DO WE NEED NOMINAL RIGIDITY?
ACCOUNTING FOR EXCHANGE RATE AND INFLATION BEHAVIOUR
WITHIN A CLASSICAL FRAMEWORK

A Thesis submitted for the
Degree of Doctor of Philosophy
of the University of Wales

by
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Non-Technical Summary

Over the last decade macroeconomists have been prolific in developing a new workhorse model for open economy analysis. The unifying feature of this literature is the introduction of nominal rigidities and market imperfections into a dynamic general equilibrium model with well-specified microfoundations. Nominal rigidities present in the models are crucial to their success in explaining empirical facts such as real exchange rate 'overshooting' and inflation persistence. The objective of this thesis is to show that real exchange rate and inflation behaviour are explicable within a classical approach — by implication there is no necessary case to add nominal rigidity. The thesis itself is divided into two sections; I explain each of them in turn.

In the first part I attempt to explain the behaviour of the real exchange rate; both the appreciation following a productivity burst and its cyclical pattern, using a dynamic stochastic general equilibrium open economy model for the UK. It is a well-established empirical fact that a burst in productivity leads to an appreciation of the currency. However, according to the 'conventional' view, if a country becomes more productive, a higher world supply of its goods should result in a relative price reduction, a depreciation of its real exchange rate. To explain the empirical finding I use the Real Business Cycle (RBC) version of the model in which prices are fully flexible, i.e. with no nominal rigidities. As the UK is a medium-sized economy, I take the world economy as given. The interaction with the rest of the world comes in the form of uncovered-real-interest-rate-parity and the current account; both of these relationships are derived explicitly from the optimising decisions of fully rational agents. As the model is highly non-linear I solve it numerically, calibrating it to UK quarterly data. A one percent productivity growth shock simulation clearly shows that the real exchange appreciates on impact and then goes back to a new depreciated equilibrium, producing a business cycle. This simulation is
very encouraging to the idea that the real exchange rate behaviour may be explicable within an RBC context.

To provide further testing of my results I use the method of 'bootstrapping.' This is a procedure of repeatedly simulating the model using the estimated model's own errors to create a large number of 'potential scenarios' of what might have happened over the period of the actual sample; these can be regarded as potential samples, i.e. 'bootstraps,' which we can use to re-estimate the same real exchange rate time series equation. The re-estimates show the sampling range within which the estimates of this equation's parameters can lie — at some specified level of probability or 'confidence,' usually 95% — if the model is correct. I can then examine whether the estimated parameters of the actual real exchange rate equation lie within this range — and so either reject or accept the model.

I find that my model tells quite a good story, the gyrations of the real exchange rate can be explained within a RBC context. I do not rule out the possibility that adding a degree of nominal rigidity, in a sort of 'stretched-RBC' framework, could also be useful. However my concern here has been to establish the basic ability of the RBC alone to provide explanatory power.

This part of the thesis also discusses simulation results for other demand and supply shocks. Given that the model's parameters are 'structural'—that is unaffected by policy regime changes—I evaluate the welfare implications of a number of alternative policy regimes using this model.

In section two of the thesis the objective is to show that the degree of inflation persistence — that is the extent to which an inflation shock does not fade away in subsequent quarters — is not an inherent fixed characteristic of an economy, but in fact depends on the stability and transparency of the monetary policy regime in place. Given the large econometric evidence of high inflation persistence for the US and other OECD countries, many macroeconomists have concluded that high inflation persistence is a 'stylised fact' and that furthermore it is evidence for a 'New Keynesian' Phillips Curve in which inflation depends to a high degree on past inflation — 'nominal rigidity.' To examine these claims for the UK, I begin by estimating regressions of inflation on its own past values for separate sample periods, for each of which the monetary policy regime was different.

Then I build a simple model based on optimising behaviour with a minimum of nominal
rigidity and examine its ability to generate the facts. I supply the values of the model parameters by 'calibration', i.e. using estimates from previous studies or given by the theoretical assumptions, for each of the regimes and solve it for the implied persistence in the inflation process. I then compare this implication with the estimated persistence for each regime. Finally I again use the bootstrapping technique described above to check whether the actual persistence coefficients lie within the 95 percent confidence limits implied by the bootstraps. I find that the very basic model 'tells quite a good story' but can not strictly generate the facts of inflation persistence. This is rather re-assuring as the model is overly simple, besides using calibrated parameters, as opposed to ones estimated on the UK data. I then re-do the exercise for the Liverpool model of the UK which is similar but uses estimated parameters.

The basic conclusion is the same that persistence varies across different regimes, being the lowest over the last decade of inflation targeting and highest during the 1970s, the period of no nominal anchor. I also plan to evaluate if a New Keynesian model would be better equipped to explain the facts; this research is still underway.

The central idea of my thesis has been to bridge the gap between theory and empirics in macroeconomics. This is a three-stage process: first, I build micro-founded theoretical models, then establish the facts — in terms of the time series properties of various macroeconomic variables and finally test the models against the stylised facts of the world using rigorous bootstrapping methodology. Bootstrapping basically involves replicating the stochastic environment to see whether the regression coefficients in the data lie within 95% confidence limits, for those coefficients, implied by the model. Given the novelty of this approach I am very enthusiastic about applying it to test alternative macroeconomic models.
Introduction

Over the last decade macroeconomists have been prolific in developing a new workhorse model for open economy analysis. The unifying feature of this literature is the introduction of nominal rigidities and market imperfections into a dynamic general equilibrium model with well-specified microfoundations. Nominal rigidities present in the models are considered crucial to their success in explaining empirical facts such as real exchange rate 'overshooting' and inflation persistence. The objective of this thesis is to show that real exchange rate and inflation behaviour are explicable within a classical approach — by implication there is no need to add nominal rigidity.

In the first part of the thesis I use a micro-founded general equilibrium open economy model based on optimising decisions of rational agents to explain the behaviour of the real exchange rate — both the appreciation after a productivity burst and the cyclical pattern found in the data. The model used is classical as prices are fully flexible, there are no market imperfections and money has no effect.\(^1\) In the second part of the thesis I turn to a new classical model derived from optimising decisions of rational agents and use it to explain the time series properties of inflation across different monetary policy regimes. The 'surprise' Phillips curve used has an information lag which gives money a brief effect. Hence both models have either no or minimal nominal rigidity; yet one explains exchange rate 'overshooting' and the other explains inflation persistence, both things that the existing literature has mostly tried to explain via nominal rigidity — starting with Dornbusch (1976) for the exchange rate, and Taylor (1980) and Calvo (1983) contracts for inflation persistence.

The thesis is organised as follows. The first chapter familiarises the reader with the under-\(^1\)The inflation tax would have an effect but we have neutralised its effect by returning it to households.
lying concepts of models embodying a real business cycle (RBC) framework which is used in subsequent chapters of the thesis to evaluate the impact of alternative monetary and fiscal policy shocks and to explain the behaviour of the real exchange rate. After describing a prototypical RBC model, I go on to trace the developments in the literature which add government expenditure, taxes, money, and open economy extensions. The main policy implication of the RBC research agenda has been that any efforts at stabilisation are likely to be counter-productive since economic fluctuations observed are not welfare-reducing deviations from \textit{natural rate} paths of an ideally efficient Walrasian economy but are optimal responses by rational agents to uncertainty regarding the rate of technological change.

In chapter two I provide theoretically coherent micro-foundations for macroeconomic models and construct an econometrically testable dynamic general equilibrium open economy model. My model is an enriched variant of a prototype RBC model embodying a representative agent framework as in McCallum (1989). The model is based on optimising decisions of rational agents, incorporating money, government and real world features like distortionary taxes and unemployment benefits. In sum I want to evaluate how the model economy will behave over time when it is hit by multiple shocks. Given that the parameters of the model are structural it can be also used to evaluate nontrivial policy changes. I calibrate the model using quarterly data for the UK and use it for quantitative analysis of policy.

Chapter three explains the solution algorithm that is used to solve the model and also reports simulation results of various demand and supply shocks. Heckman (1999) points out that recursive dynamic economic theory does not typically produce simple functional forms for estimating equations. Problems of estimating the parameters of large-scale general equilibrium models are formidable unlike the simple Keynesian models that defined the structural econometrics of the 1940s and 1950s. Most of the open economy dynamic general equilibrium models of today are highly non-linear and can not be solved analytically. The usual approach is either to take linear approximations around the steady-state-growth path or to solve them numerically, i.e. use algorithms to simulate the model economy. In solving my model, I am forced by its complexity and non-linearity to use a computer algorithm developed by the Liverpool Research Group for solving complex non-linear rational expectations models. The chapter discusses the simulation results for both demand and supply shocks with calibrated parameters.
which were used to assess the overall properties of the model. The results are consistent with my theoretical priors.

In chapter four I explore the ability of my calibrated real business cycle (RBC) model to account for the behaviour of the real exchange rate, using UK experience as my empirical focus. The continuous strength of the dollar over the 1990s fuelled interest in the relationship between productivity and exchange rates. As US productivity surged in the second half of the 1990s, the dollar began its climb against all the major currencies of the world. This has led to a large body of literature analysing the links between the real exchange rate and productivity. The ‘conventional’ view of the impact of a productivity shock on an economy is that the real exchange rate depreciates. However, this is completely at odds with the empirical findings of currency appreciation after a productivity spurt. It also fails to explain the cyclical pattern observed in the real exchange rate data. Using my RBC model I find that a one percent deterministic productivity growth shock, clearly shows that the real exchange rate appreciates on impact and then goes back to a new depreciated equilibrium, producing a business cycle — giving me the simulation properties that are needed to explain the data. I also show that the RBC model can without alterations or additions reproduce the univariate properties of the real exchange rate — by implication there is no necessary case here to add nominal rigidity.

In chapter five I estimate the model using quarterly data for the UK by Full Information Maximum Likelihood (FIML) estimation procedure. The FIML algorithm developed by Minford and Webb (2004) solves the model repeatedly in full for all sample periods for sets of parameters, choosing the likelihood maximising set. The justification for using FIML is that it increases the efficiency of the estimates compared with single equation methods or simultaneous system methods that do not impose all the model’s cross-equation restrictions. Given my small sample I can not be confident that the errors of the model are normal. Hence I use bootstrapping – a computationally intensive technique for making inference about the population characteristics — to construct confidence intervals. I start the bootstrapping procedure with the sample residuals obtained by FIML and use their distribution as an estimate of their true distribution. The bootstrap can then be regarded as a Monte Carlo study to find the true distribution of the parameter estimates on the assumption that the sample residual distributions and the parameter estimates are both the true ones. I present the results of the FIML estimation and also correct
them for bias. Finally as a robustness exercise I re-do the demand and supply shocks evaluated in chapter three using the bias-adjusted FIML estimates. The bias-corrected results are fairly similar – qualitatively – to the calibrated ones, which suggests that bias is not a major problem in the model.

The second part of the thesis begins in chapter six. In this chapter I endeavour to familiarise the reader with the concept of inflation persistence so widely discussed in the literature. The study of inflation and its dynamics is crucial for macroeconomists as inflation has far reaching implications for the economy both in terms of economic efficiency and wealth distribution. I discuss in detail the empirical literature covering both univariate and multivariate models of inflation before analysing structural models attempting to explain persistence. There are basically two sides to the story. One view upholds that inflation persistence has varied over the post WWII period for most industrialised economies; which given the substantial changes in monetary policy regimes observed supports the contention that persistence is a function of the credibility and transparency of the regime in place. The conflicting view finds that persistence has been stable over the same period even when policy has become more credible and hence monetary policy plays no role in determining the persistence properties of inflation. In other words the former believes in the non-structural nature of inflation persistence while the latter thinks that persistence is structural so must be 'hardwired' into the economic model. I also discuss the policy implications of inflation persistence in terms of design and evaluation of optimal policy.

The objective of chapter seven is to establish the facts of UK inflation persistence, allowing for breaks in monetary policy regimes. I estimate univariate processes for inflation across different time periods where these periods are carefully defined according to a priori knowledge of the UK economy. According to Perron (1990) a failure to account for such breaks could yield spuriously high estimates of the degree of persistence. My initial results clearly indicate that inflation persistence is different in different regimes, with persistence being lowest in the Inflation Targeting regime, followed by Bretton Woods and then Monetary Targeting. Persistence tends to be higher during the Deutsche Mark shadowing period as the government’s primary aim then was to defend the peg. During the Incomes Policy regime of the 1970s there was no nominal anchor; hence I get the highest persistence parameters.
In chapter eight I elucidate rather straightforward models, easily-micro-founded in a standard classical set-up to characterise each UK monetary regime of the post-war period. I am not particularly committed to these models in detail, but rather use them as a benchmark in which a minimal number of special assumptions are made — such as particular forms of 'nominal rigidity' and 'adjustment.' We may note that in any such model omitted variables will create error processes, and it is perfectly reasonable to find that these are themselves autocorrelated. These processes will then propagate themselves through all the endogenous variables and be a natural source of persistence. However, if the monetary authorities are so determined, they may partly suppress this persistence through their monetary reactions; arguably just such a determination was observed when Inflation Targeting was instituted by the Treasury in 1992 after the UK's forced exit from the Exchange Rate Mechanism (ERM). I use my structural models suitably calibrated for each of the regimes incorporating all available information about monetary policy behaviour during those periods and then solve them analytically for the implied persistence in the inflation final form equation; in order to compare this theoretical prediction with the estimated persistence for each regime — the objective of the next chapter.

In chapter nine I compare my model with the data more formally and test statistically whether my calibrated model is seriously consistent with the inflation data. Using a bootstrapping procedure, I generate bootstrap data for inflation under each regime and compute the implied sampling distribution of its AR and ARMA coefficients. I then check for each regime whether the coefficients of the actual inflation series lie within the 95% confidence interval of the AR and ARMA coefficient distribution generated by my model. This is an ambitious test for such a basic model; and perhaps surprisingly — at least for some readers — the model does quite well. As a robustness exercise I carry out the same analysis on the Liverpool Model, which is an elaborated version of the same classical structure. I find that inflation persistence is predicted by the two new classical models fairly effectively and that it varies with the monetary regime, in particular falling greatly when inflation targeting is in place.

The final chapter presents the main findings of this thesis.

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Chapter 1

Literature Review for Real Business Cycle Models

1.1 Introduction

According to Lucas (1977) “One exhibits understanding of business cycles by constructing a model in the most literal sense: a fully articulated artificial economy which behaves through time so as to imitate closely the time series behaviour of actual economics.”

The Keynesian Macroeconomic models of the 1940s were the first to attain this level of explicitness and empirical accuracy. Yet the ability of these models to imitate actual economies, has almost nothing to do with their ability to make accurate conditional forecasts — to evaluate how behaviour would have differed had certain policies been different in specified ways. This ability requires invariance of the parameters of the model under policy variation, i.e. the celebrated Lucas Critique. Now invariance of model parameters is not a property that can be assured in advance, however it seems reasonable to assume that neither tastes nor technology vary systematically with variations in policies. In contrast, agents’ decision rules will change with the change in economic environment. Any disequilibrium model, like the Keynesian models, constructed by simply codifying the decision rules that agents found useful over some previous sample period, without explaining why these rules are used, will be of no use in predicting the consequences of nontrivial policy changes.

The objective of this chapter is to familiarise the reader with the underlying concepts of
models embodying a real business cycle framework which is used in subsequent chapters of the thesis to evaluate the impact of alternative monetary and fiscal policy shocks and to explain the behaviour of the real exchange rate. The chapter is organised as follows. Section 1.2 first introduces the reader to the macroeconomic climate that led to the birth of the real business cycle (RBC) framework, it then explains the concept of business cycles and describes a prototypical RBC model. Section 1.3 elucidates the pioneering work of Kydland and Prescott (1982), Long and Plosser (1983) and others that set the stage for modern macroeconomic analysis. The role of government in the RBC framework is discussed in Section 1.4 and Section 1.5 elucidates models embodying monetary variables in the prototypical RBC model. Open economy extensions are described in Section 1.6 and the policy implications of the RBC research agenda are highlighted in Section 1.7. The conclusions are presented in Section 1.8.

1.2 Background of the Real Business Cycle Framework

In the 1950s and 1960s the Keynesian IS-LM approach was the reigning paradigm for macroeconomic analysis. Many macroeconomists believed their understanding of the economy to be nearly complete, with a theory for aggregate demand and a Phillips curve relationship providing insight to the adjustment process of wages and prices over time. In the Keynesian world inflation occurred if aggregate demand was stimulated ‘excessively’ and unemployment arose if demand was ‘insufficient.’ The only dilemma facing policy-makers was determining the most desirable position along the inflation-unemployment trade-off or the Phillips curve.

The simultaneous occurrence of high inflation and high unemployment in the 1970s led macroeconomists to question this aspect of theoretical and quantitative extant models. This experience also made economists realise that the basic Keynesian framework is not the appropriate paradigm for understanding what happens during a business cycle nor is it capable of providing the empirically correct answers to questions involving changes in the economic environment or changes in monetary or fiscal policy. As Plosser (1989) eloquently puts it “The essential flaw in the Keynesian interpretation of macroeconomic phenomenon was the absence of a consistent foundation based on the choice theoretic framework of microeconomics.” Fried-
man (1968), Lucas (1976, 1980), and Sims (1980) and Sargent (1981) forcefully demonstrate this flaw and set the stage for modern macroeconomic analysis.

Now in order to understand business cycles it is important and necessary to understand the characteristics of a perfectly working dynamic economic system. Hicks (1933) makes this point quite clearly, arguing that the "idealised state of dynamic equilibrium...give(s) us a way of assessing the extent or degree of disequilibrium." It is essential to make progress to understand the idealised state as it is logically impossible to attribute an important portion of fluctuations observed to market failure without an understanding of what sorts of fluctuations would have been observed in the absence of hypothesised market failure. Keynesian models started out assuming market failure and hence provided no such understanding. The solution then is to construct small scale dynamic general equilibrium models and attempt to understand how aggregate economic variables respond to changes in economic environment, for e.g. technology, tastes or government policies.

In the early 1980s we witnessed one of the most striking development in macroeconomics—the emergence of a substantial body of literature devoted to the 'real business cycle' approach to the analysis of economic fluctuations. This approach originated in the pioneering work of Kydland and Prescott (1982) and Long and Plosser (1983). The theory of business cycles mainly deals with approaches that describe and explain fluctuations in major economic aggregates. The main task therefore, is to establish necessary and sufficient conditions for the existence of more or less regular oscillations in a model economy. Business cycle literature from the 1890s through the 1960s concentrated mainly on the internal dynamics of the capitalistic economies. In sharp contrast, recent models of business cycles rely more on exogenous factors and much less on the internal dynamics of the system.

According to Long and Plosser (1983) "The term 'business cycle' refers to joint time series

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1The Keynesian system upheld a trade-off between inflation and unemployment. However, Friedman (1968) argues that on the basis of microeconomic principles the Phillips curve in the long-run should be vertical.
2Lucas (1976) emphasised that expectations about future policy will systematically influence current decisions and hence alter the behavioural relations exploited by the empirical implementation of the Keynesian analysis. He argued that expectations cannot be formulated or specified in an arbitrary manner and should be consistent with individual maximisation and be rational in the sense of Muth (1961). Lucas (1980) portrayed that the IS-LM model was fatally inconsistent with optimising behaviour on the part of households and firms.
3Sims (1980) and Sargent (1981) criticised the quantitative macromodels for not using micro-foundations as a guide to specification estimable equations and also for avoiding central issues of identification.
4The research programme initiated by Long and Plosser (1983) supports this view.
behaviour of a wide range of economic variables such as prices, output, employment, consumption and investment. In actual economies this behaviour seems to be characterised by at least two broad regularities: (i) Measured as deviations from trend, the ups and downs in individual series exhibit a considerable amount of persistence, (ii) Measures of various economic activities (e.g. outputs in different sectors) move together." While Haberler (1963) defines the business cycle in the general sense as "an alternation of periods of prosperity and depression, of good and bad trade."

This literature was an outgrowth of the equilibrium strategy for business cycle analysis initiated by Lucas (1972, 1973, 1975) and extended by Barro (1976, 1981), but differs from them in two critical aspects. First, real business cycle (RBC) models place much more emphasis on mechanisms involving cycle propagation, that is, spreading over time of the effects of a shock. Second, RBC models emphasise the extent to which shocks that initiate the cycles are real – as opposed to monetary – in origin. Comprehensive reviews of the RBC research by McCallum (1989) and Plosser (1989) have illustrated that despite a number of unresolved issues, the approach successfully explains some of the key empirical regularities that characterise economic fluctuations.

Real business cycle models view aggregate economic variables as the outcomes of decisions made by many individual economic agents acting to maximise their utility subject to production possibilities and resource constraints. The model asks questions like: How do rational maximising agents respond over time to changes in the economic environment? What implications do those responses have for the equilibrium outcomes of aggregate variables?

The basic framework can be described as follows. The economy is populated by many identical infinitely lived agents deriving utility from consumption and leisure over their lifetime. Each agent also has access to a constant returns to scale production technology for the single commodity in the economy. The production function requires work effort and capital, which depreciates over time. Further, production technology is assumed to be subject to temporary technological changes which provide the underlying source of variation in the economic environment to which agents must respond. The basic choices facing the consumer then are how to allocate hours between work and leisure, and how to allocate the supply of the single good between investment to augment future capital and current consumption. As soon as agents are
assumed to take future events into consideration, which may influence current decision making, they are invariably forced to form expectations about future magnitudes of importance. Thus expectations play a pivotal role in models that rely on intertemporal optimisation. The model imposes resource constraints such that the sum of time spent working and on leisure is less than or equal to some fixed amount of time in the period. Further, consumption, labour, leisure, capital and investment must all also be non-negative.

In the prototype RBC model, productivity disturbances motivate rational agents to adjust savings and investment to smooth consumption, and to adjust employment in response to changes in relative price of leisure and the productivity of labour. This behaviour is consistent with some of the stylised facts because: (i) it generates procyclical fluctuations in consumption, investment and employment, (ii) it causes investment to exhibit greater variability than output or consumption, and (iii) it produces positive persistence in all major macro-aggregates.

The greatest advantage of the RBC approach is that the behavioural equations of the model have been derived via an optimisation, so that the parameters of the model – preferences and technology – can be regarded as truly ‘structural.’ It is an equilibrium model, which is by definition constructed to predict how agents with stable tastes and technology will chose to respond to a new situation and can be used: (i) to analyse how key macroeconomic variables are likely to respond to known economic shocks or changes in the economic structure, and (ii) to identify the economic shocks and changes in economic structure underlying the observed movements in economic data. Both of these functions are important in central banks’ economic analysis, as for instance the Bank of England (1999) recognises to be for the conduct of its monetary policy. In the RBC framework alternative policies can be compared on the basis of measures of the utility benefits or costs, rather than on the basis of ad-hoc objectives. Further it allows for the analysis of policy and other shocks in the dynamic-stochastic context of a fully specified system, as called for by ‘rational expectations reasoning.’

However, McCallum (1989) expresses concern about the heavy reliance on unexplained shocks in the RBC literature to explain fluctuations in aggregate data. He argues that shocks

\[5\] Dynamic economics has over the years worked with very few forms of expectations. The assumption of static, adaptive or extrapolative expectations are rough hypothesis which were usually made because of their manageability and not because the economics profession believed them to be a correct representation of individual behaviour.
might in fact represent the influence of omitted variables, such as monetary and fiscal policy and the exchange rate rather than pure shocks to technology. In a similar spirit Mullineux and Dickinson (1992) argue that the inclusion of a random term is justified econometrically due to the problems associated with functional misspecification, measurement errors, and omitted variables. As the authors point out, functional misspecification and measurement error can not be the main source of energy driving cycles. Thus, omitted variables may in fact be the main cause for cycles in these models. According to Gabish and Lorenz (1989) construction of economic models in which results mainly rely on stochastic exogenous factors can hardly be considered as serious economic theorising.

1.3 Early Real Business Cycle Models

We begin by describing the pioneering work of Kydland and Prescott (1982). The paper integrates growth and business cycle theory with the objective to try and explain the cyclical variances of a set of economic time series for quarterly post-war US data. As Stadler (1994) argues Kydland and Prescott deviate from the prototype RBC model in two important ways. First, they assume that it takes multiple periods, specifically four quarters, to build new capital goods — hence the name time-to-build. Further it is assumed that only finished goods are part of the productive capital stock. Second, they use a non-time-separable utility function which admits greater intertemporal substitution of leisure. Implying an assumption that workers suffer from fatigue syndrome, i.e. the harder one has worked in the past, the more one treasures leisure today. This assumption is likely to amplify the effects of intertemporal substitution of leisure on the model and is required to explain aggregate movements in employment in an equilibrium business cycle model.

6 The covariances between real output and other series and the autocovariance of output.
7 According to Kydland and Prescott (1982) the primary reason for using time-to-build was that the existing neoclassical structure nor the adjustment cost technologies were adequate. The neoclassical structure was inconsistent with the positive association between the shadow price of capital and investment activity. The adjustment cost technology was consistent with this observation, however it was inconsistent with cross-sectional data and the association of investment with the lagged as well as the current capital shadow prices.
8 Capital in their model reflects all tangible capital which includes stocks of plant and equipment, consumer durables, and housing. Consumption however, does not include purchase of durables but does include the services derived from the stock of consumer durables.
9 In their formulation, the current utility of leisure depends on past leisure in an explicit way.
10 Thus making hours worked more volatile.
RBC models give rise to a non-linear dynamic system of equations and in general do not have an analytical solution. Despite the popularity of these models, there is no generally agreed procedure for solving them. As Campbell (1994) explains the difficulty arises due to the presence of fundamental non-linearity in these models that arises primarily due to the interaction between multiplicative elements such as the Cobb-Douglas production function and additive elements present in the law of motion for capital stock. This non-linearity tends to disappear only in very unrealistic cases where there is complete depreciation of capital within a single period and agents possess logarithmic utility function. In such special cases, one can obtain explicit closed-form solutions for the endogenous variables in the model. In all other cases, some form of linear approximation is required.

This led Kydland and Prescott and subsequent researchers to consider a structure for which analytical solution was possible. One such structure is a linear quadratic set-up in which the objective function is quadratic with linear constraints, and exogenous disturbances are driven by a first order Markov process. Kydland and Prescott (1982) linearise the non-linear first order conditions derived from the Lagrangian using an approximation method prior to analysing the dynamic behaviour of the model and its empirical implications. These optimal time-invariant rules are nothing but the stochastic analogue of the well-known Euler equations. They calibrated and simulated their model in order to analyse its empirical relevance. The main reason for calibrating the model was that the discounted dynamic programming problem that they proposed to explain the decision rules of the representative agent with, was too complicated to allow for a closed-form solution to be obtained. The authors were able to demonstrate that several business cycle correlations of quarterly US data can be mimicked reasonably well with an equilibrium model in which there is no role for money or government policy. It can account for much of the variability in gross national product and it can correctly predict that consumption is less variable than income while investment is more variable. The model however predicts a variability of consumption, hours worked and productivity that is too

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11 See for example Long and Plosser (1983) and McCallum (1989). It should be noted that logarithmic utility together with complete capital depreciation implies that the income and substitution effects of a wage rate change will just offset each other, leaving the leisure choice unaffected. See King, Plosser and Rebelo (1987).
13 According to Pagan (1994) calibration involves quantitative research in which a theoretical model is taken very seriously rather than a particular technique for estimating the parameters of the model. The approach is to use established empirical results to assign values to the behavioural parameters of the decision problem.
low relative to the data and a correlation between productivity and hours worked that is too high. Hansen (1985) notes the failures of the Kydland-Prescott model and suggests that they may be due to the way labour force is modelled — the assumption that individuals choose a certain number of hours per week to work.

Long and Plosser (1983) explore a model with multiple sectors to understand the comovements across sectors in response to shocks that are potentially sector specific.\textsuperscript{14} The authors interest in multiple sectors is motivated by the observation that many sectors of the economy tend to move together, however some sectors lead while others lag the general state of business activity. In the paper they demonstrate how certain very ordinary economic principles lead maximising agents to choose consumption and production plans that exhibit some important features of observed business cycles. Long and Plosser argue that although their model is not capable of explaining all observed regularities of business cycles, it provides a useful benchmark to assess the relative importance of various kinds of disturbances. They state that when present and future consumption are both normal goods in consumer preferences then it is well known that consumers will attempt to spread over many periods the consumption effects of any unanticipated wealth increment. Further in a multi-sector model consumers will also attempt to allocate their incremental savings in a way that leads to increased consumption of many different goods. Given constant relative prices, this means that persistence and comovements which are the two main characteristics of business cycles are outcomes of desired consumption plans. Long and Plosser conclude that persistence and comovement inherent in their model signify the optimal response of agents to wealth effects and are not welfare-reducing as theirs is a 'competitive theory of economic fluctuations' as discussed in Kydland and Prescott (1980) and hence the equilibria are Pareto optimal. By implication efforts to stabilise the economy can only make consumers worse off.

A lot of research in the RBC area has made attempts to expand and extend the basic neoclassical capital accumulation model to ensure a better match of the model’s prediction for hours and the actual hours worked. One approach is that of Kydland and Prescott (1982) of non-time-separable preference structure. Hansen (1985) and Rogerson (1988) explore the consequences of non-convexity, i.e. indivisibilities in the labour supply decision that requires

\textsuperscript{14}For an alternative application see Black (1987) who uses multi-sector models to explain unemployment.
the agents to either work full-time or not at all.\textsuperscript{15} This is in stark contrast to the simple model where the agents can vary the number of hours worked. The result is that volatility of hours worked in response to productivity shocks increases substantially while the estimated labour supply elasticities will remain low, as estimated by labour economists using panel data on prime age males. Another approach to enhancing the response of hours worked to match the data is to allow for heterogeneity across the agents in the model economy. Kydland (1984), Rebelo (1987), Cho and Rogerson (1988) and King, Plosser and Rebelo (1988b) use such a paradigm. All these papers suggest that there is a possibility of significant downward biases in estimates of aggregate labour supply elasticity when the agents have different skill levels.

Hansen (1985) considers a one-sector stochastic growth model with shocks to technology. The model is able to account for large fluctuations in hours worked relative to productivity without relying heavily on intertemporal substitution of leisure. Indivisible labour is modelled by assuming that agents can either work some given positive number of hours or not at all. Unlike previous models, fluctuations in aggregate employment in Hansen's model result from the agents entering and leaving the job market, rather than continuously employed agents adjusting the number of hours worked. The individuals in the model are forced to enter and exit the labour force in response to a stochastic technology shock. When one considers technology, individuals could face a production function which is convex at first and then becomes concave, i.e. marginal productivity of work effort could increase during the beginning of the day/week and then gradually subside owing to 'warm up' time required for becoming productive again. Prevalence of such a technology could lead agents to work a lot or not at all. According to McGrattan (1994) the introduction of non-convexity, allows Hansen's (1985) otherwise prototype RBC model to better mimic the variability of total hours worked than the Kydland and Prescott (1982) model. However, it can not capture the observed variability in consumption and productivity and the low correlation between productivity and hours worked.

As noted earlier, the goal of modern business cycle research in general and RBC school of thought in particular has been to develop models that mimic the cyclical patterns of aggregate

\textsuperscript{15}Non-convexity in individual preferences can arise if the utility function exhibits decreasing marginal utility of leisure at certain low levels of leisure and increasing marginal utility at higher levels of the same. Preference ordering of this form according to Hansen (1985) reflects fixed costs associated with working each period.
data. The early models in the RBC literature\textsuperscript{16} were closed economy models that assumed no externalities, taxes, government expenditure or monetary variables. Theirs was a ‘competitive theory of economic fluctuations’ and thus the equilibria were Pareto optimal. Even though these models were far removed from reality they served a purpose. As Long and Plosser (1983) state ‘...models of this type provide useful, well-defined benchmark for evaluating the importance of other factors (e.g. monetary disturbances) in actual business cycle episodes.’ Many extensions have been made to the traditional RBC models, particularly the role of government,\textsuperscript{17} role of money,\textsuperscript{18} incorporation of distortionary taxes\textsuperscript{19} and open economy extensions.\textsuperscript{20}

Today the purely real approach to business cycles no longer stands. Wage and price stickiness now play some role in virtually all business cycle theories, if only because labour market data otherwise appear inexplicable. However, one might also say that today ‘we are all real business cycle theorists.’ Most economists subscribe to a hybrid theory involving monetary shocks, real shocks, and imperfect adjustment mechanisms. All of these theories, to some extent or another, rely on the real transmission mechanisms outlined by Kydland, Prescott, and others.

1.4 Government in the RBC Framework

According to Long and Plosser (1983) it is important to emphasise that the persistence and comovements inherent in business cycle models should not be confused with welfare-reducing deviations from some ideal path. Strictly speaking, in a prototype RBC model with complete markets and the absence of any form of externality there is no stabilisation role for the government, although one could still think of a government providing public goods from the tax revenue it collects. However, many economists argue that government tax and spending policies are an important source of real disturbances to the economy. With the incorporation of govern-


\textsuperscript{19}See Braun (1994). However, McGrattan (1994a) reports that capital and income tax rate shocks do not contribute much to business cycle variability.

ment one can analyse important questions regarding changes in fiscal policies in the presence of distortionary taxes. Also variations in government spending introduces a potential source of demand side shocks to the model which is otherwise governed by supply side disturbances. The presence of distortionary taxes breaks the link between the rational agent's optimal decisions and Pareto efficiency, since welfare can be increased by removing the distortions.

The theoretical underpinnings of this line of research draws from the work of Arrow (1962), Hall (1971), Brock (1975) and Romer (1983, 1986, 1987). The logic is that in an economy with many agents, each agent takes as given the government's spending and tax policies in their choice problems. The only additional restriction is that the government's budget constraint must be satisfied. These models then provide the researcher with an artificial laboratory for answering interesting questions regarding changes in policy that is not subject to the criticism of Lucas (1976). The intuition underlying the effects of unproductive government purchases can be found in the works of Hall (1980) and Barro (1981). There are two sorts of influences. First, increasing government purchases induces a negative wealth effect that reduces consumption and raises work effort and output. Second, increasing government purchases temporarily also induces intertemporal substitution, resulting in lower consumption and investment, and higher work effort and output. The relative importance of the wealth and intertemporal substitution channels remains unresolved.

Mankiw (1989) notes that an increase in government expenditure shifts the IS curve upwards, i.e. increases the demand for goods. In order to achieve equilibrium in the market for goods, the real interest rate must rise. RBC theory emphasises the intertemporal substitution of goods and leisure, see Barro (1987). An increase in the real interest rate would lead individuals to reallocate leisure across time. At higher real interest rates, working today becomes relatively more lucrative than working in the future. This temporary increase in the supply of labour leads to a rise in equilibrium output and employment. Although Keynesian theory like the RBC theory predicts a rise in the real interest rate in response to an increase in government consumption, the impact of real interest rate on labour supply is not given prominence. Instead it is the reduction in the amount of labour unemployed or underutilised that results in an increase in employment and output.

Christiano and Eichenbaum (1992) modify a prototypical RBC model by allowing a gov-
Government expenditure shock to influence labour market dynamics. Traditionally, the litmus test for macro models has been their ability to account for the weak correlation between hours worked and returns to working. According to the authors the then existing RBC models grossly overstated this correlation\(^{21}\) while classical and Keynesian models greatly understated it. The authors argue that, by not assigning a role for government expenditure shocks in explaining labour market dynamics, existing RBC models implicitly assume that public and private consumption have the same impact on the marginal utility of private spending. In a significant departure from other RBC models, the authors relax this assumption. Since government spending is modelled as a close substitute for private consumption in the utility function, an increase in government expenditure entails a negative wealth effect which shifts labour supply. Instead of calibrating their model, Christiano and Eichenbaum use a version of Hansen's (1982) generalised method of moments (GMM) procedure for estimation. The estimation criterion is set up in such a way that, in effect, estimated parameter values equate model and sample first moments of the data. The empirical results of the paper show that when fiscal shocks are combined with shocks to technology the model's performance in the case of the US is substantially enhanced.\(^{22}\)

McGrattan (1994a) compares the contribution of technological change for a standard business cycle model that includes a public sector and fiscal disturbances to that of Prescott (1986) who estimates that technology shocks account for nearly 75% of the fluctuations in the post-war US data. McGrattan (1994a) finds that a significant proportion of the variance in aggregate consumption, investment, output, employment, and capital stock can be explained by innovations in labour and capital taxes, and government consumption. Since fiscal disturbances are shocks to labour supply, changes in fiscal variables imply a negative correlation between hours worked and wages when all other factors are held constant. When only productivity shocks are

\(^{21}\)In the RBC world the only impulses generating fluctuations in aggregate employment are stochastic shifts in the marginal product of labour. Loosely speaking, this implies that the time series on hours worked and the return on working are modelled as the intersection of a stochastic labour demand curve with a fixed labour supply curve. Hence, the unsurprising result of strong positive correlation between hours worked and the return to working.

\(^{22}\)There are two caveats to Christiano and Eichenbaum (1992) empirical results. The first is the implicit assumption that public and private capital are perfect substitutes. Some researchers such as Aschauer (1989) have argued that this assumption is empirically implausible. The second caveat is the implicit assumption that all taxes are lump-sum. The authors defend this by stating that their objective is to isolate the role of shocks to government consumption per se.
driving the fluctuations in the economy, this correlation is positive and substantially overestimated. Combining these two shocks leads the model to predict a correlation close to zero which is consistent with US data. The paper assumes that government consumption and tax rates are exogenously driven and that technology shocks, tax rates, and government consumption are correlated stochastic processes. McGrattan estimates the model using the vector autoregression procedure developed in the seminal work of Sims (1980). The main finding is that 41% of the variance of output is explained by innovations in technology, 28% by changes in government expenditure, 27% by innovations in the labour tax rate, and 4% by changes in the capital tax rate. The results reported dispute Prescott's (1986) conclusion that technology shocks account for as much as 75% of business cycle fluctuations. The model is also used to quantify the welfare costs of capital and labour taxation. The results are consistent with others in the literature that the tax on capital is more costly than the tax on labour. McGrattan finds that elimination of the taxes would yield benefits comparable to increase in the growth rate of the economy by 1%.

Braun (1994) argues that the main ingredient missing in RBC models are the factors which shift the labour supply schedule and investigates the macroeconomic effects of cyclical fluctuations in marginal tax rates. In the paper taxation causes shifts in both labour demand and labour supply, resulting in the reduction of the strong positive correlation between wages and hours worked typically found in RBC models. A fall in the effective tax rate on capital income increases the after-tax return on interest rate and leads agents to work more today. Similarly, fluctuations in the wage tax induces a large substitution effect that influences labour supply decision. The author concludes that incorporation of distortionary taxation significantly enhances the fit of the model, with most striking improvement seen in the labour market.

Cooper (1997) reviews the positive and normative aspects of ongoing research involving fiscal shocks to the basic RBC framework. On the positive aspect the author argues that periods of large government purchases are matched with large wealth-reducing taxes which in turn increase the labour supply of individual households. Increases in taxes reduce consumption expenditure and increase the marginal utility of consumption, keeping savings fixed. For the intertemporal

\footnote{The underlying idea for using this procedure is to understand how much of the variance in the aggregate data could be explained by innovations in technology, government expenditure and tax rates.}

\footnote{Technology shocks account for only 20% of fluctuations in hours of work.}
condition to hold labour supply must increase. Cooper stresses the importance of moving away from lump-sum taxation, as introduction of distortionary taxes would create additional substitution effects through the link between distortionary taxes and government spending. From the normative aspect introduction of government expenditure and distortionary taxation makes the framework rich enough to conduct a variety of tax experiments.

1.5 Money in the RBC Framework

Most of real business cycle research in the 1980s focused exclusively on models with no role for money. However, as Eichenbaum and Singleton (1986) point out this prominent omission of the models must not be interpreted as implying the literal absence of money. McCallum (1989) upholds that it would be rather doubtful that RBC proponents were supportive of the proposition that no less output would be produced in the US if there were no medium of exchange, i.e. all exchange had to be carried out by crude or sophisticated barter. However, the RBC models do imply that, to a good approximation, policy induced fluctuations in monetary variables have no affect on real variables like output, consumption, investment etc. Certainly it is true that RBC proponents do not deny that there exist correlations between money and real variables, but believe in ‘reverse causation’ according to King and Plosser (1984). This implies that the observed correlations reflect the responses by the monetary system to fluctuations induced by technology.

Given that the RBC models are a direct outgrowth of the monetary-misperception class of equilibrium models like Lucas (1972, 1973) it is quite surprising that the equilibrium business cycle programme attached an insignificant role to monetary impulses. There are theoretical as well as empirical reasons for this change in tack. First elucidating the theoretical reasons. When the non-neutrality of money is based upon expectational errors due to ‘imperfect information’ the models are relying on an apparent failure in the market for information. However, in actual developed economies data is available on monetary aggregates quite promptly to be consistent with the assumption – critical in Lucas-Barro models – that individuals are igno-

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25 Some researchers including King and Plosser (1984), Kydland (1987), Eichenbaum and Singleton (1986) and Cooley and Hansen (1989), have explored methods of incorporating money in the RBC models and have analysed its implications.
rant of cotemporeaneous monetary aggregates. King (1981) demonstrates that in Lucas' (1973) model, real output should be uncorrelated with the cotemporeaneously available monetary information. Boschen and Grossman (1982) empirically investigate this proposition and find that it is rejected by the data.

On the empirical side the first major development was that of Sims' (1980, 1982) demonstration that money stock innovations have little explanatory power for output fluctuations when a nominal interest rate variable is included in a small VAR system. According to Sims money stock innovations represent surprise policy actions by the Federal Reserve so the finding is clearly indicative of the unimportance of monetary policy actions. However, McCallum (1989) argues that throughout the post-war period US monetary policy has been implemented by means of an interest rate instrument rather than the stock of money. So Sims' finding that money stock innovations provide little explanatory power for output/employment movements can not be interpreted to imply that money policy is unimportant. In fact, the considerable explanatory power provided by interest rate innovations suggests the exact opposite.

Another influential paper at this time was Nelson and Plosser (1982) which begins with the presumption that monetary impulses can have no influence on the trend component of output and output fluctuations are dominated by trend. The authors provide statistical support for the hypothesis that real gross national product (GNP) evolves as a random walk with a drift and therefore, all changes in it are expected to be permanent. Changes in real GNP due to changes in money stock are however, likely to be temporary. The argument in the paper is related to the first-differencing of data, as its contention that output fluctuations are trend-dominated relies crucially on the analysis of output measures that have been first-differenced to remove the non-stationary trend component. However, as illustrated by McCallum (1986), the problem with this conclusion is the inability of finite-sample test procedures to clearly distinguish between $I(1)$ and $I(0)$ series.

The most influential papers incorporating money into the RBC framework are King and Plosser (1984) and Cooley and Hansen (1989). King and Plosser (1984) integrates money and banking into real business cycle theory producing a model that is capable of accounting for

\footnote{King (1981) demonstrates that this problem is not circumvented by the measurement error on the 'true' aggregate.}
the relation between money, inflation and economic activity. The primary motivation for the authors' endeavour was their dissatisfaction with the existing monetary theories of the business cycle advanced by Lucas (1973) and Fischer (1977). King and Plosser's (1984) model has two productive sectors with one intermediate and one final good. The output of the final good industry is stochastic and can be used either as a consumption good or as an input for further production. The output of the financial industry is an intermediate good, called transactions services, used both by firms in the final goods industry and by households. The demand for which arises because it economises the transactions cost of carrying out exchange. Financial services in this framework enter the representative firms production function. The production of financial services is depicted by an instantaneous production structure with constant returns to scale. Innovations in this industry result from shifts to the production structure. Deposits and real currency are treated as close substitutes. If banks are forced by law to hold non-interest-bearing reserves, the cost of bank-supplied intermediate goods goes up and that in turn raises the demand for currency.

An important implication of this model is that bank or inside money has no role in the determination of prices in a competitive unregulated banking environment. However, the banking sector influences price level determination indirectly through variations in the cost of financial services which alters the relative demands for inside and outside money. By implication, broad measures of money are likely to be procyclical due to the endogeneity of bank money which responds to a productivity shock. A rise in output due to a productivity shock causes both the demand for money as well as the interest rate to rise. As interest rates rise, financial institutions try to reduce their holdings of excess reserves, which earn no interest, by making new loans or by purchasing government securities. Given that all such loans ultimately end up as deposits at some financial institution, the broad measure of money expands in response to a productivity shock, i.e. a reverse causation. The authors conclude that in empirical work it is important to distinguish inside from outside money.

Cooley and Hansen (1989) use a cash-in-advance constraint to incorporate money in an RBC

\(^{27}\)Fischer (1977) relies on existing nominal contracts to explain the link between money and business cycles — a continuation of the Keynesian tradition that rely on implausible wage or price rigidities. Barro (1977) stresses that a key feature of Fischer's (1977) model is that agents select contracts that do not fully exploit potential gains from trade. Further, Azariadis' (1978) micro-founded model of wage employment contracts implies that perceived monetary innovations do not alter output.
model. The cash-in-advance constraint in their model is binding only in case of the consumption good while leisure and investment are treated as credit goods. Anticipated inflation in this framework causes agents to substitute away from activities that require cash to ones that do not require it, i.e. leisure and investment. Hence, if agents in this economy choose to reduce cash holdings in response to higher anticipated inflation, which arises due to an increase in the growth of money, they can only do so by reducing their consumption expenditure. In this set-up when money supply follows a constant growth rate rule individuals substitute leisure for goods and both output and investment fall. However, when money supply process is erratic the cyclical behaviour of real variables is altered — consumption becomes more variable relative to income and price level becomes volatile as well.

As has been widely recognised in literature that some form of price/wage rigidity is essential for monetary shocks to have real effects. If prices are fixed then as the central bank’s power of a monopoly over the supply of currency affects relative prices a variety of real effects readily follow. As documented by Plosser (1990), one can classify the attempts to explain monetary non-neutrality into three broad categories: (i) sticky prices, (ii) nominal wage contracts, and (iii) imperfect price information. There is much empirical evidence that many prices and wages are sticky, reacting slowly to changes in demand and to a lesser extent to supply side factors. One argument put forward in the 1970s for sticky prices came from uncertainty and costs of obtaining information about prices. However, a lack of theoretical underpinnings of this view led researchers to think in terms of menu costs, transactions cost, and informational asymmetries.28

1.6 Open Economy Extensions of the RBC Framework

As discussed above quantitative studies on closed economies suggest that a stochastic growth model with a single aggregate technology shock can replicate, among other things, the magnitude of fluctuations, relative to output, in consumption and investment and the correlations of these with output. In the analogous world economy, economies experience imperfectly correlated shocks to their technologies. Now the magnitude and character of aggregate fluctuations can in principle be substantially influenced by the interaction between the technology shocks

28See Akerlof and Yellen (1985) and Mankiw (1985) for theoretical contributions.
and the ability to borrow and lend internationally. As Backus, Kehoe and Kydland (1992) put it "In open economies, a country’s consumption and investment decisions are no longer constrained by its own production." The opportunity to share risk across countries would lead to equilibrium consumption paths that are both less variable and less closely related with the domestic output compared to a closed economy. On the other hand, domestic investment will tend to be more volatile as capital will be allocated to the country with a more favourable technology shock. The most distinguishing feature of an open economy is that it can borrow and lend in international markets by running trade surpluses and deficits.

In the world economy, most countries exhibit well-defined empirical regularities not only in the domestic indicators of economic activity, but also in key international indicators. Backus and Kehoe (1989) and Backus, Kehoe and Kydland (1992, 1994) well document the historical evidence on the international aspects of the business cycle. The significant stylised facts typical of modern open economies are: (i) national savings and investment are positively correlated,\(^{29}\) (ii) after an increase in output, the country’s net foreign asset position deteriorates, and (iii) the current account and the trade balance tend to move counter cyclically.

Mendoza (1991) explores the first two empirical regularities by extending the basic RBC framework proposed in the pioneering work of Kydland and Prescott (1982) to the case of a small open economy.\(^{30}\) A dynamic stochastic model is used to explore the interaction of domestic physical capital and foreign financial assets as an alternative vehicle for savings in an economy in which domestic productivity and the world’s real interest rate are affected by stochastic disturbances. What makes this approach novel is that it explores real business cycles in a framework in which trade in foreign assets finances trade imbalances and plays a crucial role in explaining the dynamics of savings and investment. In contrast with the RBC framework the rate of time preference is endogenously determined.\(^{31}\) Contrary to the argument presented in Feldstein and Horioka (1980) the numerical analysis undertaken indicates that the correlation between savings and investment in a small open economy can be quite high even when there is

\(^{29}\)Obstfeld (1986) shows that after persistent productivity shocks and with perfect capital mobility a non-stochastic dynamic model produces positive correlation between savings and investments. On the other hand, in an overlapping generations framework, Finn (1990) employs a two-country model to show that the correlation between savings and investments depends on the stochastic process of the underlying technology shock.

\(^{30}\)The analysis is undertaken for the Canadian economy, using annual data for the period 1946-85.

\(^{31}\)See for example Uzawa (1968) and Obstfeld (1981).
perfect capital mobility. It is shown that the model can replicate many of the stylised facts of post-war Canada with very small capital adjustment costs and minimal variability and persistence in exogenous shocks — productivity or terms of trade. Without these adjustment costs the model over exaggerates the variability in investment as the separation of savings and investment together with the absence of adjustment costs allows physical capital to be altered too easily.

Correia, Neves and Rebelo (1995) summarise the main features of business cycles in the small open economy – Portugal – and discuss the extent to which these can be rationalised on the basis of a simple dynamic stochastic general equilibrium model. The structure of their economy is very similar to Mendoza (1991) and Lundvik (1992) in which there is a single asset that can be traded with the rest of the world — an international bond that yields a real rate of return that is viewed as exogenous by agents in the economy. In order to ensure that the current account follows a trend stationary process Mendoza (1991) postulates time-non-separable preferences while the assumption of agents having finite horizon is made by Lundvik (1992). In contrast, Correia, Neves and Rebelo's (1995) model is a natural extension of closed economy RBC models as it features conventional time-separable preferences. The crucial finding of the paper is that the ability of small open economy models to mimic business cycles depends heavily on the form of the momentary utility. When the authors employed momentary utility that is standard in the closed economy literature, for e.g. Hansen's (1985) divisible labour model, they find that the model is incapable of reproducing important features of the business cycle; the volatility of consumption is much lower than in the data and the balance of trade is procyclical rather than countercyclical. However, if they adopt the momentary utility function proposed by Greenwood, Hercowitz and Huffman (1988) then the relative variability and comovement patterns found in the data for the components of the national income identity can be replicated. According to

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32 Providing support to Obstfeld (1986) and Finn (1990) argument that the intensity of the comovement between savings and investments in economies with perfect capital mobility depends on the degree of persistence of the underlying technological disturbances.

33 Dooley, Frankel and Mathieson (1987) provide evidence which suggests that even though financial markets may be fully integrated, physical capital may not be perfectly mobile.

34 These functions imply that labour effort is determined independently of the intertemporal consumption-savings choice. As noted by Harjes (1997) for these preferences to be consistent with steady-state growth, namely a constant share of time devoted to work, the disutility of work in the market has to increase with the level of technical progress.
them it is the adoption of the Greenwood, Hercowitz and Huffman (1988) momentary utility function that is fundamental to the good performance of the models of Mendoza (1991) and Lundvik (1992).\textsuperscript{35}

In another paper Mendoza (1995) uses an intertemporal general equilibrium framework to explore the effects of random shocks to productivity and the terms of trade. The stylised fact is that movements in the real exchange rate are procyclical. Also, observed terms of trade shocks are largely procyclical but persistent. Less developed countries tend to have larger cycles but both developed and developing economies have similar variability ratios, autocorrelations and gross domestic product (GDP) correlations. Mendoza's (1995) results are consistent with the empirical regularities. The paper shows that the terms of trade shocks are the main driving force behind almost half of the GDP and the real exchange rate variability. The key feature of the dynamics of the model is the persistence, magnitude and current correlation of the terms of trade and the productivity shocks and the elasticity of substitution between traded and nontraded goods. The covariance and autocorrelation structures of the shocks are very important as after any shock income effects will impinge on the optimal savings behaviour. The elasticity between traded and nontraded goods is also important since these goods are in general gross substitutes in less developed economies and gross compliments in developed economies; implying divergence for cross-price and cross-expenditure effects.

McCurdy and Ricketts (1995) extend the basic RBC model by employing stochastic international model with money.\textsuperscript{36} Using a sample of US and Canadian data to calibrate the forcing processes they are able to compute a perfectly pooled equilibrium solution for their stochastic growth model. The paper shows that changes in the rate of growth of money cause fluctuations in consumption and investment. Further the effects on goods and asset prices can be substantially different from that in endowment models. It is also shown that monetary fluctuations are transmitted to other economies via exchange rates and terms of trade adjustments.

Backus, Kehoe and Kydland (1994) employ a two-country dynamic general equilibrium

\textsuperscript{35}Devereux, Gregory and Smith (1992) argue that the Greenwood, Hercowitz and Huffman (1988) preferences are capable of resolving the 'consumption correlation puzzle' stressed by Backus, Kehoe and Kydland (1992). The puzzle consists of the fact that two-country models generally predict that the correlation of consumption between the two countries should be higher than that of output, while the reverse pattern is found in the data.

\textsuperscript{36}Their work builds on Backus, Kehoe and Kydland (1992) who allow for international borrowings in an international business cycle model in which economies experience different productivity shocks.
model to study the effects of shocks to productivity in a production economy, which uses capital and labour to produce output. The authors assume that the two countries produce imperfectly substitutable goods. Using plausible parameter values they are able to replicate key empirical regularities for eleven industrialised economies. It is shown that after an increase in domestic output the trade balance deteriorates and is negatively correlated with current as well as future movements in the terms of trade but positively correlated with past movements.\(^{37}\) The intuition for the result is as follows. A positive productivity shock increases domestic output and thus reduces its relative price leading to a terms of trade deterioration. As the productivity shock is persistent, consumption and investment also rise. The economy will therefore experience a trade deficit as the increase in consumption and investment outweigh the gains in output. This dynamic response pattern produces the countercyclical movements in the trade balance as observed in the data. It should be noted that the dynamics of investment are crucial for generating the above results. If we were to eliminate capital from their model then the trade balance will be driven by output dynamics and consumption smoothing. In that case after a positive productivity shock the trade balance will improve as preference for smooth consumption results in a smaller increase in consumption than output. The trade balance would thus be procyclical. Further, the price of the domestic output falls and terms of trade improve.

Harjes (1997) analyses whether a stochastic dynamic optimising open economy model is also capable of displaying the observed features of business cycles for a large open economy which is strongly engaged in international trade such as Germany.\(^{38}\) One approach to modelling a large economy is to use a two-country framework, applied to a specific country and to the rest of the world. However, the calibration of parameters which correspond to preferences and technology of the rest of the world is a task with serious empirical difficulties. In order to capture part of the impact of a large open economy on the world Harjes introduces a positive relationship between the price of foreign bonds and the stock of foreign bonds, that is, it is assumed that a large country is characterised by having some market power *vis-à-vis* the world economy.\(^{39}\)

\(^{37}\)Terms of trade are defined as the relative price of imports to exports and the trade balance is the ratio of net exports to output.

\(^{38}\)The model is parameterised and calibrated to West German data from 1968 to 1993.

\(^{39}\)It should be noted that small open economy models take the world interest rate as given, i.e. a small open economy is assumed to have negligible effect on world interest rate because, being a small part of the world, its effect on world saving and world investment is negligible.
In the model the world real interest rate then depends on the aggregate stock of next period's foreign bonds.\textsuperscript{40} This dependence reduces the volatility in investment and trade balance slightly which is consistent with the data.\textsuperscript{41} The model also predicts a countercyclical behaviour of the trade balance and a significant positive correlation of domestic savings and investment.

More recently, efforts have been made to develop a new workhorse model for open economy macroeconomic analysis. Obstfeld and Rogoff (1995) is commonly recognised as the contribution that launched this new wave of research. The unifying feature of this emerging literature is the introduction of nominal rigidities and market imperfections into a dynamic general equilibrium model with well-specified microfoundations. The presentation of explicit and profit maximising problems provides welcome clarity and analytic rigour compared to the models of the past, which used ad-hoc assumptions to generate features of the data such as inflation/real exchange rate persistence. Moreover, it allows the researcher to conduct welfare analysis, hence laying the groundwork for credible policy evaluation. The presence of nominal rigidities and market imperfections alters the transmission mechanism for shocks and also provides a more potent role for monetary policy. One of the goals of this new strand of literature is to provide an analytic framework that is relevant for policy analysis and offers a superior alternative to the Mundell-Fleming model that is still widely employed in policy circles as a theoretical reference point.

The choice of the exchange rate regime is a special case of the general issue of optimal monetary policy in an open economy. Researchers have been prolific using the stochastic general equilibrium paradigm to investigate the performance of alternative open economy monetary policy rules, for e.g. Benigno and Benigno (2001), the analysis of alternative exchange rate regimes in terms of macroeconomic and welfare properties, see Devereux and Engel (2001), Collard and Dellas (2002), Dellas and Tavlas (2002), and the welfare implications of different degrees on international policy coordination, Corsetti and Pesenti (2001a, 2001b), Obstfeld and Rogoff (2001), Pappa (2001), Canzoneri, Cumby and Diba (2002) to name a few. The message emerging from this literature regarding the value of the exchange rate instrument is mixed. The

\textsuperscript{40}Foreigners are not willing to borrow and lend capital at a fixed world interest rate in unlimited amounts; more the home country wishes to borrow from abroad, the higher the interest rate it has to pay, or vice versa, the more goods the home country wishes to lend to the rest of the world, the lower the interest rate payments it will receive.

\textsuperscript{41}See Baxter (1995) for evidence that larger economies have less volatility in investment and the trade balance.
results depend on the currency denomination of trade, the structure of financial markets, the type of policy rule considered and the difference in size across countries.

As Lucas (1980) notes, "One of the functions of theoretical economics is to provide fully articulated, artificial economic systems that can serve as laboratories in which policies that would be prohibitively expensive to experiment with in actual economies can be tested out at much lower cost." However, incorporating more and more features of the real world increases the complexity of models exponentially. Most of the open economy dynamic general equilibrium models of today are highly non-linear. These models can not be solved analytically, and most do not have closed form solutions. The usual approach is either to take linear approximation around the steady-state-growth path or to solve them numerically, i.e. use algorithms to simulate the model economy. As Friedman (1995) argues "Economists prefer parsimonious models. Repeated experience clearly shows that progress in economic thinking is possible only with heroic simplification." The reason being that the phenomena that economists attempt to study are very complex in comparison to the tools available at their disposal.

The challenge for modellers, therefore, has been to construct fully transparent macro models that pass the simplicity test and at the same time are reliable for forecasting and policy analysis. The model discussed in this thesis is a micro-founded general equilibrium open economy model based on optimising decisions of rational agents, incorporating money, government and distortionary taxes. The first order conditions are used to derive the behavioural equations of the model. However, as we are modelling the UK – a medium-sized open economy – so we have a full blown model only for the domestic economy taking the world economy as given.\footnote{This assumption is usually made for a small open economy.} The interaction with the rest of the world comes in the form of uncovered real interest rate parity and current account which are both explicitly micro-founded.

