sample of producer prices, the weighted average frequency of price change is 25.1%. It means that about a quarter of prices change each month. This result is similar to the estimate in Bunn and Ellis (2012b), in which they claim that an average of 26% of UK producer prices changed each month. Moreover, our result is somewhat higher than Alvarez *et al.*(2010) for Spain (21%), Cornille and Dossche (2008) for Belgium (24%), Dias *et al.* (2008) for Portugal (23%), Stahl (2006) for Germany (23%). Our result is almost the same as Gautier (2008) for France (25%), and Nakamura

and Steinsson (2008) for the U.S.(25%).[4] Above all, the producer prices are changed infrequently, and this is against a few theoretical pricing models which predict that prices would change every period, (e.g. the sticky information (Mankiw and Reis, 2002); Calvo with indexation (Smets and Wouters, 2003); Quadratic costs of adjustment (Rotemberg, 1982). As discussed in Chapter2, a menu costs model can be easily calibrated to fit the observed frequency of micro price changes.

The frequency of price changes varies substantially across product sectors. The flexibility of prices is the largest for energy sector, in which about 66% of prices change each month. The prices of intermediate goods and consumer food products change more frequently than capital goods and consumer durables. Columns 3 and 4 of Table 4.2.2 report monthly frequencies of price increases and decreases respectively, for all items and the main product groups. Column five reports the proportion of price decreases over the total number of price changes. Over 44% of price adjustments are price decreases, which gives evidence against the downward nominal rigidity hypothesis.

There is also a considerable heterogeneity in the frequency of price changes at the 2 digit industry level. As can be seen from Table 4.2.2, the prices of clothing and leather change least often among all of the 2 digit industries. While the price of petrol and secondary raw materials change far more often than that of the other 2 digit industries. Clothing is the only industry with the share of price decreases over the total number of

---

[4]However, we must notice that this is a very rough comparison. In each country's PPI data, the sampling scheme and the weight scheme are different. Furthermore, the time periods covered in each study are country specific.

Main component	All changes	Increases	Decreases	% of price decreases
Energy	65.9	39.0	26.9	40.8
Consumer food products	24.6	13.9	10.7	43.5
Consumer non-food non-durables	15.0	7.6	7.4	49.3
Consumer durables	17.7	8.9	8.8	49.7
Intermediate goods	25.1	14.1	11.0	43.8
Capital goods	18.6	10.1	8.5	45.7
All items	25.1	14.0	11.1	44.2

Table 4.1: Percentage of UK producer prices that change each month

price changes larger than 50%. In other words, we are more likely to observe price cuts in clothing industry. In sharp contrast, we are more likely to observe price increases in tobacco industry.

4.2.3 The unconditional hazard function

A price reset hazard function gives the probability of resetting a price conditional on the time elapsed since last adjustment. As discussed in Chapter2 and 3, the hazard function is important for aggregate dynamics, since it is closely related to the distribution of price spells, which in turn affects how the economy reacts to nominal disturbances.

The classic Kaplan-Meier method is widely used to estimate the unconditional hazard function, excluding all left-censored spells, keeping all right censored spells, and treat the end of a right censored data as a 'loss' (or non-price-change). This treatment of right censored spells is not a good one, because it leads to an under-estimate of the hazard for each period. Dixon *et al.* (2012) proposed two alternatives treating censored data: (a) They exclude all censored data in estimating the hazard function. (b) They treat right-censoring as a price-change ('loss is failure' or LIF), which is also a strategy used by

Industry	All changes	Increases	Decreases
Food and beverages	24.2	12.9	11.3
Tobacco	28.2	22.0	6.2
Textiles	14.6	7.9	6.7
Clothing	9.1	4.4	4.7
Leather	13.0	6.8	6.2
Wood	15.5	9.3	6.2
Pulp and paper	18.0	10.0	8.0
Media	19.3	10.0	9.3
Petrol and fuel	65.9	39.0	26.9
Chemicals	24.8	13.4	11.4
Rubber and plastic	19.5	11.4	8.1
Other non-metallic mineral products	35.3	19.0	16.3
Basic metals	39.7	23.1	16.5
Fabricated metal products	19.3	10.7	8.6
Machinery and equipment	12.8	7.6	5.3
Office machinery and computers	22.7	11.5	11.3
Electrical machinery	14.9	8.2	6.7
Radio and TV equipment	17.6	9.1	8.5
Precision instruments	15.1	8.0	7.1
Vehicles	21.2	11.2	10.0
Other transport	21.4	10.5	10.8
Furniture	16.4	8.2	8.2
Secondary raw materials	65.8	34.9	30.9

Table 4.2: Percentage of UK producer prices that change each month by 2 digit industry

Dixon and Le Bihan (2012). Because the longer spells are more likely to be censored. The method (a) is more likely to overestimate the hazard in the short term. The method (b) is the opposite extreme to the classic KM assumption and more likely to overestimate the hazard. Figure 4-1 displays the hazard functions estimated under three methods. Even though the three methods differ in the treatment of right-censored spells, they all generate similar hazard functions. There are three main characteristics in Figure 4-1:

1 All three hazard functions display a downward sloping pattern.

2 All three hazard functions exhibit significant spikes at 12 months and at 24 months.

3 All three hazard functions exhibit that a large proportion of 1-month-length price spells.

As can be seen in Figure 4-1, the hazard generated from method 'LIF' lies between the estimates from 'Uncensored' and classic 'KM'. The method 'KM' tends to underestimate the hazard, while the method 'Uncensored' is more likely to overestimate the hazard in the short term. These findings suggest that the 'LIF' method is a better method to estimate the unconditional hazard function. Dixon *et al.* (2012) also find that the approaches of using only uncensored data and treating right censoring as a price-change both result in very similar monthly cross-sectional distribution (distribution across firms). And the calibration in their paper actually uses the 'loss is failure' method.

