# Nature and Management of Financial Risk in Global Stock Markets

by

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**Abstract**

This thesis addresses three problems associated with the risk in stock markets from a global perspective. First, we investigate the empirical hedging effectiveness using index futures in six world major stock markets. A variety of econometric models including STVECM with bivariate GARCH error structure are employed. The within-sample and out-of-sample results suggest sophisticated models do not produce the best hedging strategies consistently and their usefulness has to be judged on a case-by-case basis. Second, we examine the cross hedging effectiveness of seventeen MSCI indices through a global approach of using a combination of the related index futures. A thorough comparison among strategies corresponding to different combinations of hedging instruments and econometric models is conducted for each MSCI index. The optimal hedge ratio vector is derived for each country on the basis of both within-sample and out-of-sample results. Third, we develop a global asset pricing model on the basis of Barro's rare disaster model to explain the equity risk premium puzzle. Despite the plausible analytical predictions on the expected return of the bill and equity and the equity risk premium, the global model fails to explain the scale of the equity premium observed in the data since the diversification in a global market brings down both the aggregate risk and the reward for holding risk equity. The former results in a rise in the expected return of government bills and the latter leads to a fall in the expected return of equities.

# Content

# Chapter 1

# Introduction

Financial risk is the most important fundamental factor in asset pricing. It is therefore crucial for investors to have a thorough understanding of its determinants and a firm grasp of the strategies in managing the financial risk. Its importance to practitioners has motivated numerous research efforts into both the theoretical and empirical aspect of financial risk. This thesis contributes to the existing literature by adding the empirical results of two specific strategies in managing financial risk – direct hedging and cross hedging and exploring a theoretical model on the equity risk premium determination.

Hedging using futures is a common risk management strategy adopted by market participants who want to reduce the exposure to the price movement of the financial asset in the spot market. Hedgers trade in the futures markets in order to transfer the 'unwanted' risk to speculators who are willing to bear it for the positive expected return. When investors reach the hedging decision, they would have to answer two specific questions of which hedging instrument or instruments to use and what the ratio between the position in each instrument and the asset to be hedged should be.

Direct hedging refers to the situation where the underlying of the futures contract is also the asset to be hedged. In this case, the answer to the first question is straightforward. The only hedging instrument is simply the futures contract written on the spot asset because the price of the asset only differs from the price of futures by the tiny cost of carry. No other futures contract can possibly be more closely related to the asset than the futures written on it. For example, an index fund manager should use the futures contract written on the stock index tracked by the fund for hedging. The answer to the second question is more complicated. If the goal of hedgers is to eliminate the risk completely, then the optimal hedge ratio is equivalent to the minimum variance hedge ratio, whose analytical solution is simply the covariance between the spot and futures divided by the variance of the futures. It is evident that the key to deriving an effective hedging strategy relies on the quality of the model for the covariance matrix. If a theoretically sound and empirically proven model can be fitted to the conditional second moments of the spot and futures, then the chance of obtaining an effective hedging strategy is very big.

The first part of this thesis investigates the hedging effectiveness using index futures in six of the world's major stock markets. In particular, we examine a number of direct hedging strategies suggested by different econometric models for each country. These models range from the simple single OLS regression to the sophisticated Smooth-Transition Vector Error Correction Model combined with bivariate GARCH (1, 1) error structure. Each model is first estimated using within-sample data and updated on a daily basis to generate a one-day-ahead forecast on the covariance matrix in a one-year hold-out period. Both the within- and out-of-sample results are analyzed in detail.

Cross hedging applies in the situation where the futures contract of the spot asset is either non-existent or thinly-traded. In this case, there is no evident hedging instrument but a number of possible combinations of hedging instruments. The analytical solution for the minimum variance hedge ratio *vector* is the generalization of the minimum variance hedge ratio where the covariance is replaced with the vector of the covariance between the spot asset and each futures and the variance is replaced with the covariance matrix of the multiple futures. The demand for cross hedging strategies is common among global fund managers. A considerable amount of global funds are benchmarked to MSCI indices whose corresponding index futures contract is either non-existent or thinly traded. In order to hedge their portfolio, these managers would have to employ the cross hedging strategies.

The second part of the thesis answers the question of how to cross hedge the portfolio measured by MSCI indices of seventeen countries. We assume a global approach to solve this problem. In particular, we select a block of global markets closely related to the country whose MSCI index is to be hedged. And then we apply several econometric models to the system of the spot asset and the multiple futures of the block. Both the within- and out-of-sample results are generated for each combination of hedging instruments estimated from a particular econometric model.

After investigating the empirical performance of the hedging strategies, we explore the key determinant of asset pricing – equity risk premium from a global perspective. It is well-known that the standard general equilibrium asset pricing model can explain only a fraction of the equity risk premium observed in the data. This phenomenon is discovered in Mehra and Prescott (1985) and referred to as the equity premium puzzle. Because of the importance of equity risk premium in asset pricing, a large amount of papers since Mehra and Prescott (1985) have dedicated to resolving the puzzle. A recent paper by Barro claims the puzzle can be resolved when the rare disaster scenario is taken into consideration. He modelled the disaster scenario where the output contracts in a large scale explicitly and calibrated the model using the empirical data of the twentieth century disasters in the world. The results suggest the model can produce the equity premium in line with the observed empirical counterpart. We extend Barro's model to a global setting where each of the two countries is subject to a disaster shock but the two shocks are not perfectly correlated. In other words, the global model allows the disaster risk of the equities to be partly diversifiable. In particular, we introduce a parameter to measure the correlation between the disaster shocks and a parameter to account for the output weight of each country in the world. We solve the model and analyze the implication of each disaster-related parameter in detail. We also calibrate the model using the same set of parameter values as Barro.

Chapter 2 contains a complete analysis on the direct hedging using index futures. Chapter 3 examines the cross hedging strategies using the index futures of global markets. Chapter 4 investigates the implication of disasters in the global model developed on the basis of Barro's single-country model. Chapter 5 presents the conclusion.

# Chapter 2

# Direct Hedging Effectiveness Using Index Futures

## 1. Introduction

While futures contracts are popularly viewed as instruments for speculation, the classic economic rationale for futures markets is that they facilitate hedging. Hedgers dealing in spot markets transfer the unwanted risk in price changes to speculators using futures contracts. Hedging is important to market practitioners; therefore a vast amount of research effort has been dedicated to the formulation and implementation of an optimal hedging strategy.

In a volatile and unpredictable stock market, investors can reduce the risk associated with investment by holding a well-diversified portfolio. However, the systematic risk cannot be diversified away. When the market risk is to be avoided, they would have to unwind the positions of the entire portfolio, which is very costly. Index futures contract measuring the systematic risk might seem to be an ideal tool for hedging. Since the first stock index futures contract was introduced in USA in 1982, major stock exchanges in the world have created stock index futures contracts based on different stock indices. These stock index futures contracts have become popular among institutional investors as they offer them a major trading tool in controlling market risk without changing the portfolio composition.

However, the hedging effectiveness of stock index futures cannot be taken for granted because of basis risk. The traditional one-for-one hedge replaces the risk in the spot market with the basis risk, which is greater than its minimum level in most cases. The conventional hedge is derived as the slope coefficient of the regression of spot on futures. There are two potential problems with the conventional hedge. First, if the spot and futures are cointegrated, then the simple regression is misspecified for not taking the long-term cointegration relationship into consideration. Second, the instability of the second moments of the returns implies the conventional hedge computed from the constant covariance matrix is not optimal.

We investigate the question of how effective index futures are for hedging risk in six major stock markets. We model the conditional mean of the return on spot and futures by a linear or nonlinear vector-error-correction model with the bivariate GARCH or threshold GARCH error structure. The constraint is imposed on the cointegrating vector to account for the cost of carry explicitly. In this framework, we compute the dynamic hedge ratios on a daily basis for the entire one-year hold-out period and compare hedging performance on the basis of Ederington measurement of hedging effectiveness. Section 2 contains a literature review on hedging using futures. Section 3 summarizes the hedging theories. Section 4 describes data. Section 5 explains the methodology for modelling both conditional mean and conditional variance. Section 6 presents the estimation and hedging performance results. Section 7 concludes.

## 2. Literature Review on direct hedging

The derivation of optimal hedge ratio falls into two main frameworks. One is the mean-variance framework and the other is the expected-utility framework. Under the mean-variance framework, return and risk are fully represented by the expected value and variance. The traditional theory emphasizing risk avoidance function of hedging suggests the minimum-variance hedge ratio. The portfolio theory proposed by Johnson (1960) and Stein (1961) views hedging as a strategy to enable hedgers to achieve the best trade off between expected return and risk. One way is to assume hedgers are mean-variance utility maximizers. See Ederington (1979). Given the degree of risk aversion, the optimal hedge ratio can be obtained by setting the derivative of the mean-variance utility to the hedge ratio to zero. The other way is to use the risk-return trade off (Sharpe measure) criteria. The hedge ratio which maximizes the trade off ratio is optimal. See Howard and D'Antonio (1984).

Under the more general expected-utility framework, the problem with hedging is represented by maximizing the expected utility of the end-period wealth with respect to the hedge ratio. In order to analyze within the expected-utility framework, the form of the utility function needs to be specified. The analysis under the expected utility framework using quadratic utility function is consistent with the mean-variance utility analysis. The main drawback of quadratic utility function is its implausible assumption of increasing risk aversion. To avoid this disadvantage, Cecchetti et al (1988) employed the log utility function to investigate optimal hedging strategy in the bond market. Hsin et al. (1994) assumed an exponential utility function to compare the effectiveness of hedging using futures with options in the currency market. Under normality assumption on the return, the expected utility framework is also consistent with the mean-variance framework.

Some researchers derived the necessary and sufficient conditions on the return of spot and futures markets under which the optimal hedge ratio is preference-free. These conditions are important because numerous researchers estimating minimum-variance hedge ratio through complicated techniques implicitly rely upon them for their result to be consistent with the widely accepted expected-utility maximization paradigm. Benninga et al. (1983) derived the sufficient conditions for the optimal hedge to be a fixed proportion of the cash position, regardless of the agent's utility function. Lence (1995) derived both the sufficient and necessary conditions. Rao (2000) presented the sufficient and necessary conditions in an alternative form and provided an alternative proof based on the concept of stochastic dominance.

Under a more general setting of an increasing concave utility function, the analysis under stochastic dominance theory facilitates the derivation of optimal hedge ratio which minimizes the mean extended-Gini coefficient. See Cheung et al. (1990), Kolb and Okunev (1992) and Lien and Luo (1993).

A number of econometric models have been proposed to estimate the optimal hedge ratio in practice. Conventional hedge ratio is estimated as the slope (beta) coefficient of an OLS regression with the return of spot as regressand and the return of futures and constant as regressor. See Ederington (1979). Despite its robustness, OLS regression method has a major problem. It approximates the conditional second moments by their unconditional counterparts, not making use of the available information.

In order to add information variables to this model, several more general models have been proposed. Bell and Drasker (1986) suggested a regression model to include the information variables in the alpha and beta estimates. Cita and Lien (1992) applied it and set both alpha and beta to be linear functions of the historical spot and futures returns. Myers and Thompson (1989) developed a generalized approach that allows alpha to be a linear function of explanation variables while retains beta as constant. Fama and French (1987) suggested including the current basis as an information variable. Viswanath (1993) incorporated it in estimating the MV hedge ratio.

Since the introduction of cointegration analysis by Engle and Granger (1987), the conventional model is criticized for misspecification by several researchers who suggested the correct model to use is the Error-Correction-Model (ECM). See Ghosh (1993), Chou et al. (1996) and Moosa (2003). The former two claimed the success of ECM. The latter stated the difference between ECM and conventional hedge is negligible.

Some researchers proposed to use one variant of the ECM - threshold cointegration model to investigate the relationship between the spot and futures, since the concept of threshold cointegration captures the essence of the nonlinear adjustment process possessed in economic systems.

Dwyer, Locke and Yu (1994) used minute-by-minute data on the S&P 500 futures and cash indexes to characterize the nonlinear dynamic relationship between them. Their empirical results supported the hypothesis that arbitrage activity affects the size of the responses of the futures and cash indexes to lagged variables and the values of the parameters in the error correction mechanism should depend on the regime. The threshold ECM fits significantly better than an ECM.

Martens, Kofman and Vorst (1998) proposed a Threshold Autoregressive (TAR) model for the mispricing error defined as the difference between the logarithm of actual futures price and the appropriate futures price suggested by cost-of-carry relationship. Their empirical results suggested that the impact of the mispricing error is increasing with the magnitude of that error and the information effect of lagged futures returns on index returns is significantly larger when the mispricing error is negative.

Using both minute-by-minute and daily data on the FTSE 100 stock index and index futures, Garrett and Taylor (2001) examined whether threshold models are capable of generating levels of unconditional first-order autocorrelation observed in both intraday mispricing changes and daily basis changes. Their empirical results suggest that for intraday data, microstructure effects cannot explain the observed first-order negative autocorrelation in minute-by-minute mispricing changes. However, TAR model used for the arbitrage explanation is capable of generating unconditional negative first-order autocorrelation in mispricing changes. For daily data, neither arbitrage activity nor microstructure effects could predict the basis changes.

While there are quite a few papers on the threshold cointegration that examine the relationship between stock and futures price, the implications of threshold cointegration on the minimum-variance hedge ratio and on the hedging performance have not yet been investigated.

Since the invention of the Generalized Autoregressive Conditional Heteroskedastic (GARCH) model in the late 80s, there have been a number of studies estimating the optimal hedge ratio in the GARCH framework. These studies vary in the specification of GARCH in conditional variance modelling, consideration of the cointegration relationship for conditional mean modelling and evaluation criterion for hedging performance in out-of-sample comparison.

Baillie and Myers (1991) is the first paper to employ the GARCH model in optimal hedge ratio estimation. They examined the optimal hedge ratio problem in six different commodities spot and futures markets using daily data over two futures contract periods. Starting from the univariate GARCH estimation with Student-t density on the spot and futures return series, they showed that univariate GARCH model fit the data well. Since estimating the covariance between the two returns is necessary in calculating the optimal hedge ratio, they then turned to modelling the two return series jointly. The results showed that both price levels are nonstationary, but they are not cointegrated. Consequently, the modelling of conditional mean became trivial. The vector of disturbances was assumed to have a conditional normal distribution with time-dependent covariance matrix that follows a bivariate GARCH process. Both the diagonal VECH GARCH and the full BEKK GARCH were estimated. The results showed significant gains in efficiency in modelling the cash and futures process jointly compared with a univariate analysis and the assumption on constant covariance matrix was easily rejected against both versions of GARCH. Their in-sample results indicated that GARCH hedge ratios performed best in terms of reducing portfolio variance in all six pairs of commodity markets. However, the superiority of GARCH over OLS is only marginal in some cases and the generalization is impossible. Hence it has to be judged on a case-by-case basis. Their out-of-sample comparison was carried out in a peculiar way. They estimated the model using the data of 1986 contract and then applied the estimated parameters to the data of 1982 contract. The out-of-sample results showed the superiority of GARCH to OLS in all cases.

Kroner and Sultan (1993) investigated the optimal hedge ratio problem using weekly data in five foreign currency markets. Unlike Baillie and Myers (1991), they found the price level of spot and futures were cointegrated and the cointegration parameter was approximately one. Then they cited the theoretical result in Brenner and Kroner (1995) on cointegration and imposed that the cointegration parameter is equal to one. Incorporating the error correction terms, they set up a bivariate error-correction model with a GARCH error structure. The parameterization was constant-correlation GARCH with standardized residuals normally distributed. They used both the variance and expected utility value of the hedged portfolio as criteria to compare the four hedging strategies including naive, OLS, cointegration and bivariate error-correction with GARCH error structure. To compute the expected utility value, they imposed a specific value for the agent's risk-averse parameter. To decide when to rebalance the portfolio, they imposed a value for the transaction cost. Both within and out-of-sample results suggested that the hedge ratio developed from a bivariate-GARCH model was superior to any other one, as it produced the portfolio with the smallest variance and the largest expected utility. Even when the rebalancing and transaction cost were considered, the conclusion was not changed.

Park and Switzer (1995) was the first paper to estimate the hedge ratio from a bivariate-GARCH model using the stock index spot and futures market. They used weekly data on two stock indices. Similar to Kroner and Sultan (1993), they estimated the optimal hedge ratio from a bivariate error-correction model with GARCH error structure. The parameterization of GARCH is constant-correlation version and they also imposed the constraint on the value of the cointegration parameter to be one. Unlike the spot return on commodities and currencies, return on stock indices consists of appreciation of stock indices and the dividend yield. As a result, they adjusted the error-correction term for the net difference between spot and futures price level suggested by the cost of carry relationship. However, they argued that the spot return should include the dividend because dividend-inclusive return represented the actual return on the stock index portfolios. Both their within- and out-of-sample results showed the hedge ratios estimated from bivariate GARCH models led to the smaller variance and larger expected utility of the hedged portfolio either taking transaction cost into account or not.

Choudhry (2004) is another piece of work on the optimal hedge ratio estimation using weekly stock index data. Unlike most papers in the field, it used the data on three Pacific Basin countries, including Hong Kong, Japan and Australia. Similar to Park and Switzer (1995), it took the dividend yield into consideration, however, in a different way. Firstly he computed the cost of carry as the difference between money market rate and dividend yield. Then the cost of carry was added to the spot return to form the adjusted spot return. Finally the adjusted spot return data was used in estimation. He did not check for the existence of cointegration between the spot and futures price levels and simply modelled the conditional mean as constant. The parameterization of GARCH was a bivariate diagonal VECH GARCH(1,1) with normally distributed standardized errors. In the out-of-sample forecast, he kept the values of parameters the same as those estimated from within-sample dataset, but did not update them on a weekly basis. Both of his within- and out-of-sample results suggested that the superiority of GARCH hedge ratios over OLS hedge ratios has to be judged on a case-by-case basis.

Gagnon and Lypny (1995) examined the optimal hedge ratio problem using the weekly data on Canadian interest rate markets. They showed that the interest rate and its futures were cointegrated, but both of the returns are serially autocorrelated. Thus a bivariate error correction model with MA(1) error was set up to model the conditional mean. Before setting up the model for the conditional variances, they performed the Engle and Ng (1993) sign bias test on the return series and the results showed a significant negative size bias in both series. To capture the asymmetric effect of shocks on the conditional variances, they employed the asymmetric ARCH suggested by Glosten, Jagannathan and Runkle (1993). To allow for the time-varying correlation, they used the BEKK specification of GARCH(1,1). The conditional Student-t distribution was used to represent the distribution of return errors to account for the fat tails that characterize their conditional distribution. The error-correction model with asymmetric GJR-GARCH(1,1) error structure and MA(1) term produced the best description of the data. However, both the within- and out-of-sample results suggested the hedge ratios estimated from a model with constant conditional mean and asymmetric GJR-GARCH(1,1) error structure outperformed those estimated from a more general model with MA(1) error and error correction term as the conditional mean.

Harris and Shen (2002) examined the optimal hedge ratio problem using daily FTSE 100 stock index data. They applied the rolling window method and Exponential Weighted Moving Average (EWMA) model ( also called IGARCH model ) to estimate the optimal hedge ratio and update the estimation dataset on a daily basis. Different lengths of rolling window and values of the parameters of EWMA model were used. Then they argued that although consistent the sample variance estimators would only be efficient when the errors were normally distributed. But there had been evidence that short-horizon financial asset returns were not normally distributed even conditionally. To accommodate the leptokurtosis of the distributions of returns, they assumed a Power Exponential (PE) distribution and derived the expression of variances for both rolling window and EWMA approach. They derived the conditional covariance by a simple identity involving two variance terms to avoid the problematic covariance derivation associated with PE distribution. The optimal hedge ratio estimated from models with PE distribution errors were called robust optimal hedge ratio. Their results showed that the robust estimator yielded a hedged portfolio variance marginally lower than that derived from a standard estimator and the variance of the robust hedge ratios were substantially lower than the variance of the standard optimal hedge ratio, reducing the transaction costs associated with the optimal hedge strategy.

Brook and Chong (2001) examined the cross-currency hedging performance of the implied and statistical forecasting models. They used four pairs of on-spot currencies and over-the-counter (OTC) currency options data to examine the hedging performance on one-month and three-month hedging horizons. To estimate the optimal hedge ratio, eleven models were used of which the implied model and the family of univariate GARCH were rarely used in this field. In the implied model, the variances of the two currencies were the implied volatilities derived from currency options and the covariance between the two currencies was derived via an identity that involves the implied volatility of the two currencies and their cross currency. In the univariate GARCH family models, the individual variances of currency returns were estimated from GARCH model and the covariance between the two currency returns were derived from the identity mentioned above. This identity in the cross-currency relationship enables them to apply those univariate GARCH models which normally are not adequate in deriving the optimal hedge ratio in the spot and futures hedging. Another point to notice was that they used multi-step-ahead forecast rather than one-period-ahead forecast in the out-of-sample examination. Since the variances are additive over time, the multi-step-ahead forecasts of the variance and covariance can be derived from the sum of the one-period forecasts with the parameters estimated from the daily updated within-sample dataset. Both the one-month and three-month out-of-sample results indicated that GARCH with Student-t distributed conditional error and EWMA models appeared better and the implied model and naïve model appeared worse than others.

Lien et. al (2002) provided a systematic comparison of the performances of the hedge ratios estimated from OLS and Constant-Correlation GARCH model. Their dataset covered ten pairs of markets including commodities, currencies and stock indices. The data frequency was daily and the data period was ten years. The out-of-sample period had 1,000 observations and model estimation was updated on a daily basis. The estimation consisted of two stages. In the first stage the conditional mean of each series was estimated using univariate autoregressive filters by OLS method. In the second stage the conditional variances were estimated using constant-correlation GARCH(1,1). Their results indicated that hedge ratios estimated from GARCH did not outperform those estimated from OLS. Given the transaction cost associated with the volatile hedge ratios estimated from GARCH, they concluded GARCH should not be considered for hedging purposes although they are useful for data description.

## 3. Hedging Theory

Hedging refers to the investment strategy of taking opposite positions in some investment instruments related to the existing positions. Depending on the type of hedging instrument, we can categorize it as hedging using futures and hedging using options. When the underlying of the futures is the same as the asset to be hedged, investors face the problem of finding the optimal direct hedging strategy using futures contracts. In this chapter, we examine the performance of a number of hedging strategies using futures empirically.

Despite its popularity in different types of market, the motivation for hedging is controversial. In the last fifty years, at least three theories have been advocated, each of which suggests a particular optimal hedging strategy. Traditional theory argues that investors hedge purely for risk minimization purpose and the optimal hedge ratio should always be one. Working's theory asserts that hedgers speculate on the basis rather than the price and the optimal hedge ratio depends on the expected change in basis. Portfolio theory views hedging as managing a portfolio consisting of the spot and the futures and the optimal hedge ratio corresponds to the weight of the portfolio with the best trade-off between return and risk. In this section, each of the three theories is explained in detail.

Consider the following example of a long hedge. Suppose an investor facing a one-period decision making problem has a unit of long position in an asset. Its current price is $S_1$ and the price of its corresponding futures is $F_1$. The end-period price of the spot and futures are $S_2$ and $F_2$ respectively. He or she hedges the long position in spot by taking $h$ unit of short position in the futures at time 1 and clearing it at time $2$. The change in wealth is:

$$\Delta W = S_2 - S_1 + (F_1 - F_2) \cdot h \qquad (1)$$

Define basis as spot subtracted from futures. ($B = F - S$) The change in wealth can also be written in terms of the change in basis and the futures or the change in basis and the spot.

$$\Delta W = B_1 - B_2 + (1-h)\cdot(F_2 - F_1) \tag{2}$$

$$\Delta W = h\cdot(B_1 - B_2) + (1-h)\cdot(S_2 - S_1) \tag{3}$$

### 3.1 Traditional Theory

Traditional theory argues that the only objective of hedgers is to minimize the risk associated with their spot position and they are not concerned with making profit from hedging at all. Under the assumption that the price movement in the spot parallels that in the futures market, i.e. the change in basis is zero, hedgers should always take an opposite position in the futures market of the same magnitude as the position in the spot market. In other words, the optimal hedge ratio should be one.

Under the assumption of zero change in basis, the change in wealth expressed in equation (2) reduces to:

$$\Delta W = (1-h)\cdot(F_2 - F_1) \tag{4}$$

which is random because the end-period price of the futures is unknown. Its variance is:

$$Var(\Delta W) = (1-h)^2 \cdot Var(F_2) \tag{5}$$

The hedging problem can be solved by minimizing the portfolio variance in (5) with respect to the hedge ratio. As the product of two non-negative terms, the portfolio variance reaches the minimum at zero when the hedge ratio is one.

The obvious caveat of the traditional theory is that the change in basis is unlikely to be zero. First, the cost of carry theory implies that basis tends to be smaller as futures contract approaches expiry. Second, the futures price is not only affected by the spot price but also by the expectation of the spot price at expiry which is clearly variable. Therefore, although the change in basis is small, it is very unlikely to be zero, unless the hedging period coincides with the remaining period of the futures contract. Without the assumption on zero change in basis, the optimal hedge ratio would not be one but the minimum-variance hedge ratio introduced later. Nevertheless, the change in basis is almost surely smaller than the change in the spot price. For the investors who want to reduce risk, traditional hedge is better than no hedge at all.

### 3.2 Working's Theory

Working (1953) proposed an opposite theory to the traditional theory. He argued that hedgers speculate on the basis instead of the price in one market and they are not concerned with risk at all. Working's theory is based on the implicit assumption that the spot price is unpredictable and the hedge ratio is constrained between zero and one. He concluded that long hedgers should only hedge one-for-one when the basis is expected to shrink and not hedge at all when the basis is expected to widen.

Recall the change in wealth expressed in equation (3).

$$\Delta W = h \cdot (B_1 - B_2) + (1 - h) \cdot (S_2 - S_1)$$

Because the end-period basis and spot price are unknown, the change in wealth is random with the mean as follows.

$$E_1(\Delta W) = -h \cdot (E_1 B_2 - B_1) + (1 - h) \cdot (E_1 S_2 - S_1) \tag{6}$$

Working implicitly assumed the spot price follows a martingale process, therefore the second term in (6) vanishes and the change in wealth reduces to the change in basis times minus the hedge ratio. That is,

$$E_1(\Delta W) = -h \cdot (E_1 B_2 - B_1) \tag{7}$$

When the basis is expected to broaden, i.e. the expected change in basis is positive, long hedgers should avoid the expected loss by not hedging at all. When the basis is expected to shrink, i.e. the expected change in basis is negative, long hedgers should maximize the expected gain by hedging to the maximum, i.e. hedge one-for-one in his framework.

The main difference between Working's theory and the traditional theory is that the former assumes hedgers are only concerned with expected return and the later assumes they are only concerned with risk. The portfolio theory shown next advocates that hedgers are similar to most market participants who make decision to strike the best trade-off between expected return and risk.

### 3.3 Portfolio Theory

Johnson (1960) and Stein (1961) proposed to analyze hedging in the basic portfolio mean-variance framework. Hedgers are treated as mean-variance utility maximizer, who forms the portfolio by combining the risky asset in the spot market with its futures. The efficient frontier representing the trade-off between the expected return and risk can be derived for this portfolio by varying the hedge ratio. Given the degree of risk aversion, the schedule of indifference curves can be drawn. The point at which the highest indifference curve is tangential to the efficient frontier corresponds to the optimal hedge ratio. In theory, the optimal hedge ratio can be any value. The hedge ratio greater than one is referred to as over hedge. The hedge ratio lower than one is referred to as under hedge.

Recall the expression of the change in wealth in equation (1).

$$\Delta W = S_2 - S_1 + (F_1 - F_2)\cdot h$$

Its mean and variance are as follows.

$$E_1(\Delta W) = E_1 S_2 - S_1 + h\cdot(F_1 - E_1 F_2) \tag{8}$$

$$Var_1(\Delta W) = Var_1(S_2) + h^2\cdot Var_1(F_2) - 2\cdot h\cdot Cov_1(S_2, F_2) \tag{9}$$

Hedgers' objective is to maximize the following mean-variance utility function.

$$U_1 = E_1(\Delta W) - 0.5\cdot\gamma\cdot Var_1(\Delta W) \tag{10}$$

where $\gamma$ is the degree of risk aversion.

The optimal hedge ratio can be derived by setting the F.O.C. of the utility with respect to the hedge ratio to zero.

$$\frac{\partial U_1}{\partial h} = -E_1 F_2 + F_1 - h\cdot Var_1(F_2) + h\cdot\gamma\cdot Cov_1(S_2, F_2) = 0$$

$$h = \frac{Cov_1(S_2, F_2)}{Var_1(F_2)} - \frac{E_1 F_2 - F_1}{\gamma\cdot Var_1(F_2)} \tag{11}$$

Equation (11) shows the optimal hedge ratio of a mean-variance utility maximizer. If hedgers are only concerned with risk minimization as suggested by the traditional theory, the optimal hedge ratio would reduce to the minimum-variance hedge ratio, which can be derived by setting the F.O.C. of the portfolio variance with respect to the hedge ratio to zero.

$$\frac{\partial Var_1(\Delta W)}{\partial h} = 2 \cdot h \cdot Var_1(F_2) - 2 \cdot Cov_1(S_2, F_2) = 0$$

$$h^* = \frac{Cov_1(S_2, F_2)}{Var_1(F_2)} \tag{12}$$

From (11) and (12) we can see that the minimum-variance hedge ratio is optimal when either the futures price is unpredictable or hedgers are infinitely risk averse. Under the first condition, the expected return of the futures is zero and the change in hedge ratio has no effect on the expected return, in which case, the only benefit of hedging is to reduce risk. Under the second condition, the marginal rate of substitution of return for risk is infinite, i.e. hedgers are willing to give up all the possible return for a marginal amount of risk reduction. In either case, the optimum is achieved when the portfolio variance is minimized.

In this chapter, we take the traditional view on the motivation for hedging. Hedgers participate in the market to reduce the unwanted risk. Their goal is not to maximize possible return as speculators or to achieve the best combination between return and risk as most investors. They hedge purely to minimize the risk associated with the existing positions in the spot market. Therefore the optimal hedge ratio is the minimum-variance hedge ratio.

Equation (12) presents one version of the optimal hedge ratio, which suffers from the drawback that it is difficult to estimate empirically. An alternative version of the optimal hedge ratio is presented in the next section.

### 3.4 Alternative version of the optimal hedge ratio

Hedgers' motivation is to minimize risk. If risk is measured by the variance of the change in wealth, we have the formula for the optimal hedge ratio in equation (11). If we use the variance of the *percentage* change in wealth as the indicator of risk, we reach another version of the optimal hedge ratio.

The percentage change in wealth is:

$$\frac{\Delta W}{W}=\frac{S_2-S_1+h\cdot(F_1-F_2)}{S_1}=\frac{\Delta S}{S_1}-h\cdot\frac{\Delta F}{S_1}\approx\frac{\Delta S}{S_1}-h\cdot\frac{\Delta F}{F_1}$$

where the approximation follows because the price in spot and futures are very close.

For very small $X$, we have the following approximation.

$$\frac{dX}{X}=d(\ln X)\quad\Rightarrow\quad\frac{\Delta X}{X}\approx\Delta(\ln X)$$

Apply the above to the price of the spot and futures, the proportional change in wealth can be approximated by the difference between the first-order difference of the two logarithmic price levels.

$$\frac{\Delta W}{W}\approx\Delta\ln S-h\cdot\Delta\ln F$$

The above percentage change in wealth is random with the following variance.

$$Var_1(\frac{\Delta W}{W})=Var_1(\Delta\ln S)+h^2\cdot Var_1(\Delta\ln F)-2\cdot h\cdot Cov_1(\Delta\ln S,\Delta\ln F)$$

The optimal hedge ratio can be solved from setting the F.O.C. of the variance of the percentage change in wealth with respect to the hedge ratio to zero.

$$\frac{\partial Var_1(\frac{\Delta W}{W})}{\partial h}=2\cdot h\cdot Var_1(\Delta\ln F)-2\cdot Cov_1(\Delta\ln S,\Delta\ln F)=0$$

$$h^{**}=\frac{Cov_1(\Delta\ln S,\Delta\ln F)}{Var_1(\Delta\ln F)}$$

Since the first-order difference in logarithmic price level approximates the rate of return, the alternative version of the optimal hedge ratio is the quotient of the covariance to the variance of the return.

$$h^{**}=\frac{Cov_1(\Delta\ln S,\Delta\ln F)}{Var_1(\Delta\ln F)}=\frac{Cov_1(r_s,r_f)}{Var_1(r_f)}\tag{13}$$

In the early period of the research on hedging, both versions of the optimal hedge ratio are studied and estimated. Since Engle and Granger's work on cointegration was popularized, most researchers have realized the disadvantage of the former version and only used the latter version in estimation. Specifically, the price level of spot and futures in various markets or sample periods are nonstationary but econometric estimation is only valid for stationary variables.

Following the researchers in the field, we use the second version of the optimal hedge ratio in this chapter. That is, we apply a number of econometric models with theoretical support to the data, compute the optimal hedge ratio from the corresponding covariance matrix estimates and compare the different models on the basis of hedging performance.

### 3.5 Hedging Effectiveness Measurement

Two kinds of hedging effectiveness measurement are widely used in the literature. One is the standard deviation or variance of the hedged portfolio. The other is the percentage reduction of the hedged portfolio variance compared with the spot variance. The latter was proposed by Ederington (1979) and is named as Ederington Hedging Effectiveness ( HE ) measurement. In fact, these two measurements amount to the same thing since the variance of spot is constant whichever hedging strategy is used. The ranking of the hedge ratio implied by different models is the same whether the portfolio variance or Ederington HE is used.

Hsin et al. (1994) proposed to use the change of the certainty equivalent returns relative to the spot position to measure the hedging effectiveness. Under the mean-variance utility framework, the certainty equivalent returns of the hedged position and the spot position can be computed, given an absolute risk aversion coefficient. A positive value means the hedging is effective, while a negative value indicates the hedging is ineffective. This measurement is rarely used in the literature probably because the expected return of the hedged portfolio is always insignificantly different from zero.

Some researchers took the transaction cost into consideration when comparing the hedging performances. In particular, they subtract the transaction cost from the mean-variance expected utility value and choose the strategy corresponding to the highest utility. See Kroner and Sultan (1993) and Park and Switzer (1995).

Another relevant indicator of the hedging effectiveness is the minimal return of the hedged portfolio. Since the main purpose of hedging is to reduce risk, it is beneficial to examine the hedging performance in the worst scenario when it is needed the most.

In this chapter, we follow the common practice in the literature and only look at the portfolio variance and Ederington HE measurements in evaluating the hedging strategies implied by different models.

## 4. Data

We investigate the hedging effectiveness using stock index futures in six countries. They are Australia, Germany, Japan, South Korea, UK and USA. The sample period of Germany, Japan, UK and USA covers a ten-year period between March 1995 and March 2005. The sample period of Australia starts from May 2000 and the sample period of South Korea starts from May 1996. Both of them end in March 2005. The data frequency is daily.

In order to estimate the model and perform forecasts, the whole sample period is divided into the within-sample and out-of-sample part. The former is used to estimate the model and derive the first set of parameters. For all the indices, the within-sample period starts from the first day of its full-sample period and ends on March 5$^{th}$, 2004. For Germany, Japan, UK and USA, the within-sample period lasts for 9 years. For Australia and South Korea, the time length is 3 years and 10 months and 7 years and 10 months respectively. The latter is the last year of the whole sample period. Specifically, it starts from March 7$^{th}$, 2004 and ends on March 7$^{th}$, 2005.

The data can be divided into two groups. The first group includes the two price series – the daily closing stock index and the settlement price of the corresponding index futures. The most influential index of each country is chosen. Among the corresponding index futures contracts with different maturity date, the one closest to expiry is chosen. On the first day of its expiry month, it is replaced with the nearby contract, in order to create a long series of the futures price.[1] Specifically, the stock indices used in this chapter are Australian SPI 200, German DAX 30, Japanese NIKKEI 225, South Korean KOSPI 200, UK FTSE 100 and USA S&P 500 index.

The second group includes the dividend yield corresponding to each stock index and the three-month interest rate of each country. There are two special cases. As a performance

[1] Another reason for rolling over the futures contract is to avoid the expiry effect. In the last two to three weeks of each contract, trading volume and open interest increase dramatically. These signs indicate the price process in the expiry month is probably different from that in most periods.

index DAX 30 is computed with dividend reinvestigated, therefore there is no separate DAX 30 dividend yield series. All the data is downloaded from DataStream.

In order to compute the cost of carry, we need to derive the days-to-maturity series. In the ten-year sample period, forty three-month contracts of each index futures are used. The days to maturity series are derived from counting the number of days between each day and the last trading day of the expiring or the nearby contract.

The daily return series of the spot and futures are computed by taking the difference of the logarithmic price level. Table 1 contains the descriptive statistics of the daily returns.

**Table 1: Descriptive Statistics of the daily returns on spot index and index futures**

| | *AU* | | *GM* | | *JP* | | *KR* | | *UK* | | *US* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ |
| **mean** | 0.01% | 0.01% | 0.03% | 0.03% | -0.02% | -0.02% | 0.00% | 0.00% | 0.02% | 0.02% | 0.04% | 0.04% |
| **s.d.** | 0.70% | 0.76% | 1.64% | 1.65% | 1.47% | 1.52% | 2.44% | 2.87% | 1.16% | 1.22% | 1.17% | 1.23% |
| **max** | 3.44% | 3.86% | 7.55% | 7.29% | 7.66% | 8.00% | 14.60% | 18.32% | 5.90% | 5.95% | 5.57% | 5.75% |
| **min** | -4.81% | -4.08% | -8.87% | -14.82% | -7.23% | -7.60% | -12.74% | -11.37% | -5.89% | -6.06% | -7.11% | -7.76% |
| **skewness** | -0.446 | -0.061 | -0.228 | -0.346 | 0.041 | 0.041 | 0.122 | 0.585 | -0.168 | -0.098 | -0.111 | -0.126 |
| **kurtosis** | 6.879 | 5.079 | 5.682 | 7.490 | 4.941 | 4.878 | 5.995 | 6.898 | 5.573 | 5.252 | 6.060 | 6.410 |
| **Jaque-Bera** | 662 | 181 | 724 | 2019 | 369 | 346 | 770 | 1411 | 659 | 500 | 921 | 1144 |
| **number of observation** | 1003 | 1003 | 2348 | 2348 | 2348 | 2348 | 2045 | 2045 | 2348 | 2348 | 2348 | 2348 |

Note 1: AU, GM, JP, KR, UK and US are for Australia, Germany, Japan, South Korea, UK and USA respectively.
2: $\Delta s$ , $\Delta f$ denote the return on spot index and index futures.

From the descriptive statistics in Table 1, we can see that the return series in the spot and futures market are very similar. The mean of each return is insignificantly different from zero. The standard deviation of the futures return is slightly greater than that of the spot return, indicating the futures are more volatile than the spot. None of the returns are normally distributed. Their distributions share the fat-tail feature. Among all the indices, only NIKKEI 225 has a negative average return, reflecting the prolonged recession that Japanese economy experienced in the sample period between 1995 and 2005. KOSPI 200 has the greatest standard deviation, biggest daily gain and loss even though its sample size is one-tenth less than others, indicating South Korean market is the most volatile in our data set. This is not surprising. In all six countries studied, only South Korean market can be categorized as an emerging market.

## 5. Methodology

As shown in equation (13), the optimal hedge ratio is the quotient of the covariance between spot and futures to the variance of futures. The key to derive the hedge ratio with outstanding performance is to employ the theoretically sound and empirically proven model for the covariance matrix so as to make accurate forecast on the covariance matrix. The process of finding the right model involves modelling the first and second moment of the spot and futures return in a bivariate system and simulating the hedged portfolio from the covariance matrix estimates and the actual return series. In particular, four models will be proposed and estimated. Some of them will fit the data better than others. But all of them will be evaluated according to hedging effectiveness measurement using within- and out-of-sample data.

### 5.1 Modelling the Conditional Mean

Stock index and the index futures written on it are closely related. The level of spot index represents the price investors are willing to pay for the portfolio measured by the stock index *now*. The price of the index futures represents what investors are willing to pay for the same portfolio at the futures' *expiry date*. The economic theory describing the relationship between the two is the cost of carry theory.

#### 5.1.1 Cost of Carry Theory

Cost of carry theory is the futures pricing theory based on a no-arbitrage condition. For investors willing to own an asset at the expiry date of its futures contract, there are two alternative strategies. One is to buy in the spot market with immediate delivery and 'carrying' it in inventory. The other is to buy in the futures market with deferred delivery at expiry date. In the case of stock index futures, the cost of the former strategy involves the spot price and the time value of the funds and its proceeds are the dividend. The cost of the latter strategy is the futures price. Under the no arbitrage condition, the two strategies should incur the same net cost. Denote the interest rate, dividend yield and time to maturity as $r$ , $d$ and $T-t$ respectively. The cost of carry theory implies the fair price of futures ($F_t^*$ ) as follows:

$$F_t^* = S_t \cdot e^{(r_t - d_t)\cdot(T-t)} \tag{14}$$

Taking logarithm on both sides we have a linear relationship among the stock index, the fair value of the index futures and the cost of carry.

$$\ln F_t^* = \ln S_t + (r_t - d_t)\cdot(T-t) \tag{15}$$

The cost of carry theory implies a no-arbitrage relationship among the spot, futures and cost of carry. But it does not necessarily hold at any moment in time. When the actual futures price is different from its fair value, arbitragers will enter both markets to make risk-free profit, which will result in the price movement in both markets to restore the no-arbitrage relationship. Therefore, the deviation from the no-arbitrage relationship should be short-lived and random.

If we replace the fair value of futures with its actual price on the left hand side of equation (15), then we would have to add a term to accommodate the temporary deviation. This term should be random with zero mean and no autocorrelation. That is, the cost of carry theory suggests the following relationship among the actual value of spot, futures and cost of carry.

$$\ln F_t = \ln S_t + (r_t - d_t)\cdot(T-t) + e_t \tag{16}$$

where $e_t$ is a white noise.

The deviation from the cost of carry relationship has predictive content on the price movement in both spot and futures markets. Suppose at time $t$, the futures price is lower than its fair value i.e. the deviation ( actual price minus fair value ) is negative. At time $t+1$, the undervalued futures price tends to increase. Therefore, there is a negative relationship between the deviation and the return of the futures. An undervalued futures also implies the spot index is overvalued compared with futures. The next movement in the spot tends to be a fall. Therefore, the relationship between the deviation and spot return should be positive.

The cost of carry theory implies the following model for the return in spot and futures and the deviation of the actual futures from the fair value.

$$\begin{cases} \Delta s_{t+1} = \mu_s + \delta_s \cdot (f_t - s_t - coc_t) + \varepsilon_{s,t+1} \\ \Delta f_{t+1} = \mu_f + \delta_f \cdot (f_t - s_t - coc_t) + \varepsilon_{f,t+1} \end{cases} \tag{17}$$

where $\Delta s_{t+1}, \Delta f_{t+1}$ are the difference in the logarithmic price level, i.e. the returns in the spot and futures market and $coc_t$ is the cost of carry. $\delta_s, \delta_f$ is positive and negative respectively.

The above relationship is the skeleton model of the conditional mean estimated in this chapter. It is supported not only by the cost of carry theory, but also the cointegration theory.

### 5.1.2 Cointegration relationship and VECM

Engle and Granger (1987) developed the cointegration theory with profound implications on multivariate modelling in economics and finance. Suppose there are several nonstationary variables and a linear combination of them is stationary. These variables are said to be cointegrated and the cointegration error has predictive power over the first-order difference of all the variables. More specifically, suppose $X, Y, Z$ are all $I(1)$ variables and a linear combination of them, $\alpha \cdot X + \beta \cdot Y + \gamma \cdot Z$ is $I(0)$. Then $X, Y, Z$ are cointegrated of order one. And their first-order differences can be modelled by a vector-error-correction model (VECM).

Numerous researchers have tried to find a cointegration relationship between the price of the spot and futures in different markets. The empirical test results summarized in Brenner and Kroner (1995) suggest commodity price and its futures are not cointegrated, foreign exchange rate and its futures are cointegrated, but it is unclear whether stock index and the index futures are cointegrated.

Brenner and Kroner (1995) gave some explanation for these findings. Their argument is that the spot and futures price level are cointegrated if the differential (cost of carry) is stationary, otherwise they are not cointegrated. They assumed the time to maturity of the futures contract is fixed. Therefore the differential is proportional to the difference between the two variables. In the case of commodity, it is the difference between the interest rate and the convenience cost. Since the convenience cost per period is reasonably stable and the interest rate is generally believed to be nonstationary, the spot and futures in the commodity market are unlikely to be cointegrated. In the case of foreign exchange, the differential is the difference between the interest rates of the two countries. While there is strong evidence on the nonstationarity of interest rate in the literature, the evidence on the stationarity of interest rate differential is mixed. It is very likely that the two interest rates share a common stochastic trend, in which case the difference between them is stationary. It explains why the foreign exchange rate and its futures are often proven cointegrated. In the case of stock index, the spread is between dividend yield and interest rate. There is not much evidence or theory on the stationarity of the difference between these two variables.

However, most empirical work including this chapter investigates the relationship between the contemporaneous spot and futures, i.e. the number of days to maturity of the futures contract is not fixed. The common practice is to roll over the nearest futures contract to the nearby one several days before expiry to create a long series of the futures price. Therefore the variable of days to maturity is not constant but cyclical. The differential is a product of the difference between two random variables and a cyclical deterministic variable. We will answer the question on whether the cost of carry is stationary empirically by performing a stationarity test on it. If the test results indicate stationarity, then we will have to test a cointegration relationship among variables of different order. In this circumstance, standard OLS method is invalid; therefore we have to apply the dynamic OLS method advocated by Stock and Watson (1993).

Granger representation theorem ( Granger (1983), Engle & Granger (1987) ) also suggests that if a cointegration relationship can be established among several variables, then a VECM should be used to model their first-order differences. In the current example, if the stock index, index futures and cost of carry are cointegrated and the former two variables are I(1) and the latter one is I(0), then we will estimate a two-variable VECM for the return of spot index and index futures. From the model estimates, we can derive the optimal hedge ratio as the quotient of the covariance between spot and futures to the variance of futures.

### 5.1.3 Empirical procedure

The estimation of VECM involves two steps. First, we test for and estimate the cointegration relationship among the level of stock index, index futures and cost of carry. Then, we estimate the VECM incorporating the estimated cointegrating vector.

#### 5.1.3.1 Cointegration test and cointegrating vector estimation

The first step towards establishing the cointegration relationship is to test the stationarity of all three variables in level and their first-order difference. Unit root test, specifically the Augmented Dickey-Fuller Test (ADF) will be employed on the level and the first-order difference of spot, futures and cost of carry[2]. The next step is to estimate the cointegrating

[2] The cost of carry of stock index futures is the product of the time to maturity and the difference between interest rate and dividend yield. The time to maturity is measured by the number of days between each day and

vector. We will use dynamic ordinary least squares (DOLS) method proposed by Stock and Watson (1993) and Saikkonen (1991) in estimation. Since the cost-of-carry term has already incorporated the time factor and different futures contracts are combined to form a long series, an intercept rather than time trend will be included in the regression.

The dynamic OLS method rather than standard OLS will be applied because only it is valid when cointegrated variables are of different order. As will be shown in the results section, the cost of carry is stationary, but the level of spot index and index futures are I(1). Furthermore, unlike the super consistent estimates of standard OLS, dynamic OLS estimates are asymptotically normally distributed, which makes statistical inferences on coefficient estimates possible. Therefore, cointegrating vector $(1,-1,-1)$ suggested by cost of carry theory can be tested using standard tests. The DOLS regression takes the following form.

$$f_t = \beta_0 + \beta_1 \cdot s_t + \beta_2 \cdot coc_t + \sum_{i=-k\neq 0}^{k} \gamma_i \cdot \Delta s_{t-i} + u_t$$

where $f_t$ *and* $s_t$ are the logarithm of index futures and spot index and $coc_t$ is the cost of carry. $\Delta s_{t-i}$ is the lead or lag of the first-order difference of spot. They are included to accommodate the endogenous feedback and nuisance parameters. The number of lead and lag is set at 5 because our data frequency is daily. $u_t$ is a random error.

The final part in cointegration testing is to check whether the dynamic OLS regression residual is stationary. If the nonstationary hypothesis is rejected, then the no cointegration hypothesis is also rejected, i.e. the cointegration relationship is established. Specifically, ADF test will be performed on the OLS residuals.

#### 5.1.3.2 Estimation of the VECM

While the cost of carry theory suggests the cointegrating vector to be $(1,-1,-1)$, the estimates by dynamic OLS will be $(1,-\hat{\beta}_1,-\hat{\beta}_2)$. We test the null hypothesis that they are the same. If there is no evidence overwhelmingly rejecting the null, we will impose the constraint of

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the expiry day of the corresponding futures contract. The interest rate is measured by the three-month interest rate of each country. And the dividend yield is measured by the dividend yield of the stocks index.

(1,−1,−1) on the cointegrating vector and use the corresponding error terms in the VECM. The specification of the VECM is as follows.

$$\begin{cases} \Delta s_t = \mu_s + \delta_s \cdot z_{t-1} + \sum_{i=1}^{m} \phi_{s1i} \cdot \Delta s_{t-i} + \sum_{i=1}^{n} \phi_{s2i} \cdot \Delta f_{t-i} + \varepsilon_{st} \\ \Delta f_t = \mu_f + \delta_f \cdot z_{t-1} + \sum_{i=1}^{m} \phi_{f1i} \cdot \Delta s_{t-i} + \sum_{i=1}^{n} \phi_{f2i} \cdot \Delta f_{t-i} + \varepsilon_{ft} \end{cases} \tag{18}$$

and $z_t = f_t - s_t - coc_t$

where $s_t$, $f_t$ are the logarithm of price level in the spot and futures market, $\Delta s_t$, $\Delta f_t$ are the return in the spot and futures market respectively and $z_t$ is the error term.

Comparing (18) and (17), we can see that the VECM is basically what the cost of carry theory suggests but with the extra terms of lagged returns on the right-hand side. The inclusion of these lags is to accommodate the dynamics of the model. A large number of lags in both spot and futures are included first. To achieve a parsimonious specification, we also estimate it without insignificant lags. The best specification is determined with the aide of the two frequently used information criteria – Akaike information criterion (AIC) and Schwartz Bayesian criterion (BIC). The lag structures in the spot and futures equation are kept the same.

Since the right-hand-side variables of the two equations are the same, estimating the two equations separately is equivalent to estimating the system jointly, if the two residuals are contemporaneously uncorrelated. In this part of the thesis, we make this simple but unrealistic assumption and estimate VECM using OLS method to follow the common practice in the literature. In the next part, we will specify the structure of the two residuals explicitly and estimate the conditional mean and variance jointly.

Given the cost of carry theory and Engle-Granger approach, we expect at least one of the error-correction term coefficients to be significantly different from zero. It means that as the two variables depart from the equilibrium at least one variable will adjust to restore the equilibrium. If only one variable is significantly different from zero, the information flow in the system is unidirectional, i.e. the information is reflected in the price movement of one variable. This variable is endogenous to the system. The variable insignificantly different from zero is weakly exogenous to the system, i.e. it does not react to the deviation from the stable relationship. If both of them are significant, then both variables are endogenous and the information flow is bidirectional.

#### 5.1.3.3 Conventional hedge ratio vs. VECM hedge ratio

In the early period of hedging research, there were a number of papers comparing the performances of the conventional hedge with the VECM hedge. See Myers and Thompson (1989) and Chou et al. (1996). The two hedge ratios are different because they are derived from different regression equation. Conventional hedge ratio proposed in Ederington (1979) can be estimated as the slope coefficient of the return of spot on that of futures. The regression equation of VECM hedge is similar to that of conventional hedge with the lagged error-correction term as additional regressors.

Implicitly, the conventional hedge assumes the return of spot and futures are unpredictable and their second moments are independent of time. In contrast, VECM hedge supported by Engle and Granger's cointegration theory is derived in the framework explicitly modelling the predictive power of the deviation on the returns.

Several early studies have documented that conventional hedge ratio is smaller than VECM hedge ratio. Lien (1996), Moosa (2003) and Lien (2004) explained this phenomenon by deriving the analytical conditions under which conventional hedge ratio is smaller than VECM hedge ratio and showing that these conditions are usually satisfied. In fact, this is an econometric problem. Suppose VECM is the appropriate model for the return of spot and futures. The correct regression equation should have both the lagged error-correction term and the return on futures on the right-hand side of the equation. But if we mistakenly estimate the regression without the error-correction term, as implied by conventional method, then it would lead to omitted variable bias. The conditions for positive and negative bias can be derived.

### 5.1.4 Nonlinear VECM

#### 5.1.4.1 Rational

In section 5.1.2, we proposed to model the return on spot and futures by a standard linear VECM, because it not only captures the long-run arbitrage-free relationship between the level of spot and futures but also accommodates the effect of short-term deviation on future price movement. However, linear VECM is subject to a drawback. Specifically, the coefficients of error-correction term do not change with the size of deviation, which implies the strength of the price adjustment is uniformly the same however far away from its fair level is the current price. This is counter-intuitive, as the strength of adjustment should be positively related to the size of deviation. In reality, markets are populated by heterogeneous investors with different transaction costs. As the size of deviation increases, index arbitrage becomes a profitable strategy for more investors. Therefore, the availability of index arbitrage opportunity is increasing in the size of deviation. As more investors are involved in index arbitrage, the strength of adjustment to deviation becomes stronger.

Several authors have used non-linear VECM to model the changing strength of adjustment in the context of spot and futures relationship. Yadav et al. (1994), Martens, Kofman and Vorst (1998) and Dwyer et al. (1992) used the threshold VECM. Taylor et al. (2000) employed the smooth-transition VECM (STVECM). Between the two alternatives, the STVECM is preferable. Unlike the threshold VECM that relies on the distinct classification of investors and implies an abrupt transition from no-arbitrage to arbitrage zone, STVECM allows an infinite number of investors facing different transaction cost and suggests a smooth transition between the extreme regimes. Furthermore some specifications of STVECM accommodate the threshold VECM as a limiting case. As STVECM is more general and plausible than threshold VECM, we estimate the STVEM for the conditional mean of the spot and futures return as an alternative to the standard linear VECM.

### 5.1.4.2 Specification

The standard form of a STVECM is given by the following:

$$\Delta Y_t = \left(\Phi_{1,0} + \alpha_1 z_{t-1} + \sum_{j=1}^{p} \Phi_{1j}\Delta Y_{t-j}\right) + \left(\Phi_{2,0} + \alpha_2 z_{t-1} + \sum_{j=1}^{p} \Phi_{2j}\Delta Y_{t-j}\right) \otimes G(s_t;\gamma,c) +$$

where $\Delta Y_t$ is the $k \times 1$ vector containing the change of the endogenous variables, that is, $\Delta Y_t = (\Delta Y_{1t}, \dots, \Delta Y_{kt})$; $z_{t-1} = \beta' Y_{t-1}$ for some $k \times 1$ vector $\beta$ denote the error-correction term, that is, $z_{t-1}$ is the deviation from the arbitrage-free relationship which is given by $\beta' Y_{t-1} = 0$ ; $s_t = z_{t-d}$ denote the transition variable, that is, the size of the dth lag of the deviation determines the strength of the error-correction adjustment; $G(s_t;\gamma,c)$ is the $2 \times 1$ vector of the transition function corresponding to the individual set of parameter value of $\gamma$ and $c$ for spot and futures, while they share the same transition variable and function form.

Equivalently, it can be written in the matrix form where all RHS variables are listed in a $(pk+2)\times 1$ vector $x_t$ and the corresponding parameter vectors, $\Phi_1$ and $\Phi_2$.

That is, $x_t = \left(1, z_{t-1}, \Delta Y_{1,t-1}, \ldots, \Delta Y_{1,t-p}, \ldots, \Delta Y_{k,t-1}, \ldots, \Delta Y_{k,t-p}\right)'$.

$$\Delta Y_t = \Phi_1 x_t + \Phi_2 x_t \otimes G(s_t; \gamma, c) + \varepsilon_t \qquad (19 *)$$

The transition function takes the value between zero and one. The STVECM model can be thought of as a regime-switching model that allows for two regimes, associated with the two extreme values of the transition function, $G(s_t;\gamma,c)=0$ and $G(s_t;\gamma,c)=1$, where the transition from one regime to the other is smooth. Depending on the type of asymmetry to be captured, the transition function takes one of the two forms. One type of the transition function is the first-order logistic function, which captures the asymmetric effects of positive and negative deviation from equilibrium. The first-order logistic function is shown below.

$$G(s_t;\gamma,c) = \left(1+\exp\{-\gamma(s_t-c)\}\right)^{-1}, \qquad \gamma>0 \qquad (20)$$

The parameter $\gamma$ determines the smoothness of the change in the value of the exponential function and therefore the speed of transition from one regime to the other. As $\gamma$ becomes very big, the transition between regimes becomes instant. Therefore, the STVECM with logistic transition function accommodates a two-regime threshold VECM as an extreme case.

The other type of the transition function is the exponential function, which captures the different effect of big and small deviation from the arbitrage-free relationship. It is shown as follows.

$$G(s_t;\gamma,c) = 1-\exp\{-\gamma(s_t-c)^2\}, \quad \gamma>0 \qquad (21)$$

Again, the parameter $\gamma$ determines the smoothness of the change in the value of the exponential function and therefore the speed of transition from the one regime to the other. As $\gamma$ becomes very big, the transition between regimes becomes instant. However, the STVECM with exponential transition function does not accommodate a two-regime threshold VECM as a special case because the exponential function takes the value of one for all but zero value of $s_t$. For the transition function to capture the different adjustment speed between small and big deviation and to accommodate the threshold model as a special case, another type of transition function – the second-order logistic function is proposed by Jansen and Terasvirta (1996). The second-order logistic function is of the following form.

$$G(s_t;\gamma,c) = (1 + \exp\{-\gamma(s_t - c_1)(s_t - c_2)\})^{-1}, \qquad c_1 \leq c_2, \ \gamma > 0 \tag{22}$$

The parameter $\gamma$ determines the speed of transition. When it is very big, the STVECM approaches a three-regime threshold VECM with the restriction that the outer two regimes are identical.

As our goal is to model the return on spot and futures, the exponential and the second-order logistic function are more appropriate than the first-order logistic function as the transition function is increasing in the size of the disequilibrium, but not its actual value of the deviation. In the extreme case where the lagged deviation is zero, there is no arbitrage opportunity for all investors, therefore the coefficient of adjustment is zero. In the other extreme case where the lagged deviation is very big, the arbitrage opportunity is available for a number of investors, therefore the strength of adjustment is strong and the size of adjustment coefficient is big. Between the two extreme cases, the number of investors at the position of taking the arbitrage opportunity is between those in the two extreme cases; therefore the size of the adjustment coefficient is also between those two extreme values.

### 5.1.4.3 Linearity Test

The first step towards building a nonlinear VECM is to test for nonlinearity. Since the rationale for nonlinear adjustment to disequilibrium implies the model specification of STVECM with exponential or second-order logistic function rather than first-order logistic function as transition function, we only test linearity against these two alternative specifications.

It is well-known that linearity test is complicated by the presence of unidentified nuisance parameters under the null. Specifically, under one of the two unrelated conditions on coefficients, the model is linear. Rejecting either of them does not lead to the rejection of linearity. Only if both are rejected simultaneously, the null of linearity can be rejected. Specifically, both the null $H_0: \Phi_{2,j} = 0 \text{ and } \alpha_2 = 0$ and the alternative null $H_0': \gamma = 0$ imply linearity. However, if we set up the joint of them as null, the conventional statistical theory is not available for obtaining the asymptotic null distribution of the test statistics and the critical values have to be determined by simulation.

Luukkonen, Saikkonen and Terasvirta (1988) proposed to replace the transition function by a Taylor series approximation at the point where $\gamma = 0$. In the re-parameterized equation, we can set up a null corresponding to the joint null of the original equation avoiding the identification problem and use a standard additional variable test for linearity testing.

Specifically, Sailkkonen and Terasvirta (1988) suggest approximating the exponential function by its first-order Taylor approximation around $\gamma = 0$ as follows.

$$G(s_t;\gamma,c) = G(s_t;\gamma = 0,c) + \frac{\partial G}{\partial \gamma}\Big|_{\gamma=0} \cdot \gamma + R_1 = (s_t - c)^2\,\gamma + R_1 \tag{23}$$

where $R_1$ is the difference between the approximation and the actual transition function. When $\gamma = 0$, $R_1$ is zero. Substitute (23) into (19*). The STVECM can be re-written as follows.

$$\Delta Y_t = \beta_0 x_t + \beta_1 x_t s_t + \beta_2 x_t s_t^2 + \varepsilon_t \tag{24}$$

where $\beta_0, \beta_1$ **and** $\beta_2$ are all $(pk + 2) \times 1$ vectors.

$$\beta_0 = \Phi_1 + \Phi_2(\gamma c^2 + R_1);\ \beta_1 = -2\Phi_2\gamma c;\ \beta_2 = \Phi_2\gamma\ .$$

As noted in the unidentified nuisance parameter section, the linearity corresponds to either $\Phi_2 = 0$ or $\gamma = 0$, both of which correspond to the null hypothesis that $\beta_1 = \beta_2 = 0$. The problem is solved and the linearity test in this scenario is a LM-type test because it is performed on the re-parameterization of the original model. We compute the $\chi^2$ version of the LM statistics as follows. First, estimate the model under the null of linearity and compute the residuals $\hat{\varepsilon}_t$ and the sum of squared residuals $SSR_0 = \sum_{t=1}^{T} \hat{\varepsilon}_t^2$. The regression under the null hypothesis is as follows.

$$\Delta Y_t = \beta_0 x_t + \varepsilon_t$$

Second, estimate the auxiliary regression (24). Compute the residuals $\hat{e}_t$ and the sum of squared residuals $SSR_1 = \sum_{t=1}^{T} \hat{e}_t^2$.

Last, the $\chi^2$ version of the LM statistics is $\frac{T(SSR_0 - SSR_1)}{SSR_0}$. Alternatively, we could compute the **F** version of the LM statistics which is preferable in the case of small sample. In a big sample, the two statistics are close to each other. As our sample size is quite large ( at least 1000 observations in each data set ), only the $\chi^2$ version is computed. Note that if $s_t = z_{t-1}$, then the first element of $\beta_1$ and $\beta_2$ have to be dropped to avoid multicollinearity.

If the alternative is that the transition function is a second-order logistic function, the same linearity test applies. In this case, the first-order Taylor approximation of the transition function is as follows.

$$G(s_t;\gamma,c) = G(s_t;\gamma = 0,c) + \frac{\partial G}{\partial \gamma}|_{\gamma=0} \cdot \gamma + R_1 = \frac{1}{4}(s_t - c_1)(s_t - c_2)\gamma + R_1 \qquad (23*)$$

The STVECM can be re-written as follows.

$$\Delta Y_t = \beta_0 x_t + \beta_1 x_t s_t + \beta_2 x_t s_t^2 + \varepsilon_t \qquad (24*)$$

where $\beta_0, \beta_1$ and $\beta_2$ are all $(pk + 2) \times 1$ vectors.

$$\beta_0 = \Phi_1 + \frac{1}{4}\Phi_2(\gamma c_1 c_2 + R_1);\ \beta_1 = -\frac{1}{4}\Phi_2\gamma(c_1 + c_2);\ \beta_2 = \frac{1}{4}\Phi_2\gamma.$$

The linearity hypothesis corresponds to $\beta_1 = \beta_2 = 0$. The conclusion is that the same set of LM statistics can be used to test for nonlinearity specified by a second-order logistic transition function as that with an exponential function.

#### 5.1.4.4 Estimation

The estimation of STVECM is a straightforward application of the nonlinear least squares ( NLS ) method. Same as the linearity test, the estimation is performed on the return of spot and futures respectively. Denote the vector of parameters of the return of spot and futures as $\theta_i$, where $i = 1$ or $2$. $\theta_i = (\Phi_{1i}, \Phi_{2i}, \gamma_i, c_i)'$ can be estimated as

$$\hat{\theta}_i = \mathrm{argmin}_\theta Q_T(\theta_i) = \mathrm{argmin}_\theta \sum_{t=1}^{T} (\Delta y_{it} - F(x_t;\theta_i))^2$$

where $\mathbf{F}$ is the skeleton of the model. That is, $F = \Phi_1 x_t + \Phi_2 x_t G(s_t;\gamma,c)$. The NLS estimates are quasi maximum likelihood estimates. They are consistent and asymptotically normal. While the theory of NLS is clear, the estimation in practice is problematic.