1.7 Policy Implications of RBC Models

The policy lessons of real business cycle theory are more subtle than they appear at first blush. Although the theory ascribes no ostensible role to post-war countercyclical policies, its success in
accounting for US business cycles may actually be the clearest indication yet of the effectiveness of these policies. Though, at the same time, the doubts raised by the theory about the wisdom of some policy initiatives to control business cycles may be well founded.

The main policy implication of the RBC research agenda has been that any efforts at stabilisation are likely to be counter-productive since economic fluctuations observed are not welfare-reducing deviations from 'natural rate' paths of an ideally efficient Walrasian economy but are optimal responses by rational agents to uncertainty regarding the rate of technological change. McCallum (1989) argues "...the mere existence of cyclical fluctuations is not sufficient for a conclusion that interventionist government policy is warranted." However, he also notes that these models provide no basis for concluding that the solutions generated by actual economies are Pareto optimal.

According to Mankiw (1989) a good theory needs to be both internally as well as externally consistent. An internally consistent theory is one that is parsimonious; it invokes no ad-hoc or peculiar axioms, while an externally consistent theory is one that fits the facts; it makes empirically unrefutable predictions. The real business cycle models clearly are internally consistent and to a huge degree even externally so, as they are able to replicate the main time series properties of major macroeconomic variables. However, models embodying imperfect competition, market externalities, asymmetric information and other such impediments to smooth functioning of the market invariably fail the simplicity test in an attempt to mimic the world more accurately.

1.8 Conclusions

This chapter has familiarised the reader with the underlying concepts of models embodying a real business cycle framework, which is used in subsequent chapters of the thesis to evaluate the impact of alternative monetary and fiscal policy shocks and explain the behaviour of the real exchange rate. We first introduced the reader to the macroeconomic climate - the failure of the Keynesian models in the 1970s together with the rational expectations revolution - that led to the birth of the 'real business cycle' framework. We elucidated a prototypical RBC model and then described in some detail the early models which laid the foundations for the vast body of
research that now exists embodying this framework. The early models of the RBC literature were closed economy models that assumed no externalities, taxes, government expenditure or monetary variables. Theirs was a ‘competitive theory of economic fluctuations’ and hence the equilibrium was Pareto optimal. We then traced the developments in the literature in terms of addition of government expenditure, taxes and money in the basic RBC model and their policy implications. More recently open economy extensions have been made which are vital for our understanding of international affects of business cycles and spillovers, if any. The main policy implication of the RBC research agenda has been that any efforts at stabilisation are likely to be counter-productive since economic fluctuations observed are not welfare-reducing deviations from ‘natural rate’ paths of an ideally efficient Walrasian economy but are optimal responses by rational agents to uncertainty regarding the rate of technological change.
Chapter 2

A Fully Specified Open Economy
Real Business Cycle Model for the UK

2.1 Introduction

The objective of this chapter is to specify a fully articulated model of an open economy which we propose to calibrate/estimate using quarterly data for the UK. The model presented here is an enriched variant of a prototype real business cycle (RBC) model embodying a representative agent framework as in McCallum (1989).

This chapter is organised as follows. Section 2.2 introduces some basic features of the model economy, Section 2.2.1 specifies the optimisation problem faced by a representative household, Section 2.2.2 describes the governments' role in the model economy, Section 2.2.3 specifies the optimisation problem of the representative firm and Section 2.2.4 describes the foreign sector of the economy. The conclusions are presented in Section 2.3.

2.2 Model

Consider an open economy populated by identical infinitely lived agents who produce a single good as output and use it both for consumption and investment purposes. To simplify the
notation we abstract from population growth and represent all variables in per capita terms. We assume that there are no market imperfections i.e., no frictions or transactions costs. At the beginning of each period ‘t’, the representative agent chooses: (i) the commodity bundle necessary for consumption during the period, (ii) the total amount of leisure that she would like to enjoy during the period, and (iii) the total amount of factor inputs necessary to carry out production during the period. All of these choices are constrained by the fixed amount of time available and the aggregate resource constraint that agents face. During the period ‘t’, the model economy is influenced by various random shocks.

In an open economy goods can be traded but for simplicity it is assumed that these do not enter in the production process but are only exchanged as final goods. The consumption, $C_t$ in the utility function below is composite per capita consumption, made up of agents consumption of domestic goods, $C^d_t$ and their consumption of imported goods, $C^f_t$.\(^1\) The composite consumption function can be represented as an Armington aggregator of the form

$$C_t = \left[\omega \left(C^d_t\right)^{-\rho} + (1 - \omega) \left(C^f_t\right)^{-\rho}\right]^{\frac{1}{1 - \rho}} \quad (2.1)$$

where $\omega$ is the weight of home goods in the consumption function and $\sigma$, the elasticity of substitution is equal to $\frac{1}{1 + \rho}$.\(^2\)

The consumption-based price index that corresponds to the above specification of preference,\(^3\) denoted $P_t$ is derived as

$$P_t = \left[\omega \left(P^d_t\right)^{\frac{\epsilon}{\epsilon + \rho}} + (1 - \omega) \left(P^f_t\right)^{\frac{\epsilon}{\epsilon + \rho}} \left(P^F_t\right)^{\frac{1 + \epsilon}{\epsilon + \rho}}\right]^{\frac{1}{1 + \epsilon}} \quad (2.2)$$

where $P^d_t$ is the domestic price level and $P^F_t$ is the foreign price level in domestic currency.

Given the specification of the consumption basket, the agent’s demand for home and foreign

\(^1\)It is to be noted $C^f_t$ is the same as $I\lambda_t$ used later in the chapter.

\(^2\)The derivation of $\sigma$, the elasticity of substitution can be found in the foreign sector section of the chapter.

\(^3\)The consumption-based price index $P_t$ is defined as the minimum expenditure that is necessary to buy one unit of the composite good $C_t$, given the price of the domestic good and foreign good.

\(^4\) $P^F_t = S_t P^f_t$ where $S_t$ is the nominal exchange rate and $P^f_t$ is the foreign price level in foreign prices. So, $P^F_t$ is the foreign price level in domestic prices.
goods are functions of their respective relative price and the composite consumption.

\[ C_t^d = \left( \frac{P_t^d}{\omega P_t} \right)^{-\gamma} C_t \]  

(2.3)

\[ C_t^f = \left( \frac{P_t^f}{(1 - \omega) P_t} \right)^{-\gamma} C_t \]  

(2.4)

In a stochastic environment a consumer is expected to maximise her expected utility subject to her budget constraint. Each agent's preferences are given by

\[ U = \max E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \right], \quad 0 < \beta < 1 \]  

(2.5)

where \( \beta \) is the discount factor, \( C_t \) is consumption in period 't', \( L_t \) is the amount of leisure time consumed in period 't' and \( E_0 \) is the mathematical expectations operator. An essential feature of this structure is that the agent's tastes are assumed to be constant over time and are not influenced by exogenous stochastic shocks. The preference ordering of consumption subsequences [(\( C_t, L_t \), (\( C_{t+1}, L_{t+1} \), ...)] does not depend on 't' or on consumption prior to time 't'. We assume that \( u(C, L) \) is increasing in (\( C, L \)) and concave — \( u'(C, L) > 0 \), \( u''(C, L) < 0 \). We also assume that \( u(C, L) \) satisfies Inada-type conditions: \( u'(C, L) \to \infty \) as \( c \to 0 \), and \( u'(C, L) \to 0 \) as \( c \to \infty \), \( u'(C, L) \to \infty \) as \( l \to 0 \), and \( u'(C, L) \to 0 \) as \( l \to \infty \).

In a prototype RBC model with complete markets and the absence of any form of externality there is no role for the government. Still, one could think of a government providing public goods from the tax revenue it collects, although this is not really a stabilisation role for the government. Incorporation of government expenditure, i.e. fiscal policy into the RBC framework introduces a potential source of demand side disturbance to the basic model which is otherwise governed by supply side disturbances.

Following Lucas (1980, 1987), our model assumes money has value in exchange. In order to

\footnote{For the sake of convenience we shall use consumption in place of composite consumption throughout the chapter.}
give value to money we need to introduce trading in decentralised markets. Here, to motivate the use of money a subset of consumption goods must be paid for with currency acquired in advance. The cash-in-advance model is a convenient way for representing the aspects of classical monetary theory in the context of an intertemporal model.

2.2.1 The Representative Household

The model economy is populated by a large number of identical households who make consumption, investment, and labour supply decisions overtime. Each households' objective is to choose sequences of consumption and hours of leisure that maximise its expected discounted stream of utility. The utility function is assumed to possess the following properties. The representative agent is assumed to derive positive, but diminishing marginal utility from the consumption of goods and leisure. The utility function is further assumed to be strictly concave in its arguments, i.e. consumption and leisure. In addition we postulate that consumption and leisure are normal goods, meaning that they both increase with wealth.

We assume a time-separable utility function of the form

\[ U(C_t, 1 - N_t) = \theta_0 (1 - \rho_0)^{-1} C_t^{(1-\rho_0)} + (1 - \theta_0) (1 - \rho_2)^{-1} (1 - N_t)^{(1-\rho_2)} \]  

where \( 0 < \theta_0 < 1 \), and \( \rho_0, \rho_2 > 0 \) are the substitution parameters. This sort of functional form is common in the literature for example McCallum and Nelson (1999a). The advantage of using this specification is that it does not restrict the elasticity of substitution between consumption and leisure to unity.

Barro and King (1984) note that time-separable preference ordering of this form would not restrict the sizes of intertemporal substitution effects. However, it must be noted that time-separability constrains the relative size of various responses such as those of leisure and consumption to relative price and income effects. As the authors argue, for the purpose of business cycle analysis, the presumption that departures from separability matters only for

\footnote{In cash-in-advance models, the market structure and households' constraint are altered vis-à-vis an Arrow-Debreu model in that at least some goods can be purchased only with currency accumulated in advance of shopping.}

\footnote{The Cobb-Douglas utility function is a special case of the CES utility function when \( \rho_0 = \rho_2 = 0 \).}
days and weeks and not for months or years is fully justified. As macroeconomic analysis is concerned primarily with time periods such as quarters or years, time-separability of preferences is a reasonable approximation in this context.

Individual economic agents view themselves as playing a dynamic stochastic game. Changes in expectations about future events would generally affect current decisions. Individual choices at any given point in time are likely to be influenced by what agents believe would be their available opportunity set in the future.

Each agent in our model is endowed with a fixed amount of time which she spends on leisure \( L_t \) and/or work \( N_t \). If \( H_t \), total endowment of time is normalised to unity, then it follows that

\[
N_t + L_t = 1 \text{ or } L_t = 1 - N_t
\]

Let us assume that \( \bar{l} \) is the normal amount of leisure which is necessary for an agent to sustain her productivity over a period of time. If an agent prefers more than normal amount of leisure say \( 'U_t' \) she is assumed to be unemployed \( (U_t = (1 - N_t) - \bar{l}) \) in this framework.\(^8\) An agent who chooses \( U_t \) is entitled to an unemployment benefit \( '\mu_t' \) from the state. It is assumed that \( \mu_t < v_t \), where \( v_t \) is the consumer real wage as defined below, so that there is an incentive for the agent to search for a job. With the introduction of unemployment benefits substitution between work and leisure is higher.

The representative agents budget constraint is

\[
(1 + \phi_t)C_t + \frac{b_{t+1}}{1+r_t^t} + \frac{Q_t b_t'}{(1+r_t^t)} + \frac{b_t}{P_t} + T_t =
\]

\[
(1 - \tau_{t-1})v_{t-1}N_{t-1} + \mu_{t-1}[(1 - N_{t-1}) - \bar{l}] + b_t + Q_t b_t' + \frac{(p_t + d_t)S^p_t}{P_t}
\]

where \( p_t \) denotes the present value of share, \( v_t = \frac{W_t}{P_t} \) is the real consumer wage, \( w_t = \frac{W_t}{P_t} \) is the producer real wage.\(^9\) The dividends \( d_t \) is such that the firm returns to the consumer all residual income, including any capital gains or losses on money holdings due to inflation. Further, we

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\(^8\)It should be noted that leisure, \( (1 - N_t) \) is bounded at \( \bar{l} \).  

\(^9\)\( P_t = \left[ \omega^\frac{1+r}{r} (P_t^d)^{\frac{1+r}{r}} + (1-\omega)^{\frac{1+r}{r}} (P_t^f)^{\frac{1+r}{r}} \right]^{-\frac{1+r}{r}} \) and \( W_t \) is nominal wage.

\(^{10}\)Please note that consumers take into account domestic and foreign prices while evaluating their real wages. However, producers do not, this is because they do not use imported intermediate goods.
have eliminated the effect of the expected inflation tax on the consumer’s leisure decision by assuming that the revenue from this is returned to the taxpayers.\textsuperscript{11} Consumption and labour income are taxed at rates $\phi_t$ and $\tau_t$ respectively, both of which are assumed to be stochastic processes. Also, $(1 + \phi_t) C^d_t = \frac{M^{dp}}{P_t}$, i.e. representative agent’s real demand for domestic money is equal to consumption of domestic goods inclusive of sales tax. In a similar way, the agent’s real demand for foreign money is equal to consumption of foreign goods inclusive of sales tax, $(1 + \phi_t) C^f_t = \frac{M^{fp}}{P_t}$\textsuperscript{12}. This follows from the fact that consumption in this framework is treated as a ‘cash good’, i.e. the cash-in-advance constraint is binding only in the case of consumption. Investment is treated as a credit good. $b_t^f$ denotes foreign bonds, $b_t$ domestic bonds, $S^p_t$ demand for domestic shares and $Q_t$ is the real exchange rate.\textsuperscript{13}

One way of looking at the representative household’s budget constraint is to think of time being divided into two subperiods. In the first subperiod the household receives labour income (net-of-tax) $(1 - \tau_{t-1}) v_{t-1} N_{t-1}$, bond income due from previous period, both domestic $b_t$, and foreign $Q_t b_t^f$, unemployment benefits from the government, $\mu_{t-1} [(1 - N_{t-1}) - \bar{I}]$ and dividends, $d_t$ from its investment in shares, $S^p_t$, i.e. $\left(\frac{P_t + d_t}{P_{t-1}}\right) S^p_t$. In the second subperiod she buys goods with the help of currency carried forward from the first subperiod and undertakes financial transactions, i.e. purchases shares and government bonds.\textsuperscript{14}

In a stochastic environment the representative agent maximizes her expected discounted stream of utility subject to her budget constraint. The Lagrangian associated with this problem

\textsuperscript{11}See the government budget constraint where we subtract $E_{t-1} \pi_{t+1} \cdot M^d_t$. It should be noted that the inflation tax returned to consumers is equal to the amount actually paid by them, i.e. pro rate with money holdings.

\textsuperscript{12} $M^{fp}_t = S_t M^{dp}_t$.

\textsuperscript{13} The real exchange rate is defined as $Q_t = \frac{S_t P^f_t}{P_t}$.

\textsuperscript{14}Basically consumers spend the money at the end of the period on consumption; the firms then give it back to them as wages and dividends while the government returns them the inflation tax - pro rate with money holdings. This amounts to the fact that at the end of each period firms have the money; they give it to the households during the period and then the households give it back to them in spending before the end of the period, so firms have it again.
is:

\[
U = \text{Max } E_0 \sum_{t=0}^{\infty} \beta^t \left( \left[ \theta_0 (1 - \rho_0)^{-1} C_t^{(1-\rho_0)} + (1 - \theta_0) (1 - \rho_2)^{-1} (1 - N_t)^{(1-\rho_2)} \right]
+ \lambda_t \left[ (1 - \tau_{t-1}) v_{t-1} N_{t-1} + \mu_{t-1} \left[ (1 - N_{t-1}) - \overline{L} \right] + b_t + Q_t b_t^I + \frac{(p_{t+1} + d_t) S_{t+1}^p}{P_t} - (1 + \phi_t) C_t - \frac{b_{t+1}}{1+r_t} - \frac{Q_t b_{t+1}^I}{r_t} \right] \right)
\]

(2.9)

where ‘\( \lambda \)’ is the Lagrangian multiplier, \( 0 < \beta < 1 \) is the discount factor, and \( E_0 \) is the mathematical expectations operator.

The first order conditions with respect to \( C_t, N_t, b_{t+1}, b_{t+1}^I \) and \( S_{t+1}^p \) are:\(^{15}\)

\[
(1 - \rho_0) \theta_0 (1 - \rho_0)^{-1} C_t^{\rho_0} = \lambda_t (1 + \phi_t)
\]

(2.10)

\[
(1 - \rho_2) (1 - \theta_0) (1 - \rho_2)^{-1} (1 - N_t)^{-\rho_2} = \beta E_t \lambda_{t+1} [ (1 - \tau_t) v_t - \mu_t]
\]

(2.11)

\[
\frac{\lambda_t}{1 + r_t} = \beta E_t \lambda_{t+1}
\]

(2.12)

\[
\frac{\lambda_t Q_t}{1 + r_t^I} = \beta E_t \lambda_{t+1} Q_{t+1}
\]

(2.13)

\[
\frac{\lambda_t p_t}{P_t} = \beta E_t \lambda_{t+1} \left( \frac{p_{t+1} + d_{t+1}}{P_{t+1}} \right)
\]

(2.14)

The first of the above equations equates the marginal utility of composite consumption to the shadow price of output. Note that sales tax impinges on this equation. The second equates the marginal disutility of labour to labour’s marginal product — the real wage. The marginal product of labour is affected both by tax on labour and the unemployment benefits. From the representative household’s first-order condition we know that supply of labour is positively

\(^{15}\)All future values are expected --- for convenience expectations operator is dropped.
related to the net-of-tax real wage and negatively related to the unemployment benefits. If the after-tax real wage is temporarily high, substitution effect overpowers the income effect. The increase in work effort raises employment and output. On the other hand unemployment benefits negatively impinge upon supply of work effort. These equations which are the stochastic analogue of the well known Euler equations which characterises the expected behavior of the economy, determine the time path of the economy’s values of labour, consumption and investments in financial assets.

Substituting equation (2.12) in (2.10) and ignoring second order expectational terms yields:

\[
(1 + r_t) = \left( \frac{1}{\beta} \right) \left( \frac{C_t}{C_{t+1}} \right)^{-\rho_0} \left( \frac{1 + \phi_{t+1}}{1 + \phi_t} \right) \tag{2.15}
\]

Now substituting (2.10) and (2.12) in (2.11) yields

\[
(1 - N_t) = \left\{ \frac{\theta_0 C_t^{-\rho_0} [(1 - \tau_t) v_t - \mu_t]}{(1 - \theta_0) (1 + \phi_t) (1 + r_t)} \right\} \frac{\omega^{1+\rho}}{\omega^{1+\rho} + (1 - \omega)^{1+\rho}} \tag{2.16}
\]

where \( v_t \), consumer real wage enters the labour supply equation. Now

\[
\log v_t = \log W^* - \left[ \frac{\omega^{1+\rho}}{\omega^{1+\rho} + (1 - \omega)^{1+\rho}} \log P_t^d + \frac{(1 - \omega)^{1+\rho}}{\omega^{1+\rho} + (1 - \omega)^{1+\rho}} \log P_t^F \right] \tag{2.17}
\]

Also given that \( \log W_t = \log w_t + \log P_t^d \), and using \( \log Q_t = \log P_t^F - \log P_t^d \), then

\[
\log v_t = \log w_t - \frac{(1 - \omega)^{1+\rho}}{\omega^{1+\rho} + (1 - \omega)^{1+\rho}} \log Q_t \tag{2.18}
\]

---

\(^{16}\) As noted earlier all future values are expected — for convenience expectations operator is dropped. Further it should be noted that \( E_t \left[ C_{t+1}^{-\rho_0} \right] \) is approximately equal to \( E_t \left[ C_{t+1}^{-\rho_0} \right] \), taking a Taylor series expansion of \( E_t \left[ C_{t+1}^{-\rho_0} \right] \) around \( E_t C_{t+1} \) where one ignores the \( E_t \) squared term, i.e. treats the variance as a constant given that all the errors are homoscedastic. This expansion is \( E_t \left[ C_{t+1}^{-\rho_0} \right] + E_t \left[ \rho_0 E_t C_{t+1}^{-\rho_0} \right] \left[ C_{t+1} - E_t C_{t+1} \right] + \) second order term. The middle term is zero because the expectation of the difference of \( C_{t+1} \) from its expectation is zero.

\(^{17}\) \( w_t \) is the producer real wage, \( w_t = \frac{w_t}{r_t} \).
Therefore (2.16) becomes

\[
(1 - N_t) = \left\{ \theta_0 C_t^{-\rho_0} \left( (1 - \tau_t) \exp \left( \log w_t^* - \frac{(1-\omega)^{1+r_t}}{\omega + \rho + (1-\omega)^{1+r_t}} \log Q_t \right) - \mu_t \right) \right\}^{\frac{-1}{\rho_2}}
\]

(2.19)

If each household can borrow an unlimited amount at the going interest rate, then it has an incentive to pursue a Ponzi game. The household can borrow to finance current consumption and then use future borrowing to roll over the principal and pay all of the interest. To prevent the household from playing a Ponzi game it is further assumed that the household's decision rule is subject to a transversality condition,

\[
Y_{T-1} - r_T D_T - \phi_T C_T - \tau_T v_T N_T^* - T_T = C_T
\]

(2.20)

Substituting (2.12) in (2.14) we have

\[
p_t = \left( \frac{p_{t+1} + d_{t+1}}{(1 + r_t)} \right) \frac{P_t}{P_{t+1}}
\]

(2.21)

Using \( p_{t+1} = \frac{p_{t+2} + d_{t+2}}{(1 + r_{t+1})} \frac{P_{t+1}}{P_{t+2}} \) in above, yields

\[
p_t = \left( \frac{p_{t+2} + d_{t+2}}{(1 + r_t)(1 + r_{t+1})} \right) \left( \frac{P_t}{P_{t+2}} \right) + \left( \frac{d_{t+1}}{1 + r_t} \right) \left( \frac{P_t}{P_{t+1}} \right)
\]

(2.22)

Using the arbitrage condition and by forward substitution, ignoring second order expectational terms, the above yields

\[
p_t = E_t \sum_{i=1}^{\infty} \left[ \frac{d_{t+i}}{\prod_{k=t}^{i} (1 + r_k)} \left( \frac{P_t}{P_{t+i}} \right) \right]
\]

(2.23)

The above equation states that the present value of a share is simply discounted future dividends.
In small open economy models the real exchange rate\textsuperscript{18} is taken as given because prices of both ‘home’ and ‘foreign’ goods are set in world markets which the domestic economy is too small to influence. Further, since the economy is too small to have any effect on the world real interest rate\textsuperscript{19} which is set by the balance of world saving and investment, it follows that its real exchange rate is expected to be constant and hence via uncovered interest parity its real interest rate is equal to the world real interest rate. However, we are modelling the UK a medium sized economy. In our model home goods are differentiated from foreign goods and so supplies and demands for the home good\textsuperscript{20} do affect home prices and therefore the real exchange rate. Hence expected changes in the real exchange rate drive a wedge between home and world real interest rates.

To derive the uncovered interest parity condition equation (2.12) is substituted into (2.13)

\[
\left(\frac{1 + r_t}{1 + r^f_t}\right) = \frac{Q_{t+1}}{Q_t}
\]

(2.24)

In logs this yields

\[
r_t = r^f_t + E_t \Delta \log Q_{t+1}
\]

(2.25)

2.2.2 The Government

In this framework it is assumed that the government spends current output according to a non-negative stochastic process such that $G_t \leq Y_t$ for all ‘t’. The variable $G_t$ denotes per capita government expenditure at ‘t’. Here it is also assumed that government expenditure does not enter the agent’s objective function. In case of equilibrium business cycle models embodying rational expectations, output is always at its ‘desired’ level. Given the information set, agents are maximising their welfare subject to their constraints. Since there are no distortions in this set-up government expenditure may not improve welfare through its stabilisation programme. It is for this reason government expenditure has been excluded from the representative agent’s

\textsuperscript{18}In the sense here of the relative price of home and foreign goods.

\textsuperscript{19}The economy has basically no effect on the world rate because, being a small part of the world, its affect on the world savings and investment is negligible.

\textsuperscript{20}Both at home and abroad.
utility function. As stated above the state also pays out unemployment benefits $\mu_t$ which leads to higher substitution between work and leisure.

The government finances its expenditure by collecting taxes on labour income, $\tau_t$, and taxes on consumption, $\phi_t$, which are assumed to be stochastic processes. Also, it issues debt, bonds $(b_t)$ each period which pays a return next period. Then, it collects seigniorage, i.e. $\frac{M^d_{t+1} - M^d_t}{P^d_t}$ which is assumed to act as a lump-sum tax, leaving real asset prices and allocation unaltered\(^{21}\) and is assumed to be a stochastic process.

Tax on labour income, since it reduces the after-tax return accruing to an agent from supplying labour in the market, is likely to affect her choice as to how much of labour to supply at a given point in time. By reducing the take-home wage, the labour income tax reduces the opportunity cost of leisure, and there is a tendency to substitute leisure for work. This is the substitution effect, and it tends to decrease labour supply. At the same time, the tax reduces the individual's income. Given that leisure is a normal good, this loss in income leads to a reduction in consumption of leisure, \textit{ceteris paribus}. The income effect tends to induce an individual to work more. It is the relative strengths of the income and substitution effects which would ultimately determine whether an agent would work more or less.

Tax on consumption is similar to income tax in the sense that it is imposed on flows generated in the production of current output. However, income tax is imposed on the net income received by agents whereas sales tax is imposed by the state on the sales of business firms.

The government budget constraint is:

$$G_t + b_t + \mu_t \left[ (1 - N_t) - I \right] = \tau_{t-1} v_{t-1} N_{t-1} + \phi_{t-1} C_{t-1} + \frac{b_{t+1}}{1 + \tau_t} + \frac{M^d_{t+1} - M^d_t}{P^d_t} - E_t \pi^d_{t+1} M^d_t$$

(2.26)

where $b_t$ is real bonds, $P^d_t$ is the domestic price level and $\pi^d_t$ is the domestic inflation rate. Note that $\tau_{t-1} v_{t-1} N_{t-1} + \phi_{t-1} C_{t-1}$ is the total tax revenue collected by the state. Also, the government faces a cash-in-advance constraint, i.e.:

$$P^d_t G_t \leq M^d t$$

(2.27)

\(^{21}\text{As the revenue from expected inflation is returned to the taxpayers.}\)
where $M^d_t$ is government's demand for domestic money. Here we assume that the government has home bias, i.e. it consumes only domestic goods.

### 2.2.3 The Representative Firm

Firms rent labour and buy capital inputs from households\(^{22}\) and transform them into output according to a production technology and sell consumption and investment goods to households and government. The interaction between firms and household is crucial, as it provides valuable insights for understanding the fluctuations of macroeconomic aggregates such as output, consumption and employment. The technology available to the economy is described by a constant-returns-to-scale production function:

$$Y_t = Z_t f (N_t, K_t)$$

or

$$Y_t = Z_t N_t^\alpha K_t^{1-\alpha} \quad (2.28)$$

where $0 \leq \alpha \leq 1$, $Y_t$ is aggregate output per capita, $K_t$ is capital carried over from previous period $(t - 1)$, and $Z_t$ reflects the state of technology.

Proponents of RBC theory argue that technology shock displays considerable serial correlation, with their first differences nearly serially uncorrelated. In order to introduce high persistence, they assume that technology follows a stationary Markov process, meaning that its probability distribution is independent of anything prior to time $(t - 1)$. Alternatively, one can think of technology evolving as a random walk motion without drift. The productivity term $Z_t$ reflects the state of technology. As emphasised by Stokey and Lucas (1989), the timing of information and actions taken by agents in each period is important in this context. At the beginning of period ‘t’ the current value of $Z_t$ is realised. It follows that the agents already know the value of total output based on which they take consumption and investment (end-of-period stock of capital) decisions.

It is assumed that $f(N, K)$ is smooth and concave and it satisfies Inada-type conditions, i.e. the marginal product of capital (or labour) approaches infinity as capital (or labour) goes to 0

---

\(^{22}\)Households own shares in the firms and therefore own them.
and approaches 0 as capital (or labour) goes to infinity.

\[ \lim_{K \to 0} (F_K) = \lim_{N \to 0} (F_N) = \infty \]

\[ \lim_{K \to \infty} (F_K) = \lim_{N \to \infty} (F_N) = 0 \]

The capital stock evolves according to:

\[ K_{t+1} = (1 - \delta) K_t + I_t \quad (2.29) \]

where \( \delta \) is the depreciation rate and \( I_t \) is gross investment.

In a stochastic environment the firm maximises present discounted stream, \( V \), of cash flows, subject to the constant-returns-to-scale production technology, i.e.

\[ \max V = E_t \sum_{t=0}^{T} d_t (Y_t - K_t(r_t + \delta) - w_t N_t^d) \quad (2.30) \]

subject to (2.28). Here \( r_t \) is the interest paid for the capital and \( w_t \) is the rental rate of labour inputs used by the firm, both of which are taken as given by the firm. Output of the firm depends not only on capital and labour inputs but also on \( Z_t \). The firm optimally chooses capital and labour so that marginal products are equal to the price per unit of input. The first order conditions with respect to \( K_t \) and \( N_t^d \) are as follows:

\[ K_t = \frac{(1 - \alpha) Y_t}{r_t + \delta} \quad (2.31) \]

\[ N_t^d = \left( \frac{w_t}{\alpha Z_t} \right)^{\frac{1}{\alpha-1}} K_t \quad (2.32) \]

The non-negativity constraint applies, i.e. \( K_t \geq 0 \). Firms own the capital stock and choose investment and domestic labour.

### 2.2.4 The Foreign Sector

As Obstfeld and Rogoff (1996) argue, relative prices are a central feature of open economy macroeconomics. In particular the response of the trade balance to shocks on the terms of
trade has preoccupied trade theorists for decades. In open economies a country’s investment and consumption plans are no longer constrained by its own production frontier. As in Armington (1969), demand for products in this framework are distinguished not only by their kind but also by their place of production. The Armington assumption that home and foreign goods are differentiated purely because of their origin of production has been a workhorse of empirical trade theory.

In a stochastic environment the representative agent maximises her expected discounted stream of utility subject to her budget constraint. In order to derive the real exchange rate and hence the balance of payments explicitly from micro-foundations we take into account the consumption constraint on the agent

\[ P_tC_t = P^d_tC^d_t + P^F_tC^F_t \]  
(2.33)

As noted earlier the consumption function is an Armington aggregator of the form

\[ C_t = \left[ \omega \left( C^d_t \right)^{-\rho} + (1 - \omega) \left( C^F_t \right)^{-\rho} \right]^{\frac{1}{1-\rho}} \]  
(2.34)

where \( C_t \) is composite per capita consumption, made up of \( C^d_t \), agents consumption of domestic goods and \( C^F_t \), their consumption of imported goods and \( \omega \) is the weight of home goods in the consumption function. The utility-based price index corresponding to the above consumption function is of the form

\[ P_t = \left[ \omega^{\frac{1}{1+\rho}} \left( P^d_t \right)^{\frac{1}{1+\rho}} + (1 - \omega)^{\frac{1}{1+\rho}} \left( P^F_t \right)^{\frac{1}{1+\rho}} \right]^{1+\rho} \]  
(2.35)

Now the Lagrangian associated with the agent’s maximisation subject to the budget as well as
consumption constraint is:

\[
U = \max E_0 \sum_{t=0}^{\infty} \beta^t \left[ \theta_0 (1 - \rho_0)^{-1} C_t^{(1 - \rho_0)} + (1 - \theta_0) (1 - \rho_2)^{-1} (1 - N_t)^{(1 - \rho_2)} \right] + \lambda_t \left[ (1 - \tau_{t-1}) v_{t-1} N_{t-1} + \mu_{t-1} \left[ (1 - N_{t-1}) - 1 \right] + \frac{b_t + Q_t b_t^f}{1 + r_t} \left( \frac{(p_t + \delta_t) S_t}{p_t} - (1 + \phi_t) C_t \right) - \frac{b_{t+1}}{1 + r_{t+1}} - \frac{Q_{t+1} b_{t+1}^f}{(1 + r_{t+1})} - \frac{p_t S_{t+1}}{p_{t+1}} - T_t \right] + \lambda_t^F \left[ P_t C_t - P_t^d C_t^d - P_t^F C_t^f \right]
\]

The first order conditions with respect to \( C_t^d \) and \( C_t^f \) are

\[
\theta_0 C_t^{-\rho_0} \frac{\partial C_t}{\partial C_t^d} - \lambda_t (1 + \phi_t) \frac{\partial C_t}{\partial C_t^d} - \lambda_t^F P_t \frac{\partial C_t}{\partial C_t^d} + \lambda_t^F P_t \frac{\partial C_t}{\partial C_t^f}
\]

\[ (2.37) \]

\[
\theta_0 C_t^{-\rho_0} \frac{\partial C_t}{\partial C_t^f} - \lambda_t (1 + \phi_t) \frac{\partial C_t}{\partial C_t^f} - \lambda_t^F P_t \frac{\partial C_t}{\partial C_t^f} + \lambda_t^F P_t \frac{\partial C_t}{\partial C_t^f}
\]

\[ (2.38) \]

Dividing equation (2.38) by equation (2.37), we have\(^{23}\)

\[
\frac{P_t^F}{P_t^d} = \frac{\frac{\partial C_t}{\partial C_t^d}}{\frac{\partial C_t}{\partial C_t^f}}
\]

or

\[
\frac{P_t^F}{P_t^d} = \left( \frac{1 - \omega}{\omega} \right) \left( \frac{C_t^d}{C_t^f} \right)^{1+\rho}
\]

\[ (2.40) \]

Now we can write equation (2.40) as

\[
Q_t = \left( \frac{1 - \omega}{\omega} \right) (F)^{(1+\rho)}
\]

\[ (2.41) \]

\(^{23}\)In equilibrium, terms of trade can be computed from the intra-temporal marginal rate of substitution between goods in the Armington aggregator function, see Backus, Kehoe and Kydland (1994). The marginal rate of substitution, i.e. the slope of the indifference curve is given by

\[
\frac{P_t^F}{P_t^d} = \frac{\frac{\partial C_t}{\partial C_t^d}}{\frac{\partial C_t}{\partial C_t^f}} = \left( \frac{1 - \omega}{\omega} \right) \left( \frac{C_t^d}{C_t^f} \right)^{1+\rho}
\]
where \( Q_t = \frac{P^F}{P^I} \) and \( F = \frac{C^d_t}{C^I_t} \)

Elasticity of substitution between home goods and imported foreign goods is given by

\[
\sigma = \left( \frac{\partial F}{\partial Q} \right) \left( \frac{Q}{F} \right) = \frac{1}{1+\rho}
\]  

(2.42)

Substituting (2.42) in (2.41) we have the real exchange rate

\[
Q_t = \left( \frac{1 - \omega}{\omega} \right) \left( \frac{C^d_t}{C^I_t} \right)^{\left(1\over\rho\right)}
\]  

(2.43)

To the extent that home and imported goods are not perfect substitutes, \( \sigma \) will take some finite value. The lower the estimated \( \sigma \) means the less the substitution between the two goods. In other words the greater the degree of product differentiation, the smaller the elasticity of substitution between the products.

From the real exchange rate equation we can derive import equation for our economy. Taking logs of equation (2.43) we have\(^{24}\)

\[
\log IM_t = \sigma \log \left( \frac{1 - \omega}{\omega} \right) + \log C^d_t - \log Q_t
\]  

(2.44)

To derive the import function we need to substitute out for \( \log C^d_t \). From the household’s expenditure minimisation we know

\[
C^d_t = \left( \frac{P^d_t}{\omega P_t} \right)^{-\left(1\over1+\rho\right)} C_t
\]  

(2.45)

Taking logs

\[
\log C^d_t = \sigma \log \omega + \sigma \log P_t - \sigma \log P^d_t + \log C_t
\]  

(2.46)

Now substituting equation (2.46) in equation (2.44), we have

\[
\log IM_t = \sigma \log (1 - \omega) + \log C_t - \sigma A \log Q_t
\]  

(2.47)

\(^{24}\)Note that \( IM_t = C^I_t \).
where

\[ A = \frac{\omega^{1+\rho}}{\omega^{1+\rho} + (1 - \omega)^{1+\rho}} \]

The equation states that imports into the country are positively related to the total consumption in the home country and negatively related to the real exchange rate, i.e. as \( Q_t \) increases that is the currency depreciates, import demand falls.

Now an Armington aggregator consumption function and a corresponding real exchange rate equation exists for the foreign country as well.

\[ C_t^F = \left[ \omega^F \left( C_t^H \right)^{-\sigma^F} + \left( 1 - \omega^F \right) \left( C_t^H \right)^{-\sigma^F} \right] \left( \frac{1}{\sigma^F} \right) \]  \hspace{1cm} (2.48)

\[ Q_t^F = \left( \frac{1 - \omega^F}{\omega^F} \right) \left( \frac{C_t^{Hf}}{C_t^H} \right) \left( \frac{1}{\sigma^F} \right) \]  \hspace{1cm} (2.49)

where \( C_t^F \) is the composite consumption of the foreign country, \( C_t^{Hf} \) is the foreign country's consumption of own goods, \( C_t^H \) is the foreign country's consumption of home goods, \( \omega^F \) is the weight of foreign country's own goods in its composite consumption function, \( Q_t^F \) is the real exchange rate for the foreign country, \( \sigma^F = \frac{1}{1+\rho^F} \) is the elasticity of substitution between home goods, i.e. home exports and foreign country's own goods.

Taking logs of equation (2.49)\(^{26}\)

\[ \log EX_t = \sigma^F \log \left( \frac{1 - \omega^F}{\omega^F} \right) + \log C_t^{Hf} - \sigma^F \log Q_t^F \]  \hspace{1cm} (2.50)

To derive the export function we need to substitute out for \( \log C_t^{Hf} \). As before, from the

\(^{21}\)Please note \( Q_t^F = \frac{1}{\eta^F} \).

\(^{26}\)Note that \( EX_t = C_t^{Hf} \).
foreign household’s expenditure minimisation we know

\[ C^{df}_t = \left( \frac{P^f_t}{\omega^f P^*_t} \right)^{-(\frac{1}{1+\rho^f})} C^F_t \]  

(2.51)

where \( P^*_t \) is the foreign CPI of the form

\[ P^*_t = \left[ (\omega^f)^{1+\rho^f} \left( P^f_t \right)^{\frac{\rho^f}{1+\rho^f}} + (1-\omega^f)^{\frac{1}{1+\rho^f}} \left( P^D_t \right)^{\frac{\rho^f}{1+\rho^f}} \right]^{\frac{1+\rho^f}{\rho^f}} \]  

(2.52)

where \( P^f_t \) is the foreign country’s own price level and \( P^D_t \) is the domestic price level in foreign currency.

Taking logs of equation (2.51)

\[ \log C^{df}_t = \sigma^f \log \omega^f + \sigma^f \log P^*_t - \sigma^f \log P^f_t + \log C^F_t \]  

(2.53)

Substituting equation (2.53) in equation (2.50)

\[ \log EX_t = \sigma^f \log (1-\omega^f) + \log C^F_t + \sigma^f A^f \log Q_t \]  

(2.54)

where

\[ A^f = \frac{(\omega^f)^{\frac{1}{1+\rho^f}}}{(\omega^f)^{\frac{1}{1+\rho^f}} + (1-\omega^f)^{\frac{1}{1+\rho^f}}} \]

The equation states that exports of the home country are a positive function of the total consumption in the foreign country and also a positive function of the real exchange rate. If \( Q_t \) increases, i.e. the home currency depreciates then exports will increase.

In the model home and foreign agents need foreign and home money respectively, in order to transact with each other. The foreign agents need home money to buy our exports, but get home money for imports as well as our purchase of foreign bonds. So their net supply of foreign

\(^{27}P^D_t = \frac{P^*_t}{S_t}\) where \( S_t \) is the nominal exchange rate and \( P^D_t \) is the domestic price level in domestic prices. So, \( P^D_t \) is the domestic price level in foreign prices.
money is equal to net exports plus sales of foreign bonds, i.e. the balance of payments surplus. This surplus is equal to the home agents net demand for foreign money, who get foreign money from firms exporting to foreign agents and need foreign money for imports and purchases of foreign bonds. So if home agents adjust their sales of foreign bonds then all balances. In equilibrium it is assumed that exports and imports are equal and hence the agents would have no tendency to change their asset position. In disequilibrium the changes between domestic and foreign bonds will depend upon net exports.

\[ NX_t = EX_t - IM_t \] (2.55)

Foreign bonds thus evolve over time according to the following equation

\[ b_{t+1}^f = (1 + r_t^f)b_t^f + NX_t \] (2.56)

2.3 Conclusions

This chapter provides theoretically coherent micro-foundations for macroeconomic models and constructs an econometrically testable dynamic general equilibrium open economy model\(^{28}\) which we propose to calibrate/estimate using quarterly data for the UK, and that can serve as a benchmark for quantitative analysis of policy. The model is based on optimising decisions of rational agents, incorporating money, government and real world features like distortionary taxes and unemployment benefits.\(^{29}\) The first order conditions of the household’s and firm’s optimisation problem are used to derive the behavioural equations of the model. As we are modelling the UK, a medium-sized open economy, so have a full blown model only for the domestic economy taking the world economy as given. The interaction with the rest of the world comes in the form of uncovered real interest rate parity and current account, both of which are explicitly micro-founded.

Any sort of value judgement however, necessarily rests on the model’s predictions about
the consequences of alternative policies. According to Lucas and Sargent (1979) the question of whether a model is truly structural or not is an empirical one. To quote Friedman (1953), “Normative economics and the art of economics, on the other hand, can not be independent of positive economics. Any policy conclusion necessarily rests on a prediction about the consequences of doing one thing rather than another, a prediction that must be based – implicitly or explicitly – on positive economics.” It is the empirical issue that forms the basis of our next chapter.
2. A Appendix

2. A. 1 Behavioural Equations of the RBC Model

(1) Consumption $C_t$; solves for $r_t$:

$$(1 + r_t) = \frac{1}{\beta} \left( \frac{C_t}{E_t[C_{t+1}]} \right)^{-\rho} \left( \frac{1 + \phi_{t+1}}{1 + \phi_t} \right)$$

$$r_t = \frac{1}{\beta} \left( \frac{C_t}{E_t[C_{t+1}]} \right)^{-\rho} \left( \frac{1 + \phi_{t+1}}{1 + \phi_t} \right) - 1$$

where $C_t = \omega \left( \frac{C_t}{C_t} \right)^{-\rho} + (1 - \omega) \left( \frac{C_t}{C_t} \right)^{-\rho} \left( \frac{1}{\rho} \right)$

(2) Money supply $\bar{M}_t^d$; solves for $P_t^d$:

$$\bar{M}_t^d = (1 + \phi_t)C_t^dP_t^d + G_tP_t^d$$

$$P_t^d = \frac{\bar{M}_t^d}{(1 + \phi_t)C_t^d + G_t}$$

where $P_t = \omega \left( \frac{P_t}{P_t} \right)^{\frac{\rho}{1 + \rho}} + (1 - \omega) \left( \frac{P_t}{P_t} \right)^{\frac{\rho}{1 + \rho}} \left( \frac{P_t^F}{P_t^F} \right)^{\frac{1 + \rho}{\rho}}$

(3) Demand for shares, $S_{t+1}^s$:

$$S_{t+1}^s = \bar{S}_t; b_{t+1} = b_{t+1}^s \text{ implied.}$$

(4) Present value of share:

$$p_t = E_t \sum_{t=1}^{\infty} \left[ \frac{d_{t+1}}{\prod_{k=t}^{t} \left( 1 + r_k \right)} \left( \frac{P_t}{P_{t+1}} \right) \right]$$

where $d_t$ (dividend per share), $p_t$ (present value of shares in nominal terms).

(5) Production function $Y_t$:

$$Y_t = Z_t N_t^\alpha K_t^{1-\alpha}$$

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(6) Demand for labour:

\[ N_t^d = \left( \frac{\alpha Z_t}{w_t} \right)^{\frac{1}{1-\alpha}} K_t \]

(7) Capital:

\[ K_t = (1 - \alpha) \frac{Y_t}{r_t + \delta} \]

(8) GDP identity, \( Y_t \); solves for \( C_t \):

\[ Y_t = C_t + I_t + G_t + NX_t \]

where \( NX_t \) is net exports.

(9) Investment:

\[ K_{t+1} = (1 - \delta)K_t + I_t \]

(10) \( W_t \); currently this variable is not defined.

(11) Wage \( w_t \):

\[ w_t = w_t^* \]

(12) Evolution of \( b_t \); government budget constraint:

\[ b_{t+1} = (1 + r_t)b_t + PD_t - \frac{\Delta M_t^d}{P_t^d} \]

(13) Equilibrium wage, \( w_t^* \); \( w_t^* \) is derived by equating demand for labour, \( N_t^d \), to the supply of labour \( N_t^s \), where

\[
(1 - N_t^s) = \left\{ \frac{\theta_0 C_t^{-\rho_0} \left[ (1 - \tau_t) \exp \left( \log w_t^* - \frac{(1 - \omega) 1 + p}{\omega 1 + p + (1 - \omega) 1 + p} \log Q_t \right) - \mu_t \right]}{(1 - \theta_0) (1 + \phi_t) (1 + r_t)} \right\}^{\frac{1}{\rho_2}}
\]

70
where $Q_t$ is the real exchange rate, $(1 - \omega)^{\frac{1}{1+\rho}}$ is the weight of domestic prices in the CPI index.

(14) Dividends are surplus corporate cash flow:

$$d_t S_t = Y_t - N_t^d w_t - K_t (r_t + \delta)$$

$$d_t = \frac{Y_t - N_t^d w_t - K_t (r_t + \delta)}{S_t}$$

(15) Primary deficit $PD_t$:

$$PD_t = G_t + \mu_t \left(1 - N_t^d - \bar{l}\right) + E_t \pi^d_{t+1} M^d_t - \tau_{t-1} v_{t-1} N_{t-1}^d - \phi_{t-1} C_{t-1} - T_{t-1}$$

(16) Tax $T_t$:

$$T_t = T_{t-1} + \gamma^G (PD_{t-1} + b_t r_t) + \varepsilon_t$$

(17) Exports $EX_t$:

$$\log EX_t = \sigma^f \log \left(1 - \omega^f\right) + \log C^F_t + \sigma^f A^f \log Q_t$$

where $A^f = \frac{\left(\omega^f\right)^{\frac{1}{1+\rho^f}}}{\left(\omega^f\right)^{\frac{1}{1+\rho^f}} + \left(1 - \omega^f\right)^{\frac{1}{1+\rho^f}}}$

(18) Imports $IM_t$:

$$\log IM_t = \sigma \log (1 - \omega) + \log C_t - \sigma A \log Q_t$$

where $A = \frac{\omega^{\frac{1}{1+\rho}}}{\omega^{\frac{1}{1+\rho}} + (1 - \omega)^{\frac{1}{1+\rho}}}$

(19) UIP condition:

$$r_t = r_t^f + E_t \Delta \log Q_{t+1} + \varepsilon_{UIP}$$

where $r^f$ defined the foreign real interest rate.

(20) Net exports:

$$NX_t = EX_t - IM_t$$
(21) Evolution of foreign bonds $b_t^f$:

$$b_{t+1}^f = (1 + r_t^f)b_t^f + NX_t$$

(22) Nominal exchange rate, $S_t$:

$$\log S_t = \log Q_t - \log P_t^f + \log P_t^d$$

where $P_t^f$ is foreign price and $S_t$ the nominal exchange rate.

(23) Evolution of household debt $D_{t+1}$:

$$D_{t+1} = (1 + r_t)D_t - Y_{t-1} + (1 + \phi_t)C_t + \tau_t v_t N_t^* + T_t$$

(24) Household transversality condition:

$$Y_{T-1} - r_T D_T - \phi_T C_T - \tau_T v_T N_T^* - T_T = C_T$$

(25) Government transversality condition:

$$G = 0.30$$
\[ \Delta \ln Z_t = \varepsilon_{1,t} \]
\[ \Delta \tau_t = \varepsilon_{2,t} \]
\[ \Delta \phi_t = \varepsilon_{3,t} \]
\[ \Delta \mu_t = \varepsilon_{4,t} \]
\[ \Delta \ln M_t = \varepsilon_{5,t} \]
\[ \Delta \ln P_t^f = \varepsilon_{6,t} \]
\[ \Delta \ln C_t^f = \varepsilon_{7,t} \]
\[ \Delta \ln r_t^f = \varepsilon_{8,t} \]

Table 2.1: Exogenous Processes in the RBC Model

<table>
<thead>
<tr>
<th>No.</th>
<th>Name in programme</th>
<th>Description</th>
<th>Initial value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( r^* )</td>
<td>Real Interest Rate</td>
<td>0.05266</td>
</tr>
<tr>
<td>2</td>
<td>( P )</td>
<td>Price Level</td>
<td>0.803528</td>
</tr>
<tr>
<td>3</td>
<td>( S^p )</td>
<td>Demand for Shares</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>( p )</td>
<td>Present Value of Share</td>
<td>4.6</td>
</tr>
<tr>
<td>5</td>
<td>( Y )</td>
<td>Output</td>
<td>0.867945</td>
</tr>
<tr>
<td>6</td>
<td>( N^d )</td>
<td>Demand for Labour</td>
<td>0.840382</td>
</tr>
<tr>
<td>7</td>
<td>( K )</td>
<td>Capital</td>
<td>11.71725436</td>
</tr>
<tr>
<td>8</td>
<td>( C )</td>
<td>Composite Consumption</td>
<td>0.542171914</td>
</tr>
<tr>
<td>9</td>
<td>( I )</td>
<td>Investment</td>
<td>0.129193</td>
</tr>
<tr>
<td>10</td>
<td>( v )</td>
<td>Consumer Real Wage</td>
<td>0.941375</td>
</tr>
<tr>
<td>11</td>
<td>( w )</td>
<td>Producer Real Wage</td>
<td>0.925483</td>
</tr>
<tr>
<td>12</td>
<td>( b )</td>
<td>Domestic Bonds</td>
<td>1.804726</td>
</tr>
<tr>
<td>13</td>
<td>( G )</td>
<td>Government Expenditure</td>
<td>0.196944</td>
</tr>
<tr>
<td>14</td>
<td>( w^* )</td>
<td>Equilibrium Real Wage</td>
<td>0.925483</td>
</tr>
<tr>
<td>15</td>
<td>( d )</td>
<td>Dividend per Share</td>
<td>0.285</td>
</tr>
<tr>
<td>16</td>
<td>( D )</td>
<td>Household Debt</td>
<td>4.313317</td>
</tr>
<tr>
<td>17</td>
<td>( PD )</td>
<td>Primary Deficit</td>
<td>-0.000731</td>
</tr>
<tr>
<td>18</td>
<td>( T )</td>
<td>Tax (lump-sum)</td>
<td>-0.012945</td>
</tr>
<tr>
<td>19</td>
<td>( EX )</td>
<td>Export</td>
<td>0.199159</td>
</tr>
<tr>
<td>20</td>
<td>( IM )</td>
<td>Import</td>
<td>0.197175</td>
</tr>
<tr>
<td>21</td>
<td>( Q )</td>
<td>Real Exchange Rate</td>
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</tr>
<tr>
<td>22</td>
<td>( bl )</td>
<td>Foreign Bonds</td>
<td>0.621545</td>
</tr>
<tr>
<td>23</td>
<td>( S )</td>
<td>Nominal Exchange Rate</td>
<td>1.05855</td>
</tr>
<tr>
<td>24</td>
<td>( NX )</td>
<td>Net Exports</td>
<td>0.001984</td>
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</table>

Table 2.2: Numbering of Endogenous Variables used in the RBC Model
<table>
<thead>
<tr>
<th>No.</th>
<th>Name in programme</th>
<th>Initial value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$Z$ Productivity</td>
<td>0.999105</td>
</tr>
<tr>
<td>2</td>
<td>$\tau$ (tau) Labour Income Tax</td>
<td>0.34738</td>
</tr>
<tr>
<td>3</td>
<td>$\phi$ (phi) Consumption Tax, VAT</td>
<td>0.15346</td>
</tr>
<tr>
<td>4</td>
<td>$\mu$ (mu) Unemployment Benefits</td>
<td>0.2608</td>
</tr>
<tr>
<td>5</td>
<td>$M$ Money</td>
<td>0.107535</td>
</tr>
<tr>
<td>6</td>
<td>$\bar{S}$ Supply of shares</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>$l$</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>$\bar{M}$</td>
<td>0.107535</td>
</tr>
<tr>
<td>9</td>
<td>$\epsilon_1$</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>$\epsilon_2$</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>$\epsilon_3$</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>$\epsilon_4$</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>$\epsilon_5$</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>$\epsilon_6$</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>$E_t[C_{t+1}]$</td>
<td>0.542171914</td>
</tr>
<tr>
<td>16</td>
<td>$r_{base}$</td>
<td>0.05266</td>
</tr>
<tr>
<td>17</td>
<td>$P_{base}$</td>
<td>0.803528</td>
</tr>
<tr>
<td>18</td>
<td>$Y_{base}$</td>
<td>0.867945</td>
</tr>
<tr>
<td>19</td>
<td>$N_{base}$</td>
<td>0.840382</td>
</tr>
<tr>
<td>20</td>
<td>$K_{base}$</td>
<td>11.71725436</td>
</tr>
<tr>
<td>21</td>
<td>$w^*_{base}$</td>
<td>0.925483</td>
</tr>
<tr>
<td>22</td>
<td>$d_{base}$</td>
<td>0.285</td>
</tr>
<tr>
<td>23</td>
<td>$PD_{base}$</td>
<td>-0.000731</td>
</tr>
<tr>
<td>24</td>
<td>$I_{base}$</td>
<td>0.129193</td>
</tr>
<tr>
<td>25</td>
<td>$Q_{base}$</td>
<td>0.971767</td>
</tr>
<tr>
<td>26</td>
<td>$EX_{base}$</td>
<td>0.199159</td>
</tr>
<tr>
<td>27</td>
<td>$IM_{base}$</td>
<td>0.197175</td>
</tr>
<tr>
<td>28</td>
<td>$G_{base}$</td>
<td>0.196944</td>
</tr>
<tr>
<td>29</td>
<td>$P^f$ Foreign Price Level</td>
<td>0.737074</td>
</tr>
<tr>
<td>30</td>
<td>$POP$</td>
<td>28.009</td>
</tr>
<tr>
<td>31</td>
<td>$CF$ Foreign Consumption</td>
<td>16.830648</td>
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<tr>
<td>32</td>
<td>$rf$ Foreign Interest Rate</td>
<td>0.045</td>
</tr>
<tr>
<td>33</td>
<td>$E[Q]$</td>
<td>0.971767</td>
</tr>
<tr>
<td>34</td>
<td>$C_{base}$</td>
<td>0.542171914</td>
</tr>
</tbody>
</table>

Notes: $\epsilon_i$ are parameters for defining exogenous random processes; $E_t[Y_{t+i}]$, etc., are $i$-period ahead expectations. The latter are set equal to the initial starting values.

Table 2.3: Numbering of Exogenous Variables used in the RBC Model
<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value - Single equation</th>
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</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.70</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.97</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0125</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>1.20</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>0.50</td>
</tr>
<tr>
<td>$\gamma^G$</td>
<td>0.05</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>1.00</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.70</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-0.50</td>
</tr>
<tr>
<td>$\omega^f$</td>
<td>0.70</td>
</tr>
<tr>
<td>$\rho^f$</td>
<td>-0.50</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma^f$</td>
<td>2</td>
</tr>
</tbody>
</table>

Note: the values of the coefficients used in the model have been calibrated from the recent literature.

Table 2.4: Values of Coefficients in the RBC Model
2.1.2 Data Set (Base year 1990) for the RBC Model

1. Price Level: Consumer Prices, Liverpool Model UK. Calculated by (AIIX/CAAB). Series AIIX is Nominal Total Consumption, Table 2.2 and Series CAAB is Real Total Consumption, Table 2.5 Economic Trends.


3. Primary Deficit: Public Sector Borrowing Requirement (PSBR). Series ABFP Table 6.5 Economic Trends. Calculated by dividing PSBR by the price level. Scaled as a fraction of GDP.

4. Labour Income Tax: Income Tax, Liverpool Model UK. Calculated by using the following formula (Government Expenditure-(PSBR+Real Debt Interest))/GDP. Future projections keep the average of last four periods unless there are announced tax changes.

5. Composite Consumption: Household Final Consumption, Office of National Statistics (ONS). Seasonally Adjusted. Series ABRJ Table 2.2 Economic Trends. Added Final Consumption Expenditure of Non-Profit Institutions (NPISHs), ONS. Seasonally Adjusted. Series HAYO Table 2.2 Economic Trends. Scaled as a fraction of GDP.

6. Consumption Tax (VAT): Indirect Tax Rate, Liverpool Model UK. Calculated by dividing adjustment to factor cost by GDP. Adjustment to factor cost = indirect taxes (+) subsidies.

7. Government Expenditure: General Government Final Consumption Expenditure, ONS. Seasonally Adjusted. Series NMRY Table 2.2 Economic Trends. Scaled as a fraction of GDP.

8. Investment: Gross Fixed Capital Formation, ONS. Seasonally Adjusted. Series NPQT Table 2.2 Economic Trends. Added Change in Inventories including Alignment Adjustment and Total Economy Acquisitions Less Disposals of Valuables. Both Seasonally Adjusted. Series CAFU and NPJR Table 2.2 Economic Trends, respectively. Scaled as a fraction of GDP.
9. Exports: Balance of Payments Exports Total Trade in Goods and Services, ONS. Seasonally Adjusted. Series IKBK Table A10 UK Economic Accounts. Scaled as a fraction of GDP.

10. Imports: Balance of Payments Imports Total Trade in Goods and Services, ONS. Seasonally Adjusted. Series IKBL Table A10 UK Economic Accounts. Scaled as a fraction of GDP.


13. Real Wage: Average Earnings Index for Whole Economy, Liverpool Model UK. Series DNHS Table 4.1 Economic Trends divided by the price level.


15. Domestic Bonds: Bond data, Liverpool Model UK. Scaled as a fraction of GDP.

16. Domestic Real Interest Rate: Domestic Real Short Term Interest Rate, Deposits with Local Authorities 3 months, NRS Liverpool Model UK. Series AJOI Table 7.1N Financial Statistics.

17. Foreign Real Interest Rate: Foreign Real Short Term Interest Rate, RSUS Liverpool Model UK. Average from World Model Tables.

18. Productivity: Calculated as Solow residual.

19. Money Supply: M0 Wide Monetary Base, ONS. Seasonally Adjusted. Series AVAE Table 17.5 Monthly Digest of Statistics. Scaled as a fraction of GDP.

20. Lump-sum Tax: Calculated as Residual in Primary Deficit equation.

21. Nominal Exchange Rate: Trade Weighted Exchange Rate, RXRN Liverpool Model UK. Series AJHX Table 6.1 Economic Trends. Calculated as inverse of RXRN.
22. Real Exchange Rate: Real Exchange Rate, RXR Liverpool Model UK. Calculated as log(Domestic Prices) + log(Nominal Exchange Rate) - log(Foreign Prices). Calculated as inverse of RXR.

