

Three Essays on Human Capital and Business Cycles

by

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Abstract

This thesis contains four independent chapters with all of them emphasizing the role of purposeful human capital accumulation in affecting short-run economic dynamics. Four chapters jointly are aimed to deliver two key messages: first, human capital investment is an important channel to propagate business cycle shocks; second, accounting for human capital investment decision appropriately solves two consumption puzzles ("excess sensitivity" and "excess smoothness") simultaneously. The tool used to achieve these goals is an extended version of the Uzawa-Lucas two-sector endogenous growth model. Specifically, the first chapter shows that modelling human capital formation explicitly in a business cycle framework gives rise to a strong internal propagation mechanism such that output growth is positively autocorrelated in short horizons and output has a hump-shaped impulse response. The second chapter shows that if human capital investment is counted as part of measured output (not the case in the chapter 1), the endogenous growth model in this thesis is also able to replicate the observed output dynamics via a different mechanism. The third chapter shows that taking into account people's human capital investment decision is able to reconcile the "excess sensitivity" of consumption with permanent income hypothesis. The last chapter shows that a reasonable degree of elasticity of intertemporal substitution is able to explain consumption smoothness when income process is nonstationary.

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Introduction

This thesis consists of four independent essays on the role of purposeful human capital formation in business cycle scenario. These essays jointly are aimed to deliver two messages: first, human capital investment is an important channel to propagate business cycle shocks; second, accounting for human capital accumulation solves two long-standing consumption puzzles simultaneously: "excess sensitivity" and "excess smoothness". The tool used to achieve these goals is a stochastic version of the Uzawa-Lucas two-sector endogenous growth model.

Chapter 1 is devoted to study the business cycle implications of the two-sector endogenous growth model under the assumption that human capital is nonmarket good. It shows that having a more labour-intensive technology producing human capital investment relative to that producing physical capital introduces to the line of endogenous growth business cycles research several new features that jointly change the dynamics of existing models (e.g. the one in Jones et al. (2005b)). First, changes in the relative price of human capital in terms of physical capital, interpreted as "capital gain" of human capital investment, adjusts in the direction to help equalize the rates of returns to physical and human capitals. Second, different factor intensities across two sectors give rise to *Rybczynski* effect and *Stolper-Samuelson* effect that are widely used in the literature of international economics. Third, the second sector producing human capital investment with a different technology can be interpreted as an implicit form of inter-sectoral cost of adjustment, which can be identified from the concave economy-wide production possibility frontier. Several interesting results are found. First, curtailment of working hours

when labour productivity is high arises naturally in this two-sector model as a result of agents' optimal inter-sectoral labour substitution decisions required by the no-arbitrage condition placed on the returns to two types of capitals and the *Rybczynski* effect induced by release of leisure. Therefore, the empirical finding by Gali (1999) that labour supply decreases on impact of a positive productivity shock should not be interpreted as evidence against flexible price models. Second, the two-sector model generates persistent movements in the growth rates of output and physical capital investment up to the level that matches US observations, which combined with hump-shaped impulse responses of output and physical capital investment indicates the existence of a strong interior propagation mechanism embedded in the two-sector model to spread the shock over time. Third, the two-sector model also generates greater fluctuation in working hours because of the substitution between market and nonmarket time. It is, therefore, concluded that purposeful human capital formation in a separate sector constitutes a strong interior shock propagation mechanism in business cycle framework.

Chapter 2 addresses a similar issue to that of chapter 1 under a different assumption of human capital: human capital is tradable good such that it counts as part of measured output. This chapter solves the Cogley and Nason's puzzle via a different mechanism. In particular, this part of the thesis explores the idea of changing composition of output in multi-sector models in generating hump-shaped impulse response function of output and positive autocorrelation of output growth as found in the data. On impact of a positive sector-specific shock, the representative agent relocates resources towards the sector with higher productivity, which results in higher output in one sector and lower output in the other. The overall effect on composite output on impact is hence small but still positive. In subsequent periods when the effect of a sector-specific shock dies out, the direction of inter-sectoral resource transferring reverses so that output in the sector into which factors flow increases while output in the other sector of which factors flow out decreases. Response of composite output then exhibits a "hump" because the recovery of one sector outweighs the reduction of output in the other sector for several

post-impact periods. Finally, the hump-shaped impulse response function helps give rise to positive autocorrelation of growth of composite output.

Chapter 3 shows that the two-sector endogenous growth model can reconcile the apparent "excess sensitivity" of consumption to current income found by Campbell and Mankiw (1990,1991) with permanent income hypothesis (PIH). This is because of two features of the model: nonseparability between consumption and leisure and nonmarket time in producing new human capital. The role of the first feature can be understood intuitively. When income growth is high, market wage goes up procyclically, which induces higher tendency to work and hence less time spent on leisure. Finally, changes in leisure time affect marginal utility of consumption and hence consumption growth rate via utility nonseparability. Although this is a possible mechanism to generate apparent "excess sensitivity" under permanent income hypothesis, the idea fails quantitatively. Specifically speaking, even for implausibly high values of the coefficient of relative risk aversion, nonseparability in the one-sector endogenous growth model as in Jones et al. (2005b) and the standard real business cycle (RBC) model accounts for only a small fraction of observed sensitivity. In addition to nonseparability, the two-sector stochastic endogenous growth model makes a clear distinction between market work producing physical goods and nonmarket effort producing human capital. This alternation has two impacts on the relation between consumption growth and predictable changes in current income. The first impact is direct. Existence of nonmarket time amplifies the linkage between consumption growth and predictable income growth attributed to nonseparability. The more steady state effort agents devote to producing human capital, the greater the amplification impact is. The second impact of having nonmarket time is indirect. As the two-sector model implies, changes of nonmarket time have predictive power for consumption growth, but they are omitted from the econometric model of "excess sensitivity" as Campbell and Mankiw specify. More importantly, since growth of nonmarket time is positively correlated with income growth, a researcher who is interested in estimating the mis-specified regression model is very likely to end up with high degree

of "excess sensitivity" even if the true parameter is small enough to be rationalized by utility nonseparability alone.

Chapter 4 attempts to show that relaxing the assumption of constant interest rate maintained in Deaton's original framework is able to reconcile "excess smoothness" of consumption with a modified version of PIH. To this end, first, this chapter shows that "excess smoothness" puzzle only arises in endogenous growth models but not in standard RBC models when response of consumption to changes of interest rate is restricted. This is because temporary shocks have permanent effect in endogenous growth models but only short-lasting effect in RBC models. Second, this chapter shows that when consumption is allowed to react to interest rate changes, the two-sector endogenous growth model predicts reasonable volatility of consumption growth that matches empirical evidence, but consumption growth implied by the one-sector model in Jones et al. (2005b) appears too smooth. This is because of the inter-sectoral adjustment cost interpretation of human sector in the two-sector model in which an agent cannot smooth her consumption stream as perfect as she does in the one-sector model where goods used for different purposes are perfect substitutes.

Appendix A collects the solution methods used to solve all models discussed in this thesis. Appendix B contains the details of the instrumental variable method used in chapter 3. Appendix C shows that there is a unique internal steady state for the calibrations in this thesis. Appendix D presents data description and some summary statistics.

Chapter 1

Business Cycle Persistence and Human Capital: Human Capital as Nontradable Goods

1.1 Introduction

Traditional real business cycle (RBC) models are criticized for their lack of an interior propagation mechanism to spread the effect of a shock over time. Persistence in standard RBC models is directly inherited from the persistence of exogenous shocks. The failure of RBC models in this regard is explicitly formulated by Cogley and Nason (1995) and Rotemberg and Woodford (1996). Cogley and Nason summarize two stylized facts about the dynamics of US GNP that prototypical RBC models are unable to match:

“First, GNP growth is positively autocorrelated over short horizons and has weak and possibly insignificant negative autocorrelation over longer horizons. Second, GNP appears to have an important trend-reverting component that has a hump-shaped impulse response function.” (Cogley and Nason (1995), pp. 492)

These two stylized facts about the dynamics of US output indicate the existence of a propagation mechanism internally built in the economy to spread shocks over time. As Cogley and Nason point out, in the class of models that only rely on physical capital accumulation and intertemporal substitution to spread shocks over time, the output and investment growth are often negatively and insignificantly autocorrelated over all horizons and output and investment usually have only monotonically decreasing impulse response curves following a positive technology shock. Moreover, the dynamics of output predicted by a standard exogenous growth business cycle model highly resembles the innovations that induce aggregate fluctuation. The similarity between output series and exogenous shock processes is another sign of the missing of a propagation mechanism from the standard business cycle modelling framework.

Another famous failure of traditional RBC models is the too low labour supply volatility relative to US observation. For example, the one-sector standard RBC model in King and Rebelo (1999) predicts the volatility of labour supply to be about a half of that of output. In the data, however, labour supply fluctuates nearly as much as output does.

This chapter studies a stochastic version of the Uzawa (1965) and Lucas (1988) two-sector endogenous growth model with an extension by including physical capital in human capital production. Compared with the one-sector endogenous growth model in Jones Manuelli and Siu (2005b) (JMS hereafter), the two-sector model has three new features that substantially change the dynamics of the model. First, having human capital produced in a different sector results in a material difference between physical and human capital investment: the former is a perfect substitute for consumption goods while the latter is not. Human capital investment is made of different goods from those used for consumption and investment to physical capital and its value is measured by its relative price in terms of physical goods. Changes in the relative price of human capital in terms of physical capital, interpreted as "capital gain" of human capital investment, adjusts in the direction to help equalize the rates of returns to physical and human capitals. In particular, if the net rate of return of human capital is higher (lower) than the net rate of

return of physical capital, the relative price of human capital should increase (decrease) in current period and decrease (increase) the next period to form a "capital loss (gain)" such that the no-arbitrage condition that requires two capitals are equally profitable holds. Second, different factor intensities across physical and human sectors give rise to *Rybczynski* effect and *Stolper-Samuelson* effect that are widely used in the literature of international economics. Due to the former, release of time from leisure tends to enhance the production of human capital investment and deteriorates physical goods production. Due to the latter, an increase in the relative price of human capital in terms of physical capital tends to increase the reward to labour input, which is used more intensively in human sector than physical sector, and decreases the reward to physical capital, which is used less intensively in human sector than physical sector. Third, the second sector producing human capital investment with a different technology can be interpreted as an implicit form of inter-sectoral cost of adjustment, which can be identified from the concave economy-wide production possibility frontier (PPF hereafter)¹. This results in increasing marginal cost of producing output of one sector at the expense of foregone output of the other sector. Therefore, an optimizing agent will slow down the inter-sectoral resources relocation process after shocks. These new features of the two-sector model jointly determine the dynamics of the variables in the model.

Several interesting results are found from the two sector endogenous growth model. First, curtailment of working hours when labour productivity is high arises naturally in this two-sector model. It follows as a result of agents' optimal inter-sectoral labour substitution decisions required by the no-arbitrage condition placed on the returns to two types of capitals and the *Rybczynski* effect. Therefore, the empirical finding by Gali (1999) that labour supply decreases on impact of positive productivity shock should not be interpreted as evidence against flexible price models. Second, the two-sector model generates persistent movements in the growth rates of output and physical capital investment up to the level that matches US observations, which combined with hump-shaped

¹See section IIIb in Mulligan and Sala-I-Martin (1993) for details.

impulse responses of output and physical capital investment, indicates the existence of a strong interior propagation mechanism embedded in the two-sector model to spread the shock over time. The on impact contraction of working hours and post impact labour transferring across sectors subject to an implicit form of adjustment cost account for the existence of a propagation mechanism in the endogenous growth model. Third, the two-sector model also generates greater fluctuation in working hours because of the substitution between market and nonmarket time. Inter-sectoral labour transferring enhances the substitution among different uses of time and amplifies the variability of working hours.

Two closely related works in this strand are Perli and Sakellaris (1998) and JMS². There are also other modifications built upon prototypical RBC models aiming to fix the aforementioned problems. Works to introduce an internal propagation mechanism in RBC framework to reproduce the two stylized facts include quadratic adjustment cost to capital and labour as in Cogley and Nason (1995), factor-hoarding models as in Burnside and Eichenbaum (1996) and habit formation in leisure as in Wen (1998) among others. Works aiming to increase the volatility of labour supply in the model even has a longer history. These include the indivisible labour supply model as in Hansen (1985) and models incorporating home production explicitly as in Greenwood and Hercowitz (1991) and Benhabib et al. (1991) among others.

There are two reasons to proceed in the direction of endogenous growth literature. First, works in this strand have been heavily re-motivated by the empirical findings by Dellas and Sakellaris (2003) that there is significant substitution between higher education (regarded as an important means of human capital accumulation) and competing labour market activities over business cycle frequency. In particular, they find that for the

²Both works include human capital investment and address the issues of business cycle persistence. However, the model in Perli and Sakellaris is not consistent with BGP hypothesis because of the constant elasticity of substitution aggregator of skilled and unskilled labour. Also, the assumption of the unshocked human sector hinders the generality of their results. The work by JMS modestly improves the model's prediction on the degree of persistence of output growth (0.189 in their model compared with 0.4049 in the data).

period 1968-88 one percentage increase in unemployment rate is associated with about two percentages increment in college enrolment rate (pp.149). This empirical finding gives a good reason to build up a model that incorporates the time allocation margin on different usages of non-leisure time. Second, modern macroeconomics attempts to incorporate the long-run growth and the short-run fluctuation components of economy in an integrated framework, but very few works try to build fluctuation into an endogenous growth framework relying on purposeful human capital formation. Therefore, it is meaningful to explore the role of human capital in propagating business cycle shocks given that it has enormously enhanced our understanding of economic growth.

Another strand of efforts in constructing propagation mechanism in business cycle models depends on producing “endogenous cycles”. Examples in this area include Benhabib and Farmer (1994), Farmer and Guo (1994), Schmitt-Grohe (1997) and Perli (1998). There are two major differences between the models used by these authors and that in this chapter. The first is that the equilibria of these models are different. In the class of models of “endogenous cycles”, there is a continuum of equilibria so that no unique equilibrium is determined. This is resulting from some non-standard features of these models. In general, models in this strand rely on increasing return to scale technologies or monopolistic competitions to generate indeterminacy. Technically, the eigenvalues of the coefficient matrix in the recursive solution of this class of models all lie outside or inside (depending on how the solution is presented) the unit circle so that the steady state is a “sink”. As a result, these models do not necessarily depend on fluctuations of exogenous impulses to generate business cycles. In fact, due to the existence of complex roots of the coefficient matrix that determines the impulse response functions, a one-off shock can induce cyclical responses of macro variables. This stays in sharp contrast with the monotonic convergence of macro variables in standard RBC models after technology shocks. The two-sector model in this chapter has a standard saddle path stability structure so that the equilibrium is uniquely determined. The second difference between these two types of models is their driving forces. The randomness of

models with indeterminacy is usually due to self-fulfilling beliefs (sunspots) rather than fundamental shocks, say, technology shocks in the two-sector endogenous growth model as in this chapter.

The approach to generating propagation mechanism in this chapter is preferable to that relies on “endogenous cycles” for two reasons. First, sunspot models rely on implausibly high degree of increasing returns to scale or markup to generate indeterminacy (e.g. Benhabib and Farmer (1994)). Second, when models of indeterminacy are calibrated following the standard RBC approach, they have predictions that are inconsistent with empirical observations. Two notable contradictions are the upward sloping labour demand curve as in Farmer and Guo (1994) and counter-cyclical aggregate consumption as in Benhabib and Farmer (1996a). In contrast, the two-sector endogenous growth model is able to deliver reasonable business cycle statistics without relying on any of these non-standard features of economy.

There are two different ways to view human capital in endogenous growth business cycle literature. The first and more popular view is that new human capital produced only functions to improve labour productivity and does not count as part of measured output. This approach entails the existence of a nonmarket sector producing human capital investment. Examples include Perli and Sakellaris (1998) and DeJong and Ingram (2001). The second approach to modelling human capital investment is to view it as tradable goods so that output in an economy is defined by a broader concept to include human capital investment. Example is JMS. This is an empirical issue which is hard to resolve straightforwardly. Since human capital is not as tangible as consumption goods, researchers who intend to quantify it has to use some index which is very rare in literature. Due to this empirical ambiguity on human capital measurement, I adopt two different views of human capital related variables and show how persistence can emerge under different assumptions about human capital. Since the mechanisms for persistence to arise for different views of human capital are different, I split the work on business cycle persistence into two separate chapters. In this chapter, human capital is assumed to

be nontradable goods and chapter 2 is devoted to deal with the assumption that human capital is a tradable good.

An implication of the endogenous growth model is the cyclicality of human capital investment. It is shown in this chapter that having human capital being produced by a different technology changes the way that physical and human capital investments respond to shocks compared to the one-sector model in JMS who find that agents increase investment to physical capital immediately after a positive shock and raise human capital investment with a delay. In the two-sector model when addition to human capital is produced by a different technology, the timing sequence on the responses of physical and human capital investment is the opposite. Specifically, agents tend to accumulate human capital first and increase investment to physical capital with a delay. The two-sector model predicts investments to physical and human capitals are both pro-cyclical while the one-sector model predicts physical capital investment is pro-cyclical and human capital investment is counter-cyclical. Since empirical evidence on the cyclicalities of human capital investment are mixed³, none of them should be regarded as inconsistent with data.

The rest of this chapter is organized as follows. The model's environment and a characterization of its equilibrium are in section 1.2. Section 1.3 presents a careful calibration using US data. Section 1.4 shows the numerical results of the model. Section 1.5 com-

³The view that there is significant substitution between human capital enhancement activities and competing labour market activities is supported by much evidence, but no consensus has been reached so far in literature on the cyclicalities of human capital investment. Two social activities are usually regarded as the empirical counterparts for human capital investment: formal education and job training. Dellas and Sakellaris (2003) find formal education is counter-cyclical on the ground of "opportunity cost". The opposite is found by King and Sweetman (2002). They justify their findings by quoting the intuition given by Lucas and Prescott (1974): booms are times when workers have the strongest incentive to abandon their low-productivity occupations in order to gain access to high-paying occupations, since the difference in the value of these jobs is greater in booms. Sakellaris and Splimbergo (1999) also find schooling pro-cyclical because people have stronger "ability to pay" for education in booms. Other studies on human capital investment find different human capital related activities have different cyclical features. For example, the results in Einarsson and Marquis (1999) support pro-cyclicity of job training and counter-cyclicity of formal education. Overall, no consensus has been reached on the cyclicalities of human capital investment due to at least two reasons. First, different human capital investment activities may imply opposite cyclicalities. Second, even for same human capital investment activity, uses of different proxies may lead to opposite results.

pares four variants of the two-sector model with one case nesting the one-sector model in JMS. Section 1.6 discusses the robustness of the results and Section 1.7 concludes.

1.2 Model environment

1.2.1 The model

This economy is populated with an infinite number of identical agents who live forever. They maximize their expected sum of discounted utility derived from a stream of consumptions and leisure. The momentary utility is given by:

$$U(C_t, L_t) = \frac{(C_t L_t^A)^{1-\sigma} - 1}{1-\sigma}$$

C_t and L_t are consumption and leisure at period t , respectively. A measures the relative importance of leisure in improving felicity compared to consumption. σ is the coefficient of relative risk aversion. The utility function reduces to logarithmic utility function when σ equals one: $U(C_t, L_t) = \log C_t + A \log L_t$. The momentary utility function satisfies the necessary conditions required for the existence of a balanced growth path according to King et al. (1988a).

The production of the economy takes place in two separate sectors. One sector produces physical goods used for consumption and physical capital investment while the other sector produces human capital investment. For convenience, the former sector is referred as "physical sector" and the latter as "human sector". The technology in physical sector is subject to constant return to scale in terms of effective physical and human capital inputs:

$$Y_t = F(Z_t, V_t K_t, N_t H_t) = A_g Z_t (V_t K_t)^{\phi_1} (N_t H_t)^{1-\phi_1} \quad (1.1)$$

Y_t is the output of this economy that corresponds to the notion GDP; ϕ_1 is share of physical capital in the production function; A_g is the scale parameter associated to this sector; K_t is the total physical capital stock that has been accumulated by the represen-

tative agent by the beginning of period t ; V_t is the share of the physical capital stock distributed to physical sector; N_t denotes agent's labour effort devoted to physical goods production; H_t is the representative agent's stock of human capital which is predetermined at the beginning of period t ; $N_t H_t$ represents the "effective unit of labour input". The higher the stock of human capital this representative agent has accumulated by the beginning of this period, the higher the marginal productivity schedule of her labour time. The technology shock to physical sector is assumed to evolve according to a stationary autoregressive process in log form: $\log Z_{t+1} = \rho_z \log Z_t + \varepsilon_{t+1}$. The innovations ε_{t+1} is a sequence of independently and identically distributed (i.i.d.) normal random variables with mean zero and variance σ_ε^2 . It is well-known that in models that rely on technological progress to bring about long-run growth, the technology advance has to be labour augmented. This naturally establishes a connection between endogenous growth and exogenous growth models. One can easily write down an exogenous growth counterpart of this model by simply assuming human capital stock grows at a constant rate:

$$H_{t+1} = (1 + \gamma) H_t$$

γ is the exogenous net growth rate. Hence, endogenous growth model can be easily reduced down to an exogenous one by removing human capital production and specifying a constant growth rate. To construct a two-sector endogenous growth model, human capital has to be reproducible in a separate sector. Social activities in real economy that correspond to this sector incorporate formal education, job trainings and, arguably, health cares. The aim that people devote real resources to this sector is to improve workers' stock of human capital that increases productivity. The production of human capital investment also exhibits constant return to scale in terms of effective factor inputs:

$$I_{ht} = H(S_t, (1 - V_t)K_t, M_t H_t) = A_h S_t ((1 - V_t)K_t)^{\phi_2} (M_t H_t)^{1-\phi_2} \quad (1.2)$$

I_{ht} is the new human capital produced in this period; A_h is the scale parameter assigned

to human sector; $1 - V_t$ is the rest of physical capital allocated to human sector; M_t , the learning time, represents the effort that the agent devoted to forming new human capital; S_t represents the productivity shock to human sector. This functional form of human capital technology is a generalized version of the labour-only technology in Uzawa (1965) and Lucas(1988) by taking into account the contribution of physical capital in producing new human capital. In practice, human sector employs physical assets, such as buildings and equipment, as a productive factor. In fact, Perli and Sakellaris (1998) using expenditures on education and job training estimate the contribution of labour input out of the output in human sector somewhere between 83 to 89 percentage. This implies ϕ_2 , the share of physical capital in human sector, lies between 0.11 and 0.17.

The representative agent is confined by time endowment constraint for every period t :

$$N_t + M_t + L_t = 1 \quad (1.3)$$

The laws of motions of physical and human capital both follow linear transformation rules:

$$I_{kt} = K_{t+1} - (1 - \delta_k)K_t \quad (1.4)$$

$$I_{ht} = H_{t+1} - (1 - \delta_h)H_t \quad (1.5)$$

δ_k and δ_h and denote the assumed constant depreciation rates for physical and human capital respectively.

Technically speaking, the material difference between physical and human capital is that physical capital is a perfect substitute for consumption while human capital is not. Therefore, the dynamic behaviour of consumption has direct effect on physical capital investment and indirect effect on human capital investment. This distinction between the two types of investment goods resulting from the factor intensity disparity across sectors induces several new features that were absent from the one-sector endogenous growth model as in JMS. The following of this chapter attempts to explore the role played by these new features in changing the quantitative performance of endogenous

growth business cycle model.

There is neither distortion nor externality, so the competitive equilibrium of this economy coincides with the result of the social planner problem:

$$\begin{aligned} & \underset{C_t, V_t, L_t, N_t, M_t, H_{t+1}, K_{t+1}}{MAX} & E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t L_t^A)^{1-\sigma} - 1}{1-\sigma} \\ & s.t. & (1.1), (1.2), (1.3), (1.4), (1.5) \end{aligned}$$

There are two potential problematic issues associated with human capital engineered endogenous growth model like this one with raw leisure (other than "quality leisure") entering utility function. This is because the maximization problem faced by agents is nonconcave⁴. The first issue is that there maybe multiple steady states in which growth rates are different. In appendix C, I show that the uniqueness issue of steady state in this model can be reduced down to the uniqueness of a single variable (e.g. the balanced growth rate). Furthermore, numerical check on all calibrations used in this thesis shows that there is always a unique internal steady state so that leisure time on balanced growth path (BGP hereafter) is between 0 and 1.

The second issue is that the usual sufficient second order conditions guaranteeing optimality cannot apply⁵. Nevertheless, Ladron-De-Guevara et al. (1999) show in a similar endogenous growth model with leisure that stable steady states with non-complex roots correspond to optimal solutions (theorem 3.1 pp. 614 and appendix in their paper). Using the calibration in this chapter, dynamics of the state-like variable $(\frac{K}{H})$ near the unique steady state is stable with non-complex roots, so solution of the first order conditions in this thesis should correspond to a maximum.

⁴Thanks to Professor Michael Ben-Gad who points out the problem. This nonconcavity problem results from the maintained assumption that human capital stock has asymmetric effects on different uses of time: it enhances productive time but not leisure. To see this, rewrite agents' utility function as: $U = \frac{(C_t (L_t H_t)^A H_t^{-A})^{1-\sigma} - 1}{1-\sigma}$. The objective function loses the property of joint concavity because of the term H_t^{-A} .

⁵Arrow (1968) condition is not met generically and Mangassarian (1966) condition is not met at least for the particular calibrations in this thesis.

1.2.2 Equilibrium

Definition 1 *The balanced growth path equilibrium of this model is a set of contingent plans $\{C_t, K_{t+1}, H_{t+1}, V_t, L_t, N_t, M_t\}$ that solve the central planner's maximization problem for some initial endowment $\{K_0, H_0\}$ and exogenous technology processes $\{Z_t, S_t\}$, such that $\{C_t, K_{t+1}, H_{t+1}\}$ are stationary around a common trend and $\{V_t, L_t, N_t, M_t\}$ are stationary around their constant steady-state values.*

The dynamics of the model variables are summarized by a set of first order conditions:

$$U'_{1,t} = \lambda_t \quad (1.6)$$

$$U'_{2,t} = \lambda_t F'_{2,t} H_t \quad (1.7)$$

$$U'_{2,t} = \mu_t H'_{2,t} H_t \quad (1.8)$$

$$\lambda_t F'_{1,t} = \mu_t H'_{1,t} \quad (1.9)$$

$$\lambda_t = E_t \beta \left(\lambda_{t+1} V_{t+1} F'_{1,t+1} + \mu_{t+1} (1 - V_{t+1}) H'_{1,t+1} + 1 - \delta_k \right) \quad (1.10)$$

$$\mu_t = E_t \beta \left(\mu_{t+1} M_{t+1} H'_{2,t+1} + \lambda_{t+1} N_{t+1} F'_{2,t+1} + 1 - \delta_h \right) \quad (1.11)$$

and equations (1.1),(1.2),(1.3),(1.4),(1.5). $U'_{i,t}$ is marginal utility of argument i in the utility function at time t , for $i = 1, 2$. $F'_{i,t}$ and $H'_{i,t}$ are the marginal productivity of input i at time t , for $i = 1, 2$ again. λ_t and μ_t are the co-state variables to physical and human capital respectively. Define $P_t = \frac{\mu_t}{\lambda_t}$ the relative price of human capital in terms of physical capital such that it measures the relative expensiveness of human capital given the price for physical capital; r_t and W_t the marginal productivity of physical and human capital such that $r_t = F'_{1,t}$ ⁶ and $W_t = F'_{2,t}$. Re-state the first order conditions by using the

⁶By definition, r_t is the pre-depreciation rate of return to physical capital. $r_t - \delta_k$ naturally denotes the net rate of return to physical capital.

newly defined variables and apply the functional forms of preference and technologies:

$$\frac{AC_t}{L_t} = W_t H_t \quad (1.12)$$

$$\frac{1 - \phi_1}{\phi_1} \frac{V_t K_t}{N_t H_t} = \frac{1 - \phi_2}{\phi_2} \frac{(1 - V_t) K_t}{M_t H_t} \quad (1.13)$$

$$P_t = \frac{Z_t}{S_t} \frac{A_g}{A_h} \left(\frac{\phi_1}{\phi_2} \right)^{\phi_2} \left(\frac{1 - \phi_1}{1 - \phi_2} \right)^{1 - \phi_2} \frac{V_t K_t}{N_t H_t} \quad (1.14)$$

$$1 = E_t \beta \left[\left(\frac{C_t}{C_{t+1}} \right)^\sigma \left(\frac{L_{t+1}}{L_t} \right)^{A(1-\sigma)} (r_{t+1} + 1 - \delta_k) \right] \quad (1.15)$$

$$1 = E_t \beta \left[\left(\frac{P_{t+1}}{P_t} \right) \left(\frac{C_t}{C_{t+1}} \right)^\sigma \left(\frac{L_{t+1}}{L_t} \right)^{A(1-\sigma)} \left((1 - L_{t+1}) \frac{W_{t+1}}{P_{t+1}} + 1 - \delta_h \right) \right] \quad (1.16)$$

Equation (1.12) and (1.13) are the intratemporal equilibrium conditions. The former requires the marginal rate of substitution between consumption and leisure equal to the relative price of leisure. The latter is a relation that governs the relative factor intensities across sectors such that factors devoted to two productive sectors earn the same return. Equation (1.14) expresses the relative price of human capital in terms of physical capital as a function of the factor intensity in physical sector. Equation (1.15) and (1.16) are intertemporal equilibrium conditions that link marginal utility this period to expected marginal utility next period by the net rate of return to physical capital and human capital respectively. $(1 - L_{t+1}) \frac{W_{t+1}}{P_{t+1}} - \delta_h$ is the net rate of return to human capital. $\frac{W_{t+1}}{P_{t+1}}$, the marginal productivity of human capital in human sector, is multiplied by the share of non-leisure time out of total time endowment because, by construction, utility derived from leisure does not depend on human capital⁷. Therefore, $(1 - L_{t+1})$ can be interpreted as the utilization rate of human capital, which not only varies during business cycle frequency, but affects the long-run growth rate of the economy as well⁸.

⁷Some authors, such as Collard (1999), allow human capital to enter utility function directly by specifying a momentary utility function similar to $\frac{(C(LH)^A)^{1-\sigma}-1}{1-\sigma}$. Human capital is then fully utilized such that its net return is $\frac{W}{P} - \delta_h$.

⁸The steady state net growth rate expressed in terms of the rate of return to human capital is

The dynamics of the model are summarized by two complementary sets of conditions: static equilibrium conditions that govern intratemporal resources allocations (equations (1.12), (1.13) and (1.14)) and dynamic conditions that restrict investment decisions (equations (1.15) and (1.16)). Regarding the former, it is well-known that in a two-sector production model rewards to factor inputs can be derived as functions of the relative price of outputs from the two sectors such that the supplies of factors do not affect input prices. This is known as the factor price equalization theorem in international trade literature. Specifically, by equations (1.13) and (1.14), factor rewards can be derived analytically as functions of P_t :

$$r_t = S_t^{\frac{\phi_1-1}{\phi_1-\phi_2}} Z_t^{\frac{1-\phi_2}{\phi_1-\phi_2}} \Psi_r P_t^{\frac{\phi_1-1}{\phi_1-\phi_2}} \quad (1.17)$$

$$W_t = S_t^{\frac{\phi_1}{\phi_1-\phi_2}} Z_t^{\frac{-\phi_2}{\phi_1-\phi_2}} \Psi_w P_t^{\frac{\phi_1}{\phi_1-\phi_2}} \quad (1.18)$$

Ψ_r and Ψ_w are functions of A_g, A_h, ϕ_1 and ϕ_2 and are strictly positive regardless of factor intensive ranking of the two sectors⁹. A key result from this is that the sign of the derivative of r_t and W_t with respect to P_t depends only on the factor intensity ranking. Given the assumption that technology that produces new human capital is relatively more labour intensive than what produces physical capital (i.e. $\phi_1 \succ \phi_2$), $r'_t(P_t) \prec 0$ and $W'_t(P_t) \succ 0$. This shows that given an increase (decrease) in the price of human capital relative to physical capital when productivities of two sectors remain unchanged, the reward to human capital increases (decreases) while the reward to physical capital decreases (increases). This is the result of the famous *Stolper-Samuelson* theorem: in a two-sector production model, an increase in the relative price of output of one sector benefits the reward to the factor that is used more intensively in this sector. From equation (1.17) and (1.18), one can derive the relation between the percentage change of

$[(1 + (1 - L) \frac{W}{P} - \delta_h) / (1 + \rho)]^{1/\sigma} - 1$. Therefore, the economy of less leisure enjoyment grows faster ceteris paribus.

