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Using Retinex for Point Selection in 3D Shape Registration

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Abstract

Inspired by retinex theory, we propose a novel method for selecting key points from a depth map of a 3D freeform shape; we also use these key points as a basis for shape registration. To find key points, first, depths are transformed using the Hotelling method and normalized to reduce their dependence on a particular viewpoint. Adaptive smoothing is then applied using weights which decrease with spatial gradient and local inhomogeneity; this preserves local features such as edges and corners while ensuring smoothed depths are not reduced. Key points are those with locally maximal depths, faithfully capturing shape. We show how such key points can be used in an efficient registration process, using two state-of-the-art iterative closest point variants. A comparative study with leading alternatives, using real range images, shows that our approach provides informative, expressive, and repeatable points leading to the most accurate registration results.

Keywords: Retinex; Key point; Freeform shape; Adaptive smoothing; Registration

1 Introduction

Laser scanning systems can quickly capture the surfaces of 3D shapes, with results like those in Figure 1. Since scanners have a limited field of view, and one part of the shape may occlude others, multiple datasets must be captured from different viewpoints to obtain (approximately) full coverage of the shape of interest. These datasets are in the form of *depth maps* in local scanner-centric coordinates. In order to fuse the information in these datasets to give a single complete surface, they must be brought into a single global coordinate system by *registration*, which aims to find pairwise transformations that align one dataset with another. Prior knowledge of the underlying transformation may be lacking, and furthermore, occlusion causes appearance and disappearance of points in different datasets. Registration is thus a challenging problem which has attracted attention in various disciplines, such as computer

vision, pattern recognition, computer graphics, and medical imaging. It finds applications in such areas as object recognition, quality assurance, computer aided design, medical diagnosis, and therapy planning. Many algorithms have been developed for registration of overlapping 3D freeform shapes [5, 7, 9, 10, 18, 23]. In this paper, our main concern is the detection of *key points* for *faithful representation* and *efficient registration* of 3D freeform shapes.

1.1 Previous work

The detection of key points which capture the important features of a freeform shape is of great interest, since they provide a compact alternative to the original geometry, and may be used in its place in various tasks such as shape registration, data transmission, rendering and visualisation. Various methods have been proposed for the detection of key points. They should capture both significant local details of the shape and its overall geometry. Their determination should be repeatable and they should represent intrinsic features, i.e. key points detected from one viewpoint should ideally be the same as those detected from another. Key points should also be resistant to imaging noise and resolution, and robust in the presence of occlusion of parts of the object. Finally, their detection should be computationally efficient.

Normal space sampling (NSS) was proposed in [20] as a way of sampling informative points. It calculates angles between the normal vectors at a point of interest and its neighbors, and uses these angles as a basis for sampling. More points are sampled in areas where normal vectors change more rapidly. A relevance based sampling scheme was proposed in [28], which defines the distinctiveness of a point in terms of a surrounding area having similar normal vectors. The larger the area, the less distinct the point. To address the issue of different densities of point sampling due to variations in the distance between the object and the scanner, a spherical subsampling method was proposed in [12]: instead of sampling points from regularly gridded data, it considers the scanner’s angular resolution. In [13], a Hotelling transform is first performed over the points; feature points are detected as those with the largest difference between the ranges spanned by the x and y coordinates in the transform. A supervised learning method was proposed in [3] for feature point localization within human faces. Gaussian mixture models (GMMs) are used to approximate the distribution of candidate feature points; a 3D model of feature points is built which enforces constraints on localization. Another learning method was proposed in [4] for detecting key points in faces. A face is represented using 14 landmarks. Statistical distributions of their descriptors, and weights used for linear combination, are learnt from training data. For a given face mesh, a number of descriptor maps are computed and matched against the 14 learnt ones, and combined using the learnt weights. The key points are determined as strong local maxima above a threshold of the combined values using the 14 landmark dictionary shapes. The classical Harris corner detector for 2D images was adapted in [22] to 3D meshes. Principal component analysis is first performed over local patches, then all points on a patch are transformed so that its normal vector is a local z axis, so that the points lie close to the x - y plane. A paraboloid is fitted to these transformed points, allowing x and y derivatives to be computed. To address the difference between discrete images and continuous patches, derivatives are convolved with a Gaussian function. An autocorrelation matrix is used to estimate the local corner strength. A technique called accumulative geodesic extrema (AGEX) was proposed in [19] for human body feature point detection. Key points are those with the longest distances on the surface mesh from its geodesic centroid, distances being computed using shortest geodesic paths.

The Gram matrix was analysed in [14] as a means of geometric corner detection. A set of corner points at each scale is found by searching for local spatial maxima of corner detector responses. Corners lying along edge points are pruned by thresholding the variance of the second-order partial derivatives. An

octave method of salient point detection was proposed in [2], based on the projection of the difference of the weighted averages of the points inside spheres of varying radii onto the weighted average of the normal vectors of those points inside a sphere with smaller radius. A multiscale feature extraction method was proposed in [17] based on principal component analysis, using the ratio of the resulting eigenvalues as an indicator of saliency. Multiscale analysis is also performed in [11] for key point detection in a range image. It first segments the range image to isolate each area of interest, then depth is normalized to a global average of zero and a standard deviation of 1. Multiscale Gaussian and difference of Gaussian (DoG) analysis is performed over these normalized depth values. A key point is one at a local extremum in the DoG space, larger than a threshold. Key points are detected in [26] from regions with significant variation in mean curvature. A structure tensor is first constructed for each local region as a function of the local mean curvatures and normal vectors; it is invariant with respect to sampling density. Corners are detected at the spatial extrema of the determinant of the structure tensor; spurious key points with low response, an edge response, or near to a depth discontinuity are rejected. A variant of the Laplacian of Gaussian method was proposed in [25] for key point detection. The depth gradient is first calculated in both x and y directions, and normalized taking into account the angular resolution of the range image. The second order derivative is estimated from the normalized depth gradient, yielding the magnitude of the gradient in the unit interval. A high magnitude marks a possible key point with high curvature, while spurious points are filtered by occlusion analysis and linear structural analysis. Another salient point detector was proposed by the same author in [24] which first uses principal component analysis to estimate the local surface variation at a particular point and its main direction. Key point strength is calculated by considering how much the main directions change from each other and how stable this point is on the surface. These values are smoothed over the whole image, and points with local maxima larger than a threshold are selected as key points. In [23], given an intensity image associated with a range map, an image mesh is generated and smoothed using a multi-scale bilateral filter, then the gradient at each vertex is estimated using the Laplace-Beltrami operator (LBO). Points with locally extremal gradients are filtered by thresholding their LBO response and suppressed by a non-maximal scheme to finally detect key points.

The above methods fall into two main categories, using single scale or multi-scale analysis. While the former are usually more computationally efficient, the latter are more robust to imaging noise, changes in resolution, and occlusion. They often involve the computation of second order derivatives of shape, filtering spurious salient points, and sometimes, learning. Unfortunately, second order derivatives are sensitive to occlusion, depth discontinuities, changes in sampling resolution and imaging noise. Thresholds are difficult to select and data dependent, while learning requires many representative samples which may be difficult or impossible to collect. The detection of key points from 3D freeform shapes is challenging and still remains open.

If salient key points can be reliably found, subsequent range image analysis may be simpler and more efficient. Conversely, such range image analysis results may provide an effective tool to quantitatively evaluate the extent to which useful key points have been reliably detected.

1.2 Our work

Retinex theory [6, 16] considers how brightness and reflectance behave, and investigates a computational model of color constancy: human perception of color is largely independent of illumination conditions. It shows that a captured 2D image can be decomposed into two subimages: one depends on the reflectance properties of the surface of the imaged object, while the other depends on the illumination conditions. If such a decomposition can be computed in practice, the reflectance image can be used to

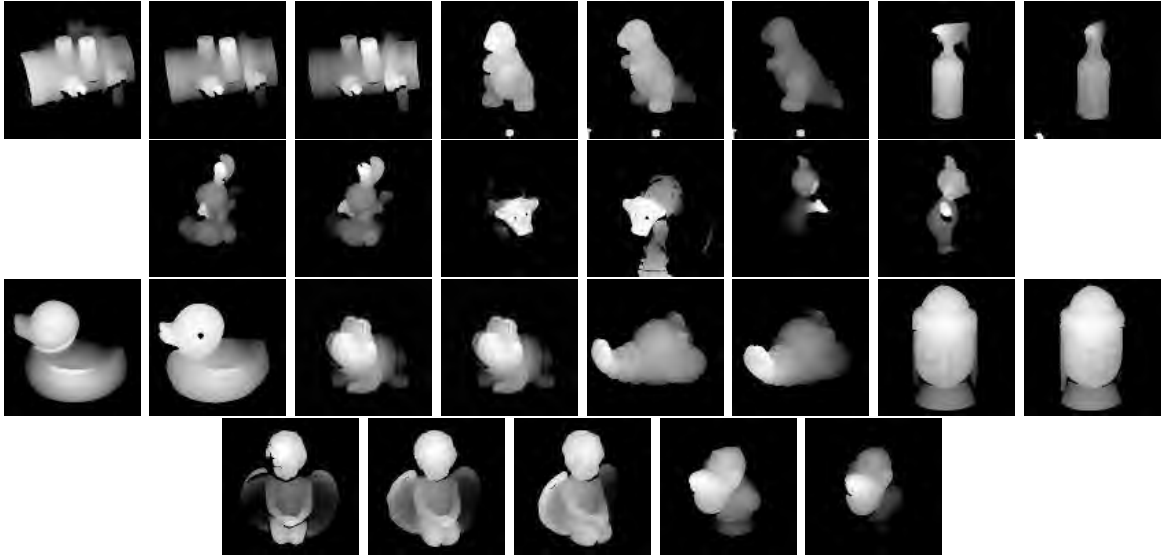


Figure 1: Real range images used. Top: valve20, valve10, valve0, dinosaur72, dinosaur36, dinosaur0, bottle0, and bottle36; Second: bunny80, bunn60, cow49, cow45, tubby120, and tubby80. Third: duck0, duck20, frog0, frog20, lobster0, lobster20, buddha0, and buddha20. Bottom: angel0, angle20, angel40, bird0, and bird20.

improve reliability of such tasks as face recognition, as this image more faithfully represents the geometrical properties of the imaged object, while the variable effects of illumination have been factored out. While such image decomposition is an ill-posed problem, various approaches have been proposed based on Gaussian smoothing [6], adaptive smoothing [16] and minimization of the sum of the first order derivatives of the illumination, and the difference between the illumination and the given image [21, 8]. Two ideas are of particular interest: adaptive smoothing [16] and reflectance inequality [21, 8], where the reflectance component R is estimated as the logarithm of the ratio of intensity F of the pixel of interest and L , that of its neighbors, as the illumination component, satisfying the constraint $R \leq 1$ and thus, $F \leq L$. Using these ideas, we propose a novel method in this paper for the detection of key points on a 3D freeform shape. To this end, depth values are first transformed and normalized so that their dependence on the particular viewpoint can be reduced. Adaptive smoothing is then applied to the normalized depths using weights defined as decreasing functions of spatial gradient and local inhomogeneity. This adaptive smoothing makes sure that local features such as corners and edges are preserved, while smoothed depths become insensitive to imaging noise, but are not reduced. Finally, key points are detected as those unaffected by the adaptive smoothing process and thus have locally maximum transformed and normalized depths.