23. Foreign Price Level: Foreign Prices, Liverpool Model. OECD Inflation Figure from the World Model.

24. Foreign Bonds: Calculated from the Foreign Bond Evolution equation. For starting value took Net International Investment Position, ONS. Series HBQC Table 8.1 UK Balance of Payments Pink Book. Scaled as a fraction of GDP.

25. Foreign Composite Consumption: Used World Trade as a proxy. World Trade calculated as Exports + Imports. Scaled as a fraction of GDP.


27. Capital Stock: Capital Stock, CS Liverpool Model UK. Calculated by CS(-1)*0.988277 + Private Sector Gross Investment.
Chapter 3

Calibration, Solution Algorithm and Simulations

3.1 Introduction

Macroeconomic analysis has come a long way since the optimising behaviour of economic agents was explicitly incorporated in models attempting to explain growth and business cycle fluctuations. The initial models were dynamic but deterministic in nature — allowing for analytical solutions, at least in the case of homogeneous agents. Then came the need to use stochastic control techniques to try and explain fluctuations in a linear quadratic set-up, again admitting an analytical solution. Subsequently, the requirement to work in a general equilibrium environment led to the development of numerical methods as the models had no analytical solution. Now for a number of years a variety of numerical methods to simulate model economies have been proposed and reviewed in the literature. This chapter discusses the issues related to calibration and to the method used to solve our macro-models.

This chapter is organised as follows. Section 3.2 first gives us the background to calibration in Section 3.2.1 and then goes onto explain the calibration of the model in Section 3.2.2. Section 3.3 explains the solution algorithm used to solve our fully specified dynamic stochastic general equilibrium model set out in Chapter 2. In Section 3.4 we show the steady-state equations of the model. In Sections 3.5 and 3.6 simulation results of both demand and supply shocks are discussed. The last Section 3.7 presents the conclusions.
3.2 Calibration

3.2.1 A Background to Calibration

Kydland and Prescott with their seminal 1982 paper pioneered ‘quantitative theory,’ also known as calibration as an alternative to traditional methods of analysing data in economics. Although originally controversial, this methodological approach is now widely used throughout the profession. The defining characteristics of the calibration approach are: (i) the parameters of the model are not estimated but instead determined by long-run, i.e. steady-state behaviour of the economy, and (ii) given these parameters, the characteristics of the model’s unconditional equilibrium distribution for the endogenous variables are compared to that of the data. Typically, this comparison involves a limited set of second moments. Due to this criticism, several alternative real business cycle models have been estimated rather than calibrated.¹ Kydland and Prescott (1991), strongly defend calibration as a scientific method.² Further, Hoover (1995) finds virtues in calibration approach on philosophical and methodological grounds.

So we see that the calibration approach attempts to formulate theoretical models in terms of parameters that are quantified from some casual empiricism or unrelated econometric studies or are chosen to guarantee that the model precisely mimics some particular features of the historical data. This is an ambitious programme of finding human behavioural constants that can be used to explain economic behaviour in wide variety of circumstances. In practice, one of the major focuses has been the reconciliation of cross-sectional and time series data, and the reconciliation of long-run time series behaviour, i.e. growth, with short-run behaviour i.e. business cycles.

Thus the real business cycle (RBC) school heralded a new methodology in macroeconomics,

¹See for example Altug (1989) which uses Maximum Likelihood while Singleton (1988) and Christiano and Eichenbaum (1992) use Generalised Methods of Moments. However, Lucas (1987), in response to Altug’s estimation and rejection of the validity of a variant of Kydland and Prescott’s (1982) RBC model writes: “...the interesting question is surely not whether [the real-business-cycle model] can be accepted as ‘true’ when nested within some broader class of models. Of course the model is not ‘true’; this much is evident from the axioms on which it is constructed. We know from the onset in an enterprise like this (I would say, in ‘any’ effort in positive economics) that what will emerge – at best – is a workable approximation that is useful in answering a limited set of questions.”

²According to them “…it is in the stage of calibration where the power of the general equilibrium approach shows up most forcefully. The insistence on internal consistency implies that parsimoniously parameterised models of the household and business sector display rich dynamic behaviour through the intertemporal substitution arising from capital accumulation and from other sources.”
whereby numerical simulation of dynamic stochastic general equilibrium (DSGE) models became an established means of evaluating macroeconomic models. Within this framework a model is specified in terms of the parameters that characterise preferences, technology, information structure and institutional arrangements. Thus, delivering macroeconomic modelling advocated by Lucas (1976). Wickens (1995) notes that the main implication of RBC analysis is that the key parameters of interest are structural — those associated with the functions to be optimised i.e., preferences, technology, policy reaction functions and any other constraints.

These models typically incorporate some intertemporal optimisation problem that produces a set of Euler equations:

$$f_j\{y_t, y_{t-1}, \ldots, y_{t-p}, y_{t+1}, \ldots, y_{t+q}, a_j, x_t\} = u_{jt}$$

or

$$E_t(f_j\{y_t, y_{t-1}, \ldots, y_{t-p}, y_{t+1}, \ldots, y_{t+q}, a_j, x_t\}) = 0$$  \hspace{1cm} (3.1)

for \(j = 1, \ldots, n\) where \(y_t\) is an \(n\)-dimensional vector of endogenous variables at time ‘\(t\)’, \(x_t\) is a vector of exogenous variables at time ‘\(t\)’, \(u_{jt}\) is a vector of stochastic shocks at time ‘\(t\)’, \(E_t\) is the mathematical expectations operator, and \(a_j\) is a parameter vector.

Driving forces of the system, \(x_t\), include shocks to technology, money, unemployment benefits, taxes and foreign variables — interest rates, consumption and prices. There are diverse endogenous variables in the system, such as output, consumption, labour and holdings of stocks and bonds. The models are ‘general equilibrium’ as there is an explicit and consistent account of the household sector as well as the business sector. To answer some research questions, one must also add a sector for the government, which is subject to its own budget constraint. It must be noted that the models are ‘economy-wide’ because monetary/fiscal policy have impact on the whole economy. Further, the rational expectations assumption requires a complete model of the economy in order to explain how expectations are formed. As agent’s expectations of the future policy affect their current decisions and because the rational expectations

\(^3\)By constructing the model from invariants, in Lucas’ usual phrase — the model can secure the benefits of a useful abstraction and generality.
hypothesis assumes that agent's expectations of the future are equal to the model's mathematical conditional expectations, dynamic macroeconomic models with rational expectations entail difference/differential equations in which both past and future differences or differentials appear.

Macroeconomic models must be dynamic primarily for two reasons: (i) in the real world there are lags in the monetary/fiscal transmission mechanism, and (ii) expectations of the future are important in financial markets. The dynamics in these models are explicit, being mainly intrinsic dynamics arising from the budget constraints and the capital accumulation equation, with additional dynamics being provided by the processes assumed to generate the exogenous i.e. forcing variables. One can further add to the dynamics by including time-to-build technology, adjustment costs, overlapping wage contracts and other forms of inertia.

However, all these additions increase the complexity of models exponentially. Heckman (1999) points that recursive dynamic economic theory does not typically produce simple functional forms for estimating equations. Problems of estimating the parameters of large-scale general equilibrium models are formidable unlike the simple Keynesian models that defined the structural econometrics of the 1940s and 1950s. Most of the open economy dynamic general equilibrium models of today are highly non-linear. These models can not be solved analytically, and most do not have closed form solutions. The usual approach is either to take linear approximation around the steady-state-growth path or to solve them numerically, i.e. use algorithms to simulate the model economy. However, as pointed out by Hansen and Heckman (1996), for large impulses or shocks, the non-linear nature of these models is potentially important. Unfortunately many 'linearised' versions of non-linear models have properties that are different from the original non-linear model. If these differences are due to the non-linearity itself, an important element of the original model is discarded by linearisation.

As Friedman (1995) argues "Economists prefer parsimonious models. Repeated experience clearly shows that progress in economic thinking is possible only with heroic simplification."
The reason being that the phenomena that economists attempt to study are very complex in comparison to the tools available at their disposal. The model presented in the previous chapter was a simple model, without any forms of additional inertia, derived from the optimisation problem of the agents in the economy — households and firms all were viewed as making their
decisions by optimising specific objective functions, subject to their constraints and subject to
the government's tax and spending decisions, treated here as exogenous.

In general, linear-quadratic optimisation problems, those in which the objective function is
quadratic and restrictions are linear, will produce a quadratic Lagrangian and hence, first order
conditions will be linear in state and decision variables. State variables are predetermined
each period, being either past decision variables or exogenous to the decision-maker. In a
deterministic set-up, first order conditions together with budget constraints and the assumed
mechanism for price formation will form a linear system each period, with as many equations
as decision variables, providing the optimal values for the decision variables as a function of the
state variables.

However, the models need to be stochastic if one wants to predict how effective policy rules
are at cushioning the economy from unanticipated shocks. In a stochastic set-up, the uncertainty
produces a higher level of difficulty. State and decision variables or their expectations, possibly
at different horizons enter into the first order conditions (FOC's) in a linear manner. Now
the system of FOC's with expectations of the future together with an endogenous expectations
formation mechanism is no longer complete.

In a general equilibrium framework one needs to simultaneously solve for the optimal values
of the decision variables of each agent in the economy as well as for equilibrium prices. Once
prices are endogenous, the budget constraints entering into the optimisation problem will no
longer be linear, since they will involve cross products of endogenous prices and decision vari-
ables. These models can not be analysed except by numerical simulation, which is the only
way to fully analyse a broad class of very relevant model economies. This technique can also
be used to analyse what happens to an economy in transition to its new steady-state from a
given initial situation. This is crucial in policy evaluation exercises, since a policy intervention
will generally take an economy outside the steady-state.

3.2.2 Model Calibration

In order to carry out model simulations, numerical values need to be assigned to the structural
parameters of the models. The calibration should be done in line with Kydland and Prescott

\footnote{See Table 2.4 for values of the parameters used, appendix Chapter 2.}
(1996) emphasis on the fact that the model economy is intended to "mimic the world along a carefully specified set of dimensions." The calibrated values, such as for example the output elasticity of the production factors, the degree of risk-aversion or the elasticity of intertemporal substitution, are taken from micro-data estimates or from some casual empirical characteristics for the economy which is to be studied. For instance Kydland and Prescott (1982) derive values for some of the remaining structural parameters so that their steady-state levels match sample averages of key economic variables, i.e. the steady-state of the model mimics the long-run features of the data.

The exogenous stochastic processes should also be calibrated. However, it is hard to find information from a real economy concerning the stochastic structure of technology shocks, shocks to preferences, error of controlling money growth or tax revenues, or the correlations among them. For this purpose, persistence properties in actual time series data can be used to calibrate some aspects of the model. For instance, in the simplest business cycle model, an AR(1) model is assumed for productivity shocks, with the coefficient generally chosen so that the simulated output series exhibits persistence similar to the gross national product series in actual economies.

A quite different strategy seeks to provide a full characterisation of the observed data series. Following Sargent (1989), a number of authors have estimated the structural parameters of DSGE models using classical Maximum Likelihood Method (MLM). These more standard econometric procedures choose values for all parameters by optimising a given criterion — the likelihood of the data, in the case of MLM. This procedure has two main advantages: (i) it avoids a possibly arbitrary selection of parameter values, and (ii) it provides a measure of dispersion that can be used to evaluate the goodness of fit of model to data.

In the ensuing analysis, as an initial starting point the parameters in the model have been calibrated to match historical data. This method has found to produce low bias. Once the model has been calibrated, the next step is to obtain its ‘solution’. By this we mean using the

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5 See, for example, the references in Ireland (1999).
6 Alternatively, a Bayesian approach can be followed by combining the likelihood function with prior distributions for the parameters of the model, to form the posterior density function. This posterior can then be optimised with respect to the model parameters either directly or by Monte-Carlo Markov-Chain (MCMC) sampling methods.
7 Once we have the tools to solve the model we are also equipped to analyse what happens to an economy
model to get the within-sample predictions of the current values of the endogenous variables, i.e. the \textit{ex post} forecasts. A solution to the Euler equation (3.1) is a stochastic process for \( y_t \). In a rational expectations difference equation system obtaining such a solution is more difficult than in a simple backward-looking difference equation system with no expectational variables. In order to obtain the practical solution of a model for the endogenous variable values over a forecast horizon an internally consistent forward-looking solution sequence has to be calculated. Furthermore, in parallel to the requirement for an initial condition when solving a conventional difference equation, there is also a need for terminal/transversality conditions that specify forecast values and expectations at the forecast horizon.

Given that we have a system of non-linear difference equations, obtaining closed-form/analytical solution for each of the endogenous variables is impossible.\(^8\) In such a case one has to resort to numerical methods to calculate the stochastic process for the variables corresponding to the solution. Iterative or indirect methods start with an initial approximation to a solution and then generate a succession of better and better approximations that tend toward an exact solution.\(^9\) Many macroeconomists have proposed algorithms to solve Euler equation (3.1) in the non-linear case.\(^10\)

Fair and Taylor (1983) propose an iterative method called the \textit{extended path} method to solve this type of non-linear model.\(^11\) In brief it works as follows: if we know the expectations of future variables in (3.1), then (3.1) is a standard system of simultaneous equations that can be solved using some non-linear method, such as the Gauss-Seidel which is used to solve conventional models.\(^12\) The solution would provide values for variables \( y_t \). The extended path method works by guessing values of these future variables. For each guess, the model is solved

\(^8\)Many papers have been written on obtaining solutions to systems like (3.1). In the case where \( f_j \) is linear, Blanchard and Kahn (1980) show how to get the solution to the deterministic part of (3.1) by finding the eigenvalues and eigenvectors of the system. Under certain conditions, the model has a unique solution.

\(^9\)In practise we are interested not only in convergence \textit{per se} but the speed of convergence as well.

\(^10\)See Taylor and Uhlig (1990) for a review.

\(^11\)A possible criticism of the \textit{extended path} method is that it removes the effect of risk on the path of the economy. However, the point is that in the model the risk terms are assumed constant on the grounds that the error terms have constant variance-covariance matrix and the model's structure is fixed and approximately linear. If these terms are constants then they do not affect the impulse responses and the equation constants are adjusted so that the calibrated model errors have a mean of zero.

\(^12\)The Gauss-Seidel method is the most commonly employed algorithm. The popularity of this method, besides its simplicity, is due to the fact that for non-explosive models convergence is always achieved.

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providing an updated guess. The model is solved again and so on.

3.3 Model Solution and Algorithm

In solving our model,\(^1\) we are forced by its complexity and non-linearity to use a computer algorithm. We must note at the outset that in a rational expectations models, the forward expectations terms tend to induce unstable roots and it is therefore necessary for a model to have a stable well-defined long-run, or saddlepath, if a solution to the model is to be obtained. The solving procedure must be therefore subject to the terminal conditions that beyond some terminal date, \(N\), all the expectational variables are set to their equilibrium values, this ensures that the algorithm will pick the unique stable path. According to Minford (1979) the justification for this is that non-convergent behaviour of the system would provoke behaviour by economic agents different from that assumed in the model. In any case, it is necessary for the terminal date to be ‘large,’ in order to reduce the sensitivity of the model to variations in the terminal date. It is also of interest to know if the model settles down to a new equilibrium following a shock. As pointed out by Matthews and Marwaha (1979), the actual value of the terminal condition can be derived from the long-run equilibrium condition of the model. In some cases, the steady-state properties of the model can be used to choose the terminal conditions of the model, although several other methods can be easily used.\(^2\)

There are several of iterative methods, but the most common is the Gauss-Seidel method. This iterative method\(^3\) is built in the programme developed by Matthews (1979) and Minford, Marwaha, Matthews and Sprague (1984) called RATEXP which has been used to get the model solution. The computer programme typically uses a backward-solving, i.e. dynamic programming technique. However, unlike the classical dynamic programming, the solution vector is approached simultaneously for all \(t = 1, 2, \ldots T\), but convergence follows a backward process. The problem lies in that the model must firstly obtain a dynamic solution for a given

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\(^1\)The model is solved ignoring second order expectational terms – in the manner of Fair and Taylor (1983) and indeed the Liverpool model for a long time – on the grounds that expected second order terms can be considered approximately constant.

\(^2\)See Whitley (1994) for a review.

\(^3\)The programme also has the option of using the Powell (1964) conjugate quadratic convergence. Despite its robustness this routine usually requires more function evaluations than the Jacobi method.
time span using initial ‘guess’ values of the expectational variables. These initial values are then adjusted in an iterative manner until convergence is obtained. After checking for equality between expectations and the solved forecasts, the initial expectations set is gradually altered until convergence is obtained. In effect this endogenises the expectational variables in that period. Our model is highly non-linear, consequently a larger number of iterations are required compared to linear models.\(^\text{16}\)

In order to understand how the algorithm works, consider a set of simultaneous non-linear structural equations written in implicit form:

\[
F\{y(t), y(t-1), x(t), u(t)\} = 0
\]  

(3.2)

where, as before \(y(t)\) is a vector of endogenous variables, \(y(t-1)\) is a vector of lagged endogenous variables, \(x(t)\) is a vector of exogenous variables, and \(u(t)\) is a vector of stochastic shocks with mean zero and constant variances. \(F(\cdot)\) represents a set of functional forms. Setting the disturbance terms equal to their expected values and solving for the reduced form, we have

\[
y_t = H\{x(t), y(t-1)\}
\]  

(3.3)

where \(H(\cdot)\) is the reduced form functional form. Partitioning equation (3.3) so as to distinguish between endogenous variables on which expectations are formed \(y(2)\) and the others \(y(1)\), we have

\[
y_1(t) = h_1\{x(t), y_1(t-1)\}
\]  

(3.4)

\[
y_2(t) = h_2\{x(t), y_2(t-1), E[y_2(t+j)/t]\}
\]  

(3.5)

where \(E[y_2(t+j)/t]\) denotes the rational expectation of \(y(2)\) for period \(t+j\) based on information available at \(t\). Our programme uses starting values for the vector \(E[y_2(t+j)/t]\) which, together with values for the ‘fully’ exogenous variables, are assumed to extend over the whole solution period. The algorithm ensures that the expectational values stored in the vector \(E[y_2(t+j)/t]\)

\(^{16}\)It should be noted that in general a non-linear model does not have a unique reduced form. Further, when a non-linear model is solved in a deterministic manner the solution values of the endogenous variables are not in general equal to their expected values. A correct solution requires stochastic simulation.
converge to the values predicted by the model for $y(2)$ in period $t + j$.

For simplicity, let us assume that the solution period extends from $t = 1, \ldots, T$, and that expectations are formed for one period ahead only. Equation (3.5) can therefore be written as

$$y_2(t) = f\{x(t), y_2(t - 1), E[y_2(t + 1)/t]\}$$  \hspace{1cm} (3.6)

where $f(\cdot)$ equals $h_2(\cdot)$ and $t = 1, \ldots, T$.

The convergence of the expectational values towards the model's predicted values follows a Jacobi algorithm, which can be described as

$$E[y_2(t, k + 1)/t - 1] = E[y_2(t, k)/t - 1] + q\{y_2(t, k) - E[y_2(t, k)/t - 1]\}$$  \hspace{1cm} (3.7)

$$0 < q < 1, \quad t = 1, 2, \ldots, T$$

for the $k^{th}$ iteration, with the objective of minimising the residual vector $R(t)$, defined as

$$R(t) = \text{abs}\{y_2(t) - E[y_2(t)/t - 1]\} < L, \quad t = 1, 2, \ldots, T$$  \hspace{1cm} (3.8)

where $q$ is the step length and $L$ is some pre-assigned tolerance level.

Since $E[y_2(t)/t - 1]$ is stored in period $t - 1$, the end period expectational variable remains undetermined. We require a value for $y_2(T + 1)$ which lies outside the domain of the solution period. The technique used in our programme consists of imposing a set of terminal conditions on the rationally expected variables.\(^{17}\)

Terminal conditions are a necessary constraint from the point of view of numerical solution and can be rigorously justified in optimisation models where transversality conditions form part of the solution.\(^{18}\) It is necessary for the terminal date to be large, in order to reduce the sensitivity of the model to variations in the terminal date. For models which possess a long-run steady-state solution, the expectational variables must be set to their long-run equilibrium values. This ensures that the algorithm will pick the unique stable path.

\(^{17}\)In a rational expectations model, the forward expectations terms tend to induce unstable roots. The use of terminal conditions has the effect of setting the starting values of the unstable roots to zero asymptotically, thereby ruling out unstable paths.

\(^{18}\)See for example Sargent (1987).
In sum, the complete algorithm can be described in the following series of steps:

**Step 1** Solve the model given initial values for the expectational variables.

**Step 2** Check for convergence.

**Step 3** Adjust expectational variables.

**Step 4** Re-solve the model given the new iterated values of the expectational variables.

### 3.4 Steady State Equations of the model

The first step in studying the properties of our model is to abstract from the presence of stochastic shocks and describe the deterministic steady-state. The steady-state of an economy is its rest point when the variances of all the shocks are zero and the levels of consumption, labour, stock of capital and inventories are constant. The study of the steady-state is important as it characterises the long-run features of the economy. We set out below the steady-state equations of our model.

\[
\beta = \frac{1}{1 + r}
\]

\[
\overline{M}^d = (1 + \phi) C^d P^d + G P^d
\]

\[
p = \frac{d}{1 + r}
\]

\[
Y = Z N^\alpha K^{(1 - \alpha)}
\]

\[
N = \frac{\alpha Y}{w^*} = \left( \frac{\alpha Z}{w^*} \right)^{\frac{1}{1 - \alpha}} K
\]

\[
K = \frac{(1 - \alpha) Y}{r + \delta}
\]
\[ Y = C + I + G + NX \]

\[ I = \delta K \]

\[ w = w^* \]

\[ rb = -PD + \frac{\Delta M^d}{P^d} \]

\[
(1 - N^*) = \left\{ \frac{\theta_0 C^{\rho_0} \left[ (1 - \tau) \exp \left( \log w^* - \frac{(1 - \omega^*)^{1 + \rho}}{\omega^{1 + \rho} + (1 - \omega^*)^{1 + \rho}} \log Q - \mu \right) \right]}{(1 - \theta_0) (1 + \phi) (1 + r)} \right\}^{-\frac{1}{\rho_2}}
\]

\[ d = \frac{Y - w^* N - K(r + \delta)}{S} \]

\[ PD = G + \mu(1 - N - \bar{N}) + \pi^d M^d - \tau N^* - \phi C - T \]

\[ \log EX = \sigma^I \log \left( 1 - \omega^I \right) + \log C^E + \sigma^I \left( \frac{(\omega^I)^{1 + \rho}}{(\omega^I)^{1 + \rho} + (1 - \omega^I)^{1 + \rho}} \right) \log Q \]

\[ \log IM = \sigma \log (1 - \omega) + \log C - \sigma \left( \frac{\omega^{1 + \rho}}{\omega^{1 + \rho} + (1 - \omega)^{1 + \rho}} \right) \log Q \]

\[ r = r^I + \Delta \log Q \]
\[ r^I b^I = N X \]

\[ \log S = \log Q - \log P^I + \log P \]

\[ Y - (1 + \phi)C - \tau v N - T = r D \]

### 3.5 Simulations

Once the model has been solved numerically, one can analyse the characteristics of the transition of the model to its steady-state. This may arise either because initially the economy is outside steady-state or because some structural change is introduced – like a policy intervention – altering the steady-state. This type of analysis is crucial, among other things, to evaluate the possible effects of changes in policy rules, i.e. of policy interventions and to assess the overall properties of the model.

Standard simulation methods consist of comparing the solution of the model with one where one or more of the exogenous variables are perturbed. Comparing the base and perturbed solutions gives an estimate of the policy multiplier(s) if the exogenous variable perturbed is a policy instrument. In other words, comparing the results of simulation experiments with those obtained in the base run provides valuable information regarding the effects of policy changes on the economy.

There is also the question of selection of the length of the simulation period. The period should be long enough for the effects of changes to work through the model. This is especially important in models which contain long lags or slow rate of adjustment. Darby, Ireland, Leith and Lewis (1999) lists two advantages of having a long simulation period: (i) when solving non-linear rational expectations models it is important to ensure that the terminal date for the simulation is sufficiently far in the future so that the simulation is unaffected by the choice of terminal date,\(^{19}\) and (ii) simulating the model over a long period makes it easier to observe the

\(^{19}\)The numerical sensitivity of the model to variations in the terminal date is one approximate way of checking the closeness of the numerical convergence date to the analytical convergence date.
long-run solution of the model.

Our simulations start in 1986:3 and end in 2000:4 using quarterly UK data. Results of our simulation exercise are reported in tabular and graphical form in the appendix. The tables and graphs show the percentage deviation of a particular variable – real output, price level, and so on – from the baseline path except in the case of interest rates where it shows percentage point deviations from the baseline.

3.6 Results

The effects of both demand and supply shocks on the behaviour of output, consumption, capital stock, investment, employment, price level, real wage, real interest rate, imports, exports, nominal and real exchange rate is examined by deterministically simulating the calibrated model using the extended path method discussed earlier. In addition to providing quantitative input to policy analysis these deterministic simulations provide useful insights into the dynamic properties of the model — equipping us to interpret more complex stochastic simulations.

For the baseline simulation — the simulation with no change in policy instruments — the endogenous variables are set so as to track the actual historical values perfectly. This is done by adding residuals to each equation in the model. The residuals are computed as if the future expectations of the endogenous variables are equal to the actual values. The residuals therefore include not only the shocks to the equations, but also the forecast errors. The endogenous variables in the base run are shown in Table 3.A.1 in the appendix.

3.6.1 5% Permanent Increase in Money Supply

Consider the case of an unanticipated 5% permanent increase in the level of the money supply relative to the historical baseline. The predictions of the model for the case of an increase in the money supply are shown in Table 3.A.2 and Figure 3.A.1 in the appendix. Although unanticipated at the time of the initial increase, the entire path of the money supply is assumed to be incorporated into agent’s forecasts as of the first quarter of the simulation. In particular, people know that the increase in money is permanent. In the very first quarter of the simulation price level increases by 5% and the nominal exchange rate also increases by 5%. Real output,
the components of real spending, real interest rates, real exchange rate, imports, exports and
bond holdings of agents are unaffected by the money expansion. In the model money is neutral,
as there are no nominal rigidities, enabling agents to make instantaneous adjustments.

3.6.2 5% Permanent Increase in Productivity

Consider the case of an unanticipated 5% permanent increase in productivity.\textsuperscript{20} The predictions
of the model are shown in Table 3.A.3 and Figure 3.A.2 in the appendix. A permanent increase
in productivity reduces marginal costs and encourages higher output and raises permanent
income. Output however can not be increased without additional labour supply and extra
capital. For the economy to move to a permanently larger capital stock – consistent with the
higher output – we observe substantial increases in investment which gradually taper off as the
economy converges to the new higher capital stock. As new ready to use capital is slow to arrive,
real interest rates rise to reduce the demand for capital to available supply and also to constrain
the consumption and investment demand in the economy. The rising real interest rate violates
uncovered real interest parity (URIP) which must be restored by a rise in the real exchange
rate relative to its expected future value. This rise is made possible by the expectation that
the real exchange rate will fall back steadily, so enabling URIP to be established consistently
with a higher real interest rate.

As the real exchange rate appreciates there is a deterioration in the terms of trade resulting
in a fall in exports and increase in imports. This pattern is reversed when the real exchange
rate depreciates. As real interest rates fall with the arrival of a stream of sufficient capital
the real exchange rate moves back to equilibrium. It must be noted however that this new
equilibrium represents a real depreciation on the previous steady-state since output is now
higher and must be sold on world markets by lowering its price. In the immediate aftermath of
the shock sufficient capital is not yet available hence the demand for labour pushes up the real
wage. Labour supply in the economy gradually reduces as higher income increases leisure time,
i.e. the wealth effect starts dominating the substitution affect. As output expands we observe
that prices in the economy fall.

\textsuperscript{20} Notice that at times there is a jump at the end of the simulation period. This is where the terminal condition
cuts in and so variables are forced to close their path down fast.
3.6.3 5% Point Permanent Increase in Unemployment Benefits

Consider the case of an unanticipated 5% point permanent increase in unemployment benefits. The predictions of the model are shown in Table 3.A.4 and Figure 3.A.3 in the appendix. A permanent unanticipated increase in unemployment benefits reduces labour supply, reduces the level of output from first the simulation period. Also investment declines causing capital stock to fall. Prices rise and currency depreciates, cancelling each other and hence having no impact on real exchange rate, imports and exports.

Benefits systems affect the decision of private agents to participate in the labour market. Unemployment benefits act as a floor on wage demands creating a downward rigidity in wages. This reduction in wage flexibility prevents adjustment, and so the labour market in response to new shocks which require wage falls — instead generates falling employment and rising unemployment. In other words, unemployment benefits have the effect of reducing the downward pressure on wages that normally would accompany an increase in unemployment.

3.6.4 5% Point Permanent Increase in Labour Income Tax

Consider the case of an unanticipated 5% point permanent increase in labour income tax. The predictions of the model are shown in Table 3.A.5 and Figure 3.A.4 in the appendix. An unanticipated permanent increase in income tax reduces labour supply, which leads to a fall in the level of output and an increase in the price level from the first simulation period. As output falls, investment in the economy reduces, leading to declining capital stock. The initial fall in the interest rates causes a real and nominal depreciation leading to improvement of the trade balance. However, as wages start declining, after the initial jump in the light of increment in labour income tax, labour supply, output and investment expand. This pushes up the real interest rates, leading ultimately to a real appreciation and a deteriorating trade balance.

An increase in income tax produces a substitution effect which outweighs the income effect of the tax. The increase in tax tends to discourage supply of labour because of the wedge it creates between pre and post-tax returns on labour. Further, the impact of changes in income taxes depends on how these tax changes are perceived. If a tax increase in the current time period is deemed permanent agents would react with the expectation that current disposable income would be smaller and thus alter consumption plans, an inward shift in the household
budget constraint.

3.6.5 5% Point Permanent Increase in Consumption Tax (VAT)

Consider the case of an unanticipated 5% point permanent increase in consumption tax. The predictions of the model are shown in Table 3.A.6 and Figure 3.A.5 in the appendix. An unanticipated permanent increase in consumption tax raises the level of unemployment and reduces the level of output. With the fall in output, investment in the economy declines and hence capital stock also reduces. In both consumption and labour income tax simulations labour supply falls as a result of substitution effect outweighing income effect. Consumption tax discourages spending and hence we observe a fall in the price level. As price level falls, the currency appreciates.

3.6.6 1% Point Permanent Increase in the Foreign Interest Rate

Consider the case of an unanticipated 1% point permanent increase in the foreign interest rate. The predictions of the model are shown in Table 3.A.7 and Figure 3.A.6 in the appendix. An unanticipated permanent increase in the foreign interest rate, through the uncovered real interest rate parity leads to a depreciation of the real as well as nominal exchange rate. This of course leads to a fall in imports and rise in exports as domestic goods are now more competitive in the world markets. As domestic interest rates catch up with the foreign rates, real and nominal exchange rates appreciate leading ultimately to a small deterioration in the trade balance. With the increase in the cost of capital, investment receives a big blow, falling by nearly 40% in the first period and then gradually coming back to equilibrium. Capital stock falls, output falls and price level increases. Movements in consumption are a mirror image of the movements in the price level.

3.6.7 5% Permanent Increase in the Foreign Price Level

Consider the case of an unanticipated 5% permanent increase in the foreign price level. The predictions of the model are shown in Table 3.A.8 and Figure 3.A.7 in the appendix. An unanticipated permanent increase in foreign prices leads to an immediate 5% appreciation of the pound sterling. The appreciation is instantaneous due to the model having a flexible exchange
rate together with absolutely no nominal rigidity. The real exchange rate does not change as the movement in opposite direction of foreign prices and nominal exchange rate cancel each other out.

3.7 Conclusions

This chapter gives the reader a background to calibration as a methodological approach used in the area of general equilibrium models. It then goes on to elaborate the calibration used in the model specified in the previous chapter. We also explain the solution method used to solve our dynamic stochastic general equilibrium model. We have discussed in detail an algorithm developed by the Liverpool Research Group which we have used to solve our complex non-linear model. Finally, the chapter has discussed the simulation results for both demand and supply shocks with calibrated parameters which were used to assess the overall properties of the model. The results are consistent with our theoretical priors.
3.A Appendix

3.A.1 Base Run

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Table 3.A.1: Base Run Simulation from 3 to 60
3. A. 2 Simulation Results in Tabular Form

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Table 3. A. 2: 5% Permanent Shock to Money Supply

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Table 3. A. 3: 5% Permanent Shock to Productivity
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Table 3.A.4: 5% Point Permanent Shock to Unemployment Benefits

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Table 3.A.5: 5% Point Permanent Shock to Labour Income Tax

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Table 3.A.6: 5% Point Permanent Shock to Consumption Tax (VAT)
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Table 3.A.7: 1% Point Permanent Shock to Foreign Interest Rate

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Table 3.A.8: 5% Permanent Shock to Foreign Prices
3.A.3 Simulation Results in Graphical Form

Figure 3.A.1: 5% Permanent Shock to Money Supply
Figure 3.A.1 cont. 5% Permanent Shock to Money Supply
Figure 3.A.2: 5% Permanent Shock to Productivity
Figure 3.A.2 cont. 5% Permanent Shock to Productivity
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Chapter 4

Real Exchange Rate Overshooting
RBC Style

4.1 Introduction

The continuous strength of the dollar over the 1990s fuelled interest in the relationship between productivity and exchange rates. As US productivity surged in the second half of the 1990s, the dollar began its climb against all the major currencies of the world. This has led to a large body of literature analysing the links between the real exchange rate and productivity. The ‘conventional’ view of the impact of a productivity shock on an economy is that the real exchange rate depreciates. However, this is completely at odds with the empirical findings of currency appreciation after a productivity spurt. It also fails to explain the cyclical pattern observed in the real exchange rate data.

In this chapter we explore the ability of a Real Business Cycle (RBC) model to account for the behaviour of the real exchange rate, using UK experience as our empirical focus.

• First, we find that a one percent deterministic productivity growth shock, shows clearly that the real exchange rate appreciates on impact and then goes back to equilibrium,\(^1\) producing a business cycle — giving us the simulation properties that we are after.

\(^1\)This equilibrium however represents a real depreciation on the previous steady state since output is now higher and must be sold on world markets by lowering its price.
• Second, we show that the RBC alone can reproduce the univariate properties of the real exchange rate — by implication there is no necessary case here to add nominal rigidity.

The chapter is organised as follows. In Section 4.2 we discuss in detail the relationship between purchasing power parity and the real exchange rate. Section 4.3 goes on to explain the links between productivity and real exchange rate behaviour. Section 4.4 establishes the facts of real exchange rate; it is integrated of order 1 and is highly persistent, the best fitting univariate process being an ARIMA(3,1,3). Section 4.5 explains the main features of the model. In Section 4.6 we calibrate the model to quarterly UK data and show the results of a 1 percent deterministic productivity growth shock, which is very encouraging to the idea that the behaviour of real exchange rates is explicable within the RBC context. In Section 4.7 we formally test our model and evaluate statistically whether our calibrated model is seriously consistent with the real exchange rate data, using bootstrapping procedure. In Section 4.8 we conclude that real exchange rate behaviour can be explained using a pure RBC model with no nominal rigidity.

4.2 The Real Exchange Rate and Purchasing Power Parity

A commonplace observation is that exchange rates appear to behave in seemingly inexplicable ways. A large number of studies have examined movements in the real exchange rate and found that they exhibit swings away from various definitions of purchasing power parity (PPP) by which is meant the longer-run equilibrium value of real exchange rate. Such an equilibrium is akin to the ‘natural rate’ of output or unemployment in a general equilibrium macroeconomic model and it may move over time for a variety of reasons — one commonly used model is that

\[ Q_t = \frac{S_t P^*_t}{P^*_t} \]

2Detailed exposition of the model can be found in Chapter 2.
3We define real exchange rate \( Q_t \) as the nominal exchange rate \( S_t \) adjusted for the ratio of foreign prices to domestic prices.
of Balassa and Samuelson\textsuperscript{4} based on differing productivity trends.\textsuperscript{5} Many studies have found definite evidence of reversion to PPP but very slow reversion. More recently studies that have allowed for non-linear adjustment\textsuperscript{6} have found that the speed of reversion is much greater, and becomes of similar order to that for other macro variables such as output and inflation.

One can think of these studies as final form equations such that as the real exchange rate moves further away from PPP the pressures of goods market arbitrage become stronger, where unspecified shocks to the economy, from demand and supply, stochastically disturb the real exchange rate away from some smoothly-moving trend. Macroeconomic models that could in principle produce such a final form range from on the one hand models with a high degree of nominal rigidity to at the other extreme real business cycle models. However models relying on nominal rigidity – with a high implicit elasticity of output to shocks – have a problem in reproducing the considerable variability of the real exchange rate exhibited by the data. It has been often remarked that these models, to take a recent example, have great difficulty in accounting for the very large swings of the dollar against the DM/Euro since 1995, first upwards and then downwards back to its 1995 starting point by early 2004.

4.2.1 Conceptual Background

It is well known that the exchange rate – arguably the single most important price in an open economy – is intimately related to the concept of purchasing power parity. The term ‘purchasing power parity’ was coined by Cassel (1918), though it has a much longer history in economics.\textsuperscript{7} Lothian and Taylor (2005) eloquently explain PPP and the real exchange rate:

\textit{The purchasing power parity (PPP) exchange rate is the exchange rate between two currencies that would equate the two relevant national price levels if expressed in a common currency at that rate, so that the purchasing power of a unit of one currency would be the same in}

\textsuperscript{4}The seeds of Balassa-Samuelson model can be found in the writings of Ricardo (1911) and Harrod (1939). Further, the use of nontradable goods in modern international economics dates from Salter (1959) and Swan (1960).

\textsuperscript{5}The crux of their analysis was identifying that productivity growth differentials between tradable and nontradable sectors is instrumental in altering a county’s internal price structure.

\textsuperscript{6}Such that as the real exchange rate moves further away from PPP the pressures of goods market arbitrage become stronger.

\textsuperscript{7}Officer (1982) provides a fascinating and scholarly account of the history of thought on purchasing power parity.
both economies.8 If the nominal exchange rate is defined simply as the price of one currency in terms of another, then the real exchange rate is the nominal exchange rate adjusted for relative national price level differences. When PPP holds, the real exchange rate is a constant, so that movements in the real exchange rate represent deviations from PPP.” While very few economists today would hold that PPP holds continuously in the real world, as Rogoff (1996) states “most instinctively believe in some variant of purchasing power parity as an anchor for long-run real exchange rate.” Indeed the implication or assumption of much reasoning in international macroeconomics – be it traditional international macroeconomic analysis9 or ‘new’ open economy models based on intertemporal optimising framework10 – is that some form of PPP holds at least as a long-run relationship. Moreover as various versions of PPP are used in a wide range of practical applications: determining the degree of misalignment of the nominal exchange rate and the appropriate policy response, choosing the right initial exchange rate for a new independent country, forecasting medium and long-term real exchange rates and the international comparison of national income levels. It is not very surprising that a large literature – academic and policy related – has evolved.

The validity of PPP has been sought in the empirical literature by examining whether the real exchange rate tends to settle down at a long-run equilibrium level — to check whether the time series appears to have been generated by a ‘mean reverting process’. Lothian and Taylor (1996), note that the professional academic opinion on the validity of PPP itself seems to display mean reversion.

Prior to the recent float of the 1970s, the dominant academic opinion seemed to assume some form of long-run PPP – the existence of a fairly stable real exchange rate – evidenced by the classic study of Friedman and Schwartz (1963) and also Galliot (1970). In the early 1970s major industrialised countries shifted to floating exchange rates. During this period the monetary approach to the exchange rate also gained dominance.11 Both of these shifted

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8This is the concept of absolute PPP. Relative PPP holds when the rate of depreciation of one currency relative to another matches the difference in aggregate price inflation between the two countries concerned.

9For example Dornbusch (1980).


11Monetary models of the exchange rate like Frenkel (1976, 1978) and Frenkel and Johnson (1978) supplement the purchasing power parity equation with money demand functions and equilibrium conditions in the money markets.
academic opinion towards continuous PPP. However, the poor empirical performance of the monetary models, together with the compelling evidence of the excess volatility of nominal exchange rates compared to macroeconomic fundamentals such as national price levels and real incomes and the high variability of the real exchange rates, led to the acknowledgment of the ‘collapse of PPP’. The monetary models also failed to explain substantial short-run variations in the exchange rate.

Dornbusch (1976a, 1976b) and Mussa (1976) explain these fluctuations by assuming that domestic nominal prices are temporarily fixed. So the prices of goods available to agents in one country change relative to prices of the same goods in another country and monetary shocks can cause a change in the exchange rate even if real supplies and demands for goods are unaffected. Dornbusch’s overshooting model provided some respite to PPP by providing rationale for short-run deviations. However, the empirical evidence against PPP was overwhelming. This might be thought as the first PPP puzzle. Using unit root tests neither Roll (1979) nor Adler and Lehman (1983) could reject the null hypothesis of random-walk behaviour in deviations from PPP and subsequent cointegration studies again found no evidence of long-run PPP. As noted by Dornbusch (1988) this led to the rather widespread belief that PPP was of little use empirically and that real exchange rate movements are highly persistent. However, more recent work on long-run PPP on major industrialised economies has been more favourable towards the long-run PPP hypothesis for the recent float.

However, Frankel (1986) noted that the tests typically employed during the 1980s may have very low power to reject the null hypothesis of real exchange rate instability when applied to data for the recent floating period alone. The basic argument is that if the real exchange

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12 See for example studies of Frenkel (1976) and Frenkel and Johnson (1978).
13 Friedman and Schwartz (1963) note that during the greenback period of 1861-79, the US-UK exchange rate varied by about 2 to 1, while the ratio of price levels varied by only 1.3 to 1.
15 See Frenkel (1981a).
16 For the recent float period Taylor (1988) and Mark (1990) reported failure of significant mean reversion. However, studies were supportive of reversion towards PPP for the interwar float e.g. Taylor and McMahon (1988), for the 1950s US-Canadian float e.g. McNown and Wallace (1989) and for the exchange rates of high inflation countries like Choudhry, McNown and Wallace (1991).
17 For example Corbae and Ouliaris (1988), Kim (1990), Cheung and Lai (1993a, 1993b) and MacDonald (1993). These results must however be taken with a pinch of salt as there is always a non-stationary representation for a time series that is arbitrarily close to any stationary representation, as contended by Cochrane (1991), Blough (1992) and Faust (1996).
rate is in fact stable in the sense that it is mean reverting over long periods of time, then the examination of just one real exchange rate series over a period of twenty years or so may not yield enough information for one to be able to detect slow mean reversion towards purchasing power parity. To circumvent the problem of the power of the test, there were two developments in research: (i) researchers sought to increase the power of the unit root test by increasing the length of the sample period under consideration\(^{19}\) — many have in fact been able to find significant evidence of mean reversion,\(^{20}\) and (ii) researchers sought to increase the power by using panel root tests applied jointly to a number of real exchange rate series over the recent float\(^{21}\) — in many of these studies, the unit root hypothesis is rejected for groups of real exchange rates.\(^{22}\) Recently, a third measure for increasing the power of unit root tests has been proposed by Cheung and Lai (1998) — by employing univariate tests based on generalised or weighted least square estimators. If one takes that the methods described above to increase the power of the tests have helped resolve the first PPP puzzle then a second puzzle arises.

As noted by Rogoff (1996), the growing empirical literature on PPP has arrived at a surprising degree of consensus on some basic facts: (i) there is fairly persuasive evidence that real exchange rates tend towards purchasing power parity in the very long-run.\(^{23}\) Consensus estimates suggest that the speed on convergence is very slow — deviations appear to damp out at a rate of roughly 15 percent per year, implying a half-life of three-five years, and (ii) short-run deviations from PPP are large and volatile.

Lothian and Taylor (2000) suggest that the Harrod-Balassa-Samuelson hypothesis may also

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\(^{19}\)However, as noted by Frankel and Rose (1996) the long samples required to generate a reasonable level of statistical power with standard univariate unit root tests may be unavailable for many currencies — generating as per Froot and Rogoff (1995) 'survivorship bias'. Further, as noted by Baxter and Stockman (1989) and Hegwood and Papell (1998) long samples may be potentially inappropriate because of differences in real exchange rate behaviour both across different historical periods and across different nominal exchange rate regimes.

\(^{20}\)For example Frankel (1986), Diebold, Husted and Rush (1991), Cheung and Lai (1993a), Lothian and Taylor (1996). However, Engel (2000) using artificial data calibrated to nominal exchange rates and disaggregated data on prices shows that tests on long-run PPP have serious size biases.

\(^{21}\)Taylor and Sarno (1998), provide Monte Carlo evidence that the null hypothesis in panel data studies is in fact a joint null hypothesis that all of the series are generated by a unit root process, so that the probability of rejection of the null might be quite high when as few as just one of the series under consideration is a realisation of a stationary process.


\(^{23}\)By implication the first PPP puzzle has been resolved.
be important in shedding light on the PPP puzzle. They allow for underlying shifts in the
equilibrium dollar-sterling real exchange rate over the past two hundred years through the use
of non-linear time trends and find that the estimated half-life of real exchange rate shocks sub-
stantially reduce even without explicit allowance for non-linear real exchange rate adjustment.
These results are similar to those of Lothian (1991) for yen-dollar and yen-sterling real exchange
rate over the period 1875-1989. Lothian finds that allowing for a linear trend in the logarithm
of real exchange rates results in a 20% reduction – depending on the exact specification of the
model – in the half-lives of adjustment in both instances.

According to Taylor, Peel and Sarno (2001) if one takes as given that real shocks can not
account for the major part of the short-run volatility of the real exchange rate\(^{24}\) and that
nominal shocks can only have substantial effects on the real economy over a time frame in
which nominal wages and prices are sticky, then the second PPP puzzle is the apparently high
degree of persistence in the real exchange rate. They seek to resolve the two PPP puzzles by
allowing for non-linearities in real exchange rate adjustment, such that the real exchange rate
behaves like a unit process closer it is to long-run equilibrium and, conversely, becomes more
mean reverting the further away from equilibrium.\(^{25}\)

To quote Rogoff (1996) “It is not difficult to rationalise slow adjustment if real shocks –
shocks to tastes and technology – are predominant. But existing models based on real shocks can
not account for short-term exchange rate volatility.” Our goal in this chapter is to show that
real shocks – in our case productivity – alone can reproduce the univariate properties of the
real exchange rate.

4.3 Real Exchange Rate and Productivity

The continuous strength of the dollar over the 1990s fuelled interest in the relationship between
productivity and exchange rates. As US productivity surged in the second half of the 1990s,

\(^{24}\) Since it seems quite implausible that shocks to real factors such as tastes and technology could be so volatile. Most explanations of short-run volatility point to financial factors like changes in portfolio preferences, short-term asset price bubbles and monetary shocks.

\(^{25}\) This may be due to greater goods arbitrage as the misalignment grows e.g. Obstfeld and Taylor (1997), or a greater likelihood of intervention by authorities to correct a misaligned exchange rate e.g. Sarno and Taylor (2001), Taylor (2003), or a growing degree of consensus concerning the appropriate or likely direction of nominal exchange rate movements among traders like Kilian and Taylor (2003).
the dollar began its climb against all the major currencies of the world. Tille, Stoffels and Gorbachev (2001) point out that between 1995 and 1999, the dollar appreciated 4.8 percent against the yen and 5.8 percent against the euro on an average annual basis. The fact that these two trends were happening together tended to suggest that productivity gains were driving the appreciation of the dollar. Further, Alquist and Chinn (2002) find that the real exchange rate is cointegrated with a broad productivity differential. Using a number of different specifications, sample periods and estimation techniques, they find that each one percentage point increase in the US-Euro area productivity differential results in between a four to five percent appreciation in the dollar/euro exchange rate.

The ‘conventional’ view of the impact of a productivity shock on an economy is that a spurt in productivity leads to an expansion of output; if this extra output has to be sold in the world markets the price of the good must fall, i.e. higher world supply of its goods should reduce their relative price. The country should experience a depreciation of the real exchange rate and a worsening of its terms of trade. However, the data is at odds with the theory. Further this view also fails to explain the cyclical pattern that we observe in actual real exchange rate data.\footnote{Figure 4.A.1 in the appendix plots the real value of the pound sterling from 1986 till date.}

Corsetti, Dedola and Leduc (2004) point out that the repeated good news about growth in the US in 1999-2000, led to a dollar appreciation with upward revision of the growth gap relative to the euro. According to them, this can be interpreted in terms of ‘crowding out,’ to borrow the terminology of the Mundell-Fleming model. The expectations of persistent productivity growth raise domestic consumption and investment much more than domestic supply. Forward looking consumers increase consumption due to expectations of higher future income and higher productivity increases expected future profits, raising investment demand. Now, in order for the markets to clear, a higher international price is needed to ‘crowd-out’ net exports. This would explain the appreciation of the dollar.

Bailey, Millard and Wells (2001) make explicit the theoretical link between profitability and the exchange rate — they argue that an increase in productivity raises future expected profits, raising equity prices and stimulating investment. This additional investment can be financed by capital inflows, enabling the domestic residents to finance the additional investment without forgoing any current consumption. These effects are consistent with other aspects of the ‘new
economy' explanation of the appreciation of the dollar against the euro, as put forward by Meredith (2001). This line of explanation argues that high returns on the US investments attracted large foreign inflows of capital, which in turn appreciated the dollar through the capital account.

In the extensive literature on the purchasing power parity theory of exchange rates, a number of studies have cited productivity differentials between the nontraded and traded sectors of economies as a prime cause of deviations between any exchange rate and its PPP value. According to the Harrod-Balassa-Samuelson (HBS) hypothesis, the real exchange rate does not respond to productivity differentials across countries; but to differences in the productivity gap between tradable and nontradable sectors of an economy. Suppose there is faster productivity growth in the tradable sector than the nontradable sector; the price of tradable goods will not change as they are tied to the world market. Productivity gains will however lead to higher wages for workers in the tradable sector. Wages will also rise in the nontradable sector as employers seek to retain their workers. However, because of lower productivity gains the firms in this sector will be unable to absorb the wage increase and will pass it on to consumers in the form of higher prices; implying an increase in the relative price of nontradables. As a result the overall price index will increase. This implies an appreciation of the real exchange rate. Thus, we see that long-run productivity differentials would lead to trend deviations from PPP. Several studies, including Balassa (1964, 1973), De Vries (1968), Clague and Tanzi (1972) and Officer (1976b) have reported empirical tests of this phenomenon. However, their conclusions about whether productivity differentials explain deviations from PPP widely differ.

An alternative way of interpreting the HBS hypothesis is that after adjusting for exchange rates, CPIs in rich countries will be high relative to those in poor countries and that CPIs in

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27 According to the law of one price the prices of tradable goods will be equalised across countries. However, this would not be the case for the nontradables where the law of one price does not hold.

28 Unless the labour market is segmented.

29 According to Froot and Rogoff (1995), most of the literature overlooks the important point that even balanced growth across the two sectors can lead to a rise in the relative price of nontradables as long as nontradable goods are relatively more labour intensive.

30 The finding that productivity gains lead to higher prices sounds puzzling. Note, however, that because the productivity increase boosts wages by more than prices, the purchasing power – wages deflated by price index – of the worker increases.

31 Officer (1976b) provides a detailed description and critique of previous empirical studies. For more general studies of PPP see Kravis, Kenesey, Heston and Summers (1975), Officer (1976a), as well as papers in the symposium on the purchasing power parity in the May 1978 issue of the Journal of International Economics.
fast-growing countries will rise relative to CPIs in slow-growing countries. Balassa (1964, 1973) finds that richer countries have higher exchange rate adjusted price levels. However, Officer (1976b) argues that Balassa's results are very sensitive to the year chosen and countries included in the analysis. Hsieh (1982) was the first to use time series rather than cross section data and provides strong evidence supporting the role of productivity differentials. Marston (1987) looks at the yen/dollar exchange rate over the period 1973-83. Using sectoral employment data the author calculates labour productivity differentials between traded and nontraded goods and argues that these variables provide an extremely plausible explanation of the long-run trend real appreciation of the yen against the dollar. Edison and Klovland (1987) examines the real exchange rate data for the British pound and Norwegian krone for the years 1874-1971. This long time series allows them to detect significant evidence of a productivity differential effect using as proxies both the real output differential and a measure of commodity/service productivity ratio differential.

The evidence of later studies is somewhat mixed. Froot and Rogoff (1991a, 1991b) look at a cross section of 22 OECD countries for the period 1950-1989. They find, both for the full sample and various subsamples, that the correlation between productivity differentials and the real exchange rates is weak at best. It must be noted however that most of the recent theoretical literature on real exchange rate has emphasised movements in nontraded goods component. Asea and Mendoza (1994) base their analysis on a dynamic two-country general equilibrium model. The conclusion of the paper is that although productivity differentials between traded and nontraded goods are extremely significant in explaining changes in the nontraded goods prices within each country, changes in nontraded goods prices account for only a small and insignificant part of the real exchange rate changes across countries.

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[32] Unlike Hsieh (1982) most previous studies used cross section data and focused as much on the level of the exchange rate as on its rate of depreciation.


[34] Asea and Mendoza (1994) take sectoral OECD data to calculate relative traded goods prices for fourteen OECD countries over the period 1975-85. They first regress the relative price of nontraded goods for each country against traded-nontraded productivity differentials and then regress cross country real exchange rates against the relative price of nontraded goods; they try both actual and estimated.

[35] Thus there is evidence of Baumol–Bowen effect. Baumol and Bowen (1966) argued that within a country, there is a broad tendency for the prices of service intensive goods like education, health care, banking etc. to rise over time.
De Gregorio, Giovannini and Wolf (1994a) present a cross country panel regression that attempts to sort the importance of demand and supply factors. They find that productivity, government spending and income variables are all highly significant and of the theoretically predicted signs. Although their model is not dynamic, De Gregorio, Giovannini and Wolf attempt to analyse whether demand factors matter in the long-run by averaging data for each country over time and then running a regression for the cross section data. They find that in the long-run productivity differentials remain extremely significant where as the effect of demand factors such as government spending and income decline in importance.

Schnatz, Vijnse & Osbat (2003) contend that an appreciation caused by the HBS hypothesis can be intensified in the medium term by demand effects. They point out that the increased productivity raises expected future income, leading to an increased demand for goods. The increase in demand for traded goods can be satisfied by running a trade balance deficit. However, the increased demand in nontraded goods will lead to an increase in their prices. Thus, demand effects lead to a relative price shift and thereby a real appreciation.

Although the HBS hypothesis explanation is quite popular, it is unable to fully explain the appreciation of the dollar. Engel (1999) measures the proportion of US real exchange rate movements that can be accounted for by movements in the relative price of nontraded goods. Using five different measures of nontraded goods prices and real exchange rates, for exchange rates of the US against a number of high income nations, Engel finds that relative prices of nontraded goods appear to account for almost none of the movement in the US real exchange rate. For the US-UK real exchange rate, the relative price of traded goods between the two, is responsible for over ninety five percent of the movement in the real exchange rate over the period 1970 to 1997. Tille, Stoffels and Gorbachev (2001) find that at most the HBS hypothesis can account for 2/3 of the dollar real appreciation in the 1990s. Further Corsetti, Dedola and Leduc (2004) point out that there are many other studies which find that nontraded goods prices explain very little about exchange rate movement; in the 1990s the US real appreciation appears to be driven mostly by improvements in the US terms of trade, rather than nontraded

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36 See for example Engel (1999) and De Gregorio and Wolf (1994a). The latter extends the analysis of De Gregorio, Giovannini and Wolf (1994a) to incorporate terms of trade shocks. It finds that terms of trade are important empirically, though productivity and government spending differentials continue to be important. However, relative incomes become insignificant once terms of trade are included – possibly relative incomes were proxying terms of trade in the 1994a paper.
prices. Furthermore, in today’s age of information technology, the nontradable sector itself is shrinking as most of the services can now be bought from across borders.

It is quite clear that there is no single factor model to determine the exchange rate. In general equilibrium, the exchange rate responds to many shocks — including productivity.

In the current chapter we explore the ability of a Real Business Cycle (RBC) model to account for the behaviour of the real exchange rate, calibrating the model to quarterly UK data. The objective of the chapter is to establish that RBC alone can reproduce the univariate properties of the real exchange rate. We however, do not rule out the possibility that adding a degree of nominal rigidity, in a sort of 'stretched-RBC' framework, could also be useful. We hope to test the impact of the explanatory power of nominal rigidity in forthcoming research.

4.4 Data Patterns

Let us begin by looking at the empirical evidence on the sterling’s real exchange rate (RXR). The path of the sterling real exchange rate from 1986 is presented in Figure 4.A.1 in the appendix.\(^{37}\)

While it appears to exhibit some sort of classically cyclical, mean reversion to a smoothly-moving trend or indeed even to a constant and has a non-zero mean, the univariate final form equation is in fact best described by a \( ARIMA(3,1,3) \) process; the series is therefore not actually mean-reverting but integrated of order 1 with pronounced serial correlation. Our main aim is to see whether our calibrated RBC model can generate the same univariate behaviour.

In this section we estimate univariate processes for the real exchange rate. There is a large body of literature that finds the real exchange rate to be non-stationary.\(^{38}\) In order to estimate non-spurious univariate processes we first check for non-stationarity. Using both the Augmented Dickey Fuller test\(^{39}\) and Phillips-Perron test,\(^{40}\) we find that the sterling’s real exchange rate is

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\(^{37}\)The real exchange rate data used is the inverse of the one in the Liverpool model, which is the ratio of UK to other OECD consumer prices adjusted for nominal exchange rate, where the nominal exchange rate is the sterling effective exchange rate.


\(^{39}\)Schwert (1989) finds in Monte Carlo simulations that when the absolute value of the moving average coefficient is close to unity, there are large size biases in the Dickey Fuller tests.

\(^{40}\)The advantage of using the PP test is that it allows in a non-parametric fashion for the possibility of serially correlated and heterogeneously distributed error terms. However, according to Phillips and Perron (1988) and Schwert (1989), the PP test statistic may be subject to distortion in the presence of a moving average components in the time series.
an $I(1)$ series.\textsuperscript{41} Table 4.1 below reports the results.\textsuperscript{42} The real exchange rate series in levels fails to reject the null hypothesis of non-stationarity at 1 percent level of significance, using both the ADF and the PP test statistic. When we test with the first difference form of the series we can easily reject the null, again at 1 percent.

<table>
<thead>
<tr>
<th>Unit Root Tests (with intercept)</th>
<th>Levels</th>
<th>First Difference</th>
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<tbody>
<tr>
<td>ADF Test Statistic</td>
<td>-2.029676*</td>
<td>-5.258587</td>
</tr>
<tr>
<td>PP Test Statistic</td>
<td>-1.785482*</td>
<td>-5.786253</td>
</tr>
</tbody>
</table>

* Acceptance at 1% level of significance using MacKinnon critical values for rejection of hypothesis of a unit root.

