Designing supply chains resilient to nonlinear system dynamics

by

Virginia Luisa Marques Spiegler
B.Eng.(Hons), M.Sc.(Dist), Pg.Dip.

A Thesis Submitted in Fulfilment of the Requirements for
the Degree of Doctor of Philosophy of
Cardiff University

Logistics Systems Dynamics Group
Logistics and Operations Management Section, Cardiff Business School
Cardiff University, Wales, United Kingdom

September 2013

Declaration

This work has not previously been accepted in substance for any degree and is not being currently submitted in candidature for any degree.

Signed: .... *(Virginia L. M. Spiegler)*
Date: .... /....... / 2013

STATEMENT 1
This thesis is being submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy

Signed: .... *(Virginia L. M. Spiegler)*
Date: .... /....... / 2013

STATEMENT 2
This thesis is the result of my own independent work/investigation, except where otherwise stated. Other sources are acknowledged by footnotes giving explicit references.

Signed: .... *(Virginia L. M. Spiegler)*
Date: .... /....... / 2013

STATEMENT 3
I hereby give consent for my thesis, if accepted, to be available for photocopying and for inter-library loan, and for the title and summary to be made available to outside organisations.

Signed: .... *(Virginia L. M. Spiegler)*
Date: .... /....... / 2013

Meinem Mann für seine Liebe und Ermunterung.

To my mentors Prof. Mohamed M. Naim, for believing in me and betting on my success; and Prof. Denis R. Towill, for his wise advices at the moment most needed.

Aos meus pais e irmão pelo carinho, apoio e torcida que apesar da distância foram sempre fortes e importantes pra mim.

To my dear friends for keeping me sane during this long journey.
Aos queridos amigos por me manterem sã durante esta jornada.

Abstract

Purpose: To propose an analytical framework for the design of supply chains that are resilient to nonlinear system dynamics. For this purpose, it is necessary to establish clearly elucidated performance criteria that encapsulate the attributes of resilience. Moreover, by reviewing the literature in nonlinear control engineering, this work provides a systematic procedure for the analysis of the impact of nonlinear control structures on systems behaviour.

Design/method/approach: The Forrester and APIOBPCS models are used as benchmark supply chain systems. Simplification and nonlinear control theory techniques, such as low order modelling, small perturbation theory and describing functions, are applied for the mathematical analysis of the models. System dynamics simulations are also undertaken for cross-checking results and experimentation.

Findings: Optimum solutions for resilience yield increased production on-costs. Inventory redundancy has been identified as a resilience building strategy but there is a maximum resilience level that can be achieved. A methodological contribution has also been provided. By using nonlinear control theory more accurate linear approximations were found for reproducing nonlinear models, enhancing the understanding of the system dynamics and actual transient responses.

Research limitations/implications: This research is limited to the dynamics of single-echelon supply chain systems and focus has been given on the analysis of individual nonlinearities.

Practical Implications: Since that the resilience performance trades-off with production, inventory and transportation on-costs, companies may consider to adjust the control parameters to the resilience 'mode' only when needed. Moreover, if companies want to invest in additional capacity in order to become more resilient, manufacturing processes should be prioritised.

Originality/value: This research developed a framework to quantitatively assess supply chain resilience. Moreover, due consideration of capacity constraints has been given by conducting in-depth analyses of systems nonlinearities.

Contents

List of Figures

List of Tables

Notation

A	Amplitude
B	mean
AI	Constant for inventory
AINV	Actual inventory level
AL	Constant specifying capacity limit
APIOBPCS	Automatic pipeline, inventory and order-based production control system
AVCON	Average consumption or smoothed demand
BACKLOG	Orders backlogged
C	General output
CLIP	Maximum or minimum cap of a variable
CODP	Customer order decoupling point
COMRATE	Completion rate
CONS	Consumption or sales demand
CP	Clerical in-process orders
CSL	Customer service level
DC	Delay clerical
DF	Delay in filling orders
DH	Delay due to minimum handling time
DI	Delay in inventory adjustment
DINV	Desired inventory
DP	Delay in production lead time
DR	Delay in smoothing requisitions
DSHIP	Desired shipment
DU	Delay, average, in unfilled orders
DWIP	Desired work in process
e	Error
EINV	Error in inventory
EOQ	Economic Order Quantity
EWIP	Error in work in process
G	General transfer function
IA	Inventory actual

IAE	Integral of absolute error
ID	Inventory desired
INSHIP	Shipment received
IOBPCS	Inventory and Order Based Production Control System
ISE	integral of square error
ITAE	Integral of time absolute error
ITSE	Integral of time square error
K	Collecting constant = $DC + DP - DH - DU + AI$
LA	Pipeline orders actual in transit
LD	Pipeline orders desired in transit
MAX[x,y]	Maximum value between x and y
MAXSHIP	Maximum shipment
MD	Manufacturing rate decision
MIN[x,y]	Minimum value between x and y
MO	Manufacturing orders
MRI	Minimum resoanable inventory
MROB	Minimum resoanable order book
MRP	Material requirement planning
MTO	Make-to-order
MTS	Make-to-stock
MW	Manufacturing rate wanted
N_A	Describing function coeficient for amplitude
N_B	Describing function coeficient for mean
OP	Orders in production
ORATE	Order rate
QR	Quick response
R	General input
RQ	Research question
RR	Requisition (orders) received
RS	Requisition (orders) smoothed
s	Laplace operator
SCM	Supplu chain management
SHIP	Actual shipment
SR	Shipment received inventory
SS	Shipment sent
ST	Shipping rate tried
T	High-order transfer function
t	Time period
T_a	Time to average incoming order

T_i	Time to recover desired inventory
T_p	Lead time
T_w	Time to recover desired work in process
T_M	Low-order transfer function
t_p	Time to peak
t_s	Time to settle
UN	Unfilled orders normal
UO	Unfilled orders
VMI	Vendor-managed inventory
WIP	Work in process
x^N	Variable x has been normalised
α	Exponential smoothing constant
γ	Time in which capacity is reached for given ω
ζ	Damping ratio
τ	Matsubara time delay
ϕ	Phase angle
ω	Angular frequency
ω_n	Natural frequency

1 Introduction

This chapter aims to briefly establish the context for this study and to outline the motivations for undertaking this research, including theoretical justification and main issues surrounding the fields of supply chain resilience and system dynamics. More detail on both of these research areas and a full review of relevant research that is found in the literature review, Chapter 2. Drawing from an overview provided in this chapter, the emerged research questions will be presented in Section 1.2. Finally in Section 1.3, the overall structure of this thesis is then outlined, showing how the chapters link with the research questions and illustrating the necessary stages for answering each question.

1.1 Research motivation

In the past few years, successful businesses have moved from mass-production to customisation and their strategies have become more market-driven instead of product-driven. Hence, providing distinctive customer value has become one of the main business drivers for companies (Datta *et al.*, 2007). In such a new business environment, individual companies are no longer able to meet the expectations of end-customers (Bhamra *et al.*, 2011). Rather, competitive advantage resides in fully-integrated supply chains.

In addition to this, modern supply chains are becoming more and more complex. With the supply chain leaning and lengthening, as a result of globalisation, supply chains are becoming more vulnerable to disruptions (Christopher and Peck, 2004). Managers have optimised supply chains by reducing holding inventory, outsourcing noncore activities, cutting the number of suppliers and sourcing globally, on the assumption that, the world market is a relatively stable and predictable place (Kearney, 2003). These resulting complex business environment has increased the importance of handling risks which can emerge from the customers' or demand side, the suppliers' side, manufacturing processes and control systems (Mason-Jones, 1998).

In this research, a supply chain perspective of risk is considered. When considering the supply chain's goal, potential risks involve any possibility of mismatch between supply and demand, as well as serving customers inefficiently. Therefore, any event that negatively affects the information and material flow between original supplier and end user should be considered as a risk of supply chain disruptions (Jüttner *et al.*, 2003).

In summary, the risk of supply chain disruptions is receiving increased attention in the business as well as the academic press (Zsidisin, 2003). Due to the current uncertain and complex environment supply chains are reviewing their strategies in order to be ahead of their competitors in delivering value to customers. For that reason, a well coordinated and well designed supply chain has become crucial. Under these circumstances, the ability of a supply chain to be resilient is vital to sustain competitiveness (Pettit *et al.*, 2010).

1.1.1 Supply chain resilience

It was following the events of the terrorist attack of 9/11 in 2001, the Asian tsunamis in 2004 and the hurricanes in North and Central Americas in 2005 that the topic of supply chain resilience emerged (Christopher and Peck, 2004; Sheffi, 2005b; Tang, 2006; Datta *et al.*, 2007). Also, the topic became very popular because of the current global financial crisis (Alsop and Armstrong, 2010; Jüttner, 2011). In particular, resilience has been used in examining responses to such major supply chain disruptions and disaster relief efforts. However, complex supply chain procedures and recent trends in the dynamics of market places (Mangan *et al.*, 2008) increased the importance of handling risks which emerge especially at the operational level (Pettit *et al.*, 2010).

Furthermore, despite the growing importance of the field of supply chain resilience, most existing studies in this area are qualitative in nature. There are still very few studies that attempt to create a quantitative framework for assessing supply chain resilience performance (Datta *et al.*, 2007; Falasca *et al.*, 2008; Ratick *et al.*, 2008; Colicchia *et al.*, 2010b; Carvalho, 2011). This is probably because there is still no consensus on the resilience definition. For instance, several other terms are linked with resilience, such as, agility, flexibility, responsiveness, adaptability, alignment, robustness (Goranson, 1999; Lee, 2003; Lummus *et al.*, 2003; Rice and Caniato, 2003; Christopher and Peck, 2004; Christopher and Rutherford, 2004; Tang, 2006; McManus *et al.*, 2007; Asbjørnslett, 2008). These terms either complement the topic of resilience or are used interchangeably with it.

Without a holistic definition for supply chain resilience, it is not possible to establish performance criteria to measure it. For this reason, this research's first motivation is to review all the existing resilience-related definitions in the supply chain literature in order to clarify its meaning. After this, an assessment framework for measuring resilience based on existing definitions and conceptual frameworks in the literature and on the dynamic behaviour of a supply chain can be created.

1.1.2 System dynamics and nonlinear models

System dynamics play a significant role in changing supply chain performance. These dynamics are normally driven by the application of different control system policies and can be considered as a source of disruption depending on the control system design (Mason-Jones, 1998; Christopher and Peck, 2004; Colicchia *et al.*, 2010b).

Despite this, the understanding of system dynamics' impact on supply chain resilience is minimal. Most existing studies on supply chain dynamics has focused on measuring and reducing demand amplification (Fransoo and Wouters, 2000; Chen *et al.*, 2000; Dejonckheere *et al.*, 2003; Disney and Towill, 2003b; Dejonckheere *et al.*, 2004; Lee and Wu, 2006) and its impact on transport operations (Potter and Lalwani, 2008; Juntunen and Juga, 2009; Marques *et al.*, 2010), financial performance (Torres and Maltz, 2010) and production operations (Bicheno *et al.*, 2001; Wikner *et al.*, 2007; Cannella *et al.*, 2008; Hamdouch, 2011).

In addition to this, only simulation methods have been recommended to analyse complex, high-order, nonlinear supply chain models as an alternative to control theory (Forrester, 1961; Wikner *et al.*, 1991; Naim and Towill, 1994; Shukla *et al.*, 2009). However, simulating complex systems without having first done some preliminary analysis can be exhaustive and unrewarding (Atherton, 1975). In supply chains, nonlinearities can naturally occur through the existence of physical and economic constraints and they cannot be disregarded in this research since that capacity flexibility has a great impact on supply chain resilience (Christopher and Peck, 2004; Sheffi, 2005b).

For this reason, there is a need to review the literature of nonlinear control engineering in order to identify suitable methods for studying different types of nonlinearity that commonly appear in supply chain systems. Hence, another motivation of this research is to provide a systematic procedure for the analysis and design of nonlinear supply chain dynamics models.

1.2 Research questions

In summary, two main gaps in the literature have been identified. The first one lies within the supply chain management theory and regards the lack of consensus on the definition of supply chain resilience and the need for clearer quantitative performance criteria to assess it. The second gap, which exists within supply chains research methodology, concerns the need to investigate complex nonlinear system dynamics models analytically to gain more insights on the impact of capacity constraints on supply chain system performance.

In order to provide a focus for this thesis, the following research questions have been formulated and are sought to be answered by this thesis:

RQ:1 Supply chain management theory research questions

RQ:1a) What are the existing resilience-related definitions in the supply chain literature?

RQ:1b) How can supply chain resilience be measured in the context of systems dynamics?

RQ:1c) How can supply chains be (re-)designed in order to be resilient against such dynamics?

RQ:2 Supply chain management methodology research questions

RQ:2a) How can we analytically study nonlinear supply chain models?

RQ:2b) How does the presence of nonlinearities impact on supply chain system responses and how is resilience affected?

Research question RQ:1a arouse out of the initial motivation to undertake this research. Given the author's background in engineering, in which the definition for resilience is well-established, some emphasis is placed on developing a quantitative measure for supply chain resilience. All the other research questions emerged during the literature review process.

1.3 Thesis structure

A brief overview of the structure of this thesis and how each chapter connects to each research question is provided in Figure 1.1. In summary, this thesis is organised in eight chapters and its contents can be summarised as:

Chapter 1: introduces the background of the fields of supply chain resilience and system dynamics and presents the initial motivation for undertaking this research. Existing gaps in the literature are introduced and research questions are then formulated.

Chapter 2: contains the literature review which provides an overview of previous research undertaken into the core themes of this thesis: supply chain resilience and system dynamics. Moreover, this chapter defines the scope of this research and provides theoretical foundation for this thesis. The first research question, which is the initial motivation of this research regarding the existing definitions of supply chain resilience, will be answered and other gaps in the literature are identified, leading to the construction of RQ:1b, RQ:1c, RQ:2a and RQ:2b.

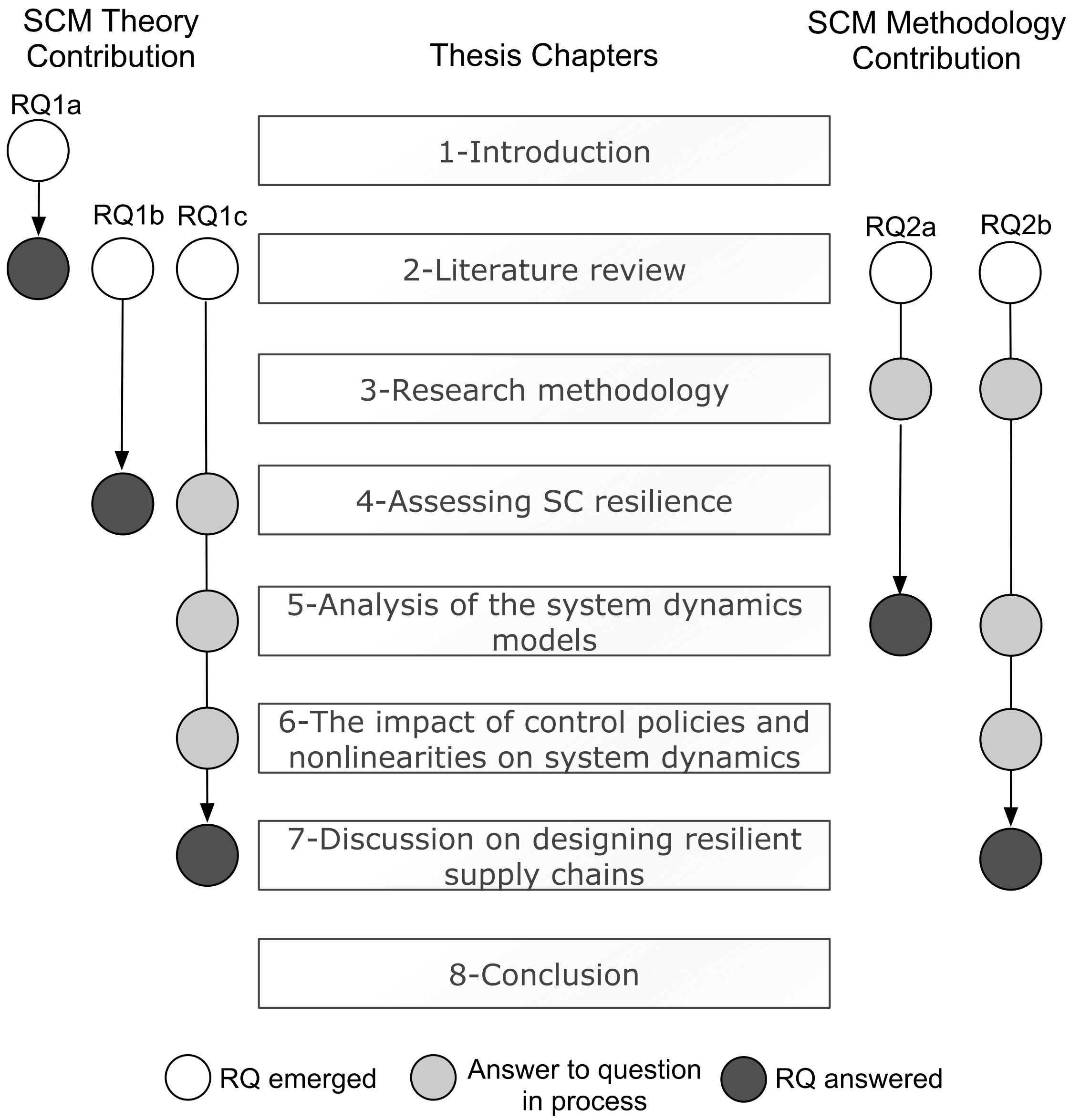


Figure 1.1: Schematic of this thesis

Chapter 3: outlines the methodology used to conduct this research including the research ontological and epistemological positions, research design, methods and tools used. An objective, holistic and value-free view, a deductive logic reasoning and a conceptual research approach are chosen for undertaking this research. Mathematical analysis and simulation are the chosen techniques and the author reviews the nonlinear control theory literature in order to identify suitable methods for the analysis of nonlinearities. Finally this chapter introduces the models used as benchmark supply chain systems to study the resilience performance and nonlinear control structures: the Automatic pipeline, inventory and order-based production control system (APIOBPCS) and the Forrester models.

Chapter 4: further explores the conceptual literature on resilience and proposes an assessment framework to measure supply chain resilience in the context of system dynamics given that the supply chain's main objective is matching supply with demand. Different composite performance indices are proposed and tested. Finally, the performance index that better encapsulates the attributes of resilience is chosen.

Chapter 5: contains analysis of the two nonlinear system dynamics models. Simplification and linearisation methods are used to estimate the nonlinear systems behaviour and responses. Also, design analysis of each model will be performed by investigating the impact of different control parameters on supply chain resilience performance. More importantly, this chapter provides initial insights into understanding the system behaviour and sets the scene for the next stage which is simulation analysis. In summary, a systematic procedure is provided for the analysis and design of nonlinear supply chain dynamics models, especially because overly simplistic linear relationship assumptions are not appropriate in this research.

Chapter 6: analyses the impact of different control policies and nonlinearities on system dynamics and resilience performance via repeated simulations. This chapter cross-checks the results obtained from Chapter 5 and further investigate unexpected or unclear findings. Moreover, trade-off and sensitivity analysis will be performed in this chapter.

Chapter 7: presents a framework on how to design supply chains resilient to nonlinear system dynamics. It discusses the insights gained from applying a conceptual literature review in the developing of an assessment framework to measure supply chain resilience and also from applying nonlinear control theory in order to mathematically analyse the behaviour of complex, nonlinear supply chain models.

Chapter 8: collates the findings from analytical and simulation stages to provide summary answers to the research questions. In this chapter, the contributions of this research to the theory, methodology and practice is summarised. Finally, the limitations and potential lines for further investigation will be discussed.

1.4 Summary

This chapter has provided background information on the research theme, motivation and the research questions to be addressed in this thesis. The structure of this thesis and a summary of the chapters' contents have been also explained. The next chapter will provide more context for the thesis through the literature review.

2 Literature Review

This chapter will provide an overview of previous research undertaken into the core themes of this thesis: supply chain resilience and system dynamics. Emphasis is particularly given to conceptual and empirical research that defines resilience and quantitative works that have attempted to measure supply chain resilience and understand supply chain dynamics. This review also identifies the gaps in the literature that led to the formulation of the research questions stated in the previous chapter.

This chapter will first give a quick guide to frequently used terms in this thesis and their definitions. Next, definitions for resilience that exist in other fields within the physical, natural and social sciences will also be addressed. In section 2.3, the research focus is narrowed down to addressing resilience in supply chain management research. Here, the causes of supply chain disruptions and strategies to improve resilience will be established. In section 2.5, the definitions of resilience and robustness are compared and contrasted in order to avoid confusion when supply chain analysis and re-design are carried out in the following chapters. Then, in section 2.6, attention is given to quantitative works that have attempted to develop measurable frameworks to reduce risk and assess resilience. Finally, the important role of system dynamics and the impact of nonlinearities on supply chain systems' performance will be discussed in section 2.7. Section 2.8 will highlight the gaps in the literature of supply chain theory and methodology.

2.1 Definitions

The aim of this section is to provide a quick overview of the key terminology and definitions that will be frequently used throughout this thesis.

2.1.1 Supply chain

In the literature there are several debates going on among logistics researchers and practitioners on what would be an acceptable definition for Supply Chain Management (SCM) (Mentzer *et al.*, 2001) and what the differences between this practice and Logistics Management are (Cooper *et al.*, 1997; Lummus *et al.*, 2001; Larson and Halldórsson, 2004).

In this thesis, a supply chain is defined as "the network of organisations that are involved, through upstream and downstream linkages, in the different processes and activities that produce value in the form of products and services delivered to the ultimate consumer" (Christopher, 1992). These upstream and downstream linkages occur via a feedforward flow of materials, a feedback flow of information (Towill, 1997b) and flow of funds (Metz, 1998). The objective of managing the supply chain is then to synchronise the requirements of the customer with the flow of material from suppliers in order to effect a balance between customer service, low inventory investment and low unit cost (Stevens, 1989); in other words, matching demand with supply in the most efficient and effective way.

2.1.2 Industrial, system and supply chain dynamics

Although the term SCM was proposed by Oliver and Webber as recently as 1982 to designate a new form of strategic logistics management, the antecedents of this field are much older and appear to have originated with physical distribution and transport and are based on the theory of Industrial Dynamics (Forrester, 1961). Forrester identified that information flow is distorted as it moves towards the upstream companies in the supply chain, and consequently the material and cash flows are amplified. Thanks to Lee *et al.* (1997a,b), this phenomenon is now known as the bullwhip effect .

After Forrester's publication, the applications of his work expanded from solving solely industrial problems to also studying phenomena in economics, public policy, environmental studies, defence and other areas, as well as the field of management. Hence, the name industrial dynamics "no longer does justice to the breadth of the field" (Richardson, 2008) and that is why the term system dynamics was created. The new term also suggests links to other systems methodologies, such as systems engineering, systems theory and systems thinking (Towill, 1992a).

Supply chain dynamics, on the other hand, is simply a term that designates system dynamics analysis in the supply chain context and expresses the need to integrate business processes and analyse supply chains holistically. A supply chain system is characterised by interfaces between suppliers and customers which comprehend complexity and dynamism. Each supply chain stage embraces the following elements (Towill, 1991) that are crucial for an understanding of this dynamic system:

- Perceived demand for products (firm orders or sales forecasts);
- Added value processes;
- Currency of information on performance;

- Machinery and processes status;
- Transmission delays in respect of information and materials;
- Current stocks, work in progress and production rates;
- Local decision rules for target inventory, new production orders and raw material orders, etc.;
- Capacity availability and requirements (Lagoudis *et al.*, 2002).

The supply chain dynamics' assumption is that the improvement of a single element in a supply chain does not necessarily imply efficiency and effectiveness of the supply chain as a whole (Towill *et al.*, 1992). "An efficient production control system can only be designed and operated if the dynamic behaviour of the constituent parts is properly understood. Only then can an optimum control law be devised which will balance...the risk of stock-out with costly fluctuations in production rate" (Towill, 1982). Through the observation of real industry cases and the modelling and simulation of scenarios (Hennet, 2009), supply chain dynamics have been used within SCM research to provide insights into supply chain behaviour and the underlying causal relationships (Wolf, 2008).

2.1.2.1 Nonlinear system dynamics

A nonlinear system is one whose performance does not obey the principle of superposition. This means that the output of a nonlinear system is not directly proportional to the input and the variables to be solved cannot be expressed as a linear combination of the independent parts (Atherton, 1975).

The real world is nonlinear and the existence of these nonlinearities makes understanding the system very difficult. In supply chain models, nonlinearities can naturally occur through the existence of physical and economic constraints, for instance fixed and variable capacity constraints in the manufacturing and shipping processes, variable delays and variable control parameters. Nonlinearities could also be intentionally introduced into the system to improve its output responses. Different types of nonlinear systems will seen in more detail in Chapter 3.

2.1.3 Risk and uncertainty

Risks and uncertainties have been studied in various business domains, for instance in the management (March and Shapira, 1987; Yates and Stone, 1992), operations (Newman *et al.*, 1993; Pagell and Krause, 1999), finance (Ashton, 1998; Chow and Denning, 1994) and distributions (Lassar and Kerr, 1996; Celly and Frazier, 1996).

In the context of purchasing and supply management, Zsidisin (2003) found that risk can be perceived as a multidimensional concept. Different companies will define risk based on their individual objectives and desired outputs. Moreover, within a company risk and uncertainty concepts among different managers may be related to different outcome variables such as commercial (e.g. inventory levels), safety (e.g. risk to life) and political issues (e.g. political ramification) (Jüttner *et al.*, 2003).

According to Kaplan and Garrick (1981), who suggested a quantitative definition for risk, the main distinction between the risk and uncertainty terms is that risk always involves some kind of loss or damage that might be received while uncertainty leads to an unknown outcome. A more recent study undertaken by Sanchez Rodrigues *et al.* (2008) reinforces that while risk can be estimated since it is a function of outcome and probability, uncertainty occurs when the outcome of an event or the probability of its occurrence cannot be estimated. Furthermore, both studies agree that risk is proportional related to uncertainty. While Kaplan and Garrick (1981) express this view by suggesting that risk is equal to uncertainty plus damage, Sanchez Rodrigues *et al.* (2008) affirm that "uncertainty increases the risk within supply chains, and risk is a consequence of the external and internal uncertainties that affect a supply chain".

In this thesis, loss or damage associated with risk is referred to as disruptions that may occur in the supply chain network. These disruptions occur due to uncertainties, or lack of information and unpredictability, in the demand and supply sides and in the control systems (Mason-Jones and Towill, 1998). This thesis will focus on evaluating the impact of risk arising from control systems and supply chain dynamics on the resilience performance. Therefore, emphasis will be given on assessing the outcome of such risks rather than determining the probability of risk occurrence. More discussion on supply chain uncertainties and different sources of risk will be presented later in Section 2.3.1.

2.1.4 Disruption

A broader definition for disruption is described as any disturbance or problem which interrupt an event, activity, or process (Collins English Dictionaries, 1995). In this work, a supply chain perspective of risk is considered. Hence, when considering the supply chain's goal, potential risks involve any possibility of a mismatch between supply and demand, as well as serving customers inefficiently. Therefore, any event that negatively affects the information and material flow between the original supplier and end user should be considered as supply chain disruption (Jüttner *et al.*, 2003). In these turbulent circumstances, the ability of a supply chain to be resilient becomes an important consideration (Pettit *et al.*, 2010).

2.1.5 Resilience

In general terms, resilience is the ability to recover from an event likely to bring about change (Collins English Dictionaries, 1995). Hence, a comprehensive definition of supply chain resilience embraced by this thesis can be defined as the ability of a supply chain to quickly respond to and recover from a change that causes disruption, maintaining or returning to its original state (based on Ponomarov and Holcomb's (2009) definition). In the following sections, a review of studies that have used this term will be undertaken.

2.2 Resilience in natural and social sciences

The existing literature on resilience spans several branches of knowledge. This multidisciplinary topic arouses interest from both natural and social scientists who have described resilience from different starting points and foci. In physics and engineering, resilience is the ability of a material to return to its original form after being bent, compressed, or stretched. In other words, it is the ability to exhibit an elastic behaviour as a result of disturbance (Pytel and Kiusalaas, 2003).

In the analysis of ecological dynamic systems, early studies started by making connections between resilience and stability (Holling, 1973). Without stability there is no return to the pre-disturbance state, hence there is an assumption of a steady ecological state in the system when evaluating its resilience. However recently, studies on sustainability and ecological-footprint analysis suggest that humanity's ecological demands already exceed what nature can supply, thus we have moved into what is termed "ecological overshoot". This situation means that we are depleting the available stock of natural capital rather than "living off the interests" (Venetoulis *et al.*, 2004). For this reason, Fiksel (2006b) argues that steady-state sustainability models are simplistic and a better understanding of the complex, dynamic, adaptive behaviour of complex systems and their resilience in the face of disruptions is needed.

In the social sciences, the study of resilience seems to have its origin in development theories of social psychology and psychiatry in which people's behaviour is examined during life course transitions and events (Kaplan, 1990). Resilience is then seen as a capability of individuals to cope successfully in the face of significant change and stress; and is a dynamic process since successful coping strengthens the individual's competence to deal with adversity in the future (Stewart *et al.*, 1997). In the fields of economics, resilience is an important concept because of the gigantic asset and business losses that could be incurred by shocks in the economic sector (Rose, 2004). Hence, this field of study should involve measurements of economic resilience in the microeconomic (individual), mesoeconomic (sector, market or cooperative group) and macroeconomic (all markets combined) levels of society since they are all affected in times of economic disturbances.

Finally, from an organisational perspective, resilience has been described as "a dynamic capacity of organisational adaptability that grows and develops over time" (Wildavsky, 1988). Later on, Weick *et al.* (1999) described that resilience is only the capacity that any organisation has to adjust and maintain desirable functions under challenging or straining conditions. An organisation that has an enhanced resilience is more likely to deal with day to day problems and those arising from a crisis. Therefore resilience is a source of competitive advantage (McManus *et al.*, 2007).

Interdisciplinary research groups are trying to encompass all these aspects of resilience through a comprehensive systems approach to study global sustainability, since industrial, social and ecological systems are closely linked. The concept of sustainability is often associated with resource constraints and maintenance of status quo and balancing economic profits with environmental and social benefits (Elkington, 1997). However, Fiksel *et al.* (2004) suggest that sustainability should also be seen as an opportunity for continued innovation, growth and prosperity, which is the characteristic of dynamic and evolving systems. Achieving sustainability requires the development of adaptive industrial and social systems that mirror the dynamic attributes of ecological systems (Holliday and Pepper, 2001; Fiksel, 2003). Hence, resilience is seen as the essence for sustainability since it reflects the "capacity of a system to tolerate disturbances while retaining its structure and function" (Fiksel, 2006a). Moreover, when studying systems sustainability it is important to take into account the system scope and complexity. System complexity is relevant for global sustainability since it establishes boundaries for the system design. For instance, a product can only be sustainable if considered in the context of the supply chain, the market and the natural environment (Fiksel, 2003).

In summary designing resilient and sustainable systems encompasses:

- Addressing multiple scales over time and space
- Capturing system dynamics and points of leverage and control
- Representing an appropriate level of complexity
- Managing variability and uncertainty
- Capturing stakeholder perspectives in various domains
- Understanding system's behaviour relative to foreseen and unforeseen stressors

While the above bulleted attributes are explored in the thesis, the focus is on understanding the behaviour of supply chains in the face of disruptions, therefore the research boundaries are delineated to the concept of supply chain resilience and not sustainability. By conducting a comprehensive literature review, Bhamra *et al.* (2011) identified that although resilience has been extensively studied from ecological, social and organisational perspectives individually, it is still an emerging topic in the supply chain management literature.

2.3 Supply chain resilience

In the supply chain literature, the idea of resilience has only recently emerged, and is essentially defined as "the ability of a system to return to its original state or move to a new, more desirable state after being disturbed" (Christopher and Peck, 2004). Sheffi (2005b), on the other hand, does not suggest that the system moves to a new state but only "bounces back" to the previous state. More recently, this definition has been grounded by Ponomarov and Holcomb (2009) using multiple disciplines, some of which were mentioned in Section 2.2. They define supply chain resilience as "the adaptive capability of the supply chain to prepare for unexpected events, respond to disruptions, and recover from them by maintaining continuity of operations at desired levels of connectedness and control over structure and function". This holistic conceptual definition will be used in Chapter 4 for constructing a quantitative framework for assessing supply chain resilience.

Greater interest in issues of security and risk management in supply chains seems to have been generated following the terrorist attack of 9/11 (Sheffi, 2001; Rice and Caniato, 2003; Barry, 2004; Spekman and Davis, 2004). Supply chain disruptions in this case were not caused by the attack itself, but by the government's response:

closing borders, shutting down air traffic, evacuating buildings (Sheffi, 2001). This event was "a wake-up call" to the uncertainty of a global environment (Barry, 2004).

Similarly, humanitarian logistics research emerged following the Asian tsunamis in 2004 (Thomas and Fritz, 2006; Kovács and Spens, 2007) and hurricanes in North and Central Americas in 2005 (Craighead *et al.*, 2007). Also, studies on warfare and peace-keeping mission logistics (Kovács and Tatham, 2009) are on the rise. Nevertheless, humanitarian logistics is not only relevant for governments. Many companies participate in humanitarian efforts and they also suffer losses when disasters interrupt their business flow. "Working to alleviate the economic impact of such disruptions makes good business sense" (Thomas and Fritz, 2006).

It was following these events that the topic of supply chain resilience emerged (Christopher and Peck, 2004; Sheffi, 2005b,a; Tang, 2006; Datta *et al.*, 2007). The topic is even more relevant today because of the current global financial crisis (Alsop and Armstrong, 2010; Jüttner, 2011). In particular, resilience has been used in examining responses to such major supply chain disruptions and disaster relief efforts. This implies the strategic planning and positioning of supply chain resources. However, recent trends in the dynamics of market places and resulting complex supply chain procedures (Mangan *et al.*, 2008) have increased the importance of handling uncertainties which are emerging at the operational level. Effectively managing operational risks directly improves financial performance (Pettit *et al.*, 2010).

Moreover, in global supply chains the longer transport distances and the more resources involved the greater the likelihood of operational disruptions (Sheffi, 2005b). Hence, resilient supply chains are capable of creating and sustaining competitive advantage (Christopher and Peck, 2004).

So far, the literature review has focused on the existing definitions of resilience.

The next point to be discussed is how supply chains achieve resilience. Before introducing the resilience strategies found in the literature, it is important to firstly address possible source of risk for supply chain disruptions. Focusing on risks has been a common practice in the literature since for each type of risk, there may be suitable resilience strategies.

2.3.1 Sources of risk

According to Mason-Jones and Towill (1998), a supply chain will normally face uncertainties originating from the customers' or demand side, the suppliers' side, manufacturing processes and control systems, the 'Uncertainty Circle' (Figure 2.1). Extending this framework, Christopher and Peck (2004) grouped supply chain risks into three categories: risks which are internal to the firm, risks which are external to the firm but internal to the supply chain and finally risks which are external to the supply chain. Similarly, Svensson (2000) argues that supply chain vulnerability is an "exposure to serious disturbance" and is caused by risks within the supply chain as well as external to it.

The management of processes and operations is a fundamental task for guaranteeing continuous flows of goods and information within a single company and within a supply chain. At a higher level though, the mismanagement of assets and infrastructure can disrupt supply chain operations (Peck, 2005) and is therefore considered a potential cause of disruptions. Other sources of internal risk found in the literature are supply chain dynamics which are normally driven by different control system policies (Mason-Jones, 1998; Christopher and Peck, 2004; Colicchia *et al.*, 2010a). By conducting multiple case studies on multinational companies, Colicchia *et al.* (2010a) found that supply chain dynamics appear in the first place among a list of

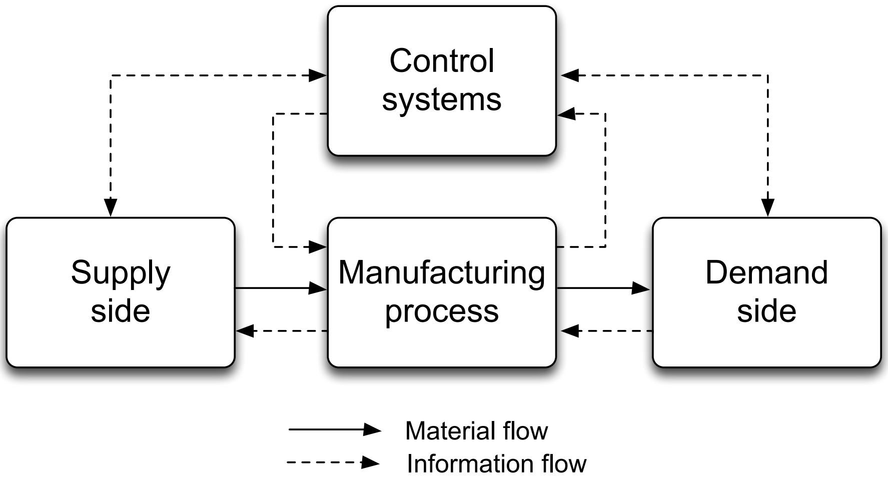


Figure 2.1: Causes of uncertainties in the supply chain
Sources: Mason-Jones and Towill (1998); Christopher and Peck (2004)

elements that cause supply chain disruption. Despite that, no research specifically on the effect of control policies and system dynamics on supply chain resilience has been found. Supply chain scholars have focused more on resilience strategies and capabilities (Bhamra *et al.*, 2011). This thesis will focus on the effect of control policies and system dynamics on supply chain resilience.

Regarding the risks that are external to the firm but internal to the supply chain, there are uncertainties arising from changes in supply and demand. Supply risk is well documented by Zsidisin (2003) who listed and classified a set of sources and outcomes of supply risk. Between the late 1980s and early 1990s, many companies in the US decreased the numbers of their suppliers due to the costly and complex task of managing multiple suppliers. Consequently, supply costs and commitment risks were increased (Tang and Tomlin, 2008). Moreover, several works presented in a special issue on improving disaster supply chain management (read editors' comment in Boin *et al.* (2010)) highlight the fact that companies with single sourcing are more prone to great losses in the event of natural disasters or terrorist attacks.

Despite recent works focusing more on risks arising from the supply side, disruptions also often occur due to changes in demand, whether it increases or decreases. In the current recession, a sharp decrease in demand has forced companies to hoard working capital by slashing inventories, idling production facilities, laying off employees, negotiating more favourable deals with suppliers and transportation providers, outsourcing more services and finding other cost efficiencies wherever possible (Alsop and Armstrong, 2010). This makes them more vulnerable to disruptions. Besides uncertainties in demand volume, demand risk encompasses changes also in demand mix, i.e. in cases of product variants (Tang and Tomlin, 2008). Demand patterns might also change which implies that the current forecast methods are no longer appropriate.

Finally, examples of external risks for a supply chain include environmental factors, political and economical policies and social and technological changes. Some of these events may be predictable, for instance those arising from regulatory changes, but most of them will not (Christopher and Peck, 2004).

2.3.2 Resilience strategies

The literature is well-supplied with advice on what is required to build supply chain resilience, i.e., redundancy, flexibility (Rice and Caniato, 2003; Christopher and Peck, 2004; Sheffi, 2005b; Sheffi and Rice, 2005; Tomlin, 2006; Ponomarov and Holcomb, 2009), situation awareness, vulnerability management (McManus *et al.*, 2007; Ponomarov and Holcomb, 2009), cultural change (Christopher and Peck, 2004; Sheffi, 2005b), demand management (Tomlin, 2006), supply chain collaboration, supply chain (re-) engineering or (re-)design (Christopher and Peck, 2004; Ponomarov and Holcomb, 2009), along with others.

Redundancy can be achieved by, for example, holding extra inventory, keeping low capacity utilisation, having many suppliers and spreading business over many locations (Rice and Caniato, 2003; Christopher and Peck, 2004; Tomlin, 2006) . This strategy works as a buffer and companies can continue operating after a disruptive event. However, since this is a very expensive measure it should only be used temporarily in situations where disruption is predictable or more likely to occur in the near future (Sheffi, 2005b). This practice might also conflict with some of the lean principles, such as eliminating waste, Just-In-Time or pull processing, building and maintaining a long relationship with suppliers and perfect first-time quality (Christopher and Peck, 2004).

In order to achieve flexibility, companies would have to invest in production systems that can accommodate multiple products and real-time changes, easy switching of suppliers, a multi-skilled work force, simultaneous rather than sequential processes and maximum postponement (Christopher and Peck, 2004; Sheffi and Rice, 2005). This strategy helps supply chains with internal disruptions or disruption caused by the demand side. Also, if there are disruptions in the upstream side, flexibility enables the company to boost production levels when goods are finally supplied after the disruptive period (Sheffi, 2005b).

Situation awareness is certainly another important factor in supply chain resilience. It refers to the organisation's awareness of existing threats as well as opportunities and their consequences (McManus *et al.*, 2007). Similarly, vulnerability management would include periodic identification, classification and mitigation of vulnerabilities (Ponomarov and Holcomb, 2009). Naturally, a cultural change is required and a collaborative risk management culture must be created. This culture should involve all levels of the supply chain since supply chain vulnerability is a network-wide concept (Christopher and Peck, 2004; Sheffi, 2005b). This idea also introduces the importance of collaboration for an effective supply chain risk management.

Demand management was briefly mentioned in the literature as a resilience strategy but it has a further importance. With demand management capabilities companies can control customer demand and shift it to alternative products that are less supply constrained during disruptions (Tomlin, 2006).

Conventionally, supply chains have been designed to optimise operational costs. Resilience should also be part of the 'objective function' when re-designing supply chains. For this, re-examining and understanding the supply chain goals and necessities is very important, for instance, in determining the degrees of cost-efficiency and resilience needed (Ponomarov and Holcomb, 2009). Supply chain re-designing does not only concern the strategical selection of suppliers and placing of distribution place. The control system also plays an important role in detecting disruption quickly and fostering speedy corrective measures for the response and recovery of the system (Christopher and Peck, 2004).

Tomlin (2006) presented some of these strategies categorised in mitigation and contingency tactics. The former implies taking actions before the disruption occurs in order to prevent an event's occurrence or to reduce its impact. Contingency strategies involve actions taken only after a disruptive event has happened. Tomlin (2006) also highlights the fact that more than one strategy can be used to manage risks. However, since some of these actions are expensive some supply chains commit to a certain degree of financial risk - it is a trade-off that needs to be taken into account.

2.4 Supply chain resilience and cost-related performances

The financial performance of a supply chain can be assessed by determining its total cost. Smooth flow of information and materials in a supply chain environment is a general strategy for reducing supply chain cost (Wikner *et al.*, 1991). Since supply chain management cuts across different functional boundaries, decision making becomes difficult since the cost in one area affects the cost in other areas (Cavinato, 1992). For example, an investment in capacity has a major impact on costs associated with inventory and order processing.

By conducting a literature survey, Gunasekaran *et al.* (2001) developed a framework for measuring the strategic, tactical and operational performance levels in a supply chain. They presented a list of key performance metrics when dealing with suppliers, delivery, customer-service, and inventory and logistics operations in a supply chain. Based on their framework, this section provides a review of relevant financial performance measures.

2.4.1 Costs associated with the ordering process and supplier's relationship

For any company, the procurement of goods is the starting point of the chain of business activities. The way the orders are generated and scheduled determines the performance of downstream activities and inventory levels. A reduction in the order cycle time leads to a reduction in the supply chain response time (Gunasekaran *et al.*, 2001). This is an important measure as well as a major source of competitive advantage (Suri, 1998) and is normally connected to resilience (Christopher and

Peck, 2004; Ponomarov and Holcomb, 2009). According to Towill (1997b), it directly influences the customer satisfaction level.

On the other hand, there is the cost of placing an order. A number of transactions are needed every time an order is placed, incurring costs to the company. These include preparing the order, communicating with suppliers, arranging for delivery, making payment, and maintaining internal records of the transaction (Slack *et al.*, 2010). Moreover, the relationship with suppliers involves costs associated with long-term association, mutual planning and problem solving efforts (Gunasekaran *et al.*, 2001).

2.4.2 Costs associated with production

Once orders are placed and the goods received, the next step is to make or assemble final products. Besides the cost of labour and raw material, the variety and volume of product and services, variations in demand, throughput time, capacity utilisation and the effectiveness of the scheduling process are some of the factors that affect production costs (Gunasekaran *et al.*, 2001).

High variations in the production rate normally leads to increased costs as the supply chain production capacity ramps up and down. Supply chains are constantly trying to balance the risk of stock-out with costly fluctuations in production rate (Towill, 1982), therefore balancing resilience and production costs.

2.4.3 Costs associated with assets and return on investment

Supply chain assets include accounts receivable, plant, property and equipment and inventories (Stewart, 1995). To measure the productivity of a firm, it is is essential to determine how the costs associated with each asset, combined with its turnover, affects the total cash flow time (Gunasekaran *et al.*, 2001). According to Stewart (1995), this can be measured as the average number of days required to transform the cash invested in assets into the cash collected from a customer.

For this reason, addressing inventory becomes a priority as it affects costs in more ways than companies can realise (Callioni *et al.*, 2005). In a supply chain, the total cost associated with inventory (Callioni *et al.*, 2005; Slack *et al.*, 2010) consists of the following:

- opportunity cost consisting of warehousing, capital and storage;
- cost associated with inventory as incoming stock level, work in progress;
- service costs, consisting of cost associated with stock management and insurance;
- cost held up as finished goods in transit;
- risk costs, consisting of cost associated with pilferage, deterioration, damage;
- cost associated with scrap and rework;
- cost associated with shortage of inventory accounting for lost sales/lost production.

2.4.4 Costs associated with delivery

The delivery channel, transport scheduling, and warehouse location play an important role in delivery performance. An increase in delivery performance is possible by selecting suitable channel, scheduling and location policies (Gunasekaran *et al.*, 2001). Over the past year many research studies have discussed the opportunities to improve supply chain financial performance by reducing lead-times in the delivery process (Blumenfeld *et al.*, 1985; Disney *et al.*, 2003; Mason and Lalwani, 2006; Wilson, 2007). Moreover, reduced delivery lead-time and increased transport flexibility is commonly associated with increased supply chain resilience (Carvalho and Cruz-Machado, 2011).

Another aspect of delivery systems is the amount of goods in transit or work in process. A large amount of goods in transit results in lower inventory turns, leading to unnecessary increases in tied-up capital. Various factors that can be attributed to this are vehicle speed and capacity, driver reliability, frequency of delivery, and the location of depots (Gunasekaran *et al.*, 2001).

2.4.5 Costs associated with customer service

Customer satisfaction is a key indicator of how likely a customer will make a purchase in the future. In a modern supply chain customers may reside locally or globally, and in either case they must be well served (Gunasekaran *et al.*, 2001). Given volatile markets and increasingly dynamic performance requirements, van Hoek *et al.* (2001) emphasised that to assess supply chain performance, supply chain metrics must centre on customer satisfaction.

A lost customer does not only mean lost sales and revenue, but also lost feedback and opportunity to improve, lost confidence and possibly lost reputation (Slack *et al.*, 2010).

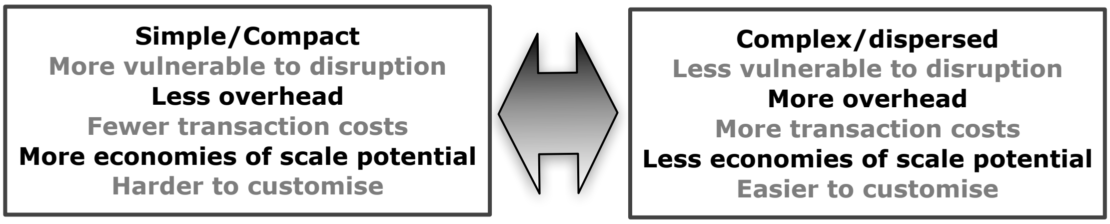
2.4.6 Existing trade-offs

The trade-off between supply chain resilience and costs is well acknowledged in the literature (Christopher and Peck, 2004; Sheffi, 2005b; Sheffi and Rice, 2005; Alsop and Armstrong, 2010). Indeed, it is costly to keep flexibility and redundancy through safety stocks, additional suppliers, extra backup sites and so on. On the other hand, lack of resilience also accounts for other cost elements: poor customer service level, vulnerability and possible loss of control (Christopher and Peck, 2004), which are more difficult to measure.

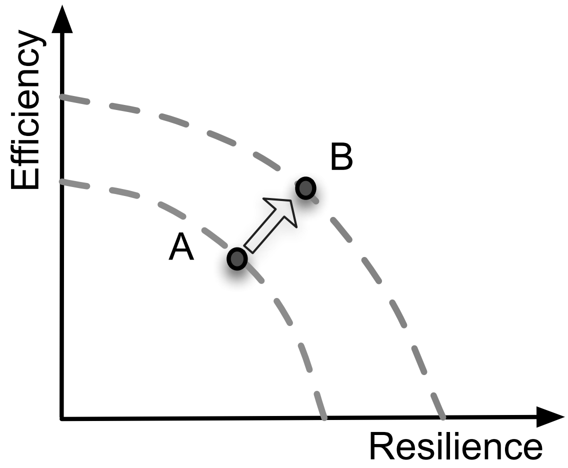
Alsop and Armstrong (2010) simplified this view by saying that there is also a trade-off between supply chains that are simple and compact and those that are complex, redundant and dispersed (Figure 2.2(a)). A more complex network can provide more discounts, excellent service and develop a stronger knowledge of processes but at the same time it is harder to manage logistics and pricing. On the other hand, the fewer and more concentrated the parties, the more likely the supply chain is to suffer from unforeseen events.

Alsop and Armstrong (2010) argue that many companies are studying ways of overcoming the efficiency-resilience trade-off (Moving from situation A to situation B in Figure 2.2(b)) by being in the middle range between simple and complex. For instance, large companies such as the Whirlpool Corp. are consolidating their brands and increasing the use of standardised components (Alsop and Armstrong, 2010).

Organisations will always have to make thoughtful choices based on their strategic objectives. Hence, it is important to consider these trade-offs when undertaking research on supply chain resilience and this research will be undertaking trade-off analyses. For this reason, transportation, production, inventory and service-related costs will be considered in this thesis when evaluating the resilience performance.



(a) Trade-offs



(b) Overcoming trade-off

Figure 2.2: Existing trade-off between simple and complex supply chain systems
Source: Alsop and Armstrong (2010)

2.5 Resilience versus robustness

In the supply chain risk management literature, several terms have been linked to resilience. For example, agility, flexibility, responsiveness, adaptability, alignment, robustness, and other similar terms frequently appear in supply chain risk research as ways of preparing for uncertainties and mitigating risks (Goranson, 1999; Lee, 2003; Lummus *et al.*, 2003; Rice and Caniato, 2003; Christopher and Peck, 2004; Christopher and Rutherford, 2004; Tang, 2006; McManus *et al.*, 2007; Asbjørnslett, 2008). These terms either complement the topic of resilience or are used interchangeably with it.

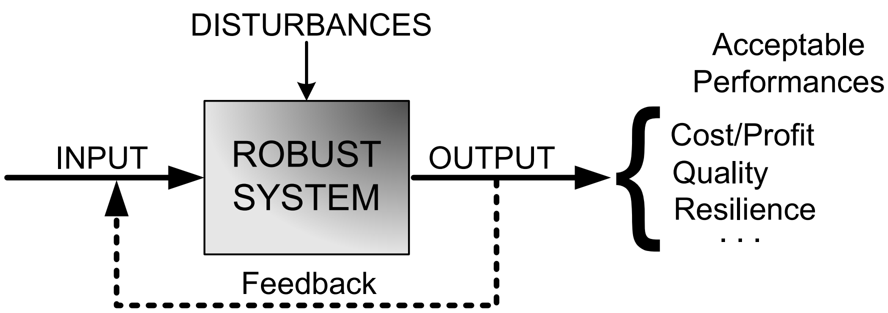
Of all these terms, robustness has been used in supply chain research interchangeably with resilience the most frequently. For instance, Asbjørnslett (2008) states that a "supply chain is robust, or resilient, with respect to a threat, if the threat is not able to produce any 'lethal' effects on the system". This means that both robustness and resilience involve post-disturbance recovery. According to Asbjørnslett, what differentiates a robust system from a resilient system is that the former has the ability to resist a disturbance and retain the same previous state. The latter has the ability to adapt and achieve a new stable situation. The latter definition is more in line with the resilience definition previously given by Christopher and Peck (2004) previously given in Section 2.3.

According to Christopher and Rutherford (2004), robustness differs from resilience by having 'Lean Thinking' as the central strategy while risk management is a key strategy to achieving a resilient supply chain. Moreover, they argue that, since a robust system is able to respond to reasonable variations and a resilient system responds to major changes in input, a resilient supply chain will be robust while the reverse is not always true.

2.5.1 Robust control systems

A control engineering definition of robustness will be used since it is consistent with the approach utilised in this thesis. A system is robust when the system has acceptable changes in performance due to model or parameter changes and moderate modelling errors (Dorf and Bishop, 1998). Hence, each system design has to define which performances should be retained in case of disturbances. In this thesis, the performances in question are supply chain resilience and system responses. So, a robust supply chain should be designed to function properly even in the presence of uncertain parameters, for instance the lead-time. Therefore, only the changes in system parameters are considered when accounting for robustness. In contrast to other supply chain authors (Christopher and Rutherford, 2004), this research has found that changes in input are not relevant in determining whether a supply chain is robust.

Figure 2.3 illustrates the difference between resilience and robustness. In Figure 2.3(a), robustness represents the characteristic of the system which should be designed to retain performance even in the case of disturbances, model inaccuracies and changes. Resilience is the performance of the output which should return to its original state after being disturbed. Since this work will be looking at uncertainties caused by control systems and supply chain dynamics, in other words changes in the systems parameters, resilience and robustness will be assessed and compared.



(a) A robust system under disturbances

Changes in :	Resilience	Robustness
System structure	✓	
Parameters	✓	✓
Input types	✓	

(b) Changes accounted by resilient and robust systems

Figure 2.3: Difference between resilience (system performance) and robustness (system characteristic)

2.6 Quantitative work on supply chain resilience

Despite the increasing number of publications on supply chain resilience, there are few studies which attempt to create a quantitative framework for assessing supply chain resilience performance (Datta *et al.*, 2007; Falasca *et al.*, 2008; Ratick *et al.*, 2008; Colicchia *et al.*, 2010b; Carvalho, 2011). Most existing studies have provided more qualitative insights into the problem and focus more on identifying sources of risks and on determining mitigation and contingency strategies. Preferred methods in the study of supply chain resilience has been theory building and case studies (Bhamra *et al.*, 2011). Quantitative researchers have focused on reducing the likelihood of the occurrence of disruptive events and/or developing means of overcoming disruptions if such events occur (Tomlin, 2006; Lodree Jr. and Taskin, 2007; Wilson, 2007; Tang and Tomlin, 2008; Mitra *et al.*, 2009; Skipper and Hanna, 2009; Wagner and Neshat, 2010; Schmidt and Singh, 2012). At no point, they explicitly state that supply chain resilience is achieved and measured. Despite that, the supply chain risk management literature refers to some of those authors as contributors to supply chain resilience research and for this reason they will be reviewed. A summary of these works can be found in Table 2.1 in which the first five rows refer to supply chain resilience-focused research and the remaining are concerned with supply chain risk-focused research.

To the author's knowledge, the first attempt to analytically assess supply chain resilience was made by Datta *et al.* (2007). The authors evaluated the impact of different strategies when considering the dynamics of demand, production and distribution functions. They considered the Customer Service Level (CSL), average inventory level and production change-over time to assess the operational resilience. In summary, they found that flexibility of production and distribution procedure is a key factor in coping with demand changes. However, their model does not consider any other factors, such as cost, that would enable trade-off analysis.

Falasca *et al.* (2008) developed a simulation-based framework for helping managers to (re-) design supply chains in order to be resilient against environmental uncertainties. Despite being only a theoretical framework, the authors addressed the necessity of minimising the immediate impact caused by disruption and the time to recover, and therefore minimising the 'resilient triangle' (Tierney and Bruneau, 2007), a concept that will be covered in Chapter 4, Section 4.1. When designing supply chains, Falasca *et al.* (2008) argue that nodes' criticality, complexity and density should be taken into account. Also looking at environmental factors and supply chain design, Ratick *et al.* (2008) developed a model which helps to allocate a cost-effective number of facilities in areas of different geographical risk factors.

Author	Method	Source of Disruption	Performance Measures	Mitig./Conting. strategies	Main findings
Datta *et al.* (2007)	Agent-based computational modelling	Demand Process (production and distribution)	Resilience is assessed by: • max. CSL • min. production change-over time • min. stockout with least stock levels	Collaboration (info. sharing) Members autonomy Early sensing Quick response Manufacturing flexibility	Flexibility of production and distribution processes is the key factor to cope with demand changes. Early sensing is a key element for agility and visibility should not be limited to central planning authorities.
Falasca *et al.* (2008)	Simulation-based framework (theoretical)	Environmental factors	Resilience can be assessed by: • min. immediate impact • min. time to recover	SC re-engineering/re-design	SC resilience depends on nodes' criticality, complexity and density. Hence SCs should be designed with the objective to minimise the 'resilience triangle'.
Ratick *et al.* (2008)	Location model formulation and linear programming	Environmental factors	Resilience is assessed by: • max. facility locations • min. total costs	Capacity redundancy (emergency backup and storage facilities) SC re-engineering/re-design	It is important to take into account potential exposure of facilities to the same geographically-related risk factors and the costs associated with opening or maintaining facilities when designing supply chains.
Colicchia *et al.* (2010b)	Monte-Carlo simulation applied to real scenario	Supply	Resilience is assessed by: • min. supply lead-time length • min. supply lead-time variation	Re-routing (bypassing some Hub & Spoke feeders and change in mode choice) Pre-booking containers Collaboration with custom authorities and shipping companies	Mitigation strategies do not influence lead-time variability but it reduces considerably lead-time average. Reducing lead-time variability will lead to resilience but not necessarily in a cost-effective way. In some cases only reducing lead-time length is sufficient.
Carvalho (2011)	Exploratory case study and structured interviews (inductive approach)	Supply Process (production capacity)	Resilience is assessed by: • max. on-time deliveries • min. damping time (time to respond) • min. recovery time	Capacity management Agility and quick response Production flexibility Re-routing Collaboration (info. sharing)	Using empirical data the authors developed a model to create a supply chain resilience index. In this way companies were compared in terms of their resilience performance.
Tomlin (2006)	Discrete time Markov process	Supply	Supplier's % uptime Length of disruptions Supplier's recovery time	Inventory redundancy Sourcing from more reliable supplier (more expensive) Re-routing	The best strategy will depend on the nature of disruption (impact x likelihood), supplier's capacity and flexibility and costs incurred by adopting each strategy.
Lodree Jr. and Taskin (2007)	Newsvendor model and its variants	Demand Environmental factors	Inventory levels CSL	Inventory management	The increasing relation between the inventory needed to cope with disruptions and the risk probability is not linear. Managers can and should maintain low probabilities of disruptions to keep high CSL.

Table 2.1: Continues...