⁹Mathematically,

$$\begin{aligned} \Psi_r &= \phi_1 A_h^{\frac{\phi_1-1}{\phi_1-\phi_2}} A_g^{\frac{1-\phi_2}{\phi_1-\phi_2}} \left(\frac{\phi_2}{\phi_1}\right)^{\frac{\phi_2(\phi_1-1)}{\phi_1-\phi_2}} \left(\frac{1-\phi_2}{1-\phi_1}\right)^{\frac{(1-\phi_2)(\phi_1-1)}{\phi_1-\phi_2}} \\ \Psi_w &= (1 - \phi_1) A_h^{\frac{\phi_1}{\phi_1-\phi_2}} A_g^{\frac{-\phi_2}{\phi_1-\phi_2}} \left(\frac{\phi_2}{\phi_1}\right)^{\frac{\phi_1\phi_2}{\phi_1-\phi_2}} \left(\frac{1-\phi_2}{1-\phi_1}\right)^{\frac{\phi_1(1-\phi_2)}{\phi_1-\phi_2}} \end{aligned}$$

relative price to the changes of factor rewards:

$$\hat{P}_t = \hat{Z}_t - \hat{S}_t + (\phi_1 - \phi_2) (\hat{W}_t - \hat{r}_t) \quad (1.19)$$

Variables with " $\hat{\cdot}$ " denotes variable's percentage deviation from its corresponding steady state value. This relation shows that if human sector is more labour intensive than physical sector ($\phi_1 > \phi_2$) and shocks to both sectors are identical ($\hat{Z}_t = \hat{S}_t$), an upswing of wage tends to raise the price of human capital while an increase in interest rate decreases it. The joint effect on the relative price of human capital in terms of physical capital, therefore, depends on the magnitudes of the responsiveness of wage and interest rate to technology shocks. In a static analysis of the two-sector model, in fact, regardless of the factor intensity ranking and so long as ϕ_1 and ϕ_2 are constant over time, simultaneous changes of \hat{Z}_t and \hat{S}_t do not affect the relative price. This is because simultaneous technology shocks in two sectors have identical impact on the returns to two types of capitals. In contrast, in a dynamic general equilibrium model as the one in this chapter, symmetric shocks to two sectors induce asymmetric responses of the returns to two capitals because of the construction of the model: physical capital investment and consumption are produced in the same sector so that physical capital investment is a perfect substitute for consumption while human capital investment is produced in a separate sector with a different technology so that substitution between human capital investment and consumption is not one-for-one. The implication of this feature of the two-sector model on movement of relative price and finally the dynamics of the model is manifested by the intertemporal optimization conditions, which will be shown later in this section.

Another intratemporal condition is the agent's decision on consumption-leisure margin captured by equation (1.12). When wage increases in productive sectors due to positive technology shocks, agents tend to increase the utilization rate of human capital (reflected by a decrease in L_t). This additional availability of human capital to productive sectors will enhance the production of the sector that uses human capital more intensively via *Rybczynski* effect, which states that an increase in the endowment of one productive

factor raises the output of the sector that employs this factor more intensively. In this model, human sector is the one that employs labour input more intensively, so release of time from leisure tends to enhance the production of human capital.

The second angle to look at the model is via the intertemporal equilibrium conditions that require the rate of return to physical capital equals the rate of return to human capital plus some form of "capital gain" of human capital investment reflected by the change of relative price in two consecutive periods. To see this, one can derive a log-linearized version of the intertemporal no-arbitrage condition by combining equation (1.15) and (1.16)¹⁰:

$$0 = E_t \left[\hat{P}_{t+1} - \hat{P}_t + \frac{r}{1+r-\delta} \left(\left(\frac{W_{t+1}}{P_{t+1}} \right) - \hat{r}_{t+1} \right) \right] \quad (1.20)$$

r is the steady state value for r_{t+1} . $\frac{W_{t+1}}{P_{t+1}}$ is the pre-depreciation rate of return to human capital while r_{t+1} the pre-depreciation rate of return to physical capital. If expected rate of return to human capital reacts more (less) than expected rate of return to physical capital in face of positive technology shocks, indicating human capital investment is expected to be more (less) profitable than physical capital investment, this intertemporal on-arbitrage condition then requires a depreciation (appreciation) of the value of human capital, which will eventually equalize the expected rates of returns to two types of capitals. Hence changes in the relative price, interpreted as "capital gain" of human capital investment, are crucial in equalizing the "effective returns" to two types of capitals. Mathematically, if $\left(\frac{W_{t+1}}{P_{t+1}} \right) - \hat{r}_{t+1}$ is expected to be positive (indicating human capital investment more profitable), $\hat{P}_{t+1} - \hat{P}_t$ should be expected to be negative (reflecting a capital loss in human capital investment). This no-arbitrage condition on physical and human capital investment casts an important restriction on the adjustment process of

¹⁰To simplify this exposition, I remove the labour-leisure margin from this analysis and assume common depreciation rate for physical and human capital ($\delta_k = \delta_h = \delta$). Adding these features complicates the analysis, but does not change the fundamentals.

the relative price of human capital. In particular, this adjustment process is stable only when human sector is more labour intensive than physical sector (i.e. $\phi_1 \succ \phi_2$). To see this, one can use equation (1.19) to get rid of $\left(\frac{w_{t+1}}{p_{t+1}}\right) - \hat{r}_{t+1}$ in (1.20):

$$\hat{P}_t = \left(1 + \frac{r}{1+r-\delta} \frac{1-(\phi_1-\phi_2)}{\phi_1-\phi_2}\right) E_t \hat{P}_{t+1} \quad (1.21)$$

The coefficient $1 + \frac{r}{1+r-\delta} \frac{1-(\phi_1-\phi_2)}{\phi_1-\phi_2}$ is greater than 1 iff $\phi_1 \succ \phi_2$ given all parameters lie within reasonable ranges¹¹. Therefore, the price adjustment process is stable in the two-sector model only if human sector is more labour intensive than physical sector.

Now having both the static and dynamic properties of the two-sector model at hand, it is ready to argue that on impact of a positive economy-wide shock, the relative price of human capital increases and resources flow from physical sector to human sector. In subsequent periods, price of human capital decreases and the direction of inter-sectoral resources transferring reverses. To prove this, suppose price of human capital does not change and factors are immobile across sectors. When the economy on a BGP is suddenly hit by a positive aggregate technology shock (i.e. simultaneous increase in \hat{Z}_t and \hat{S}_t), in the impact period, consumption increases only a little because of the tendency to smooth consumption path over time. Therefore, investment to physical capital as the perfect substitute for consumption has to jump remarkably to absorb the effect of the shock. This leads to a more than proportional increase in physical capital investment relative to human capital investment. Effectively, it is impossible for agents to keep $\frac{K_{t+1}}{H_{t+1}}$ equal to $\frac{K_t}{H_t}$ after an aggregate shock in period t . As a consequence, human capital is expected to be a relatively scarce factor compared to physical capital in next period in the economy. This potentially leads to higher expected rate of return to human capital than the rate of return to physical capital, which would violate the no-arbitrage condition in equation (1.20). What actually takes place in the model is that adjustment of the relative price of human capital and inter-sectoral resources relocations behave in the direction to

¹¹This has been point out in Barro and Sala-I-Martin (1995) and Bond et al. (1996).

balance the returns to two types of capitals. First, on impact increase and subsequent decrease of the relative price form a "capital loss" of human capital investment which reduces its "effective return". Second, on impact inflow of productive factors to human sector because of the anticipated higher return tends to increase the amount of human capital available and hence narrows the spread between returns to two capitals. This pattern of resource relocation is reinforced by *Rybczynski* effect since the release of time from leisure enhances the human capital production and hinders the physical capital production. In post-impact periods, factors flow back into physical sector to restore the equilibrium allocations as the effect of the shock dies out.

The two-sector endogenous growth model provides an alternative explanation for the inverse relation between productivity and market employment observed by Gali and Hammour (1991) who write:

"Recessions have a 'cleaning-up' effect that causes less productive jobs to be closed down. This can happen either because those jobs become unprofitable, or because recessions provide an excuse for firms to close them down in the context of formal or informal worker-firms arrangements. As a consequence, the average productivity of jobs will rise." (Gali and Hammour (1991), pp. 15).

They criticize standard RBC model for its inability to match this feature of the economy. However, the inverse relation between productivity and employment is successfully replicated by the two-sector endogenous growth model. When productivity increases, workers leave job market and retool human capital. Therefore, rather than encouraging employment, higher productivity results in a lower market employment rate. Market output still increases (but only mildly) because the positive technology effect dominates the negative effect induced by the outflow of labour force. In this flexible-price two-sector model, the inverse relation between employment and productivity arises as a natural outcome of market behaviours rather than some "formal or informal worker-firms arrangements".

1.2.3 Human capital and labour adjustment cost

Cogley and Nason (1995) has shown that models with some form of adjustment cost to labour can generate persistent movement in output growth rates. After a good technology shock, the delayed response of working hours, due to the desire to spread the extra adjustment cost over time, makes output persistently high for some periods so that the growth rates are positively autocorrelated. The mechanism of generating persistence using cost of adjustment to labour is similar to habit formation in leisure as in Wen (1998). Both approaches try to achieve persistent movement in labour supply by adding some frictions either in labour market or consumer's utility over leisure. The rest of this subsection argues that human sector in the two-sector endogenous growth model has a labour adjustment cost interpretation.

Perli and Sakellaris (1998) interpret the human sector in their model as some sort adjustment cost to human investment. This interpretation also applies to the model in this chapter where new human capital is produced by a more labour intensive technology. This can be understood from the perspective of PPF¹². For example, for the one-sector endogenous growth model in JMS, the PPF for physical goods and human capital investment is linear because technologies producing them are identical. Therefore, the relative price of human capital in terms of physical capital is always equal to one and no additional cost will be incurred when resources producing one output are transferred to the sector producing the other output. In other words, physical goods and human capital are interchangeable on the one-for-one basis. But this is not the case when new human capital is produced by a more labour intensive technology. Given a certain amount of factor endowments, the PPF for physical goods and new human capital turns to be concave so that the one-for-one relation between the two goods breaks down. In fact, the opportunity cost of producing one good in terms of foregone the other good is increasing when resource relocation is in favor of the former good. This increasing cost of transformation from the production of one good to the other slows down the inter-

¹²Please refer to Mulligan and Sala-I-Martin (1993) for a detailed discussion of this.

sectoral resources redistribution after shocks. It is optimal for agent in dynamic planning horizons to transfer labour from one sector to the other slowly so that the extra cost is more evenly spread over time.

The labour adjustment cost interpretation of human sector, together with *Rybczynski* effect discussed earlier in the paper, explains the dynamics of working hours in physical sector in both on-impact and post-impact periods and hence the dynamics of the output. On impact, *Rybczynski* effect drives down working time devoted to physical sector. As the effect of the shock dies out in subsequent periods, labour flows back slowly to this sector due to the desire to spread the adjustment cost across periods. Regarding the output of physical sector, it still increases mildly on impact despite of the relatively lower factor supply. This is because the technology effect on output is only partially offset by the outflow of resources from this sector. In subsequent periods, slow inflow of factors supports the sustainable growth of output in physical sector. The key for this mechanism to work is the on-impact outflow and post-impact slow backflow of resources in physical sector. The merit of this approach to applying the idea of labour adjustment cost is via an easy-to-interpret economic activities (human capital investment) rather than some deliberately imposed labour market friction.

1.2.4 Normalization

The characterization of the equilibria of similar two-sector endogenous growth models is in Caballe and Santos (1993) and Bond et al. (1996)¹³. Due to nonstationarity of steady state, the standard log-linearization method does not apply directly. However, if those growing variables are transformed to have stationary distributions, one can linearize the model in the neighbourhood of the stationary transformation. In this chapter, two different normalization methods handling the growth component are used. To compute

¹³The difference is the inclusion of labour-leisure choice in this model. Ladron et al. (1997). show that problem of multiple steady states with different growth rates may arise when leisure is included, but for the calibrations considered in this paper, there is always a unique growth rate in steady states.

the impulse response function of output, all non-stationary variables are discounted by their common constant BGP growth rate which is independent with the initial resource endowments. Specifically, a system involving only stationary variables can be attained by reclaiming some new variables in the following way:

$$\begin{aligned} c_t &\equiv \frac{C_t}{(1+\gamma)^t} \\ k_t &\equiv \frac{K_t}{(1+\gamma)^t} \\ h_t &\equiv \frac{H_t}{(1+\gamma)^t} \end{aligned}$$

In the nonstochastic version of the transformed model, all variables will converge to and continue to stay on a particular BGP once the initial values for the physical and human capital are given. Thus there is no matter of indeterminacy of BGP once the initial resource endowment is fixed. For the stochastic version of this growth model, however, a new BGP, in general, will be triggered when a shock occurs to the economy. In other words, the model does not converge back to the previous BGP after even a temporary shock. This is a novel property of endogenous growth models of this type: temporary shocks have permanent effects. For this reason, the first normalization method that discounts variables by a deterministic trend is only valid to attain impulse response functions that capture the reactions of variables after only one shock, rather than repeated shocks. In addition, this discounting method is even more favourable once it is viewed as a detrending method that decomposes away the deterministic growth component of series and leaving only the deviations from BGP to attention.

The second normalization method is used to simulate the model and it gains more popularity in endogenous growth literatures. Specifically, in stead of discounting non-stationary variables by a deterministic trend, one can divide all variables by the current stock of human capital such that variables in ratios are constant along nonstochastic BGP. A system involving stationary variable can be obtained by dividing variables by a

stochastic denominator:

$$\begin{aligned}c_t &\equiv \frac{C_t}{H_t} \\k_t &\equiv \frac{K_t}{H_t}, \\\gamma_{ht+1} &\equiv \frac{H_{t+1}}{H_t}\end{aligned}$$

This scaling method facilitates simulations based on the stationary solution of the transformed model. For details on solving the model numerically under two different scaling methods, please refer to Appendix A to this thesis.

1.3 Calibration

The model's performance is surely a result of calibration. The paper by Gomme and Rupert (2007) detailed a way of measurement of business cycle statistics with the inclusion of home sector. The data and results in their paper are applicable to this chapter for at least two reasons. For one thing, their data and results are fully empirically-supported and easily replicable. For the second, more importantly, the model in their paper with a distinction between market production and home production has natural connection with the two-sector endogenous growth model in this chapter. The physical sector corresponds to the market sector while the human sector corresponds to the home sector in Gomme and Rupert although the analogy of the human sector to the home sector is not essential. In the paragraphs that follow, the calibration stays as close as possible to Gomme and Rupert wherever possible. Apparently, calibration of human sector is the most difficult part of this job and there is no help received from Gomme and Rupert in this regard. Fortunately, however, the work by Perli and Sakellaris and others sheds some lights on this.

The data set used to compute the statistics to which the model's predictions are compared is provided by Gomme and Rupert on the Fed website. The data set only

refers to market variables and covers the periods from the first quarter of 1954 to the first quarter of 2004. The assumption made here is that the market sector in Gomme and Rupert corresponds to the physical sector studied in the endogenous growth model in this chapter¹⁴. For detailed description of their data set, please see Appendix D to this thesis.

All parameters stated below are on quarterly basis unless stated otherwise. The physical capital share in physical sector is 0.36, a standard value in business cycle literatures. The subjective discount factor is fixed at 0.986. After the Korean War, the US GDP, aggregate consumption and investment, on average, roughly grew at a common rate 0.42% per quarter. This pins down the balanced growth rate in the model. The consumer preference is logarithmic over consumption and leisure, a standard case in business cycle literature. The depreciate rate of physical capital is set 0.20 to match the physical capital investment to output ratio around 25.3% in steady state.

There are now a few literatures regarding the value of human capital depreciation rate. Early results by Jorgenson and Fraumeni (1989) suggest the annual depreciation rate of human capital lies between 1% and 3%. JMS estimate the lower bound for annual depreciate rate is about 1.5% using just before retirement age wage data assuming human capital investment is nearly zero towards the end of working life. They pick up an intermediate value at 2.5% yearly which corresponds to about 0.625% quarterly. DeJong and Ingram (2001) estimate it to be 0.5% per quarter. In the baseline case, the depreciation rate of human capital is 0.5%.

There is no absolute consensus regarding time allocations along the nonstochastic BGP. Labour supply accounts for about one third of total time endowment in traditional RBC models prior to the popularity of models with home production. There are exceptions. For instance, JMS calibrate labour supply attributes to only 17% of discretionary time in steady state which is well below the usual value in literatures. The introduction of home sector in RBC models necessitates the reconsideration of the issue of time allo-

¹⁴This assumption is not unique in literature. See Perli and Sakellaris (1998) for one example.

cation. According to Gomme and Rupert who reference the American Time Use Survey (2003), time spent on market working is 0.255 and time devoted to homework is 0.24. The strategy used in this paper is to choose a leisure time that is close to Gomme and Rupert (0.505) to give rise to about 30% working time in physical sector given all steady state constraints satisfied. It turns out that the leisure time along nonstochastic BGP is 0.54 and working in physical sector is 30% and time devoted to learning is 16%. Therefore, agent spends about two thirds of her non-leisure time on working and one third on either schooling or job training.

The most difficult part of calibrating an endogenous growth model with human capital production taking place in a separate sector is to estimate the input shares in human sector. This is because the output of this sector is hard to measure. Works by Perli and Sakellaris (1998) shed some light on this issue. They assume human sector in theory has its counterparts in real economy two social activities: education and job training. Regarding the former, they use data from Jorgenson and Fraumeni (1989) and calculate the contribution of physical capital to educational output at 8% implying labour's share is 92%. For the later, they simply assume the technology for job training is identical to physical goods production. Then they collect data on the relative importance of education and job training in new human capital formation and arrive at a weighted average of the share of physical capital in human capital between 11% and 17%. As is mentioned before, it seems a reasonable assumption that all human capital related social activities are labour intensive than physical goods production. Therefore, Perli and Sakellaris' results tend to over-estimate the contribution of physical capital in human sector, so the baseline value of this parameter in this chapter is set 0.11, the lower bound suggested by Perli and Sakellaris.

Constructing the technology shock to physical sector calls for calculating Solow residuals by regressing the output in this sector on all factors used in production. To the best of the author's knowledge, no previous work has estimated a production function that includes human capital stock. This is because reliable index of human capital stock is very

rare in literature. Hence, the easiest way to proceed seems to be simply omitting human capital stock in the regression to obtain Solow residuals. This is a reasonable approximation for two reasons. First, Gomme and Rupert estimate three different specifications of the technology process to obtain parameters that governs the technology shock: first, they adopt disaggregated capital stocks in addition to working hours as explanatory variables for output; second, they replace the disaggregated capital stocks by aggregated capital stock; and finally, they estimate the regression model without physical capital at all. What they find is that the last case in which physical capital stock is excluded from the regression thoroughly produces quite similar results to the first two cases. Prescott (1986) justifies this method on the basis that capital stock fluctuates much less than aggregate working hours over the business cycle because investment every period, as a flow concept, constitutes only a small fraction of capital stock. Human capital stock is also a very stable variable over the business cycle. It only depreciates with age unless some extremely unlikely accident happens, such as amnesia, paralysis. Therefore, once a worker is equipped with some knowledge or skill, her human capital stock increases permanently, from which she will benefit for a long time in her life. Second, mismeasurement of physical capital stock is a recurring theme in RBC literature. For human capital data, this problem becomes even worse. Estimating human capital stock by the wage profile over worker's life cycle will probably lead to greater measurement errors relative to physical capital stock. Therefore, there is little loss of generality to assume that fluctuations of output are largely explained by variations of aggregate working hours. The technology process can be estimated without worrying about the fairly smooth capital stocks. The resulting autocorrelation coefficient of $\log Z_t$ recovered from Solow residuals is about 0.95 and the variance of innovation $\log Z_t$ is about 0.0007, a result very close to that in Perli and Sakellaris. The scale parameter associated to physical sector (A_g) is normalized to one and A_h is implied by the BGP constraints. In the baseline parameterization, A_h is 0.0461.

In the baseline calibration, the technology shock to human sector is assumed identical

Free parameters		
β	Subjective discount factor	0.986
γ	BGP growth rate	0.0042
σ	Coefficient of relative risk aversion	1
L	Steady state leisure time	0.54
ϕ_1	Share of physical capital in physical sector	0.36
ϕ_2	Share of physical capital in human sector	0.11
δ_k	Depreciation rate of physical capital	0.02
δ_h	Depreciation rate of human capital	0.005
A_g	Scale parameter for physical sector	1
$\rho_z = \rho_s$	Persistence parameter of shock	0.95
$\sigma_z^2 = \sigma_s^2$	Variance of innovation	0.0007
Implied by BGP		
r	Steady state interest rate	0.0185
A_h	Scale parameter of human sector	0.0461
N	Steady state working time	0.3
M	Steady state learning time	0.16
A	Weight of leisure in preference	1.55
$\frac{C}{Y}$	Steady state consumption-output ratio	0.75
$\frac{I_k}{Y}$	Steady state physical investment-output ratio	0.25
V	Steady state share of physical capital in physical sector	0.89

Table 1.1: Calibration of the two-sector SEG model

to the shock to physical sector¹⁵. This is also the assumption implicitly made in the one-sector model in JMS. This seems to be a reasonable and efficient assumption to make, especially when any other effort trying to estimate the shock process only leads to large bias. A technology progress or shocks in any other form happening to physical sector are very likely to influence human sector in a similar way. One can think of the invention of internet that improves the productivity of education sector just as it does to the rest of the economy. Of course, economy-wide shock is a too extreme assumption to make. In the section of robustness check, this assumption will be relaxed to allow for sector-specific shocks. But as a benchmark parameterization, the assumption of identical shocks is kept.

The calibration of the model is reviewed in table 1.1:

¹⁵The word "identical" applies not only to the specification of shock process, but more importantly to the realizations of shocks every period. In other words, the shock processes are calibrated in a way that there is only one shock affecting the productivities in two sectors simultaneously.

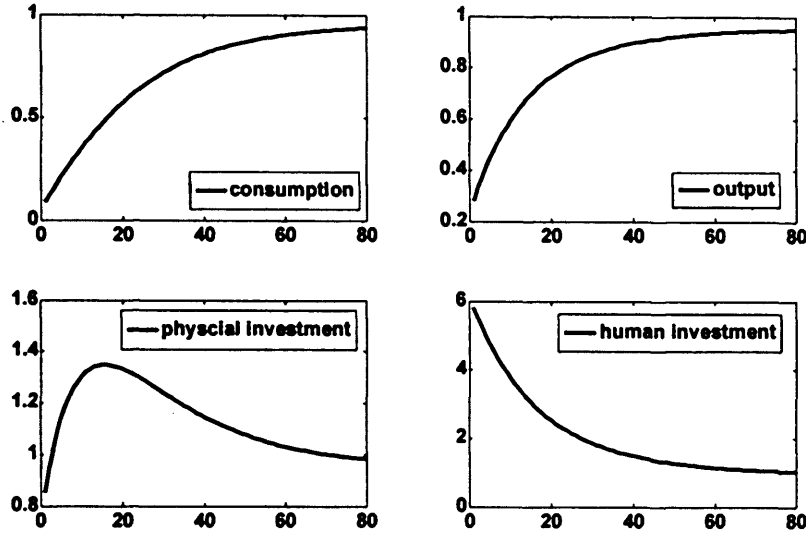


Figure 1-1: Impulse response functions to technology shock

1.4 Numerical results

1.4.1 Impulse response functions

Cogley and Nason (1995) argue that US output series appears to have a trend reverting component identifiable from a hump-shaped impulse response function. The hump, one indicator for a strong interior propagation mechanism, means that the reaction of output to a good technology shock is small on impact and continues to increase for at least a few subsequent periods. The endogenous growth model is successful in replicating this property of US time series.

Figure 1-1 shows the impulse response functions of main variables of interests to an aggregate technology shock¹⁶. An important feature of the class of endogenous growth models in this paper is that transitory shocks exert permanent impact on steady state

¹⁶All impulse responses functions presented in this chapter are reactions of variables to simultaneous increase of the productivities to physical and human sectors.

equilibrium. Specifically, a transitory technology shock, in general, will trigger a new BGP on which the growth rates are same, but the levels are different. Consequently, there is nothing to force non-stationary variables in the endogenous growth model to converge back to the previous BGP after a transitory shock. This makes it possible for output and physical investment continue to grow for some periods when the shock itself is on its downturn. It is not surprising that consumption goes up smoothly to the new BGP due to the intertemporal substitution effect required by agent's preference. Output and investment to physical capital in the endogenous growth model also exhibit smooth trajectories with small response at the begging and non-weakening reactions for at least some periods afterwards. The small reaction of output on impact is the joint effect of the price effect (which is enhanced by the *Rybczynski* effect) and the direct technology effect. Due to the former, outflow of productive factors employed by physical sector tends to reduce the output of this sector. Due to the latter, higher productivity in this sector increases output directly. The two effects finally lead to only a small increase in output on impact as part of technology effect is offset by the outflow of factors. In subsequent periods, the influence of the backflow of factors starts to dominates the decrease in technology, which sustains the long-lasting expansion in output even if technology is on the downturn. In addition, the backflow of factors to physical sector is not accomplished instantaneously. In stead, it happens slowly due to the labour adjustment cost effect. Response of physical capital investment emerges as a natural result of the hump in the output response since it is simply the difference between output and consumption. These responses are clear signs for the existence of a propagation mechanism that tends to spread the effect of a shock over its life. Consequently, the responses of variables do not need to resemble the process of the shock that induces aggregate fluctuation.

Figure 1-2 displays the responses of some stationary variables after a positive aggregate shock. Each variable in this group converge to its unique steady state value after transitory shocks. Leisure decreases on impact due to higher productivity in the productive use of time. Working hours decrease and learning time increases for the two

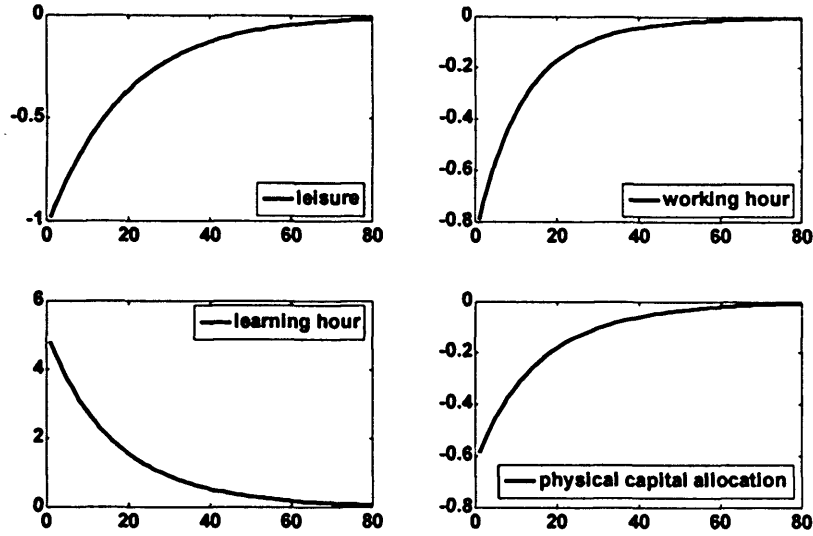


Figure 1-2: Impulse response functions to technology shock

reasons explained before. For one thing, at the early life of the shock, human capital investment is more profitable than physical capital investment due to the relative scarcity of human capital stock available. As a result, agents transfer labour from physical sector to human sector. For another, due to *Rybczynski* effect, the release of time from leisure enhances human capital production and suppresses final goods production, which reinforces labour transferring from physical sector to human sector. The decline of working hours on impact is consistent with the empirical finding by Gali (1999) who identifies a negative correlation between productivity and working hours Using VAR. He suggests that the reason for only small increase in output on impact of a positive productivity shock is the curtailment in working hours. But he regards this as evidence against neo-classical business cycle model, which predicts higher labour supply on impact of positive shocks. He further concludes that this empirical evidence is in favour of sticky price models. However, the decline in working hours on impact of a good technology shock arises as a natural result of optimal substitution between market and nonmarket work.

In endogenous growth model, an agent substitutes between market work and education. Empirical works by Dellas and Sakellaris (2003) confirm the existence of the substitution between the two activities in the data. Therefore, the model in this chapter provides a theoretical explanation, which hinges on the empirically valid substitution between education and labour market activity, for the apparent inconsistency between Gali's result and RBC framework. The message is that the observed decline in working hours in face of higher labour productivity should not be interpreted as evidence against RBC models. In contrast, decline in working hours in face of higher productivity comes as a natural outcome of agents' substitution between market work and human capital accumulation.

Inter-sectoral physical capital relocation, in general, takes place to the same direction as labour relocation¹⁷. This is because of the complementarity of the two factors in production. To see this, one can look at (1.13) that restricts the factor intensities across sectors:

$$\frac{1 - \phi_1}{\phi_1} \frac{V_t K_t}{N_t H_t} = \frac{1 - \phi_2}{\phi_2} \frac{(1 - V_t) K_t}{M_t H_t}$$

Since agents have desires to smooth leisure enjoyment over time, working hours N_t , and learning time M_t , in general, move in opposite ways after shocks. Suppose N_t goes up for some reason. V_t , the share of physical capital in physical sector, has to go up to satisfy this inter-sectoral optimality condition. Thus, physical capital and human capital switch to the same direction between sectors. This is seen from the impulse response functions in the right-hand-side half of figure 1-2.

It is already well-known that adding cost of adjustment to physical capital investment does not help generate the desired dynamics in output because physical capital investment each period only accounts for a small fraction of physical capital stock that actually enters the production function. Changing the dynamics of physical capital investment has only small impact on physical capital stock and hence output dynamics. The solution for output dynamics puzzle then relies on altering the responses of physical capital

¹⁷This statement is precisely true if labour-leisure choice is removed from the model. There would be a one-to-one positive relation between N_t and V_t .

stock itself or labour input, factors directly entering the production function. Successful efforts regarding the former include variable factor utilization rates as in Burnside and Eichenbaum (1996); those regarding the latter include adjustment cost in labour supply in Cogley and Nason (1995), habit formation in leisure in Wen (1998) and sectoral adjustment cost in Perli and Sakellaris (1998). All works belonging to the second group try to break down the post-impact period relationship between labour input and consumption in traditional RBC models. Recall the intratemporal optimality condition expressed by equation (1.12) that requires the marginal rate of substitution between consumption and leisure equal to the real wage:

$$\frac{AC_t}{L_t} = W_t H_t$$

After the impact period of a good shock, consumption continues to go up due to the intertemporal substitution effect while the marginal productivity of labour decreases as the shock dies out. The co-movement of consumption and labour productivity forces leisure to go up. If working is the only substitution for leisure, post-impact increase in leisure will suppress labour supply to fall, which drives output to fall immediately after the impact period unless capital stock responses overwhelmingly. Output falls, so does investment given consumption continue to increase for some time. The presence of intratemporal condition explains the lack of persistence output and investment in traditional RBC models. But this is not the case in this two-sector model. Although the intratemporal optimal condition still holds to force consumption and leisure to move in the same direction in post-impact periods, working hour does not necessarily fall due to the reduction of learning time. In fact, in subsequent periods after the occurrence of a good shock, release of time from learning makes working hour and leisure both increase as shown in the upper part of figure 1-2. This is impossible if working time is the only substitute of leisure as in the one-sector standard RBC models. Therefore, the presence of an additional nonmarket usage of time becomes the premise for leisure and working hours to increase simultaneously in post-impact periods given the intratemporal condition is still binding. More importantly, the labour adjustment cost effect resulting

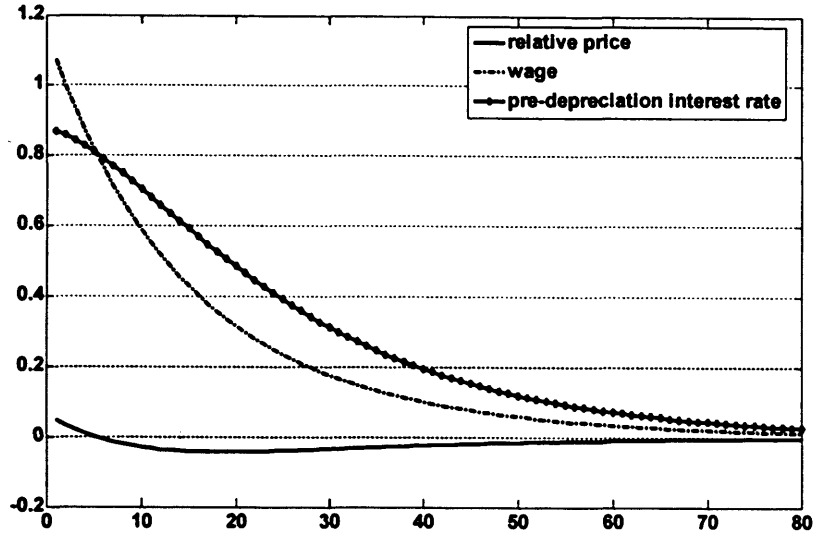


Figure 1-3: Impulse response functions to technology shock: Stolper-Samuelson effect

from the factor intensity disparity across sectors slows down the backflow of resources to physical sector and generates persistent movement in output of this sector.

Figure 1-3 shows the responses of relative price of outputs in two sectors and the prices of factors. This picture visualizes the *Stolper-Samuelson* effect as captured by equation (1.19). The relative price of human capital is higher (lower) than its steady state value when \hat{W}_t lies above (under) \hat{r}_t . Since human capital production employs more labour input relatively to physical capital production, increase in the reward to labour input raises the price of human capital while increase in the reward to physical capital reduces the the price of human capital. Positive technology shocks simultaneously increase the rewards to both inputs, so their joint effect on price of human capital depends on their relative magnitude of responses. As shown in figure 1-3, for a few subsequent periods right after the shock, \hat{W}_t reacts more than \hat{r}_t , so price of human capital lies above its steady state value. But as the effect of shock diminishes, \hat{W}_t decreases faster than \hat{r}_t , which forces the price of human capital "over-shoots" its steady state value before eventually

		$\sigma(x_t)$	$\rho(x_t, x_{t-j})$			$\rho(\gamma_{Y_t}, x_{t+j})$					
			$j =$	1	2	3	-2	-1	0	1	2
γ_{Y_t}	data	1.14		0.29	0.16	0.03	0.16	0.29	1	0.29	0.16
	model	0.82		0.29	0.27	0.25	0.27	0.29	1	0.29	0.27
γ_{C_t}	data	0.52		0.24	0.14	0.19	0.20	0.37	0.49	0.27	0.16
	model	0.43		0.78	0.75	0.73	0.35	0.38	0.83	0.50	0.49
$\gamma_{I_{kt}}$	data	2.38		0.38	0.24	0.11	0.17	0.39	0.75	0.41	0.24
	model	2.23		0.14	0.11	0.10	0.19	0.21	0.96	0.15	0.11
N_t	data	5.52		0.99	0.96	0.93	-0.22	-0.18	-0.07	0.01	0.07
	model	5.54		0.92	0.85	0.73	-0.37	-0.40	-0.73	-0.67	-0.62

Table 1.2: Business cycle statistics for baseline calibration

converging back.