These detected key points are intended for use as proxies to represent the original shape; here, we investigate whether they can represent the original shape faithfully and are suitable for solving the registration problem. If they are, then the computational efficiency of registration of overlapping 3D depth maps can be significantly improved. We carry out a comparative study, using three other state-of-the-art salient point selection methods: the octave algorithm [2], a multi-scale feature extraction (MSFE) method [17], and the normal space sampling (NSS) method [20]. The octave and MSFE methods are multi-scale methods, while NSS is a single scale method. This comparative study reveals which method can best detect informative, expressive and repeatable feature points. To determine the utility of the detected key points for downstream applications, two state-of-the-art iterative closest point

(ICP) variants, SoftICP [9] and fractional RMSD (FICP) [18], were used for registration. The former is an extension of the SoftAssign algorithm [5] which applies entropy maximization to determine weights for different tentative correspondences, then uses a two-way constraint to refine these weights before estimating the underlying transformation. To ensure robust results, these two steps are embedded in a deterministic annealing scheme. The FICP algorithm simultaneously optimizes both the size of the overlap between depth images, and the transformation parameters.

To assess the performance of the key point detection algorithms, the following measurements were made: precision and recall rates of detected key points, average e_μ and standard deviation e_σ of registration errors in millimetres for reciprocal correspondences (RCs) [9, 27], expected and estimated rotation angles θ and $\hat{\theta}$ in degrees of the underlying transformation, and the time taken for automatic key point detection and registration. While the precision and recall rates measure the repeatability of the detected key points [22, 24, 26], the corresponding registration results measure the informativeness and expressiveness of these key points. As the same registration algorithms were used in each case, differences in their performance come solely from the types of points used. The better the registration results obtained, the more representative we can consider the selected points to be of the shapes.

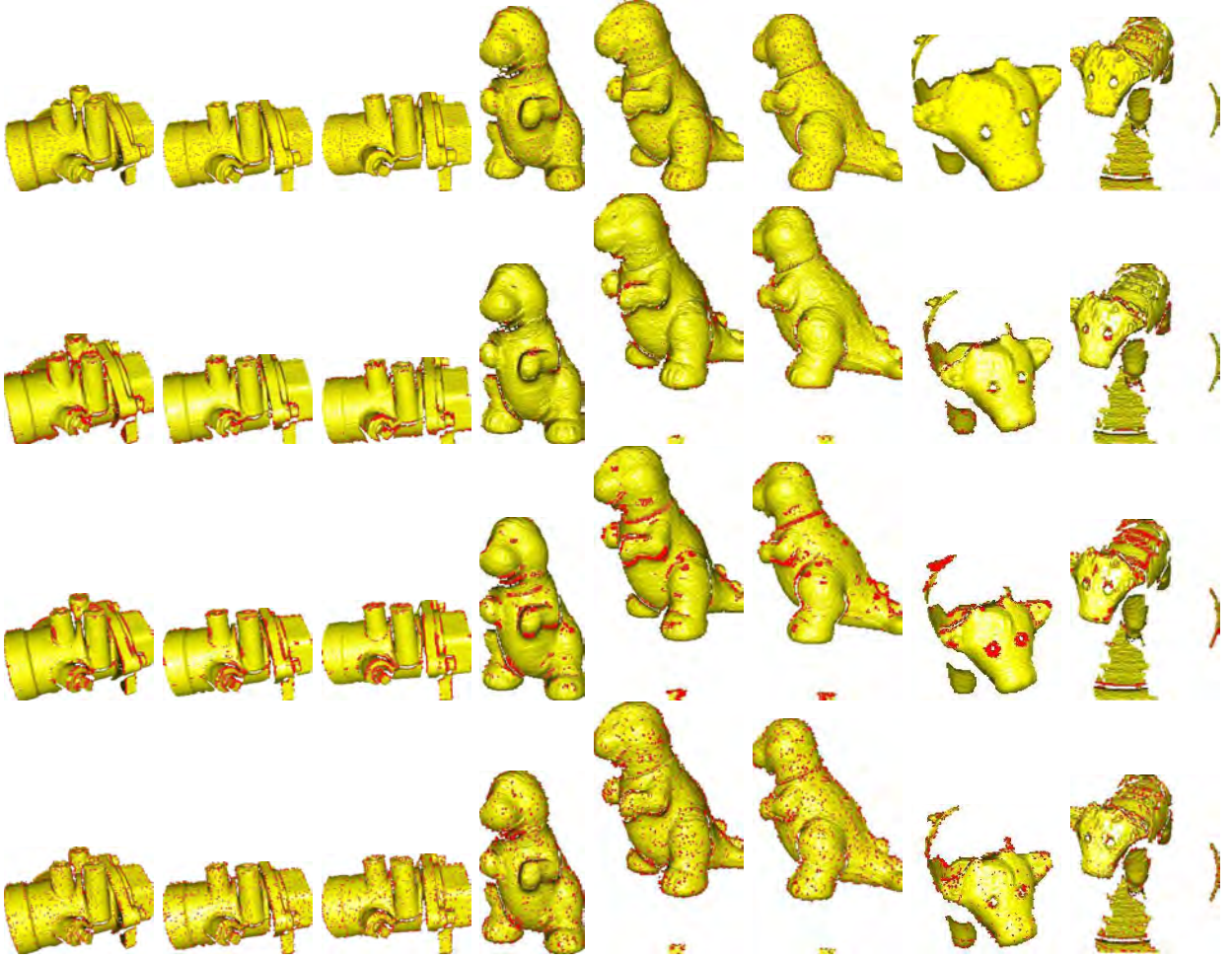


Figure 2: Key points detected in different images using different algorithms. Columns, left to right: valve20, valve10, valve0, dinosaur72, dinosaur36, dinosaur0, cow49, and cow45. Rows, top to bottom: results using our method (RKP), octave, MSFE, and NSS.

In the following, Section 2 describes our key point detection method, Section 3 presents experimental results, and Section 4 draws some conclusions and indicates future work.

2 A novel method for 3D key point detection

The following notation is used: bold face letters denote vectors, matrices, or sets, $|\cdot|$ denotes the absolute value of a scalar or the number of elements in a set, $\mathbf{a} \cdot \mathbf{b}$ denotes the dot product of vectors \mathbf{a} and \mathbf{b} , $\|\cdot\|$ denotes the Euclidean norm of a vector, and superscript T denotes the transpose of a vector.

A freeform shape as illustrated in Figure 1 is represented by four arrays, each of size of $H \times W$: a flag f , and x , y , and z coordinates. If $f(i, j) = 1$, then the point at location (i, j) is valid and is at position $(x(i, j) \ y(i, j) \ z(i, j))^T$, otherwise, it should be ignored. An invalid point is one for which the scanner determined no reliable coordinates on the imaged object surface (e.g. because the reflected signal received was either too weak or too strong due to low reflectance or specular reflection).

We now explain our key point detection method, concentrating on four main issues: depth normalization, weight estimation, adaptive smoothing, and key point extraction.

2.1 Depth normalization

Even though the depth information $z(i, j)$ directly represents the given shape, it is viewpoint dependent. To facilitate key point detection, the Hotelling transform [13] is applied to reduce such viewpoint dependence. Thus, for each valid pixel (i, j) inside the image of the given shape, all valid neighboring points inside a window of size of $s \times s$ are extracted:

$$\mathbf{A} = \{\mathbf{a}_i\} = \{(x(i+k, j+l) \ y(i+k, j+l) \ z(i+k, j+l))^T \mid f(i+k, j+l) = 1, -s/2 \leq k, l \leq s/2\}.$$

Let the centroid of all the points in \mathbf{A} be $\bar{\mathbf{a}} = \sum_{\mathbf{a}_i \in \mathbf{A}} \mathbf{a}_i / |\mathbf{A}|$. Then their covariance matrix \mathbf{C} is: $\mathbf{C} = \sum_{\mathbf{a}_i \in \mathbf{A}} (\mathbf{a}_i - \bar{\mathbf{a}})(\mathbf{a}_i - \bar{\mathbf{a}})^T$. The matrix \mathbf{C} is symmetric; its eigenvectors \mathbf{v}_i and corresponding eigenvalues λ_i can be found using the Jacobi method. Suppose that the \mathbf{v}_i are sorted in descending order of λ_i and are assembled into a matrix \mathbf{v} via $\mathbf{v} = (\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3)$. Then the point $\mathbf{p} = (x(i, j) \ y(i, j) \ z(i, j))^T$ at pixel (i, j) can be transformed to: $\tilde{\mathbf{p}} = (\tilde{x}(i, j) \ \tilde{y}(i, j) \ \tilde{z}(i, j))^T = \mathbf{v} \cdot (\mathbf{p} - \bar{\mathbf{a}})$. Next, assuming that the range of variance of the transformed depth will be similar for an object from different viewpoints, which is likely the case for many objects for viewpoints that are relatively close together, $\tilde{z}(i, j)$ is normalized over the whole image, setting

$$\tilde{z}(i, j) = 255(\tilde{z}(i, j) - \tilde{z}_{\min}) / (\tilde{z}_{\max} - \tilde{z}_{\min})$$

where \tilde{z}_{\max} and \tilde{z}_{\min} are the maximum and minimum values over the whole image. The resulting $\tilde{z}(i, j)$ is used as a normalized viewpoint independent depth for subsequent key point detection. The difference between \mathbf{p} and $\bar{\mathbf{a}}$ removes the effect of translation, while the dot product between \mathbf{v} and $\mathbf{p} - \bar{\mathbf{a}}$ removes the effect of rotation. Note, however, that computation of $\bar{\mathbf{a}}$ and \mathbf{v} may be affected by imaging noise, resolution, and occlusion. As a result, the dependence of \tilde{z} on a particular viewpoint is only somewhat reduced, not completely removed.

2.2 Weight definition

Key point detection applies an adaptive smoothing operation to the transformed and normalized depth values. This operation should preserve local features such as corners and edges. To do so, the weights for

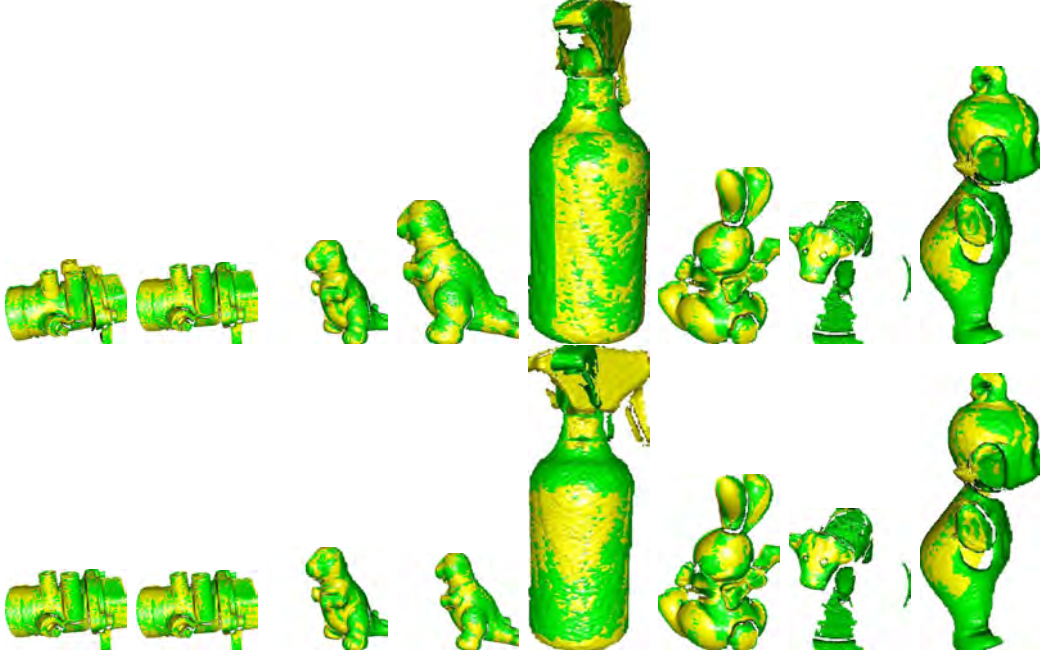


Figure 3: Registration results for different shapes using all points and different algorithms. Columns, left to right: valve20-10, valve10-0, dinosaur72-36, dinosaur36-0, bottle0-36, bunny80-60, cow49-45, and tubby120-80. Rows, top to bottom: SoftICP, FICP.

smoothing must be appropriately defined, taking into account the spatial gradient and inhomogeneity: the larger these two factors are, the smaller the weight should be. The function $w(r) = 1/(1 + \sqrt{r})$, $r \geq 0$ is used to suppress large spatial gradients and inhomogeneity; it has the following useful properties: (i) it is continuous, so does not abruptly change given a small change in r , and (ii) it decreases with r as required.