The downward sloping hazard function might reflect the "aggregation of heterogeneous price setter". There are firms with sticky pricing strategies and those with flexible
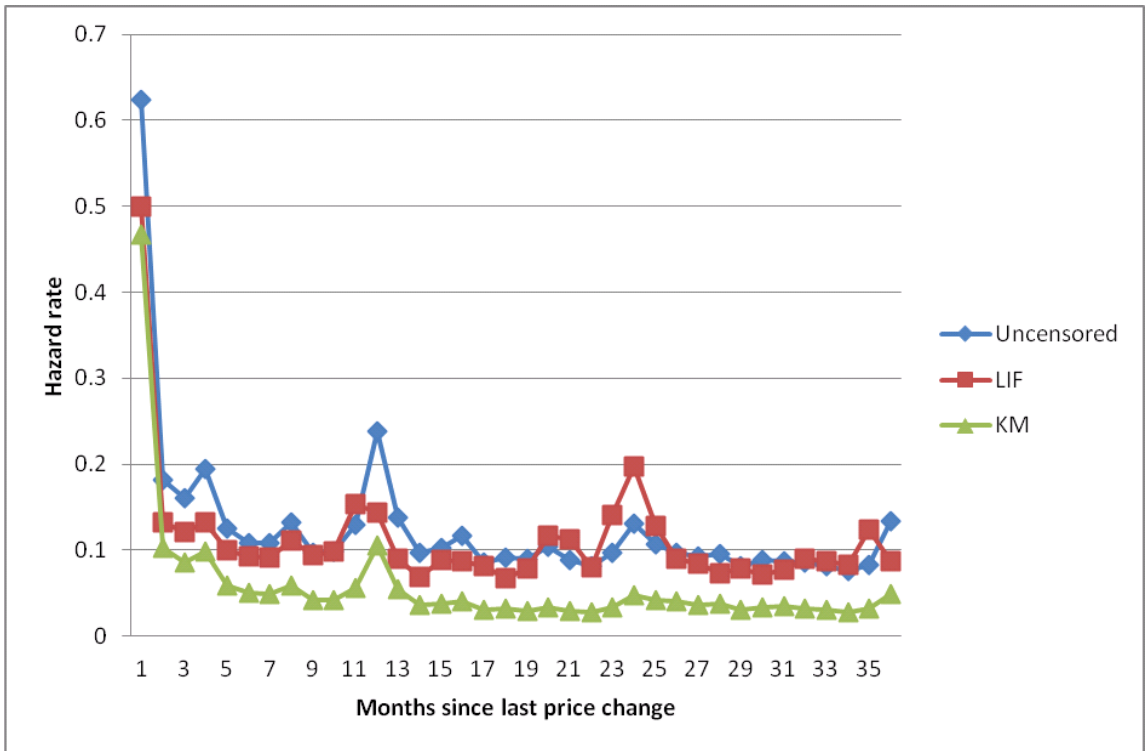
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Figure 4-1: Unconditional hazard function

pricing strategies. The firms with flexible pricing strategies are more likely to be in the "young age" zone. As firms become older, the share of price changes by firms with flexible pricing strategy will decrease. As argued in Alvarez (2008), only price changes which belong to sticky firms can be observed at high ages. In Chapter2, I extended the simple menu costs model to multiple-sector menu costs model. The simulated hazard function exhibit downward sloping pattern.

An alternative explanation to the declining hazard is the time-varing "Ss band", or the width of the inaction region (Klenow and Krytsov 2008). When a firm faces persistent idiosyncratic shock with high level, it tends to sell a large quantity under a low price. Therefore, the profit of the firm is mainly decided by choosing the right price. This will lead to a narrow Ss band. However, when the idiosyncratic shock is at low level, the firm's inaction region becomes wider. Furthermore, when Ss band is narrow and hazard rate is high, the young prices are more common; while the old prices are more common when Ss band is wider and hazard rate is lower.

4.3 Time-varying Ss band model

The decision rule of price-setting can be described as an Ss rule model. Ss rules are convenient reduced forms that can be confronted to the micro data. Sheshinski *et al.* (1981) and Dahlby (1992) firstly estimate this class of reduced form models. Recently, Fisher and Konieczny (2006) and Dhyne *et al.*(2011) estimate Ss models with random thresholds using micro price data for many categories of product. As suggested by Caballero and

Engel (1999) and Hall and Rust (2000), models assuming a random adjustment costs can rationalized the time-varying random Ss bands, which gives rise to hazard rates that vary over time for a given firm.