The responses of relative price and time allocation variables also reflect the sequence of investments to physical and human capitals in time. For some periods after a good technology shock, human capital investment increases more than investment to physical capital because of the higher rate of return to the former. And the relatively more investment in human capital drives down the relative price which is interpreted as "capital loss". This finally equates the rate of return to physical capital to the rate of return to human capital plus the "capital loss" such that the no-arbitrage condition in equation (1.20) holds. After a certain point in time, physical capital investment starts to catch up and the equilibrium ratio of the two types of capitals is restored eventually when economy arrives at a new BGP. The sequence of responses of two investments in time shows that human capital is accumulated before physical capital after good shocks.

1.4.2 Persistence and volatility

Table 1.2 ¹⁸ reports the statistics computed from US data and a simulated sample of 30,000 periods. The data set to calculate moment statistics is the same as what is used for calibration. Due to the endogenous growth component of the model, the nonstochastic steady state of the model economy is growing at an endogenously determined rate. The nonstationarity of steady state makes it a bit awkward to compute volatility statistics in the level of variables as usually done in RBC literatures. An alternative measure of economic fluctuation is to calculate moment statistics of growth rates of variables, which by construction have stationary distributions along BGP. This is also the approach proceeded in JMS. There are two reasons to work with growth rates or ratios of variables. First, many authors, such as Nelson and Plosser (1982) and Campbell and Mankiw (1987), find that real macroeconomic series are likely to follow integrated processes with order one rather than trend-stationary process, a usual way to model macro variables in exogenous growth RBC models. Second, working with the first difference of variables avoids the use of trend-removing filters, such as H-P filter.

US data suggest consumption growth rate fluctuates about a half of the fluctuation of output growth rates and investment growth rates fluctuates a bit more than twice as much as output growth rate. The third column of Table 1.2 shows that the endogenous growth model fits the volatilities of US data quite well although for some variables the model slightly underpredicts their variabilities. Specifically, the model implied volatility of output growth is 0.82 while its counterpart in the data is 1.14. The model implied volatility of consumption growth is 0.43, also smaller than 0.52 in the data. Volatility of investment growth in the data is 2.38 when the model only predicts 2.23. It is well-

¹⁸ γ_x is the growth rate of variable x while N is level of working hour in physical sector. $\sigma(x)$ measures variable's percentage deviation from the mean; $\rho(x, y)$ is the correlation coefficient of variables x and y ; The model predicts $\gamma_Y, \gamma_C, \gamma_{I_k}$ and N to have stationary distributions along BGP. Therefore, US aggregate data on Y, C, I_k are logged and first-differenced and data on working hours is in level. Unit root tests on the data suggest that the logged and first differenced series of output, consumption and physical investment are stationary, but not the level of per-capita working hours. The variability of per-capita working hours is then measured by: $\frac{\sigma(N)}{E(N)}$.

known that traditional RBC models generate too small working hour volatility compared to data. A number of works to rectify this inconsistency of RBC theory with real data has been suggested in literatures. These include indivisible labour as in Hansen (1985) and additional time allocation margin in home production literatures, such as Greenwood and Hercowitz (1991) and Benhabib et al. (1991). The endogenous growth model solves the labour volatility puzzle using the same idea as in the home production literature by adding a nonmarket time: learning. This availability of an additional margin for time use levels up the volatility of market working hours to the observed magnitude in US data. As one can see in table 1.2, the model implied volatility of hours matches its empirical counterpart extremely well (0.54 compared to 0.52).

The fifth to seventh columns of table 1.2 show the autocorrelation coefficients of the growth rates of output, consumption and physical capital investment and level of working hours. The model replicates the autocorrelation properties of output growth data strikingly well in the first order autocorrelation coefficient. This indicates that the model produces exactly the same degree of persistence observed in the data in terms of the first order autocorrelation. However, higher order autocorrelation coefficients in the data seem fall to zero quickly when those generated from the model decrease very slowly. In the data, the coefficient falls from 0.29 to 0.03 from the first order coefficient to the third order one when the model predicts 0.29 for the first one, but 0.25 for the third one, a too high value. This suggests that the endogenous growth model generates too much persistence rather than too little as in standard RBC models. For consumption, the model also exhibits too much persistence. A very interesting result is the persistence of physical capital investment in table 1.2. The autocorrelation coefficient of investment growth is usually not reported in business cycle research. But in the data, growth rate of investment is autocorrelated at even higher degree than those of output and consumption (0.38 compared to 0.29 and 0.24). Conventional RBC models fail to reproduce the persistence of investment growth for the same reason of output growth. Recall the responses of output and investment in traditional RBC models, they both fall immediately after the

impact period, which leads to the lack of persistence in both variables. This might explain why no attention has been given to this coefficient in literature. Like the way to bring about output growth persistence, a hump-shaped impulse response function of investment may be necessary. This is also evident from the work by Peril and Sakellaris (1998) who also generate hump-shaped impulse response for physical capital investment. Due to the slowed-down process of labour input relocation across sectors, investment reacts to a technology shock in a delayed fashion so that the response curve exhibits a hump. Physical capital investment in this model responds to technology shocks also with a delay for the reason discussed earlier in previous section. Following a positive shock, agents accumulate human capital quickly by delaying the accumulation of physical capital. This postponed reaction of physical capital investment explains the hump in the response curve and the positive autocorrelation coefficient in simulated output growth series. However, comparing the model-generated statistics to what is seen in the data, the model produces less investment growth persistence. For the working hours, the model generates a little less persistence than the data.

With regards to the contemporaneous correlations between output growth and other variables and the lead-and-lag pattern, in general, the model fits the data well. Consumption and investment growths are pro-cyclical both in the model and data. The part where the model's prediction is at odd with the data is the cyclical of working hour. In the data, labour supply is only slightly negatively correlation with output growth (-0.07), but the model predicts labour supply to be strongly counter-cyclical (-0.73). In addition, output growth is mildly and positively correlated with next period labour supply in the data (0.01) while the model predicts a negative value for the same correlation (-0.67).

In a nutshell, as a preliminary attempt in generating business cycle persistence relying on human capital formation, the results from the model are very promising. First, the model generates much persistence in output and investment despite of the unsatisfactoriness of too much persistence in output and consumption and less persistence in investment. Second, the model replicates quite well the pattern of relative volatilities of

main macro variables, especially in market working hours.

1.5 Comparison with Jones et al. (2005b)

1.5.1 Timing of responses

The major extension of JMS in this chapter is to allow for a more labour intensive technology to produce human capital investment. The role of factor intensity disparity across sectors in changing the dynamics of the model variables has been discussed in section 1.2.2. This section of the paper is aimed to emphasize the role again from another angle: the sequence of responses of physical and human capital investments in time.

As pointed by JMS, their model is essentially a traditional one-sector RBC model if human capital depreciates at the same rate as physical capital. The difference in depreciate rates is the only asymmetry introduced in treating physical and human capital. Therefore, all improvements they achieve in their paper over one-sector RBC models are attributed to this asymmetric treatment of the two types of capitals. However, the assumption in their paper that the two types of capitals are produced by the same technology seems too extreme and empirically implausible. To have a clearer insight into the implications of relaxing this extreme assumption, four comparable cases are discussed in this section. In the first case, physical and human capitals are treated with complete symmetry ($\phi_1 = \phi_2, \delta_k = \delta_h$). They are produced by an identical technology and depreciate at the same rate. This case corresponds to a standard one-sector RBC model. The slight difference is the higher total capital share (sum of the shares of physical plus human capitals) in the production function. In the second case, human capital is assumed to be produced by the same technology producing physical capital, but depreciates slower than physical capital ($\phi_1 = \phi_2, \delta_k > \delta_h$). The two-sector endogenous growth model parameterized in this case is essentially the same as the one-sector model in JMS. In the third case, human capital is assumed to depreciate at the same rate as physical capital. The production of human capital, however, requires a higher share

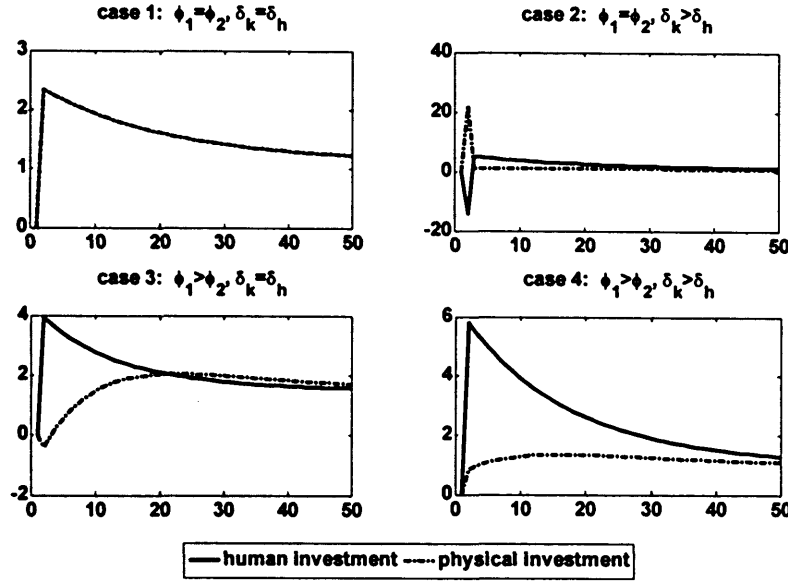


Figure 1-4: Comparing impulse response functions to technology shock: physical and human capital investments

associated with labour input than production of physical capital ($\phi_1 > \phi_2, \delta_k = \delta_h$). The final case refers to the model in this paper with two asymmetric treatments of physical and human capitals built in ($\phi_1 > \phi_2, \delta_k > \delta_h$). Human capital production function is more labour intensive and human capital depreciates slower. The final case is thought to have the most empirical relevance.

The know-how to understand the dynamics of models in all cases is via the timing sequence of the responses of physical and human capital investments to technology shocks. The impulse response functions in the upper-left quadrant of figure 1-4 correspond to case 1 where physical and human capitals are treated completely symmetric. One can see that investments to physical and human capitals react identically to a positive technology shock. Agents make no distinction between the two types of capital and always adjust them by the same amount at any point in time. The upper-right quadrant depicts the responses of physical and human investment in case 2 where the assumption of the same

depreciate rate is relaxed. When human capital depreciates much slower than physical capital, investment to physical capital jumps and human capital investment falls on impact. From the second period after the shock, the responses of investments to physical and human capitals reverse and then start to converge to the new BGP. This implies that following a positive technology shock, agents first accumulate physical capital stock and form human capital with a delay. The reason for this pattern of responses is explained in JMS. A shock has a larger impact on the net rate of return to physical capital than it does on the net rate of return to human capital because physical capital depreciates faster than human capital. A positive shock induces relative higher increase in the net rate of return to physical capital than to the net rate of return to human capital. Therefore, having relatively more investment in physical capital than human capital following a good shock will equalize the rate of return to capitals so that the no-arbitrage condition holds. It is worth noting that the relative price of human capital in terms of physical capital is always equal to one if technologies producing them are identical. Consequently, there is no room for price to adjust to balance the two returns. The only channel to equalize the returns to two capitals is through adjusting the quantity of investments, so this leads to fluctuations of capital investments on a much larger scale than the case where "capital gain" has a role to play. The lower-left quadrant of figure 1-4 shows the responses of investment decisions in the third case. The timing sequence in investments is apparently reversed. In this case, after a positive shock, agents raise human capital investment before accumulating physical capital. The reason for the reversion in the timing order is the price adjustment process and *Rybczynski* effect as explained in section 1.2.2. Holding depreciation rates of capitals identical, if the technology producing human capital is more labour intensive than that producing physical capital, a positive shock will induce a larger impact on the net rate of return to human capital than on that to physical capital. Thus, agents tend to form human capital first and accumulate physical capital with a delay. One thing that is worth noting is that there exists a hump in the response of physical capital investment when human sector is less intensive in physical capital input. This pattern of

impulse response functions indicates slow adjustment in the quantities of physical capital investment. On the contrary, this adjustment happens all of a sudden in case 1 and 2 where the factor intensities are the same across physical and human sectors. The last case is regarded most empirically relevant and corresponds to the model advocated in this paper. The results in the lower-right corner of figure 1-4 can be viewed as the joint effects of two asymmetries of treating physical and human capitals: asymmetric depreciation rates in case 2 and asymmetric factor intensities in case 3. In effect, investment to both types of capital increase on impact of a good shock although investment to human capital increases relatively more than physical capital investment does. The interpretation for this is just the opposite of JMS. After new machines are invented, firms first train workers with the skills to manage those new machines. Once workers are equipped with adequate knowledge and skills to handle new machines, firms start to renew physical stock. This change in the timing sequence of investments significantly improves model's prediction on the persistence of business cycles and some moment statistics, which will be seen in next subsection.

Figure 1-5 shows the responses of working hours, learning hours while figure 1-6 displays the responses of consumption and output following a positive technology shock in four cases considered above. There are at least three things one can learn from these pictures. First, except the first case, working hour and learning time move to different directions following a good shock. This is consistence with the empirical finding by Dellas and Sakellaris (2003) that there is significant substitution between education and competing labour activities over business cycles. Second, in all four cases, the responses of consumption are fairly smooth due to the intertemporal substitution effect. However, only in the last two cases, the trajectories for output are smooth. Recall that in last two cases, the technologies producing human capital are different from that producing physical capital. The smoothness in the responses of output in last two scenarios confirms the role of cross-sector factor intensity disparity in generating output persistence. Third, in all cases, output and working hours react quite alike. This is because physical capital

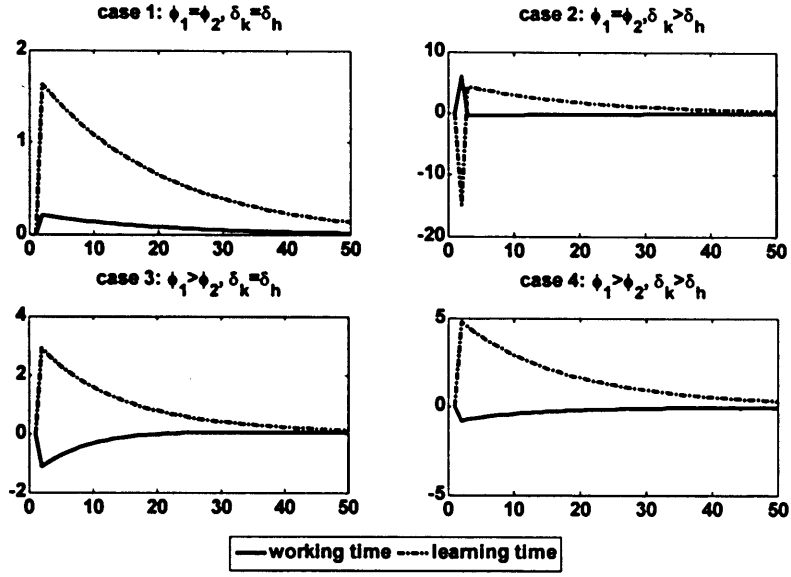


Figure 1-5: Comparing impulse response functions: working hours and learning time

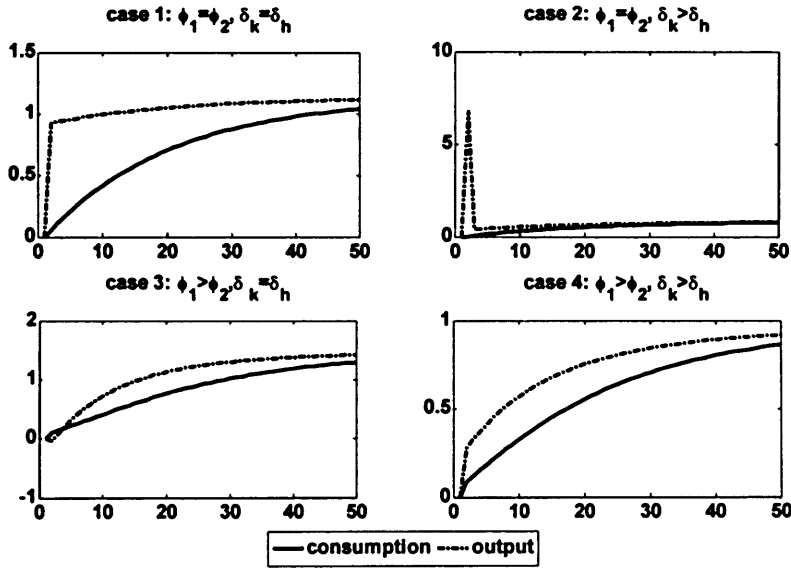


Figure 1-6: Comparing impulse response functions: consumption and output

input and labour input in either sector move to the same direction following shocks. For this reason, it should not be surprising to observe the resemblance between the dynamics of output and working hours. This resemblance also exists in one-sector RBC models because the capital input into production function as a stock variable does not vary much over business cycles. A natural implication of this resemblance is that to generate the desired dynamics of output, some twist in the labour market is necessary. As is mentioned before in this paper, the idea in this paper is to divide non-leisure time into working and learning hours. When shocks occur to two sectors at the same time, agents release some working time into learning activities. Since the effect of shocks starts to die out from the second period, working hours gradually increases to restore the equilibrium value. This ‘V’ shape response of working time, in contrast to the ‘^’ shape response in standard one-sector models, gives rise to the hump in the impulse response curve of output. The diagram in the lower-right quadrant visualizes the reasoning.

1.5.2 Persistence and volatility of some variants

Table 1.3 reports the moments statistics computed for four cases. Overall, case 4 mimics the empirical data better than any other case. When physical and human capitals are treated with absolute symmetry, the model performs very much like a traditional one-sector RBC model. The autocorrelation coefficients for output and investment growth are both very close to zero regardless of the sign. There seems no persistence at all for these two variables in the model, a well-known failure of traditional RBC models. Another major problem of the model in case 1 is the too low working hour volatility, also a well-known drawback of early RBC models. For case 2, consumption appears too smooth relative to US data. More importantly, growth rates of investment and output are negatively autocorrelated with significance. The inconsistency between model generated statistics with those from data indicate that asymmetric depreciation rates of capitals cannot generate persistence and reasonable moment statistics. Case 3 with different factor intensities across sectors appears successful in replicating moment statistics, but

		case 1	case 2	case 3	case 4
	data	$\phi_1 = \phi_2$ $\delta_k = \delta_h$	$\phi_1 = \phi_2$ $\delta_k \succ \delta_h$	$\phi_1 \succ \phi_2$ $\delta_k = \delta_h$	$\phi_1 \succ \phi_2$ $\delta_k \succ \delta_h$
$\sigma(\gamma_Y)$	1.14	2.51	25.09	0.86	0.82
$\sigma(\gamma_C)$	0.52	0.48	0.37	0.61	0.43
$\sigma(\gamma_{I_k})$	2.38	6.38	82.93	2.25	2.23
$\sigma(N)$	5.52	1.57	16.71	6.01	5.44
$\rho(\gamma_{Y_t}, \gamma_{Y_{t-1}})$	0.29	0.01	-0.50	0.86	0.29
$\rho(\gamma_{C_t}, \gamma_{C_{t-1}})$	0.24	0.95	0.95	0.81	0.78
$\rho(\gamma_{I_{kt}}, \gamma_{I_{kt-1}})$	0.38	-0.02	-0.49	0.48	0.14
$\rho(\gamma_{N_t}, \gamma_{N_{t-1}})$	0.99	0.95	-0.02	0.86	0.92
$\rho(\gamma_{Y_t}, \gamma_{C_t})$	0.49	0.35	0.02	0.68	0.83
$\rho(\gamma_{Y_t}, \gamma_{I_{kt}})$	0.75	0.99	1.00	0.87	0.96
$\rho(\gamma_{Y_t}, \gamma_{N_t})$	-0.07	0.35	0.75	-0.73	-0.73

Table 1.3: Comparing business cycle statistics for the variants

generates too much persistence. For example, output and consumption growth in the model are autocorrelated with coefficient 0.86 and 0.81 respectively while in the data the counterparts are only 0.29 and 0.24. In case 4, human capital is assumed to depreciate at a slower rate 0.005 per quarter. The results show that lowering human capital depreciation rate reduces the degree of persistence to a relatively reasonable level that is close to US observations.

1.5.3 Discussion

Case 2 discussed previously corresponds to the model used in JMS where the only asymmetry in dealing with physical and human capitals is different depreciation rates. However, the quantitative performance of case 2 is very different from that of the model in JMS. The incohesion between the results of case 2 and JMS is attributed to two implicit differences between them. The first one is the frequency on which the model is cali-

brated¹⁹. JMS use yearly frequency while case 2 adopts quarterly frequency. Recall that asymmetric treatment of capitals induces asymmetric responses of investments to capitals in time because shocks have asymmetric impacts on the rates of returns to capitals. Without the role played by relative price, investments to two types of capitals have to adjust substantially to equalize rental rates of them. Annual investment-to-capital ratio, in general, is four times as large as its quarterly counterpart. Therefore, yearly investment accounts for a much bigger fraction of capital stock than measured on quarterly basis. Thus fluctuations measured by annual data is much less than measured by quarterly data. This explains why JMS produce reasonable volatility statistics, but case 2 in this paper does not. The second difference is the definitions of "output" and "consumption". Specifically, JMS define "consumption" to include investment to human capital: $C + I_h$, and "output" the sum of consumption, investment to physical capital and investment to human capital: $C + I_k + I_h$, using the notations in the chapter. Since the model predicts that shocks have opposite effect on investments to two capitals, the variabilities of "composite consumption" and "composite output" are substantially reduced in JMS. But when variables are re-defined in the usual way by taking human capital investment as nontradable goods in case 2, the model in JMS generates unrealistic business cycle statistics. Not to mention the empirical validity JMS' definitions, by construction, their model implies consumption-output ratio fluctuates exactly as much as investment-output ratio, which is apparently inconsistent with data.

1.6 Sensitivity analysis

This section presents tests on the robustness of the results obtained previously regarding business cycle persistence and cyclical moments to alternative specifications of exogenous forces and parameters that are not strongly evidence-supported.

¹⁹This issue is also pointed out in a note by Maury and Tripier (2003) who find an earlier version of the model in JMS on quarterly basis does not perform as well as it does on yearly frequency.

	Persistence of x					Volatility of x			
	$x =$	γ_Y	γ_C	γ_{I_k}	γ_N	γ_Y	γ_C	γ_{I_k}	N
US data		0.29	0.24	0.39	0.99	1.14	0.52	2.38	5.52
Baseline ($\rho_{zs} = 1$)		0.29	0.77	0.14	0.92	0.82	0.43	2.36	5.44
$\rho_{zs} = 0.7$		-0.04	0.24	-0.06	0.87	5.33	1.11	18.14	11.62
$\rho_{zs} = 0.9$		-0.02	0.35	-0.05	0.89	3.16	0.73	10.67	8.09
$\rho_{zs} = 0.95$		0.03	0.49	-0.04	0.90	2.29	0.60	7.66	7.00
$\rho_{zs} = 0.99$		0.11	0.68	0.01	0.92	1.27	0.46	4.03	5.81
$\rho_{zs} = 0.995$		0.16	0.71	0.05	0.92	1.08	0.44	3.33	5.62

Table 1.4: Business cycle statistics for sector-specific shocks

A more generalized representation of exogenous forces in the two-sector SEG model to allow for sector-specific shocks is a vector autoregressive process:

$$\begin{bmatrix} \log Z_{t+1} \\ \log S_{t+1} \end{bmatrix} = \begin{bmatrix} \rho_z & 0 \\ 0 & \rho_s \end{bmatrix} \begin{bmatrix} \log Z_t \\ \log S_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1}^z \\ \varepsilon_{t+1}^s \end{bmatrix}$$

ε_{t+1}^z and ε_{t+1}^s are i.i.d. disturbances to $\log Z_{t+1}$ and $\log S_{t+1}$ respectively. Elements in the upper-right and lower-left positions in the autocorrelation coefficient matrix are set to zero to prohibit technology diffusion across sectors. The variance-covariance matrix of the disturbances is:

$$V \begin{bmatrix} \varepsilon_{t+1}^z \\ \varepsilon_{t+1}^s \end{bmatrix} = \begin{bmatrix} \sigma_z^2 & \sigma_{zs} \\ \sigma_{zs} & \sigma_s^2 \end{bmatrix}$$

$\sigma_{zs} = \rho_{zs}\sigma_z\sigma_s$, where ρ_{zs} is the correlation coefficient of ε_t^z and ε_t^s . Estimating the process of S_t is hard, so as a first step approximation, I will maintain the assumption that processes of Z_t and S_t have identical specification (namely, $\rho_z = \rho_s$ and $\sigma_z^2 = \sigma_s^2$). Realizations of Z_t and S_t , however, are different due to the randomness of innovations. Table 1.4²⁰ displays the implied persistence and volatility for different values of ρ_{zs} . Results show that the two-sector SEG depends on high contemporaneous correlation between sector-specific shocks to generate persistence in output growth rates. For example,

²⁰Persistence is measured by the first order autocorrelation coefficient and volatility is measured by standard deviation of growth rate.

	Persistence of x					Volatility of x			
	$x =$	γ_Y	γ_C	γ_{I_k}	N	γ_Y	γ_C	γ_{I_k}	N
US data		0.29	0.24	0.39	0.99	1.14	0.52	2.38	5.52
Baseline ($\phi_2 = 0.11$)		0.29	0.77	0.14	0.92	0.82	0.43	2.36	5.44
$\phi_2 = 0.03$		0.14	0.19	0.12	0.93	0.99	0.71	1.97	6.77
$\phi_2 = 0.05$		0.20	0.31	0.16	0.93	0.92	0.60	1.98	6.50
$\phi_2 = 0.07$		0.26	0.46	0.17	0.93	0.85	0.52	2.02	6.26
$\phi_2 = 0.09$		0.27	0.62	0.15	0.92	0.84	0.47	2.17	5.95
$\phi_2 = 0.13$		0.28	0.91	0.10	0.92	0.86	0.40	2.62	5.11
$\phi_2 = 0.15$		0.23	0.96	0.07	0.92	0.91	0.38	3.00	4.46
$\phi_2 = 0.17$		0.18	0.98	0.05	0.92	1.01	0.37	3.55	4.06

Table 1.5: Sensitivity of physical capital share in human sector

when ρ_{zs} is less than 0.9, the first order autocorrelation coefficient of output growth is slightly negative. However, as long as ρ_{zs} is above 0.95, output growth series is significantly and positively autocorrelated. With regards to the second moment property, the model's prediction is closer to data for all variables when the two shocks are more strongly correlated. It is natural to expect technologies to physical and human sectors to move to the same direction since a technology process is likely to be economy-wide. Think of the invention of internet that improves productivities of the two sectors simultaneously. Therefore, high values of ρ_{zs} should not be regarded as unusual.

Three parameters that are weakly supported by micro evidence are discussed in this section. They are the share of physical capital in human sector (ϕ_2), rate of depreciation of human capital (δ_h) and the coefficient of relative risk aversion (σ). Section 1.5 shows how alternative ways of treating physical and human capitals affect the timing order of investments to the two types of capital following shocks and hence the model's quantitative performance. It is worth knowing how sensitive the model's results are to changes of these parameters. The coefficient of relative risk aversion is also considered because JMS emphasize the remarkable impact of this coefficient on the quantitative performance of endogenous growth models. Therefore, it is meaningful to see whether this result continues to hold in a more generalized version of endogenous growth business cycle model. Table 1.5 shows the implications of increasing the share of physical capital

	Persistence of x					Volatility of x			
	$x =$	γ_Y	γ_C	γ_{I_k}	N	γ_Y	γ_C	γ_{I_k}	N
US data		0.29	0.24	0.39	0.99	1.14	0.52	2.38	5.52
Baseline ($\delta_h = 0.005$)		0.29	0.77	0.14	0.92	0.82	0.43	2.36	5.44
$\delta_h = 0.0025$		0.18	0.75	0.06	0.93	0.92	0.39	2.88	5.53
$\delta_h = 0.0075$		0.44	0.78	0.29	0.91	0.76	0.44	1.89	5.38
$\delta_h = 0.0100$		0.66	0.79	0.60	0.90	0.72	0.49	1.56	5.58
$\delta_h = 0.0125$		0.84	0.80	0.85	0.89	0.69	0.51	1.37	5.52
$\delta_h = 0.0150$		0.94	0.81	0.89	0.88	0.71	0.54	1.47	5.60

Table 1.6: Sensitivity of human capital depreciation rate

in human capital production from 0.03 to 0.17. The effects are non-monotonic for some statistics. Regarding the autocorrelation coefficient of output growth, a major indicator of persistence, the impacts are opposite when is below and above 0.11. In particular, the coefficient goes bigger as ϕ_2 increases below the threshold and starts to decrease when increases above 0.11. For all cases considered as empirically relevant, however, the lowest value of the autocorrelation coefficient is 0.14 which is still much higher than what is predicted by standard RBC models. For other statistics regarding business cycle persistence, autocorrelation coefficient of consumption growth monotonically increases when ϕ_2 increases. The degree of persistence of physical capital investment growth increases first and then decreases as ϕ_2 goes up. The autocorrelation of labour supply seems unaffected by ϕ_2 . The impact of ϕ_2 on the volatility statistics is also two-folds. Consumption growth and labour supply fluctuate less and physical capital investment growth becomes more volatile when ϕ_2 increases. Another key variable determining the results of the model in this paper is the lower depreciation rate of human capital relative to that of physical capital. The quarterly rates displayed in table 1.6 correspond to the yearly range between 1% and 6%, which is large enough to cover a majority of possible values used in literature. As δ_h gets bigger, growth rates of output, consumption and physical capital investment all becomes more autocorrelated, indicating a higher degree of persistence. For instance, autocorrelation coefficient of output growth is as high as 0.94 when δ_h is 0.015. This suggests that increasing the depreciation rate of human capital produces greater persistence

	Persistence of x				Volatility of x				
	$x =$	γ_Y	γ_C	γ_{I_k}	N	γ_Y	γ_C	γ_{I_k}	N
US data		0.29	0.24	0.39	0.99	1.14	0.52	2.38	5.52
Baseline ($\sigma = 1$)		0.29	0.77	0.14	0.92	0.82	0.43	2.36	5.44
$\sigma = 0.6$		-0.07	-0.07	0.08	0.81	40.68	49.33	19.02	66.52
$\sigma = 0.7$		0.02	0.05	0.13	0.91	6.42	6.32	7.40	24.20
$\sigma = 0.8$		0.20	0.11	0.93	0.91	1.50	1.86	1.61	10.59
$\sigma = 0.9$		0.96	0.49	0.36	0.91	0.54	0.62	1.86	7.00
$\sigma = 1.1$		0.12	0.25	0.07	0.93	1.13	0.65	2.66	4.66
$\sigma = 1.2$		0.07	0.12	0.04	0.93	1.36	0.87	2.90	4.14
$\sigma = 1.3$		0.04	0.07	0.03	0.93	1.51	1.02	3.04	3.57
$\sigma = 1.4$		0.03	0.05	0.02	0.94	1.62	1.15	3.14	3.42
$\sigma = 1.5$		0.01	0.03	0.00	0.94	1.70	1.24	3.21	3.17
$\sigma = 2.0$		-0.01	0.00	-0.02	0.94	1.95	1.53	3.43	2.48

Table 1.7: Sensitivity of coefficient of relative risk aversion

of the model variables. For volatility, growth rates of output and physical investment fluctuates less while consumption growth fluctuates more as δ_h increases. The volatility of labour supply does not seem to be affected by δ_h . JMS show the importance of σ in affecting the quantitative implications of the one-sector endogenous growth model. The sensitivity analysis of σ in a more generalized endogenous growth model reinforces JMS' result. In particular, the range of values for σ can be segmented into two regions with the cut-off value being $\sigma = 1$. For the values in the upper region, persistence in the growth rates of variables diminishes very quickly. For instance, the model generates nearly no persistence at all when σ goes up to 1.5. However, σ has little effect on the persistence of labour supply. σ also has great impact on the volatilities of variables implied by the model. For instance, volatility of output growth increases from 0.82 to 1.95 when σ goes up from 1 to 2. In the lower region when σ is less than one, the impact becomes even bigger on both persistence and volatility. First order autocorrelation coefficient of output growth decreases from 0.96 when $\sigma = 0.9$ to a negative value when $\sigma = 0.6$. Volatility of output growth when $\sigma = 0.6$ is about 50 times as large as that in logarithmic case. As the logarithmic momentary utility function is standard in RBC literature, the baseline parameterization in this paper uses $\sigma = 1$.

1.7 Conclusion

This chapter presents a two-sector stochastic endogenous growth business cycle model with new human capital produced in a separate sector. Having human capital produced with a more labour intensive technology introduces several distinctive features of the two-sector endogenous growth model compared to the one-sector model as in JMS. These new features of the two-sector model jointly determine the dynamics of the variables in the model.

Several interesting results are found from the two sector endogenous growth model. First, curtailment of working hours when labour productivity is high arises naturally in this two-sector model. It follows as a result of agents' optimal inter-sectoral labour substitution decision required by the *Rybczynski* effect and the intertemporal no-arbitrage condition. Therefore, the empirical finding by Gali (1999) that labour supply decreases on impact of positive productivity shock should not be interpreted as evidence against flexible price models. Second, the two-sector model generates persistent movements in the growth rates of output and physical capital investment up to the level that matches US data, which combined with the hump shaped impulse responses of output and physical capital investment, indicates the existence of a strong interior propagation mechanism embedded in the two-sector model to spread the shock over time. Third, the two-sector model also generates greater fluctuation in working hours because of the substitution between working in physical sector (market time) and effort in human capital production (nonmarket time). Inter-sectoral labour transferring amplifies the variability of working hours.

