Comment on "Elastoplastic Contact between Randomly Rough Surfaces"

In a recent Letter, Persson [1] suggested a new model of contact between arbitrary randomly rough surfaces. He considered also fractal surfaces which he characterized above the lower cutoff only by their dimension D_f and an additional constant. He argued that in the framework of the model, the area of real contact A in most cases is proportional to the load P. However, there are examples which show that the suggested relations are far from universality and that the fractal dimension alone cannot characterize the features of contact. So, further work is needed to create reasonable models of rough contact.

The power law of the roughness correlation function which was discovered for various surfaces [2] is usually considered as manifestation of a fractal nature of roughness. One can try to describe its complex structure as the cumulation of many simple steps [3] and one can build various rough surfaces, both fractal and nonfractal, using very simple blocks [4]. However, the contact problems with a nonfixed contact region are nonlinear and usually they cannot be solved by superposition of the solution to a simple problem. The following examples of fractal surfaces allow us to perform strict mathematical analysis of the problem: the Cantor profile (Fig. 1a) and the parametric-homogeneous (PH) functions (Fig. 1b). The former has two scaling parameters a_x and a_z such that the width L_i and the height h_i of each generation of asperities are given by $L_i = L_0 a_x^i$ and $h_i = h_0 a_z^i$, respectively [5]. There are various generalizations of the model [6]. First, two problems of contact between the Cantor punch and (i) a rigid-plastic foundation and (ii) an elastic (Winkler) foundation were solved. Only for rigid plastic solids $A \sim P$, while in the case of an elastic foundation, the load depends on not only the contact area but also the vertical distribution of material in the punch. So, two punches having the same fractal surface but situated either above or below the surface show usually different asymptotics in both load-displacement and load-area relations [5].

PH functions $b_d(\mathbf{x}; p)$ of degree d and a fixed parameter p satisfy the relation $b_d(p^n \mathbf{x}; p) = p^{nd} b_d(\mathbf{x}; p)$ for any integer n [7]. The solution to Hertz-type problem of discrete contact between a PH punch of degree $d \geq 1$ and a power law hardening solid is discrete self-similar. So, the contact region changes homothetically. The trends of both the nominal region (the convex hull of contact points) and the region of real contact are proportional to $P^{2/[2+\kappa(d-1)]}$, where κ is the exponent of hardening. For a PH parabola and a linear elastic solid $d = 2$ and $\kappa = 1$. So, for all values of fractal dimension, the areas are proportional to $P^{2/3}$. This is in accordance with the classic Hertz solution.

Although the surfaces were not random, the analysis is valid for their random generalizations [8].

The above examples show that even in geometrically linear formulations, the contact problems for elastic rough surfaces cannot be solved using just geometrical arguments and equations of elasticity should be involved.

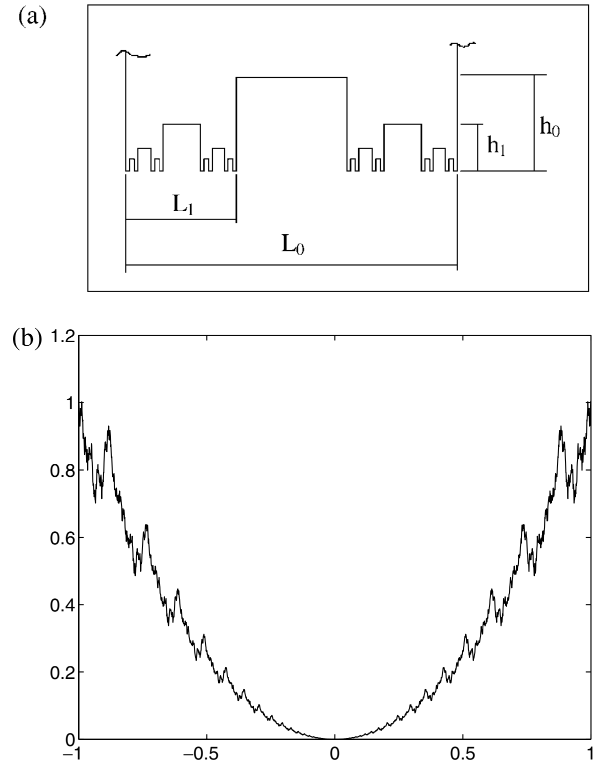

FIG. 1. Cantor profile (a) and a fractal PH parabola (b).

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