Table 4.1: Test for Non-stationarity of the Real Exchange Rate

Having established the non-stationarity of the series we now proceed to estimate the best fitting ARIMA process to the real exchange rate, using data from 1986:1 till 2000:4.\textsuperscript{43} To obtain an estimate of the ARIMA structure we use a general to specific approach, starting off with a maximum lag order of 6. We expect to find a high order ARIMA as our RBC model recommends a high order of both AR and MA as this order should reflect the number of shocks with autoregressive roots in the model — which are many. Using AIC, the information criterion proposed by Akaike (1973), we chose the lag structure. Table 4.2 below summarises our results.\textsuperscript{44} Clearly the results below indicate that RXR is a highly persistent series. An ARIMA(3,1,3) best describes the data.\textsuperscript{45}

\textsuperscript{41}The failure to find mean reversion could be due to the short length of the sample, as discussed extensively in Section 4.2.1. According to Engel (1999) in short series it is difficult to distinguish between highly persistent but stationary series and series that have a unit root.

\textsuperscript{42}For detailed results please see the appendix, Figure 4.A.2 to 4.A.5.

\textsuperscript{43}We take into account seasonal dummies as it is a well-known fact that price series are seasonal in nature.

\textsuperscript{44}For detailed results please refer to the appendix, Figure 4.A.6.

\textsuperscript{45}We all know that econometricians always try and find the best-fitting most parsimonious data generating process. For us the most parsimonious data representation was an ARIMA(1,1,1). See Figure 4.A.7 in the appendix. However, given that our model has many shocks with autoregressive roots, an ARIMA(1,1,1) has too low a order to be consistent with our theoretical priors.
<table>
<thead>
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<th></th>
<th>Coefficient</th>
<th>P-value</th>
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<tr>
<td>AR(2)</td>
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<td>AR(3)</td>
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<tr>
<td>MA(1)</td>
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<td>MA(2)</td>
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<tr>
<td>MA(3)</td>
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Table 4.2: Best Fitting RXR ARIMA

4.5 Model

We use our micro-founded stochastic general equilibrium open economy model\textsuperscript{46} based on optimising decision of rational agents; in this version we have assumed a real business cycle set-up in which prices are fully flexible and money is irrelevant. The first order conditions of households’ and firms’ optimisation problems are used to derive the behavioural equations of the model. As we are modelling the UK, an economy of only modest size with minor effects on the rest of the world, we have a full blown model only for the domestic economy, taking the world economy as given. The interaction with the rest of the world comes in the form of uncovered real interest rate parity and the current account, both of which are explicitly micro-founded.\textsuperscript{47}

4.6 Calibration and Simulation

In order to carry out model simulations, numerical values should be assigned to the structural parameters of the models.\textsuperscript{48} Once the model has been solved numerically, one can analyse the characteristics of the transition of the model to its steady-state and also test if the model is consistent with the facts.

Our simulations start in 1986:3 and end in 2000:4 using quarterly UK data. Results of our simulation exercise are reported in tabular and graphical form in the appendix. The charts show the percentage deviation of a particular variable – real output, price level, real wages and

\textsuperscript{46}Details of the model can be found in Chapter 2.
\textsuperscript{47}See appendix 4.A.1 for the list of behavioural equations.
\textsuperscript{48}See appendix 4.A.3 for values of parameters used.
so on – from the baseline path except in case of interest rates where it shows percentage point deviations from baseline.

We begin by running a deterministic productivity growth shock through our model calibrated to quarterly UK data. This deterministic simulation is done in order to establish the basic order of magnitude and shape of the response function of the RXR to the workhorse RBC technology shock. In the RBC world the workhorse shock is a burst of unanticipated productivity growth that raises the level of productivity in steady-state well above its previous path. Figure 4.A.8 shows the model simulation of such a burst: a 5-year rise of the productivity growth rate by 1% p.a. So productivity grows at 1% in the first year, 2% in the second year and so on till the fifth year when it grows at 5%. After that it is permanently 5% above the base. Results are also reported in Table 4.A.1 in the appendix.

The response profile is attractive in exhibiting a pronounced and persistent cycle. The logic behind the RXR behaviour pattern can explained as follows. The productivity burst raises permanent income and also stimulates a stream of investments to raise the capital stock in line. Output however can not be increased without increased labour supply and extra capital. The increased demand for labour pushes up the real wage. Now extra capital is slow to arrive. Thus the real interest rate must rise to reduce demand of capital to the available supply and constrain the consumption and investment demand in the economy. The rising real interest rate violates uncovered real interest parity (URIP) which must be restored by a rise in the real exchange rate relative to its expected future value. This rise is made possible by the expectation that the real exchange rate will fall back steadily, so enabling URIP to be established consistently with a higher real interest rate. As real interest rates fall with the arrival of a stream of sufficient capital and so output, the real exchange rate also moves back to equilibrium. It must be noted however that this new equilibrium represents a real depreciation on the previous steady-state since output is now higher and must be sold on world markets by lowering its price.

---

Note that the chart plots percentage deviation from the baseline path.

See Figure 4.A.9 in the appendix.
4.7 Bootstrapping

Our basic objective is to see, assuming that our model and its error processes are true, if we can generate the facts of real exchange rate such as we find them. We want to find the sampling variability implied by the model — to find the 95% confidence limits around the RXR ARIMA parameters. One approach is to linearise the model, which would allow us to map it to a VARMA and in principle compute reduced form standard errors for each parameter. However, the reliability of the standard errors would be open to question given our small sample size. Analytical computation of the confidence limits is also not possible given the non-linear nature of the model.

Comparison of our model with the ARIMA we have estimated on the actual data can not be done via deterministic simulation because the estimated equation depends on the distribution of all the shocks. What we wish to do is to replicate the stochastic environment to see whether within it our estimated ARIMA equations could have been generated. This we do via bootstrapping the RBC model with its error processes.

We take our RBC model and calibrate it to quarterly UK data to estimate the residuals in the behavioural equations and then work out their data generating process. In the model we have a general preference shock in the Euler equation for the intertemporal consumption pattern. Money supply shock in the equation for domestic prices. Productivity shock has a direct impact on the output and also an indirect impact on labour and capital. We have a productivity bias shock in the demands for labour and capital. We have an adjustment cost shock in the investment equation and a labour preference shock in the labour supply equation. We have residuals in the import and export equations coming from errors in home versus foreign preferences in the CES consumption function. In the UIP the error is basically the risk premium. In equations that are identities\textsuperscript{51} we have residuals either due to measurement errors or due to the approximations made in the model.\textsuperscript{52} Having obtained the residuals, we determine the best fitting data generating process for them,\textsuperscript{53} to obtain the i.i.d. shocks in our error

\textsuperscript{51}GDP identity, government budget constraint, primary deficit equation, evolution of lump-sum taxes, balance of payments (BOP) identity and evolution of household debt.

\textsuperscript{52}For example in the BOP identity we have taken bonds with only 1 period maturity. In the real world there are different kinds of bonds with different maturities. Our simplifying assumptions will clearly lead to residuals in the model.

\textsuperscript{53}Typically it is an AR(1) with a constant.
processes. In our model we also have exogenous processes — productivity, labour income tax, VAT, unemployment benefits, money supply, government expenditure, foreign prices, foreign consumption and foreign interest rates. To replicate the real exchange rate with its unit root we need unit root drivers in the system which are coming from the exogenous processes all of which have been modelled as random walks.

Our basic objective is to see, assuming that our model and its error processes are true, if we can generate the facts of real exchange rate as we find them. To do this we generate the sampling variability within the model by the method of bootstrapping the model’s estimated residuals; this permits us to find the 95% confidence limits around the RXR ARIMA regression parameters. The idea is to create pseudo data samples (here 500) for the real exchange rate. We draw the vectors of i.i.d. shocks in our error processes with replacement, by drawing vectors for the same time period we preserve their contemporaneous cross-correlations; we then input them into their error processes and these in turn into the model to solve for the implied path of RXR over the sample period. In Figure 4.A.10 we plot a random selection of the pseudo RXR series (shaded/dashed lines) generated by bootstrapping the model’s errors with the historical RXR data for the UK (solid line). As can be seen from the graph the model is capable of generating series with lots of cycles, very much like the true RXR series.

We run ARIMA regressions on all the samples to derive the implied 95% confidence intervals for all the coefficients. Finally we compare the ARIMA coefficients estimated from the actual data to see whether they lie within these 95% confidence intervals. The comparison both guides us on whether our model is moving the parameters in the right direction; and informs us more formally whether the data rejects the model. Table 4.3 summarises the results of this exercise.

The results in Table 4.3 clearly validate our hypothesis that real exchange rate behaviour is explicable within the RBC framework. Five out of the six ARIMA parameters, comfortably lie within the 95% confidence intervals. MA(3) lies only marginally outside the upper limit. The model also captures the direction of movement of the parameters.

\[ ^{54} \text{Note that actual data starts from 1986:1 while the model generated series start from 1986:3.} \]
### Table 4.3: Confidence Limits from our Model for RXR ARIMA

<table>
<thead>
<tr>
<th>Autoregressive Integrated Moving Average</th>
<th>95% Confidence Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARIMA(3,1,3)</td>
<td></td>
</tr>
<tr>
<td><strong>Estimated</strong></td>
<td><strong>Lower</strong></td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.350984</td>
</tr>
<tr>
<td>AR(2)</td>
<td>-0.631597</td>
</tr>
<tr>
<td>AR(3)</td>
<td>-0.469182</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.793836</td>
</tr>
<tr>
<td>MA(2)</td>
<td>0.794252</td>
</tr>
<tr>
<td>MA(3)</td>
<td>0.990501</td>
</tr>
</tbody>
</table>

### 4.8 Conclusions

The objective of this chapter has been to explain the behaviour of the real exchange rate; both the appreciation following a productivity burst and its cyclical pattern using an open economy RBC model calibrated to quarterly UK data. The model is a micro-founded general equilibrium open economy model based on optimising decisions of rational agents; in this version we have assumed a real business cycle set-up in which money is irrelevant. The first order conditions of the household and firm optimisation problem are used to derive the behavioural equations of the model. We are modelling a medium-sized open economy, so we have a full blown model only for the domestic economy taking the world economy as given. The interaction with the rest of the world comes in the form of uncovered real interest rate parity and the current account, both of which are explicitly micro-founded.

It is a well established empirical fact that a burst in productivity leads to an appreciation of the currency. However, according to the ‘conventional’ view, if a country becomes more productive, a higher world supply of its good should result in a relative price reduction. The country should experience a deterioration of the terms of trade. This is completely at odds with the data. The Harrod-Balassa-Samuelson (HBS) hypothesis, can explain the appreciation following a burst in productivity in the tradable sector. However, it still fails to explain the cycles that we observe in actual real exchange rate data.

Our deterministic simulation of a 1 percent productivity growth shock clearly shows that the real exchange appreciates on impact and then goes back to its new depreciated equilibrium. The reasoning is quite simple, yet it has an intuitive appeal. As productivity increases, permanent income expands. However, for output to increase extra labour and capital is required. Capital,
is slow to arrive, which pushes up the real interest rate in the economy so that the capital market clears. The increase in the real interest rate violates the uncovered real interest rate parity. Uncovered real interest rate parity is established by an appreciation of the pound with the expectation that it will depreciate in the future back to its equilibrium value. This equilibrium however represents a real depreciation on the previous steady-state since output is now higher and must be sold in world markets by lowering its price. This gives us a ‘business cycle’ in the real exchange rate.

Ultimately we can only settle whether our model could be consistent with the facts by asking whether it could have generated the patterns we find in the actual real exchange rate data. To do this we generate the sampling variability within the model by the method of bootstrapping the model’s estimated residuals; this permits us to find the 95% confidence limits around the real exchange rate ARIMA regression parameters. This tells us what the standard errors of these regressions are under the null hypothesis of our model — the relevant standard errors for us, rather than the usual ones which tell us whether the regression, viewed atheoretically, can reject a zero null hypothesis, a fairly uninteresting one for an economist.

We find that our model tells quite a good story, the gyrations of the real exchange rate can be explained within a RBC context. We do not rule out the possibility that adding a degree of nominal rigidity, in a sort of ‘stretched-RBC’ framework, could also be useful. However our concern here has been to establish the basic ability of the RBC alone to provide explanatory power.
4.A Appendix

4.A.1 Behavioural Equations of the RBC Model along with Residuals

(1) Consumption $C_t$; solves for $r_t$:

$$(1 + r_t) = \frac{1}{\beta} \left( \frac{C_t}{E_t[C_{t+1}]} \right)^{-\rho_o} \left( \frac{1 + \phi_{t+1}}{1 + \phi_t} \right) (\varepsilon_{rt})$$

$$r_t = \frac{1}{\beta} \left( \frac{C_t}{E_t[C_{t+1}]} \right)^{-\rho_o} \left( \frac{1 + \phi_{t+1}}{1 + \phi_t} \right) (\varepsilon_{rt}) - 1$$

where $C_t = \left[ \omega \left( C_t^d \right)^{-\rho} + (1 - \omega) \left( C_t^f \right)^{-\rho} \right]^{\frac{1}{\rho}}$

(2) Money supply $\overline{M}_t^d$; solves for $P_t^d$:

$$\overline{M}_t^d = (1 + \phi_t) C_t^d P_t^d + \overline{G}_t P_t^d$$

$$P_t^d = \left( \frac{\overline{M}_t^d}{(1 + \phi_t) C_t^d + \overline{G}_t} \right) (\varepsilon_{P_t^d})$$

where $P_t = \left[ \omega^{\frac{1}{1+p}} (P_t^d)^{\frac{1}{1+p}} + (1 - \omega)^{\frac{1}{1+p}} (P_t^F)^{\frac{1}{1+p}} \right]^{\frac{1+p}{p}}$

(3) Demand for shares, $S_{t+1}^p$:

$$S_{t+1}^p = \overline{S}_t ; b_{t+1} = b_{t+1}^p \text{ implied.}$$

(4) Present value of share:

$$p_t = E_t \sum_{i=1}^{\infty} \left[ \frac{d_{t+i}}{\prod_{k=t}^{i} (1 + r_k)} \left( \frac{P_i}{P_{t+i}} \right) \right]$$

where $d_t$ (dividend per share), $p_t$ (present value of shares in nominal terms).

(5) Production function $Y_t$:

$$Y_t = Z_t N_t^\alpha K_t^{(1-\alpha)} (\varepsilon Z_t)$$

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(6) Demand for labour:

\[ N_t^d = \left( \frac{aZ_t}{w_t} \right)^{(1-\alpha)} K_t(\varepsilon_{N_t^d}) \]

(7) Capital:

\[ K_t = \left( (1 - \alpha) \frac{Y_t}{r_t + \delta} \right) (\varepsilon_{K_t}) \]

(8) GDP identity, \( Y_t \); solves for \( C_t \):

\[ Y_t = C_t + I_t + G_t + NX_t + \varepsilon_{GDP_t} \]

where \( NX_t \) is net exports.

(9) Investment:

\[ K_{t+1} = (1 - \delta)K_t + I_t + \varepsilon_{I_t} \]

(10) \( W_t \); currently this variable is not defined.

(11) Wage \( w_t \):

\[ w_t = w_t^* \]

(12) Evolution of \( b_t \); government budget constraint:

\[ b_{t+1} = (1 + r_t) b_t + PD_t - \frac{\Delta M_t}{P_t^d} + \varepsilon_{b_t} \]

(13) Equilibrium wage, \( w_t^* \); \( w_t^* \) is derived by equating demand for labour, \( N_t^d \), to the supply of labour \( N_t^s \), where
\[
(1 - N_t^s) = \left\{ \frac{\theta_0 C_t^{s \rho_0} \left[ (1 - \tau_t) \exp \left( \log w_t^s - \frac{(1 - \omega)^{1+\rho}}{\omega^{1+\rho} + (1 - \omega)^{1+\rho}} \log Q_t - \mu_t \right) \right]}{(1 - \theta_0) (1 + \phi_t) (1 + r_t)} \right\}^{\frac{1}{\rho_2}} (\varepsilon_{N_t^s})
\]

where \( Q_t \) is the real exchange rate, \((1 - \omega)^{1+\rho}\) is the weight of domestic prices in the CPI index. 

(14) Dividends are surplus corporate cash flow:

\[
\begin{align*}
\bar{d}_t S_t &= Y_t - N_t^s w_t - K_t (r_t + \delta) \\
\bar{d}_t &= \frac{Y_t - N_t^s w_t - K_t (r_t + \delta)}{S_t}
\end{align*}
\]

(15) Primary deficit \( PD_t \):

\[
PD_t = G_t + \mu_t (1 - N_t^s - \bar{I}) + E_t \pi_{t+1}^d M_t^d - \tau_{t-1} v_{t-1} N_{t-1}^s - \phi_{t-1} C_{t-1} - T_{t-1} + \varepsilon_{PD_t}
\]

(16) Tax \( T_t \):

\[
T_t = T_{t-1} + \gamma^G (PD_{t-1} + b_t r_t) + \varepsilon_{T_t}
\]

(17) Exports \( EX_t \):

\[
\log EX_t = \sigma^f \log \left( 1 - \omega^f \right) + \log C_t^F + \sigma^f A^f \log Q_t + \varepsilon_{EX_t}
\]

where \( A^f = \frac{(\omega^f)^{1+\rho_f}}{(\omega^f)^{1+\rho_f} + (1 - \omega^f)^{1+\rho_f}} \)

(18) Imports \( IM_t \):

\[
\log IM_t = \sigma \log (1 - \omega) + \log C_t - \sigma A \log Q_t + \varepsilon_{IM_t}
\]

where \( A = \frac{\omega^{1+\rho}}{\omega^{1+\rho} + (1 - \omega)^{1+\rho}} \)
(19) UIP condition:

\[ r_t = r^f_t + E_t \Delta \log Q_{t+1} + \varepsilon_{UP,t} \]

where \( r^f \) defined the foreign real interest rate.

(20) Net exports:

\[ NX_t = EX_t - IM_t \]

(21) Evolution of foreign bonds \( b^f_t \):

\[ b^f_{t+1} = (1 + r^f_t)b^f_t + NX_t + \varepsilon_{b^f_t} \]

(22) Nominal exchange rate, \( S_t \):

\[ \log S_t = \log Q_t - \log P^f_t + \log P^d_t \]

where \( P^f_t \) is foreign price and \( S_t \) the nominal exchange rate.

(23) Evolution of household debt \( D_{t+1} \):

\[ D_{t+1} = (1 + r_t)D_t - Y_{t-1} + (1 + \phi_t)C_t + \tau_t v_t N^*_t + T_t + \varepsilon_{D_t} \]

(24) Household transversality condition:

\[ Y_{T-1} - r_T D_T - \phi_T C_T - \tau_T v_T N^*_T - T_T = C_T \]

(25) Government transversality condition:

\[ G = 0.30 \]

4.A.2 Exogenous Processes in the RBC Model

(1) \( \Delta \ln Z_t = \varepsilon_{1,t} \)
(2) $\Delta \tau_t = \varepsilon_{2,t}$
(3) $\Delta \phi_t = \varepsilon_{3,t}$
(4) $\Delta \mu_t = \varepsilon_{4,t}$
(5) $\Delta \ln M_t = \varepsilon_{5,t}$
(6) $\Delta \ln P_t^f = \varepsilon_{6,t}$
(7) $\Delta \ln C_t^f = \varepsilon_{7,t}$
(8) $\Delta \ln r_t^f = \varepsilon_{8,t}$

4.A.3 Values of Coefficients in the RBC Model

Note: the values of the coefficients used in the model have been calibrated from the recent literature.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value - Single equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.70</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.97</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0125</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>1.20</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>0.50</td>
</tr>
<tr>
<td>$\gamma^G$</td>
<td>0.05</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>1.00</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.70</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-0.50</td>
</tr>
<tr>
<td>$\omega^f$</td>
<td>0.70</td>
</tr>
<tr>
<td>$\rho^f$</td>
<td>-0.50</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma^f$</td>
<td>2</td>
</tr>
</tbody>
</table>

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Figure 4.A.1: Historical Real Exchange Rate in the UK
4.A.4 Unit Root Tests for Real Exchange Rate

**ADF Test of Levels with Intercept**

<table>
<thead>
<tr>
<th>ADF Test Statistic</th>
<th>1% Critical Value*</th>
<th>5% Critical Value</th>
<th>10% Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.02968</td>
<td>-3.5457</td>
<td>-2.9118</td>
<td>-2.5932</td>
</tr>
</tbody>
</table>

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(Q)
Method: Least Squares
Date: 02/06/05 Time: 15:35
Sample (adjusted): 1986:3 2000:4
Included observations: 58 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q(-1)</td>
<td>-0.11137</td>
<td>0.05487</td>
<td>-2.029676</td>
<td>0.0472</td>
</tr>
<tr>
<td>D(Q(-1))</td>
<td>0.307323</td>
<td>0.129739</td>
<td>2.368783</td>
<td>0.0214</td>
</tr>
<tr>
<td>C</td>
<td>0.110813</td>
<td>0.055121</td>
<td>2.010355</td>
<td>0.0493</td>
</tr>
</tbody>
</table>

R-squared | Mean dependent var | -0.00112
Adjusted R-squared | S.D dependent var | 0.031369
S.E. of regression | Akaike info criterion | -4.13413
Sum squared resid | Schwarz criterion | -4.02755
Log likelihood | F-statistic | 3.949054
Durbin-Watson stat | Prob(F-statistic) | 0.024971

Figure 4.A.2: Augmented Dickey Fuller Test of Real Exchange Rate in Levels with Intercept
### ADF Test of First Difference with Intercept

ADF Test Statistic  
1% Critical Value*  
5% Critical Value  
10% Critical Value  

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(Q,2)
Method: Least Squares
Date: 02/06/05  Time: 15:37
Sample(adjusted): 1986.4 2000.4
Included observations: 57 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>D(Q(-1))</td>
<td>-0.83359</td>
<td>0.158521</td>
<td>-5.258587</td>
<td>0</td>
</tr>
<tr>
<td>D(Q(-1),2)</td>
<td>0.118001</td>
<td>0.128942</td>
<td>0.915153</td>
<td>0.3642</td>
</tr>
<tr>
<td>C</td>
<td>-0.0021</td>
<td>0.003928</td>
<td>-0.533688</td>
<td>0.5957</td>
</tr>
</tbody>
</table>

R-squared 0.401815  Mean dependent var -0.00103
Adjusted R-squared 0.37966  S.D. dependent var 0.037589
S.E. of regression 0.029606  Akaike info criterion -4.1505
Sum squared resid 0.047331  Schwarz criterion -4.04297
Log likelihood 121.2893   F-statistic 18.13656
Durbin-Watson stat 2.080531  Prob(F-statistic) 0.000001

Figure 4.A.3: Augmented Dickey Fuller Test of Real Exchange Rate First Difference with Intercept
**PP Test of Levels with Intercept**

<table>
<thead>
<tr>
<th>PP Test Statistic</th>
<th>1% Critical Value*</th>
<th>5% Critical Value</th>
<th>10% Critical Value</th>
</tr>
</thead>
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<tr>
<td>-1.78548</td>
<td>-3.5437</td>
<td>-2.9109</td>
<td>-2.5928</td>
</tr>
</tbody>
</table>

*MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: 3
( Newey-West suggests: 3 )

Residual variance with no
correction         0.000917
Residual variance with correction      0.001294

Phillips-Perron Test Equation
Dependent Variable: D(Q)
Method: Least Squares
Date: 02/06/05   Time: 15:41
Sample(adjusted): 1986:2 2000:4
Included observations: 59 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q(-1)</td>
<td>-0.08061</td>
<td>0.055015</td>
<td>-1.465303</td>
<td>0.1483</td>
</tr>
<tr>
<td>C</td>
<td>0.079443</td>
<td>0.055229</td>
<td>1.438418</td>
<td>0.1558</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.038301</td>
<td>Mean dependent var</td>
<td>-0.00127</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.019394</td>
<td>S.D. dependent var</td>
<td>0.031119</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.030815</td>
<td>Akaike info criterion</td>
<td>-0.0883</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.054126</td>
<td>Schwarz criterion</td>
<td>-4.01787</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>122.6047</td>
<td>F-statistic</td>
<td>2.147112</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.445507</td>
<td>Prob(F-statistic)</td>
<td>0.148333</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.A.4: Phillips Perron Test of Real Exchange Rate in Levels with Intercept
PP Test of First Difference with Intercept

<table>
<thead>
<tr>
<th>PP Test Statistic</th>
<th>1% Critical Value*</th>
<th>5% Critical Value</th>
<th>10% Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5.78625</td>
<td>-3.5457</td>
<td>-2.9118</td>
<td>-2.5932</td>
</tr>
</tbody>
</table>

*MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: 3 (Newey-West suggests 3).  
Residual variance with no correction: 0.000909  
Residual variance with correction: 0.000858

Phillips-Perron Test Equation  
Dependent Variable: D(Q.2)  
Method: Least Squares  
Date: 02/06/05 Time: 15:42  
Sample(adjusYed): 1986 3 2000:4  
Included observations: 58 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>D(Q(-1))</td>
<td>-0.75497</td>
<td>0.12952</td>
<td>-5.828929</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>-0.00078</td>
<td>0.004033</td>
<td>-0.193685</td>
<td>0.8471</td>
</tr>
</tbody>
</table>

R-squared: 0.377515  
Mean dependent var: 0.000267  
Adjusted R-squared: 0.368501  
S.D. dependent var: 0.038549  
Akaike info criterion: 0.030682  
Schwarz criterion: -4.09938  
Log likelihood: 0.052719  
F-statistic: 120.795  
Prob(F-statistic): 33.97641  
Durbin-Watson stat: 1.833816  
Prob(DW statistic): 0

Figure 4.A.5: Phillips Perron Test of Real Exchange Rate First Difference with Intercept
4.A.5 Best Fitting ARIMA for Sterling Real Exchange Rate

**Best Fitting ARIMA for RXR**

Dependent Variable: D(Q)
Method: Least Squares
Date: 07/15/05 Time: 15:23
Sample (adjusted): 1987.1 2000.4
Included observations: 56 after adjusting endpoints
Convergence achieved after 35 iterations
Backcast: 1986.2 1986.4

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>@SEAS(1)</td>
<td>-0.010030</td>
<td>0.006358</td>
<td>-1.577398</td>
<td>0.1216</td>
</tr>
<tr>
<td>@SEAS(2)</td>
<td>-0.000548</td>
<td>0.006172</td>
<td>-0.088796</td>
<td>0.9296</td>
</tr>
<tr>
<td>@SEAS(3)</td>
<td>-0.006772</td>
<td>0.006270</td>
<td>-1.080044</td>
<td>0.2858</td>
</tr>
<tr>
<td>@SEAS(4)</td>
<td>0.008130</td>
<td>0.006121</td>
<td>1.328153</td>
<td>0.1907</td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.350984</td>
<td>0.132100</td>
<td>-2.659658</td>
<td>0.0108</td>
</tr>
<tr>
<td>AR(2)</td>
<td>-0.631597</td>
<td>0.120298</td>
<td>-5.250260</td>
<td>0.0000</td>
</tr>
<tr>
<td>AR(3)</td>
<td>-0.469182</td>
<td>0.123157</td>
<td>-3.809622</td>
<td>0.0004</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.793836</td>
<td>0.028323</td>
<td>28.02764</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA(2)</td>
<td>0.794252</td>
<td>0.055741</td>
<td>14.24902</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA(3)</td>
<td>0.995051</td>
<td>0.050620</td>
<td>19.56734</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared 0.367220 Mean dependent var -0.003378
Adjusted R-squared 0.243415 S.D. dependent var 0.029483
S.E. of regression 0.025645 Akaike info criterion -4.328506
Sum squared resid 0.003203 Schwarz criterion -3.966836
Log likelihood 131.1982 Durbin-Watson stat 1.898058

Inverted AR Roots .12 + .8i .12 - .8i .60
Inverted MA Roots .10 - .99i .10 + .99i .100

Figure 4.A.6: Best Fitting ARIMA for Sterling Real Exchange Rate
### Most Parsimonious ARIMA for RXR

Dependent Variable: D(Q)
Method: Least Squares
Date: 07/15/05  Time: 15:23
Sample(adjusted): 1986:3 2000:4
Included observations: 58 after adjusting endpoints
Convergence achieved after 24 iterations
Backcast: 1986:2

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>@SEAS(1)</td>
<td>-0.012876</td>
<td>0.007992</td>
<td>-1.611113</td>
<td>0.1132</td>
</tr>
<tr>
<td>@SEAS(2)</td>
<td>-0.004673</td>
<td>0.008011</td>
<td>-0.583404</td>
<td>0.5621</td>
</tr>
<tr>
<td>@SEAS(3)</td>
<td>-0.003177</td>
<td>0.007852</td>
<td>-0.404624</td>
<td>0.6874</td>
</tr>
<tr>
<td>@SEAS(4)</td>
<td>0.012034</td>
<td>0.007719</td>
<td>1.558976</td>
<td>0.1251</td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.555137</td>
<td>0.112304</td>
<td>-4.943163</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.980928</td>
<td>0.020015</td>
<td>49.01077</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

- R-squared: 0.278766
- Mean dependent var: -0.001121
- Adjusted R-squared: 0.209417
- S.D. dependent var: 0.031369
- Akaike info criterion: -4.223286
- Schwarz criterion: -4.010137
- Durbin-Watson stat: 1.903783

**Inverted AR Roots**: -0.56
**Inverted MA Roots**: -0.98

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Figure 4.A.7: Most Parsimonious ARIMA for Sterling Real Exchange Rate
4.A.6 1% p.a. Growth Shock to Productivity

Figure 4.A.8: 1% p.a. Growth Shock to Productivity
Figure 4.A.8 cont. 1% p.a. Growth Shock to Productivity

<table>
<thead>
<tr>
<th></th>
<th>Quarter 1</th>
<th>Year 1</th>
<th>Year 3</th>
<th>Long Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Interest Rate</td>
<td>0.040</td>
<td>0.100</td>
<td>0.170</td>
<td>-0.020</td>
</tr>
<tr>
<td>Price Level</td>
<td>-2.981</td>
<td>-3.768</td>
<td>-5.663</td>
<td>-7.481</td>
</tr>
<tr>
<td>Output</td>
<td>0.755</td>
<td>2.014</td>
<td>4.758</td>
<td>7.267</td>
</tr>
<tr>
<td>Labour Supply</td>
<td>-0.429</td>
<td>-0.320</td>
<td>-0.146</td>
<td>-0.108</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>0.169</td>
<td>0.805</td>
<td>2.803</td>
<td>7.647</td>
</tr>
<tr>
<td>Consumption</td>
<td>3.203</td>
<td>3.720</td>
<td>5.337</td>
<td>7.231</td>
</tr>
<tr>
<td>Investment</td>
<td>13.506</td>
<td>18.170</td>
<td>18.102</td>
<td>15.114</td>
</tr>
<tr>
<td>Real Wage</td>
<td>1.110</td>
<td>2.210</td>
<td>4.840</td>
<td>10.030</td>
</tr>
<tr>
<td>Exports</td>
<td>-1.240</td>
<td>-1.310</td>
<td>-0.970</td>
<td>0.110</td>
</tr>
<tr>
<td>Imports</td>
<td>2.040</td>
<td>2.300</td>
<td>2.680</td>
<td>3.640</td>
</tr>
<tr>
<td>Nominal Exchange Rate</td>
<td>-6.500</td>
<td>-7.360</td>
<td>-8.440</td>
<td>-7.910</td>
</tr>
<tr>
<td>Real Exchange Rate</td>
<td>-3.640</td>
<td>-3.780</td>
<td>-2.700</td>
<td>0.140</td>
</tr>
</tbody>
</table>

Table 4.A.1: 1% Growth Shock to Productivity
4.A.7 Graphs

Figure 4.A.9: Real Exchange Rate Response to 1% p.a. Productivity Growth Shock
Figure 4.A.10: Comparing Historical Real Exchange Rate with Pseudo Real Exchange Rate Series
Chapter 5

Estimation and Bootstrapping

5.1 Introduction

According to Minford and Webb (2004) large macro models are rarely estimated by full information simultaneous estimators for primarily two reasons. First, the burden of computation is large and intractable — the model must be solved across each of the sample data points for the indefinite future.\(^1\) Second, the calculation of confidence intervals around such a maximum likelihood estimate requires assumptions about the shape of the joint parameter distributions if it is to be done analytically.\(^2\) In this chapter we use a Full Information Maximum Likelihood (FIML) estimator to estimate our Dynamic Stochastic General Equilibrium (DSGE) model outlined in Chapter 2 using quarterly UK data. The FIML algorithm solves the model repeatedly in full for all sample periods for sets of parameters, choosing the likelihood maximising set. The justification for using FIML is that it reduces the bias on the estimates in comparison to both single equation and multi-equation simultaneous system estimations.

This chapter is organised as follows. Section 5.2 discusses in brief alternative techniques to estimate models. In Section 5.3 we first explain the concept of bootstrapping in Section 5.3.1 and then in Section 5.3.2 we go on to explain bootstrapping in the context of simultaneous ...

---

\(^1\) Creating the set of expectations and the currently-predicted endogenous variables and this must be done for all feasible parameter values, with those generating the highest likelihood being chosen.

\(^2\) The assumption of rational expectations implies a large number of cross-equation restrictions which makes this computationally burdensome even in the case of linear models where analytical expressions for the joint distribution are available. For non-linear models there are no analytical expressions available for the limited size samples such as ours. See Davidson (1994) for discussion.
equation systems. Section 5.4 explains the procedure used, discussing in detail the maximisation of the likelihood in Section 5.4.1. Section 5.5 deals with issues relating to convergence of bootstraps and non-cointegration. In Section 5.6 we present our results. Section 5.7 summarises the main conclusions.

5.2 Estimation

There are basically two approaches to estimate structural equations, namely, single-equation methods, also known as limited information methods and system methods, also known as full information methods. In single equation methods, each equation in the system is estimated individually, taking into account any restrictions placed on that equation such as exclusion of some variables, without worrying about the restrictions on the other equations in the system, hence the name limited information methods. In contrast in the system methods, all the equations in the model are estimated simultaneously, taking account of all restrictions on such equations by the omission or absence of some variables,\(^3\) hence the name full information methods.

Many economic relationships are of the single-equation type. In such models there is an implicit assumption that the cause and effect relationship, if any, between the variables under discussion is uni-directional: the explanatory variables are the cause and the dependent variable is the effect. However, there is no dearth of situations where there is a two-way flow of influence among economic variables; that is, one economic variable affects another economic variable(s) and, in turn affected by it (them). For example in our model interest rates are determined to a large extent by consumption. However, interest rates also have an impact on consumption demand. This leads us to consider simultaneous-equation models, models in which there is more than one regression equation, one for each interdependent variable.

To preserve the spirit of our simultaneous-equation model, we should use full information maximum likelihood method (FIML).\(^4\) However, such methods are not commonly used for a variety of reasons. First, the computational burden is enormous, even in these days of high-speed computers, not to mention the cost. Second, the systems methods, such as FIML, lead to solutions that are highly non-linear in the parameters and are therefore difficult to determine.

\(^3\)For identification such restrictions are essential.

\(^4\)For a simple discussion of this method, see Christ (1966).
Third, the difficulty with all system estimation techniques is that individual parameter estimates — by construction — are sensitive to the specification of the entire system of equations. A serious specification error in one equation can substantially affect the parameter estimates in all equations of the model. Thus, in general, the decision to use system estimation involves a trade-off between the gain in efficiency and the potential costs of specification error. According to Klein (1974) “Single equation methods, in the context of a simultaneous system, may be less sensitive to specification error in the sense that those parts of the system that are correctly specified may not be affected appreciably by errors in specification in another part.” In practice, therefore single-equation methods are often used.

The three most commonly used single-equation methods are ordinary least squares (OLS), indirect least squares (ILS) and two-stage least squares (2SLS). Although OLS in general is inappropriate in a simultaneous equation set-up, it can be applied to recursive models — where there is a definite but unidirectional cause-and-effect relationship among endogenous variables.\textsuperscript{5} The method of ILS is suitable for just or exactly identified equations and 2SLS is especially designed for over-identified equations. Both ILS and 2SLS estimates are consistent\textsuperscript{6} but the estimates may not satisfy small-sample properties such as unbiasedness and minimum variance.\textsuperscript{7}

The model that we are dealing with — dynamic stochastic general equilibrium model — whose parameters are tastes and technology and so attempts to avoid Lucas’ (1976) Critique, has sufficient restrictions and simultaneity implying that FIML estimation, which exploits the parameters interdependence in explaining the data, should undoubtedly yield gains in efficiency over single-equation methods such as 2SLS discussed above. Here we use an FIML estimator developed by the Liverpool Research Group to estimate our model using quarterly UK data. The FIML algorithm solves the model repeatedly in full for all sample periods for sets of parameters, choosing the likelihood maximising sets. We have used bootstrapping to address some practical concerns such as FIML bias and its adjustment, construction of confidence limits in the presence of non-normal errors and the vulnerability of our parameter estimates in case of non-cointegration.

\textsuperscript{5}Many argue that even though OLS is inappropriate in case of simultaneous models it can still be used — if only — as a benchmark for comparison.

\textsuperscript{6}“That is, as the sample size increases indefinitely, the estimates converge to their true population values.

\textsuperscript{7}In small samples OLS is also biased but the estimator has the virtue of being minimum variance.
5.3 Bootstrapping

5.3.1 Concept

Bootstrapping is a method of testing the reliability of the dataset. It is the creation of pseudo-replicate datasets by resampling. Bootstrapping allows one to assess whether the distribution of characters has been influenced by stochastic effects. In statistics bootstrapping is a method for estimating the sampling distribution of an estimator by resampling with replacement from the original sample. Bootstrap technique was invented by Bradley Efron (1979, 1981, 1982) and further developed by Efron and Tibshirani (1993). ‘Bootstrap’ means that one available sample gives rise to many others by resampling. While the original objective of cross-validation is to verify replicability of results and that of jackknife is to detect outliers, Efron (1981, 1982) developed bootstrap with inferential purposes.

In bootstrap, the original sample can be duplicated as many times as the computing resources allow, and then this expanded sample is treated as a virtual population. Then samples are drawn from this population to verify the estimators. Obviously the ‘source’ for resampling in bootstrap could be much larger than that in the other two. Unlike cross-validation and jackknife, the bootstrap employs sampling with replacement. Indeed, sampling with replacement in a bootstrap is more accurate than sampling without replacement in terms of simulating chance. Further, in cross-validation and jackknife, the n in the sub-sample is smaller than that in the original sample, but in bootstrap every resample has the same number of observations as the original sample. Thus, the bootstrap method has the advantage of modelling the impacts of

---

8Classical parametric tests compare observed statistics to theoretical sampling distributions. Resampling is a revolutionary methodology because it departs from theoretical distributions. Rather, the inference is based upon repeated sampling within the same sample, and that is why this school is called resampling.

9Bootstrapping alludes to a German legend about a Baron Münchhausen, who was able to lift himself out of a swamp by pulling himself up by his own hair. In later versions he was using his own boot straps to pull himself out of the sea which gave rise to the term bootstrapping.

10In cross-validation, a sample is randomly divided into two or more subsets and test results are validated by comparing across sub-samples. Readers who are familiar with the classical test theory may find some degree of resemblance between split-half reliability and simple cross-validation. Indeed, the goal of both approaches, though in different contexts, is to find out whether the result is replicable or just a matter of random fluctuations.

11Jackknife is a step beyond cross-validation. In Jackknife, the same test is repeated by leaving one subject out each time. Thus, this technique is also called leave one out. This procedure is especially useful when the dispersion of the distribution is wide or extreme scores are present in the data set.

12The principles of cross-validation, jackknife, and bootstrap are very similar, but bootstrap overshadows the others for it is a more thorough procedure in the sense that it draws many more sub-samples than the others. Mooney and Duval (1993) suggested that jackknife is of largely historical interest today.

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the actual sample size.\textsuperscript{13}

### 5.3.2 Bootstrapping in Simultaneous Models — A Brief Discussion

Economists are used to parametric tests which assume they know the distribution of the statistics being tested.\textsuperscript{14} In a linear model parameter estimates have the dimension of a mean; and for a sample with reasonable size normality of the parameter distribution is an attractive and persuasive assumption. However, this assumption does not apply in non-linear simultaneous models especially with the sample sizes that we are dealing with. Even if we assume that the multivariate error is normally distributed, parameter estimates generated are likely to be biased. The bias would not disappear even if the sample size tends to infinity.

Hence to reach an assessment for a non-linear model Monte Carlo methods\textsuperscript{15} must be used, given an estimate of the error distribution. However, in traditional Monte Carlo studies, errors are drawn from a theoretical distribution such as the normal distribution. In many cases, the asymptotic results are not reliable in small samples because the errors are not believed to be distributed normally. In such cases, traditional Monte Carlo studies are of questionable value. To deal with this problem, one must find a way of drawing errors more representative of the unknown actual-error distribution. Bootstrapping is a method for doing this. It is a computationally intensive, non-parametric technique for making inference about the population characteristic. Since we have no \textit{a priori} knowledge of the error distribution, the natural starting point is an estimate of them from the sample itself. Clearly circularity is involved since we have to choose an estimator in the first place to obtain the sample of the error distributions. The 'bootstrapping' method takes its name from this circularity.

The method of simultaneous estimation that is best suited for our model given the small

\textsuperscript{13}Through simulations Fan and Wang (1996) found that the bootstrap technique provides less biased and more consistent results than the jackknife method does.

\textsuperscript{14}This assumption is bolstered by the central limit theorem — according to which the mean of \( n \) random variables tends asymptotically to be distributed normally with the standard deviation equal to that of the random variable/the square root of \( n \).

\textsuperscript{15}In Monte Carlo simulation researchers make up data and draw conclusions based on many possible scenarios (Lunneborg (2000)). The name Monte Carlo comes from an analogy to the gambling houses on the French Riviera. Many years ago, some gamblers studied how they could maximise their chances of winning by using simulations to check the probability of occurrence for each possible case. Today Monte Carlo simulations are widely used by statisticians to study the behaviours of different statistical procedures. For example: Can the test still correctly reject the null hypothesis when the sample has unequal variances? Does the test have adequate statistical power?
sample is not known in advance. However following Minford and Webb (2004) we use the simultaneous estimator FIML designed for a multivariate normal error distribution. Given that the errors are approximately multivariate normal, the method used is an approximation to maximum likelihood.\textsuperscript{16} We start our bootstrapping procedure with our sample residuals obtained by this FIML.\textsuperscript{17} We use the distribution of the sample residuals as an estimate of their true distribution.\textsuperscript{18} The bootstrap can then be regarded as a Monte Carlo study to find the true distribution of the parameter estimates on the assumption that the sample residual distribution and the parameter estimates are both the true ones.\textsuperscript{19} We can then use the bootstrap results to generate the confidence limits from the parameter distributions yielded around the ‘true’ parameters. In general, to produce a confidence interval one finds the standard error of estimate, multiplies it by a suitable critical value taken from either the normal distribution or students’ $t$ distribution tables, and then adds and subtracts the results from the estimate. This procedure can potentially be inaccurate when the errors are not normally distributed. By bootstrapping the entire procedure, the actual sampling distribution can be estimated, allowing an appropriate confidence interval to be produced.

### 5.4 Bootstrap Procedure

In this section we describe a method for FIML estimation of simultaneous equation models developed by the Liverpool Research Group over several years.\textsuperscript{20} At the centre of this method lies a solution programme used now for over two decades to solve the Liverpool UK and multilateral world models.\textsuperscript{21} This programme has proved capable of solving large non-linear macro models in a wide variety of forecast situations — with forecasting being done quarterly for the

---

\textsuperscript{16}In maximising the likelihood of the errors we bypass the assumption of model linearity which is usual in implementation when the likelihood is solved out in terms of the model parameters. Nevertheless, should the error be non-normal we should observe the resulting problems in the parameter distributions emerging from the bootstrap. We can thus allow for these in our confidence limits based on the bootstrapped distributions.

\textsuperscript{17}It should be noted that when we refer to FIML we mean the procedure used in the context of multivariate normal errors, whether or not this context truly prevails.

\textsuperscript{18}Depending on the size of the sample, this estimate will have a coverage error — it will cover the full distribution imperfectly. However, the extent of this follows from ordinary sampling theory and can be compensated for by increasing the number of bootstraps.

\textsuperscript{19}In usual Monte Carlo studies purely theoretical error distributions are treated as true.

\textsuperscript{20}The discussion in this section follows very closely the exposition in Minford and Webb (2004).

\textsuperscript{21}See Minford, Marwaha, Matthews and Sprague (1984) and Minford, Agenor and Nowell (1986).
first decade and monthly since. The programme has a wide variety of traps built into it to prevent the solution algorithm stopping its search;\textsuperscript{22} in effect these traps restart the algorithm. The solution establishes convergence in the now-standard manner, as exemplified in Fair and Taylor (1983).

For FIML estimation along the lines described above we use a new hill-climbing algorithm to search over the parameter space. Starting from a set of parameters — calibrated or estimated by single equation methods, the algorithm varies each parameter in turn by plus or minus some percentage of its initial value, this reference value being held constant. Whichever parameter movement generates the biggest improvement in the likelihood is adopted. The operation is then repeated for the newly altered parameter set. The search process begins with + or - 10% variations in parameter values; once any improvement with such a step variation is exhausted, it tries 5% variations; then 2% then 1%, after which it stops.

As with all algorithms there is no guarantee that it will find the global maximum. However, the algorithm will climb any slope it finds itself on locally. The algorithm checks on this aspect in standard ways such as allowing the initial parameter set to be varied randomly and at random points in the search process making random perturbations in the set of parameters reached. At the end of the search for the global maximum, the algorithm restarts the search from a different, randomly chosen initial parameter values and checks that it reaches the same maximum again.\textsuperscript{23} In the final stage of our procedure we use the bootstrap to compute confidence intervals.

One of the problems in our bootstrap is the presence of autocorrelations in certain residuals. This implies that the errors can not be regarded as random and are therefore unsuitable for resampling in any order. The errors used in this stage have all been purged off autocorrelation at the original FIML estimation stage by the inclusion of appropriate autocorrelation parameters in the model itself. This is done by generating pseudo-samples from the vectors of estimated errors, with replacement on a model consisting of the estimated parameters; these are used to produce new sets of FIML estimates.

\textsuperscript{22}If for example it finds itself dividing by zero or taking the logarithm of a negative number.

\textsuperscript{23}For large non-linear macro models there are some general checks on acceptable parameter values available from the vast simulation and forecasting experience. The behaviour of large models can sharply deteriorate when parameters move far away from such acceptable values.
5.4.1 Maximising the Likelihood

Our model is non-linear and so we represent it generally as

\[
y_t = f(y_t, (L)y_t, x_t, (L)x_t, y_t^\pi, (L)y_t^\pi; \pi) + u_t \quad t = 1, 2, \ldots, n \tag{5.1}
\]
\[
x_t = A(L)x_t + \epsilon_t \tag{5.2}
\]

where \(y_t\) is the vector of \(G\) endogenous variables and \(x_t\) is a vector of exogenous variables assumed to follow a linear univariate time-series process. \(L\) is the lag operator, \(y_t^\pi\) is the vector of rational expectations projections of the model based on information at \(t\), \(\pi\) is the vector of parameters and \(u_t\) is the vector of structural equation errors.\textsuperscript{24} We can also focus on the reduced form error vector, \(u_t\). This is the error that is created when in (5.1) \(y_t\) on the right hand side is replaced by its model prediction, \(\hat{y}_t\).

\[
y_t = f(\hat{y}_t, (L)y_t, x_t, (L)x_t, y_t^\pi, (L)y_t^\pi; \pi) + v_t \quad t = 1, 2, \ldots, n \tag{5.3}
\]

Hence here we see how the model will solve for \(y_{it}\), given its solution for the other \(y_{jt}\). It clearly follows that \(v_t\) is a combination of \(u_t\).\textsuperscript{25} However, in the analysis below we focus on (5.1) as there is sufficient simultaneity built into it, given the importance of expectations in our model.\textsuperscript{26}

We assume that the errors in either case follow a multivariate normal distribution with expectation of zero and variance-covariance matrix of \(\Sigma\); and that estimation has eliminated any autocorrelation. This implies that the likelihood is:\textsuperscript{27}

\[
p(u_1, u_2, \ldots, u_n) = (2\pi)^{-nG/2} (\text{det } \Sigma)^{-n/2} \exp \left( -\frac{1}{2} \sum_{t=1}^{n} u_t' \Sigma^{-1} u_t \right) \tag{5.4}
\]

We evaluate the likelihood (5.1) directly. That is to say, for each trial set of parameters \(\pi\)

\textsuperscript{24}The errors when the endogenous variables on the right hand side are at their true values.
\textsuperscript{25}It should be noted that both \(u_t\) and \(v_t\) are observed variables conditional on the parameter set \(\pi\). Thus we can interpret the maximising process as searching for the highest likelihood that these errors are observed.
\textsuperscript{26}In (5.1) every parameter's effect on the fit of all the equations via its affect on expectations is also taken account of.
\textsuperscript{27}For case (5.1) and analogously for (5.3).
we obtain an implied set of $u_t$ for which (5.4) yields the likelihood. This is maximised by the hill-climbing algorithm described earlier. The bootstrap procedure takes the $u_t$ generated by the maximum likelihood parameter set, $\hat{\pi}$. It then resamples these vectors\(^{28}\) with replacement and inserts the sample with $\hat{\pi}$ into equation (5.1) to generate the bootstrap samples of $y_t$. The exogenous variables are held constant across bootstraps. Then the above estimation procedure is repeated on each bootstrap sample.

The bootstrap procedure can be explained in the following series of steps;

For a simple linear model with 't' observations

$$Y_t = \beta X_t + \varepsilon_t$$  \hspace{1cm} (5.5)

the residual resampling bootstrap procedure to compute standard errors for $\hat{\beta}$ is

1. Estimate $\hat{\beta}$ by OLS or 2SLS.

2. Calculate $\hat{Y}$ and the residual $\hat{\varepsilon}$ and store them.

3. Rescale the residuals as described below.

4. For the 'B' bootstrap samples, do the following:

   (a) Draw a sample of size 't' with replacement from the set of rescaled residuals $\hat{\varepsilon}^b$.

   (b) Construct new dependent variables $Y_t^b$ with the formula

   $$Y_t^b = \hat{Y}_t + \hat{\varepsilon}^b$$

   That is, for each element of $\hat{Y}$, draw an adjusted residual randomly with replacement and add it to generate a new 'Y' variable.

   (c) Regress $Y_t^b$ on $X_t$ in (5.5) and save the estimated coefficients.

5. Repeat this procedure several thousand times.

\(^{28}\)So preserving any cotemporaneous correlations.
6. Compute a 95% confidence interval i.e., compute the standard error of $\hat{\beta}$ from the sample of bootstrapped $\hat{\beta}$'s with the standard formula.

$$\hat{\sigma}_\beta = \sqrt{\frac{1}{B - 1} \sum_{i=1}^{B} \left( \hat{\beta}^i - \hat{\beta}(\cdot) \right)^2}$$

where

$$\hat{\beta}(\cdot) = \frac{1}{B} \sum_{i=1}^{B} \hat{\beta}^i$$

As already noted, it is not appropriate merely to use the residuals. When $\varepsilon_t$ is i.i.d., the OLS residuals of equation (5.5) has variance

$$E(\hat{\varepsilon}_t^2) = (1 - h_t) \sigma^2$$

where $'h_t'$ is defined as

$$h_t = X_t \left(X'X\right)^{-1} X'_t$$

Instead, one can rescale the residuals in this manner:

$$\tilde{\varepsilon}_t = \frac{\hat{\varepsilon}_t}{\sqrt{1 - h_t}} - \frac{1}{N} \sum_{s=1}^{N} \frac{\hat{\varepsilon}_s}{\sqrt{1 - h_s}}$$

The rescaled residuals are sometimes called the standardised residuals, and $'h_t'$ is the $t^{th}$ diagonal element of the hat matrix. The second term in our rescaled residual is there to ensure that the mean of the resulting residual remains zero.

5.5 Convergence, Unit Roots and Non-Cointegration

5.5.1 Convergence of Bootstrap

How many bootstraps should be used? The number of bootstraps to be used is regarded as a matter of experience from previous studies, which indicate how the margin of error around
estimates of the parameter distributions is reduced by extra bootstraps. The answer boils down to whether an additional bootstrap provides any extra information on the parameter distributions. Therefore, we compute measures of the currently estimated distributions from the bootstraps up to a point and then check whether these measures alter with extra bootstraps. Once they fail to change we conclude that further bootstraps add no useful information. As four aggregate measures the programme uses the first four moments of all the parameters, viewed as a group. As a further check we consider the upper and lower 95% confidence limit for every parameter separately. When none of these show any change – within some specified low tolerance limit – as the number of bootstraps increases we declare convergence.

What we find broadly speaking from these convergence tests is that the extra information starts to fall off at a number of bootstraps in the low hundreds rather than the thousands generally thought of as necessary — the actual number is only 175.

5.5.2 Unit roots and Non-Cointegration

The statistics literature that deals with cointegration is particularly concerned with ‘spurious’ regression. As a result many researchers now check for cointegration – stationarity of errors – between any $I(1)$ variables they wish to include in a regression. If it exists, they are protected from spurious regression.

Given the concerns that relate to non-cointegrated variables, we would ideally like all our long-run relationships between $I(1)$ variables postulated in the model to exhibit cointegration between them. However, we can not establish such cointegration beyond reasonable doubt with the sample at our disposal. Hence it can not be said with confidence that the error terms in these relationships are stationary within our sample. Nevertheless we have faith in our long-run relationships as they have strong theoretical underpinnings. So we assume that there are one or more omitted variables and model the residual as a proxy for these, usually with some autoregressive process, with the possibility open for a unit root in this. Typically we estimate stationary processes but with high roots.

However suppose that these errors, or some of them are non-stationary\textsuperscript{30} then an important


\textsuperscript{30}Even though we have estimated them as stationary.
estimation question arises — are the estimates endangered by non-stationarity of errors? Does it seriously affect the estimates or the distribution of the parameters of interest? In our model we found no evidence of non-stationarity in the equation errors — the autocorrelation roots found are comfortably less than unity. However, we would like to know whether the presence of unit roots in the errors will be detected reliably by our autocorrelation estimates and whether when this is done the main structural parameters are still estimated well.

To answer this question, we do a Monte Carlo bootstrap on our model assuming that the true parameters are those estimated by the FIML. Then we assumed that all the equation errors\textsuperscript{31} we are using in our bootstrap, instead of being random variables as estimated, are random walks. This would simulate the absence of cointegration — each equation now has an $I(1)$ process which in each sample will drive the dependent variable cumulatively away from the set of right-hand side variables.

The results are rather re-assuring. We find that if the errors are assumed to be non-stationary neither the estimates of the behavioural parameters nor their standard errors change. What this suggests is that if there is non-stationarity in the errors it should be absorbed in the coefficients of autocorrelation and should not unduly affect the main parameter estimates or their distributions. As Minford and Webb (2004) state "It is as if the model’s structural restrictions ‘anchor’ the behavioural parameters and the total error’s autocorrelation is often only modestly affected by the accumulation of what was before the purely random residual.” This is a feature of a simultaneous model where the cross-equation restrictions are created by a rational expectations model and which when exploited by a systems estimator produce a high degree of stability in estimation.

5.6 Results

We now present the FIML estimates of the model parameters based on 175 bootstraps. Though the number of bootstraps is low as compared with what is usually done in case of univariate studies, it is sufficient for the distributions to converge in fully-specified models. Broadly speaking what we find from these convergence tests is that the extra information starts to fall

\textsuperscript{31}The residual errors after allowing for the estimated autocorrelation parameters.
off at a number of bootstraps in the low hundreds rather than the thousands generally thought of as necessary; the actual number is only 175. The extra bootstraps tend to gravitate around sets of potential parameters for which the model solves sensibly; once these have been located the same ones crop up again and again.

It can be seen from Table 5.A.1 in Appendix 5.A that:

1. The system estimates are reasonably close to the limited information starting point. In this sense there is little obvious gain from system estimation.

2. The standard errors of the system estimates are generally lower, often by a large margin, than those of the limited information estimates. We also present confidence limits using the parameter distributions as related to the FIML estimate around which they are generated. In Table 5.A.1, L95 and U95 are the lower and upper 95% confidence limits, i.e. 2.5% of the distribution lies above 95% and below 95%. These are the correct measures of dispersion for hypothesis testing because the distributions are not generally normal or t-shaped and hence standard errors are not useful. Bias-adjustment is important because FIML is generally biased.\(^{32}\) So the estimates to use in our simulation are the bias-adjusted ones.\(^{33}\) The confidence limits too are considerably closer than those from the single equation t-limits. This is rather re-assuring as at the outset we expected that imposing the full constraints of the model would give better defined estimates. We think that because of the model's non-linearity, outside a fairly narrow region around our central parameter estimates the model solves poorly, that is for variable values well outside normal range. Hence, the fit is poor and the probability of such parameter estimates low — implying a narrow probability distribution around the central estimate.

3. The mean estimates of the bootstraps are mostly close to the system estimates but with some conspicuous exceptions. As explained earlier we use the gap between the FIML estimate and the bootstrap mean as an estimate of bias. One can see from Table 5.A.1 evidence of bias in certain parameters, though bias is far from general. Appendix 5.A.2

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\(^{32}\)One of its problems owing to its using every scrap of information and so reacting to outliers.

\(^{33}\)The bias is the gap between the FIML and the bootstrap mean; the point is that the bootstraps are a Monte Carlo simulation of what estimates you get in repeated samples assuming the FIML estimates are the true ones. So we see what distribution of estimates comes from the method.
shows the distribution of the parameters. These are highly non-normal in virtually all cases.

Finally, since FIML estimates are biased it may be useful to use the bootstrap procedure to correct for bias. Table 5.A.2 in appendix 5.A reports bias-adjusted FIML estimates and their confidence limits. In the present context the bias-correction procedure using the bootstrap is as follows.

1. Compute the mean value for each coefficient, and let $\tilde{\beta}$ denote the vector of the mean values. Let $\theta = \tilde{\beta} - \hat{\beta}$ denote the vector of estimated biases, where $\hat{\beta}$ is the estimated coefficient vector.

2. After the coefficient vector $\hat{\beta}$ is estimated by FIML, it has $\theta$ subtracted from it to correct for bias to give $\tilde{\beta}$ the vector FIML bias-adjusted coefficients.

The confidence limits for the bias-corrected coefficients $\tilde{\beta}$ is calculated as follows:

1. Compute $\theta_1 = (U95 \text{ or } L95) - \tilde{\beta}$ the limit band for the bias-adjusted FIML parameters.

2. After computing $\theta_1$ add it to $\tilde{\beta}$ to calculate the $U95 \text{ or } L95$ limits for the bias-adjusted coefficients.

Tables 5.A.3–5.A.10 in the appendix reports the sensitivity of our simulation results to the bias correction.\(^{34}\) Shocks to money, productivity, unemployment benefits, labour income tax, consumption tax, foreign interest rate and foreign prices are simulated with bias-adjusted FIML estimates. The bias-corrected results are fairly similar (qualitatively) to the calibrated ones, which suggests that bias is not a major problem in the model.