One key difference between the implications of the two-sector model and the one in JMS is the timing order of the responses of investments to physical and human capital to a good technology shock. In particular, the order in the two-sector model is the opposite of that in JMS. In the two-sector model, people tend to increase human capital stock immediately after a good shock and accumulate physical capital with a delay. This is because aggregate shock induces an imbalance between the rates of returns to two

capitals. Investments to two capitals then adjust differently following exogenous shocks.

The sensitivity analysis shows that the coefficient of relative risk aversion has great impact on the quantitative performance of endogenous growth model while this effect is absent in exogenous growth models. Due to unfortunate lack of data for human sector, it is difficult to obtain a reliable estimate of the contribution of each input in this sector. But the main results found in this chapter holds for a wide range of empirically relevant values. A related problem with this is the difficulty in constructing the shock to human sector. In baseline parameterization, the shock to human sector is assumed identical to the shock to physical sector. This is a too extreme assumption to make, but it helps get an insight of the internal propagation mechanism of the two-sector SEG model. For the case of sector-specific shocks, the model depends on high correlations between sector-specific shocks to produce output persistence. All of these immaturenesses in treating human capital related variables require future efforts to be exerted on the empirical works on human sector.

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Chapter 2

Output Dynamics and Human Capital: Human Capital as Tradable Goods

2.1 Introduction

This chapter addresses a similar issue to that of chapter 1 using a similar two-sector endogenous growth model, but it proceeds under a different assumption of human capital: human capital is tradable good such that it counts as part of measured output. Specifically, output in this chapter is defined by the sum of outputs from the two sectors multiplied by their respective relative prices. This broader concept of output is used by Jones Manuelli and Siu (2005b) (JMS hereafter) among others¹. According to JMS, human capital investment constitutes an important part of US GDP. It includes both private and public expenditures on health care, all forms of training, education and many other social activities that improve the quality of workers. The baseline calibration of their model implies that investment to human capital accounts up to 26.2% of aggregate output, a value close to their estimate using US data. Due to this fundamental change to

¹One more example is in *Economic Growth* (1st Eds.) by Barro and Sala-I-Martin (1995), pp. 181.

the assumption of human capital investment, the two-sector endogenous growth model replicates the observed output dynamics via a completely different mechanism. In particular, this chapter explores the idea of changing composition of output in a multi-sector model in generating the desired output dynamics that fit data.

The results of this chapter show that the two-sector stochastic endogenous growth (SEG) model with output defined by the composition of outputs of two sectors successfully replicates the fashion of autocorrelation function of US data with reasonable parameterization. This can be understood from the mechanism to generate the hump-shaped impulse response function (IRF). Although output of either individual sector does not display hump-shaped response to shocks, the composite output does. On impact of a positive shock to physical sector, the representative agent relocates labour (as well as physical capital) away from human sector, which results in higher output in physical sector and lower output in human sector. The overall effect on composite output on impact of a shock is hence small but positive. In subsequent periods when the effect of a physical sector shock gradually dies out, the agent transfers resources (both physical and human capitals) back to human sector so that output in physical sector falls while production of human capital investment recovers. The increase in human capital production outweighs the decrease in physical goods production for several periods so that the response of composite output exhibits a "hump".

An earlier work exploring the idea of changing composition of output in generating the desired output dynamics is Benhabib, Perli and Sakellaris (2006) who find that a two-sector model with consumption and investment produced in different sectors requires unrealistically high elasticity of intertemporal substitution to match properties of output dynamics. There is also other efforts in bringing about an internal propagation mechanism in business cycle modelling framework in order to reproduce the two facts. It includes quadratic adjustment cost to capital and labour as in Cogley and Nason (1995); factor-hoarding models as in Burnside and Eichenbaum (1996); different elasticities of substitution between skilled and unskilled labour across sectors in Perli and Sakellaris

(1998); and habit formation in leisure as in Wen (1998).

The empirical works by Dellas and Sakellaris (2003) heavily motivated research on cyclical implications of stochastic endogenous growth models. They find that there exists significant substitution between formal education and competing labour market activities. In economic boom, graduates tend to go to job market instead of pursuing further education because of the high opportunity cost of education. In recession, there exists a higher tendency for working people and layoffs to go back to college for more education due to the low opportunity cost of not working.

The rest of this chapter is organized as follows. The model environment is presented in section 2.2 and a careful calibration is in section 2.3. Section 2.4 and 2.5 discuss the IRF of output and autocorrelation function of output growth of the model, respectively. For comparison purpose, the results from the two-sector endogenous growth model are compared with those from other two models being the one-sector endogenous growth model in JMS and a standard one-sector exogenous growth model in King and Rebelo (1999). Section 2.6 presents sensitivity analysis and section 2.7 concludes.

2.2 Model environment

The economy consists of two separate sectors. One sector produces consumption goods and physical capital investment according to following technology:

$$C_t + I_{kt} = A_g Z_t (V_t K_t)^{\phi_1} (N_t H_t)^{1-\phi_1}$$

C_t is goods used for consumption and I_{kt} is investment to physical capital; A_g is the scale parameter associated to physical sector; K_t is the total physical capital stock that has been accumulated by the representative agent by the beginning of period t ; V_t is the share of the physical capital stock distributed to physical sector; N_t denotes agent's labour time spent in physical sector; H_t is the representative agent's stock of human capital which is predetermined for period t ; $N_t H_t$ represents the "effective labour input". The higher

the stock of human capital the representative agent has accumulated by the beginning of this period, the higher the marginal productivity schedule of her labour time. The technology shock in this sector is assumed to follow a stationary process in log form:

$$\log Z_{t+1} = \rho_z \log Z_t + \varepsilon_{t+1}^z$$

Where ε_t^z is sequence of independently and identically distributed random variables, the realization of which is observable by the agent at the beginning each period. ρ_z measures the persistence of the shock. The other sector produces investment to human capital. In this sector, the agent accumulates her human capital, which improves the efficiency of labour hours from the next period, at the cost of real resources. This sector corresponds to formal education, various types of trainings, health care and any other social activities that improve the quality of workers. The production of human capital investment allows for different factor intensities to that of physical goods production:

$$I_{ht} = A_h S_t ((1 - V_t) K_t)^{\phi_2} (M_t H_t)^{1-\phi_2}$$

This is where the one-sector stochastic endogenous growth model in JMS is extended. The separate human sector allows for different factor intensities across sectors and sector-specific shocks. And this is also a generalized version of the human capital production technology of Uzawa-Lucas by including physical capital stock. I_{ht} is the new human capital investment produced in this period; A_h is the scale parameter assigned to human sector; $(1 - V_t)$ is the rest of physical capital allocated to human sector; M_t represents the time that an agent devoted in forming new human capital; S_t is the technology shock to human sector. It follows:

$$\log S_{t+1} = \rho_s \log S_t + \varepsilon_{t+1}^s$$

ρ_s and ε_t^s have similar meanings with their respective counterpart associated to the process of Z_t . These two parallel sectors compete for limited resources (physical and human sector) such that the agent makes optimal decisions on capitals allocations across sectors.

Very few works have been done to apply the idea of human capital formation in business cycles modelling framework. Exceptions include Perli and Sakellaris (1998), DeJong and Ingram (2001) and JMS. Literatures in this strand has been heavily re-motivated by the empirical evidence found by Dellas and Sakellaris (2003) that there is significant substitution between education (regarded as important means of accumulating human capital) and competing labour market activities over the business cycle frequency. In particular, schooling or learning acts as an additional margin on agent's time allocation decision and is strongly countercyclical over the business cycles. During recession, the agent redirects time devotion away from working because of the low opportunity cost of having education. In economic boom, more graduates tend to enter the job market instead of pursuing further education probably due to the high opportunity cost for not working. Dellas and Sakellaris (2003) find that for the period 1968–88 one percentage point increase in the unemployment rate is associated with about 2 percentage increments in college enrolment rate.

Physical capital stock accumulation is subject to cost of adjustment:

$$K_{t+1} = (1 - \delta_k)K_t + \Psi \left(\frac{I_{kt}}{K_t} \right) K_t$$

δ_k is the depreciation rate of physical capital. Ψ is a function that transforms physical capital investment into useful capital stock. Function $\left(\frac{1}{\Psi'} \right)$ can be interpreted as Tobin's q , which gives the number of units of physical capital investment which must be forgone to increase the capital stock by one unit. This form of physical capital adjustment cost has been used at least by Lucas and Prescott (1971) and Baxter (1996). One merit of using this specification of adjustment cost is that near steady state analysis does not require a particular functional form for Ψ . Near the steady-state, assume that $\Psi \succ 0$, $\Psi' \succ 0$, and

$$\Psi'' < 0.$$

The law of motion of human capital follows linear transformation:

$$I_{ht} = H_{t+1} - (1 - \delta_h)H_t$$

δ_h denotes the assumed constant depreciation rates for human capital. Investment to human capital is converted into human capital stock on a one-to-one basis.

The introduction of a separate human sector gives rise to a broader concept of output in this economy which is the composition of the outputs in two sectors. This concept of output is also used by JMS. If the price for physical goods is normalized to unity and express the relative price of human capital in terms of physical goods by P_t , one can write down the composite output in this two-sector economy as:

$$Y_t = C_t + I_{kt} + P_t I_{ht}$$

The initial stocks of the two types of capitals are known to be K_0 and H_0 .

This economy is populated with an infinite number of identical agents whose momentary utility is given by:

$$U(C_t, L_t) = \frac{(C_t L_t^A)^{1-\sigma} - 1}{1 - \sigma}$$

L_t is leisure and A is consumer's subjective measurement of the relative importance of leisure to consumption good in providing utility. The agent is bound by a time constraint for every period t :

$$N_t + M_t + L_t = 1$$

There is neither distortion nor externality, so the competitive equilibrium of this economy coincides with the result of the social planner's problem:

$$\underset{C_t, V_t, N_t, M_t, I_{kt}, K_{t+1}, H_{t+1}}{MAX} \quad E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t (1 - N_t - M_t)^A)^{1-\sigma} - 1}{1 - \sigma}$$

$$s.t. \quad C_t + I_{kt} = A_g Z_t (V_t K_t)^{\phi_1} (N_t H_t)^{1-\phi_1}$$

$$K_{t+1} = (1 - \delta_k) K_t + \Psi \left(\frac{I_{kt}}{K_t} \right) K_t$$

$$H_{t+1} - (1 - \delta_h) H_t = A_h S_t ((1 - V_t) K_t)^{\phi_2} (M_t H_t)^{1-\phi_2}$$

The equilibria of this economy has been fully characterized by Caballe and Santos (1993) and Bond et al. (1996), so only a sketchy exposition of the characteristics of this model is provided here. Given the agent's preference and Cobb-Douglas technology in both sectors, this economy converges to one of the "parallel" BGPs. Regarding the set of "parallel" BGPs, the ratios remain the same although the levels of the capital stocks differ. The answer to the question that to which BGP this economy converges depends on the initial endowment of the physical and human stocks. Different combinations of the two capital stocks lead to different BGP. This property emitted from the endogenous growth model meets the observation that different countries may eventually end up with the same economic growth rate but possibly different wealth level due to the uneven initial resources endowments, physical and human stocks, in different places in the world. In steady state, variables regarding time allocation decision, relative prices and the physical capital shares in both sectors are constant while all other variables are growing at a common rate.

For comparison purpose, the one-sector stochastic endogenous growth model in JMS and a standard exogenous growth RBC model in King and Rebelo (1999) are also studied. Mathematically, the one-sector endogenous growth model in JMS is:

$$\underset{C_t, N_t, K_{t+1}, H_{t+1}}{MAX} \quad E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t (1 - N_t)^A)^{1-\sigma} - 1}{1 - \sigma}$$

$$s.t. \quad C_t + K_{t+1} - (1 - \delta_k) K_t + H_{t+1} - (1 - \delta_h) H_t = Z_t K_t^{\phi_1} (N_t H_t)^{1-\phi_1}$$

The key difference between this one-sector SEG model and the two-sector SEG model is that the former treats human the same good as physical capital while the later views human capital a different good from physical capital. The one-sector exogenous growth

model is:

$$\begin{aligned}
& \underset{C_t, N_t, K_{t+1}}{MAX} \quad E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t (1 - N_t)^A)^{1-\sigma} - 1}{1 - \sigma} \\
& s.t. \quad C_t + K_{t+1} - (1 - \delta_k) K_t = Z_t K_t^{\phi_1} (X_t H_t)^{1-\phi_1} \\
& \quad X_{t+1} = (1 + \gamma) X_t
\end{aligned}$$

X_t indicates the level of labour-augmented technology and grows at an exogenous rate γ . The difference between exogenous growth model and SEG models is what determines growth rate. If H_t in endogenous growth model is fixed to grow at rate γ instead of as a result of the agent's investment decision, the endogenous growth business models quickly reduce down to standard RBC models.

Two different normalization methods handling the growth component are used to transform the system into a stationary one. To compute the impulse response of output, all growing variables are discounted by their common constant BGP growth rate. The second normalization method used to simulate the model is to discount the growing variables by current stock of human capital such that variables in ratios are stationary around the BGP. Once the equilibrium conditions of an endogenous growth model are expressed in variables that have stationary distributions, the model is solved by log-linearization method. Appendix A contains details of the solution method.

2.3 Calibration

Calibration of the two-sector endogenous growth model in this chapter is essentially the same as those in chapter 1 and 3 except for three changes. The first change is the steady state leisure time: 0.67 in this chapter and 0.54 in previous two chapters; the second change is the physical capital share in human sector: 0.22 in this chapter and no greater than 0.17 in previous two chapters; the third change is the autocorrelation parameter of technology shocks: 0.9 in this chapter and 0.95 in proceeding chapters. The reason to have these changes in calibration is that they deliver better fit to US data in terms of IRF

of output and autocorrelation function of output growth. It is arguable that these altered values of some parameters still lie well within conventionally accepted regions. For the sake of the completeness of this chapter only, I will present the calibration procedures in adequate detail as what follows.

Gomme and Rupert's (2007) is still the main reference for the calibration. The data set associated to their paper can be found from the fed reserve website and will be used as the empirical basis of the model in this chapter. Following calibration stays closely to the values suggested by Gomme and Rupert whenever possible.

If physical goods production technology has a Cobb-Douglas form as is the case in this model, the labour's contribution to output can be measured by the share of labour income out of total income. Gomme and Rupert (2007) calibrate the labour income share to be 0.717 for US economy over the period 1954.1-2001.4 after abstracting from the government sector. This value is somewhat higher than the value (0.64) usually seen in the RBC literatures. With regard to this two-sector model, I still use 0.64 as the share of labour income out of output from physical sector. The value in Gomme and Rupert, however, is very useful reference because according to common wisdom labour income in human sector should occupy a larger fraction, which induces that 0.717 might be a weighted average of the shares of labour incomes in two sectors with the weights being the relative importance of output from individual sector in composite output.

It is considerably difficult to find a reliable estimate of the capital share in human sector because of the unfortunate lack of data on human capital. It turns out finally that this is a key parameter in determining the quantitative performance of the model. But as a preliminary step to show the potential of the model with human capital accumulation bears an internal propagation mechanism, the arbitrariness of this parameter does not seem to cause any serious trouble. It is arguable that the education sector is more intense in labour input relative to physical sector. Therefore, all cases for ϕ_2 bigger than ϕ_1 (i.e. human sector is less labour intensive than physical sector) are discarded as they are considered empirically-irrelevant. Within the empirical-relevant region, a wide range of

choices of ϕ_2 are experimented. This is done in the sensitivity analysis in section 2.6. In the baseline calibration, ϕ_2 is set to be 0.22.

In business cycle literatures, it becomes a standard approach to decompose the total capital stock into four categories: market structure, equipment and software in market sector, housing and consumer durables in home sector. Gomme and Rupert (2007) find that the series of the depreciation rates for the four types of capital stocks over the sample period 1926-2001 are all trended upwards. This nonstationarity of the depreciation rates in the data poses challenges on the assumed constant depreciation rate in nearly all RBC models². In this chapter, with no distinctions made to specific capital categories, the way wandering around the nonstationarity problem is to use the weighted average of the mean depreciation rates to various subcategories of capitals over the period 1954.1–2001.4 as calculated in Gomme and Rupert (2007), where the weight is the average share of capital in each subcategory out of the total capital stock over time. This approach can be mathematically illustrated by the equation below³:

$$\delta_k = \frac{K_s}{K}\delta_s + \frac{K_e}{K}\delta_e + \frac{K_h}{K}\delta_r + \frac{K_d}{K}\delta_d$$

The capital stocks data used for this calculation are constructed by “method 2” in Gomme and Rupert (2007). Specifically, these data are constructed from the annual capital stock data and quarterly investment data which are both converted to real 2000 dollars using Chain-type index. It covers exactly the same time period over which the depreciation rates to different types of capitals are computed. By this method, the quarterly depreciation rate of physical capital is calculated to be about 0.02 which indicates a moderately lower depreciation rate of physical capital stock compared to the value that is commonly used in RBC models.

²One of the exceptions is the work by Burnside and Eichenbaum (1996) who assume that the depreciate rate of physical capital varies according to the utilization rate of physical capital.

³ K is the total amount of capital stock (sum of all subcategories); K_s, K_e, K_r and K_d sequentially, are market capital structure, equipment and software, housing, consumer durables and $\delta_s, \delta_e, \delta_r$, and δ_d are the depreciate rates to various capital stocks as reported in Gomme and Rupert (2007).

No absolute consensus has been reached on the value of the depreciation rate of human capital stock in literatures with human capital accumulations in various forms. JMS set the yearly depreciation rate at 2.5% as the intermediate value within some range. DeJong and Ingram (2001) estimate the quarterly human capital depreciation rate to be about 0.5% using US quarterly data set from the first quarter of 1948 to the last quarter of 1995. This estimation lies in the range of annual depreciation rate between 1% to 3% found by Jorgenson and Fraumeni (1989) and some others. In the baseline case, δ_h is 0.5% in this chapter. Some other values in this region are considered in the sensitivity analysis in section 2.6.

Near steady state analysis does not necessitate a functional form for $\Psi\left(\frac{I_k}{K}\right)$. Calibration of it takes two procedures. First, Ψ is parameterized in a way that there is no adjustment cost incurred in nonstochastic steady state. This requires that (only in steady state) $\Psi = \frac{I_k}{K}$ and $\Psi' = 1$. Hence, Tobin's $q\left(\frac{1}{\Psi'}\right)$ is one in steady state. Second, all that needed to quantitatively characterize this function near steady state is one key parameter: the elasticity of investment to capital ratio $\left(\frac{I_k}{K}\right)$ to the inverse of Tobin's $q\left(\Psi'\left(\frac{I_k}{K}\right)\right)$. Denote the elasticity by η , which measures the response of $\frac{I_k}{K}$ to movement of Tobin's q ⁴. Empirical works do not find solid support for a value of η . Following Baxter (1996), η is set 200 in baseline case. Sensitivity analysis on this parameter is presented in section 2.6.

The steady state value for time devoted to leisure in this model is set to be 0.67, a very standard value in literature. Once steady state leisure devotion is pinned down, time spent in each sector is implied by the BGP constraints. In the baseline calibration, on the nonstochastic BGP, time spent in physical and human sector time are 0.23 and 0.10 respectively. Thus learning time accounts up to roughly 30% of non-leisure time.

⁴Mathematically: $\eta = -\frac{\Psi'\left(\frac{I_k}{K}\right)}{\Psi''\left(\frac{I_k}{K}\right) \cdot \frac{I_k}{K}}$. The higher the value of η , the smaller the magnitude of adjustment cost. One extreme case is when there is no cost of adjustment at all: $\Psi = \frac{I_k}{K}$, and the law of motion of physical capital reduces to the standard one: $K_{t+1} = (1 - \delta_k)K_t + I_{kt}$, and $\eta = \infty$.

The Euler equation reduces to a simple form along the nonstochastic BGP:

$$\beta R = (1 + \gamma)^\sigma$$

γ denotes the common growth rate; β is the subjective discount rate; R is the interest rate. This equation puts an important restriction to some key variables that have significant impact on this model's behaviours. Therefore, great care must be taken in calibrating these values. First of all, the common growth rate on the nonstochastic BGP is calibrated to match the real growth rate of per capita US GDP over the same sample period 1954.1-2001.4 as reported in NIPA. The per capita rate is used because this model abstracts from population growth. The quarterly value used in this paper is 0.0042 which is in line with Gomme and Rupert (2007). This is close to the annual growth rate 0.0188 in Greenwood et al. (1995) and 0.0177 in JMS. The slight difference is mainly the result of the different sample periods used.

The next variable to take care of is the coefficient of relative risk aversion. JMS demonstrates the significant effect of this parameter on the quantitative performances of the stochastic endogenous growth business cycle models. Specifically, they show the impact of σ evidently from the model's impulse response functions, second moments and cross-correlations among the key variables of interest. On the contrary, in the exogenous growth business cycle model, the coefficient of relative risk aversion has little effect on the model's predictions and this is probably why the exploration of the effect of this parameter is missing from most of the exogenous growth business cycle models and the logarithmic preference is usually adopted as a standard one. The sensitivity analysis in section 2.6 shows that the business cycle implications stemmed from different values of σ enhance the recognition of the importance of this parameter in understanding the shock propagation mechanism in RBC frameworks. The indeterminacy of this parameter exerts little discomfort to exogenous growth business cycles models, but even small changes of this intertemporal smooth parameter will have large effects on the predictions of business cycle models where growth is endogenously determined, especially on the autocorrelation

coefficients in output growth. As a result, exploring the implications of the agent's various degrees of risk aversion to the business cycle studies rises as a natural task for any endogenous growth model relying on human capital formation. Mehra and Prescott (1985) find that the bulk of micro-evidence places the value for the coefficient of relative risk aversion somewhere between 1 and 2. In the baseline calibration, logarithmic utility function is used as a comparable case to most exogenous growth models. Some other values from the range 1 and 2 are investigated and the results are shown in section 2.6.

There is considerable disagreement about the value of real interest rate in literature. This is probably due to the different assets that are used for measurement. Some authors measure the return to capital (e.g. Poterba 1998), while others use the returns in stock market (e.g. Siegel 1992). Some authors even discard the interest rate as an optional starting point for calibration and simply leave it as whatever model implies as long as the rest part of calibration fall in the "accepted region". This may lead to very implausible values (e.g. 20.5% in Greenwood et al. (1995)). Gomme and Rupert (2007) discard the use of interest rate to calibrate their model by imposing the restriction that the home and market sectors grow at the same rate. This implies the annual rate to be 13.2% from their baseline parameterization. In JMS, given the subjective discount factor fixed at 0.95, the BGP growth rate at 1.77% and coefficient of relative risk aversion at 1.4, the implied yearly interest rate is 7.13%. Most of the empirical works find the annual pre-tax interest rate lies roughly between 6% and 11%. In calibrating the model in this chapter, to avoid the problem of implausibly high interest rate arising in Greenwood et al. (1995) and others, the subjective time preference parameter β is chosen to match annual interest rate 7.4%, very much an intermediate value as one can find in the RBC literatures.

Since one advantage of endogenous growth models over their exogenous counterparts is that endogenous growth models do not need to rely on highly persistent shocks to generate fluctuation persistence, the autocorrelation coefficient of $\log Z_t$ is set 0.9, a relatively low degree of persistence in business cycle literature. The variance of innovations to $\log Z_t$

Free parameters		
β	Subjective discount factor	0.986
γ	BGP growth rate	0.0042
σ	Coefficient of relative risk aversion	1
L	Steady state leisure time	0.67
ϕ_1	Share of physical capital in physical sector	0.36
ϕ_2	Share of physical capital in human sector	0.22
δ_k	Depreciation rate of physical capital	0.02
δ_h	Depreciation rate of human capital	0.005
η	Elasticity of $\frac{I_h}{K}$ to Ψ'	200
A_g	Scale parameter for physical sector	1
ρ_z	Autocorrelation coefficient of $\log Z_t$	0.9
σ_z	Standard deviation of ε_t^z	0.0264
Implied by BGP		
r	Steady state interest rate	0.0185
A_h	Scale parameter of human sector	0.0492
N	Steady state working time	0.23
M	Steady state learning time	0.1
A	Weight of leisure in preference	2.58
$\frac{C+I_k}{Y}$	Steady state physical goods-output ratio	0.73
$\frac{P \cdot I_h}{Y}$	Steady state human capital investment-output ratio	0.27
V	Steady state share of physical capital in physical sector	0.82

Table 2.1: Calibration of the two-sector SEG model

is chosen to match the volatility of output growth. It turns out that this requires the variance of innovations to be 0.0007. Throughout this chapter, the technology shock to human sector is shut down. This is done by setting $S_t = 1$ for all t . The scale parameter associated to physical sector is normalized to one and A_h is implied by the BGP constraints. In the baseline parameterization, A_h is 0.0492.

The baseline calibration of the two-sector SEG model can be summarized in table 2.1:

JMS calibrate their model to annual frequency because the data on human capital is only available yearly. But for comparison purpose, their model is re-calibrated to equivalent quarterly frequency while keeping the original parameter values. Table 2.2 presents the quarterly equivalence:

Parameters		Annually	Quarterly
β	Discount factor	0.95	0.987
γ	BGP growth rate	0.0177	0.0044
N	Labour supply on the deterministic BGP	0.17	0.17
σ	Coefficient of relative risk aversion	1.4	1.4
ϕ_1	Share of physical capital	0.36	0.36
δ_k	Depreciation rate of physical capital	0.102	0.0265
δ_h	Depreciation rate of human capital	0.025	0.0063
A	Leisure weight in the utility function	6.36	6.36
ρ_z	Autocorrelation coefficient of the technology process	0.967	0.967
σ_z	Standard deviation of the innovation	0.0135	0.0135

Table 2.2: Calibration of the one-sector SEG model

The third column displays the original values used in JMS and the last column shows their quarterly equivalence. There are two notable differences between the calibration of the two-sector SEG and this one-sector SEG in JMS. The first one is the steady state labour supply. Leisure time in JMS is as high as 83% of time endowment which is much higher than the usual value used in the business cycle literatures. But reducing the steady state leisure time has no notable impact on the IRF of output and autocorrelation function of output growth which are the focus of this chapter. The second difference is the autocorrelation coefficient which governs the persistence of shock process. In JMS, it is as high as 0.967. It is well-known that RBC models are criticized by its heavy reliance on persistent shocks to generate persistent movement in aggregate series. To overcome this shortage, the persistence parameter of shocks in the two-sector model is set 0.9, arguably a degree that is near the lower bound any empirical work can find.

Finally, a standard RBC model (King and Rebelo (1999)) is also included in the comparison. Details about the calibration of it are already in macroeconomics textbook, so I do not present them here in this chapter.

2.4 Impulse response functions

The impact of exogenous technology shocks on endogenous variables is reflected by the IRFs. To compute the impulse responses, the first normalization method, which discounts the growing variables by their common growth rate along the nonstochastic BGP, is used. Specifically, define some new variables in the following way:

$$\begin{aligned} c_t &\equiv \frac{C_t}{(1+\gamma)^t} \\ k_t &\equiv \frac{K_t}{(1+\gamma)^t} \\ h_t &\equiv \frac{H_t}{(1+\gamma)^t} \\ i_{kt} &\equiv \frac{I_{kt}}{(1+\gamma)^t} \end{aligned}$$

Although variables in capital letters are growing in steady states, the detrended variables are stationary. Hence, the equations that capture the dynamics of SEG models can then be rearranged in terms of only stationary variables. If one removes exogenous randomness from SEG models, all variables will converge to and continue to stay on a particular BGP once the initial values for the physical and human capital are given. For the stochastic version of endogenous growth models, however, a new BGP, in general, will be triggered each time a shock hits the economy. In other words, the model does not converge back to the previous BGP even after a temporary shock⁵. For this reason, the first normalization method is only valid to attain IRFs which show the responses of variables only after one-off shocks, rather than repeated shocks. In addition, this discounting method can be viewed as a detrending method which decomposes away the long-term growth component of series and leaves only the short-term fluctuation to attention. Appendix A provides details of the log-linearization method used to solve the transformed model.

The first goal of this chapter is to show that the two-sector endogenous model has the

⁵Mathematically, this is because one of the two eigen values of the coefficient matrix in the recursive solution of state variables $[k_t, h_t]'$ is unity.

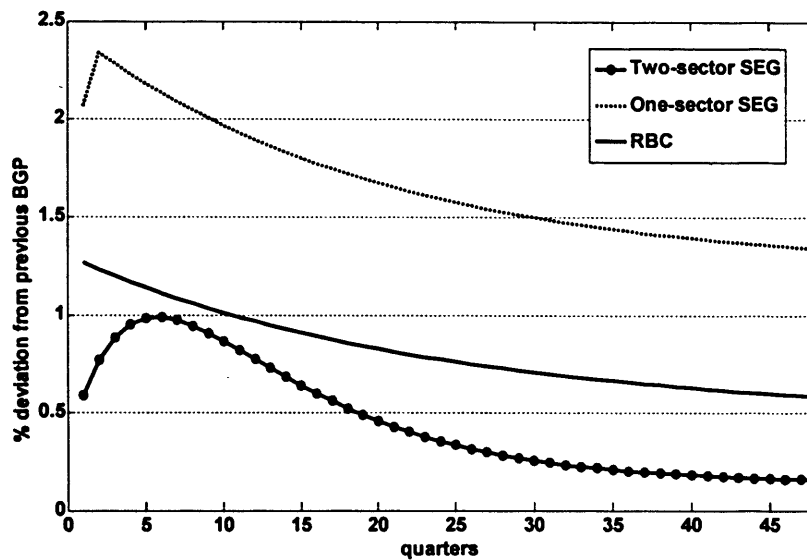


Figure 2-1: Comparing impulse response functions of composite output for different models

potential to generate output series that has a trend-reverting component identifiable by a hump-shaped IRF. This is one of the two stylized facts about the dynamics of US GNP summarized by Cogley and Nason (1995). Figure 2-1 shows the IRFs of output generated by the standard exogenous growth business cycle model by King and Rebelo (1999), the endogenous growth business cycle model in JMS and the two-sector endogenous growth business cycle model in this chapter. Apparently, the worst candidate in fitting the IRF found in the data is the exogenous growth model in which output keeps falling after the impact period of a positive shock. Output in the model of JMS goes on to increase slightly in the second period following a shock and starts to fall immediately from the third period. This short-lasting expansion of output only slightly improves the model's fitness in terms of IRF and helps to generate modest serial correlation in output growth series. The two-sector endogenous growth model, compared to the one in JMS, remarkably changes the shape of the IRF by producing a smooth hump to the observed level. Specifically, in the impact period, the composite output jumps around 0.6% above

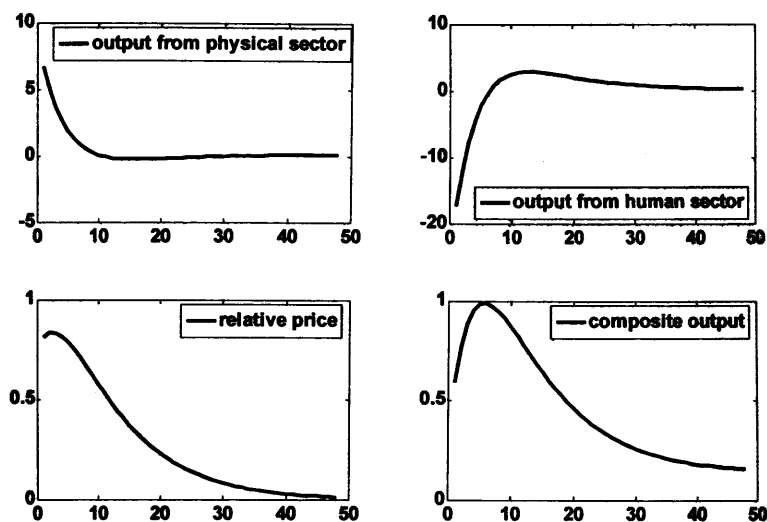


Figure 2-2: Impulse response functions of ingredients

its previous steady state value. It continues to grow for about 6 quarters before arriving at the peak at about one percent higher than the previous balanced growth path value. After the peak, it starts to fall and eventually lands on a new steady state. This pattern of impulse response curve resembles very much the one found by Cogley and Nason (1995) in US data: output continues to grow for a few periods after the impact period of a positive shock. In other words, the maximal effect of a shock comes a few periods afterwards, which stays in contrast with the exogenous growth model where maximal effect happens immediately when the shock happens. This delayed impact of a shock is the first indicator for the presence of an internal propagation structure in the business cycle model.