Let $p_{ij,t-1}$ is the actual price of a product i within the industry group j at time period $t-1$, p_{ijt}^* is the optimal price at period t. The actual price will keep the same as long as the difference between the actual price and optimal price is less or equal to the width of inaction s_t. Here we allow for time-varying pricing thresholds. Therefore, we have such a simple specification of (S,s) model, which can be written as

$$
\begin{aligned}
p_{ijt} &= p_{ij,t-1} \text{ if } |p_{ijt}^* - p_{ij,t-1}| \leq s_t \\
p_{ijt} &= p_{ijt}^* \quad \text{if } |p_{ijt}^* - p_{ij,t-1}| > s_t
\end{aligned}
\tag{4.1}
$$

As we have seen in previous section, price setting is considerably heterogeneous across industries. At the industry level, some price trajectories represented by more frequently changing prices, while others are represented by less frequently changing prices. Therefore we can extend model (4.1) to allow for time-varying and industry-specific pricing thresholds, which can be written as

$$
\begin{aligned}
p_{ijt} &= p_{ij,t-1} \text{ if } |p_{ijt}^* - p_{ij,t-1}| \leq s_{jt} \\
p_{ijt} &= p_{ijt}^* \quad \text{if } |p_{ijt}^* - p_{ij,t-1}| > s_{jt}
\end{aligned}
\tag{4.2}
$$

Let's assume such circumstance that the inflation rate is positive and steady. As

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time proceeds, $|p_{ijt}^* - p_{ij,t-\tau}|$ grows steadily[5] until it exceeds the level dictated by the rule. When the gap $|p_{ijt}^* - p_{ij,t-\tau}|$ surpasses the "adjustment threshold", the price will change. Therefore, the probability of observing a price change at time t, conditional on that the price has been kept the same for some time periods τ, will be the probability that the gap $|p_{ijt}^* - p_{ij,t-\tau}|$ is larger than the threshold s_{jt}, which is

$$\Pr\left\{|p_{ijt}^* - p_{ij,t-\tau}| > s_{jt}\right\} \tag{4.3}$$

We can use a semi-parametric survival function to develop an empirical specification of equation (4.3). However, to get a good understanding of the determinants of the hazard function, we need to analyze the factors which can affect the optimal price change.

To simplify the notation, we drop the subject i, j, and let z be the indicator of the firm (product). Different from Chapter2, we assume that a firm produce according to the CES production function (using labor and capital as inputs):

$$y_{zt} = A_{zt}\left(l_{zt}^\nu + k_{zt}^\nu\right)^{\frac{\gamma}{\nu}} \tag{4.4}$$

From this equation (4.4) , we define the following variables. The firm produce y_{zt} output in period t. In order to produce this amount of output in period t, the firm need to employ a quantity of labour as l_{zt} and a quantity of capital k_{zt}. ν is an elasticity of substitution parameter. γ is a parameter measuring return of scale. A firm-specific technology in

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[5]Caplin and Spulber (1987) assume the growth of money will raise p_{ijt}^* .

period t can be defined as A_{zt}. Differentiated goods y_{zt} can be used to produce a final consumption good Y_t. We assume the production function exhibit a CES love of variety over a continuum of differentiated goods y that are indexed by $z \in [0, 1]$:

$$Y_t = \left[\int_0^1 y_{zt}^{\frac{\eta-1}{\eta}} dz \right]^{\frac{\eta}{\eta-1}}.$$

And we assume the corresponding unit cost function P_t is:

$$P_t = \left[\int_0^1 p_{zt}^{1-\eta} dz \right]^{\frac{1}{1-\eta}}.$$

where p_{zt} denotes the nominal price the firm charges in period $t.\eta$ is the elasticity of demand. As is standard in this setup, the demand for the output of firm z is given by

$$y_{zt} = \left(\frac{p_{zt}}{P_t} \right)^{-\eta} Y_t \tag{4.5}$$

where y_{zt} denotes the quantity demanded of the firm's good. Firm's cost function can be shown as $c(y_{zt}, \kappa_t) = y_{zt}^{1/\gamma} A_{zt}^{-1/\gamma} \kappa_t$. γ is a return to scale parameter, and κ_t represents input prices. Then the firm will choose a price to maximize its profits:

$$\max_{p_{zt}} \left\{ \frac{p_{zt}}{P_t} - mc_t \right\} y_{zt} \tag{4.6}$$
$$s.t. \ y_{zt} = \left(\frac{p_{zt}}{P_t} \right)^{-\eta} Y_t$$

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where mc_t describes the firm's marginal cost function. Solving the first order condition of the model (4.6), we can get the optimal price $p_t^*(z) = \frac{\eta}{\eta-1}mc_tP_t$, which is just the markup pricing condition of monopolistic competition. If we assume the P_t grows with the inflation rate $\dot{P}_t = \pi_t$, then we can describe the gap $|p_{ijt}^* - p_{ij,t-\tau}|$ as a function of inflation and the change in marginal cost. Since the change of optimal price is a function of the inflation and firm's marginal cost, the gap $|p_{ijt}^* - p_{ij,t-\tau}|$ is also an implicit function of the inflation and firm's marginal cost, given the actual price has remain as previous optimal price for some period.

$$|p_{ijt}^* - p_{ij,t-\tau}| = \mathbb{F}(\pi_t, \dot{mc}_t) = \mathbb{Z}(t)\beta \tag{4.7}$$

The vector $\mathbb{Z}(t)$ includes all the covariates of interest, and it will be specified in next section, and also the regression coefficient vector β will be estimated.