5.7 Conclusions

In this chapter we investigate the properties of a procedure for FIML estimation of a non-linear DSGE model with rational expectations. The procedure involves solving the model for the

\(^{34}\)In other words we replace our calibrated parameters used for simulation in Chapter 3 with bias-adjusted FIML estimates to check for sensitivity of our simulation results.
sample period with parameters chosen by a hill-climbing-cum-grid algorithm. The algorithm checks the resulting likelihood within a grid ‘along the axes’ and then moves the parameters along the axis that improves the likelihood most. Having obtained the highest-likelihood set a bootstrap is performed to obtain confidence limits to ascertain the extent of bias. Bootstraps continue until there is convergence, which we interpret as no change in the mean and confidence limits of the parameters. Given the obsession in applied work on cointegration we even test the robustness of our FIML estimates to the deliberate insertion of unit roots into the error processes but maintaining their estimated variances. We found, little sensitivity, if any, of the structural parameter distribution. Overall we find the results to be fairly encouraging to the application of FIML in the context of DSGE models.
5.A Appendix

5.A.1 FIML Estimates of Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Single Equation</th>
<th>FIML Estimates</th>
<th>Bootstrap Mean</th>
<th>L95</th>
<th>U95</th>
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<tbody>
<tr>
<td>$\alpha$</td>
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<td>0.664205</td>
<td>0.595</td>
<td>0.716667</td>
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<td>$\beta$</td>
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<td>0.9306</td>
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<td>0.013339</td>
<td>0.011072</td>
<td>0.017708</td>
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<td>$\gamma^G$</td>
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<td>0.05</td>
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<td>$\rho_2$</td>
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<td>1</td>
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<td>-0.52973</td>
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<td>-0.50</td>
<td>-0.50</td>
<td>-0.50</td>
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Table 5.A.1: Full Information Maximum Likelihood Estimates of Model Parameters
5.2 Parameter Distribution

![Alpha Distribution Graph](image)

Figure 5.A.1: $\alpha$ Distribution
Figure 5.A.2: $\beta$ Distribution

Figure 5.A.3: $\delta$ Distribution

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Figure 5.A.4: \( \rho_0 \) Distribution

Figure 5.A.5: \( \theta_0 \) Distribution

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Figure 5.A.6: $\gamma^G$ Distribution

Figure 5.A.7: $\rho_2$ Distribution
Figure 5.A.8: ω Distribution

Figure 5.A.9: ρ Distribution
Figure 5.A.10: $\omega^f$ Distribution

Figure 5.A.11: $\rho^f$ Distribution
Figure 5.A.12: $\sigma$ Distribution

Figure 5.A.13: $\sigma^f$ Distribution
5.A.3 FIML Bias Corrected Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Single Equation</th>
<th>FIML Estimates</th>
<th>Bias Adjusted FIML</th>
<th>L95</th>
<th>U95</th>
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</thead>
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Table 5.A.2: Full Information Maximum Likelihood Bias Adjusted Estimates for Model Parameters

5.A.4 Simulations with FIML Bias Corrected Estimates

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<tr>
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<th>Quarter 1</th>
<th>Year 1</th>
<th>Year 3</th>
<th>Long Run</th>
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<tbody>
<tr>
<td>Real Interest Rate</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
<td>Price Level</td>
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<td>0.000</td>
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<tr>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
<td>Capital Stock</td>
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</tr>
<tr>
<td>Consumption</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Investment</td>
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<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Real Wage</td>
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<td>0.000</td>
</tr>
<tr>
<td>Exports</td>
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Table 5.A.3: 5% Permanent Shock to Money Supply
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<th>Year 3</th>
<th>Long Run</th>
</tr>
</thead>
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Table 5.A.4: 1% p.a. Growth Shock to Productivity

<table>
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<th>Year 3</th>
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<td>0.100</td>
<td>0.050</td>
<td>2.960</td>
</tr>
<tr>
<td>Nominal Exchange Rate</td>
<td>-1.170</td>
<td>-1.560</td>
<td>-2.290</td>
<td>-7.280</td>
</tr>
<tr>
<td>Real Exchange Rate</td>
<td>-2.790</td>
<td>-2.110</td>
<td>-0.450</td>
<td>0.520</td>
</tr>
</tbody>
</table>

Table 5.A.5: 5% Permanent Shock to Productivity

<table>
<thead>
<tr>
<th></th>
<th>Quarter 1</th>
<th>Year 1</th>
<th>Year 3</th>
<th>Long Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Interest Rate</td>
<td>-0.020</td>
<td>-0.020</td>
<td>0.000</td>
<td>0.010</td>
</tr>
<tr>
<td>Price Level</td>
<td>0.073</td>
<td>0.071</td>
<td>0.064</td>
<td>0.066</td>
</tr>
<tr>
<td>Output</td>
<td>-0.338</td>
<td>-0.328</td>
<td>-0.192</td>
<td>-0.166</td>
</tr>
<tr>
<td>Labour Supply</td>
<td>-0.453</td>
<td>-0.415</td>
<td>-0.191</td>
<td>-0.086</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>-0.021</td>
<td>-0.082</td>
<td>-0.181</td>
<td>-0.371</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.055</td>
<td>0.053</td>
<td>0.000</td>
<td>-0.023</td>
</tr>
<tr>
<td>Investment</td>
<td>-1.705</td>
<td>-1.634</td>
<td>-0.670</td>
<td>-0.449</td>
</tr>
<tr>
<td>Real Wage</td>
<td>0.110</td>
<td>0.080</td>
<td>0.000</td>
<td>-0.100</td>
</tr>
<tr>
<td>Exports</td>
<td>0.010</td>
<td>0.010</td>
<td>0.000</td>
<td>-0.010</td>
</tr>
<tr>
<td>Imports</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Nominal Exchange Rate</td>
<td>0.100</td>
<td>0.100</td>
<td>0.060</td>
<td>0.060</td>
</tr>
<tr>
<td>Real Exchange Rate</td>
<td>0.020</td>
<td>0.020</td>
<td>0.000</td>
<td>-0.010</td>
</tr>
</tbody>
</table>

Table 5.A.6: 5% Point Permanent Shock to Unemployment Benefits
<table>
<thead>
<tr>
<th></th>
<th>Quarter 1</th>
<th>Year 1</th>
<th>Year 3</th>
<th>Long Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Interest Rate</td>
<td>-0.090</td>
<td>-0.080</td>
<td>-0.010</td>
<td>0.030</td>
</tr>
<tr>
<td>Price Level</td>
<td>-0.049</td>
<td>0.047</td>
<td>0.287</td>
<td>0.219</td>
</tr>
<tr>
<td>Output</td>
<td>-1.284</td>
<td>-1.335</td>
<td>-1.038</td>
<td>-0.788</td>
</tr>
<tr>
<td>Labour Supply</td>
<td>-1.704</td>
<td>-1.649</td>
<td>-1.065</td>
<td>-0.614</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>-0.114</td>
<td>-0.438</td>
<td>-0.966</td>
<td>-1.269</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.673</td>
<td>0.562</td>
<td>0.046</td>
<td>-0.057</td>
</tr>
<tr>
<td>Investment</td>
<td>-9.072</td>
<td>-8.431</td>
<td>-3.816</td>
<td>0.245</td>
</tr>
<tr>
<td>Real Wage</td>
<td>0.400</td>
<td>0.310</td>
<td>0.030</td>
<td>-0.240</td>
</tr>
<tr>
<td>Exports</td>
<td>0.150</td>
<td>0.120</td>
<td>0.000</td>
<td>-0.340</td>
</tr>
<tr>
<td>Imports</td>
<td>-0.010</td>
<td>0.000</td>
<td>0.010</td>
<td>0.380</td>
</tr>
<tr>
<td>Nominal Exchange Rate</td>
<td>0.290</td>
<td>0.310</td>
<td>0.300</td>
<td>-0.150</td>
</tr>
<tr>
<td>Real Exchange Rate</td>
<td>0.330</td>
<td>0.260</td>
<td>0.000</td>
<td>-0.320</td>
</tr>
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</table>

Table 5.A.7: 5% Point Permanent Shock to Labour Income Tax

<table>
<thead>
<tr>
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<th>Quarter 1</th>
<th>Year 1</th>
<th>Year 3</th>
<th>Long Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Interest Rate</td>
<td>-0.040</td>
<td>-0.040</td>
<td>-0.010</td>
<td>0.020</td>
</tr>
<tr>
<td>Price Level</td>
<td>-3.127</td>
<td>-3.142</td>
<td>-3.204</td>
<td>-3.198</td>
</tr>
<tr>
<td>Output</td>
<td>-0.597</td>
<td>-0.613</td>
<td>-0.484</td>
<td>-0.497</td>
</tr>
<tr>
<td>Labour Supply</td>
<td>-0.798</td>
<td>-0.783</td>
<td>-0.516</td>
<td>-0.345</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>-0.041</td>
<td>-0.163</td>
<td>-0.400</td>
<td>-0.913</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.127</td>
<td>0.140</td>
<td>0.047</td>
<td>-0.046</td>
</tr>
<tr>
<td>Investment</td>
<td>-3.274</td>
<td>-3.333</td>
<td>-1.857</td>
<td>-0.817</td>
</tr>
<tr>
<td>Real Wage</td>
<td>0.190</td>
<td>0.150</td>
<td>0.030</td>
<td>-0.210</td>
</tr>
<tr>
<td>Exports</td>
<td>0.030</td>
<td>0.030</td>
<td>0.000</td>
<td>-0.100</td>
</tr>
<tr>
<td>Imports</td>
<td>-0.010</td>
<td>0.000</td>
<td>0.010</td>
<td>0.100</td>
</tr>
<tr>
<td>Nominal Exchange Rate</td>
<td>-3.030</td>
<td>-3.070</td>
<td>-3.310</td>
<td>-3.560</td>
</tr>
<tr>
<td>Real Exchange Rate</td>
<td>0.070</td>
<td>0.070</td>
<td>0.010</td>
<td>-0.100</td>
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Table 5.A.8: 5% Point Permanent Shock to Consumption Tax (VAT)

<table>
<thead>
<tr>
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<th>Year 1</th>
<th>Year 3</th>
<th>Long Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Interest Rate</td>
<td>0.010</td>
<td>0.130</td>
<td>0.380</td>
<td>0.550</td>
</tr>
<tr>
<td>Price Level</td>
<td>2.482</td>
<td>3.425</td>
<td>5.844</td>
<td>1.748</td>
</tr>
<tr>
<td>Output</td>
<td>-0.406</td>
<td>-0.799</td>
<td>-1.512</td>
<td>-3.468</td>
</tr>
<tr>
<td>Labour Supply</td>
<td>-0.333</td>
<td>-0.297</td>
<td>-0.101</td>
<td>-0.248</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>-0.579</td>
<td>-2.178</td>
<td>-5.332</td>
<td>-11.901</td>
</tr>
<tr>
<td>Consumption</td>
<td>-1.219</td>
<td>-2.369</td>
<td>-5.182</td>
<td>-1.127</td>
</tr>
<tr>
<td>Investment</td>
<td>-46.248</td>
<td>-41.569</td>
<td>-25.735</td>
<td>-0.123</td>
</tr>
<tr>
<td>Real Wage</td>
<td>-0.060</td>
<td>-0.470</td>
<td>-1.390</td>
<td>-4.390</td>
</tr>
<tr>
<td>Exports</td>
<td>3.730</td>
<td>3.570</td>
<td>2.930</td>
<td>-1.200</td>
</tr>
<tr>
<td>Imports</td>
<td>-3.470</td>
<td>-3.630</td>
<td>-4.340</td>
<td>0.870</td>
</tr>
<tr>
<td>Nominal Exchange Rate</td>
<td>10.830</td>
<td>11.230</td>
<td>12.610</td>
<td>0.510</td>
</tr>
<tr>
<td>Real Exchange Rate</td>
<td>8.150</td>
<td>7.620</td>
<td>6.130</td>
<td>-1.160</td>
</tr>
</tbody>
</table>

Table 5.A.9: 1% Point Permanent Shock to Foreign Interest Rate

176
<table>
<thead>
<tr>
<th></th>
<th>Quarter 1</th>
<th>Year 1</th>
<th>Year 3</th>
<th>Long Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Interest Rate</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Price Level</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Output</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Labour Supply</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Investment</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Real Wage</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Exports</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Imports</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Nominal Exchange Rate</td>
<td>-4.730</td>
<td>-4.760</td>
<td>-4.930</td>
<td>-5.130</td>
</tr>
<tr>
<td>Real Exchange Rate</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 5.A.10: 5% Permanent Shock to Foreign Prices
Chapter 6

Inflation Persistence Literature Review

6.1 Introduction

A large body of econometric literature has found that post-war US inflation exhibits very high persistence, approaching that of a random walk process.\(^1\) Given similar findings for other OECD countries, many macroeconomists have concluded that high inflation persistence is a \textit{‘stylised fact’} and have proposed varied microeconomic interpretations.\(^2\) However, an alternative view is that the degree of inflation persistence is not an inherent structural characteristic of industrial economies, but in fact a function of the stability and transparency of monetary policy regime in place.\(^3\) The objective of this chapter is to familiarise the reader with the literature discussing inflation persistence: definition, importance, causes, estimation and policy implications.

The chapter is organised as follows. In Section 6.2 we introduce the reader to the various definitions of inflation persistence used in the literature and also delineate the importance of the study of inflation persistence. Section 6.3 reviews the empirical estimation of the degree of inflation persistence using both univariate and multivariate approaches. Section 6.4 first discusses the origin of persistence in inflation and then addresses the question if inflation per-

\(^2\)For further discussion see Nelson (1998) and Clarida, Gali and Gertler (1999).
sistence is structural. Section 6.5 discusses the policy implications of inflation persistence and the conclusions are presented in Section 6.6.

6.2 Inflation Persistence: Definitions & Importance

6.2.1 Definitions of Inflation Persistence

Following the typology proposed by Batini (2002) and Batini and Nelson (2002), measures of inflation persistence refer, first, to positive serial correlation in inflation, second, to lags between systematic monetary policy actions and their peak effect on inflation, and third, to lagged responses of inflation to non-systematic policy actions, i.e. policy shocks. As Batini (2002) very nicely puts it "the first type of persistence is a reduced-form property of inflation that manifests simultaneously the underlying pricing process, the conduct of monetary policy and the expectations' formation process of price-setting agents. Changes of any of these three factors will influence the autocorrelation properties of inflation." According to the author the second measure of persistence, i.e. length of delay between policy action and its maximum effect on inflation "...determines the costs of disinflation", while the final measure, i.e. the lagged inflation response to monetary policy shocks, "...is often the only one consulted by economic modellers when validating models vis-à-vis the dynamics of real-world data generating process."

Willis (2003) defines persistence as the "speed with which inflation returns to baseline after a shock." This definition implies that the degree of inflation persistence basically shows the speed with which inflation converges to equilibrium after a shock hits the economy. An estimate of near 1 would correspond to extremely high persistence, while an estimate of 0 would represent no persistence. So when the value of the persistence parameter is high, inflation adjusts slowly, while when the value is low, the speed of adjustment is high. Willis also argues that, other things being equal, less persistence leads to less variability. Lower persistence is associated with faster but smaller swings in inflation over time so, in statistical terms, the overall variability of inflation reduces substantially.

In the words of Kieler (2003) persistence "is the tendency of inflation to be a slow-moving inertial variable with autocorrelations fairly close to one." Given this definition of persistence the authors are able to claim that part of the observed inflation persistence may be due to "shifts
in the nominal anchor”, i.e. to changes in the long-run level of inflation, because “persistence of inflation exceeds persistence of deviations of inflation from the estimated nominal anchor.”\textsuperscript{4} Marques (2004) defines it as the “speed with which inflation converges to equilibrium after a shock.”\textsuperscript{5} The author interprets inflation persistence as the amount of time it takes for the effect of a shock that raises inflation by 1% today to die off. If the speed is low then inflation is highly persistent while if the speed is high then inflation is not very persistent.

6.2.2 Why is measuring Inflation Persistence Important?

The study of inflation and its dynamics is crucial for macroeconomists as inflation has far reaching implications for the economy both in terms of economic efficiency and wealth distribution. At a practical level this is reflected in the mandate of price stability\textsuperscript{6} or the principal aim of keeping inflation low and stable\textsuperscript{7} of many monetary authorities. As emphasised by Clarida, Gali and Gertler (1999), understanding inflation dynamics is crucial for monetary policy. The ability to quantify and model the sluggish response of inflation to changes in monetary conditions is important for monetary policy makers because it helps them to understand how pre-emptive they should be in order to curb inflationary pressures at a minimum cost in terms of output gap variability. Documentation of the persistence properties of inflation is also important for forecasting purposes. Following a shock to the inflation process, the performance of forecasts can rely heavily on the ability of the forecaster to adequately predict the pattern of absorption of that shock. In addition the study of inflation persistence provides a useful input into the analysis of cross-country inflation differentials by helping to discern between structural and shock-induced inflation differentials and provide tools to gauge likely duration of episodes of inflation differentials.

\textsuperscript{4}See Kozicki and Tinsley (2002).

\textsuperscript{5}For similar definitions see, for instance, Andrews and Chen (1994) and Pivetta and Reis (2004).

\textsuperscript{6}Such as the ECB.

\textsuperscript{7}This is the mandate of the Bank of England, which has been very successful in maintaining annual UK retail price inflation at a level close to target since it was granted independence and responsibility of monetary policy in 1997.
6.3 Empirical Estimation of Inflation Persistence

In the literature inflation persistence is discussed under two distinct approaches. One defines and evaluates the persistence of inflation in the context of a univariate time series representation — the univariate approach; while the other uses a structural econometric model that aims at explaining the behaviour of inflation — the multivariate approach. Usually persistence is seen as referring to the duration of shocks hitting the economy.

Under the univariate approach a simple autoregressive model for inflation is assumed and the shocks are measured in the white noise component of the autoregressive process. In this approach the shocks to inflation are not identified in the sense they can not be given an economic interpretation, i.e. these shocks are basically a summary measure of all shocks affecting inflation in any given period. However, the advantage of the univariate approach is that its relative simplicity reduces the risk of specification errors. It also provides a concise easily understood measure of persistence and constitutes a useful first step in gathering information on the persistence properties of inflation series.

The multivariate approach on the other hand attempts to identify the different shocks hitting inflation and thus a shock specific analysis is very much possible. This approach implicitly or explicitly assumes a causal economic relationship between inflation and its determinants and views inflation persistence as referring to the duration of the effects on inflation of the shocks to its determinants.

6.3.1 Univariate Approach

A common practice in empirical research is to estimate univariate autoregressive (AR) time series models and measure persistence as the sum of the estimated AR coefficients. In most of the literature inflation is found to exhibit high to very high persistence over the post WWII period — persistence is found to be close to a random walk process. However, more recently

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\footnote{Such as monetary policy shocks, productivity shocks, external oil price shocks etc.}

\footnote{Usually a Phillips curve or a structural VAR model.}


there is a growing consensus that inflation persistence varies across monetary policy regimes.

One of the most influential papers of recent times is Levin and Piger (2004). The paper estimates a univariate AR model for four different price indices\textsuperscript{12} applying both classical and Bayesian econometric methods to characterise the behaviour of inflation dynamics for twelve industrial economies\textsuperscript{13} allowing for the possibility of a structural break\textsuperscript{14} at an unknown date in the inflation process\textsuperscript{15} for each country. Conditional on the break in intercept, many of the inflation series exhibit relatively low persistence;\textsuperscript{16} the median-unbiased estimate of the sum of AR coefficients is less than 0.7 for all four US inflation measures, and for 25 of the other 44 inflation series. Further, the null hypothesis of a unit root can be rejected at the 95 percent confidence level in nearly all of the cases.\textsuperscript{17} Finally, it considers rolling regressions for the US data, and shows that the results are completely consistent with low persistence and an intercept shift in the early 1990s. The authors' main conclusion is "...that high inflation persistence is not an inherent characteristic of industrial economies."

The primary drawback of Levin and Piger (2004) is that the entire analysis is univariate, thus, not taking into account the various interactions that exist in an economic system. However, in future research the authors intend to use a multivariate setting and analyse the extent to which shifts in monetary policy regime influence the dynamic behaviour of output as well as inflation. Marques (2004) questions the results of the paper as he is not sure if the high inflation persistence found in some of the countries is a real feature of the data or rather a spurious result brought about by the assumption of constant mean with a single break during the sample period.

Rose (1988) provides evidence of stationarity in inflation rates. Mixed evidence has been provided by Kirchgassner and Wolters (1993). Brunner and Hess (1993) argues that inflation rate was stationary before the 1960s, but since that time it possesses a unit root.

\textsuperscript{12}The GDP deflator, the personal consumption expenditure (PCE) price deflator, the consumer price index (CPI) and the core CPI over the period 1984-2002.

\textsuperscript{13}Australia, Canada, France, Germany, Italy, Japan, Netherlands, New Zealand, Sweden, Switzerland, the United Kingdom and the United States.

\textsuperscript{14}Failure to account for such breaks could yield spuriously high estimates of the degree of persistence, see Perron (1990) for further discussion.

\textsuperscript{15}They allow for a structural break in the slope parameter and the intercept term of the AR equation. In many cases, the authors find strong evidence for a single break in the intercept at some point in the late 1980s or early 1990s, while no evidence of a break in any of the AR coefficients.

\textsuperscript{16}An estimate of near 1 would correspond to extremely high persistence, while an estimate of 0 would represent no persistence.

\textsuperscript{17}The results of the paper are robust to the use of an alternative measure of inflation persistence, namely, the largest AR root.
The conclusion of Levin and Piger (2004) is consistent with a growing body of work documenting time variation in the level of US inflation persistence. Barksy (1987) finds that US inflation persistence was very high between 1960-79 but much lower from 1947-59. Evans and Watchel (1993) estimate a Markov-switching model for US inflation and find that during 1953-67 and 1983-93 the series was generated by a low persistence regime but during 1968-82 was generated by a random walk process.18 Bordo (1993) and Bordo and Schwartz (1999) find that inflation was more stable during the period of Bretton Woods. Brainard and Perry (2000),19 Taylor (2000)20 and Kim, Nelson and Piger (2001)21 find evidence that US inflation persistence during the Volcker-Greenspan era has been substantially lower than during the previous two decades; while according to Cogley and Sargent (2002a)22 US inflation persistence reached its post-war peak around 1979-80. Alogoskoufis and Smith (1991), analysing historical data for the US and UK inflation, emphasise that the post-Bretton Woods regime of managed floating was associated with more persistent inflation. Ravenna (2000)23 documents a large post-1990 drop in Canadian inflation persistence; Benati (2003a)24 finds that UK and US inflation had no persistence during the metallic-standard era (prior to 1914), highest persistence during the 1970s and markedly lower persistence during the last decade.25 Batini (2002) for the euro area

18 These shifts in persistence correspond well with the changes in monetary policy regimes observed in the US. Romer and Romer (2002) emphasise the success of policy in stabilising inflation during the 1950s while Clarida, Gali and Gertler (2000) consider the period post 1955 and find sufficient evidence for a shift of policy regime at the beginning of the Volcker-Greenspan era.

19 Brainard and Perry (2000) document the changes in the slope of the US Phillips curve. The curve was comparatively steeper at the time of the Great Inflation and much flatter before that, and over most of the recent period.

20 Taylor estimates two ARs, one for the period 1960-79 and the other for 1982-99. The author makes use of the 95% confidence limits of the largest AR root, proposed by Stock (1991) and also the sum of AR coefficients to compare inflation persistence across the two regimes.

21 Kim, Nelson and Piger (2001) use Bayesian tests for a structural break at an unknown break date and document structural breaks in the volatility and persistence of inflation and interest rates.

22 Cogley and Sargent (2002a) estimate time-varying AR coefficients conditional on a time-varying mean, which is specified as a random walk process. Benati (2004) also uses a similar approach. They find that the AR coefficients of inflation have dropped considerably over the last decade.

23 The paper builds a dynamic stochastic general equilibrium model for Canada and shows that after the shift to inflation targeting the dynamics of inflation changed from a near unit root to a stationary regime. It goes on to argue that inflation persistence and backward-looking Phillips curve are not structural features of the Canadian economy.

24 Benati (2003a) uses random-coefficients autoregressive representations with generalised autoregressive conditional heteroskedasticity to investigate shifts in inflation persistence over time and across monetary regimes in the US (since 1973) and in the UK (since 1662).

25 The finding that serial correlation of inflation has significantly decreased during the recent years is cited as evidence in favour of Taylor's (1998a) and Sargent's (1999) warning against 'natural rate recidivism' — that under low inflation persistence policy makers may be attracted to exploit an illusory output-inflation trade-off.
finds little evidence of an upward shift in inflation persistence in post-1980 data, while there was a substantial downward shift in the average value of inflation. In general the empirical literature suggests that inflation persistence may not be an intrinsic feature of industrialised nations, "...but rather varies with the stability and transparency of the monetary policy regime."  

Given the fact that the different countries under consideration have experienced substantial shifts in monetary policy over the past decade, particularly the widespread adoption of explicit inflation targeting, there is an opportunity to analyse the link between monetary policy regimes and inflation persistence — the main objective of our analysis.

Gadzinski and Orlandi (2004) follows the 'classical' approach of Levin and Piger (2004) paying special attention to the occurrence of a structural break in the series. The paper reports results on inflation persistence using 79 inflation series covering the EU countries, the Euro area and the US for six different inflation variables. The authors find that a structural break typically occurs at the beginning of the 90s and appears to correspond with the adoption of inflation targeting at least in the case of UK, Finland and Sweden. The general finding of the paper is that persistence is low across the board for the sample starting 1984:1. The results are reported in terms of the sum of AR coefficients as well as the half-life indicator. The analysis also suggests that the Euro area and US inflation series display comparable degree of persistence.

Osborn and Sensier (2004) study monthly RPIX inflation in the UK in the context of the change to inflation targeting in 1992. Analysing univariate models of monthly RPIX inflation since 1983, the authors find very strong statistical evidence of a break that effectively coincides with the introduction of inflation targeting. In fact persistence becomes insignificant after 1993,

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28 There is now a growing body of research supporting the view that the degree of inflation persistence is not an inherent structural characteristic of industrial economies, but in fact a function of the stability and transparency of monetary policy regime in place. See Bordo and Schwartz (1999), Sargent (1999), Goodfriend and King (2001), Erceg and Levin (2003) and Levin and Piger (2004).
29 Belgium, Germany, Denmark, Greece, Spain, France, Ireland, Italy, Luxembourg, Netherlands, Austria, Portugal, Finland, Sweden and the United Kingdom.
30 GDP inflation, CPI inflation, Core inflation, HICP inflation, private consumption inflation and services inflation.
31 This indicator measures the number of periods during which a temporary shock displays more than half of its initial impact to the process.
32 RPIX is the UK retail price index excluding mortgage payments.
which is compatible with the economic agents\textsuperscript{33} being forward-looking after this period. As a robustness exercise they also estimate a smooth transition model of the Phillips curve. Allowing for changing seasonality the evidence of change in the intercept and inflation persistence is also found in the Phillips curve model, where the change can be represented in terms of either a non-linear function of the level of inflation\textsuperscript{34} or as a structural break\textsuperscript{35} in 1992. The authors favour the latter as the change in monetary policy provides an economic explanation of why the change occurs.

Marques (2004) expresses his discontent with most of the univariate empirical estimation of inflation persistence as it assumes a constant long-run equilibrium level of inflation.\textsuperscript{36} The author argues that any estimate of inflation persistence is conditional on the assumed long-run inflation path.\textsuperscript{37} To quote "...there is a trade-off between persistence and the degree of flexibility of the assumed long-run equilibrium level of inflation: for a given series of inflation, we obtain maximum level of persistence under the assumption of a constant long-run level of inflation, but we can make persistence converge to zero if we allow enough flexibility to enter into our measure of the long-run level of inflation."

Marques analysis provides clear support to our contention that it is impossible to draw conclusions about inflation persistence without identifying a model — specifically a monetary regime. He shows, as we knew anyway, that if you have a constantly varying mean then you have no persistence and if you have a constant mean then you get maximum persistence; therefore regime identification is crucial. It should be noted that a ‘constantly varying mean’ is clearly not a regime at all.\textsuperscript{38} Further, if the mean is time-varying it is not really clear what a ‘long-run level of inflation’ could be. The question then arises whether there is one at all. Presumably

\textsuperscript{33} Including the central bank.
\textsuperscript{34} Clements and Sensier (2003) and Arghyrou, Martin and Milas (2004) find evidence of non-linearity in the models for UK inflation.
\textsuperscript{36} Exceptions are Burdekin and Syklos (1999), Bleany (2001), Bilk (2004), Levin and Piger (2004), and Osborn and Sensier (2004) which allow for the possibility of breaks in the mean of inflation.
\textsuperscript{37} Or in other words, to be able to gauge whether inflation is moving slowly or rapidly in response to a shock we need information on the likely path it would have followed had the shock not occurred as well as on the level it is to be expected to be once the effect of the shock has died off. This information is provided by a metric — the long-run equilibrium level of inflation.
\textsuperscript{38} As noted by Corvoisier and Mojon (2004) the mean of inflation is a major characteristic of a monetary policy regime and it is closely linked to the inflation objective of the central bank.
every monetary regime must have some sort of nominal anchor. What this statistical analysis
does reveal is that in the absence of identification of the regime in terms of its mean target
and its operational mode it may be impossible to identify persistence, since one may find any
degree of persistence according to one's decision about the degree of mean invariance.

The author contends that rather than assuming a constant mean or simply testing for the
possibility of some structural breaks in the mean of inflation, it is more natural to assume an
exogenous time-varying mean, as the starting null hypothesis. The paper also develops a non-
parametric measure of persistence based on the correspondence between persistence and mean
reversion. Finally, inflation persistence in the US and Euro Area is re-evaluated allowing for a
time-varying mean. The empirical evidence shows that the widespread accepted wisdom that
inflation persistence was higher in the sixties and seventies than in the last twenty years only
obtains for the US and that too for the special case of a constant mean, which itself, appears
to be a counterfactual assumption.

### 6.3.2 Multivariate Approach

Inflation is usually considered an endogenous variable, which adjusts to monetary and real
developments. However, the univariate model simply estimates inflation on its own lags, thus
ignoring various other factors which might affect the dynamics of inflation. Another major
limitation is that univariate models do not give a full account of the economically relevant
aspects of persistence, i.e. the speed at which inflation adjusts to monetary and business cycle
developments.39

The multivariate approach allows one to overcome the major shortcomings of univariate
modelling of inflation dynamics. It allows the researcher to disentangle the various shocks that
affect inflation behaviour and hence its persistence. In order to identify the shocks a full system
needs to be specified taking into account variables which affect and in turn are affected by
inflation — a simultaneous system of equations. As argued by Sims (1980) when there is true
simultaneity among a set of variables, then they all should be treated on an equal footing; there
should not be any a priori distinction between endogenous and exogenous variables. This is
the underlying concept of the VAR model developed by Sims.

39 See the typology proposed by Batini and Nelson (2002) and Batini (2002).
The typical VAR model used in the literature to analyse the behaviour of inflation includes the inflation, the deviation of output from trend and also the short-term nominal interest rate which is linked to real output and thus is essential to forming expectations of output. Also as Rudebusch (1998) observes, the interest rate equation in a VAR system has a structural interpretation as a monetary policy reaction function. One of the most influential paper using VAR has been Fuhrer and Moore (1995) which documents the persistent nature of US inflation for 1965-93 period.

More recently, Cogley and Sargent (2002a) uses a Bayesian VAR with time-varying parameters to describe the evolution of the law of motion for inflation for the US for the period 1948:1 to 2000:4. They work with a VAR(2) specification for inflation, logit of unemployment and the ex-post real interest rate. Analysing the median posterior spectrum for inflation they find that inflation was weakly persistent in the 1960s and 1990s, but strongly persistent in the late 1970s. In fact the degree of persistence peaked in 1979-80, at the same time as the peak in core inflation. Further, the evolution of the variance of inflation is closely associated with the degree of persistence. The authors find that inflation was more persistent and more variable in the 1970s, and less persistent and less variable in the 1980s and 1990s.

Corvoisier and Mojon (2004) evaluate persistence for 22 OECD countries and the Euro area by estimating 10 reduced form bi-variate models for inflation. Each model makes inflation depend on its own lags and on the lags of another macroeconomic variable, such as an indicator of monetary conditions, a real variable that typically appears in Phillips curves and an indicator of foreign or sectorial inflation shocks. The authors pay particular emphasis to identify the breaks in the mean of inflation. The paper shows that breaks in the inflation mean tend to be associated more often to breaks in the mean of nominal variables than real variables, which confirms that they are a monetary phenomenon. To the extent these breaks reflect changes in monetary policy regime, one possible interpretation is that inflation is less persistent within a regime rather then across regimes, possibly because inflation expectations adjust sluggishly to new policy regime. However, they contend that "...conditional on allowing for the breaks

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10 The study covers the period 1960-2003.
11 i.e. non-core.
12 The mean of inflation is a major characteristic of a monetary policy regime and it is closely linked to the inflation objective of the central bank.
in the mean of the inflation process, the persistence of inflation is low and stable. Hence, the persistence of inflation in the OECD countries has not been changing over time." They warn that models that ignore breaks in the mean of inflation may overrate the time it takes for inflation to adjust to shocks.

Dossche and Everaert (2005) provide concrete support to Marques (2004) contention discussed earlier. They use a structural time series approach – univariate and multivariate – to measure different sorts of inflation persistence for the US and Euro area allowing for an unobserved time-varying inflation target. The main point of the paper is that the unconditional estimates of inflation persistence, abundant in the literature, that are typically found to be close to that of a random walk process for the post WWII period are hard to interpret. The authors contend that the data generating process of inflation can be broken down into a number of components; each of them exhibiting its own degree of persistence. First permanent shifts in the central banks target lead to permanent changes in inflation. Second, private agents’ perceptions about the central bank’s inflation target can differ from the true inflation target due to asymmetric information, sticky-information\(^{44}\) or imperfect credibility.\(^{45}\) Third, slow response of inflation to various shocks hitting the economy is likely to be related to the wage- and price-setting mechanism.\(^{46}\) Fourth, persistence is determined by the persistence properties of the various macroeconomic shocks hitting inflation.\(^{47}\) The authors find that shifts in the central bank’s inflation objective induce a non-stationary component in the inflation rate. Additionally, slow adjustment of inflation expectations in response to a change in the central bank’s target of inflation and persistence of the shocks hitting inflation are important in determining the observed inflation persistence. The results clearly imply that a monetary policy regime in which the target of the central bank does not change and where public perception about the target are well anchored, inflation persistence would be relatively low.

\(^{44}\)See for example Mankiw and Reis (2002).

\(^{45}\)See Ercg and Levin (2003).

\(^{46}\)This kind of persistence is called intrinsic inflation persistence. See, Angeloni, Aucremenne, Ehrmann, Gali, Levin and Smets (2004).

\(^{47}\)This type of persistence is called extrinsic persistence. Again, see Angeloni, Aucremenne, Ehrmann, Gali, Levin and Smets (2004).
6.3.3 Anti-thesis

As the above discussion establishes the recent developments in the literature have challenged the view that inflation persistence should be viewed as a time-invariant phenomenon. Authors have argued that changes in the level of credibility of the central bank's commitment to attain their inflation target, should have an impact on the relative importance of forward-looking and backward-looking elements in inflation models such as the New Keynesian Phillips Curve. The basic idea is that the importance of lagged dependent variable terms should decline as the credibility of the central bank's commitment to low inflation increases. This in turn has an impact on the degree of inflation persistence. Relaxing the assumption of time-invariance clearly implies that the high inflation persistence observed between 1965-85 in many countries need not be an intrinsic feature of these economies. However, following conflicting results pointing to a constancy of inflation persistence over the more recent past notwithstanding the existence of changes in the monetary policy environment over the same period has led to some debate.

Batini and Nelson (2002) update and extend Friedman's (1972) evidence on the lag between monetary policy actions and the response of inflation for the UK and the US for the period 1953-2001. They reaffirm Friedman's proposition that monetary policy actions take well over a year to have their maximum effect on inflation; this result has persisted despite numerous changes in monetary policy observed in both countries. This implies that the degree of inflation persistence has been invariant across monetary policy regimes, where persistence is defined as the "lags between systematic monetary policy actions and their peak effect on inflation." The authors note that advances in information processing as well as financial market sophistication do not appear to shorten the lag. Hence, the empirical evaluation of dynamic stochastic general equilibrium (DSGE) models needs to be extended to include an assessment of these models' ability to account for the lags in monetary transmission found in the data.

Pivetta and Reis (2004) ask the question "Has persistence of inflation in the US changed since 1965?" The paper estimates the persistence over time, using different measures and estimation procedures, going on to produce confidence intervals for the estimates and formal tests for unchanged persistence. The paper basically builds on Cogley and Sargent (2002a)

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14See for example Taylor (1998a) and Sargent (1999).
Bayesian non-linear model of inflation-unemployment dynamics but differs from it in three important ways: First, Pivetta and Reis (2004) present not only point estimates as in Cogley and Sargent (2002a) but also Bayesian credible sets. Second, the paper does not restrict the parameter space solely to stationary representations; it allows for the possibility that inflation may have a unit root, a possibility that the data can not reject. Third, it computes alternative statistical measures of persistence and tries to distinguish between changes in volatility and changes in persistence. These modifications provide it with a very different conclusion: inflation persistence in the US is best described as high and approximately unchanged over the last three decades. Moreover, confidence intervals are wide enough that there remains great uncertainty on the exact value of inflation persistence at any given point in time. Hence, there is a need to model monetary policy under model uncertainty. The authors further explore the question by estimating a different model of inflation dynamics, grounded on classical theory, and again find that persistence is best characterised as being constant over time.\(^{51}\)

O'Reilly and Whelan (2004) analyse the stability of the econometric process for Euro area inflation since 1970, focussing on the sum of AR coefficients as the persistence parameter. Given the sequence of shifts in monetary policy regimes that have occurred since 1970 clearly the Lucas critique would apply, i.e. inflation regressions should exhibit substantial parameter instability, rendering them dubious for forecasting or policy analysis. However, the authors find relatively little instability. In fact for full-sample estimates the persistence parameter is close to one and results from Andrews-Ploberger unknown-breakpoint tests for structural change are consistent with the null of no change over time in this coefficient.\(^{52}\) They also estimate rolling regressions which again show that persistence estimates are stable over time and close to unity. There conclusions are also robust to specifications that include an output gap.

The authors interpret their results as providing indirect evidence against pricing models with strong forward-looking elements, such as the New Keynesian Phillips curve. This is in line with

\(^{50}\) Cogley and Sargent (2002a) truncate the parameter space to exclude non-stationary representations. This truncation strongly pushes the results towards low values of persistence. As pointed out by Sims (2001) the economic implication of this would be that tests of the natural rate hypothesis will be biased towards rejection.

\(^{51}\) Indeed, in his comment to Cogley and Sargent (2002a), Stock (2002) applies a subset of the classical methods and finds no evidence of a change in persistence.

\(^{52}\) These tests detect a break in the intercept. However, the authors contend that tests based on Andrews (1993a) critical values have poor properties when the true value of the persistence parameter is close to or equal to one. Once, they correct for this factor there is no significant evidence for an intercept break.
recent evidence for the US presented by Rudebusch (2003) which shows that the parameters of reduced-form regressions of a New Keynesian style macro-model will tend to be relatively stable even in the presence of realistic changes in monetary policy. The paper also shows that if such a structural system places relatively low weights on forward-looking expectational variables, then the inflation persistence parameter in reduced-form models will be close to one.

6.4 Structural Approach to Inflation Persistence

The last three decades have witnessed large and persistent fluctuations in the rate of inflation in most industrialised economies. After an era of relative price stability inflation started rising in the late 60s, reached double-digit levels in the mid-70s, before receding gradually in the early 80s. Since then it has remained relatively subdued, though inflationary pressures have alternated with prospects of deflation. Because these swings have been very persistent the common view is that inflation is a persistent process. This is further supported by a large body of econometric literature that finds post-war US inflation to exhibit very high persistence, approaching that of a random walk process. Given similar findings for other OECD countries, many macroeconomists take the high degree of inflation persistence as a ‘stylised fact’ that micro-founded models ought to replicate. Fuhrer (1995) speaks of a ‘...wide agreement that inflation is persistent...’ While Nelson (1998) states inflation persistence to be one of the key stylised facts that any sensible sticky-price, dynamic stochastic general equilibrium model should be capable of replicating.

6.4.1 Origin of Inflation Persistence

The logical next question is ‘Where does inflation persistence originate from?’ Benati (2003a) states that at a very general level, the stochastic properties of any macroeconomic time series can be thought of as resulting from the interaction between three separate – but not mutually exclusive – elements:

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55 The explanation of the three sources of persistence follows very closely the discussion in Benati (2003a).

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• the data generating process for the structural shocks hitting the economy

• the intrinsic structure of the economic system

• the monetary and fiscal policy regime prevailing over the sample period

First, inflation could be persistent simply because it has inherited the strong serial correlation properties of the structural shocks affecting the economy, even if a strong internal propagation mechanism or policy regime inducing persistence is absent. One of the strategies pursued by the macroeconomic profession in recent years to reproduce the strong serial correlation found in macroeconomic data is simply appealing to autocorrelated structural shocks. See for example Rotemberg and Woodford (1997), Dittmar, Gavin and Kydland (2001) and Ireland (2003). However, Fuhrer (1997) in his comment on Rotemberg and Woodford (1997) states that to the extent that the autocorrelated structural shocks capture a significant portion of the dynamics found in the data, the model is not really explaining that much or in other words to the extent that the dynamics present in the data can only be explained by things whose dynamics arise for reasons that we can not explain, the model does not really explain much.

Second, inflation could be persistent because specific structural features of the economy create a powerful propagation mechanism, converting (possibly) serially uncorrelated structural shocks into highly correlated macroeconomic time series, irrespective of the specific fiscal/monetary regime in place. A common theme in some of the recent literature is that inflation persistence is an entirely structural feature of the economy and hence has to be ‘hardwired’ into the macroeconomic model by introducing frictions of various nature.\textsuperscript{56} The literature has proposed varied microeconomic interpretations of the ‘stylised fact’ of inflation persistence.\textsuperscript{57} Roberts (1998), Ball (2000), Ireland (2000), Sims (2001), Woodford (2001) and Mankiw and Reis (2002) assume that private agents face information-processing constraints. Buiter and Jewitt (1989), Fuhrer and Moore (1995), Fuhrer (2000), Calvo, Celasun and Kumhof (2001), Christiano, Eichenbaum and Evans (2005) assume that high inflation persistence results from the structure of nominal contracts. However, as Benati (2003a) argues and we shall as well, this


\textsuperscript{57}For further discussion see Nelson (1998) and Clarida, Gali and Gertler (1999).
approach is potentially flawed as there is ample empirical evidence which strongly questions the assumption that inflation is intrinsically persistent.\footnote{See Brainard and Perry (2000), Taylor (2000), Kim, Nelson and Piger (2001) and Cogley and Sargent (2002a, 2003) for the US, Ravenna (2000) for Canada and Levin and Piger (2004) for a whole range of OECD countries.}

Third, inflation persistence could largely, or even entirely, originate from specific features of the operating rules of the monetary regime in place over the sample period. According to such an explanation, inflation persistence may not be (entirely) structural,\footnote{By structural we mean structural in the sense of Lucas (1976). See Goodfriend and King (2001) for further discussion.} and could instead be (partially or entirely) historically determined, in the sense of being the historical product of the specific way in which monetary policy has been conducted over the sample period. Changes in the conduct of monetary policy, and more generally changes in the operating rules of the underlying monetary institutions, are not the only persistence-generating devices that Benati (2003a) has in mind. In particular, it is the author's conviction that any convincing explanation of the peculiar evolution of the stochastic properties of inflation over the post-WWII must assign a central role to the learning process on the part of private economic agents about the monetary environment.\footnote{Learning about the conduct of monetary policy as a persistence-generating device is explored in the work of Sargent (1999) and Ercig and Levin (2003).}

It is now widely accepted that monetary policy has changed in important ways over time. In case of the US this is associated with the term of office of the Federal Reserve chairman.\footnote{Judd and Rudebusch (1998) were the first to document the practical importance of this.} The situation is perhaps even more clear cut in case of the UK where Nelson (2000) documents a number of 'regime-changes' in monetary policy in the period since 1972. There is also a growing body of work supporting the view that the monetary policy regime in place has a significant impact on the persistence properties of inflation. In other words the degree of inflation persistence is not an inherent structural characteristic of industrial economies, but in fact a function of the stability and transparency of monetary policy regime in place.\footnote{See Bordo and Schwartz (1999), Sargent (1999), Goodfriend and King (2001), Ercig and Levin (2003) and Levin and Piger (2004).}

Bordo and Schwartz (1999) lay emphasis on the impact of the strength and credibility of the nominal anchor provided by the monetary policy regime in place on private agents' expectation formation mechanism and through this on the actual inflation. It is a well established fact
that commodity-based monetary standards provide strong nominal anchors by eliminating the opportunity for manipulation of the money stock by the government.\footnote{See Barro (1979) and Bordo and Kydland (1996) on the Gold Standard as a commitment mechanism.} It therefore comes as no surprise that one finds low inflation persistence within such monetary systems.

Turning to the fiat money systems of today Clarida, Gali and Gertler (2000) and Cogley and Sargent (2002a, 2003) suggest a link between the degree of activism of monetary policy\footnote{A policy rule is activist, if \textit{ceteris paribus}, the central bank increases the nominal interest rate more than one-for-one in response to an increase in inflation, so that real interest rates increase. A passivist central bank, on the other hand, adjusts the nominal interest rate one-for-one or less, so that the real interest rate remains unchanged or falls as inflation rises.} and the degree of inflation persistence. Clarida, Gali and Gertler (2000) show that a passive monetary policy rule: (i) allows the economy to experience self-fulfilling fluctuations characterised by a high degree of inflation persistence — even in the absence of fundamental shocks, and (ii) can generate highly persistent fluctuations in inflation as well as the output gap in response to fundamental shocks. The authors also show that the major change in the US inflation, i.e. the Volcker disinflation, coincides with a change in the Fed’s reaction function from setting procyclical real interest rates to counter-cyclical ones. Cogley and Sargent (2002a, 2003) estimate a time-varying Taylor rule and compute time-varying spectral statistics for inflation, using a random coefficients Bayesian \textit{VAR}(2) for inflation, unemployment and the ex-post real rate, demonstrating how the extent of activism on the monetary policy rule displays a striking negative correlation with the logarithm of the spectral density of inflation\footnote{It is to be noted that the spectrum at zero frequency summarises the degree of persistence.} at zero frequency,\footnote{Stock (2002) conjectures that Cogley and Sargent’s (2002a) main result — inflation persistence in post WWII US was comparatively low in the 1960s, high in the 1970s, peaked around the beginning of the Volcker disinflation and has been decreasing ever since — crucially depends on the fact that they did not control for conditional heteroskedasticity. Benati (2003a) however provides evidence in support of Cogley and Sargent (2002a).} i.e. there is an inverse relationship between the degree of activism on one hand and core inflation and inflation persistence on the other. They show that US low frequency swings in inflation are consistent with the Federal Reserve gradually upgrading its view on the (im)possibility of exploiting an output-inflation trade-off.

More recently, over the past decade we have observed substantial shifts in the monetary policy of a number of countries, particularly the widespread adoption of explicit inflation targets.\footnote{See Bernanke, Laubach, Mishkin and Posen (1999), Johnson (2002), Mishkin and Schmidt-Hebbel (2002).} In an interesting paper Levin, Natalucci and Piger (2004) evaluate the extent to which Inflation

\footnote{See Bernanke, Laubach, Mishkin and Posen (1999), Johnson (2002), Mishkin and Schmidt-Hebbel (2002).}

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Targeting (IT) exerts a measurable influence on expectations formation and inflation dynamics. The paper compares time series data since 1994 for five IT countries\textsuperscript{68} with that of seven non-IT countries\textsuperscript{69}. It analyses the behaviour of medium- and long-term inflation expectations using Consensus Economics Inc. semiannual surveys of market forecasters, and employs the methods of Stock (1991) and Hansen (1999) to obtain median-unbiased measures of persistence for total and core consumer price inflation (CPI). For the industrialised economies, evidence indicates that IT has played a significant role in anchoring long-run inflation expectations. In case of the US and the Euro area, private-sector inflation forecasts are highly correlated with a three-year moving average of lagged inflation. In contrast, this correlation is largely absent in case of the IT countries, indicating that the central banks have been quite successful in delinking expectations from realised inflation. Further, actual inflation exhibits markedly lower persistence in IT countries.\textsuperscript{70} The paper also analyses the experience of emerging market economies using an event-study approach similar to that of Bernanke, Laubach, Mishkin and Posen (1999). The results are largely consistent with those observed for the industrialised countries.\textsuperscript{71}

Benati (2002) investigates the changes in UK economic performance post WWII using tests for multiple structural breaks at unknown points and frequency-domain techniques. The paper finds that the Phillips curve correlation between unemployment and inflation at the business cycle frequencies\textsuperscript{72} appears to have undergone significant changes over the last 50 years. It displays some instability for the Bretton Woods period, quite remarkable instability in the 1970s, slowly stabilising correlations from the beginning of 1980s and under inflation targeting the correlation exhibits by far the greatest stability. What is perhaps the most interesting finding of the paper is that for the inflation targeting period it finds that the mean, variance and persistence of inflation\textsuperscript{73} are the lowest for the post WWII era. In similar lines Benati

\textsuperscript{68} Australia, Canada, New Zealand, Sweden and the UK.
\textsuperscript{69} The US, Japan, Denmark, France, Germany, Italy and the Netherlands.
\textsuperscript{70} Siklos (1999) finds evidence of a decline in inflation persistence in some inflation targeting (IT) countries; Kuttner and Posen (2001) find evidence that IT countries experience lower inflation persistence. Corbo, Landerretche and Schmidt-Hebbel (2001) report that IT is associated with lower long-term effects of inflation innovations compared with the non-IT countries.
\textsuperscript{72} Business cycle frequencies refer to fluctuations between 1.5 to 8 years.
\textsuperscript{73} Benati (2002) uses three measures of inflation: RPIX, GDP deflator and personal consumption expenditure deflator.
(2003a) and Cogley, Morozov and Sargent (2003) find that inflation persistence for the post
WWII period was comparatively low both at the beginning of the sample and over the last
decade but markedly higher during the 1970s. Benati and Wood (2004) show that over the
last one hundred years, inflation has been notably more persistent and with a noticeable higher
mean between 1972 and 1992, a period when the Bank of England did not stress any nominal
anchor.

6.4.2 Is Inflation Persistence Structural?

Having observed the varied nature of inflation persistence the key question as found by Benati
(2003a) is "Is inflation persistence structural in the sense of Lucas (1976)?" Given the large
econometric evidence of high inflation persistence in the OECD countries a significant propor-
tion of the macroeconomic profession has concluded that persistence is a 'stylised fact' and
believes it to be structural.\(^\text{74}\)

Models treating Inflation Persistence as Structural

Fuhrer and Moore (1995) characterise inflation persistence in the US data using a vector au-
tocorrelation function relating inflation, deviations of output from trend and the short-term
nominal interest rate. The authors find that inflation is quite persistent, with positive autocor-
relations out to lags of about four years, that the output gap is somewhat less persistent and
that there exist significant positive cross-correlations between the two. They argue that the
standard sticky-price model on the lines of Phelps (1978), Taylor (1980) and also Calvo (1983)
is not consistent with the dynamic interaction of inflation and output. The authors contend
that these standard models imply that persistence in inflation derives entirely from the driving
output gap process. As a result the Phelps-Taylor-Calvo formulation "is incapable of imparting
the persistence to inflation that we find in the data." According to Fuhrer and Moore (1995)
in such a model the autocorrelation function of inflation will die out quite rapidly; this con-
tradicts the empirical autocorrelations of inflation, which decay slowly. In order to overcome

\(^{74}\)As is demonstrated by the efforts of Fuhrer and Moore (1995), Fuhrer (2000), Altig, Christiano, Eichenbaum
of various nature to generate inflation persistence.

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the shortcoming they propose a contract specification where agents care about the relative real wages over the life of the wage contract.\textsuperscript{75} This gives us a formulation where inflation depends on its past — inflation rate is significantly inertial beyond the inertia of the driving term.

Holden and Driscoll (2001) argue that if Fuhrer and Moore\textsuperscript{'} (1995) model is modified so that workers cared about contemporaneous real wages of other workers, which is a more reasonable assumption, then the model coincides with the standard formulation of Taylor (1980) and there is no persistence. Blanchard and Katz (1999) suggest that inflation persistence may be explained by taking into account the dependence of workers' reservation wages on past wages. However, Holden and Driscoll (2001) show that Blanchard and Katz (1999) conjecture turns out to imply that inflation depends negatively on itself lagged — the exact opposite of empirical regularity. A third route adopted by Roberts (1998) and Ball (2000) is to apply different varieties of near-rational expectations formation mechanism, basically a form of adaptive expectations to staggered wage-setting models. Jadresic (2000) proposes a staggered price-setting model with flexible distribution of price durations to generate inflation persistence. In a companion paper Driscoll and Holden (2001) show that persistence in inflation may be caused by coordination problems associated with workers being concerned about fair treatment, i.e. they care disproportionately more about being paid less than other workers than they do about being paid more.

Mankiw and Reis (2002) examine a model of dynamic price adjustment based on the assumption that information about macroeconomic conditions disseminates slowly throughout the population.\textsuperscript{76} This slow diffusion could arise because of either costs of acquiring information or costs of re-optimisation. Thus, in the model although prices are always changing, pricing decisions are not always based on current information. This is a sticky-information model as against the standard sticky-price model on which the New Keynesian Phillips curve is based.

The paper compares dynamics of both inflation and output in the sticky-information model with the standard sticky-price model and a backward-looking model. The main findings of the hypothetical policy experiments for the sticky-information model are: first, disinflations are always contractionary,\textsuperscript{77} although announced and credible disinflations are less costly than

\textsuperscript{75} The relative contracting specification was first introduced by Buit\-er and Jewitt (1981).

\textsuperscript{76} For earlier work on similar lines see Roberts (1997).

\textsuperscript{77} In sticky-price models, prices are sticky but inflation exhibits no inertia. Even with a sudden disinflation,
surprise ones.\(^78\) Second, monetary shocks have their maximum effect on inflation with a substantial delay.\(^79\) Inflation could well be described as inertial.\(^80\) Third, the model can explain the acceleration phenomenon that vigorous economic activity is positively correlated with rising inflation.\(^81\)

Hence, we see that the sticky-information model is more consistent with accepted views of how monetary policy works and affects the economy as against the canonical sticky-price model of inflation-output dynamics. Skeptics might argue that the sticky-information model is nothing but the revival of adaptive expectations. However, it must be noted that agents in the sticky-information model form expectations rationally, even though they do not do so often. This implies that the sticky-information model, in comparison to the adaptive model, can account for both the shift in the reduced-form Phillips curve in response to regime change and the role of credibility in reducing the costs of disinflation. In a sense it combines elements of on the one hand the sticky-price model that expectations, announcements and credibility matter for the path of inflation and output; and on the other hand of backward-looking models that disinflations consistently cause recessions rather than booms. It is possible that microfoundations of the Phillips curve may require a better understanding of bounded rationality. However, the sticky-information model offers a useful tool for the study of inflation-output dynamics.

Gali and Gertler (1999) develop and estimate a structural model of inflation that allows for a subset of firms that use a backward-looking rule of thumb to set prices. The ‘hybrid Phillips’ curve\(^82\) nests the new completely forward-looking Phillips curve as a special case, permitting the authors’ to assess the degree to which it can account for the inertia in inflation.\(^83\) In the

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\(^78\) In the sticky-price model, announced credible disinflation causes a boom. See Ball (1994a) for further discussion.

\(^79\) For econometric substantiation see Bernanke and Gertler (1995) and Christiano, Eichenbaum, and Evans (2005).

\(^80\) In sticky-price models, the greatest impact of a monetary shock on inflation occurs immediately.

\(^81\) Sticky-price models yield no association between output and the change in inflation.


\(^83\) The advantage of this set-up is that the coefficients of the hybrid Phillips curve are functions of the two key parameters: the frequency of price adjustment and the fraction of backward-looking price setters.
empirical implementation, the paper uses a measure of real marginal cost in place of the ad hoc output gap.\textsuperscript{84} as theory suggests.\textsuperscript{85} The paper goes on to show that conditional on the path of real marginal costs, the New Keynesian Phillips curve with forward-looking behaviour may provide a reasonably good description of inflation dynamics. When tested explicitly against the hybrid version, which allows a fraction of the price setters to be backward-looking, the structural estimates suggest that although this fraction is statistically significant, it is not quantitatively important. Thus, Gali and Gertler conclude that the New Keynesian Phillips curve (NKPC) provides a good first approximation to dynamics in inflation. However, they believe that sluggish behaviour of real marginal costs might help to account for the slow response of inflation to output and thus (possibly) why disinflations may entail costly output reductions.

Sbordone (2002) validates the results of Gali and Gertler (1999) by using an alternative estimation method that follows Campbell and Shiller (1988). Gali, Gertler and Lopez-Salido (2001a) suggest that the staggered contract mechanism fits the European data possibly even better that the US data and so infer that inflation in the Euro area is not intrinsically persistent and definitely less inertial than in the US. However, Rudd and Whelan (2001) question the power of the tests employed by Gali and Gertler (1999) and Gali, Gertler and Lopez-Salido (2001a). In particular they contend that instrumental variable estimates like Generalised Methods of Moments used by Gali, Gertler and Lopez-Salido (2001a) could be biased towards forward-looking inflation formation even if the true model contains no such behaviour. This issue is addressed in Gali, Gertler and Lopez-Salido (2001b). McAdam and Willman (2002), use an alternative estimation procedure\textsuperscript{86} for the Gali, Gertler and Lopez-Salido (2001a) ‘hybrid’ NKPC and support the view that Euro area inflation is strongly inertial.

\textsuperscript{84}Gali and Gertler (1999) show that real marginal cost is not well approximated by the detrended output. Specifically, detrended output and other such measures fail to account for supply shocks and other factors such as labour market frictions that affects a firm's marginal cost.

\textsuperscript{85}A clear virtue of the real marginal cost, which in the most basic case corresponds to real unit labour costs, is that it directly accounts for the influence of productivity on inflation, as well as wage pressures.

\textsuperscript{86}In the estimation procedure the instrumental variable is the weighted sum of leads of the mark-up over nominal marginal costs, as implied by Rudd and Whelan (2001), rather than the first lag of inflation.
Models treating Inflation Persistence as Non-Structural

Goodfriend and King (2001) work with New Neoclassical Synthesis model with no structural persistence and show that observed inflation persistence in the US time series is consistent with the absence of structural inflation stickiness. NNS models with staggered pricing do not exhibit structural inflation persistence: although staggered price setting makes the price level sticky, it does not make inflation sticky. Therefore an inflation shock does not need to be followed by persistently high inflation if firms expect the monetary authority to pursue a policy supportive of price stability. Hence, fully credible neutral policy that anchors inflation expectations induces firms to return quickly to zero inflation after a shock. The authors obtain simulated inflation series using the methods developed by Sbordone (1998) and Gali and Gertler (1999) and find that although the simulated series does not capture every movement of the inflation data, the pricing model without structural inflation persistence tracks actual US inflation remarkably well. The authors conjecture that the inflation persistence found in post-WWII US data may result from the way the Federal Reserve has pursued monetary policy — how it has allowed the mark-up to covary with inflation.