The best way to see the underlying mechanism generating the hump in the output response is to understand the responses of the ingredients that constitute the composite output. Figure 2-2 shows the IRFs of four variables: output of physical sector (sum of consumption and physical capital investment), human capital investment, the relative

price of human capital in terms of physical capital and the composite output. Consumption and investment to physical capital, which are the output in physical sector, both jump up in the impact period of the shock. The effect of positive shock on consumption is spread over some periods because agents tend to smooth consumption. Investment to physical capital decreases monotonically after the impact period and finally lands on a new BGP. The joint reaction of them is in the upper-left quadrant of figure 2-2. The response of the relative price of human capital is shown in the lower-left quadrant of figure 2-2. The relative price is the ratio of the shadow prices of human capital and physical goods. When a positive shock hits physical sector, consumption increases and the marginal utility of consumption goes down, which results in the lower shadow price of physical goods. As the effect of the shock dies out, the price returns to its steady state value. With regards to human sector, investment to human capital drops dramatically in the period when the shock takes place and recovers gradually in a few subsequent periods. The immediate reduction in investment to human capital on impact happens because the productive factors move away from human sector to physical sector where productivity is higher. In subsequent periods, these factors move back to human sector to restore original distribution across sectors. The key to generate a “hump” in the IRF of the composite output is that post-impact increase in human capital investment is greater than the decrease in physical goods production for some periods. Specifically, the composite output consists of two parts: output of physical sector and investment to human capital multiplied by its relative price. Mathematically, $Y_t = C_t + I_{kt} + P_t I_{ht}$. As one can see from figure 2-2, although the output in each sector does not display a hump-shape impulse response to a productivity shock, the composite output does. When the output in physical sector (i.e. $C_t + I_{kt}$) drops after the impact period of the shock, the output in the other sector (i.e. $P_t I_{ht}$) climbs up. The recovery of human sector out-performs the shrinking of physical sector in terms of magnitude for some periods after the impact period of the shock, which drives the composite output continue to grow even after the period when the impulse takes place. The response curve of composite output attained

from the two-sector SEG model peaks at around 6 quarters after the shock when the shrinking of physical goods production starts to dominate the recovery of investment to human capital. The composite output then decreases and converges to a new BGP which is modestly higher than the previous one.

The way that the composite output reacts to a productivity shock in physical sector can also be understood from the agent's perspective. This angle provides a deeper insight into the internal shock propagation mechanism of endogenous growth models relying on human capital formation. In exogenous growth models, apart from the direct positive technology effect (or direct income effect), expansions in the levels of both consumption and investment to physical capital following a positive shock are reinforced by the contemporaneous increase in labour supply. Capital stock is predetermined at the beginning of a period and hence is not adjustable after the shock. Thus changes in the labour supply acts as the only way through which the agent responds to unanticipated shocks in the impact period in standard exogenous growth models. This low degree of freedom in terms of the choice variables faced by the agent restricts her, to some extent, from allocating resources both intra and intertemporally. This lack of flexibility leads to the well-known shortcoming of most standard RBC models that they fail to generate the desired dynamics of output series that resemble the data. On the contrary, the endogenous growth model in this chapter gives room for two groups of extra margins. The first group regards the intratemporal resources distribution. After observing the shock, the agent is free to relocate time resource across three activities (leisure, working and learning) and physical capital across two sectors. This means that both physical and human capitals can be adjusted towards the sector with higher productivity. This higher degree of flexibility in resource relocation reduced the agent's loss of utility due to her inability to adjust the capital stock within period as in the standard RBC models. The second group of margins relate to the intertemporal choice faced by the agent who optimally invests in physical and human capital such that the returns to the two capital stocks are equalized. This is a well-known no-arbitrage condition in endogenous growth models with human

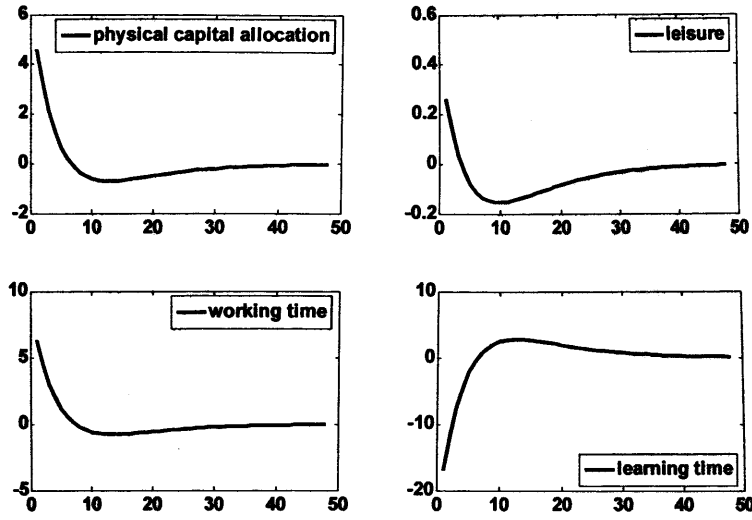


Figure 2-3: Impulse response functions of share variables

capital. These additional margins (both intra and intertemporal) embedded in endogenous growth models are potentially able to generate much richer internal dynamics that a sound business cycle model requires.

Figure 2-3 displays the responses of the “share variables” (shares of physical capital and time allocation across sectors) after a shock. These graphs show the inter-sectoral resources relocations after a shock. Notice that these variables are stationary variables by construction so that they always converge back to a unique steady state. When the productivity in physical sector goes up, the agent finds the opportunity cost of devoting time to accumulating human capital is higher and will consequently redirect time from human sector to physical sector. The intratemporal factor intensity condition requires physical capital and labour, in general, move to the same direction⁶. In other words, higher labour input in one sector must come with a higher physical capital input in the

⁶This statement is precisely true if labour-leisure choice is removed from the model. There would be a one-for-one relation between N_t and V_t . This is because of the complementarity of the two factors in

same sector. This is evident from the similarity between the responses of V_t and N_t as shown in the left half of figure 2-3. They both jump on the impact of the shock and “overshoot” their steady state values before reaching them. Leisure time fluctuates a lot less than working and learning hours. Hence learning time must behave very much the opposite to working hours as it is shown in the lower panel of figure 2-3. Following a positive shock to physical sector, working hours immediately jump while learning hours fall. From the second period after the shock, they both begin to move towards their BGP with some periods in the middle “overshooting” the steady state values. The dynamics of these “share variables” correspond to the impulse response functions as have seen in figure 2-2. One may find it surprising that leisure actually rises following a positive shock, but this comes up as a natural result when an agent is free to allocate time endowment among three different purposes (learning time as an additional margin compared to standard RBC models). In one-sector RBC models, when a positive shock occurs, substitution effect on labour supply induced by higher market wage usually dominates income effect in transitional periods. This results in higher working hours and less leisure enjoyment. But in the two-sector model where time spent on human capital accumulation as an additional margin is present, it is possible for leisure and working hours to increase simultaneously as learning time decreases sharply. In fact, if the representative agent are more willing to substitute intertemporally (e.g. when $\sigma > 2.5$), leisure would drop on the impact of a positive shock.

production. To see this, one can look at the factor intensities condition across sectors:

$$\frac{1 - \phi_1}{\phi_1} \frac{V_t}{N_t} = \frac{1 - \phi_2}{\phi_2} \frac{(1 - V_t)}{M_t}$$

Since agents have desires to smooth leisure enjoyment over time, working hours N_t , and learning time M_t , in general, move in opposite ways after shocks. Suppose N_t goes up for some reason. V_t , the share of physical capital in physical sector, has to go up to satisfy this inter-sectoral optimality condition.

2.5 Autocorrelation functions

The normalization method adopted to produce IRF in previous section does not guarantee a stationary solution: a new BGP will be triggered by a transitory shock such that all nonstationary variables do not converge back to their previous BGP values. This makes the first method invalid when one wants to simulate the model. Therefore, an alternative, but often-used, normalization method is adopted to transform the growth models into one with stationary steady state. Simulation then can be done based on the stationary solution and the autocorrelation function for the artificial output growth series can be computed. In particular, one can derive the first order conditions for the social planner's problem stated in section 2.2 and divide all growing variables by the current stock of human capital:

$$\begin{aligned}c_t &\equiv \frac{C_t}{H_t} \\k_t &\equiv \frac{K_t}{H_t} \\i_{kt} &\equiv \frac{I_{kt}}{H_t} \\\gamma_{ht+1} &\equiv \frac{H_{t+1}}{H_t}\end{aligned}$$

In stead of discounting the growing variables by the deterministic BGP growth rate, they are now expressed in ratios over a stochastic denominator. The transformed variables will have stationary distributions as long as the technology shock is not permanent. One can then log-linearize the new first order conditions locally around the unique nonstochastic BGP and finally simulate the model based on the linear recursive solutions. Since the focus of this simulation experiment is the output growth series and the model solution only permits results in ratios, one need to transform the ratios to first difference of aggregate macro variables. Appendix A contains the details. Using the recursive solution, the model is then simulated for 30,000 quarters to guarantee ergodic distributions of these variables. The autocorrelation functions are then computed based on these artificial

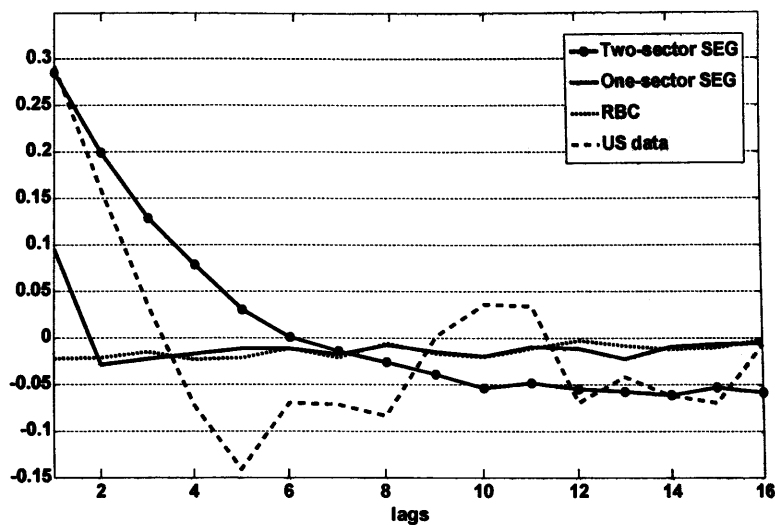


Figure 2-4: Comparing output growth autocorrelation functions of different models

series.

Figure 2-4 shows the autocorrelation functions of output growth computed from US data, the one-sector RBC model, JMS model and the two-sector endogenous growth model. In the data, the coefficients of the first two lags are significantly positive and most of the coefficients of higher orders are insignificantly negative. This pattern of the autocorrelation function indicates that US output series is strongly and positive auto-correlated over short horizons and has only weak and possibly negative autocorrelation over longer horizons. This is in accordance with the empirical findings by Cogley and Nason (1995). The question now is “which theoretical model predicts the autocorrelation function of output growth in a similar fashion?” The RBC model clearly fails the job. It predicts output growth is negatively and insignificantly autocorrelated along nearly all horizons. This is a well-known shortcoming of standard RBC models. The one-sector endogenous growth model as in JMS modestly improves the result on the first autocorrelation coefficient (from near zero to about 0.1), but its prediction on higher order

	$\rho\left(\gamma_{Y_t}, \gamma_{Y_{t-j}}\right)$			
	$j =$	1	2	3
US data		0.293	0.159	0.035
Two-sector SEG		0.286	0.199	0.128
One-sector SEG		0.095	-0.028	-0.022
RBC		-0.016	-0.016	-0.014

Table 2.3: Autocorrelation functions of different models

autocorrelation coefficients are at odds. This might be why JMS only report the first order autocorrelation coefficient. This problem has also been pointed out by Maury and Tripier (2003) to an earlier version of JMS model. On the contrary, the two-sector endogenous growth model replicates the autocorrelation pattern of output growth quite well, not only in short horizons, but also along longer horizons. In short horizon, the first order autocorrelation coefficient of output growth implied by the model is 0.286, a value only slightly lower than 0.293 in the data. This result seems to follow as a reflection of the hump-shaped IRF of output to a productivity shock. The long-lasting expansion of effect of a transitory shock after the impact period leads to the highly autocorrelated output growth series over short horizons. In the long run, autocorrelation coefficients become slightly negative as data suggest.

Table 2.3 reports the autocorrelation coefficients for the first three lags from the models discussed above. Only the first three coefficients are reported in the table because the autocorrelation of output growth is weak and insignificant after three lags both in the models and the data. Results in table 2.3 confirms the findings above: the two-sector SEG model generates greater output persistence than the other two models. For the second and third order autocorrelation coefficients, the two-sector SEG model even predicts greater persistence than the data counterparts (0.199 compared to 0.155 and 0.128 compared to 0.035). The hump-shaped IRF of output, along with the positively autocorrelation in output growth series, indicates the existence of a strong interior propagation mechanism in the two-sector SEG model.

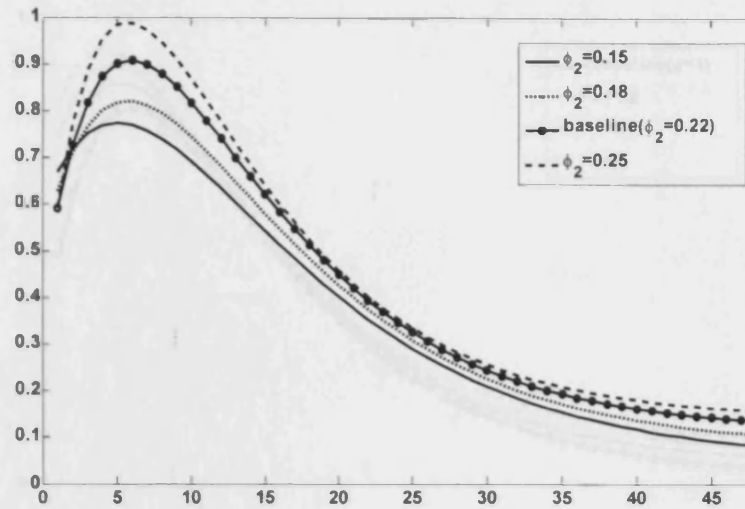


Figure 2-5: Output responses for different shares of physical capital in human sector

2.6 Sensitivity analysis

Since some parameters are poorly supported by empirical evidence, this section is devoted to explore the role of alternative calibrations of some variables on IRF of the composite output and autocorrelation function of its growth rate.

2.6.1 Impulse response functions

3.6.1.1 Sensitivity of the physical capital share in human sector (ϕ_2)

As shown in figure 2-5, the impulse response curve of the composite output becomes less hump-shaped as the capital share in human sector decreases from 0.25 to 0.15. The increasing part in the impulse response curve is attributed to the sustained recovery of investment to human capital dominating the shrinking of physical sector as the effect of the shock gradually dies out. When the physical capital share in human sector falls, the inter-sectoral factor relocation movements become less active following a shock to

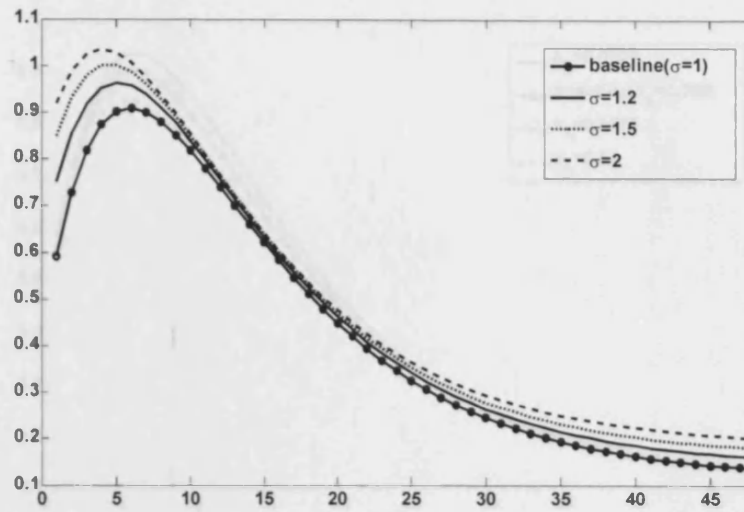


Figure 2-6: Output responses for different risk aversion parameters

physical sector. To some degree, this reduces the momentum of expanding investment to human capital in subsequent periods in keeping the composite output continue to grow. Therefore, when ϕ_2 becomes smaller, the contemporaneous impact of the shock on the composite output is larger, which is identifiable by the bigger jump of the composite output in the impact period, and the subsequent impact is smaller, which is identified by the lower peak in the impulse response curve.

3.6.1.2 Sensitivity of the coefficient of relative risk aversion (σ)

The effect of the coefficient of the relative risk aversion parameter is mainly exerted on the magnitude as we can see from figure 2-6. As σ increases, the representative agent has a higher tendency to smooth consumption path. This will suppress inter-sectoral factor flow on and after impact period so that the IRF of the composite output becomes less humped. This pattern will be translated into the autocorrelation function.

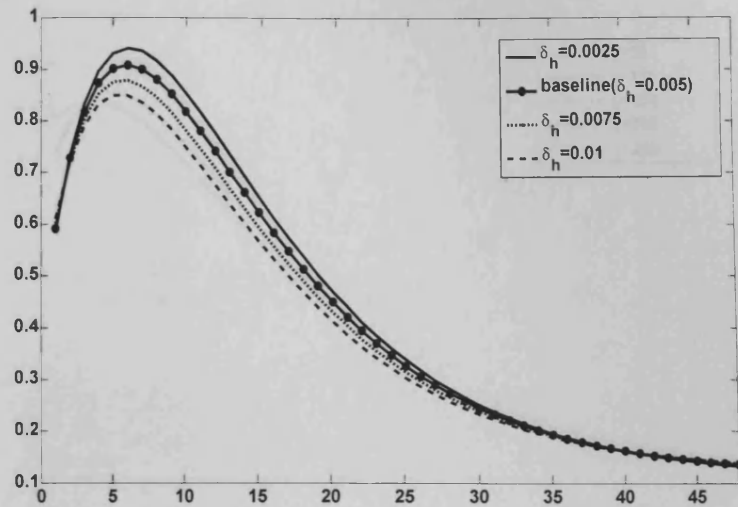


Figure 2-7: Output responses for different human capital depreciation rates

3.6.1.3 Sensitivity of the depreciation rate of human capital (δ_h)

Changing the value of depreciation rate of human capital has a similar effect as varying the capital share in human sector on the shape of the impulse response of the composite output. When human capital depreciates at a faster rate, after a positive shock to physical sector, it is optimal for the agent to relocate less labour time out of human sector into physical sector in order to maintain a certain level of investment to human capital such that physical and human capitals earn the same return. The suppressed inter-sectoral substitution restricts the expansion of the economy after the impact period of a shock. This is evident from the less humped impulse response curves in figure 2-7.

3.6.1.4 Sensitivity of adjustment cost function (η)

η measures the magnitude of physical adjustment cost. The smaller the value of η , the harder to transform physical investment into productive capital stock. As seen in figure 2-8, as η decreases, the degree of adjustment cost increases. This restricts factor

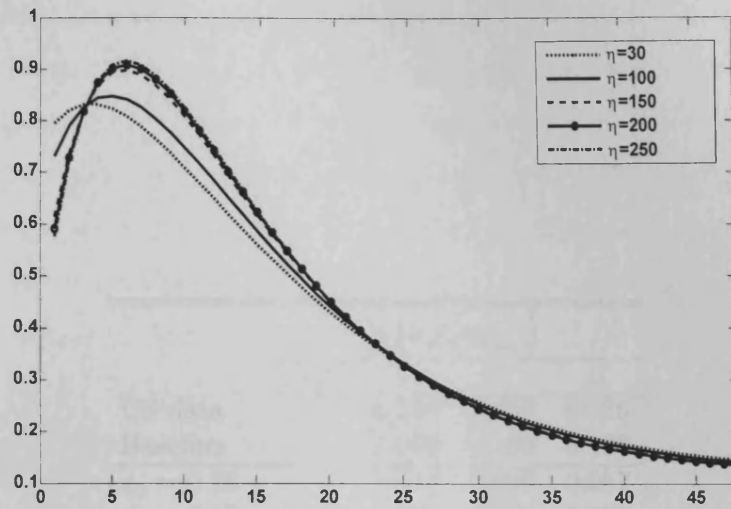


Figure 2-8: Output responses for different elasticities of adjustment cost

relocations across sectors after shocks. Hence, the impulse response of composite output becomes less humped. For example, when $\eta = 30$, the composite output goes up by about 0.8 percent, a relatively strong reaction on impact, and continues to increase only slightly for two quarters. On the other end when $\eta = 250$, indicating only mild degree of adjustment cost, output increases less than 0.6 percent on impact and keeps on increasing for 5 quarters.

The conclusion of the sensitivity analysis is that when the parameters change in a way to suppress inter-sectoral resource relocation on and after impact period, the IRF of composite output becomes less humped.

2.6.2 Autocorrelation functions

Table 2.4 displays the autocorrelation functions of the artificial output growth series under different sets of parameterizations. There are two results found from table 2.4. First, regardless of the parameterization choice, the stochastic endogenous growth model con-

	$\rho\left(\gamma_{Y_t}, \gamma_{Y_{t-j}}\right)$			
	$j =$	1	2	3
US data		0.293	0.159	0.035
Baseline		0.286	0.199	0.128
$\phi_2 = 0.15$		0.114	0.093	0.057
$\phi_2 = 0.18$		0.192	0.129	0.094
$\phi_2 = 0.25$		0.372	0.245	0.152
$\sigma = 1.2$		0.185	0.118	0.080
$\sigma = 1.5$		0.119	0.080	0.045
$\sigma = 2$		0.108	0.058	0.033
$\delta_h = 0.0025$		0.333	0.229	0.151
$\delta_h = 0.0075$		0.271	0.184	0.129
$\delta_h = 0.01$		0.230	0.158	0.104
$\eta = 30$		0.041	0.039	0.025
$\eta = 100$		0.200	0.143	0.103
$\eta = 150$		0.268	0.191	0.121
$\eta = 250$		0.328	0.219	0.145

Table 2.4: Sensitivity analysis of some key parameters

sidered in this chapter performs quite well in replicating the autocorrelation coefficients found in US output growth data. In all cases, the first two autocorrelation coefficients are significantly positive, which indicates that the model-generated output series is highly autocorrelated at least in the short run as in US data. Second, the autocorrelation function of output growth rate is clearly a reflection of the pattern of the IRF of output: there is greater autocorrelation of output growth rate when IRF of output is more humped. This is apparently evident from table 2.4. When parameters change in a way to mitigate the hump of IRF, autocorrelation coefficients of output growth rate decrease. For example, for $\phi_2 = 0.22$, the first order autocorrelation coefficient is very much the same as in the data; but when ϕ_2 reduces down to 0.15, the same coefficient is only 0.114.

2.7 Conclusion

This chapter attempts to formulate the idea of composite output in generating business cycle persistence. Following JMS, aggregate output is defined broadly to include investment to human capital. The results show that this two-sector SEG model successfully replicates the two stylized facts of output dynamics as summarized by Cogley and Nason (1995), which indicates the existence of a strong internal propagation mechanism in this model.

Due to unfortunate lack of data for human sector, some parameters are poorly evidence-supported, but the sensitivity analysis in this chapter shows that the two-sector model replicates the two observed regularities satisfactorily for wide ranges of those parameters. Except for the case where there is extremely high physical capital adjustment cost ($\eta = 30$), impulse response of output displays a clear hump following a positive shock to physical sector and simulated output growth exhibits significant persistence. An extreme assumption held throughout this chapter is the un-shocked human sector. This immaturity in treating human capital production function requires future efforts on the empirical works on human capital.

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Chapter 3

"Excess Sensitivity" Puzzle and Human Capital

3.1 Introduction

The debate on whether the Permanent Income Hypothesis (PIH) is an appropriate theory to explain aggregate consumption dynamics lasts for more than three decades ever since Hall (1978) opened the discussion in the post rational expectation revolution era. Hall's simple life-cycle version of PIH implies consumption follows a martingale process such that lagged income bears little useful information in predicting changes in current consumption. He finally concludes in favor of the theory. Subsequently, however, Flavin (1981) decisively rejects PIH because she finds current income still predicts changes in current consumption even after accounting for its impact on signaling changes in permanent income. Her findings are known as "excess sensitivity" puzzle in consumption theory, which casts serious doubt on the validity of PIH¹. More recently, Campbell and

¹However, Mankiw and Shapiro (1985) later show Flavin's findings on "excess sensitivity" of consumption to current income may be spurious results due to inappropriate detrending method that she hires. In particular, they show that if aggregate labour income series has a unit root, regressions using inappropriately detrended time series leads to seemingly "excess sensitivity" even if PIH is true. In addition, empirical evidence seems to support the assumption in Mankiw and Shapiro that US aggregate labour income data follow a difference-stationary process, other than a trend-stationary process as

Mankiw (1990, 1991) reraise the issue of "excess sensitivity" by arguing that a fraction of social income accrues to the so-called "rule-of-thumb" consumers who simply consume all their current income without worrying about their permanent income. They estimate the fraction of income that accrues to "rule-of-thumb" consumers out of total income about 50%, indicating tremendous departure from PIH. Moreover, their results appear robust across US and several European countries and are irrelevant to whether labour income has a unit root or not. Therefore, their results cast on-going challenge to PIH.

The aim of this chapter is to rationalize this consumption puzzle within the context of PIH. The model advocated in this chapter as an appropriate modern version of PIH that overcome this puzzle is a two-sector stochastic endogenous growth (SEG) model depending on purposeful human capital accumulation². For comparison purpose, I include in the analysis other two models being the one-sector SEG model in Jones Manuelli and Siu (2005b) (JMS hereafter) and a standard one-sector real business cycle (RBC) model.

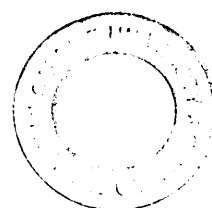
Both analytically and econometrically, I show that the apparent "excess sensitivity" between consumption growth and predictable current income growth can arise in the two-sector SEG model due to the joint existence of utility nonseparability between consumption and leisure and nonmarket time in producing human capital. The role of the first feature can be understood intuitively. When income growth is high, market wage goes up procyclically, which induces higher tendency to work and hence less time spent on leisure. Changes in leisure time affect marginal utility of consumption and hence consumption growth rate via utility nonseparability. The second feature has two impacts on the relation between consumption growth and predictable changes in current income. The first impact is direct. Existence of nonmarket time amplifies the linkage between consumption growth and predictable income growth attributed to nonseparabil-

previous thought (See Nelson and Plosser (1982) and Campbell and Mankiw (1987)). Thus, Flavin's findings of "excess sensitivity" thought as challenges to PIH should not be taken as seriously as was done in early time (See Nelson (1987), West (1988) and Diebold and Rudebusch (1991)).

²The word "purposeful" is used to distinguish the approach to modeling human capital formation in this thesis from the "learning-by-doing" approach. "Purposeful" human capital accumulation exhausts real resources while the latter approach assumes that human capital is a "by-product" of working time and accumulates automatically during working process.

ity. The more steady state effort agents devote to producing human capital, the greater the amplification effect is. The second impact of having nonmarket time is indirect. As the theory implies, changes of nonmarket time have predictive power for consumption growth, but they are omitted from the econometric model of "excess sensitivity" as Campbell and Mankiw specify. More importantly, since growth of nonmarket time is positively correlated with income growth, problem of missing variable makes estimation of the sensitivity parameter biased upward. Therefore, a researcher who is interested in estimating the mis-specified regression model is very likely to end up with high degree of "excess sensitivity" even if the true parameter is small enough to be rationalized by nonseparability alone.

There are also other efforts exerted in reconciling the consumption theory with "excess sensitivity". Most existing works rely on developing more sophisticated versions of PIH by allowing for new features that are absent in Hall's original life-cycle version. One direction to modify the simple version of PIH is to relax the assumption on constant interest rate. Example includes Michener (1984). The idea is that consumption growth reacts to interest rate in a way so that it is positively correlated with current income. Another direction is to introduce nonseparability to the utility function. Examples include nonseparability between durable and non-durable goods in Bernanke (1985); labour-leisure nonseparability in Eichenbaum et al. (1988); nonseparability between home and market consumption in Baxter and Jermann (1999); and more recently, "spirit of capitalism" in utility function in Luo et al. (2009). The idea lying under this line of research is that substitutions between consumption and other "items" that enter utility non-separately are potentially able to make consumption itself exhibit "excess sensitivity" to predictable change in current income even if the version of PIH that takes into account some sort of "nonseparability" is true in a broader sense. Models of pre-cautionary savings deviate from certainty-equivalent world by taking into account the effect of future uncertainty on consumption decision. Authors, such as Caballero (1990) and Wang (2006), show that agents accumulate pre-cautionary savings against future income eventuality by introduc-



ing heteroscedasticity in labour income process. They argue that pre-cautionary saving motive can make seemingly "excess sensitivity" of consumption arise naturally in a more generalized version of PIH.

The approach of endogenous growth is preferable to existing solutions for two reasons. First, it is a general equilibrium setup with an endogenous interest rate generating mechanism. Most previous works on consumption dynamics assume constant interest rate following Hall and therefore eliminate the possibility for consumption to adjust intertemporally due to changes of interest rate. The adjustment of consumption across periods to interest rate can be measured by elasticity of intertemporal substitution (EIS). Since empirical evidence on the magnitude of EIS is mixed³, it is pre-mature to rule out completely the interest rate channel in affecting consumption decision. The second and more important reason is that SEG model can generate income process that is difference-stationary, a characteristic of aggregate data. Previous studies on consumption puzzles make it clear the importance of specifications of income process in determining consumption property⁴. However, from a theoretical point of view, most modern macroeconomic models are stationary models such that innovations to permanent income only have temporary effect. Models in this group only generate income process that is trend-stationary or covariance-stationary. Due to the presence of human capital stock in addition to physical capital stock as a productive factor, labour income generated by SEG models is stationary in first difference. Consequently, even a temporal shock induces permanent shift in income stream.

The rest of this chapter is organized as follows. Section 3.2 describes all models discussed in this chapter. Section 3.3 briefly explains Campbell and Mankiw's regression on "excess sensitivity". Section 3.4 shows why the one-sector endogenous growth model

³Hall (1988) concludes that EIS of consumption is unlikely to be much above 0.1, and may well be zero. Some authors find the opposite. For example, Ogaki and Reinhart (1998) conclude in favor of significantly positive EIS when substitution between durable and nondurable goods is taken into account. More recently, Biederman and Goenner (2008) find EIS is likely between 0.2 and 0.8 when different specifications of preference are allowed.

⁴Recall that if income has a unit root, "excess sensitivity" might be simply a spurious econometric result so that it should not be taken seriously.

and the standard RBC model fail to reproduce "excess sensitivity". Section 3.5 explains why the two-sector endogenous growth model successfully reconciles the apparent "excess sensitivity" with PIH. Section 3.6 provides some concluding remarks.

3.2 Three models of permanent income hypothesis

This section describes two SEG models and one standard RBC model.

3.2.1 One-sector endogenous growth model

The first endogenous growth model is the one in JMS. In their model, there is a one-for-all good that is divided among consumption, investment to physical capital and investment to human capital:

$$Y_t = C_t + I_{kt} + I_{ht}$$

Y_t is the total amount of goods that are available in this economy at period t ; C_t is consumption; I_{kt} and I_{ht} are investment to physical and human capital respectively. Two things about this equation should be noticed. First, it leads to a perfect one-for-one relation among goods used for different purposes so that they are interchangeable costlessly. Second, this approach to modelling human capital entails accurate measurement of human capital investment since it constitutes part of aggregate output. However, empirical works on human capital measurement fall far behind theoretical advancements in this area. The ambiguity in accounting for human capital investment out of aggregate output makes it necessary to appeal for novel definitions of macro variables. Specifically, JMS define "measured consumption" as " $C_t + I_{ht}$ " by arguing activities to improve human capital stock, such as health care, schooling, are part of household expenditure.

The production of this economy takes Cobb-Douglas form in physical capital and "effective labour", which is the product of raw working time and human capital stock:

$$Y_t = Z_t K_t^{\phi_1} (N_t H_t)^{1-\phi_1}$$

K_t and H_t are the stocks of physical and human capital respectively at the beginning of period t ; " $N_t H_t$ " is "effective labour" input; ϕ_1 measures the contribution of physical capital in production process; Z_t stands for technology which follows an exogenous autoregressive process in log form: $\log Z_{t+1} = \rho_z \log Z_t + \varepsilon_{t+1}$. ρ_z , measuring the persistence of the shock, takes a value between 0 and 1; ε_t are a series of identically and independently distributed (i.i.d.) innovations. Two types of capitals evolve according to following laws of motions:

$$I_{kt} = K_{t+1} - (1 - \delta_k) K_t$$

$$I_{ht} = H_{t+1} - (1 - \delta_h) H_t$$

δ_k and δ_h are rates of depreciation of physical and human capital respectively. Note that, by construction, physical and human capitals are completely symmetric if δ_k equals δ_h ⁵.

An infinitely lived representative consumer maximizes her lifetime utility given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t L_t^A)^{1-\sigma} - 1}{1-\sigma}$$

E_0 is mathematical expectation at the initial period, L_t denote time spent on leisure; A measures the relative importance of leisure in providing utility compared to consumption; σ is the coefficient of relative risk aversion. When $\sigma = 1$, the momentary utility function reduces to logarithmic case: $\log C_t + A \log L_t$. The representative consumer is bound by time endowment constraint:

$$L_t + N_t = 1$$

The representative consumer in the one-sector model faces the following problem:

$$\begin{aligned} & \underset{C_t, N_t, H_{t+1}, K_{t+1}}{MAX} \quad E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t (1 - N_t)^A)^{1-\sigma} - 1}{1-\sigma} \\ & s.t. \quad C_t + K_{t+1} - (1 - \delta_k) K_t + H_{t+1} - (1 - \delta_h) H_t = Z_t K_t^{\phi_1} (N_t H_t)^{1-\phi_1} \end{aligned}$$

⁵In JMS baseline calibration, δ_k is greater than δ_h . They argue that this asymmetry between physical and human capital is important for several business cycle properties of their model.