Author	Method	Source of Disruption	Performance Measures	Mitig./Conting. strategies	Main findings
Wilson (2007)	System dynamics	Process (transport)	CSL Inventory levels Goods in transit	Collaboration (VMI)	The impacts on CSL, inventory levels and goods in transit are more severe for a traditional SC than for a collaborative system such as VMI.
Tang and Tomlin (2008)	Combination of analytical methods such as calculus and linear programming	Supply Process (manufacturing) Demand	Supply cost Increase in profit Effective sales Percentage savings	Multiple suppliers Flexible supply contract Flexible manufacturing processes Postponement Demand management	Supply, process and demand risks can be mitigated by investing in flexibility. There is no need to keep high levels of flexibility, since benefits can be perceived already with low levels of flexibility.
Mitra *et al.* (2009)	fuzzy mathematical programming	Demand Process (production)	Demand satisfaction Costs	NA	Trade-off between total cost and demand satisfaction are shown via a Pareto analysis. Scenarios with higher degrees of uncertainty results in higher cost and lower customer satisfaction.
Skipper and Hanna (2009)	multiple regression techniques	Not specified	Flexibility (function of mitigation strategies)	Top management support Goal alignment Resource alignment IT usage Information sharing Standardization Collaboration (internal and external)	Top management support, resource alignment, IT usage and external collaboration are the largest contributors to enhance flexibility, which in turn minimises supply chain risks.
Wagner and Neshat (2010)	Graph modelling	Demand Supply SC structure	Supply chain vulnerability index	NA	Supply chain vulnerability can be assessed better by understanding its drivers. Managers should use graph theory for supporting strategic planning.
Schmidt and Singh (2012)	Simulation (Arena)	Demand Supply	Customer fill rate Inventory levels	Quick response Capacity redundancy (inventory)	Reducing risk at a single location in the network may not be helpful if the rest of the supply chain is too vulnerable. Priority should be given on the weakest link.

Table 2.1: Quantitative studies on supply chain resilience and risk management

Focusing on supply uncertainties, Colicchia *et al.* (2010b) use the length and variation of the supply lead-time as indicators of supply chain resilience. They argued that a better understanding of the risk sources for specific supply chain settings can enable the design of a more resilient supply chain. Also based on the concept of the 'resilient triangle' (Tierney and Bruneau, 2007) and using exploratory case studies and empirical data, Carvalho (2011) developed a model to create a composite performance measure: the resilience index. By applying structured interviews and calculating the resilience index, the authors could compare different companies' resilience performance. However, many of the metrics used depend on qualitative perception and personal judgement from managers and are subjected to possible bias. Moreover, the measure suggested by Carvalho (2011) is suitable for analysing current state business processes but is not applicable to investigating 'what if' scenarios.

Tomlin's (2006) model considered the supplier's percentage uptime and the length of disruption which indicate the level of risk that supply chains are exposed to. While Tomlin (2006) determined economical choices of mitigation and contingency strategies in order to overcome unreliable supply, Lodree Jr. and Taskin (2007) assessed the effects for the customer side. Their work evaluated the impact of demand uncertainty and occurrence of an extreme event (such as a disaster) on inventory levels and customer service levels by finding stockout probabilities. They compared the inventory levels in the classic newsvendor solution with levels needed in the case of uncertain situations. Similarly, Schmidt and Singh (2012) undertook a simulation study to investigate how risks in both the supply and demand sides affect inventory levels and customer fill rate. Their work shows the importance of having a holistic view of the supply chain when applying mitigation strategies.

Tang and Tomlin (2008) and Skipper and Hanna (2009) demonstrated the importance of flexibility on mitigating supply, demand and process risks. Complementarily, while the latter shows through regression techniques that top management support, resource alignment, IT usage and external collaboration enhance flexibility, the former evidenced how flexible activities, such as manufacturing processes, postponement, adjustable supplier contract and demand management through flexible pricing can improve supply chain performances. Finally, both Mitra *et al.* (2009) and Wagner and Neshat (2010) developed techniques that support trade-offs visualisation and the understanding of many risk drivers.

The only work found that applied a system dynamics research method was undertaken by Wilson (2007). The author analyses how a more collaborative supply chain, such as the vendor-managed inventory (VMI) can help to overcome the impact caused by disruptions in transport processes on customer service levels, inventory levels and goods in transit.

2.7 The role of system dynamics in supply chain performance

As long as fifty years ago supply chains were recognised as a dynamic system (Forrester, 1958) and such dynamics were reported to lead to a cost increase for the supply chains. Since then many studies have investigated the causes of such dynamic behaviour in supply chains and proposed mitigating solutions. Forrester's pioneering work produced evidence of variability between production orders and actual consumer demand, encumbering the demand visibility of the last echelon. He deduced that this variability and consequential demand amplification are directly related to material and information delays, feedback loops in the decision making process and nonlinearities present in the system. Therefore, counter measures for this problem would be reducing unnecessary echelons within the system, compressing time and taking due consideration of the design of feedback systems (Wikner *et al.*, 1991).

Burbidge (1961) identified that stock control based on Economic Order Quantity (EOQ) is another source of demand distortion and amplification. Hence, while the Forrester effect is associated with structural dynamics in the supply chain, the Burbidge effect is related to operational decisions, such as scheduling, batching policies and order priorities (Towill, 1997a). Thus, Burbidge recommends the reduction of material throughput time and the use of an ordering strategy that synchronises order flow and minimises batch sizes.

Sterman (1989) demonstrated via a table top management simulator, the Beer Game, that the dynamic distortions and amplification in a supply chain are also caused by human misperceptions about inventory and demand information. His suggestion was that improving education, awareness and communication lead-time between parties would mitigate the problem.

Later, the phenomenon of demand amplification was experienced by Procter and Gamble and became widely known as the bullwhip effect (Lee *et al.*, 1997a). Unlike Sterman, they concluded that even the rational behaviour of the decision-maker can cause demand amplification. They pointed out four main causes of the bullwhip effect: demand signalling as per Forrester, order batching as per Burbidge, fluctuating prices and shortage gaming. Information sharing, lead-time reduction, single replenishment control, smart price strategies and supply conditions are some of the main counter measures proposed by them.

More recently, studies have attempted to describe and understand the distortions that also occur in the freight transport activities. This was first discussed by Holweg and Bicheno (2000) who observed through case studies an "amplified and distorted supply pattern" in a steel supply chain and referred to it as 'reverse amplification'. They affirmed that this effect was caused by supply or throughput constraints since order backlog builds up when there are supply constraints. Later, Shukla *et al.* (2009) demonstrated through simulation studies that, even under unconstrained supply, deliveries are commonly higher for the upstream company. Moreover, they further noted that shipment profiles are normally attenuated as they move downstream in the supply chain. Shukla *et al.* (2009) found that this so-called backlash effect was a reflection of the bullwhip effect, analogous to physical waveforms in a channel or pipe and can lead to high transport costs due to inefficient scheduling and premium transport rates.

Much more research has been done to measure and reduce demand amplification (Fransoo and Wouters, 2000; Chen *et al.*, 2000; Dejonckheere *et al.*, 2003; Disney and Towill, 2003b; Dejonckheere *et al.*, 2004; Lee and Wu, 2006) and its impact on supply chain performance, for instance transport operations (Potter and Lalwani, 2008; Juntunen and Juga, 2009; Marques *et al.*, 2010), financial performance (Torres and Maltz, 2010) and production operations (Bicheno *et al.*, 2001; Wikner *et al.*, 2007; Cannella *et al.*, 2008; Hamdouch, 2011).

Also, some case studies evidenced the impact of system dynamics on supply chain performance and proposed countermeasures. These include the supply chain in the grocery, food, automotive, personal care, toys, hardware and electronic industries (Edghill *et al.*, 1988; Berry and Towill, 1992; Berry and Naim, 1996; Lee *et al.*, 1997b; Mason-Jones *et al.*, 1997; Higuchi and Troutt, 2004; Georgiadis *et al.*, 2005;

Kumar and Nigmatullin, 2011).

It is claimed that to improve supply chain performance, dynamics in the system should be reduced (Torres and Maltz, 2010). Hence, there is plethora of literature researching the bullwhip effect and its effect on supply chain performance, from both a quantitative modelling perspective, either conceptually or based on empirical studies, and a descriptive perspective in the form of case studies. However, emphasis has been given to financial performance measures. For instance, most investigations have been done into the impact of supply chain dynamics on inventory, production and transport costs. Even when service levels and customer satisfaction are considered, these have been seen as service penalty costs.

2.7.1 Effects of nonlinear system dynamics

Forrester's work on industrial dynamics calls attention to the importance of considering nonlinear models to represent industrial and social processes. "Nonlinearity can introduce unexpected behaviour in a system" (Forrester, 1968), causing instability and uncertainty. Despite this, the literature still focuses on 'presumably' linear models. The reason for this is that while the linear systems theory is well established, the literature lacks a unique nonlinear theory that strives for generality and applicability (Rugh, 2002). Because there is still a debate in the literature of nonlinear systems dynamics in the natural science domain, a lack of clarity reflects the research carried out in the social sciences. Thus the research methods and predominance of sole use of simulation are very common issues in business systems dynamics studies (Forrester, 1961; Sterman, 1989; Evans and Naim, 1994; Wikner *et al.*, 2007; Shukla *et al.*, 2009; Poles, 2013).

Much of the analytical work done in nonlinear systems dynamics seems to have been undertaken in the same decade Forrester launched the World Dynamics model (Forrester, 1971). Cuypers (1973) used averaging techniques for linearising discontinuous nonlinearities in the World Dynamics model. One year later, numerical perturbation techniques and model simplification by removing variables with little variation were also explored (Cuypers and Rademaker, 1974). Ratnatunga and Sharp (1976) proposed the use of numerical analysis to linearise and reduce orders of system assuming that nonlinear associations can be approximated to a first order function. In 1980, Mohapatra established the importance of classifying the different types of nonlinearities that commonly appear in business dynamics studies in order to apply suitable techniques to each of them. Among them are CLIP functions, table functions and product operators. He also suggested some nonlinear control theory techniques including small perturbation theory and linearisation through averaging, but no implementation of such methods were carried out in his paper.

In 1992, Wikner *et al.* undertook an in-depth analysis of the complex Forrester Industrial Dynamics model. By using average techniques and block diagram manipulation, they linearised and simplified the original model and provided qualitative analytical insights into Forrester's simulation model. For instance, they highlighted the lack of feedback information fed into the manufacturing rate and a separation between 'real' and 'safety' orders. By following the same simplification and linearisation steps, Naim *et al.* (2012) achieved the same result for the discrete z-domain model, which allowed them to make analogies between Forrester's (1958) and Burns and Sivazlian's (1978) models. The latter also demonstrated the impact of the 'false order', which is a combination between 'real' and 'safety' orders that account for delays in the system. More recently, Jeong *et al.* (2000) used small perturbation theory to find state space representations of three echelons in a variant form of the industrial dynamics model. However, they do not compare the linearised model with the original one and only a simulation technique is used for the analysis of the model.

With the advance of computer technology most of the recent research has been undertaken through computer simulation. The Beer Game (Sterman, 1989), a table top board simulation game, has been translated into a computer simulation model and has been studied by many authors seeking to understand particular phenomena, such as stability, chaos, bullwhip and backlash, and to improve systems performance (de Souza *et al.*, 2000; Laugesen and Mosekilde, 2006; Hwarng and Xie, 2008; Shukla *et al.*, 2009; Marques *et al.*, 2010). Also in supply chains, other simulation models have been used to investigate the effect of capacity and batching constraints (Evans and Naim, 1994; Wikner *et al.*, 2007; Cannella *et al.*, 2008; Juntunen and Juga, 2009).

Evans and Naim (1994) used the well established inventory and order based production control system archetype to simulate eight scenarios by changing the combination of the capacity levels of each echelon in a three echelon supply chain. They concluded that capacity constraints do not always degrade the entire supply chain performance and they found 'secondary dynamics' caused by such nonlinearities. However, the sole use of simulation methods prevents such behaviours being analysed in more depth because it is hard for the designer to identify underlying relationships between variables.

In the early 1960s, Forrester concluded that nonlinearities played a central role in the dynamics of complex systems. With few exceptions, such as the Lotka-Volterra predator-prey model in biological systems (Holling, 1959), operational research, economics, and other dynamical fields were dominated by linear models at that time

(Lane and Sterman, 2011). With his experience in servo-mechanics, Forrester perceived that social, economics and industrial systems were also intrinsically nonlinear and approximating such complex systems to linear systems could be an exhaustive task. However, advances in the nonlinear control theory by engineering and mathematical sciences have been made since then and, despite this field still being an area for debate, new tools and techniques to deal with high-order, nonlinear systems with multiple loops have been further developed. Despite many analytical methods being cited and recommended by business dynamics scholars 30 years ago, they have been disregarded by recent studies, in which simulation techniques are still predominant.

2.8 Summary

In this chapter, the concept of resilience has been explored. A holistic definition for supply chain resilience was stated by Ponomarov and Holcomb (2009) and is regarded as "the adaptive capability of the supply chain to prepare for unexpected events, respond to disruptions, and recover from them by maintaining continuity of operations at desired levels of connectedness and control over structure and function". In addition to this, the term resilience has been distinguished from robustness, since they have been used interchangeably by supply chain scholars. Because this topic has its origins in other fields of research, an overview of its use in the natural and social sciences has been provided. In business, resilience studies were motivated by issues of security and risk management. Companies' performances in the global market are being threatened by terrorist attacks, environmental disasters and financial crises. The central role of control systems in synchronising demand, supply and production processes has also been pointed out as a source of risk. Hence, the failure of this operation can be devastating.

Figure 2.4 summarises what has been found in the literature regarding sources of risk and strategies to anticipate, mitigate and overcome disruptions. Moreover, the figure also lists the criteria that were used to evaluate supply chain resilience and/or disruption likelihood and their characteristics (these measures are listed followed by an asterisk). However, some of these works designed models which are more appropriate to evaluating resilience of individual companies and not the supply chain as a whole. For instance, in Figure 2.4 it is argued that resilience should be measured at the interface between the supply chain and the end customer regardless where in the supply chain the disruption occurred. Minimising risk may lead to supply chain resilience but resilience should not be assessed only by evaluating the disruption aspects. For example, the impact of a long or short disruption in the supply of raw material on customer service may be the same depending on the inventory policy chosen by the downstream companies. Of course the cost of keeping inventory sufficient to cover a long disruption is higher, but being resilient is reported to be expensive and managers have to find a balance between cost and resilience. In other words, a systems view of supply chain resilience is defended since in many cases, the effort of mitigating a type of disruption might initiate another disruption elsewhere. For instance, re-routing shipments may affect the transport available capacity.

In exploring the resilience-related supply chain literature, research question RQ:1a has been answered and evident gaps in the literature were noticed. It is clear that, from the amount of work cited in this literature review, there is a predominance of qualitative studies. Very little research has attempted to access the effectiveness of mitigation and contingency strategies employment to reduce the risk of disruption and/or increase supply chain resilience. Moreover, only five out of twelve quantitative studies actually focus on measuring resilience.

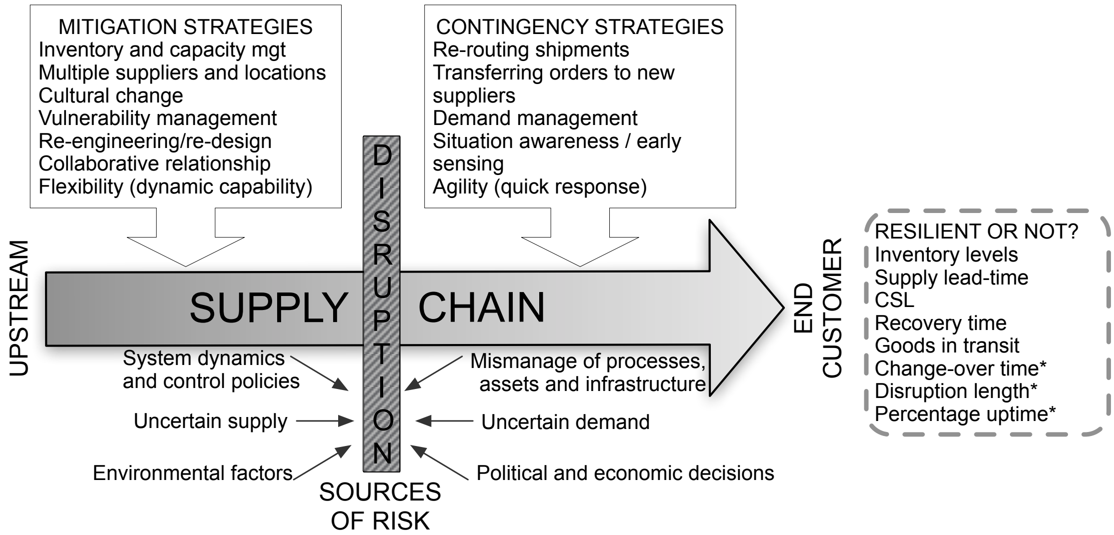


Figure 2.4: Supply chain strategies for improving resilience against disruption

Table 2.2 highlights the scope of this research and its intended contribution to the literature. No work has been found to analyse the impact of systems dynamics on supply chain resilience, despite the fact that this source of risk has been highlighted by Mason-Jones and Towill (1998) as a central activity (Figure 2.1) and observed by Colicchia *et al.* (2010a) as the most frequent reason for the occurrence of supply chain disruptions as previously stated in Section 2.3.1.

From this research, Research Questions RQ:1b and RQ:1c have been formulated. Hence, in this thesis, a system dynamics approach is taken to create an analytical framework for assessing supply chain resilience. Moreover, this work focusses on analysing the impact of system dynamics and different control policies on resilience performance. Therefore, this thesis will investigate a mitigation strategy of supply chain (re-)design, in order to find the best control policies for designing resilient supply chains. In addition to this, the use of a composite performance measure is proposed. This means that both dimensions, time and variation, are taken simul-

Author			Tomlin (2006)	Datta *et al.* (2007)	Lodree Jr. and Taskin (2007)	Wilson (2007)	Falasca *et al.* (2008)	Ratick *et al.* (2008)	Tang and Tomlin (2008)	Mitra *et al.* (2009)	Skipper and Hanna (2009)	Colicchia *et al.* (2010b)	Wagner and Neshat (2010)	Carvalho (2011)	Schmidt and Singh (2012)	This thesis
Research focus		Resilience		✔			✔	✔				✔		✔		✔
		Risk	✔		✔	✔			✔	✔	✔		✔		✔	
Source of disruption		Demand		✔	✔				✔	✔			✔		✔	
		Process		✔		✔			✔	✔				✔		
		Environment			✔		✔	✔					✔			
		Supply	✔						✔			✔	✔	✔	✔	
		System dynamics and control policies														✔
Performance measures	Separated	Time	✔	✔								✔			✔	
		Variation	✔	✔	✔	✔		✔	✔	✔	✔	✔	✔		✔	
	Composite	Time and Variation					✔*							✔**		✔
Resilience strategies	Mitigation	Invent. /capacity mgt	✔		✔			✔						✔	✔	
		Multiple supliers/locat.							✔							
		Cultural change		✔							✔					
		Vulnerab. mgt	✔									✔				
		Re-design					✔[A]	✔[A]								✔[B]
		Collaboration		✔		✔					✔	✔		✔		
		Flexibility		✔					✔		✔			✔		
	Contingency	Re-routing	✔									✔		✔		
		Demand mgt							✔							
		Early sensing		✔												
		Quick response		✔										✔	✔	

* The authors presented the simulation-based framework but did not conduct experiments
** Only applicable to case studies, specifically in automotive supply chains
[A] Re-design through strategic positioning of supply chain parties
[B] Re-design through control systems analysis

Table 2.2: Scope of this thesis - SCM theory contributions

taneously into account. The literature postulates that resilience implies not only minimising deviations from a targeted state, but also re-achieving this target as fast as possible. A unique measure will simplify and aid the process of finding the design that results in the best supply chain resilience performance.

In Table 2.2, it was evident that not much research has been undertaken in measuring supply chain resilience and, despite the importance of control systems, the impact of control policies on resilience does not seem to have been previously investigated. Finally, when reviewing the literature on supply chain dynamics, there is a predominance of studies attempting to measure and mitigate demand amplification and to link bullwhip with the increase in production, inventory and transport costs and reduction in service level. In addition to this, the author has found notable gaps in the way research is undertaken in the fields of system dynamics.

Figure 2.5 illustrates a typical process for analysing supply chain dynamics. Some research endeavours to understand supply chain systems by observing the actual physical system. This type of research serves as a base for the modelling process, since models created by supply chain researchers should reflect the real system.

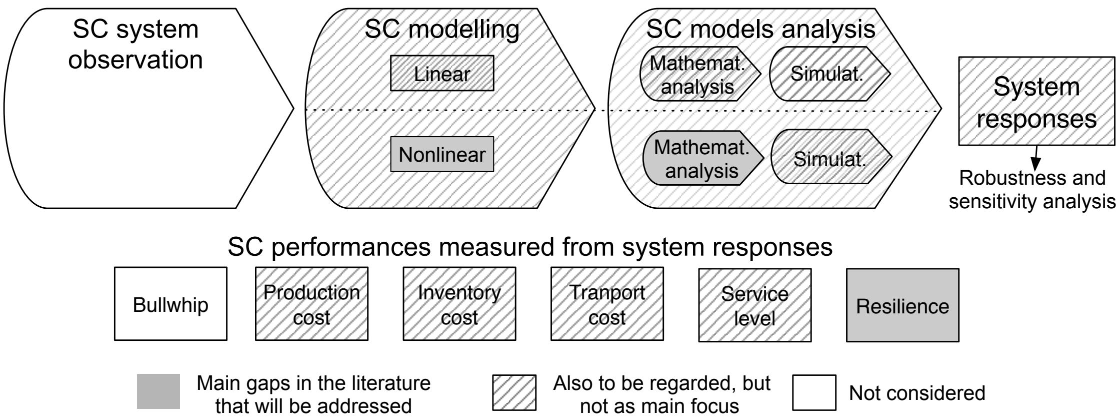


Figure 2.5: Scope of this thesis - SCM methodology contributions

Then, the modeller can choose to represent the real system in a linear or nonlinear form, depending on the accuracy needed and on his/her objectives and intentions. As this literature review has shown, supply chain systems are composed of delays, local decision rules and policies, and multiple loops with both positive and negative feedback. In the past, much research has been undertaken to investigate the impact of such complexity on system performances in a linear environment. With the advance of computer, Forrester (1958) has advocated that nonlinearities should be considered in studies of supply chain systems for the reason that they can cause instability and uncertainty in the system's behaviour; so he proposed the use of systems dynamics simulation. However, it is also suggested that simulating without previous analytical investigation can be time consuming, expensive and unrewarding (Atherton, 1975).

In order to address these gaps, the author has formulated research questions RQ:2a and RQ:2b. Firstly, the author will link the literature of nonlinear system dynamics methodology (Chapter 3) used in engineering and mathematics with the models used in supply chain research. Then, suitable methods will be chosen depending on the supply chain models and on the different types of nonlinearities involved. A comparison between analytical and simulation methods will also be undertaken to check the validity and limitations of these nonlinear system dynamics techniques on aiding the understanding on nonlinear systems. Finally, both analytical and simulation methods are used in combination to understand the impact of nonlinearities on system responses and on supply chain resilience.

3 Methodology

> *"Strange that these nonlinear phenomena that abound so widely in nature should be intractable. It is almost as if Man is to be denied a complete knowledge of the universe unless he makes a superhuman effort to solve its nonlinearities"*
>
> – Ladis D. Kovach (1960), *Life can be so nonlinear*

The previous chapters established the subject matter of this research and highlighted the relevant gaps that will be addressed through a consideration of the research questions. This chapter will explain how this research has being carried out including the research ontological and epistemological positions, research design, methods and tools used.

This chapter will first outline the ontological and epistemological underpinnings of supply chain management research and the philosophical stance considered in this thesis. Next, further details on the research methods and tools used will be provided. This includes the literature review process and a review of the analytical and simulation models used. Finally, the research design used to answer the research questions will be explained.

3.1 Research philosophies and paradigms

A research paradigm consists of an ontology, an epistemology and methodology (Blanche *et al.*, 2007). Ontology involves a set of assumptions about the "nature of reality" or the "nature of knowledge". Defined as the science of being, it questions whether reality naturally occurs, or whether reality is a construct of social interaction between individuals. In other words, it enquires whether reality is viewed from an objective or subjective perspective. Epistemology, on the other hand, refers to the assumptions and declarations made about the ways in which knowledge of the reality is obtained (Saunders *et al.*, 2009). These ontological and epistemological assumptions influence the chosen methodology, which is the means of how the knowledge of the world is gained. Methodology is the basis and rationale behind the selection of methods and collection of concepts, ideas and theories (Bryman and Bell, 2007).

When selecting a research strategy, trade-offs between control, realism and generalisation will certainly be encountered. Normally, quantitative research methods attempt to optimise control and generalisation (external validity), while qualitative research endeavours to maximise realism (internal validity) (Golicic *et al.*, 2005). Hence, it is very important to understand the implications of the epistemological considerations and the chosen methods when undertaking social science research.

In order to define the ontological and epistemological position for this research, the author has thought through what would be the nature of the phenomena to be investigated and looked in the literature for ontological and epistemological underpinnings of supply chain management research.

3.1.1 The nature of supply chain management research

Since the acknowledgement of supply chain management is an important business and research area, there has been some debate on what might constitute the philosophical nature of this field. However, the academic debate on the paradigmatic, disciplinary and theoretical state of supply chain management research is still very limited (Wolf, 2008).

In addition to this, in such a wide field that encompasses different constructs and research streams, such as leadership, intra- and inter-organizational relationships, logistics, process improvement orientation, information systems, business results and outcomes (Burgess *et al.*, 2006), marketing, strategic management, law and systems engineering (Giannakis and Croom, 2004), there is a need to understand the philosophical nature of individual theories resting within supply chain management. As pointed out by Arlbjørn and Halldórsson (2002), since researchers in logistics may have different academic backgrounds, this will lead to different epistemological perceptions of logistics problems.

3.1.1.1 Research designs and methodologies in supply chain management research

Many logistics scholars affirm that logistics and supply chain management are steeped in the positivist paradigm and that research is primarily normative and quantitative (Mentzer and Kahn, 1995; Näslund, 2002; Mangan *et al.*, 2004). However, "the problem is that this claim is not supported by comprehensive evidence, but is rather a legacy - or myth - that has been brought further" (Aastrup and Halldórsson, 2008). Most studies have only considered the two extreme positions: positivism and interpretivism or non-positivism. Hence, in order to reach their conclusions most researchers looked at whether quantitative and qualitative methods were being undertaken in supply chain management. "The relationship between epistemology and method should not be reduced to a simplistic 'quantitative versus qualitative' debate" (Duberley and Johnson, 2005).

In order to choose the most suitable methodology for this research, a comprehensive review of existing research designs and methodology in supply chain management research will be provided and debated

The two ends of the paradigmatic spectrum

Some authors used existing frameworks developed in other fields to find paradigms within supply chain management and/or logistics research. For instance, using the Meredith model (Meredith *et al.*, 1989) to identify and analyse different types of research within logistics, Dunn *et al.* (1994) suggested that logistics research can be categorised into three areas: generalised descriptions of variables (direct/natural), interpretation of informant impressions (perspective) and reconstruction of reality (artificial). However, their consideration of a rational-existential continuum forms the basis as to whether the research is inductive or deductive only. Hence, different research reasonings and intermediate ontological approaches have not been taken into account.

Burgess *et al.* (2006) reviewed a total of 100 random journal articles on supply chain management research in order to group those works into descriptive features, definitional issues, theoretical concerns and research approaches. In order to determine the paradigmatic approach of the reviewed papers, the authors consider the framework developed by Burrell and Morgan (1979). Again, in this model there is an ontological assumption of either an objective or subjective reality. Hence, Burgess *et al.* (2006) limited their search of methodologies in supply chain management research to the two ends of the paradigmatic spectrum: positivist and non-positivist approaches.

Applied methods in supply chain management research

Other studies have assessed supply chain management and/or logistics research by identifying the research methods applied. By reviewing articles published in the Journal of Business Logistics from 1978 to 1993, Mentzer and Kahn (1995) assessed the state of logistics research and found that during that period of time most published articles in that journal were mainly concerned with normative research and exploratory studies. This suggests that logisticians have found a large degree of substantive justification for further study but little theory developing and testing. Ten years later, Sacha and Datta (2005) examined the state of logistics and supply chain management research but during a later period (between 1999-2003) and found that survey methods were still the most used tools of research but supply chain management research trend had been moving from exploratory research to model building and testing.

Frankel *et al.* (2005) also claimed that logistics research is based on methods within the detached, objective, external perspective (i.e. experiments, surveys, literature/-document studies) with surveys as the primary research method. They affirm that this leaves a "white space" in understanding logistics with an involved, subjective, cognitive perspective. Aligned with this idea, Näslund (2002) points out the need for qualitative anti-positivist research in logistics research. Once more, the supply chain management understanding was summed up as objectivity versus subjectivity, quantitative versus qualitative methods, deduction versus induction and positivism versus non-positivism.

Considering an intermediate approach

Kovács and Spens (2005) searched for a new logical reasoning into supply chain management research: the abductive approach. The term abduction combines elements of deduction and induction, rationalism and empiricism (Samuels, 2000). Figure 3.1 illustrates the paths of reasoning the abductive approach in comparison with the deductive and inductive processes. The abductive approach starts in the same way as the inductive, but it makes a loop between the theoretical framework process and real-life observation before. Then, after the definition of the research questions the abductive process ends like the deductive by applying or testing the hypothesis (H) or propositions (P) and contributing to the theory. As stated by Sayer (2000), abduction is a "mode of inference in which events are explained by postulating (and identifying) mechanisms which are capable of producing them". According to Peirce (1931) this term was mistranslated from Greek and should be called retroduction instead. Hence, these two terms are used interchangeably.

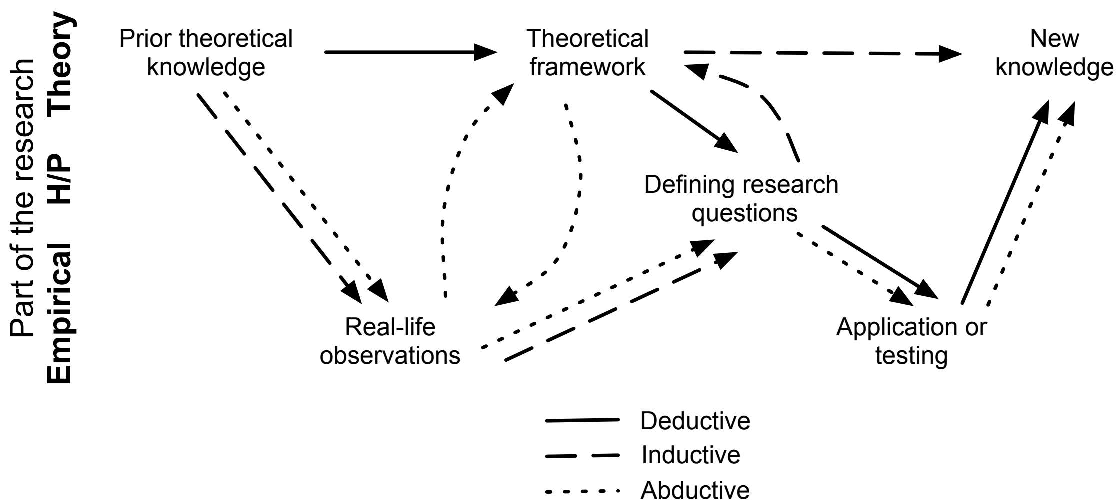


Figure 3.1: Deductive, inductive and abductive research process
Sources: Kovács and Spens (2005)

Kovács and Spens (2005) pointed out the importance of abduction for theory development in a new field like logistics that borrows theories from other scientific fields. Despite not initially finding any logistics research article referring to the abductive approach, later (in Spens and Kovács, 2006), the same authors found, by using a content analysis of the literature, that some logistics authors followed an abductive reasoning. In these studies, the theory-building is made by combining theory and empirical study and then the authors refine the theory by applying their initial findings in a different empirical study closing the abductive loop. The use of quantitative methods such as mathematical and computational modelling and cost and accounting data analysis are also found in these papers.

A work which considered an intermediate school of thought within logistics research was undertaken by Gammelgaard (2004) who used a methodological framework from Arbnor and Bjerke (1997) to categorise existing supply chain management research into three groups: analytical, systems and actors' approaches (Table 3.1). According to the analytical approach of logistics, there is an objective reality that can be decomposed into smaller elements and then studied by hypothesis development and testing. From an integrated systems perspective, such decompositions are meaningless. Researchers of this tradition strive for the holistic, as opposed to reductionistic, understanding of system parts, links, goals, and feedback mechanism in order to improve the system. Finally, for advocates of the actors' approach, reality is not objective but the result of social constructions, which means that knowledge creation, depends on the researcher's interpretation and social actors. Moreover, since the actors approach is highly contextual there is a tendency towards a more qualitative and inductive research.

Through the research questions, the author has revealed that her research is very

	Analytical approach	Systems approach	Actor's approach
Theory type	Determining cause-effect relations. Explanations, predictions. Universal, time and value free laws	Models. Recommendations, normative aspects. Knowledge about concrete systems	Interpretations, understanding. Contextual knowledge
Preferred method	Quantitative (qualitative research only for validation)	Simulation and Case studies (qualitative and quantitative)	Qualitative
Unit of analysis	Concepts and their relations	Concepts and their relations	People - and their interaction
Data analysis	Description, hypothesis testing	Mapping, modelling	Interpretation
Position of the researcher	Outside	Preferably outside	Inside - as part of the process

Source: Gammelgaard (2004)

Table 3.1: Methodological framework for supply chain management research

much related to the systems thinking approach since the nature of the phenomena to be investigated by supply chain dynamics involves questions of causality between the different elements (perceived demand, value-adding process, information currency, delays, current stocks, work in process, production rates, inventory targets, capacity) and mechanisms (how these elements in one level can affect other levels and the whole supply chain performance) in the system of interest (the supply chain). However, while the literature directly indicates the analytical approach as being related to a deductive positivism and the actor's approach related to an inductive interpretivism, the systems approach is not clearly identified as part of any of the social science schools of thought. The systems approach is also theory-driven but this theory is contextual rather than universal. The reality is still considered objective and can be susceptible to influence, and thus it is preferable that the researcher stands outside the research object.

3.1.2 Ontological position: questions of objectivity

Despite the lack of discourse given to paradigmatic issues within supply chain management research, the author is very clear in relation to her ontological position. Since the author believes that the phenomena to be investigated exist independently of her own perceptions and interpretations, the ontological assumption for this research topic will be pronounced as objective. This means that the supply chain organisation will be seen as a tangible object that obeys rules, regulations and hierarchy and adopts standardised procedures for getting things done.

3.1.3 Possible epistemological positions: schools of thought

From questions of objectivity, there are three schools of thought in social science: positivism, empiricism and critical-realism. "The empiricist school of thought believes that the facts speak for themselves and require no explanation via theoretical proposition" (May, 1997). Since not much purely empiricist research has been carried out in supply chain management, the author discarded the idea of deploying this perspective. As a matter of fact, this research turned out to be purely conceptual.

Consequently, a brief outline of the two other schools of thoughts in social science, positivism and critical-realism, will be given. The former is claimed to be the predominant philosophy in supply chain management research and because of its primary use of quantitative methods many system dynamics studies have been classified as purely and simply positivist by previous authors. The latter is an intermediate approach that seems to encompass a holistic approach very important to the understanding of supply chain dynamics.

3.1.3.1 Positivism

Positivism, a term that was coined by the nineteenth-century French philosopher Auguste Comte, declares that only verifiable allegations based on observation and experience could be considered genuine knowledge (Patton, 2002). It was a manifest against the old order of society. For positivists, the social phenomena must be studied in the same state of mind as when natural scientists explore regions of scientific domains (Durkheim, 1964 cited in May, 1997). Consequently, in their ontological view, social phenomena are independent of social actors and researchers' behaviour and perception.

Some of the positivist epistemological principles are: knowledge-claim about unobserved entities is ruled out, scientific laws are statements about general and repeatable patterns of experience, phenomena can be explained scientifically only as instances of scientific law and science must be conducted in a way that is value free (Benton and Craib, 2001). Positivists believe that scientific methods can and should be extended to the study of human mental and social life in order to establish these disciplines as social sciences. Moreover, only when reliable social scientific knowledge has been established, will controlling and regulating individual or group behaviour become possible (Benton and Craib, 2001).

In sum, positivism implies the following characteristics: objectivity or independence, value-freedom, causality or creation of wide ranging laws, hypothetico-deductive, operationalisation, cross-sectional analysis (comparison of variations across samples), reductionism, and generalisation from a sample to make universal claims (Easterby-Smith *et al.*, 1991). Mainly quantitative methods are normally used (Duberley and Johnson, 2005) and extensive research strategies.

3.1.3.2 Critical realism

Critical realism arose as an alternative to the predominance of only two basic and extreme epistemological options in social science: positivism and interpretivism (Benton and Craib, 2001); hence, adopting a methodological pluralism. Critical realists are anti-positivist but still objective. "Like positivism, realism assumes that there is an outside world that exists independent of our knowledge of it. However, unlike positivism, it assumes that the world [...] is made meaningful by our interpretations of it" (Thomas, 2004). A number of authors influenced its development (Benton, 1977; Harré, 1970; Hesse, 1966; Keat and Urry, 1975) but Bhaskar's Realist Theory of Science "has provided the most systematically developed and influential version of the approach, especially in its accounts of social science" (Benton and Craib, 2001).

The critical realism theory building is a three-stage process: collecting of evidence about patterns of observable phenomena, identifying and explaining the underlying structure or mechanism, conducting further experiments and observation assuming that these mechanisms really do exist (Benton and Craib, 2001). The reality is then stratified on three levels: the real world, which science seeks to discover, the actual level, the one produced under experimental conditions, and the empirical level.

Critical realists are opposed to reductionism, a philosophical position that considers a complex system as nothing more than the sum of its parts. Since there are mechanisms inside each part and between parts, the system is more than the sum of parts. One problem of their method is to define the stopping point to this process of penetrating behind or below the surfaces, since the world is an open system. There is a tendency to use intensive research strategies, which means that they are primarily concerned with what makes things occur in particular cases rather than demonstrating how extensive certain phenomena and patterns are in the population (Sayer, 2000). Hence, explanations are contextualised and such researchers are not interested in promoting general laws.

3.1.4 Supply chain dynamics and systems thinking: adopting a combined approach

According to Dunn *et al.*'s (1994) descriptions, both the artificial paradigm and on a direct observation of reality underlie supply chain dynamics research. The former is mainly dominated by an axiomatic and positivist approach. The latter is achieved through case studies and field experiments, and is considered a merely interpretivist method for some researchers.

In contrast, according to Gammelgaard's (2004) descriptions, this field can be related to the systems thinking approach since its main idea is the interdependency between the various elements of the supply chain. In the systems approach, theory is contextual rather than universal. Moreover, data collection and theory building seem to occur practically simultaneously. However, the reality is still considered objective and therefore it exists independently of human thoughts or beliefs.

The concern of systems dynamics is solving problems in living systems and these are characterised by dynamism and complexity. The systems approach enables the investigation of complex, dynamic feedback systems by investigating the dynamic behaviour of its elements and their interactions in different levels of the chain (Wolf, 2008). Feedback in this context means that one element might affect another and vice versa. These feedback loops need to be taken into consideration for holistic systems modelling (Forrester, 1961; Towill, 1991). Hence, the author does not identify supply chain dynamics as fitting within the reductionism in positivist principles.

Mingers (2000) affirms that "systems dynamics seems to epitomise some of the major premises of critical realism" since it is rooted in a holistic view and retroductive (or abductive) approach. This approach seems to have been used by scholars who modelled supply chains based on real world observations.

Additionally, it is not the aim of this field to generalise hypotheses and make universal claims, but to identify, through analytical experiments, largely through simulation or mathematical modelling, and empirical observations, causality between the different elements in particular systems.

On the other hand, much more time is spent on creating the model to analyse the feedback mechanisms than observing real systems and undertaking empirical analysis. Consequently, it can be argued that the systems approach might involve less intensive research than the critical realist approach, but at the same time, the research is not as extensive as in positivism. The author also agrees that there are far more published works using experimental analysis than empirical ones. An imbalance exists in the conduct and publication of rigorous qualitative research studies such as grounded theory, ethnography, phenomenology, semiotics, and historical analysis (Golicic *et al.*, 2005).

Despite analysing contextual systems, supply chain dynamics research conventionally proposes and tests theories and then provides data for the generation of scientific laws. This is a fundamental principle of positivism (Bailey, 1994). The author also alleges that the predominant view among management scientists is connected to some forms of positivism. "Positivism gives the basis for Management Science work" (Williams, 2008). Moreover, this research approach can be termed 'hard' since it emphasises the mechanisms and processes in order to analyse supply chains' performances and design (Aastrup and Halldórsson, 2008).

To summarise, the author strongly believes that the systems approach contains elements of both positivism and the critical-realism school of thoughts. Specifically in this thesis, an objective, holistic and value-free view will be taken. In relation to the logic reasoning, both deductive and abductive approach seem valid in contributing to the supply chain theory for answering Research Questions 1a, 1b and 1c. However, in tackling Research Questions 2a and 2b, the author opted to use already existent models instead of building her own model through observations, since the use of well-established models is more appropriate in answering the methodological questions. Hence, a deductive logic reasoning and a conceptual research approach were chosen.

3.2 Research methods and tools

In this section, a review of methods and tools used for the accomplishment of this thesis is presented. Following the research methodology hierarchy developed by Wolf (2008) (Figure 3.2), a research strategy can be conceptual or empirical, depending on whether field data is gathered for the generation of theory or not. This thesis follows a conceptual research strategy as already mentioned in the previous section. According to Bowen and Sparks (1998), conceptual research encourages theoretical debate and stimulates further empirical research. It does not usually rely on empirical field data, but structured tools and concepts can be used to increase reliability and validity. For instance, for research analysis, mathematical modelling, simulation and experiments can be used to generate artificial data and to refine and make theoretical models more precise (Wolf, 2008). On the other hand, a more exploratory, or unstructured, research approach can be taken by searching the literature, for instance. The term exploratory designates a type of research whose main purpose is to look for new insights, ask questions and assess phenomena in a different perspective (Adams and Schavaneveldt, 1991).

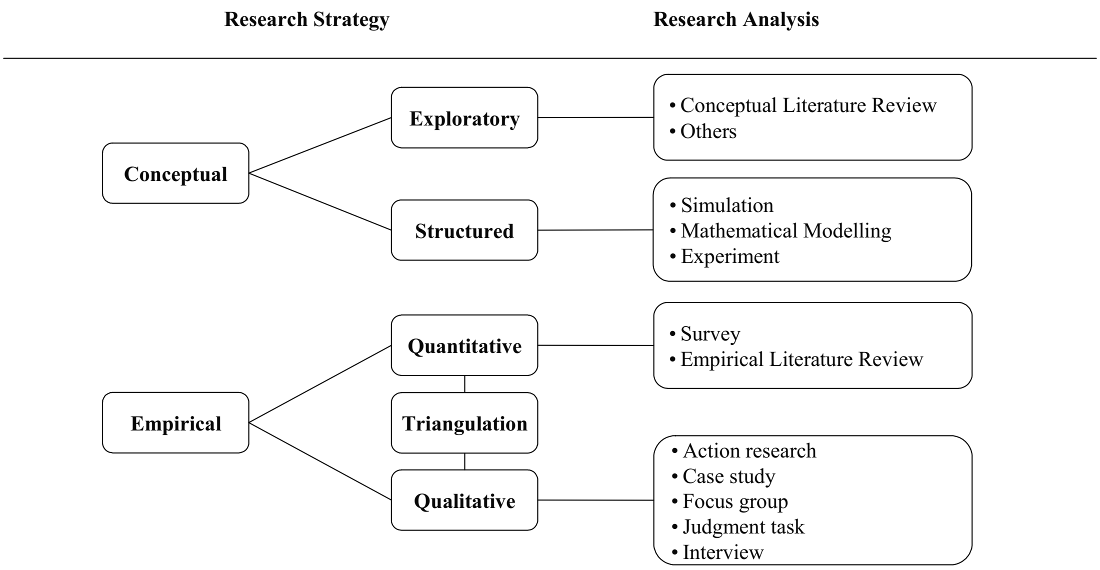


Figure 3.2: Hierarchy in Research Methodologies
Source: Wolf (2008)

The research analysis in a conceptual research can be undertaken through a conceptual literature review, mathematical modelling, simulation and experiment. The objective of a conceptual literature review is to map knowledge in a field or area in order to conceptualise models. These models can be further tested empirically or in a structured conceptual analysis (Denyer and Tranfield, 2006). Mathematical modelling is an analysis technique that uses mathematical concepts to describe the behaviour of a system (Cameron and Price, 2009). Simulations refer to "experiments on the reactions of a model through targeted manipulation of variables in an artificial environment" (Wolf, 2008). In experiments, the researcher also manipulates some variables in order to observe the resulting changes. What separates an experiment from a simulation is that the former takes place in natural settings (Saunders *et al.*, 2009).

In this thesis, both structured and exploratory conceptual research strategies are used, although the former is more predominant. The exploratory part refers to when the author searches the literatures in the natural and social sciences for the meanings of resilience and proposes a quantitative framework to assess resilience from the supply chain dynamics perspective. This assessment framework is given in Chapter 4. The structured conceptual research is presented in Chapters 5 and 6.

3.2.1 Mathematical modelling

Mathematical modelling creates models that mimic reality by using mathematical language. For the modelling of dynamic, time-dependent, and feedback systems, differential equations and control theory are normally used.

3.2.1.1 Control theory

Control theory is a branch of engineering and mathematics whose objects under study are dynamical systems. A system is composed of a set of elements connected together by information and physical links (Leigh, 2004). Since it allows a systematic evaluation of feedback systems and identification of causal relationships, its applicability for studying production distribution systems is highly recommended (Towill, 1992b).

Block diagram manipulation, state space, difference and differential equations, z- and Laplace transforms are some of the techniques used in investigating dynamical systems. In this study, block diagram manipulation is used to find the transfer functions of inventory, shipment and order responses and to simplify models whenever possible. Continuous state space, differential equations and inverse Laplace transforms, which convert signals in the frequency domain into time domain equations, will often be used to analyse systems responses. In addition to this, there will be the application of matrices and series analysis, such as Taylor and Fourier series expansions.

3.2.1.2 Discrete and continuous time domains modelling

In order to analyse any dynamic system, it is possible to consider that variables change with time discretely or continuously. A production-inventory control system may operate in either continuous, where the inventory and ordering status is reviewed continuously, or discrete times, such as in a periodic review process.

A number of studies in both discrete and continuous time production control have been undertaken. In continuous time, Simon (1952) was probably the pioneer on applying continuous control theory for investigating inventory control problems. Towill (1982), when developing a production-inventory control model, used block diagram representations in the Laplace domain. John *et al.* (1994) further developed Towill's work by implementing the pipeline feedback and their work was also in continuous time. Wikner *et al.* (1992) represented Forrester's difference equations of the industrial dynamics model into block diagram representation in the Laplace domain and Jeong *et al.* (2000) created a variant of the same in a continuous state space form. Grubbström and Huynh (2006) use Laplace transform to analyse MRP systems for basic ordering policies, such as Lot-For-Lot, Fixed Order Quantity and Fixed Period Requirements. The problem in modelling systems in continuous time is that some schedules are inherently discrete and the continuous representation of discrete delays is mathematically complicated (Naim *et al.*, 2004).

In discrete time, pure time delays are readily handled by the z-transform (Warburton and Disney, 2007) and much research was done on stability (Disney and Towill, 2002), variance amplification properties (Disney and Towill, 2003a; Disney and Grubbström, 2004), and dynamic performance (Dejonckheere *et al.*, 2003) of discrete models. The disadvantage of discrete control theory is that the mathematics involves "lengthy and tedious" algebraic manipulation (Naim *et al.*, 2004). Moreover, from the author's own experience acquired during this research, mathematics of discrete systems become even more complex when nonlinearities are considered and there are very few literature sources on the analysis of discrete nonlinear systems. For these reasons, mathematical analysis in this thesis will be undertaken in continuous time.

Although results between discrete and continuous time modelling may differ, it is argued that management insights gained from both time approaches are very similar and that their qualitative nature is essentially equivalent. Hence, either domain can be used to study supply chains (Disney *et al.*, 2006; Warburton and Disney, 2007).

3.2.1.3 Linear models

A system is linear if the principle of superposition holds, that means that the system's response given an input signal $X+Y$ is the sum of the behaviour in following signals of magnitude X and Y applied separately (Towill, 1970). Also, only linear systems can be modelled in state space representation and be represented by a single transfer function.

The linear control theory literature is well-established and there is a variety of techniques that can be used to describe the behaviour of linear systems. For this reason, previous work in supply chain dynamics has focused on 'presumably linear' models.

3.2.1.4 Nonlinear models

A nonlinear system is one whose performance does not obey the principle of superposition. This means that the output of a nonlinear system is not directly proportional to the input and the variables to be solved cannot be expressed as a linear combination of the independent parts (Atherton, 1975). While System Dynamics simulation is often used in the analysis and redesign of supply chain models that exhibit nonlinearities, quantitative analytical approaches are more often restricted to linear representations of supply chains. Hence much of the research on supply chain dynamics either takes a 'trial and error', experimental, simulation approach to redesign (Forrester, 1958, 1961; Sterman, 1989; Shukla *et al.*, 2009; Poles, 2013) or develops exact solutions of models that are already linearised approximations to the real-world situation (Towill, 1982; John *et al.*, 1994; Disney and Towill, 2005; Gaalman and Disney, 2009; Zhou *et al.*, 2010).

In this section, a classification of nonlinear systems will be given and an attempt to link the natural science descriptions and empirical evidences of nonlinearities in supply chain systems will be made. Moreover, methods for analysing nonlinear system dynamics mathematically will be gathered and where certain methods have already been used in business research this will be pointed out.

Types of nonlinearities

Since the variety of possible nonlinearities in systems dynamics models is extremely wide, it may be worthwhile to classify them into categories that suggest the types of analytical methods that can be applied. The first research found on categorisation of nonlinearities in business system dynamics models done by Mohapatra (1980) who identified three types: limiting functions, such as CLIP functions, table functions and product operators. He also recommends some techniques to deal with such properties, which include: removing redundant functions, linearisation through averaging, best-fit line approximations and small perturbation theory. However, there is no implementation of such methods in his work and for this reason, comparisons between the linearised and nonlinear models were not undertaken. In the control systems literature, nonlinearities are more extensively classified as inherent or intentional, continuous or discontinuous and single- or multiple-valued (Towill, 1970; Graham and McRuer, 1961; Vukic *et al.*, 2003), as in Figure 3.3.

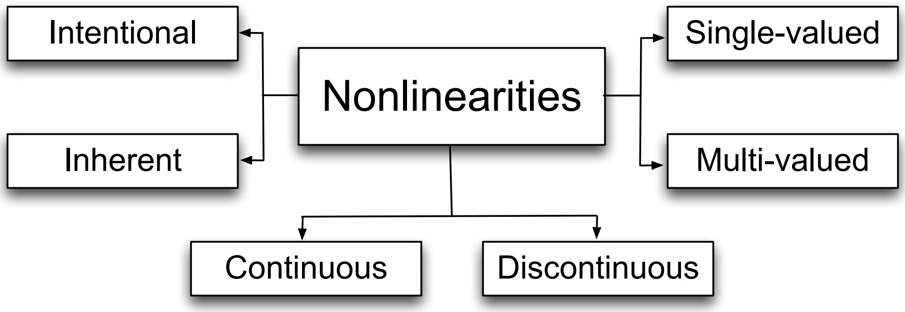


Figure 3.3: Types of nonlinearities

Inherent nonlinearities are intrinsic to the nature of the system and arise from the system's hardware and motion. They are normally undesirable and need to be compensated for by the system designer. Intentional nonlinearities are artificial and deliberately introduced by the designer in order to improve system performance (Cook, 1986). Normally in supply chain systems, nonlinearities occur naturally due to physical and economic constraints. These nonlinearities may or may not be considered in the system modelling depending on the degree of accuracy and complexity necessary for the supply chain design. On the other hand, supply chain designers may want to include nonlinearities that do not exist in reality for the sake of improving certain performance measures. This type of research has not yet been duly explored but some studies have shown that while the presence of nonlinearities may worsen some performance measures, they may improve others. For example, Evans and Naim (1994) - demand amplification versus service level, Grübbstrom and Wang (2000) - complexity of the production plan versus production cost, Wikner *et al.* (2007) - leadtime expectations versus dynamic behaviour in the system.

Continuous and discontinuous nonlinearities are associated with the rate of change in the output in relation to the input. A feature of the outputs in continuous functions is that they are smooth enough to possess convergent expansions at all points and therefore can be linearised. Examples include any adaptive control system, where certain control parameters, instead of being fixed, vary depending on the state of other variables (Cook, 1986). In Forester's industrial dynamics model this occurs with the delay in filling orders that depends on the ratio between actual and desired inventory. This effect will be further discussed in Chapter 5.

Sharp changes in output values or gradients indicate discontinuities. The most common type of discontinuous nonlinearity is the piecewise linear functions, which consist of a set of linear relations for different regions. In supply chain research, effort has been given in shaping stability regions of discontinuous, single-valued and piecewise linear supply chain systems and understanding the factors which will lead to chaotic behaviours (Larsen *et al.*, 1999; Mosekilde and Laugesen, 2007; Wang and Disney, 2012).

Single-valued nonlinearities are also called memory-less, which means that the output value does not depend on the history of the input. Multi-valued functions are often used in engineering to model hysteresis of magnetic and elastic materials and mechanical backlash of friction gears (Cook, 1986). In business studies this kind of nonlinear behaviour has been described in economics (Göcke, 2002), for instance between buying/selling states and price (Cross *et al.*, 2009) and unemployment and economy growth rate (Lang and de Peretti, 2009). In supply chain management research, multi-valued nonlinearities are not so commonly reported. They have been used to model switching of certain operation strategies depending on cost directions. Examples include investigations on changes in global sourcing (Kouvelis, 1998) and manufacturing strategies (Kogut and Kulatilaka, 1994) depending on foreign exchange rate directions. From a purely production-inventory control system perspective, which is the main focus of this work, this kind of effect has not been previously highlighted. The normal thinking is that, independent of demand growing direction, the order quantities placed to suppliers or shipped to customers will always match demand. However, when a variable capacity is put in place, these outputs can result in a complex multi-valued nonlinear behaviour. In Chapter 5, examples of this nonlinear behaviour will be addressed.

Methods for the analysis of nonlinear systems

When confronted with a nonlinear system, the first approach is to linearise it. A good justification for this is that there is a variety of techniques available in linear systems theory which is unmatched by its nonlinear counterpart (Kolk and Lerman, 1992). This is generally considered a suitable approach when the solution can be obtained in this way. While the linear system theory is well established, the literature lacks a unique nonlinear theory that strives for generality and applicability (Rugh, 2002).

Because of the confusion of terminologies and lack of detail of the research methods in the nonlinear control systems literature, the listing of all existing techniques and their applicability in the analysis of nonlinear feedback systems is a challenge. Table

3.2 presents a list of the methods that have been sufficiently acknowledged in the literature and whose full details were accessible.

	Method of Analysis	Applications	Considerations
Linearisation methods	Small perturbation theory with Taylor series expansion	Continuous Single-valued	Assumption that the amplitude of the excitation signal is small. Local stability analysis only.
	Describing function	Continuous, Discontinuous Single-valued, Multi-valued	Less accurate when nonlinearities contain higher harmonics. Analysis of systems with periodic or Gaussian random input only.
	Small perturbation theory with Volterra/Wiener series expansion	Continuous Multi-valued	Assumption that the amplitude of the excitation signal is small. Difficulty in calculating the kernels and operators of the system, making it impractical for high order systems.
	Averaging and best-fit line approximations	Continuous, Discontinuous Single-valued, Multi-valued	Gross approximation of real responses. Only when better estimates are not possible.
Graphical and simple methods	Phase plane and graphical solutions	Continuous, Discontinuous Single-valued, Multi-valued	Limited to 1^{st} and 2^{nd} order systems only.
	Point transformation method	Discontinuous Single-valued, Multi-valued	Piecewise linear systems only. For high order systems, automated numerical methods must be employed.
Exact solutions	Direct solution	Continuous Single-valued	Limited to a finite number of equations.
Stability method	Lyapunov-based stability analysis for piecewise-linear systems	Discontinuous Only single-valued examples were found	Piecewise linear systems only. Computation can be complex depending on the system.
Simulation	Numerical and simulation solution	Continuous, Discontinuous Single-valued, Multi-valued	Can be time consuming. Dependent on computer and software calculations capacity.

Table 3.2: Summary of methods used to analyse nonlinear systems

First, there are methods for system linearisation, such as small perturbation theory, describing function and averaging or best-fit line approximations. The former enables the system with continuous nonlinearities to be examined through successive approximations in the form of power series in the perturbation parameter (Kolk and Lerman, 1992). If the system can be represented by the Taylor series or Volterra series, then it can be approximated using perturbation theory (Odame and Hasler, 2008, 2010). The Volterra series is often described as a Taylor series with memory (multi-valued nonlinearities), which means that the Volterra series can represent systems where the output depends on past inputs (Elliott, 2001). The describing function method is referred to as a quasi-linearisation, since its representation of the nonlinear system is for specific inputs. For instance sinusoidal inputs are more often used since the frequency response approach is a powerful tool for the analysis and design of systems (Graham and McRuer, 1961; Towill, 1970; Atherton, 1975; Cook, 1986). Averaging and best-fit line techniques produce very gross approximations and serve as a good starting point for qualitatively understanding more complex systems (Mohapatra, 1980). However, whenever accuracy and reliability are needed these methods should be avoided (Cook, 1986).

Then there are relatively simple techniques. The phase plane analysis is a graphical method and for this reason, it is limited to second order systems (Graham and McRuer, 1961; Towill, 1970; Atherton, 1975). The point transformation method allows periodicity and stability of piecewise-linear systems to be investigated by studying the behaviour of trajectories that cross repeatedly from one region to another (Cook, 1986). There are also direct solutions for a limited number of nonlinear feedback systems with low order that can be found in Kolk and Lerman (1992).

There are more complex and sophisticated techniques such as the more recently developed method of Johansson (2003) for the stability analysis of piecewise-linear systems combining Lyapunov functions and convex optimisation techniques. Finally, there is simulation, which is a very useful but a more complementary tool to the above analytical methods. Exploratory analysis with simulation can be time consuming, expensive and unrewarding (Atherton, 1975).

3.2.1.5 System simplification

When developing a block diagram of a high-order control system, the first approach is to rearrange the blocks obtained from the conceptual model into a reduced form by identifying and eliminating redundancies, collecting constants and moving blocks to create familiar forms, such as cascade, parallel and feedback (Nise, 2000). This technique may reduce the number of variables and equations but it ensures that no causal relationships between any variables are lost.

Another characteristic of a high-order control system is the fact that it may contain poles that produce little effect on the transient response. The poles that are close to the imaginary axis on the left side of the s-plane give rise to the transient response that will decay relatively slowly, whereas the poles that are far away from the imaginary axis correspond to a fast decaying axis. If the magnitude of a pole is at least five times that of a dominant pole or pair of complex dominant poles, then the pole may be regarded as insignificant and can be neglected as far as the transient response is concerned (Nise, 2000).

However, there are better ways of approximating high-order systems to low-order ones especially when a transfer function may not have clear dominant poles (Towill, 1981; Kuo and Golnaraghi, 2003). Two methods for reducing systems order will be compared: the Towill-Matsubara (Matsubara, 1965; Towill, 1981) and Hsia methods (Hsia, 1972). The former is a method proposed by Towill (1981) when extending the time delay theorem developed by Matsubara (1965). It attempts to determine a low-order model based on the system unit step response. The latter approximates a high-order system to a low-order one by approaching their frequency responses. Although these methods are rather old, they provide the basic principles on low order modelling and are still used today (Jeong *et al.*, 2000; Kuo and Golnaraghi, 2003).

Let a high-order system be represented by a transfer function in the following form:

$$T(s) = \frac{1 + b_1 s + b_2 s^2 + \cdots + b_q s^q}{1 + a_1 s + a_2 s^2 + \cdots + a_n s^n} \tag{3.1}$$

The low order model will then be:

$$T_M(s) = \frac{1 + B_1 s + B_2 s^2 + \cdots + b_Q s^Q}{1 + A_1 s + A_2 s^2 + \cdots + A_N s^N} \tag{3.2}$$

so that $Q \leq q$ and N must be less than n.