Dittmar, Gavin and Kydland (2001) show that a flexible price, general equilibrium business cycle model with money, in which monetary policy shocks have almost no real effects can account for inflation persistence if the Fed follows a Taylor rule. The paper analyzes VARs for the actual data from 1980:2 to 2001:4 and the artificial data generated by the model.\footnote{It is to be noted that VARs by themselves are not evidence of a particular economic structure, but these time series regularities are often used in the literature to justify the assumption that systematic monetary policy actions have real effects. The identification problem is difficult and there is no widely accepted solution.} It investigates the effects of alternative monetary policy rules embedded in a neoclassical growth model with shocks to production technology, shopping time technology and monetary policy rule. Agents decisions are taken in a period only after all shocks are observed. Linearising first order conditions, transition equations, and equilibrium conditions around steady-state results in the approximation of the model used for various monetary policy experiments. The interest rate rule is calibrated to Taylor (1993) for the baseline case.\footnote{Dittmar, Gavin and Kydland (2001) use quarterly measures of inflation and interest rates as against Taylor (1993) which uses annual measures.} The main findings of the experiments are: (i) inflation persistence is not sensitive to the weight of inflation in the Taylor rule as long
as the weight is large enough to avoid indeterminacy,\(^{99}\) (ii) inflation autocorrelations are more sensitive to the weight on output then they are to the weight on inflation, and (iii) the degree of interest rate smoothing is the most important factor determining inflation persistence.

In general, inflation persistence has been considered as evidence against flexible price general equilibrium models because it is nearly impossible to generate it if the central bank follows an exogenous money supply rule. The basic objective of Dittmar, Gavin and Kydland (2001) has been to show that it is quite easy to generate inflation persistence in flexible price models if the central bank is following an interest rate rule. The key to grasp the inflation dynamics in such a paradigm is to understand how the central bank manages the short-term nominal interest rate relative to the real interest rate. Any policy that induces a persistent difference between the two will also induce inflation persistence.

Erceg and Levin (2003) explore an alternative channel to generate inflation persistence within a dynamic stochastic general equilibrium model, where persistence is not inherent in the structure of the economy, by assuming that private agents face a filtering problem about the shifting inflation target of the central bank.\(^{90}\) In particular, households and firms use optimal filtering to disentangle persistent shifts in the inflation target from transitory disturbances to the monetary policy rule. Under these circumstances, the speed at which private agents recognise a new inflation target depends on the transparency and credibility of the monetary policy authority. Thus, the signal-to-noise ratio is the key parameter determining the persistence of inflation forecast errors, and hence influencing the persistence in actual inflation and output. The central bank is assumed to react to the deviation of the inflation rate from its target and to the growth rate of real output. The key assumption is that private agents can not directly observe the central bank’s long-run inflation target although they know the form of its reaction function. Thus, agents must solve a signal extraction problem in order to forecast the future path of the inflation target, which in turn influences the outcomes of their current decisions.\(^{91}\)

\(^{99}\)According to McCallum and Nelson (1999a) indeterminacy refers to situations in which the model provides a single solution for all real variables but simply fails to determine a solution path for any nominal variable.

\(^{90}\)As in Erceg (1997), Erceg and Levin (2003) assume that labour and product markets each exhibit monopolistic competition, that wages and prices are determined by staggered four-quarter nominal contracts, and that the capital stock is endogenously determined subject to quadratic adjustment costs. Under these assumptions, a transitory money growth shock has persistent effects on output and the aggregate price level, but not on the inflation rate; see for example Chari, Kehoe, and McGratten (2000), Erceg (1997), and Edge (2000).

\(^{91}\)Erceg and Levin (2003) formulation of the information problem is formally similar to that in Brunner,
The authors show that such a mechanism accounts well for several features of the Volcker disinflation episode, such as, pronounced initial rise in the nominal interest rate, sluggish decline in the inflation, a persistent negative output gap and persistent inflation forecast errors. The calibrated model accounts quite well for the dynamics of both output and inflation and implies a sacrifice ratio of 1.6, which is very close to the estimated value.\textsuperscript{92} The paper's approach indicates that inflation persistence and substantial costs of disinflation can be generated in an optimising framework, without relaxing the assumption of rational expectations or relying on arbitrary modifications to the aggregate supply curve. Erceg and Levin contend that in such a paradigm inflation persistence is not an inherent characteristic of the economy but rather persistence arises whenever agents have to learn about the shifts in the monetary policy regime; it depends on the stability and transparency of the monetary policy regime.

Finally, Sargent's (1999) Conquest of American Inflation can be interpreted as a way for producing shifts in the stochastic properties of inflation as a result of a specific learning problem on the part of the policy maker.

6.5 Policy Implications

As discussed in Benati (2003a) there are several reasons why economists are interested in questions like: Is inflation persistence a structural feature of the economy? Has the degree of inflation persistence been basically constant or has it changed over time? What role, if any, does the monetary policy regime play in determining the degree of inflation persistence?

First, the fact that inflation persistence is structural or not has crucial implications for the design and estimation of dynamic stochastic general equilibrium (DSGE) models featuring nominal stickiness. Clearly if Goodfriend and King's (2001) analysis is correct that persistence of inflation found in post WWII US data results uniquely from the way in which the Federal Reserve pursues monetary policy, then it immediately follows: (i) all models that 'hardwire'

\textsuperscript{\textsuperscript{92}}Cukierman, and Meltzer (1980), and in Gertler (1982). The latter showed how imperfect observability of the underlying components of the money supply process could induce 'inertial' behaviour in the level of nominal wage.

\textsuperscript{\textsuperscript{93}}Ball (1994b) obtained a value of 1.8. Nevertheless, it should be recognised that estimated sacrifice ratios are somewhat sensitive to the specific measure of output gap. For example, Sachs (1985), Blinder (1987), and Mankiw (1991) obtained somewhat higher estimates of the sacrifice ratio for the Volcker disinflation.
persistence into the structure of the economy are clearly mis-specified and hence, empirical estimates of the model parameters are either wrong or very distorted. (ii) such models can not be used for evaluating alternative monetary policy frameworks or computing optimal monetary policy rules, as a pre-requisite of such analysis is that the model under consideration is structural in the sense of Lucas (1976).

Second, regarding the constancy or not of persistence in recent times, Taylor (1998a) and Sargent (1999) have warned against the dangers of 'natural rate recidivism' — that under low inflation persistence, the application of erroneous econometric tests, together with the now fading memories of the 'Great Inflation' of the 1970s, may induce policy makers to again try and exploit an illusory output-inflation trade-off. If persistence is constant as is discussed by Stock (2002), Pivetta and Reis (2004) and O'Reilly and Whelan (2004) then these dangers are over-emphasised. However, if persistence fluctuates and has reduced in the recent past as discussed by Taylor (2000), Ravenna (2000), Levin and Piger (2004) among others, then on empirical grounds the concerns of Taylor and Sargent are well justified.

Finally, regarding the role of monetary policy on inflation persistence, as emphasised by Goodfriend and King (2001), if inflation persistence is independent of the monetary policy regime, then it is not possible to design a monetary framework capable of eliminating inflation-output gap trade-off which is intrinsic in the economy. However, if persistence is *not structural* then it may be possible to design a policy framework capable of stabilising both inflation and output gap.

Angeloni, Coenen and Smets (2003) and Coenen (2003) find that the degree of inflation persistence represents a key determinant of the monetary transmission mechanism and has important ramifications for the ability of monetary policy to stabilise inflation in a country relative to its output. The authors find that it may be dangerous to rely too heavily on rules that are designed under the assumption that inflation persistence is low. If the inflation process turns out to be considerably more persistent then these rules may result in disastrous stabilisation outcomes. In contrast, rules designed and implemented under the assumption that inflation is highly persistent perform reasonably well even when it turns out to be low. Hence, a cautious monetary policy maker is well-advised to take monetary policy decisions under the assumption that inflation is substantially persistent until strong evidence to the contrary has
6.6 Conclusions

This chapter has familiarised the reader with the concept of inflation persistence so widely discussed in the literature. The study of inflation and its dynamics is crucial for macroeconomists as inflation has far reaching implication for the economy both in terms of economic efficiency and wealth distribution. We have shown that in the empirical estimation there are basically two views about the degree of inflation persistence that exists in the data. One view upholds that inflation persistence has varied over the post WWII period for most industrialised economies; which given the substantial changes in monetary policy regimes observed supports the contention that persistence is a function of the credibility and transparency of the regime in place. The conflicting view finds that persistence has been stable over the same period even when policy has become more credible and hence monetary policy plays no role in determining the persistence properties of inflation. When we look at structural models for inflation persistence again there are two camps. One believes that persistence is structural so must be ‘hardwired’ into the economic model while the other supports the non-structural nature of inflation persistence. Finally, we discussed the policy implications of inflation persistence in terms of design and evaluation of optimal policy.
Chapter 7

Empirical Analysis of UK Inflation Persistence

7.1 Introduction

The objective of this chapter is to establish the facts of UK inflation persistence, allowing for breaks in monetary policy regimes. We estimate univariate processes for inflation across different time periods where these periods are carefully defined according to a priori knowledge of the UK economy.\(^1\) Our initial results clearly indicate that inflation persistence is different in different regimes, with persistence being lowest in the Inflation Targeting regime, followed by Bretton Woods and then Monetary Targeting. Persistence tends to be higher during the Deutsche Mark shadowing period as the government’s primary aim then was to defend the peg. During the Incomes Policy regime of the 1970s there was no nominal anchor; hence we get the highest persistence parameters.

The chapter is organised as follows. Section 7.2 gives the reader a very brief overview of the monetary policy regime history for the UK. In Section 7.3 we explain the concepts of ‘net’ and ‘gross’ persistence referred to in the chapter. Section 7.4 first describes the different measures of inflation persistence used in the literature and then goes onto show the results of persistence estimates for the UK for the period 1956 to 2003. Section 7.5 summarises the main conclusions

\(^1\) According to Perron (1990) a failure to account for such breaks could yield spuriously high estimates of the degree of persistence.
of the data analysis.

7.2 UK Monetary Policy: A Brief Introduction

Nelson (2001) points out that monetary policy in the UK has undergone several regime changes over the last 50 years: from a fixed exchange rate with foreign exchange controls until 1972; to free-floating Incomes Policy with no domestic nominal anchor until 1978, followed by a system of Monetary Targeting until the mid-1980s; then back to exchange rate management, the period of ‘shadowing’ the Deutsche Mark, which finally culminated in the membership of the Exchange Rate Mechanism (ERM) from 1990-1992.\(^2\) Since, 1992 Inflation Targeting has been the official regime governing UK monetary policy, with interest rate decisions made by the Treasury up to May 1997, after which the Bank of England received its independence. Monetary Policy and interest rate decisions ever since have been made by the Monetary Policy Committee (MPC) of the Bank.

For the period as a whole, there have been large swings both in inflation and economic growth. Inflation was continuously in double digits during most of the 1970s, and returned there in the early 1980s and 1990s. Regarding economic growth Nelson (2001) documents that it was already lower in the UK in comparison to its major trading partners in the 1960s and underwent a further slowdown after 1973, with partial recovery beginning only in the 1980s. There were recessions in 1972, 1974-75, 1979-81 and 1990-92. However, the disinflation of the early 1990s has been followed by a period of low and stable inflation and reasonably stable real GDP growth.

7.3 Concept of Persistence

We note that ‘persistence’ is not entirely a clear concept. A stationary time series will typically consist of AR and MA; we confine ourselves to linear processes since the role of non-linearity seems to be basically secondary in this context. Persistence could naturally refer to the AR

\(^2\)For the first seven years of floating exchange rates foreign exchange controls continued, but were finally abolished in 1979. Thus, the absence of controls in the ERM period gave little room for monetary policy to differ, even in the short-run, from that consistent with the exchange rate target.
roots, ignoring the MA, which by construction must end sharply. However, the MA component can be inverted and turned into an infinite-order AR; this property is of course exploited in forming the widely-used VAR representation.

For a single time series we then have a pure AR of infinite order which can be truncated at some point empirically; this AR process can be used to measure persistence — in effect of all elements driving the variable. Alternatively, one could invert the AR part and turn the process into the pure MA form of the Wold decomposition; this is the natural form for the impulse reaction function, which contains the same information in principle as the AR. However, it is not so widely used and is non-parsimonious in estimation, especially here with few degrees of freedom in several regimes; therefore we use the AR in preference as a summary measure of 'net persistence.'

In the ensuing analysis we call the AR form 'net persistence' and the AR roots of the ARMA form 'gross persistence' for convenience. It is plain that the same variable can be grossly persistent — have high AR roots — and yet the MA part can somehow eliminate the shock after a period or two thus giving the roots nothing to 'work on,' so resulting in little or no 'net persistence.' Such appears to be the case with inflation in at least some of the periods we deal with.

7.4 Initial ARMA and AR Estimation

7.4.1 Introduction

We begin by estimating univariate processes for inflation across different time periods. A key aspect of our approach is to allow for structural breaks based on a priori knowledge of the UK economy, since a failure to account for such breaks could yield spuriously to high estimates of the degree of persistence.\(^3\) Our initial results clearly indicate that inflation persistence is different in different regimes, with persistence being lowest in the Inflation Targeting regime as one would expect a priori. The fixed exchange rate regime during the Bretton Woods era comes in second lowest, providing some respite to the believers of fixed exchange rate regimes. A priori one would expect Monetary Targeting to come in second lowest; however during the

\(^3\)See Perron (1990) for discussion.
money targeting regime in the UK the policy makers were constantly changing what they were targeting and the targets themselves, leading to higher persistence. Persistence tends to be higher during the Deutsche Mark (DM) shadowing period as the government’s primary aim then was to defend the peg. During the 1970s when the government introduced Incomes Policy as a means of controlling inflation, there was no nominal anchor hence we get the highest persistence parameters.

7.4.2 Measures of Persistence

As is customary in this strand of literature we shall assume that inflation follows a stationary autoregressive process of order $p$ $AR(p)$, which we write as:

$$\pi_t = \mu + \sum_{j=1}^{p} \alpha_j \pi_{t-j} + \epsilon_t$$

where $\epsilon_t$ is a serially uncorrelated but possibly heteroscedastic random error term. In order to facilitate the discussion that follows we first note that the above model may be reparameterised as:

$$\Delta \pi_t = \mu + \sum_{j=1}^{p-1} \delta_j \Delta \pi_{t-j} + (\rho - 1) \pi_{t-1} + \epsilon_t$$

where

$$\rho = \sum_{j=1}^{p} \alpha_j$$

and

$$\delta_j = -\sum_{t=1+j}^{p} \alpha_i$$

In the context of the above model persistence can be defined as the speed with which inflation converges to equilibrium after a shock in the disturbance term: given a shock that raises inflation today by 1% how long does it take for the effect of the shock to die off?

As Marques (2004) states “the concept of persistence is intimately linked to the impulse
response function (IRF) of the AR(p) process." However, being an infinite-length vector the
IRF is not a useful measure of persistence. In the literature several scalar statistics have been
proposed to measure inflation persistence. These include the 'sum of autoregressive coefficients',
the 'spectrum at zero frequency', the 'largest autoregressive root' and the 'half-life'.

Andrews and Chen (1994) argue that the cumulative impulse response function (CIRF) is
a good way of summarising the information contained in the impulse response function, and
hence a good scalar measure of persistence. In a simple AR(p) process, the CIRF is given by

\[
CIRF = \frac{1}{1 - \rho}
\]

where \( \rho \) is the 'sum of autoregressive coefficients'. As there exists a monotonic relationship
between CIRF and \( \rho \) it follows that one can simply rely on the sum of AR coefficients, \( \rho = \sum \alpha_j \),
as the best scalar measure of persistence. We must point out that all scalar measures of
persistence should be seen as giving an estimate of the 'average speed' with which inflation
converges to equilibrium after a shock to the system. The reliability of the measure would
receive a boost if the speed of convergence is more uniform throughout the convergence period.

An alternative measure of persistence widely used in the literature is given by the largest
AR root \( \gamma \), that is, the largest root of the characteristic equation

\[
\lambda^K - \sum \alpha_j \lambda^{K-j} = 0
\]

---

1. The 'spectrum at zero frequency' is a well-known measure of the low frequency autocovariance of the series. For the AR(p) process it is given by \( h(0) = \frac{\sigma^2}{(1-\rho)^2 \sigma^2} \), where \( \sigma^2 \) stands for the variance of \( \epsilon_t \). However, where one wants to test for changes in persistence over time, the use of 'spectrum at zero frequency' becomes problematic as changes in persistence will be brought about not only by changes in \( \rho \) but also by changes in \( \sigma^2 \).

2. The 'half-life' is defined as the number of periods for which the effect of a unit shock to inflation remains above 0.5. It is a very popular measure of persistence particularly in the literature that evaluates the persistence of deviations from the 'purchasing power parity equilibrium'. See for example Rossi (2001) and Murny and Papell (2002). For criticism of this measure of persistence see Pivetta and Reis (2004).

3. Impulse Response Functions (IRF) are an intuitive way to interpret measures of inflation persistence. IRF gives the response of inflation at various future dates to a shock that occurs today. CIRF as the name suggests is the concept of cumulative impact of a shock and is well documented in Hamilton (1994).

4. Andrews and Chen (1994) note that CIRF and thus \( \rho \) may not be sufficient to fully capture all the shapes in the impulse response functions. For e.g. CIRF and \( \rho \) will not be able to distinguish between two series in which one exhibits a large initial increase and then a subsequent quick decrease in the IRF while the other exhibits a relatively small initial increase followed by a subsequent slow decrease in the IRF.

5. See, for instance Stock (2002).
It is easy to show that in the distant future, the impulse response of inflation to a shock becomes increasingly dominated by the largest root, so the size of \( \gamma \) is a key determinant of how long the effects of the shock will persist. When \( \gamma = 1 \), the process is infinitely persistent since, given a shock, we expect inflation never to revert to its initial value. When \( \gamma = 0 \), inflation is white noise and there is no persistence. In between, \( 0 < \gamma < 1 \), the higher is \( \gamma \), then the longer to first approximation – it will take for inflation to come back to the original level, after a shock.

However, Phillips (1991), Andrews (1993a), Andrews and Chen (1994) and Pivetta and Reis (2004), criticise this measure of persistence. The main point of criticism is that the shape of the IRF depends on all the roots of the equation, not just the largest one. Hence, this statistic is a very poor summary of the impulse response function. According to Andrews and Chen (1994) and Marques (2004) \( \rho \) is more informative than the largest \( AR \) root as a measure of overall persistence. Despite this drawback, the largest \( AR \) root is still widely used as a measure of persistence. Levin, Natalucci and Piger (2004) argue that the largest \( AR \) root has intuitive appeal as a measure of inflation persistence, as it determines the size of the impulse response, \( \frac{b_{\pi_{t+j}}}{b_{\varepsilon_t}} \), as \( j \) grows large. The other reason being that an asymptotic theory has been developed and appropriate software is available so that it is quite easy to compute asymptotically valid confidence intervals for the corresponding estimates.\(^9\)

Levin, Natalucci and Piger (2004) also show how the volatility of inflation can be decomposed into two sources: one due to the variance of the shocks to the autoregression and the other due to the propagation of shocks through autoregressive dynamics. The measure they use is the ratio of the total variance of inflation series to the variance of shocks to the autoregression:

\[
\frac{\text{Var}(\pi_t)}{\text{Var}(\varepsilon_t)}
\]

When the ratio is only slightly above unity, then that is consistent with a white noise process for the inflation series. If the ratio is nearer or above \( 2.0 \) then it means that the volatility of inflation contains a substantial propagation component. We have not used this measure however as we have felt able to interpret the time series equation coefficients themselves adequately.

7.4.3 Estimation of Inflation Persistence

In the analysis below we measure the degree of persistence of the inflation process in terms of the sum of the AR coefficients, $\rho$; henceforth referred to as the 'persistence parameter'. We also calculate the largest AR root so that our results are comparable with others in the literature.

To obtain an estimate of $\rho$, an AR lag order $K$ must be chosen for each inflation series. For this purpose, we utilise AIC, the information criterion proposed by Akaike (1973), with a maximum lag order of $K = 4$ considered. The lag order chosen for each series is reported in Table 7.1. While not reported here, we have found that using SIC – the criterion proposed by Schwarz (1978) – does not alter any of the conclusions reached in this chapter.

<table>
<thead>
<tr>
<th>Monetary Regime</th>
<th>Lag Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Exchange Rate: US (1956:1 to 1970:4)</td>
<td>1</td>
</tr>
<tr>
<td>Incomes Policy (1971:1 to 1978:4)</td>
<td>1</td>
</tr>
<tr>
<td>Money Targeting (1979:1 to 1985:4)</td>
<td>1</td>
</tr>
<tr>
<td>Fixed Exchange Rate: Germany (1986:1 to 1992:3)</td>
<td>1</td>
</tr>
<tr>
<td>Flexible/Strict Inflation Targeting (RPI) (1992:4 to 2003:3)</td>
<td>1</td>
</tr>
<tr>
<td>Flexible/Strict Inflation Targeting (RPIX) (1992:4 to 2003:3)</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 7.1: AR Lag Order for UK Monetary Policy Regimes

Initially we ran ARs on annual inflation (year-on-year) for different regimes. However, that gave us high autoregression due to the presence of a moving average component. In the chapter inflation is calculated as quarter-on-quarter inflation annualised. Figure 7.A.1 in the appendix plots the inflation series. Further, as inflation is quarter-on-quarter one has to take into account seasonality, which we have done by using seasonal dummies.

We have estimated both 'gross' as well as 'net' persistence as explained earlier. To capture the 'gross persistence' we estimated ARMA$s$ for the different regimes.10 They are reported in Table 7.2.11 If one were to look at the AR coefficient in these results, one can however be misled about the degree of persistence. It is important to evaluate the persistence taking into account both the autoregressive as well as moving average effects that are working on inflation. In many of the regimes the MA process works against the AR process thus leading to low 'net' persistence. Compare for example the ARMA and AR results for Inflation Targeting. Using

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10 Seasonal dummies have been taken into account.
11 Details can be found in the appendix, Figure 7.A.8 to 7.A.13.
RPI the best-fitting ARMA is of order (3, 3), with the AR coefficient with quite high values, a surprising result for this regime. However, this regime also has a strong MA process working on it. The 'net' persistence that we get is 0.20 and that too not significant at conventional levels of significance. The bottom line is that persistence should be inferred from the pure ARs, however to 'understand' what is going on in the inflation time series it is imperative to analyse the ARMA.s.

<table>
<thead>
<tr>
<th>Different Monetary Regimes</th>
<th>FUS/Bretton Woods IP</th>
<th>MT</th>
<th>FGR/ERM IT (RPI)</th>
<th>IT (RPIX)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>0.414439</td>
<td>1.720229</td>
<td>0.925756</td>
<td>0.629426</td>
</tr>
<tr>
<td></td>
<td>(0.0993)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0009)</td>
</tr>
<tr>
<td>AR(2)</td>
<td>-0.353781</td>
<td>-1.707837</td>
<td>0.514976</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0221)</td>
<td>(0.0000)</td>
<td>(0.0021)</td>
<td></td>
</tr>
<tr>
<td>AR(3)</td>
<td>-0.137642</td>
<td>0.722245</td>
<td>-0.835545</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1966)</td>
<td>(0.0001)</td>
<td>(0.0000)</td>
<td></td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.968766</td>
<td>-0.997379</td>
<td>-0.761763</td>
<td>0.683187</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>MA(2)</td>
<td>0.984350</td>
<td>-0.701631</td>
<td>-0.359680</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0269)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0561)</td>
</tr>
<tr>
<td>MA(3)</td>
<td>0.759041</td>
<td>0.971221</td>
<td>0.000744</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0971)</td>
<td>(0.594662)</td>
</tr>
<tr>
<td>MA(4)</td>
<td></td>
<td></td>
<td></td>
<td>(0.0010)</td>
</tr>
</tbody>
</table>

N.B. Figures in brackets are the p values.

Table 7.2: Best Fitting ARMAs for UK Monetary Policy Regimes

To evaluate the 'net' persistence we have estimated pure ARs with seasonal dummies. The ARs are reported in the Table 7.3. As the lag order chosen for all the regimes is 1, both the measures of inflation persistence, \( \rho \) the sum of AR coefficients and \( \gamma \) the largest AR root will give us the same persistence. As noted in the opening paragraph of this section Inflation Targeting is the least persistent regime, in fact the lag is insignificant using both the RPI and RPIX. The period of the Bretton Woods era comes in second, the lag being significant only at 10 percent. Money Targeting comes in third with an AR coefficient of 0.52. The period of

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12 Detailed results can be found in the appendix, Figure 7.A.2 to 7.A.7.

13 Finding effectively zero persistence once Inflation Targeting (IT) was instituted is rather re-assuring — it implies that from the introduction of IT the economic agents may have treated monetary policy and the pursuit of the inflation target of 2.5 percent per year as plausible, and hence based their actions on expected inflation, rather than looking backwards at past inflation as a guide to the future.
DM shadowing culminating in UK joining the ERM comes in fourth with an \( AR \) coefficient of 0.63. The period with highest persistence is of course the Incomes Policy regime.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Best Fitting Autoregression</th>
<th>Estimated Coefficient for ( AR(1) )</th>
<th>( p ) value</th>
</tr>
</thead>
<tbody>
<tr>
<td>FUS/Bretton Woods</td>
<td>0.252211</td>
<td>0.0676</td>
<td></td>
</tr>
<tr>
<td>IP</td>
<td>0.735547</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>MT</td>
<td>0.516666</td>
<td>0.0090</td>
<td></td>
</tr>
<tr>
<td>FGR/ERM</td>
<td>0.629426</td>
<td>0.0009</td>
<td></td>
</tr>
<tr>
<td>IT (RPI)</td>
<td>0.202199</td>
<td>0.1999</td>
<td></td>
</tr>
<tr>
<td>IT (RPIX)</td>
<td>-0.152273</td>
<td>0.3380</td>
<td></td>
</tr>
<tr>
<td>Full Sample</td>
<td>0.743048</td>
<td>0.0000</td>
<td></td>
</tr>
</tbody>
</table>

N.B. The best fitting AR has been run taking into account seasonal dummies.

Table 7.3: Best Fitting AR for UK Monetary Policy Regimes

As demonstrated by Perron (1990), the degree of persistence of a given time series will be exaggerated if one fails to recognise the presence of a break in the mean of the process. It is therefore important to obtain formal econometric evidence about the presence or absence of structural breaks in the inflation series. This can be done using classical and Bayesian methods used to evaluate the evidence for structural breaks.\(^{14}\) However, if one possesses a priori knowledge of the break date, then one can simply estimate the univariate \( AR \) process for the inflation series over the sub-samples and then apply the breakpoint test of Chow (1960).

The Chow breakpoint test partitions data in two or more sub-samples. The test compares the sum of squared residuals obtained by fitting a single equation to the entire sample with the sum of squared residuals obtained when separate equations are fit to each sub-sample of the data. Significant differences in the estimated equations indicate a structural change in the relationship. We conducted a Chow breakpoint test to check if there were in fact structural changes in the economy. The results are reported in the Table 7.4 below. We do not accept the null of no structural change.

\[\text{Chow Breakpoint Test: } 1971:1\; 1979:1\; 1986:1\; 1992:4\]
| F-statistic      | 2.983189 | Probability | 0.000067 |
| Log likelihood ratio | 58.64247 | Probability | 0.000012 |

Table 7.4: Chow Stability Test

\(^{14}\)See Levin and Piger (2004).
Following Batini (2002), we have also estimated a regression of inflation on a constant for each of the sub-samples. Table 7.5 summarises the results of this exercise. The regression on a constant provides very useful summary statistics: its estimated parameter corresponds to the sample mean of inflation, while the residual standard error corresponds to inflation's standard deviation. If the mean of inflation has gone down, this may explain also why the variance of inflation has dropped over time. There is considerable evidence that inflation variability and the level of inflation are positively related across countries. David and Kanago (2000) review this evidence for the OECD countries.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Regression on a Constant</th>
<th>S.E. of Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>FUS/Bretton Woods</td>
<td>0.036213</td>
<td>0.033687</td>
</tr>
<tr>
<td>IP</td>
<td>0.131845</td>
<td>0.077915</td>
</tr>
<tr>
<td>MT</td>
<td>0.093772</td>
<td>0.071213</td>
</tr>
<tr>
<td>FGR/ERM</td>
<td>0.056810</td>
<td>0.041171</td>
</tr>
<tr>
<td>IT (RPI)</td>
<td>0.024828</td>
<td>0.024938</td>
</tr>
<tr>
<td>IT (RPIX)</td>
<td>0.025481</td>
<td>0.022338</td>
</tr>
<tr>
<td>Full Sample</td>
<td>0.061092</td>
<td>0.062856</td>
</tr>
</tbody>
</table>

Table 7.5: Regression on a Constant

In case of the UK we observe a clear drop in mean inflation from the high value of 13.18% p.a. during the 1970s to the low and stable value of 2.5% during the last decade. The variance of inflation has also fallen during this period. The mean inflation during the Bretton Woods era was 3.62% and DM shadowing was 5.68%. During the early 80s it was 9.4% along with a standard error of 7.12%.

7.5 Conclusions

In this chapter we look at the empirical evidence on UK inflation, carefully separating the data into periods of different monetary policy regimes. We have established that inflation persistence in the UK varies depending upon the monetary policy regimes in place. It is highest during the 1970s – the period of no nominal anchor – when the government introduced Incomes Policy as a means of controlling inflation. The lowest persistence is found in the Inflation Targeting period, as one would expect a priori. Bretton Woods period comes in next lowest, followed by Money
Targeting. *A priori* one would expect Monetary Targeting to come in second lowest; however during the Money Targeting regime in the UK the policy makers were constantly changing what they were targeting and the targets themselves, leading to higher persistence. During the DM shadowing period the government’s primary aim was to defend the peg; hence persistence tends to be higher. In what follows, to anticipate, we elucidate straightforward new classical models for the various monetary policy regimes and then test whether they can generate the facts of inflation persistence as we find them.
7.A Appendix

7.A.1 Inflation Plot

![Graph of Inflation](image)

Figure 7.A.1: Quarter-on-quarter Inflation Annualised
7. A. 2  Best Fitting ARs for UK Monetary Policy Regimes

**AR for Fixed Exchange Rate (FUS) 1956:1 till 1970:4**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI(-1)</td>
<td>0.252211</td>
<td>0.135162</td>
<td>1.865989</td>
<td>0.0676</td>
</tr>
<tr>
<td>@SEAS(1)</td>
<td>0.025167</td>
<td>0.009141</td>
<td>3.190735</td>
<td>0.0024</td>
</tr>
<tr>
<td>@SEAS(2)</td>
<td>0.046141</td>
<td>0.009181</td>
<td>5.025728</td>
<td>0.0000</td>
</tr>
<tr>
<td>@SEAS(3)</td>
<td>-0.007564</td>
<td>0.010671</td>
<td>-0.708814</td>
<td>0.4815</td>
</tr>
<tr>
<td>@SEAS(4)</td>
<td>0.039690</td>
<td>0.007339</td>
<td>5.408416</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared: 0.337852  Mean dependent var: 0.035406
Adjusted R-squared: 0.287879  S.D. dependent var: 0.033400
S.E. of regression: 0.028185  Akaike info criterion: -4.217785
Sum squared resid: 0.042103  Schwarz criterion: -4.040161
Log likelihood: 127.3158  Durbin-Watson stat: 1.947207

Figure 7.A.2: AR for Fixed Exchange Rate US
AR for Incomes Policy (IP) 1971:1 till 1978:4

Dependent Variable: PI
Method: Least Squares
Date: 09/08/04 Time: 16:12
Sample: 1971:1 1978:4
Included observations: 32

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI(-1)</td>
<td>0.735547</td>
<td>0.131656</td>
<td>5.586884</td>
<td>0.0000</td>
</tr>
<tr>
<td>@SEAS(1)</td>
<td>0.050085</td>
<td>0.023863</td>
<td>2.098825</td>
<td>0.0453</td>
</tr>
<tr>
<td>@SEAS(2)</td>
<td>0.081931</td>
<td>0.025531</td>
<td>3.209159</td>
<td>0.0034</td>
</tr>
<tr>
<td>@SEAS(3)</td>
<td>-0.047070</td>
<td>0.030082</td>
<td>-1.564702</td>
<td>0.1293</td>
</tr>
<tr>
<td>@SEAS(4)</td>
<td>0.053034</td>
<td>0.021331</td>
<td>2.486279</td>
<td>0.0194</td>
</tr>
</tbody>
</table>

R-squared 0.631629  Mean dependent var 0.131845
Adjusted R-squared 0.577056  S.D. dependent var 0.077915
S.E. of regression 0.050671  Akaike info criterion -2.984320
Sum squared resid 0.069324  Schwarz criterion -2.755299
Log likelihood 52.74912  Durbin-Watson stat 1.734798

Figure 7.A.3: AR for Incomes Policy

AR for Money Targeting Regime (MT) 1979:1 till 1985:4

Dependent Variable: PI
Method: Least Squares
Date: 09/08/04 Time: 16:13
Sample: 1979:1 1985:4
Included observations: 28

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI(-1)</td>
<td>0.516666</td>
<td>0.181012</td>
<td>2.854314</td>
<td>0.0090</td>
</tr>
<tr>
<td>@SEAS(1)</td>
<td>0.047971</td>
<td>0.025441</td>
<td>1.885575</td>
<td>0.0720</td>
</tr>
<tr>
<td>@SEAS(2)</td>
<td>0.105603</td>
<td>0.026826</td>
<td>3.936580</td>
<td>0.0007</td>
</tr>
<tr>
<td>@SEAS(3)</td>
<td>0.003096</td>
<td>0.034871</td>
<td>0.088779</td>
<td>0.9300</td>
</tr>
<tr>
<td>@SEAS(4)</td>
<td>0.020941</td>
<td>0.026443</td>
<td>0.791924</td>
<td>0.4365</td>
</tr>
</tbody>
</table>

R-squared 0.424630  Mean dependent var 0.093772
Adjusted R-squared 0.324566  S.D. dependent var 0.071213
S.E. of regression 0.058526  Akaike info criterion -2.678242
Sum squared resid 0.078783  Schwarz criterion -2.440349
Log likelihood 42.49539  Durbin-Watson stat 2.464990

Figure 7.A.4: AR for Money Targeting

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AR for Fixed Exchange Rate: Germany (FGR) 1986:1 till 1992:3

Dependent Variable: PI
Method: Least Squares
Date: 09/08/04 Time: 16:15
Sample: 1986:1 1992:3
Included observations: 27

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI(-1)</td>
<td>0.629426</td>
<td>0.163175</td>
<td>3.857369</td>
<td>0.0009</td>
</tr>
<tr>
<td>@SEAS(1)</td>
<td>0.005575</td>
<td>0.012960</td>
<td>0.430193</td>
<td>0.6712</td>
</tr>
<tr>
<td>@SEAS(2)</td>
<td>0.074814</td>
<td>0.011437</td>
<td>6.541602</td>
<td>0.0000</td>
</tr>
<tr>
<td>@SEAS(3)</td>
<td>-0.035922</td>
<td>0.018827</td>
<td>-1.907968</td>
<td>0.0696</td>
</tr>
<tr>
<td>@SEAS(4)</td>
<td>0.040522</td>
<td>0.011404</td>
<td>3.553183</td>
<td>0.0018</td>
</tr>
</tbody>
</table>

R-squared     0.692220  Mean dependent var 0.056810
Adjusted R-squared 0.636260  S.D. dependent var 0.041171
S.E. of regression 0.024831  Akaike info criterion -4.387909
Sum squared resid  0.013564  Schwarz criterion -4.147939
Log likelihood     64.23676  Durbin-Watson stat 1.933567

Figure 7.A.5: AR for Fixed Exchange Rate Germany

AR for IT (RPI) 1992:4 till 2003:3

Dependent Variable: PI
Method: Least Squares
Date: 09/08/04 Time: 16:20
Sample: 1992:4 2003:3
Included observations: 44

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI(-1)</td>
<td>0.202199</td>
<td>0.155057</td>
<td>1.304028</td>
<td>0.1999</td>
</tr>
<tr>
<td>@SEAS(1)</td>
<td>0.004205</td>
<td>0.005202</td>
<td>0.808372</td>
<td>0.4238</td>
</tr>
<tr>
<td>@SEAS(2)</td>
<td>0.058344</td>
<td>0.004341</td>
<td>13.44163</td>
<td>0.0000</td>
</tr>
<tr>
<td>@SEAS(3)</td>
<td>-0.001419</td>
<td>0.010188</td>
<td>-0.139242</td>
<td>0.8900</td>
</tr>
<tr>
<td>@SEAS(4)</td>
<td>0.018345</td>
<td>0.004400</td>
<td>4.169527</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

R-squared     0.724339  Mean dependent var 0.024828
Adjusted R-squared 0.696066  S.D. dependent var 0.024938
S.E. of regression 0.013748  Akaike info criterion -5.629174
Sum squared resid  0.007371  Schwarz criterion -5.426425
Log likelihood     128.8418  Durbin-Watson stat 2.044634

Figure 7.A.6: AR for Inflation Targeting (RPI)

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### AR for IT (RPIX) 1992:4 till 2003:3

**Dependent Variable:** PIRPIX  
**Method:** Least Squares  
**Date:** 09/08/04  
**Time:** 16:22  
**Sample:** 1992:4 2003:3  
**Included observations:** 44

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PIRPIX(-1)</td>
<td>-0.152273</td>
<td>0.156983</td>
<td>-0.969998</td>
<td>0.3380</td>
</tr>
<tr>
<td>@SEAS(1)</td>
<td>0.016681</td>
<td>0.004178</td>
<td>3.992304</td>
<td>0.0003</td>
</tr>
<tr>
<td>@SEAS(2)</td>
<td>0.062099</td>
<td>0.003347</td>
<td>18.55519</td>
<td>0.0000</td>
</tr>
<tr>
<td>@SEAS(3)</td>
<td>0.016641</td>
<td>0.009775</td>
<td>1.702345</td>
<td>0.0967</td>
</tr>
<tr>
<td>@SEAS(4)</td>
<td>0.021902</td>
<td>0.002795</td>
<td>7.836655</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

| R-squared    | 0.865861    | Mean dependent var | 0.025481 |
| Adjusted R-squared | 0.852103 | S.D. dependent var | 0.022338 |
| S.E. of regression | 0.008591 | Akaike info criterion | -6.569658 |
| Sum squared resid | 0.002878 | Schwarz criterion | -6.366909 |
| Log likelihood | 149.5325 | Durbin-Watson stat | 2.008503 |

Figure 7.A.7: AR for Inflation Targeting (RPIX)
7.A.3 Best Fitting ARMA\textsubscript{s} for UK Monetary Policy Regimes

**ARMA for Fixed Exchange Rate (FUS) 1956:1 till 1970:4**

Dependent Variable: PI  
Method: Least Squares  
Date: 09/08/04  Time: 16:39  
Sample(adjusted): 1956 4 1970 4  
Included observations: 57 after adjusting endpoints  
Convergence achieved after 15 iterations  
Backcast: 1956 1 1956 3

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI(1)</td>
<td>0.414439</td>
<td>0.152948</td>
<td>2.709681</td>
<td>0.0093</td>
</tr>
<tr>
<td>PI(2)</td>
<td>-0.353781</td>
<td>0.149547</td>
<td>-2.365678</td>
<td>0.0221</td>
</tr>
<tr>
<td>@SEAS(1)</td>
<td>0.025780</td>
<td>0.010441</td>
<td>2.469116</td>
<td>0.0172</td>
</tr>
<tr>
<td>@SEAS(2)</td>
<td>0.055665</td>
<td>0.010987</td>
<td>5.066490</td>
<td>0.0000</td>
</tr>
<tr>
<td>@SEAS(3)</td>
<td>0.000535</td>
<td>0.011954</td>
<td>0.044778</td>
<td>0.9645</td>
</tr>
<tr>
<td>@SEAS(4)</td>
<td>0.060960</td>
<td>0.011561</td>
<td>5.273132</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.137642</td>
<td>0.105100</td>
<td>-1.309629</td>
<td>0.1966</td>
</tr>
<tr>
<td>MA(2)</td>
<td>0.227487</td>
<td>0.099649</td>
<td>2.282876</td>
<td>0.0269</td>
</tr>
<tr>
<td>MA(3)</td>
<td>0.759041</td>
<td>0.100356</td>
<td>7.563482</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared | 0.522354 | Mean dependent var | 0.036269 |
Adjusted R-squared | 0.442746 | S.D. dependent var | 0.033037 |
S.E. of regression | 0.024662 | Akaike info criterion | -4.423199 |
Sum squared resid | 0.029193 | Schwarz criterion | -4.100612 |
Log likelihood | 135.0512 | Durbin-Watson stat | 2.000008 |

Inverted MA Roots: 0.46 ± 0.86i, 0.46 ± 0.86i, -0.79

Figure 7.A.8: ARMA for Fixed Exchange Rate US

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## ARMA for Incomes Policy (IP) 1971:1 till 1978:4

Dependent Variable: PI  
Method: Least Squares  
Date: 09/08/04  Time: 16:42  
Sample: 1971:1 1978:4  
Included observations: 32  
Convergence achieved after 27 iterations  
Backcast: 1970:3 1970:4

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(-1)</td>
<td>1.720229</td>
<td>0.151073</td>
<td>11.38676</td>
<td>0.0000</td>
</tr>
<tr>
<td>P(-2)</td>
<td>-1.707837</td>
<td>0.175883</td>
<td>-9.710096</td>
<td>0.0000</td>
</tr>
<tr>
<td>P(-3)</td>
<td>0.722245</td>
<td>0.151923</td>
<td>4.754016</td>
<td>0.0001</td>
</tr>
<tr>
<td>@SEAS(1)</td>
<td>-0.039412</td>
<td>0.042245</td>
<td>-0.932942</td>
<td>0.3605</td>
</tr>
<tr>
<td>@SEAS(2)</td>
<td>0.087505</td>
<td>0.033672</td>
<td>2.598759</td>
<td>0.0161</td>
</tr>
<tr>
<td>@SEAS(3)</td>
<td>-0.068058</td>
<td>0.037884</td>
<td>-1.796495</td>
<td>0.0756</td>
</tr>
<tr>
<td>@SEAS(4)</td>
<td>0.154600</td>
<td>0.031062</td>
<td>5.326690</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.968766</td>
<td>0.055369</td>
<td>-17.49659</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA(2)</td>
<td>0.984350</td>
<td>0.060160</td>
<td>16.36220</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared      0.695404  Mean dependent var 0.131845  
Adjusted R-squared 0.589457  S.D. dependent var 0.077915  
S.E. of regression 0.049923  Akaike info criterion -2.924423  
Sum squared resid 0.057322  Schwarz criterion -2.512185  
Log likelihood   55.79077  Durbin-Watson stat 1.940594  

Inverted MA Roots \(e^{-.48 + .87i}\) \(e^{-.48 - .87i}\)

---

Figure 7.A.9: ARMA for Incomes Policy
### ARMA for Money Targeting Regime (MT) 1979:1 till 1985:4

Dependent Variable: PI  
Method: Least Squares  
Date: 09/08/04 Time: 16:46  
Sample: 1979:1 1985:4  
Included observations: 28  
Convergence achieved after 17 iterations  
Backcast: 1978:4

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI(-1)</td>
<td>0.925756</td>
<td>0.037742</td>
<td>24.52848</td>
<td>0.0000</td>
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<tr>
<td>@SEAS(1)</td>
<td>0.006996</td>
<td>0.024678</td>
<td>0.283463</td>
<td>0.7795</td>
</tr>
<tr>
<td>@SEAS(2)</td>
<td>0.071150</td>
<td>0.024233</td>
<td>2.936038</td>
<td>0.0076</td>
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<td>@SEAS(3)</td>
<td>-0.057870</td>
<td>0.024673</td>
<td>-2.345483</td>
<td>0.0284</td>
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<tr>
<td>@SEAS(4)</td>
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<td>0.023397</td>
<td>-0.624055</td>
<td>0.5390</td>
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<tr>
<td>MA(1)</td>
<td>-0.997379</td>
<td>0.066516</td>
<td>-17.64775</td>
<td>0.0000</td>
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</table>

R-squared 0.674493  Mean dependent var 0.093772  
Adjusted R-squared 0.600514  S.D. dependent var 0.071213  
S.E. of regression 0.045010  Akaike info criterion -3.176442  
Sum squared resid 0.044570  Schwarz criterion -2.890669  
Log likelihood 50.47018  Durbin-Watson stat 2.120261

Figure 7.A.10: ARMA for Money Targeting

### ARMA for Fixed Exchange Rate: Germany (FGR) 1986:1 till 1992:3

Dependent Variable: PI  
Method: Least Squares  
Date: 09/08/04 Time: 18:05  
Sample: 1986:1 1992:3  
Included observations: 27

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
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<tr>
<td>PI(-1)</td>
<td>0.629426</td>
<td>0.163175</td>
<td>3.857369</td>
<td>0.0009</td>
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<td>@SEAS(1)</td>
<td>0.005575</td>
<td>0.012960</td>
<td>0.430193</td>
<td>0.6712</td>
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<tr>
<td>@SEAS(2)</td>
<td>0.074814</td>
<td>0.011437</td>
<td>6.541602</td>
<td>0.0000</td>
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<td>@SEAS(3)</td>
<td>-0.035922</td>
<td>0.018827</td>
<td>-1.907968</td>
<td>0.0696</td>
</tr>
<tr>
<td>@SEAS(4)</td>
<td>0.040522</td>
<td>0.011404</td>
<td>3.553183</td>
<td>0.0018</td>
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</table>

R-squared 0.692220  Mean dependent var 0.056810  
Adjusted R-squared 0.636260  S.D. dependent var 0.041171  
S.E. of regression 0.024831  Akaike info criterion -4.387909  
Sum squared resid 0.013564  Schwarz criterion -4.147939  
Log likelihood 64.23576  Durbin-Watson stat 1.933567

Figure 7.A.11: ARMA for Fixed Exchange Rate Germany
ARMA for IT (RPI) 1992:4 till 2003:3

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI(-1)</td>
<td>0.720498</td>
<td>0.091068</td>
<td>7.911688</td>
<td>0.0000</td>
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<tr>
<td>PI(-2)</td>
<td>0.514976</td>
<td>0.154413</td>
<td>3.335048</td>
<td>0.0021</td>
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<tr>
<td>PI(-3)</td>
<td>-0.835545</td>
<td>0.999350</td>
<td>-8.410095</td>
<td>0.0000</td>
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<tr>
<td>@SEAS(1)</td>
<td>0.040659</td>
<td>0.008304</td>
<td>4.920215</td>
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<tr>
<td>@SEAS(2)</td>
<td>0.051110</td>
<td>0.006453</td>
<td>7.919977</td>
<td>0.0000</td>
</tr>
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<td>@SEAS(3)</td>
<td>-0.021042</td>
<td>0.008941</td>
<td>-2.379818</td>
<td>0.0231</td>
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<tr>
<td>@SEAS(4)</td>
<td>-0.009923</td>
<td>0.009699</td>
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<tr>
<td>MA(1)</td>
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<td>0.047080</td>
<td>-16.18010</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-0.701631</td>
<td>0.040454</td>
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<td>0.0000</td>
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<td>MA(3)</td>
<td>0.971221</td>
<td>0.071546</td>
<td>13.57482</td>
<td>0.0000</td>
</tr>
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</table>

R-squared: 0.814881  Mean dependent var: 0.024828
Adjusted R-squared: 0.765879  S.D. dependent var: 0.024938
S.E. of regression: 0.012066  Akaike info criterion: -5.800076
Sum squared resid: 0.004950  Schwarz criterion: -5.394578
Log likelihood: 137.8017  Durbin-Watson stat: 1.975813

Inverted MA Roots: 87.49i, 87.49i, -98

Figure 7.A.12: ARMA for Inflation Targeting (RPI)
## ARMA for IT (RPIX) 1992:4 till 2003:3

Dependent Variable: PIRPIX  
Method: Least Squares  
Date: 09/08/04  Time: 18:43  
Sample: 1992:4 2003:3  
Included observations: 44  
Convergence achieved after 24 iterations  

<table>
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<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PIRPIX(-1)</td>
<td>-0.743549</td>
<td>0.115369</td>
<td>-6.444947</td>
<td>0.0000</td>
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<td>@SEAS(1)</td>
<td>0.030704</td>
<td>0.004857</td>
<td>6.321113</td>
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<td>@SEAS(2)</td>
<td>0.070139</td>
<td>0.004781</td>
<td>14.669111</td>
<td>0.0000</td>
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<tr>
<td>@SEAS(3)</td>
<td>0.051468</td>
<td>0.007977</td>
<td>6.451915</td>
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<tr>
<td>@SEAS(4)</td>
<td>0.025297</td>
<td>0.004343</td>
<td>5.825121</td>
<td>0.0000</td>
</tr>
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<td>MA(1)</td>
<td>0.683187</td>
<td>0.152514</td>
<td>4.479513</td>
<td>0.0001</td>
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<td>MA(2)</td>
<td>-0.359680</td>
<td>0.182014</td>
<td>-1.976107</td>
<td>0.0561</td>
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<tr>
<td>MA(3)</td>
<td>0.000744</td>
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<td>0.9971</td>
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<td>MA(4)</td>
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</table>

R-squared     | 0.891576 | Mean dependent var | 0.025481 |
Adjusted R-squared | 0.866794 | S.D. dependent var | 0.022338 |
S.E. of regression | 0.008153 | Akaike info criterion | -6.600668 |
Sum squared resid | 0.002326 | Schwarz criterion | -6.235720 |
Log likelihood   | 154.2147 | Durbin-Watson stat | 1.986331 |

Inverted MA Roots: 55, -55i, .55 + .55i, -.89 - .44i, -.89 + .44i

Figure 7.A.13: ARMA for Inflation Targeting (RPIX)
Chapter 8

New Classical Models for UK
Monetary Policy Regimes

8.1 Introduction

The objective of this chapter is to elucidate rather straightforward models, easily micro-founded in a standard classical set-up to characterise each UK monetary regime of the post-war period. We are not particularly committed to these models in detail, but rather use them as a benchmark in which a minimal number of special assumptions are made — such as particular forms of 'nominal rigidity' and 'adjustment'. We note that in any such model omitted variables will create error processes, and it is perfectly reasonable to find that these are themselves autocorrelated. These processes will then propagate themselves through all the endogenous variables and be a natural source of persistence. However, if the monetary authorities are so determined, they may partly suppress this persistence through their monetary reactions; arguably just such a determination was observed when Inflation Targeting was instituted by the Treasury in 1992 after the UK's forced exit from the Exchange Rate Mechanism (ERM).

The chapter is organised as follows. Section 8.2 provides a brief introduction to our modelling framework. Section 8.3 describes the model for the Bretton Woods period, Section 8.4 lays out how we model the Incomes Policy regime of the 1970s, Section 8.5 elucidates the model for the Money Targeting regime and Section 8.6 describes the model for the period of Deutsche Mark shadowing, which culminated in UK's membership of the ERM. Section 8.7 explains how we
model the more recent Inflation Targeting regime — flexible as well as strict. The conclusions are in Section 8.8.

8.2 Basic Structural Model

The models developed in this chapter are along the lines of McCallum and Nelson (1999a). As the reduced form of their model arises from a explicit optimisation analysis of the dynamic choice problems faced by rational economic agents the authors contend that it confers some degree of policy-invariance on model parameters and hence some immunity from the Lucas critique.

In all the models described in this chapter the first equation is the IS curve of the expectational variety that includes $E_i l_{t+1}$ as in Kerr and King (1996), McCallum and Nelson (1999a) and Rotemberg and Woodford (1997). This modification imparts a dynamic, forward-looking aspect to saving behaviour and leads to a model of aggregate demand that is tractable and also usable with a wide variety of aggregate supply specifications. As noted by McCallum and Nelson (1999a) this optimising IS function can be regarded as a transformation of the structural consumption Euler equation, with the market-clearing condition for output substituted into it; the error term captures stochastic movements in government spending, exports etc. In the case of fixed exchange rate regimes we have an additional expenditure switching effect in the IS curve — which routinely emerges from both Old and New Open Economy Models.

The second equation in the models is the New Classical Phillips curve — this can be regarded as the reduced form of the flex-price supply equations, assuming a one-period information lag. Finally under Inflation Targeting regimes where policy makers respond to the output gap, we take account of Orphanides (1998)-style output gap forecast errors as well.

Nelson (2000) provides estimates for the UK of the Taylor rule for several different monetary regimes in the period 1972-97; prior to the Bank of England receiving operational indepen-

---

1 Fane (1985) and Koenig (1989, 1993a, 1993b) represent previous efforts in the same direction. However, they only show that some comparative-static properties of their models are like those of an IS-LM set-up. Specifically, they do not develop dynamic equations analogous to IS and LM functions, as in McCallum and Nelson (1999a). Auerbach and Kotlikoff (1995) derive IS and LM equations from an overlapping generations framework under the very restrictive assumption of rigid prices.

2 The period of Bretton Woods and shadowing the Deutsche Mark.

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dence. His results suggest that prior to 1992, it is difficult to characterise UK monetary policy using a standard Taylor rule. During these regimes policy makers were constantly changing the rules, what they were targeting and the targets themselves. In this chapter we impose the restrictions that we think existed in those periods. Thus our models can be thought of as simple approximations of actual policy behaviour during each regime.

To ensure stationarity of output we detrend it by a Hodrick-Prescott filter. We assume the other variables to be stationary within each regime period: thus either they are assumed in the case of real variables to have constant equilibria e.g. the real exchange rate or the monetary policy regime is assumed to be aiming for some constant nominal equilibria on inflation, hence on interest rates.

8.3 Fixed Exchange Rate Regime (US) or Bretton Woods (1956:1 to 1970:4)

Our first regime is the Bretton Woods fixed exchange rate system. This is not easy to model because of its progressive deterioration in the 1960s when 'one-off' exchange rate changes became commonplace means of adjustment. Another important factor causing change was the progressive dismantling of direct controls – including a relaxation of controls on international capital flows – which, while certainly adding to the potential macro-economic benefits from international economic activity, undoubtedly made fixed exchange rates inherently more difficult to sustain. Furthermore, countries within the system came to attach different priorities to inflation and unemployment as the immediate objective of policy. There was also disagreement about how the burden of domestic policy adjustment should be shared between surplus and deficit countries, including the US, the country of the anchor currency. The system eventually collapsed under the weight of the outflows of the US dollar, which, under the parity system, had to be taken into other countries' official reserves, on such a scale that the dollar's official

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1In a famous paper, Taylor (1993) showed that US monetary policy after 1986 is well characterised by a rule for the Federal Funds rate whereby the interest rate responds to output gap and inflation deviation from target. There has subsequently been an explosion of theoretical and empirical work in this area. See for example Clarida, Gali and Gertler (2000).

2For our choice of regime dates see Minford (1993), Nelson (2000) and Budd (2002).

3See Figure 8.A.1 and 8.A.2 in the appendix.

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convertibility into gold had eventually to be formally suspended in 1971. 

Here we have made drastic simplifications, ignoring parity changes and assuming a high degree of capital mobility throughout. Equations (8.1) and (8.3) – (8.7) when put together are the IS or demand side of the model; the errors entering here from a variety of exogenous shocks, apart from $R_{FUS_t}$, are aggregated into $u_{FUS_t}$.

\[ \tilde{y}_t = \gamma(E_t \tilde{y}_{t+1}) - \alpha(R_t - E_t P_{t+1} + P_t) + \lambda(E_t N X_{t+1}) + u_{FUS_t} \]  

(8.1)

\[ \tilde{y}_t = \delta(P_t - E_{t-1} P_t) + v_{FUS_t} \]  

(8.2)

\[ N X_t = a_{FUS_0} Q_{FUS_t} + a_{FUS_1} y^F_{FUS_t} \]  

(8.3)

\[ Q_{FUS_t} = S_{FUS_t} + P^F_{FUS_t} - P_t \]  

(8.4)

\[ R_t = R^F_{FUS_t} + (E_t S_{FUS_{t+1}} - S_{FUS_t}) \]  

(8.5)

\[ S_{FUS_t} = \bar{S}_{FUS} \]  

(8.6)

\[ R^F_{FUS_t} = \rho_{FUS} R^F_{FUS_{t-1}} + \eta_{FUS} \]  

(8.7)

\[ u_{FUS_t} = \rho_{FUS_0} u_{FUS_{t-1}} + \epsilon_{FUS_t} \]  

(8.8)

\[ v_{FUS_t} = \rho_{FUS_1} v_{FUS_{t-1}} + x_{FUS_t} \]  

(8.9)
\[ y_{\text{FUS}_t}^F = \rho_{\text{FUS}_t} y_{\text{FUS}_{t-1}}^F + \theta_{\text{FUS}_t} \quad (8.10) \]

\[ P_{\text{FUS}_t}^I = \rho_{\text{FUS}_t} P_{\text{FUS}_{t-1}}^I + \kappa_{\text{FUS}_t} \quad (8.11) \]

In the equations above, \( \tilde{y}_t \) is the output gap defined as log GDP - log GDP trend, \( R_t \) the nominal interest rate is the Bank of England base rate, \( P_t \) is the price level, \( N X_t \) is net exports, \( Q_{\text{FUS}_t} \) is the real exchange rate, \( S_{\text{FUS}_t} \) is the nominal exchange rate defined as \( \mathcal{L}/\$ \), \( P_{\text{FUS}_t}^I \) is the US price level (CPI), \( y_{\text{FUS}_t}^F \) is US GDP which is used as a proxy for world income and \( R_{\text{FUS}_t}^F \) is the US federal funds rate (nominal). Equations (8.1) and (8.2) are based on aggregate demand and supply specifications that are designed to reflect rational optimising behaviour on the part of the economy’s private actors. Equation (8.1) is a forward-looking open economy IS curve. The error term \( u_{\text{FUS}_t} \) can be interpreted as the demand shock to the economy which we have modelled as an AR(1) process. Equation (8.2) is a standard New Classical Phillips Curve where \( v_{\text{FUS}_t} \) is the productivity shock modelled as an AR(1). Equation (8.3) simply puts forth the idea that the net exports of a country is a function of the real exchange rate and the world income. If the real exchange rate depreciates or the world income is higher, then there would be a greater demand for the domestic exports. Equation (8.4) is the definition of real exchange rate and equation (8.5) is the Uncovered Real Interest Rate Parity (URIP) condition. Equation (8.6) simply states that the nominal exchange rate is fixed, as we are in a fixed exchange rate regime. We have modelled the world interest rate, GDP and prices as an AR(1) process. The error terms in equation (8.7) to (8.11) are all i.i.d.