First, STVECM is a nonlinear model whose likelihood function potentially has multiple maximum. The starting values are crucial in finding the correct estimates of the model. Leybourne, Newbold and Vougas (1998) suggested to simplify the estimation problem by concentrating the sum of squares function. Specifically, when $\gamma$ **and** $c$ are fixed, the model is linear in the auto-regressive parameters $\Phi_1$ **and** $\Phi_2$. Given the value of $\gamma$ **and** $c$, $\Phi_1$ **and** $\Phi_2$ can be estimated by ordinary least squares (OLS). Denote the estimate of $\Phi$ conditional on $\gamma$ **and** $c$ as $\hat{\Phi}(\gamma,c)$. Thus, the sum of squares function $Q_T(\theta)$ can be concentrated with respect to $\Phi_1$ **and** $\Phi_2$ as

$$Q_T(\gamma,c) = \sum_{t=1}^{T} (\Delta y_t - \phi(\gamma,c) x_t(\gamma,c))^2$$

This method reduces the dimensionality of the NLS estimation to two. $Q_T(\gamma,c)$ just needs to be minimized with respect to the two parameters $\gamma$ **and** $c$ only. We do a grid search on $\gamma$ **and** $c$ and set the ones that minimize the sum of squared residuals as the starting values in the actual NLS estimation.

Second, the number of parameters is very big. For example, in the case of UK, it reaches 12. It therefore is beneficial to impose some constraints on parameter values. Recall the general form of the STVECM in (19).

$$\Delta Y_t = \left( \Phi_{1,0} + \alpha_1 z_{t-1} + \sum_{j=1}^{p} \Phi_{1j} \Delta Y_{t-j} \right) + \left( \Phi_{2,0} + \alpha_2 z_{t-1} + \sum_{j=1}^{p} \Phi_{2j} \Delta Y_{t-j} \right) \otimes G(s_t;\gamma,c) \text{ -}$$

When the transition variable – lagged disequilibrium is zero, the transition function – $G(s_t;\gamma,c)$ reduces to zero and the model reduces to

$$\Delta Y_t = \left( \Phi_{1,0} + \alpha_1 z_{t-1} + \sum_{j=1}^{p} \Phi_{1j} \Delta Y_{t-j} \right) + \varepsilon_t$$

In this extreme case, the strength of adjustment is the weakest because the lagged disequilibrium is too small to trigger any arbitrage, i.e. the adjustment to disequilibrium is zero. Since the adjustment strength of this state is measured by $\alpha_1$, we impose the constraint $\alpha_1 = 0$.

The parameter $\gamma$ measures the smoothness of transition between regimes. In order to estimate it accurately, we need to have a number of observations corresponding to the changing regime. But usually, most observations of the data set are either in one regime or another, but not in between. This makes the estimate of $\gamma$ tend to be inaccurate. Furthermore, as van Dijk, Franses and Terasvirta (2000) suggested, the t-statistics of $\gamma$ does not have the usual asymptotic t-distribution under the null that $\gamma = 0$ as a result of the unidentified nuisance parameter problem. Therefore we should not take the insignificant $\gamma$ as the evidence against nonlinearity.

Several authors including Terasvirta (1993) and Dijk, Franses and Terasvirta (2000) have proposed to adjust the transition function form by dividing $(s_t - c)$ by its standard deviation to make $\gamma$ approximately scale-free in aid of estimation. We follow their suggestions and make this change to the transition function form.

### 5.2 Modelling the Conditional Variance

In the previous section, we propose to analyze the return of spot and futures in the linear or smooth transition vector-error-correction model framework with constant conditional covariance matrix. In that setting, the OLS estimators are asymptotically efficient in the class of consistent asymptotically normal linear estimators. However, the assumption of constant covariance matrix is unrealistic for high-frequency financial returns. The history of high-frequency return series is characterized by volatile period and tranquil period, which implies the conditional second moments are autocorrelated rather than constant.

To accommodate this stylized fact, Engle (1982) and Bollerslev (1986) proposed the Autocorrelated Conditional Heteroskedasticity Model (ARCH) and the generalized ARCH (GARCH) model. In particular, under certain conditions, GARCH is a parsimonious representation of ARCH and it requires less conditions on the parameters to guarantee stability and to enforce estimation. The simplest GARCH specification – GARCH (1,1) has been proven useful and adequate by numerous researchers.

### 5.2.1 GARCH (1,1)

GARCH (1,1) models the conditional variance as the sum of the conditional variance and the squared residual in the last period. Denote the residual as $\varepsilon_t$ with the conditional variance $\sigma_t^2$. The specification of GARCH (1,1) process is the following.

$$\sigma_t^2 = \omega + \beta \cdot \sigma_{t-1}^2 + \alpha \cdot \varepsilon_{t-1}^2 \tag{25}$$

The nonnegativity of the variance requires that $\omega > 0, \alpha > 0, \beta > 0$. The standardized residual, $v_t$ ($v_t = \varepsilon_t / \sigma_t$) is assumed to be white noise with zero mean and unity variance.

It can also be written in terms of the standardized residual and the conditional variance.

$$\sigma_t^2 = \omega + (\alpha + \beta) \cdot \sigma_{t-1}^2 + \alpha \cdot (v_{t-1}^2 - 1) \cdot \sigma_{t-1}^2 \tag{26}$$

where $\alpha$ measures the extent of the impact of the last-period shock on the current volatility and $\alpha + \beta$ measures the speed of this effect dampening down.

The condition of covariance stationarity is $\alpha+\beta<1$. The condition of strict stationarity discovered by Nelson (1990) is $E[\log(\beta + \alpha \cdot v_t^2)] < 0$. A covariance stationary GARCH process is necessarily strictly stationary, but a covariance nonstationary GARCH process might be strictly stationary.[3]

The two commonly used estimation methods of GARCH process are feasible Generalized Least Squares (GLS) and Maximum Likelihood (ML), between which the Maximum Likelihood Estimators (MLEs) have better asymptotic properties. In particular, the MLEs are consistent, asymptotically normal, asymptotically efficient and invariant to one-to-one transformations of parameters.

The common practice is to assume the standardized residual is normally distributed, which justifies a bigger unconditional variance than an unconditional normal distribution. However,

[3] From Jensen's inequality, $E\log(\beta+\alpha\cdot v^2) < \log E(\beta+\alpha\cdot v^2) = \log(\beta+\alpha)$. Therefore $\beta + \alpha < 1$ implies $E\log(\beta+\alpha\cdot v^2) < 0$.

it is sometime still not enough to explain the leptokurtosis in real financial data. Some researchers propose to replace the conditional normal distribution with fat-tailed distribution, such as Student-t distribution, where the degree of freedom can be estimated along with others parameters.

Or, one can maximize the loglikelihood function as if the standardized residual were normally distributed. The estimators are termed as quasi-maximum likelihood estimators (QMLEs) and they are still consistent and asymptotically normally distributed but not asymptotically efficient provided the conditional mean and conditional variance are correctly specified.

Generally, GARCH is combined with ARMA in modelling a particular time series. When the conditional variance, $\sigma_t^2$ is a symmetric function of the residual, $\varepsilon_t$, the Hessian matrix is block-diagonal. Thus the consistent and asymptotically efficient estimates of ARMA and GARCH parameters can be derived separately, i.e. the two-step estimation method is justified. In the first step, the ARMA parameters can be estimated by least squares method. Then the residuals in this stage can be used to estimate the GARCH parameters by maximum likelihood method. When the conditional variance is asymmetric in residual, the Hessian matrix is not block diagonal and therefore the full-information method has to be employed in estimating the two groups of parameters jointly. In the case where both methods are valid, the two-step method is preferable since it involves less parameters in the loglikelihood estimation and makes the convergence easier.

### 5.2.2 Asymmetric GARCH (1,1)

Despite the apparent success of the standard GARCH, it can not capture the well-known asymmetric effect of real financial time series. The asymmetric effect discovered by Black (1976) refers to the phenomenon where negative shocks increase the predictable volatility more than positive shocks of the same magnitude. While its rationale is still under debate[4], a number of empirical studies have proven its existence. See Kroner and Ng (1998) and Brooks

[4] Black (1976) argued that negative shock leads to a decrease in the value of the firm and an increase in the debt-to-equity ratio, which leads to a higher volatility. Campbell and Hentschel (1992) argued that the arrival of news leads to an increase in volatility, which is compensated by a higher expected return and a decrease in the price. It amplifies the effect of negative shock and dampens the effect of positive shock, resulting in the asymmetric effect.

et al. (2002). Several variants of GARCH models have been proposed to accommodate the asymmetric effect, among which the most popular specification is the threshold GARCH (TGARCH).

Threshold GARCH (TGARCH or GJR-GARCH) is developed by Glosten et al. (1989) and Zakoian (1990). It accommodates the asymmetric effect simply by adding the product of a sign dummy and the past squared residual to the standard GARCH. The specification of TGARCH (1,1) is as follows:

$$\sigma_t^2 = \omega + \beta \cdot \sigma_{t-1}^2 + \alpha \cdot \varepsilon_{t-1}^2 + \gamma \cdot S_{t-1}^- \cdot \varepsilon_{t-1}^2 \tag{27}$$

where

$$S_{t-1}^- = 1 \; if \; \varepsilon_{t-1} < 0; \; S_{t-1}^- = 0 \; otherwise$$

$\omega > 0, \beta > 0 \; and \; \alpha + \frac{1}{2}\gamma > 0$ are the conditions for the nonnegativity of the conditional variance. The covariance stationarity requires $\alpha + \beta + \frac{1}{2}\gamma < 1$.

To distinguish the effect of positive and negative shock, Engle and Ng (1993) proposed the news impact curve (NIC) to examine the effect of shock at time $t-1$ on the conditional variance at time $t$, holding constant the information at time $t-2$ or before. The function corresponding to NIC of TGARCH (1,1) is as follows:

$$\sigma_t^2 = \begin{cases} A + \alpha \cdot \varepsilon_{t-1}^2, & for \; \varepsilon_t > 0 \\ A + (\alpha + \gamma) \cdot \varepsilon_{t-1}^2, & for \; \varepsilon_t < 0 \end{cases}$$

$$where \qquad A = \omega + \beta \cdot \sigma^2$$

The NIC of TGARCH (1,1) is a quadratic function of $\varepsilon_{t-1}$ centred at the origin with different slope on each side. As bigger shocks have greater impact on conditional volatility, $\alpha$ and $\alpha + \gamma$ are both positive. As negative shock has a bigger effect than positive shock of the same magnitude on the residuals, $\gamma$ is expected to be positive. Together the two conditions require $\alpha$ is positive as well. Engle and Ng (1993) compared the performance of several parameterizations of the asymmetric GARCH and they concluded that TGARCH is robust and provides the best fit to the data. As the conditional variance of TGARCH (1,1) is asymmetric in residual, the Hessian matrix of the ARMA-TGARCH parameters is not block-diagonal, invalidating the two-step estimation method. We will use the full-information maximum likelihood method to estimate TGARCH models.

### 5.2.3 Bivariate (T)GARCH (1,1)

As the optimal hedge ratio is the quotient of the covariance between the return of spot and futures to the variance of the futures, we need to model the covariance matrix of the two returns. Therefore bivariate GARCH rather than univariate GARCH is needed for our purpose. The general specification of a bivariate GARCH process is as follows:

$$\varepsilon_t = v_t \cdot \mathrm{H}_t^{1/2}$$

where $v_t$ is a two-dimensional white-noise vector with zero mean and covariance matrix equal to identity matrix, and $\varepsilon_t$ is a two-dimensional process with zero mean and covariance matrix $\mathrm{H}_t$. Three popular specifications of multivariate GARCH are VECH, BEKK and Constant-Correlation GARCH.

Engle and Kroner (1995) proposed the following VECH-GARCH representation.

$$vech(\mathrm{H}_t) = \Omega + \mathrm{B} \cdot vech(\mathrm{H}_{t-1}) + \mathrm{A} \cdot vech(\varepsilon_{t-1} \cdot \varepsilon'_{t-1})$$

where *vech* is an operation that stacks elements in the lower triangle of a matrix into a vector. For a bivariate GARCH, $\Omega$ is a $3 \times 1$ vector, $\mathrm{B}, \mathrm{A}$ are $3 \times 3$ symmetric matrices respectively.

The advantage of this parameterization is its flexibility as it allows each element of the covariance matrix to be affected by all elements of the cross-product of $\varepsilon_{t-1}$ and all elements of the lagged covariance matrix $\mathrm{H}_{t-1}$. However, the large number of parameters makes it difficult to estimate as the convergence is problematic and the stability condition is difficult to satisfy. Diagonal VECH GARCH developed by Bollerslev, Engle and Wooldridge (1988) simplifies the full VECH by restricting $\mathrm{A}$ and $\mathrm{B}$ from being symmetric to diagonal matrix.

An alternative to VECH-GARCH is developed by Bera et al. (1987), which is known as BEKK-GARCH (Bera-Engle-Kraft-Kroner). The parameterization of BEKK-GARCH (1,1) is as follows:

$$\mathrm{H}_t = \Omega \cdot \Omega' + \mathrm{B} \cdot \mathrm{H}_{t-1} \cdot \mathrm{B}' + \mathrm{A} \cdot \varepsilon_{t-1} \cdot \varepsilon'_{t-1} \cdot \mathrm{A}'$$

where $\Omega$ is a $2 \times 2$ lower triangular matrix and $\mathrm{B}, \mathrm{A}$ are $2 \times 2$ symmetric matrix. BEKK-GARCH is a special form of VECH-GARCH. As all the three terms on the right-hand-side of the covariance matrix are in quadratic form, $\mathrm{H}_t$ is guaranteed positive-definite. The restriction of BEKK also brings the number of parameters down. Similar to the diagonal VECH-GARCH, diagonal BEKK-GARCH was proposed by Engle and Kroner (1995) with the constraint that each element of covariance matrix only depends on its own lagged squared residual and its counterpart in the lagged conditional covariance matrix.

Another alternative specification is the constant-correlation GARCH proposed by Bollerslev (1990). It assumes the correlation coefficient is constant. Since an accurate estimate of conditional covariance is the key in computing dynamic hedge ratio, it would be undesirable to impose strong restriction such as constant correlation in the model.

Among the three alternative specifications of bivariate GARCH (1,1) model, we will use the diagonal BEKK-GARCH for two reasons. First, our large hold-out sample requires a robust model to guarantee successful estimation. Second, the consensus is that while models with a lot of parameters tend to fit the data well within sample but do poorly out-of-sample but parsimonious models tend to produce better forecast. In this chapter, we will look at both the within-sample and out-of-sample results with the emphasis on the latter. Therefore the most parsimonious model among all – diagonal BEKK is preferable.

Similar to the standard GARCH, TGARCH also has several alternative specifications in the multivariate context. For the same reason as the standard GARCH, only the diagonal BEKK-TGARCH (1,1) is employed in this chapter.

The specification of diagonal BEKK-TGARCH (1,1) is as follows:

$$\mathrm{H}_t = \Omega \cdot \Omega' + \mathrm{B} \cdot \mathrm{H}_{t-1} \cdot \mathrm{B}' + \mathrm{A} \cdot \varepsilon_{t-1} \cdot \varepsilon'_{t-1} \cdot \mathrm{A}' + D \cdot u_{t-1} \cdot u'_{t-1} \cdot D'$$

where $\Omega$ is a $2\times 2$ lower triangular matrix , $\mathrm{B}, \mathrm{A}$ and $D$ are $2\times 2$ matrices and $u_{t-1}$ is a two-dimensional vector with the element equal to the minimal one between the corresponding residual and zero. That is, $u_{t-1} = (u_{1,t-1}, u_{2,t-1})'$ $where\, u_{i,t-1} = \min[\varepsilon_{i,t-1}, 0]$. Like the univariate TGARCH (1,1), bivariate TGARCH (1,1) accommodates the asymmetric effect of shocks on the conditional covariance matrix.

Or equivalently, the model can be written in the form of equation, rather than matrix.

$$
\begin{aligned}
h_{11t} &= \omega_s^2 + \beta_s^2 \cdot h_{11t-1} + \alpha_s^2 \cdot \varepsilon_{st-1}^2 + d_s^2 \cdot u_{st-1}^2 \\
h_{12t} &= \omega_s\omega_3 + \beta_s\beta_f \cdot h_{12t-1} + \alpha_s\alpha_f \cdot \varepsilon_{st-1}\varepsilon_{ft-1} + d_s d_f \cdot u_{st-1}u_{ft-1} \\
h_{22t} &= \omega_f^2 + \omega_3^2 + \beta_f^2 \cdot h_{22t-1} + \alpha_f^2 \cdot \varepsilon_{ft-1}^2 + d_f^2 \cdot u_{ft-1}^2
\end{aligned}
$$

where $H_t = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$, $\Omega_t = \begin{pmatrix} \omega_s & \omega_3 \\ \omega_3 & \omega_f \end{pmatrix}$ $\mathrm{B}_t = \begin{pmatrix} \beta_s & 0 \\ 0 & \beta_f \end{pmatrix}$ $\mathrm{A}_t = \begin{pmatrix} \alpha_s & 0 \\ 0 & \alpha_f \end{pmatrix}$ $D_t = \begin{pmatrix} d_s \\ d_f \end{pmatrix}$

In summary, two models will be estimated for the second moments of the residuals in this chapter. They are bivariate diagonal BEKK-GARCH (1,1) and bivariate diagonal BEKK-TGARCH (1,1). In the actual estimation, we have to assume a distribution for the vector of standardized residuals. It is assumed to be either bivariate normal distribution or bivariate Student-t distribution. We use the two-step method to estimate the standard GARCH combined with the VECM and STVECM and the full-information maximum-likelihood method to estimate the TGARCH with the VECM.

The loglikelihood function for the bivariate normally distributed vector of residuals is:

$$
\log L(\theta) = -\frac{1}{2}[2\cdot T\cdot \log(2\pi) + \sum_{t=1}^{T}(\log|\Omega_t| + \varepsilon_t'\Omega_t^{-1}\varepsilon_t)] \tag{28}
$$

The loglikelihood function for the bivariate Student-t distributed vector of residuals is:

$$
\begin{aligned}
\log L(\theta) = &-\frac{1}{2}2\cdot T\cdot \log(\pi\cdot v) + T\cdot[\log\Gamma(\frac{v+2}{2}) - \log\Gamma(\frac{1}{2}v)] - \frac{1}{2}\sum_{t=1}^{T}(\log|\Omega_t|) \\
&+\frac{1}{2}T\cdot(v+2)\cdot\log v - \frac{1}{2}(v+2)\sum_{t=1}^{T}\log(v+\varepsilon_t'\Omega_t^{-1}\varepsilon_t)
\end{aligned} \tag{29}
$$

where $v$ is the degree of freedom parameter.

## 6. Results

### 6.1 Stationarity and Cointegration

The stationarity test results for all six countries are presented in Table 2. Unit root tests are performed on the level of logarithm of stock index and index futures, the cost of carry and the first-order difference of the logarithm of index and futures respectively. Specifically, Augmented Dickey-Fuller test is used with the number of lags selected on the basis of Schwartz Information Criterion.

**Table 2: Stationarirty test results**

| | *AU* | *GM* | *JP* | *KR* | *UK* | *US* |
|---|---|---|---|---|---|---|
| $f$ | -0.098 | -1.934 | -1.224 | -2.029 | -2.280 | -2.506 |
| $s$ | 0.141 | -1.934 | -1.306 | -1.821 | -2.290 | -2.545 |
| $coc$ | -5.233 | -9.815 | -7.774 | -6.257 | -7.068 | -6.032 |
| $\Delta f$ | -36.564 | -51.677 | -55.394 | -49.730 | -33.356 | -52.692 |
| $\Delta s$ | -35.809 | -52.127 | -53.400 | -45.299 | -33.016 | -51.780 |
| c.v. 5% | -2.864 | -2.862 | -2.862 | -2.863 | -2.862 | -2.862 |

Note: This table contains the ADF test statistics for the level of the logarithm of index futures and spot index, the cost of carry and the return of index futures and spot index. The corresponding 5% critical values are shown in the bottom row.

The conclusion is uniformly consistent for all six countries. The unit root hypothesis can not be rejected for the stock index and index futures in level at 10% significance level but it can be rejected for their first-order difference at 1% significance level, indicating the price level is $I(1)$ and the return is $I(0)$. The results also indicate that the cost of carry is $I(0)$. Therefore, we will use dynamic OLS method to estimate cointegrating vector.

Table 3 contains the estimation results for cointegrating vector and the cointegration test results. The ADF test statistics in the row above the last of Panel A together with its 5% critical value suggest that the unit root hypothesis can be rejected at 5% significance level for the residuals, indicating the level of spot index, index futures and cost of carry are indeed cointegrated. The t statistics suggest that DOLS coefficient estimates of all countries are significantly different from zero and most of them are significantly different from unity at conventional level. However, considering transaction costs not accommodated in this model, we find the evidence is not strong enough to reject the (1,-1,-1) cointegrating vector suggested by the cost of carry theory. Panel B of Table 3 presents the ADF test results on the residuals derived from imposing the cointegrating vector $(1,-1,-1)$. The test statistics for all the countries are very close to those without the constraint, indicating the constraint hardly affects the residuals. In the later VECM estimation, we will estimate the models using the cointegrating residuals derived from imposing the constraint only.

**Table 3: Cointegration test results**

Panel A: DOLS estimation result of Cointegrating vector

Cointegration equation

$$f_t = \beta_0 + \beta_1 \cdot s_t + \beta_2 \cdot coc_t + \sum_{i=-l\neq 0}^{l} \gamma_i \cdot \Delta s_{t-i} + u_t$$

ADF test equation

$$\Delta \hat{u}_t = \rho \cdot \hat{u}_t + \sum_{i=1}^{k} \gamma_i \cdot \Delta \hat{u}_{t-i} + \varepsilon_t$$

| | *AU* | *GM* | *JP* | *KR* | *UK* | *US* |
|---|---|---|---|---|---|---|
| $\beta_0$ | 0.0427 | -0.0045 | -0.0071 | -0.0994 | 0.0045 | -0.0082 |
| *sd.* | 0.0095 | 0.0022 | 0.0029 | 0.0063 | 0.0029 | 0.0010 |
| $\beta_1$ | 0.9944 | 1.0006 | 1.0008 | 1.0220 | 0.9993 | 1.0012 |
| *sd.* | 0.0012 | 0.0003 | 0.0003 | 0.0014 | 0.0003 | 0.0001 |
| $\beta_2$ | 1.5427 | 0.8734 | 1.0188 | 0.3442 | 1.1111 | 1.1152 |
| *sd.* | 0.0459 | 0.0328 | 0.1150 | 0.0388 | 0.0217 | 0.0106 |
| $t-adf$ | -7.2561 | -18.5705 | -17.1007 | -8.7956 | -6.7535 | -31.1695 |
| 5% c.v. | -2.8636 | -2.8625 | -2.8625 | -2.8626 | -2.8625 | -2.8625 |

Panel B: stationarity test results for the residual when cointegrating vector of (1,-1,-1) is imposed

| | *AU* | *GM* | *JP* | *KR* | *UK* | *US* |
|---|---|---|---|---|---|---|
| $t-adf$ | -5.8254 | -17.1656 | -14.5662 | -7.2981 | -5.7265 | -13.1445 |
| 5% c.v. | -2.8636 | -2.8625 | -2.8625 | -2.8626 | -2.8625 | -2.8625 |

Note: 1. Panel A contains the DOLS estimates for the cointegrating vector. The bottom two rows contain the ADF test statistics for the estimated residual and 5% significance level.

2. Panel B contains the ADF test statistics for the difference between the logarithmic index futures and the sum of cost of carry and the logarithmic spot index.

### 6.2 VECM estimation results

The estimation results of the linear VECM are contained in Table 4.

The estimates of error-correction coefficient, $\delta$ all have signs suggested by the theory and offer us some interesting insight of the market characteristics of these six countries. For Australia, Japan and USA, the error-correction coefficients of spot and futures are both significantly different from zero at 1% significance level and their signs are as expected, i.e. the coefficient for the futures is negative and that for the spot is positive. In these three countries, both spot index and index futures react to disequilibrium, i.e. the information flow is bi-directional. The coefficients for USA are 0.37 and -0.47 respectively, indicating strong symmetric response in both spot and futures market. The coefficients for Australia are 0.19 and -0.21 respectively, suggesting symmetric but weaker response in both markets. The coefficients for Japan are 0.45 and -0.27 respectively, indicating strong and weak response in spot and futures market when there is an arbitrage opportunity. The results for Germany are very different from the above three countries. While the error-correction coefficient in the futures market is insignificantly different from zero at 10% significance level indicating no response from the futures market; the coefficient in the spot equation is -0.86 at 1% level, suggesting a strong feedback in the spot market. When there is disequilibrium between the German spot and futures markets, the former reacts strongly but the latter does not change at all, i.e. the information flow is unidirectional. In contrast to the German results, UK results suggest spot market does not react to disequilibrium but futures market responds reasonably strongly with a coefficient of 0.24 at 1% level. For South Korea, both coefficients are small and the coefficient in the futures equation is only significant at 10% level. The response to disequilibrium in the South Korean futures market is negligible and that in the spot market is also weak.

The null hypothesis of no autocorrelation of $5^{th}$ and $15^{th}$ order can not be rejected at even 10% level for any residual. It indicates all the residuals are white noise and the modelling of conditional mean of the returns is probably adequate. The null of no autocorrelation of $5^{th}$ and $15^{th}$ order can be rejected at 1% for all the squared residuals, indicating GARCH effect in all residuals. The Jaque-Bera statistics are all significantly different from zero, indicating the residuals are not normally distributed. Together they suggest the need to use more complicated model than VECM for the bivariate system. One way is to model the conditional second moments by GARCH. The other way is to replace linear VECM by nonlinear VECM for the conditional mean. The estimation results of these two models are shown later.

**Table 4: VECM estimation results**

VECM specification

$$\Delta s_t = \mu_s + \delta_s \cdot z_{t-1} + \sum_{i=1}^{m} \varphi_{s1i} \cdot \Delta s_{t-i} + \sum_{i=1}^{n} \varphi_{s2i} \cdot \Delta f_{t-i} + \varepsilon_{st}$$

$$\Delta f_t = \mu_f + \delta_f \cdot z_{t-1} + \sum_{i=1}^{m} \varphi_{f1i} \cdot \Delta s_{t-i} + \sum_{i=1}^{n} \varphi_{f2i} \cdot \Delta f_{t-i} + \varepsilon_{ft}$$

$$z_t = f_t - s_t - coc_t$$

| | *AU* | | *GM* | | *JP* | | *KR* | | *UK* | | *US* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ |
| $\delta$ | 0.188 | -0.210 | 0.860 | -0.023 | 0.449 | -0.271 | 0.078 | -0.067 | 0.111 | -0.241 | 0.373 | -0.465 |
| $p-value$ | 0.008 | 0.005 | 0.000 | 0.818 | 0.000 | 0.016 | 0.003 | 0.090 | 0.182 | 0.007 | 0.003 | 0.001 |
| $\mu$ | 0.000 | 0.000 | 0.001 | 0.000 | 0.000 | 0.000 | 0.001 | -0.001 | 0.000 | 0.000 | 0.000 | 0.001 |
| $p-value$ | 0.356 | 0.783 | 0.013 | 0.381 | 0.149 | 0.989 | 0.125 | 0.364 | 0.216 | 0.891 | 0.614 | 0.005 |
| $\varphi_{12}$ | | | | | | | | | -0.057 | -0.062 | | |
| $p-value$ | | | | | | | | | 0.084 | 0.066 | | |
| $\varphi_{13}$ | | | | | | | | | -0.090 | -0.091 | | |
| $p-value$ | | | | | | | | | 0.005 | 0.005 | | |
| $\varphi_{15}$ | | | | | | | | | -0.062 | -0.066 | | |
| $p-value$ | | | | | | | | | 0.027 | 0.024 | | |
| $\varphi_{16}$ | | | -0.059 | -0.045 | | | | | | | | |
| $p-value$ | | | 0.012 | 0.069 | | | | | | | | |
| $\varphi_{18}$ | | | 0.053 | 0.062 | | | | | | | | |
| $p-value$ | | | 0.105 | 0.057 | | | | | | | | |
| $\varphi_{21}$ | | | | | -0.050 | -0.066 | 0.074 | -0.028 | | | | |
| $p-value$ | | | | | 0.020 | 0.002 | 0.002 | 0.324 | | | | |
| $\varphi_{25}$ | | | | | | | -0.051 | -0.072 | | | | |
| $p-value$ | | | | | | | 0.065 | 0.043 | | | | |
| AIC | -7.081 | -6.924 | -5.460 | -5.377 | -5.608 | -5.537 | -4.602 | -4.266 | -6.086 | -5.991 | -6.067 | -5.972 |
| SIC | -7.071 | -6.915 | -5.450 | -5.367 | -5.600 | -5.530 | -4.591 | -4.255 | -6.074 | -5.978 | -6.062 | -5.967 |
| $Q(5)$ | 2.715 | 2.503 | 5.013 | 5.597 | 2.083 | 1.671 | 5.713 | 4.044 | 0.525 | 1.055 | 7.457 | 7.237 |
| $p-value$ | 0.744 | 0.776 | 0.414 | 0.347 | 0.838 | 0.893 | 0.335 | 0.543 | 0.991 | 0.958 | 0.189 | 0.204 |
| $Q(15)$ | 7.072 | 8.958 | 20.796 | 17.449 | 10.847 | 7.768 | 15.320 | 21.776 | 21.211 | 15.243 | 22.311 | 19.533 |
| $p-value$ | 0.956 | 0.880 | 0.143 | 0.293 | 0.763 | 0.933 | 0.429 | 0.114 | 0.130 | 0.434 | 0.100 | 0.191 |
| $Q^2(5)$ | 141 | 66 | 759 | 351 | 111 | 102 | 177 | 496 | 740 | 639 | 282 | 259 |
| $p-value$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $Q^2(15)$ | 174 | 132 | 1854 | 790 | 266 | 255 | 359 | 1161 | 1694 | 1432 | 563 | 507 |
| $p-value$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Jaque-Bera | 792 | 173 | 981 | 1939 | 439 | 333 | 571 | 1124 | 684 | 477 | 945 | 1130 |
| $p-value$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

Note: 1. Table 4 contains the coefficient estimates of the VECM for the return on spot index and index futures and their P-values.
2. AIC and BIC are the value of Akaike information criterion (AIC) and Schwartz Bayesian criterion (BIC) of the VECM.
3. Q(5) and Q(15) are the 5th and 15th Ljung-Box Q-statistics of the VECM residuals.
4. $Q^2(5)$ and $Q^2(15)$ are the 5th and 15th Ljung-Box Q-statistics of the squared VECM residuals.

## 6.3 Nonlinear VECM results

### 6.3.1 Linearity Test Results

Table 5 summarizes the linearity test results of the six pairs of returns. For each country, five transition variables from the first to the fifth lag of the error correction term are used in testing. The structure of lags is carried forward from that of the linear model. The linearity hypothesis can be rejected when at least one of the five lagged disequilibrium variables is used as the transition variable for all six pairs of returns. Apart from USA, more than one transition variable suggest nonlinearity. In the case of USA, only if the third lag is chosen as the transition variable, the LM statistics are significant. The choice of transition variable for USA is therefore straightforward. In the case of South Korea and UK, all five lagged deviations suggest nonlinearity. As the LM test is most powerful when the true transition variable is chosen, the transition variable that corresponds to the lowest p-value is most likely to be true. And as our model requires estimating the two returns simultaneously, intuitively, the transition variable for both returns should be the same. Based on these reasons, we chose the fourth, third, first, first, second and third lagged error-correction term as the transition variable for Australia, Germany, Japan, South Korea, UK and USA respectively. Their results are highlighted red.

**Table 5: LM-type tests for STVECM nonlinearity for daily return on spot index and index futures**

| transition variable (s(t)) | AU | | GM | | JP | | KR | | UK | | US | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | S | F | S | F | S | F | S | F | S | F | S | F |
| z(t-1) | 4.80 | 0.45 | 28.56 | 26.18 | 16.70 | 18.42 | 33.84 | 64.25 | 27.13 | 25.08 | 4.12 | 8.69 |
| | 0.09 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.01 |
| z(t-2) | 23.00 | 4.66 | 25.55 | 17.69 | 8.23 | 6.24 | 37.59 | 52.49 | 47.34 | 33.46 | 8.63 | 8.19 |
| | 0.00 | 0.32 | 0.00 | 0.02 | 0.22 | 0.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.08 |
| z(t-3) | 23.39 | 14.41 | 51.83 | 42.92 | 12.93 | 12.40 | 31.30 | 39.06 | 38.01 | 31.54 | 14.15 | 12.05 |
| | 0.00 | 0.01 | 0.00 | 0.00 | 0.04 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.02 |
| z(t-4) | 36.28 | 20.19 | 8.18 | 5.21 | 8.92 | 8.17 | 34.41 | 25.23 | 34.61 | 31.63 | 6.84 | 7.75 |
| | 0.00 | 0.00 | 0.42 | 0.73 | 0.18 | 0.23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.14 | 0.10 |
| z(t-5) | 29.58 | 14.93 | 19.48 | 21.18 | 19.16 | 17.85 | 21.12 | 30.28 | 31.78 | 34.04 | 3.10 | 4.48 |
| | 0.00 | 0.00 | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.54 | 0.34 |

Note: This table contains the statistics and p-values of LM variants of the LM-type tests for STVECM nonlinearity of the daily return of spot index and index futures of six countries. The tests are applied in a VECM model for the first differences. The transition variables that correspond to the lowest p-value for each country are highlighted. The LM statistics are based on the auxiliary regression models given in (24).

### 6.3.2 Grid Search Results

Figure 1 to 6 in the appendix show the grid search results of the sum of squared residuals (SSR) corresponding to the exponential transition function. The parameter values corresponding to the minimal SSR are set as the starting values in the estimation. In those 3-D graphs, the horizontal axis pointing right is for the parameter c and the horizontal axis pointing inwards is for the parameter $\gamma$. The vertical axis is for the sum of squared residuals (SSR). For all but one country, the minimal SSR corresponds to the value of c around zero. The implication is that when the past deviation is close to zero, the returns behave according to the inner-regime part of the model and the model is nearly symmetric around zero. Only in the case of South Korea, the minimal SSR is reached when c is at its minimum, which is difficult to explain. The grid search for the second-order logistic function is not performed as it involves three parameters and no illustrative graph can be drawn. The same set of starting values for the STVECM with the exponential function is used for that with the second-order logistic function and the extra parameter of c is set to zero. These graphs also show that SSR is not sensitive to the change in $\gamma$. It is consistent with the big standard deviation of $\gamma$ in the estimation part.

### 6.3.3 Estimation Results

Table 6 – 10 contain the STVECM estimation results for Australia, Germany, Japan, United Kingdom and United States. The results for South Korea are not presented because the estimation fails. This is not surprising as its SSR graphs show that the minimal SSR is reached only when the transition variable is at extreme value. In other words, its loglikelihood function is ill-defined. For the remaining five countries, the NLS estimation succeeds.[5] The results for both the STVECM with exponential and quadratic logistic transition function are presented.

[5] Four of five transition variables suggested by the linearity test results are used in the estimation as they result in successful estimation. In the case of Australia, while the linearity test suggests the fourth lag as the transition variable, the model fails to converge when it is used. When the third lag is set as the transition variable, the estimation is successful and the estimates are intuitive. Therefore the reported results for Australia correspond to the third lag instead of the fourth lag as the transition variable.

## Table 6: STVECM Estimation results for Australia

$$\Delta y_t = \phi_{0,1} + (\phi_{0,2} + \phi_{1,2} z_{t-1}) G(z_{t-3}; \gamma, c) + \varepsilon_t$$

Exponential: $G(z_{t-3}; \gamma, c) = 1 - \exp[-\gamma (z_{t-3} - c)^2 / var(z_t)]$

Quadratic Logistic: $G(z_{t-3}; \gamma, c, c^*) = \{1 + \exp[-\gamma (z_{t-3} - c)(z_{t-3} - c^*) / var(z_t)]\}^{-1}$

| | Exponential | | Quadratic Logistic | |
|---|---|---|---|---|
| | Spot | Futures | Spot | Futures |
| $\phi_{0,1}$ | -0.0047 | 0.0029 | -0.0031 | 0.0067 |
| s.d | 0.0038 | 0.0012 | 0.0015 | 0.0118 |
| $\phi_{0,2}$ | 0.0052 | -0.0034 | 0.0036 | -0.0072 |
| s.d | 0.0037 | 0.0013 | 0.0015 | 0.0118 |
| $\phi_{1,2}$ | 0.2772 | -0.2915 | 0.2996 | -0.2974 |
| s.d | 0.0766 | 0.0989 | 0.0706 | 0.0961 |
| $\gamma$ | 2.3755 | 8.6086 | 1.3586 | 11.6105 |
| s.d | 1.4598 | 6.6247 | 9134 | 12.6146 |
| c | 0.0089 | 0.0043 | 0.0054 | 0.0043 |
| s.d | 0.0012 | 0.0003 | 0.0001 | 16.3175 |
| $c^*$ | -- | -- | 0.2358 | 0.0043 |
| s.d | -- | -- | 1542.0000 | 16.3177 |
| SSR | 0.0482 | 0.0566 | 0.0478 | 0.0565 |
| | [0.0492] | [0.0575] | | |
| Log-likelihood | 3560 | 3479 | 3563 | 3480 |
| | [3553] | [3475] | | |
| AIC | -7.0950 | -6.9334 | -7.1001 | -6.9332 |
| | [-7.0807] | [-6.9243] | | |
| BIC | -7.0705 | -6.9089 | -7.0707 | -6.9038 |
| | [-7.0709] | [-6.9145] | | |
| Q(10) | 6.6252 | 3.5602 | 6.7627 | 3.3523 |
| $Q^2(10)$ | 155.2800 | 86.9680 | 140.6800 | 86.3670 |

Note: This table contains the estimates of STVECM model with exponential and quadratic logistic transition function for Australia using nonlinear least squares method. The coefficient estimates and the Newey-West HAC standard errors are presented, followed by the sum of squared residuals (SSR) of each equation, log-likelihood value and Akaike information criterion and Schwarz information criterion. The Q-statistics and Q-squared-statistics of order 10 are in the last two rows.

From the results in the left two columns of Table 6, we can see that the STVECM with exponential transition function fits the Australian data well. The sum of squared residuals of both spot and futures are around 2% lower than their counterparts of linear VECM. This is enough to compensate for the increase in the number of parameters for the STVECM, which is preferred to linear VECM on the basis of AIC. But the former is marginally worse than the latter by BIC that penalizes parameter number more severely. $\phi_{1,2}$ of both spot and futures are significantly different from zero at 1% level with anticipated positive sign for the spot and negative sign for the futures and slightly larger size than the linear counterpart. This is reasonable as no-arbitrage inner regime implies a zero error-correction coefficient and the coefficient of the linear model is in effect the weighted average of the coefficient in outer and inner regime.

The threshold coefficient, c is positive and significantly different from zero for both spot and futures. It implies there would be no arbitrage for every investor when the lagged level of futures is greater than the spot by the sum of cost of carry and c. The coefficient measuring the speed of transition $\gamma$ is 2.37 and 8.60 for spot and futures respectively, suggesting the change in the adjustment of futures to disequilibrium is more abrupt as arbitrage opportunity becomes available to more people. This is probably because the transaction cost of futures is smaller than the spot. The diagnostic test results suggest the STVECM residuals are white noise but conditional heteroskedastic, therefore the need for GARCH model for the residuals. The results of STVECM with quadratic logistic transition function are worse than those with exponential function. One of the two estimates of threshold coefficients in the spot equation is insignificantly from zero, suggesting poor fit of the model. And the two in the futures equation are almost the same, implying possible multicollinearity.

## Table 7: STVECM Estimation results for Germany

$$\Delta y_t = \phi_{0,1} + \phi_{2,1}\Delta y_{t-6} + \phi_{3,1}\Delta y_{t-8} + \left(\phi_{0,2} + \phi_{1,2}z_{t-1} + \phi_{2,2}\Delta y_{t-6} + \phi_{3,2}\Delta y_{t-8}\right)G(z_{t-3};\gamma,c) + \varepsilon_t$$

$$\text{Exponential: } G(z_{t-3};\gamma,c) = 1 - \exp[-\gamma(z_{t-3} - c)^2/\text{var}(z_t)]$$

$$\text{Quadratic Logistic: } G(z_{t-3};\gamma,c,c^*) = \{1 + \exp[-\gamma(z_{t-3} - c)(z_{t-3} - c^*)/\text{var}(z_t)]\}^{-1}$$

| | Exponential | | Quadratic Logistic | |
|---|---|---|---|---|
| | Spot | Futures | Spot | Futures |
| $\phi_{0,1}$ | 0.001 | -0.001 | 0.0021 | 0.0001 |
| s.d. | 0.003 | 0.0014 | 0.0028 | 0.0004 |
| $\phi_{2,1}$ | 0.2372 | -0.2832 | 0.2011 | -0.0609 |
| s.d. | 0.2883 | 0.1543 | 0.3031 | 0.028 |
| $\phi_{3,1}$ | 0.5907 | 0.1858 | 0.6752 | 0.0669 |
| s.d. | 0.2428 | 0.1245 | 0.367 | 0.0332 |
| $\phi_{0,2}$ | -0.0002 | 0.0014 | -0.0013 | 0.0248 |
| s.d. | 0.003 | 0.0015 | 0.0028 | 0.0077 |
| $\phi_{1,2}$ | 0.8675 | -0.069 | 0.8679 | 1.1148 |
| s.d. | 0.093 | 0.1037 | 0.0931 | 0.4547 |
| $\phi_{2,2}$ | -0.2999 | 0.2601 | -0.2633 | 2.4901 |
| s.d. | 0.2919 | 0.1562 | 0.3068 | 0.2455 |
| $\phi_{3,2}$ | -0.5454 | -0.1378 | -0.6305 | 0.2924 |
| s.d. | 0.2462 | 0.1331 | 0.3728 | 0.2841 |
| $\gamma$ | 12.1908 | 32.9423 | 23.2529 | 0.2769 |
| s.d. | 10.5974 | 41.7971 | 28.4365 | 0.168 |
| c | 0.011 | 0.0032 | 0.0096 | -0.0205 |
| s.d. | 0.0006 | 0.0004 | 0.0006 | 0.0014 |
| $c^*$ | -- | -- | 0.0123 | 0.0299 |
| s.d. | -- | -- | 0.001 | 0.0081 |
| SSR | 0.5781 | 0.6276 | 0.578 | 0.6178 |
| | [0.5807] | [0.6311] | | |
| Log-likelihood | 6401 | 6305 | 6401 | 6323 |
| | [6392] | [6295] | | |
| AIC | -5.4608 | -5.3786 | -5.4602 | -5.3935 |
| | [-5.4601] | [-5.3769] | | |
| BIC | -5.4387 | -5.3564 | -5.4356 | -5.3689 |
| | [-5.4503] | [-5.3671] | | |
| Q(10) | 7.318 | 6.3459 | 7.3556 | 6.8201 |
| $Q^2(10)$ | 1371.5 | 619.7 | 1368.4 | 539.98 |

See the note of Table 6.

STVECM with quadratic logistic transition function provides a better fit and more insights to the German data than exponential transition function. The former corresponds to lower sum of squared residuals and higher log-likelihood for the futures than both the latter and linear model and is preferred by both AIC and BIC. The two threshold coefficients of the futures are -0.02 and 0.02 respectively, meaning the no-arbitrage inner regime is large and symmetrical around zero for the futures. As shown in Figure 7, most deviations fall into the inner regime. That is, the opportunity for arbitrage using futures is very rare.

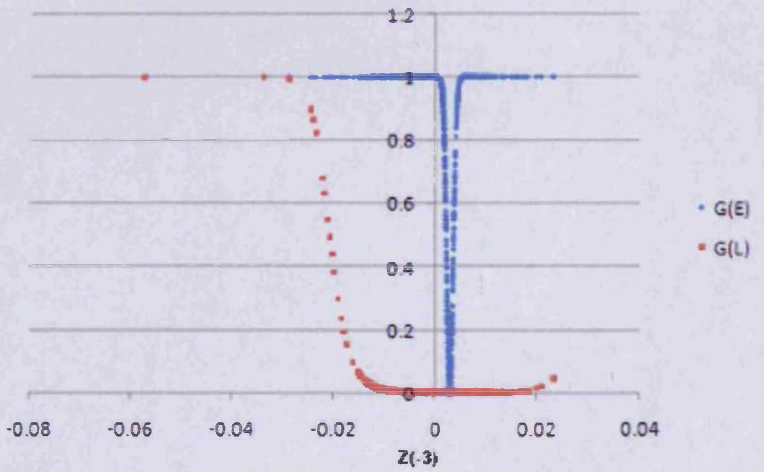


Figure 7

Figure 7 plots the estimated transition functions agains the transition variable for Germany. G(E) is the exponential transtition function. G(L) is the quadratic logistic transition function.

Combining the estimates of threshold coefficient of the futures with its adjustment coefficient ( $\phi_{1,2}$ is 1.11), we can see that futures rarely respond to disequilibrium, but respond very strongly when the deviation is very large. In contrast, the estimates of threshold coefficient suggest a small no-arbitrage range for the spot, meaning most deviations fall into the outer-regime. Therefore, when spot and futures are so far apart that there are arbitrage opportunities, spot is more likely to move to restore equilibrium. The estimate of $\gamma$ for spot is very big at 23.25. As the deviation increases, the adjustment to disequilibrium in the spot increases abruptly. The diagnostic test results indicate the same conclusion for German data as Australia. That is, we need to model the second moments to deal with the conditional heteroskedasticity.

## Table 8: STVECM Estimation results for Japan

$$\Delta y_t = \phi_{0,1} + \phi_{2,1}\Delta y_{t-1} + (\phi_{0,2} + \phi_{1,2} z_{t-1} + \phi_{2,2}\Delta y_{t-1}) G(z_{t-1}; \gamma, c) + \varepsilon_t$$

$$\text{Exponential: } G(z_{t-1}; \gamma, c) = 1 - \exp[-\gamma(z_{t-1} - c)^2 / var(z_t)]$$

$$\text{Quadratic Logistic: } G(z_{t-1}; \gamma, c, c^*) = \{1 + \exp[-\gamma(z_{t-1} - c)(z_{t-1} - c^*)/var(z_t)]\}^{-1}$$

| | Exponential | | Quadratic Logistic | |
|---|---|---|---|---|
| | Spot | Futures | Spot | Futures |
| $\phi_{0,1}$ | 0.0359 | 0.0185 | 0.0324 | 0.038 |
| | 0.0127 | 0.0074 | 0.717 | 0.0975 |
| $\phi_{2,1}$ | -0.8493 | 0.9748 | 0.0553 | 1.8479 |
| | 0.4393 | 0.2488 | 0.9648 | 4.1102 |
| $\phi_{0,2}$ | -0.0363 | -0.0186 | -0.2041 | -0.0381 |
| | 0.0127 | 0.0074 | 4.2429 | 0.0976 |
| $\phi_{1,2}$ | 0.4057 | -0.3569 | -8.9109 | -0.3475 |
| | 0.1128 | 0.1356 | 143.6709 | 0.1413 |
| $\phi_{2,2}$ | 0.7999 | -1.0502 | -0.622 | -1.9219 |
| | 0.4385 | 0.243 | 6.0947 | 4.1145 |
| $\gamma$ | 145.8734 | 0.1938 | 0.0067 | 0.2432 |
| | 109.0049 | 0.0548 | 0.1083 | 0.1983 |
| c | 0.0088 | 0.0224 | 0.1659 | 0.0225 |
| | 0.0001 | 0.0005 | 3.6342 | 69.332 |
| $c^*$ | -- | -- | -0.0198 | 0.0225 |
| | -- | -- | 0.1337 | 69.3325 |
| SSR | 0.5003 | 0.5361 | 0.4998 | 0.5361 |
| | [0.5029] | [0.5396] | | |
| Log-likelihood | 6593 | 6512 | 6594 | 6512 |
| | [6584] | [6501] | | |
| AIC | -5.61 | -5.541 | -5.6102 | -5.54 |
| | [-5.6077] | [-5.5374] | | |
| BIC | -5.5928 | -5.5238 | -5.5906 | -5.5203 |
| | [-5.6004] | [-5.5301] | | |
| Q(10) | 7.3065 | 4.6028 | 6.6187 | 4.6388 |
| $Q^2(10)$ | 208.57 | 207.45 | 194.23 | 208.3 |

See the note of Table 6.

The estimation results for Japan indicate that STVECM with both exponential and quadratic logistic transition function fits the data better than linear VECM by SSR, Log-likelihood and AIC, but they are marginally worse by BIC. Between the two specifications of STVECM, the exponential is better for having more significant coefficient estimates. The adjustment coefficients $\phi_{1,2}$ for both spot and futures are significant at 1% level and have the anticipated sign and similar size. The speed of transition coefficient $\gamma$ is very high for the spot and low for the futures. It implies the speed of transition is faster in the spot. Since the coefficient measuring the strength of adjustment is similar between spot and futures, the strength of adjustment is greater in the spot than the futures when the deviation from equilibrium is small in Japan. This is in contrast to Australia.

The estimation results of United Kingdom indicate that STVECM with exponential transition function is preferred to linear VECM and STVECM with quadratic logistic transition function. Its SSR is nearly 2% lower than the counterpart of the linear model. Despite having six extra explanatory variables, it is preferred by AIC for both spot and futures and also by BIC for the futures. STVECM with quadratic logistic transition function does not provide a good fit to the futures with four key coefficients insignificantly different from zero. The estimates of $\phi_{1,2}$ suggest the strength of adjustment in spot is close to that in futures. The estimates of c suggest no-arbitrage inner range centres near zero, meaning the actual difference between the level of spot and futures is very close to the measured cost of carry. The estimate of $\gamma$ is very big for the futures, indicating fast speed of transition in the futures, which is similar to Australia and opposite to Japan.

## Table 9: STVECM Estimation results for United Kingdom

$$\Delta y_t = \phi_{0,1} + \phi_{2,1}\Delta y_{t-2} + \phi_{3,1}\Delta y_{t-3} + \phi_{4,1}\Delta y_{t-5}$$
$$+ (\phi_{0,2} + \phi_{1,2}z_{t-1} + \phi_{2,2}\Delta y_{t-2} + \phi_{3,2}\Delta y_{t-3} + \phi_{4,2}\Delta y_{t-5})G(z_{t-2};\gamma,c) + \varepsilon_t$$

Exponential: $G(z_{t-2};\gamma,c) = 1 - \exp[-\gamma(z_{t-2} - c)^2/var(z_t)]$

Quadratic Logistic: $G(z_{t-2};\gamma,c,c^*) = \{1 + \exp[-\gamma(z_{t-2} - c)(z_{t-2} - c^*)/var(z_t)]\}^{-1}$

| | Exponential | | Quadratic Logistic | |
|---|---|---|---|---|
| | Spot | Futures | Spot | Futures |
| $\phi_{0,1}$ | -0.0021 | -0.0067 | -0.0015 | 0.0025 |
| s.d | 0.0016 | 0.0041 | 0.0026 | 0.0037 |
| $\phi_{2,1}$ | -0.0731 | 0.7872 | -0.1301 | 0.041 |
| s.d | 0.1412 | 0.2505 | 0.1238 | 0.2111 |
| $\phi_{3,1}$ | 0.1982 | 1.7495 | 0.1399 | 0.4657 |
| s.d | 0.1645 | 0.2145 | 0.2658 | 0.9854 |
| $\phi_{4,1}$ | 0.2093 | 0.3184 | 0.1882 | 0.0003 |
| s.d | 0.0707 | 0.2012 | 0.2148 | 0.2477 |
| $\phi_{0,2}$ | 0.0028 | 0.0067 | 0.0022 | -0.0071 |
| s.d | 0.0017 | 0.0041 | 0.0027 | 0.0116 |
| $\phi_{1,2}$ | 0.2285 | -0.2267 | 0.2346 | -0.7167 |
| s.d | 0.0961 | 0.0892 | 0.0946 | 0.9786 |
| $\phi_{2,2}$ | 0.0185 | -0.8583 | 0.0823 | -0.2695 |
| s.d | 0.1512 | 0.2539 | 0.1355 | 0.4182 |
| $\phi_{3,2}$ | -0.3212 | -1.8538 | -0.2637 | -1.4187 |
| s.d | 0.1717 | 0.2163 | 0.2756 | 1.8596 |
| $\phi_{4,2}$ | -0.3067 | -0.3875 | -0.2878 | -0.1495 |
| s.d | 0.0766 | 0.2049 | 0.2224 | 0.5619 |
| $\gamma$ | 3.3009 | 801.8717 | 5.9495 | 0.0791 |
| s.d | 2.7159 | 339.458 | 8.6511 | 0.1354 |
| c | 0.0044 | 0.0039 | 0.0061 | -0.0064 |
| s.d | 0.0006 | 0 | 0.0022 | 0.0159 |
| $c^*$ | -- | -- | 0.0024 | 0.0122 |
| s.d | -- | -- | 0.001 | 0.0173 |
| SSR | 0.3059 | 0.3346 | 0.3062 | 0.3385 |
| | [0.3106] | [0.3418] | | |
| Log- | 7157 | 7051 | 7155 | 7038 |
| | [7135] | [7023] | | |
| AIC | -6.0968 | -6.0071 | -6.095 | -5.9946 |
| | [-6.0862] | [-5.9906] | | |
| BIC | -6.0698 | -5.98 | -6.0656 | -5.9652 |
| | [-6.0739] | [-5.9783] | | |
| Q(10) | 15.02 | 10.494 | 14.22 | 11.339 |
| $Q^2(10)$ | 1150.5 | 1116.1 | 1150.2 | 976.99 |

See the note of Table 6.

## Table 10: STVECM Estimation results for Unite States

$$\Delta y_t = \phi_{0,1} + (\phi_{0,2} + \phi_{1,2} z_{t-1}) G(z_{t-3}; \gamma, c) + \varepsilon_t$$

**Exponential:** $G(z_{t-3}; \gamma, c) = 1 - \exp[-\gamma(z_{t-3} - c)^2/\text{var}(z_t)]$

**Quadratic Logistic:** $G(z_{t-3}; \gamma, c, c^*) = \{1 + \exp[-\gamma(z_{t-3} - c)(z_{t-3} - c^*)/\text{var}(z_t)]\}^{-1}$

| | Exponential | | Quadratic Logistic | |
|---|---|---|---|---|
| | Spot | Futures | Spot | Futures |
| $\phi_{0,1}$ | 0.0041 | 0.0042 | 0.0073 | 0.0038 |
| | 0.0033 | 0.0016 | 0.0152 | 0.0024 |
| $\phi_{0,2}$ | -0.0052 | -0.0039 | -0.0082 | -0.0034 |
| | 0.0036 | 0.0016 | 0.0153 | 0.0026 |
| $\phi_{1,2}$ | 0.5069 | -0.4848 | 0.4682 | -0.4937 |
| | 0.2255 | 0.1492 | 0.18 | 0.1474 |
| $\gamma$ | 0.1135 | 0.7402 | 0.2046 | 2.1695 |
| | 0.1912 | 0.7619 | 0.3113 | 2.8301 |
| $c$ | -0.0081 | -0.0046 | -0.0069 | -0.0021 |
| | 0.0063 | 0.0013 | 440.4154 | 0.001 |
| $c^*$ | -- | -- | -0.0069 | -0.0073 |
| | -- | -- | 440.4086 | 0.0027 |
| SSR | 0.3169 | 0.3484 | 0.3168 | 0.3483 |
| | [0.3182] | [0.3498] | | |
| Log-likelihood | 7126 | 7014 | 7126 | 7015 |
| | [7124] | [7013] | | |
| AIC | -6.068 | -5.9731 | -6.0674 | -5.9725 |
| | [-6.0667] | [-5.9720] | | |
| BIC | -6.0557 | -5.9608 | -6.0527 | -5.9578 |
| | [-6.0618] | [-5.9671] | | |
| Q(10) | 12.123 | 8.6211 | 12.512 | 8.5497 |
| $Q^2(10)$ | 478.02 | 441.38 | 476.87 | 439.53 |

See the note of Table 6.

The results of United States show that STVECM fits the data slightly better than linear VECM with marginally improved SSR and Log-likelihood. On the basis of AIC, it is slightly better than linear VECM. On the basis of BIC, it is slightly worse. Since the two threshold coefficient estimates take the same value in STVECM with quadratic logistic transition function, it is clearly inferior. The estimate of $\phi_{1,2}$ is around 0.5 for both spot and futures, indicating their strength of adjustment are both pretty strong. The estimate of $\gamma$ is small for both spot and futures, which implies the strength of adjustment does not change abruptly with the size of deviation. Same as the other four countries, the diagnostic test results of United States also suggest heteroskedastic residuals.

### 6.4 Bivariate GARCH(1,1) results

Table 11 and 12 contain the results of linear VECM combined with bivariate Garch (1,1) with normally and Student-t distributed residuals respectively.

Comparing the diagnostic test results in Table 11 and12 with Table 4, we can see that there is much less remaining GARCH effect in the standardized residuals of the GARCH models. Furthermore, all the GARCH coefficients are highly significantly different from zero. In particular, the estimates of $\alpha_{ii}^2 + \beta_{ii}^2$ are all very close to unity, implying strong persistence in conditional second moments. It is evident that GARCH models are highly effective in accounting for the conditional heteroskedasticity of the data.

However, the stationarity statistics reveal that GARCH process with normally distributed residuals is probably nonstationary. As shown in the bottom two rows of Table 5, the covariance stationarity condition $\alpha_{ii}^2 + \beta_{ii}^2 < 1$ and the strict stationarity condition $E[\log(\alpha_{ii} \cdot v_t^2 + \beta_{ii}^2)] < 0$ are barely satisfied in the case of Germany, South Korea, UK and USA. In contrast, the results in Table 12 suggest that GARCH with Student-t distributed residuals is stationary. The conditions of both covariance stationarity and strict stationarity are comfortably satisfied. The log likelihood and the information criteria in Table 11 and 12 suggest that GARCH with Student-t distributed residuals is slightly better than GARCH with normally distributed residuals. Furthermore the degree of freedom parameter of the former is highly significant for all six countries. All these results imply that the GARCH with Student-t distributed residuals is better than the GARCH with normally distributed residuals.

Closely examining the diagnostic test statistics of GARCH with Student-t residuals, we can see that there is evidence of remaining GARCH effect in all countries, particularly in UK and USA. It suggests that both variants of GARCH (1, 1) model are probably not adequate to model our data, which indicates the need of searching for better models. In the next section we report the results of the bivariate TGARCH models.

**Table 11: Estimation Results for VECM combined with Bivariate Garch (1,1) with normally distributed standardized residuals**

Bivariate Garch(1,1) specification:

$$\begin{pmatrix}\varepsilon_{st}\\ \varepsilon_{ft}\end{pmatrix}\Omega_{t-1} \sim N(0,H_t)$$

$$h_{11t} = \omega_s^2 + \beta_s^2 \cdot h_{11t-1} + \alpha_s^2 \cdot \varepsilon_{st-1}^2$$
$$h_{12t} = \omega_s\omega_3 + \beta_s\beta_f \cdot h_{12t-1} + \alpha_s\alpha_f \cdot \varepsilon_{st-1}\varepsilon_{ft-1}$$
$$h_{22t} = \omega_f^2 + \omega_3^2 + \beta_f^2 \cdot h_{22t-1} + \alpha_f^2 \cdot \varepsilon_{ft-1}^2$$

| | *AU* | | *GM* | | *JP* | | *KR* | | *UK* | | *US* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *s* | *f* | *s* | *f* | *s* | *f* | *s* | *f* | *s* | *f* | *s* | *f* |
| $\omega$ | 0.002 | 0.001 | 0.002 | 0.002 | -0.003 | -0.003 | 0.003 | 0.003 | 0.001 | 0.001 | 0.001 | 0.001 |
| $z-stats$ | 7.83 | 7.37 | 13.93 | 12.50 | -16.11 | -15.89 | 13.00 | 13.14 | 11.67 | 11.19 | 7.97 | 7.93 |
| $\omega_3$ | 0.000 | | 0.001 | | 0.001 | | 0.001 | | 0.000 | | 0.000 | |
| $z-stats$ | 3.80 | | 20.39 | | 12.15 | | 6.21 | | 17.53 | | -14.01 | |
| $\beta$ | 0.944 | 0.960 | 0.947 | 0.945 | 0.952 | 0.946 | 0.956 | 0.964 | 0.974 | 0.973 | 0.980 | 0.981 |
| $z-stats$ | 97 | 130 | 288 | 305 | 259 | 226 | 377 | 556 | 715 | 616 | 895 | 982 |
| $\alpha$ | 0.229 | 0.197 | 0.307 | 0.312 | 0.246 | 0.254 | 0.277 | 0.252 | 0.209 | 0.210 | 0.189 | 0.185 |
| $z-stats$ | 14.75 | 11.59 | 31.76 | 36.52 | 28.77 | 28.55 | 35.27 | 38.78 | 32.83 | 31.32 | 37.54 | 38.08 |
| $\log l$ | 8042 | | 16106 | | 16697 | | 11538 | | 18191 | | 18283 | |
| *AIC* | -16.04 | | -13.77 | | -14.23 | | -11.31 | | -15.53 | | -15.57 | |
| *SIC* | -16.00 | | -13.75 | | -14.21 | | -11.29 | | -15.51 | | -15.56 | |
| $Q(5)$ | 2.03 | 2.40 | 3.80 | 0.97 | 0.86 | 1.07 | 0.94 | 6.10 | 5.73 | 6.01 | 7.02 | 9.35 |
| $p-value$ | 0.85 | 0.79 | 0.58 | 0.97 | 0.97 | 0.96 | 0.97 | 0.30 | 0.33 | 0.31 | 0.22 | 0.10 |
| $Q(15)$ | 3.91 | 6.02 | 16.66 | 8.79 | 6.46 | 4.61 | 7.64 | 18.41 | 16.15 | 15.26 | 18.75 | 18.81 |
| $p-value$ | 1.00 | 0.98 | 0.34 | 0.89 | 0.97 | 1.00 | 0.94 | 0.24 | 0.37 | 0.43 | 0.23 | 0.22 |
| $Q^2(5)$ | 29.59 | 8.06 | 8.33 | 5.62 | 7.98 | 3.72 | 0.23 | 6.92 | 35.69 | 24.48 | 19.44 | 30.00 |
| $p-value$ | 0.00 | 0.15 | 0.14 | 0.35 | 0.16 | 0.59 | 1.00 | 0.23 | 0.00 | 0.00 | 0.00 | 0.00 |
| $Q^2(15)$ | 33.85 | 18.54 | 22.18 | 13.51 | 21.97 | 14.20 | 3.81 | 9.20 | 52.37 | 36.97 | 23.83 | 37.56 |
| $p-value$ | 0.00 | 0.24 | 0.10 | 0.56 | 0.11 | 0.51 | 1.00 | 0.87 | 0.00 | 0.00 | 0.07 | 0.00 |
| Jaque-Bera | 277 | 45 | 121 | 218 | 223 | 199 | 506 | 256 | 106 | 130 | 523 | 549 |
| $\alpha_{ii}^2+\beta_{ii}^2$ | 0.943 | 0.961 | 0.990 | 0.990 | 0.966 | 0.960 | 0.991 | 0.993 | 0.993 | 0.992 | 0.996 | 0.996 |
| $E[\log(\alpha_{ii}^2 \cdot v_t^2+\beta_{ii}^2)]$ | -0.062 | -0.041 | -0.020 | -0.021 | -0.040 | -0.048 | -0.020 | -0.014 | -0.009 | -0.010 | -0.005 | -0.005 |

Note: 1. This table contains the estimation results of bivariate GARCH (1,1) model. The dependent variables are the residuals of the spot and futures equation of estimated VECM shown in Table 4.

2. The coefficient estimates and their z-statistics are followed by the loglikehood value, Akaike information criteron and Schwartz Bayesian Information criteron.

3. The diagnostic test statistics include Q-statistics and Q-squared-statistics of order 5 and 15 and Jaque-Bera statistics. The significant ones are highlighted.

4. The last two rows contain the statistics for covariance and strict stationarity condition. The statistics suggesting possible failure of stationarity condition are highlighted.