4.4   Empirical specification

In this section we develop an estimable model consisting of empirical versions of the equation(4.3) and (4.7). It is well known that OLS is not a good method to analyze survival data. Because it assumes the residuals to be distributed normally, which is equivalent to say that time to an event(failure) is assumed to follow a normal distribution. For example, if we are thinking about an case of Calvo pricing which the instantaneous risk of price changing is constant over time. Then the distribution of time (duration)

would follow an exponential distribution. Moreover, the duration (time to failure) is always positive, while theoretically, the normal distribution is supported on the entire real line. Therefore, we will choose survival analysis (duration model). Similar approaches have been adopted in previous studies, such as Aucremmne and Dyhne (2005), Dias *et al.* (2007),Fougere *et al.* (2007), Nakamura and Steinsson (2008), Matsuoka (2010), and Vasquez-Ruiz (2011).At its core, survival analysis concerns nothing more than making a substitution for the normality assumption characterized by OLS with some more appropriate for the problem at hand.

We first recall that the general definition of hazard function. The hazard function, in our context, investigates the probability of a price change conditional on the elapsed duration of a price spell. The hazard function can be defined as $h(t) = \frac{f(t)}{S(t)}$, where $S(t)$ is the survival function, and $f(t)$ is the density function. The survival function can be defined as $S(t) = \Pr(T \geq t) = 1 - F(t)$ where $F(t)$ is the distribution function of the duration variable T, and $F(t) \in [0, 1]$. It is always a source of concern that the results of analyses are being determined by the assumption. We would prefer a method that does not require assumptions about the distribution of failure times. Cox (1972) provided such an option, so called Cox model. In Cox model, the effect of the exogenous variable is specified as multiplying a baseline hazard function by a function that depends on the exogenous variable. We can define the hazard function of the ith cluster for the kth failure type as

$$h_{ki}(t) = h_0(t) g(\mathbb{Z}, \beta)$$

where $h_0(t)$ is the baseline hazard function. The function $g(\mathbb{Z}, \beta)$ should be non-negative, and it can be specified as:

$$g(\mathbb{Z}, \beta) = \exp(\mathbb{Z}\beta)$$

Recall that the probability of observing a price change at time t, conditional on that the price has been kept the same for some time periods τ, will be the probability that the change in the gap between the optimal price and actual price, $|p^*_{ijt} - p_{ij,t-\tau}|$, is larger than the threshold s_{jt}. Therefore, we have

$$
\begin{aligned}
\Pr\left\{|p^*_{ijt} - p_{ij,t-\tau}| > s_{jt}\right\} &= h_0(t) \exp\left\{\mathbb{Z}_{ki}(t)\beta + \psi_j\right\} \\
&= \exp(\psi_j) h_0(t) \cdot \exp(\mathbb{Z}_{ki}(t)\beta)
\end{aligned}
\tag{4.8}
$$

where ψ_j captures the variation of thresholds among industries. $h_0(t)$ can be seen as a term which is implicitly affected by the time-varying threshold. In Cox model, the baseline hazard function can be estimated separately[6] by performing an analysis at each failure and only concerning with the order in which the failures occurred. No assumption is made about the distribution of time to failure. We can obtain the maximum likelihood estimates of β from Cox's partial likelihood function, $L(\beta)$. As proved by Lin (1994), the estimator $\hat{\beta}$ is a consistent estimator and asymptotically normal as long as the marginal models are correctly specified.

It may be too restricted to assume that the baseline hazard function is the same

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[6]We drop all the left and double censored spells. And we apply the "LIF" method when right-censoring is treated.

across different industries. An alternative specification would be to assume that there are industry-specified baseline functions $h_{j0}(t)$. Therefore, we have following so-called stratified-Cox model

$$h_{ki}(t) = h_{j0}(t) \cdot \exp\left(\mathbb{Z}_{ki}(t)\beta\right) \tag{4.9}$$

In order to account for unobservable heterogeneity, we follow Nakamura and Steinsson (2008) and Matsuoka (2010) to build a semiparametric hazard model with shared frailty. At the observation level, frailty is introduced as an unobservable multiplicative effect α on the hazard function. And the frailty α is a random positive quantity. For purposes of model identifiability, α is assumed to have mean one and variance θ. In line with Nakamura and Steinsson(2008), we specify the unobserved heterogeneity as being common to all observations within the same product. In another word, we assume that the heterogeneities are not specific to a price spell, but are shared along the same price trajectory. Frailty model can be written as

$$h_{ki}(t) = \alpha_i \cdot h_{j0}(t) \cdot \exp\left(\mathbb{Z}_{ki}(t)\beta\right) \tag{4.10}$$

where α_i follow a gamma distribution. We can test the existence of unobserved heterogeneity by using a likelihood-ratio test of $H_0 : \theta = 0$.