Towill-Matsubara method

This method initially involves choosing the poles nearest to the imaginary axis to determine $T_M(s)$. However, the Matsubara time delay theorem is also incorporated to compensate for inaccuracies in the low order model. This gives us the following model:

$$T_M(s) = e^{-\tau s} \left(\frac{1 + B_1 s + B_2 s^2 + \cdots + b_Q s^Q}{1 + A_1 s + A_2 s^2 + \cdots + A_N s^N} \right) \tag{3.3}$$

where τ is a time delay in the response which is determined by matching the system and model step responses according to the integral of error from time zero to infinity. In other words, the area between the input and output lines in the system, $T(s)$, should match the respective area in the low order model, $T_M(s)$ plus the area caused by this time delay.

Full detail on this method can be found on Appendix A

Hsia method

The approximation method proposed by Hsia (1972) is based on selecting A_i and B_i, in such way that $T_M(s)$ has a frequency (ω) response very close to that of $T(s)$. In order words, the magnitude of the frequency function $T(i\omega)/T_M(i\omega)$ is required to deviate the least amount from unity for various frequencies. Hence, the following relation should be satisfied as closely as possible:

$$\frac{|T(i\omega)|^2}{|T_M(i\omega)|^2} = 1, \text{ for } 0 \leq \omega \leq \infty \tag{3.4}$$

Full detail on this method can be found on Appendix A

3.2.2 Simulation

Simulation offers a "middle ground between pure mathematical modelling, empirical observation and experiments for strategic issues in supply chain research" (Größler and Schieritz, 2005). Its advantages are that simulation does not require specific mathematical forms that are analytically solvable and where an optimal solution exists, because simulations proceed step-for-step using numerical approximation methods. In addition to this, some simulation approaches provide the possibility to include estimations of factors that are difficult to measure, such as 'soft variables'

(Wolf, 2008). For instance, in this thesis an assessment framework is developed to quantify, by some means, a soft supply chain performance measure: resilience.

There are many kinds of simulation techniques to evaluate dynamic systems, for example system dynamics, discrete-event and agent-based simulations. The latter is concerned with representing the actions and interactions of autonomous agents and assumes that a global system control does not exist (Gröβler and Schieritz, 2005). This technique has not been found useful since control systems are the subject of this research. Moreover, since control policies are the focal point, it has been assumed that supply chain managers and employees will adopt standardised procedures before making decisions.

Discrete-event simulation models the operation of a system as a discrete sequence of sample paths that characterise its behaviour (Fishman, 2001). This sort of simulation is used to understand the system's behaviour and infrastructure and help in making decisions, such as on jobs assignment and resources allocation (Allen, 2011), for instance, determining the number of machines and workers necessary to cope with the current demand. Hence, discrete-event simulation is less suitable to answer this thesis research.

System dynamics simulation is suited for representing situations where feedback relations play a significant role in understanding the system's dynamic behaviour (Akkermans and Dellaert, 2005). This method has been advocated by Forrester (1961) and involves translating the behaviours between variables into a causal loop diagram, converting these relations into differential equations, subjecting the system to a disturbance and then studying the output responses to understand the cause and effect relations.

There are four important elements to be considered when formulating system dynamics simulation models: levels, flow rates, decision functions and information channels (Forrester, 1961). Levels describe the accumulations within the system and represent the current value of the variables. They are a normally function of inflow and outflow rates. In production-inventory control systems, inventories are levels that depend on production, receipt or delivery rates. Flow rates are the instantaneous flows between the levels in the system. For instance, the production rate will transfer productions from raw material to finished goods inventories. Decision functions are the differential or algebraic equations that state the policies used to control the rates between levels. Finally, information channels connect the information known about the levels with the decision functions. For instance, in a production-inventory system the levels of inventory and work in process can be used to determine the order rate.

Many authors suggest that system dynamics simulation involves calculations only in continuous time. However, continuous equations can be discretised into difference equations by considering $\Delta t = 1$. The advantage of this discretisation is that the simulation time can be reduced since fewer points are needed to find numerical solutions. In this thesis, both continuous and discrete simulations are used. Continuous simulation techniques will be particularly used to directly compare simulation results with the mathematical models in Chapter 5. Discrete simulations will be used in Chapter 6 as a complementing approach for the understanding of the impact of nonlinear system dynamics on supply chain resilience.

3.2.3 Software package and tools

Several tools and specialist software packages are available for undertaking simulations and aiding on mathematical analyses of system dynamics models. The following software packages have been used in the research presented in this thesis and the advantages and disadvantages of using each of these tools will be informed.

- *Microsoft Excel*: used for undertaking individual discrete simulation of linear and nonlinear models.

  Advantages: Instantaneous visualisation of systems responses to different parameters (no need for compiling). Errors in the modelling process can be easily spotted. No need for programming skills for creating models. Good starting point.

  Disadvantages: Limited amount of cells. Slow when undertaking repetitive and automated simulations. Large number of calculations can cause failures and consequently loss of data. Very few graphical options.

- *MATLAB®:* used for undertaking repetitive and automated discrete simulations of linear and nonlinear models and drawing graphs.

  Advantages: Quick and efficient for undertaking repetitive simulations. Control engineering functions are available (in a separate package). No limit on amount of data to be stored (matrices can have any dimensions). It has a secure way for saving data. Plenty of graphical options.

  Disadvantages: Difficulty in spotting mistakes in the modelling process (normally a calibration with models made in Excel is made first). Need for compiling when any change is made to the model. Need for programming skills.

- *Wolfran Mathematica®*: used for manipulating mathematical expressions, finding responses in the linear models through Laplace transforms and undertaking continuous simulations for nonlinear models.

  Advantages: Excellent for manipulating symbolic mathematical expressions.

Finds exact solutions for responses of linear models. It has control engineering functions integrated to it. Many graphical options.

Disadvantages: Language is not so intuitive. 'Black box': not possible to access or change settings when finding numerical solutions in continuous time. Not enough memory for simulating nonlinear behaviour for certain frequency and amplitude conditions.

- *Simulink®*: used for undertaking continuous simulations for linear and nonlinear models.

  Advantages: Simulation settings can be accessed and changed. Models are built in block diagram format instead of entering differential equations. Integration with MATLAB, therefore it has the same advantages. Able to simulate linear and nonlinear models for a wide range of frequencies and amplitudes. It can be used for hybrid (discrete and continuous) simulations if required (not applied in this thesis).

  Disadvantages: Settings are not always intuitive and manual uses technical language. Integration with MATLAB requires that users keep swapping between windows causing slow operation.

There are many other software packages available to undertake system dynamics simulations, such as Vensim®, Stella® and iThink®. However, these packages lack flexibility in accessing calculation settings and work normally as a 'black box' and for this reason have not been considered.

3.2.4 Alternative research methods

In Section 3.1.4 the author explained that the systems approach contains elements of both positivism and the critical-realism school of thoughts and that this thesis will follow an objective, holistic and value-free view. To answer research questions 1a, 1b and 1c, the author believes that an empirical research through conducting case studies in combination with system dynamics modelling in an abductive approach would be an alternative research method to contribute to the supply chain theory in evaluating the resilience performance. Moreover, via case-studies, new supply chain dynamics models can be introduced to the supply chain literature.

However, the methodological research questions 2a and 2b imply the use of mathematical modelling in a deductive and conceptual approach. Hence, the choice of methodology made by the author seemed the most appropriate one for answering both the theoretical and methodological questions in the same research project.

Another choice made by the author was the use of deterministic control theory, which means which no randomness is involved in the development of future states of the system. Deterministic system inputs are normally used to analyse nonlinearities in models (Atherton, 1975; Towill, 1981; Cook, 1986) since it is easier to analyse the output responses. After having a better understanding of the models' deterministic behaviour, stochastic input behaviour may be incorporated and statistical analysis could be undertaken.

3.3 Research design

The research design that encompasses the procedure employed to conduct this research is illustrated in Figure 3.4. As for every deductive research, this work started by scanning the literature. In this thesis, both supply chain theory (Chapter 2) and methodology (Chapter 3) literatures have been explored in order to establish the research questions. Next, from the literature, a quantitative assessment framework (Chapter 4) for measuring resilience has been created. This framework has been explored via both analytical (Chapter 5) and simulations models (Chapter 6) and has always been referred back to the literature to check whether the proposed measure is suitable. Finally, the results from the investigation of the models have made contributions back to the theory (a better understanding of supply chain resilience) and methodology (how to better investigate nonlinear models) in Chapter 7, closing the loop.

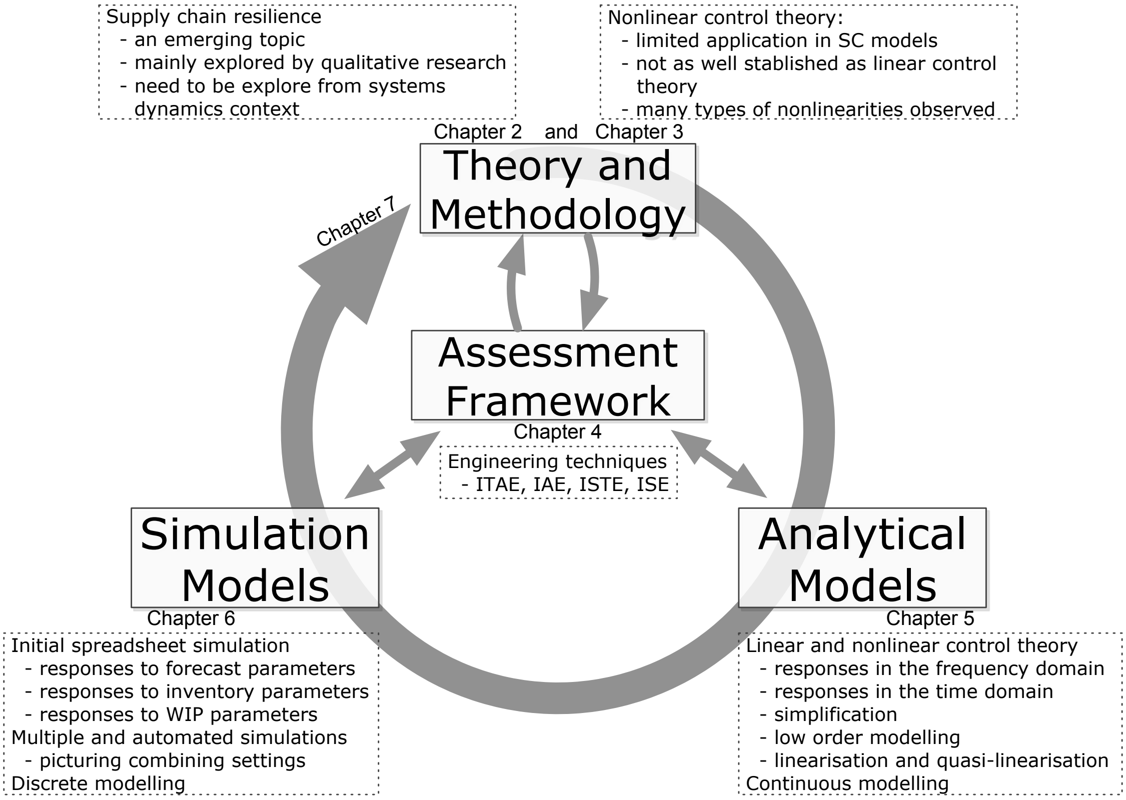


Figure 3.4: Research design

Next, more detail on how the literature process and the assessment framework were undertaken and a review of the supply chain models used will be provided.

3.3.1 Literature review process

The part of the literature chapter that looks at supply chain resilience was based on an exploratory literature review process, which was initiated by conducting keyword searches in multiple databases, such as ABI/INFORM Global, EBSCOHost, Scopus, ScienceDirect and Emerald. Google Scholar was also found to be useful to locate conference papers and technical reports. Among the keywords searched, the author started with 'supply chain risk' in order to map out the research outlines of this field. In parallel, the keywords 'resilience' and 'robustness' were searched alone so as to identify the various fields using these concepts. Later, the search was narrowed by combining 'supply chain' with 'resilience', 'uncertainty', 'disruption' and 'robustness'. After this last search stage, the author collated all quantitative studies and qualitative studies that were relevant to developing the supply chain resilience assessment framework, which will be presented in Chapter 4.

Regarding the methodological aspects, searches of the same sources and for a combination of the words 'nonlinear' and 'supply chain' with 'system dynamics', 'systems', 'control engineering' and 'control theory' were made. The result of these searches revealed that the supply chain literature is dominated by the use of numerical and simulation methods. Moreover, the very few analytical studies found do not clearly state the research methods and theories applied. On the other hand, in the engineering and mathematics domain, the research methods applied are expressed with clarity. However, since the academic papers in these fields are constrained by the number of pages, details on the description of the methods are normally not included. For this reason, the author made use of textbooks in the field of nonlinear control theory.

3.3.2 Assessment framework

In order to develop the framework for assessing supply chain resilience, further investigation of the conceptual literature on resilience has been undertaken. A customer's perspective and the supply chain's main objective have also been considered. Hence, based on the existing literature a performance index to measure supply chain resilience has been proposed and tested in Chapter 4.

3.3.3 Analytical and simulation models

In this subsection, an introduction of the models used to investigate both the theoretical and methodological questions will be provided. The author chose two very well-established models, Forrester's industrial dynamics model (Forrester, 1958) and the automatic pipeline, inventory and order based production control system - APIOBPCS model (John *et al.*, 1994).

The choice of these models was made during the development of the resilience assessment framework, as presented in Chapter 4. It will be shown that the study of resilience require models which represent inventory, backlog or unfilled order and shipment behaviour more precisely. "In constructing a useful dynamic model of corporate behaviour it is essential to have clearly in mind the purpose of the model ... and model variables should be selected to correspond to those in the system being represented" (Forrester, 1961).

The reason for choosing the APIOBPCS model is that this control system is representative of the Beer Game table top simulator (Sterman, 1989), which is often used to demonstrate the effects of information distortion in supply chains. John *et al.* (1994) created a linearised version of the Beer Game by setting the ordering rule as function of demand, inventory and pipeline states but disregarding capacity constraints. Later, Shukla *et al.* (2009) inserted nonlinearities back to the model in order to evaluate the effects of demand amplification on transport responses. However, Shukla *et al.* (2009) only used simulation to study this model.

The motivation for selecting Forrester's model is that it encompasses many nonlinearities and complex behaviour observed in real supply chains. Moreover, this model includes a great number of variables and equations that may better represent a typical production-distribution system. Despite being well-known and often quoted in the literature it has not been studied in-depth through analytical methods.

3.3.3.1 Forrester's Industrial Dynamics model of a production-distribution system

In the mid 1950s, a team of academics at the Massachusetts Institute of Technology (MIT) discussed the importance of bringing together mathematical and scientific methods to solve problems in industry. By then, most of the work done in operations research was focusing on open-loop processes, meaning that the inputs to the decision process were assumed to be uninfluenced by the decisions themselves (Forrester, 1968). With the advances of computing technology and the possibility of undertaking low-cost simulation experiments, the concept of feedback systems from engineering was then introduced into social science (Richardson, 2011).

The Forrester model is a representation of a production-distribution system whose objective is "the examination of possible fluctuations or unstable behaviour arising from principal organisational relationships and management policies at the factory, distributor and retailer" (Forrester, 1961). Principal time delays in the flows of orders and material, inventory and pipeline policies, forecasting and trend exploration methods are represented in the form of levels (accumulation within the system), rates (flow between levels), delays and decision functions or rate equations (statements or policies that determine how available information about levels leads to decisions).

While the original Forrester's supply chain model is often quoted as the embodiment of the bullwhip effect it has had little exposure with respect to its use as a benchmark for applying supply chain analysis and redesign methods. Two notable exceptions are the research findings of Wikner *et al.* (1992) and Jeong *et al.* (2000). The former explore a two-stage approach to understanding the causes of the bullwhip effect. While attempting to maintain model equivalence in terms of transient responses the first stage develops a linear representation of the original Forrester model before looking for opportunities to simplification, by eliminating redundant variables. The approach used makes the model mathematically tractable and it is easy to determine the transfer function of a single echelon. Importantly Wikner *et al.* (1992) found that in the linearised and simplified form the model has no state feedback and that the bullwhip effects caused by a differentiator term in the numerator of the transfer function. Such insights gained from the final simplified version of the model are then used to test new supply chain designs (Wikner *et al.*, 1991; Towill *et al.*, 1992; Wikner *et al.*, 1992) .

Figure 3.5 represents one echelon of Forrester's model in a Laplace-domain block diagram representation developed by Wikner *et al.* (1992). This figure suggests a complex behaviour featured by the presence of nonlinearities, given by multiplication and division (represented by $\textcircled{\Pi}$) between variables and CLIP functions (represented by $\boxed{}$), multiple loops and seemingly high order. The manufacturing rate decision (MD) seems to be dependent on the requisition order (RR) from the downstream party, actual and normal unfilled orders (UO and UN, respectively), actual and desired pipeline orders (LA and LD, respectively), actual and desired inventories

(IA and ID, respectively) and the manufacturing capacity (AL). There are several constant delays in the system, including in the production lead-time (DP), in smoothing requisitions (DR), in the inventory and pipeline adjustments (DI) and others. In addition, there is one variable delay: the delay in filling orders (DF) which is proportional to the ratio between actual and desired inventories. Complete set of equations used in the DYNAMO programmed can be found on Appendix B.

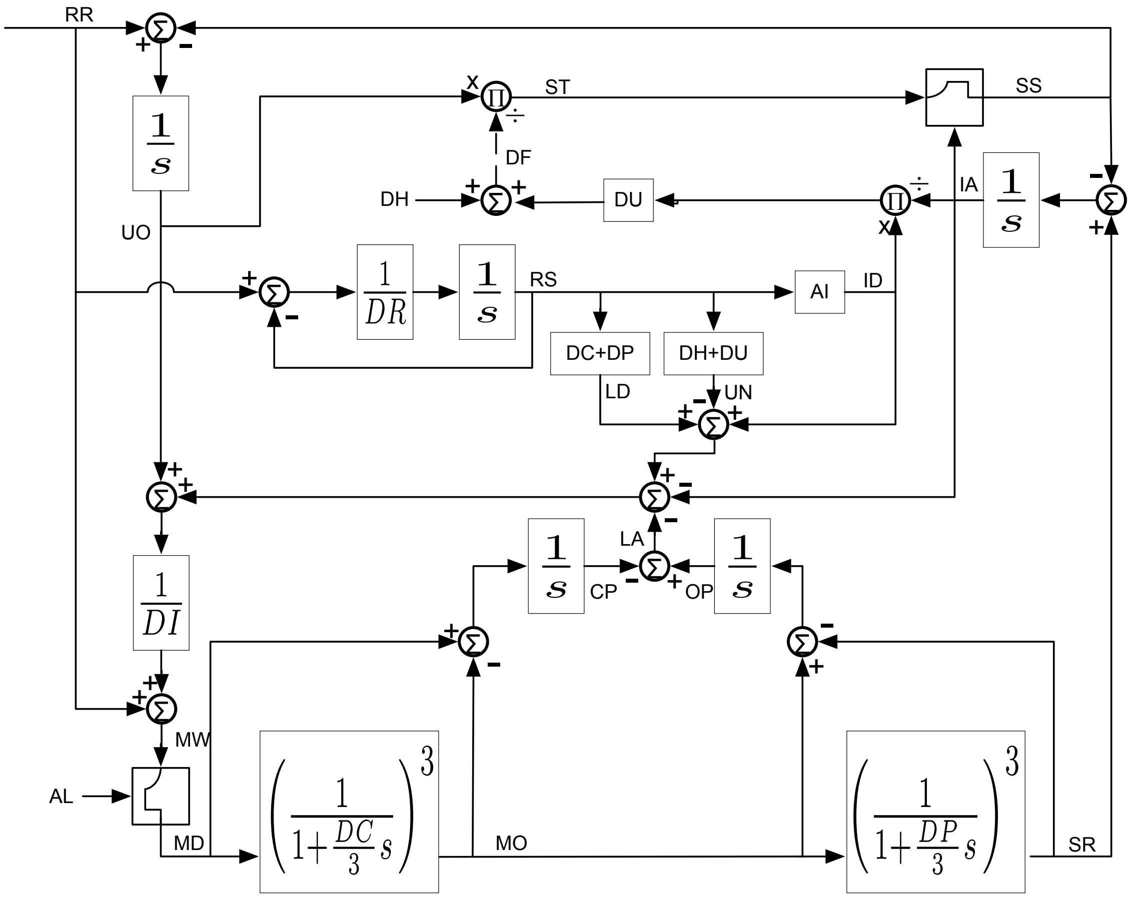


Figure 3.5: Block diagram representations of Forrester's industrial dynamics model
Source: Wikner *et al.* (1992)

Jeong *et al.* (2000), on the other hand, utilises the Forrester model to show the potential of a control algorithm, consisting of state feedback and filtering with an integral controller, in achieving a stable response when the system saturates. They utilise Matsubara time delay theorem and small perturbation theory to find a state space representation of three echelons in a variant form of Forrester model. However, they ignored one of the two capacity constraints and third order delays present in the original model.

In Chapter 5, it will be shown that the Forrester model can be simplified by block diagram manipulation, low order modelling and linearisation methods. Block diagram manipulation and linearisation through averaging techniques had already been attempted by Wikner *et al.* (1992) who highlighted the lack of feedback information fed into the manufacturing rate, exposing a division between real and safety orders. Hence, they showed that the so-called 'Forrester effect', in which orders are amplified from sink to source, is not due to linear feedback control but due to a first-order derivative term in the feedforward path. However, the main problem with their model is that while accuracy is kept for analysing the manufacturing orders, their linearised and simplified model is unreliable for analysing inventory and shipment responses, which are the main focus of this research. Small perturbation theory has been used by Jeong *et al.* (2000) to linearise the Forrester model. However, they do not compare the linearised model with the original one and despite their efforts to linearise part of the model, they use solely simulation methods to analyse the effect of different capacity levels in the factory's production rate on unfilled orders through the chain.

3.3.3.2 The APIOBPCS model

The APIOBPCS model belongs to the IOBPCS family (Figure 3.6), a range of production planning and inventory control systems developed by the Logistics Systems Dynamics Group at Cardiff University. This range of models may take into account the following supply chain elements: a demand forecasting method, production and distribution lead-times, an inventory feedback loop, a WIP feedback loop and target inventory levels. Moreover, these models have been well-acknowledged in the the supply chain literature (Wilson, 2007; Cannella *et al.*, 2008; Aggelogiannaki *et al.*, 2008; Eshlaghy and Razavi, 2011)

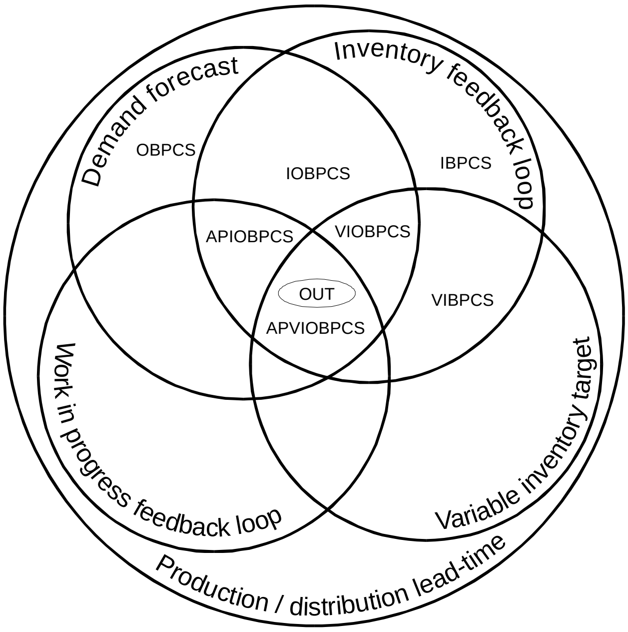


Figure 3.6: The IOBPCS family
Source: Lalwani *et al.* (2006)

The IOBPCS model, the basic of these production-inventory control algorithms, was introduced by Towill (1982) after converting the system dynamics model studied by Coyle (1977) into a control engineering format (Disney and Towill, 2005). It was then extended by Edghill and Towill (1990), who included variable desired inventory levels (VIOBPCS). In this model, the inventory target levels are a function of observed demand multiplied by a factor which depends on the lead-time. Later, John *et al.* (1994) further improved the model performance by introducing a WIP feedback loop into the ordering rule, creating the APIOBPCS. They concluded that

APIOBPCS was able to manage noise present in the consumption data, but it has the following limitation: estimated pipeline lead-time must be equivalent to the actual pipeline lead-time, otherwise a problem with inventory balance will occur. Hence, further advances were made with the inclusion of variable inventory targets and the APVIOBPCS was created (Disney and Towill, 2005).

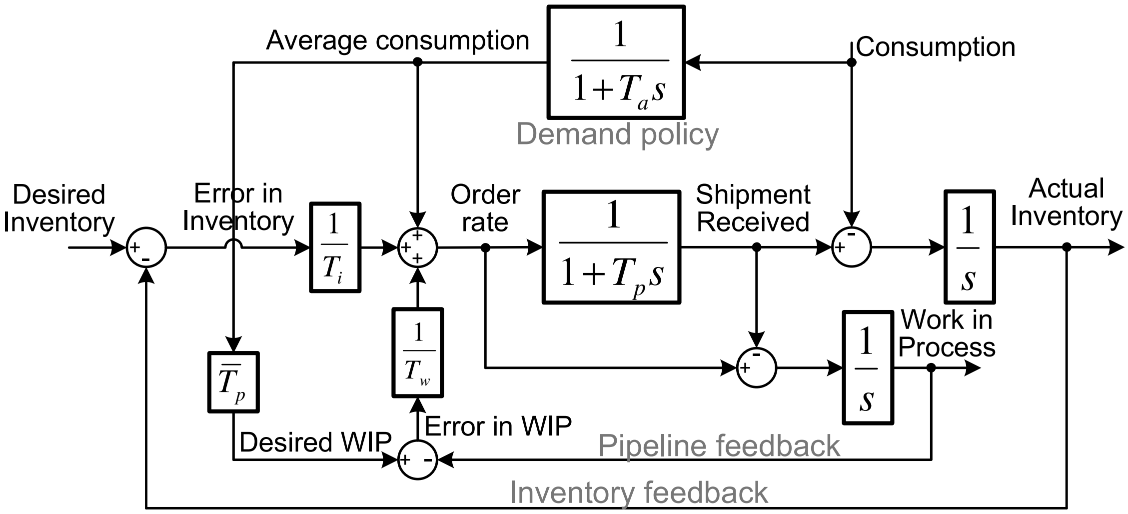
Since variable inventory targets will be explored by the investigation of the Forrester model, the author decided to investigate the APIOBPCS model, in which the inventory feedback control is made by comparing the actual inventory level with a fixed inventory target. Also, the APIOBPCS structure is a linear representation of Sterman's Beer Game algorithm (Naim and Towill, 1995; Mason-Jones *et al.*, 1997; Disney *et al.*, 2000). Figure 3.7(a) illustrates the Laplace-domain block diagram form of the ordering system incorporating automatic pipeline feedback developed by John *et al.* (1994).

Demand policy

The value of the current demand is exponentially smoothed which can be represented by a first order lag. Hence, the parameter T_a represents the time to average demand so that the exponential smoothing function $\alpha = 1/(1 + T_a/\Delta t)$, where Δt is the sample time interval.

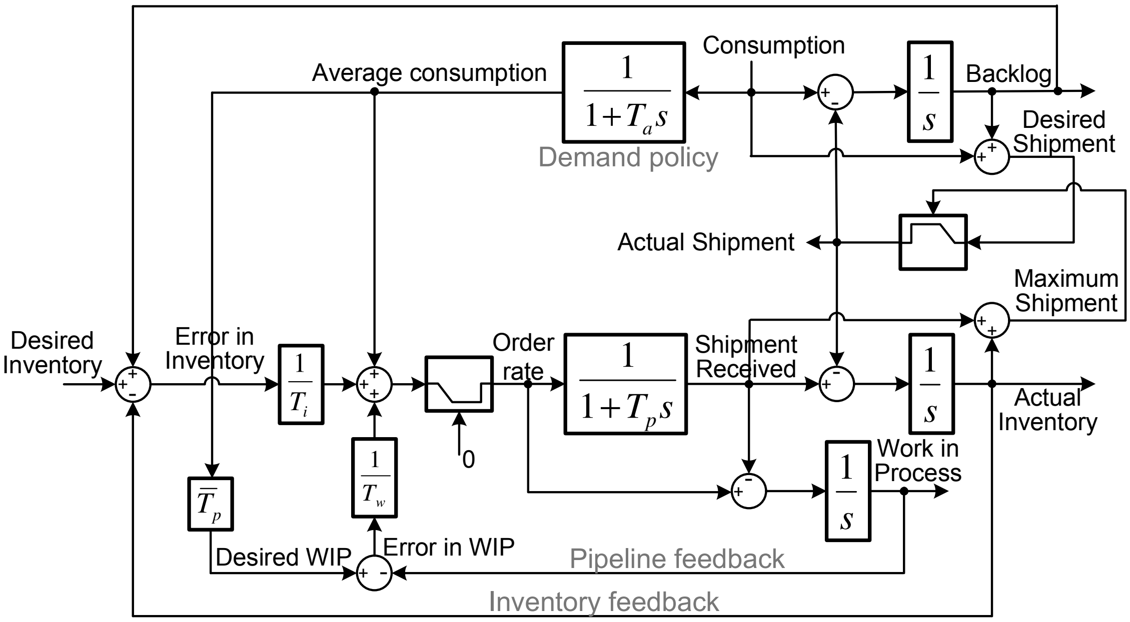
Inventory and pipeline policies

The inventory and pipeline policies are characterised by feedback loops. The inventory control is concerned with the rate $(1/T_i)$ at which a deficit in inventory is recovered. This policy is responsible for reducing the discrepancy between desired and actual inventory. The pipeline policy considers the actual work in process (WIP) and the time (T_w) it takes to recover to target levels. While the desired



(a) Linear Model

Source: John *et al.* (1994)



(b) Nonlinear Model

Figure 3.7: Block diagram representations of APIOBPCS

inventory is a constant value, the desired WIP is function of the expected lead-time ($\bar{T}_p$) and the forecasted demand.

Ordering rule

Finally the orders placed onto the supplier, or production process, will take into account the forecasted demand and the errors in inventory and WIP. The receipt of material is represented by a first order lag with a lead-time T_p. The combined policies result in a third-order system. Its transfer functions will be presented later in Chapter 5.

The limitation of the linear control model is that regardless of the actual inventory level the customer will always receive goods. From Figure 3.7(a), it can be observed that the customer consumption is subtracted from the inventory even if no products are available. In addition to this, order rates can be negative when the errors in inventory and/or WIP are negative. This can occur when the actual inventory or WIP is greater than desired. This negative order rate implies that goods can be returned back to the supplier. Both characteristics outlined are unrealistic traits of the linear model.

For these reasons, the author uses a nonlinear model, Figure 3.7(b), to stop the returning of goods to the supplier and to investigate how the backlog situation will affect the shipments to customers. Moreover, this nonlinear model is better representative of the Beer Game. Difference equations of this nonlinear model is in Appendix C. These nonlinearities are represented as in Wikner *et al.* (1992) where the authors managed to translate the clip function (⧄) used by Forrester's DYNAMO program into a block diagram representation as discussed Section 3.3.3.1. Here, the clip function in the order rate means that the minimum possible value of the order rate is zero while in the actual shipment the clip function denotes that the maximum possible value for the shipments sent is the sum of the actual inventory and shipment received. When the shipments sent are not equal to the customer demand, then backlog builds up. Hence, the desired shipment is the customer demand plus any backlog. Note that, as represented in Figure 3.7(b), backlog and inventory will not occur simultaneously and will not be negative because of the clip function. Hence, the error in inventory will then be the desired inventory level minus the holding inventory plus the backlog, since the backlog represents a negative inventory level.

In 2007, Wikner *et al.* (2007) introduced another model to the IOBPCS family in order to represent make-to-order (MTO) supply chain systems (Figure 3.8). After receiving the customer order, a company typically faces two options for the MTO portion of their supply chain. On the one hand, it can follow a chase strategy with the goal of keeping the delivery lead times at a steady level, leading to the need for flexibility in production capacity. On the other hand, the company can follow a level planning strategy, keeping a fixed capacity level and letting the order backlog fluctuate with the demand levels thus affecting the delivery lead-time.

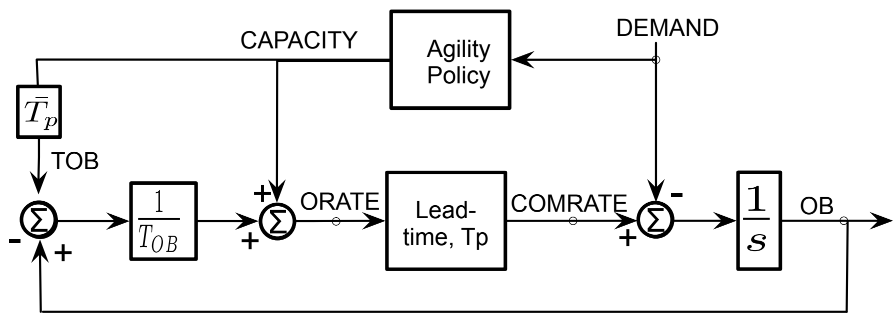


Figure 3.8: Block diagram of order-book-based MTO system
Source: Wikner *et al.* (2007)

Wikner *et al.* (2007) then defined the order book (OB) as "the aggregate number of order-based backorders in the system, that is, customer orders or commitments that are unfilled and, thus, awaiting to be released to production, work-in-process, products awaiting delivery". For this reason, the important role of the production control system in MTO environments is to manage the order book so as to satisfy customers' demands in terms of delivery lead-time. Decision-making in such a system represents a complex trade-off, as the acceptance of a large number of customer orders increases the revenue but also increases the order book and, therefore, also the workload in the system and possible customer dissatisfaction (Weng, 1999). From an aggregated production control perspective, an increase in the order book can be managed either by employing a flexible capacity or by letting the delivery lead-times fluctuate (Wikner *et al.*, 2007). The delivery lead-time can be obtained from the model in Figure 3.8 by dividing OB by the completion rate (COMRATE).

Wikner *et al.* (2007) noted the similarities of their MTO model with the IOBPCS family of archetypes hence having similar dynamic properties that are already well understood. Hence, this thesis' analysis of the non-linear behaviours of the API-OBPCS model may be easily extended in the future to other members of the model family.

3.4 Summary

This chapter has explained how this research has being carried out including the research ontological and epistemological positions, research design, methods and tools used. An objective, holistic and value-free ontological perspective has been taken and a deductive logic reasoning and a systems and conceptual epistemological research approach were chosen.

Details on the research methods and tools used have been provided. This included a review of the analytical and simulation models available and used. In summary this research uses nonlinear control theory combined with repeated simulation techniques in both continuous and discrete time domains to analyse the resilience of nonlinear supply chain models. Moreover, a discussion on alternative research methods has been provided.

Finally, the research design used to answer the research questions has been explained. This included the literature review process, the assessment framework and the analytical and simulation models used. In summary, two nonlinear production-distribution models, the Forrester model and the APIOBPCS model, have been selected to investigate the research questions.

4 Assessing supply chain resilience

"The oak fought the wind and was broken, the willow bent when it must and survived."

– Robert Jordan (1993), *The fires of heaven*

This chapter further explores the conceptual literature presented in Chapter 2 on resilience and proposes an assessment framework to measure supply chain resilience in the context of system dynamics given the supply chain's main objective: matching supply with demand.

Firstly, the most important conceptual frameworks suggested in the literature are introduced, illustrated and further discussed. Secondly, this chapter discusses the potential supply chain measurable performances that can represent resilience. The choice of a measurable performance will be dependent on the production planning processes adopted by different supply chains, such as for instance make-to-order (MTO) and make-stock (MTS) production planning systems. Then, composite performance indices to assess supply chain resilience are proposed and investigated. Finally, after comparing the different performance indices, a decision on how to measure supply chain resilience will be made based on the effect of each performance index on the output responses. Moreover, the selectivity property of the performance indices will also be taken into account.

4.1 Holistic conceptual framework for SC resilience

When reviewing the supply chain literature on resilience, some contradictions were found in relation to the terminology used, such as robustness being used interchangeably with resilience, and also in relation to the given definitions. For instance, according to Christopher and Peck (2004) resilience is not only the ability of a system to respond and recover to its original state, but also to achieve a new more desirable state. Other authors (Sheffi, 2005b; Rice and Caniato, 2003; Tierney and Bruneau, 2007), however, define resilience only as the ability to recover to its original state by quickly reacting to disruptions. Moreover, the literature is dominated by qualitative aspects which make supply chain resilience difficult to measure. In this way, several metrics have been used by quantitative researchers to assess resilience, for example inventory levels, lead-times, customer service levels, recovery time and disruption length. It is important to develop a single measure of resilience to ensure consistency and repeatability in results. In order to achieve this, a clearer and exact concept of resilience is needed.

As already mentioned in Chapter 2, Ponomarov and Holcomb (2009) developed a holistic conceptual framework for supply chain resilience using theory building and borrowing concepts from other disciplines. They defined supply chain resilience as: "the adaptive capability of the supply chain to prepare for unexpected events, respond to disruptions, and recover from them by maintaining continuity of operations at desired levels of connectedness and control over structure and function". Hence, this definition implies achieving the three following properties:

1. *Readiness:* being prepared or available for service.

   The implication of this definition is whether the supply chain can continue providing goods or services at reasonable costs according to the end customer requirements. For instance, Grünwald and Fortuin (1992) highlighted the dangers when companies strive for zero inventory and just-in-time. Hence, they introduced the concept of a Minimum Reasonable Inventory (MRI) and claimed that this more realistic goal should be aimed for by supply chains instead of trying to reduce stocks to zero. Towill (1996), observed that when stock control dynamics is poorly understood, supply chain managers should focus on reducing flow times to achieve MRI. In this way, wasteful inventory will generally be reduced in proportion to the lead-time improvement achieved.

Analogous to the Minimum Reasonable Inventory (MRI) in MTS systems, there is a notion of a Minimum Reasonable Order Book (MROB) in MTO systems. Order book is defined as the "aggregate number of order-based back-orders in the system, that is, customer orders or commitments that are unfilled and, thus, awaiting to be released to production, work-in-process, products awaiting delivery, etc" (Wikner *et al.*, 2007). Both target inventory and order book should be estimated based on forecasted sales and should accommodate (be ready for) demand fluctuations so that frequent changes in capacity are not necessary.

2. *Response:* reaction to a specific stimulus.

   Quick Response (QR) is not a new term and finds its roots in a strategy used by Japanese companies in the 1980s. The original idea of this concept was to focus mainly on manufacturing processes to provide 'modern' customers with newer, better and more customised products much faster. This approach drives supply chain lead-time reduction and is also known as Time-Based Competition (Suri, 1998).

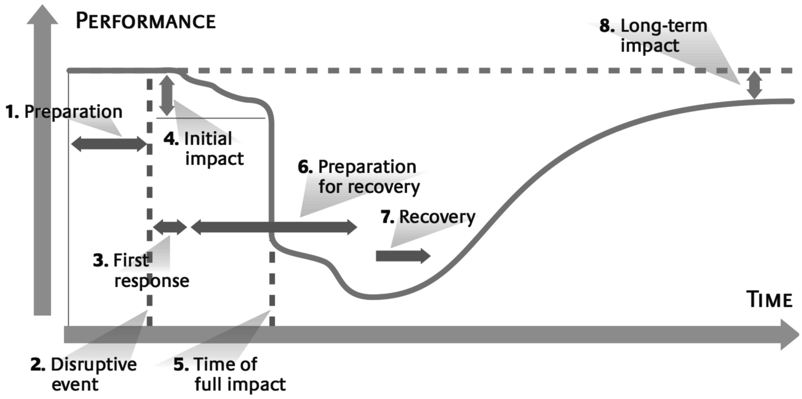
Reducing lead-times has been reported as a possible measure of resilience (Colicchia *et al.*, 2010b) and quick response a proposed resilience strategy (Datta *et al.*, 2007; Carvalho, 2011; Schmidt and Singh, 2012). However, quick response for resilience implies not only reducing average delivery lead-times but also suggests that, in times of uncertainty, supply chains should minimise the time to react to disruptions and begin the recovery stage quickly.

3. *Recovery:* a return to 'normal' stable or steady state conditions.

   Recovery is a cornerstone of supply chain resilience. As in the ability to respond, the recovery is also time-based and this process should be as quick as possible to ensure supply chain resilience. Sometimes the literature refers to the recovery process as a combination of responding and recovering (Pettit *et al.*, 2010).

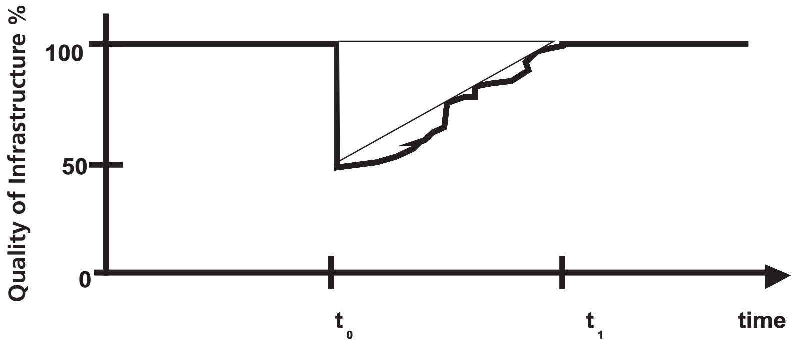
   Although there have been efforts to recommend an effective recovery plan for reducing the impact of disruptions (Sheffi and Rice, 2005; Tang, 2006), attention has been given to recovery after the disruption has occurred. When designing supply chains that are resilient to system dynamics source of risk, a mitigation strategy can be implemented to control and minimise the recovery time.

Sheffi and Rice (2005) outlined an illustration of how disruptions would affect company's performance which can be measured by sales, production levels, profits or customer service (Figure 4.1(a)). Additionally, their illustration demonstrates different phases of the system's performance response: after a disruption the performance decreases but as actions are taken the system's performance will be gradually restored. Similarly, Tierney and Bruneau (2007) and Asbjørnslett (2008) highlight the



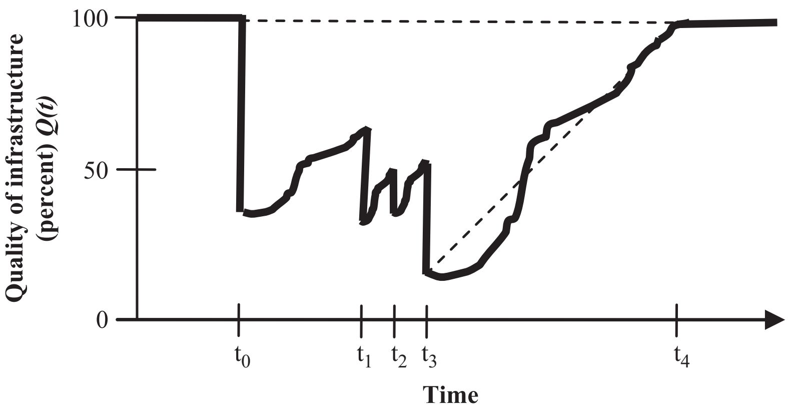
(a) Disruption profile

Source: Sheffi and Rice (2005)



(b) Resilience triangle

Source: Tierney and Bruneau (2007)



(c) Multi-event resilience

Source: Zobel and Khansa (2011)

Figure 4.1: Resilience profiles

relation between a disruptive event and business indicators. Tierney and Bruneau (2007) call this loss of functionality from disruption followed by a gradual recovery the 'resilient triangle' (Figure 4.1(b)). According to them, this triangle should be minimised. More recently, Zobel and Khansa (2011) extended this idea of the resilient triangle by introducing multiple related disaster or emergency events. Figure 4.1(c) illustrates the resilience profile of a system that has not had the chance to fully recover by the time the next disruption occurs.

The problem with the disruption profile suggested by Sheffi and Rice (2005) in Figure 4.1(a) is that the authors do not state which of the supply chain performances the figure represents. Different performances may behave in different ways where disruption occurs. For instance, the production sector can be disrupted for a while but inventory levels may be sufficient to guarantee customer service and, therefore, sales may be uninterrupted. Moreover, all of the representations in Figure 4.1 assume that after recovery, the performance will gradually and smoothly recover to its target without overshooting. What if the response and recovery of the chosen performance does not have a triangular shape? This question will be addressed in Section 4.2.3.

4.2 Finding a supply chain performance metric related to resilience

When deciding which supply chain performance metric should be analysed, the supply chain's objective with regard to satisfying customers should be considered. The way a supply chain targets customer satisfaction will depend on the nature of its business with two different cases being MTO and MTS supply chain systems.

4.2.1 Make-to-order

As the name suggests, planning and scheduling in an MTO system only occur after the order is received. Hence, no efforts are directed towards production until a order is confirmed. This production planning system is normally common among companies with high product variety, high holding costs, low volumes and irregular demand.

The key implication of this method is that it results in a "long planning and execution window for order delivery" (Mahadevan, 2007). Hence, this approach creates additional waiting time for customers before receiving the product, but may also allow a certain degree of customisation when compared to obtaining products from retailers' shelves. In this way, MTO supply chains are concerned with delivering the orders in a minimum reasonable time (Wikner *et al.*, 2007).

4.2.2 Make-to-stock

At the other end of the production planning spectrum, the purpose of MTS systems is the replenishment of inventory to a target level. This approach is particularly relevant to companies with lower product holding costs, standardised and high volume products with regular and predictable demand. The starting point for planning production in these companies is the estimated demand during the planning period. Then, depending on the available inventory of finished goods and goods already in the production pipeline, production and raw material order quantities are calculated. The MTS system has the objective of "effectively responding to the depletion of finished goods inventory through the planning system" (Mahadevan, 2007).

In summary in an MTS system, products are produced based on a demand forecast while maintaining minimum reasonable inventory (Grünwald and Fortuin, 1992)

It is also important to mention that MTS may also refer to a part of the system situated in the upstream from the customer order decoupling point (CODP), whereas MTO would be in the downstream from the CODP. Hence both can be part of one production system (Wikner *et al.*, 2007).

4.2.3 Dimensions for assessing supply chain resilience

Figure 4.2 illustrates two different dimensions, quantity and time, that can be used to measure the supply chain resilience of MTS and MTO systems. Moreover, the figure reveals which measures concern the customers and which performances interest supply chain operations. For instance, while for MTS supply chains the inventory cover time is more relevant from a control perspective, the customer is more interested in the amount of inventory still available. In MTO systems, supply chains are concerned with the aggregate number of order-based backorders awaiting production and delivery - the order book, whereas the customer's perceived measure is the lead-time between the placement of his order and receiving the product. In this context, time and quantity can be said to be two sides of the same coin since they are connected by the following relation with demand: $time = {}^{quantity}/_{demand}$.

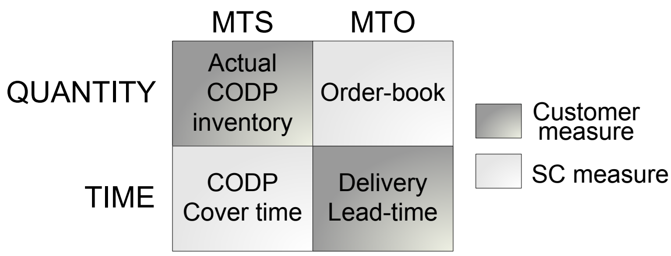
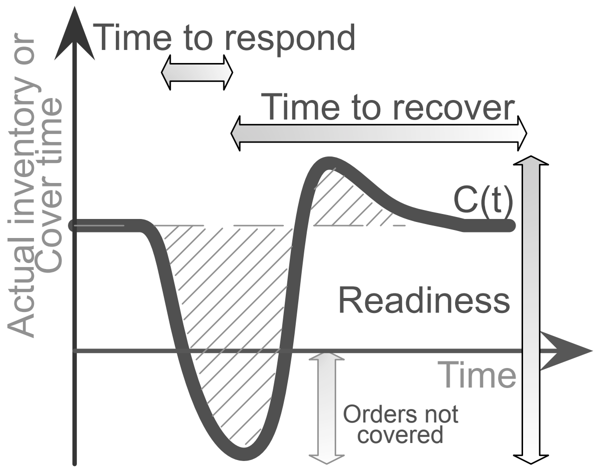


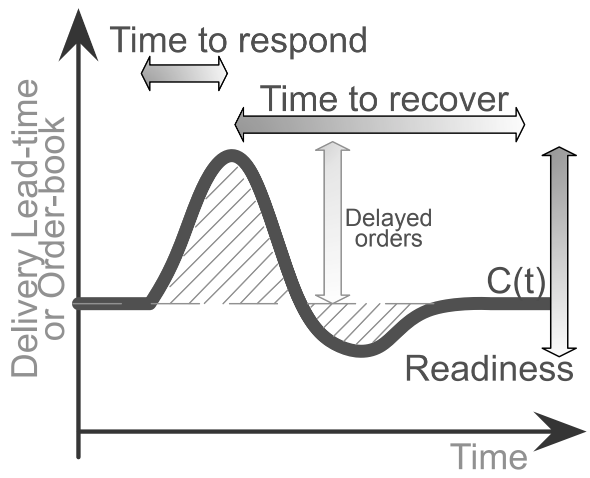
Figure 4.2: Different dimensions for assessing supply chain resilience performance

Building on Sheffi and Rice (2005), Asbjørnslett (2008), Tierney and Bruneau (2007), Zobel and Khansa (2011) and using the supply chain resilience definition of Ponomarov and Holcomb (2009), Figure 4.3 is presented. This figure in combination with Figure 2.4 suggests that a key indicator of supply chain resilience is the impact any disturbance has on the end customer, no matter where in the supply chain that disruption occurs. Therefore resilience may be measured by evaluating the output response, $C(t)$, at the interface between the supply chain and the end customer.

Based on the foregoing literature synthesis, Figure 4.3 represents the proposed system dynamics metrics for assessing supply chain resilience. The actual inventory or cover time in the MTS and the delivery lead-time or the order-book in the MTO system should be monitored and evaluated as surrogates of the customer service level. After a disturbance, both systems show signs of decline in service level until a point when they start to improve again. This corresponds to the response time. Then, the recovery process starts and lasts until the service level again achieves the desired target. The readiness is represented by the maximum peak to trough vertical displacement. The smaller the vertical displacement is, the more prepared or available for service, in other words, the more ready the supply chain may be said to be. Note that, in both MTS and MTO systems, the performance is considered poor if different from the targeted value, and therefore, it does not matter if the actual performance is above or below the target. In the case of inventory levels, this statement is valid because lower inventory levels threaten customer service and higher inventory levels increase supply chain costs. For lead-times, many may think that delivering goods in a shorter time than that promised will satisfy the customer. However, the literature shows that consistency in delivery lead-times is equally important (Gosling *et al.*, 2012) because customers may not be prepared to receive



(a) MTS



(b) MTO

Figure 4.3: Assessing supply chain resilience: readiness, response and recovery

goods before the scheduled date.

Taking all the attributes of the system curve into account, it is proposed that the smaller the region between the actual response and the target level, as highlighted in Figure 4.3, and the faster the response and recovery are, the more resilient the supply chain can be said to be. This follows the same reasoning as minimising the resilient triangle; however in the approach suggested here it is considered that the output may overshoot and/or undershoot before recovering, hence not assuming a triangular shape but some form of oscillatory behaviour.

This thesis focuses only on the MTS supply chain system as a conscious decision to narrow this research down. Therefore, from now on the author will only refer to actual inventory responses when investigating supply chain resilience. However, the lessons learned in this research process can be certainly applied to investigate resilience in MTO systems.

The next section compares and contrasts suitable composite performance measures that encompass the three properties for resilience as given in Section 4.1.

4.3 Finding a composite performance measure

As discussed before, a design of a resilient supply chain control system is an attempt to meet a set of specifications which defines the overall resilience performance of the supply chain in terms of certain measurable characteristics. Figure 4.3 illustrates these measures in respect of the dynamic responses of inventory and lead-time to a single disturbance, or single step input. These measures include minimising time to respond, time to recover and the vertical displacement which must be satisfied simultaneously. The problem of having separated measure when evaluating the system responses is that the design process may become a trial and error procedure. However, if a composite performance index could be established on the basis that it might describe the resiliency of the supply chain system, then the design process could become more logical and straightforward.

In control engineering, a number of composite performance indices are used to evaluate system responses. The most common of these performance indices are:

the integral of time absolute error (ITAE), the integral of absolute error (IAE), the integral of time square error (ITSE) and the integral of square error (ISE).

4.3.1 Integral of absolute error

The integral of the absolute magnitude of the error or IAE simply integrates the absolute error over time and assumes the following form:

$$IAE = \int_0^\infty |e(t)|\,\mathrm{d}t = \lim_{\delta t \to 0} \sum_{t=0}^{\infty} |e(t)|\delta t \tag{4.1}$$

where $e(t)$ is the error in the customer service related measure, i.e., the difference between the targeted value and the actual performance response.

This formula puts equal weight on small and large errors occurring whether sooner or later in time. A system designed to minimise IAE tends to produce a slow response and usually with a small sustained oscillation (continued oscillation due to insufficient damping); in other words it yields a fairly good underdamped system (Shinners, 1998).

Since this performance index simply calculates the area between the targeted and current performance values, it is analogous to the calculation of the resilient triangle proposed by Tierney and Bruneau (2007) and represented in Figure 4.1(b). A variant form of this performance measure, IAE^3, has also been used by system dynamics scholars to estimate the supply chain's production on-costs (Stalk and Hout, 1990; Towill *et al.*, 1992).

4.3.2 Integral of time absolute error

The ITAE or integral of time multiplied by the absolute value of error is used to emphasise long duration errors and is recommended for the analysis of systems which require fast settling time (Dorf and Bishop, 1998). The ITAE is given by:

$$ITAE = \int_0^\infty t.|e(t)|\, \mathrm{d}t = \lim_{\delta t \to 0} \sum_{t=0}^{\infty} t.|e(t)|\delta t \tag{4.2}$$

By using this performance index, two dimensions of time, which can be related to response and recovery times, and one dimension of variation, or readiness, are taken into account. Hence, more weight is given to time than to variation. Moreover, errors that exist after a long time are weighted much more heavily than those at the start of the response.

ITAE tuning produces systems which recover very rapidly. Nevertheless, the downside of this performance index is that it also produces systems with a slow-moving initial response which is necessary to avoid sustained oscillation (Shinners, 1998).

4.3.3 Integral of square error

The ISE, is represented by the following equation:

$$ISE = \int_0^\infty e^2(t)\, \mathrm{d}t = \lim_{\delta t \to 0} \sum_{t=0}^{\infty} e^2(t)\delta t \tag{4.3}$$

This performance index penalises large errors more than smaller ones, since the square of a large error becomes much greater and the square of a small error becomes much smaller.

Control systems specified to minimise ISE will tend to eliminate large errors quickly, but will tolerate small errors persisting for a long period of time. Often this leads to a fast initial response and low amplitudes but a slow recovery and sustained oscillation since the system is very underdamped (Dorf and Bishop, 1998).

4.3.4 Integral of time square error

Another control system performance index is the ITSE or the integral of time multiplied by the square of the errors which is given by:

$$ITSE = \int_0^\infty t.e^2(t)\,\mathrm{d}t = \lim_{\delta t \to 0} \sum_{t=0}^{\infty} t.e^2(t)\delta t \tag{4.4}$$

This performance index continues penalising large errors more than small ones. Like the ISE but also penalises long duration errors. Hence the system recovery will be very quick for high amplitude values but it may maintain low amplitude errors for a longer period. The disadvantage of this index is that its handling can be very difficult (Shinners, 1998).

Figure 4.4 graphically represents the simulated calculations of IAE, ITAE, ISE and ITSE given a response $inventory(t)$ and its target value, $Tinv$. For all the performance indices, the error $e(t)$ can be found by making $Tinv - inventory(t)$. In order to consider both positive and negative errors, IAE and ITAE consider the absolute value of this error, $|e(t)|$, while ISE and ITSE consider its squared value, $e^2(t)$. Then, for ITAE and ITSE, the absolute and squared values of the error are multiplied by time. Finally, the integrations of $|e(t)|$, $t.|e(t)|$, $e^2(t)$ and $t.e^2(t)$ result

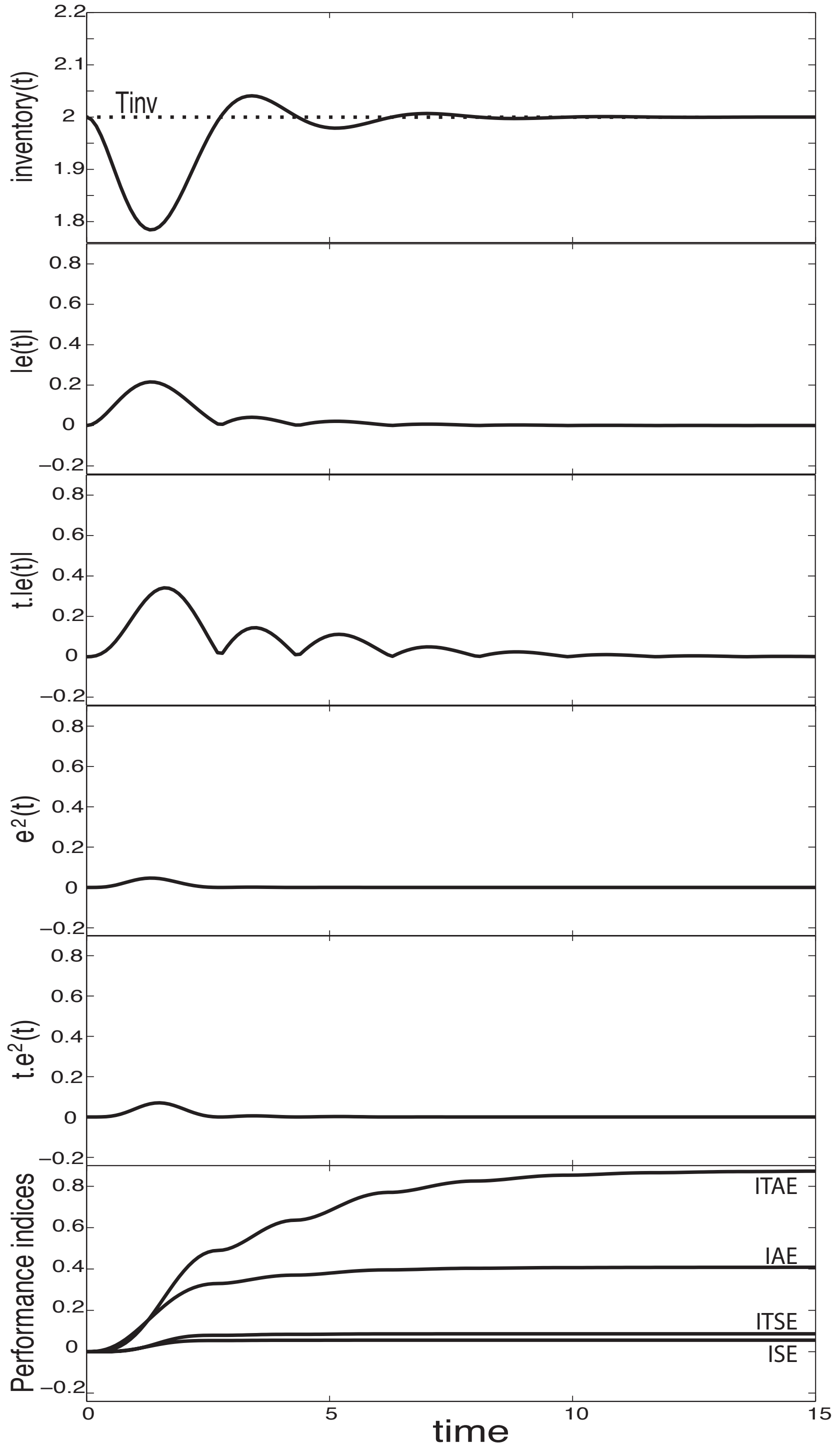


Figure 4.4: Calculating ITAE, IAE, ITSE and ISE

in IAE, ITAE, ISE and ITSE performance indices, respectively. In this example, the error values are small, and for this reason IAE and ITAE are greater than ISE and ITSE, respectively. Hence, the former two indices are more sensitive to small errors and penalise all errors equally.