**Table 12: Estimation Results for VECM combined with Bivariate Garch (1,1) with student-t distributed standardized residuals**

Bivariate Garch(1,1) specification:

$$\begin{pmatrix}\varepsilon_{st}\\ \varepsilon_{ft}\end{pmatrix}\Omega_{t-1} \sim t(0,H_t,v)$$

$$h_{11t} = \omega_s^2 + \beta_s^2 \cdot h_{11t-1} + \alpha_s^2 \cdot \varepsilon_{st-1}^2$$
$$h_{12t} = \omega_s\omega_3 + \beta_s\beta_f \cdot h_{12t-1} + \alpha_s\alpha_f \cdot \varepsilon_{st-1}\varepsilon_{ft-1}$$
$$h_{22t} = \omega_f^2 + \omega_3^2 + \beta_f^2 \cdot h_{22t-1} + \alpha_f^2 \cdot \varepsilon_{ft-1}^2$$

| | AU | | GM | | JP | | KR | | UK | | US | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | s | f | s | f | s | f | s | f | s | f | s | f |
| $\omega$ | 0.001 | 0.001 | 0.001 | 0.001 | 0.002 | 0.002 | 0.002 | 0.002 | 0.001 | 0.001 | 0.001 | 0.001 |
| $z-stats$ | 6.34 | 5.93 | 7.31 | 7.05 | 9.38 | 9.02 | 7.07 | 7.23 | 9.24 | 8.92 | 6.78 | 7.09 |
| $\omega_3$ | 0.000 | | 0.000 | | 0.001 | | 0.000 | | 0.000 | | 0.000 | |
| $z-stats$ | 2.49 | | 5.81 | | 9.20 | | -4.62 | | 8.36 | | -5.19 | |
| $\beta$ | 0.951 | 0.967 | 0.964 | 0.965 | 0.959 | 0.957 | 0.971 | 0.973 | 0.963 | 0.963 | 0.976 | 0.977 |
| $z-stats$ | 89.4 | 127.6 | 297.3 | 309.4 | 169.8 | 154.4 | 288.4 | 347.3 | 260.8 | 254.5 | 382.4 | 399.6 |
| $\alpha$ | 0.192 | 0.163 | 0.222 | 0.214 | 0.190 | 0.193 | 0.175 | 0.171 | 0.203 | 0.203 | 0.165 | 0.160 |
| $z-stats$ | 9.39 | 8.40 | 20.33 | 19.92 | 14.70 | 14.51 | 16.14 | 17.12 | 18.65 | 18.51 | 17.52 | 17.68 |
| $d.f.$ | 9.305 | | 6.602 | | 7.658 | | 4.751 | | 6.244 | | 5.447 | |
| $z-stats$ | 7.33 | | 13.30 | | 10.99 | | 11.83 | | 13.69 | | 14.08 | |
| $\log l$ | 8082 | | 16320 | | 16807 | | 11715 | | 18394 | | 18523 | |
| $AIC$ | -16.116 | | -13.948 | | -14.321 | | -11.483 | | -15.701 | | -15.778 | |
| $SIC$ | -16.077 | | -13.928 | | -14.301 | | -11.461 | | -15.681 | | -15.758 | |
| $Q(5)$ | 2.20 | 2.56 | 3.62 | 0.65 | 0.73 | 0.92 | 1.22 | 4.52 | 5.50 | 5.80 | 6.96 | 9.08 |
| $p-value$ | 0.82 | 0.77 | 0.61 | 0.99 | 0.98 | 0.97 | 0.94 | 0.48 | 0.36 | 0.33 | 0.22 | 0.11 |
| $Q(15)$ | 4.12 | 6.23 | 16.62 | 8.89 | 6.55 | 4.54 | 7.80 | 17.13 | 15.69 | 14.87 | 19.07 | 18.95 |
| $p-value$ | 1.00 | 0.98 | 0.34 | 0.88 | 0.97 | 1.00 | 0.93 | 0.31 | 0.40 | 0.46 | 0.21 | 0.22 |
| $Q^2(5)$ | 32.39 | 9.64 | 14.34 | 5.87 | 8.27 | 2.97 | 1.99 | 14.09 | 21.00 | 13.07 | 16.74 | 26.47 |
| $p-value$ | 0.00 | 0.09 | 0.01 | 0.32 | 0.14 | 0.71 | 0.85 | 0.02 | 0.00 | 0.02 | 0.01 | 0.00 |
| $Q^2(15)$ | 36.51 | 21.07 | 29.58 | 12.86 | 25.01 | 15.51 | 3.90 | 16.02 | 33.41 | 21.84 | 20.42 | 33.15 |
| $p-value$ | 0.00 | 0.14 | 0.01 | 0.61 | 0.05 | 0.42 | 1.00 | 0.38 | 0.00 | 0.11 | 0.16 | 0.00 |
| Jaque-Bera | 285 | 49 | 174 | 350 | 235 | 207 | 521 | 278 | 99 | 119 | 513 | 547 |
| $\alpha_{ii}^2+\beta_{ii}^2$ | 0.942 | 0.962 | 0.978 | 0.978 | 0.956 | 0.952 | 0.972 | 0.975 | 0.969 | 0.969 | 0.981 | 0.980 |
| $E[\log(\alpha_{ii}^2 \cdot v_t^2+\beta_{ii}^2)]$ | -0.052 | -0.032 | -0.010 | -0.010 | -0.036 | -0.039 | -0.014 | -0.011 | -0.019 | -0.019 | -0.009 | -0.009 |

See the note of Table 11.

## 6.5 Bivariate TGARCH (1,1) results

As TGARCH is asymmetric in residuals, the information matrix of VECM combined with TGARCH is not block diagonal. The two-step method is not applicable in this case. Therefore we use the full-information maximum likelihood method in estimation. The results for TGARCH with the assumption of normally and Student-t distributed standardized residuals are shown in Table 13 and Table 14 respectively.

First, the second row from the bottom of Table 13 and Table 14 contains the number of iterations to achieve convergence. In the normal case, five of six countries do not have the convergence problem. Despite several starting values having been tried, convergence can not be achieved for USA. In the Student-t case, convergence is achieved for all countries.

Second, the last row shows the stationarity statistics. Similar to the results for standard GARCH, the results indicate that TGARCH with normal residuals is probably nonstationary, but TGARCH with Student-t residuals is stationary.

Third, most coefficient estimates are significantly different from zero in both cases. In particular, the TGARCH coefficients measuring the asymmetric effect are highly significant, indicating negative shocks have a bigger effect on the conditional volatility than positive shocks in all six countries. Therefore standard GARCH is misspecified as it fails to capture this effect. This may explain the failure of the standard GARCH. However, to achieve convergence, we have to impose the constraint that both the intercept and alpha are zero for Australia and alpha is zero for USA.

Fourth, the loglikelihood of each country with Student-t distribution is higher than the counterpart with normal distribution, and the information criteria, AIC and BIC also suggest models with Student-t distribution fit the data better.

Last, the diagnostic test results suggest there is no remaining GARCH effect when normal is assumed. Compared with the results in Table 12, the remaining GARCH effect is trivial when Student-t is assumed.

On the whole, the results of TGARCH with Student-t distributed residuals are satisfactory. And it is by far the best model among the five models we have examined for the data. In the out-of-sample period, we will estimate it 261 times to simulate the dynamic optimal hedge ratio for all six countries.

**Table 13: Estimation Results for VECM combined with BEKK-Tgarch (1,1) and normally distributed standardized residuals**

Conditional Mean

$$\Delta s_t = \mu_s + \delta_s \cdot z_{t-1} + \sum_{i=1}^{m} \varphi_{s1i} \cdot \Delta s_{t-i} + \sum_{i=1}^{n} \varphi_{s2i} \cdot \Delta f_{t-i} + \varepsilon_{st} \quad \text{where} \quad z_t = f_t - s_t - coc_t$$

$$\Delta f_t = \mu_f + \delta_f \cdot z_{t-1} + \sum_{i=1}^{m} \varphi_{f1i} \cdot \Delta s_{t-i} + \sum_{i=1}^{n} \varphi_{f2i} \cdot \Delta f_{t-i} + \varepsilon_{ft}$$

Conditional Variance

$$\begin{pmatrix} \varepsilon_{st} \\ \varepsilon_{ft} \end{pmatrix} \Big| \Omega_{t-1} \sim N(0, H_t) \qquad \begin{aligned} h_{1t} &= a_s^2 + \beta_s^2 \cdot h_{1t-1} + \alpha_s^2 \cdot \varepsilon_{st-1}^2 + d_s^2 \cdot u_{st-1}^2 \\ h_{12t} &= \omega_s \omega_3 + \beta_s \beta_f \cdot h_{12t-1} + \alpha_s \alpha_f \cdot \varepsilon_{st-1} \varepsilon_{ft-1} + d_s d_f \cdot u_{st-1} u_{ft-1} \\ h_{2t} &= \omega_f^2 + \omega_3^2 + \beta_f^2 \cdot h_{2t-1} + \alpha_f^2 \cdot \varepsilon_{ft-1}^2 + d_f^2 \cdot u_{ft-1}^2 \end{aligned} \qquad \begin{aligned} u_{st} &= \min(\varepsilon_{st}, 0) \\ u_{ft} &= \min(\varepsilon_{ft}, 0) \end{aligned}$$

| | *AU* | | *GM* | | *JP* | | *KR* | | *UK* | | *US* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ |
| $\delta$ | 0.142 | -0.227 | 0.901 | 0.059 | 0.426 | -0.296 | 0.026 | -0.137 | 0.080 | -0.173 | 0.532 | -0.229 |
| $z-stats$ | 2.948 | -4.150 | 22.080 | 1.222 | 5.878 | -3.897 | 1.262 | -5.982 | 1.518 | -3.023 | 7.364 | -2.988 |
| $\mu$ | na | na | 0.001 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 |
| $z-stats$ | na | na | 5.164 | 3.202 | -0.585 | 1.585 | -0.041 | -0.803 | 1.735 | 0.374 | 1.920 | 4.063 |
| $\varphi_{12}$ | | | | | | | | | -0.024 | -0.030 | | |
| $z-stats$ | | | | | | | | | -1.246 | -1.496 | | |
| $\varphi_{13}$ | | | | | | | | | -0.051 | -0.047 | | |
| $z-stats$ | | | | | | | | | -2.688 | -2.328 | | |
| $\varphi_{15}$ | | | | | | | | | -0.070 | -0.073 | | |
| $z-stats$ | | | | | | | | | -3.769 | -3.720 | | |
| $\varphi_{16}$ | | | -0.052 | -0.044 | | | | | | | | |
| $z-stats$ | | | -2.772 | -2.239 | | | | | | | | |
| $\varphi_{18}$ | | | 0.022 | 0.027 | | | | | | | | |
| $z-stats$ | | | 1.201 | 1.414 | | | | | | | | |
| $\varphi_{21}$ | | | | | -0.026 | -0.046 | 0.093 | 0.032 | | | | |
| $z-stats$ | | | | | -1.181 | -2.050 | 4.884 | 1.478 | | | | |
| $\varphi_{25}$ | | | | | | | -0.047 | -0.044 | | | | |
| $z-stats$ | | | | | | | -2.692 | -2.237 | | | | |
| $\omega$ | 0.001 | 0.001 | 0.002 | 0.002 | 0.003 | 0.003 | 0.003 | 0.003 | 0.001 | 0.001 | 0.001 | 0.001 |
| $z-stats$ | 8.593 | 7.229 | 14.291 | 13.654 | 17.766 | 16.589 | 12.700 | 12.873 | 12.294 | 11.249 | 12.035 | 11.300 |
| $\omega_3$ | 0.001 | | 0.001 | | 0.001 | | 0.000 | | 0.000 | | 0.000 | |
| $z-stats$ | 6.480 | | 9.245 | | 8.958 | | 4.362 | | 13.085 | | 17.901 | |
| $\beta$ | 0.953 | 0.962 | 0.945 | 0.944 | 0.941 | 0.936 | 0.961 | 0.963 | 0.973 | 0.973 | 0.968 | 0.970 |
| $z-stats$ | 119 | 128 | 280 | 282 | 203 | 176 | 427 | 499 | 623 | 546 | 501 | 566 |
| $\alpha$ | na | na | 0.283 | 0.271 | 0.249 | 0.242 | 0.184 | 0.208 | 0.144 | 0.158 | na | na |
| $z-stats$ | na | na | 22.81 | 19.07 | 25.40 | 24.28 | 14.38 | 20.00 | 11.71 | 12.96 | na | na |
| d | 0.297 | 0.242 | 0.167 | 0.219 | 0.073 | 0.131 | 0.266 | 0.214 | 0.209 | 0.195 | 0.327 | 0.321 |
| $z-stats$ | 12.43 | 11.21 | 6.51 | 9.23 | 2.71 | 5.04 | 18.66 | 13.51 | 14.51 | 12.93 | 34.90 | 36.54 |
| $\log l$ | 8049 | | 16158 | | 16733 | | 11611 | | 18233 | | 18348 | |
| *AIC* | -16.048 | | -13.802 | | -14.252 | | -11.372 | | -15.554 | | -15.626 | |
| *BIC* | -16.004 | | -13.760 | | -14.216 | | -11.325 | | -15.507 | | -15.599 | |
| $Q(5)$ | 1.031 | 1.877 | 3.931 | 1.502 | 0.553 | 0.602 | 2.238 | 2.718 | 2.920 | 2.928 | 6.060 | 8.444 |
| $p-value$ | 0.960 | 0.866 | 0.559 | 0.913 | 0.990 | 0.988 | 0.815 | 0.743 | 0.712 | 0.711 | 0.300 | 0.133 |
| $Q(15)$ | 3.373 | 5.689 | 16.495 | 9.282 | 6.225 | 4.219 | 9.054 | 18.844 | 13.065 | 11.132 | 18.719 | 20.022 |
| $p-value$ | 0.999 | 0.985 | 0.350 | 0.862 | 0.976 | 0.997 | 0.875 | 0.221 | 0.597 | 0.743 | 0.227 | 0.171 |
| $Q^2(5)$ | 9.311 | 3.632 | 8.657 | 9.192 | 7.709 | 4.058 | 0.619 | 4.819 | 13.285 | 9.800 | 5.393 | 8.966 |
| $p-value$ | 0.097 | 0.603 | 0.124 | 0.102 | 0.173 | 0.541 | 0.987 | 0.438 | 0.021 | 0.081 | 0.370 | 0.110 |
| $Q^2(15)$ | 14.863 | 15.052 | 25.704 | 20.496 | 22.479 | 14.687 | 5.236 | 9.089 | 24.211 | 18.559 | 9.996 | 15.214 |
| $p-value$ | 0.461 | 0.448 | 0.041 | 0.154 | 0.096 | 0.474 | 0.990 | 0.873 | 0.062 | 0.234 | 0.820 | 0.436 |
| Jaque-Bera | 121 | 26 | 111 | 148 | 210 | 181 | 435 | 184 | 84 | 102 | 279 | 350 |
| Number of Iteration | Convergence achieved after 15 iterations | | Convergence achieved after 85 iterations | | Convergence achieved after 33 iterations | | Convergence achieved after 22 iterations | | Convergence achieved after 40 iterations | | Failure to improve Likelihood after 20 iterations | |
| Stationarity | 0.952 | 0.956 | 0.988 | 0.996 | 0.950 | 0.948 | 0.993 | 0.984 | 0.990 | 0.986 | 0.991 | 0.992 |

See the note of Table 11.

**Table 14: Estimation Results for VECM combined with BEKK-Tgarch (1,1) and Student-t distributed standardized residuals**

Conditional Mean

$$\Delta s_t = \mu_s + \delta_s \cdot z_{t-1} + \sum_{i=1}^{m} \varphi_{s1i} \cdot \Delta s_{t-i} + \sum_{i=1}^{n} \varphi_{s2i} \cdot \Delta f_{t-i} + \varepsilon_{st} \qquad \text{where } z_t = f_t - s_t - coc_t$$

$$\Delta f_t = \mu_f + \delta_f \cdot z_{t-1} + \sum_{i=1}^{m} \varphi_{f1i} \cdot \Delta s_{t-i} + \sum_{i=1}^{n} \varphi_{f2i} \cdot \Delta f_{t-i} + \varepsilon_{ft}$$

Conditional Variance

$$\begin{pmatrix} \varepsilon_{st} \\ \varepsilon_{ft} \end{pmatrix} \Omega_{t-1} \sim t(0, H_t, v)$$

$$h_{11t} = \omega_s^2 + \beta_s^2 \cdot h_{11t-1} + \alpha_s^2 \cdot \varepsilon_{st-1}^2 + d_s^2 \cdot u_{st-1}^2$$

$$h_{12t} = \omega_s \omega_3 + \beta_s \beta_f \cdot h_{12t-1} + \alpha_s \alpha_f \cdot \varepsilon_{st-1} \varepsilon_{ft-1} + d_s d_f \cdot u_{st-1} u_{ft-1} \qquad u_{st} = \min(\varepsilon_{st}, 0)$$

$$h_{22t} = \omega_f^2 + \omega_3^2 + \beta_f^2 \cdot h_{22t-1} + \alpha_f^2 \cdot \varepsilon_{ft-1}^2 + d_f^2 \cdot u_{ft-1}^2 \qquad u_{ft} = \min(\varepsilon_{ft}, 0)$$

| | AU | | GM | | JP | | KR | | UK | | US | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ | $\Delta s$ | $\Delta f$ |
| $\delta$ | 0.110 | -0.247 | 0.901 | 0.043 | 0.454 | -0.261 | 0.061 | -0.059 | 0.102 | -0.147 | 0.514 | -0.232 |
| $z-stats$ | 2.20 | -4.43 | 22.30 | 0.95 | 6.19 | -3.42 | 2.81 | -2.35 | 1.89 | -2.53 | 6.27 | -2.71 |
| $\mu$ | na | na | 0.001 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.001 |
| $z-stats$ | na | na | 6.550 | 4.382 | -0.881 | 1.373 | 0.864 | -0.030 | 2.522 | 0.756 | 2.791 | 4.697 |
| $\varphi_{12}$ | | | | | | | | | -0.047 | -0.052 | | |
| $z-stats$ | | | | | | | | | -2.464 | -2.623 | | |
| $\varphi_{13}$ | | | | | | | | | -0.058 | -0.055 | | |
| $z-stats$ | | | | | | | | | -3.109 | -2.828 | | |
| $\varphi_{15}$ | | | | | | | | | -0.069 | -0.069 | | |
| $z-stats$ | | | | | | | | | -3.728 | -3.545 | | |
| $\varphi_{16}$ | | | -0.055 | -0.044 | | | | | | | | |
| $z-stats$ | | | -3.102 | -2.400 | | | | | | | | |
| $\varphi_{18}$ | | | 0.019 | 0.023 | | | | | | | | |
| $z-stats$ | | | 1.066 | 1.269 | | | | | | | | |
| $\varphi_{21}$ | | | | | -0.045 | -0.068 | 0.079 | 0.016 | | | | |
| $z-stats$ | | | | | -2.177 | -3.189 | 4.090 | 0.749 | | | | |
| $\varphi_{25}$ | | | | | | | -0.028 | -0.027 | | | | |
| $z-stats$ | | | | | | | -1.678 | -1.426 | | | | |
| $\omega$ | 0.001 | 0.001 | 0.001 | 0.001 | 0.003 | 0.003 | 0.002 | 0.002 | 0.001 | 0.001 | 0.001 | 0.001 |
| $z-stats$ | 6.700 | 5.670 | 7.238 | 6.996 | 10.790 | 10.567 | 7.815 | 8.051 | 9.558 | 9.316 | 7.756 | 7.182 |
| $\omega_3$ | 0.001 | | 0.000 | | -0.001 | | 0.000 | | 0.000 | | 0.000 | |
| $z-stats$ | 5.645 | | 4.279 | | -8.206 | | 0.584 | | 7.870 | | 9.153 | |
| $\beta$ | 0.958 | 0.964 | 0.962 | 0.963 | 0.948 | 0.944 | 0.969 | 0.969 | 0.962 | 0.963 | 0.975 | 0.976 |
| $z-stats$ | 109 | 105 | 286 | 301 | 139 | 126 | 283 | 311 | 258 | 268 | 397 | 419 |
| $\alpha$ | na | na | 0.209 | 0.199 | 0.174 | 0.162 | 0.130 | 0.150 | 0.152 | 0.141 | na | na |
| $z-stats$ | na | na | 14.972 | 13.971 | 10.236 | 8.837 | 8.653 | 11.548 | 9.743 | 8.980 | na | na |
| d | 0.248 | 0.215 | 0.121 | 0.135 | 0.149 | 0.179 | 0.171 | 0.135 | 0.192 | 0.203 | 0.239 | 0.234 |
| $z-stats$ | 9.526 | 8.431 | 4.230 | 5.061 | 5.351 | 6.783 | 8.511 | 6.144 | 9.814 | 10.955 | 18.619 | 19.077 |
| v * | 9.869 | | 6.641 | | 8.144 | | 4.584 | | 6.267 | | 5.802 | |
| $z-stats$ | 6.716 | | 12.781 | | 10.381 | | 11.538 | | 13.553 | | 13.489 | |
| $\log l$ | 8082 | | 16333 | | 16826 | | 11776 | | 18434 | | 18556 | |
| $AIC$ | -16.112 | | -13.951 | | -14.330 | | -11.533 | | -15.725 | | -15.802 | |
| $SIC$ | -16.063 | | -13.906 | | -14.291 | | -11.484 | | -15.676 | | -15.773 | |
| $Q(5)$ | 1.13 | 2.03 | 3.69 | 1.02 | 0.60 | 1.30 | 1.03 | 1.40 | 3.11 | 1.99 | 6.70 | 8.92 |
| $p-value$ | 0.95 | 0.85 | 0.60 | 0.96 | 0.99 | 0.94 | 0.96 | 0.92 | 0.68 | 0.85 | 0.24 | 0.11 |
| $Q(15)$ | 3.46 | 5.90 | 16.08 | 8.82 | 6.49 | 5.04 | 7.59 | 13.26 | 13.06 | 9.76 | 18.57 | 19.74 |
| $p-value$ | 1.00 | 0.98 | 0.38 | 0.89 | 0.97 | 0.99 | 0.94 | 0.58 | 0.60 | 0.84 | 0.23 | 0.18 |
| $Q^2(5)$ | 11.53 | 3.74 | 10.20 | 4.96 | 7.83 | 3.47 | 1.55 | 9.13 | 6.77 | 4.05 | 7.48 | 12.30 |
| $p-value$ | 0.04 | 0.59 | 0.07 | 0.42 | 0.17 | 0.63 | 0.91 | 0.10 | 0.24 | 0.54 | 0.19 | 0.03 |
| $Q^2(15)$ | 17.11 | 15.52 | 27.32 | 13.55 | 24.13 | 15.10 | 4.12 | 11.41 | 16.37 | 11.49 | 10.70 | 17.24 |
| $p-value$ | 0.31 | 0.42 | 0.03 | 0.56 | 0.06 | 0.44 | 1.00 | 0.72 | 0.36 | 0.72 | 0.77 | 0.31 |
| Jaque-bera | 128 | 26 | 165 | 233 | 214 | 188 | 459 | 225 | 87 | 110 | 326 | 388 |
| Num (iteration) | Convergence achieved after 17 iterations | | Convergence achieved after 19 iterations | | Convergence achieved after 21 iterations | | Convergence achieved after 18 iterations | | Convergence achieved after 21 iterations | | Convergence achieved after 25 iterations | |
| Stationarity | 0.950 | 0.952 | 0.977 | 0.980 | 0.940 | 0.938 | 0.970 | 0.966 | 0.967 | 0.971 | 0.979 | 0.979 |

* v is the degree of freedom parameter of student-t distribution

See the note of Table 11.

**Table 15: Results of STVECM combined with Bivariate Garch(1,1) and Student-t distribution**

**Bivariate Garch(1,1) specification:**

$$\begin{pmatrix}\varepsilon_{st}\\ \varepsilon_{ft}\end{pmatrix}\Omega_{t-1} \sim t(0, H_t, v)$$

$$h_{11t} = \omega_s^2 + \beta_s^2 \cdot h_{11t-1} + \alpha_s^2 \cdot \varepsilon_{st-1}^2$$
$$h_{12t} = \omega_s \omega_3 + \beta_s \beta_f \cdot h_{12t-1} + \alpha_s \alpha_f \cdot \varepsilon_{st-1} \varepsilon_{ft-1}$$
$$h_{22t} = \omega_f^2 + \omega_3^2 + \beta_f^2 \cdot h_{22t-1} + \alpha_f^2 \cdot \varepsilon_{ft-1}^2$$

| | *AU* | | *GM* | | *JP* | | *UK* | | *US* | |
|---|---|---|---|---|---|---|---|---|---|---|
| | *s* | *f* | *s* | *f* | *s* | *f* | *s* | *f* | *s* | *f* |
| $\omega$ | 0.001 | 0.001 | 0.001 | 0.001 | 0.002 | 0.002 | 0.001 | 0.001 | 0.001 | 0.001 |
| $z-stats$ | 6.241 | 6.031 | 8.384 | 8.004 | 9.664 | 9.134 | 11.185 | 10.845 | 6.997 | 7.388 |
| $\omega_3$ | 0.000 | | 0.000 | | 0.001 | | 0.000 | | 0.000 | |
| $z-stats$ | 2.049 | | 6.730 | | 9.303 | | 9.875 | | 5.111 | |
| $\beta$ | 0.952 | 0.969 | 0.958 | 0.960 | 0.956 | 0.955 | 0.953 | 0.953 | 0.975 | 0.976 |
| $z-stats$ | 86.7 | 129.5 | 268.0 | 281.0 | 160.1 | 149.3 | 220.9 | 219.0 | 364.4 | 386.8 |
| $\alpha$ | 0.192 | 0.157 | 0.230 | 0.222 | 0.193 | 0.193 | 0.216 | 0.215 | 0.166 | 0.160 |
| $z-stats$ | 8.695 | 8.046 | 20.429 | 20.145 | 14.785 | 14.485 | 18.763 | 18.681 | 17.456 | 17.564 |
| v | 9.724 | | 5.845 | | 7.117 | | 5.234 | | 5.264 | |
| $z-stats$ | 7.322 | | 14.042 | | 11.773 | | 16.087 | | 14.170 | |
| $\log l$ | 8090 | | 16214 | | 16753 | | 18145 | | 18455 | |
| *AIC* | -16.163 | | -13.857 | | -14.275 | | -15.489 | | -15.733 | |
| *SIC* | -16.124 | | -13.837 | | -14.256 | | -15.469 | | -15.713 | |
| $Q(5)$ | 2.15 | 1.23 | 4.20 | 1.60 | 0.90 | 1.11 | 6.63 | 4.93 | 5.11 | 3.44 |
| $p-value$ | 0.83 | 0.94 | 0.52 | 0.90 | 0.97 | 0.95 | 0.25 | 0.42 | 0.40 | 0.63 |
| $Q(15)$ | 4.01 | 3.88 | 17.10 | 10.56 | 7.10 | 5.06 | 15.61 | 14.27 | 16.07 | 13.07 |
| $p-value$ | 1.00 | 1.00 | 0.31 | 0.78 | 0.96 | 0.99 | 0.41 | 0.51 | 0.38 | 0.60 |
| $Q^2(5)$ | 22.22 | 10.11 | 11.86 | 3.14 | 7.66 | 3.59 | 10.55 | 5.94 | 15.92 | 23.62 |
| $p-value$ | 0.00 | 0.07 | 0.04 | 0.68 | 0.18 | 0.61 | 0.06 | 0.31 | 0.01 | 0.00 |
| $Q^2(15)$ | 26.64 | 21.89 | 26.37 | 10.70 | 24.19 | 15.61 | 21.35 | 14.90 | 20.11 | 30.98 |
| $p-value$ | 0.03 | 0.11 | 0.03 | 0.77 | 0.06 | 0.41 | 0.13 | 0.46 | 0.17 | 0.01 |
| $J-B$ | 268 | 47 | 146 | 308 | 227 | 205 | 90 | 63 | 514 | 565 |
| $\alpha_{ii}^2 + \beta_{ii}^2$ | 0.942 | 0.963 | 0.971 | 0.971 | 0.952 | 0.950 | 0.955 | 0.955 | 0.979 | 0.979 |
| $E[\log(\alpha_{ii}^2 \cdot v_t^2 + \beta_{ii}^2)]$ | -0.052 | -0.032 | -0.014 | -0.013 | -0.040 | -0.041 | -0.029 | -0.029 | -0.010 | -0.010 |

**Note: The dependent variables are the STVECM residuals of spot and futuress of each country. See the note of Table 11.**

### 6.6 Results of Bivariate GARCH combined with Nonlinear VECM

As the diagnostic test results in Table 6 – 10 suggest, there is strong volatility clustering effect in the residuals of STVECM. Therefore, we combine the STVECM for the conditional mean of the returns with a bivariate GARCH (1, 1) modelling the residuals. The results for conditional covariance matrix are presented in Table 15, where the distribution of the standardized residuals is assumed to be Student-t. The results corresponding to normality assumption are not reported because the estimation failed in three of five cases and there is evidence of nonstationarity in the rest two cases. From Table 15, we can see that the estimation is successful in all five cases and none of them risks being nonstationary. Furthermore, the Q squared stats indicate that there is no remaining ARCH effect, in contrast to the results of linear VECM. As STVECM combined with bivariate GARCH (1, 1) with Student-t distributed errors provides a good fit to the data, we derive its corresponding within-sample hedge ratio and compare the performance with the other strategies.

## 6.7 Results on the hedging effectiveness

### 6.7.1 Within-sample results

As shown in the part on hedging theory, the optimal hedge ratio is simply the quotient of the covariance between the return of spot index and index futures to the variance of the futures. In order to implement a hedging strategy, we need to have a model for the covariance matrix of the spot and futures, from which the covariance and variance estimates can be extracted and the hedge ratios can be computed. In this chapter, we adopted the framework of Vector-Error-Correction Model for the mean of these returns and bivariate GARCH model for the second moments. In particular, the models we estimate include VECM, VECM with TGARCH (1, 1) and STVECM with GARCH (1, 1). From the estimates of each model, we derive a series of hedge ratios and simulate the corresponding hedged portfolio in the within-sample period. The hedging strategies corresponding to VECM, VECM with TGARCH (1, 1) and STVECM with GARCH (1, 1) are termed as ECM, dynamic and nonlinear respectively. The conventional OLS hedge is also computed as the slope coefficient of the return of spot on the return of futures for comparison. The descriptive statistics of these portfolio's daily returns are presented in Table 16. For comparison purpose, the corresponding statistics for the return of unhedged spot position and that of the naïve one-for-one hedged portfolio are also shown.

The fifth column of Table 16 shows the hedging effectiveness of each strategy. The hedging effectiveness (HE) measures the proportional reduction of variance of the hedged portfolio from the unhedged. The bigger is it, the more effective is the strategy. The results in Table 16 demonstrate that all strategies can reduce the risk of spot position dramatically, with more than 90% improvement of all strategies in 3 out of 6 countries and 69.30% as the smallest improvement suggested by naïve hedge for South Korea.

Since the HE is too high in all cases, it is difficult to see the difference among different strategies. Following Baillie and Myers (1991), we compute the percentage reduction in the variance of the hedged portfolio suggested by the nonlinear strategy to naïve, OLS, ECM and dynamic strategy. These statistics tell us precisely how much better or worse the nonlinear hedging strategy is relative to the other strategies in reducing portfolio variances. And we can easily see the order of preference of different strategies for the same country.

The within-sample results show that naïve hedge is the worst by a long way in all countries. All the other three strategies can reduce the variance of naïve hedged portfolio by around 10%. In the extreme case of South Korea, dynamic hedge reduces it by more than one third.

For all countries, OLS outperforms ECM marginally. It supports the theoretical results in Lien (2005), where he argued that the OLS should always outperform the other constant hedge ratios by HE measurement using within-sample data, because OLS hedge corresponds to the minimal unconditional variance of the hedged portfolio that inversely related to HE measurement.

From the comparison statistics for dynamic and nonlinear hedge we can see that the performances of these two strategies are very similar. In the five countries where the estimation of nonlinear models is successful, the nonlinear hedge outperforms dynamic hedge in three and underperforms in two. The reduction or increment of the portfolio variance is less than 2% in all cases. Therefore, the difference between these two strategies is tiny.

Since the difference between OLS and ECM hedge and that between dynamic and nonlinear hedge are tiny, we can categorize them into two groups – the simple and complicated. The comparison results are mixed. In Germany, South Korea and United Kingdom, the complicated strategies perform much better than the simple ones. The reduction in variance ranges between 5% and 9%. However, in Australia, Japan and United States, the complicated ones either perform marginally better or marginally worse than the simple ones with the change in variance between -1% and 2%.

On the whole, these results are in line with the common results in this field. Complicated hedging strategies outperform the simple ones in most cases, with tiny improvement in some countries and big improvement in others. But they are also likely to be worse than the simple ones. It is very difficult to reach a clear-cut conclusion on the performance of any hedging strategy. The hedging effectiveness has to be investigated on a case-by-case basis.

## Table 16: Within-Sample hedge ratio and portfolio return

| | | E(rt) | s.d.(rt) | Max(rt) | Min(rt) | HE | E(hr) | s.d.(hr) | Max(hr) | Min(hr) | comparison |
|---|---|---|---|---|---|---|---|---|---|---|---|
| AU | spot | 0.01% | 0.70% | 3.44% | -4.81% | | | | | | |
| | naïve | 0.00% | 0.32% | 1.70% | -1.37% | 78.83% | | | | | 14.16% |
| | ols | 0.00% | 0.30% | 1.73% | -1.39% | 81.95% | 0.84 | 0.00 | 0.84 | 0.84 | -0.67% |
| | ecm | 0.00% | 0.30% | 1.72% | -1.32% | 81.91% | 0.85 | 0.00 | 0.85 | 0.85 | -0.47% |
| | dynamic | 0.00% | 0.30% | 1.67% | -1.21% | 81.51% | 0.83 | 0.07 | 1.24 | 0.70 | 1.71% |
| | nonlinear | 0.00% | 0.30% | 1.68% | -1.33% | 81.83% | 0.84 | 0.05 | 1.16 | 0.72 | |
| GM | spot | 0.03% | 1.64% | 7.55% | -8.87% | | | | | | |
| | naïve | 0.00% | 0.69% | 5.94% | -5.62% | 82.30% | | | | | 11.14% |
| | ols | 0.00% | 0.67% | 4.57% | -4.97% | 83.17% | 0.91 | 0.00 | 0.91 | 0.91 | 6.56% |
| | ecm | 0.00% | 0.67% | 4.62% | -4.99% | 83.17% | 0.91 | 0.00 | 0.91 | 0.91 | 6.58% |
| | dynamic | 0.02% | 0.65% | 6.01% | -4.27% | 84.20% | 0.87 | 0.14 | 1.15 | 0.40 | 0.46% |
| | nonlinear | 0.02% | 0.65% | 6.23% | -4.24% | 84.27% | 0.86 | 0.14 | 1.14 | 0.37 | |
| JP | spot | -0.02% | 1.47% | 7.66% | -7.23% | | | | | | |
| | naïve | 0.00% | 0.45% | 3.31% | -2.07% | 90.87% | | | | | 7.53% |
| | ols | 0.00% | 0.43% | 3.31% | -2.16% | 91.48% | 0.92 | 0.00 | 0.92 | 0.92 | 0.88% |
| | ecm | 0.00% | 0.43% | 3.31% | -2.08% | 91.46% | 0.94 | 0.00 | 0.94 | 0.94 | 1.11% |
| | dynamic | 0.00% | 0.43% | 3.31% | -2.18% | 91.59% | 0.94 | 0.04 | 1.08 | 0.75 | -0.37% |
| | nonlinear | 0.00% | 0.43% | 3.31% | -2.11% | 91.56% | 0.94 | 0.04 | 1.07 | 0.78 | |
| KR | spot | 0.00% | 2.44% | 14.60% | -12.74% | | | | | | |
| | naïve | 0.00% | 1.35% | 9.87% | -13.24% | 69.30% | | | | | 34.14% |
| | ols | 0.00% | 1.15% | 8.60% | -9.93% | 77.97% | 0.75 | 0.00 | 0.75 | 0.75 | 8.19% |
| | ecm | 0.00% | 1.15% | 8.65% | -10.07% | 77.96% | 0.76 | 0.00 | 0.76 | 0.76 | 8.25% |
| | dynamic | 0.00% | 1.10% | 8.08% | -8.29% | 79.78% | 0.83 | 0.12 | 1.16 | 0.46 | |
| | nonlinear | - | - | - | - | - | - | - | - | - | - |
| UK | spot | 0.02% | 1.16% | 5.90% | -5.89% | | | | | | |
| | naïve | 0.00% | 0.29% | 2.38% | -2.25% | 93.82% | | | | | 15.00% |
| | ols | 0.00% | 0.27% | 2.40% | -2.30% | 94.45% | 0.92 | 0.00 | 0.92 | 0.92 | 5.27% |
| | ecm | 0.00% | 0.27% | 2.40% | -2.29% | 94.45% | 0.93 | 0.00 | 0.93 | 0.93 | 5.34% |
| | dynamic | 0.00% | 0.26% | 2.33% | -2.22% | 94.80% | 0.92 | 0.06 | 1.19 | 0.75 | -1.06% |
| | nonlinear | 0.00% | 0.27% | 2.35% | -2.25% | 94.75% | 0.91 | 0.06 | 1.18 | 0.71 | |
| US | spot | 0.04% | 1.17% | 5.57% | -7.11% | | | | | | |
| | naïve | 0.00% | 0.31% | 1.87% | -2.55% | 93.01% | | | | | 11.08% |
| | ols | 0.00% | 0.29% | 1.87% | -2.29% | 93.68% | 0.92 | 0.00 | 0.92 | 0.92 | 1.70% |
| | ecm | 0.00% | 0.29% | 1.87% | -2.33% | 93.66% | 0.93 | 0.00 | 0.93 | 0.93 | 1.96% |
| | dynamic | 0.01% | 0.29% | 1.87% | -2.39% | 93.71% | 0.94 | 0.06 | 1.07 | 0.80 | 1.16% |
| | nonlinear | 0.00% | 0.29% | 1.87% | -2.27% | 93.78% | 0.94 | 0.05 | 1.04 | 0.79 | |

Note: 1. This table contains the within-sample hedging performances of each strategy. Naïve strategy is always to hedge one for one. OLS strategy derives its hedge ratio from a simple OLS regression of spot return on futures return. ECM strategy is computed from the estimated VECM. Dynamic strategy corresponds to the estimated VECM combined with threshold GARCH (1,1). Nonlinear strategy derives its hedge ratio from the estimated STVECM combined with GARCH (1,1).

2. In Column 1 to 4, the descriptive statistics of the hedged portfolio return implied by each strategy are presented. Column 6 to 9 contain the descriptive statistics of the hedge ratios.

3. The hedging effectiveness (HE) measurement is presented in Column 5. It is the proportional reduction of portfolio variances suggested by each hedging strategy compared with spot return.

4. The last column contains the proportion of reduction of the portfolio variance suggested by nonlinear model to that of the other four models. In the case of South Korea, it is the reduction of portfolio variance suggested by dynamic model to that of the others.

### 6.7.2 Out-of-sample results

The out-of-sample estimation is carried out on a daily basis for the entire one-year holdout period. We estimate the simple regression, VECM and VECM combined with TGARCH (1, 1) every day using data up to that day. The STVECM combined with GARCH (1, 1) is not used for the out-of-sample comparison, since its success can not be guaranteed in a large number of estimations. From the estimated models, we make a one-day-ahead forecast on the covariance matrix and compute the next-day hedge ratio. Combining the simulated hedge ratio series with the return in the spot and futures, we compute the series of the return of the hedged portfolio for each strategy. The descriptive statistics of these portfolio returns are presented in Table 17.

The estimation is done by programming in Eviews4.0. The full-information method is used for VECM combined with TGARCH (1, 1). In each estimation, the starting value is set to be the estimates from the last estimation. The iterative method is the popularly used one in GARCH estimation – BHHH[6]. The convergence criterion is that the change in the norm of parameters is less than 1e-05. Convergence is achieved in most cases. However, because the likelihood function is flat, the convergence is not achieved all the times and failure to improve the likelihood happens sometimes. When it occurs, the log-likelihood value of that day is compared with the one before. If it increases, the estimates are kept otherwise the estimates when convergence is achieved last is used instead.

The out-of-sample results are presented in Table 17. Similar to the within-sample results, the hedging strategies are very effective. The hedging effectiveness is higher than 90% in four of six cases and the smallest improvement of 69.92% in Australia.

Different from the within-sample results which show dramatic improvement of other hedging strategies over naive hedge, the out-of-sample results are mixed. While in Australia and South Korea, naïve hedge is inferior to others by a long way, it is only marginally better than the naïve in Germany and Japan. And the evidence shows that naïve hedge is the best among all strategies in UK and USA. Recall that naïve hedge is supported by traditional theory which assumes the change in basis is zero. The results of UK and USA may simply reflect the change in their basis is very small. We show the descriptive statistics of the change in basis in

[6] BHHH stands for Berndt, Hall, Hall, and Hausman. BHHH algorithm substitutes the outer product of the gradients for the observed negative Hessian matrix. See Berndt et al. (1974).

the out-of-sample period in Table 18. The standard deviation of the change in basis of UK and USA is indeed much smaller than that of others.

**Table 17: Out-of-Sample hedge ratio and portfolio return**

| | | E(rt) | s.d.(rt) | Max(rt) | Min(rt) | HE | E(hr) | s.d.(hr) | Max(hr) | Min(hr) | comparison |
|---|---|---|---|---|---|---|---|---|---|---|---|
| AU | spot | 0.09% | 0.43% | 1.22% | -1.27% | | | | | | |
| | naïve | 0.00% | 0.23% | 0.61% | -0.81% | 69.92% | | | | | 27.77% |
| | ols | 0.01% | 0.20% | 0.49% | -0.69% | 77.66% | 0.83 | 0.00 | 0.84 | 0.83 | 2.76% |
| | ecm | 0.01% | 0.20% | 0.51% | -0.69% | 77.21% | 0.85 | 0.00 | 0.85 | 0.85 | 4.66% |
| | dynamic | 0.02% | 0.20% | 0.48% | -0.69% | 78.27% | 0.77 | 0.03 | 0.85 | 0.71 | |
| GM | spot | 0.03% | 0.95% | 2.59% | -3.52% | | | | | | |
| | naïve | 0.00% | 0.45% | 1.01% | -1.43% | 77.91% | | | | | 4.42% |
| | ols | 0.00% | 0.43% | 1.01% | -1.45% | 79.20% | 0.91 | 0.00 | 0.91 | 0.91 | -1.53% |
| | ecm | 0.00% | 0.43% | 1.01% | -1.44% | 79.19% | 0.91 | 0.00 | 0.91 | 0.91 | -1.48% |
| | dynamic | 0.01% | 0.44% | 1.02% | -1.73% | 78.89% | 0.88 | 0.04 | 0.97 | 0.74 | |
| JP | spot | 0.02% | 1.04% | 2.76% | -4.97% | | | | | | |
| | naïve | 0.00% | 0.29% | 0.96% | -0.83% | 92.32% | | | | | 5.06% |
| | ols | 0.00% | 0.28% | 0.94% | -0.92% | 92.59% | 0.92 | 0.00 | 0.93 | 0.92 | 1.61% |
| | ecm | 0.00% | 0.28% | 0.94% | -0.90% | 92.64% | 0.94 | 0.00 | 0.94 | 0.94 | 0.92% |
| | dynamic | 0.00% | 0.28% | 0.94% | -0.84% | 92.71% | 0.95 | 0.02 | 1.03 | 0.87 | |
| KR | spot | 0.03% | 1.51% | 4.90% | -6.07% | | | | | | |
| | naïve | 0.00% | 0.45% | 1.69% | -2.10% | 91.02% | | | | | 16.18% |
| | ols | 0.01% | 0.46% | 1.21% | -1.78% | 90.48% | 0.75 | 0.00 | 0.76 | 0.75 | 20.95% |
| | ecm | 0.01% | 0.46% | 1.23% | -1.74% | 90.79% | 0.76 | 0.00 | 0.77 | 0.76 | 18.26% |
| | dynamic | 0.01% | 0.41% | 1.56% | -1.70% | 92.47% | 0.90 | 0.04 | 1.01 | 0.79 | |
| UK | spot | 0.04% | 0.63% | 1.93% | -2.32% | | | | | | |
| | naïve | 0.00% | 0.14% | 0.44% | -0.51% | 95.28% | | | | | -4.00% |
| | ols | 0.00% | 0.14% | 0.43% | -0.42% | 94.73% | 0.93 | 0.00 | 0.93 | 0.92 | 6.82% |
| | ecm | 0.00% | 0.14% | 0.43% | -0.43% | 94.82% | 0.93 | 0.00 | 0.93 | 0.93 | 5.25% |
| | dynamic | 0.00% | 0.14% | 0.44% | -0.50% | 95.09% | 0.97 | 0.02 | 1.03 | 0.92 | |
| US | spot | 0.02% | 0.69% | 1.62% | -1.65% | | | | | | |
| | naïve | 0.00% | 0.16% | 0.60% | -0.50% | 94.46% | | | | | -0.99% |
| | ols | 0.00% | 0.16% | 0.45% | -0.47% | 94.34% | 0.92 | 0.00 | 0.92 | 0.92 | 1.10% |
| | ecm | 0.00% | 0.16% | 0.47% | -0.46% | 94.45% | 0.94 | 0.00 | 0.94 | 0.93 | -0.74% |
| | dynamic | 0.00% | 0.16% | 0.47% | -0.46% | 94.41% | 0.96 | 0.02 | 1.04 | 0.90 | |

Note: 1. This table contains the out-of-sample hedging performances of all apart from nonlinear strategy.
2. The last column shows the proportional reduction of the variance of dynamic portfolio to that of the others.
3. See note 1, 2 and 3 of Table 16.

The out-of-sample comparison results between OLS and ECM is very different from the within-sample counterpart. While the within-sample results suggest OLS is always better than ECM, the out-of-sample results suggest this is true only in Australia and Germany. But in Japan, South Korea, UK and USA, ECM is slightly better than OLS. The implication is that ECM is not always dominated by OLS and it should be a candidate hedging strategy for evaluation.

Similar to the within-sample results, the out-of-sample results also show that the complicated dynamic strategy does not outperform others consistently across countries. Only in South Korea, it beats all the others by a long way. It reduces the portfolio variance suggested by other strategies by around 20%. In Australia, although dynamic hedge outperforms naïve hedge by 27.77%, its improvement over OLS and ECM hedge is less than 5%. In Japan, the superiority of dynamic hedge is only evident when compared with naïve hedge, but marginal when compared with the other two more complicated ones. In Germany, dynamic strategy is marginally worse than OLS and ECM. In USA, it is only marginally different from all the others. In UK, dynamic hedge is beaten by naïve hedge by a long way. The variance of dynamic hedge is 4% greater than that of naïve hedge.

On the whole, the out-of-sample results support the view that dynamic hedge outperforms other strategies in some countries, but the extent of improvement varies. The usefulness of dynamic hedge has to be judged on a case-by-case basis.

**Table 18: Descriptive Stats for the change in basis in the out-of-sample period**

| | AU | GM | JP | KR | UK | US |
|---|---|---|---|---|---|---|
| Mean | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% |
| Median | -0.01% | 0.01% | 0.01% | 0.00% | -0.01% | 0.00% |
| Maximum | 0.64% | 1.44% | 1.44% | 2.09% | 0.50% | 0.50% |
| Minimum | -0.60% | -1.01% | -1.01% | -1.94% | -0.44% | -0.60% |
| Std. Dev. | 0.23% | 0.44% | 0.44% | 0.45% | 0.13% | 0.16% |
| Skewness | 19.89% | 23.55% | 23.55% | 29.50% | 25.44% | 6.55% |
| Kurtosis | 3.01 | 3.38 | 3.38 | 5.96 | 3.99 | 3.77 |
| Jarque-Bera | 2 | 4 | 4 | 99 | 13 | 7 |

Note: The basis is the difference between the return of spot and futures. The out-of-sample period starts from March 08, 2004 and ends at March 05, 2005. The data frequency is daily.

### 6.8 Transaction cost and alternative measurement of hedging success

Throughout this chapter, the transaction cost related to portfolio rebalance is ignored. In reality, the transaction cost is an important factor in determining the investors' behaviour. It would affect the conclusion of this chapter in two ways. First, hedging strategies which imply more volatile hedge ratio such as dynamic hedge would be penalized more because more transaction costs would be incurred when the hedge ratio is adjusted too much too frequently. Second, when the transaction cost is taken into account, the objective function would be different and result in less frequent rebalancing. It is possible that more risk reduction might be achieved.

In this chapter, the hedging effectiveness is only based on the reduction in hedged portfolio variance. However, in practice, it may be relevant to investigate what would happen in the worse scenario particularly. The fourth column of Table 9 and 10 contains the minimal rate of return of hedged portfolio of all six countries. There is no clear pattern in the results. The dynamic hedge gives the highest minimal return in the case of South Korea and USA, but the lowest minimal return in the case of Germany and something in the middle in the rest cases.

## 7. Conclusion

This chapter investigates the hedging effectiveness using index futures in six major stock markets. Under the mean-variance framework, the minimum-variance hedge ratio is optimal for hedgers aiming at risk reduction. The vector-error-correction model (VECM) implied by the cost of carry theory and the empirical data is assumed to model the conditional mean of the return on spot and futures where the error correction term is derived from imposing the constraint on the cointegrating vector. The smooth transition VECM ( STVECM ) that captures the changing strength of the response to deviation is also estimated within the sample. The bivariate threshold GARCH (1,1) and standard GARCH (1,1) are fitted to the residuals from the VECM and STVECM respectively with Student-t distribution assumed for the standardized residuals. The estimation results suggest that these models provide adequate and statistically satisfactory fit to the data. However, their implied hedging strategies do not always correspond to the best hedging performance for each country in the within and out-of-sample period. In some cases, sophisticated models are beaten by naïve hedge or simple OLS hedge. The superiority of the sophisticated strategy has to be judged on a case-by-case basis. These results are in line with the general results in the literature. They seem to be related to the typical results in forecasting – sophisticated models with a large number of parameters tend to fit the data well within sample but do badly out-of-sample.

One main drawback of this chapter is that the transaction cost is assumed zero throughout. However, in reality transaction cost is an important factor in making investment decision. The volatile hedge ratios implied by the sophisticated models should be penalized to make the analysis realistic. When the transaction cost is taken into account, the objective function would be different and result in less frequent rebalancing. It is possible that more risk reduction might be achieved.

This chapter investigates the performance of the minimum-variance hedge ratio. However, it is possible that hedgers are not only concerned with the average variation of portfolio return measured by the portfolio variance but also the portfolio return in extremely bad scenarios. We will have to model the return on spot and futures in extreme events explicitly and set up the objective functions accordingly.

# Appendix

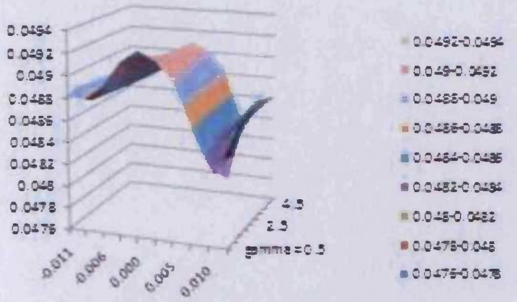

Figure 1 (a): SSR AU Spot

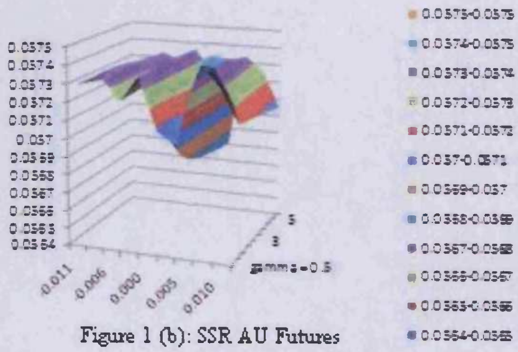

Figure 1 (b): SSR AU Futures

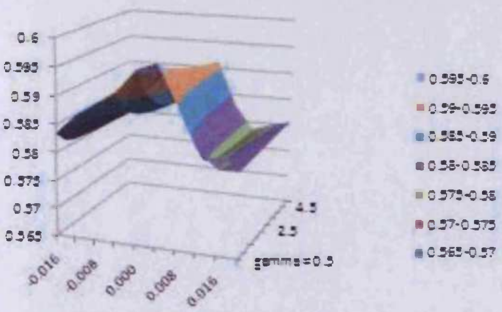

Figure 2 (a): SSR GM Spot

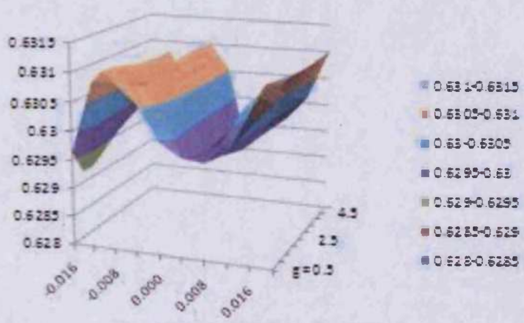

Figure 2 (b): SSR GM Futures

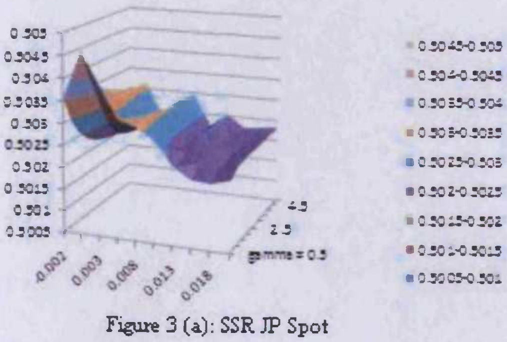

Figure 3 (a): SSR JP Spot

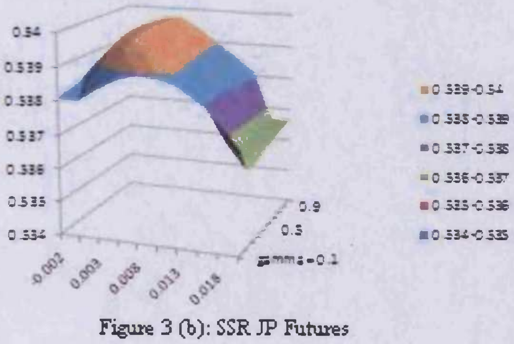

Figure 3 (b): SSR JP Futures

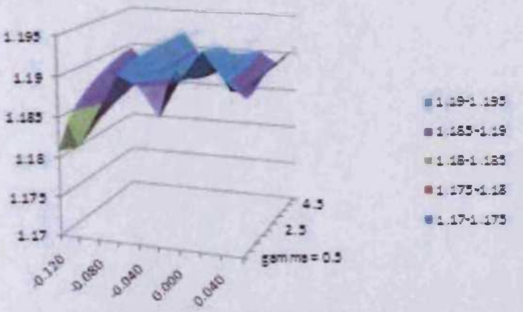

Figure 4 (a): SSR KR Spot

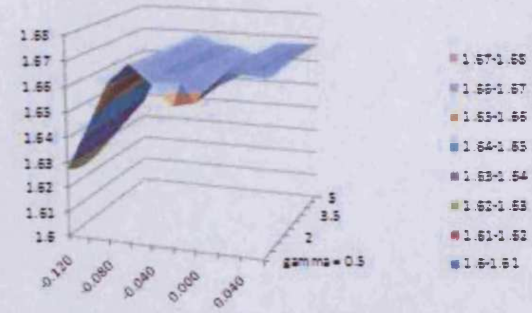

Figure 4 (b): SSR KR Futures

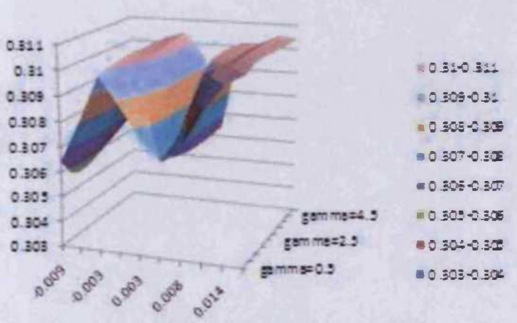

Figure 5 (a): SSR UK Spot

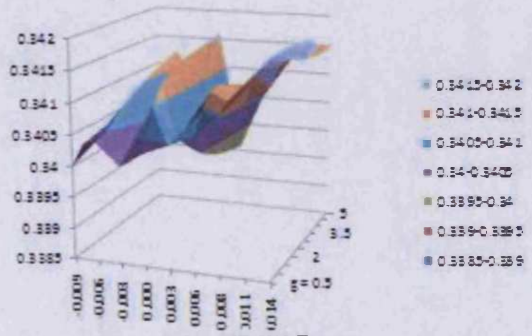

Figure 5 (b): SSR UK Futures

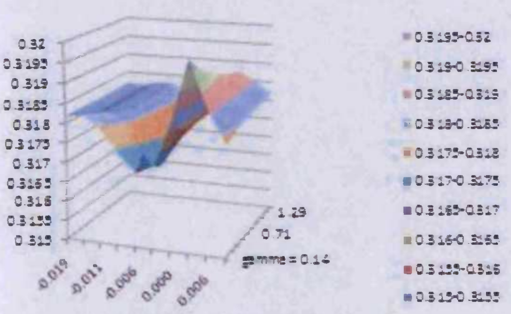

Figure 6 (a): SSR US Spot

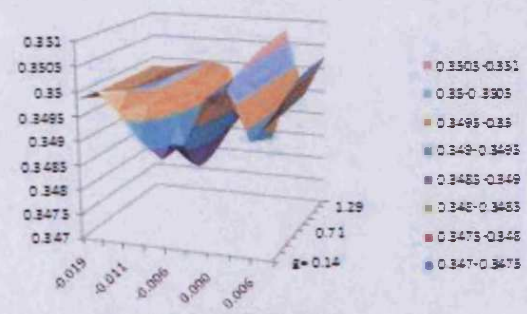

Figure 6 (b): SSR US Futures

# Chapter 3

# Cross Hedging Effectiveness Using Index Futures

## 1. Introduction

Cross hedging is the strategy to reduce risk when no futures contract corresponding to the spot asset to be hedged is available. The demand for cross hedging strategies is widespread among different market classes. In the commodity market, to hedge positions in commodities that have no futures contract, investors have to use the futures of related commodities. In the foreign exchange market, investors usually hedge positions in minor currencies with the futures of major currencies. To hedge the risk in a stock portfolio, investors often make the use of the futures of the related market indices.

Compared with direct hedging, cross hedging is more complicated on two counts. First, there are a number of futures contracts potentially suitable as cross-hedging instruments, in contrast to the unique futures contract for direct hedging. Investors must make a decision on which hedging instrument to use. Second, it is possible that a combination of several futures contracts is more effective than a single futures contract in reducing portfolio risk. It is therefore necessary to generalize the analytical solution to the optimal hedge ratio for direct hedging to the optimal hedge ratio *vector* for cross hedging and compare the hedging performance across all the alternatives empirically.

This chapter answers the question of how to hedge a hypothetical stock portfolio measured by Morgan Stanley Capital International (MSCI) index of seventeen countries. MSCI global equity indices have been compiled since 1969 covering all the major stock markets in the world. Assets benchmarked to MSCI have reached 3 trillion dollars in 2007. MSCI indices are used by 22 of 25 largest firms managing assets globally. It is of practical interest to study the hedging effectiveness of portfolio that tracks MSCI stock index. However, the futures contracts based on MSCI indices are either non-existent or thinly-traded. In the case where direct hedging is almost impossible it is natural to study the effectiveness of cross hedging.

Section 2 contains a brief literature review on cross hedging. Section 3 describes the data. Section 4 presents the derivation of the generalized solution to the optimal cross hedge ratio vector. Section 5 explains the model and outlines the procedure. Section 6 presents the estimation and hedging performance results. The last section concludes.

## 2. Literature Review

Anderson and Danthine (1981) investigated the theoretical aspect of cross hedging. Rooted in their pioneering work, a number of researchers studied the hedging effectiveness of cross hedging in different markets including commodities, currencies, bonds and equities empirically.

Anderson and Danthine (1981) set up a model to characterize the principal features of optimal decisions by a variety of market participants including long, short hedgers and speculators. The agent makes simultaneous decision on the number of cash and futures position to maximize the mean variance utility. Given the cash position, the optimal futures position can be decomposed into a pure speculation part and a pure hedge part. The former depends on the expected prices of multiple futures contracts, the covariance matrix of multiple futures prices and the investors' degree of risk aversion. The latter depends on the covariance between spot asset to be hedged and multiple futures and the covariance matrix of the multiple futures prices.

A number of researchers have investigated the effectiveness of cross hedging the holding in various commodities using the futures of the related commodities.

Miller (1985) investigated the simple and multiple cross-hedging of mill feeds using the futures for oats, corn, soybean meal and wheat. He set up a multivariate regression with the cash price of mill feeds on the left-hand side and the futures price of related commodities on the right-hand side. The monthly data set covering the period between January 1972 and December 1982 consists of the four-year estimation and six-year forecasting period. The cross-hedge ratios were first estimated by OLS method and re-estimated every month as time moves on. The results for the portfolio risk measured by the average forecasts errors (AFE) and the root-mean-square forecast errors (RMSFE) show that the corn futures are the best single cross-hedging instrument and the combination of corn and soybean meal futures is the best combination of multiple cross-hedging futures contracts that outperforms the single corn futures contract.

Vukina and Anderson (1993) designed a multi-period hedging model that allows the futures position to be revised within the cash position holding period. Investors determine the time path of the futures position to maximize the expected utility of their end-period wealth. The analytical solution to the sequence of cross-hedge ratios in three-period and five-period model is derived. To estimate the model, they used a state-space forecasting approach to approximate the expectations on prices and the covariance matrix of forecasts errors. Routine, static and dynamic hedge were applied to the cross-hedging of fish meal using the futures of soybean meal for a weekly data set covering June 1986 to May 1991. The empirical results suggest both static and dynamic hedged portfolio are more volatile for less risk-averse investors as expected. For less risk-averse investors, the static hedge leads to lower mean return, which is attributed to the static model's inadequacy in modelling an intrinsically dynamic decision process. The dynamic model on the other hand can improve the ability of a weakly risk-averse investor, even if a fixed transaction cost is taken into consideration.

Hayenga, Jiang and Lence (1996) compared the hedging performance of the common practice of the industry with the traditional hedge and the hedge derived from a regression with lagged basis as explanatory variables. They showed that the common practice of taking the historical price ratio between the commodity to be delivered and the futures price is a special case of the traditional hedge with the restriction not necessarily satisfied. The traditional OLS regression is a special case of the hedge with lagged basis as regressor. Specifically, they investigated the simple cross-hedging of pork and beef product using live hog and cattle futures contracts for the daily data sample covering 1986 to 1995. The results showed that hedge ratios derived from regression with lagged basis performed the best and the common practice of the industry the worst.

There is also a vast amount of literature addressing the problem of cross hedging the exposure in minor currencies using the futures of major currencies or commodities.

Eaker and Grant (1987) presented extensive empirical evidence on the effectiveness of cross-hedging for nine foreign currencies, among which four have no futures contracts. They divided the monthly data set covering November 1976 to November 1983 into two parts. The hedge ratios estimated using multivariate regressions from the first half were applied to the second half to generate the naïve out-of-sample portfolio returns. The results showed that cross hedging is less effective and more variable compared with direct hedging; the strong economic relations between countries is loosely correlated to hedging effectiveness; multiple futures hedging is more effective than single future hedging. They also tested the cross-hedging using gold futures for currencies, but got negative results.

Braga, Martin and Meike (1989) studied the effectiveness of cross hedging the Italian Lira/US Dollar exchange rate with Deutsch Mark futures and compared it with the hedging using the Italian Lira/US Dollar forward contract. They gathered the weekly exchange rate data of spot and futures in the period between January 1982 and September 1986 and simulated the cross-hedging portfolio for one, two and four-week hedging horizon respectively. In the out-of-sample period, they re-estimated the model and updated the hedge ratio every eight weeks. The general results suggest hedging effectiveness increases when hedging horizon lengthens; it increases when nearby instead of mid-distant contract is used; it increases when optimal hedge instead of naïve hedge strategy is adopted. Their results also indicate the average cost of all cross hedge strategies for a short US Dollar position is substantially lower than that of a traditional forward market hedge but the former is subject to big variations. Investors therefore face a trade-off between lower average cost with higher risk and higher average cost with zero risk.

Benet (1990) investigated the commodity futures cross hedging of foreign exchange exposure. Based on the 'flow' theory of exchange rate determination that suggests a positive correlation between exchange rate and export commodity price, the hypothesis of 'primary export commodity' was proposed. Under the general form of this hypothesis, the export commodity futures cross hedges should be successful in reducing foreign exchange risk. Under its strict form, the intra-country ranking of exporting commodity should be positively related to the hedging effectiveness using different commodity futures. The data set consists of the monthly spot exchange rates of thirteen minor currencies, five major currencies and fifteen 'primary export commodity' futures contracts in the period between August 1973 and December 1985. The within sample results support the general but not the strict form of the hypothesis. The out-of-sample results showed a big 'drop' of hedging performance. The mean hedging effectiveness of commodity and currency strategies were both negative. He attributed the negative out-of-sample results to the instability of hedge ratios.