3.2.2 Two-sector endogenous growth model

The second SEG model is different from previous one in terms of the way of modelling human capital production and hence the treatment of human capital investment. In particular, other than being produced in the same sector as what produces physical goods, human capital is accumulated in a separate sector with possibly a different technology which employs more labour input relative to that producing physical goods. This modification of the approach to treating human capital results in a two-sector SEG model. For convenience, the sector producing human capital investment is referred as "human sector" and the other sector producing consumption and physical capital investment is called "physical sector" throughout this chapter. Assume that human capital production also takes a form of Cobb-Douglas function⁶:

$$I_{ht} = A_h S_t ((1 - V_t) K_t)^{\phi_2} (M_t H_t)^{1-\phi_2}$$

ϕ_2 is the share of physical capital in this sector. According to common cognition, ϕ_2 is likely to be smaller than ϕ_1 , indicating, human sector is more labour intensive than physical sector. $(1 - V_t)$ is the share of physical capital allocated to this sector; M_t is the time devoted to improving human capital. For simplicity, M_t is referred as "learning time" although it may include other social activities that go beyond its literal meaning. S_t is the technology shock to this sector and it follows an exogenous autoregressive process in log form: $\log S_{t+1} = \rho_s \log S_t + \varepsilon_{t+1}^s$. ρ_s takes a value between 0 and 1; ε_t^s are i.i.d. innovations. A_h is a scale parameter. Technology in physical sector is:

$$C_t + I_{kt} = A_g Z_t (V_t K_t)^{\phi_1} (N_t H_t)^{1-\phi_1}$$

A_g is the scale parameter of this sector and other variables have their meanings as before.

⁶Gomme and Rupert (2007) show that in a two sector production economy, within the family of constant elasticity of substitution (CES) production functions, Cobb-Douglas is the only case consistent with BGP hypothesis.

One apparent benefit of having a separate technology producing human capital is that instead of being obliged to count human capital investment as part of GDP, one can treat it as nontradable goods such that other macro variables are defined canonically. Precisely, the notion of aggregate output in the two-sector model refer to the output of physical sector⁷: $Y_t = C_t + I_{kt}$. M_t is nonmarket time and N_t is market hours. Another advantage of the two-sector model is to relax the assumption implicitly made in the one-sector model that technology producing human capital investment is identical to that producing physical goods. Usual observations of some counterparts of human sector (education, job training) in real economy seem conformable with the argument that human sector is more labour intensive than physical sector. This feature is precluded from the one-sector model by construction.

Capitals accumulate according to:

$$I_{kt} = K_{t+1} - (1 - \delta_k) K_t$$

$$I_{ht} = H_{t+1} - (1 - \delta_h) H_t$$

Representative consumer maximizes her lifetime utility given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t L_t^A)^{1-\sigma} - 1}{1-\sigma}$$

and is subject to time constraint:

$$L_t + N_t + M_t = 1$$

Hence, the two-sector SEG model can be formulated as the following optimization

⁷This definition is also adopted by Perli and Sakellaris (1998) and DeJong and Ingram (2001)

problem:

$$\begin{aligned}
& \underset{C_t, N_t, M_t, H_{t+1}, K_{t+1}}{MAX} \quad E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t (1 - N_t - M_t)^A)^{1-\sigma} - 1}{1 - \sigma} \\
& s.t. \quad C_t + K_{t+1} - (1 - \delta_k) K_t = A_g Z_t (V_t K_t)^{\phi_1} (N_t H_t)^{1-\phi_1} \\
& \quad H_{t+1} - (1 - \delta_h) H_t = A_h S_t ((1 - V_t) K_t)^{\phi_2} (M_t H_t)^{1-\phi_2}
\end{aligned}$$

3.2.3 One-sector standard RBC model

I also include in the analysis of consumption puzzles one standard exogenous growth one-sector RBC model as in King et al. (1988a). Although, as argued before, stationary models are inappropriate context to study "excess smoothness" puzzle within, they provide useful comparisons with SEG models and help to get a deeper insight into the mechanism hidden in endogenous growth models. The model is summarized as:

$$\begin{aligned}
& \underset{C_t, N_t, K_{t+1}}{MAX} \quad E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t (1 - N_t))^{1-\sigma} - 1}{1 - \sigma} \\
& s.t. \quad C_t + K_{t+1} - (1 - \delta_k) K_t = Z_t K_t^{\phi_1} (X_t N_t)^{1-\phi_1} \\
& \quad X_{t+1} = (1 + \gamma) X_t
\end{aligned}$$

X_t is labour-augmented technology progress and grows at exogenous rate γ . Notice the connection between endogenous growth model and exogenous growth model. If one lets H_t in the endogenous growth model grow at a constant exogenous rate γ , other than via purposeful accumulation, the endogenous growth models will collapse down to a standard RBC model.

3.3 Campbell and Mankiw's regression

Campbell and Mankiw propose an alternative theory to PIH to explain observed consumption dynamics. They assume that a fraction of total income accrues to "rule-of-

thumb" consumers who do not care about intertemporal substitution and simply exhaust their current income. The rest of total income accrues to permanent-income consumers. To test the "rule-of-thumb" hypothesis against PIH, they estimate the following equation:

$$\Delta c_t = \mu + \lambda \Delta y_t + \theta r_{t-1}^f + \varepsilon_t \quad (3.1)$$

Where Δc_t and Δy_t are aggregate consumption growth and income growth, respectively; r_{t-1}^f is risk-free interest rate in previous period; ε_t is the revision of agents' assessment of their permanent income due to new information that becomes available only in period t . Campbell and Mankiw interpret λ the fraction of total income that accrues to "rule-of-thumb" consumers and θ the reciprocal of coefficient of relative risk aversion. They then test PIH against "rule-of-thumb" hypothesis by setting up the null $\lambda = 0$ and the alternative $\lambda > 0$. If λ is significantly greater than zero, PIH is rejected on the whole and a favorable theory to explain aggregate consumption movement would take into account compositions of "rule-of-thumb" consumers and permanent-income consumers. Their estimate of λ turns out to be about 50%, a result interpreted as large departure from PIH.

The goal of this chapter is to show that a researcher, who is interested in estimating the econometric model in equation (3.1), can wrongly interpret her result in favor of "rule-of-thumb" hypothesis because estimator of λ using Campbell and Mankiw approach can be seriously biased upward due to omitted variables problem when utility is nonseparable between consumption and leisure. The missing variable from the regression model is the nonmarket time spent on accumulating human capital. To achieve this goal, first, I show why utility nonseparability alone is unable to generate "enough" apparent "excess sensitivity" through two examples: the one-sector SEG model and the standard RBC model. Second, I show why the two-sector SEG model with human capital produced in a different sector successfully rationalizes high estimates of λ within the context of PIH.

3.4 One-sector endogenous growth model and the RBC model

The reason to organize these two one-sector models together is that they have similar implication on "excess sensitivity" issue. This will be seen later.

3.4.1 Analytical results

Here I show analytically that utility nonseparability between consumption and leisure can give rise to positive relation between consumption growth and predictable current income growth, but it seems unable to generate the "excess sensitivity" to the magnitude that matches Campbell and Mankiw's observation.

The first order conditions of the one-sector SEG model are⁸:

$$\frac{AC_t}{L_t} = W_t H_t \quad (3.2)$$

$$1 = E_t \beta \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{L_{t+1}}{L_t} \right)^{A(1-\sigma)} (1 + r_{t+1}) \right] \quad (3.3)$$

$$W_t H_t = (1 - \phi_1) \frac{Y_t}{N_t} \quad (3.4)$$

$$r_t = \phi_1 \frac{Y_t}{K_t} - \delta_k \quad (3.5)$$

$$L_t + N_t = 1 \quad (3.6)$$

Y_t , the measured aggregate output in this economy, is equal to $C_t + I_{kt} + I_{ht}$, which has to include the hard-to-measure investment to human capital due to the one-sector setup. Equation (3.2) is the intratemporal optimization condition that equates the marginal

⁸The one-sector SEG model is different from its exogenous counterpart only in the presence of human capital stock. However, for the purpose (to derive a relation between consumption growth and anticipated income growth) pursued in this section, whether human capital is present or not plays no role. Thus, the analytical results obtained from the one-sector SEG model also apply to its exogenous growth counterpart.

rate of substitution between consumption and leisure to their relative price. Equation (3.3) is the intertemporal condition that requires the ratio of marginal utility of current consumption and next-period consumption equal to the relative price of consumption in two consecutive periods, being interest rate $1 + r_{t+1}$. Equation (3.4) and (3.5) are the firm's optimal conditions that require the marginal returns to factors equal their respective cost. Equation (3.6) is the time endowment constraint. This system of equations is essentially the same with those of the standard RBC model. Denote a variable in lower-case the log form of that in capital letter except interest rate⁹: $x \equiv \log(X)$, so the growth rate of variable X in time $t+1$ from previous period is approximately $\Delta x_{t+1} \equiv x_{t+1} - x_t$ for all modelled variables. Thus the system of equations (3.2) (3.3) (3.4) (3.5) (3.6), can be rearranged in terms of the first-difference of all variables except r_t by using two approximations: $\log E_t(\cdot) \approx E_t \log(\cdot)$ and $\log(1 + \cdot) \approx \cdot$:

$$\Delta c_t = \Delta w_t + \Delta h_t + \Delta l_t \quad (3.7)$$

$$E_t[-\sigma \Delta c_{t+1} + A(1 - \sigma) \Delta l_{t+1} + r_{t+1}] = \rho \quad (3.8)$$

$$\Delta y_t = \Delta w_t + \Delta h_t + \Delta n_t \quad (3.9)$$

$$\Delta n_t = -\frac{L}{N} \Delta l_t \quad (3.10)$$

N and L are the steady state values of labour supply and leisure. $\rho = \frac{1-\beta}{\beta}$, being the time preference parameter. Above equations can be reduced down to a single equation that relates expected consumption growth to anticipated changes in current income:

$$\begin{aligned} E_t \Delta c_{t+1} &= \xi_1 E_t \Delta y_{t+1} + \tau_1 (r_t^f - \rho) \\ \xi_1 &= \frac{AN(\sigma - 1)}{\sigma + AN(\sigma - 1)} \\ \tau_1 &= \frac{1}{\sigma + AN(\sigma - 1)} \end{aligned} \quad (3.11)$$

⁹Since r_t is approximately $\log(1 + r_t)$.

Where r_t^f is risk-free interest rate. Equation (3.11) is essentially the theoretical counterpart of equation (3.1). The first order conditions of the model show that a positive ξ_1 is a natural implication of utility nonseparability rather than indicating the existence of "rule-of-thumb" consumers. In particular, if $\sigma = 1$, consumption growth of permanent income consumers is not associated with current income growth. Hence, empirically high estimate of ξ_1 has to be explained by some other attribute of consumers, such as "rule-of-thumb" as suggested by Campbell and Mankiw. But if $\sigma > 1$, indicating nonseparability between consumption and leisure in the utility function, a positive ξ_1 can naturally arise within this PIH framework. Given nonseparability as an alternative explanation for the apparent "excess sensitivity", there are two questions to answer: 1. can nonseparability alone QUANTITATIVELY account for the observed magnitude of sensitivity between consumption and current income with reasonable parameterization? 2. is this theoretical explanation for "excess sensitivity" empirically valid?

A quick check on the ability of this one-sector SEG model to replicate "excess sensitivity" can be implemented by looking at the expression of ξ_1 , the value of which is positive as long as σ is greater than one. Moreover, ξ_1 is an increasing function of σ because $\frac{\partial \xi_1}{\partial \sigma} > 0$ if $A > 0$ and $N > 0$, conditions always met in economics models. Therefore, consumption growth reacts to changes in current income more rigorously when the degree of nonseparability increases. However, utility nonseparability alone is inadequate to generate "excess sensitivity" to the observed degree (about 50% as reported in Campbell and Mankiw) even when σ is unrealistically high. This can be seen from an example. When $A = 1$, $N = 0.3$ and $\sigma = 5$, $\xi_1 = 0.19$, a value well below 0.5. In fact, $\lim_{\sigma \rightarrow \infty} \xi_1 = \frac{AN}{1+AN}$, for the example given before, the largest value for ξ_1 to achieve when σ approaches infinity is only 0.23. Thus, an immediate look at the expression of ξ_1 shows that although nonseparability can generate positive linkage between consumption growth and current income growth, it fails quantitatively.

The next question to answer is whether this alternative interpretation of high estimation of λ that hinges on the nonseparability between consumption and leisure is

empirically valid. A test on this was carried out by Campbell and Mankiw. They estimate a more general version of PIH that takes into account explicitly changes of labour supply, but unfortunately their results reject decisively the idea that variations in labour supply predict consumption growth. Hence, even if the nonseparability built in the one-sector SEG model is able to produce positive estimates of λ , empirical evidence offers little support for this explanation.

3.4.2 Econometric results

To provide a more rigorous answer to the question whether high estimates of λ can be recovered by the mechanism of nonseparability, I estimate the model in equation (3.1) using artificial samples generated by these one-sector models. The logic is that if estimate of λ using simulated series is high and significant, then "excess sensitivity" found by Campbell and Mankiw is explained by an alternative mechanism that hinges on utility nonseparability rather than the existence of "rule-of-thumb" consumers, and vice versa.

Estimating equation (3.1) using model generated series requires calibrating and solving the models beforehand. The one-sector SEG model is calibrated following JMS with special attention given to the parameter σ which governs utility nonseparability. JMS' original calibration is on yearly frequency because the human capital series that are available to them is annual. However, standard business cycle research uses data on quarterly frequency. Therefore, to facilitate comparison, I adopt the quarterly equivalence of their original calibration with the only exception being the shock process¹⁰. The results are summarized in table 3.1:

Calibration of a one-sector RBC model is fairly standard now in literature, so I will display it straightaway in table 3.2:

¹⁰It is a bit awkward to find quarterly counterparts of parameters associated to a shock on yearly basis directly, so the shock in the one-sector model is assumed to follow the same process as the shock in the two-sector model. This assumption has little impact on the issues discussed in this chapter anyway.

	Parameters	Annually	Quarterly
β	Discount factor	0.95	0.987
γ	BGP growth rate	0.0177	0.0044
N	Labour supply on the deterministic BGP	0.17	0.17
σ	Coefficient of relative risk aversion	1-10	1-10
ϕ_1	Share of physical capital	0.36	0.36
δ_k	Depreciation rate of physical capital	0.102	0.0265
δ_h	Depreciation rate of human capital	0.025	0.0063
A	Leisure weight in the utility function	6.36	6.36
ρ_z	Persistence parameter of shock	0.967	0.95
σ_z^2	Variance of the innovation	0.0001	0.0007

Table 3.1: Calibration of the one-sector SEG model

	Parameters	
β	Discount factor	0.987
γ	BGP growth rate	0.0042
N	Labour supply on the deterministic BGP	0.33
σ	Coefficient of relative risk aversion	1-10
ϕ_1	Share of physical capital	0.36
δ_k	Depreciation rate of physical capital	0.025
A	Leisure weight in the utility function	1.65
ρ_z	Persistence parameter of shock	0.95
σ_z^2	Variance of the innovation	0.0007

Table 3.2: Calibration of the one-sector RBC model

Since σ is the parameter that governs the degree of nonseparability between consumption and leisure, I adopt a wide range of values for it in experiments. The solution method to solve all models that are discussed in this chapter are collected in the technical Appendix A to this thesis.

Since changes in current income, as explanatory variables on the right hand side of equation (3.1), is correlated with disturbance, I estimate the model using instrumental variable (IV), an approach adopted by Campbell and Mankiw, Baxter and Jermann (1999). Generally speaking, this method amounts to find a set of variables (instruments) that are correlated with explanatory variables and uncorrelated with the error term. Then one can regress the dependent variable on the linear projections of the explanatory variables on instruments. Details of this econometric approach and implementation of it in this context are in Appendix B. It is natural to think of the lagged consumption or income growth as valid instruments for current income growth. r_{t-1}^f is risk-free interest rate and is pre-determined for period t , so it is not necessary to instrument r_{t-1}^f in theory. In practice, however, authors, such as Campbell and Mankiw, Baxter and Jermann (1999), also instrument r_{t-1}^f by its own lagged values. As a check on the robustness of the results, I adopt different sets of instruments in the regression¹¹.

Table 3.3 and 3.4 reports estimations of λ and θ using simulated samples of 30,000 periods from the one-sector SEG model and the standard RBC model, respectively. The first columns of these tables contain different values of coefficient of relative risk aversion used for simulations. The second and third columns report estimates of λ and θ by ordinary least square (OLS) method. The fourth and fifth columns contain estimates using instrumental variables approach. The last two columns display adjusted R^2 of regressing explanatory variables on all instruments, which indicates the quality of instruments¹².

¹¹Four instrument sets are used. They are $\{1, \Delta y_{t-1} \dots \Delta y_{t-3}, r_{t-1}^f\}$; $\{1, \Delta c_{t-1} \dots \Delta c_{t-3}, r_{t-1}^f\}$; $\{1, \Delta c_{t-1} \dots \Delta c_{t-3}, r_{t-2}^f \dots r_{t-4}^f\}$; $\{1, \Delta y_{t-1} \dots \Delta y_{t-3}, r_{t-2}^f \dots r_{t-4}^f\}$. The first two instrument sets do not include lag interest rates while the last two sets include lagged interest rates to stay in line with existing literature even if interest rate as explanatory variable is uncorrelated with the disturbance. However, uses of different instruments do not produce significantly different results. Therefore, table 3.3 and 3.4 do not report the instrument list used to produce the results under IV approach.

¹²Low R^2 is a common problem in using first-differenced lagged variables as instruments. Please see

	OLS		IV			
	λ	θ	λ	θ	R_y^2	R_r^2
$\sigma = 1$	0.026	0.886	0.028	0.862	0.03	0.807
$\sigma = 1.2$	0.212	0.639	0.092	0.596	0.02	0.815
$\sigma = 1.4$	0.315	0.509	0.179	0.434	0.02	0.817
$\sigma = 1.6$	0.381	0.443	0.236	0.399	0.01	0.825
$\sigma = 1.8$	0.426	0.384	0.262	0.289	0.01	0.835
$\sigma = 2$	0.460	0.353	0.297	0.276	0.01	0.838
$\sigma = 5$	0.624	0.194	0.311	0.063	0.004	0.849
$\sigma = 10$	0.706	0.124	0.314	0.045	0.003	0.870

Table 3.3: Regression results using simulated data from the one-sector SEG model

	OLS		IV			
	λ	θ	λ	θ	R_y^2	R_r^2
$\sigma = 1$	0.263	0.996	0.025	0.908	0.005	0.845
$\sigma = 1.2$	0.329	0.813	0.096	0.692	0.004	0.846
$\sigma = 1.4$	0.371	0.672	0.194	0.584	0.003	0.856
$\sigma = 1.6$	0.399	0.560	0.210	0.463	0.003	0.863
$\sigma = 1.8$	0.418	0.496	0.224	0.412	0.003	0.870
$\sigma = 2$	0.434	0.421	0.254	0.350	0.003	0.883
$\sigma = 5$	0.496	0.152	0.297	0.142	0.002	0.936
$\sigma = 10$	0.505	0.061	0.308	0.060	0.002	0.965

Table 3.4: Regression results using simulated data from the RBC model

As one can see, results in table 3.3 and 3.4 are quite alike. This is because the reduced forms of the two models impose exactly the same constraint on the relation among consumption growth, income growth and real interest rate. Although OLS results are known to be biased, they carry useful information on the role of nonseparability in affecting the correlation between consumption growth and income growth. For the complete separable utility case ($\sigma = 1$), regression result shows that variation in consumption growth is mostly explained by changes of interest rate. Income growth explains little of consumption growth. As σ increases, the role of income growth in explaining consumption movement strengthens while that of interest rate weakens. Estimations by IV method for different values of σ has a similar pattern to those by OLS. In general, nonseparability in the one-sector SEG model and the RBC model only generates moderate degree of "excess sensitivity" which is notably smaller than the observed magnitude. Notice that estimate of λ monotonically goes up as σ increases. According to JMS, their model fits the data best in terms of business cycle measurements when σ is 1.4. However, estimate of λ that comes out from the SEG model when σ is 1.4 is only 0.179, indicating too small "excess sensitivity". The highest estimate of λ obtained when $\sigma = 10$ is about 0.3, only slightly more than a half of the empirical counterpart found by Campbell and Mankiw. Moreover, such a high value of σ is quite unusual in macroeconomics. To summarize, nonseparability alone in the one-sector SEG model cannot produce adequate "excess sensitivity" even for too high values of σ .

3.5 Two-sector endogenous growth model

The two-sector SEG model assumes human capital investment takes place in a different sector from the sector in which consumption goods and physical capital investment are produced. More importantly, by assumption, human sector is a nonmarket sector so that time spent on producing new human capital is nonmarket time and output of this

Campbell and Mankiw (1990) and Baxter and Jermann (1999).

sector does not count as measured aggregate output. The setup of human sector is an analogue to home sector in the household production literature with the difference being the functions of the outputs in these nonmarket sectors: home sector produces home consumption that improves utility directly while human sector accumulates new human capital that improves productivity from next period onwards. The goal of this subsection is to show that this feature of the two-sector model together with nonseparability in an agent's utility function is able to produce apparent "excess sensitivity" up to the observed magnitude under reasonable parameterization of σ .

3.5.1 Analytical results

The first order conditions of the two-sector model are as follows:

$$\frac{AC_t}{L_t} = W_t H_t \quad (3.12)$$

$$1 = E_t \beta \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{L_{t+1}}{L_t} \right)^{A(1-\sigma)} (1 + r_{t+1}) \right] \quad (3.13)$$

$$W_t H_t = (1 - \phi_1) \frac{Y_t}{N_t} \quad (3.14)$$

$$r_t = \phi_1 \frac{Y_t}{K_t} - \delta_k \quad (3.15)$$

$$L_t + N_t + M_t = 1 \quad (3.16)$$

Recall that M_t is time spent on human capital production, or "learning time" for simplicity. The system is similar as before, but there are two changes to notice. The first one is measurement of aggregate output: Y_t in the two-sector model is defined by the sum of consumption and physical capital investment. Human capital investment is treated as nontradable goods and does not constitute part of Y_t . Second, total labour supply is disaggregated into time spent on producing physical goods and time spent on producing new human capital. The distinction between the two usages of non-leisure time is crucial

because "learning time" is treated as nonmarket time due to the nature of human sector by assumption. Therefore, "working time" in the two-sector model refers to time spent in physical sector which is the concept relevant to labour supply.

Using the approximation techniques again, the system is rearranged in terms of growth rates:

$$\Delta c_t = \Delta w_t + \Delta h_t + \Delta l_t \quad (3.17)$$

$$E_t [-\sigma \Delta c_{t+1} + A(1 - \sigma) \Delta l_{t+1} + r_{t+1}] = \rho \quad (3.18)$$

$$\Delta y_t = \Delta w_t + \Delta h_t + \Delta n_t \quad (3.19)$$

$$\Delta n_t = -\frac{L}{N} \Delta l_t - \frac{M}{N} \Delta m_t \quad (3.20)$$

M is the steady state fraction of time endowment spent on accumulating human capital stock. One observes that the only change in the two-sector model is the presence of Δm_t . Again, the system can be reduced to a single equation governing the relation between consumption growth and expected change in current income:

$$\begin{aligned} E_t \Delta c_{t+1} &= \xi_2 E_t (\Delta y_{t+1} + \Delta m_{t+1}) + \tau_2 (r_t^f - \rho) \\ \xi_2 &= \frac{AN(\sigma - 1)}{\sigma(1 - M) + AN(\sigma - 1)} \\ \tau_2 &= \frac{1}{\sigma(1 - M) + AN(\sigma - 1)} \end{aligned} \quad (3.21)$$

There are two changes when equation (3.21) is compared to equation (3.11). First, ξ_2 is greater than ξ_1 , other things being equal, due to the presence of $(1 - M)$ in the denominator of ξ_2 . The higher the fraction of time allocated to form human capital, the higher the coefficient associated to $E_t \Delta y_{t+1}$. The second change, which turns out to be more quantitatively important, is the presence of Δm_{t+1} in equation (3.21). If the two-sector SEG model is an appropriate version of PIH, the model estimated by Campbell and Mankiw is mis-specified for omitting time spent on improving human capital

stock¹³. More importantly, if Δm_{t+1} is positively correlated with Δy_{t+1} , estimator of λ in Campbell and Mankiw's model using data generated by equation (3.21) is biased upward. This is indeed what actually happens in the two-sector model. When productivity is high, agents improve human capital stock by increasing learning time. Hence, strong pro-cyclicality of human capital investment results in significant positive contemporaneous correlation between output growth and learning time growth. As a consequence, researchers who run a regression model specified by equation (3.1) when the true data generating process obeys equation (3.21) tend to over-estimate λ .

3.5.2 Econometric results

To justify quantitatively that high estimations of λ are spurious results due to omitted variable problem, I calibrate the two-sector SEG model exactly the same as what is done in section 1.3 in chapter 1. But for completeness of this chapter, I briefly review the calibration below.

All parameters stated below are on quarterly basis. The share of physical capital in physical sector is 0.36, a standard value in business cycle literatures. The subjective discount factor is fixed at 0.986. After the Korean War, US GDP, aggregate consumption and investment, on average, roughly grew at a common rate 0.42% per quarter. This pins down the deterministic balanced growth rate in the model. The depreciation rate of physical capital is set 0.02 to match the physical capital investment to output ratio (25.3%). The depreciation rate of human capital is 0.5%. Physical capital share in human sector is set 0.11 as benchmark. When the scale parameter of physical sector is normalized to one, BGP constraints pin down A_h to be 0.0461. The leisure time needed to give rise to 0.3 working time in physical sector is 0.54.

In the baseline calibration, the technology shock to human sector is assumed identical to the shock to physical sector. This is also the assumption implicitly made in the one-

¹³A similar critique to Campbell and Mankiw's specification is raised by Baxter and Jermann (1999) in whose model the omitted variables are unobservable factors associated with home production.

sector model in JMS. This seems to be a reasonable and efficient assumption to make, especially when any other effort trying to estimate the shock process only leads to large bias. A technology progress or shock in any form happening to physical sector is very likely to influence human sector in a similar way. One can think of the invention of internet that improves the productivity of education sector just as it does to the rest of the economy. Of course, economy-wide shock is a too extreme assumption to make. In the section of robustness check, this assumption will be relaxed to allow for sector-specific shocks. But in the baseline parameterization, the assumption of identical shocks is kept.

The coefficient of relative risk aversion is the parameter that captures the degree of nonseparability between consumption and leisure. Since nonseparability is the premise for the apparent "excess sensitivity" to arise, a wide range of the value of σ is used. For the two-sector SEG model, σ is chosen between 1 to 5. When $\sigma = 1$, momentary utility function reduces to logarithmic case, indicating separability between consumption and time spent on leisure. When $\sigma > 1$, marginal utility of consumption depends on leisure. Results on "excess sensitivity" are shown for various degrees of σ . The calibration of the model is summarized in table 3.5:

Equation (3.1) is then estimated using simulated series of 30,000 periods from the two-sector model. Table 3.6 summarizes the results for the two-sector model. The sixth column reports the contemporaneous correlation between output growth and changes in learning time for different values of σ . As it is shown, Δy_t and Δm_t are strongly contemporaneously correlated for all choices of σ . A regression that precludes the changes in learning time in predicting consumption growth is likely to over-state the responses of consumption growth to predictable output growth and consequently leads to an inappropriate rejection of PIH.

The second and third columns display OLS estimations of λ and θ . Estimator of λ increases as σ increases. Column four and five report IV estimates. The results show that even slight increase in σ exerts notable impact on estimations of λ and θ . In particular, estimator of λ increases from 0.036 to as high as 0.701 and estimator of θ decreases

Free parameters		
β	Subjective discount factor	0.986
γ	BGP growth rate	0.0042
σ	Coefficient of relative risk aversion	1-5
L	Steady state leisure time	0.54
ϕ_1	Share of physical capital in physical sector	0.36
ϕ_2	Share of physical capital in human sector	0.11
δ_k	Depreciation rate of physical capital	0.02
δ_h	Depreciation rate of human capital	0.005
A_g	Scale parameter of physical sector	1
$\rho_z = \rho_s$	Persistence parameter of shock	0.95
$\sigma_z^2 = \sigma_s^2$	Variance of innovation	0.0007
Implied by BGP when $\sigma = 1$		
r	Steady state interest rate	0.0185
A_h	Scale parameter of human sector	0.0461
N	Steady state working time	0.3
M	Steady state learning time	0.16
A	Weight of leisure in preference	1.55
$\frac{C}{Y}$	Steady state consumption-output ratio	0.75
$\frac{I_k}{Y}$	Steady state physical investment-output ratio	0.25
V	Steady state share of physical capital in physical sector	0.89

Table 3.5: Calibration of the two-sector SEG model

	OLS		IV				
	λ	θ	λ	θ	$\rho(\Delta y_t, \Delta m_t)$	R_y^2	R_r^2
$\sigma = 1$	0.306	0.683	0.036	0.913	0.832	0.15	0.94
$\sigma = 1.2$	0.625	0.293	0.071	0.523	0.959	0.01	0.92
$\sigma = 1.4$	0.703	0.221	0.270	0.312	0.979	0.003	0.91
$\sigma = 1.6$	0.742	0.189	0.527	0.194	0.985	0.002	0.90
$\sigma = 1.8$	0.766	0.171	0.633	0.159	0.988	0.001	0.89
$\sigma = 2$	0.784	0.157	0.701	0.149	0.989	0.0007	0.89
$\sigma = 5$	0.887	0.061	0.856	0.056	0.991	0.0006	0.88

Table 3.6: Regression results using simulated data from the two-sector SEG model

from 0.913 to 0.149 rapidly as σ goes up from 1 to 2. The sensitive reliance of the results on value of σ does not happen in the one-sector models (e.g. estimators of λ and θ in the one-sector SEG model go up and down from near 0 to 0.297 and from 0.862 to 0.276, respectively). This is due to the presence of the nonmarket time in the two-sector model and, more importantly, the strongly positive correlation between learning time growth and output growth. When $\sigma = 1.6$, a reasonable degree of risk aversion in macroeconomics, estimated λ is 0.527, a value very close to the empirical finding by Campbell and Mankiw. Therefore, the two-sector SEG model successfully rationalizes the apparent "excess sensitivity" within the context of PIH.

3.5.3 Robustness check

This section presents tests on the robustness of the results on "excess sensitivity" puzzle obtained in previous section. I first generalize the exogenous forces in the two-sector SEG model to allow for sector-specific shocks. The joint process of the sector-specific shocks can be collected in a vector autoregressive representation:

$$\begin{bmatrix} \log Z_{t+1} \\ \log S_{t+1} \end{bmatrix} = \begin{bmatrix} \rho_z & 0 \\ 0 & \rho_s \end{bmatrix} \begin{bmatrix} \log Z_t \\ \log S_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1}^z \\ \varepsilon_{t+1}^s \end{bmatrix}$$

ε_{t+1}^z and ε_{t+1}^s are i.i.d. disturbances to $\log Z_{t+1}$ and $\log S_{t+1}$, respectively. Elements in the upper-right and lower-left positions in the autocorrelation coefficient matrix are set to zero to prohibit technology diffusion across sectors. The variance-covariance matrix of the innovations is:

$$V \begin{bmatrix} \varepsilon_{t+1}^z \\ \varepsilon_{t+1}^s \end{bmatrix} = \begin{bmatrix} \sigma_z^2 & \sigma_{zs} \\ \sigma_{zs} & \sigma_s^2 \end{bmatrix}$$

$\sigma_{zs} = \rho_{zs}\sigma_z\sigma_s$, where ρ_{zs} is the correlation coefficient of ε_t^z and ε_t^s . Estimating the process of S_t is hard, so as a first step approximation, I will maintain the assumption that processes of Z_t and S_t have identical specification (namely, $\rho_z = \rho_s$ and $\sigma_z^2 = \sigma_s^2$). Realizations of Z_t and S_t , however, are different due to the randomness of innovations.

	Instrumental Variable		
	λ	θ	$\rho(\Delta y_t, \Delta m_t)$
$\rho_{zs} = 0$			
$\sigma = 1$	0.039	0.911	-0.922
$\sigma = 1.5$	-0.044	0.610	-0.911
$\sigma = 2$	-0.048	0.487	-0.913
$\sigma = 5$	-0.061	0.258	-0.899
$\rho_{zs} = 0.5$			
$\sigma = 1$	0.051	0.920	-0.794
$\sigma = 1.5$	-0.021	0.547	-0.778
$\sigma = 2$	-0.031	0.383	-0.773
$\sigma = 5$	0.014	0.169	-0.744
$\rho_{zs} = 0.9$			
$\sigma = 1$	0.040	0.914	-0.346
$\sigma = 1.5$	0.069	0.331	-0.154
$\sigma = 2$	0.171	0.215	-0.115
$\sigma = 5$	0.207	0.026	-0.041
$\rho_{zs} = 0.95$			
$\sigma = 1$	0.031	0.912	0.065
$\sigma = 1.5$	0.085	0.315	0.199
$\sigma = 2$	0.313	0.191	0.219
$\sigma = 5$	0.510	0.073	0.286
$\rho_{zs} = 0.99$			
$\sigma = 1$	0.032	0.915	0.390
$\sigma = 1.5$	0.374	0.253	0.745
$\sigma = 2$	0.774	0.169	0.768
$\sigma = 5$	0.875	0.080	0.796

Table 3.7: Regression results using simulated data for sector-specific shocks

Table 3.7 displays IV estimations of λ and θ using artificial samples for different values of ρ_{zs} . As one can see, when innovations to two shocks are uncorrelated at all, simulated Δy_t and Δm_t are negatively correlated and the model cannot produce positive estimate of λ for whatever value of σ . This is also the case when the correlation coefficient of innovations is 0.5. Recall that one important reason that the two-sector SEG model can produce significant positive estimate of λ is that the omitted growth rate of learning time is positively correlated with output growth. This omitted variable problem makes estimation of λ biased upward so that the apparent "excess sensitivity" arises. In the

Instrumental Variable ($\sigma = 1.6$)			
	λ	θ	$\rho(\Delta y_t, \Delta m_t)$
baseline($\phi_2 = 0.11, \delta_h = 0.005$)	0.527	0.194	0.985
$\phi_2 = 0.08$	0.411	0.194	0.982
$\phi_2 = 0.13$	0.456	0.268	0.984
$\phi_2 = 0.15$	0.620	0.295	0.986
$\phi_2 = 0.17$	0.290	0.335	0.989
$\phi_2 = 0.20$	0.057	0.247	0.992
$\delta_h = 0.0025$	0.684	0.273	0.987
$\delta_h = 0.0075$	0.146	0.267	0.976

Table 3.8: Regression results using simulated data for different values of some key parameters

simulation experiments, for low values of ρ_{zs} , the omitted variable and output growth are negatively correlated, so estimation of λ is biased to the opposite direction. This is why "excess sensitivity" does not emerge in the first two cases. When ρ_{zs} goes above 0.9, indicating innovations to two shocks are strongly and positively correlated, output growth and growth rate of learning time become positively correlated. For example, when ρ_{zs} is 0.95, correlation coefficient of Δy_t and Δm_t is always positive for all choices of σ , which biases estimator of λ upward to the observed magnitude. Specifically, when ρ_{zs} is as high as 0.99, estimation of λ reaches 0.774 when σ is 2, a value much higher than the findings in Campbell and Mankiw. Empirically, it is natural to expect technologies to physical and human sectors to move to the same direction since a technology progress is likely to be economy-wide. Therefore, high values of ρ_{zs} should not be regarded unusual.