The vector $\mathbb{Z}(t)$ includes some regressors varying with time which economic theory suggests may be relevant factors in explaining the conditional probability of price change over time. From previous section, the derivation of the time-varying Ss band model

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suggests that $\mathbb{Z}(t)$ should includes[7]: a) Inflation rate, which is measured as the monthly growth rate of the producer price index. It can be expected that the inflation rate will have a positive and significant effect on the hazard rate of price changes. The lead and lag of inflation rate could also affect the probability of price change, these should also be taken into consideration. b) Interest rate, the three-month Libor rate is chosen. Because the aggregate demand is more responsive to the Libor rate than to the base rate as it is the benchmark interest rate that influences the interest rate at which the private sector, both corporate and personal, can borrow. c) Oil price, a Brent series from Bloomberg (Ticker:CO1 Comdty) is used. To construct the monthly series, daily closing prices for all trading days are averaged within the month. It is suggested that the sharp increase of oil prices is more like past supply shocks. And high oil price may change inflation expectations. d) Industrial production index. The industrial production index has been used as a proxy to measure demand pressure. And previous finding suggests that the probability of changing prices varies positively with the industry sales growth. e) Nominal effective exchange rate. It represents the relative value of a home country's currency compared to the other major currencies being traded. Two nominal effective exchange rate series (pound vs. U.S. dollar, pound vs. euro) are used. A higher nominal effective exchange rate means that the pound is worth more than an imported currency.

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[7]Our model suggests that unit labor cost could be an explanatory variable. However, no wage micro data is available and the if I use the average earning index as an approximate of the unit labor cost. The regression subjects to the serious multicolinearity. The coefficient on wage and most coefficients on the other covariates become insignificant. Hence, the unit labor cost is dropped from our estimation. I argue that though this may lead to bias in the estimation result, the effect won't be too much. Because the other covariates may implicitly cover part of the change in the unit labor cost.

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The change in the effective exchange rate would have both supply side and demand side effect.

4.5 Estimates

Figure 4-2 presents the aggregate baseline hazard function estimated from model 4.8. It is very similar to the unconditional hazard function. It shows that the probability of a firm to change its price after one month is about 60%. This probability drops sharply to a level lower than 20% for the second and third month. The hazard rate jumps above 20% at the 12th month. Afterwards, the hazard function becomes relatively flat. After 60 months, the hazard function becomes more and more volatile. Because large amount of price spells are either ended with price change or censored. All price spell will definitely ends before or at the end of our sample period. Therefore, the baseline hazard rate equals to 1 at the end of sample period.

Figure4-3 shows the sectoral baseline hazard functions. The baseline hazard function in each product group (main sector) displays a downward sloping pattern which is similar to the aggregate baseline hazard. Our finding is consistent with the finding in Nakamura and Steinsson (2008). We can find that the 12-month spikes in baseline hazard are quite significant in all sectors, except for the energy. The baseline hazard function for energy goods differs greatly from the other sectors. In particular, the spike at 1-month is more pronounced for energy sector. In the energy sector, the firms change their price more frequently. Furthermore, the energy sector is characterised by very short durations and

within this sector, very few price spells are observed with duration longer than 18 months, which makes the estimation of the hazard rates for longer durations very imprecise. There is no price spell in energy sector with duration longer than 36 months. We also conduct the log rank test to see whether the baseline hazard functions are the same across 6 main sectors. The test result rejects the null hypothesis that

$$H_0 : h0_1(t) = h0_2(t) = ... = h0_6(t)$$

Table4.6 reports the main estimation results under different specification of the Cox model. The column (1) and (2) report the estimation from the equation (4.8). The column (3) and (4) shows the estimated hazard rate model for price changes using the stratified Cox model (equation 4.9). The last two column (5) and (6) report the estimation result for the shared frailty model. The table 4.6 report the estimated hazard ratio rather than coefficient β. The hazard ratio equivalent to $\exp(\beta)$. Hence, if a hazard ratio is greater than one, it means that the relevant variable has a positive effect on the hazard rate of price changes. While a hazard ratio less than one implies that the variable has a negative effect on the hazard rate of price changes. Above all, hazard ratio equals to one when variable has no effect on the hazard rate of price changes.

It can be seen that the estimated hazard ratio for the inflation variable are highly significant across all specifications. And all hazard ratios for inflation variable are relatively larger than one. It shows that the PPI inflation rate positively and significantly affect the hazard rate of price changes. Specifically, if the monthly PPI inflation rate

increases by 1%, it will raise the probability that a firm will change its price about 7% (hazard ratio lies with a range from 5% to about 9%), given that the price remains the same until that time. Our result is economically large in magnitude comparing with the previous findings. For example, Cecchetti (1986) find that a 5% increase in the inflation rate raises the instantaneous probability of price changes by 10%. However, Cecchetti (1986) only have price data on several magazines. In our research data set, there are over 240,000 products. Moreover, Cecchetti's research focus on the retail shop, while our study focus on the factory gate. The firms at an earlier point in the supply chain may be more sensitive to the changes in aggregate price level. However, we also find that, neither the change of one-period lagged inflation rate nor the change of one-period ahead inflation rate has a significant effect on the probability of price changes. It is worth noting that the coefficient on inflation is highly significant ($p<.01$). We are quite confident that the increase in PPI inflation will increase the hazard rate significantly. Fourgere et al. (2007) find that the impact of the inflation on the probability of a price change is significant. Moreover, their finding suggests an even larger impact of the inflation.