Now substituting equation (8.6) in (8.5) yields:

\[ R_t = R_{\text{FUS}_t}^F = \frac{\eta_{\text{FUS}_t}}{1 - \rho_{\text{FUS}_L}} \quad (8.12) \]

Leading equation (8.3) and (8.4) forward one-period, and taking expectations we have

\[ E_t N X_{t+1} = a_{\text{FUS}_0} E_t Q_{\text{FUS}_{t+1}} + a_{\text{FUS}_1} E_t y_{\text{FUS}_{t+1}}^F \quad (8.13) \]

*Figure 8.A.3 in the appendix plots the nominal exchange rate.*

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\[ E_t Q_{FUS_{t+1}} = \overline{S}_{FUS} + E_t P_{FUS_{t+1}}^t - E_t P_{t+1} \]  

(8.14)

Substituting equation (8.14) in (8.13) and using equation (8.10) and (8.11) we have

\[ E_t N_{X_{t+1}} = a_{FUS_0} \overline{S}_{FUS} + a_{FUS_0} \left( \frac{\rho_{FUS_1}}{1 - \rho_{FUS_3} L} \right) \kappa_{FUS_t} - a_{FUS_0} E_t P_{t+1} + a_{FUS_1} \left( \frac{\rho_{FUS_2}}{1 - \rho_{FUS_4} L} \right) \theta_{FUS_t} \]  

(8.15)

Now substituting equation (8.15) for \( E_t N_{X_{t+1}} \) and equation (8.2) for \( \tilde{y}_t \), using \( E_t \tilde{y}_{t+1} = \rho_{FUS_1} v_{FUS_t} \) and equation (8.12) in equation (8.1) yields:

\[ \delta (P_t - E_{t-1} P_t) + \frac{x_{FUS_t}}{1 - \rho_{FUS_1} L} = \gamma \left( \frac{\rho_{FUS_1}}{1 - \rho_{FUS_1} L} \right) x_{FUS_t} - \alpha \left[ \left( \frac{\eta_{FUS_t}}{1 - \rho_{FUS} L} \right) - E_t P_{t+1} + P_t \right] + \lambda \left[ a_{FUS_0} \overline{S}_{FUS} + a_{FUS_0} \left( \frac{\rho_{FUS_1}}{1 - \rho_{FUS_3} L} \right) \kappa_{FUS_t} \right] + \left( \frac{\theta_{FUS_t}}{1 - \rho_{FUS_4} L} \right) \]  

(8.16)

Now using the Muth Method, the Wold decomposition for \( P_t \) in this model is:

\[ P_t = Cons + \sum_{i=0}^{\infty} q_i \varepsilon_{FUS_{t-i}} + \sum_{i=0}^{\infty} r_i x_{FUS_{t-i}} + \sum_{i=0}^{\infty} s_i \eta_{FUS_{t-i}} + \sum_{i=0}^{\infty} d_i \kappa_{FUS_{t-i}} + \sum_{i=0}^{\infty} l_i \theta_{FUS_{t-i}} \]  

(8.17)

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Using the Wold decomposition equation (8.16) can be written as

\[
\delta(q_0\varepsilon_{FUS_t} + r_0x_{FUS_t} + s_0\eta_{FUS_t} + d_0\kappa_{FUS_t} + l_0\theta_{FUS_t}) + \frac{x_{FUS_t}}{1 - \rho_{FUS_t}L} = \gamma \left( \frac{\rho_{FUS_t}}{1 - \rho_{FUS_t}L} \right)x_{FUS_t} - \alpha \left( \frac{\eta_{FUS_t}}{1 - \rho_{FUS}L} \right) + (\alpha - a_{FUS_0}\lambda) \\
\left( \sum_{i=1}^{\infty} q_i\varepsilon_{FUS_{t-i}} + \sum_{i=1}^{\infty} r_i x_{FUS_{t-i}} + \right) \\
\left( \sum_{i=1}^{\infty} s_i \eta_{FUS_{t-i}} + \sum_{i=1}^{\infty} d_i \kappa_{FUS_{t-i}} + \sum_{i=1}^{\infty} l_i \theta_{FUS_{t-i}} \right) \\
- \alpha \left( \sum_{i=1}^{\infty} q_i\varepsilon_{FUS_{t-i}} + \sum_{i=1}^{\infty} r_i x_{FUS_{t-i}} + \right) \\
\left( \sum_{i=1}^{\infty} s_i \eta_{FUS_{t-i}} + \sum_{i=1}^{\infty} d_i \kappa_{FUS_{t-i}} + \sum_{i=1}^{\infty} l_i \theta_{FUS_{t-i}} \right) + \lambda a_{FUS_0} S_{FUS} + \lambda a_{FUS_0} \left( \frac{\rho_{FUS}}{1 - \rho_{FUS}L} \right) \kappa_{FUS_t} + a_{FUS_1}\lambda \left( \frac{\rho_{FUS_t}}{1 - \rho_{FUS_t}L} \right) \theta_{FUS_t} + \frac{x_{FUS_t}}{1 - \rho_{FUS_t}L}
\]

\quad (8.18)

Solving equation (8.18) the price level under fixed exchange rate regime is:

\[
P_t = Cons + q_0\varepsilon_{FUS_t} + q_1\varepsilon_{FUS_{t-1}} + \left( \frac{q_2}{1 - \rho_{FUS_t}L} \right) \varepsilon_{FUS_{t-2}} + r_0x_{FUS_t} + \\
\left( \frac{r_2}{1 - \rho_{FUS_t}L} \right)x_{FUS_{t-2}} + s_0\eta_{FUS_t} + s_1\eta_{FUS_{t-1}} + \\
\left( \frac{s_2}{1 - \rho_{FUS_t}L} \right) \eta_{FUS_{t-2}} + d_0\kappa_{FUS_t} + d_1\kappa_{FUS_{t-1}} + \left( \frac{d_2}{1 - \rho_{FUS_t}L} \right) \kappa_{FUS_{t-2}} + l_0\theta_{FUS_t} + l_1\theta_{FUS_{t-1}} + \left( \frac{l_2}{1 - \rho_{FUS_t}L} \right) \theta_{FUS_{t-2}}
\]

\quad (8.19)
Using equation (8.19) we can work out \( \pi_t \) under fixed exchange rate regime:⁷

\[
\pi_t = Cons + q_0 \varepsilon_{FUS_t} + (q_1 - q_0) \varepsilon_{FUS_{t-1}} + \left( \frac{q_2}{1 - \rho_{FUS_0} L} - q_1 \right) \varepsilon_{FUS_{t-2}}
\]

\[
- \left( \frac{r_2}{1 - \rho_{FUS_0} L} \right) \varepsilon_{FUS_{t-3}} + r_0 x_{FUS_t} + (r_1 - r_0) x_{FUS_{t-1}} + \left( \frac{r_3}{1 - \rho_{FUS_1} L} - r_1 \right) x_{FUS_{t-2}}
\]

\[
- \left( \frac{s_2}{1 - \rho_{FUS_0} L} \right) \eta_{FUS_{t-3}} + s_0 \eta_{FUS_t} + (s_1 - s_0) \eta_{FUS_{t-1}} + \left( \frac{s_3}{1 - \rho_{FUS_1} L} - s_1 \right) \eta_{FUS_{t-2}}
\]

\[
- \left( \frac{d_2}{1 - \rho_{FUS_0} L} \right) \kappa_{FUS_{t-3}} + d_0 \kappa_{FUS_t} + (d_1 - d_0) \kappa_{FUS_{t-1}} + \left( \frac{d_3}{1 - \rho_{FUS_1} L} - d_1 \right) \kappa_{FUS_{t-2}}
\]

\[
- \left( \frac{l_2}{1 - \rho_{FUS_0} L} \right) \theta_{FUS_{t-3}} + l_0 \theta_{FUS_t} + (l_1 - l_0) \theta_{FUS_{t-1}} + \left( \frac{l_3}{1 - \rho_{FUS_1} L} - l_1 \right) \theta_{FUS_{t-2}}
\]

(8.20)

The theoretical implied form for inflation is an ARMA(5,6).⁸

### 8.4 Incomes Policy Regime (1971:1 to 1978:4)

Sterling was floated in June 1972.⁹ 1972 was also the year of the Heath government's 'U-turn' in macroeconomic policy. The view of the government was that it could stimulate output and employment through expansionary monetary and fiscal policies, while at the same time keeping inflation under control through statutory wage and price controls.¹⁰ The opinion of the day was that the break-out of inflation in the 1970s largely reflected autonomous wage and price movements, and that the appropriate policy response was to take actions that exerted downward pressure on specific products, rather than to concentrate on a monetary policy response. Examples of non-monetary attempts to control inflation included statutory incomes policy announced in November 1972 and the voluntary incomes policy pursued by the Labour government from 1974; the extension of food subsidies in March 1974 budget; and the cuts in

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⁷We have not worked out the exact values of the Muth undetermined coefficients as the purpose of solving the models analytically is primarily to get the ARMA form of the inflation process implied by theory.

⁸See Figure 8.4.7 in the appendix for the theoretical implied ARMA(5,6) estimated on the inflation data for this regime.

⁹The float of the exchange rate was announced on the 23 June 1972. See Bank of England (1972).

¹⁰From 1973 to 1980, the government periodically used the Supplementary Special Deposits Scheme, called the 'Corset', as a quantitative control on the expansion of the banks' balance sheets and therefore of the £M3 monetary aggregate.
indirect taxation in the July 1974 mini-Budget.\textsuperscript{11}

From late 1973 policy makers did start paying heed to the growing criticism of rapid money growth that they had permitted. However, there was an unwillingness to make the politically unpopular decision of raising nominal interest rates. The Bank of England was given instructions from the Government that the growth of broad money, the Sterling M3 aggregate, was to be reduced — however, the nominal interest rates must not be increased. The result was the 'Corset', the introduction of direct quantitative control on £M3, which imposed heavy marginal reserve requirements if increases in banks' deposits exceeded a limit. While this control did result in a reduction in the observed £M3 growth, it did so largely by encouraging the growth of deposit substitutes, distorting £M3 as a monetary indicator and weakening its relationship with future inflation.\textsuperscript{12} For the rest of the 1970s monetary policy often looked restrictive as measured by £M3 growth, but loose as measured by interest rates or monetary base growth.

In July 1976 targets were announced for £M3 monetary aggregate.\textsuperscript{13} From then on UK had a monetary policy that reacted to monetary growth and to the exchange rate.\textsuperscript{14} Depreciation of the exchange rate in 1976 was a major factor that triggered a tighter monetary policy during 1976-1979. However, we must not over emphasise the monetary tightness as the nominal interest rate was cut aggressively — by more than 900 basis points from late 1976 to early 1978 — ahead of the fall in inflation from mid-1977 to late 1978. Reflecting the easier monetary policy, money base (£M0) growth, which had been reduced to single digits in the late 1977, rose sharply and peaked at more than 18% in July 1978; inflation troughed at 7.6% in October 1978 and continued to rise until May 1980, when it was 21%. Furthermore, the nominal Treasury bill rate from July 1976 to April 1979 averaged 9.32%. In real terms it was well below zero, indicating the continued tendency of the policy makers until 1978 to hold nominal interest rates well below the actual and prospective inflation rate.\textsuperscript{15}

\textsuperscript{12}As Nelson (2000) points out it is likely that this served principally as a device for restricting artificially the measured growth of £M3 without changing the monetary base or interest rates, rather than as a genuinely restrictive monetary policy measure. See also Minford (1993).
\textsuperscript{13}The value of this target was 11% from May 1976 to April 1978 and 10% from May 1978 to April 1980. These are the mid-points of the successive targets announced for the annual £M3 growth.
\textsuperscript{14}For discussions of the development of UK monetary policy in the 1970s, see Bank of England (1984), Goodhart (1989) and Minford (1993).
\textsuperscript{15}Judd and Rudolph (1998) report average real interest rate for the US for the period 1970-78 to be 2 basis points. Hence, the phenomenon of low or negative real interest rates in the 1970s was more pronounced in the
Nelson (2000) finds that the estimated long-run response of the nominal interest rate to inflation was well below unity during the 1970s. Moreover, the real interest rate was permitted to be negative for most of the period. These results suggest that UK monetary policy failed to provide a nominal anchor in the 1970s. However, we note that there was a determinate inflation rate during this period, even though there was clearly no orthodox monetary anchor. What we have chosen to do from a modelling viewpoint is treat Incomes Policy as the determinant of inflation and to assume that interest rates ‘fitted in’ with what the model dictated was necessary to achieve that inflation rate and the accompanying output rate. Plainly this is a drastic over-simplification since interest rates were independently set at quite inappropriate levels; however, introducing such contradictory monetary policy poses too much of a modelling challenge for this exercise -- it could well be that there was such monetary indeterminacy, and incomes policy so incredible, that we were here in a ‘non-Ricardian’ period where fiscal policy was left to determine inflation. However exploring such possibilities lies well beyond the scope of this chapter.

\[ \tilde{y}_t = \gamma (E_t \tilde{y}_{t+1}) - \alpha (R_t - E_t \pi_{t+1}) + u_{IP_t} \]  \hspace{1cm} (8.21)

\[ \tilde{y}_t = \delta (\pi_t - E_{t-1} \pi_t) + v_{IP_t} \]  \hspace{1cm} (8.22)

\[ \pi_t = \pi_{t-1} (1 - \rho) + \tau_{IP_t} \]  \hspace{1cm} (8.23)

\[ u_{IP_t} = \rho_{IP_0} u_{IP_{t-1}} + \varepsilon_{IP_t} \]  \hspace{1cm} (8.24)

\[ v_{IP_t} = \rho_{IP_1} v_{IP_{t-1}} + \xi_{IP_t} \]  \hspace{1cm} (8.25)

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\[ \tau_{IP_t} = \rho_{IP_3} \tau_{IP_{t-1}} + \zeta_{IP_t} \] (8.26)

In the equations above, \( \pi_t \) is the inflation quarter-on-quarter annualised and \( c \) is the incomes policy constraint. As before equation (8.21) is a forward-looking IS curve and equation (8.22) is a New Classical Phillips curve. Equation (8.23) states that inflation at time \( t \) is set by incomes policy at some fraction of the actual inflation in period \( t - 1 \) but subject to an error, the 'break-down' of policy, which we have modelled as an AR(1). During this period there was a serious credibility problem. So, if the government came along and announced that it would cut inflation by 80 percent that simply would not be believable. However, if the government announced that it would cut inflation by say 20 percent then that would definitely be more credible and policy makers would be in a position to gradually get inflation expectations and hence inflation under control. Furthermore, it should be remembered that during this period there were no explicit targets. However, from policy makers' behaviour we do know that there existed implicit targets, and \( c \) helps us operationalise that. We have modelled the IS and PP forecast error as AR(1) processes. \( \varepsilon_{IP_t} \), \( x_{IP_t} \) and \( \zeta_{IP_t} \) are all i.i.d.

Substituting equation (8.26) in (8.23) the solution for \( \pi_t \) in this regime is:

\[ \pi_t = (\rho_{IP_3} + 1 - c)\pi_{t-1} - \rho_{IP_3}(1 - c)\pi_{t-2} + \zeta_{IP_t} \] (8.27)

The theoretical implied form for inflation is an AR(2).\(^{16}\)

8.5 Money Targeting Regime (1979:1 to 1985:4)

In 1979 inflation was rising rapidly from an initial rate of over 10 percent. The policy of wage controls that had been used to hold down inflation in 1978 had crumbled in the 'winter of discontent' of that year when graves went undug and rubbish piled up in the streets. The budget was in crisis, the deficit already up to 5 percent of GDP and headed to get worse due to large public sector pay increases promised by the previous government. Milton Friedman (1980) advised a gradual reduction in the money supply growth rate and a cut in taxes in order to stimulate output. The first part was accepted, but the opinion was that tax rates needed

\(^{16}\) See appendix Figure 8 A.8 for estimates.
to remain high to try and reduce the deficit which was important in conditioning financial confidence.

As mentioned earlier monetary aggregate targeting was introduced in the UK in 1976 in conjunction with the International Monetary Fund (IMF) support arrangement. Figure 8.A.4 in the appendix plots the growth of £M0 from 1970 till the end of 2003 and of £M3 from 1979 to 1985. The previous government was quite successful in shrinking the Public Sector Borrowing Requirement (PSBR) from 10 percent in 1975 to less than 4 percent in 1977. However, the policies lacked long-term durability. To achieve durability policy was cast in the form of a Medium Term Financial Strategy (MTFS), a monetary and fiscal policy programme announced by the Conservative Government in its annual budget in 1980. This strategy consisted first of a commitment to a five-year rolling target for gradually decelerating £M3. Second, controls were removed, including the ‘Corset’, exchange controls and incomes policy. Third, the monetary commitment was backed up by a parallel reduction in the PSBR/GDP ratio.

Large misses of the £M3 target were permitted as early as mid-1980, with the MTFS being heavily revised in 1982. In October 1985 £M3 targeting was abandoned. It was however clear prior to the abandonment that key policy makers did not regard overshoots of the £M3 target as intolerable, as long as other measures of monetary conditions, such as interest rates or monetary base growth, were not indicating that monetary policy was loose.¹⁷ Formally, monetary targets continued to be a part of the MTFS right until 1996. However, by 1988, the targets had been so de-emphasised in monetary policy formation that Nigel Lawson, the Chancellor of the Exchequer could say “As far as monetary policy is concerned, the two things perhaps to look at are the interest rate and the exchange rate.”¹⁸

Even though the logic behind the MTFS was well developed, it failed not only to command credibility, but also to be carried out in its own literal terms. Policy turned out to be more fiercely contractionary than gradualism had intended. As Minford (1993) very succinctly puts it “The paradox was: tougher yet less credible policies, apparently the worst of both worlds.”

¹⁷See Goodhart (1989) and Minford (1993).
¹⁸Testimony, 30 November 1988, in Treasury and Civil Service Committee.
\[
\tilde{y}_t = \gamma (E_t \tilde{y}_{t+1}) - \alpha r_t + u_{MT_t} \tag{8.28}
\]
\[
\tilde{y}_t = \delta (P_t - E_{t-1} P_t) + v_{MT_t} \tag{8.29}
\]
\[
M_t = P_t - \beta_{MT_t} R_t + \beta_{MT_t} (E_t \tilde{y}_{t+1}) + \xi_{MT,0_t} \tag{8.30}
\]
\[
\Delta M_t = \overline{M} + \xi_{MT,1_t} \tag{8.31}
\]
\[
R_t = r_t + E_t P_{t+1} - P_t \tag{8.32}
\]
\[
u_{MT_t} = \rho_{MT_0} u_{MT_{t-1}} + \varepsilon_{MT_t} \tag{8.33}
\]
\[
v_{MT_t} = \rho_{MT_1} v_{MT_{t-1}} + \xi_{MT_t} \tag{8.34}
\]
\[
\xi_{MT,0_t} = \rho_{MT_2} \xi_{MT,0_{t-1}} + \varepsilon_{MT_t} \tag{8.35}
\]

In the equations above, \( r_t \) is the real interest rate and \( M_t \) is the money demand (or supply). Equation (8.28) and (8.29) are the IS and Phillips curve, respectively. Equation (8.30) is the LM curve sets out a standard money demand schedule. The shock to money demand is persistent as seen in equation (8.35). Growth in money supply equals an exogenously specified target \( \overline{M} \) and a random shock (equation (8.31)). Equation (8.32) is the definition of nominal interest rate in the model. As before the IS and PP curve shocks have been modelled as AR(1) processes. \( \varepsilon_{MT_t}, x_{MT_t} \) and \( \xi_{MT_t} \) are all i.i.d.

\(^{19}\text{We assume } \overline{M} = 0 \text{ for convenience.}\)
Leading equation (8.29) one period and taking expectations yields:

\[ E_t \tilde{y}_{t+1} = \rho_{MT_t} v_{MT_t} = \left( \frac{\rho_{MT_t}}{1 - \rho_{MT_t} L} \right) x_{MT_t} \]  
(8.36)

Substituting equation (8.32) and (8.36) in equation (8.28) yields:

\[ \tilde{y}_t = \gamma \left( \frac{\rho_{MT_t}}{1 - \rho_{MT_t} L} \right) x_{MT_t} - \alpha [R_t - (E_t P_{t+1} - P_t)] + u_{MT_t} \]  
(8.37)

Further, substituting equation (8.31) in (8.30) yields:

\[ R_t = \left( \frac{1}{\beta_{MT_t}} \right) P_t + \left( \frac{\beta_{MT_t}}{\beta_{MT_t}} \right) E_t \tilde{y}_{t+1} - \left( \frac{1}{\beta_{MT_t}} \right) \frac{M}{1 - L} + \left( \frac{1}{\beta_{MT_t}} \right) \xi_{MT,0_t} - \left( \frac{1}{(1 - L)\beta_{MT_t}} \right) \xi_{MT,1_t} \]  
(8.38)

Substituting equation (8.38) and (8.29) in equation (8.37) yields the semi-reduced form solution for \( P_t \):

\[ \delta(P_t - E_{t-1} P_t) + v_{MT_t} = \gamma \left( \frac{\rho_{MT_t}}{1 - \rho_{MT_t} L} \right) \]  
(8.39)

Now using the Muth Method, the Wold decomposition for \( P_t \) in this model is:

\[ P_t = Cons + \sum_{i=0}^{\infty} a_i \xi_{MT,-i} + \sum_{i=0}^{\infty} b_i x_{MT,-i} + \sum_{i=0}^{\infty} c_i \epsilon_{MT,-i} + \sum_{i=0}^{\infty} d_i \xi_{MT,1,-i}. \]  
(8.40)
Using the Wold decomposition equation (8.39) can be written as

\[
\delta(a_0 \epsilon_{MT_t} + b_0 x_{MT_t} + c_0 \epsilon_{MT_t} + d_0 \xi_{MT,1,t}) + \frac{x_{MT_t}}{1 - \rho_{MT_t} L} = \frac{\gamma \rho_{MT_t} x_{MT_t}}{1 - \rho_{MT_t} L} - \frac{\alpha}{\beta_{MT_1}} \left( C_{ns} + \sum_{i=0}^{\infty} a_i \epsilon_{MT_{t-i}} + \sum_{i=0}^{\infty} b_i x_{MT_{t-i}} + \sum_{i=0}^{\infty} c_i \epsilon_{MT_{t-i}} + \sum_{i=0}^{\infty} d_i \xi_{MT,1,t-i} \right)
\]

\[
- \left( \frac{\alpha}{\beta_{MT_4}} \right) \left( \frac{\rho_{MT_t} x_{MT_t}}{1 - \rho_{MT_t} L} \right) + \left( \frac{\alpha}{\beta_{MT_4}} \right) \left( \frac{M}{1 - L} \right) - \left( \frac{\alpha}{\beta_{MT_4}} \right) \left( \frac{\epsilon_{MT_t}}{1 - \rho_{MT_t} L} \right)
\]

\[
+ \left( \frac{\alpha}{(1 - L) \beta_{MT_4}} \right) \xi_{MT,1,t} + \alpha \left( C_{ns} + \sum_{i=1}^{\infty} a_i \epsilon_{MT_{t-i+1}} + \sum_{i=1}^{\infty} b_i x_{MT_{t-i+1}} + \sum_{i=1}^{\infty} c_i \epsilon_{MT_{t-i+1}} + \sum_{i=1}^{\infty} d_i \xi_{MT,1,t-i+1} \right)
\]

\[
- \alpha \left( C_{ns} + \sum_{i=0}^{\infty} a_i \epsilon_{MT_{t-i}} + \sum_{i=0}^{\infty} b_i x_{MT_{t-i}} + \sum_{i=0}^{\infty} c_i \epsilon_{MT_{t-i}} + \sum_{i=0}^{\infty} d_i \xi_{MT,1,t-i} \right)
\]

\[
+ \left( \frac{\epsilon_{MT_t}}{1 - \rho_{MT_t} L} \right)
\]

(8.41)

Solving equation (8.41) the price level in money targeting regime is:

\[
P_t = C_{ns} + a_0 \epsilon_{MT_t} + \left( \frac{a_1}{1 - \rho_{MT_t} L} \right) \epsilon_{MT_{t-1}} + b_0 x_{MT_t} + \left( \frac{b_1}{1 - \rho_{MT_t} L} \right) x_{MT_{t-1}}
\]

\[
+ c_0 \epsilon_{MT_t} + \left( \frac{c_1}{1 - \rho_{MT_t} L} \right) \epsilon_{MT_{t-1}} + d_0 \xi_{MT,1,t} + \left( \frac{d_1}{1 - L} \right) \xi_{MT,1,t-1}
\]

(8.42)

Using equation (8.42) we can work out \( \pi_t \) under money targeting regime:

\[
\pi_t = C_{ns} + a_0 \epsilon_{MT_t} + \left( \frac{a_1}{1 - \rho_{MT_t} L} - a_0 \right) \epsilon_{MT_{t-1}} - \left( \frac{a_1}{1 - \rho_{MT_t} L} \right) \epsilon_{MT_{t-2}}
\]

\[
+ b_0 x_{MT_t} + \left( \frac{b_1}{1 - \rho_{MT_t} L} - b_0 \right) x_{MT_{t-1}} - \left( \frac{b_1}{1 - \rho_{MT_t} L} \right) x_{MT_{t-2}}
\]

\[
+ c_0 \epsilon_{MT_t} + \left( \frac{c_1}{1 - \rho_{MT_t} L} - c_0 \right) \epsilon_{MT_{t-1}} - \left( \frac{c_1}{1 - \rho_{MT_t} L} \right) \epsilon_{MT_{t-2}}
\]

\[
+ d_0 \xi_{MT,1,t} + \left( \frac{d_1}{1 - L} - d_0 \right) \xi_{MT,1,t-1} - \left( \frac{d_1}{1 - L} \right) \xi_{MT,1,t-2}
\]

(8.43)

The theoretical implied form for inflation is an ARMA(4,5).\(^{20}\)

\(^{20}\)See Figure 8.A.9 in the appendix for the estimated ARMA(4,5).
8.6 Fixed Exchange Rate Regime (Germany) or ERM (1986:1 to 1992:3)

The next regime largely consists of informal linking of the Sterling to the Deutsche Mark (DM). This includes not only the ‘shadowing’ of the Mark in 1986-88, but also the period from 1989-1990 during which UK was a formal member of the Exchange Rate Mechanism (ERM). The idea essentially was that, just as the other major European currencies were successfully aiming to hold inflation down by anchoring their currencies to the DM within the ERM, the UK too could lock in to Germany’s enviable record of sustained low inflation even without actually joining the mechanism. The approach was never formally announced, but it became clear in practice that the Sterling DM exchange rate, which had depreciated very sharply from DM 4 in July 1985 to DM 2.74 in early 1987, was not subsequently allowed to appreciate above DM 3 even though this meant a massive increase in UK foreign exchange reserves, and a reduction of interest rates from 11 percent to a trough of 7 percent during 1987 to prevent the appreciation. This had the effect of accommodating and aggravating the inflationary consequences of the earlier depreciation.

In the Spring of 1988, the exchange rate cap was lifted but by then the boom was already entrenched. Interest rates were pushed up to 15 percent by the Autumn of 1989 to bring the situation under control. A year later the UK also formally joined the ERM. The episode produced a painful recession in which inflation which had risen to over 7% fell back sharply. According to Nelson (2000) from 1987-1990, the Bundesbank’s monetary policy, rather than a domestic variable, served as UK monetary policy’s nominal anchor.

At the time of ERM entry UK policy needs appeared to coincide with those of its partners. In principle it seemed possible that with the enhanced policy credibility that ERM membership was expected to bring, UK could hope to complete the domestic economic stabilisation programme with lower interest rates than otherwise, and so at less cost in terms of loss of output. There was also a very strong non-monetary consideration, that the UK would have little influence on the outcome of the European Inter-Government Conference if it was not in the ERM.

However, things did not go as planned. German reunification meant that Germany needed to maintain a tight monetary policy at a time when the domestic situation in a number of
ERM countries, including the UK, required monetary easing. Parity adjustment was against the ERM rules and seemed inconsistent with maintaining policy credibility. The UK was then confronted with a situation where tightening policy by raising rates made no economic sense in terms of domestic conditions. It then sought to maintain the parity through intervention in the hope that the pressures in Germany would abate. In reality those pressures did not ease soon enough and after heavy intervention, and a last bout of interest rate increases, the UK had no choice but to withdraw from the ERM on September 1992.

The model we use here is the same as the Bretton Woods model with the exception that Germany replaces the US throughout.

\[ \tilde{y}_t = \gamma (E_t \tilde{y}_{t+1}) - \alpha (R_t - E_t P_{t+1} + P_t) + \lambda (E_t NX_{t+1}) + u_{FGR_t} \]  
(8.44)

\[ \tilde{y}_t = \delta (P_t - E_{t-1} P_t) + v_{FGR_t} \]  
(8.45)

\[ NX_t = a_{FGR_0} Q_{FGR_t} + a_{FGR_1} y_{FGR_t}^F \]  
(8.46)

\[ Q_{FGR_t} = S_{FGR_t} + F_t - P_t \]  
(8.47)

\[ R_t = R_{FGR_t}^F + (E_t S_{FGR_{t+1}} - S_{FGR_t}) \]  
(8.48)

\[ S_{FGR_t} = \overline{S}_{FGR} \]  
(8.49)

\[ R_{FGR_t}^F = \rho_{FGR} R_{FGR_{t-1}}^F + \eta_{FGR_t} \]  
(8.50)

\[ u_{FGR_t} = \rho_{FGR_0} u_{FGR_{t-1}} + \varepsilon_{FGR_t} \]  
(8.51)
\[ v_{FGR_t} = \rho_{FGR_1} v_{FGR_{t-1}} + x_{FGR_t} \]  
(8.52)

\[ y_{FGR_t} = \rho_{FGR_2} y_{FGR_{t-1}} + \theta_{FGR_t} \]  
(8.53)

\[ P_{FGR_t} = \rho_{FGR_2} P_{FGR_{t-1}} + \kappa_{FGR_t} \]  
(8.54)

Here \( S_{FGR_t} \) is the nominal exchange rate \( $/DM \), \( R_{FGR_t}^E \) is the German nominal interest rate (day-to-day money rate) and \( P_{FGR_t}^I \) is the German price level (CPI).

The solution for inflation under fixed exchange rate regime against Germany is

\[
\pi_t = \text{Cons} + q_0 \varepsilon_{FGR_t} + (q_1 - q_0) \varepsilon_{FGR_{t-1}} + \left( \frac{q_2}{1 - \rho_{FGR_0} L} - q_1 \right) \varepsilon_{FGR_{t-2}} - \left( \frac{q_2}{1 - \rho_{FGR_0} L} \right) \varepsilon_{FGR_{t-3}} + \eta_{FGR_{t-1}} \varepsilon_{FGR_{t-1}} + \left( \frac{r_2}{1 - \rho_{FGR_1} L} - r_1 \right) x_{FGR_{t-2}} - \left( \frac{r_2}{1 - \rho_{FGR_1} L} \right) x_{FGR_{t-3}} + s_0 \eta_{FGR_{t-1}} + \left( s_1 - s_0 \right) \eta_{FGR_{t-1}} + \left( \frac{s_2}{1 - \rho_{FGR_1} L} - s_1 \right) \eta_{FGR_{t-2}} - \left( \frac{s_2}{1 - \rho_{FGR_1} L} \right) \eta_{FGR_{t-3}} + d_0 \kappa_{FGR_{t-1}} + \left( d_1 - d_0 \right) \kappa_{FGR_{t-1}} + \left( \frac{d_2}{1 - \rho_{FGR_2} L} - d_1 \right) \kappa_{FGR_{t-2}} - \left( \frac{d_2}{1 - \rho_{FGR_2} L} \right) \kappa_{FGR_{t-3}} + l_0 \theta_{FGR_{t-1}} + \left( l_1 - l_0 \right) \theta_{FGR_{t-1}} + \left( \frac{l_2}{1 - \rho_{FGR_3} L} - l_1 \right) \theta_{FGR_{t-2}} - \left( \frac{l_2}{1 - \rho_{FGR_3} L} \right) \theta_{FGR_{t-3}} \]

(8.55)

The theoretical implied form for inflation is as with Bretton Woods an ARMA(5,6).\(^{22}\)

### 8.7 Inflation Targeting Regime (1992:4 to 2003:3)

Immediately following the UK’s exit from the Exchange Rate Mechanism (ERM) in September 1992, inflation expectations were between 5 percent and 7 percent at maturities 10 to 20 years ahead – well above the inflation target of 1-4 percent at the time. Five years into the regime, by April 1997, inflation expectations had ratcheted down to just over 4 percent. A credibility gap

\[^{21}\text{Figure 8 A.5 in the appendix plots the nominal exchange rate.}\]

\[^{22}\text{See Figure 8 A.10 in the appendix for the ARMA(5,6) estimated for this period.}\]
still remained but it had narrowed markedly. The announcement of operational independence for the Bank of England in May 1997\(^{23}\) caused a further decline in inflation expectations by around 50 basis points across all maturities. By the end of 1998, inflation expectations were around the UK's 2.5 percent inflation target, at all maturities along the inflation term structure. They have remained at that level since then.

Using the inflation target as a reference point for expectations is important during the transition to low inflation as the target then serves as a means of guiding inflation expectations downwards over time. It is widely thought, though not a feature of our models here, that lags in policy mean that inflation targeting needs to have a forward-looking dimension. According to Haldane (2000) a successful inflation targeting regime must have 'ghostbusting' as an underlying theme, by which he means that policy makers take seriously the need to be pre-emptive in setting monetary policy, offsetting incipient inflationary pressures.\(^{24}\) Nevertheless within our model here a forward element makes no sense and in fact causes indeterminacy; so we have framed interest rate policy in terms of current inflation and output.

\[
\bar{y}_t = \gamma (\bar{y}_{t+1}) - \alpha (R_t - E_t \pi_{t+1}) + u_{FIT_t}
\]  
\[\bar{y}_t = \delta (\pi_t - E_t \pi_t) + v_{FIT_t} \]  
\[R_t = \beta_{FIT_0} + \beta_{FIT_1} R_{t-1} + \beta_{FIT_2} (\pi_t - \pi^*) + \beta_{FIT_3} (\bar{y}_t) - \beta_{FIT_3} (w_{FIT_t}) \]  
\[u_{FIT_t} = \rho_{FIT_0} u_{FIT_{t-1}} + \epsilon_{FIT_t} \]

\(^{23}\)Autonomy of the Bank is enshrined in the Bank of England Act of 1998. This act confers instrument-independence on the Bank, though the government still sets the goals of policy. In the jargon, there is goal-independence but instrument independence.

\(^{24}\)Haldane (2000) goes on to say "Like ghosts, these pressures will be invisible to the general public at the time policy measures need to be taken. Claims of sightings will be met with widespread derision and disbelief. But the central bank's job is to spot the ghosts and to exorcise them early. A successful monetary policy framework is ultimately one in which the general public is not haunted by inflationary shocks."
\[ v_{FIT_i} = \rho_{FIT_1} v_{FIT_{i-1}} + x_{FIT_i} \quad (8.60) \]

\[ w_{FIT_i} = \rho_{FIT_2} w_{FIT_{i-1}} + z_{FIT_i} \quad (8.61) \]

In the equations above all variables are as defined earlier, with the exception of inflation where we use RPIX rather than the RPI since the regime is defined in terms of this variable; \( \pi^* \) is the inflation target of the Bank of England. \( R_t \) the nominal interest rate is the Bank of England base rate is plotted in Figure 8.A.6 in the appendix. As before equation (8.56) and (8.57) are the IS and Phillips curve, respectively. Equation (8.58) is a Taylor rule with interest rate smoothing. We also take into account Orphanides (1998) style output gap forecast error. As before the IS, PP and output gap forecast error have all been modelled as AR(1) processes. \( \varepsilon_{FIT_i}, x_{FIT_i} \) and \( z_{FIT_i} \) are all i.i.d.

Leading equation (8.57) one period and taking expectations yields:

\[ E_t \tilde{y}_{t+1} = \rho_{FIT_1} v_{FIT_{i}} = \left( \frac{\rho_{FIT_1}}{1 - \rho_{FIT_1} L} \right) x_{FIT_i} \quad (8.62) \]

Now using the Muth Method, the Wold decomposition for \( \pi_t \) in this model is:

\[ \pi_t = Cons + \sum_{i=0}^{\infty} q_i \varepsilon_{FIT_{i+1}} + \sum_{i=0}^{\infty} r_i x_{FIT_{i+1}} + \sum_{i=0}^{\infty} s_i z_{FIT_{i+1}} \quad (8.63) \]

Using equation (8.57), (8.58), (8.62) and (8.63) in equation (8.56) we have

\[ \delta(q_0 \varepsilon_{FIT_t} + r_0 x_{FIT_t} + s_0 z_{FIT_t}) + \frac{x_{FIT_t}}{1 - \rho_{FIT_1} L} = \left( \frac{\gamma \rho_{FIT_1}}{1 - \rho_{FIT_1} L} \right) x_{FIT_t} \]

\[ -\alpha \left( \frac{\beta_{FIT_0} + \beta_{FIT_2}(\pi_t - \pi^*) + \beta_{FIT_1}(\tilde{y}_t) - \beta_{FIT_3}(w_{FIT_t})}{1 - \beta_{FIT_1} L} \right) + \]

\[ \alpha \left( Cons + \sum_{i=1}^{\infty} q_i \varepsilon_{FIT_{i+1}} + \sum_{i=1}^{\infty} r_i x_{FIT_{i+1}} + \sum_{i=1}^{\infty} s_i z_{FIT_{i+1}} \right) + u_{FIT_t} \quad (8.64) \]
or

\[
\begin{align*}
((1 - \beta_{FIT_1}L) + \alpha \beta_{FIT_3}) \left( \theta (q_0 \varepsilon_{FIT_t} + r_0 x_{FIT_t} + s_0 z_{FIT_t}) + \frac{x_{FIT_t}}{1 - \rho_{FIT_1} L} \right) = (1 - \beta_{FIT_1}L) \left[ \left( \frac{\gamma \rho_{FIT_3}}{1 - \rho_{FIT_1} L} \right) x_{FIT_t} + \frac{\varepsilon_{FIT_t}}{1 - \rho_{FIT_3} L} \right] \\
+ \alpha \left( \text{Cons} + \sum_{i=1}^{\infty} q_i \varepsilon_{FIT_{t-i}} + \sum_{i=1}^{\infty} r_i x_{FIT_{t-i}} + \sum_{i=1}^{\infty} s_i z_{FIT_{t-i}} \right)
- \alpha \beta_{FIT_1} \left( \text{Cons} + \sum_{i=1}^{\infty} q_i \varepsilon_{FIT_{t-i}} + \sum_{i=1}^{\infty} r_i x_{FIT_{t-i}} + \sum_{i=1}^{\infty} s_i z_{FIT_{t-i}} \right) - \alpha \beta_{FIT_0} + \alpha \beta_{FIT_2} \pi^* \\
- \alpha \beta_{FIT_2} \left( \text{Cons} + \sum_{i=0}^{\infty} q_i \varepsilon_{FIT_{t-i}} + \sum_{i=0}^{\infty} r_i x_{FIT_{t-i}} + \sum_{i=0}^{\infty} s_i z_{FIT_{t-i}} \right) \\
+ \alpha \beta_{FIT_3} \left( \frac{z_t}{1 - \rho_{FIT_2} L} \right)
\end{align*}
\]

(8.65)

The solution for \( \pi_t \) under flexible inflation targeting regime is:

\[
\begin{align*}
\pi_t &= \text{Cons} + q_0 \varepsilon_{FIT_t} + q_1 \varepsilon_{FIT_{t-1}} + \left( \frac{q_2}{1 - \rho_{FIT_0} L} \right) \varepsilon_{FIT_{t-2}} + r_0 x_{FIT_t} + r_1 x_{FIT_{t-1}} + \\
& \quad \left( \frac{r_2}{1 - \rho_{FIT_1} L} \right) x_{FIT_{t-2}} + s_0 z_{FIT_t} + s_1 z_{FIT_{t-1}} + \left( \frac{s_2}{1 - \rho_{FIT_3} L} \right) z_{FIT_{t-2}} \\
& \quad \left( \frac{z_t}{1 - \rho_{FIT_2} L} \right)
\end{align*}
\]

(8.66)

The theoretical implied form for inflation under flexible inflation targeting is an ARMA(3,4).\(^{25}\)

One can analyse a strict inflation targeting regime by simply changing equation (8.58) to:

\[
R_t = \beta_{SIT_0} + \beta_{SIT_1} R_{t-1} + \beta_{SIT_2} (\pi_t - \pi^*)
\]

(8.67)

Now the Central Bank, in the words of Mervyn King, is an 'Inflation nutter'. Using this interest rate rule the reduced form for \( \pi_t \) is:

\[
\begin{align*}
\pi_t &= \text{Cons} + q_0 \varepsilon_{SIT_t} + q_1 \varepsilon_{SIT_{t-1}} + \left( \frac{q_2}{1 - \rho_{SIT_0} L} \right) \varepsilon_{SIT_{t-2}} + r_0 x_{SIT_t} \\
& \quad + r_1 x_{SIT_{t-1}} + \left( \frac{r_2}{1 - \rho_{SIT_1} L} \right) x_{SIT_{t-2}} \\
& \quad + \beta_{SIT_2} (\pi_t - \pi^*)
\end{align*}
\]

(8.68)

\(^{25}\)See Figure 8.A.11 and 8.A.12 for the ARMA(3,4) regression estimates.
The theoretical implied form for inflation under strict inflation targeting is an $ARMA(2, 3)$.\(^{26}\)

### 8.8 Conclusions

The aim of this chapter has been essentially qualitative: to see whether a basic classical model, calibrated in a standard way, can generate predictions that persistence will depend on the monetary regime in the direction of the stylised facts. We use our structural models suitably calibrated for each of the regimes incorporating all available information about monetary policy behaviour during those periods and then solve them analytically for the implied persistence in the inflation final form equation; in order to compare this theoretical prediction with the estimated persistence for each regime — the objective of the next chapter.

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\(^{26}\)See Figure 8.A.13 and 8.A.14 for the estimated results.
8.A  Appendix

8.A.1  Data Set (Base year 2000)


2. UK Gross Domestic Product: Gross Domestic Product (GDP) at Factor Cost £ millions. Seasonally Adjusted. Series YBHH Table 1.1 Monthly Digest of Statistics.

3. UK M0: Wide Monetary Base. Seasonally Adjusted. Series AVAE Table 6.2 Economic Trends.


7. UK RPIX Price Index : Retail Price Index All items excluding Mortgage Payments, Office of National Statistics (ONS). Not Seasonally Adjusted. Series CHMK Table 3.1 Economic Trends.


11. US Interest Rate: Federal Funds Rate %, Federal Reserve. Series USFEDFUN in DataStream.


14. German Interest Rate: Day-to-day Money Rate %, Eurostat. Series BDESSFRT in DataStream.


8.A.2 Graphs

![Graph](image)

Figure 8.A.1: Log of GDP and Trend of GDP Logged
Figure 8.A.2: Output Gap

Figure 8.A.3: Nominal Exchange Rate Sterling/Dollar
Figure 8.A.4: Growth Rate of M0 and M3 Monetary Aggregates

Figure 8.A.5: Nominal Exchange Rate Sterling/Deutsche Mark
Figure 8.A.6: Bank of England Base Rate
8.A.3 Theoretical ARMAAs for UK Monetary Policy Regimes

Theoretical ARMA for Fixed Exchange Rate (FUS)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pt(-1)</td>
<td>0.369189</td>
<td>0.201363</td>
<td>1.833452</td>
<td>0.0744</td>
</tr>
<tr>
<td>Pt(-2)</td>
<td>0.171348</td>
<td>0.189620</td>
<td>0.902684</td>
<td>0.3722</td>
</tr>
<tr>
<td>Pt(-3)</td>
<td>0.310143</td>
<td>0.120899</td>
<td>2.387576</td>
<td>0.0219</td>
</tr>
<tr>
<td>Pt(-4)</td>
<td>-0.201346</td>
<td>0.139460</td>
<td>-1.443753</td>
<td>0.1568</td>
</tr>
<tr>
<td>Pt(-5)</td>
<td>0.386006</td>
<td>0.143975</td>
<td>2.647308</td>
<td>0.0105</td>
</tr>
<tr>
<td>@SEAS(1)</td>
<td>0.001728</td>
<td>0.013779</td>
<td>0.125384</td>
<td>0.9009</td>
</tr>
<tr>
<td>@SEAS(2)</td>
<td>0.031112</td>
<td>0.011945</td>
<td>2.604535</td>
<td>0.0130</td>
</tr>
<tr>
<td>@SEAS(3)</td>
<td>-0.053013</td>
<td>0.015826</td>
<td>-3.349695</td>
<td>0.0018</td>
</tr>
<tr>
<td>@SEAS(4)</td>
<td>0.026297</td>
<td>0.014657</td>
<td>1.758218</td>
<td>0.0866</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.140974</td>
<td>0.205473</td>
<td>0.686093</td>
<td>0.4967</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-0.273203</td>
<td>0.124791</td>
<td>-2.189283</td>
<td>0.0346</td>
</tr>
<tr>
<td>MA(3)</td>
<td>0.021023</td>
<td>0.071950</td>
<td>0.292194</td>
<td>0.7717</td>
</tr>
<tr>
<td>MA(4)</td>
<td>0.453756</td>
<td>0.084775</td>
<td>5.352506</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA(5)</td>
<td>-0.734413</td>
<td>0.141294</td>
<td>-5.197755</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA(6)</td>
<td>-0.657025</td>
<td>0.195730</td>
<td>-2.845876</td>
<td>0.0070</td>
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</table>

R-squared          0.733605  Mean dependent var  0.036284
Adjusted R-squared 0.637977  S.D. dependent var  0.033872
S.E. of regression  0.020380  Akaike info criterion -4.718361
Sum squared resid   0.018199  Schwarz criterion  -4.165865
Log likelihood      142.3957  Durbin-Watson stat  2.104830

Inverted MA Roots  
99  -48.87i  -48.87i  -57
-76.54i  -76.54i  

Figure 8.A.7: Theoretical ARMA for Fixed Exchange Rate US
Theoretical ARMA for Incomes Policy (IP)

Dependent Variable: PI
Method: Least Squares
Date: 07/14/05 Time: 13:59
Sample: 1971:1 1978:4
Included observations: 32

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI(-1)</td>
<td>0.861013</td>
<td>0.195330</td>
<td>4.408002</td>
<td>0.0002</td>
</tr>
<tr>
<td>PI(-2)</td>
<td>-0.169693</td>
<td>0.194315</td>
<td>-0.872771</td>
<td>0.3908</td>
</tr>
<tr>
<td>@SEAS(1)</td>
<td>0.049412</td>
<td>0.023982</td>
<td>2.060367</td>
<td>0.0495</td>
</tr>
<tr>
<td>@SEAS(2)</td>
<td>0.084904</td>
<td>0.025869</td>
<td>3.282064</td>
<td>0.0029</td>
</tr>
<tr>
<td>@SEAS(3)</td>
<td>-0.046669</td>
<td>0.030219</td>
<td>-1.544336</td>
<td>0.1346</td>
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<td>@SEAS(4)</td>
<td>0.073130</td>
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<td>2.325152</td>
<td>0.0281</td>
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R-squared                  0.642114
Adjusted R-squared         0.573290
S.E. of regression        0.050896
Akaike info criterion    -2.950696
Schwarz criterion        -2.675871
Durbin-Watson stat      1.964546

Figure 8.A.8: Theoretical ARMA for Incomes Policy
Theoretical ARMA for Money Targeting Regime (MT)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI(-1)</td>
<td>0.286295</td>
<td>0.648767</td>
<td>0.441291</td>
<td>0.6653</td>
</tr>
<tr>
<td>PI(-2)</td>
<td>0.554090</td>
<td>0.711317</td>
<td>0.778963</td>
<td>0.4481</td>
</tr>
<tr>
<td>PI(-3)</td>
<td>0.107269</td>
<td>0.811942</td>
<td>0.175293</td>
<td>0.8632</td>
</tr>
<tr>
<td>PI(-4)</td>
<td>-0.152762</td>
<td>0.622285</td>
<td>-0.245485</td>
<td>0.8094</td>
</tr>
<tr>
<td>SEAS(1)</td>
<td>0.002207</td>
<td>0.051554</td>
<td>0.042818</td>
<td>0.9664</td>
</tr>
<tr>
<td>SEAS(2)</td>
<td>0.089966</td>
<td>0.058575</td>
<td>1.536412</td>
<td>0.1453</td>
</tr>
<tr>
<td>SEAS(3)</td>
<td>-0.009002</td>
<td>0.054845</td>
<td>-0.164738</td>
<td>0.8713</td>
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<tr>
<td>SEAS(4)</td>
<td>-0.047685</td>
<td>0.055448</td>
<td>-0.859993</td>
<td>0.4033</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.168212</td>
<td>0.758192</td>
<td>-0.221859</td>
<td>0.8274</td>
</tr>
<tr>
<td>MA(2)</td>
<td>0.061228</td>
<td>0.739206</td>
<td>0.082829</td>
<td>0.9351</td>
</tr>
<tr>
<td>MA(3)</td>
<td>-0.239690</td>
<td>0.352232</td>
<td>-0.680488</td>
<td>0.5066</td>
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<td>MA(4)</td>
<td>-0.189694</td>
<td>0.347840</td>
<td>-0.545349</td>
<td>0.5935</td>
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<tr>
<td>MA(5)</td>
<td>-0.453990</td>
<td>0.423384</td>
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</table>

R-squared 0.738736
Adjusted R-squared 0.529725
S E of regression 0.048836
Sum squared resid 0.035774
Log likelihood 53 54815
Durbin-Watson stat 2.229760

Inverted MA Roots 1.00 19 - 83i 19+ 83i - 61+ 51i - 61 - 51i

Figure 8.A.9: Theoretical ARMA for Money Targeting
### Theoretical ARMA for Fixed Exchange Rate: Germany (FGR)

Dependent Variable: PI
Method: Least Squares
Date: 07/14/05  Time: 14:54
Sample: 1986 1 1992 3
Included observations: 27
Convergence achieved after 40 iterations
Backcast: 1984 3 1985 4

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI(-1)</td>
<td>0.428356</td>
<td>0.861649</td>
<td>0.497136</td>
<td>0.6281</td>
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<tr>
<td>PI(-2)</td>
<td>0.723307</td>
<td>0.595016</td>
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<tr>
<td>PI(-3)</td>
<td>0.257794</td>
<td>0.800482</td>
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<tr>
<td>PI(-4)</td>
<td>-0.619766</td>
<td>0.283003</td>
<td>-2.189963</td>
<td>0.0490</td>
</tr>
<tr>
<td>PI(-5)</td>
<td>-0.125797</td>
<td>0.644575</td>
<td>-0.195163</td>
<td>0.8485</td>
</tr>
<tr>
<td>SEAS(1)</td>
<td>0.005264</td>
<td>0.029054</td>
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<td>0.8593</td>
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<tr>
<td>SEAS(2)</td>
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<tr>
<td>SEAS(3)</td>
<td>-0.034824</td>
<td>0.089178</td>
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<td>0.7030</td>
</tr>
<tr>
<td>SEAS(4)</td>
<td>0.003438</td>
<td>0.057634</td>
<td>0.059623</td>
<td>0.9534</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.102058</td>
<td>0.727395</td>
<td>-0.140306</td>
<td>0.8907</td>
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<tr>
<td>MA(2)</td>
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<td>0.423769</td>
<td>-2.265776</td>
<td>0.0405</td>
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<tr>
<td>MA(3)</td>
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<td>1.040809</td>
<td>-0.797397</td>
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<td>MA(4)</td>
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<td>MA(5)</td>
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</tr>
<tr>
<td>MA(6)</td>
<td>0.315285</td>
<td>0.449498</td>
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<td>0.4964</td>
</tr>
</tbody>
</table>

- R-squared: 0.829031
- Adjusted R-squared: 0.629568
- S.E. of regression: 0.025058
- Sum squared resid: 0.005355
- Log likelihood: 72.17350
- Durbin-Watson stat: 2.158795

Inverted MA Roots: 99+ 0i, 99 - 0i, -15 + 55i, -15 - 55i, -79 + 58i, -79 - 58i

Figure 8.A.10: Theoretical ARMA for Fixed Exchange Rate Germany
**Theoretical ARMA for FIT (RPI)**

Dependent Variable: PI  
Method: Least Squares  
Date: 07/14/05  
Time: 14:03  
Sample: 1992.4 2003.3  
Included observations: 44

Convergence achieved after 114 iterations  
Backcast: OFF (Roots of MA process too large)

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<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
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<tr>
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<td>0.0462</td>
</tr>
<tr>
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<td>0.013195</td>
<td>-0.082985</td>
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</tr>
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</table>

R-squared 0.846019  Mean dependent var 0.024828  
Adjusted R-squared 0.799359  S.D. dependent var 0.024938  
S.E. of regression 0.011170  Akaike info criterion -5.938792  
Sum squared resid 0.004118  Schwarz criterion -5.492745  
Log likelihood 141.6534  Durbin-Watson stat 1.769624  

Inverted MA Roots 0.68  0.28+ 0.62  0.28 - 0.62  -1.28

Estimated MA process is noninvertible

Figure 8.A.11: Theoretical ARMA for Flexible Inflation Targeting (RPI)
### Theoretical ARMA for FIT (RPIX)

**Dependent Variable**: PIRPIX  
**Method**: Least Squares  
**Date**: 07/14/05  
**Time**: 14:04  
**Sample**: 1992 4 2003 3  
**Included observations**: 44  
**Convergence achieved after**: 18 iterations  
**Backcast**: 1991 4 1992 3

<table>
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<th>Variable</th>
<th>Coefficient</th>
<th>Std Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PIRPIX(1)</td>
<td>-0.519678</td>
<td>0.237874</td>
<td>-2.184673</td>
<td>0.0361</td>
</tr>
<tr>
<td>PIRPIX(2)</td>
<td>0.439440</td>
<td>0.212002</td>
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<td>0.0465</td>
</tr>
<tr>
<td>PIRPIX(3)</td>
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<td>0.185283</td>
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<td>0.1715</td>
</tr>
<tr>
<td>SEAS(1)</td>
<td>0.008097</td>
<td>0.012632</td>
<td>0.862661</td>
<td>0.3875</td>
</tr>
<tr>
<td>SEAS(2)</td>
<td>0.055804</td>
<td>0.008427</td>
<td>6.217172</td>
<td>0.0000</td>
</tr>
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<td>SEAS(3)</td>
<td>0.027343</td>
<td>0.015811</td>
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</tr>
<tr>
<td>SEAS(4)</td>
<td>-0.008678</td>
<td>0.015701</td>
<td>-0.525333</td>
<td>0.6034</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.400444</td>
<td>0.196892</td>
<td>2.033524</td>
<td>0.0501</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-0.795947</td>
<td>0.233611</td>
<td>-3.407147</td>
<td>0.0017</td>
</tr>
<tr>
<td>MA(3)</td>
<td>-0.149047</td>
<td>0.194298</td>
<td>-0.767103</td>
<td>0.4485</td>
</tr>
<tr>
<td>MA(4)</td>
<td>0.833311</td>
<td>0.174781</td>
<td>4.823258</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

- **R-squared**: 0.899099  
- **Mean dependent var**: 0.025481  
- **S.D. dependent var**: 0.022338  
- **Akaike info criterion**: -6.581666  
- **Schwarz criterion**: -6.135619  
- **Durbin-Watson stat**: 1.966112

**Inverted MA Roots**: 
- 68 - 44i  
- 68+ 44i  
- - 88+ 42i  
- - 88 - 42i

---

Figure 8.A.12: Theoretical ARMA for Flexible Inflation Targeting (RPIX)
Theoretical ARMA for SIT (RPI)

Dependent Variable: PI
Method: Least Squares
Date: 07/14/05 Time: 14.10
Sample: 1992 4 2003 3
Included observations: 44
Convergence achieved after 24 iterations
Backcast: 1992 1 1992 3

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI(-1)</td>
<td>0.040961</td>
<td>0.186712</td>
<td>0.218846</td>
<td>0.8280</td>
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<tr>
<td>PI(-2)</td>
<td>0.840205</td>
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<td>0.0009</td>
</tr>
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<td>@SEAS(1)</td>
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<td>0.006645</td>
<td>0.371492</td>
<td>0.7125</td>
</tr>
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<td>0.002522</td>
<td>7.138738</td>
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</tr>
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<td>0.012031</td>
<td>0.233986</td>
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<td>@SEAS(4)</td>
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</tr>
<tr>
<td>MA(1)</td>
<td>0.191495</td>
<td>0.220178</td>
<td>0.869726</td>
<td>0.3904</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-0.748301</td>
<td>0.154775</td>
<td>-4.834757</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA(3)</td>
<td>-0.383194</td>
<td>0.168307</td>
<td>-2.276762</td>
<td>0.0290</td>
</tr>
</tbody>
</table>

R-squared: 0.761896
Adjusted R-squared: 0.707472
S.E. of regression: 0.013488
Akaike info criterion: -5.593621
Sum squared resid: 0.006367
Schwarz criterion: -5.229873
Log likelihood: 132.0641
Durbin-Watson stat: 1.959398

Inverted MA Roots: .98 - .58 - .23i - .58 + .23i

Figure 8.A.13: Theoretical ARMA for Strict Inflation Targeting (RPI)
### Theoretical ARMA for SIT (RPIX)

Dependent Variable: PIRPIX  
Method: Least Squares  
Date: 07/14/05  Time: 14:11  
Sample: 1992 4 2003 3  
Included observations: 44  
Convergence achieved after 25 iterations  
Backcast: 1992 1 1992 3

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PIRPIX(-1)</td>
<td>1.164900</td>
<td>0.189971</td>
<td>6.131996</td>
<td>0.0000</td>
</tr>
<tr>
<td>PIRPIX(-2)</td>
<td>-0.312258</td>
<td>0.174781</td>
<td>-1.786570</td>
<td>0.0827</td>
</tr>
<tr>
<td>@SEAS(1)</td>
<td>-0.007281</td>
<td>0.005423</td>
<td>-1.342536</td>
<td>0.1881</td>
</tr>
<tr>
<td>@SEAS(2)</td>
<td>0.051438</td>
<td>0.004968</td>
<td>10.35491</td>
<td>0.0000</td>
</tr>
<tr>
<td>@SEAS(3)</td>
<td>-0.059054</td>
<td>0.008302</td>
<td>-7.113288</td>
<td>0.0000</td>
</tr>
<tr>
<td>@SEAS(4)</td>
<td>0.030130</td>
<td>0.008373</td>
<td>3.598394</td>
<td>0.0010</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-1.595308</td>
<td>0.172973</td>
<td>-9.222878</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA(2)</td>
<td>0.702709</td>
<td>0.269526</td>
<td>2.607201</td>
<td>0.0133</td>
</tr>
<tr>
<td>MA(3)</td>
<td>0.164706</td>
<td>0.166715</td>
<td>0.987947</td>
<td>0.3300</td>
</tr>
</tbody>
</table>

R-squared: 0.889449  
Mean dependent var: 0.025481  
S.E. of regression: 0.007851  
Akaikes info criterion: -6.976052  
Sum squared resid: 0.002157  
Schwarz criterion: -6.311105  
Log likelihood: 155.8732  
Durbin-Watson stat: 1.938130

Inverted MA Roots: 88.47i, 88.47i, -1.17

---

Figure 8.A.14: Theoretical ARMA for Strict Inflation Targeting (RPIX)
Chapter 9

Calibration and Bootstrapping

9.1 Introduction

In the previous chapter we established that our basic classical model, calibrated in a standard way, can generate predictions that persistence will depend on the monetary regime in the direction of the stylised facts. As we have seen the analytical solution for inflation in each of the regimes\textsuperscript{1} has an $ARMA(p,q)$ representation. It is clear from the reduced-form solution for inflation that, under all the regimes persistence is the product of the forcing processes interacting with the monetary regime in place.