In a nutshell, the assumption of identical shock maintained in the main body of this chapter is not essential for the two-sector SEG model to generate apparent "excess sensitivity". Using sector-specific shock is also able to reproduce high estimation of λ so long as innovations to sector-specific shocks are highly correlated, which should not be regarded as empirically unexpected.

Other parameters that are not strongly evidence-supported and are important to the quantitative performance of the model are the share of physical capital in human sector (ϕ_2) and depreciation rate of human capital (δ_h). Previous results on "excess sensitiv-

ity" suggest that the two-sector SEG model generates apparent "excess sensitivity" that matches empirical evidence the best when $\sigma = 1.6$. Hence, table 3.8 only displays estimations of λ and θ for σ equals 1.6. It is hard to recover values of ϕ_2 by calculating the share of income that returns to physical capital in human sector, so a relatively wide range of ϕ_2 is adopted in the sensitivity analysis. As table 3.8 suggests, for ϕ_2 is less than 0.2, the model implies estimation of λ that are in line with empirical evidence. However, as ϕ_2 increases, estimation of λ goes near zero, indicating nearly no "excess sensitivity". Hence, for the two-sector SEG model to rationalize the apparent "excess sensitivity", share of physical capital in human sector is required not to be too high. This does not seem to overcloud the prospect of the two-sector SEG model as an explanation for the apparent "excess sensitivity" because according to Perli and Sakellaris (1998), ϕ_2 probably has a value between 0.11 and 0.17.

Depreciation rate of human capital is now more standard in endogenous growth literature. Most works use 1% to 3% as a range for yearly depreciation rate. Estimations of λ and θ using simulated samples for different values of δ_h are in the last two rows of table 3.8. The results show that estimator of λ is very sensitive to this parameter. When δ_h reduces to 0.0025, estimated λ is 0.684, indicating high degree of "excess sensitivity". But when δ_h increases to 0.0075, implied λ is only 0.146. Hence, a more accurate calibration of δ_h is asked for in future empirical works on human capital research.

3.6 Conclusion

For a long time, "excess sensitivity" puzzle is regarded as evidence against permanent income hypothesis. This chapter of my thesis attempts to reconcile this puzzle with PIH by a version of stochastic endogenous growth model. The class of SEG models is arguably an appropriate context to study the consumption puzzles because labour incomes generated by this class of models follow difference-stationary process.

Campbell and Mankiw explain the apparent sensitive response of consumption to

changes of current income by the presence of "rule-of-thumb" consumers who simply consume all current income. However, I show in the first part of this chapter that apparent "excess sensitivity" can naturally arise in the two-sector SEG model for two reasons. First, consumption growth is mildly correlated with current income growth due to utility nonseparability between consumption and leisure. Second and more importantly, Campbell and Mankiw's econometric model is mis-specified by omitting changes of time spent on forming new human capital which is positively correlated with income growth. Therefore, their estimates over-state the parameter that captures the relation between consumption and current income. Simulation experiments show that the two-sector model successfully reproduces the apparent "excess sensitivity" which maybe wrongly interpreted as evidence against PIH.

Due to the lack of reliable measurement of human capital stock in literature, the technology shock to human sector is assumed identical to that to physical sector in benchmark calibration. This is an efficient assumption that helps to save unnecessary efforts beside the focus of this chapter, but obviously an extreme one. Although sensitivity analysis shows that the results of interest in this chapter also hold for sector-specific shocks so long as the two shocks are highly correlated, more efforts should be exerted to empirical works on human capital in the future to strengthen all arguments made in this chapter.

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Chapter 4

"Excess Smoothness" Puzzle and Human Capital

4.1 Introduction

Deaton (1987) finds that when labour income follows an integrated process (a characterization appears to be supported by evidence), the permanent income theory (PIH) predicts too much volatility of aggregate consumption. This is known as "Deaton's Paradox" or "excess smoothness" puzzle. His findings are understandable intuitively. When labour income series has a unit root, temporary shocks have permanent effect on income stream. Consequently, consumption that follows permanent income adjusts strongly to new information and hence exhibits too much variability. Therefore, PIH fails fundamentally in terms of the very first reason to construct it: explaining aggregate consumption smoothness.

Unfortunately, the role of variable interest rate in affecting consumption dynamics is left unexplored in Deaton's original framework. There are two possible reasons for this lack of attention paid to the role of variable interest rate in affecting consumption dynamics. The first and maybe more important one is the mixed empirical evidence on whether agents defer current consumption when interest rate is high. The responsiveness

of consumption growth to variations of interest rate is measured by EIS. Some empirical studies only find very low degree of EIS indicating changes in interest rate hardly affect agents' consumption decision. For example, Hall (1988) finds "the elasticity is unlikely to be much above 0.1, and may well be zero"¹. Given such a low degree of intertemporal substitution induced by changes of interest rate, maintaining the assumption of constant rate does not seem to jeopardize fundamentally any inference drawn based on that assumption. The second reason may be the difficulty in generating output series that follow difference-stationary process in a general equilibrium framework in which interest rate varies endogenously. Most modern macroeconomics models are trend-stationary models, which are not the context for "excess smoothness" puzzle to arise within. It is, therefore, interesting to see whether building in an endogenous interest rate generating mechanism while keeping the nonstationary property of labour income process solves Deaton's puzzle.

This chapter attempts to show that relaxing the assumption of constant interest rate maintained in Deaton's original framework is able to reconcile the puzzle with a modified version of PIH. To this end, this chapter first shows that when consumption is restricted to react to interest rate changes², "excess smoothness" puzzle arises in endogenous growth models but not in the standard RBC model. This is because temporary technology shocks induce permanent shift in an agent's income stream in endogenous growth models but only short-lasting effect on one's income in exogenous growth models. Second, this chapter shows that when interest rate plays a role in changing consumption allocations across periods, the two-sector SEG model produces reasonable consumption growth volatility due to the adjustment cost interpretation of human sector while the two

¹Some empirical works find the opposite as quoted in footnote 3 in chapter 3. And there are also works that attempt to reconcile low EIS in empirical literature and high EIS that are necessary in theoretical framework. For example, Guvenen (2006) associates high EIS to the rich whose behaviors determine properties of macro variables and low EIS to the poor whose decisions mainly affect aggregate consumption. Certainly, the topic of EIS is not a issue of this chapter, so a too high value of σ is regarded as empirially irrelevant.

²This has a similar effect to keeping interest rate constant. Technically, this is done by deliberately setting a low degree of elasticity of intertemporal substitution on consumption.

one-sector models (the one-sector SEG model and the standard RBC model) only predict over-smoothed consumption. In one-sector models, goods used for different purposes are of perfect one-for-one relationships so that consumption goods can be easily transformed into its substitutes. When interest rate is high, agents defer current consumption in favor of investments for future benefit without incurring any extra cost. Therefore, consumption is easily smoothed intertemporally. In contrast, substitution between consumption and human capital investment is subject to inter-sectoral cost of adjustment in the two-sector SEG model. This is because human sector employs labour input more intensively than physical sector. As a result, the economy-wide production possibility frontier (PPF) for outputs from two different sectors is concave, which indicates increasing marginal cost of producing one unit of one good in terms of foregone the other good. Therefore, an agent cannot smooth her consumption stream as perfect as she does when goods for different usages are perfect substitutes. Consumption growth in the two-sector model exhibits more variability than in the one-sector models.

There are other works in explaining the "excess smoothness" puzzle of consumption. Quah (1990) decomposes labour income into permanent and transitory component without altering the difference-stationary property of labour income series. He assumes agents have superior information advantages about their future income over econometricians such that they can distinguish between permanent and temporary shifts in income. Thus consumption adjusts to income changes less strongly than what an econometrician may find by assuming all innovations have only permanent effect. Diebold and Rudebusch (1991) find PIH predicts volatility of consumption growth in accordance with empirical evidence using fraction integrated representation of labour income process. Models of pre-cautionary savings deviate from certainty-equivalent world by taking into account the effect of future uncertainty on consumption decision. Authors, such as Caballero (1990) and Wang (2006), show that agents accumulate pre-cautionary savings against future income eventuality by introducing heteroscedasticity in labour income process. They argue that accounting for pre-cautionary saving motive solves "excess smoothness"

puzzle.

The approach of endogenous growth is preferable to existing solutions for two reasons. First, it is a general equilibrium setup with an endogenous interest rate generating mechanism. Most previous works on consumption dynamics assume constant interest rate following Hall and therefore eliminate the possibility for consumption to adjust intertemporally due to changes of interest rate. The adjustment of consumption across periods to interest rate can be measured by elasticity of intertemporal substitution (EIS). Since empirical evidence on the magnitude of EIS is mixed, it is pre-mature to rule out completely the interest rate channel in affecting consumption decision. The second and more important reason is that SEG model can generate income process that is difference-stationary, a characteristic of aggregate data. Previous studies on consumption puzzles make it clear the importance of specifications of income process in determining consumption property³. However, from a theoretical point of view, most modern macroeconomic models are stationary models such that innovations to permanent income only have temporary effect. Models in this group only generate income process that is trend-stationary or covariance-stationary. Due to the presence of human capital stock in addition to physical capital stock as a productive factor, labour income generated by SEG models is stationary in first difference. Consequently, even a temporal shock induces permanent shift in income stream.

In Deaton's original framework, "excess smoothness" regards the second moment property of consumption growth relative to that of innovation to permanent income. In modern general equilibrium framework when income process is endogenous, this notion, arguably, can be transformed to mean the relative volatility of consumption growth to that of income growth. The logic is that, given a general equilibrium model, a researcher can freely pick up a value for the variance of the innovation that induces aggregate fluctuation to match the volatility of income growth and compute the implied volatility of consumption growth. This is a valid approach to measure "smoothness" of consumption

³Recall that "excess smoothness" puzzle exists only when labour income process is nonstationary.

growth, because, in a certainty-equivalent world, the variance of innovation does not affect the relative volatility of consumption growth to income growth. If the model implied volatility of consumption growth in relative terms greater than the empirical counterpart, "excess smoothness" puzzle arises. Numerous empirical evidence has shown that volatility of consumption growth is about a half of income growth. Therefore, according to the "definition" above, "excess smoothness" puzzle arises when a model predicts that consumption growth fluctuates significantly more than a half of income growth.

In this chapter, three models are studied in the context of "excess smoothness": the one-sector SEG model, the two-sector SEG model and a standard RBC model. They are the same models as those in section 3.2 and calibrations of these models are also exactly the same as in chapter 3. Notice that since the case of identical shock in the two-sector SEG model is not significantly different from the case of sector-specific shocks as long as innovations to two shocks are highly correlated, the assumption of aggregate shock is maintained in the scenario of "excess smoothness" puzzle.

The rest of chapter is organized as follows. Section 4.2 and 4.3 display results on relative volatility of consumption growth for different degrees of EIS. Section 4.4 presents sensitivity analysis of the two-sector model. Section 4.5 concludes this chapter.

4.2 Low elasticity of intertemporal substitution

Since "excess smoothness" puzzle was born in partial equilibrium models in which interest rate is assumed constant, it is useful to inspect whether the same puzzle exists in general equilibrium models when the mechanism for consumption growth to respond to interest rate is largely shut down. This is done by deliberately setting a very low value of EIS. In SEG and most macro models, EIS is approximately equal to the reciprocal of the coefficient of relative risk aversion σ . Hence, modeling low degree of EIS requires a high value of σ . Table 4.1 reports the volatility of consumption growth and physical

	One-sector SEG model		Two-sector SEG model	
	$\frac{\sigma(\gamma_C)}{\sigma(\gamma_Y)}$	$\frac{\sigma(\gamma_{I_k})}{\sigma(\gamma_Y)}$	$\frac{\sigma(\gamma_C)}{\sigma(\gamma_Y)}$	$\frac{\sigma(\gamma_{I_k})}{\sigma(\gamma_Y)}$
US data	0.46	2.09	0.46	2.09
Baseline($\delta_h = 0.005, \phi_2 = 0.11$)	0.70	8.83	0.94	1.45
$\delta_h = 0.0025$	0.68	10.62	0.93	1.56
$\delta_h = 0.0075$	0.71	8.31	0.95	1.35
$\delta_h = 0.01$	0.72	7.16	0.97	1.25
$\phi_2 = 0.13$	-	-	0.93	1.58
$\phi_2 = 0.15$	-	-	0.91	1.66
$\phi_2 = 0.17$	-	-	0.90	1.79

Table 4.1: Consumption volatilities of SEG models when the risk aversion parameter is 10

capital investment growth relative to output growth implied by the one-sector and two-sector SEG models when σ equals 10 (although the focus is the models' implications on the volatility of consumption growth, volatility of physical investment growth is also reported alongside for information). The second row of table 4.1 reports the relative volatility of consumption growth and investment growth in US data. The data set used to calculate these statistics is from Gomme and Rupert (2007). As this data set suggests, consumption growth fluctuates about a half of output growth while investment growth fluctuates about twice as much as output growth. Many studies on the first difference of aggregate series come up with similar results (e.g. King et al. 1988b and JMS).

The first column of 4.1 contains various values of δ_h and ϕ_2 as a robustness check. The subsequent columns show the relative volatilities of consumption growth and investment growth. The results show that, for all values of δ_h and ϕ_2 ⁴, consumption growth rates implied by both models exhibit too much volatility relative to their empirical counterparts. For example, for the baseline calibration, the relative volatility of consumption growth rate is 0.70 in the one-sector model and 0.94 in the two-sector model, both larger than that in the US data (0.46). This indicates that when response of consumption to changes of interest rate is restricted by low degree of EIS, both SEG models successfully replicate

⁴By construction, ϕ_2 in the one-sector SEG model is equal to ϕ_1 .

	$\frac{\sigma(\gamma_C)}{\sigma(\gamma_Y)}$	$\frac{\sigma(\gamma_{I_k})}{\sigma(\gamma_Y)}$
US data	0.46	2.09
$\sigma = 1$	0.28	3.79
$\sigma = 5$	0.50	2.88
$\sigma = 10$	0.51	2.85
$\sigma = 15$	0.51	2.85
$\sigma = 20$	0.51	2.85

Table 4.2: Consumption volatilities of the one-sector RBC model for different risk aversion parameters

"excess smoothness" puzzle. One thing to notice is the magnitude of excessive volatility implied by two models. The two-sector SEG model produces excessive volatility of consumption growth on a much greater scale than does the one-sector model. Specifically, consumption growth implied by the two-sector model is roughly as volatile as output growth, but consumption growth volatility implied by the one-sector is about two thirds of output growth volatility, a value greater than the empirical evidence but smaller than its counterpart implied by the two-sector model. Regarding investment growth, the one-sector model predicts far too much variability in investment growth. This is because the role of adjustable human capital price in equalizing the returns to two capitals is missing from the one-sector SEG model (for a discussion on this, please refer to section 1.5.3).

For comparison purpose, I also calculate the moment statistics of consumption growth implied by the standard RBC model for various degree of σ . Table 4.2 presents the results. The range of σ spans from the logarithmic utility to 20, an extremely high degree of risk aversion in literature. The results show, even for high values of σ (*e.g.* 10, 15, 20), volatility of simulated consumption growth is only about a half of that of simulated output growth, a prediction very close to observation. Moreover, the relative volatility of consumption growth does not seem to increase when σ exceeds 10. On the other hand, modelled consumption appears too smooth for low degree of risk aversion ($\frac{\sigma(\gamma_C)}{\sigma(\gamma_Y)}$ equals 0.28 for logarithmic utility). In sum, the exogenous growth model is unable to produce excessive volatility of consumption growth for whatever values of σ . Instead, consumption is too

smooth in RBC models.

The failure of exogenous growth model to produce excess volatility can be understood from the effect of income shocks in two classes of models. In standard RBC models, exogenous shocks have only temporary effect on BGP as long as shocks *per se.* are not permanent. In particular, increase in income of a representative consumer due to a temporary upswing of technology lasts for only a few periods depending on the persistence of the shock. The permanent income of a consumer only goes up a little even if current income increases substantially. Therefore, knowing increase in current income is only temporary, a permanent-income consumer rationally smooth her consumption path over time such that current increase in consumption is small. On the contrary, when agents accumulate human capital in addition to physical capital, a feature captured by SEG models, even temporary shocks have permanent effect on consumers' income stream. Consequently, knowing good shocks shift up her permanent income as well as current income, a consumer adjusts current consumption more aggressively than does in exogenous growth models.

From the econometric point of view, "excess smoothness" puzzle arises when income is characterized by difference-stationary process and does not emerge if income is trend-stationary. This is exactly the difference between SEG models and exogenous growth RBC models. Simulated output is stationary in first difference in SEG models while output in exogenous growth model is stationary apart from a deterministic trend. Thus, SEG models form a natural laboratory to study "excess smoothness" paradox for their nonstationarity property.

4.3 High elasticity of intertemporal substitution

As the results in previous section show, "excess smoothness" puzzle arises naturally in SEG models when responses to interest rate are largely restricted by high values of σ , so the second step to take naturally is to find out how will lowering σ in SEG

	One-sector SEG model		Two-sector SEG model	
	$\frac{\sigma(\gamma_C)}{\sigma(\gamma_Y)}$	$\frac{\sigma(\gamma_{I_k})}{\sigma(\gamma_Y)}$	$\frac{\sigma(\gamma_C)}{\sigma(\gamma_Y)}$	$\frac{\sigma(\gamma_{I_k})}{\sigma(\gamma_Y)}$
US data	0.46	2.09	0.46	2.09
Baseline($\delta_h = 0.005, \phi_2 = 0.11$)	0.09	23.01	0.48	2.72
$\delta_h = 0.0025$	0.08	27.10	0.43	3.12
$\delta_h = 0.0075$	0.10	19.33	0.59	2.48
$\delta_h = 0.01$	0.11	15.12	0.68	2.14
$\phi_2 = 0.13$	-	-	0.46	3.08
$\phi_2 = 0.15$	-	-	0.43	3.29
$\phi_2 = 0.17$	-	-	0.38	3.47

Table 4.3: Consumption volatilities of SEG models when the risk aversion parameter is 1

models to allow consumption to react to interest rate affect results on the volatility of consumption growth. It is pointless to do this in the context of the exogenous growth model because it always predicts too much smoothness in consumption growth. Table 4.3 shows consumption volatility when utility function is logarithmic. Consumption growth rate implied by the one-sector model exhibits "too much" smoothness when σ equals one. This is evident from the second column of table 4.3. For all cases, the model implied consumption volatilities only account for less than a quarter of the US counterpart. For example, for the baseline calibration, the relative volatility of consumption growth rate implied by the one-sector model is only 0.09. The last two columns of table 4.3 reports the relative volatility of consumption growth implied the two-sector model. Generally speaking, the two-sector model predicts much more volatile consumption growth than the one-sector model. One thing worth mentioning is that when $\phi_2 = 0.13$, the two-sector fits the data precisely (0.46 compared to 0.46).

In general equilibrium models, income and interest rate always move shoulder-to-shoulder, but their impacts on agents' decisions on current consumption are opposite. Specifically, higher income encourages current consumption while higher interest rate makes one defer current consumption. Therefore, a reasonable consumption theory must take into account both factors. Deaton's original work only considered income effect on consumption and ignored interest rate effect completely, so he found excess volatility of

consumption growth. The two SEG models take into account both elements, but the interest rate effect in the one-sector model seems too overwhelming so that consumption becomes too smooth. On the other hand, thanks to the implicit adjustment cost via human sector in the two-sector model, consumption becomes reasonably volatile. The next subsection is devoted to explain the different implications of the two SEG models on consumption growth volatility by the inter-sectoral adjustment cost interpretation of human sector.

According to Mulligan and Sala-I-Martin (1993), in a two-sector production economy, the economy-wide production possibility frontier (PPF) for two goods produced in two different sectors is concave as long as the two sectors have different factor intensities⁵. The two-sector SEG model in this chapter satisfies this condition since, by assumption, human capital production is more labour intensive than physical goods production. Therefore, the opportunity cost for producing one good in terms of foregone the other good is increasing when resource relocation is in favor of the former good. This requires optimizing agent to spread this extra cost incurred by inter-sectoral resources transferring over time. The use of different factor intensities across sectors to give rise to inter-sectoral adjustment cost here is similar to Perli and Sakellaris (1998).

The inter-sectoral adjustment cost interpretation of human sector can account for the notable difference between the implications of the one- and two-sector SEG models in terms of the relative volatility of consumption growth to output growth. If goods used for consumption and those for investments are perfect substitutes, a feature in the one-sector model, agents will defer current consumption in favor of future consumption when return to investment is high. This perfect intratemporal substitution among goods used for different purposes smoothes the effect of a positive shock on consumption over time. As a result, consumption appears to be over-smoothed in the one-sector model. To produce more variable movement in consumption, the one-sector model has to resort to some other channel. This is achieved by reducing the sensitivity of consumption

⁵PPF is linear when two technologies are identical.

to changes of interest rate, effectively, lowering EIS of consumption. This is why the one-sector model relies heavily on high values of σ to replicate the observed degree of consumption variability. In contrast, the two-sector SEG model does not necessitate the use of low EIS to generate high fluctuation of consumption growth because of a naturally built-in mechanism via inter-sectoral adjustment cost, which makes substitution between consumption and investments, especially human capital investment, harder. In particular, when interest rate is high, agents are willing to smooth consumption over time. However, they are partially restricted to do so because redirecting resources in producing consumption goods to human sector is subject to increasing marginal cost. Consequently, agents does not substitute consumption intertemporally as much as they do when consumption and human capital investment are of perfect one-for-one relationship. The presence of inter-sectoral adjustment cost explains why the two-sector SEG model does not depends on high value of σ to make consumption volatile.

4.4 Robustness test

This section presents results on the second moment of consumption growth for the case where exogenous forces in the two-sector model are disaggregated into sector-specific shocks and innovations to shocks are highly correlated. As before, the shock to human sector is assumed to follow the same process as the shock to physical sector. Mathematically, it amounts to set $\rho_z = \rho_s$ and $\sigma_z^2 = \sigma_s^2$ in following representation of shocks:

$$\begin{bmatrix} \log Z_{t+1} \\ \log S_{t+1} \end{bmatrix} = \begin{bmatrix} \rho_z & 0 \\ 0 & \rho_s \end{bmatrix} \begin{bmatrix} \log Z_t \\ \log S_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1}^z \\ \varepsilon_{t+1}^s \end{bmatrix}$$

$$V \begin{bmatrix} \varepsilon_t^z \\ \varepsilon_t^s \end{bmatrix} = \begin{bmatrix} \sigma_z^2 & \sigma_{zs} \\ \sigma_{zs} & \sigma_s^2 \end{bmatrix}$$

Let ρ_{zs} denote correlation coefficient of innovations to two shocks: $\rho_{zs} = \frac{\sigma_{zs}}{\sigma_z \sigma_s}$. Table 4.4 shows the results for different degrees of ρ_{zs} when consumption is restricted to react to

	$\frac{\sigma(\gamma_C)}{\sigma(\gamma_Y)}$	$\frac{\sigma(\gamma_{I_k})}{\sigma(\gamma_Y)}$
US data	0.46	2.09
baseline($\rho_{zs} = 1$)	0.94	1.45
$\rho_{zs} = 0.7$	0.56	6.15
$\rho_{zs} = 0.8$	0.63	5.69
$\rho_{zs} = 0.9$	0.75	4.68
$\rho_{zs} = 0.95$	0.83	3.75
$\rho_{zs} = 0.99$	0.92	2.24

Table 4.4: Consumption volatilities of the two-sector SEG model for sector-specific shocks when the risk aversion parameter is 10

	$\frac{\sigma(\gamma_C)}{\sigma(\gamma_Y)}$	$\frac{\sigma(\gamma_{I_k})}{\sigma(\gamma_Y)}$
US data	0.46	2.09
Baseline($\rho_{zs} = 1$)	0.48	2.72
$\rho_{zs} = 0.7$	0.21	3.40
$\rho_{zs} = 0.8$	0.22	3.39
$\rho_{zs} = 0.9$	0.23	3.38
$\rho_{zs} = 0.95$	0.26	3.34
$\rho_{zs} = 0.99$	0.36	3.17

Table 4.5: Consumption volatilities of the two-sector model for sector-specific shocks when the risk aversion parameter is 1

changes of interest rate. When ρ_{zs} is as low as 0.7, the two-sector model still predicts volatility of consumption growth about 20% above the empirical observation. As ρ_{zs} increases, the model predicted volatility of consumption growth soars up very quickly. For ρ_{zs} equals 0.99, consumption growth fluctuates nearly as much as output growth, strong evidence of "excess smoothness".

Table 4.5 shows the relative volatilities of consumption when σ is equal to 1. Although consumption growth appears too smooth for all cases of sector-specific shocks, it still fluctuates much more than in the one-sector model. The sensitivity analysis confirms the adjustment cost interpretation of human sector which, to some extent, hinders consumption smoothing, but it seems that the two-sector model with sector-specific shocks still needs a slightly lower degree of EIS (higher σ) to produce consumption volatility that matches evidence.

4.5 Conclusion

This chapter shows that embedding an endogenous interest rate generating mechanism into PIH lowers the prediction of the theory on consumption growth volatility in line with empirical observations. Simulation results show that if consumers are prohibited from substituting consumption intertemporally by using high degree of risk aversion, the standard one-sector RBC model does not produce excessive volatility of consumption growth, but SEG models do. This is because a temporary shock in endogenous growth models has permanent effect on one's lifetime income stream so that consumption growth exhibits too much variability, but the same shock in a standard RBC model only has short-lasting effect so that consumption that follows permanent income adjusts mildly. For a reasonable value of the coefficient of relative risk aversion, the one-sector SEG model predicts too much smoothness, but predictions of the two-sector model match observations. This is because the inter-sectoral adjustment cost that results from factor intensity disparity across sectors in the two-sector model obstructs intertemporal substitution of consumption. To summarize, the two-sector SEG model as a modern version of PIH with endogenous interest rate generating mechanism is able to match the observed volatility of consumption growth with reasonable parameterization.

Due to the lack of reliable measurement of human capital stock in literature, the technology shock to human sector is assumed identical to that to physical sector in benchmark calibration. This is an efficient assumption that helps to save unnecessary efforts beside the focus of this chapter, but obviously an extreme one. Although sensitivity analysis shows that the results of interest in this chapter also hold for sector-specific shocks so long as the two shocks are highly correlated, more efforts should be exerted to empirical works on human capital in the future to strengthen all arguments made in this chapter.

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Appendix A

Solution method

Appendix A is devoted to illustrate the method used to solve all models discussed in my thesis in adequate detail. The solution method is log-linearization in Uhlig (1999). It involves linearizing the system of equations (with one or more of them in expectational form) that characterize the equilibrium of the optimization problem around its steady state and then solving the expectational difference equations using the method of undetermined coefficients. The solution is characterized in the form of two sets of recursive equations with the first set being evolutions of state variables and the second set being equations that express current control variables in terms of current state variables. Based on the solution, impulse response functions can be computed and the model can be simulated.

A.1 Solving the RBC model

The first order conditions of the one-sector exogenous growth model are:

$$\frac{AC_t}{1 - N_t} = (1 - \phi_1) Z_t \left(\frac{K_t}{X_t N_t} \right)^{\phi_1} X_t \quad (\text{A.1})$$

$$1 = E_t \beta \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{L_{t+1}}{L_t} \right)^{A(1-\sigma)} \times \left[\phi_1 Z_{t+1} \left(\frac{K_{t+1}}{X_{t+1} N_{t+1}} \right)^{\phi_1 - 1} + 1 - \delta_k \right] \right\} \quad (\text{A.2})$$

$$Z_t K_t^{\phi_1} (N_t X_t)^{1-\phi_1} = C_t + K_{t+1} - (1 - \delta_k) K_t \quad (\text{A.3})$$

The exogenous processes of Z_t and X_t are:

$$Z_{t+1} = Z_t^{\rho_z} e^{\varepsilon_{t+1}} \quad (\text{A.4})$$

$$X_{t+1} = (1 + \gamma) X_t \quad (\text{A.5})$$

Z_t captures the stochastic component of productivity with its persistence is measured by ρ_z , a value between 0 and 1 by assumption. ε_t is identically and independently distributed innovation with zero mean and standard deviation σ_z . X_t is the deterministic component of productivity with the net growth rate being γ .

The system characterizing the equilibrium of the model consists of three equations (i.e. equation (A.1) to (A.3)) in terms of three endogenous variables: C_t , K_t , N_t . But the system is non-stationary because of X_t . One can define stationary variables $c_t \equiv \frac{C_t}{X_t}$ and $k_t \equiv \frac{K_t}{X_t}$. N_t is stationary because the income and substitution effects on labour supply due to changes in wage exactly cancel out¹. Hence, N_t does not need to be divided by

¹To see this, consider a static model in which agents maximize $U(C, L) = \frac{(CL^\lambda)^{1-\sigma} - 1}{1-\sigma}$ subject to $C = W(1 - L)$ where W denotes market wage. The first order condition with respect to L reduces to: $L = \frac{A}{1-A}$ so that wage has no effect on labour supply.

X_t . Use the scaled variables to rewrite equation (A.1) to (A.3):

$$\frac{Ac_t}{1 - N_t} = (1 - \phi_1) Z_t \left(\frac{k_t}{N_t} \right)^{\phi_1} \quad (\text{A.6})$$

$$1 = E_t \beta (1 + \gamma)^{-\sigma} \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} \left(\frac{1 - N_{t+1}}{1 - N_t} \right)^{A(1-\sigma)} \times \left[\phi_1 Z_{t+1} \left(\frac{k_{t+1}}{N_{t+1}} \right)^{\phi_1 - 1} + 1 - \delta_k \right] \right\} \quad (\text{A.7})$$

$$c_t + (1 + \gamma) k_{t+1} - (1 - \delta_k) k_t = Z_t k_t^{\phi_1} N_t^{1-\phi_1} \quad (\text{A.8})$$

The steady-state solution to the problem can be found by removing time subscripts from the stationary version of first order conditions. This amounts to find solution to the following static system of three simultaneous equations in c, k, N , which stand for the steady-state values of c_t, k_t, N_t , respectively:

$$\frac{Ac}{1 - N} = (1 - \phi_1) \left(\frac{k}{N} \right)^{\phi_1} \quad (\text{A.9})$$

$$1 = \beta (1 + \gamma)^{-\sigma} \left[\phi_1 \left(\frac{k}{N} \right)^{\phi_1 - 1} + 1 - \delta_k \right] \quad (\text{A.10})$$

$$c + (\gamma + \delta_k) k = k^{\phi_1} N^{1-\phi_1} \quad (\text{A.11})$$

Notice that the steady-state value for Z_t is one. Then one can calibrate the model to fit certain aspects of a real economy and calculate the rest of model using equations (A.9) to (A.11).