Note that if the system does not reach the steady state or has a steady state error, IAE, ITAE, ISE and ITSE will tend to infinity implying a significant lack of resilience, since there is no recovery. Accordingly, a system designed to minimise the values of ITAE, IAE, ITSE and ISE would provide the best response and recovery with the lowest deviation from the target, or best readiness. However, each of the performance indices gives a different weight to each of the resilience measures. In the next section these composite performance measures will be compared and the implication of choosing each of them will be discussed.

4.4 Comparing the different performance indices

In order to compare the performance indices, standard transfer functions will be used to evaluate the selectivity of each performance index and its impact on the system response. Since the objective is to design a system that minimises the values of IAE, ITAE, ISE or ITSE, the selectivity approach is undertaken to evaluate the sensibility of each index to changes in the parameter settings. After finding the parameter settings that provide the best response according to each performance index, the impact of the chosen index on the system response will be investigated. Finally, the performance index that best represents resilience will be chosen.

4.4.1 Standard transfer function representing order rate response

In control systems, it is usually desirable to relate any transfer function to a standard form so that well-known relationships can be used to describe the system's behaviour without sketching responses all the time (Nise, 2000). In Chapter 3, it has been seen that the APIOBPCS model used in this thesis gives third order transfer functions. Hence, in order to compare the different performance indices given in the previous section, a standard third order transfer function, $G(s)$, is firstly used to represent the relation between an output transform, $C(s)$ and an input transform, $R(s)$. Third-order transfer functions can be re-written as a product of a first-order and a second-order systems (Srivastava *et al.*, 2009) resulting in the following standard third order transfer function:

$$G_1(s) = \frac{C_1(s)}{R(s)} = \frac{p.\omega_n^2}{(s+p)(s^2+2\zeta\omega_n s+\omega_n^2)} \tag{4.5}$$

where ω_n represents the natural frequency and this determines how fast the system oscillates during the transient response, while ζ, the damping ratio, describes how oscillations in the system decay with time. The term p represents a real pole of the system.

In order to have an idea of the behaviour of this standard transfer function to a step input, initial and final value theorems can be used.

<u>Initial value theorem</u>

This theorem allows frequency domain expressions to be related to the time domain behaviour as time approaches zero. For the standard transfer function defined as in 4.5, the initial value is given by:

$$\lim_{x\to 0} C_1(t) = \lim_{x\to\infty} s.C_1(s) = \lim_{x\to\infty} s.\frac{p.\omega_n^2}{(s+p)(s^2+2\zeta\omega_n s+\omega_n^2)}.\frac{1}{s} = 0 \tag{4.6}$$

The term $1/s$ at the end of Equation 4.6 indicates that the transfer function is being submitted to a unit step change in input, $R(s) = 1/s$.

Final value theorem

Similarly, this theorem allows us to find the value of the response as time reaches infinity. For Equation 4.5, the final value is:

$$\lim_{x\to\infty} C_1(t) = \lim_{x\to 0} s.C_1(s) = \lim_{x\to 0} s.\frac{p.\omega_n^2}{(s+p)(s^2+2\zeta\omega_n s+\omega_n^2)}.\frac{1}{s} = 1 \tag{4.7}$$

In summary, given a unit step input, the standard third order transfer function, $G(s)$, will produce a response that will start with value 0 and end with value 1, independent of the values given to the parameters ω_n and ζ. The positive sign of the transfer function also indicates that, as soon as the input increases, the output response will also increase. Hence, this standard output response resembles the response of an order rate in a production-inventory control system. In the next section, a standard form of a third order transfer function that provides an output that resembles an inventory response with a fixed target will be proposed.

4.4.2 Transfer function representing inventory response

Since the numerator of a transfer function defines the amplitude of the steady state and the transient response of the system, a change can be made to it in order to provide the output behaviour needed. Consequently, the following transfer function is used to represent an inventory response given an input:

$$G_2(s) = \frac{C_2(s)}{R(s)} = \frac{-p.\omega_n^2.s}{(s+p)(s^2+2\zeta\omega_n s+\omega_n^2)} \tag{4.8}$$

Again, the initial and final value theorems can give a clue of how the output response will behave given a unit step input.

<u>Initial value theorem</u>

$$\lim_{x\to 0} C_2(t) = \lim_{x\to\infty} s.C_2(s) = \lim_{x\to\infty} s.\frac{-p.\omega_n^2.s}{(s+p)(s^2+2\zeta\omega_n s+\omega_n^2)}.\frac{1}{s} = 0 \tag{4.9}$$

<u>Final value theorem</u>

$$\lim_{x\to\infty} C_2(t) = \lim_{x\to 0} s.C_2(s) = \lim_{x\to 0} s.\frac{-p.\omega_n^2.s}{(s+p)(s^2+2\zeta\omega_n s+\omega_n^2)}.\frac{1}{s} = 0 \tag{4.10}$$

Hence, given an unit step input, the transfer function given by Equation 4.8 will produce a response that will start and end with value 0 and the negative sign in the numerator indicates an opposite relationship between input and output, which means that as input increases, the output response decreases.

4.4.3 Selectivity

The selectivity property evaluates how selective the performance indices are when changes in the parameters occur. Since the objective is to design a system that minimises the values of IAE, ITAE, ISE or ITSE, the most selective performance index is the one whose minimum point can be easily identified.

In order to determine the selectivity of the performance indices, both transfer functions, $G_1(s)$ and $G_2(s)$, one representing the order rate response $C_1(s)$ and the other representing the inventory response $C_2(s)$, will be considered. Fixing the values of ω_n and p and varying the damping ratio, ζ, it is possible to check the selectivity of each performance index.

Figure 4.5(a) illustrates the selectivity of the performance indices when varying the damping ratio of the standard transfer function that represents an order rate response, $C_1(s)$ to a unit step input $R(s)$. The parameters ω_n and p have been set equal to 1. An inspection of these curves reveals that ISE is the least sensitive to parameter changes since the curve is rather flat near the point where the performance index reaches its minimum value (ISE is minimised for $\zeta = 0.354$). Therefore the selectivity of this performance index is poor. Similarly, IAE reaches minimum values when $\zeta = 0.505$ and it has slightly better selectivity than ISE. Again, ITSE (minimised for $\zeta = 0.429$) is somewhat more sensitive than the former performance indices but, as already mentioned, it is not easy to calculate.

ITAE is found to be the most sensitive of all performance measures and has its minimum value more clearly identified when $\zeta = 0.606$. Another measure pointed out by this graph is the IAE^3. This measure shows that simply finding the cube of IAE makes this index more selective. Since this index is not much concerned about time of recovery and considers mainly the difference between targeted and actual values, it has been often used to represent and estimate the production on-cost (Stalk and Hout, 1990; Towill *et al.*, 1992). Hence, this measure will be taken into account in Chapter 6 when evaluating the trade-off between resilience and production on-costs.

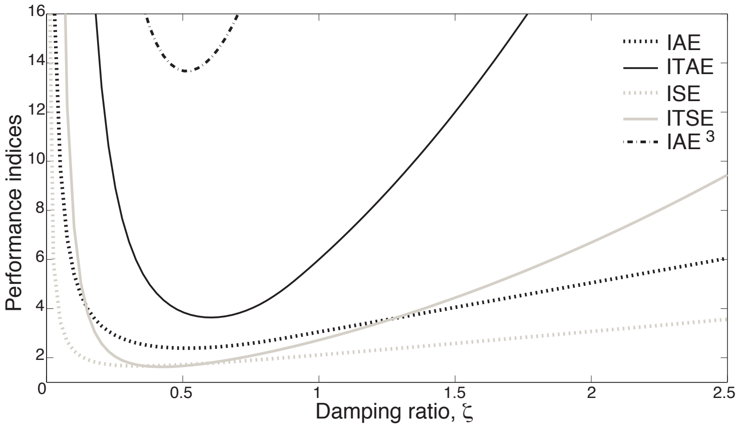
When considering the transfer function that better represents the inventory response, $C_2(s)$, which is the performance response proposed in this thesis to evaluate supply chain resilience in an MTS system, the selectivity of the performance indices becomes even clearer (Figure 4.6(a)). Only the ITAE index is capable of identifying a minimum value when the damping parameter changes. In other words, IAE and its cubic counterpart, ISE and ITSE completely lack selectivity.

Next, the time responses produced by both transfer functions, $G_1(s)$ and $G_2(s)$ for different damping parameter values, ζ, are discussed.

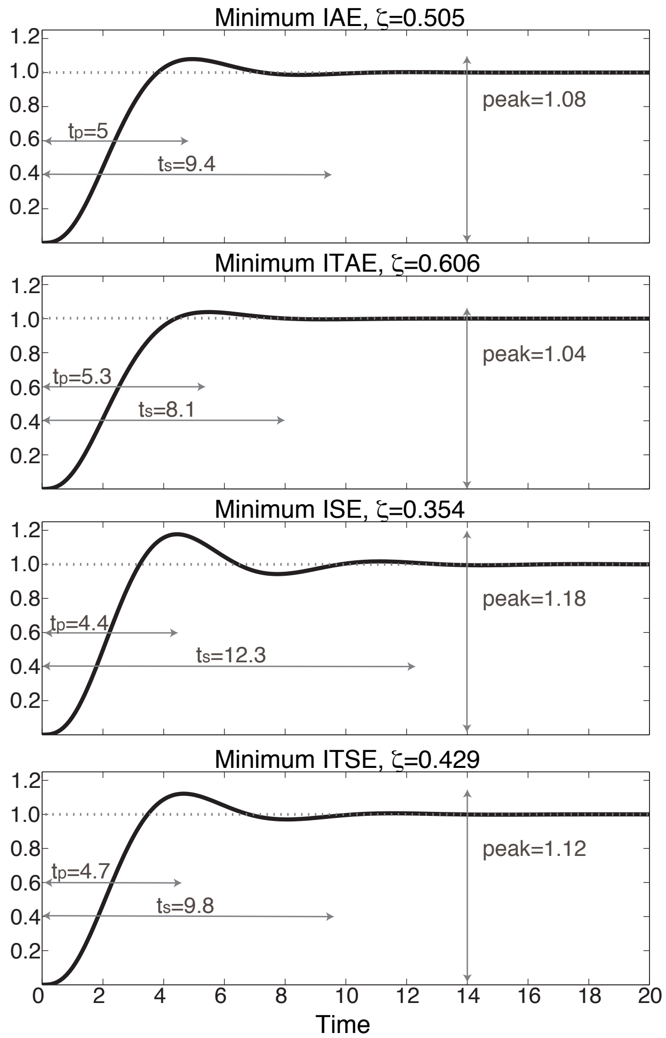
4.4.4 Effect on system's output responses

Figures 4.5(b) and 4.6(b) illustrate the unit step time responses yielded by transfer functions $G_1(s)$ and $G_2(s)$ presented in Equations 4.5 and 4.8 when submitted to different values of damping ratio, ζ and when $\omega_n = 1$ and $p = 1$. In Figure 4.5(b), the time responses of the standard third order transfer function, $G_1(s)$, are plotted for the parameters that yield the minimum performance indices illustrated in Figure 4.5(a). In order to evaluate these systems' responses in Figure 4.5(b), the nomenclature and properties used in control engineering are employed, which are: peak time (t_p), settling time within 2% of tolerance fraction (t_s) and peak or overshoot value. The overshoot is the value where a response exceeds its target the most. Hence, the peak time is the time required for the response to reach the first peak of the overshoot. Finally, the settling time is the time at which the output has entered, and remained within, a specified error band (tolerance fraction). Note that these measures relate significantly with the resiliency properties of response, recovery and readiness.

As shown in Figure 4.5(b) despite the minimum ITAE resulting in a slower peak time, it reaches the steady state much more quickly and has less deviation from the

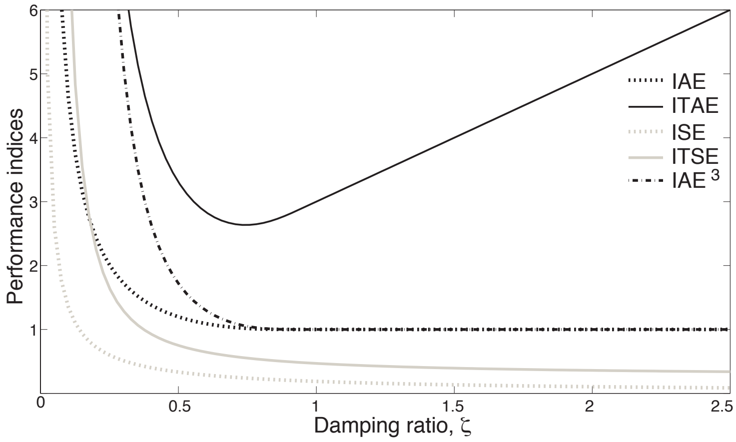


(a) Selectivity of performance indices

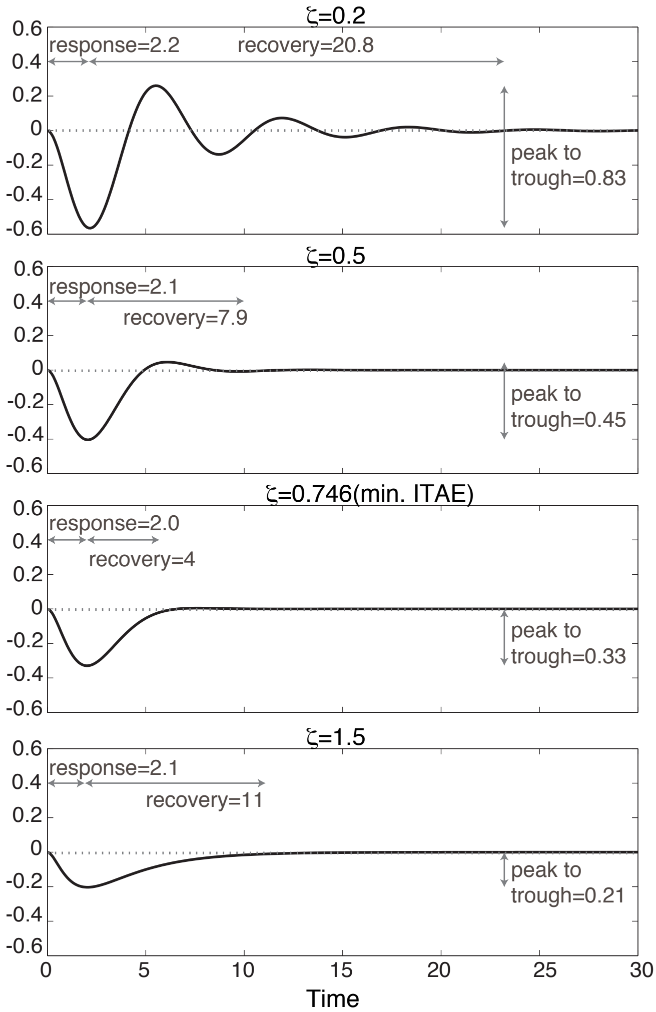


(b) Responses to a single step input

Figure 4.5: Comparing performances indices for order rate, $C_1(t)$



(a) Selectivity of performance indices



(b) Responses to a single step input

Figure 4.6: Comparing performances indices for inventory, $C_2(t)$

target and a lower peak than the other performance indices. This delay in rising time is compensated by a lack of sustained oscillations, which are particularly present for the ISE and ITSE minimums.

By examining Figure 4.6(b), it is possible to do an analogy between the standard transfer function with the control engineering properties and the resilience performance criteria. Although the only performance index that is able to determine a minimum value is the ITAE, the figure demonstrates the outputs for different damping ratios including the ITAE minimum, which is $\zeta = 0.746$. For very underdamped systems, the recovery time is very slow, sustained oscillations are observed and peak to trough values are high. The response times for the four presented outputs are in the same range, but this time the ITAE minimum result in a slightly better response time, which corresponds to the peak time in the previous figure (Figure 4.5(b)). In the case of overdamped systems, the peak to trough value is better than the ITAE minimum, but the response and, especially, the recovery times are very slow. Hence, there is a trade-off between recovery and readiness in this control system. Moreover, for $\zeta > 0.76$, all outputs have the same area between the target and current performance resulting in the same IAE index (see Figure 4.6(a)). The reason is, as ζ increases, the deviation from the target decreases but the recovery increases proportionally. And even ISE and ITSE indices are unable to capture the differences in the system's behaviour when overdamped. Zobel (2011) has also observed this effect when analysing the impact of earthquakes on the quality of infrastructure and buildings. Zobel (2011) points out that "if resilience is considered to be a function only of the area of the resilience triangle, then very different combinations of initial loss and recovery time can correspond to exactly the same resilience value". Hence, depending on different circumstances, the decision maker will have to opt between readiness or recovery. However, the literature tends to draw attention to the importance of responding and recovering fast, especially when there is a probability of multiple related disruption events (Zobel and Khansa, 2011).

4.5 Summary

This chapter has further explored the conceptual frameworks for assessing supply chain resilience. In particular, this work uses the definition of supply chain resilience developed by Ponomarov and Holcomb (2009) to translate their qualitative description of resilience into measurable properties: readiness, response and recovery. By stating that the objective of the supply chain is matching supply with demand, and hence, satisfying customers, this chapter has established that actual inventory or cover time responses at an MTS supply chain system and order book or delivery lead-time at an MTO system should be considered the performance indicators for resilience.

An important aspect of the proposed assessment framework is that supply chain resilience should be measured at the interface between the supply chain and the end customer because of the supply chain's goal to satisfy customers. This goes against the many scholars who have developed frameworks more suitable to assessing supply chain resilience at a local level (Datta *et al.*, 2007; Colicchia *et al.*, 2010b; Carvalho, 2011). Moreover, a resilience profile and an assessment framework have been constructed based on the works of Sheffi (2005b); Asbjørnslett (2008); Tierney and Bruneau (2007) and Zobel and Khansa (2011). However, since a system dynamics approach has been taken, the resilience profile suggested by this thesis considers an oscillatory behaviour of the performance responses to disruptions instead of a triangular shape.

When determining a composite performance measure to assess supply chain resilience, the author has utilised techniques from control engineering. The performance indices IAE, ITAE, ISE and ITSE have been pointed out as possible measures to design systems with minimum response and recovery times and vertical displacement. Each of these indices gives different weight to specific properties of the performance response. The use of IAE, which is simply the calculation of the area between the target value and the current response, has previously been suggested (Tierney and Bruneau, 2007; Asbjørnslett, 2008; Zobel and Khansa, 2011) to minimise the resilience triangle. However, as already shown, the area between the target performance and the current one is not the only way to minimise the resilience triangle. So, the author used standard transfer functions representing order rate and inventory responses to compare and contrast the different performance indices. In the selectivity analysis, the ITAE and IAE[3] indices demonstrated to be the most sensitive among the other performance indices to control order rate responses. However for the control of inventory responses, ITAE was the only index to provide a selectivity. Hence, this index facilitates the choice of parameter settings that yield a resilient supply chain system. In addition to this, the output response produced by ITAE minimum parameters does not have sustained oscillations and seems to respond and recover much faster than the other responses. In the inventory response, a trade-off between readiness and recovery for overdamped systems has been found. This trade-off has also been supported by another study (Zobel, 2011) but more weight should be given to the recovery time given its importance (Zobel and Khansa, 2011), so ITAE is the most suitable index of all the indices considered. Resilience is sometimes regarded as "the speed with which the company returns to normal performance levels" (Sheffi, 2005b).

Further investigation into designing resilient supply chains in this thesis will only consider MTS systems. This is due to the time constraint of the research and also the need for an in-depth analysis of the supply chain models to answer the methodological research questions already established. However, the author believes that evaluating resilience in MTO supply chains offers a future research opportunity, especially when considering the MTS-CODP-MTO combination.

Adapting from Figure 2.4, Figure 4.7 is given to illustrate the supply chain resilience framework that will be considered in this thesis. The source of risk in question is the disturbances arising from system dynamics and control policies. Only one mitigation strategy will be considered, which is that of proposing a re-design of the supply chain control parameters that yield a resilient supply chain. Since only MTS production systems are taken into account, actual inventory responses at the interface between the supply chain and the end customer will be evaluated. Moreover, the ITAE index will be used to find the parameter settings that result in a resilient system.

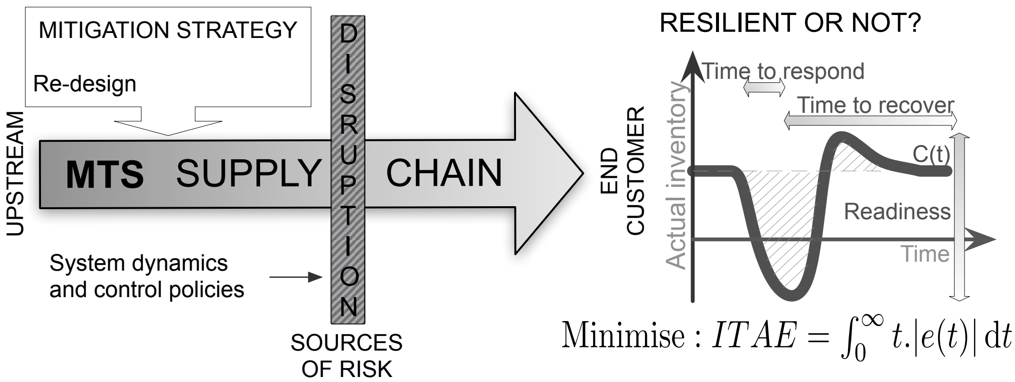


Figure 4.7: Assessing supply chain resilience: thesis framework

5 Analysis of the system dynamics models

"Nonlinear systems engineering is regarded not just as a difficult and confusing endeavor; it is widely viewed as dangerous to those who think about it for too long."

– Wilson J. Rugh (2002), *Nonlinear system theory*

This chapter contains analysis of the two nonlinear system dynamics models: Forrester's (Part I) and the APIOBPCS (Part II) models. A detailed and step-by-step description of suitable simplification and linearisation methods for the analysis of these models is provided. Simplification methods include block diagram manipulation and low-order modelling and linearisation methods which encompass small perturbation theory for continuous nonlinearities and describing functions for discontinuous nonlinearities. For each method deployed, a comparison between the original model and the resulting one has been made. At the end of each part, the simplified and linearised models are used for estimating system behaviour and responses. Finally, a design analysis of each model will be performed by investigating the impact of different control parameters on supply chain resilience performance.

Part I: Forrester's model

The first model to be investigated is the well-known production-distribution model proposed by Forrester (1958; 1961). This benchmark model, which is often quoted synonymously with the bullwhip effect, is one of the few production and inventory control system representations that has inherent continuous, discontinuous, single- and multi-valued nonlinearities. In addition to this, Forrester's model has already been translated from the DYNAMO language into a differential equations form (Wikner *et al.*, 1992; Jeong *et al.*, 2000).

However, very little previous research used Forrester's model as a benchmark for applying supply chain analysis and redesign methods. In this section, simplification and linearisation techniques will be applied to analyse the Forrester model.

5.1 Previous simplification and linearisation

As already mentioned in Section 3.3.3.1, Wikner *et al.* (1992) have endeavoured to gain more insights into the Forrester model by a two-stage linearisation and simplification approach. More importantly, their work translated the Forrester DYNAMO equations into a Laplace domain block diagram form to improve visibility of the model's system structure.

Figure 5.1 illustrates the block diagram representation of one echelon of the Forrester model and the steps taken by Wikner *et al.* (1992) in simplifying the original model. Appendix B contains the listing and explanation of constants, variables and equations.

The first step taken by Wikner *et al.* (1992) was to translate the original DYNAMO equations into a control engineering block diagram representation and to identifiy any nonlinearities. Then they removed the discontinuous nonlinearities, represented by the CLIP functions ( ), by assuming that the capacity limitations in the manufacturing (AL) and shipping (IA) are never attained under 'normal' operating conditions, so that the actual manufacturing rate equals the wanted manufacturing rate ($MD = MW$) and that the shipment sent equals the shipment tried ($SS = ST$) at all times. Note that while AL is a fixed capacity constraint IA is a variable.

The continuous nonlinearity, caused by the nonlinear comparator ratio ( ) between actual (IA) and target (ID) inventories in defining the delay in filling orders (DF), was considered the average value of this delay and then kept fixed as shown in Step 2 of Figure 5.1. Lastly in Step 3, by block diagram manipulation Wikner *et al.* collected constants by making $K = AI - DH - DU + DC + DP$, and eliminated redundancies in the original model, resulting in the final block diagram of Figure 5.1. Step 3 is further explained in Section 5.3.

The resulting model highlights the lack of information feedback into the manufacturing rate (MD) and exposes a separation between real and safety orders. Hence, Wikner *et al.* showed that demand amplification is not due to linear feedback control but due to a first order derivative term in the feedforward path. As their study was focussed on the bullwhip effect, greater accuracy is achieved for the analyses of manufacturing order rates. However, the main problem with their model is that their linearised and simplified model is unreliable for analysing inventory (IA) and shipments' (SS) responses due to the use of averaging techniques for linearisation and a disregard for CLIP functions.

In the following sections, it will be shown that it is possible to use the Taylor series to represent some of the nonlinearities present in the original model and linearise it

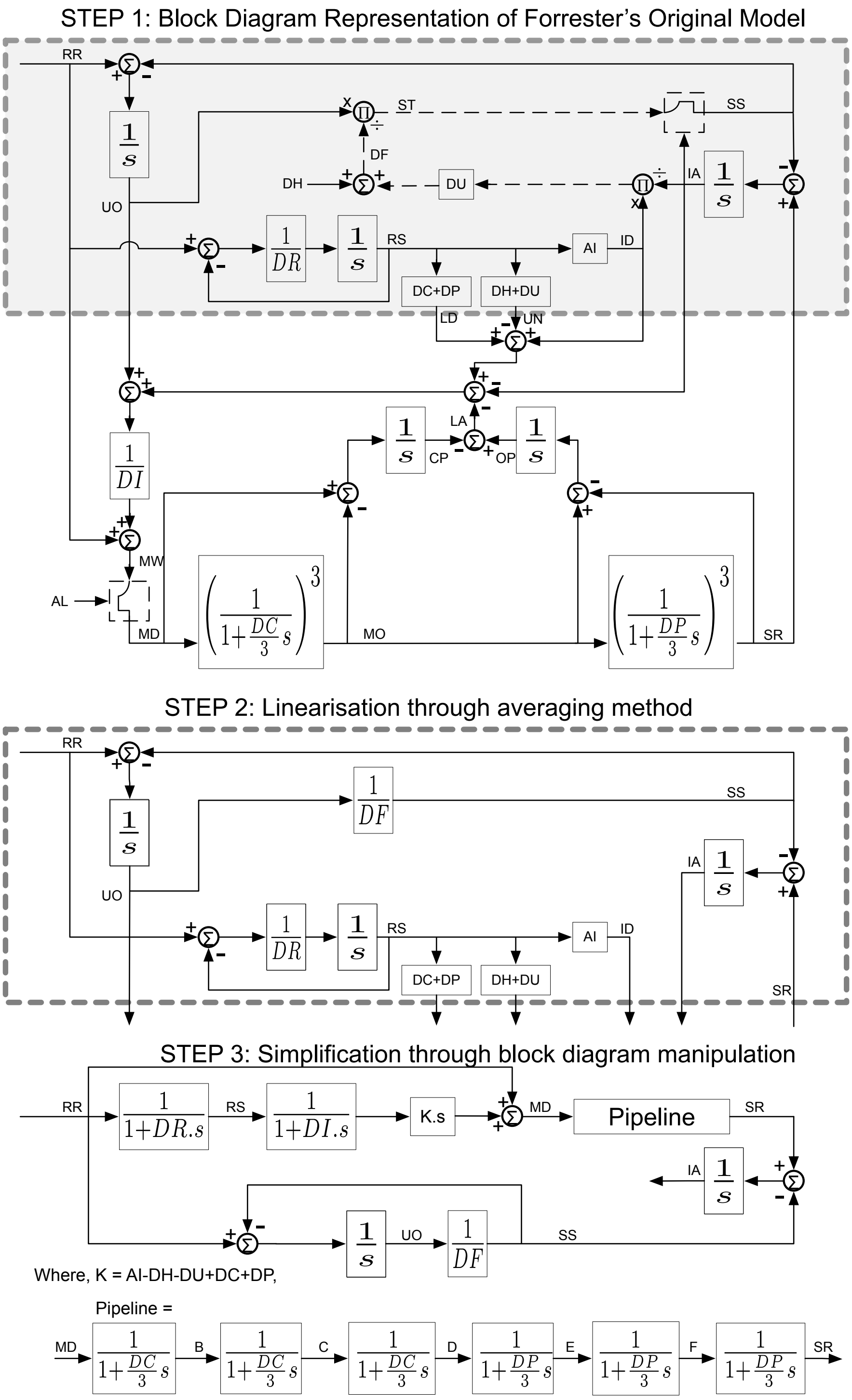


Figure 5.1: Wikner et al.'s approach for simplifying the Forrester model

with small perturbation theory to get better accuracy. Moreover, the presence of CLIP functions are not ignored and their effect on the overall response of the model will be investigated together with describing function technique.

5.2 This thesis' approach to simplification and linearisation

Wikner *et al.*'s (1992) approach consisted of conducting linearisation before simplification. This tendency has been seen in the system dynamics literature (Cuypers, 1973; Cuypers and Rademaker, 1974; Jeong *et al.*, 2000; Naim *et al.*, 2012). However, causal relationships between certain variables may be lost during the linearisation process.

Here, it is proposed that system dynamics models should be simplified first, by eliminating all redundancies whenever possible. Having a clearer view of the model will enable better analysis and synthesis of the nonlinear elements. These two potential routes to linearisation and simplification are shown in Figure 5.2.

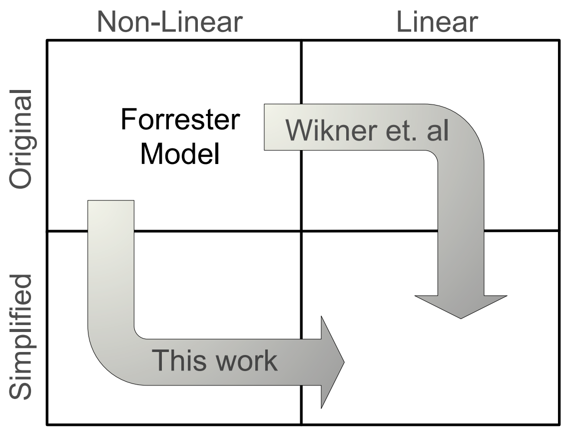


Figure 5.2: Different steps for obtaining a simplified and linearised version of non-linear models

5.3 Simplification

This section explains in detail how block diagram manipulation and low-order modelling techniques can be used to further simplify the Forrester model.

5.3.1 Block diagram manipulation

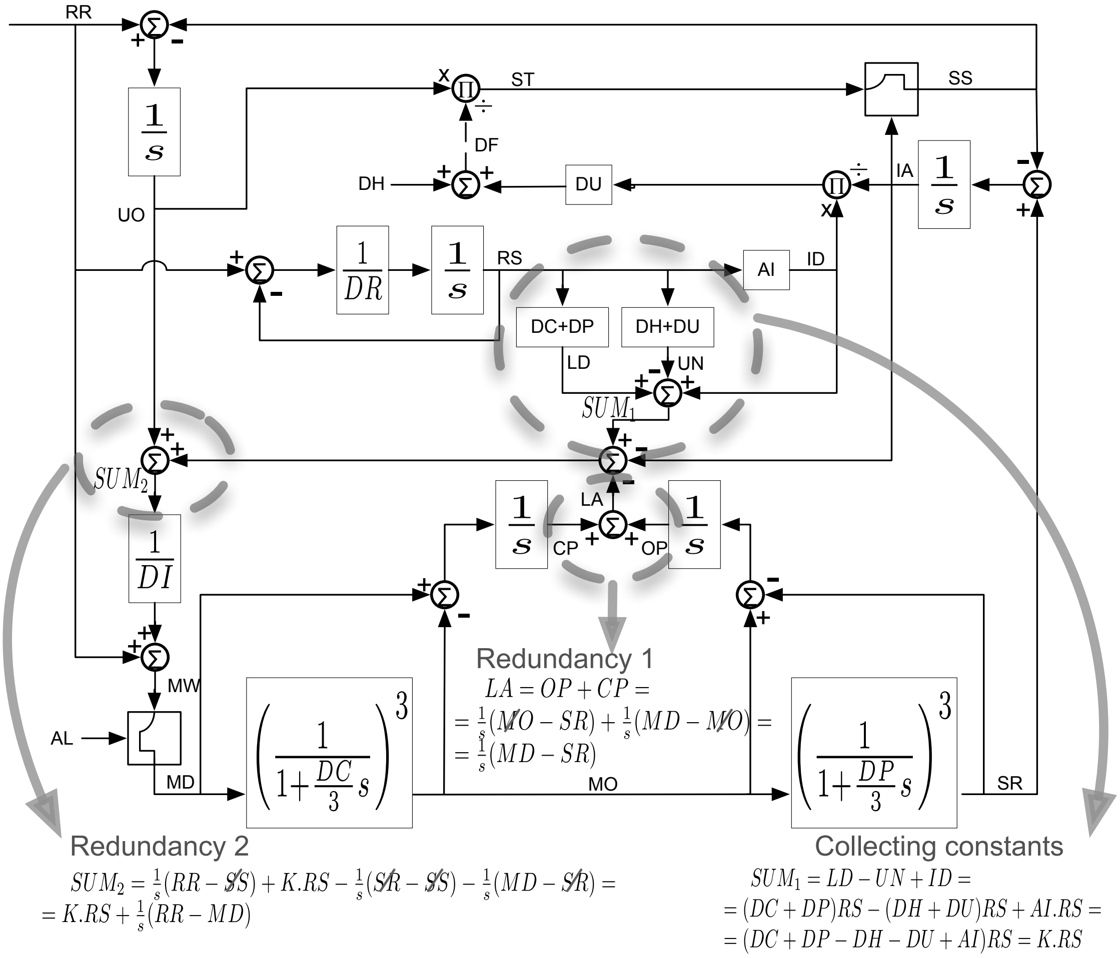
The original block diagram of Figure 5.1 can be manipulated so that redundancies are removed and constants are collected as given in Figure 5.3(a). The sequence of steps taken were:

1. **Redundancy 1:** calculation of actual pipeline orders in transit (LA). The figure shows that the information about manufacturing orders (MO) is being added and reduced at the same time when calculating LA; therefore the information about MO is redundant in the determination of LA.

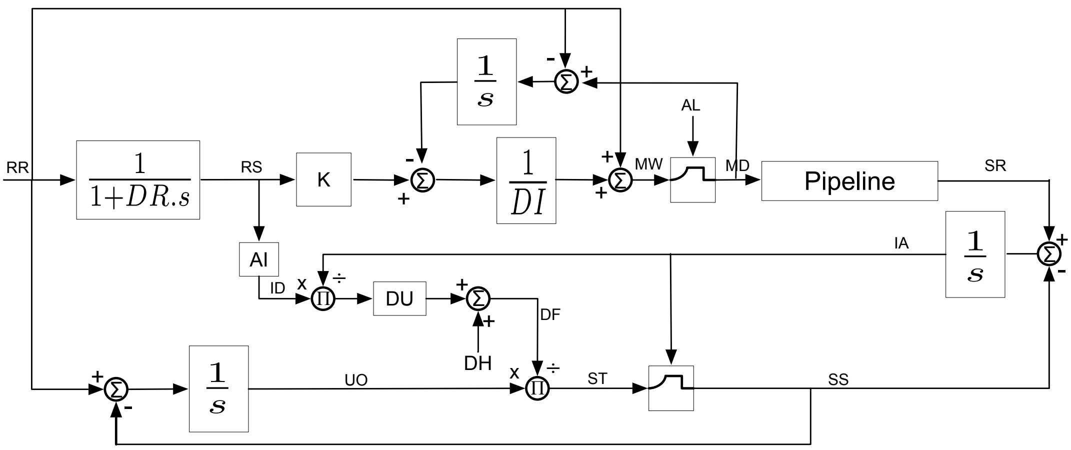
2. **Collecting Constants:** gathering constants from the output variable in the summing comparator SUM_1. In the SUM_1 expression, all the constants that multiply the variable smoothed requisition orders (RS) were combined together and called K, which is equal to $DC + DP - DH - DU + AI$.

3. **Redundancy 2:** calculation of the output variable in the summing comparator SUM_2. In the calculation of SUM_2 both shipment received (SR) and shipment sent (SS) information is found to be redundant; therefore, they can be removed at that summation point.

After removing all the redundancies and combining the constants in K, the block diagram in Figure 5.3(b) is then presented. Note that the resulting simplified nonlinear model in Figure 5.3(b) provides exactly the same responses as in the original



(a) Forrester's model: Original - removing redundancies and collecting constants



(b) Forrester's model: simplified and nonlinear

Figure 5.3: Simplification process of Forrester's model

model. No variable interactions were lost in this simplification process. Moreover, it can now be seen that the Forrester model contains a feedback loop in the manufacturing order rate (MD) but only to provide information regarding the manufacturing capacity. If the system cannot manufacture the amount wanted in a particular time period, this information is then fed back so that these orders can be produced later. However, if the capacity limitation is never reached, this feedback information is not needed and hence it can be ignored. As also evidenced by Wikner *et al.* (1992) and Naim *et al.* (2012), inventory information is not fed back into manufacturing orders. Therefore, bullwhip occurs due to orders being placed as a combination of 'real' plus 'safety' orders.

5.3.2 Low-order modelling

A high-order control system often contains poles that produce little effect on the transient response. For instance, in the Forrester pipeline, which is represented by the sixth-order transfer function $\left(\frac{1}{1+\frac{DC}{3}s}\right)^3 \cdot \left(\frac{1}{1+\frac{DP}{3}s}\right)^3$, Forrester's parameter values ($DC = 1$ and $DP = 6$) demonstrate that the delay DC has little impact in the transient response of the pipeline as demonstrated by Figure 5.4. This is due to the position of the poles in the s-plane. If the magnitude of a pole is at least 5 times that of a dominant pole or pair of complex dominant poles, then the pole may be regarded as insignificant and can be ignored as far as the transient response is concerned (Nise, 2000).

However, Towill (1981) and Kuo and Golnaraghi (2003) argue that there are better ways of approximating high-order models to low-order ones especially when a transfer function may not have clear dominant poles. In the case of the Forrester pipeline, if the delay DC increases or DP decreases it would be more difficult to

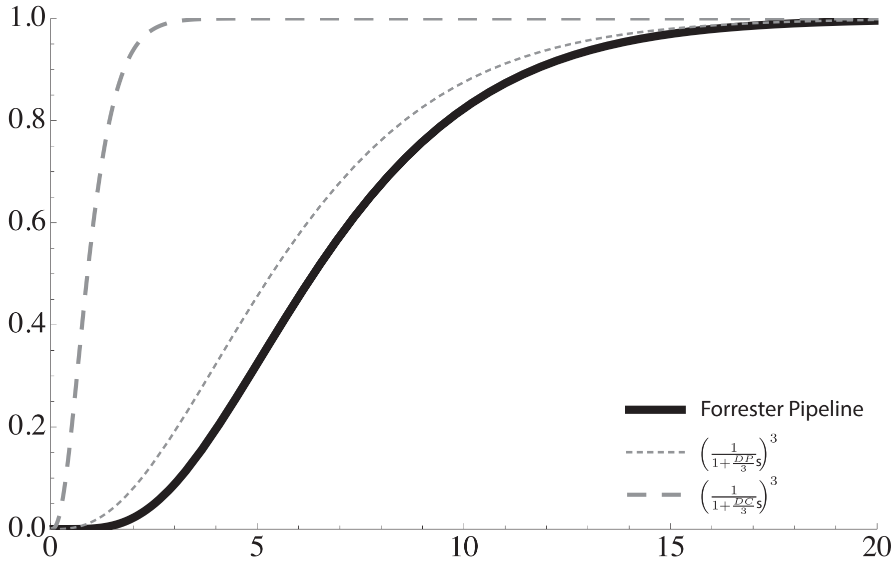


Figure 5.4: The Forrester pipeline in comparison with the effect caused by each delay element

determine the dominant and insignificant poles.

The reader can refer to Appendix A to review the methods suggested by Towill (1981) and Kuo and Golnaraghi (2003). The former method is extended from the time delay theorem developed by Matsubara (1965). This method attempts to determine a low-order model based on the system unit step response and has already been used by Jeong *et al.* (2000) to approximate high-order delays in the Forrester model. Kuo and Golnaraghi (2003) recommend a method proposed by Hsia (1972) that approximates a high-order system to a low-order model by approaching their frequency responses.

If a system is represented by a transfer function in the following form:

$$T(s) = \frac{1 + b_1 s + b_2 s^2 + \cdots + b_q s^q}{1 + a_1 s + a_2 s^2 + \cdots + a_n s^n} \tag{5.1}$$

then, the low-order model can be represented as:

$$T_M(s) = \frac{1 + B_1 s + B_2 s^2 + \cdots + b_Q s^Q}{1 + A_1 s + A_2 s^2 + \cdots + A_N s^N} \tag{5.2}$$

given that $Q \leq q$ and N must be less than n.

In the case of Forrester's model, the high-order pipeline transfer function is:

$$\begin{aligned} T(s) =& \frac{SR}{MD} = \left(\frac{1}{1+\frac{DC}{3}s}\right)^3 \cdot \left(\frac{1}{1+\frac{DP}{3}s}\right)^3 = \\ &= 1 \Big/ \Big[1 + (DC+DP)s + \left(\tfrac{DC^2}{3} + DC.DP + \tfrac{DP^2}{3}\right)s^2 + \left(\tfrac{DC^3}{27} + \tfrac{DC^2 DP}{3} + \tfrac{DC.DP^2}{3} + \tfrac{DP^3}{27}\right)s^3 + \\ &+ \left(\tfrac{DC^3 DP}{27} + \tfrac{DC^2 DP^2}{9} + \tfrac{DC.DP^3}{27}\right)s^4 + \left(\tfrac{DC^3 DP^2}{81} + \tfrac{DC^2 DP^3}{81}\right)s^5 + \left(\tfrac{DC^3 DP^3}{729}\right)s^6\Big] \end{aligned} \tag{5.3}$$

5.3.2.1 Matsubara time delay theorem

In order to approximate the transfer function system given in Equation 5.3 to a first order model using the Matsubara's (1965) time delay theorem, firstly one pole has to be selected. This is normally the pole with the least magnitude. Assuming that DP is always greater than DC then $\frac{-3}{DP}$ can be chosen as an initial pole. Hence, the initial low-order model will be:

$$T_{M'}^{(1)}(s) = \frac{1}{1 + \frac{DP}{3}s} \tag{5.4}$$

To compensate for the error between the original system in Equation 5.3 and the low-order model in Equation 5.4, Matsubara has proposed the addition of a pure delay, τ, to the low-order model, so that $\tau = (a_1 - b_1) - (A_1 - B_1)$ (Check

Appendix A for more detail). From Equations 5.1, 5.2 and 5.3, it is found that $b_1 = 0$, $a_1 = DC + DP$, $B_1 = 0$ and $A_1 = \frac{DP}{3}$. Hence, the time delay will be $\tau = DC + \frac{2}{3}DP$ and the low-order system can be represented by:

$$T^{(1)}_{M\tau}(s) = e^{-\left(DC+\frac{2}{3}DP\right)s}\left[\frac{1}{1+\frac{DP}{3}s}\right] \tag{5.5}$$

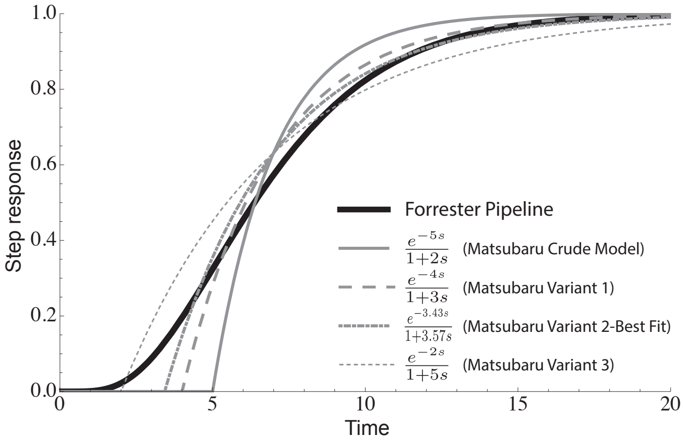
Repeating the processes above, it is found that the second and third order approximations would, respectively, be

$$T^{(2)}_{M\tau}(s) = e^{-\left(DC+\frac{DP}{3}\right)s}\left[\frac{1}{(1+\frac{DP}{3}s)^2}\right] \tag{5.6}$$

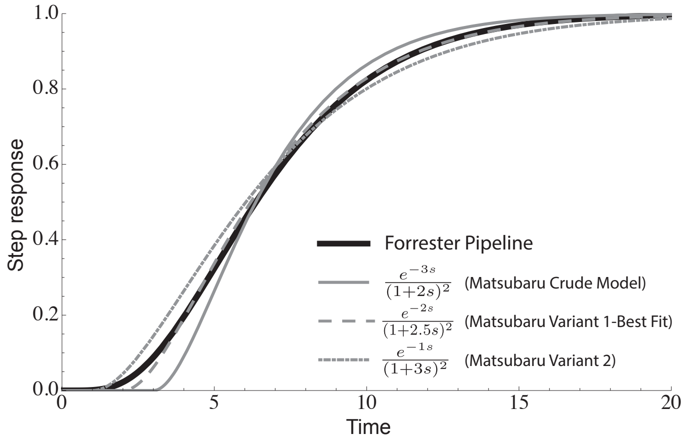
$$T^{(3)}_{M\tau}(s) = e^{-(DC)s}\left[\frac{1}{(1+\frac{DP}{3}s)^3}\right] \tag{5.7}$$

Note that for the third order model, a response caused by the delay DP with a shift in time of DC seconds is considered. Figure 5.4 shows the effect of the delays DC and DP. In the same figure, if the response caused by the delay $DP = 6$ is simply shifted by $DC = 1$ to the right, the pipeline and the model would match very closely.

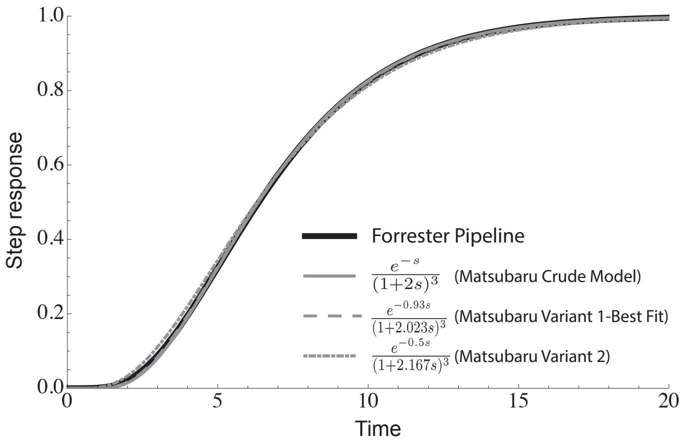
Figures 5.5(a), 5.5(b) and 5.5(c) illustrate the first, second and third order responses of Equations 5.5, 5.6 and 5.7, respectively, comparing them with the sixth order Forrester pipeline. As previously observed, the choice of poles and consequently time delay τ is initially recommended to be equal to the poles nearest to the imaginary axis. But the advantage of this method is that the choice of poles and τ can be varied in order to find a better fit between the system and the low-order model responses as long as the relationship $(a_1 - b_1) = (A_1 - B_1) + \tau$ is maintained.



(a) First order delay approximations



(b) Second order delay approximations



(c) Third order delay approximations

Figure 5.5: The Matsubara time delay approximations for Forrester's pipeline

By using MATLAB programming, the value of τ which provides the best-fit response model has been found in the order of 0.01 time unit. The objective function is to minimise the absolute error between the system output of the sixth-order Forrester pipeline and the low-order models. Figures 5.5(a), 5.5(b) and 5.5(c) also illustrate the best-fit responses.

Note that the first order approximation is less effective because it does not produce a response with an s-shape. On the other hand, the second order responses fit better with the pipeline response. In particular, the best fit is found when the model assumes a DP value equal to 7.5 instead of 6 (the original value for DP) and τ equal to 2.

If for any reason it is necessary to avoid the time delay model $T_{M\tau}(s)$ and a low-order model in the form $T_M(s)$ is preferable, it is possible to adjust the model coefficients by placing a 'dummy' pole so that $(A_1 - B_1) = (a_1 - b_1)$ (Towill, 1981). However, by placing this 'dummy' pole, a minimum of a second order model will be necessary. Hence, the first, second and third order low-order time delay models will become, respectively, the following second, third and fourth order low-order models:

$$T_M^{(1)}(s) = \frac{1}{\left(1 + \frac{DP}{3}s\right).\left[1 + (DC + \frac{2}{3}DP)s\right]} \tag{5.8}$$

$$T_M^{(2)}(s) = \frac{1}{\left(1 + \frac{DP}{3}s\right)^2.\left[1 + (DC + \frac{DP}{3})s\right]} \tag{5.9}$$

$$T_M^{(3)}(s) = \frac{1}{\left(1 + \frac{DP}{3}s\right)^3.\left[1 + DCs\right]} \tag{5.10}$$

5.3.2.2 Hsia Method

The approximation method proposed by Hsia (1972) is based on selecting A_i and B_i (see Equation 5.2), in such a way that $T_M(s)$ has a frequency response very close to that of $T(s)$. In other words, the magnitude of the frequency function $\frac{T(j\omega)}{T_M(j\omega)}$ is required to deviate the least amount from unity for various frequencies. Hence, the following relation should be satisfied:

$$\frac{|T(i\omega)|^2}{|T_M(i\omega)|^2} = 1, \text{ for } 0 \leq \omega \leq \infty \tag{5.11}$$

When applying this method to find a first order approximation of Forrester's pipeline, the value of coefficient A_1 of the following low-order model is to be found.

$$T_M^{(1)}(s) = \frac{1}{1 + A_1 s} \tag{5.12}$$

Then, the next step is to find the ratio $\frac{T(s)}{T_M(s)}$:

$$\frac{T(s)}{T_M(s)} = \frac{1 + m_1 s}{1 + l_1 s + l_2 s^2 + l_3 s^3 + l_4 s^4 + l_5 s^5 + l_6 s^6} \tag{5.13}$$

where the coefficients l_i correspond to the coefficients a_i of the system $T(s)$ of Equation 5.3 and m_1 is equal to the coefficient A_1 of the low-order model in Equation 5.12. Hence,

$$m_1 = A_1 \tag{5.14}$$

$$l_1 = DC + DP \tag{5.15}$$

$$l_2 = \frac{DC^2}{3} + DC.DP + \frac{DP^2}{3} \tag{5.16}$$

$$l_3 = \frac{DC^3}{27} + \frac{DC^2DP}{3} + \frac{DC.DP^2}{3} + \frac{DP^3}{27} \tag{5.17}$$

$$l_4 = \frac{DC^3DP}{27} + \frac{DC^2DP^2}{9} + \frac{DC.DP^3}{27} \tag{5.18}$$

$$l_5 = \frac{DC^3DP^2}{81} + \frac{DC^2DP^3}{81} \tag{5.19}$$

$$l_6 = \frac{DC^3DP^3}{729} \tag{5.20}$$

The magnitude ratio between the system and the model will then be:

$$\begin{aligned} &\frac{|T(j\omega)|^2}{|T_M(j\omega)|^2} = \frac{T(s)T(-s)}{T_M(s)T_M(-s)} = \\ &= \frac{1+m_1s}{1+l_1s+l_2s^2+l_3s^3+l_4s^4+l_5s^5+l_6s^6} . \frac{1-m_1s}{1-l_1s+l_2s^2-l_3s^3+l_4s^4-l_5s^5+l_6s^6} \\ &= \frac{1+e_2s^2}{1+f_2s^2+f_4s^4+f_6s^6+f_8s^8+f_{10}s^{10}+f_{12}s^{12}} \end{aligned} \tag{5.21}$$

In order to satisfy the condition of Equation 5.11, Hsia's (1972) method suggests that at least $e_2 = f_2$. If the chosen low-order model was of second order or above, then the equalities $e_4 = f_4$, $e_6 = f_6$, . . . , and so on should also be respected (refer to Appendix A for more detail). Hence, the magnitude ratio between the system and the first order model in Equation 5.21 will have residual errors caused by coefficients f_4, f_6, f_8, f_{10} and f_{12}. The coefficients e_2 and f_2 can be found by solving the multiplication in Equation 5.21, resulting in:

$$\begin{cases} e_2 = & f_2 = -m_1^2 = -A_1^2 \\ f_2 = & 2l_2 - l_1 = \frac{1}{3}(-DC^2 - DP^2) \end{cases} \tag{5.22}$$

By replacing the second equation in the first one, the first order model can be determined as:

$$T_M^{(1)}(s) = \frac{1}{1 + \frac{\sqrt{DC^2+DP^2}}{\sqrt{3}} s} \tag{5.23}$$

Repeating the processes above to find the second order approximation, the following system of equations has to be solved:

$$\begin{cases} e_2 = & f_2 = 2m_2 - m_1^2 = 2A_2 - A_1 \\ e_4 = & f_4 = m_2^2 = A_2^2 \\ f_2 = & 2l_2 - l_1 = \frac{1}{3}(-DC^2 - DP^2) \\ f_4 = & 2l_4 - 2l_1 l_3 + l_2^2 = \frac{1}{27}(DC^4 + 3DC^2DP^2 + DP^4) \end{cases} \tag{5.24}$$

resulting in the following second order model

$$T_M^{(2)}(s) = \frac{1}{1+\frac{1}{3}\sqrt{3DC^2+3DP^2+2\sqrt{3}\sqrt{DC^4+3DC^2DP^2+DP^4}}s+\left(\frac{\sqrt{DC^4+3DC^2DP^2+DP^4}}{3\sqrt{3}}\right)} \tag{5.25}$$

For obtaining the third order approximation, it is necessary to solve a lengthy system of equations. Moreover, the resulting low-order model is a large and complex equation and there is no need in demonstrating it here. Via Wolfram Mathematica a numerical solution was found for the third order approximation when $DC = 1$

and $DP = 6$, which is illustrated in Figure 5.6 together with the other low-order approximations.

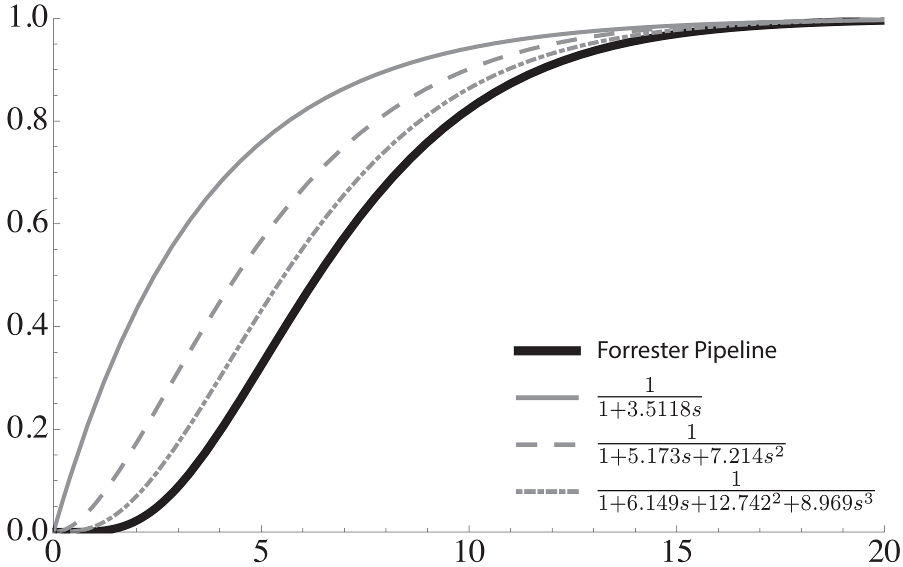


Figure 5.6: The Hsia first, second and third order approximations to Forrester's pipeline

5.3.2.3 Comparing the proposed methods

Comparing the methods recommended by Towill (1981) (Matsubara method) and Kuo and Golnaraghi (2003) (Hsia method), it is verified that the latter, although very sophisticated and complex, yields a poorer approximation for low-order modelling of the Forrester pipeline. Figure 5.7 demonstrates that even for the frequency response, which was the main interest of the Hsia method, the Matsubara low-order models provide better results. Although the Hsia method for the second and third order approximations can maintain an amplitude nearly the same as in the original system response, the approximations always lag behind the system response. For this reason, the Matsubara method has been chosen in the simplification process of the Forrester model. Hence, in the following analysis the Matsubara first order pipeline approximation in the form of $T_M^{(1)}(s)$ given by Equation 5.8 will be considered.

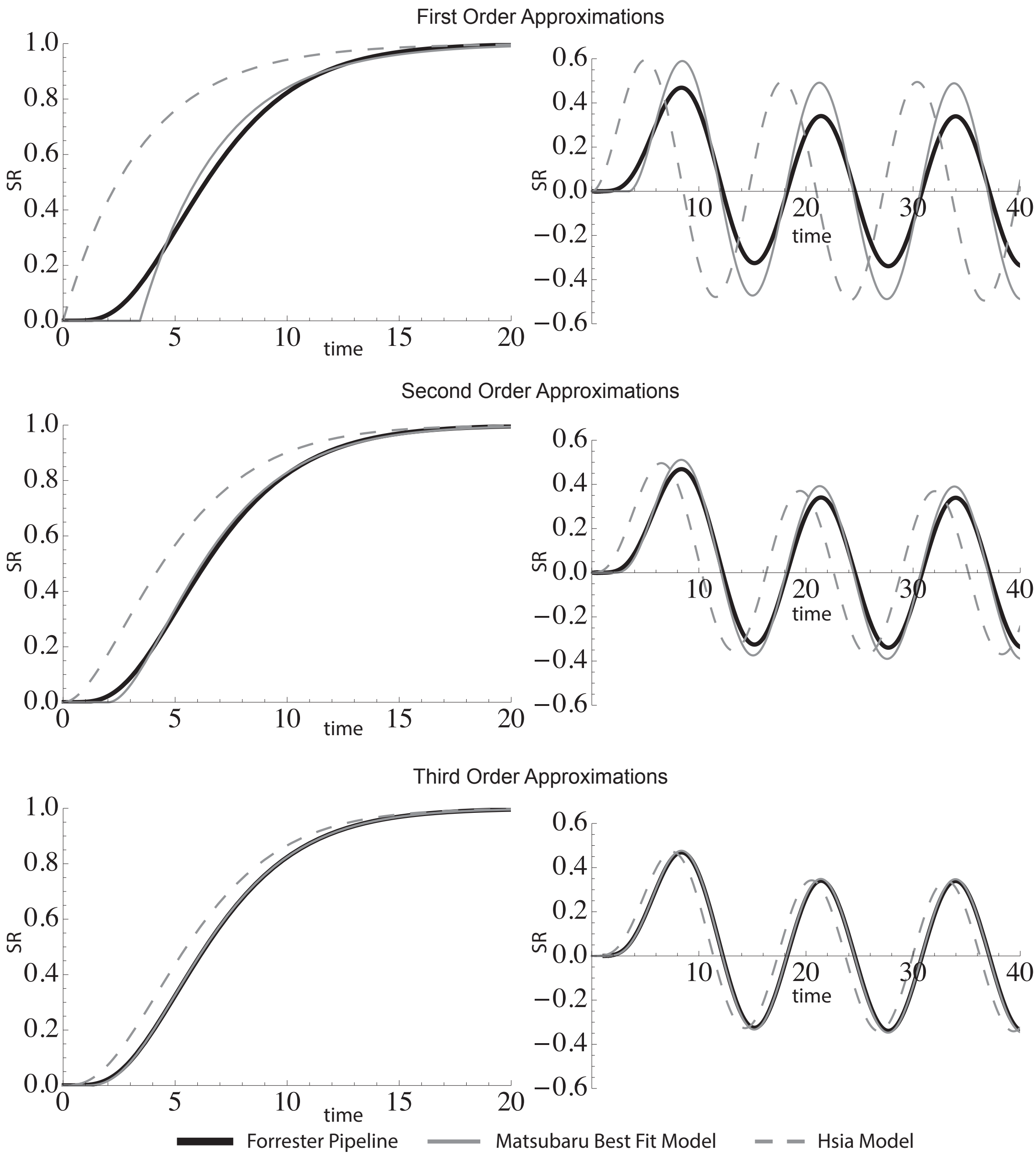


Figure 5.7: Comparing step and frequency responses of the Hsia and Matsubara methods of low-order modelling

5.4 Linearisation and quasi-linearisation

In this section, an analysis of the nonlinearities present in Forrester's model will be undertaken. Both continuous and discontinuous nonlinearities will be analysed separately since each of them requires different linearisation methods.