The estimates show that the change in the interest rate will significantly affect the hazard rate of price changes. A 1% increase in interest rate (Libor) rate, will lead to about 4% to 8% increase in hazard rate of price changes. The change in oil price has a significant but very small positive effect on the probability of changing prices. Moreover, the change in industrial production and effective exchange rate do have significant effect on the hazard rate of price changes.

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We capture the unobservable heterogeneity by using frailty model. Notice that regardless of the choice of frailty distribution, the frailty model reduces to non-frailty model when variance of frailty equals to zero. That is to say, if $\theta = 0$, then $h_\theta(t) = h(t)$. The last two columns of table 4.6 report the estimation of θ. The likelihood ratio test suggests that the null hypothesis that there is no heterogeneity present is strongly rejected. The estimated hazard ratios from frailty model are generally higher than the estimates from the other two models. This facts indicate that failure to account for the unobservable heterogeneity may result in an underestimate of hazard ratio.

Overall, it is important to stress a point that the coefficients associated to the time varying regressors, which measure the state of the economy, are in general individually significant, using the likelihood ratio test, the null hypothesis that the included time varying regressors are not jointly significant is strongly rejected. Further more, even controlling for different sources of heterogeneity, coefficients associated to the time varying regressors are statistically very significant, suggests that the state dependent models are likely to proved a reasonable approximation to the micro price data underlying the UK PPI.

4.6 Conclusion

This study documents the main stylized facts of price-setting behaviour of British firms over the period January 1998 to February 2008. We develop a time-varying Ss band model and use the individual prices underlying the UK PPI to analyze the factors which

can affect the hazard rate of price changes through a semiparametric survival analysis model that fully capture observable and unobservable heterogeneities among the individual firms. Instead of assuming the distribution for the baseline hazard function, we let "data speak" and avoid the situation that the results of analyses are being determined by the assumptions and not the data. The study presents statistically significant evidence that the economic environment affects the hazard rate of price changes, which is consistent with the predictions in state-dependent pricing models. We can summarize the key empirical findings as follows.

First, producer prices are moderately sticky. The weighted average frequency of price change is 25.1%. The frequency of price changes varies substantially across product sectors. There are about 44% of price adjustments are price decreases, which gives evidence against the downward nominal rigidity hypothesis.

Second, the unconditional hazard function displays a downward sloping pattern with annual spikes. The hazard rate is quite high at the first month, which indicates that a large proportion of firms reset their price in short period. After correcting for firm's heterogeneity and estimate a semiparametric duration model, the baseline hazard functions still exhibit a downward slope with relatively large 12-month spike. The downward sloping hazard can be explained by a time-varying Ss band model with persistent strong idiosyncratic shock.

Third, the inflation rate affects the instantaneous probability of price change conditional on that the price has been kept constant until that time period. Specifically, a

1% increase in the inflation rate significantly increase the hazard rate of price change by about 7%. This result is consistent with the analysis of the pricing behaviour of firms using qualitative surveys, and previous probabilistic and non-parametric studies.

Fourth, the factors that can affect firm's cost or demand will significantly affect the hazard rate of price change, but in different magnitude. The change in interest rate will have a large effect on the hazard rate of price change. While the change in oil price, industrial production, and exchange rate only have very small effect on the probability of changing prices.

Five, the unobservable heterogeneity is captured by using frailty model. Given the significance level of the likelihood-ratio test, we reject the null hypothesis that no such heterogeneity present.