DeMaskey (1997) presented evidence on cross hedging of three minor European and three minor Asian currencies using single and multiple major currency futures. The monthly data set covering the period between 1983 and 1992 was divided into three reflecting the changes in the international exchange rate regimes. The within-sample results support the cross-hedging strategy using single contract in all minor currencies except Hong Kong Dollar. Germany Mark futures and Japanese Yen futures are most effective for the cross-hedging of European and Asian minor currency respectively. Multiple futures cross-hedging strategies are generally more successful in reducing foreign exchange risk, but the number of futures in the hedged portfolio does not have a significant effect on the hedging performance. The out-of-sample approach was performed by implementing prior sub-period hedge ratios into the subsequent hedge sub-period. The out-of-sample performance is worse for all cross-hedging strategies, with the decrease more dramatic for Asian minor currencies. The stability test results imply instability in the hedge ratios.

Some authors investigated the cross hedging issue in bond markets. In particular, the relatively safe government bills or bonds were often chosen to hedge the exposure in the risky private bonds.

Kuberek and Pefley (1983) outlined a procedure for evaluating the cross-hedging effectiveness of interest-rate futures and applied it to hedge the price risk of corporate debt using the Treasury bond futures. The monthly data set covering the period between 1977 and 1981 includes the realized return of two separate corporate bond portfolios with different grade and the Treasury bond futures prices with six different delivery period. They derived the unexpected spot returns by subtracting the one-month Treasury bill return from the realized spot returns and used the realized futures return as unexpected return on the basis of 'zero drift' assumption. The complete sample was divided into two reflecting the change of monetary policy of the Federal Reserve in 1979. To accommodate the different variances in the two sub-samples, they used the generalized least squares (GLS) estimation method. The results showed that cross hedging of corporate debt using Treasury bond futures is highly effective; for each contract maturity, it is more effective for the higher-quality bond portfolio; for both bond portfolios, hedging effectiveness declines as contract maturity lengthens.

There is a strong case for a non-constant hedge ratio based on conditional moments (see the previous chapter section 5.2). This argument applies as much to cross- as direct-hedges, as is clear from the work of Koutmos, Kroner and Pericli (1998).

Koutmos, Kroner and Pericli (1998) investigated the dynamic cross hedging with mortgage-backed securities (MBS) by 10-year Treasury bond futures. They used the daily data of MBS with different coupon rates and 10-year Treasury bond futures in the period between July 1992 and August 1995. The error-correction model with bivariate GARCH residuals (EC-GARCH) provides the best fit to the data. The usual hedged portfolio variance is used to measure hedging effectiveness. Both within and out-of-sample results support the dynamic hedge ratio suggested by EC-GARCH. When the expected utility is used to measure the hedging performance with the transaction cost deducted, EC-GARCH also beats the other models. Their stationarity test results suggest dynamic hedge ratios are nonstationary and are cointegrated with the 30-year Treasury bond rate, indicating the dynamic hedging strategy subsumes the prepayment risk linked to the market rate.

The application of cross hedging strategies in stock markets has been carried out by several authors mainly on using the market index futures to hedge individual stocks.

Butterworth and Holmes (2001) provided the evidence of hedging effectiveness of investment trust companies (ITCs) using FTSE-100 and FTSE-mid 250 index futures. They collected the daily return on thirty-two ITCs and four indices for the period of February 1994 to December 1996 and used the FTSE-100 index futures and FTSE-mid 250 index futures to hedge the cash positions. Both the OLS and Least Trimmed Squares (LTS) approach were used in estimation. Four hedging strategies including unity hedge, beta hedge, Minimum-Variance (MV) hedge and composite hedge were compared on the basis of within-sample performance. The composite hedge is derived by forming synthetic index futures with fixed weight on FTSE-100 and FTSE-mid 250. The results showed that unity and beta hedge performed the worst. MV-hedge using FTSE-mid 250 performs better than MV-hedge using FTSE-100 for hedging ITCs. But the superiority is far less when cash portfolios are broad market indexes. The composite hedge showed minor improvement over FTSE-mid 250. The hedge ratios estimated by LTS suggested similar results to OLS.

Brooks, Davies and Kim (2006) investigated the cross hedging with single stock futures (SSF) in USA. They proposed to use SSF to hedge the risk of stocks on which no option or exchange-traded futures contract are written and introduced three matching methods to select the SSF on the basis of historical correlation, cross-sectional matching characteristics and both. Multiple hedging with up to three contracts was also performed. Their daily data set covering a period from September 2003 to March 2005 includes 86 stocks and 350 SSFs. The out-of-sample results showed that the best hedging performance is achieved through a portfolio hedged with market index futures and a SSF matched both by historical return correlation and cross-sectional matching characteristics, keeping the chosen SSF contract unchanged for the whole out-of-sample period and updating the optimal hedge ratio for each rolling window after estimating the model.

## 3. A general approach to deriving the optimal hedge ratios of multiple futures contracts

The main purpose of hedging is to reduce risk. Therefore, it is important to select a hedging instrument the return of which approximates the movement of the asset being hedged so that the variation of the hedged portfolio return is kept to a minimum.

When the underlying of the futures contract is the same as the asset being hedged, the choice of hedging instrument is straightforward. The complication lies in the choice of the econometric models from which the hedging ratios can be generated. As shown in the previous chapter, direct hedging in the context of a stock market index requires the use of the index futures contract of the underlying market index and the comparison of several econometric models on the ground of fitness and hedging performance. When the asset being hedged does not have a corresponding futures contract, the hedging decision is more complicated since hedgers not only face the choice of econometric models but also the choice of hedging instruments. Unlike direct hedging, cross hedging theory does not suggest a unique hedging instrument. Those assets closely related to the asset being hedged are all potential hedging instruments. Moreover, it is also possible that a combination of several futures is superior to any single futures contract in reducing the portfolio risk. To derive the optimal hedging strategy in the cross hedging context, we need to compare the performances of all possible strategies empirically. But at first, it is necessary to generalize the analytical optimal hedge ratio in the direct hedging context to the vector of optimal hedge ratios in the context of cross hedging with multiple futures contracts.

Suppose an investor holds a portfolio with $\mathbf{M}$ assets, where the return of asset $\mathbf{i}$ is $\mathbf{s}_i$. The return of all assets can be summarized by the $\mathbf{M}$-dimensional vector $\mathbf{s}$, where $\mathbf{s} = (\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_M)'$. The holding of asset $\mathbf{i}$ is $\mathbf{x}_i$. The portfolio composition can be represented by the M-dimensional vector x, where $\mathbf{x} = (x_1, x_2, \ldots, x_M)'$. The total return of the investor on the spot portfolio is $\mathbf{s}'\mathbf{x}$.

Suppose the investor uses N futures contracts to hedge this portfolio. The underlying of the futures are different from the spot assets and the number of futures is possibly different from that of the assets in the portfolio. The return of the futures is summarized by the N-dimensional vector f, where $\mathbf{f} = (f_1, f_2, \ldots, f_N)'$. The positions in futures are represented by the N-dimensional vector y, where $\mathbf{y} = (y_1, y_2, \ldots, y_N)'$. The total return on the futures position is $\mathbf{f}'\mathbf{y}$.

The return on the hedged portfolio is:

$$\pi = \mathbf{s}'\mathbf{x} - \mathbf{f}'\mathbf{y} \tag{1}$$

with expectation:

$$E(\pi) = E(\mathbf{s}')\mathbf{x} - E(\mathbf{f}')\mathbf{y} \tag{2}$$

and variance:

$$V(\pi) = \mathbf{x}'\Sigma_{ss}\mathbf{x} + \mathbf{y}'\Sigma_{ff}\mathbf{y} - 2\mathbf{x}'\Sigma_{sf}\mathbf{y} \tag{3}$$

where $\Sigma_{ss}$ is the $M \times M$ covariance matrix of s, $\Sigma_{ff}$ is the $N \times N$ covariance matrix of f and $\Sigma_{sf}$ is the $M \times N$ covariance matrix between s and f.

Like in Chapter 2, we assume investors are mean-variance utility maximizer. That is, they want their portfolio to have a high return and low risk. In order to do that, they choose the optimal cross hedge ratios to achieve the best trade-off between the expected return and risk, i.e. to maximize the mean-variance utility. This problem can be expressed mathematically by maximizing the mean-variance utility function of the portfolio return with respect to the futures positions, given the positions of the spot portfolio.

$$\text{Max } U(\pi) = E(\pi) - \frac{1}{2}\gamma V(\pi) \quad \text{w.r.t. } \mathbf{y} \tag{4}$$

where $\gamma$ is the parameter of degree of risk aversion.

The F.O.Cs give the optimal solutions of the futures position[1]:

$$\mathbf{y}^* = -\frac{1}{\gamma}\Sigma_{ff}^{-1}E(f) + \Sigma_{ff}^{-1}\Sigma_{sf}^{'}\mathbf{x} \tag{5}$$

where $\mathbf{y}^*$ is a **N**-dimensional vector whose **jth** element $y_j^*$ is the optimal position on the **jth** futures contract.

Suppose the prices of futures are subject to a martingale process, i.e. the return on futures is unpredictable. The first term in the above expression disappears. The vector of optimal positions in futures reduces to:

$$\mathbf{y}^* = \Sigma_{ff}^{-1}\Sigma_{sf}^{'}\mathbf{x} \tag{6}$$

which is the general solution to the optimal cross hedging vector.

Direct hedging can be thought of as a special case of cross hedging where both **M and N** are equal to one and the asset is the same as the underlying of the futures. In this case, optimal hedge ratio $(\mathbf{y}^*/\mathbf{x})$ is the quotient of the covariance between the return of asset and its futures to the variance of futures.

In the special case where there is only one asset in the spot portfolio $(\mathbf{M} = 1)$, $\Sigma_{ff}^{-1}\Sigma_{sf}^{'}$ is a N-dimensional vector, $\mathbf{x}$ is a scalar and $\mathbf{y}^*$ is the optimal vector of multiple hedge ratios. This formula is the same as what Anderson and Danthine (1981) derived. Miller (1985), Eaker and Grant (1987) and Butterworth and Holmes (2001) have applied it to commodity, currency and stock futures hedging respectively. Specifically, they run a multivariate regression where the LHS variable is the return of the asset being hedged and the RHS variables are the return of **N** futures. The coefficient estimates are the optimal position in the futures.

In the general case where there are **M** assets in the spot portfolio, $\Sigma_{ff}^{-1}\Sigma_{sf}^{'}$ is a $\mathbf{N} \times \mathbf{M}$ matrix, the column vector of which contains the optimal positions in the multiple futures required by each spot asset if it were the only asset in the portfolio; $\mathbf{x}$ is a **M**-dimensional vector containing the weight of each asset in the spot portfolio; $\mathbf{y}^*$ is a **N**-dimensional vector containing the optimal positions in the multiple futures corresponding to the complete portfolio. The relationship between Anderson and Danthine (1981) and the generalized

[1] See the derivation in the appendix.

approach is clear. When the spot portfolio consists of more than one asset, the optimal position of the multiple futures is the weighted average of those required by each spot asset. Given the weight of each asset and the optimal futures positions required by each asset, we can derive the optimal futures positions for the complete portfolio by computing the weighted average.

The multivariate regression approach used to derive the optimal positions in multiple futures of Danthine and Anderson (1981) has to be modified. Instead of a multivariate OLS regression, a system of **M** multivariate regression equations is needed. The regressand of the **ith** equation is the return of the **ith** asset and the return of **N** futures are the common regressors of all equations. The **M** equations should be estimated simultaneously. The optimal position of the **jth** futures in the complete portfolio is the weighted average of the coefficient estimates of the **ith** asset on the **jth** futures over **M** assets.

Despite the derivation of the general approach in hedging a portfolio of **M** assets using **N** instruments, we do not attempt to apply it to the data and compute the optimal positions for the portfolio made up of **M** assets.

The example we consider in this chapter is the problem of hedging the MSCI index for a number of the world's largest markets. This problem frequently confronts institutions whose portfolios are benchmarked against MSCI indices. In order to reduce the portfolio risk, they have the incentive to hedge. Ideally, they would like to use the futures contracts underlying MSCI index to avoid mismatch. However, only a few such contracts exist and the majority of them are thinly traded in the market. Since most funds benchmarked against MSCI indices invest in global assets and are of considerable size, thinly traded instruments are not very useful to them. Their participation in such markets would cause abrupt movement in price. A more realistic approach is to use the most heavily traded futures underlying the related market indices to cross hedge the spot portfolio measured by MSCI indices.

Our objective is to find the single or multiple index futures contracts best suited to hedge each MSCI index. In other words, we are dealing with cross hedging in a situation where **M** is one and **N** is to be determined. We use more sophisticated and theoretically sound models than simple OLS for estimation and thoroughly compare the performances of a variety of alternative strategies.

## 4. Data

We investigate the hedging effectiveness of MSCI stock index of 17 countries using the most heavily traded index futures contracts of the same country and other related countries.

The 17 MSCI indices cover both developed and emerging countries. The developed countries are Australia, Canada, France, Germany, Hong Kong, Japan, Italy, Netherlands, Spain, Sweden, Switzerland, UK and USA. The emerging countries are Brazil, South Korea, South Africa and Taiwan. MSCI computes price index, total return index with dividend reinvested at the minimum tax rate and net return index with dividend reinvested at the maximum tax rate. The price index is used here and the issue of dividend is dealt with separately.

The most heavily traded futures contracts of these 17 countries are the index futures of SPI 200 of Australia, TSE 60[2] of Canada, CAC 40 of France, DAX 30 of Germany, Hang Seng Index of Hong Kong, Nikkei 225 of Japan, MIB 40[3] of Italy, AEX of Netherlands, OMXS 30 of Sweden, SMI of Switzerland, FTSE 100 of UK, S&P 500 of USA, Bovespa[4] of Brazil, KOSPI 200 of South Korea, JSE 40 of South Africa and TAIEX of Taiwan. The month-end MSCI indices and the month-end settlement price of the futures contracts are downloaded from DataStream[5].

All the MSCI indices of developed markets start from December 1969, those of Brazil, South Korea and Taiwan start from December 1987 and that for South Africa starts from December 1992. Each futures series start from its first trading month. The continuous futures series are derived by joining the settlement price of the contract closest to expiration with the next on the first day of the expiration month. The monthly return is computed by taking the log difference of the month-end data.

2 TSE 60 replaced TSE 35 as the benchmark index of Canada in September 1999. Since the two indices are very close in the two overlapping months, we joined the two futures settlement price series in September 1999 to create a complete series.

3 MIB 40 replaced MIB 30 as the benchmark index of Italy in September 2004. Since the two indices are very close in the six overlapping months, the two series of futures settlement price are joined in September 2004 to create a complete series.

4 Because of the hyper-inflation of Brazil in the early 1990s, the data for Bovespa index futures before March 1994 is discarded.

5 The only special case is Taiwan, where TAIEX futures data is downloaded from Taiwan Futures Exchange's website, because Taiwanese futures data is unavailable in DataStream.

In order to derive the cost of carry of futures, the 3-month interbank interest[6] rate of each country and the dividend yield series of the corresponding spot index are also downloaded from DataStream. The dividend yield data of Brazil, Germany, Italy and Sweden are unavailable. The dividend yield of the stock index of ten out of the rest twelve countries starts at the same time as the corresponding index futures. Canadian and South African dividend yield series start in January 2002 and February 1999 respectively several years after the induction of their index futures. The number of months to maturity of each futures contract is computed from studying the lasting period of all the ever-existent futures contracts.

Since gold price has a significant effect on the performance of the stock market of South Africa, gold futures is selected as an alternative hedging instrument for South Africa. The end-month price for one-month gold futures contract traded in COMEX division of New York Mercantile Exchange (NYMEX) is also downloaded from DataStream.

# 5. Methodology

## 5.1 Cointegration relationship

From the analytical solution to the optimal cross hedging vector we can see that the key to find the best cross hedging strategy is to estimate the second moments of the return of MSCI and various index futures accurately. To estimate the second moments, we must model the first moments first. Similar to the cointegration relationship among spot and futures in direct hedging, a cointegration relationship can be established among the level of MSCI and the related index futures, which justifies the Vector-Error-Correction Model (VECM) for the mean of returns.

The cost of carry theory in pricing stock index futures implies a long-term equilibrium relationship represented by a cointegration equation among the level of stock index, the corresponding index futures and the cost of carry. In the previous chapter on direct hedging, its existence is proved in six major index futures markets empirically by the Augmented Dickey-Fuller (ADF) test results of the Engle-Granger approach. In the context of cross hedging, the implication of cost of carry theory is not straightforward. As will be shown

6 For South Korea, Brazil and Taiwan, the 91-Day deposit rate, CDB up to 30 days and 3 month Deposit rate are used as the short-term interest rate.

below, it suggests a long-term equilibrium relationship among the asset being hedged, its own futures, cost of carry and the futures of other related markets.

Denote the MSCI price index of country one as $\mathbf{MSCI_1}$, the price of the most heavily traded index futures contract of country one as $\mathbf{F_1}$ and the price of its underlying as $\mathbf{S_1}$.

MSCI index and the underlying of the index futures of the same country are closely related because they measure the performance of the same market. On the one hand, there is an overlapping group of stocks that are components of both indices. On the other hand, the rest of the stocks in the two indices are different due to the different criteria in stock selection between MSCI and the body that compiles the underlying index of the futures. Furthermore, the weighting methods are different between the two. For example, MSCI indices have become free-float adjusted since June 2002. The free-float adjustment was made to most local stock indices more recently and some indices are still not free-float-adjusted even now. In order to relate the two indices, we introduce a difference factor -- $\mathbf{DF_1}$ to measure the difference between the two. That is,

$$\mathbf{MSCI_1 = S_1 \cdot DF_1} \tag{7}$$

Taking logarithm on both sides gives the following linear relationship.

$$\mathbf{lnMSCI_1 = lnS_1 + lnDF_1} \tag{8}$$

Since the constituent lists of MSCI indices are unavailable, detailed comparison between the components of the two stock indices can not be made. It is therefore impossible to model the difference factor explicitly. Nevertheless, the difference factor can be approximated by some observable variables shown later.

The standard cost of carry theory implies the following relationship among the level of index futures of country one and its underlying index.

$$\mathbf{F_1 = S_1 e^{(r-d)(T-t)}} \tag{9}$$

where $\mathbf{r, d}$ **and** $\mathbf{(T-t)}$ are the interest rate, dividend yield and time to maturity respectively. This theory simply states that the contemporary price of index futures is equal to the price of the underlying plus the cost of interest rate minus the dividend yield.

Denote the cost of carry of the stock index futures of market one as $\mathbf{coc_1}$. That is, $\mathbf{coc_1} = (\mathbf{r} - \mathbf{d})(\mathbf{T} - \mathbf{t})$. The above equation can be written as the following linear relationship among the logarithm of $\mathbf{S_1}$ and $\mathbf{F_1}$ and $\mathbf{coc_1}$.

$$\mathbf{lnF_1 = lnS_1 + coc_1} \tag{10}$$

Combining equation (8) and (10), we get the following equation for $\mathbf{MSCI_1, F_1, coc_1}$ and $\mathbf{DF_1}$.

$$\mathbf{lnMSCI_1 = lnF_1 - coc_1 + lnDF_1} \tag{11}$$

The above equation represents a long-term relationship among one country's MSCI index, index futures and its cost of carry. The last term represents the difference between the MSCI index and the underlying of the index futures. Since it is well-known that the level of MSCI index and index futures are I(1) variable, it is very likely that there is a cointegration relationship among them, cost of carry and other variables measuring the difference factor.

In order to test for cointegration, we have to use some variables to approximate the difference factor. Since the difference factor incorporates the information of either one country's stocks in the list of MSCI index or the underlying index, it is closely related to the stock market performance of this country. Apart from its own market index, the performance of one market is likely to be closely related to that of other related markets since less barrier of capital flow and enhanced globalization of financial markets enable information to be reflected in stock prices in different markets. A number of studies have documented the partial integration of developed and emerging markets to the world market. See, for example, Bekaert, Harvey and Ng (2004), Karolyi and Stulz (2003) and etc. Consequently, the difference factor may be well approximated by the stock indices of the related countries[7]. For example, MSCI USA may be cointegrated with S&P 500 index futures, cost of carry of S&P 500, FTSE 100 index futures and DAX 30 index futures. MSCI Hong Kong may be cointegrated with Hang Seng Index futures, Nikkei 225 futures and TAIEX futures.

7 In the case of South Africa, the gold price has a significant effect on the MSCI index. Therefore a gold future is chosen as an alternative hedging instrument as well.

For simplicity, suppose the index futures of market two $(F_2)$ and three $(F_3)$ can explain a lot of the difference factor of market one. Algebraically, the difference factor can be approximated by the following product.

That is,

$$DF_1 = a \cdot F_2^b \cdot F_3^c \qquad (12)$$

where $a, b$ and $c$ are constant.

Substituting (12) into equation (11), we get a linear relationship among $\ln MSCI_1, \ln F_1,$ $\ln F_2, \ln F_3$ and $coc_1$.

$$\ln MSCI_1 = \ln F_1 - coc_1 + b \cdot \ln F_2 + c \cdot \ln F_3 + \ln a \qquad (13)$$

The above equation represents the long-term relationship among the MSCI index and the settlement price of several index futures contracts. It can be tested for cointegration. If the cointegration relationship exists, the deviation from it should be temporary and have predictive power on future movement of the variables in level. If the stock index and index futures of the two countries as in this example are nonstationary and cointegrated, then we should model their returns by VECM.

To implement the above analysis, we need to find the country or countries whose index futures can approximate the difference factor. Intuitively, stock markets of the countries that are trading partners or are geographically close to each other are very likely to be related and stock market of big countries tends to affect that of small countries. For example, the stock market of a European country is more likely to be affected by other European markets, but less so by Asian markets. US market tends to be influential to all markets.

On the basis of economic intuition, a block of relevant markets is selected for each country. Dynamic OLS cointegration test is performed on MSCI, the index futures of the same country, the cost of carry of the underlying of the same country and the index futures of some other countries in the block. Dynamic OLS method is selected for three reasons. First, as it will be shown in the result section, the logarithm of the level of MSCI index and index futures are I(1) but the cost of carry term is stationary for some countries, therefore invalidating the standard Engle-Granger approach. Second, the estimates of dynamic OLS coefficients are asymptotically normally distributed, therefore making statistical inferences possible. Third, dynamic OLS method has been employed in testing the cointegration relationship in the context of direct hedging in chapter 2. It is consistent to use the same method in related areas of this thesis. Given the theoretical relationship among MSCI index, the index futures of related countries and cost of carry in equation (13), the dynamic OLS equation takes the following form.

$$\begin{aligned}\ln MSCI_{1t} &= \beta_0 + \beta_1 \ln F_{1t} + \beta_2 \ln F_{2t} + \beta_3 \ln F_{3t} + \beta_4 coc_t \\ &+ \sum_{i=-k\neq 0}^{k} \gamma_{1i} \Delta \ln F_{1,t-i} + \sum_{i=-k\neq 0}^{k} \gamma_{2i} \Delta \ln F_{2,t-i} + \sum_{i=-k\neq 0}^{k} \gamma_{3i} \Delta \ln F_{3,t-i} + \sum_{i=-k\neq 0}^{k} \gamma_{4i} \Delta coc_{t-i} \\ &+ u_t\end{aligned}$$

where $\mathbf{lnMSCI_{1t}}$, $\mathbf{lnF_{1t}}$ and $\mathbf{coc_{1t}}$ are the MSCI index, index futures and cost of carry of one country and $\mathbf{lnF_{2t}}$ **and** $\mathbf{lnF_{3t}}$are the index futures of two related countries. ADF test will be used to test the stationarity of the estimated residual. If the hypothesis of non-stationarity is rejected, then these variables are cointegrated; otherwise, the cointegration relationship can not be established.

In the cases where cointegration relationship can be found for the MSCI index and a group of index futures, we keep the residuals from the estimated cointegrating vector as the error-correction term in VECM for the return of MSCI index and the futures. This two-step method is valid as long as the information matrix is block-diagonal. In the next section, we model the covariance matrix of the VECM residuals.

### 5. 2. Multivariate GARCH

Recall the analytical solution to the optimal hedge vector in equation (6)

$$\mathbf{h^* = y^*/x = \Sigma_{ff}^{-1}\Sigma_{sf}'}$$

which is the product of the inverse of the covariance matrix of the return of futures and the transpose of the vector of the covariance between the return of the asset to be hedged and the futures. To estimate them, we have to model the second moments of the return of spot and futures jointly.

Most empirical studies in the literature estimate the cross hedging ratios from a multivariate OLS regression. Implicitly, they assume the second moments are constant over time. However, it is well-known that the second moments of asset returns are subject to volatility clustering effect, i.e. big shocks are more likely to be followed by big shocks and small shocks are more likely to be followed by small shocks. In other words, the second moments are very likely to be time-dependent and probably autocorrelated. If this effect does exist in a particular data set, then it would invalidate the use of OLS estimation method.

In order to accommodate the feature of changing conditional volatility, several econometric models have been proposed, among which the class of Generalized Autoregressive Conditionally Heteroskedastic Model (GARCH) is the most frequently used in the literature.

Autoregressive Conditionally Heteroskedastic Model (ARCH) proposed in Engle (1982) explicitly models conditional variance by a moving average process of past squared innovations, so that past shocks have a direct effect on future conditional variance. Despite its conceptual innovation, ARCH of high order turns out to be difficult to estimate due to the non-negativity condition on all the coefficients.

Bollerslev (1986) developed the Generalized Autoregressive Conditionally Heteroskedastic Model (GARCH) that specifies conditional variance as an ARMA process with past squared innovations in the moving average part. It is proved that GARCH is equivalent to ARCH of infinite order with a certain constraint on the coefficients. Researchers using data of different frequency have demonstrated that the simplest form of GARCH -- GARCH (1,1) is successful in measuring and forecasting volatility. For example, Bollerslev and Baillie (1991), Baillie and Myers (1991), Jacobsen and Dannenburg (2003), Giovannini and Jorion (1989) and Bollerslev, Engle and Wooldridge (1988) implemented ARCH or GARCH to model the intraday foreign exchange rate, to derive the optimal hedge ratio in the commodity market using a daily data set, to test International CAPM using a weekly data set, to model monthly stock return and to model the quarterly equity risk premium respectively.

Another important development in this field is the invention of the class of asymmetric GARCH model. The asymmetric effect of conditional variance states that not only the size of shocks but also the sign of shocks have significant impact on future volatility. Intuitively, an increase in volatility leads to an increase in the required asset returns, therefore decreasing the asset prices. This volatility feedback effect amplifies negative shocks and dampens positive shocks, making conditional volatility asymmetric. Nelson (1993) and Glosten and et al. (1993) developed Exponential GARCH ( EGARCH ) and threshold GARCH ( TGARCH ) respectively to accommodate the asymmetric effect. Engle and Ng (1993) compared the two models empirically and concluded that the former is too sensitive to extreme values and the latter is robust.

Similar to the rapid development in the field of univariate GARCH model, various specifications of multivariate GARCH model such as VECH, BEKK and constant-correlation GARCH have been developed by Bera and et al. (1987), Bollerslev, Engle and Wooldridge (1988) and Engle and Kroner (1995) respectively.

While the majority studies prove the success of GARCH model in describing high-frequency data from intraday to weekly, GARCH is also suitable for the data at monthly frequency. Drost and Nijman (1993) show that GARCH process is closed on temporal aggregation. In other words, if a high-frequency series is a GARCH process, then its corresponding low-frequency series is also a GARCH process. In the preliminary study[8] of this chapter, volatility clustering effect is discovered in the daily returns, which indicates that the use of GARCH model for the monthly return is appropriate. Besides, the usual empirical test results shown later also suggest that there is indeed volatility clustering effect in the residuals of several monthly returns. Furthermore, Drost and Nijman (1993) and Hafner and Rombouts (2006) demonstrate that quasi maximum likelihood (QML) estimates of the aggregated GARCH process are consistent in univariate and multivariate case respectively. Therefore the usual maximum likelihood estimation method is valid.

Our goal in this chapter is to find the best strategy to cross hedge a spot portfolio measured by MSCI index of country j (j=1, 2 ... 17). Since the theory on cross hedging does not suggest a unique cross hedging instrument or combination of instruments, finding the best strategy relies on the empirical comparison of the performances of different strategies. The model that fits the data well and produces consistent outstanding hedging performance both within- and out-of-sample has the strongest support from the data.

8 The results are not included due to the lack of space.

To make the comparison as thorough as possible, the hedge ratios suggested by a variety of possible strategies are estimated. For country $j$, the block made up of country $m, \dots, p$ is decided on the basis of economic relevance[9]. Their most frequently traded index futures are the possible instruments to hedge $\mathbf{MSCI_j}$. Since the index futures of country $j$ -- $f_j$ is obviously related to $\mathbf{MSCI_j}$, it is added to the block. Hedging strategies involving one, two and three instruments are formed, where the instruments are selected from $f_j, f_m, \dots, f_p$.

Another way in explaining the model is as follows. For each MSCI index, we try to model a eighteen-element vector consisting of the return of its MSCI index and the seventeen index futures. For a particular country $j$, the block consisting of country $m, \dots, p$ is selected. Only elements -- $\mathbf{MSCI_j}, f_j, f_m, \dots$ **and** $\mathbf{f_p}$ of the vector are kept for estimation and the rest are set to zero. In fact, only a particular subset of the eighteen returns is modelled and this subset varies across countries.

When a cointegration relationship is identified for a country, the deviation from it is fitted to the mean of returns because of its predictive power. The second moments of their residuals are modelled by constant ( OLS ), multivariate GARCH (1,1) or TGARCH (1,1). In particular, for cross hedging with $\mathbf{k}$ instruments $(\mathbf{k} = 1,2,3)$, the $(\mathbf{k}+1) \times (\mathbf{k}+1)$ covariance matrix of the return of the asset being hedged and the $\mathbf{k}$ futures are modelled by a multivariate GARCH(1,1) or TGARCH(1,1) process.

Take UK as an example. Suppose the block of UK involves UK, France, Germany and USA and MSCI UK is cointegrated with FTSE 100, S&P 500 and DAX 30 futures. The one instrument strategy only involves FTSE 100 index futures. The two instrument strategy involves FTSE 100 and either CAC 40 or DAX 30 or S&P 500 index futures. The three instrument strategy involves FTSE 100, S&P 500 and either CAC 40 or DAX 30 index futures. For each set of instruments, the optimal hedge ratio or ratios are estimated from models such as OLS, multivariate GARCH (1,1) and TGARCH (1,1) with and without the deviation from the cointegration relationship fitted to the mean.

Two technical questions concerning estimation are the specification of multivariate GARCH and the assumption on the standardized residuals.

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9 For South Africa, gold futures is also a possible hedging instrument.

We select the diagonal VECH specification for three reasons.[10] First, it is intuitive to model the conditional second moments by their own lags and the squared residuals rather than those of the return of others. Second, it is more important to save parameters in the case of trivariate and quadrivariate GARCH model than bivariate GARCH. Therefore the diagonal VECH is preferable to the full VECH. Third, compared with diagonal BEKK, diagonal VECH is less restrictive. In cases where estimation is successful for both, the latter is preferable.

The standardized residuals are assumed to be multi-normally distributed. As shown in the chapter on direct hedging, the assumption on the distribution of residuals has little effect on the estimation results, therefore Student-t distribution is not used here.

The specification of the model is as follows. The residuals of VECM or the deviations of returns from the mean are modelled by the following GARCH and TGARCH process.

When there are $\mathbf{k}$ hedging instruments, the $(\mathbf{k}+\mathbf{1}) \times (\mathbf{k}+\mathbf{1})$ dimensional covariance matrix

$\mathbf{H_t}$ has the following standard GARCH specification.

$$\mathrm{vech}(H_t) = \Omega + B \cdot \mathrm{vech}(H_{t-1}) + A \cdot \mathrm{vech}(\varepsilon_{t-1} \cdot \varepsilon_{t-1}') \qquad (14)$$

$\varepsilon_{t-1}$ is a vector with $\mathbf{k+1}$ elements, i.e. $\varepsilon_{t-1} = (\varepsilon_{1,t-1}, \varepsilon_{2,t-1}, \dots . \varepsilon_{k+1,t-1})$;

$\Omega$ is a vector with $\frac{(1+k)(2+k)}{2}$ elements;

$\mathbf{A}$ and $\mathbf{B}$ are both $\frac{(1+k)(2+k)}{2} \times \frac{(1+k)(2+k)}{2}$ matrices.

The GARCH standardized residuals are subject to a $(\mathbf{k}+\mathbf{1}) \times (\mathbf{k}+\mathbf{1})$ multi-variate normal distribution.

Or $\mathbf{H_t}$ has the following TGARCH specification.

[10] See the specification of diagonal VECH in equation (14).

$$\mathrm{vech}(H_t) = \Omega + B \cdot \mathrm{vech}(H_{t-1}) + A \cdot \mathrm{vech}(\varepsilon_{t-1} \cdot \varepsilon_{t-1}^{'}) + D \cdot \mathrm{vech}(u_{t-1} \cdot u_{t-1}^{'}) \quad (15)$$

$u_{t-1}$ is a vector with $k+1$ elements, i.e. $u_{t-1} = (u_{1,t-1}, u_{2,t-1}, \dots, u_{k+1,t-1})$, where $u_{i,t-1} = \min[\varepsilon_{i,t-1}, 0]$.

$A, B$ and $D$ are all $\frac{(1+k)(2+k)}{2} \times \frac{(1+k)(2+k)}{2}$ matrices and the standardized residuals are assumed

to have a multi-normal distribution.

The entire sample is divided into within-sample and out-of-sample. The former ends in December 2005 and the latter ends in May 2007. The models are first estimated using the within-sample data. The within-sample hedge ratios are computed from the estimates of the second moments. Then one-month forecast of the covariance matrices is generated from the model estimates each month in the out-of-sample period. And the one-step-ahead hedge ratios are computed accordingly. The usual measurement of hedging effectiveness is computed as one minus the quotient of the variance of the hedged portfolio to that of the spot portfolio.

## 6. Results

Table 1 contains the unit root test results for the logarithm and return of MSCI and most heavily traded index futures and the cost of carry of seventeen countries. In particular, the Augmented Dickey-Fuller (ADF) test is performed for each series, where the number of lag is selected on the basis of AIC. The first line of each country contains ADF test statistics and the second line contains the corresponding P-values. The unit root null hypothesis is not rejected for all level variables but rejected for all returns. It is clear that the logarithm of all indices are I(1), therefore justifying the search for cointegration relationship. In contrast to the stationarity test result in chapter 2 on direct hedging, the ADF test results suggest the cost of carry term is I(1) for ten of eleven countries, with the exception of Japan. This is probably due to the difference in data frequency. Because we do not have the dividend yield data for Germany, Italy, Sweden and Brazil and the dividend yield series of Canada and South Africa are shorter than that of index futures, ADF test can not be performed on the cost of carry of these six countries.

**TABLE 1: Stationarity test results for the level and return of MSCI index and index futures and the cost of carry of the index futures**

| | au | br | ca | fr | gm | hk | it | jp | kr |
|---|---|---|---|---|---|---|---|---|---|
| lnMSCI | -0.42 | -1.17 | 0.22 | -0.94 | -0.98 | -1.41 | -2.42 | -2.23 | -1.08 |
| p-value | 0.90 | 0.69 | 0.97 | 0.77 | 0.76 | 0.58 | 0.14 | 0.20 | 0.72 |
| lnF | -0.88 | -0.52 | -0.22 | -0.86 | -0.95 | -1.55 | -1.45 | -1.70 | -1.15 |
| p-value | 0.79 | 0.88 | 0.93 | 0.80 | 0.77 | 0.50 | 0.55 | 0.43 | 0.69 |
| R(MSCI) | -18.36 | -6.23 | -17.27 | -16.36 | -17.10 | -10.54 | -17.05 | -16.96 | -14.19 |
| p-value | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| R(futures) | -19.43 | -6.97 | -12.90 | -14.35 | -13.56 | -15.21 | -13.33 | -14.86 | -10.00 |
| p-value | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| COC | -2.27 | NA | NA | -1.44 | NA | -2.58 | NA | -6.06 | -2.54 |
| p-value | 0.18 | NA | NA | 0.56 | NA | 0.10 | NA | 0.00 | 0.11 |
| | ne | so | sp | sw | sz | tw | uk | us | |
| lnMSCI | -1.36 | -0.39 | -0.80 | -1.84 | -0.52 | -3.62 | -1.93 | -0.87 | |
| p-value | 0.60 | 0.91 | 0.82 | 0.36 | 0.88 | 0.01 | 0.32 | 0.80 | |
| lnF | -1.10 | 0.39 | -0.84 | -0.58 | -1.36 | -2.19 | -1.97 | -1.72 | |
| p-value | 0.72 | 0.98 | 0.80 | 0.87 | 0.60 | 0.21 | 0.30 | 0.42 | |
| R(MSCI) | -17.12 | -13.26 | -16.30 | -15.68 | -7.19 | -13.84 | -13.93 | -17.85 | |
| p-value | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | |
| R(futures) | -14.29 | -15.05 | -13.15 | -12.85 | -5.32 | -5.97 | -12.81 | -17.47 | |
| p-value | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | |
| COC | -2.42 | NA | -1.64 | NA | -2.71 | -1.69 | -1.61 | -2.50 | |
| p-value | 0.14 | NA | 0.46 | NA | 0.07 | 0.43 | 0.47 | 0.12 | |

Note: 1. This table contains the ADF test statistics and their p-values of the logarithm and return of monthly MSCI index and index futures of 17 countries and the cost of carry term of 11 index futures. lnMSCI is the logarithm of the level of MSCI. lnF is the logarithm of the level of futures. R(MSCI) and R(futures) are the returns of MSCI and index futures. COC is the cost of carry.

2. au, br, ca, fr, gm, hk, it, jp, kr, ne, so, sp, sw, sz, tw, uk and us are short for Australia, Brazil, Canada, France, Germany, Hong Kong, Italy, Japan, South Korea, Netherlands, South Africa, Spain, Sweden, Switzerland, UK and USA repectively.

Table 2 contains the results for cointegration test using dynamic OLS method. The test equation is shown in the first row where the dependent variable is the logarithm of MSCI index of a particular country and the independent variables are the logarithm of index futures of the same country and the other one or two related countries. The leads and lags of the first-order difference of these variables are also included on the right-hand side. Note that the cost of carry is not an independent variable in this equation. In the preliminary tests where it is included, the coefficient estimate of the cost of carry is implausibly big, though it should be unity in theory. Given the results on cost of carry are incomprehensible; we exclude it in testing for cointegration relationship. The number of leads and lags is chosen to be two because others seem to be insignificantly different from zero in the preliminary test results.

## Table 2: Cointegration test results

Cointegration equation

$$\ln MSCI_{1t} = \beta_0 + \beta_1 \ln F_{1t} + \beta_2 \ln F_{2t} + \beta_3 \ln F_{3t} + \sum_{i=-k}^{k} \gamma_{1i} \Delta \ln F_{1,t-i} + \sum_{i=-k}^{k} \gamma_{2i} \Delta \ln F_{2,t-i} + \sum_{i=-k}^{k} \gamma_{3i} \Delta \ln F_{3,t-i} + u_t$$

ADF test equation

$$\Delta \hat{u}_t = \rho \cdot \hat{u}_t + \sum_{i=1}^{k} \gamma_i \cdot \Delta \hat{u}_{t-i} + \varepsilon_t$$

| | BR | CA | FR | GM | HK | IT | JP | KR |
|---|---|---|---|---|---|---|---|---|
| $\beta_0$ | 16.44 | 0.40 | -1.49 | -1.05 | 1.25 | -2.73 | -3.34 | 2.91 |
| sd. | 0.19 | 0.03 | 0.04 | 0.04 | 0.19 | 0.09 | 0.16 | 0.15 |
| $\beta_1$ | 0.95 | 0.98 | 0.85 | 1.11 | 0.57 | 0.89 | 0.77 | 1.11 |
| sd. | 0.02 | 0.02 | 0.01 | 0.02 | 0.04 | 0.03 | 0.01 | 0.01 |
| $\beta_2$ | -0.33 | 0.01 | 0.23 | -0.27 | 0.40 | 0.07 | 0.05 | -0.07 |
| sd. | 0.03 | 0.01 | 0.01 | 0.02 | 0.04 | 0.04 | 0.02 | 0.05 |
| $\beta_3$ | 0.18 | 0.04 | | | | | 0.29 | -0.23 |
| sd. | 0.02 | 0.01 | | | | | 0.02 | 0.03 |
| t-adf | -7.06 | -4.17 | -2.99 | -2.32 | -4.53 | -2.07 | -3.93 | -4.62 |
| 1% c.v. | -3.47 | -3.47 | -3.46 | -3.46 | -3.50 | -3.48 | -3.48 | -3.49 |
| 5% c.v. | -2.88 | -2.88 | -2.88 | -2.88 | -2.89 | -2.88 | -2.88 | -2.89 |
| 10% c.v. | -2.58 | -2.58 | -2.57 | -2.57 | -2.58 | -2.58 | -2.58 | -2.58 |
| | **NE** | **SP** | **SO** | **SW** | **TW** | **UK** | **US** | |
| $\beta_0$ | 1.34 | -1.45 | -2.60 | 1.98 | -2.71 | -0.98 | -0.59 | |
| sd. | 0.22 | 0.14 | 0.12 | 0.09 | 0.20 | 0.06 | 0.04 | |
| $\beta_1$ | 0.89 | 1.09 | 0.85 | 1.20 | 0.96 | 0.97 | 0.98 | |
| sd. | 0.03 | 0.03 | 0.02 | 0.02 | 0.03 | 0.01 | 0.01 | |
| $\beta_2$ | 0.02 | 0.35 | 0.00 | -0.15 | -0.01 | 0.04 | 0.12 | |
| sd. | 0.04 | 0.03 | 0.02 | 0.03 | 0.03 | 0.01 | 0.01 | |
| $\beta_3$ | | -0.50 | 0.05 | | | -0.04 | -0.04 | |
| sd. | | 0.03 | 0.01 | | | 0.01 | 0.01 | |
| t-adf | -2.30 | -4.10 | -5.05 | -2.98 | -2.65 | -3.67 | -3.74 | |
| 1% c.v. | -3.46 | -3.47 | -3.47 | -3.46 | -3.50 | -3.46 | -3.47 | |
| 5% c.v. | -2.88 | -2.88 | -2.88 | -2.88 | -2.89 | -2.88 | -2.88 | |
| 10% c.v. | -2.57 | -2.58 | -2.58 | -2.57 | -2.58 | -2.57 | -2.57 | |

Note: 1. This table contains the DOLS cointegration test results for 15 countries. For each country, the dependent variable is the logarithm of MSCI index and the independent variables are the logarithm of the index futures of the same country and one or two futures of the other countries. The cointegrating vector estimates and their standard deviations are presented. The residuals are subject to ADF test. The ADF test statistics and their critical values are shown below the coefficient estimates.

2. The extra index futures instruments apart from that of their own for Brazil (BR), Canada (CA), France (FR), Germany (GM), Hong Kong (HK), Italy (IT), Japan (JP), South Korea (KR), Netherlands (NE), Spain (SP), South Africa (SO), Sweden (SW), Taiwan (TW), UK and US are the index futures of US and South Africa, US and Brazil, US, US, Singapore, US, US and Australia, US and Japan, UK, US and UK, US and Brazil, Germany, Hong Kong, US and France and UK and Germany.

The extra futures contracts for hedging each MSCI index are chosen on the basis of plausibility. See note 2 of Table 2 for the list of countries whose index futures contract are chosen for the MSCI of each country. From the ADF test statistics for the estimated residuals we can see that the null hypothesis of non-stationarity can be rejected at 1% level for nine countries including Brazil, Canada, Hong Kong, Japan, South Korea, Spain, South Africa, UK and USA, at 5% for two countries including France and Sweden and at 10% level for Taiwan. In the case of Germany, Italy and Netherlands, the ADF test statistics are not significant at conventional level, though they are close to 10% critical level. These results support the view that there is a cointegration relationship among the MSCI index and related index futures in these fifteen countries. This table does not report the test results for Australia and Switzerland since there is hardly any evidence of cointegration for their MSCI index with related index futures. Examining the coefficient estimates and their standard deviations, we can see that most coefficient estimates are highly significantly different from zero. Together with the ADF test statistics they support the view that there is a long-term relationship represented by the estimated cointegrating vector for the logarithm of MSCI index and that of the related index futures. The deviation from this relationship can help to predict the future movement of these variables to a degree, i.e. a VECM can be established to model the return of these indices with the lagged residual of the estimated equations above as error-correction term.

In the next four tables, we present the hedging performance results of around forty strategies for each country. As explained in the methodology part of this chapter, cross hedging theory does not imply which hedging instruments should be used for hedging each MSCI index. Instead, it suggests a range of combinations of index futures contracts of related countries as candidate instruments for cross hedging. For example, when facing the problem of hedging MSCI UK, we can use FTSE 100 index futures with either DAX 30 or S&P 500. The question of which combination is the best can only be answered empirically. Furthermore, we not only face the choice of hedging instruments, but also the econometric models of the return of these instruments. As in Chapter 2, the candidate models vary from simple OLS regression to complicated VECM combined with GARCH. For example, if we decide to use FTSE 100 index futures with the index futures of DAX 30 to hedge MSCI UK, we can model the three returns by simple OLS, VECM, VECM with trivariate GARCH or VECM with trivariate TGARCH, each of which corresponds to an estimated covariance matrix. From the covariance matrix estimates, we can not only compute the within-sample cross hedge ratios but also generate the one-step-ahead forecasts of covariance matrix and compute the out-of-sample cross hedge ratios. The combination of hedging instruments and econometric model of their return corresponds to a hedging strategy. All the strategies can be simulated and evaluated on the basis of hedged portfolio variance and hedging effectiveness. The one corresponding to low portfolio variance and high hedging effectiveness in both within and out-of-sample period is considered the best.

Specifically, Table 3 - 6 contain the descriptive statistics of the return and hedging effectiveness of a variety of strategies for each country. Each strategy is based on a model of the return of several hedging instruments. If we were to present the estimation results of all the models, we would end up with several hundreds of extra tables in this thesis. Since these models are rather similar and the more important aim of this chapter is to find the best strategy for each country, we decide not to include most of these tables but only present the ones of GARCH class corresponding to the best strategy for a particular country in the appendix. All of the estimated GARCH models shown in the appendix are stationary, adequate and successfully estimated.

In Table 3 - 6, the statistics of both within- and out-of-sample hedged portfolio returns are shown, where the former are in the row starting with 'IN' and the latter in the row starting with 'OUT'. Naïve and OLS with the index futures of the same country are the benchmarks. Their standard deviation and hedging effectiveness are highlighted in blue and grey respectively. The hedging strategies with lower standard deviation and higher hedging effectiveness are better than the benchmarks and highlighted in yellow. The best strategy outperforms the benchmarks in both within and out-of-sample period and is highlighted in red.

The seventeen countries are divided into four groups on the basis of portfolio volatility and degree of hedging effectiveness. Brazil, Hong Kong and South Africa are in the first group because of their volatile returns in both within- and out-of-sample period. As shown in Part

A, B and C of Table 3, the standard deviation of their hedged monthly return suggested by naïve and simple OLS are around or above 2% and their hedging effectiveness are around or below 90% in both periods. In particular, the out-of-sample hedging effectiveness of Hong Kong and South Africa are less than 75%. The low hedging effectiveness leaves plenty of scope for improvement by adopting sophisticated strategies.

The results in Table 3 demonstrate that the improvement of sophistication is indeed big and universal. From the within-sample results we can see that thirty-two of thirty-five strategies outperform the benchmarks in Brazil, thirty-seven of forty-seven strategies outperform in Hong Kong and thirty-eight of forty-seven strategies outperform in South Africa.

The best strategy for Brazil involves BOVESPA and JSE 40 index futures modelled by VECM and trivariate GARCH (1,1). It reduces the standard deviation from 3.73% of naïve and 3.32% of OLS to 2.89% and improves the hedging effectiveness from 86.94% of naïve and 89.68% of OLS to 92.17%. The out-of-sample result for this strategy is consistent with its within-sample counterpart. The hedging effectiveness is improved to 85.79% from 85.63% of naïve and 84.62% of OLS in the out-of-sample period.

# TABLE 3 ( Part A ): Hedging Performance results for MSCI Brazil

| | | With One Hedging Instrument | | | | | | | With Three Hedging Instruments | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | BOVESPA | | | | | | | BOVESPA S&P 500 SPI 200 | | | | | | BOVESPA S&P 500 JSE 40 | | | | | |
| Method | | naïve | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.10 | 0.14 | 0.14 | 0.11 | 0.10 | 0.10 | 0.10 | 0.14 | 0.13 | 0.08 | 0.07 | 0.07 | 0.08 | 0.14 | 0.14 | 0.08 | 0.09 | 0.08 | 0.08 |
| OUT | | 0.04 | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 | 0.06 | 0.06 | 0.04 | 0.05 | 0.04 | 0.05 | 0.06 | 0.06 | 0.05 | 0.04 | 0.05 | 0.05 |
| IN | Min | -0.12 | -0.08 | -0.08 | -0.08 | -0.08 | -0.08 | -0.08 | -0.08 | -0.08 | -0.07 | -0.08 | -0.08 | -0.07 | -0.08 | -0.08 | -0.07 | -0.07 | -0.08 | -0.08 |
| OUT | | -0.05 | -0.04 | -0.04 | -0.05 | -0.05 | -0.05 | -0.05 | -0.04 | -0.04 | -0.05 | -0.05 | -0.73 | -0.13 | -0.04 | -0.04 | -0.04 | -0.05 | -0.07 | -0.11 |
| IN | S. D. | 3.73 | 3.32 | 3.32 | 3.06 | 2.96 | 3.07 | 2.94 | 3.31 | 3.31 | 3.08 | 3.05 | 3.04 | 3.06 | 3.29 | 3.29 | 2.96 | 2.92 | 3.04 | 2.97 |
| OUT | (%) | 2.14 | 2.22 | 2.22 | 2.19 | 2.60 | 2.20 | 2.74 | 2.23 | 2.23 | 2.37 | 2.38 | 17.68 | 3.86 | 2.17 | 2.17 | 2.28 | 2.35 | 2.86 | 3.41 |
| IN | Skew- | -0.06 | 0.34 | 0.34 | 0.05 | 0.00 | 0.05 | 0.04 | 0.27 | 0.26 | -0.06 | -0.11 | -0.12 | -0.09 | 0.38 | 0.36 | 0.05 | 0.04 | 0.05 | 0.00 |
| OUT | ness | 0.14 | 0.76 | 0.76 | 0.11 | -0.06 | 0.17 | 0.15 | 0.65 | 0.62 | -0.08 | -0.01 | -3.65 | -1.77 | 0.74 | 0.70 | 0.56 | -0.16 | -0.22 | -1.37 |
| IN | Kur- | 3.59 | 4.71 | 4.71 | 3.58 | 3.62 | 3.55 | 3.47 | 4.39 | 4.30 | 3.01 | 2.95 | 2.87 | 3.11 | 4.62 | 4.47 | 2.92 | 3.13 | 3.02 | 3.11 |
| OUT | tosis | 3.00 | 4.80 | 4.81 | 3.30 | 2.74 | 3.44 | 2.82 | 4.61 | 4.55 | 2.53 | 2.56 | 14.60 | 6.86 | 4.38 | 4.31 | 3.38 | 2.37 | 3.43 | 5.85 |
| IN | HE | 86.94 | 89.68 | 89.68 | 91.21 | 91.78 | 91.16 | 91.90 | 89.75 | 89.74 | 91.12 | 91.26 | 91.35 | 91.23 | 89.87 | 89.86 | 91.77 | 92.01 | 91.37 | 91.73 |
| OUT | (%) | 85.63 | 84.62 | 84.60 | 85.01 | 78.79 | 84.84 | 76.44 | 84.48 | 84.44 | 82.37 | 82.32 | NE | 53.49 | 85.28 | 85.27 | 83.74 | 82.77 | 74.40 | 63.66 |

| | | With Two Hedging Instruments | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | BOVESPA SPI 200 | | | | | | BOVESPA JSE 40 | | | | | | BOVESPA S&P 500 | | | | | |
| Method | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.14 | 0.14 | 0.08 | 0.08 | 0.08 | 0.08 | 0.15 | 0.15 | 0.08 | 0.08 | 0.08 | 0.07 | 0.13 | 0.13 | 0.09 | 0.09 | 0.09 | 0.11 |
| OUT | | 0.06 | 0.06 | 0.05 | 0.06 | 0.05 | 0.07 | 0.06 | 0.06 | 0.04 | 0.05 | 0.04 | 0.84 | 0.06 | 0.06 | 0.05 | 0.10 | 0.05 | 0.08 |
| IN | Min | -0.08 | -0.08 | -0.07 | -0.07 | -0.07 | -0.07 | -0.08 | -0.08 | -0.08 | -0.07 | -0.08 | -0.07 | -0.08 | -0.08 | -0.08 | -0.08 | -0.08 | -0.08 |
| OUT | | -0.04 | -0.04 | -0.04 | -0.09 | -0.04 | -0.06 | -0.04 | -0.04 | -0.04 | -0.12 | -0.04 | -0.80 | -0.04 | -0.04 | -0.05 | -0.04 | -0.05 | -0.04 |
| IN | S. D. | 3.32 | 3.32 | 3.07 | 3.03 | 3.09 | 3.07 | 3.31 | 3.31 | 2.89 | 2.93 | 2.89 | 2.93 | 3.31 | 3.31 | 3.11 | 2.81 | 3.16 | 3.03 |
| OUT | (%) | 2.23 | 2.23 | 2.25 | 3.32 | 2.23 | 2.82 | 2.18 | 2.18 | 2.11 | 4.21 | 2.13 | 30.11 | 2.23 | 2.23 | 2.30 | 3.25 | 2.25 | 2.96 |
| IN | Skew- | 0.30 | 0.30 | -0.10 | 0.03 | -0.08 | 0.03 | 0.45 | 0.45 | 0.03 | -0.05 | 0.04 | -0.06 | 0.25 | 0.23 | -0.05 | 0.05 | -0.03 | 0.22 |
| OUT | ness | 0.75 | 0.75 | 0.38 | -0.64 | 0.37 | 0.50 | 0.85 | 0.85 | 0.55 | -1.31 | 0.56 | 0.20 | 0.65 | 0.63 | 0.26 | 1.26 | 0.18 | 0.74 |
| IN | Kur- | 4.57 | 4.56 | 3.06 | 3.08 | 2.97 | 3.08 | 5.06 | 5.06 | 2.94 | 3.08 | 2.93 | 3.08 | 4.34 | 4.22 | 3.14 | 3.44 | 3.24 | 3.84 |
| OUT | tosis | 4.83 | 4.84 | 3.51 | 3.94 | 3.46 | 3.81 | 4.64 | 4.65 | 3.02 | 4.33 | 3.03 | 7.42 | 4.68 | 4.65 | 4.00 | 5.15 | 3.87 | 3.59 |
| IN | HE | 89.68 | 89.68 | 91.14 | 91.40 | 91.05 | 91.17 | 89.74 | 89.74 | 92.17 | 91.97 | 92.16 | 91.97 | 89.75 | 89.74 | 90.93 | 92.60 | 90.64 | 91.42 |
| OUT | (%) | 84.50 | 84.48 | 84.19 | 65.55 | 84.50 | 75.14 | 85.16 | 85.15 | 86.12 | 44.49 | 85.79 | NE | 84.48 | 84.41 | 83.52 | 66.95 | 84.17 | 72.58 |

Note: 1. This table contains the performance results of 37 strategies for hedging MSCI Brazil. The strategies involving one hedging instrument are shown in the top left block of the table and this instrument is BOVESPA index futures of Brazil. The strategies involving three instruments are shown in the other two top blocks. They are BOVESPA with S&P 500 of US and SPI 200 of Australia and BOVESPA with S&P 500 of US and JSE 40 of South Africa respectively. The strategies invoving two instruments are shown in the bottom three blocks. They are BOVESPA with SPI 200, BOVESPA with JSE 40 and BOVESPA with S&P 500 respectively.

2. Six econometric models are used to generate hedge ratios for each set of hedging instruments. 'ols' corresponds to the regression of the return of MSCI Brazil on the left-hand side and that of the hedging instruments on the right-hand side. 'ci-ols' corresponds to the 'ols' regression with the lagged error-correction term as additional variable. 'garch' corresponds to multivarate GARCH (1,1) for the returns of MSCI Brazil and its hedging instrument(s). 'tgarch' corresponds to multivariate TGARCH (1,1) for the same set of variables. 'ci-garch' corresponds to VECM combined with multivariate GARCH (1,1). 'ci-tgarch' corresponds to VECM combined with multivariate TGARCH (1,1).

3. The rows starting with 'IN' contain the within-sample results and those starting with 'OUT' contain the out-of-sample results.

4. 'HE' stands for hedging effectiveness.

5. The naive and single-instrument OLS strategy are highlighted in blue and grey respectively. The strategies that outperform both of them are highlighted in yellow. The best strategy is highlighted in red.

6. MSCI Brazil, BOVESPA futures, S&P 500 futures and JSE 40 futures are cointegrated.

7. NE is short for negative value.

The improvement in Hong Kong is pronounced. The best strategy involves Hang Seng, MSCI Singapore and TAIEX index futures and is estimated by OLS method with error-correction term fitted to the mean. It improves hedging effectiveness from around 85% to 87.88% within the sample and from slightly less than 75% to 82.40% in the out-of-sample period.

The performance of cross hedging strategy in South Africa is also impressive. The best strategy suggested by the data involves JSE 40 and BOVESPA modelled by trivariate TGARCH (1,1) combined with error-correction model. The hedging effectiveness is improved from less than 92% to 93.33% within-sample and from less than 72% to 74.79% in the out-of-sample period. However, the coefficient estimates of TGARCH are insignificant, suggesting the model is unreliable. Therefore the strategy involving JSE and gold estimated by OLS method is selected instead. Although its within-sample improvement is not big, it produces the best out-of-sample results among forty-nine alternatives, with the hedging effectiveness at 75.44%.

In Sample (IN): 1998M12 2005M12

Out of Sample (OUT): 2006M01 2007M05

**TABLE 3 ( Part B ): Hedging Performance results for MSCI Hong Kong**

With One Hedging Instrument

| Hedging Instrument | | Hang Seng | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Method | | naïve | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| OUT | | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.05 | 0.02 |
| IN | Min | -0.08 | -0.09 | -0.09 | -0.09 | -0.09 | -0.09 | -0.09 |
| OUT | | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.05 |
| IN | S. D. | 2.52 | 2.45 | 2.45 | 2.44 | 2.49 | 2.49 | 2.52 |
| OUT | (%) | 1.57 | 1.56 | 1.56 | 1.73 | 1.54 | 1.93 | 1.82 |
| IN | Skew- | -0.26 | -0.53 | -0.56 | -0.58 | -0.60 | -0.70 | -0.66 |
| OUT | ness | -0.39 | -0.42 | -0.43 | -0.46 | -0.31 | 0.71 | -0.90 |
| IN | Kur- | 3.81 | 4.61 | 4.70 | 4.87 | 4.91 | 5.33 | 5.13 |
| OUT | tosis | 1.61 | 1.81 | 1.86 | 1.76 | 1.60 | 4.18 | 2.92 |
| IN | HE | 84.63 | 85.56 | 85.54 | 85.65 | 85.09 | 85.00 | 84.68 |
| OUT | (%) | 74.57 | 74.76 | 74.68 | 69.10 | 75.31 | 61.44 | 65.54 |

With Three Hedging Instruments

| Hedging Instrument | | Hang Seng MSCI Singapore Nikkei 225 | | | | | |
|---|---|---|---|---|---|---|---|
| Method | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.06 | 0.05 | 0.06 | 0.06 | 0.07 | 0.07 |
| OUT | | 0.01 | 0.01 | 0.05 | 0.02 | 0.02 | 0.02 |
| IN | Min | -0.06 | -0.05 | -0.06 | -0.06 | -0.07 | -0.07 |
| OUT | | -0.03 | -0.03 | -0.04 | -6.60 | -0.05 | -0.05 |
| IN | S. D. | 2.15 | 2.17 | 2.35 | 2.16 | 2.67 | 2.66 |
| OUT | (%) | 1.36 | 1.35 | 2.44 | 160 | 1.91 | 2.15 |
| IN | Skew- | -0.06 | 0.02 | 0.33 | 0.26 | 0.32 | 0.35 |
| OUT | ness | -0.47 | -0.47 | 0.20 | -3.75 | -0.93 | -0.68 |
| IN | Kur- | 3.11 | 3.01 | 3.03 | 3.21 | 3.22 | 3.28 |
| OUT | tosis | 2.14 | 2.17 | 2.46 | 15.06 | 2.90 | 2.43 |
| IN | HE | 88.85 | 88.65 | 86.70 | 88.70 | 82.85 | 82.88 |
| OUT | (%) | 80.99 | 81.14 | 38.29 | NE | 62.08 | 52.08 |

| Hedging Instrument | | Hang Seng MSCI Singapore TAIEX | | | | | |
|---|---|---|---|---|---|---|---|
| Method | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| OUT | | 0.01 | 0.01 | 0.03 | 0.02 | 0.02 | 0.02 |
| IN | Min | -0.06 | -0.05 | -0.05 | -0.05 | -0.06 | -0.06 |
| OUT | | -0.03 | -0.03 | -0.10 | -0.07 | -0.05 | -0.05 |
| IN | S. D. | 2.22 | 2.24 | 2.15 | 2.05 | 2.37 | 2.21 |
| OUT | (%) | 1.31 | 1.30 | 2.85 | 2.20 | 1.81 | 1.93 |
| IN | Skew- | 0.06 | 0.10 | 0.02 | -0.14 | 0.22 | 0.09 |
| OUT | ness | -0.54 | -0.56 | -1.64 | -1.11 | -0.40 | -0.83 |
| IN | Kur- | 3.08 | 2.93 | 3.00 | 3.00 | 2.71 | 2.80 |
| OUT | tosis | 2.09 | 2.13 | 6.45 | 4.08 | 3.03 | 3.00 |
| IN | HE | 88.10 | 87.88 | 88.89 | 89.86 | 86.48 | 88.25 |
| OUT | (%) | 82.16 | 82.40 | 16.16 | 49.69 | 66.24 | 61.50 |

| Hedging Instrument | | Hang Seng MSCI Sinapore S&P 500 | | | | | |
|---|---|---|---|---|---|---|---|
| Method | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.06 | 0.05 | 0.05 | 0.05 | 0.06 | 0.07 |
| OUT | | 0.01 | 0.01 | 0.06 | 0.03 | 2.00 | 0.04 |
| IN | Min | -0.06 | -0.06 | -0.05 | -0.05 | -0.06 | -0.07 |
| OUT | | -0.03 | -0.03 | -0.07 | -0.10 | -0.23 | -0.12 |
| IN | S. D. | 2.20 | 2.22 | 2.17 | 2.13 | 2.82 | 3.10 |
| OUT | (%) | 1.39 | 1.37 | 3.03 | 3.40 | 49.24 | 4.27 |
| IN | Skew- | 0.07 | 0.16 | 0.21 | 0.13 | 0.33 | 0.28 |
| OUT | ness | -0.44 | -0.46 | -0.04 | -1.23 | 3.64 | -1.15 |
| IN | Kur- | 3.30 | 3.11 | 3.01 | 3.07 | 2.50 | 2.54 |
| OUT | tosis | 1.98 | 2.05 | 3.37 | 4.19 | 14.59 | 3.67 |
| IN | HE | 88.29 | 88.08 | 88.64 | 89.08 | 80.81 | 76.76 |
| OUT | (%) | 79.89 | 80.65 | 5.21 | NE | NE | NE |

With Two Hedging Instruments

| Hedging Instrument | | Hang Seng MSCI Singapore | | | | | |
|---|---|---|---|---|---|---|---|
| Method | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| OUT | | 0.01 | 0.01 | 0.04 | 0.20 | 0.02 | 0.02 |
| IN | Min | -0.06 | -0.05 | -0.05 | -0.05 | -0.05 | -0.06 |
| OUT | | -0.03 | -0.03 | -0.13 | -0.04 | -0.05 | -0.06 |
| IN | S. D. | 2.22 | 2.24 | 2.18 | 2.17 | 2.19 | 2.23 |
| OUT | (%) | 1.38 | 1.36 | 3.57 | 5.30 | 1.65 | 1.90 |
| IN | Skew- | 0.00 | 0.08 | 0.08 | 0.13 | 0.08 | 0.13 |
| OUT | ness | -0.46 | -0.47 | -2.01 | 3.11 | -1.25 | -1.08 |
| IN | Kur- | 3.08 | 2.93 | 2.83 | 3.01 | 2.90 | 2.87 |
| OUT | tosis | 2.04 | 2.12 | 7.91 | 12.22 | 3.93 | 4.08 |
| IN | HE | 88.07 | 87.86 | 88.50 | 88.63 | 88.46 | 88.03 |
| OUT | (%) | 80.39 | 80.87 | NE | NE | 71.79 | 62.73 |

| Hedging Instrument | | Hang Seng Nikkei 225 | | | | | |
|---|---|---|---|---|---|---|---|
| Method | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 |
| OUT | | 0.02 | 0.02 | 0.02 | 0.03 | 0.02 | 0.03 |
| IN | Min | -0.08 | -0.08 | -0.07 | -0.07 | -0.08 | -0.09 |
| OUT | | -0.03 | -0.03 | -0.03 | -0.03 | -0.11 | -0.03 |
| IN | S. D. | 2.44 | 2.44 | 2.34 | 2.36 | 2.33 | 2.48 |
| OUT | (%) | 1.51 | 1.51 | 1.83 | 1.67 | 3.10 | 1.60 |
| IN | Skew- | -0.42 | -0.45 | -0.20 | -0.34 | -0.51 | -0.64 |
| OUT | ness | -0.37 | -0.38 | -0.44 | 0.32 | -1.87 | 0.04 |
| IN | Kur- | 4.40 | 4.50 | 3.78 | 3.94 | 4.25 | 4.98 |
| OUT | tosis | 1.81 | 1.85 | 1.70 | 2.34 | 6.68 | 2.36 |
| IN | HE | 85.63 | 85.61 | 86.80 | 86.62 | 86.94 | 85.17 |
| OUT | (%) | 76.45 | 76.24 | 65.48 | 71.13 | 0.45 | 73.58 |

| Hedging Instrument | | Hang Seng TAIEX | | | | | |
|---|---|---|---|---|---|---|---|
| Method | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 |
| OUT | | 0.02 | 0.02 | 0.02 | 0.07 | 0.02 | 0.04 |
| IN | Min | -0.09 | -0.09 | -0.09 | -0.09 | -0.09 | -0.10 |
| OUT | | -0.03 | -0.03 | -0.03 | -1.17 | -0.02 | -0.16 |
| IN | S. D. | 2.40 | 2.41 | 2.38 | 2.43 | 2.44 | 2.48 |
| OUT | (%) | 1.44 | 1.44 | 1.52 | 28.33 | 1.31 | 4.66 |
| IN | Skew- | -0.51 | -0.53 | -0.61 | -0.70 | -0.72 | -0.73 |
| OUT | ness | -0.45 | -0.46 | -0.45 | -3.69 | -0.36 | -2.22 |
| IN | Kur- | 4.75 | 4.82 | 5.45 | 5.57 | 5.57 | 5.62 |
| OUT | tosis | 1.89 | 1.94 | 2.11 | 14.81 | 1.99 | 7.71 |
| IN | HE | 86.07 | 86.05 | 86.37 | 85.82 | 85.68 | 85.16 |
| OUT | (%) | 78.60 | 78.55 | 76.22 | NE | 82.29 | NE |

| Hedging Instrument | | Hang Seng S&P 500 | | | | | |
|---|---|---|---|---|---|---|---|
| Method | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.06 | 0.05 | 0.06 | 0.06 | 0.06 | 0.06 |
| OUT | | 0.02 | 0.02 | 0.16 | 0.03 | 0.02 | 0.02 |
| IN | Min | -0.09 | -0.09 | -0.09 | -0.09 | -0.09 | -0.10 |
| OUT | | -0.03 | -0.03 | -0.04 | -0.03 | -0.06 | -0.03 |
| IN | S. D. | 2.43 | 2.43 | 2.38 | 2.42 | 2.42 | 2.44 |
| OUT | (%) | 1.59 | 1.58 | 4.48 | 1.97 | 2.13 | 1.53 |
| IN | Skew- | -0.62 | -0.62 | -0.77 | -0.75 | -0.90 | -0.86 |
| OUT | ness | -0.45 | -0.45 | 2.78 | 0.14 | -1.23 | -0.10 |
| IN | Kur- | 4.81 | 4.84 | 5.80 | 5.96 | 6.34 | 6.35 |
| OUT | tosis | 1.84 | 1.87 | 10.84 | 1.72 | 4.45 | 2.26 |
| IN | HE | 85.73 | 85.70 | 86.38 | 85.92 | 85.90 | 85.61 |
| OUT | (%) | 73.67 | 74.02 | NE | 59.67 | 52.90 | 75.82 |

Note: 1. This table contains the performance results of 49 strategies for hedging MSCI Hong Kong. The strategies involving one hedging instrument are shown in the top left block of the table and this instrument is Hang Seng indexfutures of Hong Kong. The strategies involving three instruments are shown in the other two top blocks. They are Hang Seng Index with MSCI Singapore of Singapore andNikkei 225 of Japan, Hang Seng Index with Nikkei 225 of Japan and TAIEX of Taiwan and Hang Seng Index with Nikkei 225 of Japan and S&P 500 of US respectively. The strategies invoving two instruments are shown in the bottom four blocks. They are Hang Seng Index with MSCI Singapore, Hang Seng Index with Nikkei 225, Hang Seng Index with TAIEX and Hang Seng Index with S&P 500 respectively.