5.4.1 Analysis of continuous nonlinearities

Focusing on the continuous nonlinearities, the analysis starts by temporarily assuming that the CLIP functions () are not active. In other words, the manufacturing rate decision will be equal to the manufacturing rate wanted, $MD = MW$, and the shipment sent will be the same as the shipment tried, $SS = ST$, independent of actual inventory levels.

Hence, the system in Figure 5.3(b) without discontinuities is represented in Figure 5.8 and can be described by the system of differential equations (Equations 5.26-5.32), where $\dot{rs}$, $\dot{rs}_s$, $\dot{md}_d$, $\dot{sr}$, $\dot{uo}$ and $\dot{ia}$ are the state variables, $\dot{x} = f(x, u)$, of the system and rr is the system input, u. Note that, since the six-order pipeline was replaced by the lower-order equation in Equation 5.8, four other states have been excluded. The state variables rs_s and md_d, representing dummy variables RS_S and MD_D respectively, have been added to help to derive the state variable equations below

$$\dot{rs} = f_1(x, u) = \frac{rr - rs}{DR} \tag{5.26}$$

$$\dot{rs}_s = f_2(x, u) = \frac{rs - rs_s}{DI} \tag{5.27}$$

$$\dot{md}_d = f_3(x,u) = \frac{3K.rs}{DI.DP} - \frac{3K.rs_s}{DI.DP} - \frac{3md_d}{DP} + \frac{3rr}{DP} \tag{5.28}$$

$$\dot{sr} = f_4(x,u) = \frac{3(md_d - sr)}{3DC + 2DP} \tag{5.29}$$

$$\dot{uo} = f_5(x,u) = rr - ss = rr - \frac{uo.ia}{AI.DU.rs + DH.ia} \tag{5.30}$$

$$\dot{ia} = f_6(x,u) = sr - ss = sr - \frac{uo.ia}{AI.DU.rs + DH.ia} \tag{5.31}$$

$$rs(0) = a(0) = b(0) = sr(0) = rr(0), ia(0) = AI.rr(0),$$

$$uo(0) = (DH + DU)rr(0) \tag{5.32}$$

The outputs, $y = g(x,u)$, of interest are the manufacturing rate, MD, the actual inventory levels, IA, and shipment sent, SS. In addition to these outputs, it is interesting to know how the time-varying parameter DF will be affected after the linearisation.

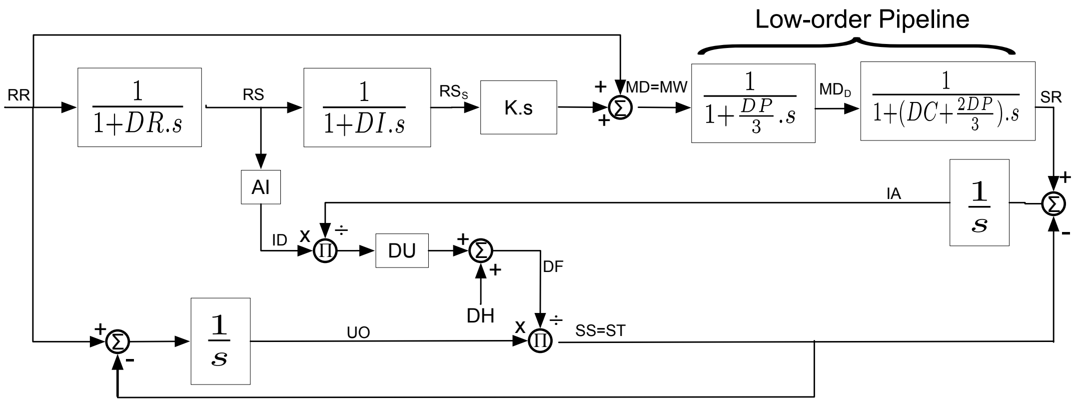


Figure 5.8: Forrester's model: simplified with only continuous nonlinearities (no presence of CLIP functions)

$$md = g_1(x, u) = rr + \frac{K.rs}{DI} - \frac{K.rs_s}{DI} \tag{5.33}$$

$$ss = g_2(x, u) = \frac{uo.ia}{AI.DU.rs + DH.ia} \tag{5.34}$$

$$ia = g_3(x, u) = ia \tag{5.35}$$

$$df = g_4(x, u) = \frac{AI.rs}{ia}.DU + DH \tag{5.36}$$

The mathematical model given by Equations 5.26 - 5.36 is nonlinear due to the presence of nonlinear algebraic differential equations, which in the block diagram are represented by the symbol $\textcircled{\Pi}$. The overall model can be linearised about a nominal operating state space x^* and for a given input u^* by using small perturbation theory with Taylor series expansion since these continuous nonlinearities are single-valued or memoryless. The linearisation process involved in this approach is such that departures from a steady state point are small enough to produce transfer function coefficients. Hence, by assuming a small amplitude of the excitation signal, the nonlinear differential equations are replaced by a set of linearised differential equations with coefficients dependent upon the steady state operating point.

The first order Taylor series approximation of the nonlinear state derivatives leads to the following linearised function:

$$\Delta\dot{x} = A\Delta x + B\Delta u \tag{5.37}$$

$$\Delta y = C\Delta x + D\Delta u \tag{5.38}$$

where $\Delta x = x - x^*$, $\Delta y = y - y^*$, $\Delta u = u - u^*$ and A, B, C, D can be found through the following partial derivatives:

$$\left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right) = \left(\begin{array}{ccc|c} \frac{\partial f_1(x^*,u^*)}{\partial rs} & \cdots & \frac{\partial f_1(x^*,u^*)}{\partial ia} & \frac{\partial f_1(x^*,u^*)}{\partial rr} \\ \vdots & \ddots & \vdots & \vdots \\ \frac{\partial f_6(x^*,u^*)}{\partial rs} & \cdots & \frac{\partial f_6(x^*,u^*)}{\partial ia} & \frac{\partial f_6(x^*,u^*)}{\partial rr} \\ \hline \frac{\partial g_1(x^*,u^*)}{\partial rs} & \cdots & \frac{\partial g_1(x^*,u^*)}{\partial ia} & \frac{\partial g_1(x^*,u^*)}{\partial rr} \\ \vdots & \ddots & \vdots & \vdots \\ \frac{\partial g_4(x^*,u^*)}{\partial rs} & \cdots & \frac{\partial g_4(x^*,u^*)}{\partial ia} & \frac{\partial g_4(x^*,u^*)}{\partial rr} \end{array}\right) \tag{5.39}$$

Firstly, the equilibrium or resting points (x^*, u^*) need to be determined. Considering a step increase in sales, Forrester (1961) defines the input or requisition rate as a function of an initial value (RRI) and a requisition step change ($STEP$). Hence, the final requisition value, $rr(\infty) = RRI + STEP$, will be the steady state operating point for the input, $u^* = rr(\infty)$. Then, Mathematica has been used to solve the system of equations where all state derivatives are equal to zero to find the equilibrium point for the state variables, which are:

$$rs^* = rr_s^* = md_d^* = sr^* = rr(\infty) = RRI + STEP \tag{5.40}$$

$$uo^* = (DH + DU)(RRI + STEP) \tag{5.41}$$

$$ia^* = AI(RRI + STEP) \tag{5.42}$$

Finding the partial derivatives and replacing them with the steady state point will result in the matrix given by Equation 5.43, which can then be converted back to a block diagram representation as in Figure 5.9. Note that, in the resulting matrix only DF is input-dependent, and hence it could not be represented in the block diagram of Figure 5.9.

$$\left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right) = \left(\begin{array}{cccccc|c}
\frac{-1}{DR} & 0 & 0 & 0 & 0 & 0 & \frac{1}{DR} \\
\frac{1}{DI} & \frac{-1}{DI} & 0 & 0 & 0 & 0 & 0 \\
\frac{3K}{DP.DI} & \frac{-3K}{DP.DI} & \frac{-3}{DP} & 0 & 0 & 0 & \frac{3}{DP} \\
0 & 0 & \frac{3}{(3DC+2DP)} & \frac{-3}{(3DC+2DP)} & 0 & 0 & 0 \\
\frac{DU}{DU+DH} & 0 & 0 & 0 & \frac{-1}{DU+DH} & \frac{-DU}{AI(DU+DH)} & 1 \\
\frac{DU}{DU+DH} & 0 & 0 & 1 & \frac{-1}{DU+DH} & \frac{-DU}{AI(DU+DH)} & 0 \\
\hline
\frac{K}{DI} & \frac{-K}{DI} & 0 & 0 & 0 & 0 & 1 \\
\frac{-DU}{DU+DH} & 0 & 0 & 1 & \frac{1}{DU+DH} & \frac{DU}{AI(DU+DH)} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\frac{-DU}{RRI+STEP} & 0 & 0 & 0 & 0 & \frac{-DU}{AI(RRI+STEP)} & 0
\end{array}\right) \tag{5.43}$$

When comparing Figures 5.8 and 5.9, it can be seen that after linearisation the product functions (Ⓟ) are replaced by summing comparators (Σ) and also the location for some variable and parameters is affected.

Figure 5.10 illustrates unit step and sinusoidal responses in manufacturing rate (ΔMD), inventory (ΔIA) and shipment sent (ΔSS) from their initial states and the delay in filling orders (DF), comparing the output responses of the original model

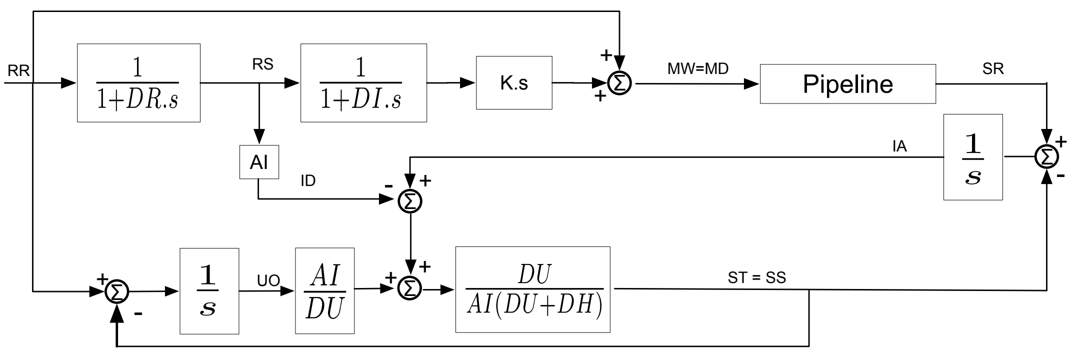


Figure 5.9: Forrester's model: Simplified and linearised with small perturbation

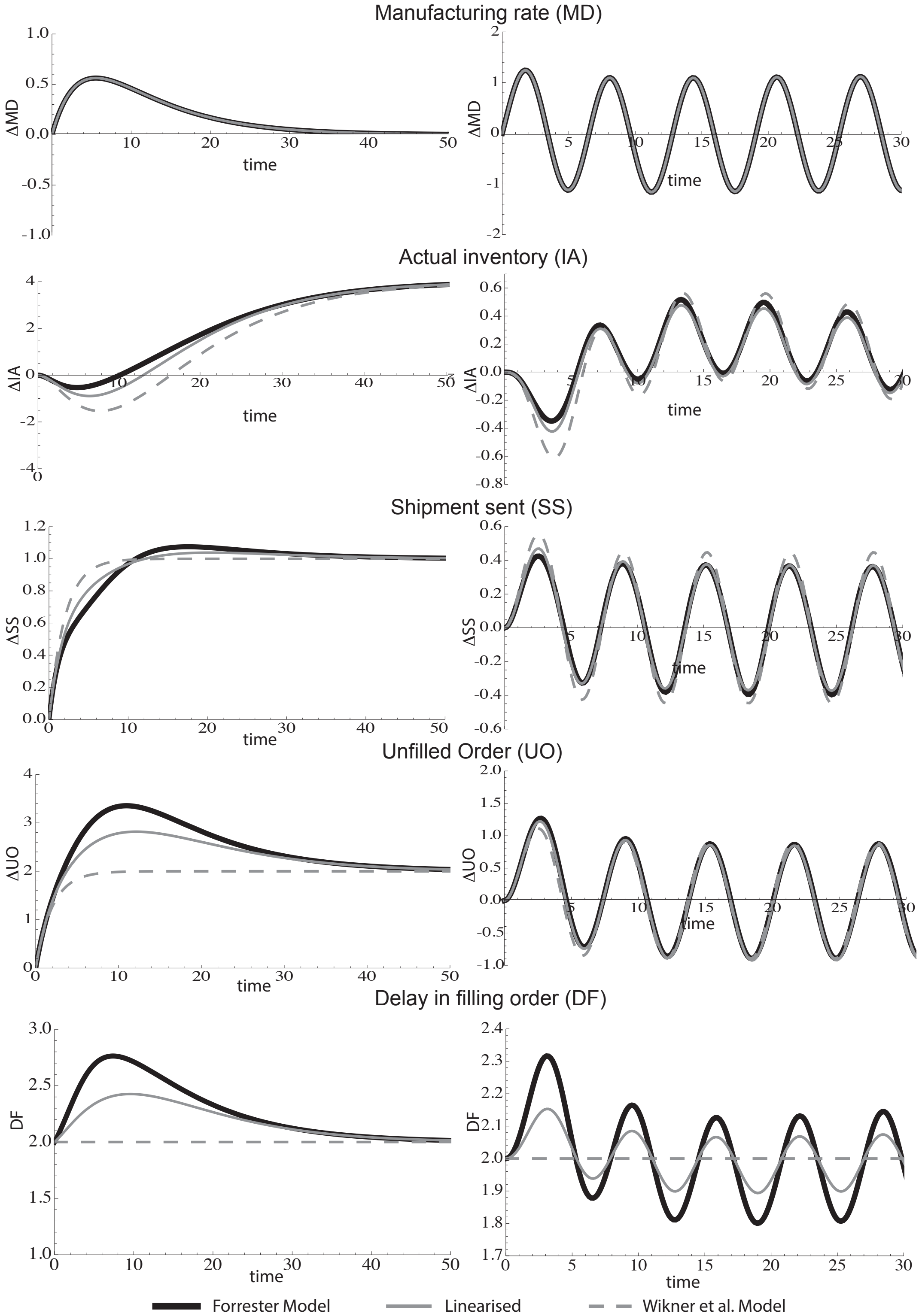


Figure 5.10: Comparing the Forrester, Wikner et al. and linearised model using small perturbation theory

with averaging (Wikner *et al.*, 1992) and small perturbation linearisation techniques. Figure 5.10 demonstrates the power of small perturbation theory for the analysis of continuous nonlinearities. Accuracy is increased as the amplitude of the input signals is decreased although in all cases the small perturbation theory approach gives a better approximation than the averaging technique used by Wikner *et al.* (1992).

5.4.2 Analysis of discontinuous nonlinearities

The CLIP functions are re-inserted back into the linearised model resulting in Figure 5.11. Note that when re-inserting the CLIP functions, two feedback loops have also been re-instated as shown in the figure. Each of the discontinuous nonlinearities are separately analysed.

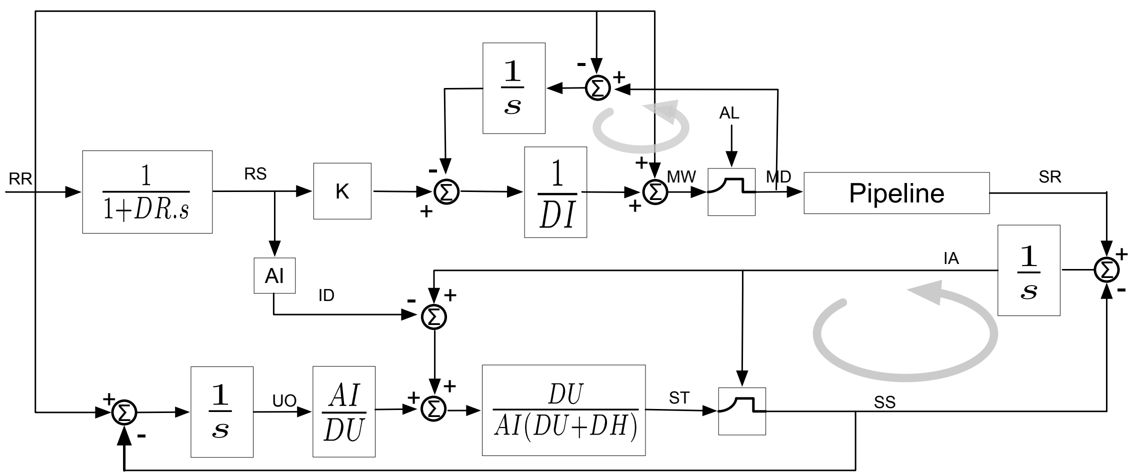
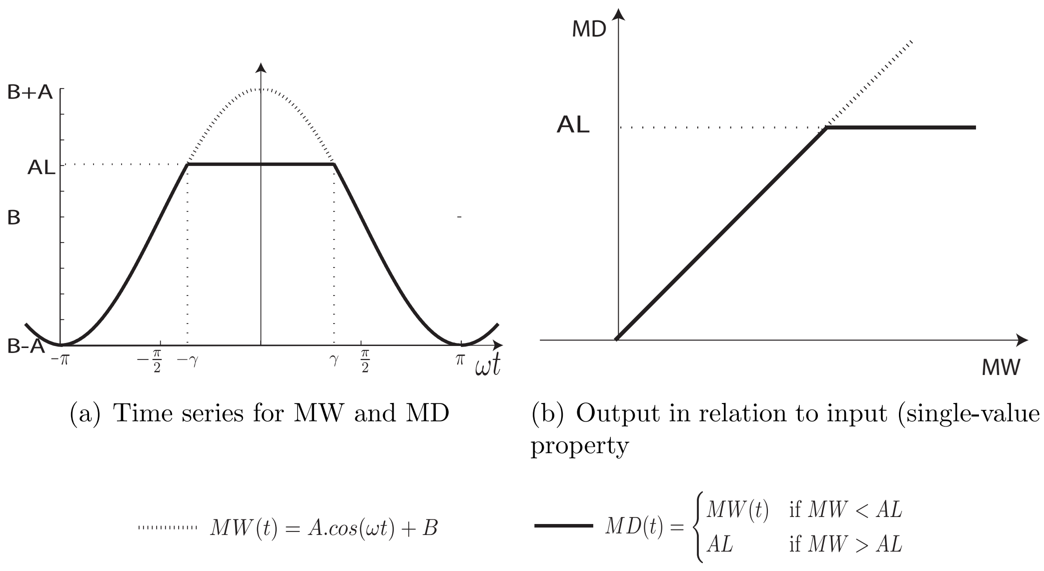


Figure 5.11: Forrester's model: Simplified, containing only discontinuous nonlinearities

5.4.2.1 Manufacturing constraint: maximum capacity

When introducing a manufacturing capacity constraint (AL) the system will behave as in Figure 5.12 in an open loop form. A sinusoidal input, MW, to the nonlinearity, which represents a saturation function of a maximum limit value AL, will produce an output MD of the same frequency but different amplitudes and mean (Figure 5.12(a)). Figure 5.12(b) illustrates the single-valued property of this nonlinearity. The output MD does not depend on the past values of the input MW, but it varies according to the actual state of MW. Although the function is nonlinear, it can be represented by two piecewise linear equations:

$$MD(t) = \begin{cases} MW(t) & \text{if } MW < AL \\ AL & \text{if } MW \geq AL \end{cases} \tag{5.44}$$



(a) Time series for MW and MD

(b) Output in relation to input (single-value property

Figure 5.12: Asymmetric output saturation in relation to sinusoidal input MW

By investigating again Figure 5.11, it is clear that if the output MD differs from demand RR, this error will be accumulated due to the presence of an integrator $^1/_s$ in the feedback loop. Hence, the manufacturing rate (MD) will only align with the demand if the manufacturing capacity (AL) is at least equal to the average demand. If manufacturing capacity is less than the required demand (RR), then the manufacturing wanted (MW) will increase exponentially and the system will never stabilise.

Other effects that this discontinuous nonlinearity can cause are periodic oscillations. For an autonomous system, when the state vector returns to one of its previous values, it must necessarily repeat this motion and so the response will keep recurring indefinitely without reaching steady state (Cook, 1986). These oscillations, also known as limit cycles, may occur as the output of the nonlinearity switches from one region to another in the piecewise function plans.

In order to investigate discontinuous nonlinear feedback systems, the describing function method can be used. This method is a quasi-linear representation for a nonlinear element subjected to a sinusoidal input. This is a method that attempts to predict limit cycles occurrence and properties, such as frequency, amplitude and stability (Atherton, 1975).

The basic idea of the describing function is to represent a nonlinear element by a type of transfer function, or gain, derived from its effects on a sinusoidal input signal. Figure 5.12 illustrates the asymmetric saturation in the manufacturing rate when the manufacturing wanted rate (MW) is greater than its capacity (AL). For asymmetric nonlinearities, or symmetric nonlinearities subjected to biased inputs, at least two terms of the describing function are needed: one that describes the change in the output amplitude (N_A) as the input amplitude increases or the saturation value decreases, and another term that determines the change in the output mean (N_B). This leads to the so-called dual-input describing function (Vukic *et al.*, 2003; Cook, 1986) or sinusoid plus bias describing function (Atherton, 1975). Another effect caused by this type of nonlinearity is the possible change in phase angle (ϕ) of the output response in relation to its input. Hence, given the input:

$$MW(t) = A.cos(\omega t) + B \tag{5.45}$$

where ω is the angular frequency and is equal to $\omega = 2\pi/T$. The output MD can be approximated to:

$$MD(t) = N_A.A.cos(\omega t + \phi) + N_B.B \tag{5.46}$$

In order to determine the terms of the describing function (N_A, N_B and ϕ) the series have to be expanded and its first harmonic coefficients must be determined. The Fourier series expansion method is used to represent the output MD as a series such as:

$$\begin{aligned} MD(t) &\approx b_0 + a_1 cos(\omega t) + b_1 sin(\omega t) + a_2 cos(2\omega t) + b_2 sin(2\omega t) + \cdots = \\ &\approx b_0 + \sum_{k=1}^{\infty} [a_k cos(k.\omega t) + b_k sin(k.\omega t)] \end{aligned} \tag{5.47}$$

where the coefficients are given by:

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} MD(t)cos(k.\omega t)d\omega t \tag{5.48}$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} MD(t)sin(k.\omega t)d\omega t \tag{5.49}$$

$$b_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} MD(t)d\omega t \tag{5.50}$$

and MD is the piecewise linear function:

$$MD(t) = \begin{cases} A.cos(\omega t) + B & \text{if } -\pi < \omega t < -\gamma \\ AL = A.cos(\gamma) + B & \text{if } -\gamma < \omega t < \gamma \\ A.cos(\omega t) + B & \text{if } \gamma < \omega t < \pi \end{cases} \tag{5.51}$$

The advantage of the Fourier series in the analysis of discontinuous nonlinearities is that the series can converge to the correct value at every point where the function is linear. At the points of discontinuities it converges to the average of the two values obtained by taking the limit of the MD as it approaches this point from each side.

For the describing function, only the first, or fundamental, harmonic is usually used to approximate the periodic series. This is appropriate for symmetric systems because they contain only odd harmonics; therefore higher harmonics will be attenuated by the linear dynamics of the system (Vukic *et al.*, 2003). However, in the case of asymmetric nonlinearities, the second harmonic also occurs. For this reason describing function techniques tend to be less accurate than those for symmetrical system and the complementary use of simulation is recommended (Atherton, 1975). As can be seen from Figure 5.12 the manufacturing constraint in the Forrester model is an asymmetric nonlinearity.

If the piecewise linear output MD is approximated to the first harmonic, it results in:

$$MD(t) = b_0 + a_1 cos(\omega t) + b_1 sin(\omega t) = b_0 + \sqrt{a_1^2 + b_1^2}.cos(\omega t + \phi) \tag{5.52}$$

where, $\phi = arctan\left(\frac{b_1}{a_1}\right)$

In this way the two terms of the describing function can be determined as:

$$N_A = \frac{\sqrt{a_1^2 + b_1^2}}{A} \tag{5.53}$$

$$N_B = \frac{b_0}{B} \tag{5.54}$$

For single-valued nonlinearities the coefficient b_1, the imaginary part, will be equal to zero and therefore the phase angle will be also zero. Hence, for the asymmetric saturation in the Forrester system it is found that:

$$N_A = \frac{-\gamma + \pi + cos\gamma.sin\gamma}{\pi} \tag{5.55}$$

$$N_B = \frac{B.\pi + A.\gamma.cos\gamma - A.sin\gamma}{B.\pi} \tag{5.56}$$

where $\gamma = cos^{-1}\left(\frac{AL-B}{A}\right)$.

Figure 5.13 illustrates how the coefficients of the describing function for the manufacturing capacity vary as the amplitude of manufacturing wanted rate, A_{MW}, increases. For amplitudes lower than the capacity AL, the system behaves as linear and output MD will be equal to the input MW corresponding to a describing function gain equal to 1. However, when MW hits the capacity AL only a fraction of this rate will actually be manufactured. The mean also decreases as the amplitude increases in relation to the capacity AL.

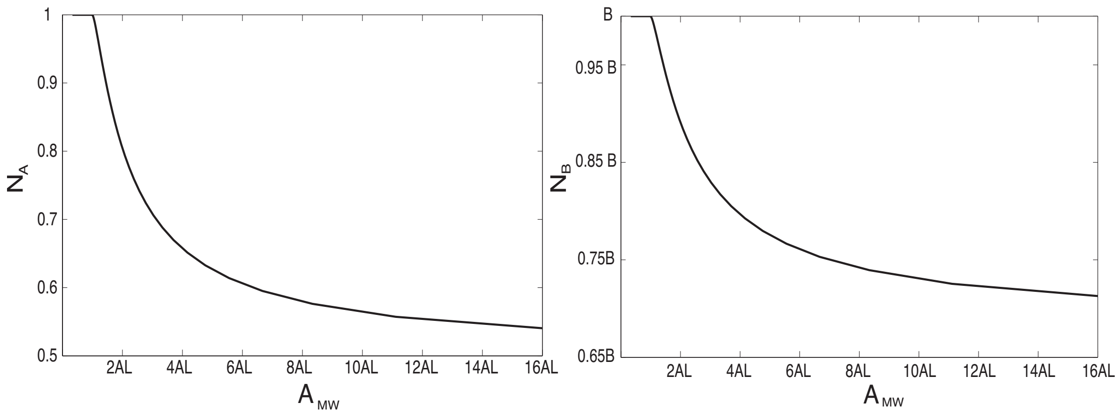


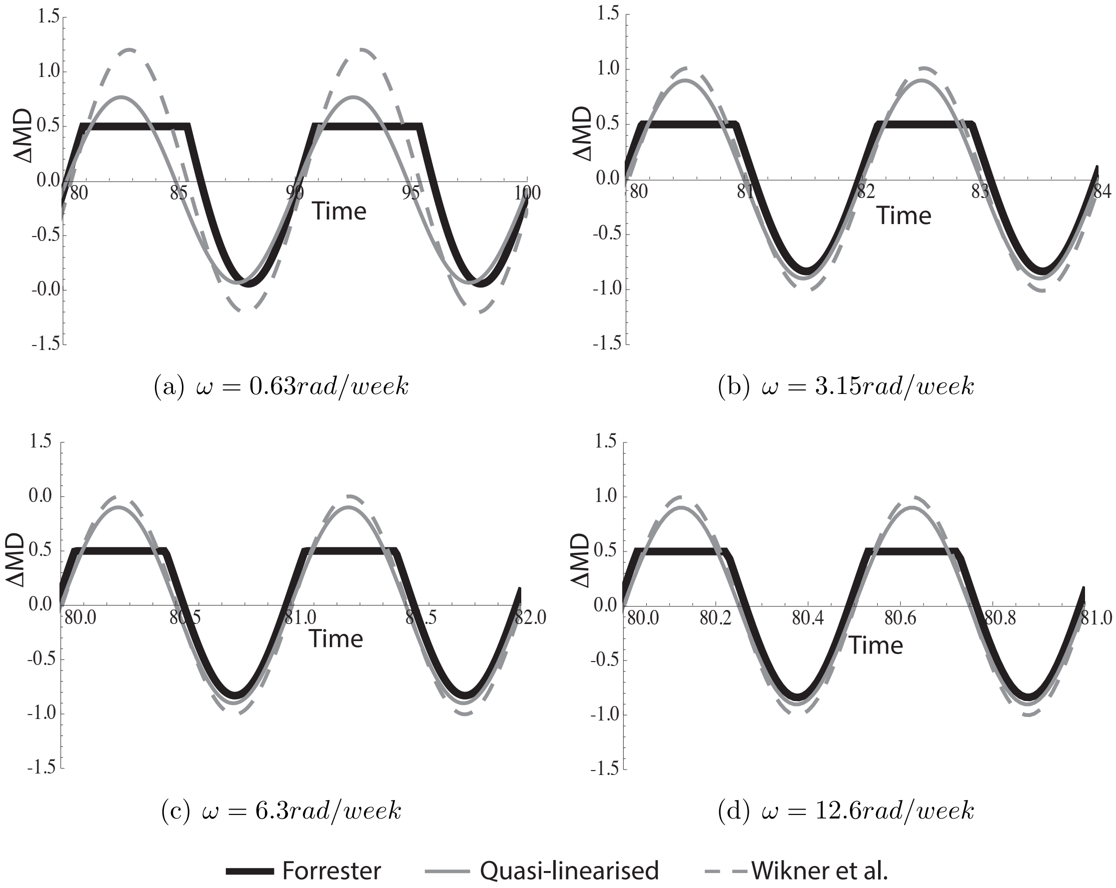
Figure 5.13: Terms of describing function for asymmetric saturation in Forrester's model

Figure 5.14 demonstrates the sinusoidal responses comparing the original model with the Wikner *et al.* model and the describing function method for different frequencies. Again, a better linear approximation for the original model has been found.

5.4.2.2 Shipment constraint

The CLIP function in the shipment system is used to avoid any shipments being made to customers if no inventory is actually available. Hence, shipments sent (SS), will be equal to shipment tried (ST), only if actual inventory (IA) is greater than ST.

Different from the discontinuity in the manufacturing system, this second nonlinearity is found to be not only amplitude-dependent but also frequency-dependent. Figure 5.15 illustrates a set of system responses for inventory and shipments given different amplitudes and frequencies. In the example in Figure 5.15, it seems that higher frequencies and lower amplitudes (Figure 5.15(a)) result in a linear beha-



(a) $\omega = 0.63 rad/week$

(b) $\omega = 3.15 rad/week$

(c) $\omega = 6.3 rad/week$

(d) $\omega = 12.6 rad/week$

Figure 5.14: Comparing the Forrester, Wikner et al. and quasi-linearised model using a Describing Function for MD

viour. Hence, SS will be equal to ST, corresponding to a describing function of 1. However, for lower frequencies and increased amplitude (Figure 5.15(d)), the inventory capacity is reduced and a complex nonlinear behaviour is observed. Figure 5.15 also illustrates that this nonlinearity is multi-valued (see subfigures on the top right corner of each figure and compare with Figure 5.12(b)). For a given input ST the output SS can assume different values depending on the past states of ST.

Since this discontinuous nonlinearity is frequency-dependent, there will be one describing function for each frequency. Matlab combined with Simulink has been

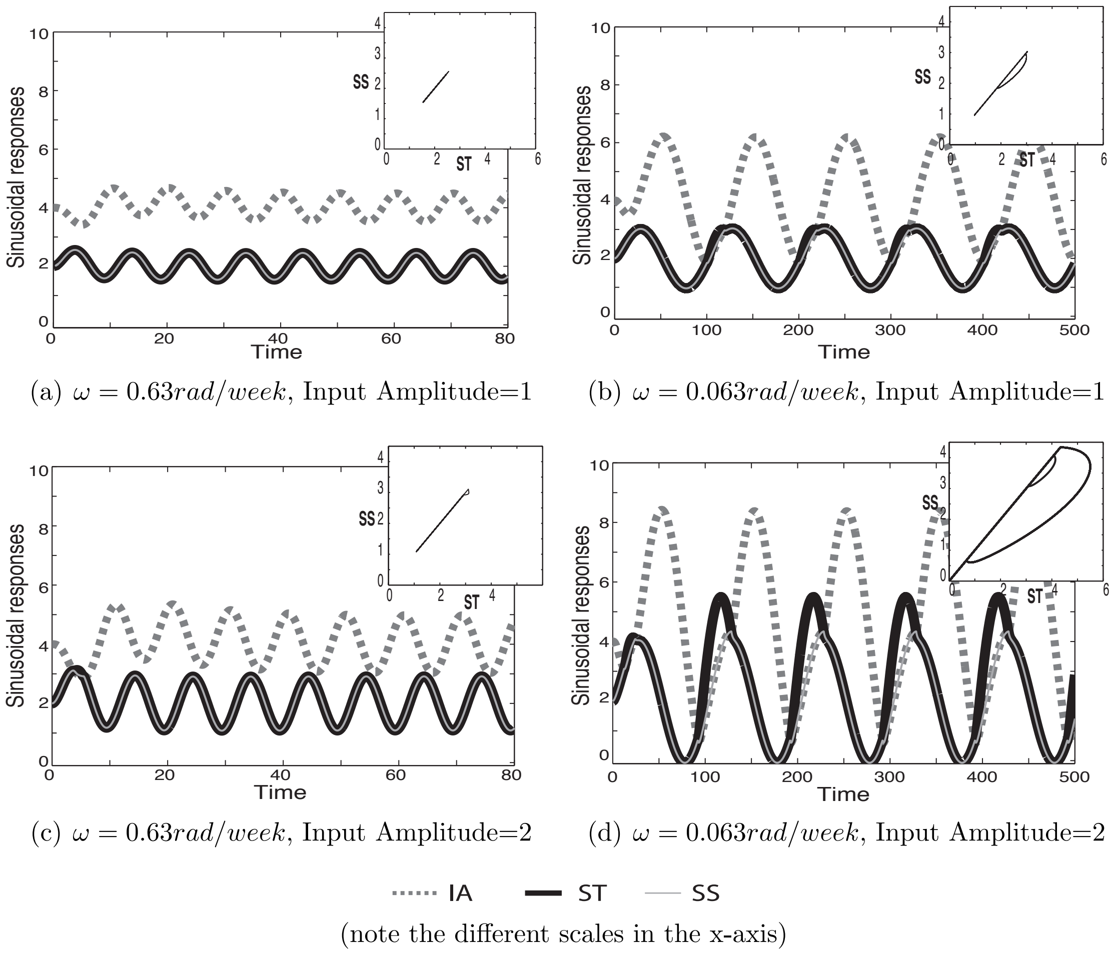


Figure 5.15: Actual inventory (IA), shipment tried (ST) and sent (SS) responses to different amplitudes and frequencies

used to find the describing function corresponding to the amplitude gain and also to identify the phase shift, resulting in Figure 5.16.

Figure 5.16 confirms that the nonlinearity in the shipment process only occurs for very low frequencies and high amplitudes. Another important factor is regarding the inventory constant AI. Just as the manufacturing capacity (AL) in the first CLIP function, the inventory capacity (IA) is required to be at least equal to the average demand, otherwise the error between shipment (SS) and required demand (RR) will increase exponentially and the system will never stabilise, as seen in Figure 5.11.

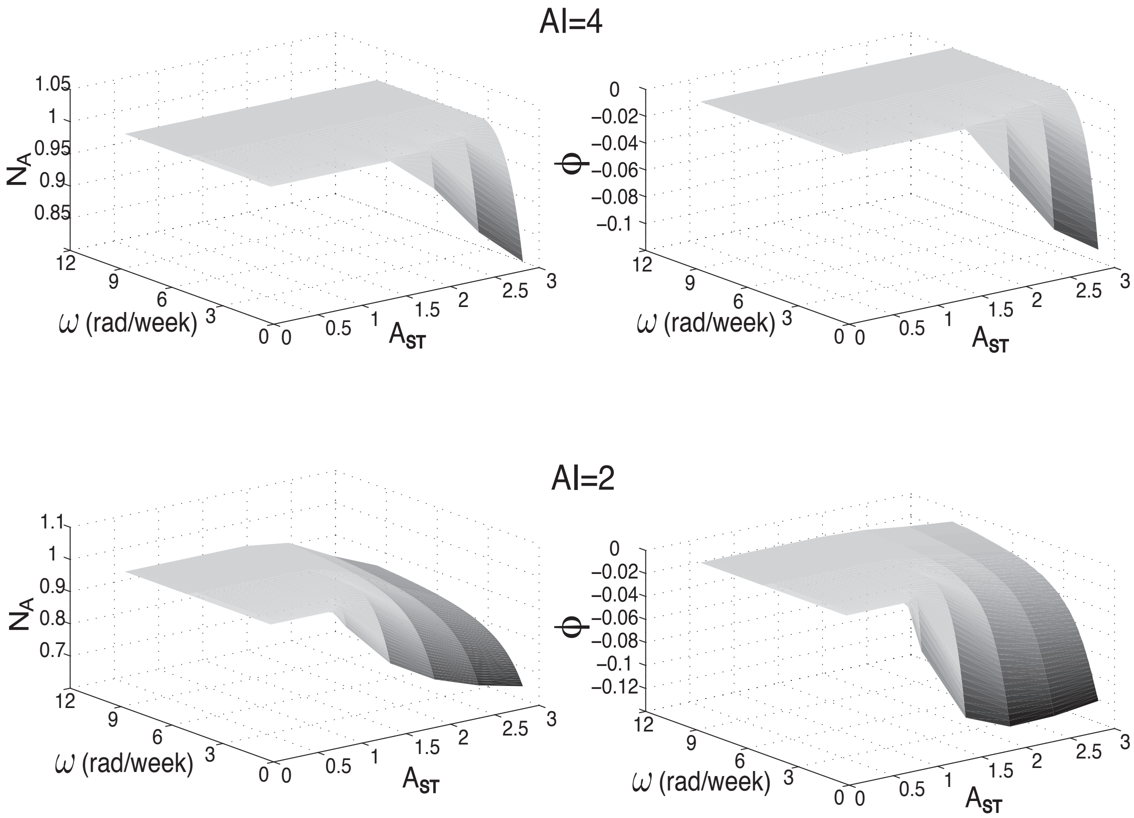
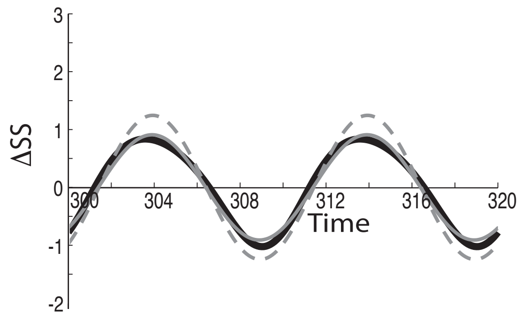


Figure 5.16: Describing function amplitude gain and phase in relation to ST amplitude and frequency

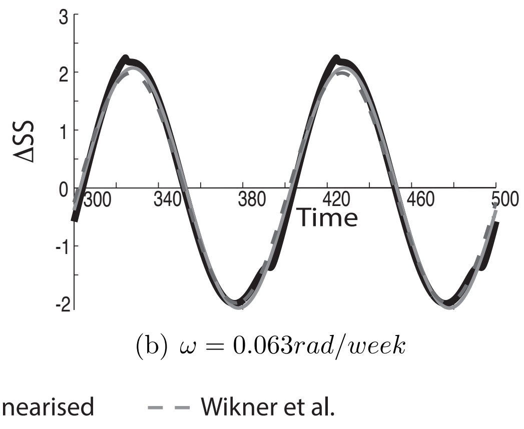
Hence, the inventory constant AI has to be at least one. Moreover, as inventory levels decrease, the nonlinearity takes effect more regularly.

When considering that inventory target values in the Forrester model were 4 weeks at the Factory, 6 weeks at the distributor and 8 weeks at the retail, the approximation made by Wikner *et al.* (1992) is reasonable because the CLIP function will only take effect for extremely low frequencies and high amplitude demands. The system designer does not have to be concerned with the shipment constraint when demand has medium to high frequencies and low amplitudes.

Figure 5.17 compares the change in shipment sent from its original states, ΔSS, of the original Forrester model with the Wikner *et al.* model and the describing function method when the system reaches steady state. Different frequencies are compared while the demand amplitude is fixed at 2 units. Note that for this comparison the first CLIP function (manufacturing constraints) was kept inactive. For both responses in Figures 5.17(a) and 5.17(b), the describing function method provided a better approximation to the Forrester model. Although these differences between Wikner *et al.*, Forrester and the quasi-linearised models are not so significant, the describing function method provided a better understanding of the shipment constraint in relation to its effect on the output phase and amplitude shift for certain input frequencies and amplitudes.



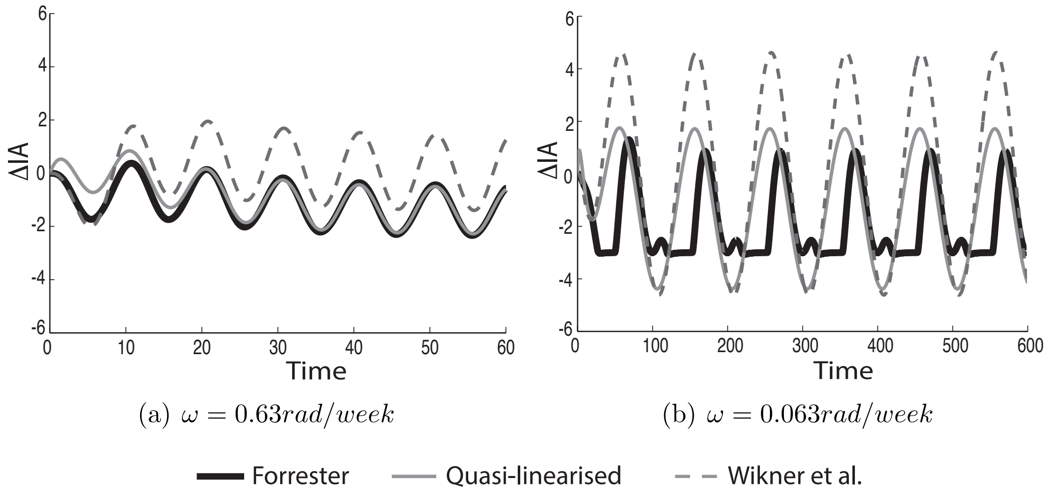
(a) $\omega = 0.63 rad/week$



(b) $\omega = 0.063 rad/week$

Figure 5.17: Comparing the Forrester, Wikner et al. and quasi-linearised models using a describing function for SS

Finally, Figure 5.18 illustrates the change in inventory (ΔIA) when all discontinuous nonlinearities, or CLIP functions, are effective. It is very clear that, in the example used ($AI = 2$ and $AL = 1$ and demand amplitude=2) in Figure 5.18, the inventory response represented by Forrester's model has a shift in the mean value. The quasi-linear model was able to track this behaviour, while the Wikner *et al.* model did not since it ignores the CLIP functions. As the demand frequency decreases and amplitude increases, the nonlinear responses in the original Forrester



(a) $\omega = 0.63 rad/week$ (b) $\omega = 0.063 rad/week$

Figure 5.18: Comparing the Forrester, Wikner et al. and quasi-linearised models for IA when all CLIP functions are active

model become more and more complicated (Figure 5.18(b)). Hence, the linearised model becomes less and less accurate.

5.5 Response analysis for resilience

So far, we have only been concerned with the simplification and linearisation process of the nonlinear Forrester model and accuracy in the response. For design work it is necessary to estimate the effect of input signals on the performance of nonlinear systems. More specifically this section is concerned with supply chain resilience.

5.5.1 Transfer functions

From the linearised model represented in the block diagram of Figure 5.9, it is possible to determine the transfer function of the actual inventory (IA), the key indicator for supply chain resilience as identified in Section 4.2.3, in relation to the input demand or requisitions (RR). To do so, firstly the transfer functions of the manufacturing orders (MD), shipments received (SR), shipment sent (SS) and unfilled orders (UO) are found as follows:

$$\frac{MD}{RR} = 1 + \frac{Ks}{(1+DIs)(1+DRs)} \tag{5.57}$$

$$\frac{SR}{RR} = \frac{9\Big(1+DIs+DRs+Ks+DIDRs^2\Big)}{(1+DIs)(3+DPs)(1+DRs)(3+(3DC+2DP)s)} \tag{5.58}$$

$$\frac{SS}{RR} = \frac{AI(1+DIs)(3+DPs)(3+(3DC+2DP)s)(1+(DR-DU)s)+9DU(1+(DI+DR+K)s+DIDRs^2))}{(1+DIs)(3+DPs)(1+DRs)(3+(3DC+2DP)s)(AI+DU+AI(DH+DU)s)} \tag{5.59}$$

$$\begin{aligned}\frac{UO}{RR} =& \frac{1}{s}(RR-SS) = \Big[DU\Big(-9K+9DP(1+DIs)(1+DRs)+2DP^2s(1+DIs)(1+DRs)+3DC(1+\\ & DIs)(3+DPs)(1+DRs)\Big)+AI(1+DIs)(3+DPs)(3+3DCs+2DPs)(DH+DHDRs+DU\\ & (2+DRs))\Big]\Big/\Big((1+DIs)(3+DPs)(1+DRs)(3+(3DC+2DP)s)(AI+DU+AI(DH+DU)s)\Big)\end{aligned} \tag{5.60}$$

$$\begin{aligned}\frac{IA}{RR} =& \frac{1}{s}(SR-SS) = AI\Big[-9DC+9DH-9DP+18DU+9K-(9DCDI-9DHDI+3DCDP+\\ & 9DIDP+2DP^2+9DCDR-9DHDR+9DPDR-9DCDU-18DIDU-9DPDU-9DRDU\\ & -9DHK-9DUK)s-(3DCDIDP+2DIDP^2+9DCDIDR-9DHDIDR+3DCDPDR+\\ & 9DIDPDR+2DP^2DR-9DCDIDU-3DCDPDU-9DIDPDU-2DP^2DU-9DIDRDU)s^2\\ & -(3DCDIDPDR+2DIDP^2DR-3DCDIDPDU-2DIDP^2DU)s^3)\Big]\Big/\Big((1+DIs)(3+DPs)\\ & (1+DRs)(3+(3DC+2DP)s)(AI+DU+AI(DH+DU)s)\Big)\end{aligned} \tag{5.61}$$

where $K = AI - DH - DU + DC + DP$.

Although Forrester's model has been simplified and its order reduced, the resulting simplified and linearised model is fifth-order. Its characteristic equation is given by $((1+DIs)(3+DPs)(3+3DCs+2DPs)(1+DRs)(AI+DU+AIDHs+AIDUs)$

and, assuming that physical delays cannot be negative, system stability is reached when the parameters DR, DI and AI are greater than zero.

5.5.2 Inventory step response and ITAE

In order to obtain the inventory time equation through inverse Laplace Transform, the author has decided to separate the control parameters from the parameters that the supply chain designer cannot select or control, such as physical parameters like delivery and manufacturing lead-times. Other system dynamics researchers have done the same when equations become large and therefore difficult to interpret (Towill, 1992a; Wikner *et al.*, 1992; Disney and Towill, 2003b; Jeong *et al.*, 2000). In the Forrester model, the delay in smoothing requisitions (DR), the delay in inventory/pipeline adjustment (DI) and the constant for inventory (AI) are the control parameters and all the other parameters occur due to physical conditions.

By substituting the physical parameters with actual values (in Appendix B) in Equation 5.61 and inserting a unit step change in the customer's requisition (RR), the actual inventory will have the following response:

$$IA = -\frac{AI\,(-1 - AI - 5s + 4DIs + 5DRs - 5DIs^2 + 5DIDRs^2)\,.\frac{1+AI}{10AI.DI.DR}}{(\frac{1}{5}+s)(\frac{1+AI}{2AI}+s)(\frac{1}{DI}+s)(\frac{1}{DR}+s)s} \tag{5.62}$$

where the new pole ($s = 0$) represents the step input and the term $\frac{1+AI}{10AI.DI.DR}$ indicates that the equation has been normalised according to the leading coefficient of the denominator. Note that the system's order has been reduced. This is due to the position of the pole $\frac{-3}{DP}$ coinciding with the position of a zero.

Under the assumption that the poles in Equation 5.62 differs from each other, simple partial fraction expansion can be applied. In the case of repeated poles a special case of the partial fraction expansion method has to be used. Hence, Equation 5.62 can now be rewritten as:

$$IA = \frac{A}{(\frac{1}{5}+s)} + \frac{B}{(\frac{1+AI}{2AI}+s)} + \frac{C}{(\frac{1}{DI}+s)} + \frac{D}{(\frac{1}{DR}+s)} + \frac{E}{s} \tag{5.63}$$

From Equation 5.63, it is found that the coefficient of the transient response is given by A, B, C and D, while the steady state of the system will be equal to E. By solving the partial fraction expansion, the coefficients can be determined as:

$$A = \frac{25AI(-5(AI+DI)+(DI-5)DR)}{(5+3AI)(DI-5)(DR-5)} \tag{5.64}$$

$$B = \frac{2AI^2(5DI+AI(-10+4AI+13DI)-5(AI(DI-2)+DI)DR)}{(5+3AI)(AI(DI-2)+DI)(AI(DR-2)+DR)} \tag{5.65}$$

$$C = \frac{AI(5+AI)DI^3}{(DI-5)(AI(DI-2)+DI)(DR-DI)} \tag{5.66}$$

$$D = \frac{AIDR(DI(DR-5)-DR(-5+(6+AI)DR))}{(DR-5)(DR-DI)(AI(DR-2)+DR)} \tag{5.67}$$

$$E = AI \tag{5.68}$$

The time function of the actual inventory can be obtained by undertaking the inverse Laplace transform:

$$IA(t) = A.e^{-\frac{t}{5}} + B.e^{-\frac{(1+AI).t}{2AI}} + C.e^{-\frac{t}{DI}} + D.e^{-\frac{t}{DR}} + E \tag{5.69}$$

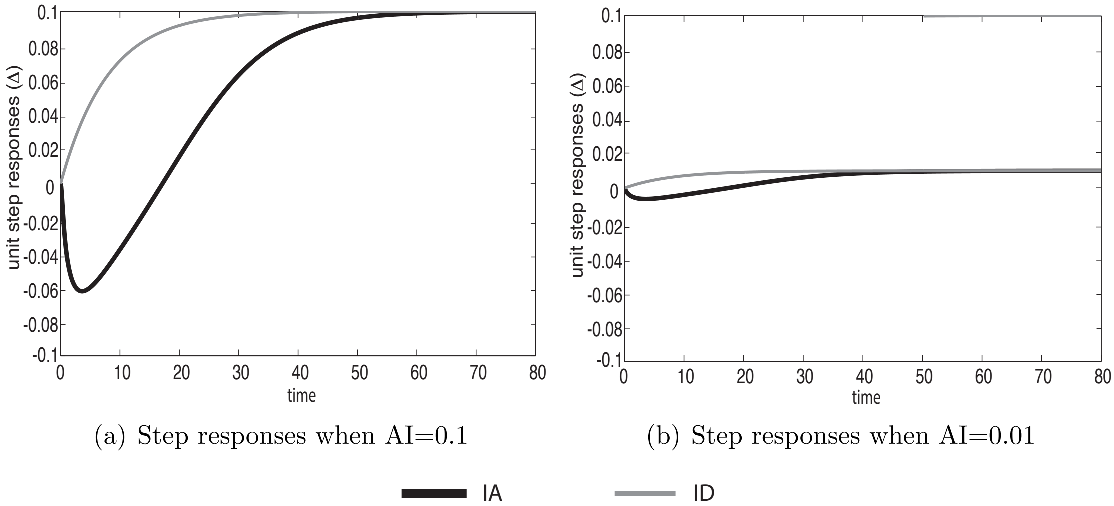
In order to calculate ITAE of the inventory response, the error between the target and actual inventories is needed. Investigating Figures 5.8 and 5.9 again, it is found that the inventory position is always being compared with the variable ID (desired inventory). Hence the target inventory in Forrester's model is variable instead of fixed. Hence, the error in the inventory is the difference between ID and IA. The unit step response of ID is easily determined by finding the inverse Laplace transform of $\frac{AI}{(1+DR.s)s}$, which is:

$$ID(t) = -AI.e^{-\frac{t}{DR}} + AI \tag{5.70}$$

However, since the ITAE involves the integral of an absolute function, there are some aspects of the actual inventory function to be considered. After the step change, if the inventory amount drops and recovers without overshooting the target inventory, then the ITAE can be calculated as:

$$\begin{aligned} ITAE_{(IA)} &= AI.DR^2 - \left(A.5^2 + B.\left(\tfrac{2AI}{(1+AI)}\right)^2 + C.DI^2 + D.DR^2\right) \\ &= \frac{AI\big(5\big(DI^2+DI(5+DR)+DR(5+DR)\big)+AI^2\big(39+DI^2+7DR+DI(7+DR)\big)+AI\big(25+6DI^2+42DR+5DR^2+DI(40+6DR)\big)\big)}{(1+AI)^2} \end{aligned} \tag{5.71}$$

Equation 5.71 suggests that the ITAE of inventory error will be minimised when the constant of inventory (AI) is equal to zero. From the transfer function in Equation 5.61 it can be observed that the linearised model of Figure 5.9 will result in a null transfer function for inventory when $AI = 0$. In order to confirm if this assumption holds true for the nonlinear model of Figure 5.8, numerical examples are given in Figure 5.19. Because Forrester's model cannot be simulated when $AI = 0$, due to the presence of divisions, Figure 5.19 illustrates the inventory (IA) and its target (ID) responses when $AI = 0.1$ and $AI = 0.01$. As AI decreases, the error



(a) Step responses when AI=0.1

(b) Step responses when AI=0.01

Figure 5.19: Step responses of Forrester's model containing only continuous nonlinearities (see Figure 5.8)

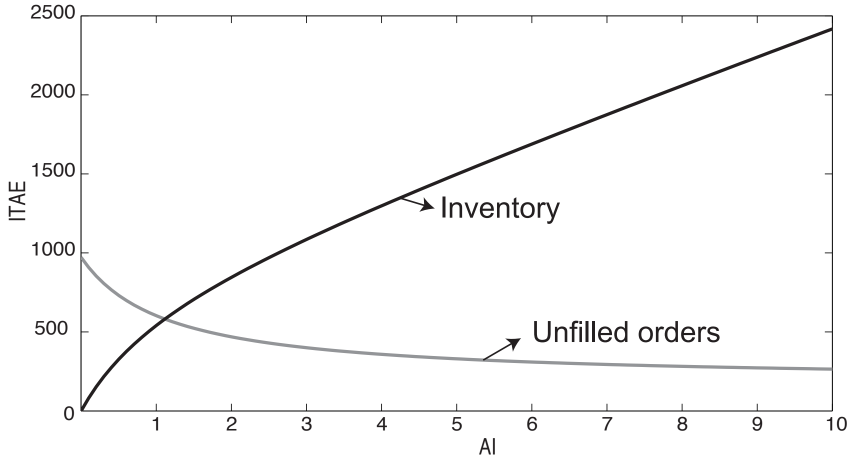
area between ID and AI also decreases, suggesting that ITAE will tend to zero as AI tends to zero.

This is an unexpected result since it is normally predicted that higher target inventories will provide better supply chain resilience. Besides, this may be a limitation of using a variable target inventory control system since changes in the target inventory mean the system take longer to reach its desired level. However, when reinvestigating Forrester's model represented by the linear block diagram in Figure 5.9, it is found that the target inventory will have an influence on the amount shipped to customer (SS), which in turn affects unfilled orders (UO). The unfilled orders can be interpreted as a backlog and they should also be minimised in order to maintain customer service levels. For this reason, the inventory response is not the only indicator of resilience in Forrester's model and the unfilled order response should also be considered. By repeating the previous steps (see Appendix D) on the determination of ITAE, we find that the ITAE of unfilled orders can be approximated to:

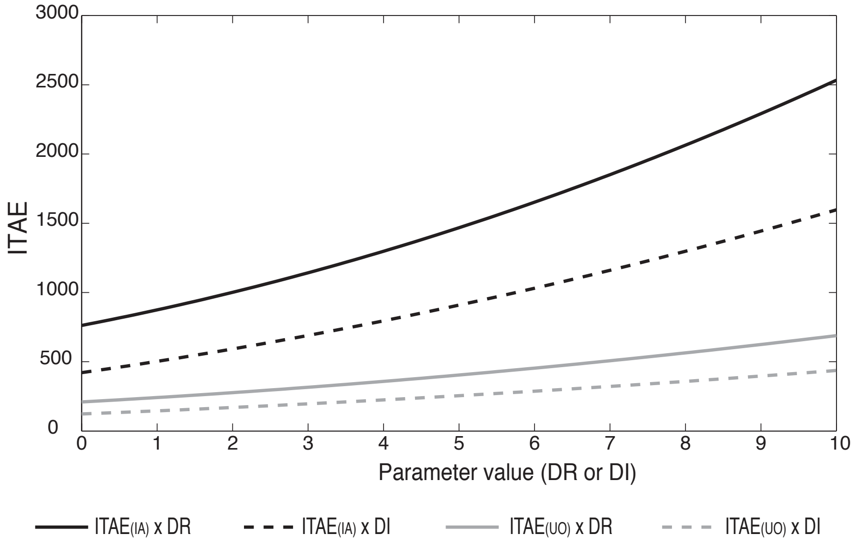
$$ITAE_{(UO)} = \frac{-8+5DI^2+5DI(7+DR)+5DR(7+DR)+AI^2(45+7DR+DI(9+DI+DR))+AI\big(23+6DI^2+DR(52+5DR)+DI(52+6DR)\big)}{(1+AI)^2} \tag{5.72}$$

Note that in the Forrester model there is a target unfilled order, which is equal to $(DH + DU)RR$. Hence this target has been considered when calculating the ITAE in Equation 5.72. Figure 5.20(a) compares the ITAE indices for actual inventory and unfilled order responses as the constant of inventory (AI) increases. The values of DR and DI have been fixed at 8 and 4, respectively. As the constant of inventory (AI) increases, the target inventory increases and the longer it takes the inventory to reach this target. Hence, the higher the ITAE values are for the inventory response. On the other hand, as inventory levels increase more customer orders can be met, and therefore the error in unfilled orders decreases rapidly. There seem to be a break-even point when AI is slightly above 1 unit. After this point, the decreasing rate of ITAE in unfilled orders is very slow in comparison to the increase in the inventory ITAE. Figure 5.20(b) on the other hand, shows that as the parameters DR and DI increase both ITAEs in inventory and unfilled orders will also increase.

So far the CLIP functions have been put aside in the analysis but they have a great impact on resilience performance. We have seen in the previous section that the minimum capacity of both CLIP functions should be at least equal to the average demand, meaning that $AL_{min} = RR$ and $IA_{min} = RR$. While AL is a fixed capacity determined by the infrastructure in the manufacturing system, IA is a variable capacity influenced by the control parameter AI which should not be less than one to maintain the IA_{min} condition. In the next subsection, how these discontinuous nonlinearities affect the system responses and the resilience performance is examined.