The objective of this chapter is to compare our model with the data more formally and test statistically whether our calibrated model is seriously consistent with the inflation data. Using a bootstrapping procedure, we generate bootstrap data for inflation under each regime and compute the implied sampling distribution of its $AR$ and $ARMA$ coefficients. We then check for each regime whether the coefficients of the actual inflation series lie within the 95\% confidence interval of the $AR$ and $ARMA$ coefficient distribution generated by our model. This is an ambitious test for such a basic model; and perhaps surprisingly – at least for some readers – the model does quite well. As a robustness exercise we carry out the same analysis on the Liverpool Model, which is an elaborated version of the same classical structure. We find that inflation persistence is predicted by the two new classical models fairly effectively and that

\textsuperscript{1}Except in case of the incomes policy regime where the solution for inflation has a pure $AR(2)$ representation as the demand and supply shocks $u_t$ and $r_t$ do not enter the solution.

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it varies with the monetary regime, in particular falling greatly when inflation targeting is in
place.

The chapter is organised as follows. In Section 9.2 we explain the choice of the calibrated
parameters and also the estimation procedure to get the remaining parameters. In Section
9.3 we bootstrap our classical models for the various regimes to test whether our models can
capture the persistence properties of inflation and in Section 9.4 we do the same analysis with
the Liverpool model. The conclusions are presented in Section 9.5.

9.2 Calibration and Estimation

In order to operationalise the models described in the previous chapter we use calibrated values
of various parameters. Several of these values are borrowed from closed economy models in
Rudebusch and Svensson (1999) and McCallum (2001) or are based on evidence discussed there.
For the open economy we use parameter values reported in Ball (1999) and Batini and Haldane
(1999). The calibrated parameter values can be found in the appendix Table 9.A.1.

We also estimate some of the parameters in the models. The estimated parameters can
be found in the appendix Table 9.A.2. In each of the models we estimate the $AR$ coefficients
of the IS and PP shocks. In doing so we initially intended to use the solution implied by
the model for $y_t$, $P_t$ or $\pi_t$. This means that the expectation of a variable would have been
model derived rather than being simply the realised value. However, as the solution itself is a
function of the errors, clearly there is circularity. What we do from a practical view point is
get a first approximation of the errors by simply plugging in the calibrated parameter values
along with the data in the IS/PP curve equation. For the expectational variables we get a first
approximation using McCallum’s MSV approach but ignoring the errors that appear in the
solution.\footnote{This is clearly not ideal. However, it enables us to get some starting values of the errors.}
Once we have the shock data we run $AR(1)$ on it, to get our first estimates of $\rho_0$ and
$\rho_1$ in the various models.\footnote{Please note that we have omitted the subscripts specific to each regime, e.g. $\rho_{PU\text{S},t}$ etc.}

To work out the ‘true’ errors and rhos we have used a rolling forecast programme. The pro-
grame works as follows. Our first estimates of the rhos enable it to work out the expectational

\footnote{This is clearly not ideal. However, it enables us to get some starting values of the errors.}
\footnote{Please note that we have omitted the subscripts specific to each regime, e.g. $\rho_{PU\text{S},t}$ etc.}
variables in the model. Using the expectational variables the model solves for the endogenous variables for the current period and all periods in the future. The new error then is simply the difference between the left hand side and right hand side of the original equation where actual data is plugged in for current and lagged endogenous variables and the expected terms are from the current rolling forecast. Then it estimates AR(1) on these new errors to get the new rhos, which can then be used to work out the new expectational variables. The model then solves again to get the new endogenous variables and then gets yet again a new set of errors. This iterative procedure is repeated till the errors and rhos converge to their 'true' values — as if the expectations that are model derived were used in the first place.

Steps of the rolling forecast explained in case of the PP curve:

1. Input first approximation of $\rho_1$ to allow the model to solve for the endogenous variables.

2. New error in the PP curve will be

$$v_{t}^{new} = \hat{y}_{t} - \delta(P_t - E_{t-1}P_t)$$

(9.1)

where $\hat{y}_t$ and $P_t$ will be actual data while $E_{t-1}P_t$ will be the model implied expectation from the current rolling forecast.

3. Estimate AR(1) on $v_{t}^{new}$

$$v_{t}^{new} = \rho_1^{new}v_{t-1}^{new} + x_{t}^{new}$$

(9.2)

4. Use $\rho_1^{new}$ in place of $\rho_1$ and repeat steps till the rhos converge.

In addition to the IS/PP shocks the money targeting regime has a money demand shock and the inflation targeting regime has Orphanides (1998) forecast error. Both of these are modelled as AR(1)'s. As we again face the circularity problem, we use the rolling forecast procedure described above to converge to the 'true' errors. Once we get them we simply estimate an AR(1) on the money demand shock and Orphanides forecast error data to get $\rho_6$ and $\rho_2$.\footnote{We omit regime specific subscripts.}

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Further, the foreign interest rate, foreign GDP and foreign prices in the fixed exchange rate regimes are modelled as $AR(1)$’s. We estimated the coefficients, $\rho$, $\rho_4$ and $\rho_5$ by running an $AR(1)$ on the foreign interest rate, foreign GDP, foreign prices data.

9.3 Bootstrapping

The models presented in the previous chapter are all linear and hence can be solved analytically. The reduced form solution of the models imply a data generating process for inflation, i.e. our model maps into an exact description of the data. So a natural way to test the model is to use the ARMA form implied by it. However, comparison of our models with the ARs and ARMA we have estimated on the actual data can not be done via deterministic simulation because the estimated equations depend on the distributions of all the shocks, each of them with rather different impulse response functions because of different MA processes. What we wish to do is to replicate the stochastic environment to see whether within it our estimated AR and ARMA equations could have been generated. This we do via bootstrapping the models with their error processes.

The idea is to create pseudo data samples – here 1000 – for inflation. Within each regime we draw the vectors of i.i.d. shocks in our error processes with replacement;9 we then input them into their error processes and these in turn into the model to solve for the implied path of inflation over the sample period. We then run AR and ARMA regressions on all the samples to derive the implied 95% confidence intervals for all the coefficients. Finally we compare the AR and ARMA coefficients estimated from the actual data7 to see whether they lie within these 95% confidence intervals. The comparison both guides us on whether our models are moving the parameters in the right direction; and informs us more formally whether the data rejects the models. Table 9.1 summarises the results of this exercise. In our analysis we shall refer to the $AR(1)$ as the measure of ‘net persistence’ that interests us most; the ARMA parameters, hard to interpret as they are in terms of their net effect, we refer to in the context of the formal rejection test.

---

7 Again we have omitted subscripts for specific regimes.
8 By drawing vectors for the same time period we preserve their contemporaneous cross-correlations.
9 The ARMA form being the model implied for the various regimes.
**Fixed Exchange Rate (US) - Bretton Woods**

Autoregressive Moving Average (ARMA)

<table>
<thead>
<tr>
<th>ARMA(5,6)</th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>0.369189</td>
<td>-0.435870  0.832419</td>
</tr>
<tr>
<td>AR(2)</td>
<td>0.171348</td>
<td>-0.736690  0.339484</td>
</tr>
<tr>
<td>AR(3)</td>
<td>0.310143</td>
<td>-0.094970  0.934679</td>
</tr>
<tr>
<td>AR(4)</td>
<td>-0.201350</td>
<td>-0.415650  0.682341</td>
</tr>
<tr>
<td>AR(5)</td>
<td>0.386906</td>
<td>-0.119370  0.775123</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.140974</td>
<td>-1.000110  0.496279</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-0.273200</td>
<td>-0.992090  0.456181</td>
</tr>
<tr>
<td>MA(3)</td>
<td>0.021023</td>
<td>-1.463660  0.341461</td>
</tr>
<tr>
<td>MA(4)</td>
<td>0.453756</td>
<td>-1.210480  0.590593</td>
</tr>
<tr>
<td>MA(5)</td>
<td>-0.734410</td>
<td>-1.067740  0.849136</td>
</tr>
<tr>
<td>MA(6)</td>
<td>-0.557030</td>
<td>-0.734190  0.924609</td>
</tr>
</tbody>
</table>

Autoregression

| AR(1)   | 0.252211  | 0.092357  0.469074    |

**Incomes Policy**

Autoregressive Moving Average (ARMA)

<table>
<thead>
<tr>
<th>ARMA(2,0)</th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>0.861013*</td>
<td>0.228295  0.847431</td>
</tr>
<tr>
<td>AR(2)</td>
<td>-0.169590</td>
<td>-0.253890  0.427912</td>
</tr>
</tbody>
</table>

Autoregression

| AR(1)    | 0.735547  | 0.330924  0.784238    |

**Money Targeting**

Autoregressive Moving Average (ARMA)

<table>
<thead>
<tr>
<th>ARMA(4,5)</th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>0.286295</td>
<td>-0.185210  1.087224</td>
</tr>
<tr>
<td>AR(2)</td>
<td>0.554090</td>
<td>-0.521350  0.645124</td>
</tr>
<tr>
<td>AR(3)</td>
<td>0.107269</td>
<td>-0.310950  0.862329</td>
</tr>
<tr>
<td>AR(4)</td>
<td>-0.152760</td>
<td>-0.403790  0.680318</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.168210</td>
<td>-1.339540  0.807581</td>
</tr>
<tr>
<td>MA(2)</td>
<td>0.061228</td>
<td>-1.711810  1.489934</td>
</tr>
<tr>
<td>MA(3)</td>
<td>-0.239690</td>
<td>-2.044730  0.847799</td>
</tr>
<tr>
<td>MA(4)</td>
<td>-0.189690</td>
<td>-2.063560  0.921117</td>
</tr>
<tr>
<td>MA(5)</td>
<td>-0.453990</td>
<td>-1.507360  1.197630</td>
</tr>
</tbody>
</table>

Autoregression

| AR(1)    | 0.516666* | 0.519515  0.830059    |

Table 9.1: Confidence Limits from our Model for Theoretical ARMAs
### Fixed Exchange Rate (Germany) - ERM

Autoregressive Moving Average (ARMA)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>0.428356</td>
<td>-0.303170</td>
<td>1.050878</td>
</tr>
<tr>
<td>AR(2)</td>
<td>0.723307*</td>
<td>-0.990920</td>
<td>0.279975</td>
</tr>
<tr>
<td>AR(3)</td>
<td>0.257794</td>
<td>-0.142510</td>
<td>1.115138</td>
</tr>
<tr>
<td>AR(4)</td>
<td>-0.619770</td>
<td>-0.679960</td>
<td>0.593272</td>
</tr>
<tr>
<td>AR(5)</td>
<td>-0.125800*</td>
<td>-0.117420</td>
<td>0.917180</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.102060</td>
<td>-1.624150</td>
<td>0.870962</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-0.972880</td>
<td>-2.014400</td>
<td>2.082863</td>
</tr>
<tr>
<td>MA(3)</td>
<td>-0.829780</td>
<td>-4.609630</td>
<td>0.823029</td>
</tr>
<tr>
<td>MA(4)</td>
<td>0.458327</td>
<td>-4.299220</td>
<td>1.679433</td>
</tr>
<tr>
<td>MA(5)</td>
<td>0.183196</td>
<td>-4.022460</td>
<td>1.764819</td>
</tr>
<tr>
<td>MA(6)</td>
<td>0.315285</td>
<td>-4.230640</td>
<td>2.238993</td>
</tr>
</tbody>
</table>

**Autoregression**

AR(1) 0.629426 0.260355 0.647129

### Flexible Inflation Targeting (RPI)

Autoregressive Moving Average (ARMA)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>0.418438</td>
<td>-0.720640</td>
<td>1.192160</td>
</tr>
<tr>
<td>AR(2)</td>
<td>0.379242</td>
<td>-0.735370</td>
<td>0.937082</td>
</tr>
<tr>
<td>AR(3)</td>
<td>-0.456180*</td>
<td>-0.247320</td>
<td>0.938262</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.034027</td>
<td>-1.511640</td>
<td>0.760795</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-0.747690</td>
<td>-1.295610</td>
<td>1.134739</td>
</tr>
<tr>
<td>MA(3)</td>
<td>0.770969*</td>
<td>-1.291620</td>
<td>0.584936</td>
</tr>
<tr>
<td>MA(4)</td>
<td>-0.401650</td>
<td>-0.552310</td>
<td>0.926373</td>
</tr>
</tbody>
</table>

**Autoregression**

AR(1) 0.202199 -0.023440 0.598286

Table 9.1 cont. Confidence Limits from our Model for Theoretical ARMA
### Flexible Inflation Targeting (RPIX)

<table>
<thead>
<tr>
<th>Autoregressive Moving Average (ARMA)</th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARMA(3,4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.519680</td>
<td>-0.681860 1.257508</td>
</tr>
<tr>
<td>AR(2)</td>
<td>0.438440</td>
<td>-0.733350 0.949475</td>
</tr>
<tr>
<td>AR(3)</td>
<td>0.258987</td>
<td>-0.256630 0.946301</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.400444</td>
<td>-1.494870 0.748731</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-0.795950</td>
<td>-1.237780 1.081012</td>
</tr>
<tr>
<td>MA(3)</td>
<td>-0.149050</td>
<td>-1.300500 0.661509</td>
</tr>
<tr>
<td>MA(4)</td>
<td>0.633311</td>
<td>-0.506660 0.920978</td>
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<tr>
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<td></td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.152273</td>
<td>-0.459867 0.119078</td>
</tr>
</tbody>
</table>

### Strict Inflation Targeting (RPI)

<table>
<thead>
<tr>
<th>Autoregressive Moving Average (ARMA)</th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARMA(2,3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.040861</td>
<td>-0.011890 1.069581</td>
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<tr>
<td>AR(2)</td>
<td>0.640205</td>
<td>-0.099710 0.976303</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.191495*</td>
<td>-1.872590 -0.131300</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-0.748300</td>
<td>-1.206100 0.856868</td>
</tr>
<tr>
<td>MA(3)</td>
<td>-0.383190*</td>
<td>-0.136060 0.980095</td>
</tr>
<tr>
<td>Autoregression</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.202199</td>
<td>-0.207010 0.293517</td>
</tr>
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</table>

### Strict Inflation Targeting (RPIX)

<table>
<thead>
<tr>
<th>Autoregressive Moving Average (ARMA)</th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARMA(2,3)</td>
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<td></td>
</tr>
<tr>
<td>AR(1)</td>
<td>1.164900*</td>
<td>-0.05212 1.003118</td>
</tr>
<tr>
<td>AR(2)</td>
<td>-0.312260*</td>
<td>-0.07417 0.976858</td>
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<tr>
<td>MA(1)</td>
<td>-1.595310</td>
<td>-1.78878 -0.079260</td>
</tr>
<tr>
<td>MA(2)</td>
<td>0.702709</td>
<td>-0.99176 0.792745</td>
</tr>
<tr>
<td>MA(3)</td>
<td>0.164706</td>
<td>-0.13604 0.978132</td>
</tr>
<tr>
<td>Autoregression</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.152273</td>
<td>-0.616226 -0.191988</td>
</tr>
</tbody>
</table>

Table 9.1 cont. Confidence Limits from our Model for Theoretical ARMAs

Taking each regime in turn, we find that the model has considerable success under the
Bretton Woods regime. Here it predicts that net persistence, the $AR(1)$ parameter, will be moderate — within a range of 0.09 and 0.47 — against an actual estimate of 0.25. All eleven $ARMA$ parameters comfortably lie within the 95% confidence intervals implied by the model's bootstraps.

With the Incomes Policy period net persistence rose substantially in the actual data — from 0.25 to 0.74. The model’s net persistence range similarly moves upwards to a range of 0.33 to 0.78 with the actual estimate clearly lying within. The theoretical implied $ARMA$ form is an $AR(2)$. The model does not perform all that badly as the $AR(1)$ parameter is only marginally outside; however it is clearly under-predicting the degree of persistence. Thus the data formally rejects the model for this period but does at least get the direction of change right; it is perhaps not surprising the model fails given our discussion of its inadequacies in the previous chapter.

We observe that net persistence during Money Targeting (MT) and Fixed Exchange Rate (Germany)/ERM remained high at similar levels to the Incomes Policy period; viz 0.52 for MT and 0.63 for ERM. In the Money Targeting regime the model predicts a net persistence range of 0.52 to 0.83. The $AR(1)$ parameter being 0.517 – after rounding – just manages to lie within the limits. All the $ARMA$ parameters, comfortably lie within the 95% confidence intervals. Thus, the data accepts the model. Moving onto the ERM period, the actual estimate of net persistence is 0.63 which lies within the model’s range of 0.26 to 0.65. Analysing the inflation $ARMA$ we find that two out of the eleven coefficient lie outside; one of them quite a lot so.\footnote{Clearly the problem is that the theoretical implied $ARMA$'s are so large that their standard errors rise — it's harder to pin each parameter down.}

In both cases the model also fails to capture the direction of movement; hence the model in total is formally rejected. Again perhaps, given the turbulence of this period with repeated changes of tack in the emerging formulation of monetary policy, and also the fitful progress in inflation itself towards the official targets, the model's failure is not surprising. It does at least accurately gauge the continued presence of substantial persistence, as well as its level.

We get the most interesting results when we turn to the Inflation Targeting regime. In the case of Flexible Inflation Targeting (FIT) with RPI the model is formally rejected. The net persistence $AR(1)$ parameter lies within the model’s limits; however two of the seven $ARMA$ parameters lie quite a lot outside. When we use RPIX to calculate inflation the model is re-
assuringly accepted — reassuringly because RPIX, not RPI, is the regime target. All seven 
ARMA coefficients comfortably lie within the bootstrap confidence limits. The model also 
captures the more parsimonious AR(1) form of representing persistence; it predicts a range of 
-0.46 to 0.12 with the actual estimate -0.15 comfortably lying within.

We think it is logical to concentrate on RPIX for this period, since with interest rate 
targeting the RPI itself became highly influenced by monetary policy in a misleading way. 
Certainly the persistent movements of interest rates would have imparted spurious persistence 
to RPI inflation as can be seen by the positive net persistence coefficient estimated.

In case of Strict Inflation Targeting (SIT) with RPI the model again is rejected. Although 
like FIT: RPI the net persistence parameter lies between the model’s 95% confidence limits, two 
of the five ARMA parameters lie outside — both of them quite a lot outside. In case of MA(1) 
the model gets the direction totally wrong; it predicts a negative band while the actual estimate 
is positive. When we use RPIX the model is again rejected; but the rejection is not as severe 
as in the case of SIT: RPI. It predicts that the range of net persistence falls to -0.62 to -0.19.
The actual AR(1) parameter falls to -0.15, lying within this range. All the MA coefficient are 
within the 95% confidence intervals while both the AR are outside.

What we find therefore is that the model does capture some directional characteristics of 
the data. The data say that both Bretton Woods and the Inflation Targeting regimes generated 
quite low persistence, while the Incomes Policy, Money Targeting and ERM periods all exhibited 
quite high persistence. This is mirrored accurately by the model. However, in formal terms the 
model is rejected for two out of the five regimes – representing the period of no nominal anchor 
from 1971 to 1978 and the Deutsche Mark shadowing period from 1986 to 1992 – when UK 
inflation went through its wildest gyrations; plainly and not surprisingly the model needs more 
careful dynamic specification for these periods.

9.3.1 Robustness Exercise: Our New Classical Model

As a robustness test of our model, we would like to check whether the coefficients of the best-
fitting ARMA representation of the inflation data lie within the model’s confidence limits.
We all know that econometricians always try and find the best-fitting most parsimonious data 
generating process. According to our null-hypothesis model within which we organise our
thinking; the best-fitting ARMA is in fact a mis-specification. However, it must be noted that
the model implied ARMA and the best-fitting ARMA are simply different ways of representing
the same data. Hence, if a model is in fact ‘true’ then all coefficients of any representation should
in principle lie within the model’s confidence limits.

Theoretically any specification of the data can be tested by our model. If we take a repre-
sentation of the data then the question is ‘How does a particular representation fare in terms
of the standard errors implied by the model generated data?’ In other words, a model, if cor-
rect, generates data in such a way that it should still accept or reject, as the case may be, the
best-fitting representation. Table 9.2 reports the results of this exercise.

<table>
<thead>
<tr>
<th>Fixed Exchange Rate (US) - Bretton Woods</th>
</tr>
</thead>
<tbody>
<tr>
<td>AutoRegression Moving Average (ARMA)</td>
</tr>
<tr>
<td>ARMA(2,3)</td>
</tr>
<tr>
<td>Estimated</td>
</tr>
<tr>
<td>AR(1)</td>
</tr>
<tr>
<td>AR(2)</td>
</tr>
<tr>
<td>MA(1)</td>
</tr>
<tr>
<td>MA(2)</td>
</tr>
<tr>
<td>MA(3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Incomes Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>AutoRegression Moving Average (ARMA)</td>
</tr>
<tr>
<td>ARMA(3,2)</td>
</tr>
<tr>
<td>Estimated</td>
</tr>
<tr>
<td>AR(1)</td>
</tr>
<tr>
<td>AR(2)</td>
</tr>
<tr>
<td>AR(3)</td>
</tr>
<tr>
<td>MA(1)</td>
</tr>
<tr>
<td>MA(2)</td>
</tr>
</tbody>
</table>

Table 9.2: Confidence Limits from our Model for Best-fitting ARMAs
### Money Targeting

**Autoregressive Moving Average (ARMA)**

<table>
<thead>
<tr>
<th></th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARMA(1,1)</td>
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<td></td>
</tr>
<tr>
<td>AR(1)</td>
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<td>0.383017</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.997379</td>
<td>0.708031</td>
</tr>
</tbody>
</table>

### Fixed Exchange Rate (Germany) - ERM

**Autoregressive Moving Average (ARMA)**

<table>
<thead>
<tr>
<th></th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARMA(1,0)</td>
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<td></td>
</tr>
<tr>
<td>AR(1)</td>
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<td>0.260355</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.647129</td>
</tr>
</tbody>
</table>

### Flexible Inflation Targeting (RPI)

**Autoregressive Moving Average (ARMA)**

<table>
<thead>
<tr>
<th></th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARMA(3,3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.720498</td>
<td>-0.687580</td>
</tr>
<tr>
<td>AR(2)</td>
<td>0.514975</td>
<td>-0.689790</td>
</tr>
<tr>
<td>AR(3)</td>
<td>-0.835545*</td>
<td>-0.306820</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.761763</td>
<td>-1.567670</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-0.701631</td>
<td>-1.044250</td>
</tr>
<tr>
<td>MA(3)</td>
<td>0.971221*</td>
<td>-1.017430</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.756928</td>
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</tbody>
</table>

### Flexible Inflation Targeting (RPIX)

**Autoregressive Moving Average (ARMA)**

<table>
<thead>
<tr>
<th></th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARMA(1,4)</td>
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</tr>
<tr>
<td>AR(1)</td>
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<td>-0.651710</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.683187</td>
<td>-1.630600</td>
</tr>
<tr>
<td>MA(2)</td>
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<td>-0.393600</td>
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<tr>
<td>MA(3)</td>
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<tr>
<td>MA(4)</td>
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<td>-0.524310</td>
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<tr>
<td></td>
<td></td>
<td>0.924447</td>
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</tbody>
</table>

Table 9.2 cont. Confidence Limits from our Model for Best-fitting ARMAs
**Strict Inflation Targeting (RPI)**

Autoregressive Moving Average (ARMA)

<table>
<thead>
<tr>
<th>ARMA(3,3)</th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tr>
<tr>
<td>AR(1)</td>
<td>0.720498</td>
<td>-0.438527</td>
</tr>
<tr>
<td>AR(2)</td>
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</tr>
<tr>
<td>AR(3)</td>
<td>-0.835545*</td>
<td>-0.140024</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.761763</td>
<td>-1.781348</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-0.701631</td>
<td>-0.988100</td>
</tr>
<tr>
<td>MA(3)</td>
<td>0.971221*</td>
<td>-0.952990</td>
</tr>
</tbody>
</table>

**Strict Inflation Targeting (RPIX)**

Autoregressive Moving Average (ARMA)

<table>
<thead>
<tr>
<th>ARMA(1,4)</th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower</td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.743549</td>
<td>-0.868180</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.683187</td>
<td>-2.005190</td>
</tr>
<tr>
<td>MA(2)</td>
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<td>-0.311950</td>
</tr>
<tr>
<td>MA(3)</td>
<td>0.000744</td>
<td>-0.392200</td>
</tr>
<tr>
<td>MA(4)</td>
<td>0.594662</td>
<td>-0.570560</td>
</tr>
</tbody>
</table>

Table 9.2 cont. Confidence Limits from our Model for Best-fitting ARMA's

What we find is that this exercise more or less confirms the results we found using the theoretical ARMA representations earlier. Using the best-fitting ARMA's the model is comprehensively accepted in the case of the Bretton Woods and Money Targeting regime and rejected for Incomes Policy, just as in the theoretical ARMA's. For the ERM period under the theoretical representation the model was marginally rejected, though marginally accepted using the best-fitting representation, a simple AR(1). For our preferred regime of Flexible Inflation Targeting with RPIX, the model is accepted under the theoretical representation and very marginally rejected – on one parameter – for the best-fitting one. For the other IT regimes all versions are rejected under both; though the Strict IT regime with RPIX also comes close under both.

All in all we find that our very basic model 'tells quite a good story' even if it can not strictly generate the facts of inflation persistence; this is in a way rather re-assuring because it is merely a classroom model with calibrated parameters; the rejection does not seem so bad
that some dynamic enrichment could not fix it.

9.4 Substituting a Fully-specified version of the Basic New Classical Model — Bootstrapping the Liverpool Model

It seemed worthwhile to test our conclusion from the results with this basic model by running the same regimes on a fully-specified model derived from the same micro-foundations, viz the Liverpool Model of the UK.\(^9\) Plainly considerable efforts have been made in this 20-year-old model to address the dynamic issues that appear to lie behind the stochastic short-comings of our benchmark model. In Table 9.3 that follows we report the results of applying the regimes to the equivalent Liverpool versions of the model set out above. We are unable to run the model over the 1956-71 period as the errors used below – which were those available from the model’s fitting procedures – relate to the period 1986-2002. We have however assumed these could be applied to the 1971-86 period, as this was a period of floating exchange rates as well.

<table>
<thead>
<tr>
<th>Incomes Policy</th>
<th>Autoregressive Moving Average (ARMA)</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ARMA(2,0)</td>
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<tr>
<td>AR(1)</td>
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</tr>
<tr>
<td>Autoregression</td>
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</tr>
<tr>
<td>AR(1)</td>
<td>0.735547</td>
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</table>

Table 9.3: Confidence Limits from the Liverpool Model for Theoretical ARMAs

\(^9\)Appendix 9.B carries some details of the model.
## Money Targeting

### Autoregressive Moving Average (ARMA)

<table>
<thead>
<tr>
<th></th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
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</tr>
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<td>-0.845800 0.508746</td>
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<td>AR(3)</td>
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<td>-0.492720 0.742031</td>
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<td>-0.617990 0.526714</td>
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<td>MA(2)</td>
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<td>-1.013140 0.548595</td>
</tr>
<tr>
<td>MA(5)</td>
<td>-0.453990</td>
<td>-0.470160 0.968168</td>
</tr>
</tbody>
</table>

**Autoregression**

| AR(1)  | 0.516666     | 0.035179 0.593107        |

## Fixed Exchange Rate (Germany) - ERM

### Autoregressive Moving Average (ARMA)

<table>
<thead>
<tr>
<th></th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
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</tr>
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</tr>
<tr>
<td>AR(3)</td>
<td>0.257794</td>
<td>-0.943070 1.210251</td>
</tr>
<tr>
<td>AR(4)</td>
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<td>-0.954318 0.632320</td>
</tr>
<tr>
<td>AR(5)</td>
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</tr>
<tr>
<td>MA(1)</td>
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<td>-1.017260 1.015541</td>
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<tr>
<td>MA(2)</td>
<td>-0.972880*</td>
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</tr>
<tr>
<td>MA(3)</td>
<td>-0.829780*</td>
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</tr>
<tr>
<td>MA(4)</td>
<td>0.458327</td>
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<td>MA(5)</td>
<td>0.183196</td>
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</tr>
<tr>
<td>MA(6)</td>
<td>0.315285</td>
<td>-0.751777 0.945495</td>
</tr>
</tbody>
</table>

**Autoregression**

| AR(1)  | 0.629426*    | 0.826362 0.985212        |

---

Table 9.3 cont. Confidence Limits from the Liverpool Model for Theoretical ARMAs
**Flexible Inflation Targeting (RPIX)**

<table>
<thead>
<tr>
<th>Autoregressive Moving Average (ARMA)</th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARMA(3,4)</td>
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<td></td>
</tr>
<tr>
<td>AR(1)</td>
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</tr>
<tr>
<td>AR(2)</td>
<td>0.438440</td>
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</tr>
<tr>
<td>AR(3)</td>
<td>0.258987</td>
<td>-0.794090 0.932990</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.400444</td>
<td>-1.741530 0.945705</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-0.795950</td>
<td>-1.289680 1.944119</td>
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</tr>
<tr>
<td>MA(4)</td>
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<td>-0.503310 0.942000</td>
</tr>
<tr>
<td>Autoregression</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.152273*</td>
<td>-0.558027 -0.199794</td>
</tr>
</tbody>
</table>

**Strict Inflation Targeting (RPIX)**

<table>
<thead>
<tr>
<th>Autoregressive Moving Average (ARMA)</th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARMA(2,3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(1)</td>
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<td>-1.71052 0.476371</td>
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<tr>
<td>AR(2)</td>
<td>-0.312260</td>
<td>-0.96927 0.484217</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-1.595310*</td>
<td>-1.31977 1.177825</td>
</tr>
<tr>
<td>MA(2)</td>
<td>0.702709</td>
<td>-0.90165 1.261976</td>
</tr>
<tr>
<td>MA(3)</td>
<td>0.164706</td>
<td>-0.96865 0.592074</td>
</tr>
<tr>
<td>Autoregression</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.152273*</td>
<td>-0.605031 -0.306878</td>
</tr>
</tbody>
</table>

Table 9.3 cont. Confidence Limits from the Liverpool Model for Theoretical ARMAs

Again taking each regime in turn, we find that the Liverpool model has some success in case of Incomes Policy. The net persistence parameter and both the ARMA coefficients comfortably lie within the limits. We get similar results for Money Targeting with the net and gross persistence parameters within the model's 95% limits — AR(2) is only marginally outside. In case of ERM the model clearly overpredicts persistence. The actual estimate is 0.63 while the model implied confidence band is 0.83 to 0.99. Analysing the inflation ARMA we find that three out of the eleven parameters lie outside the models confidence limits; hence the data formally rejects the model for this period.

The model also slightly under-predicts the persistence under Flexible Inflation Targeting: RPIX the actual estimate being -0.15 while the model implies a band of -0.56 to -0.20. In
formal terms the data accepts the model as all the ARMA parameters are within the 95% limits implied by the model’s bootstraps. Re-assuringly the data rejects Strict Inflation Targeting: RPIX as two of the ARMA parameters lie outside; one of them quite a lot so. The net persistence parameter also lies way outside the model’s confidence limits.

We see that the Liverpool model also captures the directional characteristics of the inflation data. Like the data it says that persistence is high in case of Incomes Policy, Money Targeting and ERM period and it falls dramatically as we move to the Inflation Targeting regime. In formal terms the model is rejected in one out of the four regimes,\(^{10}\) representing the period of 1986 to 1992 when there were repeated changes of tack in the emerging formulation of monetary policy, and also the fitful progress in inflation itself towards the official targets, the model’s failure is not surprising. As with the benchmark model, the same message comes through: that changes in persistence come about through regime changes.

### 9.4.1 Robustness Exercise: Liverpool Model

As before, we would like to check whether the coefficients of the best-fitting ARMA representation of the inflation data lie within the Liverpool model’s confidence limits. Table 9.4 below summarises the results of this analysis.

<table>
<thead>
<tr>
<th>Incomes Policy</th>
<th>Autoregressive Moving Average (ARMA)</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARMA(3,2)</td>
<td>Estimated</td>
<td>Lower</td>
</tr>
<tr>
<td>AR(1)</td>
<td>1.720229</td>
<td>-0.380451</td>
</tr>
<tr>
<td>AR(2)</td>
<td>-1.707837*</td>
<td>-1.662390</td>
</tr>
<tr>
<td>AR(3)</td>
<td>0.722245</td>
<td>-0.642377</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.968766</td>
<td>-1.257318</td>
</tr>
<tr>
<td>MA(2)</td>
<td>0.984350</td>
<td>-0.969748</td>
</tr>
</tbody>
</table>

Table 9.4: Confidence Limits from the Liverpool Model for Best-fitting ARMAs

\(^{10}\)At least when shocks are applied from the 1986-2003 period; strictly we should apply to each regime the shocks from that regime only.
### Money Targeting

**Autoregressive Moving Average (ARMA)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>0.925756</td>
<td>-0.66046 - 1.022038</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.997379*</td>
<td>-0.988756 - 0.997495</td>
</tr>
</tbody>
</table>

### Fixed Exchange Rate (Germany) - ERM

**Autoregressive Moving Average (ARMA)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>0.629426*</td>
<td>0.826362 - 0.985212</td>
</tr>
</tbody>
</table>

### Flexible Inflation Targeting (RPIX)

**Autoregressive Moving Average (ARMA)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>-0.743549</td>
<td>-1.001816 - 0.993012</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.683187*</td>
<td>-1.645765 - 0.624049</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-0.359680</td>
<td>-0.631137 - 0.863117</td>
</tr>
<tr>
<td>MA(3)</td>
<td>0.000744</td>
<td>-0.348020 - 0.445033</td>
</tr>
<tr>
<td>MA(4)</td>
<td>0.594662*</td>
<td>-0.196350 - 0.542150</td>
</tr>
</tbody>
</table>

### Strict Inflation Targeting (RPIX)

**Autoregressive Moving Average (ARMA)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>-0.743549</td>
<td>-0.982940 - 0.850013</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.683187*</td>
<td>-1.820788 - 0.390057</td>
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<tr>
<td>MA(2)</td>
<td>-0.359680</td>
<td>-0.949071 - 0.907622</td>
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<tr>
<td>MA(3)</td>
<td>0.000744</td>
<td>-0.407236 - 0.480447</td>
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<tr>
<td>MA(4)</td>
<td>0.594662*</td>
<td>-0.351638 - 0.471365</td>
</tr>
</tbody>
</table>

Table 9.4 cont. Confidence Limits from the Liverpool Model for Best-fitting ARMAs

In case of Incomes Policy four out of the five ARMA coefficients comfortably lie within the limits; the fifth one just outside. So the data more or less accepts the model. We get similar results for Money Targeting with the gross persistence parameters within the model's 95% limits — MA(1) is only marginally outside. The best-fitting ARMA for the ERM period is

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an $AR(1)$. As discussed earlier the model clearly overpredicts persistence for the ERM regime. In formal terms the model is rejected.

In case of Flexible Inflation Targeting: RPIX the model is rejected. However, the rejection is not very comprehensive as both $MA(1)$ and $MA(4)$ are very close to the upper bound implied by the model. In case of Strict Inflation Targeting: RPIX the model is outrightly rejected. As can be seen $MA(1)$ and $MA(4)$ quite a lot outside; even the direction of movement being incorrect.

Again the robustness exercise provides further support to the Liverpool model, which in comparison to our basic model is capable of capturing the persistence properties of inflation under the Incomes Policy regime. Like our model it also fails to capture the ERM period. This in a way is re-assuring as this was the famous period when the Chancellor and the Economic adviser resigned on the same night. The basic story that comes across from the above analysis is that one can model inflation persistence using a new classical set-up and that persistence changes with the monetary policy regime in place.

### 9.5 Conclusions

The facts of UK inflation persistence appear to be inconsistent with the common practice of building persistence into the Phillips Curve; this is because persistence clearly varies across the post-war sample. Our contention here is that, as suggested by a basic New Classical model of the economy as well as by a more elaborate version in practical use, it varies mainly with the monetary regime. The roots of the inflation $ARMA$ process may not alter too much over time as they depend in this model on the autoregressive coefficients of the exogenous error processes. Monetary regimes can however ‘close this persistence down’ for inflation if they choose, this closing-down showing in the $MA$ part of the process. Our benchmark and also our full New Classical model with a non-persistent Phillips Curve are both capable of replicating a fair amount of this regime-dependence qualitatively, generating higher persistence in the earlier regimes than under inflation targeting regimes; they each strictly fail to capture the dynamics of particular periods. Nevertheless the rejections do not seem to invalidate the basic conclusion that persistence is likely to depend on the monetary regime. We conclude in short that inflation
persistence is a phenomenon of policy not of nature.
9.A Our New Classical Model: Calibrated and Estimated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibrated Value</th>
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</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.4</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1</td>
</tr>
<tr>
<td>$a_{FUS,0}$</td>
<td>2</td>
</tr>
<tr>
<td>$a_{FUS,1}$</td>
<td>0.65</td>
</tr>
<tr>
<td>$a_{FGR,1}$</td>
<td>2</td>
</tr>
<tr>
<td>$a_{FGR,1}$</td>
<td>0.65</td>
</tr>
<tr>
<td>$c$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\beta_{MT_4}$</td>
<td>0.15</td>
</tr>
<tr>
<td>$\beta_{MT_5}$</td>
<td>1</td>
</tr>
<tr>
<td>$\beta_{IT_0}$</td>
<td>0.025</td>
</tr>
<tr>
<td>$\beta_{IT_1}$</td>
<td>0.85</td>
</tr>
<tr>
<td>$\beta_{IT_2}$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\beta_{IT_3}$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 9.A.1: Calibrated Parameters

<table>
<thead>
<tr>
<th>Regime</th>
<th>Estimated Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Exchange Rate: US (FUS)</td>
<td>$\rho$  $\rho_0$  $\rho_1$  $\rho_2$  $\rho_3$  $\rho_4$  $\rho_5$</td>
</tr>
<tr>
<td>Incomes Policy (IP)</td>
<td>0.958062  0.521085  0.717961  0.833633  1.002312</td>
</tr>
<tr>
<td>Money Targeting (MT)</td>
<td>0.575105  -0.05911  -0.211389  0.908337</td>
</tr>
<tr>
<td>Fixed Exchange Rate: Germany</td>
<td>0.990000  0.650013  0.722195  0.478226  1.001455</td>
</tr>
<tr>
<td>Flexible Inflation Targeting</td>
<td>0.853722  0.030236  -0.32902</td>
</tr>
<tr>
<td>(FIT) RPI</td>
<td></td>
</tr>
<tr>
<td>Flexible Inflation Targeting</td>
<td>0.855848  0.041632  -0.41007</td>
</tr>
<tr>
<td>(FIT) RPIX</td>
<td></td>
</tr>
<tr>
<td>Strict Inflation Targeting</td>
<td>0.858897  0.000518</td>
</tr>
<tr>
<td>(SIT) RPI</td>
<td></td>
</tr>
<tr>
<td>Strict Inflation Targeting</td>
<td>0.849514  0.073353</td>
</tr>
<tr>
<td>(SIT) RPIX</td>
<td></td>
</tr>
</tbody>
</table>

Table 9.A.2: Estimated Parameters
9.B The Liverpool Model of the UK

9.B.1 Liverpool Model: Overview

The Liverpool Model of the UK is an open economy version of a rational expectations IS-LM model, such as can be derived from a micro-founded model by suitable approximations\(^\text{11}\) — thus for example the Liverpool Model IS curve has the expectation of future output in it, the hallmark of this approximation. The model’s Phillips or Supply curve assumes overlapping 4-quarter wage contracts — thus real wages are affected by the 4-quarter-moving average of inflation surprises. The labour market underpinning it is explicit and the model solves for equilibrium or natural rates of output, unemployment and relative prices. In recent work a new full information maximum likelihood (FIML) algorithm developed in Cardiff University\(^\text{12}\) has been used to re-estimate the model parameters: it turns out that the new estimates are little different from the model’s original ones, based partly on single equation estimates, partly on calibration from simulation properties.

The model has been used in forecasting continuously since 1979, and is now one of only two in that category. The other is the National Institute of Economic and Social Research (NIESR) model, which however has been frequently changed in that 20-year period: the only changes in the Liverpool Model were the introduction of the explicit natural rate supply-side equations in the early 1980s and the shift from annual data to a quarterly version in the mid-1980s. In an exhaustive comparative test of forecasting ability over the 1980s, Andrews, Minford and Riley (1996) showed that out of three models extant in that decade — Liverpool, NIESR, and London Business School (LBS) — the forecasting performance of none of them could ‘reject’ that of the others in non-nested tests, suggesting that the Liverpool Model during this period was, though a newcomer, at least was not worse than the major models of that time. For 1990s forecasts no formal test is available, but the LBS model stopped forecasts and in annual forecasting post-mortem contests the NIESR came top in two years, Liverpool in three. In terms of major UK episodes, Liverpool model forecasts successfully predicted the sharp drop in inflation and the good growth recovery of the early 1980s. From the mid-1980s they rightly predicted that

\(^{11}\)See McCallum and Nelson (1999a).
\(^{12}\)See Minford and Webb (2004).
the underlying rate of unemployment was coming down because of supply-side reforms and that unemployment would in time fall steadily in consequence. Then they identified the weakness of UK membership of the Exchange Rate Mechanism (ERM) and its likely departure because of the clash between the needs of the UK economy and those of Germany leading the ERM at the time of German Reunification. After leaving the ERM they forecast that inflation would stay low and that unemployment would fall steadily from its ERM-recession peak back into line with the low underlying rate — as indeed was the case. Thus the Liverpool Model has a reasonable forecasting record; its capacity to replicate the dynamics of the data is currently being examined, in work similar to this.
9.B.2 Behavioural Equations

\[ \log(EG_t) = \log(ESTAR_t) + A39 \log(Y_t/YSTAR_t) \]  
(9.B.1)

\[ XVOL_t = A40YSTAR_t\{A27 \log(WT_t) + A28 \log(Y_t) + A47 + \\
A29 \{ESTAR_t + 0.6 \{RXR_t - ESTAR_t\}\} + \\
A30 \{XVOL_t - 1/\{A40YSTAR_t - 1\}\}\} \]  
(9.B.2)

\[ XVAL_t = XVAL_{t-4} + \{XVOL_t - XVOL_{t-4}\} + A31 \]
\[ \{0.32YSTAR_t\{RXR_t - RXR_{t-4} - ESTAR_t + ESTAR_{t-4}\}\} + \\
A32XVAL_{res_{t-1}} \]  
(9.B.3)

\[ \log(M0_t) = A44 + A13 \log(M0_{t-1}) + A14\{\log(Y_t) + \\
\log(1 - TAX_{t-1})\} + A16TREND_t + A17NRS_t + A18VAT_t \]  
(9.B.4)

\[ \log(U_t) = A42 + A3 \log(Y_t) + A4\{\log(RW_t) + \log(1.0 + BO_t) + \\
\log(1.0 + VAT_t)\} + A5TREND_t + A6 \log(U_{t-1}) + A36U_{res_{t-1}} \]  
(9.B.5)

\[ \log(G_t) = A45 + A19RL_t + A20\{\log(G_{t-1}) - \log(FIN_{t-1})\} + \\
A21\{\log(G_{t-1}) - \log(G_t - 2)\} + \log(G_{t-1}) \]  
(9.B.6)

\[ \log(CON_t) = A46 + A22RL_t + A23 \log(W_t) + A24QEXP_t + \\
A25 \log(CON_{t-1}) \]  
(9.B.7)

\[ \log(RW_t) = A43 + A7UNR_t + A8\{\log(UB_t) + \log(1.0 + LO_t)\} + \\
A9 \log(U_t) + A37 \log(RW_{t-1}) + \{.995\}UNR_t\{-A10\} + \\
A10 \log(RW_{t-2}) + A11ETA_t + A12ETA_{t-1} \]  
(9.B.8)

\[ RXR_t = A41 + 0.000 + A1\{\log(RW_t) + \log(1.0 + BO_t)\} + \\
A53\{\log(P_t) - \log(P_{t-4})\} + \{1. + A1\} \log(1. + VAT_t) + \\
A2TREND_t + A35RXR_{res_{t-1}} \]  
(9.B.9)
9.B.3 Identities and Calibrated Relationships

\[ RS_t = \{RXR_t - EEX_t\} + RSUS_t \]  \hspace{1cm} (9.B.10)

\[ NRS_t = P \exp_r + RS_t \]  \hspace{1cm} (9.B.11)

\[ RL_t = \{RXR_t - EEXL_t\}/5.0 + RLUS_t \]  \hspace{1cm} (9.B.12)

\[ NRL_t = RL_t + PEXL_t \]  \hspace{1cm} (9.B.13)

\[ Y_t = GINV_t + CON_t + EG_t + XVOL_t - AFC_t \]  \hspace{1cm} (9.B.14)

\[ INF_L = \log(MON_t) - \log(MON_{t-4}) - \log(M0_t) \]
\[ + \log(M0_{t-4}) \]  \hspace{1cm} (9.B.15)

\[ \log(P_t) = \log(P_{t-4}) + INF_t \]  \hspace{1cm} (9.B.16)

\[ W_t = FIN_t + G_t \]  \hspace{1cm} (9.B.17)

\[ BDEF_t = EG_t - 2.0 \times TAX_t \times Y_t + TAX_0 \times Y_0 \]  \hspace{1cm} (9.B.18)

\[ AFC_t = Y_t\{0.6588318\{AFC_{t-1}/Y_{t-1}\} + 0.1966416\{AFC_{t-3}/Y_{t-3}\} + 0.1454006\{AFC_{t-4}/Y_{t-4}\}\} \]  \hspace{1cm} (9.B.19)

\[ PSBR_t = BDEF_t + RDI_t \]  \hspace{1cm} (9.B.20)

\[ RDI_t = -0.5\{NRL_{t-1}/4.0\}FIN_{t-1}\{\{P_t/P_{t-1}\}^{0.66} - 1.0\} + \]
\[ PSBR_t\{0.32\{NRS_t/4.0\} + 0.5\{NRL_t/4.0\}\} + \]
\[ 0.32\{NRS_t/4.\}FIN_{t-1} - 0.32\{NRS_{t-1}/4.\}FIN_{t-1} + RDI_{t-1} \]  \hspace{1cm} (9.B.21)

\[ GINV_t = G_t - G_{t-1} + A38G_{t-1} \]  \hspace{1cm} (9.B.22)

\[ FIN_t = EG_t - Y_t \times \{TAX_t\} + XVAL_t + A54 \times FIN_{t-1} + \]
\[ \{1. - A54\} \times \{FIN_{t-1} \times \{P_{t-1}/P_t\}^{0.66}\}\} \]
\[ \{1.0 - 0.155 \times \{NRL_t/NRL_{t-1}\} - 1.0\} + res\_FIN_t + RDI_t \]  \hspace{1cm} (9.B.23)

9.B.4 Equilibrium Variables (star)

The star variables YSTAR, USTAR, ESTAR and WSTAR are the equilibrium values of Y, U, RXR and RW respectively, found by solving equations (9.B.2), (9.B.5), (9.B.8) and (9.B.9) under the conditions that XVOL=0 and exogenous variables maintain their current values;
EGSTAR is the value of EG that would produce a constant debt/GDP ratio with Y=YSTAR.

<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
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<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.528</td>
<td>-0.003</td>
<td>-2.150</td>
<td>0.792</td>
<td>0.010</td>
<td>0.804</td>
<td>0.470</td>
<td>0.210</td>
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</tr>
<tr>
<td>-0.290</td>
<td>0.189</td>
<td>0.870</td>
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<td>0.640</td>
<td>-0.215</td>
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<td>0.153</td>
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<td>0.000</td>
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<td>0.103</td>
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<td>1.000</td>
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<td>-0.125</td>
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<td>0.170</td>
<td>25.262</td>
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<td>-0.337</td>
<td>0.013</td>
<td>0.666</td>
<td>11.503</td>
<td>-0.016</td>
<td>-0.011</td>
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<tr>
<td>0.011</td>
<td>0.750</td>
<td>-0.750</td>
<td>0.300</td>
<td>-1.000</td>
<td>-1.000</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Table 9.B.1: Liverpool Model Coefficient Values in order A1–56

9.B.5 Model Notation

Endogenous Variables

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Y</td>
<td>GDP at Factor Cost</td>
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<td></td>
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<td></td>
<td></td>
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<tr>
<td>P</td>
<td>Consumer Price Level</td>
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<td></td>
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<td></td>
<td></td>
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<tr>
<td>INFL</td>
<td>Percentage Growth Rate of P (year-on-year)</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>MON</td>
<td>Nominal Money Stock (M0)</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>RW</td>
<td>Real wages (Average Earnings/Price)</td>
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<td>U</td>
<td>Unemployment</td>
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<td></td>
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<td>Q</td>
<td>Output Deviation from Trend (Y/YSTAR)</td>
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<td>AFC</td>
<td>Adjustment to Factor Cost</td>
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<td>EG</td>
<td>Real Government Spending on Goods and Services</td>
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<td>BDEF</td>
<td>Interest-exclusive Budget Deficit (deflated by CPI)</td>
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<td>PSBR</td>
<td>Public Sector Borrowing Requirement (deflated by CPI)</td>
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<td>XVAL</td>
<td>Real Current Account of Balance of Payments</td>
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<td>XVOL</td>
<td>Same, at Constant Terms of Trade</td>
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<td>RS(RL)</td>
<td>Real Short Term (log term) Interest Rate</td>
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<td>NRS(NRL)</td>
<td>Nominal Short Term (long term) Interest Rate</td>
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<td>G</td>
<td>Real Private Stock of Durable Goods, including Inventories</td>
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<td>W</td>
<td>Real Private Stock of Wealth</td>
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<td>Real Private Stock of Financial Assets (net)</td>
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<td>Real Private Non-Durable Consumption</td>
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<td>RXR</td>
<td>Real Exchange Rate (relative CPI, UK v. ROW)</td>
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<td>RDI</td>
<td>Real Debt Interest</td>
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<td>GINV</td>
<td>Gross Private Investment in Durables Plus Stockbuilding</td>
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**Exogenous Variables**

\[ \text{MTEM} \quad \text{Temporary growth of money supply} \]
\[ \text{PEQ} \quad \text{Growth of money supply} \]
\[ \text{BO} \quad \text{Employers national insurance contributions} \]
\[ \text{UNR} \quad \text{Trade Unionisation rate} \]
\[ \text{LO} \quad \text{Average amount lost in taxes and national insurance} \]
\[ \text{TREND} \quad \text{Time trend} \]
\[ \text{WT} \quad \text{World Trade} \]
\[ \text{TAX} \quad \text{Overall tax rate} \]
\[ \text{UB} \quad \text{Unemployment benefit rate (in constant pounds)} \]
\[ \text{EUNRS} \quad \text{Euro nominal short-term interest rates} \]
\[ \text{EURXR} \quad \text{Euro real exchange rate index} \]
\[ \text{EUCPI} \quad \text{Euro CPI} \]
\[ \text{RSUS} \quad \text{US real short-term interest rate} \]

**Exogenous variables (e= error term)**

\[ \text{RSUS} = c + 0.899 \text{RSUS}(-1) + e \]
\[ \text{EUNRS} = c + 0.977 \text{EUNRS}(-1) + e \]
\[ D \log \text{WT} = c + e \]
\[ D \text{BO} = c + e \]
\[ D \text{VAT} = c - 0.286 \text{VAT}(-1) + e \]
\[ D \text{UNR} = c + 0.869 \text{UNR}(-1) + e \]
\[ D \text{UB} = c + e \]
\[ D \text{LO} = c + e \]
\[ D \text{TAX} = c - 0.365 \text{TAX}(-1) + e \]
\[ D \log \text{EURXR} = c + 0.235 D \log \text{EURXR}(-1) + e \]
\[ D \log \text{EUCPI} = c + 0.503 D \log \text{EUCPI}(-1) + e \]
Chapter 10

Conclusions

Most open economy dynamic general equilibrium models of today incorporate some form of nominal rigidity and market imperfections. Nominal rigidities present in these models are considered crucial to their success in explaining empirical facts such as real exchange rate ‘overshooting’ and inflation persistence. The objective of this thesis has been to show that real exchange rate and inflation behaviour are explicable within a classical approach — by implication there is no need to add nominal rigidity. The main conclusions of the study are broadly reviewed here.

Chapter one provides an overview of the real business cycle (RBC) modelling strategy and reviews the relevant literature. The review of RBC models suggests that the important contribution of RBC models to date has been the reminder they provide that monetary factors are not the sole source of economic fluctuations. A complete understanding of business cycles requires a broader theory which relies both on ‘real’ as well as ‘monetary’ shocks.

In chapter two I build a model that is an enriched variant of a prototype RBC model embodying a representative agent framework as in McCallum (1989). Notably, I examine the effects of distortionary taxation, unemployment benefits, shocks to money and foreign variables like interest rates and prices on the model economy. In sum, I build an econometrically testable business cycle model that can serve as a benchmark for quantitative analysis of policy.

Chapter three explains the solution algorithm used to solve my model. Given the highly non-linear and complex nature of my model I solve it using computer algorithm developed by the Liverpool Research Group for solving complex non-linear rational expectations models. I
also discuss the challenges of this solution method and the simulation results for simple shocks with calibrated parameters. The effects of both demand and supply shocks on the behaviour of output, consumption, capital stock, investment, employment, price level, real wage, real interest rate, nominal exchange rate, real exchange rate and imports and exports are examined by deterministically simulating the calibrated model. In addition to providing quantitative input to policy analysis these deterministic simulations provide useful insights into the dynamic properties of the model.

In chapter four I explain the behaviour of the real exchange rate; both the appreciation following a productivity burst and its cyclical pattern using an open economy RBC model calibrated to quarterly UK data. It is a well established empirical fact that a burst in productivity leads to an appreciation of the currency. However, according to the ‘conventional’ view, if a country becomes more productive, a higher world supply of its good should result in a relative price reduction. The Harrod-Balassa-Samuelson (HBS) hypothesis, can explain the appreciation following a burst in productivity in the tradable sector. However, it still fails to explain the cycles that we observe in actual real exchange rate data. Using my RBC model I find that a one percent deterministic productivity growth shock, clearly shows that the real exchange rate appreciates on impact and then goes back to a new depreciated equilibrium, producing a business cycle — giving me the simulation properties that are needed to explain the data. I also show that the RBC model can without alterations or additions reproduce the univariate properties of the real exchange rate — by implication there is no necessary case here to add nominal rigidity.

In chapter five I estimate the model by Full Information Maximum Likelihood (FIML) estimation procedure, using quarterly data for the UK. The procedure involves solving the model for the sample period with parameters chosen by a hill-climbing-cum-grid algorithm. The algorithm checks the resulting likelihood within a grid ‘along the axes’ and then moves the parameters along the axis that improves the likelihood most. Having obtained the highest-likelihood set a bootstrap is performed to obtain confidence limits to ascertain the extent of bias. I also present the results of estimating my model with this new bootstrap procedure. It is shown that the distributions of parameters are far from normal in most cases; while the FIML estimator is based on the assumption that they are normal. There is evidence of bias in certain
parameters, though bias is far from general. After correcting the FIML estimates for bias I use them to deterministically simulate the model. The bias-corrected results are fairly similar to the calibrated ones, which suggests that bias is not a major problem in the model.

Chapter six provides an overview of the growing body of research analysing the links between the degree of inflation persistence and the credibility and transparency of the monetary policy regime in place. There are basically two sides to the story. One view upholds that inflation persistence has varied over the post WWII period for most industrialised economies; which given the substantial changes in monetary policy regimes observed supports the contention that persistence is a function of the credibility and transparency of the regime in place — so it is non-structural. The conflicting view finds that persistence has been stable over the same period even when policy has become more credible and hence monetary policy plays no role in determining the persistence properties of inflation, i.e. persistence is structural so must be ‘hardwired’ into the economic model.

In chapter seven I establish the facts of UK inflation persistence, carefully separating data into periods of different monetary policy regimes based on a priori knowledge of the economy. My results indicate that inflation persistence is different in different regimes, with persistence being lowest in the Inflation Targeting regime, followed by Bretton Woods and then Monetary Targeting. Persistence tends to be higher during the Deutsche Mark shadowing period as the government's primary aim then was to defend the peg. During the Incomes Policy regime of the 1970s there was no nominal anchor; hence I get the highest persistence parameters.

In chapter eight I elucidate rather straightforward models, easily-micro-founded in a standard classical set-up incorporating all available information about monetary policy behaviour during those periods to characterise each UK monetary regime of the post-war period. I am not particularly committed to these models in detail, but rather use them as a benchmark in which a minimal number of special assumptions are made — such as particular forms of ‘nominal rigidity’ and ‘adjustment.’ I solve the models analytically for the implied persistence in the inflation final form equation, in order to compare this theoretical prediction with the estimated persistence for each regime. The aim in this section is essentially qualitative: to see whether a basic classical model, calibrated in a standard way, can generate predictions that persistence will depend on the monetary regime in the direction of the stylised facts. On the whole I find
that this is so. In truth this is probably all one could expect from such a basic model.

In chapter nine I compare my model with the data more formally and test statistically whether my calibrated model is seriously consistent with the inflation data. Using a bootstrapping procedure, I generate bootstrap data for inflation under each regime and compute the implied sampling distribution of its AR and ARMA coefficients; in case of the ARMA for the theoretical implied form as well as the best-fitting ARMA representation of the data. I then check for each regime whether the coefficients of the actual inflation series lie within the 95% confidence interval of the AR and ARMA coefficient distribution generated by my model. This is an ambitious test for such a basic model; and perhaps surprisingly – at least for some readers – the model does quite well. As a robustness exercise I carry out the same analysis on the Liverpool Model, which is an elaborated version of the same classical structure. I find that inflation persistence is predicted by the two new classical models fairly effectively and that it varies with the monetary regime, in particular falling greatly when inflation targeting is in place.

The present study opens a number of avenues for future research. I plan to use my RBC model to try and explain the behaviour of the dollar, its continuous strength over the 1990s and its more recent downward ride. It would be interesting to build the model further to evaluate the additional explanatory power - if any - of adding a degree of nominal rigidity, in a sort of ‘stretched-RBC’ framework. I also plan to replicate the analysis of inflation persistence and monetary policy regimes using the US and other OECD countries as my empirical focus. It would be of considerable interest to examine the performance of a more elaborate, dynamically enriched version of my classical model. I also plan to evaluate if a New Keynesian model would be better equipped to explain the facts of inflation persistence; this research is still underway.

The central idea of my thesis has been to bridge the gap between theory and empirics in macroeconomics. This is a three-stage process: first, I build micro-founded theoretical models, then establish the facts – in terms of the time series properties of various macroeconomic variables and finally test the models against the stylised facts of the world using rigorous bootstrapping methodology. Bootstrapping basically involves replicating the stochastic environment to see whether the regression coefficients in the data lie within 95% confidence limits, for those coefficients, implied by the model. Given the novelty of this approach I am also very enthusiastic
about applying it to test alternative macroeconomic models.
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