2. MSCI Hong Kong, Hang Seng futures and MSCI Singapore futures are cointegrated.

3. See the note 2 - 5 and 7 of Table 3 (part A).

In Sample (IN): 1994M04 2005M12 | Out of Sample (OUT): 2006M01 2007M05

## TABLE 3 ( Part C ): Hedging Performance results for MSCI South Africa

| | | With One Hedging Instrument | | | | | | | With Three Hedging Instruments | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | JSE 40 | | | | | | | JSE 40 BOVESPA TSE 60 | | | | | | JSE 40 BOVESPA S&P 500 | | | | | | JSE 40 BOVESPA GOLD | | | | | |
| Method | | naive | ols | ci-ols | garch | tgarch | ci- | ci- | ols | ci-ols | garch | tgarch | ci- | ci- | ols | ci-ols | garch | tgarch | ci- | ci- | ols | ci-ols | garch | tgarc | ci- | ci- |
| IN | Max | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.05 | 0.04 | 0.05 | 0.04 | 0.04 | 0.05 | 0.04 | 0.04 | 0.04 |
| OUT | | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 | 0.03 | 0.03 |
| IN | Min | -0.05 | -0.04 | -0.04 | -0.05 | -0.05 | -0.05 | -0.05 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.05 | -0.04 | -0.05 | -0.05 | -0.05 | -0.06 | -0.06 | -0.04 | -0.05 | -0.04 | -0.05 | -0.05 | -0.04 |
| OUT | | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.07 | -0.06 | -0.07 | -0.05 | -0.04 | -0.04 | -0.07 | -0.07 | -0.05 | -0.07 |
| IN | S. D. | 1.95 | 1.84 | 1.84 | 1.81 | 1.78 | 1.79 | 1.79 | 1.80 | 1.81 | 1.73 | 1.70 | 1.72 | 1.78 | 1.82 | 1.83 | 1.76 | 1.84 | 1.74 | 1.84 | 1.82 | 1.83 | 1.75 | 1.80 | 1.76 | 1.73 |
| OUT | (%) | 2.32 | 2.34 | 2.34 | 2.38 | 2.34 | 2.38 | 2.35 | 2.26 | 2.25 | 2.29 | 2.26 | 2.26 | 2.25 | 2.28 | 2.27 | 2.47 | 2.38 | 2.59 | 2.35 | 2.17 | 2.17 | 2.48 | 2.46 | 2.33 | 2.51 |
| IN | Skew- | 0.23 | -0.14 | -0.14 | -0.13 | -0.10 | -0.15 | -0.10 | 0.01 | -0.05 | 0.02 | 0.03 | -0.02 | -0.10 | -0.13 | -0.18 | -0.04 | -0.05 | -0.41 | -0.20 | -0.10 | -0.16 | -0.03 | -0.08 | -0.19 | -0.20 |
| OUT | ness | 0.14 | 0.11 | 0.11 | 0.16 | 0.07 | 0.17 | 0.14 | 0.07 | 0.07 | 0.06 | 0.02 | 0.05 | 0.00 | 0.06 | 0.04 | -0.63 | -0.50 | -0.56 | 0.00 | -0.02 | 0.00 | -0.71 | -0.77 | -0.20 | -0.75 |
| IN | Kur- | 3.03 | 2.54 | 2.54 | 2.56 | 2.59 | 2.58 | 2.62 | 2.51 | 2.45 | 2.74 | 2.62 | 2.68 | 2.58 | 2.54 | 2.56 | 3.00 | 3.17 | 3.24 | 3.32 | 2.64 | 2.62 | 2.85 | 2.80 | 2.82 | 2.84 |
| OUT | tosis | 1.80 | 1.94 | 1.94 | 1.81 | 1.74 | 1.82 | 1.73 | 1.94 | 1.95 | 1.92 | 2.03 | 2.12 | 2.05 | 1.98 | 2.00 | 3.37 | 3.14 | 3.12 | 2.16 | 1.85 | 1.85 | 2.79 | 3.00 | 2.11 | 3.32 |
| IN | HE | 91.01 | 91.93 | 91.93 | 92.26 | 92.51 | 92.35 | 92.42 | 92.28 | 92.21 | 92.89 | 93.15 | 92.99 | 92.49 | 92.11 | 92.03 | 92.65 | 91.97 | 92.84 | 91.97 | 92.13 | 92.05 | 92.73 | 92.27 | 92.61 | 92.91 |
| OUT | (%) | 71.87 | 71.38 | 71.36 | 70.26 | 71.30 | 70.18 | 70.95 | 73.22 | 73.35 | 72.45 | 73.17 | 73.17 | 73.53 | 72.68 | 73.08 | 67.91 | 70.40 | 64.70 | 71.02 | 75.40 | 75.34 | 67.72 | 68.18 | 71.65 | 67.09 |

| | | With Two Hedging Instruments | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | JSE 40 BOVESPA | | | | | | JSE 40 TSE 60 | | | | | | JSE 40 S&P 500 | | | | | | JSE 40 GOLD | | | | | |
| Method | | ols | ci-ols | garch | tgarch | ci- | ci- | ols | ci-ols | garch | tgarch | ci- | ci- | ols | ci-ols | garch | tgarch | ci- | ci- | ols | ci-ols | garch | tgarc | ci- | ci- |
| IN | Max | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.07 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |
| OUT | | 0.03 | 0.03 | 0.03 | 0.07 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.05 | 0.04 | 0.04 | 0.03 | 0.03 | 0.03 | 0.04 | 0.03 | 0.03 | 0.03 | 0.02 | 0.03 | 0.03 |
| IN | Min | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.05 | -0.05 | -0.06 | -0.06 | -0.06 | -0.06 | -0.04 | -0.05 | -0.05 | -0.04 | -0.06 | -0.05 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.05 |
| OUT | | -0.04 | -0.04 | -0.04 | -0.05 | -0.04 | -0.05 | -0.04 | -0.04 | -0.06 | -0.05 | -0.06 | -0.05 | -0.04 | -0.04 | -0.07 | -0.09 | -0.04 | -0.19 | -0.04 | -0.04 | -0.06 | -0.04 | -0.04 | -0.05 |
| IN | S. D. | 1.81 | 1.81 | 1.74 | 1.75 | 1.71 | 1.69 | 1.84 | 1.84 | 1.86 | 1.77 | 1.81 | 1.79 | 1.84 | 1.85 | 1.81 | 1.63 | 1.86 | 1.70 | 1.82 | 1.82 | 1.74 | 1.77 | 1.74 | 1.73 |
| OUT | (%) | 2.25 | 2.25 | 2.21 | 2.98 | 2.26 | 2.19 | 2.30 | 2.27 | 2.31 | 2.34 | 2.38 | 2.90 | 2.34 | 2.33 | 2.81 | 3.43 | 2.29 | 5.26 | 2.16 | 2.19 | 2.50 | 2.34 | 2.28 | 2.34 |
| IN | Skew- | -0.06 | -0.06 | -0.06 | -0.08 | 0.01 | -0.04 | -0.16 | -0.20 | 0.11 | -0.20 | -0.27 | -0.31 | -0.12 | -0.17 | -0.15 | -0.04 | -0.19 | -0.23 | -0.11 | -0.12 | -0.08 | -0.08 | -0.13 | -0.15 |
| OUT | ness | 0.07 | 0.06 | -0.01 | 0.33 | -0.02 | -0.26 | 0.07 | 0.05 | -0.52 | -0.34 | -0.41 | -0.11 | 0.11 | 0.10 | -0.47 | -0.57 | 0.06 | -2.13 | -0.01 | 0.02 | -0.41 | -0.23 | 0.05 | -0.59 |
| IN | Kur- | 2.46 | 2.45 | 2.45 | 2.46 | 2.60 | 2.50 | 2.55 | 2.57 | 4.06 | 3.10 | 3.10 | 3.37 | 2.54 | 2.56 | 2.58 | 2.77 | 2.86 | 2.98 | 2.64 | 2.60 | 2.70 | 2.79 | 2.69 | 2.80 |
| OUT | tosis | 1.95 | 1.95 | 2.13 | 3.94 | 2.09 | 2.52 | 1.96 | 1.99 | 3.01 | 2.25 | 2.73 | 2.16 | 1.94 | 1.95 | 2.73 | 2.51 | 2.08 | 7.75 | 1.84 | 1.86 | 2.58 | 1.81 | 1.72 | 2.38 |
| IN | HE | 92.19 | 92.19 | 92.63 | 92.75 | 93.04 | 93.22 | 91.98 | 91.94 | 91.76 | 92.53 | 92.24 | 92.36 | 91.95 | 91.86 | 92.21 | 93.67 | 91.82 | 93.11 | 92.13 | 92.11 | 92.78 | 92.59 | 92.77 | 92.89 |
| OUT | (%) | 73.36 | 73.40 | 74.41 | 53.45 | 73.27 | 74.79 | 72.34 | 72.98 | 71.92 | 71.31 | 70.40 | 55.94 | 71.17 | 71.57 | 58.70 | 38.22 | 72.54 | NE | 75.44 | 74.84 | 67.16 | 71.27 | 72.63 | 71.33 |

Note: 1. This table contains the performance results of 49 strategies for hedging MSCI South Africa. The strategies involving one hedging instrument are shown in the top left block of the table and this instrument is JSE 40 indexfutures of South Africa. The strategies involving three instruments are shown in the other two top blocks. They are JSE 40 with BOVESPA of Brazil and TSE 60 of Canada, JSE 40 with BOVESPA and S&P 500 of US and JSE 40 with BOVESPA and gold respectively. The strategies invoving two instruments are shown in the bottom four blocks. They are JSE 40 with BOVESPA, JSE 40 with TSE 60, JSE 40 with S&P 500 and JSE 40 with gold respectively.

2. MSCI South Africa, JSE 40 futures, BOVESPA and S&P 500 are cointegrated.

3. See the note 2 - 5 and 7 of Table 3 (part A).

The second group includes Spain, Taiwan, Canada, France, South Korea, Switzerland, Sweden, Netherlands and Australia. The volatility of their hedged returns is moderate in both within and out-of-sample period, leaving small scope for improvement. Nevertheless, sophisticated models bring moderate and consistent improvement over the benchmarks for all eight of nine countries in the group.

In Sample (IN): 1992M07 2005M12 Out of Sample (OUT): 2006M01 2007M05

## TABLE 4 ( Part A ): Hedging Performance results for MSCI Spain

| | | With One Hedging Instrument | | | | | | | With Three Hedging Instruments | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | IBEX 35 | | | | | | | IBEX 35 S&P 500 CAC 40 | | | | | | IBEX 35 S&P 500 FTSE 100 | | | | | |
| Method | | naïve | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.03 | 0.03 | 0.03 | 0.03 | 0.04 | 0.04 | 0.03 | 0.03 | 0.03 | 0.03 |
| OUT | | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| IN | Min | -0.04 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.02 | -0.03 | -0.04 | -0.04 | -0.02 | -0.03 |
| OUT | | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| IN | S. D. | 1.16 | 1.13 | 1.13 | 1.06 | 1.03 | 1.04 | 1.04 | 1.08 | 1.08 | 1.09 | 1.09 | 1.08 | 1.04 | 1.08 | 1.08 | 1.09 | 1.09 | 1.00 | 1.02 |
| OUT | (%) | 0.57 | 0.56 | 0.56 | 0.61 | 0.60 | 0.59 | 0.55 | 0.47 | 0.48 | 0.51 | 0.50 | 0.53 | 0.53 | 0.46 | 0.47 | 0.57 | 0.54 | 0.60 | 0.58 |
| IN | Skew- | 0.55 | 0.41 | 0.39 | 0.47 | 0.47 | 0.46 | 0.45 | 0.24 | 0.26 | 0.10 | 0.10 | 0.08 | 0.22 | 0.28 | 0.28 | 0.04 | 0.04 | 0.29 | 0.06 |
| OUT | ness | 0.72 | 0.35 | 0.29 | 0.64 | 0.73 | 0.90 | 1.12 | -0.03 | -0.02 | 0.15 | 0.04 | 0.12 | 0.25 | -0.02 | 0.00 | 0.28 | 0.16 | 0.24 | -0.05 |
| IN | Kur- | 5.78 | 5.15 | 5.07 | 4.76 | 4.83 | 4.85 | 4.72 | 3.59 | 3.69 | 3.91 | 3.91 | 3.50 | 3.47 | 3.47 | 3.59 | 3.88 | 3.88 | 3.35 | 3.33 |
| OUT | tosis | 2.61 | 2.27 | 2.26 | 2.60 | 2.69 | 2.87 | 3.52 | 1.77 | 1.80 | 1.84 | 1.68 | 1.87 | 1.83 | 1.65 | 1.72 | 1.89 | 1.81 | 1.81 | 1.86 |
| IN | HE | 96.56 | 96.72 | 96.72 | 97.12 | 97.26 | 97.21 | 97.24 | 97.02 | 97.01 | 96.97 | 96.97 | 97.00 | 97.25 | 97.03 | 97.02 | 96.93 | 96.93 | 97.44 | 97.32 |
| OUT | (%) | 96.75 | 96.86 | 96.85 | 96.20 | 96.33 | 96.52 | 96.98 | 97.72 | 97.65 | 97.38 | 97.48 | 97.12 | 97.10 | 97.81 | 97.71 | 96.65 | 97.09 | 96.34 | 96.56 |

| | | With Two Hedging Instruments | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | IBEX 35 CAC 40 | | | | | | IBEX 35 FTSE 100 | | | | | | IBEX 35 S&P 500 | | | | | |
| Method | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.03 | 0.04 |
| OUT | | 0.01 | 0.01 | 0.01 | 0.14 | 0.01 | 0.15 | 0.01 | 0.01 | 0.01 | 0.05 | 0.01 | 0.06 | 0.01 | 0.01 | 0.00 | 0.21 | 0.01 | 0.08 |
| IN | Min | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.04 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.02 | -0.03 |
| OUT | | -0.01 | -0.01 | -0.01 | -0.10 | -0.01 | -0.09 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.02 | -0.01 | -0.04 |
| IN | S. D. | 1.11 | 1.11 | 1.12 | 1.12 | 1.04 | 1.04 | 1.10 | 1.10 | 1.11 | 1.11 | 1.08 | 1.06 | 1.08 | 1.08 | 1.03 | 1.03 | 1.00 | 1.00 |
| OUT | (%) | 0.53 | 0.53 | 0.55 | 4.92 | 0.58 | 4.91 | 0.50 | 0.51 | 0.72 | 1.38 | 0.63 | 2.35 | 0.47 | 0.48 | 0.49 | 5.26 | 0.49 | 2.65 |
| IN | Skew- | 0.28 | 0.30 | 0.36 | 0.36 | 0.36 | 0.22 | 0.35 | 0.35 | 0.09 | 0.09 | 0.12 | 0.09 | 0.28 | 0.28 | 0.14 | 0.14 | 0.15 | 0.08 |
| OUT | ness | 0.14 | 0.14 | 0.27 | 1.06 | 0.71 | 1.17 | 0.15 | 0.18 | 0.32 | 2.32 | 0.88 | 0.58 | -0.02 | 0.00 | 0.07 | 3.51 | 0.26 | 1.02 |
| IN | Kur- | 4.90 | 4.92 | 4.07 | 4.07 | 4.38 | 4.26 | 4.06 | 4.24 | 3.92 | 3.92 | 4.17 | 4.07 | 3.53 | 3.65 | 3.64 | 3.64 | 3.23 | 3.35 |
| OUT | tosis | 1.90 | 1.95 | 2.04 | 6.02 | 2.35 | 5.88 | 1.69 | 1.76 | 2.60 | 8.94 | 3.14 | 2.15 | 1.75 | 1.81 | 1.56 | 13.98 | 1.65 | 5.64 |
| IN | HE | 96.83 | 96.82 | 96.76 | 96.76 | 97.20 | 97.25 | 96.89 | 96.88 | 96.82 | 96.82 | 97.03 | 97.10 | 97.00 | 97.00 | 97.25 | 97.25 | 97.45 | 97.45 |
| OUT | (%) | 97.17 | 97.11 | 96.91 | NE | 96.58 | NE | 97.45 | 97.35 | 94.80 | 80.54 | 95.94 | 44.07 | 97.74 | 97.66 | 97.59 | NE | 97.60 | 28.87 |

Note: 1. MSCI Spain, IBEX 35 futures, S&P 500 futures and FTSE 100 futures are cointegrated.
2. See the note 2 - 5 and 7 of Table 3 (part A).

Part A of Table 4 contains the results for Spain. Despite of the high hedging effectiveness of more than 96.5% in both within and out-of-sample period, nineteen of thirty-five sophisticated strategies outperform the benchmarks consistently. The best of them involves IBEX 35 and S&P 500 modelled by VECM combined with trivariate GARCH (1,1). It improves hedging effectiveness by nearly 1% in both periods.

Part B of Table 4 contains the results for Taiwan. Though Taiwan is categorized as emerging market by MSCI, its volatility is moderate and the naïve hedging is highly effective, which is in contrast to Hong Kong in the category of developed market. In particular, its naïve hedge's within-sample hedging effectiveness reaches 96.03%. Nevertheless, the sophistication makes further improvement. The best strategy for Taiwan involves TAIEX, Hang Seng and S&P 500 index futures and is estimated by OLS method. It reduces the within-sample standard deviation by 0.12% to 1.55% and the out-of-sample standard deviation by 0.07% to 0.97%. Or equivalently, the hedging effectiveness is improved by around 0.6% to 96.61% within sample and around 1% to 93.12%.

In Sample (IN): 1998M10 2005M12 Out of Sample (OUT): 2006M01 2007M05

## TABLE 4 ( Part B ): Hedging Performance results for MSCI Taiwan

| | | With One Hedging Instrument | | | | | | | With Three Hedging Instruments | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | TAIEX | | | | | | | TAIEX Hang Seng Nikkei 225 | | | | | | TAIEX Hang Seng S&P 500 | | | | | |
| Method | | naïve | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.04 | 0.04 | 0.04 | 0.066 | 0.046 | 0.15 | 0.492 | 0.03 | 0.03 | 0.084 | 0.105 | 4.42 | 3.68 | 0.04 | 0.04 | 0.08 | 0.08 | 2.98 | 3.272 |
| OUT | | 0.01 | 0.01 | 0.01 | 0.035 | 0.02 | 0.04 | 0.088 | 0.01 | 0.01 | 0.115 | 0.046 | 0.03 | 0.54 | 0.01 | 0.01 | 0.041 | 0.68 | 0.08 | 0.696 |
| IN | Min | -0.04 | -0.05 | -0.04 | -0.09 | -0.08 | -0.19 | -0.52 | -0.04 | -0.04 | -0.07 | -0.1 | -3.59 | -3.01 | -0.04 | -0.04 | -0.1 | -0.08 | -3.47 | -3.538 |
| OUT | | -0.02 | -0.02 | -0.02 | -0.02 | -0.05 | -0.06 | -0.06 | -0.03 | -0.03 | -0.07 | -0.05 | -4.86 | -4.95 | -0.02 | -0.02 | -1.57 | -0.02 | -2.58 | -2.61 |
| IN | S. D. | 1.68 | 1.67 | 1.67 | 2.66 | 2.07 | 6.38 | 20.79 | 1.58 | 1.58 | 3.23 | 4.33 | 119 | 104 | 1.55 | 1.56 | 2.86 | 2.73 | 97.52 | 100.42 |
| OUT | (%) | 1.07 | 1.04 | 1.04 | 1.42 | 1.55 | 2.17 | 2.93 | 1.03 | 1.03 | 3.65 | 2.22 | 118 | 122 | 0.97 | 0.98 | 37.92 | 16.68 | 62.56 | 90.10 |
| IN | Skew- | 0.04 | -0.04 | -0.03 | -0.5 | -0.38 | -0.35 | -0.22 | -0 | -0.01 | 0.219 | 0.095 | 0.38 | 0.24 | 0.13 | 0.13 | 0.132 | 0.19 | -0.14 | -0.056 |
| OUT | ness | -0.1 | -0.15 | -0.15 | 0.692 | -0.88 | -0.11 | 1.439 | -0.4 | -0.36 | 1.591 | 0.309 | -3.75 | -3.63 | -0.19 | -0.16 | -3.74 | 3.71 | -3.74 | -1.627 |
| IN | Kur- | 2.53 | 2.71 | 2.68 | 4.299 | 4.118 | 3.63 | 3.036 | 2.54 | 2.55 | 3.141 | 2.832 | 5.93 | 5.85 | 2.5 | 2.52 | 4.25 | 3.4 | 5.65 | 5.765 |
| OUT | tosis | 2.41 | 2.41 | 2.41 | 3.393 | 4.214 | 3.82 | 7.037 | 2.96 | 2.84 | 7.617 | 3.388 | 15.1 | 14.6 | 2.53 | 2.52 | 15.01 | 14.9 | 15 | 4.182 |
| IN | HE | 96.03 | 96.12 | 96.11 | 90.12 | 94.00 | 42.99 | NE | 96.51 | 96.49 | 85.37 | 73.77 | NE | NE | 96.61 | 96.60 | 88.55 | 89.52 | NE | NE |
| OUT | (%) | 91.72 | 92.16 | 92.11 | 85.26 | 82.57 | 65.86 | 37.35 | 92.27 | 92.28 | 2.87 | 63.95 | NE | NE | 93.12 | 93.03 | NE | NE | NE | NE |
| Hedging Instrument | | | With Two Hedging Instruments | | | | | | | | | | | | | | | | | |
| | | | TAIEX Hang Seng | | | | | | TAIEX Nikkei 225 | | | | | | TAIEX S&P 500 | | | | | |
| Method | | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | | 0.04 | 0.04 | 0.051 | 0.053 | 0.13 | 0.716 | 0.03 | 0.03 | 0.078 | 0.079 | 0.56 | 0.25 | 0.04 | 0.04 | 0.062 | 0.07 | 0.09 | 0.189 |
| OUT | | | 0.01 | 0.01 | 0.05 | 0.022 | 0.15 | 0.02 | 0.01 | 0.01 | 0.041 | 0.062 | 0.03 | 0.03 | 0.01 | 0.01 | 0.097 | 0.39 | 0.09 | 0.036 |
| IN | Min | | -0.04 | -0.04 | -0.07 | -0.06 | -0.35 | -0.75 | -0.04 | -0.04 | -0.08 | -0.1 | -1.08 | -0.27 | -0.04 | -0.04 | -0.07 | -0.08 | -0.15 | -0.3 |
| OUT | | | -0.03 | -0.03 | -0.05 | -0.1 | -0.04 | -0.19 | -0.03 | -0.03 | -0.1 | -0.05 | -0.47 | -0.03 | -0.02 | -0.02 | -0.03 | -2.36 | -0.02 | -0.056 |
| IN | S. D. | | 1.61 | 1.61 | 2.00 | 2.28 | 7.41 | 26.18 | 1.61 | 1.61 | 3.26 | 3.76 | 30.38 | 11.71 | 1.56 | 1.56 | 2.10 | 2.58 | 4.16 | 6.32 |
| OUT | (%) | | 1.00 | 1.01 | 2.31 | 3.01 | 3.99 | 4.79 | 1.06 | 1.05 | 3.00 | 2.83 | 11.32 | 1.78 | 0.97 | 0.98 | 2.86 | 58.53 | 2.56 | 2.23 |
| IN | Skew- | | 0.08 | 0.07 | -0.17 | 0.131 | -1.55 | -0.18 | -0.09 | -0.07 | -0.06 | -0.2 | -0.71 | 0.04 | 0.12 | 0.12 | 0.126 | 0.1 | -0.7 | -1.171 |
| OUT | ness | | -0.32 | -0.28 | 0.452 | -1.46 | 2.94 | -2.91 | -0.32 | -0.28 | -1.75 | 0.812 | -3.58 | 0.16 | -0.12 | -0.12 | 2.518 | -3.55 | 2.63 | -0.004 |
| IN | Kur- | | 2.81 | 2.78 | 3.942 | 2.745 | 7.75 | 3.968 | 2.43 | 2.46 | 3.365 | 2.801 | 4.21 | 2.64 | 2.47 | 2.5 | 3.715 | 3.4 | 4.46 | 9.048 |
| OUT | tosis | | 2.54 | 2.5 | 4.303 | 5.446 | 11.6 | 11.06 | 2.79 | 2.69 | 7.271 | 3.422 | 14.3 | 2.23 | 2.49 | 2.5 | 9.588 | 14.2 | 10.1 | 2.941 |
| IN | HE | | 96.38 | 96.36 | 94.38 | 92.74 | 23.04 | NE | 96.37 | 96.36 | 85.11 | 80.22 | NE | NE | 96.60 | 96.59 | 93.83 | 90.66 | 75.70 | 43.99 |
| OUT | (%) | | 92.66 | 92.57 | 61.12 | 34.18 | NE | NE | 91.87 | 91.96 | 34.50 | 41.49 | NE | 76.82 | 93.09 | 93.01 | 40.54 | NE | 52.17 | 63.63 |

Note: 1. MSCI Taiwan, TAIEX futures and Hand Seng Index futures are cointegrated.
2. See the note 2 - 5 and 7 of Table 3 (part A).

Part C of Table 4 contains the results for Canada. Six of twenty-five strategies bring consistent improvement, among which the best involves TSE 60 and S&P 500 index futures estimated by VECM combined with trivariate GARCH (1,1). It decreases the standard deviation by 0.06% and 0.04% and increases the hedging effectiveness by 0.49% and 0.68% within and out-of-sample respectively.

In Sample (IN): 1994M04 2005M12 Out of Sample (OUT): 2006M01 2007M05

## TABLE 4 ( Part C ): Hedging Performance results for MSCI Canada

| | | With One Hedging Instrument | | | | | | | With Two Hedging Instruments | | | | | | | | | | | | | With Three Hedging Instruments | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | TSE 60 | | | | | | | TSE 60 BOVESPA | | | | | | TSE 60 S&P 500 | | | | | | TSE 60 BOVESPA S&P 500 | | | | | |
| Method | | naïve | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.06 | 0.04 | 0.04 | 0.035 | 0.035 | 0.03 | 0.03 | 0.04 | 0.04 | 0.037 | 0.036 | 0.03 | 0.03 | 0.04 | 0.04 | 0.04 | 0.04 | 0.03 | 0.031 | 0.04 | 0.04 | 0.04 | 0.043 | 0.036 | 0.035 |
| OUT | | 0.01 | 0.01 | 0.01 | 0.01 | 0.009 | 0.01 | 0.008 | 0.01 | 0.01 | 0.008 | 0.042 | 0.01 | 0.08 | 0.01 | 0.01 | 0.016 | 0.02 | 0.01 | 0.051 | 0.01 | 0.01 | 0.01 | 0.01 | 0.015 | 0.012 |
| IN | Min | -0.05 | -0.05 | -0.05 | -0.03 | -0.03 | -0.03 | -0.03 | -0.05 | -0.05 | -0.03 | -0.03 | -0.03 | -0.03 | -0.05 | -0.05 | -0.04 | -0.04 | -0.05 | -0.041 | -0.05 | -0.05 | -0.03 | -0.03 | -0.03 | -0.04 |
| OUT | | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.06 | -0.02 | -0.03 | -0.02 | -0.02 | -0.01 | -0.31 | -0.02 | -0.33 | -0.02 | -0.02 | -0.01 | -0.01 | -0.01 | -0.01 |
| IN | S. D. | 1.05 | 1.00 | 1.00 | 0.97 | 0.97 | 0.97 | 0.97 | 1.00 | 1.01 | 0.99 | 0.99 | 0.97 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.94 | 0.96 | 0.98 | 0.99 | 1.07 | 1.05 | 0.93 | 0.94 |
| OUT | (%) | 0.69 | 0.77 | 0.79 | 0.69 | 0.66 | 0.66 | 0.65 | 0.77 | 0.78 | 0.70 | 2.14 | 0.68 | 2.31 | 0.83 | 0.84 | 0.65 | 7.83 | 0.65 | 8.35 | 0.84 | 0.84 | 0.59 | 0.62 | 0.76 | 0.74 |
| IN | Skew- | 0.7 | -0.15 | -0.24 | 0.024 | 0.016 | -0.1 | -0.08 | -0.15 | -0.18 | 0.275 | 0.25 | 0.04 | 0.11 | -0.32 | -0.39 | 0.155 | 0.15 | -0.52 | -0.16 | -0.36 | -0.34 | 0.72 | 0.619 | 0.205 | 0.045 |
| OUT | ness | -0.85 | -0.71 | -0.68 | -0.73 | -0.6 | -0.99 | -0.81 | -0.71 | -0.71 | -0.79 | -0.87 | -0.83 | 2.25 | -0.79 | -0.77 | 0.699 | -3.43 | -0.35 | -3.42 | -0.79 | -0.77 | -0.27 | -0.22 | 0.021 | -0.21 |
| IN | Kur- | 14.6 | 8.71 | 8.06 | 5.158 | 5.128 | 4.89 | 4.857 | 8.71 | 7.96 | 4.678 | 4.654 | 4.35 | 4.49 | 9.74 | 9.07 | 6.048 | 5.91 | 7.05 | 5.723 | 9.78 | 8.82 | 6.06 | 5.691 | 4.773 | 5.822 |
| OUT | tosis | 3.52 | 3.27 | 3.22 | 3.2 | 3.081 | 3.71 | 3.483 | 3.26 | 3.18 | 2.907 | 5.245 | 3.23 | 8.68 | 3.54 | 3.47 | 3.071 | 13.5 | 2.58 | 13.62 | 3.55 | 3.41 | 2.15 | 1.951 | 2.5 | 2.297 |
| IN | HE | 95.07 | 95.58 | 95.57 | 95.83 | 95.85 | 95.84 | 95.84 | 95.58 | 95.51 | 95.68 | 95.68 | 95.84 | 95.77 | 95.71 | 95.70 | 95.81 | 95.82 | 96.07 | 95.95 | 95.72 | 95.65 | 94.98 | 95.10 | 96.19 | 96.06 |
| OUT | (%) | 94.46 | 93.14 | 92.90 | 94.60 | 94.63 | 94.99 | 95.10 | 93.15 | 92.94 | 94.44 | 47.18 | 94.71 | 38.59 | 92.05 | 91.82 | 95.07 | NE | 95.14 | NE | 91.96 | 91.97 | 95.98 | 95.64 | 93.34 | 93.63 |

Note: 1. MSCI Canada, TSE 60 futures, S&P 500 futures and BOVESPA futures are cointegrated.
2. See the note 2 - 5 and 7 of Table 3 (part A).

The results for Switzerland and France are very similar. Their hedged portfolio returns are the least volatile and their naïve and simple OLS hedge are the most effective among countries in this group. Part D of Table 4 displays the results for Switzerland. Its benchmark hedging effectiveness is slightly less than 98%. The best strategy involves SMI, S&P 500 and AEX index futures estimated by quadrivariate TGARCH (1,1) and the corresponding hedging effectiveness are above 98% in both samples. Part E of Table 4 contains the results for France. The best strategy involves CAC 40, S&P 500 and DAX 30 index futures estimated by a quadrivariate GARCH (1,1). It improves hedging effectiveness by 0.16% to 98.42% within the sample and by 0.33% to 97.69% out of the sample.

In Sample (IN): 1990M12 2005M12 Out of Sample (OUT): 2006M01 2007M05

## TABLE 4 ( Part D ): Hedging Performance results for MSCI Switzerland

| | | With One Hedging | | | | With Two Hedging Instruments | | | | | | | | | With Three Hedging Instruments | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | SMI | | | | SMI DAX 30 | | | SMI AEX | | | SMI S&P 500 | | | SMI S&P 500 DAX 30 | | | SMI S&P 500 AEX | | |
| Method | | naïve | ols | garch | tgarch | ols | garch | tgarch | ols | garch | tgarch | ols | garch | tgarch | ols | garch | tgarch | ols | garch | tgarch |
| Max | IN | 0.03 | 0.03 | 0.04 | 0.03 | 0.03 | 0.04 | 0.04 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 |
| | OUT | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.02 | 0.01 | 0.01 | 0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| Min | IN | -0.02 | -0.03 | -0.03 | -0.03 | -0.03 | -0.02 | -0.02 | -0.03 | -0.02 | -0.03 | -0.03 | -0.02 | -0.03 | -0.03 | -0.02 | -0.02 | -0.03 | -0.02 | -0.02 |
| | OUT | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.02 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| S. D. (%) | IN | 0.74 | 0.70 | 0.71 | 0.67 | 0.70 | 0.70 | 0.71 | 0.70 | 0.68 | 0.69 | 0.69 | 0.69 | 0.69 | 0.69 | 0.68 | 0.68 | 0.69 | 0.68 | 0.67 |
| | OUT | 0.42 | 0.43 | 0.46 | 0.46 | 0.42 | 0.53 | 0.50 | 0.42 | 0.50 | 0.65 | 0.42 | 0.41 | 0.53 | 0.42 | 0.46 | 0.44 | 0.43 | 0.50 | 0.36 |
| Skew-ness | IN | 0.52 | 0.04 | 0.64 | 0.19 | 0.04 | 0.78 | 0.89 | 0.04 | 0.35 | 0.56 | 0.08 | 0.27 | 0.41 | 0.08 | 0.28 | 0.54 | 0.10 | 0.12 | 0.00 |
| | OUT | -0.26 | -0.22 | -0.08 | -0.10 | -0.22 | -1.55 | -0.08 | -0.21 | 0.47 | 0.49 | -0.28 | -0.15 | -0.65 | -0.28 | -0.59 | -0.28 | -0.29 | -0.35 | -0.19 |
| Kur-tosis | IN | 5.78 | 6.30 | 7.48 | 5.75 | 6.28 | 7.63 | 8.03 | 6.31 | 6.32 | 6.76 | 6.09 | 5.77 | 6.70 | 6.15 | 5.66 | 6.77 | 6.02 | 5.42 | 5.28 |
| | OUT | 2.17 | 1.67 | 1.85 | 1.91 | 1.66 | 5.57 | 2.41 | 1.66 | 2.84 | 3.40 | 1.62 | 2.05 | 2.54 | 1.64 | 2.17 | 1.94 | 1.66 | 2.64 | 2.51 |
| HE (%) | IN | 97.54 | 97.82 | 97.76 | 97.97 | 97.82 | 97.78 | 97.77 | 97.82 | 97.93 | 97.85 | 97.88 | 97.89 | 97.89 | 97.88 | 97.94 | 97.93 | 97.88 | 97.93 | 98.01 |
| | OUT | 97.71 | 97.71 | 97.37 | 97.28 | 97.71 | 96.40 | 96.86 | 97.71 | 96.86 | 94.59 | 97.73 | 97.85 | 96.42 | 97.71 | 97.27 | 97.53 | 97.69 | 96.88 | 98.35 |

Note: See the note 2 - 5 and 7 of Table 3 (part A).

In Sample (IN): 1990M12 2005M12 Out of Sample (OUT): 2006M01 2007M05

## TABLE 4 ( Part E ): Hedging Performance results for MSCI France

| | | With One Hedging Instrument | | | | | | | With Three Hedging Instruments | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | CAC 40 | | | | | | | CAC 40 S&P 500 AEX | | | | | |
| Method | | naïve | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| Max | IN | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| | OUT | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| Min | IN | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.01 | -0.02 | -0.01 | -0.01 |
| | OUT | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| S. D. (%) | IN | 0.80 | 0.71 | 0.71 | 0.74 | 0.72 | 0.72 | 0.72 | 0.68 | 0.68 | 0.73 | 0.74 | 0.72 | 0.72 |
| | OUT | 0.42 | 0.47 | 0.47 | 0.46 | 0.46 | 0.46 | 0.47 | 0.43 | 0.44 | 0.43 | 0.48 | 0.43 | 0.44 |
| Skew-ness | IN | -0.32 | -0.39 | -0.39 | -0.22 | -0.20 | -0.23 | -0.23 | -0.23 | -0.22 | 0.07 | 0.08 | 0.07 | 0.00 |
| | OUT | -0.04 | -1.33 | -1.36 | -1.01 | -1.18 | -1.25 | -0.85 | -1.06 | -1.08 | -1.01 | -0.94 | -0.99 | -0.78 |
| Kur-tosis | IN | 3.42 | 3.16 | 3.17 | 3.47 | 3.29 | 3.40 | 3.35 | 2.84 | 2.83 | 2.92 | 2.95 | 2.84 | 2.81 |
| | OUT | 5.35 | 6.52 | 6.56 | 6.13 | 6.41 | 6.31 | 6.09 | 5.42 | 5.45 | 5.52 | 4.89 | 5.11 | 4.92 |
| HE (%) | IN | 97.80 | 98.28 | 98.28 | 98.15 | 98.21 | 98.21 | 98.22 | 98.43 | 98.43 | 98.17 | 98.10 | 98.21 | 98.24 |
| | OUT | 97.34 | 96.70 | 96.66 | 96.80 | 96.84 | 96.75 | 96.69 | 97.16 | 97.12 | 97.22 | 96.59 | 97.17 | 97.15 |

| | | With Three Hedging Instruments | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | CAC 40 S&P 500 DAX 30 | | | | | | CAC 40 S&P 500 FTSE 100 | | | | | |
| Method | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| Max | IN | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| | OUT | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| Min | IN | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 |
| | OUT | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| S. D. (%) | IN | 0.69 | 0.69 | 0.68 | 0.70 | 0.68 | 0.74 | 0.69 | 0.69 | 0.72 | 0.72 | 0.72 | 0.72 |
| | OUT | 0.43 | 0.43 | 0.39 | 0.43 | 0.42 | 0.44 | 0.43 | 0.43 | 0.38 | 0.37 | 0.39 | 0.40 |
| Skew-ness | IN | -0.27 | -0.27 | -0.22 | -0.09 | -0.20 | -0.05 | -0.34 | -0.34 | -0.22 | -0.20 | -0.19 | -0.14 |
| | OUT | -1.30 | -1.33 | -0.97 | -0.34 | -0.97 | -0.43 | -1.18 | -1.21 | -0.32 | -0.17 | -0.51 | -0.03 |
| Kur-tosis | IN | 3.04 | 3.02 | 3.07 | 3.03 | 2.97 | 3.02 | 3.10 | 3.10 | 3.19 | 3.15 | 3.35 | 3.10 |
| | OUT | 5.91 | 5.95 | 5.35 | 3.80 | 5.26 | 4.54 | 5.76 | 5.80 | 2.93 | 2.95 | 3.97 | 2.95 |
| HE (%) | IN | 98.39 | 98.39 | 98.42 | 98.31 | 98.42 | 98.15 | 98.35 | 98.35 | 98.24 | 98.20 | 98.25 | 98.25 |
| | OUT | 97.21 | 97.18 | 97.67 | 97.17 | 97.36 | 97.05 | 97.19 | 97.16 | 97.83 | 97.91 | 97.75 | 97.55 |

| | | With Two Hedging Instruments | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | CAC 40 AEX | | | | | | CAC 40 DAX 30 | | | | | |
| Method | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| Max | IN | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.03 | 0.02 | 0.02 | 0.02 |
| | OUT | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 |
| Min | IN | -0.02 | -0.02 | -0.02 | -0.02 | -0.01 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 |
| | OUT | -0.01 | -0.01 | -0.01 | -0.09 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| S. D. (%) | IN | 0.66 | 0.68 | 0.73 | 0.73 | 0.72 | 0.72 | 0.69 | 0.69 | 0.73 | 0.73 | 0.68 | 0.72 |
| | OUT | 0.45 | 0.45 | 0.47 | 2.29 | 0.45 | 0.45 | 0.44 | 0.44 | 0.40 | 0.37 | 0.47 | 0.39 |
| Skew-ness | IN | -0.20 | -0.20 | 0.13 | 0.16 | 0.15 | 0.10 | -0.24 | -0.23 | 0.19 | 0.09 | -0.24 | -0.03 |
| | OUT | -1.00 | -1.03 | -0.68 | -3.45 | -1.10 | 0.13 | -1.27 | -1.31 | -0.78 | -0.11 | -0.59 | -0.15 |
| Kur-tosis | IN | 2.76 | 2.76 | 2.95 | 2.98 | 2.80 | 2.88 | 3.01 | 2.99 | 3.68 | 3.26 | 2.85 | 3.12 |
| | OUT | 5.31 | 5.35 | 5.09 | 13.63 | 5.65 | 1.86 | 5.88 | 5.92 | 4.65 | 2.05 | 3.88 | 2.75 |
| HE (%) | IN | 98.41 | 98.41 | 98.15 | 98.18 | 98.23 | 98.21 | 98.38 | 98.38 | 98.17 | 98.17 | 98.42 | 98.23 |
| | OUT | 96.99 | 96.96 | 96.67 | 20.91 | 96.89 | 96.97 | 97.09 | 97.05 | 97.56 | 97.88 | 96.71 | 97.72 |

| | | With Two Hedging Instruments | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | CAC 40 FTSE 100 | | | | | | CAC 40 S&P 500 | | | | | |
| Method | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| Max | IN | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| | OUT | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.02 | 0.01 | 0.01 | 0.01 | 0.02 | 0.01 | 0.01 |
| Min | IN | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 |
| | OUT | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.02 | -0.01 | -0.02 |
| S. D. (%) | IN | 0.70 | 0.70 | 0.73 | 0.71 | 0.70 | 0.72 | 0.70 | 0.70 | 0.71 | 0.73 | 0.70 | 0.73 |
| | OUT | 0.44 | 0.44 | 0.42 | 0.56 | 0.48 | 0.83 | 0.44 | 0.45 | 0.43 | 0.89 | 0.39 | 0.92 |
| Skew-ness | IN | -0.33 | -0.33 | -0.10 | -0.14 | -0.02 | -0.43 | -0.40 | -0.39 | -0.23 | -0.14 | -0.22 | -0.18 |
| | OUT | -1.08 | -1.11 | -0.15 | -0.02 | -0.52 | 0.35 | -1.37 | -1.40 | -0.80 | 0.10 | -1.31 | -0.99 |
| Kur-tosis | IN | 3.06 | 3.05 | 3.25 | 2.95 | 2.98 | 3.36 | 3.20 | 3.20 | 3.49 | 3.29 | 3.47 | 3.17 |
| | OUT | 5.51 | 5.56 | 2.78 | 2.59 | 3.37 | 1.98 | 6.49 | 6.52 | 4.50 | 3.11 | 5.09 | 3.63 |
| HE (%) | IN | 98.34 | 98.34 | 98.19 | 98.26 | 98.34 | 98.23 | 98.33 | 98.33 | 98.30 | 98.17 | 98.30 | 98.19 |
| | OUT | 97.08 | 97.05 | 97.37 | 95.19 | 96.59 | 89.70 | 97.03 | 97.00 | 97.23 | 87.93 | 97.66 | 87.29 |

Note: 1. MSCI France, CAC 40 futures and S&P 500 are cointegrated.
2. See the note 2 - 5 and 7 of Table 3 (part A).

Part F of Table 4 presents the results for South Korea. The difference in the standard deviation of within and out-of-sample period (2.5% compared with 0.7%) indicates that the hedged portfolio return has changed from volatile to moderate. Unlike Brazil, Hong Kong and South Africa, its volatile within-sample returns correspond to high hedging effectiveness of above 93%. The cross hedging strategy involving KOSPI 200, S&P 500 and Nikkei 225 modelled by VECM and estimated by OLS method improves both within-sample and out-of-sample hedging effectiveness by 0.23% and 0.21% to 94.08% and 96.72% respectively.

In Sample (IN): 1998M08 2005M12

Out of Sample (OUT): 2006M01 2007M05

**TABLE 4 ( Part F ): Hedging Performance results for MSCI Korea**

| | | With One Hedging Instrument | | | | | | | With Three Hedging Instruments | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | KOSPI 200 | | | | | | | KOSPI 200 S&P 500 Nikkei 225 | | | | | | KOSPI 200 S&P 500 TAIEX | | | | | |
| Method | | naïve | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.12 | 0.08 | 0.08 | 0.09 | 0.09 | 0.11 | 0.12 | 0.09 | 0.07 |
| OUT | | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.01 | 0.02 | 0.01 | 0.01 | 0.02 | 0.14 | 0.07 | 0.16 | 0.02 | 0.02 | 0.02 | 0.09 | 0.06 | 0.07 |
| IN | Min | -0.13 | -0.13 | -0.12 | -0.13 | -0.13 | -0.12 | -0.12 | -0.12 | -0.12 | -0.12 | -0.14 | -0.11 | -0.12 | -0.12 | -0.12 | -0.11 | -0.13 | -0.12 | -0.12 |
| OUT | | -0.01 | -0.01 | -0.01 | -0.02 | -0.06 | -0.03 | -0.01 | -0.01 | -0.01 | -0.05 | -0.09 | -0.06 | -0.02 | -0.01 | -0.01 | -0.90 | -0.10 | -0.20 | -0.03 |
| IN | S. D. | 2.51 | 2.46 | 2.46 | 2.54 | 2.50 | 2.47 | 2.41 | 2.43 | 2.43 | 2.57 | 3.85 | 2.47 | 2.45 | 2.40 | 2.40 | 2.52 | 2.96 | 2.57 | 2.45 |
| OUT | (%) | 0.76 | 0.71 | 0.71 | 1.00 | 1.82 | 1.04 | 1.04 | 0.69 | 0.68 | 1.96 | 4.53 | 2.79 | 4.58 | 0.76 | 0.76 | 21.63 | 3.56 | 5.30 | 2.16 |
| IN | Skew- | -0.46 | -0.60 | -0.62 | -0.38 | -0.44 | -0.70 | -0.71 | -0.45 | -0.49 | -0.26 | -0.33 | -0.44 | -0.47 | -0.80 | -0.86 | 0.17 | -0.05 | -0.37 | -0.84 |
| OUT | ness | 0.68 | 0.72 | 0.72 | 0.29 | -1.40 | -0.96 | 0.49 | 0.62 | 0.54 | -0.89 | 1.36 | 0.79 | 2.05 | 0.76 | 0.75 | -3.71 | -0.12 | -2.65 | 1.11 |
| IN | Kur- | 13.97 | 13.06 | 12.76 | 13.32 | 14.73 | 11.79 | 12.32 | 11.91 | 11.60 | 9.13 | 4.49 | 8.61 | 9.40 | 12.49 | 12.43 | 10.53 | 8.36 | 9.81 | 10.22 |
| OUT | tosis | 2.34 | 2.50 | 2.52 | 2.61 | 5.01 | 4.81 | 2.18 | 2.71 | 2.66 | 3.37 | 7.64 | 4.62 | 6.53 | 2.87 | 2.87 | 14.86 | 6.44 | 10.71 | 6.00 |
| IN | HE | 93.67 | 93.95 | 93.94 | 93.53 | 93.75 | 93.86 | 94.16 | 94.09 | 94.08 | 93.35 | 85.17 | 93.90 | 93.98 | 94.22 | 94.21 | 93.63 | 91.23 | 93.36 | 93.97 |
| OUT | (%) | 95.81 | 96.41 | 96.42 | 92.78 | 76.42 | 92.30 | 92.27 | 96.58 | 96.72 | 72.40 | NE | 44.09 | NE | 95.83 | 95.88 | NE | 9.39 | NE | 66.48 |

| | | With Two Hedging Instruments | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | KOSPI 200 Nikkei 225 | | | | | | KOSPI 200 TAIEX | | | | | | KOSPI 200 S&P 500 | | | | | |
| Method | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.11 | 0.10 | 0.11 | 0.13 | 0.09 | 0.09 | 0.10 | 0.10 | 0.12 | 0.13 | 0.08 | 0.08 | 0.10 | 0.10 | 0.10 | 0.11 | 0.08 | 0.09 |
| OUT | | 0.01 | 0.01 | 0.03 | 0.02 | 0.04 | 0.03 | 0.01 | 0.01 | 0.05 | 0.03 | 0.10 | 0.04 | 0.01 | 0.01 | 0.02 | 0.08 | 0.02 | 0.04 |
| IN | Min | -0.13 | -0.12 | -0.13 | -0.13 | -0.12 | -0.12 | -0.13 | -0.13 | -0.12 | -0.13 | -0.13 | -0.13 | -0.12 | -0.12 | -0.12 | -0.13 | -0.11 | -0.11 |
| OUT | | -0.01 | -0.01 | -0.03 | -0.05 | -0.02 | -0.05 | -0.01 | -0.01 | -0.03 | -0.07 | -0.31 | -0.03 | -0.01 | -0.01 | -0.02 | -0.06 | -0.02 | -0.04 |
| IN | S. D. | 2.46 | 2.46 | 2.64 | 3.36 | 2.50 | 2.48 | 2.44 | 2.44 | 2.64 | 3.08 | 2.46 | 2.47 | 2.43 | 2.43 | 2.47 | 2.95 | 2.32 | 2.35 |
| OUT | (%) | 0.70 | 0.69 | 1.31 | 2.14 | 1.34 | 2.02 | 0.74 | 0.74 | 1.77 | 2.37 | 8.16 | 1.85 | 0.71 | 0.71 | 1.25 | 2.74 | 1.10 | 1.98 |
| IN | Skew- | -0.61 | -0.63 | -0.45 | -0.06 | -0.77 | -0.78 | -0.92 | -0.97 | -0.01 | 0.06 | -1.27 | -1.18 | -0.46 | -0.50 | -0.39 | -0.18 | -0.64 | -0.55 |
| OUT | ness | 0.65 | 0.59 | 0.65 | -0.52 | 1.64 | -0.59 | 0.72 | 0.72 | 0.90 | -0.80 | -3.02 | 0.48 | 0.75 | 0.74 | 0.46 | 1.28 | -0.13 | 0.32 |
| IN | Kur- | 12.94 | 12.38 | 10.92 | 6.45 | 10.06 | 9.95 | 13.41 | 13.10 | 11.34 | 8.84 | 11.74 | 11.64 | 12.25 | 12.17 | 11.32 | 7.55 | 9.68 | 10.48 |
| OUT | tosis | 2.46 | 2.44 | 4.11 | 2.25 | 6.03 | 2.95 | 2.54 | 2.57 | 4.29 | 3.35 | 12.42 | 2.19 | 2.74 | 2.73 | 1.95 | 6.92 | 1.95 | 3.23 |
| IN | HE | 93.95 | 93.93 | 93.00 | 88.65 | 93.72 | 93.82 | 94.05 | 94.04 | 92.99 | 90.50 | 93.91 | 93.89 | 94.06 | 94.06 | 93.88 | 91.27 | 94.59 | 94.47 |
| OUT | (%) | 96.48 | 96.60 | 87.64 | 67.20 | 87.10 | 70.82 | 96.06 | 96.03 | 77.69 | 59.74 | NE | 75.39 | 96.35 | 96.41 | 88.83 | 46.42 | 91.35 | 71.96 |

Note: 1. MSCI Korea, KOSPI 200 futures, S&P 500 futures and Nikkei 225 futures are cointegrated.

2. See the note 2 - 5 and 7 of Table 3 (part A).

Part G and H of Table 4 contain the results for Netherlands and Sweden. Both of them have a moderate within-sample volatility ( standard deviation around 1.30% ) and a slightly smaller out-of-sample volatility ( standard deviation around 1.10% ). The sophisticated model further brings them down by around 0.06% in both countries. The best strategy for Netherlands involves AEX, FTSE 100 and SMI index futures modelled by quadrivariate TGARCH (1,1). The one for Sweden involves OMXS and FTSE 100 index futures modelled by VECM and trivariate GARCH (1,1). Surprisingly, the two seemingly loosely related European countries – Switzerland and Netherlands supply an important hedging tool to each other.

In Sample (IN): 1990M12 2005M12 Out of Sample (OUT): 2006M01 2007M05

## TABLE 4 ( Part G ): Hedging Performance results for MSCI Netherlands

| | | With One Hedging Instrument | | | | | | | With Three Hedging Instruments | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | AEX | | | | | | | AEX FTSE 100 DAX 30 | | | | | | AEX FTSE 100 S&P 500 | | | | | | AEX FTSE 100 SMI | | | | | |
| Method | | naïve | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.04 | 0.04 | 0.04 | 0.038 | 0.038 | 0.04 | 0.038 | 0.04 | 0.04 | 0.033 | 0.037 | 0.03 | 0.04 | 0.04 | 0.04 | 0.038 | 0.04 | 0.04 | 0.038 | 0.04 | 0.04 | 0.04 | 0.037 | 0.038 | 0.038 |
| OUT | | 0.02 | 0.02 | 0.02 | 0.021 | 0.021 | 0.02 | 0.022 | 0.02 | 0.02 | 0.02 | 0.023 | 0.02 | 0.02 | 0.02 | 0.02 | 0.024 | 0.02 | 0.02 | 0.024 | 0.02 | 0.02 | 0.02 | 0.023 | 0.023 | 0.022 |
| IN | Min | -0.04 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.04 | -0.03 | -0.04 | -0.03 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.037 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 |
| OUT | | -0.01 | -0.02 | -0.02 | -0.02 | -0.02 | -0.01 | -0.01 | -0.01 | -0.01 | -0.02 | -0.01 | -0.02 | -0.01 | -0.01 | -0.01 | -0.02 | -0.02 | -0.02 | -0.017 | -0.01 | -0.01 | -0.01 | -0.01 | -0.02 | -0.02 |
| IN | S. D. | 1.41 | 1.27 | 1.27 | 1.28 | 1.28 | 1.27 | 1.26 | 1.21 | 1.21 | 1.19 | 1.21 | 1.20 | 1.26 | 1.21 | 1.21 | 1.18 | 1.24 | 1.21 | 1.24 | 1.21 | 1.21 | 1.20 | 1.21 | 1.24 | 1.24 |
| OUT | (%) | 1.06 | 1.18 | 1.18 | 1.07 | 1.09 | 1.13 | 1.14 | 1.22 | 1.22 | 1.13 | 1.12 | 1.24 | 1.20 | 1.22 | 1.22 | 1.19 | 1.24 | 1.22 | 1.27 | 1.20 | 1.20 | 1.11 | 1.00 | 1.20 | 1.19 |
| IN | Skew- | -0.05 | -0.03 | -0.02 | -0 | 0.036 | 0.07 | 0.071 | -0.08 | -0.08 | -0.01 | 0.041 | 0.03 | -0.01 | -0.09 | -0.09 | 0.053 | 0.13 | -0.04 | -0.083 | -0.08 | -0.08 | -0.12 | -0.13 | -0.13 | -0.14 |
| OUT | ness | 0.06 | 0.04 | 0.06 | 0.013 | 0.01 | 0.08 | 0.055 | 0.08 | 0.09 | -0.15 | 0.082 | -0.23 | 0.01 | 0.1 | 0.1 | 0.009 | -0.19 | -0.21 | -0.18 | 0.12 | 0.12 | 0.1 | 0.096 | -0.08 | -0.08 |
| IN | Kur- | 3.66 | 3.6 | 3.61 | 3.767 | 3.792 | 3.72 | 3.721 | 3.42 | 3.42 | 3.355 | 3.605 | 3.35 | 3.48 | 3.52 | 3.52 | 3.621 | 3.8 | 3.54 | 3.665 | 3.47 | 3.47 | 3.46 | 3.389 | 3.659 | 3.563 |
| OUT | tosis | 2 | 2.1 | 2.09 | 2.074 | 2.035 | 2.02 | 2.011 | 1.96 | 1.95 | 2.18 | 2.042 | 2.01 | 1.8 | 1.99 | 1.99 | 2.159 | 2.35 | 2.13 | 2.059 | 1.97 | 1.97 | 2.2 | 2.214 | 2.051 | 2.071 |
| IN | HE | 92.74 | 94.11 | 94.10 | 94.05 | 94.06 | 94.10 | 94.21 | 94.69 | 94.69 | 94.88 | 94.69 | 94.74 | 94.24 | 94.72 | 94.72 | 94.92 | 94.42 | 94.69 | 94.41 | 94.65 | 94.65 | 94.73 | 94.66 | 94.41 | 94.38 |
| OUT | (%) | 88.73 | 85.93 | 86.05 | 88.41 | 88.19 | 87.19 | 87.04 | 84.99 | 85.00 | 87.25 | 87.44 | 84.50 | 85.63 | 85.14 | 85.10 | 85.70 | 84.57 | 85.14 | 83.90 | 85.51 | 85.46 | 87.70 | 90.07 | 85.55 | 85.69 |

| | | With Two Hedging Instruments | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | AEX DAX 30 | | | | | | AEX FTSE 100 | | | | | | AEX S&P 500 | | | | | | AEX SMI | | | | | |
| Method | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.04 | 0.04 | 0.04 | 0.036 | 0.04 | 0.036 | 0.04 | 0.04 | 0.038 | 0.038 | 0.04 | 0.04 | 0.04 | 0.04 | 0.038 | 0.04 | 0.04 | 0.038 | 0.04 | 0.04 | 0.04 | 0.037 | 0.039 | 0.037 |
| OUT | | 0.02 | 0.02 | 0.022 | 0.033 | 0.02 | 0.023 | 0.02 | 0.02 | 0.024 | 0.028 | 0.02 | 0.05 | 0.02 | 0.02 | 0.023 | 0.03 | 0.02 | 0.641 | 0.02 | 0.02 | 0.02 | 0.043 | 0.023 | 0.027 |
| IN | Min | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.04 | -0.04 | -0.03 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.04 | -0.038 | -0.04 | -0.04 | -0.03 | -0.03 | -0.03 | -0.03 |
| OUT | | -0.02 | -0.02 | -0.01 | -0.18 | -0.01 | -0.03 | -0.01 | -0.01 | -0.01 | -0.21 | -0.01 | -0.25 | -0.02 | -0.02 | -0.01 | -0.02 | -0.01 | -0.012 | -0.02 | -0.02 | -0.01 | -0.04 | -0.02 | -0.01 |
| IN | S. D. | 1.26 | 1.26 | 1.29 | 1.26 | 1.29 | 1.28 | 1.21 | 1.21 | 1.18 | 1.19 | 1.19 | 1.18 | 1.24 | 1.24 | 1.18 | 1.17 | 1.24 | 1.24 | 1.27 | 1.27 | 1.24 | 1.24 | 1.23 | 1.23 |
| OUT | (%) | 1.21 | 1.20 | 1.14 | 4.79 | 1.15 | 1.37 | 1.21 | 1.21 | 1.17 | 5.34 | 1.08 | 6.48 | 1.17 | 1.16 | 1.11 | 1.21 | 1.14 | 15.50 | 1.21 | 1.21 | 1.09 | 2.44 | 1.17 | 1.36 |
| IN | Skew- | -0.07 | -0.07 | 0.041 | 0.019 | 0 | 0.049 | -0.06 | -0.06 | -0.02 | -0.01 | 0.08 | -0.02 | -0.11 | -0.11 | -0.07 | -0.05 | -0.05 | -0.089 | -0.06 | -0.06 | -0.06 | -0.09 | -0.03 | -0.07 |
| OUT | ness | 0.02 | 0.04 | -0.15 | -3.29 | -0.04 | -0.39 | 0.11 | 0.11 | 0.033 | -3.26 | 0.1 | -3.1 | 0.09 | 0.11 | 0.152 | 0.21 | 0.11 | 3.715 | 0.06 | 0.07 | -0.06 | -0.3 | -0.14 | 0.177 |
| IN | Kur- | 3.51 | 3.52 | 3.738 | 3.472 | 3.59 | 3.609 | 3.46 | 3.47 | 3.629 | 3.689 | 3.64 | 3.88 | 3.55 | 3.56 | 3.557 | 3.56 | 3.41 | 3.519 | 3.57 | 3.59 | 3.5 | 3.486 | 3.562 | 3.518 |
| OUT | tosis | 2.09 | 2.08 | 1.97 | 13 | 1.89 | 2.705 | 1.94 | 1.94 | 1.989 | 12.86 | 2.03 | 12.1 | 2.17 | 2.16 | 2.223 | 2.32 | 2.1 | 14.9 | 2.1 | 2.09 | 1.67 | 2.339 | 2.146 | 1.906 |
| IN | HE | 94.20 | 94.20 | 93.95 | 94.23 | 93.99 | 94.08 | 94.65 | 94.65 | 94.92 | 94.88 | 94.84 | 94.91 | 94.43 | 94.43 | 94.90 | 95.02 | 94.39 | 94.38 | 94.17 | 94.16 | 94.43 | 94.43 | 94.47 | 94.48 |
| OUT | (%) | 85.37 | 85.53 | 87.03 | NE | 86.70 | 81.20 | 85.37 | 85.33 | 86.23 | NE | 88.30 | NE | 86.34 | 86.47 | 87.75 | 85.21 | 87.06 | NE | 85.25 | 85.33 | 88.13 | 40.12 | 86.27 | 81.44 |

Note: 1. MSCI Netherlands, AEX futures and FTSE 100 are cointegrated.
2. See the note 2 - 5 and 7 of Table 3 (part A).