(a) ITAE of inventory and unfilled order in relation to AI



(b) ITAE of inventory and unfilled order in relation to DR and DI

Figure 5.20: Effect of parameter values on resilience performance

5.5.3 Effect of CLIP functions

This section therefore will investigate the effects of CLIP functions or discontinuous nonlinearities on the system responses and resilience.

5.5.3.1 Investigating limit cycles

One of the effects that discontinuous nonlinearities can cause are periodic oscillations. For an autonomous system, when the state vector returns to one of its previous values, it must necessarily repeat this motion and so the response will keep recurring indefinitely without reaching steady state (Cook, 1986). These oscillations, also known as limit cycles, may occur as the output of the nonlinearity switches from one state to another as shown in the piecewise function. In order to guarantee a resilient system, these oscillations should be avoided. In this section, the presence or not of limit cycles in Forrester's model due to the presence of CLIP functions is identified. For this, Appendix E gives details of how describing functions can be used to find limit cycle areas.

In the simplified Forrester model of Figure 5.11, it possible to see the presence of feedback loops. The system closed loop transfer functions can be found by replacing the CLIP functions with their respective describing functions. Note that both of the describing functions had two elements: N_A, which represents the amplitude gain, given by the real term, and the phase shift, given by the imaginary term, in the output response and N_B, which corresponds to the change in mean. To simplify the calculation of the transfer functions in Figure 5.11 it is assumed that the change in mean does not occur and the term N_B will be dismissed. This will not have an impact on finding limit cycles since, as shown in Appendix E, only the changes in amplitude can provoke oscillations in the system.

To determine whether oscillations occur in the IA and UO, the main resilience indicators, the outputs MD and SS are investigated since oscillations in these responses will in turn cause oscillations in IA and UO. Let $N_{A(MW)}$ and $N_{A(ST)}$ be the gain caused by the discontinuities in the manufacturing and shipment processes,respectively. The transfer function for MD is therefore

$$\frac{MD}{RR} = \frac{N_{A(MW)}\,(1 + (5 + AI + DI + DR)s + DIDRs^2)}{(N_{A(MW)} + DIs)(1 + DRs)} \tag{5.73}$$

Note that this is the closed loop transfer function. To find the open loop transfer function $\overline{MD}(s)$ we can make

$$\frac{\overline{MD}}{1 + \overline{MD}} = \frac{N_{A(MW)}\,(1 + (5 + AI + DI + DR)s + DIDRs^2)}{(N_{A(MW)} + DIs)(1 + DRs)} \tag{5.74}$$

$$\overline{MD} = -\frac{N_{A(MW)}\,(1 + (5 + AI + DI + DR)s + DIDRs^2)}{s((5 + AI)N_{A(MW)} + DI(-1 + N_{A(MW)})(1 + DRs))} \tag{5.75}$$

By replacing $s = i\omega$ in $\overline{MD}(s)$ equation, we can find the values of frequency ω that makes $\overline{MD}(i\omega) = -1$. Mathematica has been used to solve this expression. It has been found that oscillations in MD do not occur. Repeating the same procedure for SS, which is influenced by both CLIP functions $N_{A(MW)}$ and $N_{A(ST)}$, no oscillations are found either.

5.5.3.2 Predicting system behaviour

In order to understand the effect of the discontinuities present in the manufacturing and shipment processes, root locus techniques can be used to predict how these discontinuous nonlinearities affect the system responses. In Figure 5.11, the system transfer functions can be found by replacing the CLIP functions with their respective describing functions. Because the describing function element may change the initial and final value of step responses, ITAE cannot be directly calculated. Nevertheless, root locus analysis can be made to examine how the roots of the system change with the variation of these describing function gains. Therefore we have the new system characteristic equation:

$$(1+2s)(1+5s)(1+DRs)\big((1+AI)N_{A(ST)}+2AIs\big)(N_{A(MW)}+DIs) \qquad (5.76)$$

Note that only two poles of the characteristic equation are affected by the discontinuous nonlinearities. Since many high-order systems can be represented by a series of second and first order transfer functions (Srivastava *et al.*, 2009), the characteristic equation in Equation 5.76 can be re-arranged as:

$$(1+2s)(1+5s)(1+DRs)\Big(N_{A(MW)}N_{A(ST)}+AI.N_{A(MW)}N_{A(ST)}+(2AI.N_{A(MW)}+DIN_{A(ST)}+AI.DI.N_{A(ST)})s+2AI.DIs^2\Big) \qquad (5.77)$$

In this way, it is possible to determine the damping ratio, ζ, and the natural frequency, ω_n of the second order term in Equation 5.77 as:

$$\omega_n=\sqrt{\frac{(1+AI)N_{A(WM)}N_{A(ST)}}{2AI.DI}}, \qquad \zeta=\frac{2AI.N_{A(WM)}+(1+AI)DI.N_{A(ST)}}{4AI.DI\sqrt{\frac{(1+AI)N_{A(WM)}N_{A(ST)}}{2AI.DI}}} \qquad (5.78)$$

We have seen in Chapter 4, that ω_n determines how fast the system oscillates during the transient response, while ζ describes how much the system oscillates as the response decays toward steady state. Note that, since ITAE can only be measured on step or impulse responses, it is not possible to calculate an expression of ITAE using the describing function technique. For this reason, natural frequency and damping ratio are used to estimate the system's resilience performance.

By keeping the values of DI and AI fixed ($DI = 4$ and $AI = 4$, as given by Forrester), Table 5.1 illustrates the values of natural frequency and the damping ratio as both input amplitudes to the nonlinearities, A_{MW} and A_{ST}, increase. Figures 5.13 and 5.16 can be referred in order to check on the values of the describing functions when an input amplitude to the manufacturing and shipment constraints are given. To determine the input amplitude and corresponding describing function values in the shipment constraint (A_{ST} and $N_{A(ST)}$) an input frequency of 0.1 Hz was chosen since the shipment nonlinearity is frequency dependent.

Table 5.1 shows us that the value of the natural frequency decreases as both input amplitudes (A_{MW} and A_{ST}) increase. Regarding the damping ratio, the system is slightly overdamped, $\zeta = 1.107$, when linear ($N_{A(MW)} = 1$ and $N_{A(ST)} = 1$). As the CLIP function becomes active and the gain in the manufacturing constraint, $N_{A(MW)}$, decreases the system becomes more overdamped. For instance, when the CLIP function in the shipment process is inactive ($N_{A(ST)} = 1$) and the manufacturing capacity, AL is too low, the system has a damping ratio of 1.342. On the other hand, as the input amplitude in the shipment constraint (A_{ST}) increases, the system in practice becomes critically damped with $\zeta = 1.006$. Note that when both capacity constraints produce the same describing function gain ($N_{A(MW)} = N_{A(ST)}$), the system's damping ratio is the same as in the linear case.

In other words, when investigating the impact of the CLIP functions on supply chain resilience it seems that both nonlinearities cause a negative impact on the system's oscillation speed by decreasing its natural frequency. When it comes to the damping effect, the system never overshoots because the values of ζ are always greater than 1. However as seen in Chapter 4, overdamped systems have a sluggish recovery, which means that supply chains become less resilience. In this way, the nonlinearity present in the shipment process is not much of a concern for the supply chain designer since its presence causes a decrease in the system's damping ratio.

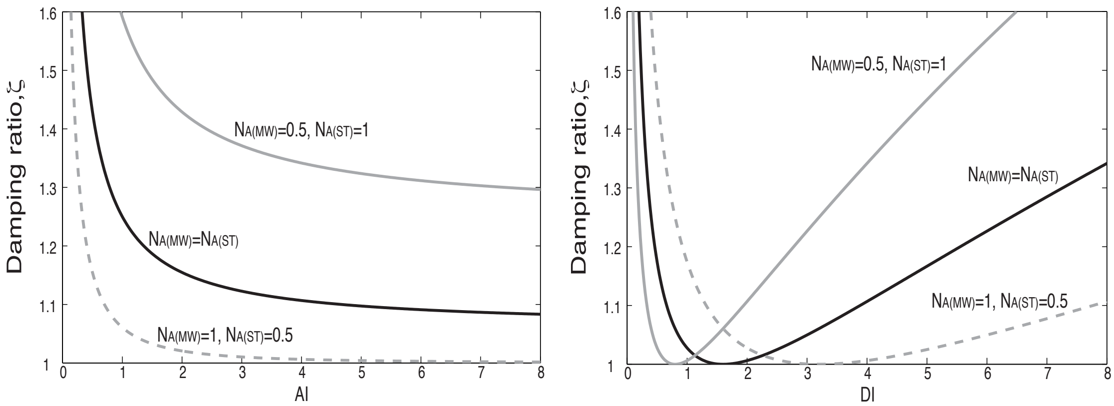
		A_{ST}*	≤1.15	1.55	2.08	2.99	5.36	∞
A_{MW}	$N_{A(MW)}$	$N_{A(ST)}$**	1	0.9-0.083i	0.8-0.115i	0.7-0.114i	0.6-0.083i	0.5
≤AL	1	ω_n	0.395	0.375	0.354	0.331	0.306	0.280
		ζ	1.107	1.083	1.061	1.039	1.021	1.006
1.45AL	0.9	ω_n	0.375	0.356	0.335	0.314	0.290	0.265
		ζ	1.133	1.107	1.081	1.056	1.033	1.014
2.03AL	0.8	ω_n	0.354	0.335	0.316	0.296	0.274	0.250
		ζ	1.167	1.137	1.107	1.078	1.050	1.025
3.12AL	0.7	ω_n	0.331	0.314	0.296	0.277	0.256	0.234
		ζ	1.209	1.175	1.141	1.107	1.073	1.042
6.29AL	0.6	ω_n	0.306	0.290	0.274	0.256	0.237	0.217
		ζ	1.266	1.226	1.187	1.147	1.107	1.068
∞	0.5	ω_n	0.280	0.265	0.250	0.234	0.217	0.198
		ζ	1.342	1.296	1.250	1.203	1.155	1.107

* Based on input frequency of 0.1Hz
** Imaginary parts cause little impact on the poles position and will be disregarded for further calculations

Table 5.1: Effect of the CLIP functions on system's natural frequency and damping ratio

On the other hand, the manufacturing capacity has a significant impact on damping the system's response and making it slower.

Figure 5.21 illustrates the effect of increasing the inventory constant, AI, and the delay in inventory adjustment, DI, on the system's damping ratio. Three possible combinations of describing function gains have been considered in this plot: when $N_{A(MW)} = N_{A(ST)}$ (the black solid line), when the inventory capacity is infinity and the manufacturing capacity is very low ($N_{A(MW)} = 0.5$ and $N_{A(ST)} = 1$, the grey solid line) and when manufacturing capacity is infinity and inventory capacity very low ($N_{A(MW)} = 1$ and $N_{A(ST)} = 0.5$, the grey dashed line). Figure 5.21(a) shows that, independent of capacity availability, the increase in AI will result in a reduced damping ratio. Morever, for AI values greater than 4 the change in damping ratio is gradual and may not justify the increase in inventory levels. In contrast, Figure 5.21(b) illustrates that depending on the system's capacity configuration the system's damping ratio is minimised for different values of DI. The trade-off DI value



(a) Effect of AI (b) Effect of DI

Figure 5.21: Effect of inventory constant and delay in inventory adjustment on the system's damping ratio

lies between 1.2 and 2, which may be an ideal setting for a supply chain system with capacity uncertainties.

5.6 Summary

In this part of Chapter 5, Forrester's model has been investigated in considerable depth using available techniques to mathematically analyse high-order and nonlinear models. Building on previous research (Wikner *et al.*, 1992; Jeong *et al.*, 2000), this complex model was translated from DYNAMO equations into differential equations and represented in Laplace-domain block diagrams. This thesis has contributed to achieving a more accurate simplified and linearised model. Block diagram manipulation enabled the reduction in the number of variables from 18 to 10 without compromising in any way the system responses subjected to this study. Then, by using the Matsubara time delay theorem to approximate the pipeline, the resulting model has been reduced from ninth to fifth order. Among the linearisation methods deployed, small perturbation theory was used to linearise the continuous nonlinearities represented by divisions and multiplications in Forrester's equations. With this method a transfer function could be obtained and an estimate of the stability region near the nominal operating points was made. The describing function enabled the understanding of discontinuous nonlinearities' amplitude and frequency dependency, single- and multi-value characteristics and their effect on the output's mean, amplitude and phase. Moreover, this method was used to investigate the occurrence of limit cycles which have not been encountered in Forrester's model.

Another contribution of this analysis was to identify that inventory response is not the only indicator of resilience in Forrester's model, especially when investigating the impact of the inventory constant AI on supply chain resilience. Unfilled orders also have to be considered. With the derivation of time responses and approximate calculations of their ITAE performances, an analysis on possible parameter settings that yield resilience has been made and the findings will be cross checked with the simulation results in Chapter 6.

More importantly, the analysis done in this section will serve as a guideline for undertaking the repeated simulation in Chapter 6 as it has provided a holistic understanding of Forrester's model. The reduction of variables and collection of constants have made it much clearer which parameters the supply chain designer should be focusing on. For instance, the control parameters DR, DI and AI were separated from the several physical delays represented in this model. Moreover, the linearisation methods enabled the determination of transfer functions, the estimation of ITAE equations and an evaluation of the impact of each control parameter on resilience and the assessment of the systems' damping ratios and natural frequencies. Without this knowledge, the simulation analysis would be time consuming and unproductive.

Part II: APIOBPCS model

The second model chosen to represent an MTS supply chain system is the Automatic Pipeline Inventory and Order Based Production Control System (APIOBPCS). This decision support system, in contrast with Forrester's model, considers feedback information of inventories both on-hand and in process. In addition to this, measures between target and actual inventories occur by a linear comparator, which obtains the difference between the two signals. In Forrester's model, this comparator was based on the ratio instead of the difference. Hence, the APIOBPCS model does not have any continuous nonlinearity.

In order to make this model more representative of the Beer Game dynamic behaviour, Shukla *et al.* (2009) inserted CLIP functions to avoid order rates reaching negative values and shipments being made without an on-hand inventory. This is diagrammatically represented by Figure 5.22. Negative order rates imply the return of goods back to suppliers. This nonlinear version of APIOBPCS is analogous to the

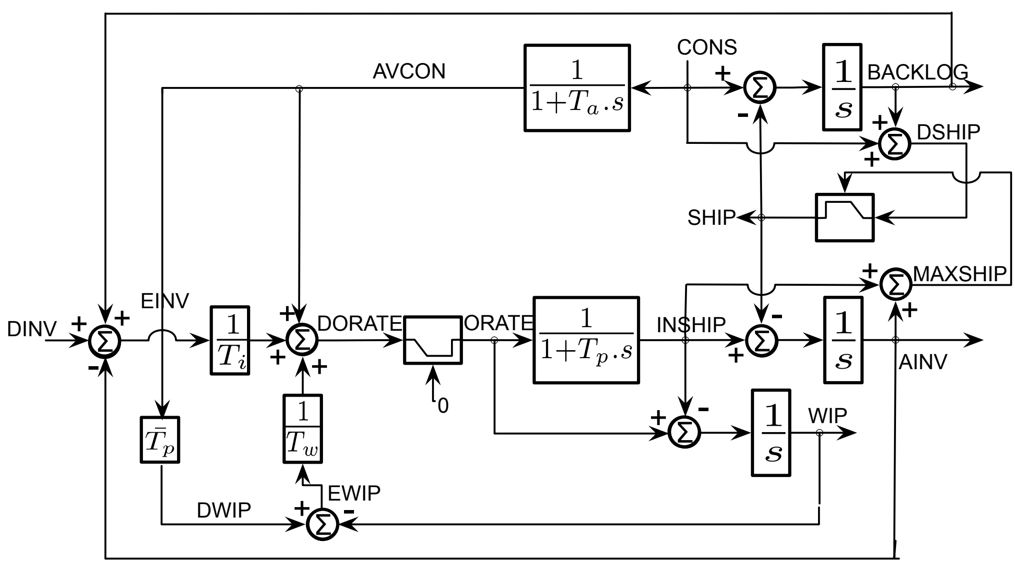


Figure 5.22: Nonlinear representation of the APIOBPCS model

Beer Game developed by Sterman (1989) and the reader can also refer to Appendix C which contains the difference equations that are in line with those previously used by Shukla *et al.* (2009). This system is characterised by three control parameters (T_a, T_i and T_w) and a physical parameter, the actual lead-time T_p. The expected lead-time, $\bar{T}_p$, is assumed to be equal to the actual lead-time.

In this section the APIOBPCS is explored to understand some of its output behaviour especially in relation to supply chain resilience. Quasi-linearisation or describing function method will be used to understand the effect of CLIP functions in the APIOBPCS model but no comparison between the linearised and actual models will be made since this has already been demonstrated in Section 5.4. Following the same steps in analysing Forrester's model, the effect of these nonlinearities on the system response can be analysed.

5.7 Quasi-linearisation

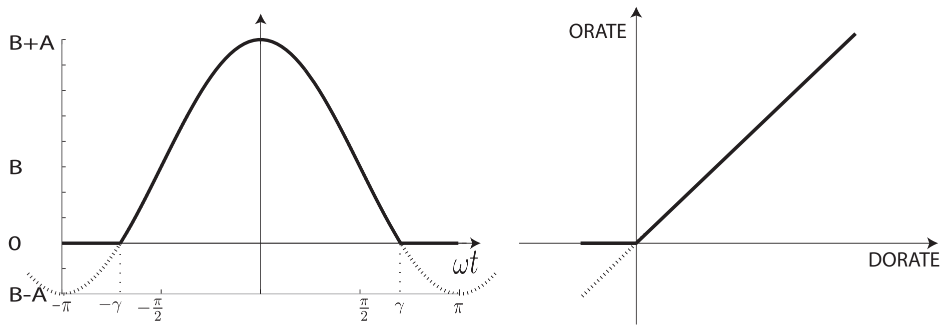
In this section, the analysis of the nonlinearities present in the APIOBPCS model will be undertaken.

5.7.1 Analysis of discontinuous nonlinearities

Since the APIOBPCS model contains only discontinuous nonlinearities, describing function techniques are used to find the quasi-linear representation of these nonlinearities. The first discontinuity to be analysed is the CLIP function present in the ordering system which prevents orders of negative values. Then, the constraint in the shipments will be investigated.

5.7.1.1 Manufacturing constraint: non-negative order rate

When introducing the manufacturing constraint, in which ORATE has to be greater than zero, in an open loop case the system will behave as in Figure 5.23. A sinusoidal input, the desired order rate (DORATE), to the nonlinearity, which represents a saturation function of a minumin limit of zero, will produce an output ORATE of the same frequency but different amplitudes and mean. Figure 5.23(b) illustrates the single-valued property of this nonlinearity. The output ORATE does not depend on the past values of the input DORATE, but it varies according to the actual state of DORATE. ORATE is described by two piecewise linear equations in the figure's legend.



(a) Time series for DORATE and ORATE (b) Output in relation to input (single-value property

$DORATE(t) = A.cos(\omega t) + B$ $\quad ORATE(t) = \begin{cases} DORATE(t) & \text{if } DORATE > 0 \\ 0 & \text{if } DORATE < 0 \end{cases}$

Figure 5.23: Asymmetric output saturation in relation to sinusoidal input DORATE

This nonlinearity is analogous to the manufacturing constraint found in Forrester's model in Section 5.4.2.1 and Figure 5.12. Following the same steps as in Section 5.4.2.1, the elements of the describing function can be found as:

$$N_A = \frac{\gamma - cos\gamma.sin\gamma}{\pi} \tag{5.79}$$

$$N_B = \frac{B.\pi + A.(-\gamma + \pi)cos\gamma + A.sin\gamma}{B.\pi} \tag{5.80}$$

where γ, in this case, is equal to $cos^{-1}\left(\frac{-B}{A}\right)$ and the phase $\phi = 0$ since this is a single-valued nonlinearity. Figure 5.24 illustrates how the coefficients of the describing function vary as the amplitude of the desired order rate, A_{DORATE}, increases. For amplitudes lower than the mean B, the system behaves as linear and output $ORATE$ will be equal to the input $DORATE$ corresponding to a describing function gain equal to 1. However, when the amplitude A_{DORATE} increases only a fraction of this rate will actually be ordered. So, although the gain describing function, N_A, differs from the one found in Forrester's model, the numerical result is the same: the gain describing function varies from 0.5 to 1.

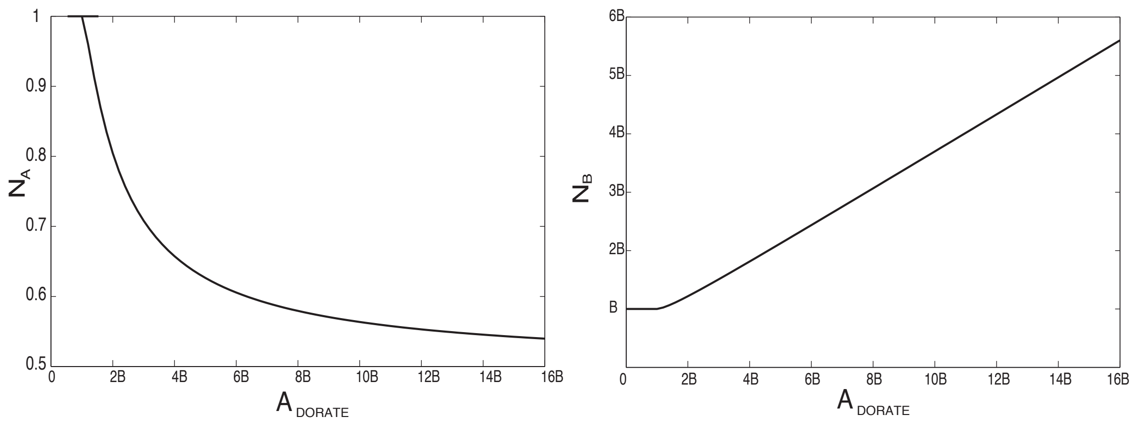


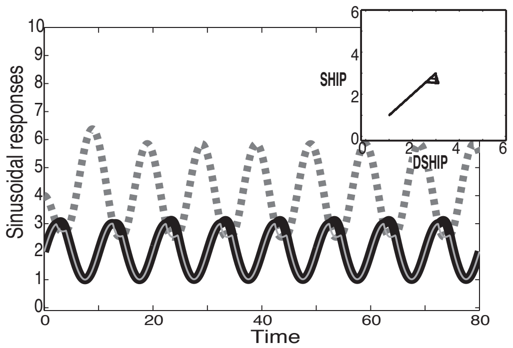
Figure 5.24: Terms of describing function for asymmetric saturation in the API-OBPCS model

However, the change in the output mean differs from Forrester's model. The mean increases as the amplitude increases because in this case, the limit value of the order rate is not in its maximum value but in its minimum value.

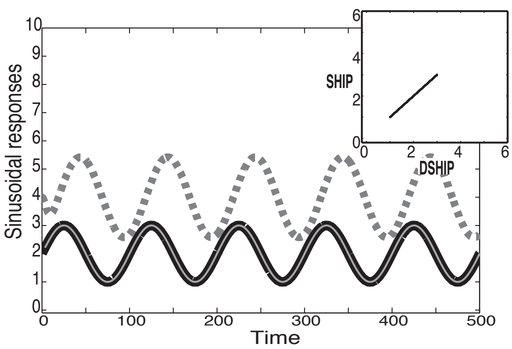
5.7.1.2 Shipment constraint

The CLIP function in the shipment element of APIOBPCS is also used to avoid any shipments being made to customers if no inventory is actually available. Hence, shipments sent ($SHIP$), will be equal to desired shipment tried ($DSHIP$), only if the sum of actual inventory ($AINV$) and current shipment received ($INSHIP$) results in conditions sufficient for the shipment.

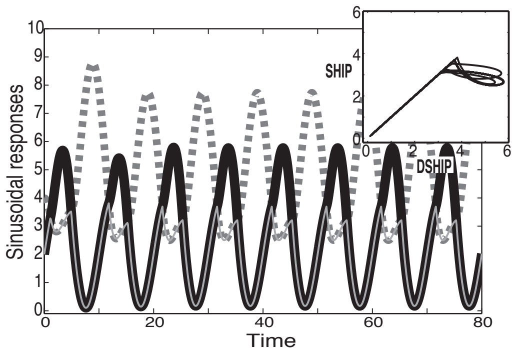
As in Forrester's model, this nonlinearity in the APIOBPCS is amplitude-dependent and frequency-dependent. Figure 5.25 demonstrates a set of system responses for the



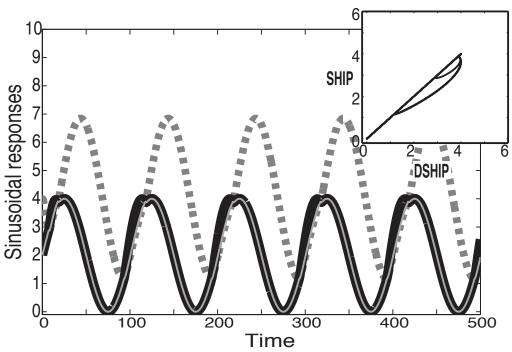
(a) $\omega = 0.63 rad/week$, Input Amplitude=1



(b) $\omega = 0.063 rad/week$, Input Amplitude=1



(c) $\omega = 0.63 rad/week$, Input Amplitude=2



(d) $\omega = 0.063 rad/week$, Input Amplitude=2

MAXSHIP DSHIP SHIP

(note the different scales in the x-axis)

Figure 5.25: Maximum shipment (MAXSHIP), desired shipment(DSHIP) and shipment (SHIP) responses to different amplitudes and frequencies

maximum, desired and actual shipments given different amplitudes and frequencies. While in Forrester's model lower frequencies would necessarily increase the chance that the output would behave nonlinearly, in the APIOBPCS shipment system Figure 5.25 illustrates that this is not always true. As frequencies are decreased, the system output, $SHIP$, moves from a linear behaviour to a nonlinear one and to linear again.

This effect is again confirmed by Figure 5.26, where the describing functions elements, N_A and ϕ of this nonlinearity are plotted against different amplitudes and frequencies.

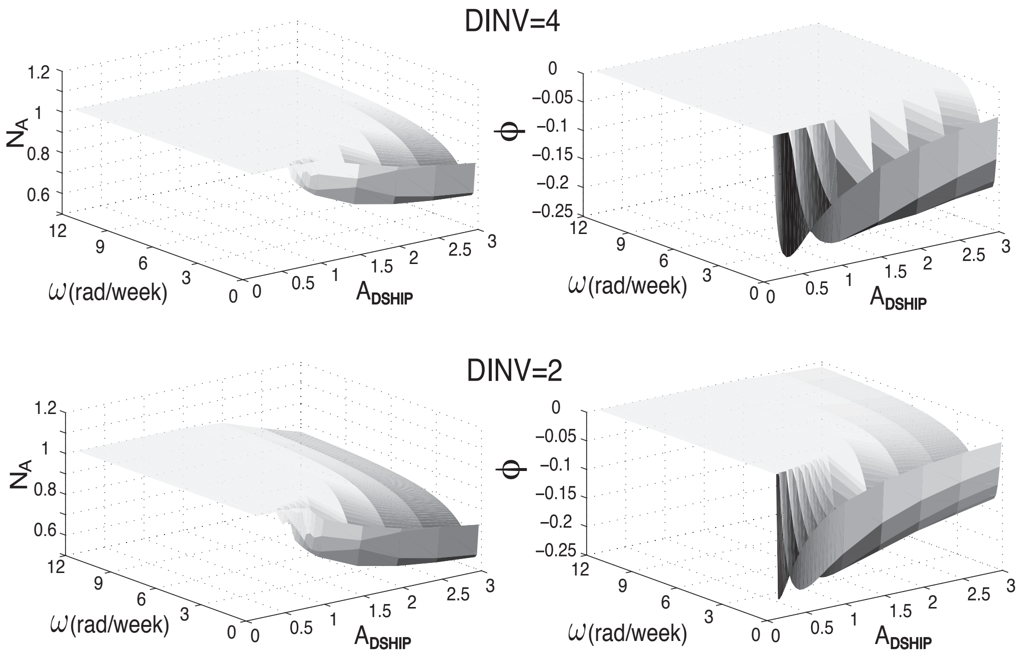


Figure 5.26: Describing function amplitude gain and phase in relation to DSHIP amplitude and frequency

5.8 Response analysis for resilience

In this section, an analysis of the APIOBPCS resilience performance will be undertaken. Transfer functions, step response and root locus techniques will be used to estimate the effect of input signals on the performance of nonlinear systems..

5.8.1 Transfer Functions

By temporarily assuming that the CLIP functions ( ) are not active, the transfer functions of the APIOBPCS model can be determined. In other words, the manufacturing rate decision will be equal to the desired one, ORATE = DORATE, and the shipment sent will be the same as the desired shipment, SHIP = DSHIP, independent of actual inventory levels. This linear representation of APIOBPCS is illustrated in Figure 5.27.

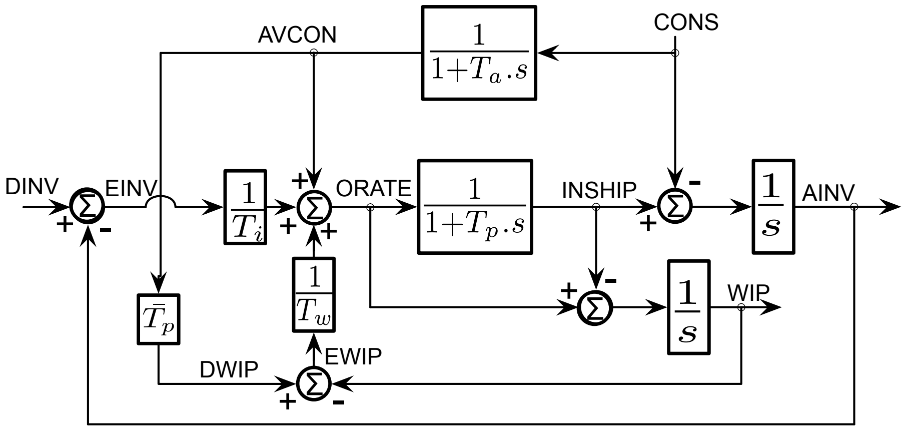


Figure 5.27: Linear representation of the APIOBPCS model

From the block diagram in Figure 5.27, it is possible to determine the actual inventory (AINV) and order rate (ORATE) transfer functions in relation to the input demand or consumption (CONS), resulting in the following third order equations:

$$\frac{AINV}{CONS} = \frac{(T_i\bar{T}_p - T_iT_p) - (T_iT_wT_p + T_aT_iT_p + T_aT_iT_w)s - (T_aT_iT_wT_p)s^2}{T_w + (T_iT_w + T_iT_p + T_aT_w)s + (T_iT_wT_p + T_aT_iT_w + T_aT_iT_p)s^2 + (T_aT_iT_wT_p)s^3} \tag{5.81}$$

$$\frac{ORATE}{CONS} = \frac{T_w + (T_aT_w + T_iT_w + T_wT_p + T_i\bar{T}_p)s + (T_aT_wT_p + T_iT_wT_p + T_iT_p\bar{T}_p)s^2}{T_w + (T_iT_w + T_iT_p + T_aT_w)s + (T_iT_wT_p + T_aT_iT_w + T_aT_iT_p)s^2 + (T_aT_iT_wT_p)s^3} \tag{5.82}$$

The third order equations in Equation 5.81 and in Equation 5.82 can be re-written as a product of a first order and a second order systems (Srivastava *et al.*, 2009) which will assume the following standard form:

$$\frac{AINV}{CONS} = \frac{[(T_i\bar{T}_p - T_iT_p) - (T_iT_wT_p + T_aT_iT_p + T_aT_iT_w)s - (T_aT_iT_wT_p)s^2] . \frac{1}{T_aT_iT_wT_p}}{(s + \frac{1}{T_a})(s^2 + (\frac{1}{T_p} + \frac{1}{T_w})s + \frac{1}{T_iT_p})} \tag{5.83}$$

$$\frac{ORATE}{CONS} = \frac{[(T_w + (T_aT_w + T_iT_w + T_wT_p + T_i\bar{T}_p)s - (T_aT_wT_p + T_iT_wT_p + T_iT_p\bar{T}_p)s^2] . \frac{1}{T_aT_iT_wT_p}}{(s + \frac{1}{T_a})(s^2 + (\frac{1}{T_p} + \frac{1}{T_w})s + \frac{1}{T_iT_p})} \tag{5.84}$$

Comparing the equations above with the standard third order system equation (refer back to Equation 4.5), the expressions for ω_n and ζ can be found as given in Equation 5.85. Note that if any of the parameters, T_i, T_w, T_i and T_p, are equal to zero then the ω_n and ζ equations in Equation 5.85 are not valid. This is because the system will no longer follow the third order standard form and will assume a second-order or even first order nature.

$$\omega_n = \sqrt{\frac{1}{T_iT_p}}, \qquad \zeta = \frac{(T_w + T_p)T_i}{2T_w} . \sqrt{\frac{1}{T_iT_p}} \tag{5.85}$$

We find that the natural frequency depends only on two parameters, T_i and T_p, and this relationship is inverse. Hence, the longer the lead-time and the inventory recovery time are, the slower the system response will be. The damping ratio also depends on T_w and as this parameter approaches zero the damping ratio approaches positive and negative infinity as shown in Figure 5.28. This figure illustrates the relationship of the damping ratio and T_i and T_w when T_p is fixed. The relationship between ζ and T_i is positive for T_i values greater than zero. This means that as T_i increases, ζ also increases. When T_i is negative, the ζ will assume an imaginary value.

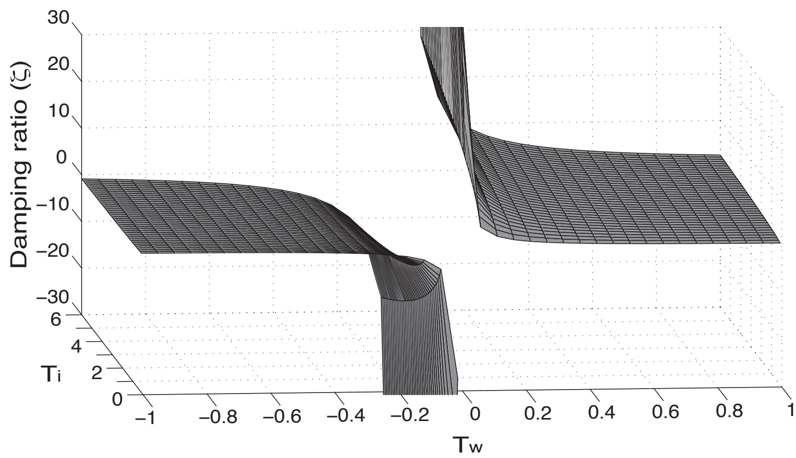
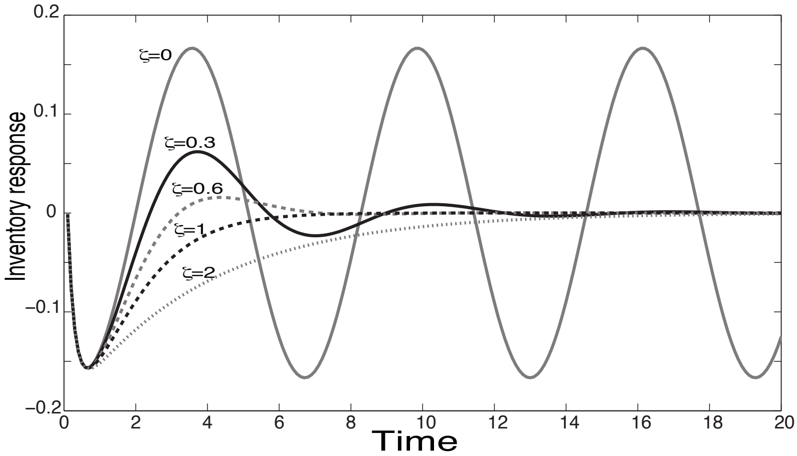
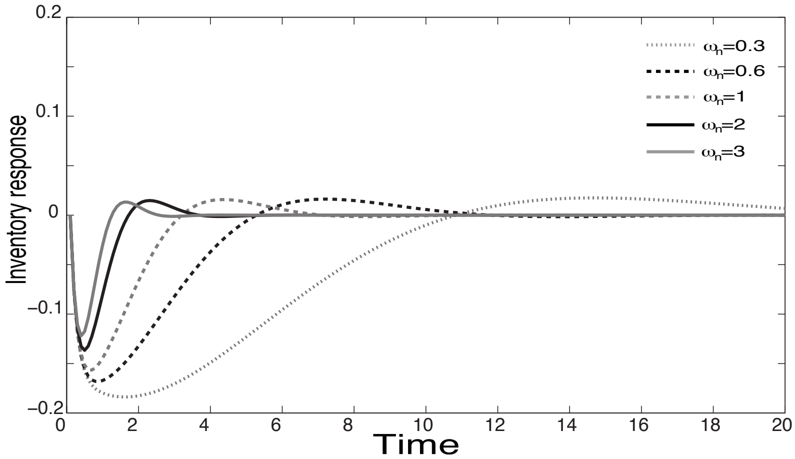


Figure 5.28: Damping ratio in relation to T_i and T_w

Figure 5.29 illustrates the impact of different damping ratios and natural frequency values on inventory responses of APIOBPCS. For obtaining Figure 5.29(a), ω_n, T_a and T_p have been fixed at 1 rad/week, 6 weeks and 1 week, respectively. When the damping ratio is large, for instance $\zeta = 2$, the vertical displacement of the inventory response will be the same but the inventory level will take longer to respond and recover when compared with $\zeta = 1$. As the value of T_a decreases the vertical replacement of the inventory response is normally smaller when damping ratios are large. There is normally a trade off between displacement and recovery as seen in Chapter 4. For damping ratios lower than one, the system oscillates for a longer time. Figure 5.29(b) illustrates the different natural frequencies' impact on inventory response while ζ is kept equal to 0.6. As ω_n decreases the system response and recovery becomes slower and its vertical displacement becomes greater. Hence, no trade-off is found and when accounting for supply chain resilience the natural frequency needs to be maximised.



(a) The effect of ζ



(b) The effect of ω_n

Figure 5.29: The effects of damping ratio and natural frequency on inventory response (AINV)

Table 5.2 contains the transfer functions and the system characteristics when the parameters are set to zero. When one of the parameters of the APIOBPCS system is equal to zero the characteristic equation might not be of third order and new natural frequencies and damping ratios should be considered. In the case of first order models, the time constant τ describes the system behaviour.

When	$\frac{AINV}{CONS}$	$\frac{ORATE}{CONS}$	System characterised by
$T_p = 0$	$\frac{-(T_a T_i T_w)s}{(T_a T_i T_w)s^2+(T_i T_w+T_a T_w)s+T_w}$	$\frac{T_w+(T_a T_w+T_i T_w)s}{(T_a T_i T_w)s^2+(T_i T_w+T_a T_w)s+T_w}$	$\omega_n = \sqrt{\frac{1}{T_a T_i}}$ $\zeta = \frac{T_a+T_i}{2}\sqrt{\frac{1}{T_a T_i}}$
$T_i = 0$	$\frac{0}{(T_a T_w)s+T_w} = 0$	$\frac{T_w+(T_a T_w+T_w T_p)s+(T_a T_w T_p)s^2}{(T_a T_w)s+T_w}$	$\tau = T_a$
$T_w = 0$	$\frac{-(T_a T_i T_p)s}{(T_a T_i T_p)s^2+(T_i T_p)s}$	$\frac{(T_i T_p)s+(T_i {T_p}^2)s^2}{(T_a T_i T_p)s^2+(T_i T_p)s}$	$\omega_n = 0$ $\zeta = 0$
$T_a = 0$	$\frac{-(T_i T_w T_p)s}{(T_i T_w T_p)s^2+(T_i T_w+T_i T_p)s+T_w}$	$\frac{T_w+(T_i T_w+T_w T_p+T_i T_p)s+(T_i T_w T_p+T_i {T_p}^2)s^2}{(T_i T_w T_p)s^2+(T_i T_w+T_i T_p)s+T_w}$	$\omega_n = \sqrt{\frac{1}{T_i T_p}}$ $\zeta = \frac{(T_w+T_p)T_i}{2T_w}\sqrt{\frac{1}{T_i T_p}}$

Table 5.2: Changes in the APIOBPCS system when parameters are equal to zero

5.8.2 Inventory step response and ITAE

The location of the system's poles can give a qualitative assessment of the system's state and an idea of its transient response. Since the characteristic equation of the APIOBPCS system is a third order differential equation we expect to find three poles.

The first pole is easily identified from Equations 5.83 and 5.84 and corresponds to the root of the first order term in the system characteristic equation, or $-\frac{1}{T_a}$. Since the values of exponential smoothing coefficient, α, and the values of T_a will always be a real number, this first pole will always be real. Moreover, positive values of T_a are needed for the system's stability.

The other two poles are equal to the roots (p_1 and p_2) of the second order term in the denominator:

$$p_{1,2} = \frac{-T_iT_w - T_iT_p \pm \sqrt{{T_i}^2({T_w}^2 + {T_p}^2) + 2T_iT_wT_p(T_i - 2T_w)}}{2T_iT_wT_p} \tag{5.86}$$

By evaluating Equation 5.86, it is possible to determine which values of T_i, T_w and T_p generate real or complex poles. Furthermore, by determining which set of parameters results in positive real roots, the unstable region can be delimited. The roots will be real when the value of the discriminant is greater than or equal to zero. The discriminant is given by $\zeta^2 - 1$, and thus when $\zeta^2 > 1$ the values inside the square root will be positive making the roots real. Complex roots will occur if the discriminant is negative or $\zeta^2 < 1$.

It is important to identify purely imaginary roots because the system's response will be oscillatory. This will happen when the discriminant is negative and the real coefficient is zero. The real term will be zero when $T_w = -T_p$. The area between the line where $T_w = 0$ and the imaginary roots line is the area where roots have positive real part. Consequently, the system will not stabilise if the chosen T_w is lower than zero and greater than $-T_p$. More details on stability and performance of the IOBPCS (Inventory and Order Based Production Control System) family can be found in Disney and Towill (2002); Disney *et al.* (2006); Disney and Grubbström (2004) and Wang and Disney (2012).

Given the inventory transfer function Equation 5.83 and the two poles (p_1 and p_2) of the second order term Equation 5.86, the time function for the actual inventory can be finally determined as:

$$ainv(t) = A \cdot e^{-\frac{t}{T_a}} + B \cdot e^{p_1 t} + C \cdot e^{p_2 t} \tag{5.87}$$

where A, B and C are coefficients related to the system poles and D is the coefficient of the step input pole ($s = 0$). Assuming that the expected lead-time is equal to the actual lead-time ($\bar{T}_p = T_p$), the coefficient values are:

$$\begin{aligned}
A &= \frac{-T_i {T_a}^3 (T_w + T_p)}{(T_a p_1 + 1)(T_a p_2 + 1)} \cdot \frac{1}{T_a T_i T_w T_p} \\
B &= \frac{-T_i (T_a T_w + T_a T_p + T_w T_p + p_1 T_a T_w T_p)}{(p_1 - p_2)(p_1 + 1/T_a)} \cdot \frac{1}{T_a T_i T_w T_p} \\
C &= \frac{-T_i (T_a T_w + T_a T_p + T_w T_p + p_2 T_a T_w T_p)}{(p_2 - p_1)(p_2 + 1/T_a)} \cdot \frac{1}{T_a T_i T_w T_p} \\
D &= \frac{T_a}{p_1 p_2} (T_i \bar{T}_p - T_i T_p) \cdot \frac{1}{T_a T_i T_w T_p} = 0
\end{aligned} \tag{5.88}$$

The second term, which is a division by $T_a T_i T_w T_p$, indicates that the results were normalised according to the leading coefficient of the denominator.

Finally, the ITAE expression for the actual inventory time equation is determined. Firstly the target inventory is considered as being equal to zero. Hence, the error in the inventory ($EINV$) is the difference between zero and the actual inventory. However, since the ITAE involves the integral of an absolute function, there are some aspects of the actual inventory function to be considered. After the step change, if the inventory amount drops and recovers without overshooting again, then the ITAE can be calculated as:

$$ITAE_{einv} = -\left(A.T_a^2 + \frac{B}{{p_1}^2} + \frac{C}{{p_2}^2} \right) = \frac{T_i (T_p + T_w)\left({T_a}^2 T_w + T_i T_p T_w + T_a T_i (T_p + T_w) \right)}{{T_w}^2} \tag{5.89}$$

Equation 5.89 suggests that ITAE in the inventory will be minimised when T_i is zero. This result is expected and implies that this single-echelon supply chain would review the inventory continuously and that the supplier would replenish material continuously as well. Hence, supply chain resilience would be guaranteed. It is also possible to see in Equation 5.89 that there is a positive relationship between ITAE and the control parameter T_a and the physical delay T_p. In other words as both parameters increase, the resilience performance decreases. On the other hand, the control parameter T_w has a negative relationship with ITAE. Figure 5.30 illustrates these relations when varying each of the control parameters while keeping the others equal to a unity. Lead-time was also fixed at one week.

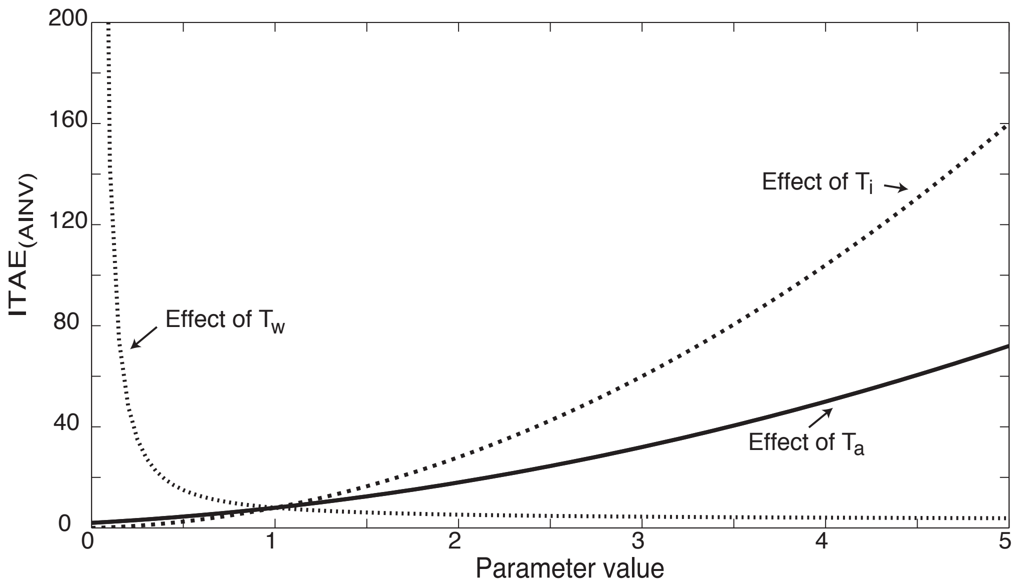


Figure 5.30: Effect of the control parameters and the physical delay on ITAE

Experiments show that if T_w increases significantly in relation to the lead-time, the system outputs start to overshoot and Equation 5.89 is no longer valid. In the case of an overshoot, there is a need to determine the zeros of the function $einv(t)$ and calculate the integral by parts, considering the absolute value of each part. Alternatively, the generic ITAE Equation 4.2 can be used with $\delta t = 0.05$ to run a simulation for a long time period.

5.8.3 Effect of CLIP functions

In Section 5.8.2, the resilience performance was investigated by assuming inactive CLIP functions. In this section, limit cycles and the effect of these discontinuous nonlinearities on system behaviour will be investigated.

5.8.3.1 Investigating limit cycles

In order to investigate the presence of oscillation in the inventory and shipment responses of the nonlinear APIOBPCS model we need to find the open loop transfer functions for $\overline{ORATE}$ and $\overline{SHIP}$. By replacing the CLIP functions of Figure 5.22 with the gains $N_{A(DORATE)}$ in the ordering system and $N_{A(DSHIP)}$ in the shipment system, we find that the open loop transfer functions are

$$\overline{ORATE} = \frac{N_{A(DORATE)}(1+T_p s)(T_w+s(T_a T_w+T_i(T_p+T_w)))}{s((1+sT_a)T_i(1+sT_p)T_w+N_{A(DORATE)}(-(T_i+T_p)T_w-sT_p(T_a(-T_i+T_w)+T_i(T_p+T_w))))} \tag{5.90}$$

$$\overline{SHIP} = \frac{N_{A(DSHIP)}(1+s)}{s(1-N_{A(DSHIP)})} \tag{5.91}$$

By replacing $s = i\omega$ in the equations above and using Mathematica to find the values of ω that makes $\overline{ORATE} = -1$ and $\overline{SHIP} = -1$ we find that oscillations will occur only for the nonlinearity in the manufacturing and this will be when:

$$T_w = -N_{A(DORATE)}.T_p \tag{5.92}$$

Since the values of $N_{A(DORATE)}$ vary from 0.5 to 1, limit cycles occur when

$$-T_p < T_w < -0.5T_p \tag{5.93}$$

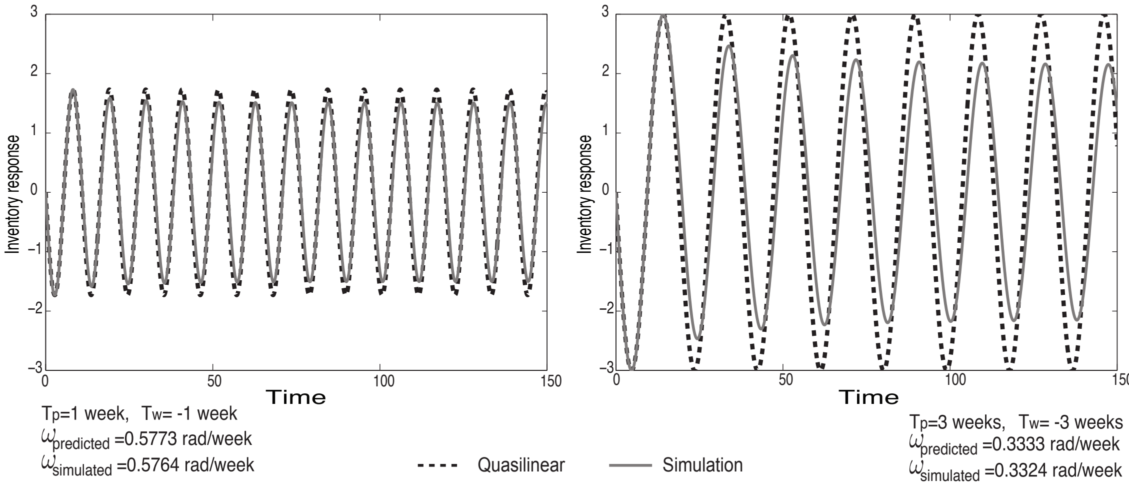
and the frequency of oscillation is predicted to be

$$\omega = -\frac{\sqrt{T_i \left(4N_{A(DORATE)}T_pT_w{}^2 - T_i(N_{A(DORATE)}T_p + T_w)^2\right)}}{2T_iT_pT_w} \tag{5.94}$$

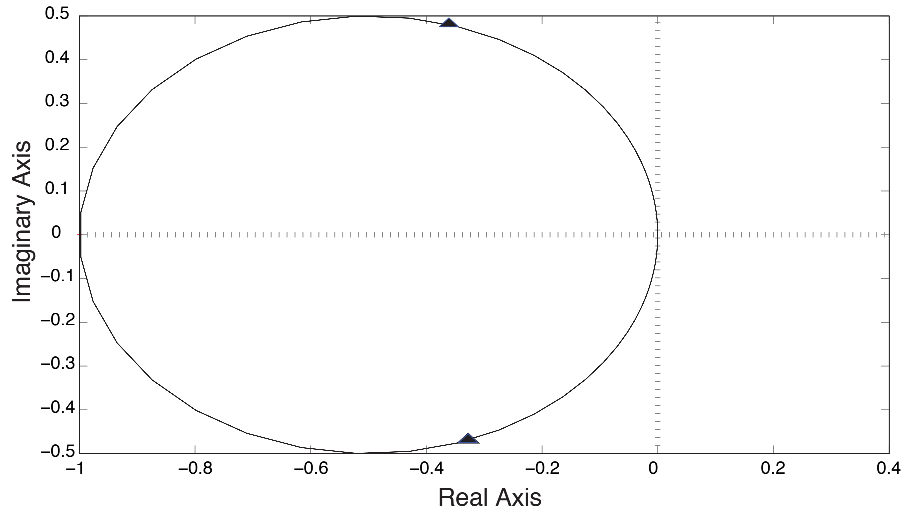
The nonlinearity in shipments does not cause any oscillations. Figure 5.31 illustrates two examples of limit cycle comparing the quasilinear model with the simulation model using Matlab/Simulink. Figure 5.31(a) shows the step responses for $T_p = 1$ and $T_p = 3$. T_a and T_i have been fixed at 6 and 3 weeks, respectively. The Figure confirms that the accuracy in the prediction of the oscillation frequency is very high. Figure 5.31(b) illustrates the Nyquist diagram for both cases of Figure 5.31(a). As seen in the figure the loci of the open loop transfer function intercepts the real axis at -1, confirming the existence of a limit cycle.

5.8.3.2 Predicting system behaviour

In this section, root locus techniques are again used to predict how the discontinuous nonlinearities affect the system responses in the APIOBPCS model. By replacing the CLIP functions of Figure 5.22 with the gains $N_{A(DORATE)}$ in the ordering system and $N_{A(DSHIP)}$ in the shipment system, we find that the new system



(a) Time responses



(b) Nyquist plot

Figure 5.31: Limit cycle examples

characteristic equation is found to be:

$$\left(\frac{1}{T_a}+s\right)\left(s^2+\Big(\frac{N_{A(DORATE)}}{T_w}+\frac{1}{T_p}\Big)s+\frac{N_{A(DORATE)}}{T_iT_p}\right) \tag{5.95}$$

In this way the effect on the system natural frequency and damping ratio can be calculated as:

$$\omega_n = \sqrt{\frac{N_{A(DORATE)}}{T_i T_p}}, \qquad \zeta = \frac{(T_w + N_{A(DORATE)}.T_p)T_i}{2N_{A(DORATE)}.T_w}.\sqrt{\frac{N_{A(DORATE)}}{T_i T_p}} \tag{5.96}$$

When analysing the results in Equation 5.96 we find that the nonlinearity will always decrease the value of the natural frequency ω_n, which consequently causes a negative impact on supply chain resilience. On the other hand, the damping ratio depends on the combining values of other parameters. For instance, Figure 5.32 illustrates that when T_w values are greater or equal to lead-time T_p then the damping ratio increases as the nonlinearity takes effect. T_i and T_p have been fixed and are equal to 3 weeks. The choices of these parameter settings were recommended by previous research (John *et al.*, 1994; Sterman, 1989; Shukla *et al.*, 2009) to design 'optimum' behaviour in the system dynamics of APIOBPCS. However, when $T_w = T_p$ the system changes from critically damped ($\zeta = 1$) to overdamped ($\zeta > 1$), which means that the system does not overshoot but takes longer to recover as the nonlinearity takes effect. But when $T_w = 2.T_p$, then the system damping ratio goes from underdamped ($\zeta = 0.75$) to less underdamped ($\zeta = 0.88$). This may have a positive impact on supply chain resilience since the oscillations and errors will decay quicker. On the other hand, when $T_w = {}^{T_p}/_2$ the nonlinearity assists in decreasing the system's damping ratio.

Note that the nonlinearity present in the shipment system ($N_{A(DSHIP)}$) has no effect on the system's characteristic equation. Hence, it does not influence the system inventory and order rate responses. The only impact that this nonlinearity

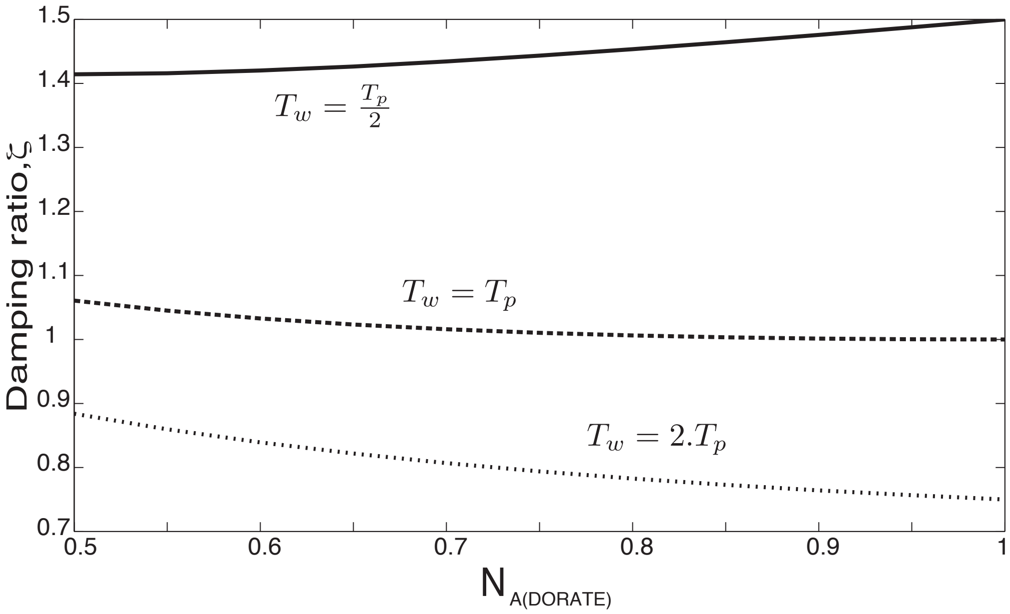


Figure 5.32: Effect of manufacturing constraints (non-negative order rate) on the system damping ratio

has is in the shipment rate given by the following transfer function:

$$\frac{SHIP}{CONS} = \frac{N_{A(DSHIP)}(1+s)}{N_{A(DSHIP)}+s} \tag{5.97}$$

For this reason, as in Forrester's model where it is necessary to take into account the actual inventory and unfilled orders as an indicator of supply chain resilience, in the APIOBPCS both actual inventory and shipment have to be considered. Since Equation 5.97 initial and final values are not changed by the describing function element, we are able to predict its impact on ITAE. Figure 5.33 demonstrates the negative impact on shipments when the nonlinearity in the shipment takes effect. When $N_{A(DSHIP)} = 1$ the nonlinearity is inactive, which means that the maximum shipment capacity (actual inventory plus shipment received) is never reached and ITAE of shipment is zero. When this shipment capacity is reached, shipments to customers will be cut and the supply chain will be less resilient.

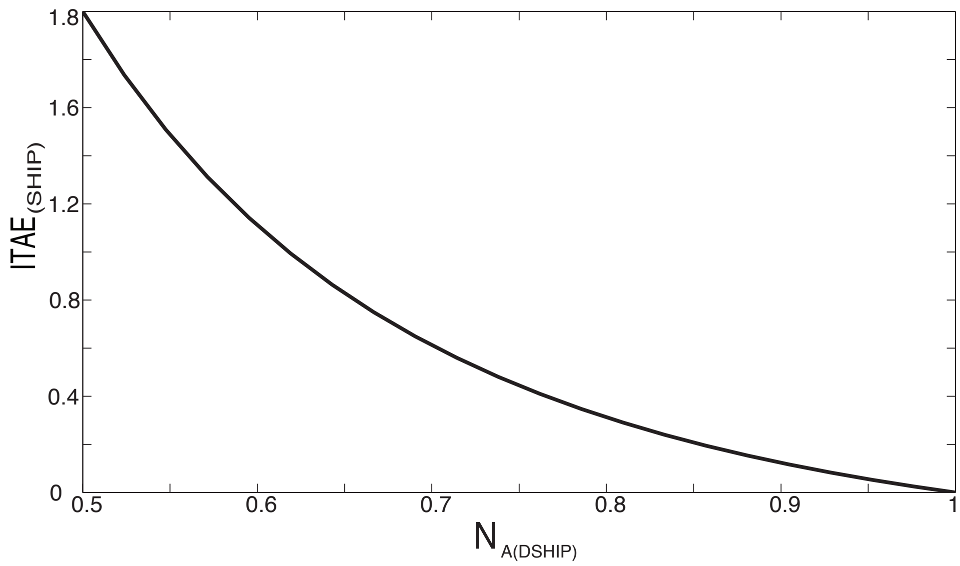


Figure 5.33: Effect of shipment constraint on ITAE

5.9 Summary

In this section the APIOBPCS model, which was originally developed by John *et al.* (1994) in an effort to create a linear form of the nonlinear Sterman Beer Game ordering rule, has been investigated. A nonlinear block diagram representation containing the CLIP functions from Shukla *et al.*'s nonlinear difference equations has been established here to allow a comprehensive analysis of the nonlinear elements.