Finally, our estimation results of hazard ratio are quite robust under different specifications of the empirical semiparametrical duration models.
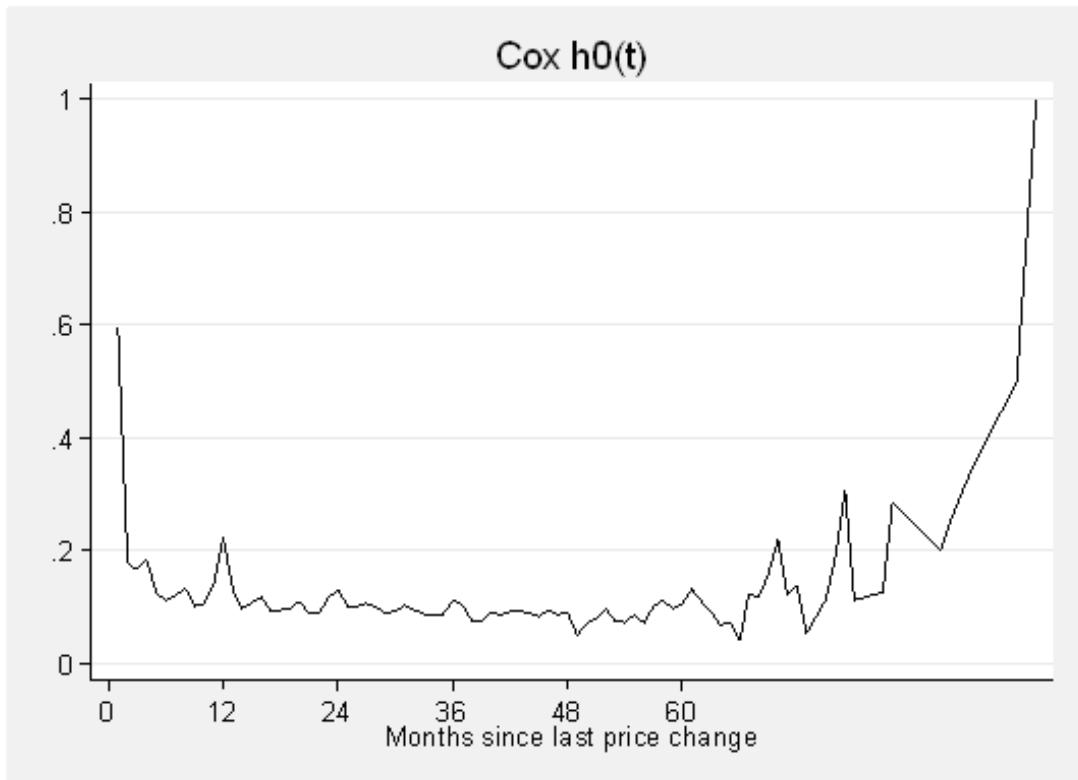
Figure 4-2: Aggregate baseline hazard function
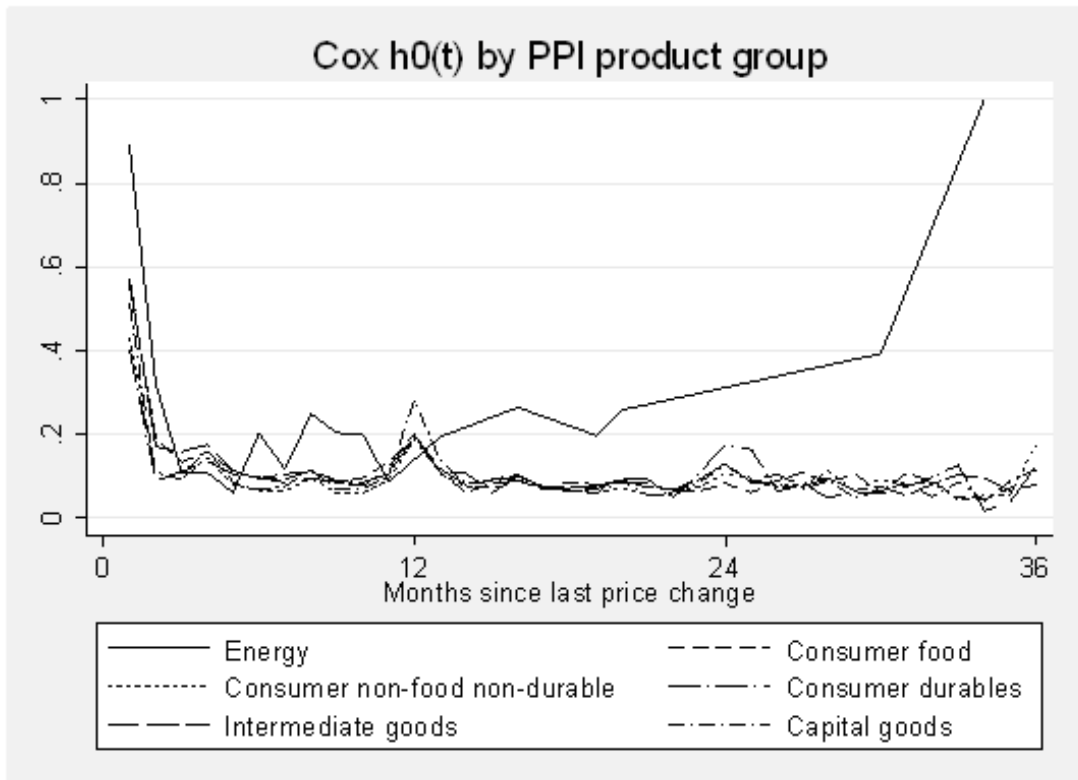
Figure 4-3: Sectoral baseline hazard function.

	(1)	(2)	(3)	(4)	(5)	(6)
Inflation	1.075***	1.05***	1.077***	1.059***	1.089***	1.078***
Interest	1.039***	1.049***	1.053***	1.061***	1.083***	1.079***
Oil_price	1.003***	1.004***	1.003***	1.005***	1.006***	1.005***
Industry_production	1.002*	1.007*	1.003*	1.005*	1.004*	1.006*
eff_exchange_euro	1.005	1.007*	1.006*	1.07*	1.012**	1.011**
eff_exchange_us	0.994*	0.996*	1.002**	1.001**	1.004***	1.003***
Inflation(t+1)		0.965		0.964		0.986
Inflation(t-1)		1.007		1.006		1.003
Theta					0.089***	0.059***

Table 4.3: Hazard rate for price changes

*significant at 10%; **significant at 5%; ***significant at 1%.

Chapter 5

Conclusion

5.1   Main findings

In Chapter 2 "Frequency, hazard function, and distribution across firms", I propose a unified framework to assess the price rigidity. The frequency, hazard function, and the distribution across firms are three different perspectives to look at the same thing. I prove that these three methods can give the consistent estimates if we choose the proper treatment on censoring price spells. I also compare the different assumption on the right-censoring, and how robust our results are.

In CPI micro- data set, price changes are frequent (about 21% on average) and the frequency of price changes varies a lot between different sectors. Excluding the sales from CPI data, the frequency of price changes becomes much lower.