In Sample (IN): 1990M12 2005M12 Out of Sample (OUT): 2006M01 2007M05

## TABLE 4 ( Part H ): Hedging Performance results for MSCI Sweden

| | | With One Hedging Instrument | | | | | | | With Three Hedging Instruments | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | OMXS 30 | | | | | | | OMXS 30 DAX 30 FTSE 100 | | | | | | OMXS 30 S&P 500 S&P 500 | | | | | |
| Method | | naïve | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 |
| OUT | | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| IN | Min | -0.05 | -0.04 | -0.04 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.04 | -0.04 | -0.03 | -0.04 | -0.04 | -0.04 | -0.05 | -0.04 | -0.05 | -0.05 |
| OUT | | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.03 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.03 | -0.02 | -0.02 | -0.02 | -0.03 | -0.03 | -0.03 |
| IN | S. D. | 1.35 | 1.29 | 1.29 | 1.21 | 1.21 | 1.22 | 1.22 | 1.26 | 1.27 | 1.19 | 1.20 | 1.19 | 1.23 | 1.27 | 1.28 | 1.21 | 1.22 | 1.22 | 1.22 |
| OUT | (%) | 1.18 | 1.15 | 1.15 | 1.17 | 1.17 | 1.17 | 1.21 | 1.16 | 1.17 | 1.15 | 1.17 | 1.15 | 1.19 | 1.13 | 1.14 | 1.19 | 1.21 | 1.24 | 1.26 |
| IN | Skew- | -0.26 | -0.31 | -0.30 | -0.19 | -0.20 | -0.19 | -0.21 | -0.20 | -0.23 | -0.04 | -0.07 | -0.04 | -0.22 | -0.27 | -0.30 | -0.28 | -0.29 | -0.34 | -0.38 |
| OUT | ness | -0.05 | -0.20 | -0.18 | -0.32 | -0.29 | -0.34 | -0.45 | -0.15 | -0.14 | -0.57 | -0.59 | -0.48 | -0.63 | -0.21 | -0.19 | -0.47 | -0.45 | -0.56 | -0.62 |
| IN | Kur- | 4.74 | 4.38 | 4.41 | 4.23 | 4.19 | 4.25 | 4.21 | 4.25 | 4.27 | 3.92 | 3.95 | 3.80 | 4.13 | 4.42 | 4.44 | 4.15 | 4.03 | 4.24 | 4.31 |
| OUT | tosis | 2.78 | 2.99 | 2.98 | 3.06 | 3.03 | 3.07 | 3.24 | 3.14 | 3.07 | 3.19 | 3.14 | 3.24 | 3.31 | 3.15 | 3.07 | 3.33 | 3.28 | 3.50 | 3.49 |
| IN | HE | 96.53 | 96.80 | 96.80 | 97.22 | 97.22 | 97.18 | 97.18 | 96.95 | 96.94 | 97.25 | 97.24 | 97.30 | 97.14 | 96.90 | 96.89 | 97.20 | 97.14 | 97.18 | 97.16 |
| OUT | (%) | 92.63 | 93.09 | 93.07 | 92.78 | 92.77 | 92.79 | 92.29 | 92.90 | 92.83 | 93.05 | 92.77 | 93.10 | 92.58 | 93.27 | 93.21 | 92.59 | 92.35 | 91.94 | 91.69 |

| | | With Two Hedging Instruments | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | OMXS 30 DAX 30 | | | | | | OMXS 30 FTSE 100 | | | | | | OMXS 30 S&P 500 | | | | | |
| Method | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.04 |
| OUT | | 0.02 | 0.02 | 0.02 | 0.03 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.33 | 0.02 | 0.05 | 0.02 | 0.02 | 0.02 | 0.06 | 0.02 | 0.06 |
| IN | Min | -0.04 | -0.04 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.04 | -0.05 | -0.04 | -0.04 | -0.04 | -0.04 | -0.05 | -0.05 | -0.05 | -0.06 |
| OUT | | -0.02 | -0.02 | -0.03 | -0.04 | -0.08 | -0.04 | -0.02 | -0.02 | -0.02 | -0.06 | -0.02 | -0.10 | -0.02 | -0.02 | -0.02 | -0.14 | -0.02 | -0.66 |
| IN | S. D. | 1.28 | 1.28 | 1.21 | 1.20 | 1.25 | 1.21 | 1.29 | 1.29 | 1.21 | 1.22 | 1.22 | 1.22 | 1.29 | 1.30 | 1.21 | 1.20 | 1.23 | 1.29 |
| OUT | (%) | 1.13 | 1.14 | 1.20 | 1.87 | 2.27 | 2.00 | 1.16 | 1.16 | 1.12 | 8.45 | 1.11 | 4.36 | 1.15 | 1.15 | 1.18 | 4.28 | 1.19 | 16.36 |
| IN | Skew- | -0.24 | -0.26 | -0.12 | -0.13 | -0.09 | -0.15 | -0.31 | -0.30 | -0.23 | -0.22 | -0.21 | -0.22 | -0.30 | -0.32 | -0.28 | -0.31 | -0.33 | -0.82 |
| OUT | ness | -0.23 | -0.21 | -0.31 | -0.15 | -2.17 | -0.15 | -0.17 | -0.14 | -0.55 | 3.00 | -0.63 | -0.72 | -0.20 | -0.17 | -0.39 | -2.11 | -0.38 | -3.61 |
| IN | Kur- | 4.44 | 4.46 | 4.32 | 4.49 | 4.32 | 4.48 | 4.31 | 4.29 | 4.17 | 4.28 | 4.15 | 4.07 | 4.38 | 4.40 | 4.46 | 4.39 | 4.43 | 6.16 |
| OUT | tosis | 3.18 | 3.12 | 3.42 | 1.97 | 8.22 | 1.84 | 2.96 | 2.92 | 3.27 | 11.85 | 3.35 | 2.73 | 3.01 | 2.95 | 3.22 | 8.30 | 3.18 | 14.40 |
| IN | HE | 96.89 | 96.88 | 97.22 | 97.27 | 97.02 | 97.22 | 96.81 | 96.80 | 97.22 | 97.16 | 97.16 | 97.16 | 96.80 | 96.80 | 97.19 | 97.27 | 97.12 | 96.85 |
| OUT | (%) | 93.23 | 93.17 | 92.42 | 81.68 | 72.93 | 79.06 | 92.99 | 92.88 | 93.45 | NE | 93.55 | NE | 93.09 | 93.06 | 92.68 | 3.85 | 92.53 | NE |

Note: 1. MSCI Sweden, OMXS 30 futures and DAX 30 futures are cointegrated.
2. See the note 2 - 5 and 7 of Table 3 (part A).

Part I of Table 4 shows the results for Australia. Compared with others in this group, the return of Australia is stable and the hedging effectiveness is low. But sophistication only brings tiny improvement in the within-sample period and no improvement in the out-of-sample period. The best hedge involves SPI 200 and Hang Seng index futures estimated by OLS method. Its performance is only slightly better than the simple OLS.

In Sample (IN): 1988M10 2005M12

Out of Sample (OUT): 2006M01 2007M05

**TABLE 4 ( Part I ): Hedging Performance results for MSCI Australia**

| | | With One Hedging | | | | With Two Hedging Instruments | | | | | | | | | | | | With Three Hedging Instruments | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | SPI 200 | | | | SPI 200 Hang Seng | | | SPI 200 Nikkei 225 | | | SPI 200 FTSE 100 | | | SPI 200 S&P 500 | | | SPI 200 S&P 500 Hang Seng | | | SPI 200 S&P 500 Nikkei 225 | | | SPI 200 S&P 500 FTSE 100 | | |
| Method | | naive | ols | garch | tgarch | ols | garch | tgarch | ols | garch | tgarch | ols | garch | tgarch | ols | garch | tgarch | ols | garch | tgarch | ols | garch | tgarc | ols | garch | tgarch |
| IN | Max | 0.04 | 0.04 | 0.03 | 0.03 | 0.04 | 0.03 | 0.03 | 0.04 | 0.03 | 0.04 | 0.04 | 0.03 | 0.03 | 0.04 | 0.04 | 0.03 | 0.04 | 0.03 | 0.03 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.03 |
| OUT | | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 1.48 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| IN | Min | -0.03 | -0.03 | -0.03 | -0.02 | -0.02 | -0.03 | -0.02 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.02 | -0.02 | -0.02 | -0.02 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 |
| OUT | | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.02 | -0.02 | -0.01 | -0.01 | -0.03 | -0.01 | -0.01 | -0.29 | -0.01 | -0.01 | -0.04 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| IN | S. D. | 1.05 | 1.03 | 1.03 | 1.02 | 1.02 | 1.01 | 1.00 | 1.03 | 1.04 | 1.07 | 1.03 | 1.04 | 1.02 | 1.03 | 1.05 | 1.02 | 1.02 | 1.01 | 1.02 | 1.03 | 1.06 | 1.04 | 1.03 | 1.05 | 1.01 |
| OUT | (%) | 0.55 | 0.56 | 0.60 | 0.61 | 0.56 | 0.94 | 0.79 | 0.56 | 0.61 | 0.92 | 0.56 | 0.62 | 37.34 | 0.56 | 0.63 | 1.13 | 0.57 | 0.66 | 0.68 | 0.56 | 0.61 | 0.59 | 0.57 | 0.58 | 0.58 |
| IN | Skew- | 0.32 | 0.27 | 0.24 | 0.24 | 0.25 | 0.19 | 0.14 | 0.28 | 0.24 | 0.29 | 0.27 | 0.25 | 0.11 | 0.27 | 0.27 | 0.19 | 0.25 | 0.28 | 0.32 | 0.27 | 0.40 | 0.39 | 0.27 | 0.25 | 0.08 |
| OUT | ness | -0.14 | -0.08 | -0.23 | -0.08 | -0.13 | 0.37 | 0.31 | -0.07 | -0.04 | -0.98 | -0.08 | -0.18 | 3.46 | -0.09 | -0.32 | -1.77 | -0.17 | 0.36 | 0.71 | -0.08 | 0.13 | 0.52 | -0.09 | -0.27 | -0.06 |
| IN | Kur- | 4.03 | 3.59 | 3.43 | 3.37 | 3.52 | 3.22 | 3.00 | 3.60 | 3.48 | 3.81 | 3.58 | 3.55 | 3.49 | 3.66 | 3.73 | 3.18 | 3.68 | 3.64 | 3.39 | 3.66 | 4.12 | 3.88 | 3.65 | 3.90 | 3.73 |
| OUT | tosis | 2.28 | 2.19 | 2.13 | 2.08 | 2.21 | 2.04 | 2.94 | 2.20 | 2.12 | 3.88 | 2.19 | 2.03 | 13.86 | 2.20 | 2.24 | 6.85 | 2.23 | 2.63 | 3.53 | 2.20 | 2.41 | 2.72 | 2.17 | 2.34 | 2.16 |
| IN | HE | 92.48 | 92.72 | 92.75 | 92.91 | 92.85 | 92.96 | 93.20 | 92.72 | 92.63 | 92.13 | 92.72 | 92.53 | 92.85 | 92.74 | 92.50 | 92.87 | 92.92 | 92.95 | 92.85 | 92.74 | 92.30 | 92.65 | 92.75 | 92.41 | 93.02 |
| OUT | (%) | 94.52 | 94.37 | 93.60 | 93.25 | 94.33 | 84.05 | 88.95 | 94.41 | 93.28 | 84.90 | 94.36 | 93.04 | NE | 94.30 | 92.89 | 77.31 | 94.15 | 92.27 | 91.72 | 94.31 | 93.38 | 93.69 | 94.20 | 93.93 | 94.01 |

Note: See the note 2 - 5 and 7 of Table 3 (part A).

The third group includes Germany, UK and USA. These three countries have stable hedged portfolio return both within- and out-of-sample.

In Sample (IN): 1990M12 2005M12

Out of Sample (OUT): 2006M01 2007M05

TABLE 5 ( Part A ): Hedging Performance results for MSCI Germany

| | | With One Hedging Instrument | | | | | | | With Three Hedging Instruments | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | DAX 30 | | | | | | | DAX 30 S&P 500 AEX | | | | | | DAX 30 S&P 500 SMI | | | | | | DAX 30 S&P 500 FTSE 100 | | | | | |
| Method | | naïve | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.02 | 0.02 | 0.02 | 0.03 | 0.03 | 0.02 | 0.02 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 | 0.02 | 0.04 | 0.03 | 0.02 | 0.03 | 0.02 | 0.02 | 0.02 | 0.03 | 0.02 | 0.02 |
| OUT | | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| IN | Min | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.04 | -0.04 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.04 | -0.04 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 |
| OUT | | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 |
| IN | S. D. | 0.98 | 0.97 | 0.97 | 1.00 | 1.00 | 0.97 | 0.97 | 0.96 | 0.96 | 0.99 | 1.01 | 0.99 | 0.97 | 0.96 | 0.96 | 1.03 | 1.01 | 1.00 | 1.01 | 0.96 | 0.96 | 0.99 | 1.12 | 0.99 | 1.03 |
| OUT | (%) | 0.72 | 0.74 | 0.74 | 0.77 | 0.79 | 0.76 | 0.77 | 0.73 | 0.73 | 0.73 | 0.74 | 0.72 | 0.74 | 0.76 | 0.76 | 0.76 | 0.76 | 0.76 | 0.75 | 0.75 | 0.74 | 0.74 | 0.78 | 0.75 | 0.78 |
| IN | Skew- | -0.51 | -0.52 | -0.52 | -0.32 | -0.32 | -0.44 | -0.44 | -0.52 | -0.52 | -0.11 | -0.11 | -0.11 | -0.22 | -0.52 | -0.53 | 0.01 | -0.29 | -0.42 | -0.40 | -0.53 | -0.53 | -0.29 | -0.25 | -0.32 | -0.33 |
| OUT | ness | -0.87 | -1.07 | -1.03 | -1.05 | -0.99 | -0.98 | -0.98 | -1.13 | -1.09 | -1.11 | -0.89 | -0.99 | -0.92 | -1.14 | -1.11 | -0.89 | -0.83 | -0.75 | -0.73 | -1.16 | -1.12 | -1.01 | -0.75 | -0.96 | -0.96 |
| IN | Kur- | 3.55 | 3.55 | 3.56 | 3.53 | 3.53 | 3.51 | 3.48 | 3.79 | 3.80 | 3.45 | 3.38 | 3.28 | 3.21 | 3.46 | 3.47 | 4.95 | 3.92 | 3.61 | 3.72 | 3.56 | 3.57 | 3.25 | 3.46 | 3.33 | 3.42 |
| OUT | tosis | 2.64 | 3.36 | 3.23 | 3.37 | 3.12 | 3.14 | 3.16 | 3.54 | 3.41 | 3.43 | 2.99 | 3.22 | 3.02 | 3.59 | 3.47 | 2.89 | 2.74 | 2.42 | 2.42 | 3.63 | 3.48 | 3.28 | 2.55 | 3.04 | 2.96 |
| IN | HE | 97.52 | 97.60 | 97.60 | 97.41 | 97.41 | 97.58 | 97.56 | 97.63 | 97.63 | 97.47 | 97.38 | 97.50 | 97.58 | 97.64 | 97.64 | 97.29 | 97.39 | 97.41 | 97.38 | 97.62 | 97.62 | 97.50 | 96.75 | 97.48 | 97.27 |
| OUT | (%) | 94.77 | 94.51 | 94.57 | 94.06 | 93.82 | 94.28 | 94.02 | 94.66 | 94.73 | 94.75 | 94.53 | 94.78 | 94.47 | 94.22 | 94.31 | 94.21 | 94.25 | 94.28 | 94.38 | 94.41 | 94.49 | 94.61 | 93.98 | 94.34 | 93.96 |

| | | | With Two Hedging Instruments | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | | DAX 30 AEX | | | | | | DAX 30 SMI | | | | | | DAX 30 FTSE 100 | | | | | | DAX 30 S&P 500 | | | | | |
| Method | | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | | 0.02 | 0.02 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 | 0.02 | 0.03 | 0.03 | 0.02 | 0.02 | 0.02 | 0.02 | 0.03 | 0.03 | 0.02 | 0.02 | 0.02 | 0.02 | 0.03 | 0.03 | 0.02 | 0.02 |
| OUT | | | 0.01 | 0.01 | 0.01 | 0.23 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.05 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.02 | 0.01 | 0.09 |
| IN | Min | | -0.04 | -0.04 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.04 | -0.04 | -0.03 | -0.04 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 |
| OUT | | | -0.02 | -0.02 | -0.02 | -0.03 | -0.02 | -0.03 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.04 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.42 |
| IN | S. D. | | 0.96 | 0.97 | 0.94 | 1.00 | 0.95 | 0.98 | 0.96 | 0.96 | 1.04 | 1.04 | 0.98 | 1.02 | 0.96 | 0.96 | 1.07 | 1.06 | 0.95 | 0.97 | 0.96 | 0.96 | 1.05 | 1.05 | 0.99 | 0.99 |
| OUT | (%) | | 0.73 | 0.73 | 0.73 | 5.82 | 0.72 | 1.05 | 0.76 | 0.76 | 0.76 | 0.87 | 0.76 | 0.82 | 0.75 | 0.74 | 0.77 | 1.99 | 0.76 | 0.78 | 0.74 | 0.74 | 0.77 | 1.08 | 0.78 | 10.71 |
| IN | Skew- | | -0.53 | -0.53 | -0.19 | -0.11 | -0.25 | -0.20 | -0.51 | -0.52 | -0.22 | -0.22 | -0.41 | -0.34 | -0.52 | -0.53 | -0.27 | -0.27 | -0.29 | -0.31 | -0.52 | -0.52 | -0.28 | -0.28 | -0.39 | -0.43 |
| OUT | ness | | -1.08 | -1.04 | -1.12 | 3.58 | -0.97 | -1.11 | -1.12 | -1.09 | -1.00 | -0.19 | -0.95 | -0.78 | -1.16 | -1.12 | -1.01 | 0.78 | -0.86 | -0.83 | -1.11 | -1.07 | -0.95 | -0.04 | -0.92 | -3.42 |
| IN | Kur- | | 3.69 | 3.69 | 3.21 | 3.44 | 3.21 | 3.31 | 3.43 | 3.44 | 3.93 | 3.92 | 3.46 | 3.68 | 3.53 | 3.54 | 3.56 | 3.55 | 3.21 | 3.18 | 3.58 | 3.59 | 3.42 | 3.42 | 3.50 | 3.37 |
| OUT | tosis | | 3.38 | 3.25 | 3.55 | 14.27 | 3.24 | 4.14 | 3.53 | 3.40 | 3.16 | 3.84 | 3.10 | 2.69 | 3.62 | 3.47 | 3.42 | 4.14 | 2.82 | 3.06 | 3.49 | 3.36 | 3.02 | 2.54 | 3.06 | 13.69 |
| IN | HE | | 97.61 | 97.60 | 97.70 | 97.44 | 97.70 | 97.54 | 97.64 | 97.63 | 97.20 | 97.20 | 97.54 | 97.34 | 97.62 | 97.61 | 97.08 | 97.08 | 97.68 | 97.56 | 97.62 | 97.61 | 97.14 | 97.14 | 97.49 | 97.46 |
| OUT | (%) | | 94.64 | 94.71 | 94.66 | NE | 94.87 | 89.04 | 94.20 | 94.28 | 94.25 | 92.50 | 94.17 | 93.36 | 94.40 | 94.47 | 94.14 | 60.43 | 94.30 | 93.99 | 94.48 | 94.55 | 94.16 | 88.34 | 93.88 | NE |

Note: 1. MSCI Germany, DAX 30 futures and S&P 500 are cointegrated.
2. See the note 2 - 5 and 7 of Table 3 (part A).

In Sample (IN): 1990M12 2005M12 Out of Sample (OUT): 2006M01 2007M05

**TABLE 5 ( Part B ): Hedging Performance results for MSCI UK**

| | | With One Hedging Instrument | | | | | | | With Three Hedging Instruments | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | FTSE 100 | | | | | | | FTSE 100 S&P 500 CAC 40 | | | | | | FTSE 100 S&P 500 DAX 30 | | | | | | |
| Method | | naïve | ols | ci-ols | garch | tgarch | ci- | ci- | ols | ci-ols | garch | tgarch | ci- | ci- | ols | ci-ols | garch | tgarch | ci- | ci- |
| IN | Max | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| OUT | | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.01 | 0.01 |
| IN | Min | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.02 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.04 | -0.04 | -0.04 |
| OUT | | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| IN | S. D. | 0.74 | 0.72 | 0.72 | 0.74 | 0.73 | 0.74 | 0.73 | 0.70 | 0.70 | 0.68 | 0.69 | 0.69 | 0.69 | 0.70 | 0.70 | 0.72 | 0.74 | 0.73 | 0.73 |
| OUT | (%) | 0.24 | 0.25 | 0.25 | 0.26 | 0.26 | 0.26 | 0.26 | 0.26 | 0.26 | 0.27 | 0.27 | 0.27 | 0.26 | 0.24 | 0.24 | 0.27 | 0.26 | 0.26 | 0.27 |
| IN | Skew- | -0.63 | -0.42 | -0.39 | -0.66 | -0.64 | -0.63 | -0.57 | -0.33 | -0.31 | -0.14 | -0.43 | -0.21 | -0.30 | -0.26 | -0.23 | -0.64 | -1.07 | -0.87 | -1.02 |
| OUT | ness | -0.27 | -0.06 | -0.07 | -0.30 | -0.14 | -0.21 | -0.16 | -0.23 | -0.25 | -0.10 | -0.06 | -0.33 | -0.12 | -0.15 | -0.19 | -0.13 | -0.16 | -0.17 | -0.31 |
| IN | Kur- | 5.72 | 5.04 | 4.96 | 5.88 | 5.45 | 5.59 | 5.19 | 4.68 | 4.67 | 4.45 | 5.51 | 4.52 | 4.86 | 4.64 | 4.61 | 6.05 | 8.27 | 7.18 | 7.68 |
| OUT | tosis | 2.31 | 2.31 | 2.40 | 2.68 | 2.32 | 2.23 | 2.22 | 2.19 | 2.26 | 2.30 | 2.30 | 2.17 | 2.24 | 2.07 | 2.15 | 2.24 | 1.83 | 2.17 | 2.33 |
| IN | HE | 96.61 | 96.80 | 96.80 | 96.63 | 96.72 | 96.59 | 96.66 | 96.95 | 96.95 | 97.11 | 97.03 | 97.06 | 97.03 | 96.93 | 96.92 | 96.75 | 96.57 | 96.68 | 96.65 |
| OUT | (%) | 98.69 | 98.54 | 98.50 | 98.39 | 98.39 | 98.38 | 98.43 | 98.41 | 98.37 | 98.33 | 98.33 | 98.31 | 98.46 | 98.65 | 98.60 | 98.33 | 98.43 | 98.36 | 98.24 |

| | | With Two Hedging Instruments | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | FTSE 100 CAC 40 | | | | | | FTSE 100 DAX 30 | | | | | | FTSE 100 S&P 500 | | | | | |
| Method | | ols | ci-ols | garch | tgarch | ci- | ci- | ols | ci-ols | garch | tgarch | ci- | ci- | ols | ci-ols | garch | tgarch | ci- | ci- |
| IN | Max | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| OUT | | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.01 | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 | 0.01 |
| IN | Min | -0.03 | -0.03 | -0.02 | -0.03 | -0.02 | -0.03 | -0.03 | -0.03 | -0.03 | -0.04 | -0.02 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 |
| OUT | | 0.00 | 0.00 | -0.01 | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | -0.01 |
| IN | S. D. | 0.71 | 0.71 | 0.68 | 0.69 | 0.69 | 0.69 | 0.71 | 0.71 | 0.70 | 0.73 | 0.69 | 0.71 | 0.71 | 0.71 | 0.75 | 0.72 | 0.75 | 0.74 |
| OUT | (%) | 0.25 | 0.25 | 0.31 | 0.49 | 0.31 | 0.52 | 0.23 | 0.24 | 0.26 | 0.29 | 0.33 | 0.30 | 0.26 | 0.26 | 0.26 | 0.61 | 0.27 | 0.43 |
| IN | Skew- | -0.28 | -0.27 | -0.20 | -0.38 | -0.33 | -0.41 | -0.27 | -0.26 | -0.61 | -0.93 | -0.40 | -0.46 | -0.45 | -0.41 | -0.91 | -0.52 | -0.87 | -0.76 |
| OUT | ness | -0.09 | -0.10 | 0.16 | -0.13 | -0.18 | -0.19 | 0.02 | 0.00 | -0.15 | -0.44 | -0.46 | -0.31 | -0.17 | -0.20 | -0.21 | -0.94 | -0.32 | -0.56 |
| IN | Kur- | 4.47 | 4.47 | 4.35 | 4.63 | 4.19 | 4.52 | 4.57 | 4.56 | 5.57 | 7.04 | 4.64 | 4.58 | 5.29 | 5.20 | 7.15 | 5.52 | 6.70 | 6.09 |
| OUT | tosis | 2.33 | 2.42 | 3.47 | 3.17 | 3.34 | 3.28 | 2.34 | 2.43 | 2.28 | 2.21 | 2.28 | 1.81 | 2.14 | 2.20 | 2.13 | 2.99 | 2.17 | 2.31 |
| IN | HE | 96.89 | 96.88 | 97.11 | 97.03 | 97.05 | 97.03 | 96.85 | 96.85 | 96.92 | 96.66 | 97.01 | 96.90 | 96.83 | 96.82 | 96.54 | 96.76 | 96.51 | 96.55 |
| OUT | (%) | 98.56 | 98.53 | 97.68 | 94.38 | 97.71 | 93.54 | 98.73 | 98.69 | 98.43 | 98.02 | 97.38 | 97.87 | 98.41 | 98.35 | 98.36 | 91.12 | 98.23 | 95.73 |

Note: 1. MSCI UK, FTSE 100 futures, S&P 500 futures and DAX 30 futures are cointegrated.
2. See the note 2 - 5 and 7 of Table 3 (part A).

Part A and B of Table 5 show the results for Germany and UK respectively. Both of them have very low volatility in the two periods. In particular, the out-of-sample standard deviation of UK ( 0.24% ) is the lowest among eighteen countries. Not surprisingly, the improvement of sophisticated models is tiny. For Germany, the best strategy involving DAX 30 and AEX index futures modelled by VECM and trivariate GARCH (1,1) improves the hedging effectiveness by 0.10% in both periods. For UK the best strategy involving FTSE 100 and DAX 30 index futures estimated by OLS improves the hedging effectiveness by 0.05% in the two periods.

Part C of Table 5 contains the results for USA. Similar to Germany and UK, USA has stable volatility in both periods. In particular, its within-sample standard deviation ( 0.49% ) is the lowest among all countries. The improvement of sophistication is tiny and no strategy beats naïve hedge in both periods. Therefore naïve hedge is the best strategy for USA.

In Sample (IN): 1991M04 2005M12 Out of Sample (OUT): 2006M01 2007M05

**TABLE 5 ( Part C ): Hedging Performance results for MSCI USA**

| | | With One Hedging Instrument | | | | | | | With Three Hedging Instruments | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | S&P 500 | | | | | | | S&P 500 FTSE 100 DAX 30 | | | | | | S&P 500 FTSE 100 Nikkei 225 | | | | | |
| Method | | naive | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 | 0.02 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 | 0.02 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 | 0.01 |
| OUT | | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| IN | Min | -0.01 | -0.01 | -0.01 | -0.02 | -0.02 | -0.02 | -0.02 | -0.01 | -0.01 | -0.02 | -0.02 | -0.02 | -0.02 | -0.01 | -0.01 | -0.02 | -0.02 | -0.01 | -0.02 |
| OUT | | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| IN | S. D. | 0.49 | 0.49 | 0.49 | 0.49 | 0.49 | 0.49 | 0.49 | 0.48 | 0.48 | 0.50 | 0.50 | 0.50 | 0.50 | 0.49 | 0.49 | 0.49 | 0.49 | 0.49 | 0.49 |
| OUT | (%) | 0.44 | 0.44 | 0.44 | 0.46 | 0.46 | 0.45 | 0.45 | 0.45 | 0.45 | 0.44 | 0.43 | 0.45 | 0.45 | 0.45 | 0.45 | 0.43 | 0.43 | 0.44 | 0.45 |
| IN | Skew- | -0.26 | -0.25 | -0.26 | -0.10 | -0.10 | -0.16 | -0.17 | -0.19 | -0.20 | -0.09 | -0.09 | -0.15 | -0.15 | -0.21 | -0.22 | -0.05 | -0.05 | -0.09 | -0.12 |
| OUT | ness | -0.54 | -0.51 | -0.52 | -0.43 | -0.43 | -0.41 | -0.36 | -0.53 | -0.54 | -0.48 | -0.47 | -0.36 | -0.36 | -0.52 | -0.52 | -0.48 | -0.49 | -0.49 | -0.43 |
| IN | Kur- | 3.91 | 3.92 | 3.91 | 4.46 | 4.46 | 4.27 | 4.36 | 3.89 | 3.85 | 3.88 | 3.88 | 3.84 | 3.84 | 3.79 | 3.81 | 3.87 | 3.87 | 3.75 | 3.74 |
| OUT | tosis | 1.90 | 1.88 | 1.89 | 1.84 | 1.84 | 1.86 | 1.85 | 1.90 | 1.89 | 1.99 | 1.98 | 1.91 | 1.91 | 1.89 | 1.88 | 1.95 | 1.92 | 2.06 | 1.98 |
| IN | HE | 98.57 | 98.58 | 98.57 | 98.55 | 98.55 | 98.55 | 98.57 | 98.61 | 98.60 | 98.51 | 98.51 | 98.52 | 98.52 | 98.59 | 98.59 | 98.54 | 98.54 | 98.58 | 98.56 |
| OUT | (%) | 94.38 | 94.41 | 94.39 | 93.95 | 93.97 | 94.09 | 94.09 | 94.27 | 94.17 | 94.52 | 94.62 | 94.34 | 94.34 | 94.15 | 94.10 | 94.61 | 94.71 | 94.43 | 94.32 |
| Hedging Instrument | | | With Two Hedging Instruments | | | | | | | | | | | | | | | | | |
| | | | S&P 500 DAX 30 | | | | | | S&P 500 Nikkei 225 | | | | | | S&P 500 FTSE 100 | | | | | |
| Method | | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 | 0.02 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 | 0.02 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 | 0.02 |
| OUT | | | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| IN | Min | | -0.01 | -0.01 | -0.02 | -0.02 | -0.02 | -0.02 | -0.01 | -0.01 | -0.02 | -0.02 | -0.02 | -0.02 | -0.01 | -0.01 | -0.02 | -0.02 | -0.01 | -0.02 |
| OUT | | | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| IN | S. D. | | 0.48 | 0.48 | 0.50 | 0.50 | 0.50 | 0.50 | 0.49 | 0.49 | 0.49 | 0.49 | 0.49 | 0.49 | 0.49 | 0.49 | 0.49 | 0.49 | 0.49 | 0.49 |
| OUT | (%) | | 0.44 | 0.44 | 0.47 | 0.44 | 0.46 | 0.48 | 0.44 | 0.44 | 0.46 | 0.46 | 0.46 | 0.47 | 0.45 | 0.45 | 0.44 | 0.54 | 0.43 | 0.54 |
| IN | Skew- | | -0.20 | -0.22 | -0.11 | -0.11 | -0.15 | -0.15 | -0.25 | -0.26 | -0.15 | -0.15 | -0.18 | -0.19 | -0.22 | -0.22 | -0.04 | -0.04 | -0.01 | -0.09 |
| OUT | ness | | -0.54 | -0.54 | -0.42 | -0.62 | -0.34 | -0.70 | -0.52 | -0.53 | -0.42 | -0.41 | -0.44 | -0.34 | -0.51 | -0.52 | -0.47 | -0.30 | -0.52 | -0.26 |
| IN | Kur- | | 3.92 | 3.91 | 4.13 | 4.13 | 4.09 | 4.09 | 3.86 | 3.90 | 4.20 | 4.20 | 4.25 | 4.19 | 3.85 | 3.82 | 4.01 | 4.01 | 4.00 | 3.92 |
| OUT | tosis | | 1.91 | 1.91 | 1.85 | 2.10 | 1.85 | 2.32 | 1.90 | 1.90 | 1.85 | 1.88 | 1.85 | 1.89 | 1.87 | 1.87 | 1.90 | 1.82 | 2.02 | 1.84 |
| IN | HE | | 98.61 | 98.60 | 98.53 | 98.53 | 98.52 | 98.52 | 98.58 | 98.58 | 98.57 | 98.57 | 98.58 | 98.58 | 98.59 | 98.58 | 98.54 | 98.54 | 98.56 | 98.55 |
| OUT | (%) | | 94.36 | 94.35 | 93.72 | 94.35 | 93.92 | 93.37 | 94.39 | 94.38 | 93.94 | 94.01 | 93.88 | 93.80 | 94.17 | 94.11 | 94.58 | 91.57 | 94.72 | 91.59 |

Note: 1. MSCI USA, S&P 500 futures, FTSE 100 futures and DAX 30 are cointegrated.
2. See the note 2 - 5 and 7 of Table 3 (part A).

The last group includes Italy and Japan. Part A and B of Table 6 contain the results for Italy and Japan respectively. Both of them show significant changes in hedging effectiveness in the two periods. For Italy, hedging is highly effective in the within-sample period with the hedging effectiveness above 96% but much less effective in the out-of-sample period with the standard deviation around 81%. The situation in Japan is the other way around. Hedging effectiveness increases from around 83% to above 91%. For both countries, several sophisticated strategies improve within-sample results but none improve the out-of-sample results. The change in hedging effectiveness across two sample periods indicates a structural change in the data, which explains why the cross hedging strategies that perform well within the sample do badly in the out-of-sample period. Within and out-of-sample results together suggest the best strategy for them only involve the index futures of the same country. For Italy, it is estimated by OLS method. For Japan, it is estimated by VECM and bivariate GARCH (1,1).

In Sample (IN): 1994M12 2005M12 Out of Sample (OUT): 2006M01 2007M05

**TABLE 6 ( Part A ): Hedging Performance results for MSCI Italy**

| | | With One Hedging Instrument | | | | | | | With Three Hedging Instruments | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | MIB | | | | | | | MIB S&P 500 CAC 40 | | | | | | MIB S&P 500 DAX 30 | | | | | |
| Method | | naïve | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.03 | 0.03 | 0.03 | 0.03 | 0.04 | 0.03 | 0.03 | 0.03 | 0.03 | 0.04 | 0.04 | 0.03 | 0.03 |
| OUT | | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.04 | 0.04 | 0.02 | 0.02 | 0.04 | 0.05 | 0.05 | 0.03 |
| IN | Min | -0.05 | -0.03 | -0.03 | -0.02 | -0.02 | -0.04 | -0.03 | -0.03 | -0.03 | -0.02 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 |
| OUT | | -0.02 | -0.02 | -0.02 | -0.01 | -0.02 | -0.15 | -0.02 | -0.01 | -0.01 | -0.04 | -0.03 | -0.02 | -0.02 | -0.01 | -0.01 | -0.04 | -0.03 | -0.02 | -0.02 |
| IN | S. D. | 1.13 | 1.04 | 1.04 | 1.02 | 1.06 | 1.05 | 1.05 | 1.03 | 1.03 | 1.11 | 1.27 | 1.16 | 1.18 | 1.03 | 1.03 | 1.10 | 1.20 | 1.15 | 1.17 |
| OUT | (%) | 0.93 | 0.91 | 0.91 | 0.93 | 0.93 | 4.97 | 1.14 | 0.94 | 0.94 | 1.17 | 1.10 | 1.30 | 1.26 | 0.93 | 0.93 | 1.45 | 1.54 | 1.51 | 1.31 |
| IN | Skew-ness | -0.78 | -0.40 | -0.41 | -0.19 | -0.19 | -0.50 | -0.27 | -0.45 | -0.46 | -0.15 | -0.16 | -0.14 | -0.11 | -0.42 | -0.43 | 0.10 | -0.18 | -0.26 | -0.24 |
| OUT | | 1.13 | 0.86 | 0.87 | 0.90 | 0.89 | -2.20 | 0.41 | 0.94 | 0.95 | -0.90 | -0.89 | 1.61 | 1.81 | 0.94 | 0.95 | -0.08 | 1.38 | 1.74 | 1.36 |
| IN | Kur-tosis | 4.55 | 3.40 | 3.42 | 2.78 | 2.67 | 3.83 | 3.26 | 3.57 | 3.60 | 2.88 | 3.12 | 2.71 | 2.70 | 3.40 | 3.44 | 3.13 | 3.24 | 3.02 | 3.00 |
| OUT | | 3.79 | 3.38 | 3.39 | 3.33 | 3.30 | 6.22 | 3.09 | 3.49 | 3.50 | 5.65 | 5.58 | 5.82 | 6.85 | 3.48 | 3.50 | 6.05 | 6.99 | 6.50 | 4.29 |
| IN | HE | 96.68 | 97.22 | 97.21 | 97.30 | 97.07 | 97.17 | 97.15 | 97.25 | 97.25 | 96.80 | 95.80 | 96.49 | 96.36 | 97.26 | 97.26 | 96.87 | 96.30 | 96.55 | 96.46 |
| OUT | (%) | 80.92 | 81.64 | 81.63 | 80.89 | 80.88 | NE | 70.97 | 80.54 | 80.59 | 69.86 | 73.14 | 62.35 | 65.04 | 80.90 | 80.89 | 53.05 | 47.60 | 49.49 | 62.08 |

| | | With Two Hedging Instruments | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | MIB CAC 40 | | | | | | MIB DAX 30 | | | | | | MIB S&P 500 | | | | | |
| Method | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.03 | 0.03 | 0.02 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.04 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 |
| OUT | | 0.02 | 0.02 | 0.02 | 0.02 | 0.11 | 0.13 | 0.02 | 0.02 | 0.02 | 0.05 | 0.04 | 0.04 | 0.02 | 0.02 | 0.02 | 0.01 | 0.02 | 0.03 |
| IN | Min | -0.03 | -0.03 | -0.03 | -0.02 | -0.04 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.04 | -0.02 | -0.03 | -0.03 | -0.02 | -0.03 | -0.04 | -0.03 |
| OUT | | -0.01 | -0.01 | -0.03 | -0.03 | -0.03 | -0.08 | -0.01 | -0.01 | -0.03 | -0.03 | -0.03 | -0.05 | -0.01 | -0.01 | -0.05 | -0.03 | -0.02 | -0.04 |
| IN | S. D. | 1.03 | 1.03 | 1.09 | 1.09 | 1.06 | 1.12 | 1.03 | 1.03 | 1.20 | 1.23 | 1.06 | 1.06 | 1.03 | 1.03 | 1.02 | 1.05 | 1.06 | 1.08 |
| OUT | (%) | 0.94 | 0.94 | 1.14 | 1.04 | 3.02 | 4.30 | 0.93 | 0.93 | 1.16 | 1.74 | 1.69 | 1.98 | 0.92 | 0.92 | 1.33 | 0.96 | 1.10 | 1.54 |
| IN | Skew-ness | -0.45 | -0.45 | -0.33 | -0.22 | -0.38 | -0.07 | -0.41 | -0.42 | -0.17 | -0.19 | -0.33 | -0.22 | -0.43 | -0.45 | -0.11 | -0.18 | -0.39 | -0.23 |
| OUT | | 0.94 | 0.95 | -0.74 | 0.06 | 2.58 | 1.22 | 0.94 | 0.95 | -0.15 | 1.39 | 1.24 | -0.38 | 0.89 | 0.91 | -1.81 | -0.81 | 0.25 | 0.18 |
| IN | Kur-tosis | 3.57 | 3.58 | 2.88 | 2.77 | 3.63 | 2.67 | 3.39 | 3.41 | 2.68 | 2.56 | 3.50 | 2.87 | 3.48 | 3.52 | 3.06 | 3.20 | 3.99 | 2.98 |
| OUT | | 3.49 | 3.50 | 5.11 | 3.55 | 9.96 | 7.00 | 3.48 | 3.50 | 4.52 | 6.51 | 4.40 | 4.93 | 3.42 | 3.44 | 7.75 | 3.85 | 2.92 | 3.39 |
| IN | HE | 97.25 | 97.25 | 96.94 | 96.91 | 97.09 | 96.72 | 97.26 | 97.26 | 96.29 | 96.08 | 97.09 | 97.08 | 97.24 | 97.23 | 97.32 | 97.12 | 97.11 | 96.98 |
| OUT | (%) | 80.44 | 80.46 | 71.39 | 75.90 | NE | NE | 80.91 | 80.90 | 70.12 | 32.62 | 36.90 | 13.23 | 81.33 | 81.30 | 60.65 | 79.37 | 73.05 | 47.20 |

Note: 1. MSCI Italy, MIB futures and S&P 500 futures are cointegrated.

2. See the note 2 - 5 and 7 of Table 3 (part A).

In Sample (IN): 1996M06 2005M12 Out of Sample (OUT): 2006M01 2007M05

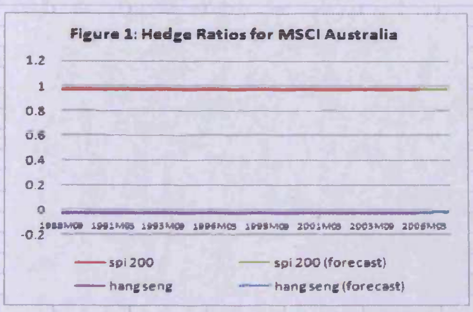
**TABLE 6 ( Part B ): Hedging Performance results for MSCI Japan**

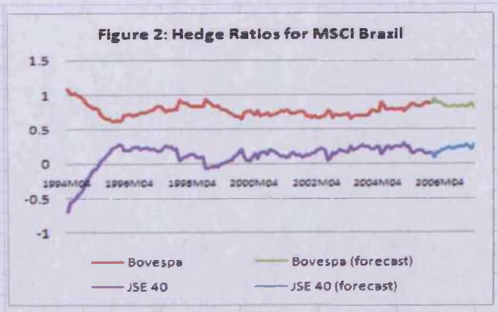
| | | With One Hedging Instrument | | | | | | | With Three Hedging Instruments | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | Nikkei 225 | | | | | | | Nikkei 225 S&P 500 SPI 200 | | | | | | Nikkei 225 S&P 500 KOSPI 200 | | | | | |
| Method | | naïve | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.09 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.06 | 0.07 | 0.07 | 0.06 | 0.06 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 |
| OUT | | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.01 | 0.01 | 1.03 | 1.21 | 0.03 | 0.04 | 0.02 | 0.02 | 0.45 | 1.06 | 0.05 | 0.18 |
| IN | Min | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.04 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 |
| OUT | | -0.01 | -0.01 | -0.01 | -0.06 | -0.14 | -0.01 | -0.01 | -0.02 | -0.02 | -0.07 | -0.03 | -2.26 | -0.02 | -0.01 | -0.01 | -0.03 | -0.03 | -0.12 | -0.04 |
| IN | S. D. | 2.20 | 1.85 | 1.85 | 1.86 | 1.85 | 1.84 | 1.84 | 1.78 | 1.79 | 1.77 | 1.80 | 1.74 | 1.78 | 1.84 | 1.84 | 1.85 | 1.83 | 1.82 | 1.82 |
| OUT | (%) | 0.90 | 0.94 | 0.98 | 1.67 | 3.55 | 0.90 | 0.98 | 0.95 | 0.98 | 25.00 | 29.35 | 54.86 | 1.64 | 0.98 | 1.02 | 10.99 | 25.60 | 4.40 | 4.57 |
| IN | Skew- | 0.63 | 0.51 | 0.47 | 0.60 | 0.56 | 0.53 | 0.49 | 0.57 | 0.55 | 0.70 | 0.48 | 0.58 | 0.54 | 0.57 | 0.53 | 0.41 | 0.47 | 0.54 | 0.43 |
| OUT | ness | 0.18 | 0.14 | 0.09 | -2.22 | -3.36 | 0.18 | 0.21 | -0.05 | -0.06 | 3.69 | 3.73 | -3.74 | 1.02 | 0.10 | 0.05 | 3.40 | 3.73 | -1.86 | 2.79 |
| IN | Kur- | 4.51 | 4.62 | 4.59 | 5.20 | 4.91 | 4.74 | 4.81 | 4.68 | 4.57 | 5.21 | 5.08 | 4.83 | 4.80 | 4.57 | 4.50 | 4.93 | 4.97 | 4.69 | 4.95 |
| OUT | tosis | 1.84 | 1.98 | 1.91 | 8.51 | 13.28 | 2.12 | 1.90 | 2.06 | 2.01 | 14.79 | 14.96 | 15.01 | 3.91 | 1.97 | 1.89 | 13.36 | 14.95 | 5.54 | 11.06 |
| IN | HE | 79.82 | 85.80 | 85.72 | 85.62 | 85.78 | 85.93 | 85.94 | 86.78 | 86.67 | 87.00 | 86.51 | 87.37 | 86.90 | 85.94 | 85.86 | 85.80 | 86.15 | 86.21 | 86.31 |
| OUT | (%) | 92.58 | 91.77 | 91.07 | 74.16 | NE | 92.51 | 91.15 | 91.70 | 91.17 | NE | NE | NE | 75.21 | 91.10 | 90.31 | NE | NE | NE | NE |

| | | With Two Hedging Instruments | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hedging Instrument | | Nikkei 225 SPI 200 | | | | | | Nikkei 225 KOSPI 200 | | | | | | Nikkei 225 S&P 500 | | | | | |
| Method | | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch | ols | ci-ols | garch | tgarch | ci-garch | ci-tgarch |
| IN | Max | 0.07 | 0.06 | 0.07 | 0.07 | 0.06 | 0.06 | 0.07 | 0.07 | 0.06 | 0.07 | 0.07 | 0.06 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.06 |
| OUT | | 0.02 | 0.02 | 0.25 | 0.06 | 0.02 | 0.01 | 0.02 | 0.02 | 0.13 | 0.04 | 0.04 | 0.04 | 0.02 | 0.02 | 0.32 | 0.02 | 0.02 | 0.02 |
| IN | Min | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.04 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 |
| OUT | | -0.02 | -0.02 | -0.03 | -0.07 | -0.02 | -0.03 | -0.01 | -0.01 | -0.02 | -0.07 | -0.01 | -0.03 | -0.01 | -0.01 | -0.09 | -0.05 | -0.08 | -0.03 |
| IN | S. D. | 1.79 | 1.80 | 1.77 | 1.76 | 1.76 | 1.74 | 1.84 | 1.85 | 1.83 | 1.80 | 1.84 | 1.83 | 1.85 | 1.85 | 1.82 | 1.80 | 1.78 | 1.80 |
| OUT | (%) | 0.98 | 1.01 | 6.05 | 3.45 | 1.02 | 1.39 | 0.97 | 1.01 | 3.27 | 2.48 | 1.22 | 1.72 | 0.96 | 1.01 | 8.37 | 1.57 | 2.39 | 1.53 |
| IN | Skew- | 0.61 | 0.59 | 0.72 | 0.83 | 0.67 | 0.71 | 0.56 | 0.52 | 0.49 | 0.58 | 0.62 | 0.55 | 0.53 | 0.50 | 0.47 | 0.44 | 0.47 | 0.39 |
| OUT | ness | -0.06 | -0.08 | 3.53 | -0.60 | -0.06 | -0.39 | 0.11 | 0.06 | 3.06 | -0.99 | 0.89 | 0.33 | 0.10 | 0.05 | 3.05 | -1.80 | -2.25 | -0.10 |
| IN | Kur- | 4.57 | 4.47 | 4.82 | 5.49 | 4.83 | 4.91 | 4.60 | 4.53 | 4.55 | 4.87 | 4.85 | 4.66 | 4.57 | 4.53 | 5.07 | 5.01 | 4.77 | 4.81 |
| OUT | tosis | 2.02 | 1.96 | 14.08 | 2.65 | 1.97 | 2.12 | 1.99 | 1.91 | 11.78 | 4.70 | 3.62 | 2.68 | 1.95 | 1.87 | 12.39 | 6.86 | 8.25 | 1.94 |
| IN | HE | 86.66 | 86.55 | 86.95 | 87.10 | 87.19 | 87.35 | 85.93 | 85.85 | 86.13 | 86.51 | 85.97 | 86.07 | 85.85 | 85.77 | 86.25 | 86.53 | 86.89 | 86.58 |
| OUT | (%) | 91.20 | 90.59 | NE | NE | 90.36 | 82.10 | 91.24 | 90.50 | 1.24 | 43.18 | 86.31 | 72.55 | 91.42 | 90.64 | NE | 77.18 | 47.29 | 78.32 |

Note: 1. MSCI Japan, Nikkei 225 futures, S&P 500 futures and SPI 200 futures are cointegrated.
2. See the note 2 - 5 and 7 of Table 3 (part A).

Figure 1 - 16 display the hedge ratio or ratios suggested by the best model of each country in the within-sample and out-of-sample period. The hedge ratios implied by OLS are more stable than those implied by GARCH or TGARCH. The hedge ratio of the index futures of the same country as MSCI is close to one. And the others are close to zero.



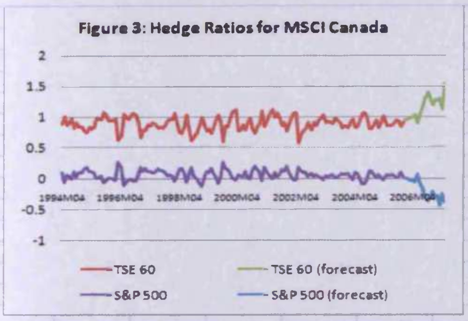


Figure 3: Hedge Ratios for MSCI Canada

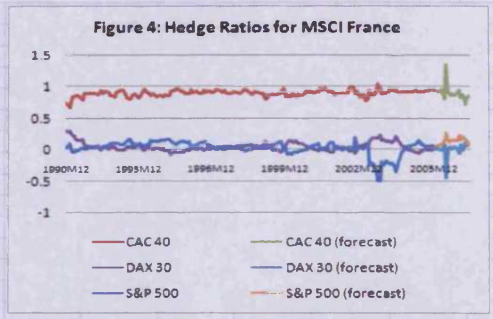


Figure 4: Hedge Ratios for MSCI France

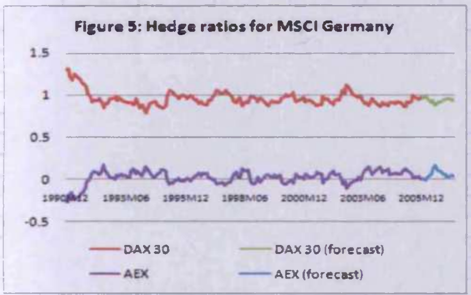


Figure 5: Hedge ratios for MSCI Germany

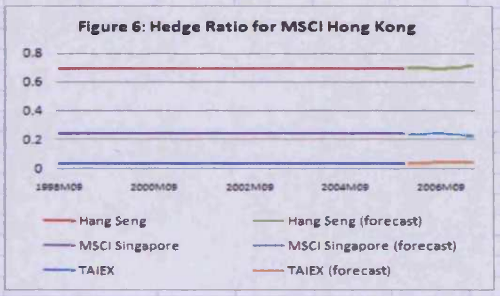


Figure 6: Hedge Ratio for MSCI Hong Kong

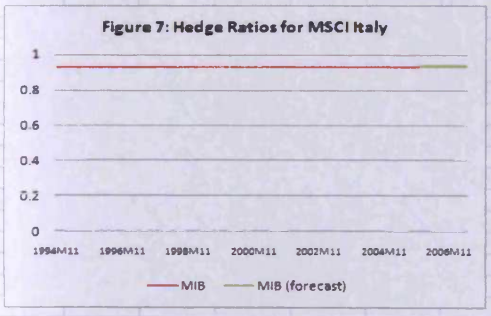


Figure 7: Hedge Ratios for MSCI Italy

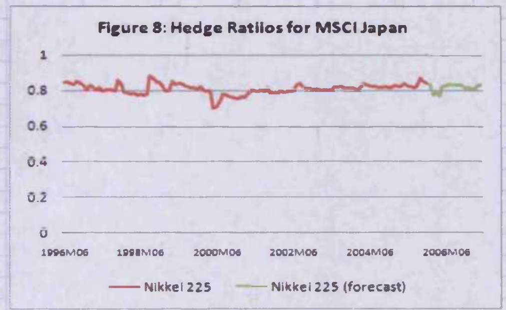


Figure 8: Hedge Ratiios for MSCI Japan

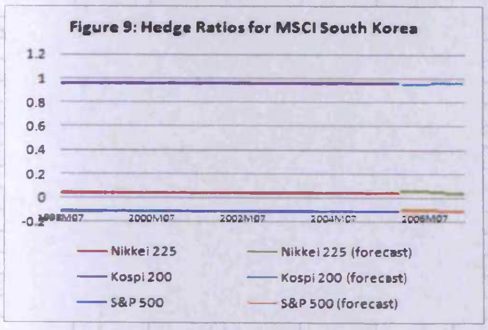


Figure 9: Hedge Ratios for MSCI South Korea

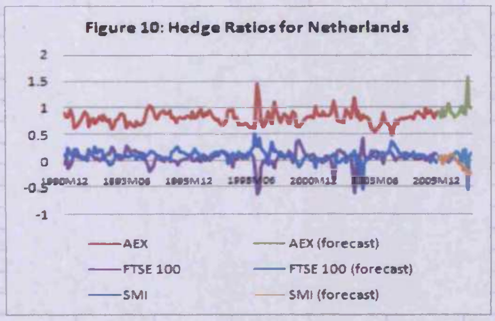


Figure 10: Hedge Ratios for Netherlands

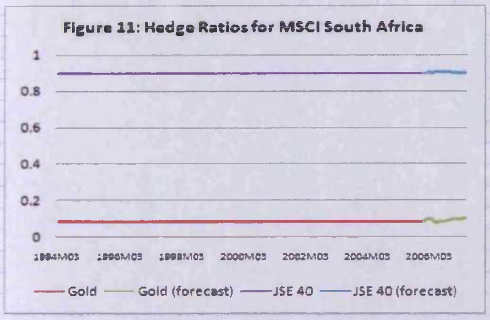


Figure 11: Hedge Ratios for MSCI South Africa

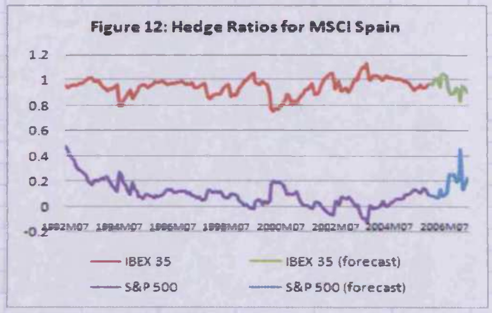


Figure 12: Hedge Ratios for MSCI Spain

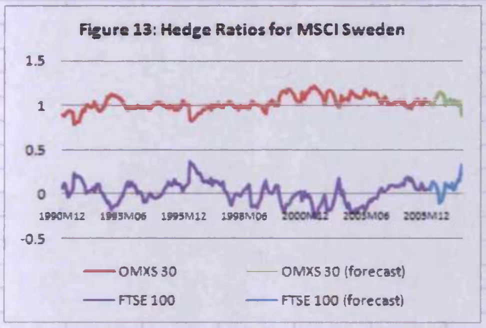


Figure 13: Hedge Ratios for MSCI Sweden

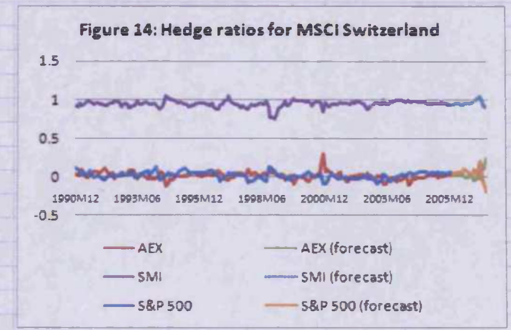


Figure 14: Hedge ratios for MSCI Switzerland

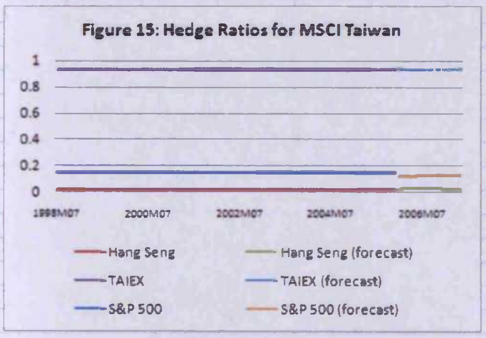


Figure 15: Hedge Ratios for MSCI Taiwan

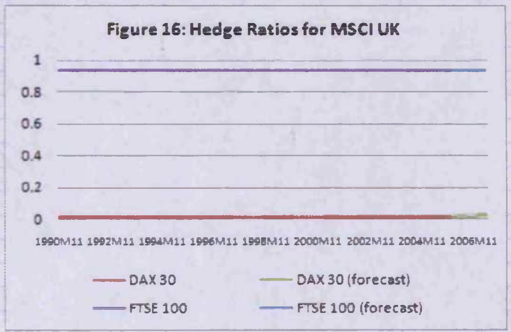


Figure 16: Hedge Ratios for MSCI UK

## 7. Conclusion

Cross hedging is an important strategy for investors in situations where the spot asset to be hedged has no actively traded futures contract. It is widely used in a variety of markets such as commodity, currency, interest rate and equity. In this chapter, we investigated the particular cross hedging problem confronted by institutional investors whose portfolio is benchmarked to MSCI indices.

First, we derived the analytical solution to the optimal cross hedge ratio vector for a spot portfolio with multiple assets. Second, we studied the empirical question of how to cross hedge a spot portfolio measured by one of the seventeen MSCI indices with the most heavily traded index futures of the related markets. The main results are as follows.

Most MSCI indices are not cointegrated with the most frequently traded index futures of the same country, but are cointegrated with the multiple index futures. In most cases, sophisticated cross hedging strategies involving several index futures improve the hedging effectiveness upon naïve or simple OLS hedge consistently in both within- and out-of-sample periods. In particular, for countries whose returns are volatile in both sample periods, the improvement is pronounced; for countries with moderate volatility, there is small and consistent improvement; for countries with stable return, the improvement is tiny; for countries with completely different volatility across two sample periods, the strategies that work well within the sample cannot beat simple strategies out-of-the-sample. For some countries, the VECM combined with multivariate GARCH (1, 1) or TGARCH (1, 1) fits the data well and produces the best hedging performance among all the alternatives. For the other countries, the best strategies are implied by simpler models.

The cross hedging effectiveness is measured by the reduction in portfolio variance. However, one problem of hedging, both direct and cross, is that it is unclear how far it can reduce the risk associated with sharp movements in markets. In the case of international cross-hedging, the answer seems in part to depend on the correlation between markets in extreme events, which is the subject of the next chapter.

## Appendix

A1: The derivation for the optimal cross hedge ratio vector in (5).

The problem is to maximize the expected utility of the portfolio return with respect to the futures position. That is,

$$\text{Max } U(\pi) = E(\pi) - \frac{1}{2}\gamma V(\pi) \quad \text{w.r.t. } y \tag{4}$$

Substitute the expression for the expected return and variance of the portfolio return in (4).

$$U(\pi) = E(s')x - E(f')y - \frac{1}{2}\gamma[x'\Sigma_{ss}x + y'\Sigma_{ff}y - 2x'\Sigma_{sf}y]$$

The derivative to the futures is as follows.[11]

$$\frac{\partial U}{\partial y} = -E(f') - \frac{1}{2}\gamma[(\Sigma_{ff} + \Sigma_{ff}')y - 2x'\Sigma_{sf}]$$

Because the covariance matrix of futures is symmetric, the above can be reduced to the following.

$$\frac{\partial U}{\partial y} = -E(f') - \gamma[\Sigma_{ff}y - x'\Sigma_{sf}]$$

Setting it to zero gives the solution for the optimal hedge ratio vector in (5).

$$y^* = -\frac{1}{\gamma}\Sigma_{ff}^{-1}E(f) + \Sigma_{ff}^{-1}\Sigma_{sf}'x$$

[11] Two rules of matrix derivatives are used here.

$$\frac{\partial x'Ax}{\partial x} = (A + A')x \quad \textit{where } x \textit{ is a vector and } A \textit{ is a matrix}$$

$$\frac{\partial ax}{\partial x} = a \quad \textit{where } a \textit{ and } x \textit{ are both vector}$$

A2: This part of the appendix contains the estimation results for the countries whose best strategy is modelled by GARCH or TGARCH. The VECM combined with a trivariate GARCH (1,1) is chosen for Brazil, Spain, Canada, Sweden and Germany, whose results are presented in Table A1. A quadrivariate GARCH (1,1) is selected for France, whose results are shown in Table A2. The VECM combined with a bivariate GARCH (1,1) is chosen for Japan, whose results are in Table A3. The quadrivariate TGARCH (1,1) is fitted to Netherlands and Switzerland with results shown in Table A4.

All the GARCH and TGARCH processes are stationary. The insignificant Q-statistics and Q-squared-statistics of the standardized residuals demonstrate that the residuals are free of autocorrelation or conditional heteroskedasticity, therefore the models are adequate. The insignificant Q-statistics of the VECM residuals indicate no autocorrelation. Although the Q-statistics of the VECM residuals of Brazil, Spain, Canada, Sweden and Japan are insignificant at 5% level, the theoretical results on temporal aggregation justifies the use of GARCH on the monthly return data set. The Q-squared-statistics of the VECM residuals significant different from zero at 5% level for Germany, France, Netherlands and Switzerland clearly indicate the need of model the conditional second moments explicitly.