In contrast to Forrester's model, the APIOBPCS model has been extensively studied previously. Much is already known about its stability and many researchers have proposed different designs especially to avoid demand amplification. In this thesis, the focus is specifically on supply chain resilience performance. Hence shipment and inventory responses have been studied in order to enhance their response and recovery times

Since only discontinuous nonlinearities or CLIP functions are present in the APIOBPCS model, the describing function technique has been employed. Once again, examples of single- and multi-valued and amplitude- and frequency-dependent nonlinearities were observed. Using the describing function method, a limit cycle behaviour was identified. This effect was caused by the nonlinearity present in the manufacturing system. Unstable and limit cycle regions should be avoided since this is a prerequisite for resilience as the system would not be able to recover. An estimation of the system behaviour, through natural frequency and damping ratio calculations, enabled the researcher to understand how different parameter settings and nonlinearities affect supply chain resilience.

Finally, the results obtained in this chapter will help in conducting the simulations in the next chapter. For instance with the transfer function analysis the simulation process will be initially undertaken within the pre-determined stable boundaries and will only be focusing on important parameter values for achieving supply chain resilience. Moreover, the describing function technique has provided insights into understanding the impact of the different capacity constraints found in the APIOBPCS model. Hence, the simulations will be used only to check whether the analysis in this chapter has given correct insights and greater simulation efforts will be directed to check some surprising results, such as the fact that the shipment constraint does not cause any impact on other system responses.

6 The impact of control policies and nonlinearities on system dynamics

"That first inventory control system with pencil and paper simulation was the beginning of system dynamics."

– Jay W. Forrester (1989), *The beginning of system dynamics.*

In this chapter, repeated simulation technique is used to determine the impact of different control policies and nonlinearities on system dynamics and resilience performance. Moreover, this chapter will cross-check the results obtained from the analysis undertaken in the previous chapter and will further investigate unexpected or unclear findings.

A trade-off analysis between the resilience performance and production and holding inventory on-costs will also be performed. Finally, a sensitivity analysis will be undertaken to check the system robustness of any given supply chain design due to possible changes in lead-time.

6.1 Systems policies

In this section repeated simulation will be used to investigate the impact of systems design policies. After the nonlinear control theory analysis conducted in Chapter 5, a number of commonalities and differences between Forrester's and the APIOBPCS models were found (Table 6.1). For instance, both models contain inventory and demand policies. However, while the APIOBPCS model considers inventory both on-hand and in process, Forrester's model only considers actual inventory levels. In this way, both models contain a control parameter that determines the time to recover inventory: DI for Forrester's and T_i for APIOBPCS.

System characteristics	Forrester	APIOBPCS
Inventory policy	• On-hand inventory • No information feedback to manufacturing orders • Controls: DI	• On-hand and WIP inventories • Information feedback to manufacturing orders • Controls: T_i and T_w
Demand policy	• Exponential smoothing • Controls: DR	• Exponential smoothing • Controls: T_a
Backlog or unfilled order policy	• Target unfilled order is different from zero	• Target backlog is equal zero
Manufacturing capacity	• Maximum capacity (AL)	• Minimum capacity (zero)
Shipment capacity	• Maximum capacity (IA)	• Maximum capacity (MAX-SHIP)

Table 6.1: Differences and commonalities between the Forrester and APIOBPCS models

Regarding the demand policy, both models use exponential smoothing to forecast demand. Hence the delay in smoothing requisitions DR and the time to average demand T_a are also analogous.

6.1.1 The effect of inventory controllers

In the Forrester model the only parameter used to control inventory levels is DI, which is placed in the feedforward path to control the rate of manufacturing orders placed. Section 5.5.2 and Figure 5.20(b) demonstrated that as this parameter increases, ITAE also increases. Here, this relationship is crossed-checked through simulation.

A nominal setting, where the total pipeline lead-time is equal to 8 weeks ($DC + DP = 8$), has been set and ITAE values of inventory and unfilled orders were normalised in relation to their respective minimised values. After the simplification of Forrester's model, it was found that the total pipeline lag time is dependent on both production lead-time DP and clerical delay DC. The results in Figure 6.1 confirm that ITAE values are minimised when $DI = 0$ independent of the pipeline lead-times. The figure also shows that the ITAE of inventory increases more rapidly in relation to the ITAE of unfilled orders as DI increases.

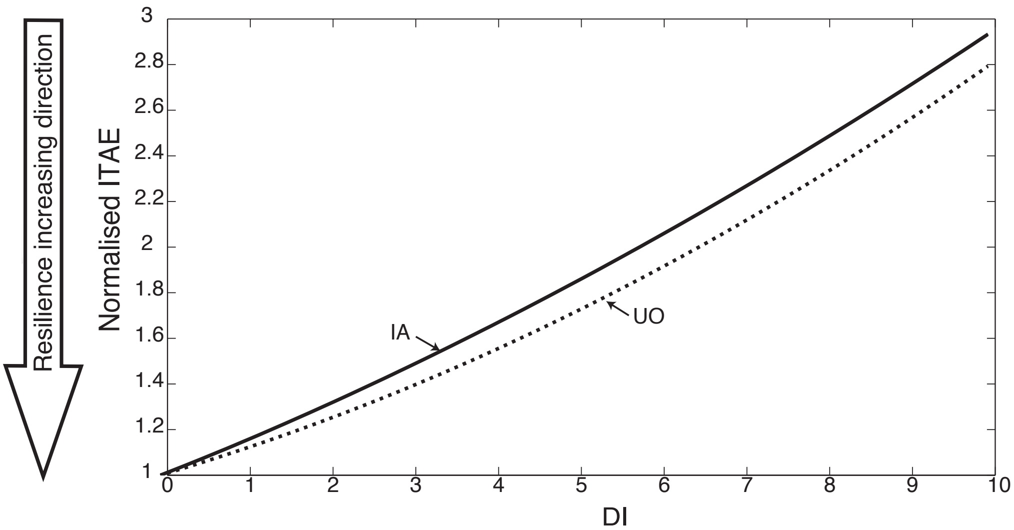


Figure 6.1: Impact of DI on the resilience performance of Forrester's model

On the other hand, in the APIOBPCS model inventory control is made by using feedback information of inventories both on-hand and in process. Hence, two control parameters, T_i and T_p, are used to maintain inventory levels. In the last chapter it was found that these parameters make conflicting contributions to resilience (see

Figure 5.30). For this reason, repeated simulation will be used to identify parameter combinations that yield better supply chain resilience.

The forecasting constant T_a is kept positive (to keep the system stable) and initially fixed at 6 weeks. The influence of this consumption averaging constant on supply chain resilience will be discussed later in Section 6.1.2. Hence, by varying the control parameters T_i and T_w as a function of T_p, the resilience area and the production on-costs for a one-echelon supply chain could be illustrated (Figure 6.2). The ratios $^{T_p}/_{T_i}$ and $^{T_p}/_{T_w}$ are used since different minimum points are found depending on the value of lead-time. The darker area of the greyscale image represents the parameter settings which result in smaller ITAE values for inventory. It will be demonstrated in Section 6.4 that this region of maximum resilience for inventory responses coincides with the region of minimum ITAE values of shipments when a backlog situation occurs.

The scenario where lead-time is equal to 8 weeks was set as the nominal setting. The minimum ITAE performance index found for this scenario is when $^{T_p}/_{T_w}$ is 11.67 and $^{T_p}/_{T_i}$ is 14.78. This is the point of maximum resilience that the system can achieve. In order to normalise the results, all the ITAE performance indices were divided by the minimum index value. In this way, the change in resilience performance can be discussed in a percentage relation to the maximum resilience point of the nominal scenario. For instance, the area where the normalised ITAE values are equal to 1.2 in Figure 6.2 means that the resilience performance dropped by 20% when changed to this set of parameters.

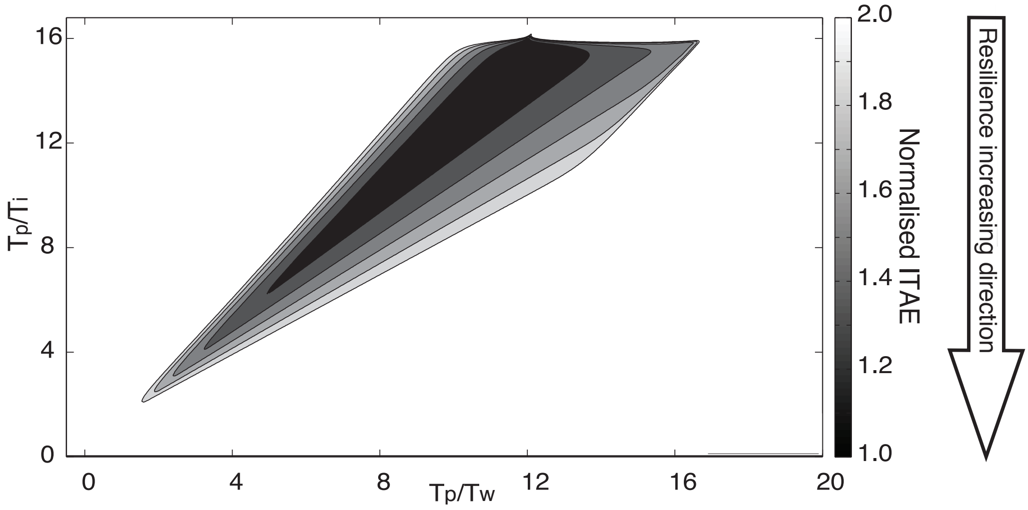


Figure 6.2: Resilient region in the APIOBPCS model

6.1.1.1 Feedback versus feedforward inventory control

Although the Forrester and APIOBPCS models have certain equivalent parameters, other policies are very different. The inventory control systems of both models, for instance, are very different. For this reason it is very hard to compare both models and identify which inventory control system, whether with feedback or feedforward controller, provides better resilience.

The author has evaluated the impact that a fixed change in the parameters T_i, T_w and DI has on the ITAE values. For instance, a change of ± 0.1 in DI would result in a change of between 1-1.15% in both ITAE values of IA and UO. While the same change in T_w alone would provoke a change in the inventory ITAE of between 7-20%. T_i alone, in turn, can affect ITAE more drastically (between 20-36%). Both T_i and T_w combined can make changes of up to 43% in the ITAE value.

Given these results, it is possible to presume that the feedback inventory control system is much more sensitive when compared to feedforward control systems. A small change in its controllers can help companies to increase resilience quickly but it can also make the system respond very slowly or even become unstable.

6.1.2 The effect of demand forecasting policy

The demand smoothing parameters T_a and DR of the APIOBPCS and Forrester model, respectively, can be considered equivalent and they represent the time to average or smooth demand so that the exponential smoothing function $\alpha = 1/(1 + T_a/\Delta t)$ or $\alpha = 1/(1 + DR/\Delta t)$.

When considering T_i=T_w=T_p=8, $DI = (DC + DP) = 8$ and α values varying between 0 and 1, it is found that minimum ITAE values are achieved when $T_a = 0$ and $DR = 0$ or $\alpha = 1$ (see Figure 6.3). This means that resilience can be improved when forecasts are not taken into account. Order rate is then based on the incoming demand and inventory controllers only, i.e. the supply chain substitutes a production levelling strategy with a chase. This finding is consistent with Christopher and Peck's (2004) observation that "forecast-driven" organisations are more prone to vulnerabilities than "demand-driven" organisations. It should be noted, however, that this approach yields a considerable peak in order requirements which might increase production costs. This will be investigated later in Section 6.3.1.

From Figure 6.3, it can also be observed that as α reaches zero, the ITAE approaches infinity since the system will be in the marginally stable region. With this parameter choice, the order rate will only be based on forecasts and will not consider changes in demand, which implies a lack of resilience.

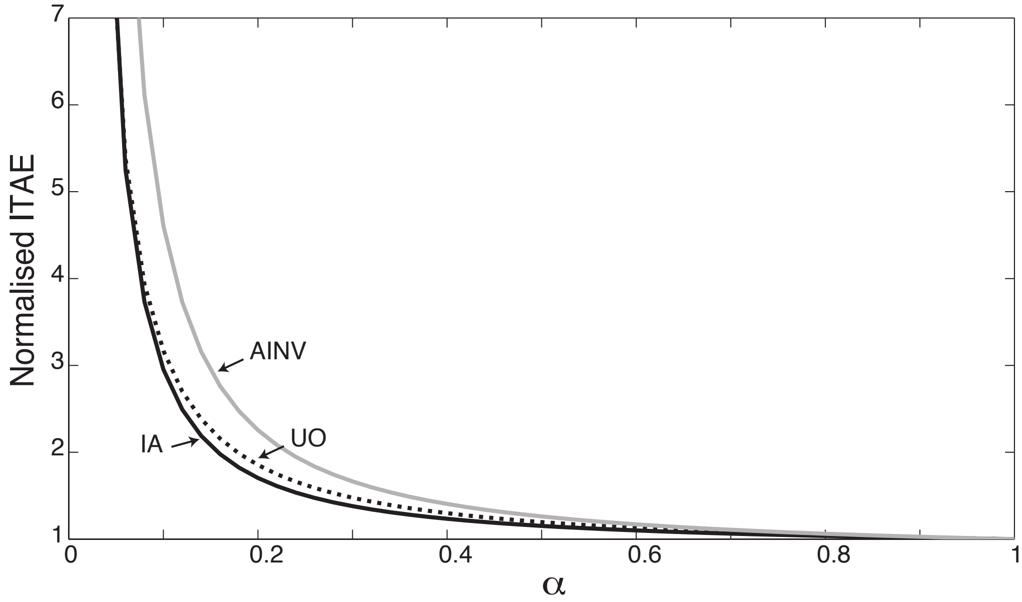


Figure 6.3: The effect of demand forecasting policy

6.2 Effect of capacity constraints

In Sections 5.5.3 and 5.8.3, the effect of CLIP functions on supply chain resilience has been analytically investigated using the describing function method and root locus techniques. However, since the describing function method prevents the analysis of step responses, the impact of discontinuous nonlinearities was predicted given the change in the system poles positions and, consequently, the change in the natural frequency and damping ratio.

In this section, simulation is used to plot step responses and to calculate the impact of discontinuous nonlinearities on supply chain resilience. The results will be crossed checked with the previous analytical predictions.

6.2.1 Manufacturing constraints

Manufacturing constraints are represented by maximum manufacturing capacity in the Forrester model and by non-negative production in the APIOBPCS. It was seen in Chapter 5 that the nonlinearity in the Forrester model will always increase the system damping ratio independent of the chosen control parameters and for this reason a negative impact on supply chain resilience was predicted.

In the Forrester model, the effect of the nonlinearity can be easily assessed by decreasing the value of AL, the manufacturing capacity. Figure 6.4 illustrates four different examples of inventory and unfilled order responses when AL=2, 1.5, 1.3 and 1.1. The parameters used were the same as Forrester suggested and are in

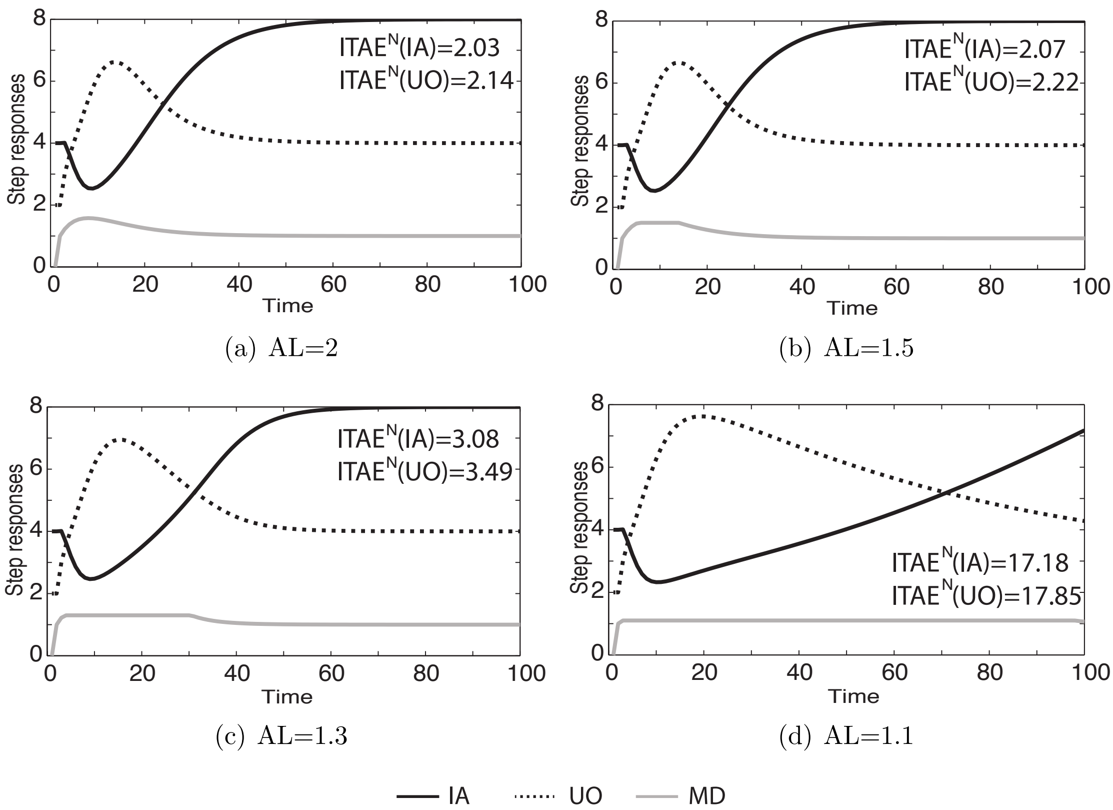


Figure 6.4: The effect of manufacturing capacity on supply chain resilience

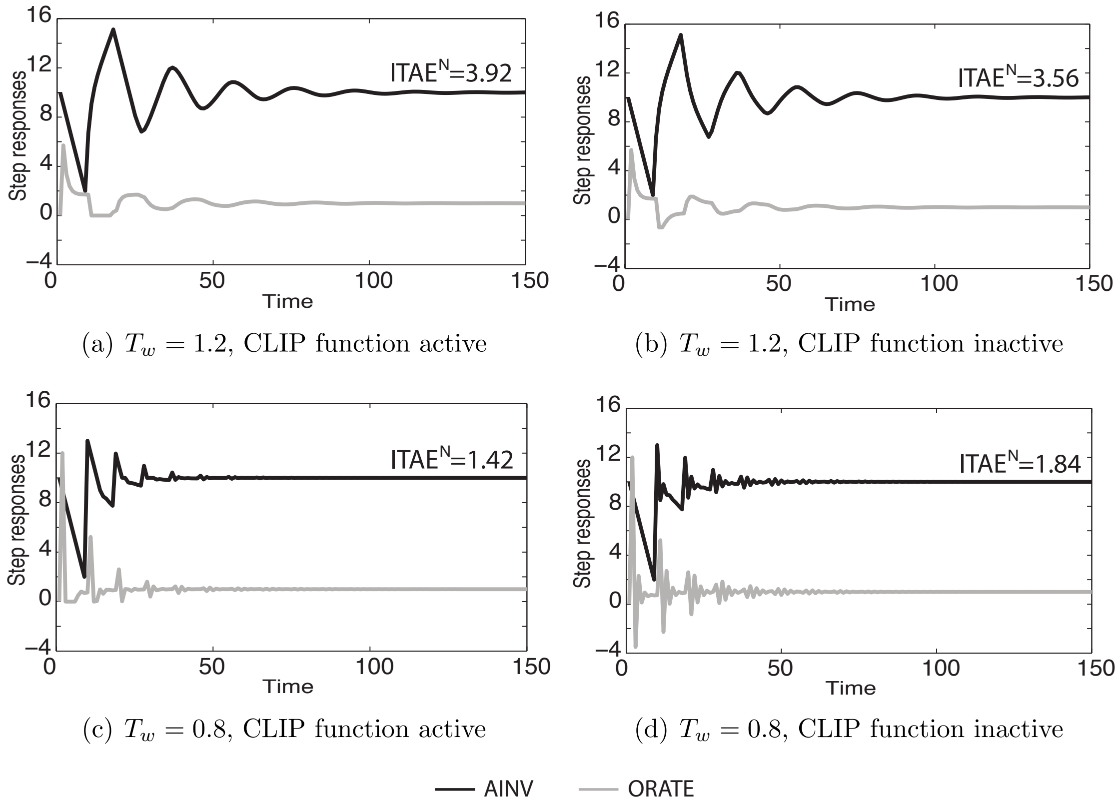
Appendix B. The figure shows that as AL decreases both inventory and unfilled order responses take more time to recover. Hence, ITAE values increase. ITAE values were normalised in relation to the nominal setting given in the previous section. AL cannot be smaller than the demand (1 unit) otherwise the system will never stabilise.

When AL=2, the order rate does not reach its capacity and the CLIP function does not take effect. The peak in order rate is dependent on the parameter values of DI and DR and can be determined by the transfer function given in Equation 5.57. In this way, the supply chain designer can control the peak of the order rate depending on the available manufacturing capacity.

In the APIOBPCS model, on the other hand, the non-negativity nonlinearity is more complex to predict because it depends on the input step size and direction (increase or decrease), on the control parameter combination (T_a, T_i, T_w) and lead-time (T_p). For a unit step increase, the APIOBPCS order rate may reach zero if the system is underdamped and subjected to oscillations. Figure 6.5 illustrates two different examples when the manufacturing CLIP function is active and inactive and the parameter T_w is greater or less than 1. The other parameters are $T_a = 6$ and $T_i = T_p = 8$. In the example in Figure 6.5, the analysis results in Section 5.8.3.2 and Figure 5.32 are confirmed. Figure 6.5 shows that depending on the values of the control parameters, the CLIP function in the manufacturing process of the APIOBPCS model may or may not be beneficial to resilience.

6.2.2 Shipment constraints

In both the Forrester and APIOBPCS models, CLIP functions are used to constrain the transportation system and allow the shipment of goods only if there is



(a) $T_w = 1.2$, CLIP function active (b) $T_w = 1.2$, CLIP function inactive

(c) $T_w = 0.8$, CLIP function active (d) $T_w = 0.8$, CLIP function inactive

Figure 6.5: The effect of non-negative manufacturing constraint on supply chain resilience

sufficient inventory. The available capacity is, therefore, given by the inventory response, which is a variable capacity and depends on the desired inventory constants (AI for Forrester and DINV for APIOBPCS). In this way, this nonlinearity will take place depending on these constants.

Although the shipment constraint representations of the Forrester and APIOBPCS models are similar, the effects caused by them are very different. In Section 5.5.3, it was found that for the Forrester model this constraint does not cause a significant detriment to the system damping ratio in comparison to the manufacturing constraint. However it causes a decrease in the natural frequency, suggesting that the system will oscillate much more slowly which may affect supply chain resilience.

Using simulation and the parameters suggested by Forrester, the impact of the Forrester shipment constraint on supply chain resilience is illustrated in Figure 6.6. The reader can refer back to Figure 5.20(a), where analysis of ITAE values were made for the linearised model without discontinuities. It is evident that there is a similarity between Figures 6.6 and 5.20(a). The major difference between the results is that for $AI \leq 1$ the system never reaches steady state and for this reason ITAE goes to infinity. This has previously been highlighted in Section 5.4.2.2, page 156.

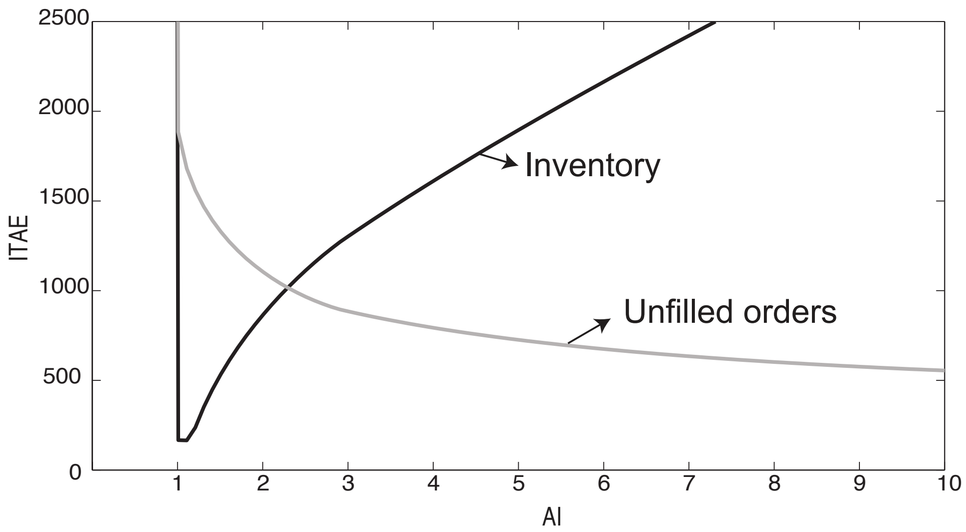


Figure 6.6: Effect of Forrester's shipment constraint on supply chain resilience

Moreover, in the example given in Figure 6.7, the CLIP function is active only for AI values bellow 3. For this reason more significant changes in both inventory and unfilled order ITAE values are observed when AI is between 1 and 3. Figure 6.7 illustrates the step responses of inventory, shipment tried, actual shipment and unfilled orders when AI =3, 2.5, 2 and 1.5. As inventory levels go down, the shipments are constrained and the number of unfilled orders increase substantially. On the other hand, inventory errors given by the vertical displacement become smaller. This CLIP function trades-off inventory and unfilled order performances, and therefore the resilience performance may not be so badly affected when taking both responses equally into consideration.

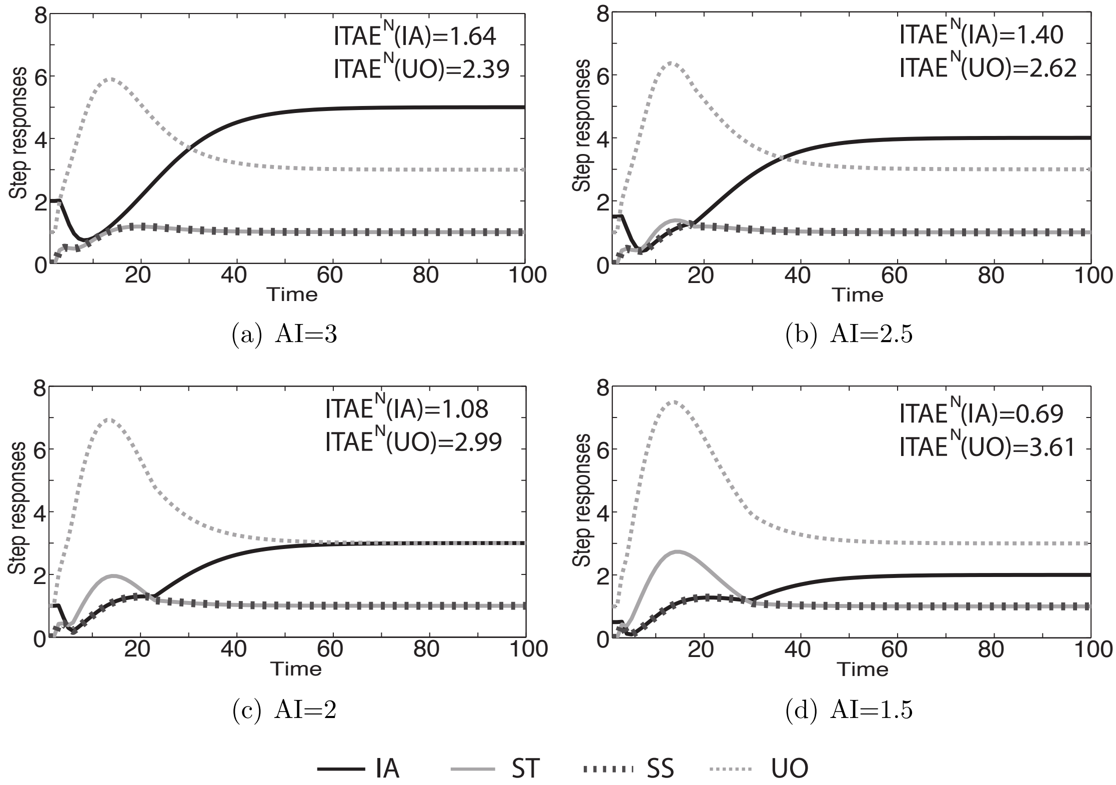


Figure 6.7: Effect of Forrester's shipment constraint on inventory and unfilled order responses

For the APIOBPCS model, Section 5.8.3.2 highlighted the fact that the shipment constraint does not cause any impact on other responses besides the shipment itself. Using simulation and the control parameters setting as $T_a = 6$ and $T_i = T_w = T_p = 8$, Figure 6.8 illustrates the impact of desired inventory constant (DINV) on inventory and shipment ITAE values. It is clear that inventory levels only affect the shipment response confirming the analysis of Section 5.8.3.2.

Figure 6.9 illustrates this phenomenon better by presenting the inventory and shipment responses for different DINVs. Note that the inventory profile does not

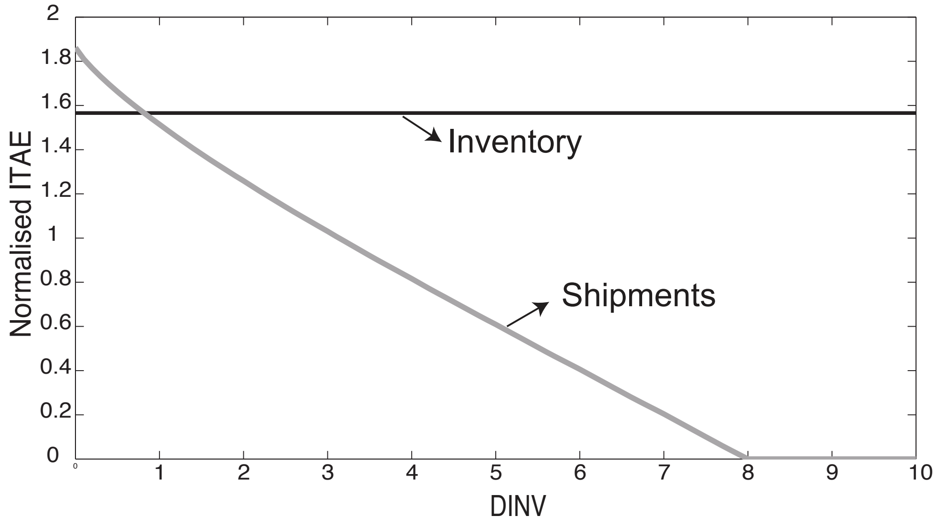
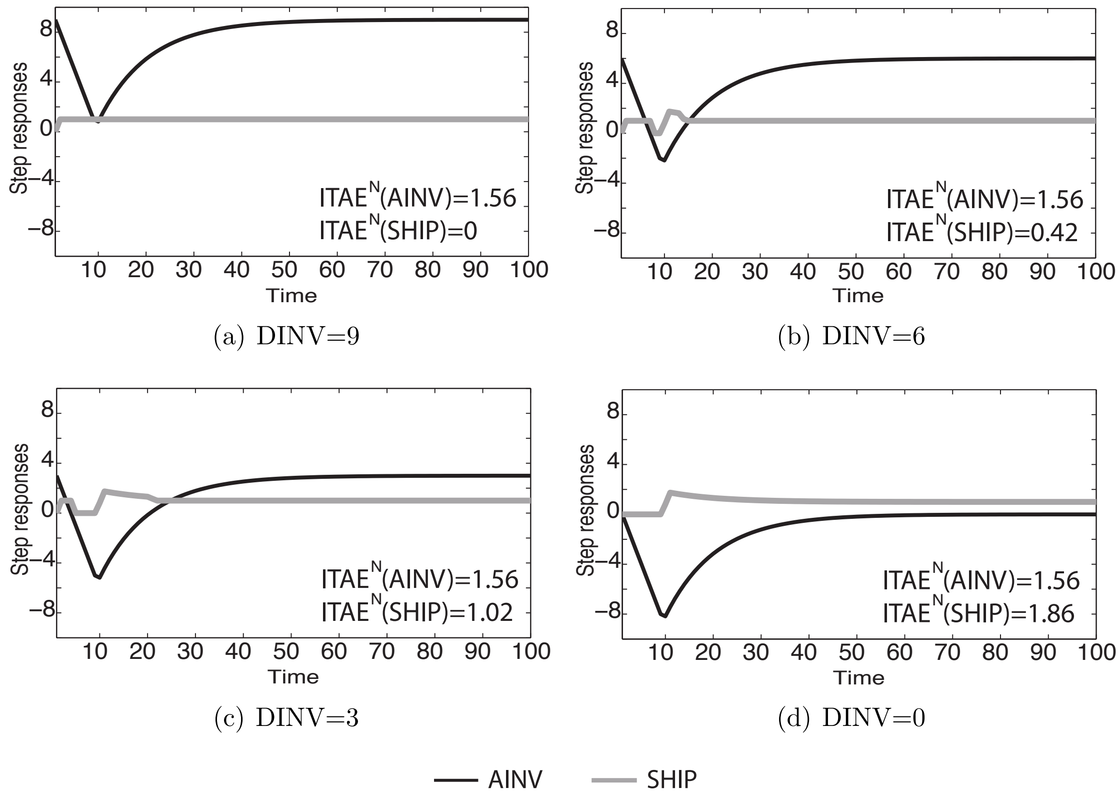


Figure 6.8: Effect of APIOBPCS shipment constraint on supply chain resilience



(a) DINV=9

(b) DINV=6

(c) DINV=3

(d) DINV=0

Figure 6.9: Effect of APIOBPCS shipment constraint on inventory and shipment responses

change as the target inventory goes down. Basically, the inventory response simply shifts the position down but no changes on its vertical displacement and recovery time are observed. However, as inventory crossed the zero line and backlogs start to build up, the shipments profile becomes distorted and different from the demand. Not only does the vertical displacement increase but the time taken to recover target shipments also increases. Hence, supply chain resilience is badly affected by the decrease of inventory levels but only when a backlog situation occurs.

6.3 Trade-off analysis

In this section, a trade-off analysis between supply chain resilience and production and inventory on-costs is undertaken.

6.3.1 Production on-costs versus supply chain resilience

The production on-costs represent the increase of production overheads due to system dynamics and they are functions of the chosen set of parameters and the lead-time. The production on-costs are estimated to be “proportional to the cubic function of the area between the oscillation output [order rate] and the neutral axis” (Stalk and Hout, 1990; Towill *et al.*, 1992). As seen in Chapter 4, this is equivalent to finding the IAE^3 of order rate.

Figure 6.10 illustrates the resilience region for the Forrester model inventory response (IA) when considering different combinations of α and DI control parameter values, which have been investigated individually in the previous sections. The same region is found when investigating the unfilled order responses (UO). The black contour lines correspond to the increase in production overheads due to the system

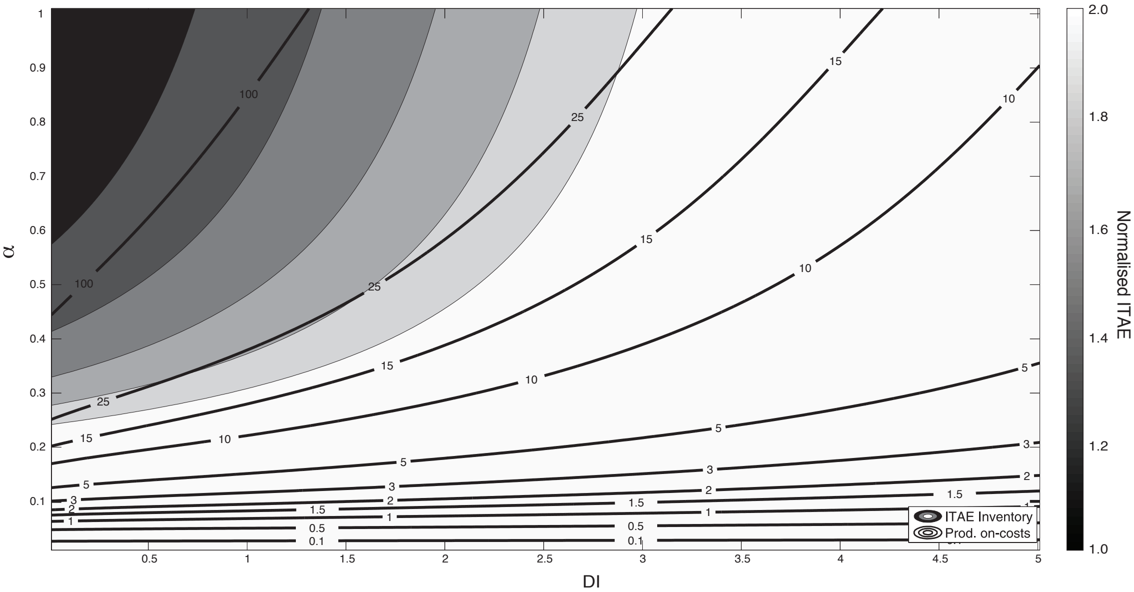


Figure 6.10: Trade-off between resilience and production on-cost in the Forrester model

dynamics. The formulation assumes that if the actual order rate response is equal to demand, the production costs would not be evident, and therefore the increase in production overheads would be equal to zero. Note that when the system is designed to quickly respond and recover its inventory and unfilled order targets, production on-costs will reach their maximum.

In the APIOBPCS model, Figure 6.11 demonstrates that the resilience area does not lie in the region of maximum production on-costs. But it is also not located within the region of lowest production on-costs either. Note that the white areas and where there are no on-costs contour lines in Figure 6.11 , that is where ITAE and on-costs approach infinity, represent the unstable regions of the APIOBPCS model.

Both model outputs represented by Figures 6.10 and 6.11 indicate a trade-off between supply chain resilience and production on-costs. The set of parameters which improve response and recovery time and minimise the deviation from the

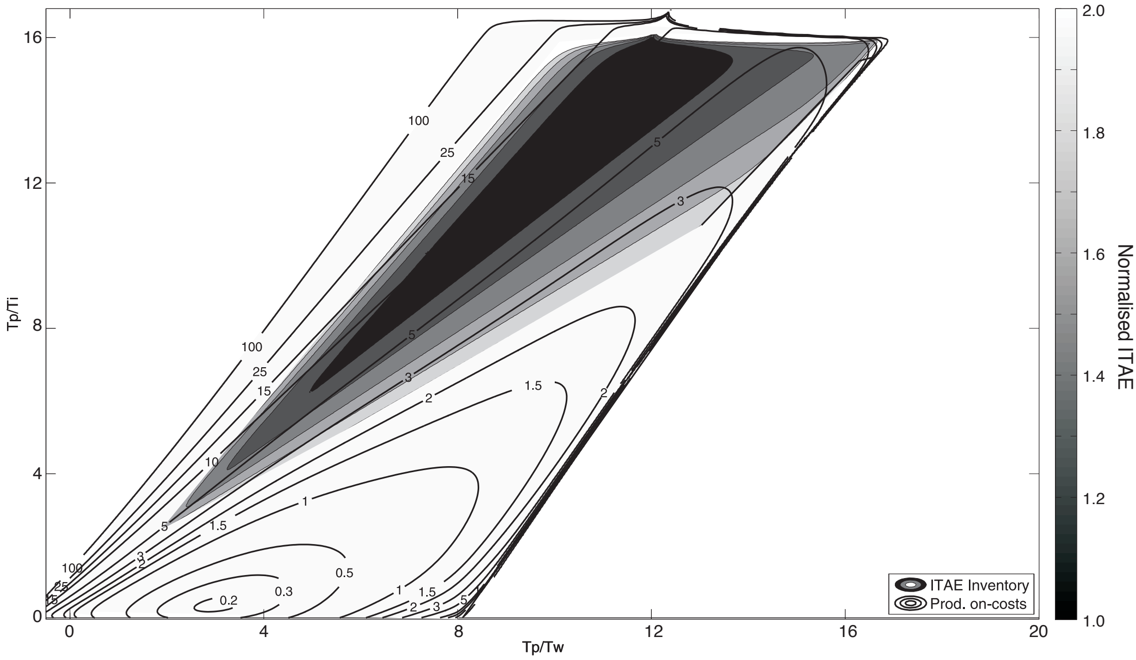


Figure 6.11: Trade-off between resilience and production on-cost in the APIOBPCS model

target inventory imply increased variation in the production schedule. This is consistent with the literature in which authors have described how increased resilience through flexibility and agility would lead to increased operational costs (Christopher and Peck, 2004; Sheffi, 2005b; Sheffi and Rice, 2005). Hence, supply chain managers may want to adjust control parameters depending on how uncertain conditions evolve and how resilient they want to become. While the model and associated simulation do not explicitly measure other related costs such as poor customer service level, vulnerability and possible loss of control due to non-resilience (Christopher and Peck, 2004), they are implicit in the ITAE measure of resilience.

6.3.2 Holding inventory versus supply chain resilience

Here, the costs arising from keeping redundancy such as inventory are discussed. In this section, considering that the inventory holding costs are proportional to the average inventory held during a certain period of time, it is possible to investigate what its impact on resilience is.

In the Forrester model, as target inventory (AI) increases the ITAE of inventory also increases because of its variable target inventory control system which makes the system less responsive as illustrated in Figures 6.6 and 6.7. Thus, the only benefit of increasing inventory levels will be a quicker response and the recovery of unfilled orders. Moreover, independent of the weight given to inventory and unfilled order recovery, there will always be a point at which the increase of inventory does not improve supply chain resilience any further.

In the APIOBPCS model though, the increase of target inventory (DINV) causes no impact on inventory responses (see Figures 6.8 and 6.9). On the other hand, ITAE value of shipments, which is another resilience-related performance, is negatively affected if average inventory levels are kept low. Hence, a trade-off between resilience and inventory can be seen. However, as observed in the Forrester model, there is a point at which increasing inventory levels will only increase costs without enhancing supply chain resilience.

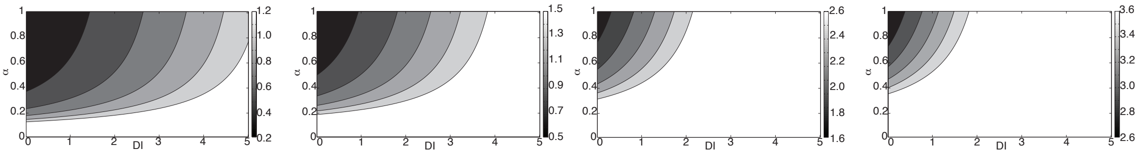
The findings here reported are consistent with previous qualitative research (Christopher and Peck, 2004; Sheffi, 2005b; Sheffi and Rice, 2005), in which supply chain researchers claimed that increased redundancy improves supply chain resilience. However, it has here been analytically and numerically demonstrated that there is a maximum resilience level and increasing redundancy beyond this point will only incur costs and bring no further improvements in service levels assuming supply chain dynamics as a source of risk.

6.4 Sensitivity analysis and robustness

Any supply chain design, involving the selection of control parameters, is based on the assumption of a known and given lead-time. By undertaking a sensitivity analysis, it is possible to check on the robustness of any given supply chain design due to possible changes in lead-time. The lead-time is an important physical parameter that a supply chain designer cannot select or control.

In the Forrester model, many lead-times are considered to represent a production-distribution system. For instance, besides the production delay (DP), there are also the clerical delay (DC), delay due to minimum handling (DH) and delay in unfilled orders (DU). Returning to the simplified model of Figure 5.3(b), it can be seen that both DC and DP cause the same impact on the pipeline since they both affect the lag time of the production orders. In this way, a combination of these delays will be considered when investigating the impact of changes in the pipeline lead-time. On the other hand, DU and DH affect the delay in filling orders (DF), which is a variable delay in the shipment system. Since DU is a delay in the form of a gain and DH is a delay added to the inventory error, the system is certainly more sensitive to changes in DU than in DH. For this reason, changes in DU will be investigated.

In the APIOBPCS model, only the pipeline lead-time (T_p) is considered in the model and changes of this physical parameter will be examined.



(a) $DC + DP = 4$ (b) $DC + DP = 6$ (c) $DC + DP = 10$ (d) $DC + DP = 12$

Figure 6.12: Assessing robustness on inventory responses due to changes in the pipeline lead-time of Forrester's model

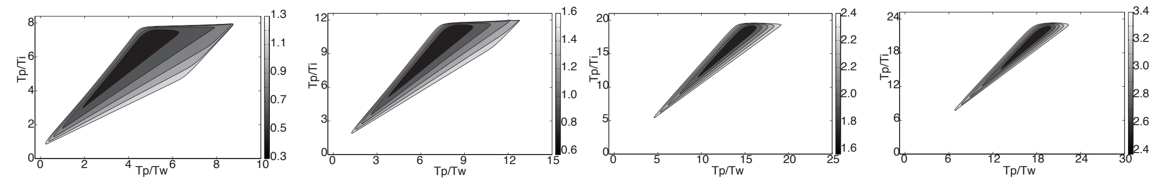
6.4.1 Production/pipeline lead-time uncertainty

Given the nominal scenarios of the Forrester and APIOBPCS models, the impact of ±25% and ±50% changes in lead-time on the system performance is evaluated. As the lead-time increases, not only does the resilience area become smaller but also the minimum ITAE values increase (See Figures 6.12 and 6.13). This means that managers must be careful with their choices of parameters because a change in lead-time can move their inventory response and recovery out of the resilience area.

Tables 6.2 and 6.3 contain the results of the robustness test of the inventory responses. After determining the parameter settings that minimised the ITAE index value ($\alpha = 1$, $DI = 0$ for Forrester and $T_i = 0.54$, $T_w = 0.69$ for APIOBPCS) in the nominal scenario the akin ITAE values of the other scenarios were compared.

The results suggest that when the system is resilient to systems dynamics, it is not robust to uncertainties in lead-time, especially when lead-time increases. With increases of 25% and 50% in the lead-time, the resilience performance would worsen 68% and 161% in the Forrester model and 69% and 176% in the APIOBPCS model, respectively.

In order to determine whether the percentage of changes in the resilience performance is considered high or low, a comparison has been made between these results and non-optimum resilient regions. The design suggested by Forrester (1968) and



(a) $T_p = 4$ (b) $T_p = 6$ (c) $T_p = 10$ (d) $T_p = 12$

Figure 6.13: Assessing robustness on inventory responses due to changes in the pipeline lead-time of the APIOBPCS model

$DC + DP$	ITAE	α	DI	% Change in Performance
4	0.23	1	0	-77%
6	0.53	1	0	-47%
8	1	1	0	NA
10	1.68	1	0	68%
12	2.61	1	0	161%

Table 6.2: Robustness test for inventory responses in the Forrester model

T_p	ITAE	T_p/T_w	T_p/T_i	% Change in Performance
4	0.32	5.83	7.39	-68%
6	0.59	8.75	11.08	-41%
8	1	11.67	14.78	NA
10	1.69	14.58	18.47	69%
12	2.76	17.51	22.17	176%

Table 6.3: Robustness test for inventory responses in the APIOBPCS model

APIOBPCS designs suggested by John *et al.* (1994); Sterman (1989) and Shukla *et al.* (2009)($T_a = 16, 4, 16$; $T_i = 8, 8, 8$; $T_w = 16, 8, 6$; respectively) were considered. It has been found that ± 25% and ±50% changes in lead-time would normally provoke changes of around ±25% and ±50% in the ITAE values. Hence it is found that a less resilient design, which also yields a lower production on-cost, has the advantage of being more robust.

It is valid to emphasise that, since the ITAE penalises long duration errors, when lead-time is increased the ITAE value will significantly increase non-linearly.

In summary, Figures 6.14 and 6.15 demonstrate different regions of parameter settings which correspond to high resilience (Region A), high robustness (Region B) and low production on-costs (Region C). Region D was chosen as a possible trade-off between the three other regions. Figures 6.14 and 6.15 also illustrate system responses to a step change in demand for different lead-times in these regions. In the resilient region A, a quick inventory response and recovery is observed for the nominal scenario in both Forrester and APIOBPCS models. However, as lead-time changes, considerable changes in step response characteristics are observed: the error between target and actual values becomes larger and especially the time to recover inventory increases. In other words, since this parameter setting provides a quick response in inventory, an increase in lead-time provokes overshoots (only in the APIOBPCS) and longer duration errors. For the robust Region B, changes in lead-time do not greatly affect the time of inventory recovery. Nevertheless, with this setting, the system responds and recovers more slowly and is less ready to serve as the trough values are greater. Another observation is that peaks for order rate in the robust Region B are lower, implying that robust systems yield lower production on-cost. In the region where the increase of production overheads due to system dynamics is lower (Region C), the recovery of inventory is even slower. As expected, keeping production orders smooth results in a lack of supply chain resilience. Region D yields a response that is less resilient but more robust to changes in lead-time when compared to Regions A and B. From the inventory and order rate responses, it is observed that when lead-time increases, neither system responses over or undershoots.

In the Forrester model it is also important to examine the unfilled orders when evaluating resilience but the same regions found in Figure 6.14 hold true if considering unfilled orders instead of inventory responses. Figure 6.14 shows that in the resilient Region A, not only is a quick inventory recovery observed, but also a fast recovery and smaller vertical displacement of unfilled orders are observed.

In the APIOBPCS model though, it is important not only to examine the variation in inventory but also the outbound shipment profiles for evaluating the resilience performance. When there is sufficient inventory and shipment capacity, the shipments to the customers will be the same as the demand. On the other hand, when there is stockout and orders are backlogged, the amount of goods delivered to customers will vary over time. The target inventory determines whether orders will be backlogged and, consequently, whether shipments will be disturbed. Considering that the target inventory is equal to zero, after a step change in demand all customer orders will be immediately backlogged. With these considerations, Figure 6.16 illustrates which set of parameters minimise the ITAE on shipments. This figure shows the quicker response and recovery regions (Regions A and E) for shipments together with the robustness region (Region B), the low production on-costs (Region C) and the trade-off Region D. Since Region E does not appear in the inventory result, the plots of both shipments and inventory responses are provided in Figure 6.16 for this region only.

Note that there are two regions of parameter settings that minimise the ITAE values in shipments, Regions A and E. However, Region E corresponds to the unstable region. From the inventory and shipment responses plots, also in Figure 6.16, it is possible to visualise how shipments recover very quickly but at the expense of a steady state error in inventory. In order to evaluate the supply chain resilience in the case of backlog situations both shipment and inventory responses must be taken into account. Hence, the intersection regions between Figures 6.14 and 6.16 should be considered. Advantageously, minimum ITAE values for shipments within

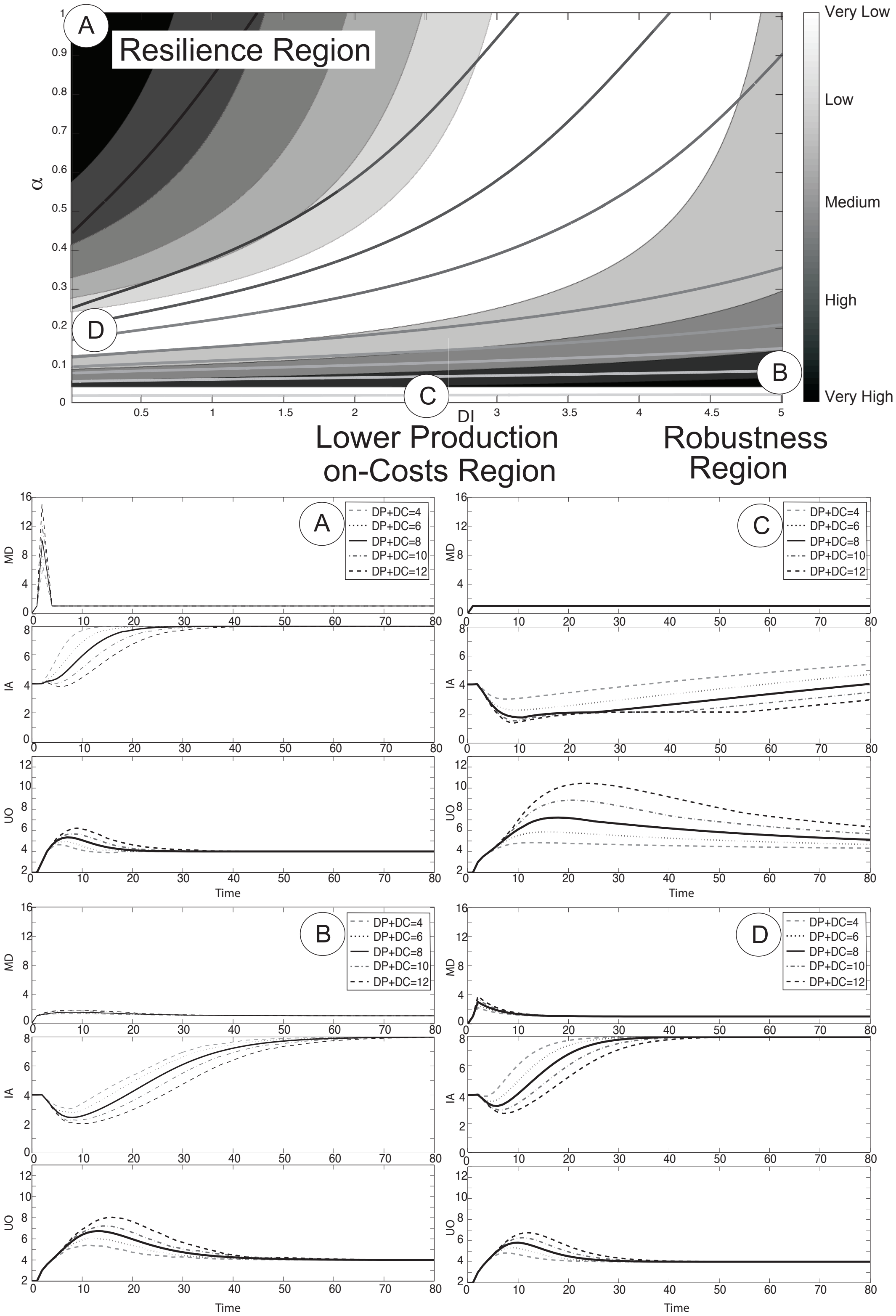


Figure 6.14: Robustness, Resilience and Production on-costs regions for inventory of the Forrester model

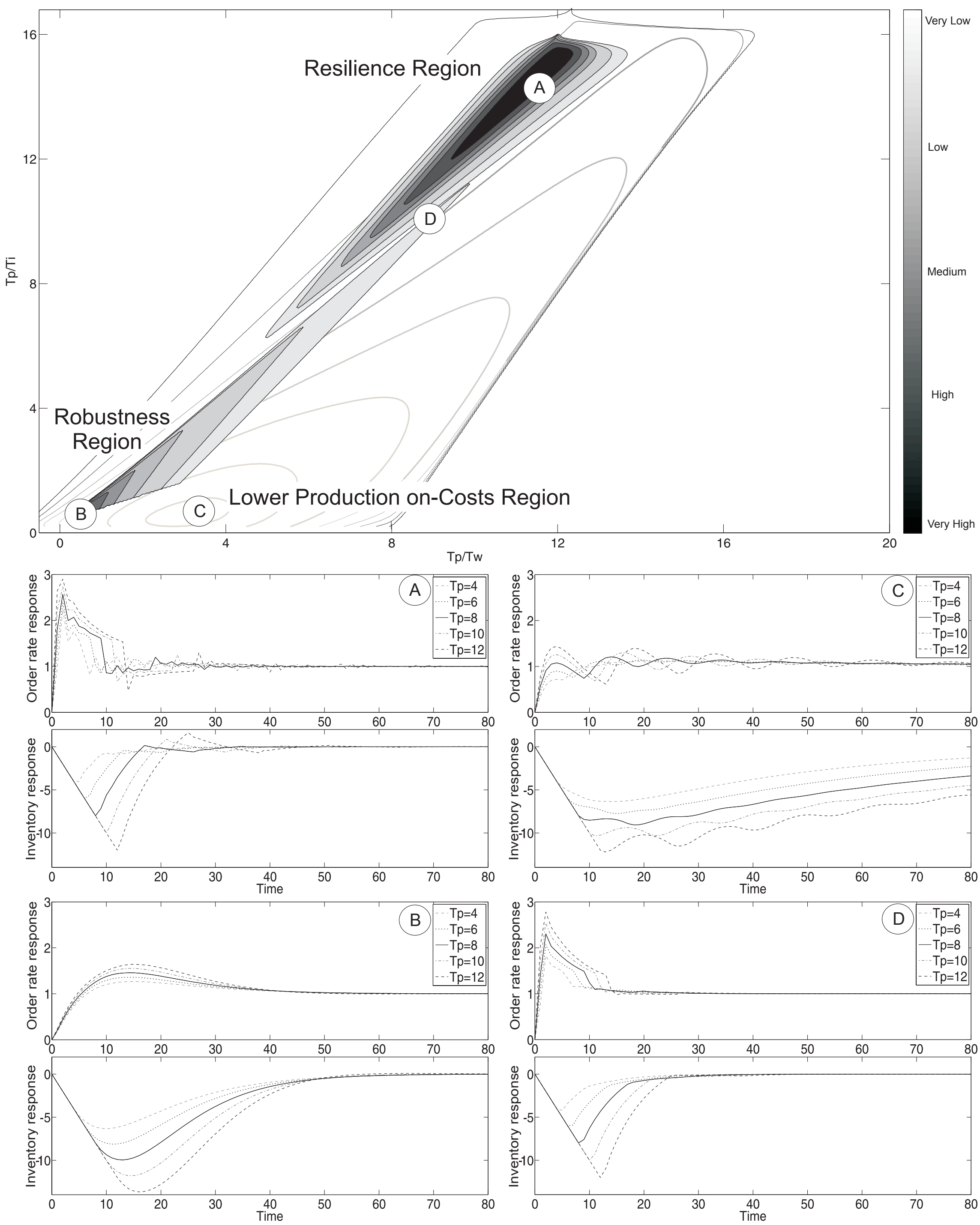


Figure 6.15: Robustness, Resilience and Production on-costs regions for inventory of APIOBPCS

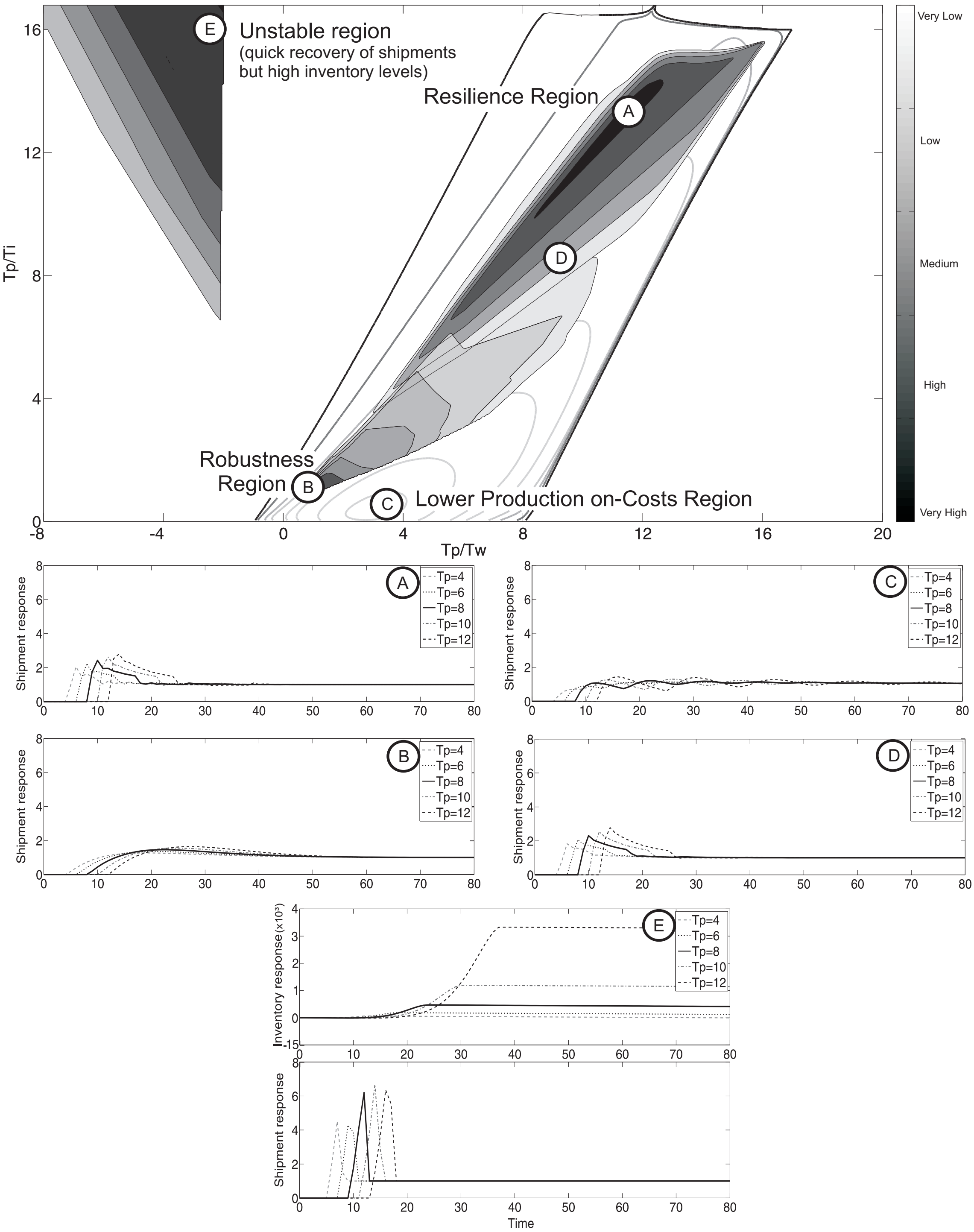


Figure 6.16: Robustness, Resilience and Production on-costs regions for shipments of APIOBPCS when DINV=0

the stable regions are located in similar region as for inventory. As a matter of fact, the responses in Region A provide a quick response and recovery of both inventory and deliveries. The responses illustrate that, since the initial inventory is equal to zero, no deliveries are initially made. At this point, the orders placed to suppliers are increased so as to recover the error in inventory. Later, practically all the materials received are dispatched to the customer, causing peaks of shipments. On the other hand, in the robust Region B and lower production on-cost Region C, the shipments and inventory take longer to recover. However, the smoother response on shipments would imply lower transportation costs. In other words, another trade-off, between transportation costs and resilience, was found.