The local spatial gradient $g(i, j)$ at pixel (i, j) is the square root of the sum of the squares of the differences in depths of pixels in horizontal and vertical directions (thus using the L_2 norm):

$$g(i, j) = \sqrt{g_u^2(i, j) + g_v^2(i, j)}$$

where $g_u(i, j) = \tilde{z}(i + 1, j) - \tilde{z}(i - 1, j)$ and $g_v(i, j) = \tilde{z}(i, j + 1) - \tilde{z}(i, j - 1)$. Suppose the average gradient over the whole image is \bar{g} . Then the weight $w_g(i, j)$ for the gradient component is set to $w_g(i, j) = 1/(1 + \sqrt{0.1g(i, j) \exp(0.1\bar{g})})$.

Given a valid pixel at location (i, j) , its valid neighbors are in: $\mathbf{N} = \{(i + k, j + l) | f(i + k, j + l) = 1, -s/2 \leq k, l \leq s/2\}$. The local inhomogeneity $h(i, j)$ at pixel (i, j) considers the difference of depth between neighboring pixels and is set to the average of the absolute values of these differences (using the L_1 norm):

$$h(i, j) = \frac{1}{|\mathbf{N}|} \sum_{(m, n) \in \mathbf{N}} |\tilde{z}(m, n) - \tilde{z}(i, j)|.$$

Then $h(i, j)$ is normalized using: $h'(i, j) = (h(i, j) - h_{\min})/(h_{\max} - h_{\min})$ where h_{\min} and h_{\max} are the minimum and maximum inhomogeneity over the whole image. To further suppress large inhomogeneities, $h'(i, j)$ is transformed to: $\hat{h}(i, j) = \sin(h'(i, j)\pi/2)$. Finally, the weight $w_h(i, j)$ for the inhomogeneity component is set to: $w_h(i, j) = 1/(1 + \sqrt{10\hat{h}(i, j) \exp(10\bar{h})})$ where \bar{h} is the average of

$\hat{h}(i, j)$ over the whole image.

While the spatial gradient in the L_2 norm is likely to be dominated by noisy pixels or pixels with large depth discontinuities, the spatial inhomogeneity in the L_1 norm is useful to capture small changes in depths of pixels along edges or at corners. Combining the weights $w_g(i, j)$ and $w_h(i, j)$ for the spatial gradient and inhomogeneity components together thus captures both coarse and fine features in the underlying shape, giving the final weight $w(i, j)$ for each valid pixel (i, j) : $w(i, j) = w_g(i, j)w_h(i, j)$.

2.3 Adaptive smoothing

Once the weights $w(i, j)$ have been defined, adaptive smoothing is performed. Following the idea used to estimate the illumination component of an image in retinex theory [16, 21], we assume that the smoothed depth cannot be smaller than the original depth. Thus, adaptive smoothing is performed using the following steps:

Initialize the smoothed depth $s_0(i, j) = \tilde{z}(i, j)$, the number of iterations $I = 0$, and the maximum number of iterations I_{\max}

While $I < I_{\max}$, do:

$I \leftarrow I + 1$

If the pixel at location (i, j) is valid, extract all its valid neighbors:

$\mathbf{N} = \{(i + k, j + l) \mid f(i + k, j + l) = 1, -s/2 \leq k, l \leq s/2\}$.

Compute the weighted average $\hat{s}(i, j)$ of smoothed depths $s_0(m, n)$ at pixels (m, n) in \mathbf{N} using weights $w(m, n)$: $\hat{s}(i, j) = \sum_{(m, n) \in \mathbf{N}} s_0(m, n)w(m, n) / \sum_{(m, n) \in \mathbf{N}} w(m, n)$

Ensure the smoothed depths are non-decreasing: $s_1(i, j) = \max(\hat{s}(i, j), s_0(i, j))$

Prepare for the next round: $s_0(i, j) \leftarrow s_1(i, j)$

The parameter I_{\max} affects the number and location of keypoints to be selected; unless otherwise stated, we set $I_{\max} = 30$.

2.4 Key point detection

Pulling all the ingredients defined in the previous sections, our novel key point detection algorithm may be summarized as follows:

Initialize window size to $s \times s$; unless otherwise stated, $s = 3$

Normalize the depth values of the given range image

Estimate the weight of each valid pixel

Perform adaptive smoothing

Transform the smoothed depths $s_1(i, j)$ to $\tilde{b}(i, j)$ using a logarithmic operation:

$\tilde{b}(i, j) = \log((\tilde{z}(i, j) + 1)/(s_1(i, j) + 1))$.

Normalize the logarithmically transformed depths:

$\bar{b}(i, j) = (\tilde{b}(i, j) - \tilde{b}_{\min})/(\tilde{b}_{\max} - \tilde{b}_{\min})$

where \tilde{b}_{\min} and \tilde{b}_{\max} are the minimum and maximum of $\tilde{b}(i, j)$ over the whole image.

Table 1: Precision Pre and recall Rec rates of the detected keypoints, the average e_μ and standard deviation e_σ of registration errors in millimetres based on RCs, expected and estimated rotation angles θ and $\hat{\theta}$ in degrees, and registration time t in seconds for different algorithms and different freeform shapes.

Shapes	Algo.	Pre (%)	Rec (%)	e_μ (mm)	e_σ (mm)	θ (°)	$\hat{\theta}$ (°)	t (s)
valve20-10	SoftICP	76.70	84.88	0.40	0.22	10	10.10	97
	FICP	76.95	85.16	0.40	0.22		10.12	77
valve10-0	SoftICP	86.16	86.75	0.38	0.21	10	10.11	66
	FICP	86.56	87.16	0.39	0.21		10.11	45
dinosaur72-36	SoftICP	60.52	51.51	0.62	0.87	36	35.15	41
	FICP	61.00	51.92	0.63	0.85		35.02	34
dinosaur36-0	SoftICP	63.35	61.02	0.56	0.54	36	35.72	66
	FICP	63.58	61.24	0.56	0.54		35.66	50
bottle0-36	SoftICP	59.08	62.14	0.67	0.33	36	32.68	31
	FICP	70.91	74.59	0.67	0.58		3.88	30
bunny80-60	SoftICP	76.76	71.87	0.22	0.11	20	19.94	29
	FICP	76.49	71.61	0.22	0.11		19.87	20
cow49-45	SoftICP	47.81	21.36	0.71	0.70	40	41.30	26
	FICP	46.95	20.98	0.71	0.71		41.59	28
tubby120-80	SoftICP	50.60	44.42	0.25	0.16	40	39.09	23
	FICP	50.57	44.39	0.26	0.17		39.02	30

Extract key points as those $(x(i, j) \ y(i, j) \ z(i, j))^T$ for which $\bar{b}(i, j) = 1$.

We call the above algorithm the *retinex key point* (RKP) algorithm. It has a computational complexity of $O(n)$ for depth transformation, normalization, and weight estimation, $O(I_{\max}n) = O(n)$ for adaptive smoothing, and $O(n)$ for key point extraction. Overall, it thus has linear computational complexity in the number n of valid points in the shape. This algorithm has the following property:

Property 1 *Detected key points have locally maximal transformed and normalized depths.*

Proof: The definition of a key point as one satisfying $\bar{b}(i, j) = 1$ means that $\tilde{b}(i, j) = \tilde{b}_{\max}$. The non-decreasing constraint on depth during adaptive smoothing implies that $\tilde{z}(i, j) \leq s_1(i, j)$ and thus $\tilde{b}(i, j) \leq 0$. Consequently, $\tilde{b}_{\max} = 0$ and also $\tilde{b}(i, j) = 0$, so $\tilde{z}(i, j) = s_1(i, j)$. From the initial values of $s_0(i, j) = \tilde{z}(i, j)$ and the non-decreasing constraint on smoothed depth, $s_1(i, j) = \max(\hat{s}(i, j), s_0(i, j))$, we have $\tilde{z}(i, j) = \max(\hat{s}(i, j), \tilde{z}(i, j))$. Thus, pixel (i, j) has a locally maximal transformed and normalized depth, a property which is unaffected by adaptive smoothing. The adaptively smoothed depth determines whether a point has a locally maximal depth. ■

3 Experimental results

In this section, we use real data to demonstrate the utility of our algorithm for key point detection and its application for efficient registration of overlapping 3D freeform shapes. The detected key points are

directly used for registration in two state-of-the-art iterative closest point (ICP) variants: SoftICP [9] and Fractional RMSD (FICP) [18]. The RKP algorithm usually selects around 10% points in the shape, so to enable a fair comparison in registration, we have also ensured that the octave, MSFE and NSS methods all select 10% of the points from the shape.

For each pair of overlapping shapes, we refer to the first as the *data shape*, and the second as the *reference shape*. All real data in Figure 1 were downloaded from [15]. They were captured using a Minolta Vivid 700 range camera with a resolution of 200×200 . The performance of the algorithm is measured using the following parameters: the precision *Pre* and recall *Rec* rates as a percentage of the detected keypoints, the average and standard deviation of registration errors of reciprocal correspondences (RCs) [9, 27] between the whole data and reference shapes, the rotation angle θ in degrees of the estimated transformation, and the time in seconds used for key point detection and registration (KDR). The data files used encode the rotation angles θ of the transformations in the filenames, giving ground truth for evaluation algorithm performance.

In pattern recognition and information retrieval [29], precision (also called positive predictive value) is the fraction of retrieved instances that are relevant, while recall (also known as sensitivity) is the fraction of relevant instances that are retrieved. Both precision and recall are therefore based on an understanding and measure of relevance. When detecting keypoints in the context of registration of overlapping data and reference shapes, the goal of the keypoint detector is to repeatably detect any keypoints in the data shape that have also been detected in the reference shape, taking into account that only some will be present due to change of view and occlusion. The overlap of reference and data shapes is defined in terms of their reciprocal correspondences (RCs). Because RCs represent correct correspondences and thus the same points on the object of interest, they characterize the repeatability of the detected keypoints in the data and reference shapes. Thus, here, precision (*Pre*) and recall (*Rec*) rates of the detected keypoints are defined as: $N/n_1 \times 100\%$ and $N/n_2 \times 100\%$, where N , n_1 and n_2 are the number of RCs and the numbers of detected keypoints in the data and reference shapes respectively. Our definition of recall rate agrees with that in [1].