The aggregate hazard function is downward sloping with large annual spikes. The

downward sloping aggregate hazard function can be seen as a result from the mixture of heterogeneous price-setters.

I estimate the distribution across firms from the hazard function. Comparing with the common used distribution of duration across contracts, I find that the DAF (the desired distribution) is much flatter than the distribution across contracts. The single sector Calvo distribution is far different from the actual DAF, suggesting that the Calvo probability itself is not enough to generate the whole picture of price rigidity.

I solve and simulate two benchmark models, the basic Calvo model and menu costs model. Both models can match the empirical findings in frequency of prices changes and the proportion of prices increases. However, these two benchmark models cannot meet the empirical findings in hazard functions . Moreover, the implied DAFs are different from empirical one. Furthermore, I solve and simulate a multiple-sector Calvo model and a multiple-sector menu costs model. The results suggest that adding heterogeneity to price-setting model can improve its fitness on micro evidence. As shown in Dixon and Kara (2010), allowing for a distribution of durations can take us a long way to solving the puzzle of inflation persistence. In other words, an explicit modelling of the distribution of durations can help the DSGE model to match macro data.

In Chapter 3 "What we can learn about the behaviour of firms from the average monthly frequency of price-changes in CPI data: an application to the UK CPI micro data", we find that there is no simple way to map the frequency to the cross-sectional distribution: to uncover this we need to make additional assumptions. However, we are

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able to say what the theoretical minimum cross-sectional mean duration is consistent with an observed frequency: it is the mean duration of a price-spell which occurs when all price-spells in the sector have the same or almost the same length. However, the cross-sectional mean can be much longer.

When we look at the UK data using an estimated hazard function, we find that the UK data is long way from the distribution implied by the theoretical minimum. Looking at different levels of disaggregation, we find that whilst the minimum theoretical mean duration is around 5.5-6.7 months, the actual data reveals a mean of almost 11 months. We also find that the more disaggregated the data, the closer are both the median and mean to the true values. We also look at the aggregate data using the hypothesis that the sectoral frequencies are generated by a Calvo distribution. The mean and median of the hypothetical Calvo distribution can be close to the actual values. Above all, DSGE models under the Calvo distribution hypothesis behave in a very similar way to models calibrated with the microdata. This suggests that we can use the disaggregated frequency data to calibrate DSGE models when we do not have reliable hazard function estimates or access to the price microdata.

In Chapter 4 "The Ss pricing rule: an application to the UK PPI micro data", We find that producer prices are moderately sticky, and the unconditional hazard function displays a downward sloping pattern with annual spikes. The hazard rate is quite high at the first month, which indicates that a large proportion of firms reset their price in short period. After correcting for firm's heterogeneity and estimate a semiparametric duration

model, the baseline hazard functions still exhibit a downward slope with relatively large 12-month spike. The downward sloping hazard can be explained by a time-varying Ss band model with persistent strong idiosyncratic shock.

The estimation result of the duration model suggests that the inflation rate affects the instantaneous probability of price change conditional on that the price has been kept constant until that time period. Specifically, a 1% increase in the inflation rate significantly increase the hazard rate of price change by about 7%. This result is consistent with the analysis of the pricing behaviour of firms using qualitative surveys, and previous probabilistic and non-parametric studies. Moreover, the factors that can affect firm's cost or demand will significantly affect the hazard rate of price change, but in different magnitude. The change in interest rate will have a large effect on the hazard rate of price change. While the change in oil price, industrial production, and exchange rate only have very small effect on the probability of changing prices.

The last but not the least, the unobservable heterogeneity is captured by using frailty model. Given the significance level of the likelihood-ratio test, we reject the null hypothesis that no such heterogeneity present.

5.2 Limitations of this research

No research can cover all aspects of the subject area it intends to investigate, and this thesis is no exception to this rule.

Firstly, the micro data underlying the UK CPI and PPI are collected at monthly

frequency. It is impossible to check whether the price has been changed more than once within a month. This may lead to an overestimation of price rigidity for some goods, for example, the fuel, fresh fruits, and etc. The weekly data or daily data would be preferred.

Secondly, this research only covers episodes of low inflation in the UK. It is obscure about the relationship between inflation and consumer/producer price setting during high inflation episodes. A natural hypothese raised from this issue is how different the price-setting behaviors exhibit in low- and high-inflation economies.

Thirdly, this research only focuses on the price micro data. The wage micro dataset, however, to our knowledge, is not public available. Furthermore, there is no easy mapping between the wage micro data and price micro data. The lacking of wage micro data prevents us to investigating the pass-through of unit labor cost on to price.

5.3   Future research

Threre are three directions in which the research as a whole could be further extended. Firstly, more investigation could be carried out along similar research lines to this, but involving wage micro data. As shown in Carlsson and Skans (2012), using detailed data on product prices and unit labor cost merged at the firm level can help to evaluate competing sets of assmptions regarding firm's price-setting behviour. Secondly, a DSGE model with different price-setting strategy can be test statistically. Especially, an indirect inference test will be considered. Thirdly, the research can be extended to the policy issue such as how monetary policy should respond to the aggregate shocks and/or sectoral shocks.

Furthermore, the future research can be extended to investigate the consequences of employing models that are inconsistent with the micro data for optimal monetary policy design.

Chapter 6

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