Given the steady-state values for scaled consumption, physical capital and labour supply, one can define new variables that measure their percentage deviations from corresponding steady-state values: $\hat{\chi}_t \equiv \frac{\chi_t - \bar{\chi}}{\bar{\chi}}$, for $\chi_t = c_t, k_t$ and N_t . Take first-order Taylor expansion of equation (A.6) to (A.8) in the neighbourhood of c, k, N^2 and obtain

² Apply the formula of first-order Taylor approximation for multi-variable functions around $(\bar{x}_1, \bar{x}_2, \dots)$: $f(x_1, x_2, \dots) = f(\bar{x}_1, \bar{x}_2, \dots) + f'_1(\bar{x}_1, \bar{x}_2, \dots)(x_1 - \bar{x}_1) + f'_2(\bar{x}_1, \bar{x}_2, \dots)(x_2 - \bar{x}_2) + \dots$

a linearized system of equations involving \hat{c}_t , \hat{k}_t and \hat{N}_t :

$$\hat{c}_t + \left(\frac{N}{1-N} + \phi_1 \right) \hat{N}_t = \hat{Z}_t + \phi_1 \hat{k}_t \quad (\text{A.12})$$

$$\frac{AN(\sigma-1)}{1-N} \hat{N}_t - \sigma \hat{c}_t = E_t \left[\begin{aligned} & \left(\frac{AN(\sigma-1)}{1-N} + \frac{\phi_1^3 K^{\phi_1-1} N^{1-\phi_1} \beta}{(1+\gamma)^\sigma} \right) \hat{N}_{t+1} - \sigma \hat{c}_{t+1} + \\ & \frac{\phi_1 K^{\phi_1-1} N^{1-\phi_1} \beta}{(1+\gamma)^\sigma} \hat{Z}_{t+1} - \left(\frac{(\phi_1^3 \beta + \phi_1^2 \beta) K^{\phi_1-1} N^{1-\phi_1}}{(1+\gamma)^\sigma} \right) \hat{k}_{t+1} \end{aligned} \right] \quad (\text{A.13})$$

$$\begin{aligned} C \hat{c}_t + (1+\gamma) K \hat{k}_{t+1} - (\phi_1 K^{\phi_1} N^{1-\phi_1} + 1 - \delta_k) \hat{k}_t = \\ \phi_1 K^{\phi_1} N^{1-\phi_1} \hat{N}_t + \phi_1 K^{\phi_1} N^{1-\phi_1} \hat{Z}_t \end{aligned} \quad (\text{A.14})$$

Three equations can be divided into two subgroups: deterministic equations including (A.12) and (A.14); expectational equation (A.13). One can then reorganize the system in vector autoregressive form by collecting \hat{c}_t and \hat{N}_t in a vector and letting the endogenous state variable \hat{k}_t and exogenous state variable \hat{Z}_t be themselves. Express the system recursively in following format:

$$0 = A \hat{k}_{t+1} + B \hat{k}_t + D \begin{bmatrix} \hat{c}_t \\ \hat{N}_t \end{bmatrix} + F \hat{Z}_t \quad (\text{A.15})$$

$$0 = E_t \left(G \hat{k}_{t+1} + H \hat{k}_t + J \begin{bmatrix} \hat{c}_{t+1} \\ \hat{N}_{t+1} \end{bmatrix} + L \begin{bmatrix} \hat{c}_t \\ \hat{N}_t \end{bmatrix} + M \hat{Z}_{t+1} \right) \quad (\text{A.16})$$

Where A, B, F are 2×1 vectors; D is a 2×2 matrix; G, H, M are scalars; and J, L are 1×2 vectors. Equation (A.15) summarizes the two deterministic equations and equation (A.16) represents the expectational equation. Elements in $A, B, D, F, G, H, J, L, M$ are given numerically by the values of exogenous parameters and the steady-state solution of the model.

The system of equation (A.15) and (A.16) is solved by the method of undermined

coefficients. First, represent the solution by two recursive law of motions:

$$\hat{k}_{t+1} = P\hat{k}_t + Q\hat{Z}_t \quad (\text{A.17})$$

$$\begin{bmatrix} \hat{c}_t \\ \hat{N}_t \end{bmatrix} = R\hat{k}_t + S\hat{Z}_t \quad (\text{A.18})$$

P and Q are scalars and R and S are 2×1 vectors. The first equation captures the evolution of the endogenous state variable \hat{k}_t and second equation is the policy function that determines the control variables given current state. To solve for P, Q, R, S , substitute the recursive equilibrium law of motions back into equations (A.15) and (A.16):

$$0 = (AP + B + DR)\hat{k}_t + (AQ + DS + F)\hat{Z}_t \quad (\text{A.19})$$

$$\begin{aligned} 0 &= (GP + H + JRP + LR)\hat{k}_t + \\ &\quad (GQ + JRQ + JS\rho_z + LS + M\rho_z)\hat{Z}_t \end{aligned} \quad (\text{A.20})$$

Since these two equations hold for any value of \hat{k}_t and \hat{Z}_t , the coefficients associated to them should be zero:

$$0 = AP + DR + B \quad (\text{A.21})$$

$$0 = (G + JR)P + H + LR \quad (\text{A.22})$$

$$0 = AQ + DS + F \quad (\text{A.23})$$

$$0 = GQ + JRQ + JS\rho_z + LS + M\rho_z \quad (\text{A.24})$$

Combine equation (A.21) and (A.22) to eliminate R , one can end up with a quadratic equation in P :

$$0 = -JD^{-1}AP^2 + (G - JD^{-1}B - LD^{-1}A)P + H - LD^{-1}B \quad (\text{A.25})$$

D must be nonsingular to get this equation. The recursive equilibrium law of motion is

stationary *iff* the solution for P lies within unit circle. R can be calculated by

$$R = -D^{-1}(AP + B) \quad (\text{A.26})$$

Q and S can be found by combining equation (A.23) and (A.24):

$$Q = [G + JR - (J\rho_z + L)D^{-1}A]^{-1} [(J\rho_z + L)D^{-1}F - M\rho_z] \quad (\text{A.27})$$

$$S = -D^{-1}(AQ + F) \quad (\text{A.28})$$

Again, the matrix D must be nonsingular.

Now the solution of the model can be fully characterized by three equations:

$$\hat{k}_{t+1} = P\hat{k}_t + Q\hat{Z}_t \quad (\text{A.29})$$

$$\begin{bmatrix} \hat{c}_t \\ \hat{N}_t \end{bmatrix} = R\hat{k}_t + S\hat{Z}_t \quad (\text{A.30})$$

$$\hat{Z}_{t+1} = \rho_z\hat{Z}_t + \varepsilon_{t+1} \quad (\text{A.31})$$

The solution can be used to evaluate the implications of the model. Two commonly adopted techniques to examine the performance of the model are impulse response functions (IRF) and Monte Carlo simulation, both of which can be performed based on this solution. IRF measures the reactions of variables to one percent increase in ε_t in a particular period. This can be done by setting $\varepsilon_1 = 1$ and $\varepsilon_t = 0$ for $t > 1$, and calculating the rest of the model recursively onwards given \hat{k}_0 and \hat{Z}_0 being zero. Simulation is done by first randomly generating realizations of ε_t for as many times as desired from a normal distribution with zero mean and given standard deviation. Second, given the initial state, one can compute the rest of the model recursively onwards according to the equilibrium law of motions. This artificial sample of the modelled variables is used to calculate some statistics of interest, such as volatilities, autocorrelations and cross-correlations.

A.2 Solving the endogenous growth models

Log-linearization method is only applicable to stationary models in which all variables are constant in deterministic steady state. SEG models do not have stationary steady state so that log-linearization method cannot be applied directly. However, if nonstationary variables in SEG models are normalized somehow to achieve stationarity, the method will be valid for the transformed system. In this thesis, two different normalization methods are used for different purposes. The first method (referred as deterministic discounting) that is to discount nonstationary variables by deterministic balanced growth rate is used to compute impulse response functions. The second method (referred as stochastic discounting) that is to discount nonstationary variables by current stock of human capital is used to simulate the model. In the rest of this appendix, I will show how these two normalization methods are applied to the two-sector SEG model, and this approach is easily replicable to other SEG models, such as the one-sector model in JMS and the two-sector model with cost of adjustment to physical capital investment.

A.2.1 Stochastic discounting

The first order conditions of the two-sector model and the constraints are:

$$\frac{AC_t}{1 - N_t - M_t} = (1 - \phi_1) Z_t \left(\frac{V_t K_t}{N_t H_t} \right)^{\phi_1} H_t \quad (\text{A.32})$$

$$\frac{(1 - \phi_1) V_t}{\phi_1 N_t} = \frac{(1 - \phi_2) (1 - V_t)}{\phi_2 M_t} \quad (\text{A.33})$$

$$P_t = \left(\frac{\phi_1}{\phi_2} \right)^{\phi_2} \left(\frac{1 - \phi_1}{1 - \phi_2} \right)^{1 - \phi_2} \left(\frac{V_t K_t}{N_t H_t} \right)^{\phi_1 - \phi_2} \quad (\text{A.34})$$

$$1 = E_t \beta \left\{ \left[\left(\frac{C_t}{C_{t+1}} \right)^\sigma \left(\frac{1 - N_{t+1} - M_{t+1}}{1 - N_t - M_t} \right)^{A(1 - \sigma)} \times \right. \right. \\ \left. \left. \phi_1 Z_{t+1} \left(\frac{V_{t+1} K_{t+1}}{N_{t+1} H_{t+1}} \right)^{\phi_1 - 1} + 1 - \delta_k \right] \right\} \quad (\text{A.35})$$

$$1 = E_t \beta \left\{ \left(\frac{P_{t+1}}{P_t} \right) \left(\frac{C_t}{C_{t+1}} \right)^\sigma \left(\frac{1-N_{t+1}-M_{t+1}}{1-N_t-M_t} \right)^{A(1-\sigma)} \times \left[(N_{t+1} + M_{t+1}) (1 - \phi_2) S_{t+1} \left(\frac{(1-V_{t+1})K_{t+1}}{M_{t+1}H_{t+1}} \right)^{\phi_2} + 1 - \delta_h \right] \right\} \quad (\text{A.36})$$

$$C_t + K_{t+1} - (1 - \delta_k)K_t = A_g Z_t (V_t K_t)^{\phi_1} (N_t H_t)^{1-\phi_1} \quad (\text{A.37})$$

$$H_{t+1} - (1 - \delta_h)H_t = A_h S_t [(1 - V_t)K_t]^{\phi_2} (M_t H_t)^{1-\phi_2} \quad (\text{A.38})$$

And Z_t and S_t are governed by an exogenous vector autoregressive process:

$$\begin{bmatrix} \log Z_{t+1} \\ \log S_{t+1} \end{bmatrix} = N \begin{bmatrix} \log Z_t \\ \log S_t \end{bmatrix} + \varepsilon_{t+1} \quad (\text{A.39})$$

Where N is $\begin{bmatrix} \rho_z & 0 \\ 0 & \rho_s \end{bmatrix}$ and ε_t is $\begin{bmatrix} \varepsilon_t^z \\ \varepsilon_t^s \end{bmatrix}$.

The system that consists of seven equations in terms of seven endogenous variables ($C_t, K_{t+1}, H_{t+1}, V_t, N_t, M_t, P_t$) is non-stationary because C_t, K_t and H_t are growing in steady-state. To achieve stationarity, define new variables in the following way:

$$\begin{aligned} c_t &\equiv \frac{C_t}{H_t} \\ k_t &\equiv \frac{K_t}{H_t} \\ \gamma_{ht+1} &\equiv \frac{H_{t+1}}{H_t} \end{aligned}$$

γ_{ht} is the gross growth rate of human capital stock. Rewrite the system in terms of stationary variables:

$$\frac{Ac_t}{1 - N_t - M_t} = (1 - \phi_1) Z_t \left(\frac{V_t k_t}{N_t} \right)^{\phi_1} \quad (\text{A.40})$$

$$\frac{(1 - \phi_1) V_t}{\phi_1 N_t} = \frac{(1 - \phi_2) (1 - V_t)}{\phi_2 M_t} \quad (\text{A.41})$$

$$P_t = \frac{Z_t}{S_t} \left(\frac{\phi_1}{\phi_2} \right)^{\phi_2} \left(\frac{1 - \phi_1}{1 - \phi_2} \right)^{1-\phi_2} \left(\frac{V_t k_t}{N_t} \right)^{\phi_1 - \phi_2} \quad (\text{A.42})$$

$$1 = E_t \beta \left\{ \left(\frac{c_t}{c_{t+1}} \right)^\sigma \gamma_{ht+1}^{-\sigma} \left(\frac{1-N_{t+1}-M_{t+1}}{1-N_t-M_t} \right)^{A(1-\sigma)} \times \left[\phi_1 Z_{t+1} \left(\frac{V_{t+1}k_{t+1}}{N_{t+1}} \right)^{\phi_1-1} + 1 - \delta_k \right] \right\} \quad (\text{A.43})$$

$$1 = E_t \beta \left\{ \left(\frac{P_{t+1}}{P_t} \right) \left(\frac{c_t}{c_{t+1}} \right)^\sigma \gamma_{ht+1}^{-\sigma} \left(\frac{1-N_{t+1}-M_{t+1}}{1-N_t-M_t} \right)^{A(1-\sigma)} \times \left[(N_{t+1} + M_{t+1}) (1-\phi_2) S_{t+1} \left(\frac{(1-V_{t+1})k_{t+1}}{M_t} \right)^{\phi_2} + 1 - \delta_h \right] \right\} \quad (\text{A.44})$$

$$c_t + k_{t+1} \gamma_{ht+1} - (1 - \delta_k) k_t = A_g Z_t (V_t k_t)^{\phi_1} N_t^{1-\phi_1} \quad (\text{A.45})$$

$$\gamma_{ht+1} - 1 + \delta_h = A_h S_t [(1 - V_t) k_t]^{\phi_2} M_t^{1-\phi_2} \quad (\text{A.46})$$

The next step is to rewrite these equations in steady state and calibrate the model to fit some major macro economy facts given the steady state constraints are binding. Log-linearization method is now applicable to this transformed system. First, apply first order Taylor expansion for each individual equation around the steady state. Although this is a straightforward exercise, it is a bit awkward to display all linearized equations due to the length of some equations. To summarize the result in word, the linearized system involves seven difference equations in seven variables: $\hat{c}_t, \hat{V}_t, \hat{N}_t, \hat{M}_t, \hat{P}_t, \hat{k}_t, \hat{\gamma}_{ht}$. Next, condense the system in vector form with distinction made between deterministic equations and expectational equations. To simplify notation, let $y_t = [\hat{c}_t, \hat{V}_t, \hat{N}_t, \hat{M}_t, \hat{P}_t]'$, a vector collecting all control variables; and $x_t = [\hat{k}_t, \hat{\gamma}_{ht}]'$, containing two endogenous state variables³; and $u_t = [\hat{Z}_t, \hat{S}_t]'$, containing exogenous state variables. Thus, the system is reorganized as follows:

$$0 = Ax_{t+1} + Bx_t + Dy_t + Fu_t \quad (\text{A.47})$$

$$0 = E_t (Gx_{t+1} + Hx_t + Jy_{t+1} + Ly_t + Mu_{t+1}) \quad (\text{A.48})$$

Where A, B, F are 5×2 matrices; D is a 5×5 matrix; G, H, M are 2×2 matrices; and J, L

³ Although $\hat{\gamma}_{ht}$ is named an endogenous state variable here, the policy function does not depend on this variable. This is because γ_{ht} is not present in the system of equations from A.40 to A.46 (only γ_{ht+1} exists). Therefore, the only effective state variable is \hat{k}_t .

are 2×5 matrices. Equation (A.47) summarizes five deterministic equations and equation (A.48) represents two expectational equations. Elements in $A, B, D, F, G, H, J, L, M$ are given numerically by the values of exogenous parameters and the steady state solution of the model. As before, represent the solution to this system by two equilibrium recursive law of motions:

$$x_{t+1} = Px_t + Qu_t \quad (\text{A.49})$$

$$y_t = Rx_t + Su_t \quad (\text{A.50})$$

Where P and Q are 2×2 matrices and R and S are 5×2 matrices. Substituting the two recursive equations back into equation (A.47) and (A.48) and equating coefficient matrices associated to x_t and u_t to zero lead to four simultaneous matrix equations in P, Q, R and S . Solving these matrix equations will complete characterizing the solution. According to Uhlig (1999),

- P satisfies the matrix quadratic equation

$$0 = -JD^{-1}AP^2 + (G - JD^{-1}B - LD^{-1}A)P + H - LD^{-1}B \quad (\text{A.51})$$

Notice that since there are two endogenous state variables (\hat{k}_t and $\hat{\gamma}_{ht}$) in this case, P is a 2×2 matrix, other than a scalar in the one-sector RBC model. Hence, solving for P requires solving this matrix quadratic equation. Again, a necessary condition for this quadratic equation to make sense is matrix D is nonsingular.

- R is given by

$$R = -D^{-1}(AP + B) \quad (\text{A.52})$$

- Q satisfies

$$\left(-N' \otimes JD^{-1}A + I_2 \otimes (JR + G - LD^{-1}A)\right) Vec(Q) = \quad (A.53)$$

$$Vec\left((JD^{-1}F - M)N + LD^{-1}\right) \quad (A.54)$$

Where $Vec(\cdot)$ is column-wise vectorization; \otimes is Kronecker product; I_2 is identity matrix of size 2×2 .

- S is given by

$$S = -D^{-1}(AQ + F) \quad (A.55)$$

The crucial part in deriving the solution is to solve the matrix quadratic equation in (A.51). For detailed illustration of the method, please refer to Uhlig (1999). To have a stationary recursive solution, one should pick up the solution for P whose eigenvalues are both smaller than one. Once P is solved, the rest of the solution is not hard to derive.

Since this thesis focuses on properties of growth rates of macro aggregate series, variables expressed in the form of ratios over human capital stock need to be transformed into first difference. The method to do this is shown through an example of consumption. Recall that $c_t \equiv \frac{C_t}{H_t}$, so the growth rate of aggregate consumption can be calculated as below:

$$\begin{aligned} \gamma_{c,t+1} &= \log C_{t+1} - \log C_t \\ &= \log c_{t+1} - \log c_t + \log H_{t+1} - \log H_t \\ &= (\log c_{t+1} - \log c) - (\log c_t - \log c) + \log \frac{H_{t+1}}{H_t} \\ &= \hat{c}_{t+1} - \hat{c}_t + (\log \gamma_{ht+1} - \log \gamma_h) + \log \gamma_h \\ &= \hat{c}_{t+1} - \hat{c}_t + \hat{\gamma}_{ht+1} + \log \gamma_h \end{aligned}$$

Where c, γ_h are steady-state values of c_t and γ_{ht} . Growth rates of other variables can be derived similarly.

A.2.2 Deterministic discounting

The second normalization method defines stationary variables as follows:

$$\begin{aligned} c_t &\equiv \frac{C_t}{(1+\gamma)^t} \\ k_t &\equiv \frac{K_t}{(1+\gamma)^t} \\ h_t &\equiv \frac{H_t}{(1+\gamma)^t} \end{aligned}$$

The system consisting of equations (A.32) to (A.38) changes to:

$$\frac{Ac_t}{1 - N_t - M_t} = (1 - \phi_1) Z_t \left(\frac{V_t k_t}{N_t h_t} \right)^{\phi_1} h_t \quad (\text{A.56})$$

$$\frac{(1 - \phi_1) V_t}{\phi_1 N_t} = \frac{(1 - \phi_2) (1 - V_t)}{\phi_2 M_t} \quad (\text{A.57})$$

$$P_t = \left(\frac{\phi_1}{\phi_2} \right)^{\phi_2} \left(\frac{1 - \phi_1}{1 - \phi_2} \right)^{1 - \phi_2} \left(\frac{V_t k_t}{N_t h_t} \right)^{\phi_1 - \phi_2} \quad (\text{A.58})$$

$$1 = E_t \beta \left\{ \left(\frac{c_t}{c_{t+1}} \right)^{\sigma} (1 + \gamma)^{-\sigma} \left(\frac{1 - N_{t+1} - M_{t+1}}{1 - N_t - M_t} \right)^{A(1-\sigma)} \times \left[\phi_1 Z_{t+1} \left(\frac{V_{t+1} k_{t+1}}{N_{t+1} h_{t+1}} \right)^{\phi_1 - 1} + 1 - \delta_k \right] \right\} \quad (\text{A.59})$$

$$1 = E_t \beta \left\{ \left(\frac{P_{t+1}}{P_t} \right) \left(\frac{c_t}{c_{t+1}} \right)^{\sigma} (1 + \gamma)^{-\sigma} \left(\frac{1 - N_{t+1} - M_{t+1}}{1 - N_t - M_t} \right)^{A(1-\sigma)} \times \left[(N_{t+1} + M_{t+1}) (1 - \phi_2) S_{t+1} \left(\frac{(1 - V_{t+1}) k_{t+1}}{M_t h_{t+1}} \right)^{\phi_2} + 1 - \delta_h \right] \right\} \quad (\text{A.60})$$

$$c_t + (1 + \gamma) k_{t+1} - (1 - \delta_k) k_t = A_g Z_t (V_t k_t)^{\phi_1} (N_t h_t)^{1 - \phi_1} \quad (\text{A.61})$$

$$(1 + \gamma) h_{t+1} - (1 - \delta_h) h_t = A_h S_t [(1 - V_t) k_t]^{\phi_2} (M_t h_t)^{1 - \phi_2} \quad (\text{A.62})$$

The system can then be log-linearized and expressed in percentage deviations:

$$0 = Ax_{t+1} + Bx_t + Dy_t + Fu_t \quad (\text{A.63})$$

$$0 = E_t (Gx_{t+1} + Hx_t + Jy_{t+1} + Ly_t + Mu_{t+1}) \quad (\text{A.64})$$

Where $y_t = [\hat{c}_t, \hat{V}_t, \hat{N}_t, \hat{M}_t, \hat{P}_t]'$, a vector collecting all control variables; and $x_t = [\hat{k}_t, \hat{h}_t]'$, containing two endogenous state variables; and $u_t = [\hat{Z}_t, \hat{S}_t]'$, containing exogenous state variables. The model is then solved by method of undetermined coefficients and the solution is characterized by two recursive equations:

$$x_{t+1} = Px_t + Qu_t \quad (\text{A.65})$$

$$y_t = Rx_t + Su_t \quad (\text{A.66})$$

P, Q, R and S satisfy the conditions listed in appendix A.2.1. Responses of variables collected in y_t and x_t to innovations to u_t can then be calculated.

A.3 MATLAB code

The MATLAB code to implement the algebra and simulations is also provided by Uhlig which is downloadable from <http://www2.wiwi.hu-berlin.de/institute/wpol/html/toolkit.htm>. The modified code that solves all the models presented in this thesis are available upon request.

Bibliography

- [1] Uhlig, H., 1999. "A toolkit for analyzing nonlinear dynamic stochastic models easily".
In: Marimon, R., Scott, A. (Eds.), Computational Methods for the Study of Dynamic
Economics, Oxford University Press, Oxford, pp. 30–61.

Appendix B

Instrumental variable method

The model to estimate has the following specification:

$$\Delta c_t = \mu + \lambda \Delta y_t + \theta r_{t-1}^f + \varepsilon_t \quad (\text{B.1})$$

Since $E(\Delta y_t \varepsilon_t) \neq 0$, OLS estimator of λ is biased. Following a standard approach, equation (B.1) is estimated by instrumental variable method. It is natural to use lagged output growth or consumption growth as instruments for Δy_t because they are correlated with Δy_t and uncorrelated with the current innovation. The following example demonstrates how IV estimators of μ , λ and θ in equation (B.1) can be constructed using lag output growth rates as instruments for Δy_t . Estimators using other instrument sets can be constructed similarly. For notation convenience, the model in equation (B.1) can be condensed in a vector form as:

$$\Delta c_t = x_t' \varphi + \varepsilon_t \quad (\text{B.2})$$

Where $x_t' = [1, \Delta y_t, r_{t-1}^f]$ and $\varphi' = [\mu, \lambda, \theta]$. Collect all instruments in a vector $z_t' = [1, \Delta y_{t-1}, \Delta y_{t-2}, \Delta y_{t-3}, r_{t-1}^f]$, where constant 1 and the risk-free interest rate r_{t-1}^f are instruments for μ and r_{t-1}^f respectively. The moment conditions are:

$$E(z_t \varepsilon_t) = E\left[z_t \left(\Delta c_t - x_t' \varphi\right)\right] = 0 \quad (\text{B.3})$$

Since the number of moment conditions is greater than the number of parameters to estimate, φ is over-identified. Hence, φ is chosen to minimize a quadratic function:

$$Q(\varphi) = \left[\frac{1}{T} \sum_{t=1}^T z_t (\Delta c_t - x_t' \varphi) \right]' W \left[\frac{1}{T} \sum_{t=1}^T z_t (\Delta c_t - x_t' \varphi) \right] \quad (\text{B.4})$$

Where T is the number of simulated periods and W is a 5×5 weighting matrix which satisfies $W = \left(\frac{1}{T} \sum_{t=1}^T z_t z_t' \right)^{-1}$. The resulting IV estimator of φ is:

$$\hat{\varphi}_{IV} = \left[\left(\sum_{t=1}^T x_t z_t' \right) \left(\sum_{t=1}^T z_t z_t' \right)^{-1} \left(\sum_{t=1}^T z_t x_t' \right) \right]^{-1} \left(\sum_{t=1}^T x_t z_t' \right) \left(\sum_{t=1}^T z_t z_t' \right)^{-1} \sum_{t=1}^T z_t \Delta c_t \quad (\text{B.5})$$

x_t, z_t and Δc_t for $0 < t < T$ are simulated samples.

Appendix C

Uniqueness of steady state

Express the first order conditions and constraints of the two-sector SEG model by variables' long-run values (variables with no time subscript denote their long-run values and A_g is normalized to unity):

$$\frac{AC}{(1 - N - M)H} = (1 - \phi_1) \left(\frac{VK}{NH} \right)^{\phi_1} \quad (\text{C.1})$$

$$\frac{1 - \phi_1}{\phi_1} \frac{VK}{NH} = \frac{1 - \phi_2}{\phi_2} \frac{(1 - V)K}{MH} \quad (\text{C.2})$$

$$(1 + \gamma)^{-\sigma} = \frac{1 + \phi_1 \left(\frac{VK}{NH} \right)^{\phi_1 - 1} - \delta_k}{1 + \rho} \quad (\text{C.3})$$

$$(1 + \gamma)^{-\sigma} = \frac{1 + (N + M) A_h (1 - \phi_2) \left(\frac{(1 - V)K}{MH} \right)^{\phi_2} - \delta_h}{1 + \rho} \quad (\text{C.4})$$

$$\gamma = \left(\frac{VK}{NH} \right)^{\phi_1 - 1} V - \delta_k - \frac{C}{K} \quad (\text{C.5})$$

$$\gamma = A_h \left(\frac{(1 - V)K}{MH} \right)^{\phi_2} M - \delta_h \quad (\text{C.6})$$

Where $\rho = \frac{1-\beta}{\beta}$ and γ is the balanced growth rate. Define $f_k \equiv \frac{VK}{NH}$ and $f_h \equiv \frac{(1-V)K}{MH}$. The simultaneous equation system can then be rearranged in 6 unknowns ($f_k, f_h, N, M, \gamma, \frac{C}{K}$):

$$\frac{A}{1-N-M} \frac{C}{K} (Nf_k + Mf_h) = (1-\phi_1) f_k^{\phi_1} \quad (C.7)$$

$$\frac{1-\phi_1}{\phi_1} f_k = \frac{1-\phi_2}{\phi_2} f_h \quad (C.8)$$

$$(1+\gamma)^{-\sigma} = \frac{1 + \phi_1 f_k^{\phi_1-1} - \delta_k}{1+\rho} \quad (C.9)$$

$$(1+\gamma)^{-\sigma} = \frac{1 + (N+M) A_h (1-\phi_2) f_h^{\phi_2} - \delta_h}{1+\rho} \quad (C.10)$$

$$\gamma = f_k^{\phi_1-1} \left(\frac{Nf_k}{Nf_k + Mf_h} \right) - \delta_k - \frac{C}{K} \quad (C.11)$$

$$\gamma = A_h f_h^{\phi_2} M - \delta_h \quad (C.12)$$

The exogenous information set is $(A, \rho, \sigma, \phi_1, \phi_2, \delta_k, \delta_h, A_h)$. Next, I will show the uniqueness of the solution to the above system of equations can be reduced down to the uniqueness of variable γ . To see this, one can solve for $f_k, f_h, N, M, \frac{C}{K}$ in terms of γ using equations C.8 to C.12 :

- from equation C.9, $f_k = \left(\frac{(1+\gamma)^{-\sigma}(1+\rho)-1+\delta_k}{\phi_1} \right)^{\frac{1}{\phi_1-1}}$
- from equation C.8, $f_h = \left(\frac{(1-\phi_1)\phi_2}{(1-\phi_2)\phi_1} \right) f_k$
- from equation C.10, $N + M = \frac{(1+\gamma)^{-\sigma}(1+\rho)-1+\delta_h}{A_h(1-\phi_2)} f_h^{-\phi_2}$
- from equation C.12, $M = \frac{\gamma+\delta_h}{A_h} f_h^{-\phi_2}$
- from equation C.11, $\frac{C}{K} = f_k^{\phi_1-1} \left(\frac{Nf_k}{Nf_k + Mf_h} \right) - \delta_k - \gamma$

Substitute all these into equation C.7 to obtain a highly nonlinear function in γ : $\Theta(\gamma) = 0$. Then one can find the zeros of $\Theta(\gamma)$ for the baseline calibration of exogenous parameters: $A = 1.5455, \rho = 0.0142, \sigma = 1, \phi_1 = 0.36, \phi_2 = 0.11, \delta_k = 0.02, \delta_h = 0.005, A_h = 0.0461$. The numerical solution shows that there is only one set of internal solution that

satisfies $0 \prec L \prec 1$:

$$\gamma^* = 0.0042, L^* = 0.542, N^* = 0.298, M^* = 0.160, \left(\frac{K}{H}\right)^* = 11.06$$

Appendix D

Data description and summary statistics

The data set covering from the first quarter of 1954 to the first quarter of 2004 is downloadable from <http://clevelandfed.org/research/Models/rbc/index.cfm>. According to Gomme and Rupert (2007), output (Y) is measured by real per capita GDP less real per capita Gross Housing Product. They argue that income in home sector should be removed when calculating market output using NIPA data set. The price deflator is constructed by dividing nominal expenditures on nondurables and services by real expenditures. Population are measured by civilians aged 16 and over. Consumption (C) is measured by real personal expenditures on nondurables and services less Gross Housing Product. Gomme and Ruppert report four types of investments: market investment to nonresidential structure, market investment to equipment and software, household investment to residential product and household investment to nondurables. Investment (I) in this thesis corresponds to the simple sum of the aforementioned four types of investments. Working hours (N) is measured as per capita market time. Figure D-1 depicts growth rates of output, consumption and investment over the periods from 1954.1 to 2004.1. Several observations are reflected in this picture:

1. Output growth fluctuates more than consumption growth; investment growth fluctuates

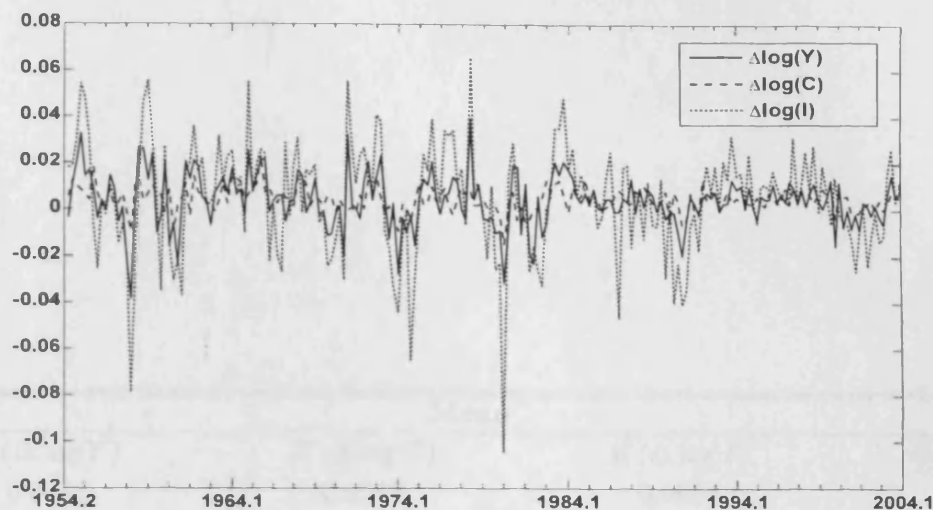


Figure D-1: Plot of US data from 1954.1 to 2004.1

tuates more than output growth.

2. Consumption growth and investment growth are strongly procyclical.
3. Economy fluctuates substantially less after 1980s¹.

Table D.1² summarizes the observed business cycle properties numerically. The first panel of table D.1 shows that output, consumption and investment grow at similar rate over time. This is in line with the balanced growth path hypothesis. The second panel reflects the relative order of variabilities of main macro variables in figure D-1. The third panel shows that growth rates of variables are all positively autocorrelated. The last panel confirms that consumption and investment growth rates are procyclical and working hours are slightly countercyclical.

¹Although "Great Moderation" is not a topic of this thesis, it is clearly seen from the picture.

²The second moment results are actually the standard deviation of net growth rate multiplied by 100. For example, standard deviation of net output growth ($\Delta \log Y$) is 0.0114. Since standard deviation of net growth rate equals that of gross growth rate, this number (when multiplied by 100) can be interpreted as percentage deviation of gross output growth from its mean.

Mean			
$E(\Delta \log Y)$	$E(\Delta \log C)$	$E(\Delta \log I)$	$\frac{E(N)}{E(N)}$
0.0042	0.0047	0.0045	1
Fluctuation			
$\sigma(\Delta \log Y)$	$\sigma(\Delta \log C)$	$\sigma(\Delta \log I)$	$\frac{\sigma(N)}{E(N)}$
1.14	0.52	2.38	5.6
Autocorrelation			
$\rho(\Delta \log Y_t, \Delta \log Y_{t-1})$	$\rho(\Delta \log C_t, \Delta \log C_{t-1})$	$\rho(\Delta \log I_t, \Delta \log I_{t-1})$	$\rho(N_t, N_{t-1})$
0.29	0.24	0.39	0.98
Cross-correlation			
$\rho(\Delta \log Y_t, \Delta \log Y_t)$	$\rho(\Delta \log Y_t, \Delta \log C_t)$	$\rho(\Delta \log Y_t, \Delta \log I_t)$	$\rho(\Delta \log Y_t, N_t)$
1	0.49	0.75	-0.07

Table D.1: Business cycle statistics in US data from 1954.1 to 2004.1