Such definitions of precision and recall can be understood from two points of view:

1. Keypoints in the data and reference shapes can be detected and compared individually with some ground truth, showing the extent to which the detected keypoints are relevant to the ground truth. In this case, keypoints in the data and reference shapes are equally treated and play the same role in representing the ground truth. From this viewpoint, *Pre* and *Rec* rates essentially measure the precision of detected keypoints in the data and reference shapes respectively.
2. The keypoints to be detected in the data shape should agree with those detected in the reference shape. The precision rate shows the extent to which the detected keypoints in the data shape are repeatable and relevant to those in the reference shape. The recall rate shows the extent to which the keypoints in the reference shape are successfully detected and retrieved by those in the data shape. In this case, the distinction between precision and recall rates lies in that they distinguish different roles played by the keypoints in the data and reference shapes: the former should reproduce and maximize overlap with the latter; the latter is regarded as ground truth.

It can be seen that the second interpretation is more suitable for keypoint detection in a registration context, since no independent ground truth is available for performance measurement. The data and reference shapes are closely tied to the *differences* in representation of the geometry of the object of interest from different viewpoints, and the size of their overlap is defined through registration of the keypoints in the data shape with those in the reference shape. Thus, we have adopted this approach

throughout this paper.

An experimental study was carried out to evaluate six aspects of the proposed RKP algorithm, as detailed below: key point detection, reference registration, point selection from the data shape and both shapes, window size and the optimal number of iterations for adaptive smoothing. To facilitate visualization, the estimated transformation was applied to the whole data shape, rather than the key-points only. Experimental results are presented in Figures 3–10, and Tables 1–7. In Figures 4–8 and Figure 10, yellow represents the transformed data shape, while green represents the reference shape. All experiments were carried out on a Pentium IV, 2.8GHz computer with 504MB RAM with unoptimized code written in Microsoft Visual C++ 6.0.

3.1 Key point detection

In this section, we use real data to demonstrate the detected key points. To this end, the valve20, valve10, valve0, dinosaur72, dinosaur36, dinosaur0, cow49, and cow45 shapes in Figure 1 were selected; results are presented in Figure 2. The key points detected are represented by red plus signs.

The points detected by the proposed RKP algorithm are distributed relatively evenly over the whole shapes, yet characterize the main features and details of the valve, dinosaur and cow shapes. In particular, more key points were detected on the ports of the valve, the head, belly, toes and tail of the dinosaur, and the ears, eyes and mouth of the cow. Even though the number of key points varies from one shape to another, typically around 10% points in the original shape are detected as key points. This means that around 10% points can be used for a faithful representation of the geometry and details of each 3D freeform shape of interest.

In contrast, both the octave and the MSFE methods selected points mainly in areas with depth discontinuities, since points in these areas usually vary significantly in normal vector and eigenvalues of the local covariance matrix. The NSS method sampled points from both depth discontinuous and planar areas, although more points were selected from depth discontinuous areas with larger changes in normal vector. It is usually difficult to distinguish foreground objects of interest from the cluttered background without prior knowledge, but they were treated equally, and thus key points were selected from both. The RKP and NSS methods provide visually more accurate representations of the overall geometry and details for the original valve, dinosaur and cow shapes.

While key point detection typically took under 2s for the RKP, MSFE and NSS methods, the octave method took up to 30s, since it was designed to operate over point clouds, rather than structured range images. This observation is consistent with their computational complexity: RKP, MSFE, and NSS methods have linear computational complexity in terms of the number of points in the shape, while the octave method has quadratic computational complexity as it depends on computation of interpoint distances to find points within a threshold distance of each point of interest.

3.2 Reference registration

We next used the complete sets of points in the original shapes for registration to provide a performance baseline, allowing evaluation of the extent to which the selecting key points accelerate registration and affect its accuracy. To this end, the valve20-10, valve10-0, dinosaur72-36, dinosaur36-0, bottle0-36, bunny80-60, cow49-45, and tubby120-80 shape pairs in Figure 1 were selected. Experimental results are presented in Figure 3 and Table 1. The valve, dinosaur, bunny, and tubby shapes were accurately registered by both the SoftICP and FICP algorithms. The estimated rotation angles for the underlying transformations are close to the ground truths. The transformed data shapes fit onto the reference shapes

perfectly. The bottle0-36 and cow59-45 shapes are challenging to register since the former includes a simple cylindrical shape leading to rotational ambiguity in the underlying transformation, while the latter has a cluttered background, complicating evaluation of the quality of tentative correspondences established. Nevertheless, such shapes are useful to reveal the true performance of different key point detection techniques. The SoftICP algorithm is more accurate than the FICP algorithm: the latter registered the dominant bottle body, but not the bottle handle—the probabilistic SoftICP algorithm is more powerful than the threshold based FICP algorithm.

3.3 Point selection from the data shape

We next investigated different point selection techniques applied to the data shapes \mathbf{P} only, as was done in [7]. In this case, registration was performed between the sampled points in the data shape and *all* points in the reference shape; in this case only the precision rate of the detected key points is defined, but not the recall rate. Experimental results are presented in Figures 4 and 5, and Tables 2 and 3. It can be seen that the proposed RKP algorithm always detected key points with superior precision to the octave, MSFE, and NSS methods by as much as 43%, 36%, and 34% respectively. This higher precision for the RKP key points carries through to more accurate registration results. The SoftICP algorithm accurately registered points selected from 7 out of the 8 data shapes: the key points selected by our RKP approach provide a sound basis for registration. In contrast, using key points generated by the other methods, the SoftICP algorithm inaccurately registered the valve20, dinosaur36, and bunny80 shapes, and failed to register the points selected by either the octave or MSFE method for the cow49 shape and the points selected by the NSS method for the tubby120 shape. For bottle0-36, all methods produced inaccurate results as measured by either average registration error or rotation angle of the underlying transformation. The proposed RKP algorithm produced a larger average registration error and established 2% more RCs than the other methods. This shows that the simple geometry of the bottle leads to an ill-posed registration problem, and any registration algorithm is likely to converge to a local minimum, giving inaccurate results. Over all 8 pairs of overlapping shapes, the increase in average registration error compared to using full data without point sampling was 6%, 31%, 29%, and 16% for the RKP, octave, MSFE, and NSS methods respectively.

The octave, MSFE, and NSS methods have reduced precision compared to the proposed RKP algorithm for detection of key points by up to 26%, 45%, and 30% respectively: the key points detected by the former are less repeatably placed and thus less useful as a basis for registration. While the FICP algorithm failed to register the points selected by all four methods for the bottle0 shape, it successfully registered the points selected by our proposed RKP algorithm for all 7 other shapes. In contrast, it failed to register the points selected by the octave, MSFE, and NSS methods for the dinosaur72, dinosaur36, and cow49 shapes, and points selected by the MSFE method for the data tubby120 shape. These registration failures can be seen in the legs of the dinosaur and the head of the cow in shapes dinosaur72, dinosaur36, and cow49 relative to the references dinosaur36, dinosaur0, and cow45 respectively. Over all 8 pairs of overlapping shapes, the average registration error was increased compared to using full data without point sampling by 6%, 60%, 116%, and 71% by the RKP, octave, MSFE, and NSS methods respectively.

The above analysis shows that sampling around 10% points using the RKP algorithm has little effect on the registration accuracy for overlapping 3D freeform shapes, especially when using the FICP algorithm. The octave, MSFE, and NSS methods do not perform as well. We believe this is because they estimate quantities based on second-order derivatives of the discrete range data, such as normal vectors and eigenvalues of the local covariance matrix, and these are sensitive to imaging noise, occlusion, and

Table 2: The precision Pre rate of the detected keypoints, the average e_μ and standard deviation e_σ of registration errors in millimetres based on RCs, expected and estimated rotation angles θ and $\hat{\theta}$ in degrees, and KDR time t in seconds for key points selected from the data shape using different algorithms, registered using the SoftICP algorithm.

Image	Algo.	Pre (%)	e_μ (mm)	e_σ (mm)	θ ($^\circ$)	$\hat{\theta}$ ($^\circ$)	t (s)
valve20-10	RKP	83.85	0.42	0.23	10	10.08	9
	octave	75.90	0.43	0.22		9.94	11
	MSFE	64.10	0.50	0.24		9.71	13
	NSS	63.32	0.46	0.23		9.87	17
valve10-0	RKP	94.06	0.39	0.20	10	10.14	5
	octave	89.44	0.40	0.21		10.09	16
	MSFE	86.46	0.39	0.20		10.04	5
	NSS	81.30	0.40	0.20		10.07	4
dinosaur72-36	RKP	74.08	0.64	0.85	36	35.32	4
	octave	64.93	0.76	0.84		34.45	4
	MSFE	55.63	0.74	0.88		34.08	6
	NSS	50.14	0.64	0.85		34.86	7
dinosaur36-0	RKP	78.30	0.59	0.54	36	35.45	8
	octave	63.87	0.60	0.55		35.58	7
	MSFE	55.38	0.72	0.58		35.22	19
	NSS	51.55	0.68	0.56		34.83	9
bottle0-36	RKP	80.08	0.78	0.47	36	29.84	3
	octave	71.75	0.68	0.34		32.70	4
	MSFE	63.51	0.67	0.33		33.32	5
	NSS	78.07	0.68	0.36		32.63	3
bunny80-60	RKP	90.88	0.23	0.11	20	20.07	4
	octave	71.36	0.27	0.12		19.22	4
	MSFE	78.02	0.24	0.11		19.51	4
	NSS	65.32	0.28	0.14		18.76	4
cow49-45	RKP	70.25	0.74	0.70	40	40.92	3
	octave	24.76	1.57	2.61		91.39	5
	MSFE	22.22	1.31	2.85		62.61	4
	NSS	42.54	0.96	0.98		41.77	4
tubby120-80	RKP	79.07	0.26	0.18	40	38.86	4
	octave	61.78	0.28	0.16		38.64	15
	MSFE	52.62	0.38	0.23		38.02	4
	NSS	52.62	0.34	0.22		29.04	3

Table 3: The precision Pre rate of the detected keypoints, the average e_μ and standard deviation e_σ of registration errors in millimetres based on RCs, expected and estimated rotation angles θ and $\hat{\theta}$ in degrees, and KDR time t in seconds for key points selected from the data shape using different algorithms, registered using the FICP algorithm.