**TABLE A1: Estimation Results for Brazil, Spain, Canada, Sweden and Germany**

| | Brazil | | Spain | | Canada | | Sweden | | Germany | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Mean specification: VECM<br>return (MSCI) = C(1) + C(4)*COINT<br>return (futures_1) = C(2) + C(5)*COINT<br>return( futures_2) = C(3) + C(6)*COINT | | | | | | | | | |
| | Covariance specification: Diagonal VECH<br>GARCH = M + A1.*RESID(-1)*RESID(-1)' + B1.*GARCH(-1) | | | | | | | | | |
| futures_1 | BOVSPA | | IBEX 35 | | TSE 60 | | OMXS 30 | | DAX 30 | |
| futures_2 | JSE 40 | | S&P 500 | | S&P 500 | | FTSE 100 | | AEX | |
| | Coef | S.E. | Coef | S.E. | Coef | S.E. | Coef | S.E. | Coef | S.E. |
| C(1) | 1.89E-02 | 8.98E-03 | 1.18E-02 | 4.61E-03 | 9.38E-03 | 2.69E-03 | 1.17E-02 | 3.86E-03 | 5.90E-03 | 3.97E-03 |
| C(4) | 1.13E-02 | 3.63E-02 | -2.05E-01 | 1.07E-01 | -3.20E-01 | 1.12E-01 | -6.22E-02 | 7.73E-02 | 3.04E-01 | 9.29E-02 |
| C(2) | 2.06E-02 | 1.03E-02 | 1.12E-02 | 4.57E-03 | 9.35E-03 | 2.64E-03 | 1.17E-02 | 3.72E-03 | 7.20E-03 | 4.03E-03 |
| C(5) | 5.76E-03 | 3.95E-02 | -1.62E-01 | 1.12E-01 | -2.80E-01 | 1.14E-01 | -4.61E-02 | 7.70E-02 | 3.35E-01 | 9.34E-02 |
| C(3) | 1.24E-02 | 6.19E-03 | 6.65E-03 | 3.06E-03 | 8.87E-03 | 3.68E-03 | 5.32E-03 | 2.52E-03 | 7.49E-03 | 3.79E-03 |
| C(6) | 1.55E-03 | 2.39E-02 | -1.69E-01 | 7.69E-02 | -1.30E-01 | 1.28E-01 | -8.26E-02 | 5.90E-02 | 2.35E-01 | 9.68E-02 |
| M(1,1) | 1.40E-03 | 5.14E-04 | 3.60E-04 | 1.56E-04 | 3.03E-04 | 1.53E-04 | 5.53E-04 | 2.12E-04 | 4.14E-04 | 1.40E-04 |
| M(1,2) | 1.11E-03 | 4.77E-04 | 3.98E-04 | 1.73E-04 | 3.25E-04 | 1.62E-04 | 5.07E-04 | 2.00E-04 | 4.06E-04 | 1.34E-04 |
| M(1,3) | 2.96E-04 | 1.84E-04 | 7.90E-05 | 4.41E-05 | 2.00E-04 | 9.41E-05 | 2.00E-04 | 9.38E-05 | 3.89E-04 | 1.36E-04 |
| M(2,2) | 9.11E-04 | 4.97E-04 | 4.41E-04 | 1.95E-04 | 3.66E-04 | 1.76E-04 | 4.64E-04 | 1.94E-04 | 4.15E-04 | 1.39E-04 |
| M(2,3) | 2.04E-04 | 1.69E-04 | 7.56E-05 | 4.17E-05 | 2.39E-04 | 9.54E-05 | 1.89E-04 | 8.65E-05 | 4.09E-04 | 1.45E-04 |
| M(3,3) | 7.70E-04 | 8.31E-04 | 6.78E-05 | 5.42E-05 | 2.28E-04 | 1.29E-04 | 1.89E-04 | 1.07E-04 | 5.23E-04 | 2.09E-04 |
| A1(1,1) | 7.63E-02 | 3.77E-02 | 6.06E-02 | 2.30E-02 | 3.59E-01 | 9.80E-02 | 1.78E-01 | 5.12E-02 | 1.32E-01 | 3.72E-02 |
| A1(1,2) | 6.91E-02 | 3.47E-02 | 7.23E-02 | 2.38E-02 | 4.27E-01 | 9.98E-02 | 1.85E-01 | 5.32E-02 | 1.29E-01 | 3.76E-02 |
| A1(1,3) | 1.33E-02 | 2.03E-02 | 4.56E-02 | 2.27E-02 | 1.69E-01 | 7.11E-02 | 8.39E-02 | 4.13E-02 | 1.07E-01 | 3.75E-02 |
| A1(2,2) | 7.04E-02 | 3.43E-02 | 8.84E-02 | 2.49E-02 | 5.16E-01 | 1.05E-01 | 1.90E-01 | 5.62E-02 | 1.28E-01 | 3.91E-02 |
| A1(2,3) | 2.08E-02 | 2.05E-02 | 4.95E-02 | 2.42E-02 | 1.90E-01 | 7.87E-02 | 1.06E-01 | 4.46E-02 | 1.12E-01 | 3.98E-02 |
| A1(3,3) | 8.46E-02 | 6.85E-02 | 9.80E-02 | 5.55E-02 | 1.48E-01 | 8.36E-02 | 1.58E-01 | 6.63E-02 | 1.42E-01 | 4.97E-02 |
| B1(1,1) | 7.52E-01 | 7.31E-02 | 8.28E-01 | 3.83E-02 | 5.58E-01 | 8.77E-02 | 7.08E-01 | 5.95E-02 | 7.44E-01 | 5.72E-02 |
| B1(1,2) | 8.00E-01 | 6.64E-02 | 8.05E-01 | 4.15E-02 | 5.21E-01 | 8.10E-02 | 7.06E-01 | 5.95E-02 | 7.50E-01 | 5.52E-02 |
| B1(1,3) | 9.09E-01 | 6.37E-02 | 8.99E-01 | 4.69E-02 | 6.81E-01 | 9.80E-02 | 7.93E-01 | 6.70E-02 | 7.31E-01 | 6.88E-02 |
| B1(2,2) | 8.38E-01 | 6.13E-02 | 7.80E-01 | 4.55E-02 | 4.77E-01 | 7.72E-02 | 7.10E-01 | 6.10E-02 | 7.52E-01 | 5.58E-02 |
| B1(2,3) | 9.28E-01 | 5.56E-02 | 8.97E-01 | 4.56E-02 | 6.48E-01 | 9.52E-02 | 7.77E-01 | 6.29E-02 | 7.22E-01 | 7.37E-02 |
| B1(3,3) | 7.43E-01 | 2.28E-01 | 8.60E-01 | 7.46E-02 | 7.22E-01 | 1.28E-01 | 7.32E-01 | 9.55E-02 | 6.64E-01 | 8.84E-02 |
| Log likelihood | 631 | | 1094 | | 1027 | | 1169 | | 1252 | |
| SIC | -8.10 | | -12.75 | | -13.73 | | -12.22 | | -13.15 | |
| pv-Q(6) | 0.49 | | 0.40 | | 0.75 | | 0.32 | | 0.66 | |
| | 0.58 | | 0.17 | | 0.81 | | 0.35 | | 0.69 | |
| | 0.27 | | 0.36 | | 0.71 | | 0.52 | | 0.89 | |
| pv-Qsq(6) | 0.93 | | 0.93 | | 0.96 | | 0.82 | | 0.54 | |
| | 0.85 | | 0.95 | | 0.95 | | 0.90 | | 0.51 | |
| | 0.85 | | 0.99 | | 0.97 | | 0.98 | | 0.09 | |
| pv-Q(6) of VECM | 0.07 | | 0.32 | | 0.61 | | 0.19 | | 0.49 | |
| | 0.16 | | 0.14 | | 0.52 | | 0.44 | | 0.50 | |
| residual | 0.07 | | 0.08 | | 0.44 | | 0.49 | | 0.71 | |
| pv-Qsq(6) of VECM | 0.53 | | 0.25 | | 0.73 | | 0.36 | | 0.00 | |
| | 0.50 | | 0.23 | | 0.87 | | 0.35 | | 0.00 | |
| residual | 0.47 | | 0.16 | | 0.22 | | 0.05 | | 0.00 | |

**TABLE A2: Estimation Results for France**

| | | |
|---|---|---|
| | Mean specification: constant<br>return (MSCI) = C(1)<br>return (futures_1) = C(2)<br>return( futures_2) = C(3)<br>return( futures_3) = C(4) | |
| | Covariance specification: Diagonal VECH<br>GARCH = M + A1.*RESID(-1)*RESID(-1)'<br>+ B1.*GARCH(-1) | |
| futures_1 | CAC 40 | |
| futures_2 | S&P 500 | |
| futures_3 | DAX 30 | |
| | Coef | S.E. |
| C(1) | 1.03E-02 | 4.03E-03 |
| C(2) | 9.37E-03 | 4.22E-03 |
| C(3) | 7.83E-03 | 2.67E-03 |
| C(4) | 1.16E-02 | 5.01E-03 |
| M(1,1) | 3.91E-04 | 2.22E-04 |
| M(1,2) | 4.30E-04 | 2.31E-04 |
| M(1,3) | 3.80E-05 | 2.82E-05 |
| M(1,4) | 1.29E-04 | 5.46E-05 |
| M(2,2) | 4.61E-04 | 2.36E-04 |
| M(2,3) | 4.46E-05 | 3.06E-05 |
| M(2,4) | 1.46E-04 | 5.83E-05 |
| M(3,3) | 7.19E-04 | 4.13E-04 |
| M(3,4) | 5.08E-05 | 2.09E-05 |
| M(4,4) | 2.06E-04 | 1.05E-04 |
| A1(1,1) | 3.87E-02 | 2.40E-02 |
| A1(1,2) | 4.30E-02 | 2.43E-02 |
| A1(1,3) | 7.97E-03 | 6.25E-03 |
| A1(1,4) | 2.65E-02 | 1.36E-02 |
| A1(2,2) | 4.86E-02 | 2.48E-02 |
| A1(2,3) | 1.12E-02 | 7.04E-03 |
| A1(2,4) | 2.99E-02 | 1.45E-02 |
| A1(3,3) | 1.01E-01 | 5.45E-02 |
| A1(3,4) | 1.71E-02 | 6.60E-03 |
| A1(4,4) | 4.01E-02 | 1.91E-02 |
| B1(1,1) | 8.10E-01 | 8.77E-02 |
| B1(1,2) | 7.98E-01 | 8.54E-02 |
| B1(1,3) | 9.65E-01 | 2.45E-02 |
| B1(1,4) | 9.23E-01 | 3.09E-02 |
| B1(2,2) | 7.92E-01 | 8.06E-02 |
| B1(2,3) | 9.57E-01 | 2.51E-02 |
| B1(2,4) | 9.15E-01 | 3.13E-02 |
| B1(3,3) | 4.43E-01 | 2.88E-01 |
| B1(3,4) | 9.54E-01 | 1.61E-02 |
| B1(4,4) | 9.05E-01 | 4.13E-02 |
| Log likelihood | 1697.34 | |
| SIC | -17.78 | |
| pv-Q(6) | 0.83<br>0.88<br>0.54<br>0.74 | |
| pv-Qsq(6) | 0.07<br>0.11<br>0.48<br>0.48 | |
| pv-Q(6) of VECM residual | 0.79<br>0.82<br>0.46<br>0.53 | |
| pv-Qsq(6) of VECM residual | 0.00<br>0.00<br>0.02<br>0.01 | |

**TABLE A3: Estimation Results for Japan**

| | | |
|---|---|---|
| | Mean specification: VECM<br>return (MSCI) = C(1) + C(3)*COINT<br>return (futures_1) = C(2) + C(4)*COINT | |
| | Covariance specification: Diagonal VECH<br>GARCH = M + A1.*RESID(-1)*RESID(-1)'<br>+ B1.*GARCH(-1) | |
| futures_1 | Nikkei 225 | |
| | Coef | S.E. |
| C(1) | 1.48E-03 | 4.39E-03 |
| C(3) | -3.72E-01 | 1.13E-01 |
| C(2) | -5.40E-04 | 5.51E-03 |
| C(4) | -2.96E-01 | 1.43E-01 |
| M(1,1) | 4.69E-04 | 2.46E-04 |
| M(1,2) | 8.25E-04 | 2.54E-04 |
| M(2,2) | 8.61E-04 | 5.53E-04 |
| A1(1,1) | 6.72E-02 | 6.58E-02 |
| A1(1,2) | 6.91E-02 | 6.04E-02 |
| A1(2,2) | 6.16E-02 | 5.80E-02 |
| B1(1,1) | 7.80E-01 | 1.08E-01 |
| B1(1,2) | 6.80E-01 | 6.68E-02 |
| B1(2,2) | 7.25E-01 | 1.36E-01 |
| Log likelihood | 481.44 | |
| SIC | -7.84 | |
| pv-Q(6) | 0.90<br>0.86 | |
| pv-Qsq(6) | 0.78<br>0.62 | |
| pv-Q(6) of VECM residual | 0.94<br>0.81 | |
| pv-Qsq(6) of VECM residual | 0.76<br>0.65 | |

**TABLE A4: Estimation Results for Switzerland and Netherlands**

| | Switzerland | | Netherlands | |
|---|---|---|---|---|
| | Mean specification: constant<br>return (MSCI) = C(1)<br>return (futures 1) = C(2)<br>return (futures 2) = C(3)<br>return (futures 3) = C(4)<br>Covariance specification: Diagonal VECH<br>GARCH = M + A1.*RESID(-1)*RESID(-1)' + D1.*(RESID(-1)*(RESID(-1)<0))*(RESID(-1)*(RESID(-1)<0))' + B1.*GARCH(-1) | | | |
| futures_1 | SMI | | AEX | |
| futures_2 | S&P 500 | | FTSE 100 | |
| futures_3 | AEX | | SMI | |
| C(1) | 9.35E-03 | 3.85E-03 | 8.88E-03 | 3.80E-03 |
| C(2) | 9.40E-03 | 4.02E-03 | 8.99E-03 | 3.92E-03 |
| C(3) | 6.91E-03 | 2.62E-03 | 5.48E-03 | 3.17E-03 |
| C(4) | 8.69E-03 | 4.10E-03 | 9.85E-03 | 4.13E-03 |
| M(1,1) | 1.14E-03 | 3.70E-04 | 8.37E-04 | 1.67E-04 |
| M(1,2) | 1.05E-03 | 3.52E-04 | 7.61E-04 | 1.46E-04 |
| M(1,3) | 1.79E-04 | 1.06E-04 | 4.66E-04 | 1.54E-04 |
| M(1,4) | 6.67E-04 | 2.45E-04 | 5.85E-04 | 1.93E-04 |
| M(2,2) | 9.72E-04 | 3.43E-04 | 7.41E-04 | 1.61E-04 |
| M(2,3) | 1.46E-04 | 9.21E-05 | 4.07E-04 | 1.40E-04 |
| M(2,4) | 6.18E-04 | 2.42E-04 | 5.52E-04 | 1.87E-04 |
| M(3,3) | 5.69E-05 | 5.20E-05 | 5.29E-04 | 2.10E-04 |
| M(3,4) | 1.34E-04 | 8.40E-05 | 3.92E-04 | 1.78E-04 |
| M(4,4) | 6.51E-04 | 3.29E-04 | 6.63E-04 | 3.87E-04 |
| A1(1,1) | 1.75E-01 | 2.01E-02 | 3.39E-01 | 1.19E-01 |
| A1(1,2) | 1.77E-01 | 4.83E-03 | 3.21E-01 | 9.74E-02 |
| A1(1,3) | 1.56E-01 | 9.37E-02 | 1.62E-01 | 9.43E-02 |
| A1(1,4) | 2.44E-01 | 9.19E-02 | 2.18E-01 | 9.79E-02 |
| A1(2,2) | 1.79E-01 | 2.20E-02 | 3.19E-01 | 8.81E-02 |
| A1(2,3) | 1.56E-01 | 9.27E-02 | 1.55E-01 | 9.49E-02 |
| A1(2,4) | 2.45E-01 | 8.22E-02 | 2.16E-01 | 8.97E-02 |
| A1(3,3) | 1.31E-01 | 1.31E-01 | 1.76E-01 | 1.30E-01 |
| A1(3,4) | 2.13E-01 | 1.18E-01 | 1.61E-01 | 1.03E-01 |
| A1(4,4) | 3.47E-01 | 1.64E-01 | 1.83E-01 | 1.17E-01 |
| D1(1,1) | 3.05E-04 | 8.62E-02 | -2.22E-02 | 1.66E-01 |
| D1(1,2) | -1.40E-04 | 8.35E-02 | -1.14E-02 | 1.53E-01 |
| D1(1,3) | 8.00E-04 | 1.26E-01 | 1.19E-01 | 1.53E-01 |
| D1(1,4) | -8.69E-04 | 1.22E-01 | -2.33E-02 | 1.45E-01 |
| D1(2,2) | 2.49E-04 | 8.45E-02 | -8.05E-03 | 1.49E-01 |
| D1(2,3) | -7.73E-04 | 1.21E-01 | 1.24E-01 | 1.50E-01 |
| D1(2,4) | 5.67E-04 | 1.16E-01 | -3.22E-03 | 1.34E-01 |
| D1(3,3) | -8.55E-03 | 1.30E-01 | 2.59E-01 | 2.05E-01 |
| D1(3,4) | 6.52E-03 | 1.38E-01 | 5.43E-02 | 1.50E-01 |
| D1(4,4) | -3.77E-03 | 1.81E-01 | -3.29E-02 | 1.35E-01 |
| B1(1,1) | 3.00E-01 | 1.62E-01 | 3.45E-01 | 4.32E-02 |
| B1(1,2) | 3.51E-01 | 1.53E-01 | 3.97E-01 | 1.55E-02 |
| B1(1,3) | 5.30E-01 | 1.69E-01 | 4.29E-01 | 1.37E-01 |
| B1(1,4) | 3.90E-01 | 1.38E-01 | 4.69E-01 | 1.21E-01 |
| B1(2,2) | 4.02E-01 | 1.48E-01 | 4.41E-01 | 3.28E-02 |
| B1(2,3) | 5.79E-01 | 1.50E-01 | 4.93E-01 | 1.25E-01 |
| B1(2,4) | 4.28E-01 | 1.42E-01 | 4.93E-01 | 1.12E-01 |
| B1(3,3) | 8.25E-01 | 9.43E-02 | 3.81E-01 | 1.68E-01 |
| B1(3,4) | 6.34E-01 | 1.11E-01 | 5.08E-01 | 1.75E-01 |
| B1(4,4) | 4.66E-01 | 1.55E-01 | 5.54E-01 | 2.04E-01 |
| Log likelihood | 1694.56 | | 1619.35 | |
| SIC | -17.46 | | -16.63 | |
| pv-Q(6) | 0.14 | | 0.77 | |
| | 0.19 | | 0.65 | |
| | 0.56 | | 0.80 | |
| | 0.65 | | 0.19 | |
| pv-Qsq(6) | 0.93 | | 0.74 | |
| | 0.89 | | 0.37 | |
| | 0.98 | | 0.84 | |
| | 0.41 | | 0.85 | |
| pv-Q(6) of VECM | 0.16 | | 0.84 | |
| | 0.18 | | 0.68 | |
| | 0.46 | | 0.90 | |
| residual | 0.68 | | 0.18 | |
| pv-Qsq(6) of VECM | 0.00 | | 0.00 | |
| | 0.00 | | 0.00 | |
| | 0.02 | | 0.00 | |
| residual | 0.00 | | 0.00 | |

# Chapter 4

# Rare Disaster Model in the Two-Country World

## 1. Introduction

In this chapter, we explore another aspect of stock market risk – the theoretical prediction of the equity risk premium. The equity premium puzzle is the contradiction between the prediction of the general equilibrium model and its empirical counterpart. Specifically, the equity premium implied by the theory is far smaller than the equity premium actually observed. The issue is clearly important because the future equity premium is a key factor in deciding the long term allocation of portfolios between shares and bonds.

In the most recent of many attempts to explain the equity premium paradox, Barro (2006) set up a Lucas-tree type general equilibrium model and combined it with a process featuring rare disasters where output contracts drastically. The model predicts a high equity premium and a low expected return of government bill in line with the data. The rationale is that the potential of rare disasters increases the aggregate risk in the economy, which leads to a strong demand for safe assets and widens the gap between the expected returns of risky and safe assets.

It is realistic to take rare disasters into consideration. Despite the rare occurrences, disasters do have a significant impact on financial decision making. However, disasters are not only rare but also non-simultaneous in different countries. For example, when Europe was in World War II, Latin America was in peace. It is therefore interesting to investigate the implications of the interaction between countries in disaster on the equity premium.

In our extension to Barro's model, there are two equities and two government bills. The endowment of both trees is modeled by the process that incorporates disasters as in Barro. We then introduce a parameter to measure the interaction between the two outputs in disasters to allow for joint disaster and single disasters. We solve the model for the expected return of equity and bill and produce calibration results. The analytic solution is similar to that of Barro's – the potential disaster leads to high equity premium and low expected return of the bill. However, the aggregate risk in a two-country world is much smaller than that in a one-country world, which leads to an increase in the expected return of the government bill. And in a two-country world, part of the equity risk becomes diversifiable, which leads to a decrease in the expected return of equity. Together, these effects in the two-country world results in a much lower equity premium. We calibrate the model using a wide range of the parameter values based on Barro (2006) and fail to produce predictions in line with the data. We conclude that the prospect of rare disasters is unlikely to be the true explanation of the equity risk premium puzzle.

## 2. Literature Review

Lucas (1978) introduced the endowment economy approach in asset pricing. In his 'tree' model, the economy is populated with agents each endowed with a 'tree' producing a stream of perishable fruit. The endowment is stochastic, exogenous and differs among agents. There is a complete market where agents buy and sell the shares of trees to mutually insure each other, the idiosyncratic risk is diversified away and the aggregate risk is allocated efficiently. The heterogeneous agents can be aggregated into a representative utility maximizer with time-additive utility function.[1] The Euler equations of the representative agent link the asset prices to the endowment processes. The equilibrium asset prices can be derived given the aggregate and individual endowment processes.

Mehra and Prescott (1985) applied a variation of the Lucas tree model to derive the risk-free rate and the equity risk premium for the U.S. securities in the period of 1889 to 1979. They found the model suggested a much higher risk-free rate and it can explain only a fraction of the observed risk premium. The average risk-free rate of the period is 0.8%, in contrast to the theoretical value of 3.7%. The empirical risk premium averages around 6%, but the counterpart

[1] For example, if Rubinstein (1974) condition is satisfied, the aggregation is possible. That is, if all agents have utilities in the HARA class with a common cautiousness, then the representatives' utility does not depend on the distribution of income. Furthermore, if all agents share the same time preference, then the representative shares this common time preference.

suggested by the model is at most 0.39%. The failure of the general equilibrium asset pricing theory in explaining the observed data is termed as the equity premium puzzle. In particular, they assumed the representative agent maximizes a time-additive iso-elastic utility. The endowment growth is modelled as a binominal Markov process. They calibrated the model so that the mean, variance and the first-order serial correlation of their binominal model fits the empirical counterpart. They assumed there is a risk-free bond and a Lucas-tree equity whose payoff equals state-contingent per capital consumption. Since the endowment growth is quite stable with a standard deviation of 3.6%, the variation of the stochastic discount factor is small, which implies a big risk-free rate and a small risk premium. For the theoretical risk premium to be in line with the empirical observations, investors have to be extremely risk-averse. If so, the iso-elastic utility function would imply a huge risk-free rate.

Siegel (1992), (1998) and (1999) extended the data set of Mehra and Prescott (1985) to cover the last two-hundred-year period for the U.S. He found the equity return is stable at around 7% in the last two centuries, but the short-term bill rate has fallen from 5.1% between 1802 and 1870 to 3.2% between 1871 and 1925 and to 0.7% between 1926 and 1998. He mentioned the possibility that the ex-post real short-term bill rate in the recent period is a biased estimate of the expected real rate because of the unanticipated surge of inflation after World War II and in the seventies. In other words, the ex-ante real rate should be higher than its ex-post counterpart. But there is little uncertainty about the inflation over a short period such as three months. This is precisely why government bill rather than bond is chosen to measure the risk-free rate. Nevertheless, Siegel's average risk premium for the U.S. in the last two hundred years is 4.1%, which is still ten times higher than what the standard model in Mehra and Prescott (1985) can explain.

The equity premium puzzle is a contradiction between the standard theory and the observed data. One way in resolving it is to argue the data does not measure the variables we are interested in. Brown, Goetzmann and Ross (1995) asserted that the long-term return series of the U.S. are subject to survivorship bias. While the U.S. stock market has prospered for a long time, a lot of stock markets in the other part of the world have experienced temporary or permanent breakdown. The experience of the U.S. market is not representative, but only the result of good luck. The risk faced by an investor before the uncertainty was resolved is much greater than that reflected in the ex-post U.S. data. This problem is similar to the 'peso problem' in the foreign exchange market, where peso forward rates appeared to be biased forecasts of future spot rates over short sample periods, essentially because they account for a nonzero probability of devaluation that is not observed. They modelled the stock prices by a diffusion process with an absorbing lower bound. They showed that survival could induce an observed equity risk premium substantially greater than the true one. For instance, if the probability of survival is 50%, then a 4% risk-free rate and a zero risk premium would suggest an observed risk premium of 4%.

Jorion and Goetzmann (1999) collected the stock return data excluding dividend for 39 markets in the period 1921 to 1996. The U.S. return is the highest at 4.3%, but the median is only 0.8%. Since there is no particular pattern for dividend return among different countries, their data showed the high stock market return of U.S. is not the norm but the exception. However, equity risk premium is the *difference* between the return of stock and bill. Although the U.S. stock market outperformed others, the U.S. equity premium is not necessarily greater than others for almost all countries that experienced serious disruptions of stock markets also experienced periods of high inflation. In some cases, such as Germany between 1922 and 1923 and Japan after World War II, the hyper-inflation wiped out the value of government bill or bond completely. In other words, not only the expected return of equity but also that of government bill is subject to survivorship bias. The overall effect of survivorship on the risk premium is unclear.

Similar to the survivorship explanation, Rietz (1988) argued the standard interpretation of data is inappropriate. Unlike the standard model that assumes a symmetric process for the endowment growth, Rietz modelled the endowment growth by a three-state Markov process. In the first and second state, the endowment grows or falls slightly around the steady-state rate, similar to Mehra and Prescott (1985). In the third state, it falls dramatically as in depression. The occurrence of the third state is possible but unlikely and that is why it is rarely observed. His model shows that the potential of a depression-like state implies a high risk and justifies a small risk-free rate and a high risk premium. Unlike the survivorship explanation, which relies on the inconsistency between the ex-post and ex-ante data, the potential disaster explanation by Rietz admits the observed return data as proper reflection of the true data generation process and maintains the framework of the general equilibrium asset pricing model.

Mehra and Prescott (1988) raised strong criticisms of Rietz (1988). They pointed out in a depression-like state government tends to partially default on its debts through unanticipated inflation and therefore it is unrealistic to assume the government bill is risk-free in depression-like state. They also challenged the parameter values Rietz used in the calibration. For instance, Rietz showed if the coefficient of risk aversion is 8.85 and the time preference is 0.999, a 1.4% probability that the consumption falls by 25% per year would imply consistent values for the risk-free rate and risk premium. But the usual value for the coefficient of risk aversion is between 1 and 4. A coefficient of 8.85 is still too high. During the last 100 years, the biggest annual consumption drop is 8.8% in the U.S. A 25% drop per annum is too extreme.

Barro (2006) built on the idea proposed by Rietz (1988) and improved it by modelling the partial default of government bill in depression-like state and deriving the empirical distribution for the size of output contraction in disaster from the 20$^{th}$ century cross-country data. In particular, he modelled the growth rate of endowment as a random walk with drift and two i.i.d. shocks. One shock measures the slight fluctuation of endowment in non-disaster state. Its function is similar to the binominal Markov process in Mehra and Prescott (1985). The other shock measures the drastic contraction of output in disaster, similar to the third state in Rietz (1988). With the probability close to one, the second shock takes the value of zero. With the probability close to zero, it takes a negative value corresponding to the size of the output contraction in disaster. His model also accommodated the partial default scenario. It was assumed that with a constant probability the government partially defaults on its debt in disaster state and the percentage loss is of the same size as the output contraction. The calibration results showed that the model can produce theoretical values in line with the observed data.

The other way in resolving the puzzle is to modify the theory of representative agent to accommodate the historical facts. Several authors have proposed to use preferences different from the time-additive iso-elastic utility function.

Epstein and Zin (1989) and (1991) argued the restriction of the time-additive iso-elastic utility function may result in its empirical failure. The function form of the standard utility is such that the coefficient of relative risk aversion is also the reciprocal of the elasticity of intertemporal substitution, i.e. one parameter measures two different aspects of the utility. The consensus is that people are moderately risk averse, but very averse to intertemporal consumption variation. The coefficient of relative risk aversion is between 1 and 4. The elasticity of intertemporal substitution is very small[2], i.e. its reciprocal is huge. Epstein and Zin proposed a generalized expected utility preference to disentangle risk aversion and intertemporal substitution. It implies the risk-free rate is only affected by the elasticity of intertemporal substitution, but not the risk aversion. And the risk premium is only influenced by the risk aversion, but not the elasticity of intertemporal substitution. However, it can not explain the equity premium puzzle because of the moderate risk aversion. Furthermore, it creates a 'risk-free rate puzzle' (Weil (1989)) because the low elasticity of intertemporal substitution suggests a much higher risk-free rate than observed.

Standard preferences assume individuals derive utility from absolute consumption only. Duesenberry (1949) suggested that relative consumption also affects utility. People compare themselves with a benchmark measured by either their own consumption in the past ( internal habit formation ), or the society's consumption ( external habit formation ). If they are relatively better off, then they get positive utility; otherwise, they get negative utility.

Abel (1990) used a combined multiplicative internal and external habit formation preference. The internal benchmark is the individual consumption in the last period. The external benchmark is the aggregated consumption in the last period. The model effectively increases the patience and therefore lowers the risk-free rate without increasing the risk aversion. Intuitively, people want to consume more tomorrow than today in a growing economy (time preference could be negative), because they want to keep up with others. If their consumption falls through time, they get negative utility not only from the falling consumption but also from

[2] Hall(1988) showed that the estimated slope coefficient is very small in a regression of consumption growth on expected real interest rates.

the gap with others. To avoid it, they have a greater incentive to save, which keeps the risk-free rate low.

Campbell and Cochrane (1999) used an additive external habit formation preference. It makes the habit as a subsistence level. As consumption falls toward habit, people become less willing to tolerate further falls in consumption. In other words, the model predicts a time-varying risk aversion – people are more risk averse in recession and less risk averse in boom. They are reluctant to hold stocks not because of the risk, but because stocks tend to pay less in recession when people are most risk averse.

Dumas (1989) explored the possibility of heterogeneous preferences. He assumed there are two agents with different CRRA utility. Similar to Campbell and Cochrane (1999), his model also implies a time-varying risk premium. Intuitively, the relatively less risk-averse agent is exposed to more aggregate risk, therefore owns a large share of wealth in boom and a small share of wealth in recession. It makes the representative agent with average wealth less risk-averse in boom and more risk-averse in recession. This counter-cyclical risk aversion makes stocks less desirable and increases the equity premium.

Some authors tried to resolve the puzzle by assuming some kind of market failure. Mankiw (1986) assumed the market is incomplete. In the financial market, only aggregate risk can be traded, but not idiosyncratic risks. In such an economy, agents can partially insure each other by self-insurance through borrowing and lending among each other, if the idiosyncratic risk is not highly persistent. The desire for saving explains a low risk-free rate. But it can not explain the high risk premium because self-insurance results in almost efficient allocations of risks and the stock prices are similar to those in the complete market.

Constantinides, Donaldson and Mehra (2002) used an overlapping generation model with credit constraints to explain the puzzle. Three generations exist simultaneous. The young generation can not borrow against their future income because of the credit constraints and therefore can not buy equity either. Effectively, the young generation is excluded from financial market. The middle-aged generation buy bond and equity from the old generation to smooth intertemporal consumption. The calibration results suggest a smaller risk-free rate and a higher equity risk premium, but not to the extent of the observed data.

In this chapter, we try to resolve the equity premium puzzle by potential economic disasters, as Rietz (1988) and Barro (2006). We extended Barro's model to a two-country world, where there are two equities and two government bills. Each equity entitles its owner to a share of the endowment of a country. The state-contingent endowments are different between the two countries. In particular, when one country is in disaster, the other country is not necessarily in disaster. The investors of one country can diversify both non-disaster and disaster risk by holding the equity of the other country. Compared with Barro's one-country world, the two-country world is less risky, therefore implying a much higher risk-free rate. And, in a two-country world diversification makes part of the risk in holding equity diversifiable, therefore implying a lower equilibrium expected return of equity. Together they imply a low equity premium in the two-country model.

## 3. Stochastic discount factor and risk-free interest rate

### 3.1 Stochastic discount factor in the presence of possible disaster

In this two-country one-good representative-agent fruit-tree model, the agents of two countries allocate their consumptions between present and different states in the future by trading assets in a complete market. The objective of the agents is to maximize the intertemporal utility with respect to their budget constraints.

Suppose the agents of two countries share the same time-additive iso-elastic utility function. All the agents of the two countries can be aggregated into a group of representative agents. Furthermore, suppose there is no transaction cost. The aggregate consumption equals the aggregate endowment, which is the same as aggregate output.

$$MaxE_t U = U(C_t) + \sum_{i=1}^{\infty} e^{-\rho \cdot i} E_t[U(C_{t+i})] \qquad (1)$$

$$MaxE_tU^* = U(C_t^*) + \sum_{i=1}^{\infty} e^{-\rho \cdot i} E_t[U(C_{t+i}^*)] \tag{1*}$$

$$\text{where } U(C) = \frac{1}{1-\theta} C^{1-\theta} \tag{2}$$

$$\begin{aligned} s.t.\, C_t + C_t^* &= A_t + A_t^* = A_t^W \\ C_{t+i}(s) + C_{t+i}^*(s) &= A_{t+i}(s) + A_{t+i}^*(s) = A_{t+i}^W(s) \end{aligned} \tag{3}$$

$C, C^*, A$ , $A^*$ and $A^W$ are the consumption of country one and two, the endowment of country one and two and the world respectively. The complete market enables agents in both countries to reach Arrow-Debreu equilibrium where the marginal rates of substitution between consumption at different time and in different state are equal between them. They face the same set of Arrow-Debreu prices and there is a common stochastic discount factor for pricing any asset. The one-period stochastic discount factor is:

$$M_{st+1}(s) = e^{-\rho} \cdot \frac{u'(C_{t+1}(s))}{u'(C_t)} = e^{-\rho} \cdot \frac{u'(C_{t+1}^*(s))}{u'(C_t^*)} = e^{-\rho} \cdot (\frac{A_{t+1}^W(s)}{A_t^W})^{-\theta} \tag{4}$$

Assume the growth rate of endowments follow a random walk with drift process, i.e. the growth rate of endowment varies around its steady-state rate.

$$g_{t+1} = \ln A_{t+1} - \ln A_t = \gamma + u_{t+1} + v_{t+1} \tag{5}$$

$$g_{t+1}^* = \ln A_{t+1}^* - \ln A_t^* = \gamma^* + u_{t+1}^* + v_{t+1}^* \tag{5*}$$

$\gamma$ and $\gamma^*$ are the steady-state endowment growth rate. $u_{t+1}$ and $u_{t+1}^*$ represent the endowment shocks not associated with disaster. They are subject to a bi-variate normal distribution with zero means, standard deviation $\sigma$ and $\sigma^*$ respectively and correlation coefficient $\kappa$ .

$v_{t+1}$ and $v_{t+1}^*$ model the output contraction in the disaster state. The idea of having two shocks, $u$ and $v$ is that they accommodate both the relatively small fluctuations in normal periods and the drastic contraction in disaster periods. Same as in Barro (2006), $u$ and $v$ are assumed to be independent for each country. In non-disaster states for country one, $v_{t+1}$ takes a value of zero; in a disaster state, it takes the value of $\ln(1-b)$. Furthermore, we make similar assumptions for the shocks of country two. In non-disaster states of country two, $v^*_{t+1}$ is zero; in disaster state of country two, it is $\ln(1-b^*)$.

$b$ and $b^*$ represent the size of endowment contraction of country one and two due to disasters respectively. Since the disaster size is unknown at the starting time, they are assumed to be random. Assume they have the same probability distribution denoted as $b$ and they are independent of each other in the state in which both countries are in disaster. In other words, the size of contraction of country one is not related to that of country two in the joint disaster state, even though they are drawn from the same probability distribution.

The joint probability distribution of $v_{t+1}$ and $v^*_{t+1}$ is assumed to be symmetric. In other words, the conditional probability of the disaster shocks of two countries are the same, i.e. the distribution of the disaster shock in country two, given country one being or not being in disaster, is the same as the distribution of the disaster shock in country one, given country two being or not being in disaster.

Same as Barro (2006), the probability of individual country in disaster per unit of time is assumed to be $p$. As the length of period approaches zero, the individual disaster probability approximates $1-e^{-p}$. Furthermore, we introduce a conditional probability to describe the relationship between the two countries in disasters. The probability of one country in disaster *conditioned* on the fact that the other country is also in disaster is $\eta$. It ranges from zero to one. The higher is it, the more correlated are the two countries in disaster.

The joint probability distribution of $v_{t+1}$ and $v^*_{t+1}$ is summarized in Table 1.

*Table 1: Joint probability distribution of v and v**

*Part. a: Probability matrix*

| | | Country 1 | | |
|---|---|---|---|---|
| | | No disaster | Disaster | |
| Country 2 | No disaster | $(e^{-p}-1)(1-\eta)+e^{-p}$ | $(1-e^{-p})(1-\eta)$ | $e^{-p}$ |
| | Disaster | $(1-e^{-p})(1-\eta)$ | $(1-e^{-p})\eta$ | $1-e^{-p}$ |
| | | $e^{-p}$ | $1-e^{-p}$ | 1 |

*Part. b: Value of $v$ and $v^*$*

| | | Country 1 | |
|---|---|---|---|
| | | No disaster | Disaster |
| Country 2 | No disaster | $v=0; v^*=0$ | $v=\ln(1-b); v^*=0$ |
| | Disaster | $v=0; v^*=\ln(1-b^*)$ | $v=\ln(1-b); v^*=\ln(1-b^*)$ |

Given the above assumptions on the growth rates of output, we can express the stochastic discount factor in terms of the growth rates and the proportions of each country's output in the current period. The output in the next period is the product of the growth rate and the current output.

$$A_{t+1} \approx A_t(1+g_{t+1}) \tag{6}$$

$$A_{t+1}^* \approx A_t^*(1+g_{t+1}^*) \tag{6*}$$

The growth rate of world output is the average of the two individual growth rates weighted by the proportion of each country's output in the current period.

$$\begin{aligned}\frac{A_{t+1}^W}{A_t^W} &= \frac{A_{t+1}+A_{t+1}^*}{A_t+A_t^*} \approx \frac{A_t(1+g_{t+1})+A_t^*(1+g_{t+1}^*)}{A_t+A_t^*} \\ &= 1+\frac{A_t}{A_t+A_t^*}g_{t+1}+\frac{A_t^*}{A_t+A_t^*}g_{t+1}^* \\ &= 1+mg_{t+1}+(1-m)g_{t+1}^* \\ &\approx \exp[mg_{t+1}+(1-m)g_{t+1}^*]\end{aligned} \tag{7}$$

Substitute the expression of world growth rate into the stochastic discount factor.

$$M_{st+1}(s) = e^{-\rho} \cdot (\frac{A_{t+1}^{W}(s)}{A_{t}^{W}})^{-\theta} \approx \exp[-\rho - \theta m g_{t+1}(s) - \theta(1-m) g_{t+1}^{*}(s)] \tag{8}$$

The common stochastic discount factor shown in equation (8) is one of the key elements in pricing the assets in the model. Before going to those assets, we use it to price the simple asset – one-period completely risk-free government bill, which pays one unit of consumption good in the next period even if there is a disaster. The rate of return of this asset is the *risk-free interest rate* in this two-country world. This non-existent bill is equivalent to the government bill in Rietz (1988). Although we do not assume such an asset exists in the model, it helps to explain the feature of the model and the method used in pricing actual assets later.

### 3.2 Risk-free interest rate in the presence of possible disaster

The usual first-order condition of representative agents' intertemporal utility maximization implies

$$1 = E_{t}(M_{st+1} \cdot R_{t+1}) \tag{9}$$

where $M_{st+1}$ is the one-period stochastic discount factor and $R_{t+1}$ is the return of any one-period asset. Equation (9) is the formula for pricing all one-period assets. The one-period risk-free interest rate is simply the inverse of the expected stochastic discount factor.

$$R_{t+1}^{risk-free} = \frac{1}{E_{t}(M_{st+1})} \tag{10}$$

Combining the stochastic discount factor shown in (8) and the endowment processes in (5) and (5*), we can derive the solution for the risk-free interest rate shown below.[3]

$$\ln R_{t+1}^{risk-free} \approx \Phi_{1} - \Phi_{2} \tag{11}$$

3 By assuming i.i.d. shocks for the two endowment processes, we effectively make the term structure flat. The real interest rate of any period is the same as that implied by the above one-period risk-free bill.

where

$$\Phi_1 = \rho + \theta m\gamma + \theta(1-m)\gamma^* - \frac{1}{2}(\theta m\sigma)^2 - \frac{1}{2}[\theta(1-m)\sigma^*]^2 - \theta^2 m(1-m)\kappa\sigma\sigma^*$$

$$\Phi_2 = p\{(1-\eta)[E(1-b)^{-\theta m} - 1 + E(1-b)^{-\theta(1-m)} - 1] + \eta[E(1-b)^{-\theta m} \cdot E(1-b)^{-\theta(1-m)} - 1]\}$$

$\Phi_1$ is the risk-free interest rate if the probability of disaster is zero.

$\Phi_2$ represents the component of the risk-free interest rate attributable to the prospect of a disaster.

In the extreme case where country one or two accounts for the whole world output, we reach Barro's one-country model. The risk-free interest rate of Barro's model is:

$$\ln R_{t+1}^{risk-free}(one) \approx \Phi_1{}' - \Phi_2{}' \qquad (11^*)$$

where $\Phi_1{}' = \rho + \theta\gamma - \frac{1}{2}(\theta\sigma)^2$

$\Phi_2{}' = p[E(1-b)^{-\theta} - 1]$

### 3.2.1 Effect of the non-disaster related factors

$\Phi_1$ represents the standard components of risk-free interest rate, attributable to time preference, output growth and ordinary shock.

The risk-free interest rate is *increasing* in the time preference ($\rho$), since the more impatient people are, the higher the risk-free interest rate has to be to persuade them to save rather than consume. It is also *increasing* in the world output growth ($\theta m\gamma + \theta(1-m)\gamma^*$). The faster the world economy grows, the lower the marginal rate of substitution for future consumption, the less demand for risk-free asset, the greater the risk-free interest rate.

The risk-free interest rate is *decreasing* in the volatility of the ordinary world output shock ($(\theta m\sigma)^2 + [\theta(1-m)\sigma^*]^2 + \theta^2 m(1-m)\kappa\sigma\sigma^*$). The more volatile the world output, the bigger the aggregate risk in the economy, the bigger the demand for risk-free asset, the lower the risk-free interest rate. It is also *decreasing* in the correlation coefficient between the ordinary shocks in two countries ($\kappa$). The more correlated are the two outputs, the more difficult is to diversify the day-to-day risk, the stronger is the demand for risk-free asset, the lower is the risk-free interest rate.

When the endowment process of the two countries are the same, $\Phi_1$ reduces to $\Phi_1^*$.

$$\Phi_1^* = \rho + \theta\gamma - \frac{1}{2}(\theta m\sigma)^2 - \frac{1}{2}[\theta(1-m)\sigma]^2 - \theta^2 m(1-m)\kappa\sigma\sigma$$

which is greater than its counterpart in the one-country model ($\Phi_1^{'}$). Other things equal, the risk-free interest rate in a two-country world is bigger than its one-country counterpart. The reason is that investors can *diversify* the ordinary risk in a two-country world, therefore the aggregate risk is smaller and the demand for risk-free asset is weaker.

### 3.2.2 Effect of disaster-related factors

$\Phi_2$ represents the components of the risk-free interest rate attributable to a disaster, such as the probability of disaster ($p$) and the conditional probability of joint disaster ($\eta$).

$$\begin{aligned}\Phi_2 &= p\{(1-\eta)[E(1-b)^{-\theta m} - 1 + E(1-b)^{-\theta(1-m)} - 1] + \eta[E(1-b)^{-\theta m} \cdot E(1-b)^{-\theta(1-m)} - 1]\} \\ &= p\{(1-\eta)[(E\beta - 1) + (E\beta^* - 1)] + \eta(E\beta \cdot E\beta^* - 1)\}\end{aligned}$$

where: $\beta = (1-b)^{-\theta m}$ $\qquad\qquad\qquad$ $\beta^* = (1-b^*)^{-\theta(1-m)}$

are the value of the stochastic discount factor ( SDF ) in those states of the world when country one and country two suffer a disaster respectively.

$\Phi_2$ is the risk return adjustment for disasters. It is the part of the expected SDF related to the disaster. Specifically, it is the sum of the difference between SDF in each of the three types of disaster states: single disaster in country one ( $\beta$ ), single disaster in country two ( $\beta^*$ ) and joint disaster ( $\beta\beta^*$ ) and the SDF in non-disaster state (unity), weighted by their probability $p(1-\eta)$, $p(1-\eta)$ and $p\eta$[4].

Since output in disaster states is less than that in the states without it, the output in disaster states is more valuable, therefore the SDFs of those states are bigger, i.e. $\beta$ , $\beta^*$ and $\beta\beta^*$ are all greater than one. Hence, $\Phi_2$ must be positive, reflecting the fact that disaster risks increase the attraction of risk-free asset. $\Phi_2$ represents the expected return investors are willing to give up due to the potential disasters.

Moreover, the higher is $p$ , the greater is the probability of the state in which either one or both countries suffer disaster, so $\Phi_2$ is increasing in $p$ . Intuitively, the more likely is disaster, the riskier is the world, and the more valuable is the risk-free asset.

The higher is $\eta$, the greater is the probability of the joint disaster state but the less is the probability of the single disaster state. As a result, the effect of $\eta$ on $\Phi_2$ is not as straightforward as $p$ . A simple mathematic deduction on the derivative of $\Phi_2$ to $\eta$ reveals that its effect on the joint disaster state dominates the sum of those on the two single disaster states. Therefore, $\Phi_2$ is also increasing in $\eta$ . Intuitively, the more likely is a joint disaster, the more difficult is to diversify the disaster risk, the greater is the aggregate risk, and the more attractive is the risk-free asset.

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[4] In fact, the SDF of the joint disaster state is $\beta\beta^*$ . The part of the expected SDF related to it should be $E(\beta\beta^*)$ . Since we assume the size of output contraction of one country is independent of that of the other, $E(\beta\beta^*)$ is the same as $E\beta \cdot E\beta^*$ , as shown in the last term of $\Phi_2$ .

### 3.2.3 Effect of output weight

In the one-country model, i.e. when $m = 1$ or $m = 0$, $\Phi_2$ reaches its maximum level:

$$\Phi_2 = p(E\beta - 1) \text{ or } \Phi_2 = p(E\beta^* - 1)$$

so that the risk-free interest rate is minimized, because the risk is totally non-diversifiable and the aggregate risk is at the maximum level. In this case, the demand for the risk-free asset is the strongest other things equal. It is interesting to note that the risk-free interest rate in the one-country model is lower than its two-country counterpart *even if* the conditional probability of joint disaster is one ($\eta = 1$). The reason is the sizes of output contraction in the joint disaster state are independent. Even if the two countries always enter disaster state simultaneously, people still can diversify the idiosyncratic risk associated with the size of output contraction in disasters, therefore the aggregate risk in a two-country world is smaller than its one-country counterpart.

On the other hand, when $m = \frac{1}{2}$, $\Phi_2$ is at its minimum level:

$$\Phi_2 = p\{2(1-\eta)[E(1-b)^{-\frac{\theta}{2}} - 1] + \eta[E(1-b)^{-\frac{\theta}{2}} - 1]^2\}$$

so that the risk-free interest rate is maximized, because the diversification opportunity is the greatest when the countries are of equal size, minimizing the aggregate risk and thereby the demand for risk-free asset. Market clearing requires the interest rate is at the maximum level.

### 3.2.4 Effect of the degree of risk aversion on the risk-free interest rate

The effect of $\theta$ on the risk-free interest rate is mixed. In the power utility function, $\theta$ describes two distinct aspects of the utility, which have opposite implication to the risk-free interest rate. On the one hand, the lower is the intertemporal elasticity of substitution (higher $\theta$), the *higher* does the risk-free interest rate have to be to motivate people to save. This is reflected in the positive relationship between the component of the risk-free interest rate attributable to non-disaster related factors ($\Phi_1$) and $\theta$. On the other hand, the more risk averse they are (higher $\theta$), the more return they are willing to give up due to the potential disaster, the *lower* the risk-free interest rate. This is reflected in the positive relationship between the component of the risk-free interest attributable to disasters ($\Phi_2$) and $\theta$. The overall effect of $\theta$ on the risk-free rate is therefore ambiguous.

- **Summary on the effects of various factors on the risk-free interest rate**

In section 3.2, we derived the risk-free interest rate of the model, explored the implication of various factors and compared it with its one-country counterpart. The main results can be summarized as follows. In general, allowing for disasters increases the attraction of the risk-free asset and decreases the risk-free interest rate. However, the effect of disaster on the risk-free interest rate in a two-country model is smaller than its one-country counterpart, because both the ordinary risk and disaster risk can be diversified and the aggregate risk is smaller in the two-country world. Specifically, diversification opportunities are on both fronts. As long as the ordinary shocks are less than perfectly correlated across the two countries, the world ordinary shock can be reduced. As to the disaster shocks, even in the extreme case where the two countries always enter disaster simultaneously, there is still gain to be made from diversification because the size of output contractions in disaster state are not necessarily the same in the two countries.

## 4. Government bills in the presence of disaster and partial default

Equity risk premium is a measurement of the value of risk. The standard method to estimate it is to compute the spread between the rate of return of risky equity and safe government bill. However, government bill is only risk-free in real terms if there is no unanticipated inflation. While in non-disaster state, inflation in a short period of time can be forecasted reasonably accurately, in disaster state, inflation even in a short period of time can be highly unpredictable. For instance, in German hyperinflation period, the value of government bill and bond were wiped off completely. Therefore it would be unrealistic to assume government bill is risk-free in all possible circumstances.

As in Barro (2006), we assume there is no risk-free asset but only government bill in country one and two promising a unit of consumption in the next period. The face return of the bill issued by the government of country one and two is $R^f$ and $R^{f^*}$ respectively. In non-disaster states, the actual return is the same as its face return. In the state where country one suffers a disaster, its government may partially default through unanticipated inflation with the probability of $q$. Similarly, in the state where country two is in disaster state, its government may default with probability of $q^*$. In the event of a default, the shortfall is a percentage of the face value equal to the scale of output contraction, $b$ or $b^*$[5]. Default has no effect on output. The government proceeds from default are returned to representative agents through lump-sum transfer. Moreover, the defaults are assumed to be independent events, i.e. whether the government of country one (two) defaults has no effect on whether government two (one) defaults.

The assumptions made for the government one's bill can be summarized as follows. With probability $e^{-p}$, no disaster occurs in country one and therefore no default either; with probability $(1-e^{-p})(1-q)$, a disaster occurs but no default; with probability $(1-e^{-p})q$, default occurs with the payoff $1-b$. The characteristics of government one's bill are summarized in Table 2.

[5] This assumption is to avoid further complication of the model. Barro gave empirical evidence to support it.

*Table 2: Joint probability distribution of the payoff of country one's government bill*

*Part. a: Probability matrix*

| | | Country 1 | | | |
|---|---|---|---|---|---|
| | | No disaster | Disaster | | |
| | | | Default | No Default | |
| Country 2 | No disaster | $(e^{-p}-1)(1-\eta)+e^{-p}$ | $(1-e^{-p})(1-\eta)q$ | $(1-e^{-p})(1-\eta)(1-q)$ | $e^{-p}$ |
| | Disaster | $(1-e^{-p})(1-\eta)$ | $(1-e^{-p})\eta q$ | $(1-e^{-p})\eta(1-q)$ | $1-e^{-p}$ |
| | | $e^{-p}$ | $(1-e^{-p})q$ | $(1-e^{-p})(1-q)$ | 1 |

*Part. b: Value of v and $v^*$*

| | | Country 1 | |
|---|---|---|---|
| | | No disaster | Disaster |
| Country 2 | No disaster | $v=0; v^*=0$ | $v=\ln(1-b); v^*=0$ |
| | Disaster | $v=0; v^*=\ln(1-b^*)$ | $v=\ln(1-b); v^*=\ln(1-b^*)$ |

*Part c: payoff of government one's bill*

| | | Country 1 | | |
|---|---|---|---|---|
| | | No disaster | Disaster | |
| | | | Default | No default |
| Country 2 | No disaster | $R^f$ | $(1-b)R^f$ | $R^f$ |
| | Disaster | $R^f$ | $(1-b)R^f$ | $R^f$ |

### 4.1 Face return of the government one's bill

The solution for the face rate of return of the government one's bill is:

$$\begin{aligned}
&\ln R_{t+1}^f \approx \Phi_1 - \mathrm{A} - \mathrm{B} \\
&\textit{where} \\
&\mathrm{A} = p(1-q)(1-\eta)(E\beta-1) + p(1-q)\eta E(\beta\beta^*-1) + p(1-\eta)(E\beta^*-1) \\
&\mathrm{B} = pq(1-\eta)[E(1-b)\beta-1] + pq\eta E[(1-b)\beta\beta^*-1]
\end{aligned} \quad (14)$$

$\Phi_1$ is the component of the government one's face return attributable to non-disaster related factors, which is the same as the $\Phi_1$ in equation (11). The implication is that the effect of non-disaster related factors on the face return and the risk-free interest rate are the same. This is not surprising as the default prospect of government one's bill is dependent on the occurrence of disasters.

Compared with non-disaster state, the payment in disaster state is more valuable, because there is less output available in such state. Output contraction leads to higher marginal rate of substitution for the output in disaster state, i.e. higher SDF. Therefore, the possibility of disaster can potentially make government bill more attractive. This rationale also explains why the risk-free interest rate in the presence of disaster possibility is lower than its counterpart in the model without disaster possibility. However, unlike the risk-free asset, the government bill is subject to default risk. In some disaster states, a percentage of the promised payment will not be delivered to the bill holders. Compared with non-disaster state where the payment is guaranteed, the payment prospect in disaster state is undesirable. Therefore, the possibility of disaster can also potentially make the bill less attractive overall.

By writing the component attributable to disaster separately as $\mathrm{A}$ and $\mathrm{B}$, we can see the above two effects more clearly. $\mathrm{A}$ is the risk-adjusted return in the scenarios where country one is not in default and either country one or two or both are in disaster, weighted by the corresponding probability. In particular, the first term corresponds to the state where country one is in disaster but not in default and country two is not in disaster. The SDF and the probability of such state is one and $p(1-q)(1-\eta)$ respectively. The second term corresponds to the state where both are in disaster and country one not in default. Its SDF and probability is one and $p(1-q)\eta$ respectively. The third term corresponds to the state where country one is not in disaster and country two is in disaster. Its SDF is also one and the probability is $p(1-\eta)$. $\mathrm{A}$ is positive, meaning the prospect of disaster without default clearly adds value to the government one's bill. $\mathrm{B}$ is the risk-adjusted return in the scenarios where country one is in default and country two may or may not be in disaster, weighted by the probability. The payment in such state is desirable because of the high SDF, but the amount of the payment is less as a result of default.

Therefore the sign of B is ambiguous, depending on the parameter values. The overall effect of the payment in such state on the government bill is ambiguous. As A is positive and B is unclear, the overall effect of the disaster on the face return is unclear.

### 4.2 Expected return of the government one's bill

#### 4.2.1 Effect of the probability of disaster

The expected return of the government bill is less than the face return by the expected loss in default. The solution for the expected return of the government bill is:

$$\begin{aligned}
&\ln E_t R^b_{t+1} \approx \Phi_1 - \mathrm{A} - \mathrm{X} \\
&\textit{where} \\
&\mathrm{A} = p(1-q)(1-\eta)(E\beta - 1) + p(1-q)\eta E(\beta\beta^* - 1) + p(1-\eta)(E\beta^* - 1) \\
&\mathrm{X} = pq(1-\eta)E[(1-b)\beta - (1-b)] + pq\eta E[(1-b)\beta\beta^* - (1-b)]
\end{aligned} \tag{15}$$

where $\Phi_1$ and A are the same as those in (14), meaning the effect of non-default related factors on the expected return and the face return of the bill are the same. The expected return is less than the face return by the expected loss in default, $pqEb$, because it is the weighted average of the face return and the return in default. Unlike B, X is always positive, meaning the disaster prospect will always bring down the expected return of the bill, in the presence of default probability.

#### 4.2.2 Effect of the conditional probability of a joint disaster

The expected return of the bill can also be expressed as the sum of the risk-free interest rate and the default-risk premium. That is,

$$\ln E_t R^b_{t+1} \approx \ln R^f_{t+1} + pq[(1-\eta)Eb(\beta - 1) + \eta Eb(\beta\beta^* - 1)] \tag{16}$$

The expression in the bracket is the risk premium for the default in the two possible states where it can occur. With probability $pq(1-\eta)$, default occurs in country one but country two is not in disaster simultaneously, where the SDF is $\beta$. With probability $pq\eta$, the default occurs in country one and country two is also in disaster, where the SDF is $\beta\beta^*$. The former SDF is less than the latter, because the marginal rate of substitution for output in country-one-only disaster state is less than that in joint disaster state. Nevertheless, both SDFs are greater than one, therefore the default premium is positive. As the default premium is positively related to the probability of default ($q$), the expected return of the bill is also positively related to the probability of default. The more likely is the government to default, the higher is the expected return of its bill.

In the extreme case where the probability of joint disaster is zero ($\eta = 0$), the government bill needs to offer a risk premium for the default in country-one-only disaster state. The expected return of the bill is shown in (17).

$$\ln E_t R^b_{t+1|\eta=0} \approx \ln R^f_{t+1} + pqEb(\beta - 1) \tag{17}$$

In the other extreme case where the two countries always enter disaster state simultaneously ($\eta = 1$), the default risk premium needs to compensate for the default in joint disaster state. The expected return is as follows.

$$\ln E_t R^b_{t+1|\eta=1} \approx \ln R^f_{t+1} + pqEb(\beta\beta^* - 1) \tag{18}$$

The expected return in the second extreme case is higher. Since the payment loss in a joint disaster state is more valuable than that in a country-one-only disaster state, the government bill has to offer a higher expected return to compensate for it.

As the probability of joint disaster ($\eta$) increases, the chance of a default in the joint disaster state rises and that in the single disaster state falls. The payoff pattern becomes less desirable and the default risk premium needs to increase to compensate. That is, the default risk premium is increasing in the probability of joint disaster. This point can be seen in the second term in the bracket of (16). However, as the output of the two countries become more correlated, it is also more difficult to diversify the disaster risk and the aggregate risk increases. The higher is the aggregate risk, the more desirable is asset in general, which implies a tendency of a decrease in the government bill's expected return. That is, the risk-free interest rate is decreasing in the probability of joint disaster. This point can be seen in the first term in (16). In summary, the overall effect of $\eta$ on the expected return of the bill is ambiguous, depending on the parameter values.

### 4.2.3 Effect of the output weight

The output weight affects both components of the expected return of the government bill. As the output weight of country one increases from zero to half, the world becomes more diversified and the aggregate risk in the economy decreases, which brings up the risk-free interest rate. As it increases further from half to one, the world output becomes more focused on country one and the aggregate risk increases, therefore decreasing the risk-free interest rate. The relationship between the risk-free interest rate and the output weight is symmetric around a half.

One the other hand, as the output weight of country one increases from zero to one, country one becomes more dominant and its idiosyncratic risk becomes more correlated to the aggregate risk. The diversification of the risk of government one's bill turns more difficult and the non-diversifiable part of the risk turns bigger, the expected return has to increase to compensate for the additional idiosyncratic risk, therefore pushing up the default risk premium.

As the output weight of country one increases from zero to half, both the risk-free interest rate and the default risk premium increase. Together they result in an increase in the expected return of the government one's bill. As the output weight of country one increases further from half to one, the risk-free interest rate goes down and the default risk premium continues going up, the sum of the two – the expected return of government one's bill may go up or down, depending on which force dominates.

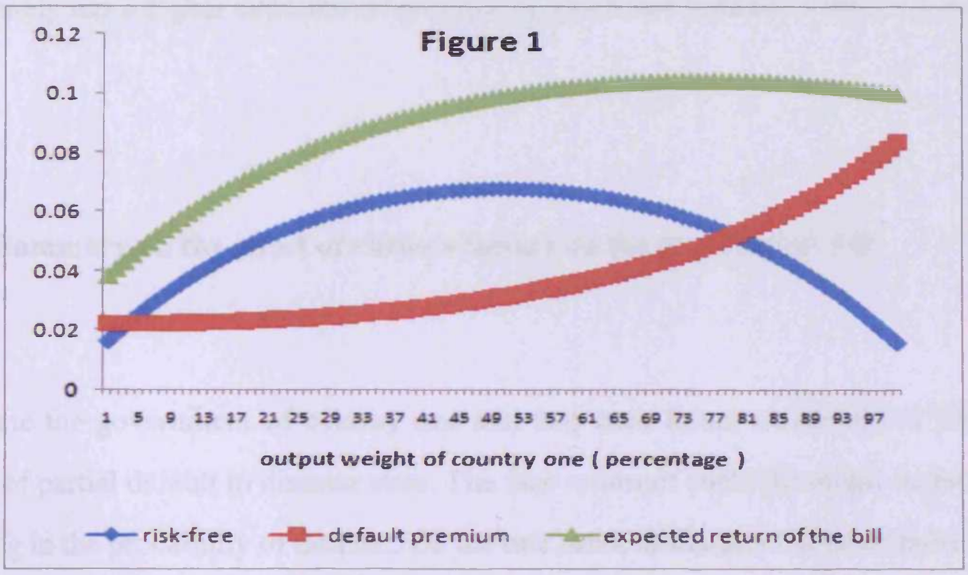



Figure 1 illustrates the expected return of the government one's bill and its two components. The blue curve symmetrical around 50 percentage point represents the risk-free interest rate. The red curve monotonically increasing represents the default risk premium. The green curve representing the expected return of the bill is derived simply by adding the other two up. It is definitely increasing from 0 to 50 percentage point. But we can not be sure whether it decreases eventually and if so at which point it starts decreasing.

The two-country model is a generalization of Barro's model. We can derive the expected return of the government bill in Barro's model simply by setting $m$ to unity, i.e. when country one accounts for the whole world output. The expected return of the bill of Barro's model is represented by the right-end point of the green curve in Figure 1. In Barro's model the risk-free interest rate is lower because the aggregate risk is greater and the default premium is higher because it is completely non-diversifiable. Therefore it is unclear whether the one-country expected return of the bill is higher than its counterpart in the two-country model.

Finally, consider two extreme cases. When country one is extremely small ($m \approx 0$), the scope for diversification is very big, therefore the default risk premium is very small. When country one is extremely big ($m \approx 1$), the scope for diversification is very small, therefore the default risk premium is very big. Since the aggregate risks in these two cases are the same, the former corresponds to a lower expected return of the government bill than the later. In other words, in a world with an extremely big and an extremely small country, the government bill of the big one definitely has a higher expected return than the small one because it bears more aggregate risk.

### ❖ Summary on the effect of various factors on the government bill

We assume the government of country one and two each issues a one-period bill with the prospect of partial default in disaster state. The face return of such bill might be increasing or decreasing in the probability of disaster. On the one hand, as disaster becomes more likely, the government bill becomes more attractive since it promises payment in disaster state where the output is valuable. One the other hand, it becomes less attractive since it may default in such state.

While the overall effect of disaster on the face return is ambiguous, its effect on the expected return of the bill is definitely negative, because the expected return of the bill is the weighted average of the face return and the return in default. The effect of the probability of joint disaster on the expected return is ambiguous, because of its opposite effect on the two components of the expected return. On the one hand, as the two disaster shocks become more correlated, the aggregate risk becomes bigger and the risk-free interest rate falls. On the other hand, as the probability of joint disaster increases, the payoff pattern shifts towards the worst scenario where default coincides with joint disaster, the default risk premium has to increase to compensate. As the sum of the risk-free interest rate and default risk premium, the expected return of the bill might be increasing or decreasing in the probability of joint disaster.

Compared with the one-country model, the expected return of the government bill of the two-country model might be higher or lower. One the one hand, the risk-free interest rate is higher in the two-country model because of the smaller aggregate risk. On the other hand, the default risk premium is lower since part of the default risk can be diversified away, therefore a smaller amount of default risk is compensated by an increase in the expected return. The overall effect is determined by the dominant force.

## 5. Price and Expected Return of the Equity

As far as equities are concerned, we assume the agents trade two classes of shares, which entitle them to a dividend stream equal to the output of either country one or two's tree. We derive the solution for the price and expected return of these equities and investigate the implication of potential disasters.

### 5.1. Equity Price

In this model, the tree asset represents the claim on the dividend stream equal to the output of a country. Since the output is assumed to follow a random walk with drift process and independent shocks, the price of the tree asset ( $P_t$ ) is closely related to that of a simple one-period equity claim ( $P_{t1}$ ) on the dividend of the next period.