Image	Algo.	Pre (%)	e_μ (mm)	e_σ (mm)	θ ($^\circ$)	$\hat{\theta}$ ($^\circ$)	t (s)
valve20-10	RKP	83.63	0.42	0.23	10	10.10	5
	octave	76.11	0.42	0.22		10.07	5
	MSFE	63.32	0.45	0.23		9.89	7
	NSS	63.11	0.44	0.23		9.93	17
valve10-0	RKP	93.61	0.39	0.21	10	10.16	4
	octave	89.51	0.39	0.21		10.07	4
	MSFE	85.68	0.39	0.20		10.07	3
	NSS	83.72	0.45	0.20		9.83	3
dinosaur72-36	RKP	75.31	0.71	0.84	36	34.14	3
	octave	46.33	1.19	1.01		22.11	2
	MSFE	34.51	1.17	0.78		16.05	4
	NSS	35.07	1.23	1.02		18.59	4
dinosaur36-0	RKP	78.30	0.60	0.55	36	35.35	5
	octave	41.50	1.44	1.27		11.42	7
	MSFE	28.47	1.30	1.20		9.38	5
	NSS	39.47	1.46	1.29		14.74	5
bottle0-36	RKP	80.49	0.70	0.47	36	9.88	4
	octave	73.51	0.75	0.69		21.28	3
	MSFE	26.14	2.78	3.30		36.01	5
	NSS	77.89	0.80	0.66		25.85	4
bunny80-60	RKP	90.37	0.23	0.11	20	20.11	4
	octave	70.59	0.28	0.13		19.43	3
	MSFE	77.71	0.25	0.11		19.17	3
	NSS	66.25	0.32	0.17		18.30	3
cow49-45	RKP	68.46	0.74	0.70	40	40.73	4
	octave	23.81	1.30	1.71		76.52	4
	MSFE	12.38	1.29	1.63		30.67	3
	NSS	26.34	1.44	1.41		31.64	3
tubby120-80	RKP	78.49	0.27	0.18	40	38.83	3
	octave	58.38	0.38	0.17		37.94	3
	MSFE	28.27	0.68	0.59		41.15	3
	NSS	55.76	0.43	0.28		25.50	3



Figure 4: Registration results using the SoftICP algorithm for various shapes with key points selected from the data shape using different algorithms. Rows, top to bottom: RKP, octave, MSFE, and NSS. Columns, left to right: valve20-10, valve10-0, dinosaur72-36, dinosaur36-0, bottle0-36, bunny80-60, cow49-45, and tubby120-80.

appearance and disappearance of points. In contrast, the RKP algorithm employs adaptive smoothing and first order derivatives, and thus is more robust. The adaptive smoothing operation provides a

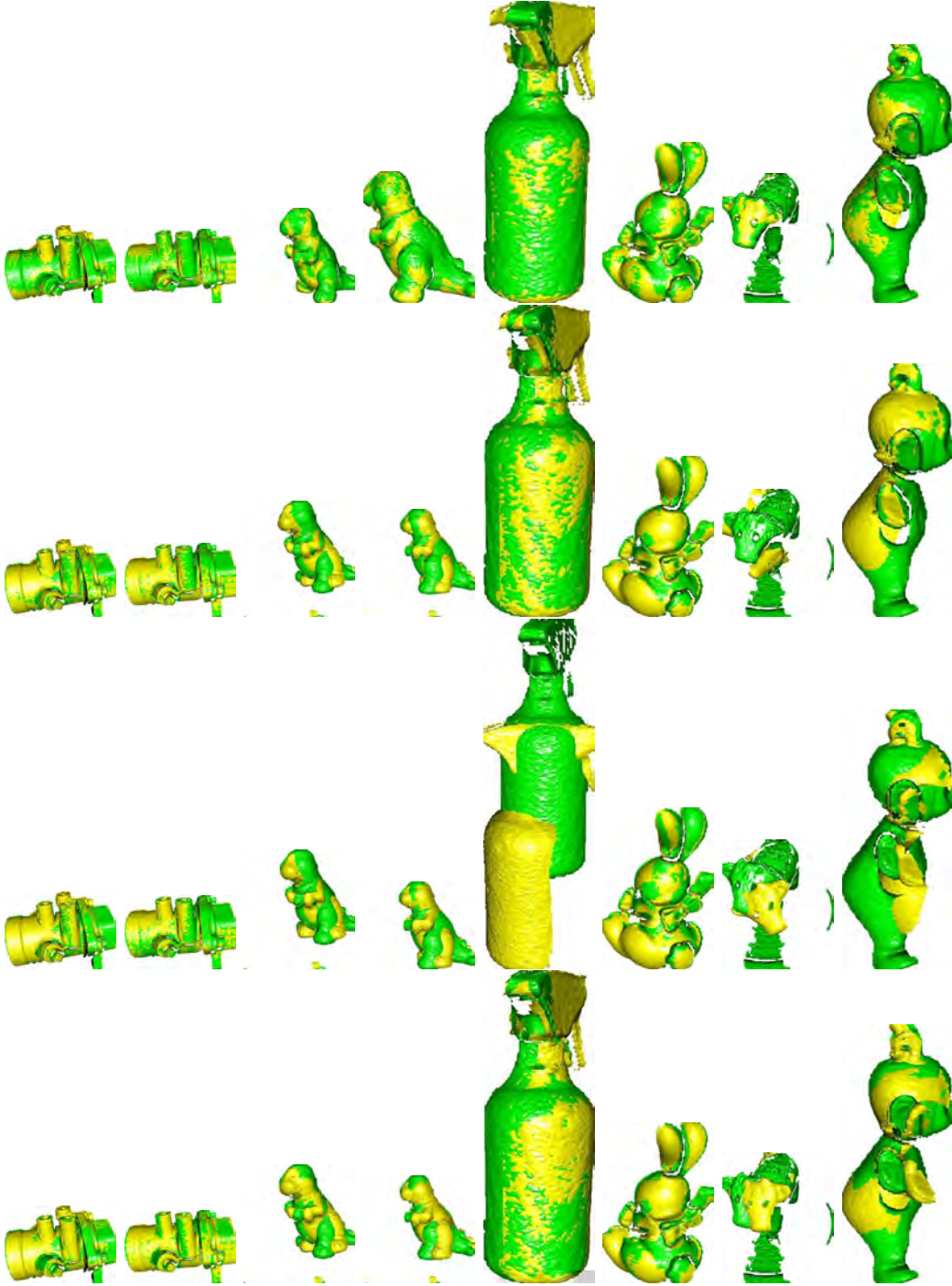


Figure 5: Registration results using the FICP algorithm for various shapes with key points selected from the data shape using different algorithms. Rows, top to bottom: RKP, octave, MSFE, and NSS. Columns, left to right: valve20-10, valve10-0, dinosaur72-36, dinosaur36-0, bottle0-36, bunny80-60, cow49-45, and tubby120-80.

reliable reference for the RKP algorithm to select key points as local maxima.

Comparing Tables 2 and 3 with Table 1, it can be seen that the SoftICP algorithm is more accurate than

the FICP algorithm. Both algorithms produce worse results when using point sampling, as might be expected. The FICP algorithm is more sensitive to use of point sampling than the SoftICP algorithm. This is because the entropy maximisation principle, the two-way constraint, and the deterministic annealing scheme in the SoftICP algorithm provide a powerful probabilistic framework for weighting the tentative correspondences established, while the FICP algorithm has difficulty in defining the quality of tentative correspondences and thus in rejecting outliers. The FICP algorithm is slightly more efficient than the SoftICP algorithm, but it usually converges prematurely, producing inaccurate registration results. While all registration results are slightly worse when sampling, the time needed for registration is reduced by as much as 78% using point sampling and registration by the RKP algorithm. Detecting key points speeds up registration without significant loss of accuracy.

3.4 Point selection from both shapes

In the previous experiments, only the data shapes were sampled with key points. Here, we sample both data and reference shapes and again perform registration using SoftICP and FICP. Such sampled points are more challenging to register, since they are less likely to correspond to exactly the same points on the shapes, and failure to robustly choose identical key points will be apparent. For accurate registration, the sampled points should faithfully represent the geometry and details of the shapes of interest from any viewpoint, and should reliably represent the original shapes from different viewpoints. Our experimental results are presented in Figures 6 and 7 and Tables 4 and 5. It can be seen that the precision rate of the RKP algorithm is always higher than for other algorithms, and its recall rate is also higher except for the valve10-0 and tubby120-80 shape pairs, where they are similar to those produced by the MSFE method. Over all 8 pairs of overlapping shapes, the proposed RKP algorithm had better precision and recall rates compared to the octave, MSFE, and NSS methods on average by as much as 68%, 55%, 19%, 13%, 36%, and 26% respectively, showing that the key points detected by the RKP method are more representative of the underlying geometry.

The higher precision and recall rates of these key points are confirmed by the registration results. The SoftICP algorithm accurately registered the points sampled by the proposed RKP method with the average error increasing by as little as 7%, even though both the data and reference shapes were sampled, reducing time by up to 90%. In contrast, point sampling by the octave, MSFE, and NSS methods increased errors by up to 46%, 51%, and 52% respectively. Even when sampling both shapes by the RKP algorithm, good registration can be achieved, and all overlapping shapes were brought into accurate alignment with each other. In contrast, the sampled points from the octave, MSFE, and NSS methods proved less useful for registration; the transformed valve20, dinosaur72 and tubby120 shapes are displaced in 3D space with respect to the valve10, dinosaur36 and tubby80 shapes respectively.

When performing registration with the FICP algorithm, again the RKP algorithm produced the highest precision and recall rates of any key point selection method for every case, except for the result for the valve10-0 pair produced by the MSFE method. This shows that the MSFE method can produce good results, but is not as reliable as the proposed RKP method. The RKP algorithm is better than the octave, MSFE, and NSS methods for the detection of key points in the sense of precision and recall rates by as much as 88%, 76%, 44%, 39%, 51%, and 43% respectively. These remarkable results show that out of these methods, the RKP method detected key points most closely representing the geometry and details of the original shapes, providing the best registration results. Even though point sampling by the octave, MSFE, and NSS methods increased the average registration error significantly, by as much as 85%, 89%, and 97% respectively, for the RKP method it increased by just 13%. While the FICP algorithm failed to register the points sampled by the octave, MSFE, and NSS methods, causing

Table 4: Precision Pre and recall Rec rates for detected keypoints, the average e_μ and standard deviation e_σ of registration errors in millimetres based on RCs, expected and estimated rotation angles θ and $\hat{\theta}$ in degrees, and KDR time t in seconds, for key points selected from both shapes using various algorithms and registered using the SoftICP algorithm.

Image	Algo.	Pre (%)	Rec (%)	e_μ (mm)	e_σ (mm)	θ ($^\circ$)	$\hat{\theta}$ ($^\circ$)	t (s)
valve20-10	RKP	48.45	59.13	0.41	0.22	10	10.08	5
	octave	35.62	39.43	0.44	0.23		10.08	6
	MSFE	45.86	50.78	0.47	0.23		9.86	6
	NSS	40.71	45.07	0.47	0.23		9.91	6
valve10-0	RKP	59.04	55.08	0.40	0.21	10	10.01	4
	octave	42.80	43.07	0.41	0.20		10.09	5
	MSFE	57.51	57.87	0.40	0.20		10.04	7
	NSS	53.52	53.85	0.39	0.20		10.07	7
dinosaur72-36	RKP	39.88	35.71	0.65	0.86	36	35.04	6
	octave	25.63	21.77	0.67	0.86		34.95	6
	MSFE	34.65	29.42	0.73	0.84		34.75	4
	NSS	30.56	25.96	1.11	1.01		31.21	4
dinosaur36-0	RKP	42.58	39.14	0.59	0.54	36	35.45	8
	octave	25.71	24.77	0.61	0.55		35.33	4
	MSFE	29.07	27.99	0.69	0.57		36.00	4
	NSS	31.94	30.76	0.89	0.65		33.68	4
bottle0-36	RKP	42.91	43.27	0.75	0.32	36	34.80	3
	octave	26.31	27.62	0.82	0.72		20.94	3
	MSFE	27.54	28.91	1.85	1.42		32.43	4
	NSS	25.44	26.70	0.88	0.65		26.13	3
bunny80-60	RKP	53.21	50.16	0.23	0.11	20	19.91	4
	octave	25.54	23.91	0.31	0.13		19.53	3
	MSFE	51.08	47.83	0.26	0.11		19.52	3
	NSS	41.02	38.40	0.31	0.14		18.86	3
cow49-45	RKP	39.43	15.99	0.79	0.88	40	40.38	3
	octave	9.21	4.09	1.93	3.00		79.78	3
	MSFE	21.90	9.73	0.91	1.28		51.83	2
	NSS	7.94	3.53	1.29	2.41		104.81	3
tubby120-80	RKP	34.59	28.54	0.28	0.18	40	39.06	2
	octave	22.77	20.00	0.40	0.21		39.70	2
	MSFE	32.98	28.96	0.45	0.30		37.49	2
	NSS	31.94	28.05	0.47	0.34		29.43	4

Table 5: Precision Pre and recall Rec rates for the detected keypoints, the average e_μ and standard deviation e_σ of registration errors in millimetres based on RCs, expected and estimated rotation angles θ and $\hat{\theta}$ in degrees, and KDR time t in seconds for key points selected from both shapes using different algorithms, and registered using the FICP algorithm.

Image	Algo.	Pre (%)	Rec (%)	e_μ (mm)	e_σ (mm)	θ (°)	$\hat{\theta}$ (°)	t (s)
valve20-10	RKP	48.67	59.40	0.42	0.23	10	10.02	4
	octave	36.11	39.98	0.45	0.23		10.12	4
	MSFE	45.94	50.86	0.44	0.22		10.04	4
	NSS	38.43	43.66	0.55	0.28		10.03	5
valve10-0	RKP	58.95	54.99	0.40	0.20	10	9.97	3
	octave	42.95	43.22	0.43	0.20		10.12	4
	MSFE	57.35	57.71	0.39	0.21		10.07	4
	NSS	52.82	53.15	0.41	0.21		10.07	5
dinosaur72-36	RKP	39.26	35.16	0.66	0.87	36	35.20	4
	octave	18.87	16.03	1.19	0.91		22.24	2
	MSFE	16.90	14.35	1.19	1.01		22.70	3
	NSS	19.44	16.51	1.23	0.97		19.87	3
dinosaur36-0	RKP	42.44	39.01	0.60	0.55	36	35.35	6
	octave	15.55	14.97	1.51	1.31		8.72	4
	MSFE	11.12	10.71	1.48	1.13		8.53	4
	NSS	22.49	21.65	1.33	1.26		14.09	4
bottle0-36	RKP	44.15	44.51	0.87	0.58	36	19.72	3
	octave	24.21	25.41	1.09	0.61		23.36	3
	MSFE	24.21	25.41	2.11	1.62		27.42	2
	NSS	23.50	24.68	1.44	0.80		20.19	3
bunny80-60	RKP	53.21	50.16	0.27	0.13	20	18.82	2
	octave	25.70	24.06	0.31	0.14		19.34	2
	MSFE	52.01	48.69	0.25	0.12		18.76	3
	NSS	39.16	36.67	0.36	0.19		18.30	3
cow49-45	RKP	39.78	16.13	0.85	0.85	40	44.01	3
	octave	6.98	3.10	1.72	1.20		75.27	3
	MSFE	22.86	10.15	0.78	0.90		45.01	3
	NSS	11.43	5.08	1.60	1.41		34.86	3
tubby120-80	RKP	34.01	28.06	0.29	0.18	40	38.45	3
	octave	21.20	18.62	0.43	0.22		40.04	3
	MSFE	19.63	17.24	0.64	0.42		30.58	3
	NSS	30.37	26.67	0.67	0.37		29.67	4

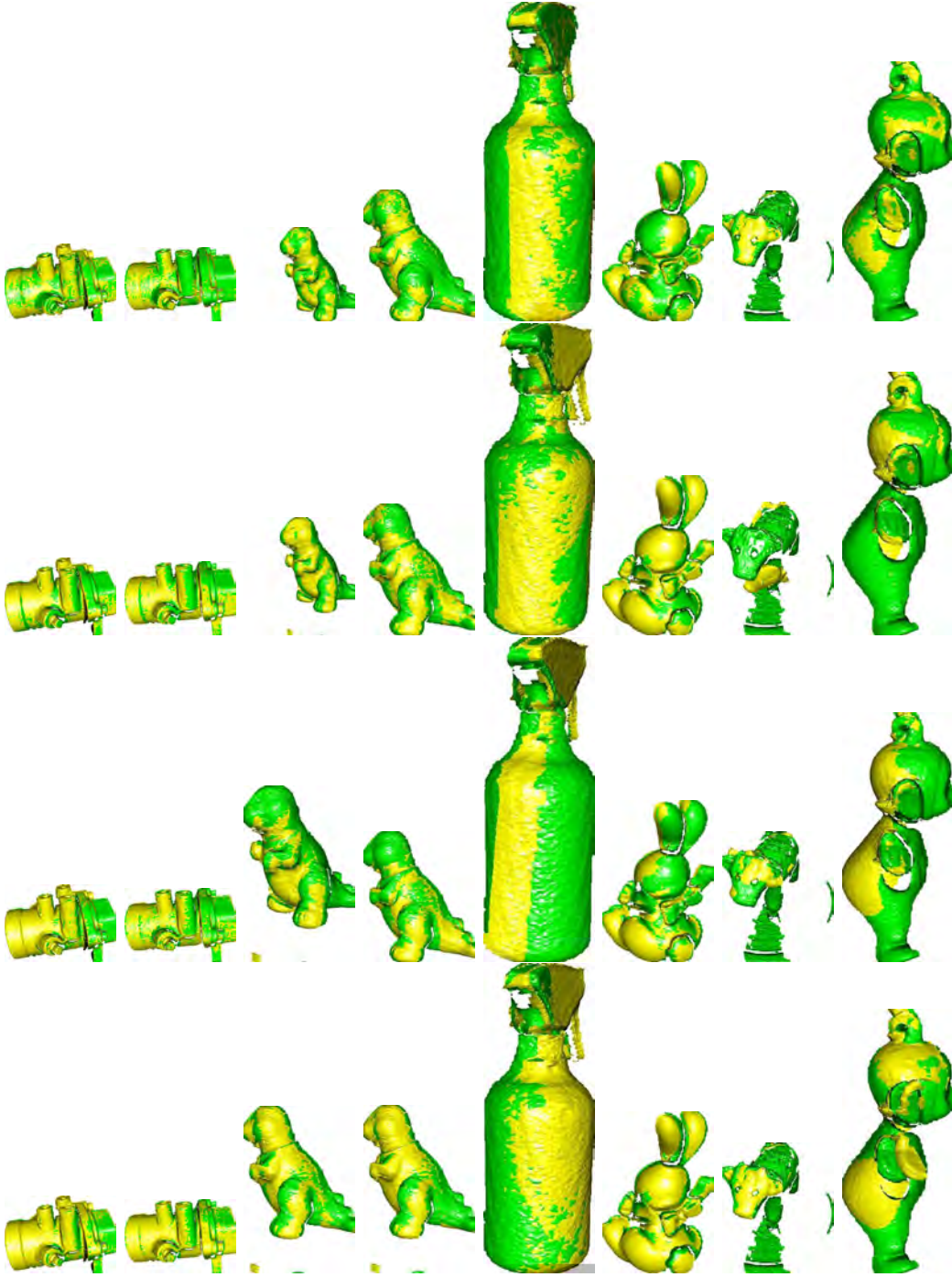


Figure 6: Registration results using the SoftICP algorithm for various shapes using key points selected from *both* shapes by different algorithms. Rows, top to bottom: RKP, octave, MSFE, and NSS. Columns, left to right: valve20-10, valve10-0, dinosaur72-36, dinosaur36-0, bottle0-36, bunny80-60, cow49-45, and tubby120-80.

the transformed dinosaur72, dinosaur36, cow49, and tubby120 shapes to mismatch the dinosaur36, dinosaur0, cow45, and tubby80 shapes respectively, it successfully registered the points sampled by the

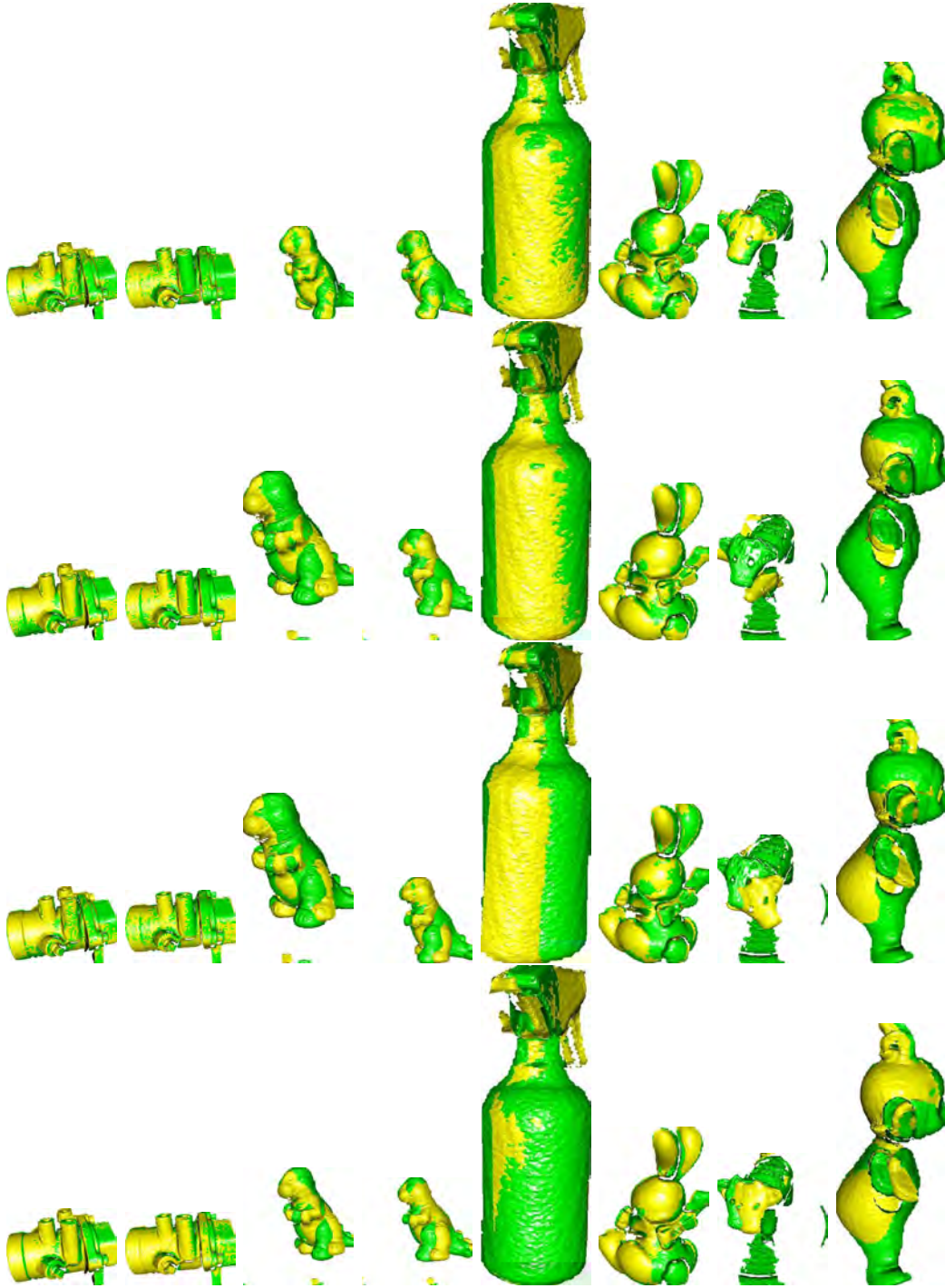


Figure 7: Registration results using the FICP algorithm for various shapes using key points selected from *both* shapes by different algorithms. Rows, top to bottom: RKP, octave, MSFE, and NSS. Columns, left to right: valve20-10, valve10-0, dinosaur72-36, dinosaur36-0, bottle0-36, bunny80-60, cow49-45, and tubby120-80.

RKP algorithm and brought all the overlapping shapes into accurate alignment, except for the bottle0-36

pair, which was more challenging to register due to the previously mentioned cylindrical ambiguity.

The SoftICP algorithm exhibited similar behavior for the registration of points sampled from both data and reference shapes by the RKP, octave, and MSFE methods to the case when the data shape only was sampled, but it produced worse results when registering points sampled by the NSS method. The repeatability of points sampled by the NSS method is worse due to the random nature of its sampling. The FICP algorithm is more sensitive to the choice of point sampling method.