The price of the equity claim on the next-period dividend is:

$$P_{t1} = E_t(M_{st+1} \cdot A_{t+1}) \tag{19}$$

where $M_{st+1}$ is the SDF of the next-period consumption and $A_{t+1}$ is the dividend in the next period. Combining the SDF shown in equation (8) and the output shown in equation (5), we can derive the solution for the price of this one-period equity claim and the price to earning ( P-E ratio ) of the equity as follows[6]:

$$P_{t1} \approx A_t \cdot \exp[-\rho + (1-\theta m)\gamma - \theta(1-m)\gamma^* + \frac{1}{2}(1-\theta m)^2\sigma^2 + \frac{1}{2}\theta^2(1-m)^2\sigma^{*2} - (1-\theta m)\theta(1-m)\sigma\sigma^*\kappa]$$
$$\cdot \{p\eta[E(1-b)\beta\beta^* - 1] + p(1-\eta)[E(1-b)\beta - 1] + p(1-\eta)(E\beta^* - 1) + 1\} \tag{20}$$

and

$$P_t / A_t = (\Phi_3 - \Phi_4)^{-1} \tag{21}$$

where

[6] See the derivation in the appendix.

$$\Phi_3 = \rho - (1-\theta m)\gamma + \theta(1-m)\gamma^* - \frac{1}{2}(1-\theta m)^2\sigma^2 - \frac{1}{2}\theta^2(1-m)^2\sigma^{*2} + (1-\theta m)\theta(1-m)\sigma\sigma^*\kappa$$

$$\Phi_4 = p\{\eta[E(1-b)\beta\beta^* - 1] + (1-\eta)[E(1-b)\beta - 1 + E\beta^* - 1]\}$$

### 5.1.1 Effect of non-disaster related factors on the equity price

$\Phi_3$ represents the component of the P-E ratio attributable to non-disaster-related factors. The equity price is *decreasing* in time preference. The more impatient are the people (higher $\rho$), the less demand is for asset in general, the lower is the equity price. It is also *decreasing* in the growth rate of country two. The higher is the steady-state growth rate of country two (higher $\gamma^*$), the lower is the marginal rate of substitution for future consumption. The lower is the stochastic discount factor, the lower is the equity price.

The effect of the steady-state growth rate of country one ($\gamma$) on the equity one's price is two-fold. On the one hand, the higher is the growth rate of country one, the more dividend is available from tree one, the stronger is the demand and the higher is the price. On the other hand, the higher is the growth rate of country one, the more world output is available in the future, the lower is the marginal rate of substitution for future consumption, the lower is the demand for asset in general and the lower is the price.

The overall effect depends on which force dominates. If $\theta m < 1$, the first force dominates, i.e. the price of equity one is *increasing* in the growth rate of country one. Intuitively, when country one is small enough, its growth rate does not affect the stochastic discount factor much, but it does affect its own dividend stream significantly. An increase in the growth rate leads to a higher payoff and a not much lower stochastic discount factor, therefore higher price. If $\theta m > 1$, the second force dominates, i.e. the price of equity one is decreasing in its own growth rate. Intuitively, when country one is big enough, the impact of its growth rate on the stochastic discount factor overwhelms that on its own future payoff. An increase in the growth rate leads to a higher payoff and a much lower stochastic discount factor, therefore a decrease in price.

The effect of the correlation coefficient between the ordinary shocks ( $\kappa$ ) is also mixed. On the one hand, the more correlated are the two ordinary shocks, the more correlated is equity one's payoff to the world output, the worse is equity one as a diversification tool, the lower is the price. On the other hand, the more correlated are they, the bigger is the aggregate risk, the more demand for asset in general, implying a higher price. If $\theta m < 1$, the price is decreasing in $\kappa$. Intuitively, when country one is small enough, it can hardly influence the stochastic discount factor and the correlation between the two shocks is close to the correlation between the payment of equity one and the world output. The more correlated are they, the bigger is the beta, the higher is the expected return, the lower is the price. If $\theta m > 1$, the price is increasing in $\kappa$. Intuitively, when country one is big enough, there is hardly the scope for diversification. The more correlated is it with a small country, the riskier is the world, the more demand for asset in general, the higher is the price.

### 5.1.2 Effect of disaster related factors on the equity price

$\Phi_4$ represents the component of the equity price attributable to disaster-related factors.

$$\Phi_4 = p\{\eta[E(1-b)\beta\beta^* - 1] + (1-\eta)[E(1-b)\beta - 1] + (1-\eta)(E\beta^* - 1)\}$$

In particular, the three terms in $\Phi_4$ are the difference between the dividend payoff of the equity in the joint-disaster, country-one-only-disaster and country-two-only-disaster state each valued by the SDF and the payoff in non-disaster state. They represent the difference made by these three types of disaster to the equity price.

In the joint-disaster state, the dividend payoff of equity one is lower than its counterpart in non-disaster state by $b$ as the result of output contraction when country one is in disaster. The actual payoff is $1-b$ valued by the SDF of this state, $\beta\beta^*$. Compared with the payoff in non-disaster state, the amount is less but the value of each unit is bigger, therefore the total effect of the joint disaster on the price is ambiguous.

Similarly, in the country-one-only disaster state, the actual payoff is $1-b$ valued by the SDF, $\beta$, which is also ambiguous relative to the value of the dividend received in non-disaster state. Compared with the payoff in the joint-disaster state, the payoff in the country-one-only disaster state is less valuable because per unit value of the payment is smaller and the size of payment is the same.

In the country-two-only disaster state, the actual payoff is valued by the SDF, $\beta^*$. In this state, there is no loss in the dividend payment but per unit value of the payment is higher due to the global output contraction caused by the disaster in country two. As this state becomes more likely, the price of equity one should increase.

As the probability of disaster ($p$) increases, the probability of the above three states all increase. Since the effect of two of them on the equity price is unclear, the effect of $p$ on equity price is ambiguous.

The effect of the conditional probability of joint disaster ($\eta$) on the price of equity one can be derived from examining the derivative of $\Phi_4$ to $\eta$.

$$\frac{\partial \Phi_4}{\partial \eta} = p\{[E(1-b)\beta\beta^* - (1-b)\beta + 1 - \beta^*]\} = pE\{[(1-b)\beta - 1](\beta^* - 1)\}$$

When $\theta m < 1$, the above term is negative, implying the equity price is decreasing in $\eta$.

When $\theta m > 1$, the above term is positive, implying the equity price is increasing in $\eta$.

Not surprisingly, these conditions are the same as those for $\kappa$, since both $\kappa$ and $\eta$ measure the correlation between the shocks of country one and two. When country one is small enough, a higher probability of joint disaster leads to a worse payoff prospect ( joint disaster is worse than single disaster because of the high SDF ) and not much different SDF, therefore decreasing the equity price. When it is big enough, a higher probability of the joint disaster results in a slightly worse payoff prospect and a much higher SDF, therefore increasing the equity price.

### 5.2 Expected return of equity

Since the shocks of different time periods are independent, the expected return of the one-period equity claim is the same as that of the equity itself. The expected return of the one-period equity claim is simply the expected dividend of the next period divided by its current price.

$$\ln E_t R^{1e}_{t+1} = \ln E_t ({A_{t+1}}/{P_{t1}}) = \ln E_t A_{t+1} - \ln P_{t1} \tag{22}$$

The solution for the expected return of the equity and the one-period equity claim is:[7]

$$\begin{aligned}
&\ln(E_t R^{e}_{t+1}) = \ln(E_t R^{1e}_{t+1}) \approx \Phi_5 - \Phi_6 \\
&\Phi_5 = \rho + \theta m\gamma + \theta(1-m)\gamma^* + \theta m\sigma^2 - \frac{1}{2}\theta^2 m^2\sigma^2 - \frac{1}{2}\theta^2(1-m)^2\sigma^{*2} + (1-\theta m)\theta(1-m)\sigma\sigma^*\kappa \\
&\Phi_6 = p\big(\eta[E(1-b)\beta\beta^* - (1-b)] + (1-\eta)[E(1-b)\beta - (1-b)] + (1-\eta)(E\beta^* - 1)\big)
\end{aligned} \tag{23}$$

#### 5.2.1 Effect of factors unrelated to disasters on the expected equity return

$\Phi_5$ summarizes the effect of factors unrelated to disasters on the expected return of equity. Since the expected return of equity is determined by the expected dividend and the current price, the factors that do not affect the dividend have the opposite effect on the expected return to that on the equity price. Specifically, the expected return of the equity is increasing in the time preference and the steady-state growth rate of country two. It is increasing in the correlation between the two ordinary shocks when $\theta m < 1$ and decreasing in it otherwise.

As shown in the previous section, the price of equity might be increasing or decreasing in its own steady-state growth rate ($\gamma$) depending on $\theta m$. However, the expected return of equity one is always increasing in it. As $\gamma$ increases, the expected dividend growth of equity one

[7] See the derivation and the proof of the equivalence to the expected return of the tree asset in the appendix

increases. This positive effect of $\gamma$ on the expected dividend growth overwhelms that on the price. Therefore, the equity return is increasing in $\gamma$, whether the equity price is increasing or decreasing in it.

### 5.2.2 Effect of factors related to disasters on the expected equity return

$$\Phi_6 = p\left(\eta[E(1-b)\beta\beta^* - (1-b)] + (1-\eta)[E(1-b)\beta - (1-b)] + (1-\eta)(E\beta^* - 1)\right)$$

$\Phi_6$ represents the component of the expected equity return attributable to disaster-related factors. The three terms of $\Phi_6$ correspond to the risk-adjusted payoff of equity one in the joint disaster state, country-one-only disaster state and country-two-only disaster state respectively, weighted by the probability.

In the former two states, the output of country one decreases, damaging the dividend prospect, which implies a potential decrease in the demand for equity, thereby decreasing the price. In the meantime, the payoffs in these states are more valuable than that in non-disaster states, implying a potential increase in the demand and the price. Together the equity price might be increasing or decreasing in the prospect of these two types of disaster. However, the disaster has a direct negative influence on the expected dividend growth and this effect overwhelms the ambiguous effect on the price. Therefore the total effect of joint and country-one-only disaster on the equity price is negative.

The last term corresponds to the country-two-only disaster state. Since the dividend of equity one does not drop in this state, it has no effect on the expected dividend growth of equity one, but only the positive effect on the price. Since the SDF in this state is higher, i.e. the dividend payoff is valued more, the prospect of this state makes the equity more attractive, therefore increasing the price and decreasing the expected return. As the weighted average of the risk-adjusted payoff in these three states, the expected equity return is decreasing in the probability of a disaster.

The conditional probability of joint disaster has no effect on the expected dividend growth of equity one, therefore its effect on the expected equity return is exactly opposite to that on the equity price. That is, if $\theta m < 1$ the expected return is increasing in the conditional probability of joint disaster and it is decreasing in it otherwise.

The expected return can also be written as the sum of risk-free interest rate and the 'true' risk premium[8].

$$\ln(E_t R_{t+1}^e) = \ln R_{t+1}^{risk-free} + \theta m\sigma^2 + \theta(1-m)\sigma\sigma^*\kappa + p\left(\eta Eb(\beta\beta^* - 1) + (1-\eta)Eb(\beta - 1)\right) \tag{24}$$

Equation (24) tells us that, in addition to the standard component attributable to variance and covariance between ordinary shocks to output growth, the excess return of equity over risk-free interest rate is the risk-adjusted value of output loss in the two states where country one is in disaster and country two either is or is not in disaster, weighted by the respective probabilities. In other words, the expected equity return is the sum of the risk-free interest rate, the ordinary risk premium and the disaster risk premium.

While the disaster prospect has a negative effect on the risk-free interest rate, it has a positive effect on the disaster risk premium. The overall effect on the expected equity return is negative since the former dominates the latter.

As the probability of joint disaster increases, the pattern of the dividend payment turns worse because the loss in a joint-disaster state is more valuable than that in single-disaster state. This is reflected in the positive relationship between the disaster risk premium and the probability of joint disaster. However, the prospect of joint disaster state also increases the aggregate risk and decreases the risk-free interest rate, therefore the overall effect on the expected equity return is ambiguous. Examining the derivative that, if $\theta m < 1$, the increase in the excess return overwhelms the decrease in the risk-free interest rate and the overall effect is positive. Otherwise, the expected equity return is decreasing in the probability of joint disaster.

[8] The risk premium we can measure is the spread between equity return and the government bill return. Since the latter is subject to default risk, the spread does not reflect the true risk premium of the equity.

It is worth noting that the terms in the bracket of equation (24) are the same as those of equation (16). It implies that the disaster risk premium and the default risk premium only differ by the conditional probability of default, $q$. This results from the assumption on the equality between percentage loss in default and the scale of output contraction in disaster.

### 5.2.3 Effect of output weight on the expected equity return

$$\ln(E_t R_{t+1}^e) = \ln R_{t+1}^{risk-free} + \theta m\sigma^2 + \theta(1-m)\sigma\sigma^* \kappa + p\left(\eta Eb(\beta\beta^* - 1) + (1-\eta)Eb(\beta - 1)\right) \tag{24}$$

The expected return of equity is the sum of the risk-free interest rate, ordinary risk premium and the disaster risk premium. The risk-free interest rate is increasing in the output weight of country one when it is below 50 percent and decreasing in it otherwise, because the aggregate risk is at the minimum level when the two countries are of equal size. The disaster risk premium is increasing in the output weight of country one because it becomes increasingly difficult to diversify the disaster risk as the share of country one's output in the world turns bigger. The relationship between the ordinary risk premium shown below and the output weight is more complicated.

$$ordinary\ risk\ premium = \theta m\sigma^2 + \theta(1-m)\sigma\sigma^* \kappa \tag{25}$$

The ordinary risk premium is the reward for the non-diversifiable idiosyncratic ordinary risk. When country one is extremely small ($m \approx 0$), it reduces to $\theta\sigma\sigma^* \kappa$, which is equivalent to the risk premium in a standard CAPM. In this case, country two accounts for the whole world output and underlies the SDF. The risk premium of equity one is proportional to the correlation between the output of country one and the output of the world (same as country two's output). When country one is extremely big ($m \approx 1$), it reduces to $\theta\sigma^2$, which is equivalent to the risk premium in a standard one-country Lucas tree model, such as Mehra and Prescott (1985). In this case, country one accounts for the whole world output and its idiosyncratic risk is also the aggregate risk, whose reward is increasing in the size of volatility and the degree of risk aversion. In the general case, $m$ lies between zero and one and this part of the risk premium is the weighted average of the two extremes. The relationship between $m$ and the ordinary risk premium depends on which extreme value is bigger. In particular, if $\sigma^* \kappa$ is bigger than $\sigma$, the ordinary risk premium is decreasing in $m$; otherwise, it is increasing in $m$.

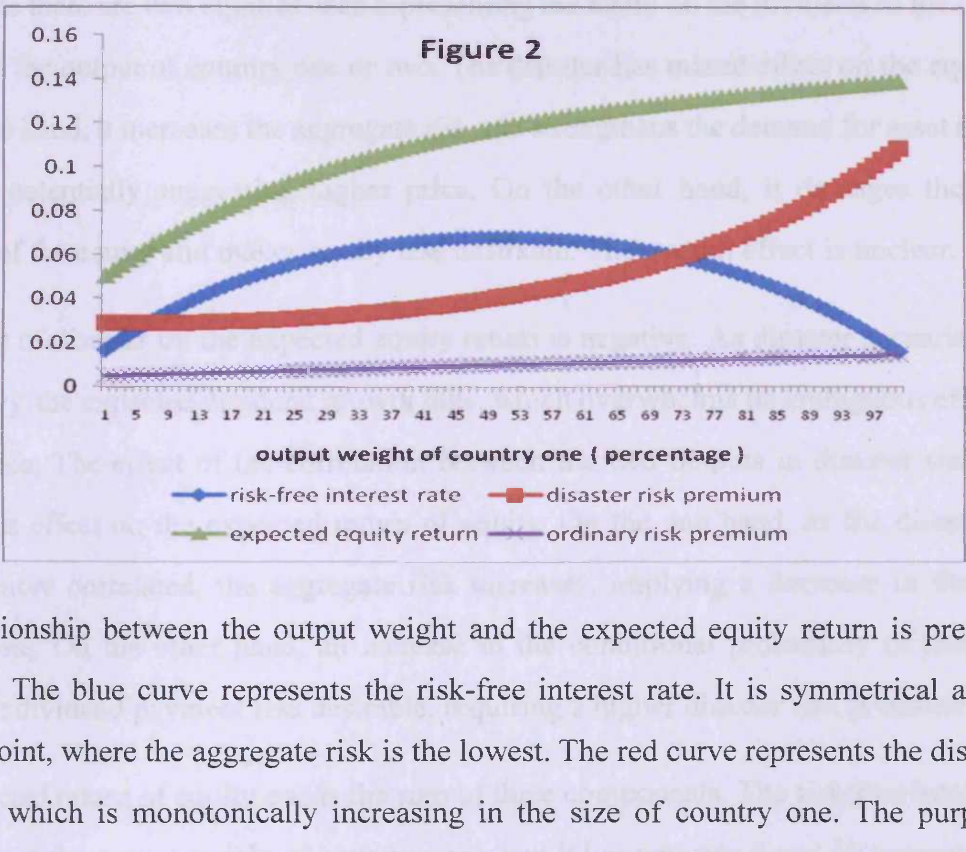


Figure 2

The relationship between the output weight and the expected equity return is presented in Figure 2. The blue curve represents the risk-free interest rate. It is symmetrical around 50 percent point, where the aggregate risk is the lowest. The red curve represents the disaster risk premium which is monotonically increasing in the size of country one. The purple curve represents the ordinary risk premium in the case where $\sigma^* \kappa$ is smaller than $\sigma$. It is monotonically increasing as well. The green line represents the expected equity return derived from adding the other three up. It is increasing in the weight of country one before 50 percent point. Its shape after 50 percent point is unclear.[9] The expected equity return in Barro's model is represented by the right-end point on the green curve. Because its risk-free interest rate is lower and the disaster risk premium is higher than the counterpart in the two country model but

[9] The non-decreasing curve representing the expected equity return in Figure 2 is drawn for illustration purpose. It only demonstrates one possibility where the expected equity return is monotonically increasing in the weight. It could well be increasing first and then decreasing.

the comparison between its ordinary risk premium and the two-country counterpart is unclear, we can not be sure how it compares with the expected return in the two-country model.

- **Summary on the effect of various factors on the equity price and the expected return of equity**

We assume there are two equities each representing the claim on the dividend of the Lucas tree asset with the output of country one or two. The disaster has mixed effect on the equity price. On the one hand, it increases the aggregate risk and strengthens the demand for asset in general, therefore potentially suggesting higher price. On the other hand, it damages the dividend prospect of the equity and makes equity less desirable. The overall effect is unclear.

The effect of disaster on the expected equity return is negative. As disaster scenario becomes more likely, the expected dividend growth falls, which overwhelms its ambiguous effect on the equity price. The effect of the correlation between the two outputs in disaster states has an ambiguous effect on the expected return of equity. On the one hand, as the disaster shocks become more correlated, the aggregate risk increases, implying a decrease in the risk-free interest rate. On the other hand, an increase in the conditional probability of joint disaster makes the dividend payment less desirable, requiring a higher disaster risk premium.

The expected return of equity one is the sum of three components. The risk-free interest rate is increasing in the output weight of country one when it lies between 0 and 50 percent point and decreasing in it otherwise. The ordinary risk premium is a weighted average of two extreme values. The relationship between it and the output weight depends on the size ordinary risk and the correlation between the two ordinary risks. The disaster risk premium is increasing in the weight of country one. As the sum of the three components, the expected return of equity might be increasing or decreasing in the output weight. The expected return of equity in Barro's one country model might be higher or lower than its two-country counterpart.

## 6. Equity Risk Premium

The equity risk premium is defined as the difference between the expected return on the risky equity and that on the safe government bill. In practice, it has two possible definitions. One is the observable gap between the return on government debt and the return on equity. The other is the unobservable gap between the true risk-free interest rate and the equity return. Since our goal is to explain the equity risk premium reflected in the data, the latter definition is adopted. It is clearly driven by two sorts of factors: the ordinary shocks, which impinges solely on the expected return of equity, and disaster shocks, which likewise affect equity and leave government bill untouched, unless there is a default, in which case bills and equities are affected equally.

Combining equation (16) and (24), we can derive the solution for the equity risk premium.

$$\begin{aligned} \textit{equityrisk premium} &= \ln(E_t R^e_{t+1}) - \ln(E_t R^b_{t+1}) \\ &= \left(\theta m \sigma^2 + \theta(1-m)\sigma\sigma^* \kappa\right) + p(1-q)\left(\eta E b(\beta\beta^* - 1) + (1-\eta) E b(\beta - 1)\right) \end{aligned} \tag{26}$$

The terms in the first bracket are the reward for bearing the ordinary risks. Since the government bill only defaults when there is a disaster and the disaster shock is independent of the ordinary shock, the government bill bears no ordinary risk. This part of the equity risk premium is solely the result of carrying the ordinary risk of the equity.

When the weight of country one's output is close to zero, the output of country two is almost the world output. The ordinary risk of equity one is rewarded for the correlation to the ordinary shock of country two. The more correlated are they, the worse is equity one as a diversification tool, the higher is its expected return and the equity risk premium. The ordinary equity risk premium in this case is $\theta\sigma\sigma^*\kappa$. When country one accounts for the whole world output, it is impossible to diversify any risk and its ordinary risk is fully compensated by the premium of $\theta\sigma^2$. In the general case, this part of the equity premium is the weighted average of the two extreme values.

The terms in the second bracket are the reward for carrying the disaster risks. It is the expected value of the loss valued by SDF in the state where there is disaster but no default. Clearly the loss in the state where there is a disaster and also a default is rewarded equally in the return on bills and equities.

The disaster risk premium is increasing in the probability of disaster and the conditional probability of joint disaster and decreasing in the probability of default. The more likely is a disaster, the more loss is expected, the greater the risk premium. The more likely is the joint disaster, the worse is the payoff prospect, the higher is the risk premium. The more likely is a default, the higher is the expected return on the bills. But the default prospect has no effect on the expected return of equity. Therefore, the risk premium is decreasing in the probability of default.

The other factors including the steady-state growth rate of country one and two and time preference have the same effect on the expected return on the bills and equities, therefore no impact on the risk premium.

The part of the equity risk premium associated with disasters is the difference between the disaster risk premium and the default premium of the equity. It is proportional to the other two premiums, therefore increasing in the output weight of country one. As the weight of country one's output increases, it becomes more difficult to diversify the disaster risk of the equities and the default risk of the bills, both expected returns increase. Since the disaster is more likely to happen than default, the disaster risk is greater than the default risk and more difficult to diversify. As a result, the expected return of equity increases more than the expected return of the bill, i.e. the equity risk premium widens. Compared with Barro's model, where the output weight of country one is at the maximum level, the two-country model suggests a lower equity risk premium.

### ❖ The implications of two-country rare disaster models on the expected asset return and equity risk premium

There are several interesting implications of the analysis of the two-country rare disaster model for the potential disasters on asset returns and equity risk premium.

*First, the probability of disaster is inversely related to the expected return of both equity and government bill.*

As a disaster scenario becomes more likely an inter-temporal utility maximizer would demand more assets in general to transfer more wealth to future disaster states. This tendency would increase the attraction of equity and government bill, i.e. the equity price would tend to rise and the face return of government bill would tend to fall in response. But since the riskiness of both equity and government bills are increasing in the probability of a disaster, their attraction to a risk-averse individual also tends to decrease. The overall effect of the probability of a disaster on the attraction of assets is ambiguous, i.e. we can not be sure about the effect of the probability of a disaster on the price of equity and the face return of government bills. Since the expected asset returns are positively related to their payoff prospect and negatively related to their attraction and the former becomes worse as a result of an increased disaster probability, the overall effect on expected asset return is negative.

*Second, the probability of disaster is positively related to the equity risk premium.*

As a disaster scenario becomes more likely, the amount of aggregate risk increases. A risk-averse individual would demand higher expected return for bearing risk. Other things being equal, relatively risky assets become less attractive and have to offer higher premium over relatively safe assets. Although government bills are subject to default risk, they are still safer than equity because their payoff would not contract in a disaster in which the government does not default. As a result, the equity risk premium is increasing in the probability of disaster.

*Third, the probability of a joint disaster is positively related to the equity risk premium.*

As a joint disaster becomes more likely, the two-country world becomes more risky, i.e. the aggregate risk gets bigger. Other things being equal, the relatively more risky equity has to offer a higher premium over relatively safe government bills.

*Fourth, the probability of a default conditioned on a disaster is positively related to the expected return of government bills.*

As default becomes more likely, government bills become more risky and have to offer a higher expected return. In the extreme case where the probability of default is zero, government bills are risk-free and their expected return reaches the minimum.

*Fifth, the bigger the difference in size between two countries the lower the risk free interest rate is.*

Other things being equal, the closer two countries are in size, the smaller is the aggregate risk and risk free assets are less attractive. And in a more polarized world the aggregate risk is bigger and the risk-free interest rate is lower.

*Sixth, the size of a country is positively related to its equity risk premium.*

The more dominant is a country, the more difficult it is to diversify the risk of its assets and the higher is the expected asset return. Between equity and government bills, the riskiness of the former increases faster than the latter as a country becomes increasingly dominant. As a result, the gap between two expected returns (equity risk premium) is increasing in the size of the country.

### ❖ Implications of rare disasters on hedging

The potential of rare disasters has an important role to play in hedging because the aim of hedging is to reduce or eliminate risk. Since stock prices fall dramatically in a disaster state, hedgers are particularly interested in preserving their wealth. In Chapters 2 and 3, we employed the standard VECM for conditional mean and GARCH for conditional covariance matrix to model the returns in spot and futures. Since this type of model cannot accommodate extreme values, we implicitly assumed the disaster state away. An improvement on empirical front would be to model the returns in normal and disaster states separately and assign a probability to each state. Specifically, a regime-switching model is probably more appropriate to accommodate the disaster scenario. The hedge ratios generated from this model would do particularly well in a disaster state.

On the theoretical front, potential disasters affect hedging in two ways. First, if disasters cannot be regarded as trivial then we cannot claim the return of futures follows a martingale process. Without the martingale assumption the minimum-variance hedge ratio would be different from the optimal hedge ratio that corresponds to the highest mean-variance utility value. If the return of futures is unpredictable in normal states, then the unconditional return of the futures would be negative and the optimal hedge ratio would be greater than the minimum-variance hedge ratio. Intuitively, if there is a considerable probability of disaster then hedgers would tend to overhedge to achieve positive expected return. Second, the correlation coefficient between spot and futures in disaster states is different from that in normal states. Since the minimum-variance hedge ratio is positively related to the unconditional correlation coefficient between spot and futures, it would be higher or lower than that computed from returns in normal states only depending on whether the correlation coefficient is bigger in disaster states than normal states.

## 7. Leverage

So far, we have assumed there are only two types of assets. One is the default-possible government bill and the other is the equity share representing the claim to the *entire* endowment. In this case, equity payment and consumption are equal in all periods. We have shown that risk premium is high when the probability of disaster and default are taken into consideration. However, if the ratio of consumption to equity payment is pro-cyclical, i.e. equity payment is particularly low in disasters, and then the risk premium will become even higher. One way to generate the pro-cyclical equity payment to consumption is to introduce leverage, i.e. to assume the ownership of the tree includes both fixed claims and equities.

As in Barro (2006), we assume there is a one-period equity claim that represents a claim on the entire endowment of a country. Also assume there is a one-period private bond, whose characteristics are the same as the government bill of the corresponding country. The private bond promises the same rate of return as the government bill. In non-disaster state, the actual payment is the same as promised. In disaster state, with probability of $q$, only part of the face return is realized and with probability $1-q$, the face return is realized. Same as the government bill, the default of private bond is carried out through unanticipated inflation. Therefore, the issuers of the private bonds can avoid bankruptcy despite of defaulting on its bond. Same as the government bill, the default size of the private bond is the same as the endowment contraction. The proceeds from default are returned to representative agents through lump-sum transfer.

With the equity and the private bond, the ownership structure is as follows.

In the period $t$, the tree owner issues an equity claim on the entire output of country one in the next period, $A_{t+1}$, at the price $P_t$. Then the owner issues $\beta_t$ unit of one-period private bond and gives the proceeds to the equity holder. The net price paid for the equity is $P_t - \beta_t$. The debt-to-equity ratio is:

$$\lambda = \frac{\beta_t}{P_t - \beta_t} \tag{27}$$

In the period $t+1$, the output is $A_{t+1}$ and the promised payment to the private bondholders is $\beta_t \cdot R^f$. In non-disaster state, the actual payment to the bond holders is equal to the promised. In disaster state, the actual payment is reduced to $(1-b)\beta_t \cdot R^f$. In any case, the remaining proceeds go to the equity holders.

By the theory of Modigliani and Miller (M&M), the value of equity is not affected by the capital structure of the firm. Therefore the price of equity one is still given by equation (21). However, leverage *does affect* the expected return of equity. The rate of return of levered equity is:

$$R_{t+1}^{e\_lev} = \frac{A_{t+1} - \beta_t R_{t+1}^b}{P_t - \beta_t} \tag{28}$$

The solution for the expected return of levered equity is:

$$\ln E_t R_{t+1}^{e\_lev} = (1+\lambda)\ln E_t R_{t+1}^e - \lambda \ln E_t R_{t+1}^b \tag{29}$$

The expected return of equity with leverage is the weighted average of the expected return of equity without leverage (24) and the expected return of the government bill (16) where the former weights $(1+\lambda)$ and the latter weights $(-\lambda)$.

Combining the levered expected return of equity with the expected return of government bill, we can derive the solution for the levered risk premium.

$$\ln RP_t^{lev} = (1+\lambda)\cdot \ln RP_t \tag{30}$$

The levered risk premium is just the unlevered risk premium multiplied by the leverage ratio ( the ratio of asset to equity ). Similar to the unlevered risk premium, the levered risk premium is increasing in the probability of disaster ( $p$ ) and the conditional probability of joint disaster ( $\eta$ ) and decreasing in the conditional probability of default ( $q$ ).

## 8. Calibration

By definition disaster is an event in which output contracts drastically in a short period of time. It could reflect an economic event (Great Depression, financial crisis), wartime destruction, natural disaster and epidemic disease. The data in these events are hard to find and to access, therefore we rely as far as possible on the parameter values used in Barro (2006) based on the 20th century history of economic disasters covering the two world wars, the Great Depression and various financial crises.

In the model, disaster corresponds to GDP contraction in a *small unit* of time. However, in reality, it usually takes several years for a disaster to complete. In the process, annual change in GDP is negative for several consecutive periods. This raises the question of what we should use to measure the size of disaster – the annual decrease or the accumulative decrease of GDP. As it is realistic to consider the size of disaster is more important than the duration, we use the cumulative decrease of GDP to measure the size of disaster, which is also what Barro (2006) used. People are assumed to be aware of the probability distribution of the ultimate output contraction in disaster, but not the duration or the actual outcome of the disaster.

In our two-country model, we make the same assumptions on the parameter values as Barro on the output process, degree of risk aversion, time preference, disaster size and the probability of default and disaster. We also introduce three critical additional parameters – the correlation coefficient between ordinary shocks, $\kappa$, the conditional probability of joint disaster, $\eta$, and the composition of current world output, $m$. Since $\kappa$ has little impact on the expected returns and equity risk premium, it is fixed at 0.5. In the case of the other two parameters, we examine a range of values. Before analyzing the calibration results, it is worth noting the theoretical results on the importance of parameters.

As far as $m$ is concerned, its impact on the aggregate risk is symmetrical around 50 percent point. When it is at 50 percent, the benefit of diversification is fully achieved, the aggregate risk is at the minimum level and the risk-free interest rate is at the maximum level. Between 0 and 50 percent, the higher is $m$, the greater is the benefit of diversification and the lower is the aggregate risk, therefore the risk-free interest is increasing in $m$. Between 50 and 100 percent, the higher is $m$, the less is the benefit of diversification and the higher is the aggregate risk, therefore the risk-free interest rate is monotonically decreasing in $m$. This suggests the expected return of bills should be hump-shaped. The default and disaster risk premiums are increasing in $m$. The bigger is country one, the more difficult it is to diversify its risk, including both the default risk and the disaster risk, the higher are the premiums. This suggests the expected return of bills and equities should be higher when $m$ is one than when $m$ is zero. The equity risk premium is also increasing in $m$. As $m$ rises, the diversification of the idiosyncratic risk becomes more difficult, especially for the disaster risk associated with equities because it is more tied up with the market price of risk, therefore the expected return of equities has to increase more than that of bills, resulting in an increase in the equity risk premium. This suggests the equity risk premium is monotonically increasing in $m$.

The effect of $\eta$ on the expected return of equities and bills are similar. On the one hand, the higher is $\eta$, the greater is the aggregate risk, implying a lower risk-free interest rate. It is worth noting that this effect impinges equally on both the expected return of equities and bills, therefore $\eta$ has no effect on the equity risk premium. One the other hand, the higher is $\eta$, the more likely is the joint disaster scenario, the worse is the payoff pattern of equities and bills, the higher is the disaster and default risk premium. Since the disaster probability is bigger than the default probability, the equities are hit harder than the bills when the payoff pattern turns bad, therefore the equity premium is increasing in $\eta$. When $m < \theta^{-1}$, the expected return on equities is increasing in $\eta$ but its implication on the bills is unclear.

The calibration results are summarized in Table 1 and 2 and shown in more detail in Figure 3 to 7. In these figures, we plot the expected return of equities and bills and the equity risk premium against the weight of country one's output ranging from 0 to 100 percent. The value of the expected return of equities and bills and the equity risk premium of Barro's model correspond to the right-end point of each curve. The parameter values are taken from Barro's baseline model. That is, we set the steady-state growth rate of both countries ($\gamma$) at 0.025, the standard deviation of ordinary shock ($\sigma$) at 0.02, the time preference ($\rho$) at 0.03, the degree of risk aversion ($\theta$) at 4 and the debt-to-equity ratio ($\lambda$) at 0.5.

**TABLE 1**

**Expected return of equity (ERe); Expected return of government bill (ERb); Equity premium (EPrem)**

Assumptions: $\theta = 4; \sigma = 0.02; \rho = 0.03; \gamma = 0.025; p = 0.017; \alpha = 0.4$

| $w$ | w = 0.1 | | | w = 0.5 | | | w = 0.9 | | | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\eta$ | 0.1 | 0.5 | 0.9 | 0.1 | 0.5 | 0.9 | 0.1 | 0.5 | 0.9 | (Barro) |
| E(Re) | 4.52% | 5.49% | 6.46% | 9.72% | 9.52% | 9.32% | 9.64% | 9.49% | 9.33% | 8.96% |
| E(Rb) | 4.16% | 4.09% | 4.02% | 8.36% | 7.45% | 6.54% | 5.65% | 5.23% | 4.81% | 3.59% |
| E(Prem) | 0.36% | 1.40% | 2.44% | 1.36% | 2.07% | 2.78% | 4.00% | 4.26% | 4.52% | 5.38% |

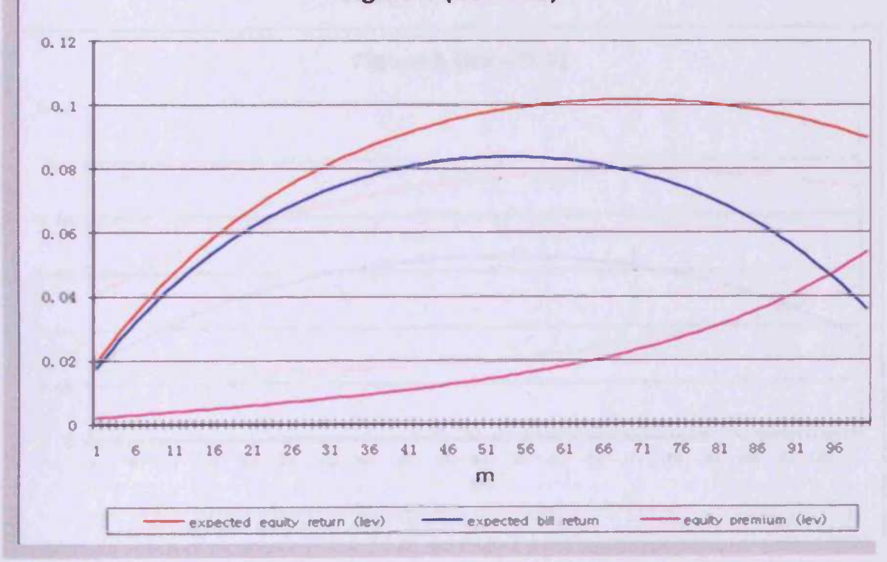
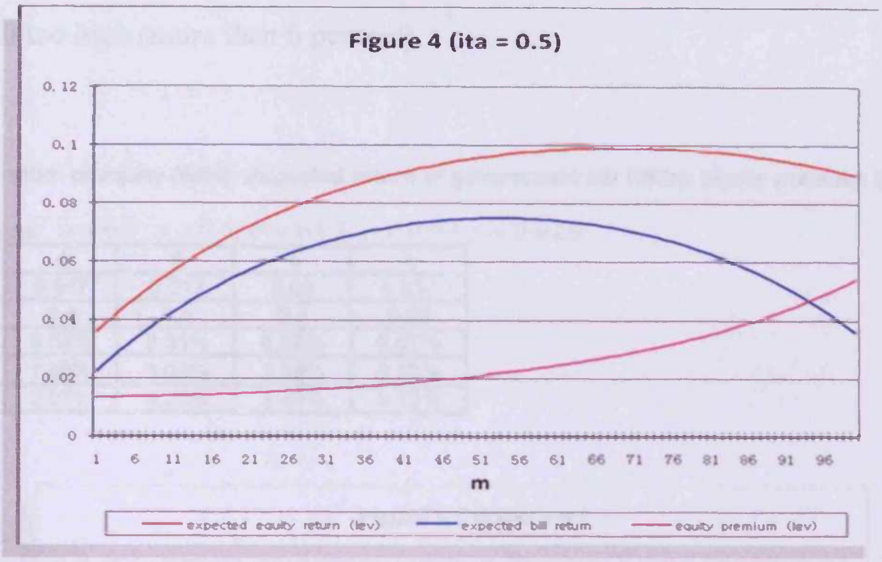



Figure 3 (ita = 0.1)



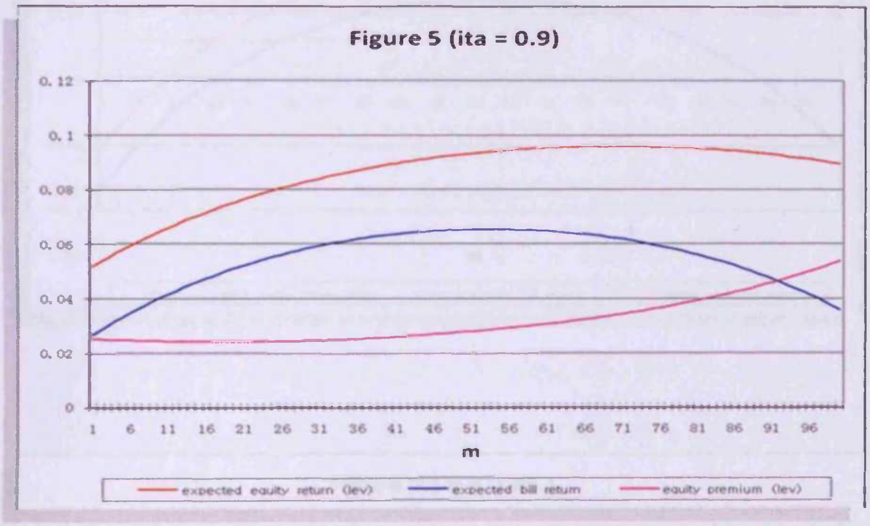



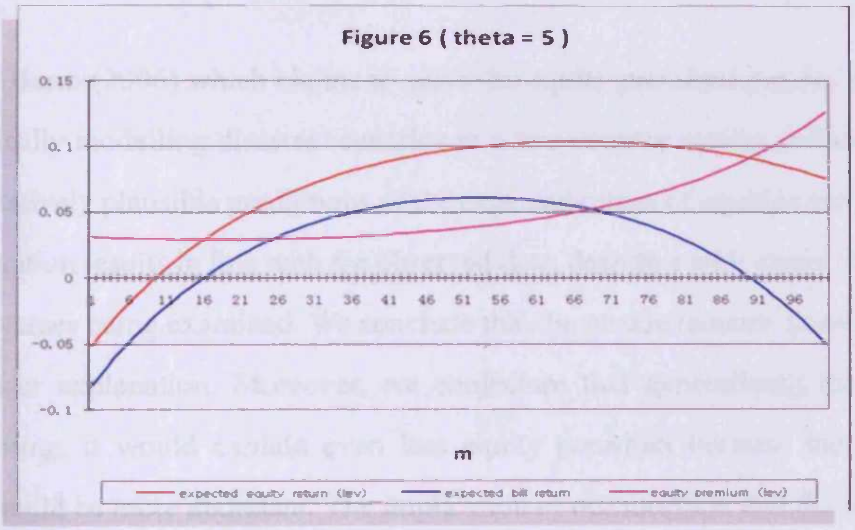

Figure 3 is derived under the assumption of low correlation between the two disaster shocks ($\eta$= 0.1). It shows the calibration results are consistent with those predicted by the theories. In particular, the expected return of bills has a hump shape and the corresponding expected return is lower when country one is extremely small than when it is extremely big. The expected return of bills is far too high (more than 8 percent) when the two countries are of similar size. The equity risk premium is monotonically increasing in the output weight of country one, but the value (less than 2 percent) is too low compared with the empirical data when the two countries are of similar size. Raising the value of $\eta$ to 0.5 or 0.9 does help increase the equity risk premium to between 2 and 3 percent, as shown in Figure 4 and 5. But the expected return of bills is still too high (more than 6 percent).

**TABLE 2**

**Expected return of equity (ERe); Expected return of government bill (ERb); Equity premium (EPrem)**

**Assumptions:** $w = 0.5$; $\eta = 0.5$; $\sigma = 0.02$; $\rho = 0.03$; $\gamma = 0.025$

| $\theta$ | 4 | 5 | 4 | 4 |
|---|---|---|---|---|
| $p$ | 0.017 | 0.017 | 0.03 | 0.017 |
| $q$ | 0.4 | 0.4 | 0.4 | 0.05 |
| E(Re) | 9.52% | 9.81% | 6.82% | 9.87% |
| E(Rb) | 7.45% | 6.08% | 3.39% | 6.76% |
| E(Prem) | 2.07% | 3.73% | 3.42% | 3.10% |



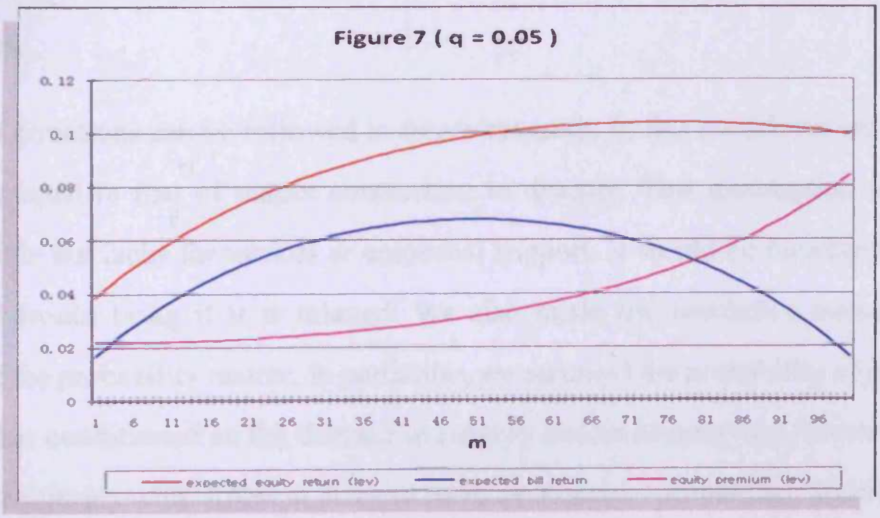




Figure 6 shows raising the degree of risk aversion from 4 to 5 does increase the equity risk premium to around 4 percent when the two countries are of similar size. But it also corresponds to far too high expected return of equities (around 10 percent) and bills (more than 5 percent).

Figure 7 shows decreasing the conditional probability of default from 0.4 to 0.05 does increase the equity risk premium to around 3 percent when the two countries are of similar size. But again the expected returns are far too high. The expected return of equities and bills are around 10 percent and 6 percent levels respectively.[10]

## 9. Conclusion

Starting from Barro (2006) which claims to solve the equity premium puzzle, we have built a model specifically modelling disaster scenarios in a two country setting. Although the model leads to qualitatively plausible predictions of the expected return of equities and bills, it fails to produce calibration results in line with the observed data, despite a wide range of combinations of parameter values being examined. We conclude that the puzzle remains unsolved, at least by the rare disaster explanation. Moreover, we conjecture that generalizing the model to the N-country setting, it would explain even less equity premium because the diversification opportunity would be more abundant. The implication of our model is that the high equity risk premium observed in the last two hundred years remains unexplainable and therefore investors should not form their expectations of future equity returns simply on the basis of historical performances.

A number of directions can be followed in future research. In this model, we assumed the size of default is equal to that of output contraction in disaster. This assumption is to make the model tractable but lacks theoretical or empirical support. It would be interesting to see what difference it would bring if it is relaxed. We also made the restrictive assumption on the symmetry of the probability matrix. In particular, we assumed the probability of joint disaster is the same either conditioned on the disaster in country one or country two. However, it is more realistic for countries with different sizes to have asymmetric probability matrix. It would be interesting to replace it in the future research.

---

[10] More combinations of parameter values of p and q are tried, but none results in values in accordance to the data.

# Appendix

A1: Derivation of the risk-free interest rate in equation (11).

$$
\begin{aligned}
& E_t M_{st+1} \\
& \approx E\{\exp[-\rho - \theta m g_{t+1} - \theta(1-m) g^*_{t+1}]\} \\
& = E\{\exp[-\rho - \theta m(\gamma + u_{t+1} + v_{t+1}) - \theta(1-m)(\gamma^* + u^*_{t+1} + v^*_{t+1})]\} \\
& = \exp[-\rho - \theta m\gamma - \theta(1-m)\gamma^*] \cdot \exp[\frac{1}{2}(\theta m\sigma)^2 + \frac{1}{2}(\theta(1-m)\sigma^*)^2 + \theta^2 m(1-m)\kappa\sigma\sigma^*] \\
& \qquad \cdot E_t\{\exp[-\theta m v_{t+1} - \theta(1-m) v^*_{t+1}]\} \\
& = \exp[-\rho - \theta m\gamma - \theta(1-m)\gamma^* + \frac{1}{2}(\theta m\sigma)^2 + \frac{1}{2}(\theta(1-m)\sigma^*)^2 + \theta^2 m(1-m)\kappa\sigma\sigma^*] \\
& \quad \cdot \{(e^{-p} - 1)(1-\eta) + e^{-p} + (1-e^{-p})(1-\eta)[E(1-b)^{-\theta m} + E(1-b^*)^{-\theta(1-m)}] \\
& \qquad + (1-e^{-p})\eta E(1-b)^{-\theta m} \cdot E(1-b^*)^{-\theta(1-m)}\} \\
& \approx \exp[-\rho - \theta m\gamma - \theta(1-m)\gamma^* + \frac{1}{2}(\theta m\sigma)^2 + \frac{1}{2}(\theta(1-m)\sigma^*)^2 + \theta^2 m(1-m)\kappa\sigma\sigma^*] \\
& \quad \cdot \{1 + p(\eta - 2) + p(1-\eta)[E(1-b)^{-\theta m} + E(1-b)^{-\theta(1-m)}] + p\eta E(1-b)^{-\theta m} \cdot E(1-b)^{-\theta(1-m)}\}
\end{aligned}
$$

$$
\begin{aligned}
& \ln R^{risk-free} = -\ln E_t M_{st+1} \\
& \approx \rho + \theta m\gamma + \theta(1-m)\gamma^* - \frac{1}{2}(\theta m\sigma)^2 - \frac{1}{2}(\theta(1-m)\sigma^*)^2 - \theta^2 m(1-m)\kappa\sigma\sigma^*] \\
& \quad - \ln\{1 + p(\eta - 2) + p(1-\eta)[E(1-b)^{-\theta m} + E(1-b^*)^{-\theta(1-m)}] + p\eta E(1-b)^{-\theta m} \cdot E(1-b^*)^{-\theta(1-m)}\} \\
& = \rho + \theta m\gamma + \theta(1-m)\gamma^* - \frac{1}{2}(\theta m\sigma)^2 - \frac{1}{2}(\theta(1-m)\sigma^*)^2 - \theta^2 m(1-m)\kappa\sigma\sigma^*] \\
& \quad - p\{(\eta - 2) + (1-\eta)[E(1-b)^{-\theta m} + E(1-b^*)^{-\theta(1-m)}] + \eta E(1-b)^{-\theta m} \cdot E(1-b^*)^{-\theta(1-m)}\} \\
& = \rho + \theta m\gamma + \theta(1-m)\gamma^* - \frac{1}{2}(\theta m\sigma)^2 - \frac{1}{2}(\theta(1-m)\sigma^*)^2 - \theta^2 m(1-m)\kappa\sigma\sigma^*] \\
& \quad - p\{(\eta - 2) + (1-\eta)[E(1-b)^{-\theta m} + E(1-b)^{-\theta(1-m)}] + \eta E(1-b)^{-\theta m} \cdot E(1-b)^{-\theta(1-m)}\}
\end{aligned}
$$

A2: Derivation of the face rate of return of government one's bill in equation (14).

$$1 = E_t(M_{st+1} \cdot R_{t+1})$$

$$\begin{aligned}
1 &\approx E_t\{\exp[-\rho - \theta m g_{t+1} - \theta(1-m)g^*_{t+1}] \cdot R_{t+1}\} \\
&= E_t\{\exp[-\rho - \theta m\gamma - \theta(1-m)\gamma^* - \theta m u_{t+1} - \theta(1-m)u^*_{t+1} - \theta m v_{t+1} - \theta(1-m)v^*_{t+1}] \cdot R_{t+1}\} \\
&= \exp[-\rho - \theta m\gamma - \theta(1-m)\gamma^* + \frac{1}{2}\theta^2 m^2\sigma^2 + \frac{1}{2}\theta^2(1-m)^2\sigma^{*2} + \theta^2 m(1-m)\sigma\sigma^*\kappa] \\
&\quad \cdot E_t\{\exp[-\theta m v_{t+1} - \theta(1-m)v^*_{t+1}] \cdot R_{t+1}\} \\
&= \exp[-\rho - \theta m\gamma - \theta(1-m)\gamma^* + \frac{1}{2}\theta^2 m^2\sigma^2 + \frac{1}{2}\theta^2(1-m)^2\sigma^{*2} + \theta^2 m(1-m)\sigma\sigma^*\kappa] \\
&\quad \cdot \{[(e^{-p}-1)(1-\eta) + e^{-p}] + [(1-e^{-p})(1-\eta)(1-q)E(1-b)^{-\theta m}] \cdot R^{1f}_{t+1} \\
&\qquad + [(1-e^{-p})(1-\eta)qE(1-b)^{1-\theta m}] \cdot R^{1f}_{t+1} \\
&\quad + [(1-e^{-p})(1-\eta)E(1-b^*)^{-\theta(1-m)}] \cdot R^{1f}_{t+1} + [\eta(1-e^{-p})(1-q)E(1-b)^{-\theta m} \cdot E(1-b^*)^{-\theta(1-m)}] \cdot R^{1f}_{t+1} \\
&\quad + [\eta(1-e^{-p})qE(1-b)^{1-\theta m} \cdot E(1-b^*)^{-\theta(1-m)}] \cdot R^{1f}_{t+1}\} \\
&\approx \exp[-\rho - \theta m\gamma - \theta(1-m)\gamma^* + \frac{1}{2}\theta^2 m^2\sigma^2 + \frac{1}{2}\theta^2(1-m)^2\sigma^{*2} + \theta^2 m(1-m)\sigma\sigma^*\kappa] \\
&\quad \cdot \{[-p(1-\eta) + 1 - p] \cdot R^{1f}_{t+1} + [p(1-\eta)(1-q)E(1-b)^{-\theta m}] \cdot R^{1f}_{t+1} + [p(1-\eta)qE(1-b)^{1-\theta m}] \cdot R^{1f}_{t+1} \\
&\quad + [p(1-\eta)E(1-b)^{-\theta(1-m)}] \cdot R^{1f}_{t+1} + [\eta p(1-q)E(1-b)^{-\theta m} \cdot E(1-b^*)^{-\theta(1-m)}] \cdot R^{1f}_{t+1} \\
&\quad + [\eta pqE(1-b)^{1-\theta m} \cdot E(1-b^*)^{-\theta(1-m)}] \cdot R^{1f}_{t+1}\} \\
&= \exp[-\rho - \theta m\gamma - \theta(1-m)\gamma^* + \frac{1}{2}\theta^2 m^2\sigma^2 + \frac{1}{2}\theta^2(1-m)^2\sigma^{*2} + \theta^2 m(1-m)\sigma\sigma^*\kappa] \\
&\quad \cdot \{[1 - p(2-\eta)] \cdot R^{1f}_{t+1} + [p(1-\eta)(1-q)E(1-b)^{-\theta m}] \cdot R^{1f}_{t+1} + [p(1-\eta)qE(1-b)^{1-\theta m}] \cdot R^{1f}_{t+1} \\
&\quad + [p(1-\eta)E(1-b)^{-\theta(1-m)}] \cdot R^{1f}_{t+1} + [\eta p(1-q)E(1-b)^{-\theta m} \cdot E(1-b)^{-\theta(1-m)}] \cdot R^{1f}_{t+1} \\
&\quad + [\eta pqE(1-b)^{1-\theta m} \cdot E(1-b)^{-\theta(1-m)}] \cdot R^{1f}_{t+1}\}
\end{aligned}$$

$$\begin{aligned}
0 &= -\rho - \theta m\gamma - \theta(1-m)\gamma^* + \frac{1}{2}\theta^2 m^2\sigma^2 + \frac{1}{2}\theta^2(1-m)^2\sigma^{*2} + \theta^2 m(1-m)\sigma\sigma^*\kappa \\
&\quad + \ln\{1 - p(2-\eta) + [p(1-\eta)(1-q)E(1-b)^{-\theta m}] + [p(1-\eta)qE(1-b)^{1-\theta m}] \\
&\quad + [p(1-\eta)E(1-b)^{-\theta(1-m)}] + [\eta p(1-q)E(1-b)^{-\theta m} \cdot E(1-b)^{-\theta(1-m)}] \\
&\quad + [\eta pqE(1-b)^{1-\theta m} \cdot E(1-b)^{-\theta(1-m)}]\} + \ln R^{1f}_{t+1}
\end{aligned}$$

$$\begin{aligned}
0 &\approx -\rho - \theta m\gamma - \theta(1-m)\gamma^* + \frac{1}{2}\theta^2 m^2\sigma^2 + \frac{1}{2}\theta^2(1-m)^2\sigma^{*2} + \theta^2 m(1-m)\sigma\sigma^*\kappa \\
&\quad - p(2-\eta) + [p(1-\eta)(1-q)E(1-b)^{-\theta m}] + [p(1-\eta)qE(1-b)^{1-\theta m}] \\
&\quad + [p(1-\eta)E(1-b)^{-\theta(1-m)}] + [\eta p(1-q)E(1-b)^{-\theta m} \cdot E(1-b)^{-\theta(1-m)}] \\
&\quad + [\eta pqE(1-b)^{1-\theta m} \cdot E(1-b)^{-\theta(1-m)}] + \ln R^{1f}_{t+1}
\end{aligned}$$

$$\ln R_{t+1}^{1f} = \rho + \theta m\gamma + \theta(1-m)\gamma^{*} - \frac{1}{2}\theta^2 m^2\sigma^2 - \frac{1}{2}\theta^2(1-m)^2\sigma^{*2} - \theta^2 m(1-m)\sigma\sigma^{*}\kappa$$
$$+ p(2-\eta) - [p(1-\eta)(1-q)E(1-b)^{-\theta m}] - [p(1-\eta)qE(1-b)^{1-\theta m}]$$
$$-[p(1-\eta)E(1-b)^{-\theta(1-m)}] - [\eta p(1-q)E(1-b)^{-\theta m}\cdot E(1-b)^{-\theta(1-m)}]$$
$$-[\eta pqE(1-b)^{1-\theta m}\cdot E(1-b)^{-\theta(1-m)}]$$

$$\ln R_{t+1}^{1f} \approx \rho + \theta m\gamma + \theta(1-m)\gamma^{*} - \frac{1}{2}\theta^2 m^2\sigma^2 - \frac{1}{2}\theta^2(1-m)^2\sigma^{*2} - \theta^2 m(1-m)\sigma\sigma^{*}\kappa$$
$$- p[\eta - 2 + (1-\eta)(1-q)E(1-b)^{-\theta m} + (1-\eta)qE(1-b)^{1-\theta m} + (1-\eta)E(1-b)^{-\theta(1-m)}$$
$$+ (1-q)\eta E(1-b)^{-\theta m}\cdot E(1-b)^{-\theta(1-m)} + \eta qE(1-b)^{1-\theta m}\cdot E(1-b)^{-\theta(1-m)}]$$

A3: Derivation of the expected rate of return of government one's bill in equation (15).

$$E_t R_{t+1}^{1b} = [e^{-p} + (1-e^{-p})(1-q)]R_{t+1}^{1f} + (1-e^{-p})qE(1-b)R_{t+1}^{1f}$$
$$= e^{-p}\{[1+(e^{p}-1)(1-q)] + (e^{p}-1)qE(1-b)\}R_{t+1}^{1f}$$

$$\ln E_t R_{t+1}^{1b} = -p + \ln\{[1+(e^{p}-1)(1-q)] + (e^{p}-1)qE(1-b)\} + \ln R_{t+1}^{1f}$$
$$\approx -p + \ln[1 + p(1-q) + pqE(1-b)] + \ln R_{t+1}^{1f}$$
$$\approx -p + p(1-q) + pqE(1-b) + \ln R_{t+1}^{1f}$$
$$= -pqEb + \ln R_{t+1}^{1f}$$

$$\ln E_t R_{t+1}^{1b} \approx \rho + \theta m\gamma + \theta(1-m)\gamma^{*} - \frac{1}{2}\theta^2 m^2\sigma^2 - \frac{1}{2}\theta^2(1-m)^2\sigma^{*2} - \theta^2 m(1-m)\sigma\sigma^{*}\kappa$$
$$- p[\eta - 2 + (1-\eta)(1-q)E(1-b)^{-\theta m} + (1-\eta)qE(1-b)^{1-\theta m} + (1-\eta)E(1-b)^{-\theta(1-m)}$$
$$+ (1-q)\eta E(1-b)^{-\theta m}\cdot E(1-b)^{-\theta(1-m)} + \eta qE(1-b)^{1-\theta m}\cdot E(1-b)^{-\theta(1-m)} + qEb]$$

A4: Derivation of the price of equity one in equation (20).

$$
\begin{aligned}
P_{t1} &= E_t[M_{st+1} \cdot A_{t+1}] \\
&\approx E_t\{\exp[-\rho - m\theta g_{t+1} - (1-m)\theta g^*_{t+1}] \cdot A_t \exp(g_{t+1})\} \\
&= A_t E_t\{\exp[-\rho + (1-m\theta) g_{t+1} - (1-m)\theta g^*_{t+1}]\} \\
&= A_t E_t\{\exp[-\rho + (1-m\theta)(\gamma + u_{t+1} + v_{t+1}) - (1-m)\theta(\gamma^* + u^*_{t+1} + v^*_{t+1})]\} \\
&= A_t E_t\{\exp[-\rho + \gamma - m\theta\gamma - (1-m)\theta\gamma^*] \cdot \exp[(1-m\theta)u_{t+1} - (1-m)\theta u^*_{t+1}] \\
&\qquad \cdot \exp[(1-m\theta)v_{t+1} - (1-m)\theta v^*_{t+1}]\} \\
&= A_t \cdot \exp[-\rho + \gamma - m\theta - (1-m)\theta\gamma^* + \frac{1}{2}(1-m\theta)^2\sigma^2 + \frac{1}{2}(1-m)^2\theta^2\sigma^{*2} - (1-m\theta)(1-m)\theta\kappa\sigma\sigma^*] \\
&\quad \cdot E_t\{\exp[(1-m\theta)v_{t+1} - (1-m)\theta v^*_{t+1}]\} \\
&= A_t \cdot \exp[-\rho + \gamma - m\theta - (1-m)\theta\gamma^* + \frac{1}{2}(1-m\theta)^2\sigma^2 + \frac{1}{2}(1-m)^2\theta^2\sigma^{*2} - (1-m\theta)(1-m)\theta\kappa\sigma\sigma^*] \\
&\quad \cdot \{(e^{-p} - 1)(1-\eta) + e^{-p} + [(1-e^{-p})(1-\eta)] \cdot [E(1-b)^{1-\theta m} + E(1-b)^{-\theta(1-m)}] \\
&\qquad + (1-e^{-p})\eta E(1-b)^{1-\theta m} \cdot E(1-b)^{-\theta(1-m)}\} \\
&= A_t \cdot \exp[-\rho + (1-\theta m)\gamma - \theta(1-m)\gamma^* + \frac{1}{2}(1-\theta m)^2\sigma^2 + \frac{1}{2}\theta^2(1-m)^2\sigma^{*2} - (1-\theta m)\theta(1-m)\sigma\sigma^*\kappa] \\
&\quad \cdot \{p\eta[E(1-b)^{1-\theta m} - 1] \cdot [E(1-b)^{-\theta(1-m)} - 1] + 1 + p[E(1-b)^{1-\theta m} - 1] + p[E(1-b)^{-\theta(1-m)} - 1]\}
\end{aligned}
$$

A5: Derivation of the expected return of equity one in equation (22).

$$
\begin{aligned}
&\ln E_t(R^{e1}_{t+1}) = \ln E_t(A_{t+1}/P_{t1}) \\
&= \ln E_t(A_{t+1}) - \ln P_{t1} \\
&= \ln E_t[A_t \cdot \exp(\gamma + u_{t+1} + v_{t+1})] - \ln P_{t1} \\
&= \ln\{A_t \cdot \exp(\gamma + \frac{1}{2}\sigma^2) \cdot [e^{-p} + (1-e^{-p}) \cdot E(1-b)]\} - \ln P_{t1} \\
&\approx \ln A_t + \gamma + \frac{1}{2}\sigma^2 - p + p \cdot E(1-b) - \\
&\left(\begin{aligned}
&\ln A_t + [-\rho + (1-\theta m)\gamma - \theta(1-m)\gamma^* + \frac{1}{2}(1-\theta m)^2\sigma^2 \\
&\qquad + \frac{1}{2}\theta^2(1-m)^2\sigma^{*2} - (1-\theta m)\theta(1-m)\sigma\sigma^*\kappa] \\
&+ \ln\{p\eta[E(1-b)^{1-\theta m} - 1] \cdot [E(1-b)^{-\theta(1-m)} - 1] + 1 + p[E(1-b)^{1-\theta m} - 1] + p[E(1-b)^{-\theta(1-m)} - 1]\}
\end{aligned}\right) \\
&= \rho + \theta m\gamma + \theta(1-m)\gamma^* + \theta m\sigma^2 - \frac{1}{2}\theta^2 m^2\sigma^2 - \frac{1}{2}\theta^2(1-m)^2\sigma^{*2} + (1-\theta m)\theta(1-m)\sigma\sigma^*\kappa \\
&\quad - p\{Eb + \eta[E(1-b)^{1-\theta m} E(1-b)^{-\theta(1-m)} - 1] + (1-\eta)[E(1-b)^{1-\theta m} - 1] + (1-\eta)[E(1-b)^{-\theta(1-m)} - 1]\}
\end{aligned}
$$

# Chapter 5

# Conclusion

In this thesis, we look at three problems related to the financial risk of stock markets. In particular, the effectiveness of direct and cross hedging using index futures is examined empirically. A theoretical model explaining the equity premium puzzle by Barro (2006) is extended to a two-country setting. The findings are as follows.

We examine a variety of hedging strategies supported by econometric models from simple OLS to complicated VECM with TGARCH error and STVECM with GARCH error using both the within-sample and out-of-sample data. We find the sophisticated models fit the data well but do not produce consistently and significantly better performance over unity and simple OLS hedge. The usefulness of the sophisticated models has to be judged on the case-by-case basis. It demonstrates the typical difficulty in forecasting where complicated models fit the within-sample data well but perform badly out-of-sample. In future research, we may explicitly model the transaction cost to penalize the volatile hedge ratios or look at other indicator other than Ederington measurement to assess the hedging effectiveness.

We investigated the effectiveness of cross hedging the portfolios each benchmarked to one of the seventeen MSCI indices using the related index futures. We find that most MSCI indices are not cointegrated with the most frequently traded index futures of the same country, but are cointegrated with the multiple index futures. A variety of combinations of hedging instruments and econometric models are tried for each country. For countries whose returns are volatile in both within- and out-of-sample period, the sophisticated models can improve the hedging effectiveness significantly; for countries with moderate volatility, the improvement is small and consistent; for countries with stable return, the improvement is tiny; for countries with completely different volatility across two sample periods, no strategy performs better than unity or simple OLS consistently.

Our extension of Barro's rare disaster model leads to intuitive predictions of the expected return of equities and bills but fails to explain the scale of the observed equity risk premium, despite the fact that a large range of parameter values has been examined. The conclusion is that the rare disaster explanation cannot resolve the equity risk premium puzzle when extended to a more realistic setting. In future research, we may relax several constraints on the model such as the symmetric probability matrix and the assumption on the same size of the loss in disaster and default.

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