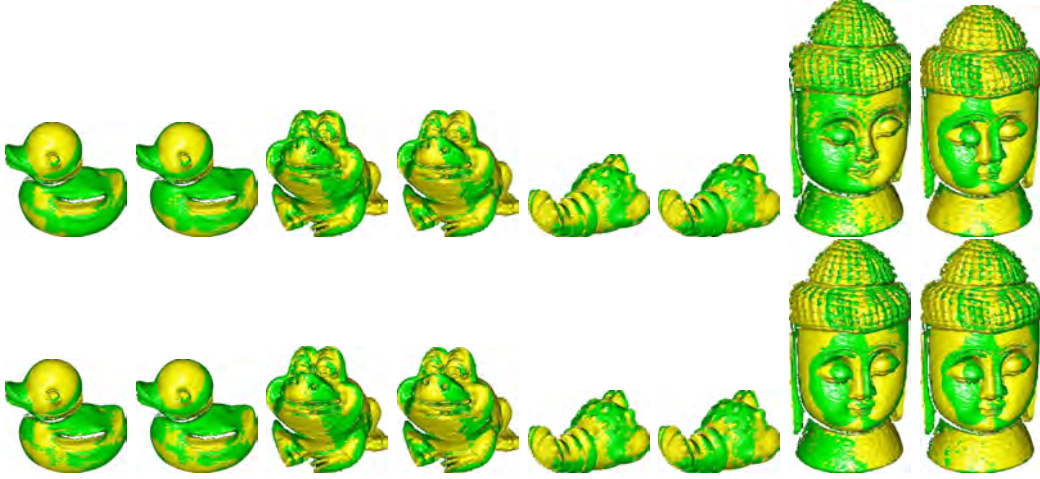


Figure 8: Registration results of different algorithms using the RKP algorithm to select keypoints from both shapes, using different window sizes. Rows, top to bottom: 3×3 , 7×7 windows. Models, left to right: duck0-20, frog0-20, lobster0-20, and buddha0-20. Odd columns: SoftICP; even columns: FICP.

3.5 Window size

Elsewhere, when using the RKP algorithm, we set the window size to 3×3 for neighbor detection and adaptive smoothing. In this section, we experimentally investigate whether this is a good option and consider window sizes from 3×3 to 7×7 , using the new duck0-20, frog0-20, lobster0-20, and buddha0-20 shape pairs illustrated in Figure 1. Both data and reference shapes were sampled and registered using both the SoftICP and FICP algorithms. The experimental results are presented in Figure 8 and Table 6. A larger window size usually decreases the precision and recall rates of the detected key points registered by the SoftICP algorithm on average by 7% and 10%, and by the FICP algorithm by 9% and 12% respectively. A larger window size results in less reliable key points, because it blurs local features and thus makes key point detection harder. This conclusion is confirmed by the registration results. A larger window size of 7×7 usually produces slightly worse results than 3×3 ; the rotation angle of the estimated transformation is close to the ground truth. Significantly worse results are obtained when registering the buddha0-20 pair using the SoftICP algorithm: the nose, eyes, and ears in the transformed data buddha0 and reference buddha20 shapes are clearly displaced in 3D space with the average error being increased by as much as 45%. Thus, a window size of 3×3 is suggested for key point detection.

Table 6: Precision Pre and recall Rec rates for detected keypoints, the average e_μ and standard deviation e_σ of registration errors in millimetres based on RCs, expected and estimated rotation angles θ and $\hat{\theta}$ in degrees, and KDR time t in seconds for key points selected from both shapes using the RKP algorithm and different window sizes, d registered by different algorithms.

Image	Size	Algo.	Pre (%)	Rec (%)	e_μ (mm)	e_σ (mm)	θ ($^\circ$)	$\hat{\theta}$ ($^\circ$)	t (s)
duck0-20	3×3	SoftICP	52.00	52.20	0.41	0.29	20	18.99	4
		FICP	51.11	51.30	0.41	0.26		18.69	5
	7×7	SoftICP	44.34	44.51	0.44	0.27		18.33	5
		FICP	45.08	45.24	0.47	0.30		16.39	5
frog0-20	3×3	SoftICP	40.71	49.15	0.31	0.16	20	19.40	4
		FICP	41.14	49.67	0.32	0.17		18.71	3
	7×7	SoftICP	39.87	45.82	0.32	0.16		19.14	4
		FICP	40.65	46.72	0.34	0.18		18.47	3
lobster0-20	3×3	SoftICP	50.80	54.40	0.38	0.23	20	19.17	4
		FICP	50.80	54.40	0.41	0.25		18.44	3
	7×7	SoftICP	47.34	47.69	0.38	0.24		19.22	4
		FICP	47.20	47.54	0.40	0.26		18.50	3
buddha0-20	3×3	SoftICP	56.17	54.10	0.60	0.24	20	19.88	6
		FICP	50.57	48.70	0.94	0.57		8.07	4
	7×7	SoftICP	53.68	48.88	0.87	0.50		13.33	16
		FICP	42.83	39.00	0.88	0.50		13.21	6

3.6 Maximum number of iterations for adaptive smoothing

In this section, we investigate a suitable value for the maximum number (I_{\max}) of iterations used for adaptive smoothing in our RKP algorithm. We considered three possibilities: 10, 30, and 50. The new free form shapes angel0, angle20, angle40, bird0, and bird20 illustrated in Figure 1 were selected for the experiments, with point sampling applied to both shapes which again were registered using both the SoftICP and FICP algorithms. The experimental results are presented in Figures 9 and 10 and Table 7.

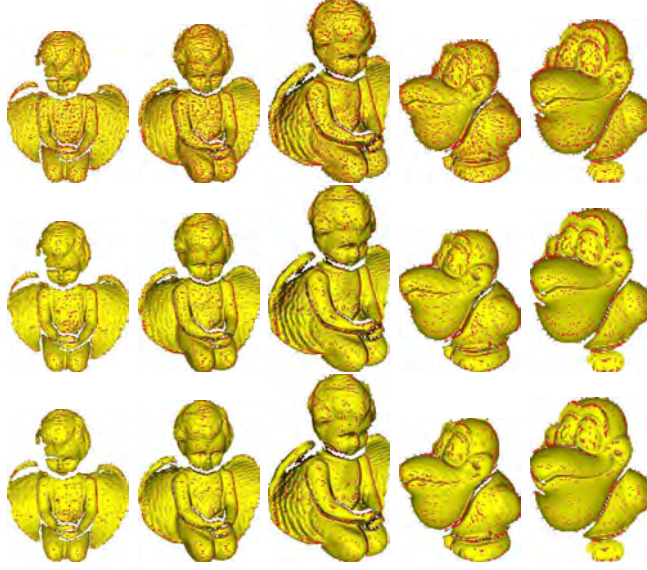


Figure 9: Keypoints detected by the RKP algorithm with the parameter I_{\max} taking different values. Rows, top to bottom: $I_{\max}=10$, 30, and 50. Models, left to right: angel0, angle20, angle40, bird0 and bird20.

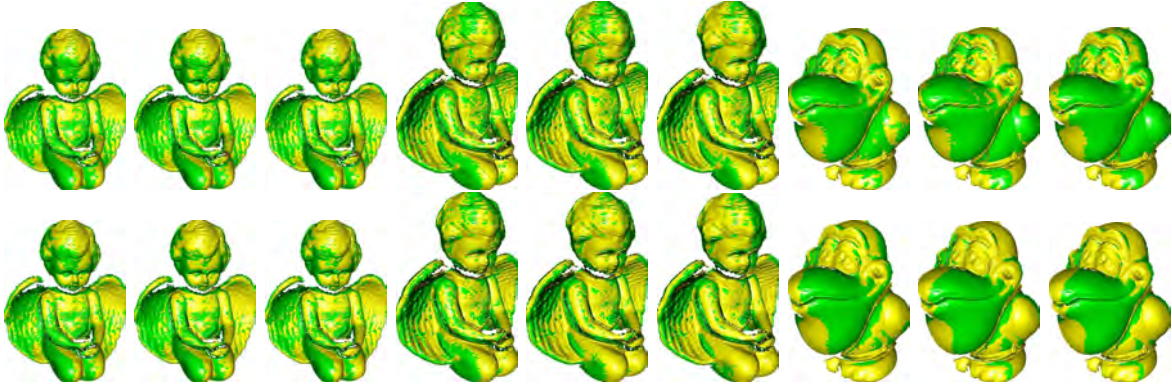


Figure 10: Registration results for different algorithms using keypoints selected from both shapes by the RKP algorithm with the parameter I_{\max} taking different values. Rows, top to bottom: SoftICP and FICP. Models, left three: angel0-20; Middle three: angel20-40; Right three: bird0-20. Columns 1, 4, and 7: $I_{\max}=10$; Columns 2, 5, and 8: $I_{\max}=30$; Columns 3, 6, and 9: $I_{\max}=50$.

Figure 9 and Table 7 show that the larger the maximum number of iterations used for adaptive smoothing, the fewer keypoints the RKP algorithm detects. This is because when the maximum number of iterations is small, 10, for example, the smoothing operation has mainly a local effect, leading points to

Table 7: Precision Pre and recall Rec rates for detected keypoints, the average e_μ and standard deviation e_σ of registration errors in millimetres based on RCs, expected and estimated rotation angles θ and $\hat{\theta}$ in degrees, and KDR time t in seconds for keypoints selected from both shapes by the proposed RKP algorithm with the parameter I_{\max} taking different values and registered by different algorithms. n_1 and n_2 are the numbers of the detected key points in the data and reference shapes respectively.

Image	I_{\max} (n_1, n_2)	Algo.	Pre (%)	Rec (%)	e_μ (mm)	e_σ (mm)	θ ($^\circ$)	$\hat{\theta}$ ($^\circ$)	t (s)
angel0-20	10 (2365, 2549)	SoftICP	54.29	50.37	0.49	0.26	20	20.52	7
		FICP	54.04	50.14	0.50	0.25		20.48	3
	30 (1301, 1429)	SoftICP	50.34	45.84	0.50	0.26		20.48	4
		FICP	50.65	46.12	0.52	0.26		20.34	2
	50 (989, 1107)	SoftICP	49.95	44.62	0.50	0.26		20.54	4
		FICP	50.05	44.71	0.51	0.27		20.42	2
angel20-40	10 (2549, 2166)	SoftICP	46.72	54.99	0.54	0.34	20	20.35	6
		FICP	46.92	55.22	0.55	0.34		20.22	4
	30 (1429, 1223)	SoftICP	45.55	53.14	0.55	0.34		20.35	4
		FICP	45.91	53.55	0.57	0.34		20.21	2
	50 (1107, 940)	SoftICP	43.00	50.64	0.55	0.34		20.34	3
		FICP	43.54	51.28	0.59	0.34		20.10	3
bird0-20	10 (1538, 1183)	SoftICP	43.17	56.13	0.29	0.12	20	19.56	4
		FICP	42.52	55.28	0.34	0.13		19.04	4
	30 (883, 656)	SoftICP	39.75	53.51	0.30	0.13		19.67	3
		FICP	40.09	53.96	0.39	0.15		18.15	3
	50 (667, 507)	SoftICP	38.23	50.29	0.32	0.13		19.79	3
		FICP	39.73	52.27	0.45	0.19		17.68	3

be compared in a small area, and thus more points to be depth maxima and hence selected as keypoints. In contrast, when the number is large, 50, for example, the smoothing operation propagates local information from one region to another, enabling comparison of points over a larger area, leading to fewer points being selected as keypoints having maximal depth. Figure 10 and Table 7 show that while a large number of points takes more time to process and register, they tend to produce more accurate registration results, with increased precision and recall rates for the detected keypoints: they describe the geometry and details of the underlying shapes more faithfully. In contrast, a smaller number of points leads to more computationally efficient registration, but usually produces larger registration errors, as fewer points less well characterize the geometry. This observation is demonstrated by the fact that the FICP algorithm superimposes the transformed bird0 shape over the bird20 shape with less inter-penetration through each other, and increases the average error when $I_{\max} = 10$ by as much as 32%; the number of the detected keypoints drops rapidly by 58% and 59% in the angel0 and angel20 shapes as the maximum number of iterations for adaptive smoothing increases from 10 to 50, while the drop in the precision and recall rates is less serious, just 8% and 11% for the SoftICP algorithm with a gain of 42% in overall time. These results show that the proposed RKP method: (i) can flexibly detect varying numbers of keypoints as required by controlling the value of I_{\max} , and (ii) can stably detect repeatable key points. $I_{\max} = 30$ is recommended as a good compromise between registration accuracy and computational efficiency.

4 Conclusions

While the latest laser scanners enable fast and affordable capture of depth maps and shapes of interest, multiple views must be registered to form complete models. Registration is a costly process, and in this paper, we show to significantly reduce this cost using a sampling method based on a novel way of selecting key points. Our contributions can be summarized as follows.

- Inspired by retinex theory [6, 16], we have proposed a novel method of key point detection using adaptive smoothing. This operation suppresses the impact of points in depth discontinuous regions, and all points unaffected by smoothing are selected as key points. Key points have locally maximal transformed and normalized depth. The method is easy to implement. This appears to be the first time that retinex theory has been adapted from 2D image enhancement for application in 3D key point analysis and detection.
- The detected key points have been shown to be useful for efficient registration of overlapping 3D freeform shapes. No matter whether key points are selected from just one or both shapes, accurate results are obtained, with registration being up to 20 times faster than when using unsampled shapes. This is a significant improvement in computational efficiency without sacrificing registration accuracy. The reason why our proposed RKP algorithm works well in this context is that the smoothed depths provide a reliable reference for the judgment of whether a point is a key point. Existing methods detect salient points in an ad hoc way, while the proposed RKP method guarantees that the detected points are locally highest after normalization and adaptive smoothing.
- A comparative study has been performed between our proposed key point detection method and three other state-of-the-art methods, using real depth images. It shows that it is feasible to apply ICP variants to directly register key points as long as they are informative, expressive, and repeatable enough. The precision and recall rates of the key points detected by the proposed

RKP method are usually higher than those for the selected competitors, in one extreme case by as much as 88% and 76%. RKP sampling of both shapes increases the average registration error by as little as 7%, while for other selected state-of-the-art methods it increases by as much as 52%.

Our proposed RKP method is a powerful approach to key point detection. The detected key points are useful in the context of registration, and with direct application of ICP variants, they can produce accurate registration results with significantly improved computational efficiency. Future research will investigate the similarity between the shapes defined by these key points and the original complete point sets, how the detected key points can be applied for the generation of levels of detail for efficient data transmission, rendering and visualization, and how the detected points can be incorporated into feature extraction and matching methods [7, 23] for applications such as registration.

Acknowledgments

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References

- [1] H. Aanæs, A.L. Dahl, K.S. Pedersen. Interesting interest points. *IJCV* 97(2012) 18-35.
- [2] U. Castellani, M. Cristani, S. Fantoni and V. Murino. Sparse points matching by combining 3D mesh saliency with statistical descriptors. *Computer Graphics Forum* 27(2008) 643-652.
- [3] L. Chen, L. Zhang, H. Zhang, M. Abdel-Mottaleb. 3D Shape constraint for facial feature localization using probabilistic-like output. *Proc. the Sixth IEEE Int. Conf. Automatic Face and Gesture Recognition*, pp. 302-307, 2004.
- [4] C. Creusot, N. Pears, J. Austin. Automatic keypoint detection on 3D faces using a dictionary of local shapes. *Proc. 3DIMPVT*, pp. 204-211, 2011.
- [5] S. Gold, A. Rangarajan, C.-P. Lu, S. Pappu, E. Mjølhus. New algorithms for 2-D and 3-D point matching: pose estimation and correspondence. *Pattern Recognition* 31(1998) 1019-1031.
- [6] D.J. Jobson, Z. Rahman, G.A. Woodwell. A multiscale Retinex for bridging the gap between color images and the human observation of scenes. *IEEE Trans. Image Processing* 6(1997) 965-976.
- [7] A. Johnson and M. Hebert. Using spin images for efficient object recognition in cluttered 3D scenes. *IEEE Transactions on Pattern Analysis and Machine Intelligence (PAMI)* 21(1999) 433-449.
- [8] R. Kimmel, M. Elad, D. Shaked, and R. Keshet A variational framework for Retinex. *International Journal of Computer Vision*, 52(1), 7-23, 2003
- [9] Y. Liu. Automatic 3d freeform shape matching using the graduated assignment algorithm. *Pattern Recognition*, 38(2005) 1615-1631.
- [10] Y. Liu. Constraints for closest point finding. *Pattern Recognition Letters*, vol. 29, no. 7, pp. 841-851, 2008.

- [11] T.R. Lo, and J.P. Siebert. Local feature extraction and matching on range images: 2.5D SIFT. *Computer Vision and Image Understanding*, 113 (2009) 1235-1250.
- [12] A. Mandow, J.L. Martínez, A.J. Reina, Jesús Morales. Fast range-independent spherical subsampling of 3D laser scanner points and data reduction performance evaluation for scene registration. *Pattern Recognition Letters* 31(2010) 1239-1250.
- [13] A.S. Mina, M. Bennamoun, R. Owens. Keypoint detection and local feature matching for textured 3D face recognition. *IJCV* 79(2008) 1-12.
- [14] J. Novatnack and K. Nishino. Scale-dependent/invariant local 3D shape descriptors for fully automatic registration of multiple sets of range images. *Proc. of the 10th European Conference on Computer Vision (ECCV)*, pp. 440-453, 2008.
- [15] OSU(MSU/WSU) range image database. <http://sampl.ece.ohio-state.edu/data/3DDB/RID/index.htm>, 2010.
- [16] Y.K. Park, S.L. Park, J.K. Kim. Retinex method based on adaptive smoothing for illumination invariant face recognition. *Signal Processing* 88(2008) 1929-1945.
- [17] M. Pauly, R. Keiser, M. Gross. Multi-scale feature extraction on point-sampled surfaces. *Computer Graphics Forum*, vol. 22, no.3, pp. 281-289, 2003.
- [18] J.M. Phillips, R. Liu, C. Tomasi. Outlier robust ICP for minimizing fractional RMSD. *Proc. 3DIM*, pp. 427-434, 2007.
- [19] C. Plagemann, V. Ganapathi, D. Koller, S. Thrun,. Real-time identification and localization of body parts from depth images. *Proc. ICRA*, pp. 3108-3113, 2010.
- [20] S. Rusinkiewicz, M. Levoy. Efficient variants of the ICP algorithm. *Proc. International Conference on 3D Digital Imaging and Modeling (3DIM)*, pp. 145-152, 2001.
- [21] C.-T. Shen, W.-L. Hwang. Color image enhancement using retinex with robust envelope. In *Proc. ICIP*, 2009, pp. 3141-3144.
- [22] I. Sipiran¹ and B. Bustos. A robust 3D key points detector based on Harris operator. *Proc. Eurographics Workshop on 3D Object Retrieval*, pp. 7-14, 2010.
- [23] E.R. Smith, R.J. Radke, C.V. Stewart. Physical scale keypoints: matching and registration for combined intensity/range images. *International Journal of Computer Vision*, 97(2012) 2-17.
- [24] B. Steder, R.B. Rusu, K. Konolige, W. Burgard. Point feature extraction on 3D range scans taking into account object boundaries. *Proc. ICRA*, pp. 2601-2608, 2011.
- [25] B. Steder, G. Grisetti, and W. Burgard. Robust place recognition for 3D range data based on point features. *Proc. of the IEEE Int. Conf. on Robotics and Automation (ICRA)*, pp. 1400-1405, 2010.
- [26] J. Stuckler and S. Behnke. Key point detection in depth images through scale-space surface analysis. *Proceedings of IEEE International Conference on Robotics and Automation (ICRA)*, pp. 3568-3574, 2011
- [27] R. Toldo, A. Beinat, F. Crosilla. Global registration of multiple point clouds embedding the Generalized Procrustes analysis into an ICP framework. *Proc. 3DPVT*, 2010.

- [28] A. Torsello, E. Rodolà, and A. Albarelli. Sampling relevant points for surface registration. *Proc. 3DIMPVT*, pp. 290-295, 2011.
- [29] Precision and recall. http://en.wikipedia.org/wiki/Precision_and_recall

Responses to the comments from reviewers

Reviewer #1

Q P6 — The technique described in section 2.1 on depth normalization implicitly assumes that the range of variance will be similar for an object for different viewpoints. While this is likely the case for many objects and for viewpoints that are relatively close together, it is easy to imagine shapes and situations where this does not work. Some mention of this would be beneficial.

A Done. Thanks for this very good point.

Q P10 — The definition of Precision and Recall is not traditional. While its use here is fully described and unambiguous, the redefinition of these terms does not match with their standard definitions. I see no reason why different terminology than Precision and Recall should not be used here to denote N/n_1 and N/n_2 .

A We believe that these definitions are still in agreement with the traditional definitions, with a particular emphasis on the different roles played by different shapes in the process of registration of overlapping shapes: one is used as a data shape, the other is used as a reference shape. Such context is used for the definition of Precision and Recall.

Q P12 — It's not clear to me that a higher value of N/n_1 necessarily is a measure of superior performance. It may simply mean a smaller value of n_1 , which after a threshold could be detrimental. Displaying in Table 2 the values of n_1 for each of the techniques (RKP, octave, MSFE, and NSS) would be interesting.

A A smaller value of n_1 means that fewer key points were detected. In such cases, these key points poorly characterize the geometry of a free form shape and thus causes registration problems, leading to poor registration results. As discussed in Section 3.1, all methods detected around 10% of the original points as keypoints. In this case, (1) n_1 cannot be too small; and (2) it is almost constant, and thus is worth tabulating. The relative ratio shows the extent to which the detected key points in the data shape really find correspondents in the reference shape, and thus shows the performance of the algorithms in the sense of detecting repeatable and relevant key points for the definition of the registration problem.

Q P12 — It is stated that "the key points detected by the former are less reputably placed." This is inferred and has not been shown directly. The statement should either be modified as such, or more direct evidence presented to support it.

A The statement is inferred from two considerations: (1) registration results, in the sense of both average registration errors and the precision rates of the detected key points, (2) Figure 2, in which it can be observed that the proposed method tends to detect salient and feature points as key points, while the competitors usually detect either points at depth and orientation discontinuities, or random points, as key points. A careful analysis shows that the key points detected

by the former are more repeatable and relevant for representing the geometry of free form shapes independently of viewpoint, and for defining registration between overlapping shapes.

Q P13 — For bottle0-36, e_μ and e_σ are reported as being higher for RKP than the other techniques. This poorer performance, while spurious, should be commented on in the text.

A Done.

Q P19 — In Table 5, for valve10-0, the Recall is superior for the MSFE method than for RKP. This should be commented on in the text.

A Done.

Q P26 — Given the above, the statement that "The precision and recall rates of the key points detected by the proposed RKP method are better than those for the selected competitors ." should be qualified and moderated.

A Done.

Reviewer #2

No questions were raised.