In summary, five different regions for a supply chain design have been identified. Tables 6.4 and 6.5 summarises the results obtained for both the Forrester and APIOBPCS models as shown in Figures 6.14, 6.15 and 6.16. The tables provide numerical values which illustrate the difference between the different regions. If the supply chain is designed with parameter settings as in Region A, the system response will be highly resilient but also costly and not robust. This is evidenced by a reasonably short time to recover inventory, unfilled orders and shipments and shallow troughs in inventory responses, high percentage changes in order rate, inventory, unfilled orders and shipment performances and high peaks in order rate. The peak in shipments is relatively high as well, but the resilience of the response is guaranteed by short recovery times. This design is recommended for supply chains whose cost of not meeting customer expectations is very high. Also, in order to cope with low robustness the supply chain should make efforts to keep lead-times constant.

Designing the supply chain within Region B will provide robust responses, with low to medium production on-costs but low resilience. This is evidenced by less sensitive responses in the case of a sudden increase in lead-time. However, the recovery times for unfilled order, shipments and inventory are longer than in design Region A and inventory troughs are deeper. This design is recommended for supply

Region	Res.	Rob.	Cost		Sensit. Analysis +25% T_p	Sensit. Analysis +50% T_p	Peaks/ Troughs	Time to recover	Comments
A	Hi	Lo	Hi	O	53%	124%	11.00	4	This system yields quick response and recovery of inventory and unfilled orders. However, this resilience performance is gained at the expense of increased on-cost due to high peak in the order rate. The three responses are very sensitive to change in lead-time; therefore are not robust.
				I	68%	162%	4.00	26	
				U	70%	166%	5.31	23	
B	Lo	Hi	Lo	O	16%	27%	1.49	25	Responses within this region are less sensitive to changes in lead-time which makes this system more robust. The lower order rate peaks reduce costs. On the other hand, longer response and recovery in inventory and unfilled orders increase the risk of disruption.
				I	35%	72%	2.44	60	
				U	37%	79%	6.73	47	
C	Lo	Lo*	Lo	O	0%*	0%*	1.00	0	The system yields a very low resilience performance due to very slow responses in inventory and unfilled orders. These two responses are also relatively sensitive to changes in lead-time. There is no addition in production cost since order rate is equal demand
				I	41%	82%	1.72	470	
				U	43%	87%	7.20	427	
D	Med	Med*	Med	O	37%*	72%*	2.60	12	This is a trade-off region where inventory and unfilled order responses and recoveries are less quick but also less sensitive to changes in T_p when compared with Region A. Medium peaks in order rate demonstrate a compromise in cost as well.
				I	53%	109%	3.73	33	
				U	57%	114%	5.81	30	

O-Order rate response I-Inventory response U-Unfilled order response

*Cost performance is robust since order rates are not sensitive to changes in $DP + DC$.

Table 6.4: Summary of Figure 6.14

Region	Res.	Rob.	Cost		Sensit. Analysis +25% T_p	+50% T_p	Peaks/ Troughs	Time to recover	Comments
A	Hi	Lo	Hi	O	80%	198%	+2.64	72	This system yields quick response and recovery of inventory and shipments. However, this resilience performance is gained at the expense of increased on-cost due to high peak in the order rate. The three responses are very sensitive to change in T_p; therefore are not robust
				I	69%	176%	-8.00	40	
				S	57%	113%	+2.44	40	
B	Lo	Hi	Med	O	16%	33%	+1.48	59	Responses within this region are less sensitive to changes in T_p which makes this system more robust. The lower shipment and order rate peaks reduce costs. On the other hand, longer response and recovery in inventory and shipments and deeper troughs in inventory increase the risk of disruption.
				I	28%	65%	-9.22	84	
				S	29%	62%	+1.78	68	
C	Lo	Lo*	Lo	O	19%*	47%*	+1.27	208	The system yields a very low resilience performance due to very slow responses in inventory and shipments. These two responses are also relatively sensitive to changes in T_p. Low cost is achieved by low peaks and low variability in order rate.
				I	54%	124%	-9.16	527	
				S	36%	85%	+1.23	246	
D	Med	Med*	Med	O	19%*	46%*	+2.23	37	This is a trade-off region where inventory and shipment response and recovery are less quick but also less sensitive to changes in T_p when compared with Region A. Medium peaks in order rate demonstrate a compromise in cost as well.
				I	45%	103%	-8.00	57	
				S	42%	97%	+2.23	44	
E	None	Lo	Hi	O	NA	NA	+69	∞	This is a unstable region as order rate and inventory responses never reach steady state. The shipment response is very quick, but at the expense of high inventory levels and high required shipment capacity.
				I	NA	NA	+470	∞	
				S	47%	106%	+6.10	12	

O-Order rate response I-Inventory response S-Shipment response

*Cost performance is robust since order rates are not sensitive to changes in T_p.

Table 6.5: Summary of Figures 6.15 and 6.16

chains whose lead-times may vary significantly, but these supply chains will be more vulnerable in the case of disruption, especially due to external factors.

Region C is a risky region to be in. Supply chains designed within this set of parameters will not be able to cope with any kind of uncertainty. Demand must be steady, production lead-times must be precise and suppliers must be committed due to the very slow response in inventory and shipments, as the time to recover inventory is near 500 weeks. The advantage of this design is the low production on-costs achieved by low peaks in order rate. Another observation is that, in this region, the order rate response is robust to changes in lead-time. This implies that the production on-cost performance would not change significantly in the case of lead-time increases. However, there is the imminent risk of low customer satisfaction.

The trade-off region, Region D, is perhaps the best design for supply chains willing to perform fairly resiliently against many sources of risks and, at the same time, keeping this resilience performance relatively strong in the case of lead-time changes. Hence, the system has medium robustness and medium resilience. In relation to production on-costs, the supply chain has to compromise as well. However, according to the sensitivity analysis, production on-cost performance is robust against an uncertain lead-time.

Finally, the parameter settings in Region E in the APIOBPCS model are highly undesirable. While shipment recovery is very fast, only 12 weeks, the high peak of +6.1 implies that the supply chain would need high shipment capacity. In addition to this, the supply chain would not maintain a Minimum Reasonable Inventory since the inventory levels never recover from 470 units. For this reason, ITAE values reach infinity meaning that the system has no resilience and the sensitivity analysis is not applicable.

6.4.2 Delay in unfilled order uncertainty

When verifying the impact of changes in delay in unfilled orders (DU) on resilience in the Forrester model it has been found that the system is robust across all sets of parameters (α and DI combination) when considering both inventory and unfilled order responses. Hence, it is expected that with $\pm 25\%$ and $\pm 50\%$ changes in DU, ITAE values will be affected with $\pm 25\%$ and $\pm 50\%$ of changes in their values.

6.5 Summary

In this chapter the repeated simulation technique has been used to further investigate the impact of control policies and nonlinearities on system dynamics, and consequently, on supply chain resilience performance.

Among the system policies here investigated, the inventory controllers play the most important role in achieving supply chain resilience. For both the supply chain models here investigated, the low values of inventory control parameters make responses quicker. When investigating the demand forecasting policy, it has been found that forecast-driven organisations are less resilient than demand-driven ones. Hence, if supply chains replace a levelling strategy with a chase, resilience is improved.

Capacity limitations have also been investigated. In both models, the manufacturing capacities decrease the amplitude of the output responses. In Forrester's model, as the maximum manufacturing capacity decreases or the input amplitude increases, the system becomes slower and the supply chain resilience performance is negatively affected. This shows that when production achieves a maximum capacity constraint the system becomes less resilient. On the other hand, in APIOBPCS model only the minimum capacity constraint has been studied. As the input amplitude increases, the resilience performance may improve or degrade with the presence of the manufacturing capacity depending on the chosen control parameter settings. Hence, minimum capacity constrains in manufacturing are not so critical.

Regarding the shipment capacities, in the Forrester model this nonlinearity provokes an increase in the unfilled orders response and recovery times. In the API-OBPCS model, the nonlinearity in the shipment process does not cause any changes in the system's inventory response. However, when inventories are negative shipments will be distorted and the resilience performance will be negatively affected.

In the trade-off analysis, it has been found that the set of parameters which improve resilience yields increased variation in the production schedule increasing operational costs. Moreover, inventory redundancy has also been identified as a resilience building strategy. However, it was demonstrated that there is a maximum resilience level and increasing redundancy beyond this point will only incur costs and lead to no further improvements in service levels.

Finally, in the sensitivity analysis different design regions have been identified giving the opportunity for supply chain managers to prioritise change programmes according to the supply chain objective.

7 Designing resilient supply chains

"Far better to get an approximate answer to the right question rather than the exact answer to the wrong question."

– John Tukey (1962), *The future of data analysis*

This chapter summarises the insights gained from the research process. More specifically, it brings together the assessment procedure to measure supply chain resilience developed in Chapter 4 and the nonlinear control theory approach to mathematically analyse the behaviour of complex, nonlinear supply chain models, as given in Chapters 5 and 6.

Finally, based on these insights and on previous supply chain design research, a framework to design supply chains resilient to nonlinear system dynamics is proposed.

7.1 Insights gained from the conceptual literature review

One of the main challenges of this research was to develop an assessment framework for supply chain resilience that could be used in the context of system dynamics. In order to create a measurable performance indicator out of a qualitative variable, a comprehensive conceptual literature review has been undertaken, as presented in Chapters 2 and 4. Figure 7.1 illustrates the steps taken for building the performance index. Not only has the organisational resilience concept been explored, but also resilience from the ecological, engineering, physical, psychological and economic perspectives has been taken into account (Sections 2.2-2.6 and 4.1). By using these multiple disciplines, a grounded definition of supply chain resilience was used to discuss the potential supply chain metrics that could be used to represent supply chain resilience (Section 4.2). Finally, a performance index has been suggested and tested (Section 4.3). An evaluation of this index was made by analysing its degree of selectivity and the resulting output responses (Section 4.4). Moreover, throughout the research process the results have always been compared with the supply chain literature information on resilience.

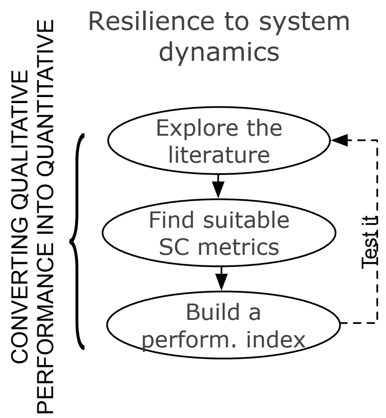


Figure 7.1: Application of conceptual literature review in this research

In summary, a conceptual literature review was used to critically examine the existing literature and to map knowledge in the area of supply chain resilience in order to conceptualise a framework. Many insights were gained when using this approach:

- Areas of controversy in the literature regarding the definition of supply chain resilience, such as confusion between robustness and resilience and use of different supply chain performance metrics to measure it, were identified and addressed by adopting well-established concepts in the natural and social sciences domain (Sections 2.2-2.6).

- The resilience assessment framework was evolved from a combination of previous conceptual frameworks, such as the 'resilient triangle' and control theory performance indices (Section 4.1).

- A debate on the way supply chains target customer satisfaction and on which supply chain performance metric should be used to indicate resilience were raised and addressed (Section 4.2).

- A new supply chain resilience profile or appearance was highlighted. Instead of assuming a triangular shape ('resilient triangle'), performance response and recovery can also assume an oscillatory shape since overshoots may occur when considering the system dynamic behaviour (Section 4.2.3).

- The review of engineering literature revealed the suitability of employing controlled system performance indices to measure supply chain resilience (Sections 4.3-4.4).

7.2 Insights gained from nonlinear control theory

This thesis also conducted an extensive literature search and review on the specific topic of nonlinear control theory, as presented in Chapter 3. To date, simulation techniques have mainly been used to deal with complex, nonlinear supply chain systems. However, this research suggests a more rigourous approach that permits mathematical analysis of nonlinearities (Figure 7.2) as precursor for simulation experiments.

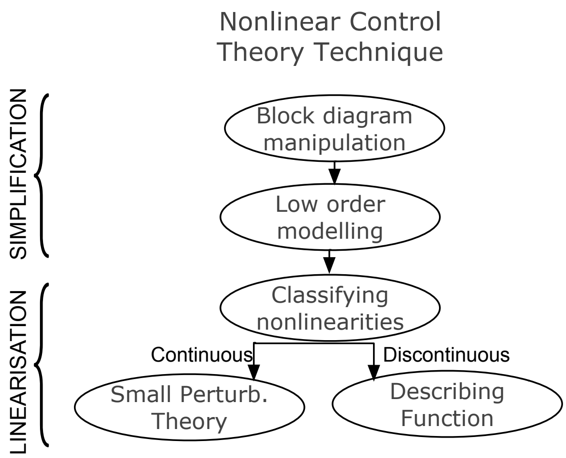


Figure 7.2: Application of nonlinear control theory in this research

Firstly, simplification methods should be used to eliminate unnecessary complexities in the model and reveal the underlying relationship between the variables. Then, some of the linearisation methods presented in Table 3.2 were used to analytically investigate common nonlinearities present in a supply chain system. The choice of each technique was made with regards to its complexity and suitability for different types of nonlinearities.

The use of this approach brought a number of insights to bear on the understanding of the system dynamics behaviour and how each nonlinearity affects responses. Tables 7.1 and 7.2 summarise the analytical insights obtained with the use of nonlinear control theory, the resulting simulation experiments and the implications of not conducting a mathematical analysis before simulation.

In Forrester's model (Table 7.1), all the techniques to analysis nonlinearities (Figure 7.2) have been applied. The simplification techniques, such as block diagram manipulation and low order modelling, have contributed to providing a better visualisation and understanding of the variable interactions in the model. Moreover, by reducing the number of equations and orders, these techniques supported the application of small perturbation theory and describing functions methods. Finally, these linearisation techniques provided further insights since they made possible the calculation of the system transfer functions and local stability boundaries and the understanding of how different capacity constraints impact on the system's resilience behaviour.

In the APIOBPCS model (Table 7.2), similar insights were gained. Since the original model was already simple and no continuous nonlinearities were found, only describing function techniques were applied to understand the impact of capacity constraints on the manufacturing and shipment processes. Transfer functions and stability boundaries were obtained by temporarily deactivating the discontinuous nonlinearities.

7.3 Insights gained from trade-off analysis

This thesis conducted a trade-off analysis in order to determine the implications of designing a resilient supply chain on its cost performance. In chapter 2, it has been pointed out that researchers widely recognised the existence of a trade-off between cost and supply chain resilience (Christopher and Peck, 2004; Sheffi, 2005b; Sheffi and Rice, 2005; Alsop and Armstrong, 2010). This is related to the fact that resilience strategies suggests companies to create flexibility and redundancy through safety stocks, additional suppliers and extra backup sites. Moreover, a complex supply chain network can provide more discounts, excellent service and develop a stronger knowledge of processes (Alsop and Armstrong, 2010).

On the other hand, the fewer and more concentrated the parties, the more likely

	Analytical insights	Resulting simulation experiments	If not carried out
Block diagram manipulation	• Better visualisation and understanding of the model's relevant variables, constants and their relationship.	• System policies were identified and simulations were carried out for the control parameters involved.	• Simulation would have been a slow process since the relationships between variables were not well understood.
	Example 1: It has been identified that no inventory information is fed back to the ordering policy and that demand amplification is not caused by feedback loops. **Example 2:** The only feedback information in the ordering process is due to the manufacturing capacity **Example 3:** It has been identified that both IA and UO should be accounted for assessing supply chain resilience in the Forrester model **Example 4:** It has been identified that in the Forrester model there is a target value for unfilled orders.		
	• Reduction of equations which enabled the application of small perturbation theory and describing functions techniques.	• Simulations were undertaken with original equations to compare results with analytical, simplified and linearised models.	• The linearisation process of the system would be very difficult given the amount of equations.
	Example 1: Eight variables were eliminated from the original Forrester model without compromising the system responses subjected to study.		
	• Better understanding of the delays involved in the shipment receipt	• Sensitivity analysis was carried out to check the impact of increased delays on system's performance	• Unnecessary simulations would have been conducted.
	Example 1: It was found that both DC and DP have the same effect on the shipment receipt delay. Hence there was no need to simulate different values for each parameters separately.		
Low order modelling	• Reduction of order in the pipeline which also contributes to the application of small perturbation theory and describing functions techniques.	• Simulations were undertaken with original equations to compare results with analytical, simplified and linearised models.	• Greater effort would be needed to apply small perturbation theory. High order models may also demotivate the researcher.
	Example 1: Three orders have been eliminated in this process without much loss of accuracy		
Small perturbation theory	• Possibility to find system transfer functions and ITAE estimated equations	• Simulations focused only on important parameters for achieving supply chain resilience	• The understanding of the impact of each control parameter using only simulation would be time-consuming.
	Example 1: The parameter AI was found to be the most important control parameter that defines ITAE values and provokes conflicting impact on inventory and unfilled order responses. Hence it has been investigated in greater depth in the simulation process. **Example 2:** Small values of DI and DR will always benefit resilience. Simulations results also confirmed that without much effort.		
	• It was possible to find local stability boundaries	• Simulations were undertaken only within the pre-determined stability boundaries	• Unnecessary simulations would have been carried out
	Example 1: It has been pre-determined that DR and DI should be positive control parameters in order to reach stability		
Describing functions	• Understanding the impact of the different capacity constraints (manufacturing and shipment) and input amplitudes on system's damping ratio and natural frequency.	• Simulations were undertaken to check whether the analysis were giving correct insights and more effort has been given to check unexpected results.	• The understanding of capacity constraints, especially of shipments, would be very difficult with sole use of simulation.
	Example 1: Analysis showed that manufacturing constraints always provoke negative impact on resilience performance. For this reason, not much effort in the simulation process was needed to confirm this effect. **Example 2:** Analysis showed that shipment constraint worsens the system's natural frequency but slightly improves the damping ratio. Simulations showed that only unfilled order responses are worsened by this capacity constraint but not the inventory responses.		
	• Understanding the impact of different input frequencies on system's behaviour	• Simulations were undertaken only to confirm analytical insights.	• Several simulation experiments would have been necessary to gain the same insights

Table 7.1: Table of insights: Forrester's model

<table>
<tr><th></th><th>Analytical insights</th><th>Resulting simulation experiments</th><th>If not carried out</th></tr>
<tr><td rowspan="4">Transfer function analysis</td><td>• Possibility to find system transfer functions and ITAE estimated equations</td><td>• Simulation process focused on important parameters for achieving supply chain resilience</td><td>• A better understanding of each control parameter's influence on resilience was achieved by using both analytical and simulation techniques.</td></tr>
<tr><td colspan="3">Example 1: The parameters T_i and T_w were found to be important control parameters for resilience. They provoke opposite impacts on ITAE values. Hence they have been investigated more in-depth in the simulation process.
Example 2: Small values of T_a will always benefit resilience. Simulations confirmed that a chase strategy is preferable.</td></tr>
<tr><td>• It was possible to find stability boundaries</td><td>• Initial simulations were undertaken only within the pre-determined stability boundaries</td><td>• Unnecessary simulations would have been carried out</td></tr>
<tr><td colspan="3">Example 1: It has been pre-determined that T_i should be positive and T_w should not be between $-T_p$ and zero</td></tr>
<tr><td rowspan="5">Describing functions</td><td>• Understanding the impact of the different capacity constraints (manufacturing and shipment) and input amplitudes on system's damping ratio and natural frequency.</td><td>• Simulations were undertaken to check whether the analysis gave correct insights and more effort has been given to check unexpected results.</td><td>• The understanding of capacity constraints would be very difficult and some results would have been missed when using only simulation techniques.</td></tr>
<tr><td colspan="3">Example 1: Analysis showed that manufacturing constraint (non-negative order rate) may cause a positive or negative impact on resilience depending on control parameters. This effect may have never been discovered when using only simulation.
Example 2: The shipment constraint does not cause any impact on other system's responses. This effect was easily pointed out by describing function techniques.</td></tr>
<tr><td>• Understanding the impact of different input frequencies on system's behaviour</td><td>• Simulations were undertaken only to confirm analytical insights.</td><td>• Several simulation experiments would have been necessary to gain the same insights</td></tr>
<tr><td>• Predicting limit cycle caused by nonlinearities</td><td>• Simulations have been undertaken within the regions of limit cycle and instability for further investigation.</td><td>• Finding limit cycles would have been a 'trial' and 'error' approach</td></tr>
<tr><td colspan="3">Example 1: Within the limit cycle region, resilience is not achieved since none of the responses recovers.
Example 2: When investigating the unstable region with simulation, it was found that shipment recovery can be minimised at the expense of steady state error in inventory.</td></tr>
</table>

Table 7.2: Table of insights: APIOBPCS model

the supply chain is to suffer from unforeseen events. Hence, the lack of resilience also accounts for costs associated with poor customer service level, vulnerability and possible loss of control (Christopher and Peck, 2004), which are more difficult to assess but are implicit in the ITAE measure of resilience used in this research.

Because many companies are studying ways of overcoming the efficiency-resilience trade-off, this research studied the implication on the transportation, production and inventory on-costs when designing resilient supply chains.

By using an analytical approach this thesis has shown that indeed this trade-off exists and detailed results were provided in Sections 6.3 and 6.4. When designing resilient supply chains the production on-costs, which represent the increase of production overheads due to system dynamics, is increased. This is due to the fact that the set of parameters that triggers fast recovery of resilience-related responses also yields oscillatory and high amplitude behaviour of order rates, in other words, increased bullwhip.

Regarding inventory holding cost, it has been found that, initially, increasing target inventories improves resilience. However, there is a point at which an increase in inventory does not improve supply chain resilience any further. This suggests that if supply chain managers conduct a risk analysis and can predict the impact of any disruptions, only enough a certain maximum level of inventory need be held to overcome such disruptions.

Although transportation costs have not been explicitly modelled in this research, there was an indication of how it would be a effected via the sensitivity analysis undertaken in Section 6.4. Figure 6.16 illustrates a possible trade-off between transportation on-costs and resilience. When backlogs occur, a large quantity of goods needs to be shipped in a short period of time in order to recover inventory targets as rapidly as possible. Hence, transportation costs increase with the need for increased spare capacity or the hiring of third party logistics providers with associated premium freight rates.

In summary, this thesis has not only analytically shown the existence of an efficiency-resilience trade-off, but also provided better insights on how such a trade-off occurs. In this way companies may reflect better on the implication of achieving resilience when making decisions based on their strategic objectives. For example, supply chains may aim to attain high levels of resilience only if the cost related to poor customer service surpasses the increase in operational costs.

7.4 Proposed framework to design resilient supply chains

In this section, a framework to design supply chains resilient to nonlinear system dynamics is proposed. In 1994, Naim and Towill developed a framework that uses system dynamics modelling, analysis and simulation aids in the decision making process to design supply chain systems according to their management objectives. "This methodology is a direct offshoot of the pioneering works of Jay Forrester" (Bechtel and Jayaram, 1997) and it has been advocated, utilised and adapted by many authors (Tibben-Lembke, 1998; Hong-Minh *et al.*, 2000; Kumar and Yamaoka, 2007; Raj and Lakshminarayanan, 2008; Kumar and Nigmatullin, 2011; Bhatti *et al.*, 2012) to design efficient supply chains, re-engineer processes and analyse supply chains' dynamic behaviour.

Based on Naim and Towill's (1994) method, a framework to design resilient supply chains is presented in Figure 7.3. The main difference between Naim and Towill's (1994) framework and the one presented Figure 7.3 is the replacement of linear control theory technique with the nonlinear one. In this method, there are two distinct, but overlapping, phases of analyses. In the qualitative phase, both the objective of the study and the key drivers are identified through an intuitive and conceptual modelling process. Then, the relationships among key drivers are represented in a block diagram. The second phase is the quantitative analysis, which is associated with the development of mathematical and simulation models.

Figure 7.3 also highlights the steps taken in this research and the main contributions to the framework. Although the research in this thesis did not involve real world observation, it has addressed a gap in the supply chain literature by examin-

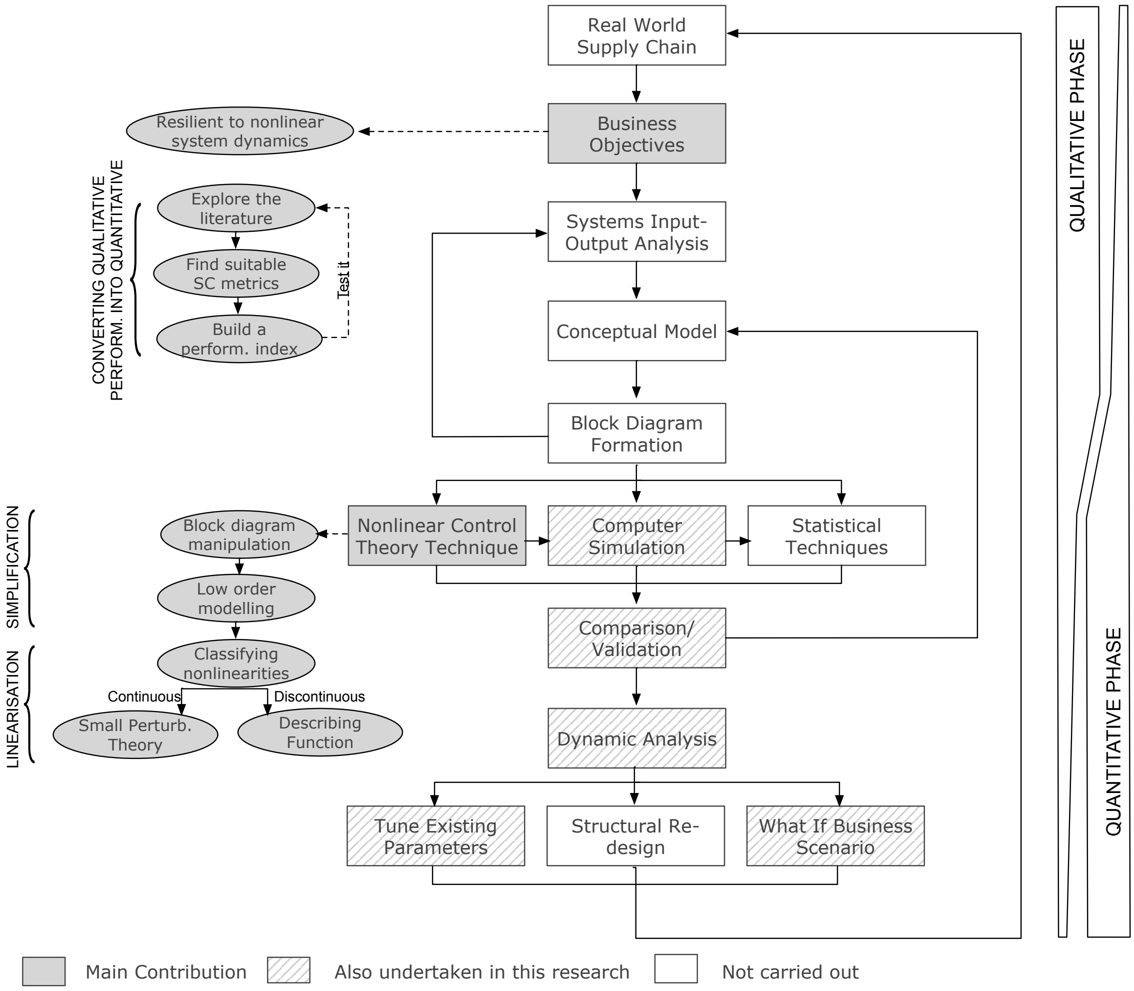


Figure 7.3: Framework for designing resilient supply chains
Extended from: Naim and Towill (1994)

ing a particular business objective: to be resilient to nonlinear system dynamics. This qualitative performance objective was then be converted into a quantitative measures in order to use the proposed framework in Figure 7.3. In addition to this, this thesis has addressed the gap in Naim and Towill's (1994) framework which considered only linear control theory techniques to investigate 'presumably linear' models.

Next, a detailed explanation of the phases involved in the supply chain design process is given.

7.4.1 Qualitative phase

This phase starts by exploring a **particular supply chain system** and defining its boundaries and interfaces. For that, knowing the **business objectives** is very important. Forrester (1961) also indicated that in designing a model of an organisation the elements that must be included arise directly from the questions that are to be answered or objectives that are to be achieved. Moreover, since there is no all-inclusive model, different models should be created to address different questions about the same system and models can be extended or altered so that new objectives are achieved.

Naim and Towill (1994) suggested that four main business objectives can be evaluated using their framework. These are: inventory reduction target, controlled service levels, minimum variance in material flow and minimum total cost of operations and procurement. In this research, a fifth objective has been included: increased supply chain resilience. Moreover, organisations should be aware that there are trade-offs between these objectives and different weighting may be given to each of them.

The resilience term, which has mainly been described in qualitative aspects, was converted into a measurable form by exploring the literature of natural and social sciences. Then, supply chain metrics were chosen to represent this qualitative performance and an index, the ITAE, was found to epitomise the resilience attributes. It was important that, before implementing this newly proposed resilience performance index, tests were made to verify whether this index could provide results consistent with the descriptions in the literature.

The next step is to describe how the material and information flows occur and how the production control is made. This **input-output analysis** will point out any material and information delays, production and logistics constraints, how information is processed and how planning and scheduling operations are carried out. The information obtained from this step supports the development of a suitable **conceptual model**, which can be illustrated in the form of causal loop diagrams, cognitive maps or other appropriate soft system methodology tools (Wilson, 2001). These illustrative diagrams are also reported to help in communicating with the relevant people in the supply chain and extracting more information to refine the model (Naim and Towill, 1994; Hicks, 2004).

Finally, as the operations and control procedures become known, the soft system diagrams can be converted into **block diagram** form. The latter contains mathematical descriptions of the relationships between the various interacting variables in the conceptual model. Each block in the block diagram establishes a relationship by including a mathematical expression that, for example, may represent delays. At this stage, considerable insights into how supply chains work are attained.

In this thesis, two pre-existing models have been used: the Forrester and APIOBPCS models. The former was developed by Forrester (1961) in a slightly different way. His conceptual model was created by establishing the business objectives and by generating a flow diagram for each supply chain echelon. However, Forrester (1961) has never translated his model into block diagram form since the only method advocated by him was computer simulation. Hence, he advanced from a flow diagram to the description of the system equations that need to be inputted in the simulation programme. Wikner *et al.* (1992) translated Forrester's model into block diagram form. APIOBPCS, on the other hand, was developed by John *et al.* (1994) using the same qualitative steps in the framework of Figure 7.3.

In summary, after creating an assessment framework for supply chain resilience, this research utilised the existing formulated block diagrams given by John *et al.* (1994) and Wikner *et al.* (1992) to analyse the two supply chain models and design them to be resilient to nonlinear system dynamics. Both models were suitable for investigating this thesis' research questions and implementing the resilience quantitative framework created in Chapter 4. The only adjustments made were in relation to the supply chain metric used. In Chapter 4, the inventory response was highlighted as a suitable metric to analyse resilience in MTS systems. However, when analysing the models, it has been found that, besides inventory response, unfilled order response in the Forrester model and shipment sent response in the APIOBPCS model should also be accounted for when measuring resilience.

7.4.2 Quantitative phase

According to Naim and Towill (1994), the first step of the quantitative phase is choosing one or more, of three possible techniques for analysing the supply chain: control theory, computer simulation and statistical analysis. In contrast to Naim and Towill's (1994) framework, the present author strongly recommends the sequence of analysis as pointed out in Figure 7.3 whenever possible. This recommendation follows on the author's own experience when conducting this research. The choice of each method may also depend on the degree of complexity involved in the setting up of a mathematical model, the volume of data available for analysis and the analytical skills of the supply chain designer.

Nonlinear control theory

The author firstly recommends the use of nonlinear control theory techniques before undertaking simulation and statistical analysis. This is due to the fundamental insights and understanding that this technique provides, as discussed in section 7.2. Simulation methods have been used to analyse complex, high-order, nonlinear models as an alternative to control theory (Forrester, 1961; Wikner *et al.*, 1991). However, this research has shown that there are many techniques for simplifying and analysing complex nonlinear models.

The first step for the analysis of complex, high-order models is to undertake **simplification**. If the system can be simplified that is when underlying control mechanisms are revealed (Wikner *et al.*, 1992). Moreover, because the simplification process provides a clearer view of the model it also aids in the analysis and synthesis of any nonlinear elements. For this reason it is recommended that simplification is undertaken before solving nonlinearity problems.

There are two techniques to conduct simplification. The first one is **block diagram manipulation**, which is simply the rearrangements of the block diagram obtained from the conceptual model into a reduced form by identifying and eliminating redundancies, collecting constants and moving blocks to create familiar forms (cascade, parallel and feedback) in the model (Nise, 2000). Although this technique may reduce the number of variables and equations, it ensures that no causal relationships between variables are lost. The second technique to simplify models is the **low order modelling**. This method consists of reducing the order of the model by eliminating poles that produce little effect on the system's transient responses and rearranging the remaining poles and zeros so that the maximum accuracy is kept. In this thesis, Hsia (1972) and Matsubara's (1965) methods have been introduced which are still being used in system dynamics research (Jeong *et al.*, 2000; Kuo and Golnaraghi, 2003)

The second step is to analyse the effects of nonlinearities present in the system. In section 3.2.1.4, several methods for the analysis of nonlinear models have been discussed. In particular, the linearisation methods are recommended whenever a solution can be obtained in this way because there is a variety of techniques available in linear systems theory. It has been stated that averaging and best-fit approximations may be utilised to replace nonlinear elements with linear representations (Mohapatra, 1980; Wikner *et al.*, 1992). However, this thesis has shown that accuracy and reliability between linearised and nonlinear responses can be improved when using **small perturbation theory** for continuous nonlinearities and **describing function techniques** for discontinuous nonlinearities.

Computer simulation

After having a better understanding of the system's behaviour and its underlying structures, single or repeated simulations can be carried out to confirm the insights acquired in the previous step and to obtain a more exact result of the system responses. The advantage of simulations is that the original conceptual model can be studied without simplification or linearisation, but from experience gained in this research process it is very hard to gain insights from only simulating complex models. Moreover, previous researchers stated that simulation on its own can be sometimes deceiving (Atherton, 1975; Towill, 1981; Rugh, 2002) as this "guess and check" approach may overlook underlying mechanisms and dynamic behaviour.

Statistical techniques

Finally, statistical techniques can be used to analyse real data if sufficient volume of data is available for the purpose of analysis. Such techniques may involve detrending, smoothing, range analysis, auto- and cross-correlations to identify features in the data, such as degree of scatter, short/long term trends, cyclical variation and exogenous events (Naim and Towill, 1994). In this thesis, this technique has not been applied because of the unavailability of data and the fact that thesis focuses fundamentally on undertaking research on system's design. Moreover, the ITAE index is suitable to evaluate system responses to determinist step inputs.

Comparison and validation of the model normally involves consultation with the interested parties in the supply chain to ensure correctness of the model. Then, real data is inputted from the supply chain system into the model and validation is obtained by comparing the outputs from the model with the output of the real system. In this research, this type of validation was not carried out since the author used pre-existing and well-established supply chain models. Hence, it is assumed that both the works of Forrester (1958) and John *et al.* (1994) had been formerly validated with real data. However, this thesis went through a validation procedure when comparing the simplified and linearised models with the original ones. For each step taken in the simplification and linearisation process, frequency and/or step output responses have been used for comparison and validation.

Following the validation process, the model can be subjected to extensive **dynamic analysis**. The objective of this stage is to determine the dynamic performance of the supply chain by subjecting the model to severe test inputs. In this thesis the supply chain resilience performance, which is given by the minimisation of the ITAE indices of customer service-related responses, was investigated by making a sharp, step change in the customer demand. Moreover, changes in damping ratios and natural frequencies have also been used as an estimation of the resilience performance.

Finally, the supply chain models can be further inspected by changing the control procedure, creating various scenarios and undertaking sensitivity analysis to reveal how vulnerable the supply chain is. For this type of analysis, computer simulation methods can be used for generating results relatively easily and quickly. Naim and Towill (1994) suggest a structured approach to exploit supply chain models:

- **Tuning existing parameters**: supply chains can be redesigned by maintaining the original supply chain structure but varying the control parameters to improve performance. One of the contributions of this thesis was finding the resilience regions of the different parameter settings.

- **Structural redesign**: this involves altering the model's structure, such as removing an echelon or including a feedback information into the control system. In this thesis, this has not been done since the author decided to start with a relatively simple supply chain system. Moreover re-engineering processes, such as the inclusion of new feedback control systems, were beyond the scope of this research.

- **'What if?' business scenarios**: this involves testing how the supply chain would perform for alternative business propositions or unexpected changes in the business scenario. This thesis has tested the impact of expected changes in physical parameters, such as lead-times. This sensitivity analysis was important to determine the regions of the parameter settings where the supply chain would be less vulnerable to changes.

7.5 Summary

This chapter has highlighted the insights gained by combining the conceptual literature review with nonlinear control theory. Moreover, insights gained from trade-off analysis has also been reflected. In the conceptual literature review process, the author critically examined the existing literature and mapped knowledge in the area of supply chain resilience. A qualitative performance indicator, supply chain resilience, has been converted into a quantitative measure. In the same way, nonlinear control theory provided many insights into understanding the system's behaviour and the impact of nonlinearities on system response. Finally, the trade-off analysis enabled the researcher to understand the cost implications of designing supply chains resilient to system dynamics.

This thesis has added to the previous framework (Naim and Towill, 1994) on designing supply chain systems according to the management objectives. Here, optimising supply chain resilience was the main objective and for this reason a suitable resilience performance measure has been developed.

More importantly, this thesis has contributed in providing a systematic procedure for the analysis of the impact of nonlinear control structures on systems behaviour. The previous framework developed by Naim and Towill (1994) and other authors (Forrester, 1958, 1961; Sterman, 1989; Shukla *et al.*, 2009; Poles, 2013) suggested that nonlinearities could be only analysed by undertaking simulation experiments. By adopting nonlinear control theory, this thesis has found more accurate linear approximations for reproducing nonlinear models, enhancing the understanding of the system dynamics and actual transient responses. Moreover, the analytical phase was found to be an important precursor for undertaking simulations.

8 Conclusion

This chapter will relate the findings back to the research questions that emerged from the preliminary investigation for this research and from the literature review process. In addition, the contributions of this research to theory, methodology and practice will be summarised. Finally, the limitations and potential areas for further investigation will be discussed.

8.1 Contribution to theory (RQ:1)

A great interest in supply chain resilience has arisen from the issues of security and risk management at the beginning of the 21^{st} century following major supply chain disruptions caused by political, economical and environmental instability (Sheffi, 2001; Rice and Caniato, 2003; Christopher and Peck, 2004; Spekman and Davis, 2004; Barry, 2004; Thomas and Fritz, 2006; Kovács and Spens, 2007; Alsop and Armstrong, 2010; Jüttner, 2011). In particular, greater research focus has been placed on the strategic planning and positioning of supply chain parties in order to improve responses to major supply chain disruption and disaster relief efforts.

On the other hand, handling the uncertainties which are emerging at the operational level has become very important given the recent trends in the dynamics of market places and the resulting complex supply chain procedures. In global supply chains the longer the distances between parties and the more resources involved the greater the likelihood of operational disruptions (Sheffi, 2005b). Moreover, system dynamics and control policies have been pointed to as a central activity in the management of material and information flows (Mason-Jones and Towill, 1998) and as major sources of supply chain disruption (Colicchia *et al.*, 2010a). Due to the lack of literature regarding this research topic, this thesis has addressed the impact of system dynamics and control policies on supply chain resilience.

Moreover, since there was no consensus on the supply chain management definition of resilience, this research also explored the existing resilience-related literature to establish clearly elucidated performance criteria that encapsulate the attributes of resilience. In summary, the supply chain management theory research questions (RQ:1a-c) formulated at the beginning of this thesis can essentially be answered as follows:

RQ:1a) What are the existing resilience-related definitions in the supply chain literature?

This research question arose out of the initial motivation for this research. Coming from an engineering background, the author had very clear definitions for resilience from a a physical science perspective. However, while consulting the business and more specifically the supply chain literatures, it was found that there are different interpretations for resilience and an interchangeable use of this term with robustness.

In Chapter 2, it was demonstrated that the existing definitions of resilience are often contradictory and confusing. Moreover, initial definitions were extensive and lacked theoretical justification giving only fragmental perspectives of the phenomenon. Examples of the contradictions found would be:

- **Adaptability**: A resilient supply chain may be adaptable, as the desired state in many cases is different from the original one. For instance, Christopher and Peck (2004) described supply chain resilience as the ability of a system not only to return to its original state but to move to a new, more desirable one after being disrupted. This notion of adaptability has not been considered by other supply chain scholars (Sheffi, 2005b; Rice and Caniato, 2003; Tierney and Bruneau, 2007).

- **Resilience versus robustness**: Some authors have used the term robustness to describe any system that accomplishes post-disturbance recovery (Tang, 2006; Asbjørnslett, 2008). Asbjørnslett (2008) declared that what differentiates a robust system from a resilient one is the adaptability characteristic which is only seen in resilient systems. In contrast, Christopher and Rutherford (2004) stated that robustness differs from resilience by having a 'Lean Thinking' strategy while risk management is a key strategy for achieving a resilient supply chain.

- **Measuring resilience**: Although there are many recommended strategies to increase resilience (Rice and Caniato, 2003; Christopher and Peck, 2004; Sheffi, 2005b; McManus *et al.*, 2007; Tomlin, 2006), very few supply chain scholars have attempted to measure it. Some of these works designed models which are more appropriate to evaluating the resilience of individual companies (Datta *et al.*, 2007; Colicchia *et al.*, 2010b; Carvalho, 2011) while others designed frameworks to measure the resilience of the supply chain as a whole. Moreover, several supply chain metrics have been used as surrogates of indicated resilience: inventory levels, supply lead-time, CSL, recovery time, amount of goods in transit, change over time, disruption length and percentage uptime.

In order to justify the need for resilient supply chains, an operational definition of the resilience phenomenon as well as an understanding of the key elements and capabilities that characterise it is needed. In this way, Ponomarov and Holcomb (2009) undertook an extensive review of the literature using multiple disciplines to develop an integrated perspective on resilience. Their outcome was a multidimensional and multidisciplinary definition for supply chain resilience which was described as "the adaptive capability of the supply chain to prepare for unexpected events, respond to disruptions, and recover from them by maintaining continuity of operations at desired levels of connectedness and control over structure and function". This holistic conceptual definition implies an adaptability and has been used in this thesis to build an assessment framework.

To differentiate robustness from resilience, the author relied on the engineering definition given by Dorf and Bishop (1998): a system is robust when the system has acceptable changes in performance due to model or parameter changes and moderate modelling errors.

RQ:1b) How can supply chain resilience be measured in the context of systems dynamics?

In Chapter 4, the holistic concept of resilience given by Ponomarov and Holcomb (2009) has been extended and the conceptual literature on supply chain resilience has been further explored. In summary the steps taken to create an assessment framework for resilience in the context of supply chain dynamics was:

Identifying properties of resilience (Section 4.1)

From the multidimensional and multidisciplinary definition adopted by this research, three properties of resilience have been identified and expanded as follows:

- Readiness: implies whether the supply chain can continue providing goods or services at reasonable cost according to the end customer requirements.
- Response: implies not only reducing average delivery lead-times but also suggests that, in times of uncertainty, supply chains should minimise the time to react to disruptions and begin the recovery stage quickly
- Recovery: implies the return to 'normal' stable or steady state conditions. In the definition, adaptability is implied, and therefore the system's steady state may change after disruption.

Relating resilience properties to other conceptual frameworks (Section 4.1)

In parallel with identifying the three properties of resilience, the author has found four important conceptual frameworks that illustrate how disruptions would affect companies' performance (Sheffi and Rice, 2005; Tierney and Bruneau, 2007; Asbjørnslett, 2008; Zobel and Khansa, 2011). In their illustrative framework of the resilience profile, the three resilience properties (readiness, response and recovery) became more visible and clear: after a disruption the performance decreases but as actions are taken the system's performance will be gradually restored. Tierney and Bruneau (2007) call this loss of functionality from disruption followed by a gradual recovery the 'resilient triangle'. Sheffi and Rice (2005); Tierney and Bruneau (2007); Asbjørnslett (2008); Zobel and Khansa (2011) argue that the area of this triangle should be minimised.

Finding a suitable supply chain performance metric (Section 4.2)

What the works of Sheffi and Rice (2005); Tierney and Bruneau (2007); Zobel and Khansa (2011) did not specify was which supply chain performance metric should be used to measure resilience. By stating that the objective of the supply chain is matching supply with demand, and hence, satisfying customers, it was established in section 4.2 that actual inventory or cover time responses of a MTS supply chain system and order book or delivery lead-time of a MTO system should be considered the performance indicators for resilience.

Adapting previous frameworks to the context of system dynamics (Section 4.2)

A resilience profile and an assessment framework have been constructed based on the works of Sheffi and Rice (2005); Tierney and Bruneau (2007); Asbjørnslett (2008); Zobel and Khansa (2011). However, since a system dynamics approach has been taken, the resilience profile suggested by this thesis considers an oscillatory behaviour of the responses to disruptions instead of a triangular shape.

More importantly, the proposed assessment framework considers that supply chain resilience should be measured at the interface between the supply chain and the end customer because of the former's goal to satisfy the latter.

Determining a composite performance measure (Section 4.3)

The author has brought techniques from control engineering in determining a composite performance measure to assess supply chain resilience. The performance indices IAE, ITAE, ISE and ITSE have been considered as possible measures to design systems with minimum response and recovery times, and vertical displacements. Each of these indices gives a different weight to specific properties of the performance response.

Investigation and choice of performance index (Section 4.4)

Finally, the author used standard transfer functions to represent order rate and inventory responses and to compare and contrast the different performance indices. Selectivity and output responses were the criteria used to determine the most suitable index. In summary, ITAE was the chosen index to measure supply chain resilience in the context of system dynamics. This index facilitates the choice of parameter settings that yield output responses with minimised sustained oscillations and that result in fast response and recovery times.

RQ:1c) How can a supply chain be (re-)designed in order to be resilient against such dynamics?

The framework for measuring supply chain resilience created in Chapter 4 was then implemented in the analytical models of Chapter 5 and in the simulation models of Chapter 6.

In order to undertake a mathematical analysis of Forrester's and the APIOBPCS nonlinear models applying the proposed resilience framework, simplification and linearisation techniques were used. This was necessary because ITAE values can only be analytically estimated if transfer functions are determined. Where an ITAE estimation was not possible due to a change in the initial and final values of the step responses, root locus techniques were utilised to estimate the system's natural frequency and damping ratio. These two properties can be used to estimate the system responses and consequently they provide a qualitative insight into how resilient the supply chain's performance is.

During the analysis process, it has been found that, in both Forrester's (Section 5.5.2 on page 164) and the APIOBPCS (Section 5.8.3.2 on page 191) models, inventory responses are not the only supply chain metric that indicates resilience. In the APIOBPCS model, both inventory (on-hand and backlogged) and shipment responses were considered to evaluate resilience. In Forrester's model, the on-hand inventory and unfilled orders were utilised for the resilience analysis. The shipment response in Forrester's model was not useful because it reflected the minimum value between inventory and a fraction of the unfilled orders. In other words, when inventory levels are high, the ITAE in shipments is small because unfilled orders are low. When inventory levels are low, the ITAE in shipments will also be small because its response will be close to the inventory response. This brings attention to the fact that different supply chain performance metrics may be used depending on how the system is modelled.

With a better understanding of the system's behaviour and with an indication of which parameter settings minimise ITAE values, simulations have been carried out to confirm the insights acquired in the analytical phase and to obtain more exact results of the system responses. Across Sections 6.1-6.2, a detailed analysis on the effect of each system policy and each capacity constraint on the resilience performance was undertaken via simulation. In Section 6.3, trade-off analyses between supply chain resilience, production on-cost and holding inventory levels were undertaken. Trade-off analysis draws attention to the fact that the designer may not achieve all business objectives at the same time. For instance, it has been demonstrated that a system with improved resilience will have increased on-costs. However, it was also demonstrated that although resilience is achieved with increased redundancy, there is a maximum resilience level that a supply chain can reach. Therefore, increasing redundancy beyond this point will only incur costs and no further improvements in service levels. In Section 6.4, a sensitivity analysis was used to compare and contrast the resilient, robust and lower production on-cost regions. Hence, the relative benefits of different designs have been identified, giving the opportunity to supply chains to prioritise change programmes given the cost benefits.

Finally, Chapter 7 provided a framework for the steps necessary to design supply chains resilient to nonlinear system dynamics based on the insights gained during the research process. This framework, which has been adapted and extended from Naim and Towill (1994), starts from observing the real supply chain system to generate conceptual models that can be analysed using nonlinear control theory, computer simulation and statistical techniques. After the validation and dynamics analysis steps, tuning existing parameters, making an structural re-design and using 'what if?' scenarios can be used to (re-)design supply chains according to their business objective: to be resilient to nonlinear system dynamics.

In summary, the answer to this research question involves consideration of the following points:

1. Supply chain dynamics play an important role in resilience due to delays and feedback information in the system. For a given control policy it has been found that the choice of decision parameters affects the degree of resilience and robustness that the system has.

2. This thesis analytically demonstrates the trade-off between production on-costs and supply chain resilience. Three main factors in the ordering policy which resulted in increased resilience but high on-costs have been identified: *a)* decreasing the times to recover inventory (DI for Forrester's and T_i for APIOBPCS) and WIP (T_w in the APIOBPCS only) in the ordering control algorithm; *b)* moving from a levelling strategy to a chase strategy ($DR = 0$ for Forrester's and $T_a = 0$ for APIOBPCS) and *c)* increasing target inventory

levels (AI for Forrester's and $DINV$ for APIOBPCS). Regarding the former, as managers make efforts to recover inventory more quickly and hence achieve resilience, the variation in the order rates will rise leading to increased costs as the supply chain production capacity ramps up and down. On the other hand, by increasing DI, T_i and T_w and having a higher degree of smoothing, there may be some compromise in resilience but this will result in a considerable decrease in production on-costs.

3. In the APIOBPCS model, a trade-off between transportation on-costs and resilience has been identified. When backlogs occur, a large quantity of goods needs to be shipped in a short period of time in order to recover inventory targets as rapidly as possible. Hence, transportation costs increase with the need for increased spare capacities or the hiring of third party logistics providers with associated premium freight rates. In Forrester's model, this effect is not shown since there is a mechanism to delay the orders filling process (DF) and to gradually ship goods to the customer as inventory targets recover.

4. Using engineering definitions and tools for measuring resilience and robustness and applying them to supply chain design, it has been found that these two desired performances are not always achieved simultaneously. In fact a resilient design yields responses that are very sensitive to changes in lead-time. The lower the lead-time, the more resilient the supply chain is. However, any unexpected increases in lead-time will result in considerable deviation from nominal performance.

5. By investigating different control policies, supply chain design has been explored as a mitigation strategy. The literature suggests many designs which

yield decreased production costs and robust system responses. However, no previous system dynamics research on inventory and ordering control systems design that considers supply chain resilience has been found. Hence, this research has filled this gap in the literature.

8.2 Contribution to methodology (RQ:2)

The real world is nonlinear and the existence of such nonlinearities makes the understanding of system dynamics difficult. For this reason, previous work specifically on supply chain dynamics has focused on 'presumably linear' models (Towill, 1982; John *et al.*, 1994; Disney and Towill, 2005; Gaalman and Disney, 2009; Zhou *et al.*, 2010) or has taken a 'trial and error', or experimental, simulation approach (Forrester, 1958, 1961; Sterman, 1989; Shukla *et al.*, 2009; Poles, 2013).

For this reason this research identified and categorised the different types of nonlinearities that commonly appear in supply chain dynamics models in order to suggest suitable analytical methods for investigating each type of nonlinearity. Moreover, simplification techniques have also been used to reduce model complexity and to assist in gaining system dynamics insights. Hence, another outcome of this thesis was the development of a methodological framework to obtain more accurate simplified linear representations of complex nonlinear supply chain models by using nonlinear control theory. The application of these nonlinear control methods also enhanced the understanding of how each nonlinearity affects the system behaviour and transient responses.

The well-known Forrester and APIOBPCS models have been used as benchmark supply chain systems to study nonlinear control structures and to experiment with the application of small perturbation and describing function methods. Performances of the linearised models have been compared with numerical solutions of the original Forrester and APIOBPCS models. Moreover, these techniques have also been compared to simple averaging and best-fit line approximation advocated and applied by the previous research (Cuypers, 1973; Mohapatra, 1980; Wikner *et al.*, 1992; Naim *et al.*, 2012).

A systematic procedure has been provided for the analysis and design of nonlinear supply chain dynamics models. Hence, the supply chain management methodology research questions (RQ:2) can be answered as follows:

RQ:2a) How can we analytically study nonlinear supply chain models?

Until now, simulation methods have been recommended to analyse complex, high-order, nonlinear supply chain models as an alternative to control theory (Forrester, 1961; Wikner *et al.*, 1991; Naim and Towill, 1994; Shukla *et al.*, 2009). However, this research has shown that there are analytical techniques for simplifying and analysing complex nonlinear models.

In Chapter 3, a review of mathematical and simulation methods that can be used to analyse system dynamics models has been provided. The author has extensively searched within the nonlinear control theory literature for suitable methods that can be used in the analysis of nonlinear system dynamics models. Although the literature on nonlinear system dynamics is still emerging and there is still not a well-established and unified theory even in the physical science domain, this thesis brings together much of the existing knowledge and research available on nonlinear systems. Section 3.2.1.4 provides a list of methods recommended to analyse different types of nonlinearities.

In Chapter 5, both Forrester's and APIOBPCS models were used to test some of the proposed methods given in Section 3.2.1.4 (see Table 3.2). The choice of linearisation methods was made since that, after linearisation, linear systems theory techniques can be explored for the system's analysis and design.

A sequence of steps has been followed to analyse each model. In the Forrester model, given the complexity and the high-order of the model, simplification techniques were applied first in order to decrease the number of equations and variables in the model. This procedure helps in revealing the underlying relationship between variables and encourages the supply chain analyst to further explore the models with other nonlinear control theory techniques. For instance, the application of small perturbation theory (Section 5.4.1) would have been too elaborate if the number of equations had not been reduced (the matrix in Equation 5.43 would have been of greater dimensions).

After the simplification process, the analysis of the nonlinear elements of the model starts by identifying and categorising the different types of nonlinearities . As seen in Section 3.2.1.4, nonlinearities can be intentional or inherent, continuous or discontinuous and single- or muti-valued. Small perturbation theory can only be applied to investigate continuous nonlinearities. Then, the Taylor series expansion is used when these continuous nonlinearities are single-valued and the Volterra series are used if they are multi-valued. In the case of Forrester's model, the continuous nonlinearities were single-valued and for this reason the Taylor series expansion was applied. Although describing function technique is suitable to analyse any type of nonlinearity, the author chose this procedure for the analysis of discontinuous nonlinearities only. The reason for this is that this technique is used to analyse systems submitted to sinusoidal or random inputs only. In other words, this technique does not provide a solution in which the system's transfer functions can be calculated after linearisation.

In the APIOBPCS model, simplification is not needed since the model is already third-order. Moreover, continuous nonlinearities does not exist in this model since the signal comparisons are made by summing comparators (Σ) instead of product comparators (Π). In this way, only describing functions have been applied to understand the effect of CLIP functions.

In summary, more accurate simplified linear representations of complex nonlinear supply chain models can be obtained by the use of simplification and linearisation techniques. Moreover, these techniques provide more insights into understanding the systems behaviour and how each nonlinearity affects responses. Figure 7.2 summarises the steps necessary to employ nonlinear control theory for analytically studying nonlinear supply chain models. Finally, when combining the use of nonlinear control theory with computer simulation and statistical techniques, it is possible to properly analyse supply chains' dynamic behaviour, effectively design the supply chain system and re-engineer processes (Figure 7.3).

RQ:2b) How does the presence of nonlinearities impact on supply chain system responses and how is resilience affected?

Forrester's and the APIOBPCS models represent production-inventory control systems in different ways and with different foci. Hence when considering both models, the intention of this research was not to discuss the competence and relevance of each model but to increase the portfolio of supply chain representations and solutions.

Model	Type of Nonlinearity	Application	Impact on SC system response	Impact on SC resilience performance	Sections
Forrester's	Continuous Single-valued	Calculation of DF	• Provokes an abrupt and sharp change in the inventory, unfilled orders and shipment responses depending on the inventory constant AI and the ratio between desired and actual inventories ($^{ID}/_{IA}$). • Does not affect manufacturing orders. (See Figure 5.10 to compare the original nonlinear response with linearised ones)	• Does not affect the resilience performance because it causes opposite impacts on inventory and unfilled orders. • When $^{ID}/_{IA}$ increases, inventory recovery is fast, but DF increases and the number of unfilled orders increases (Figures 5.10) • When AI decreases, ITAE of inventory decreases but ITAE of unfilled orders increases (Figures 5.20(a) or 6.6).	5.4.1 5.5.1 5.5.2
Forrester's	Discontinuous Single-valued	Manufact. capacity (AL)	• Decreases the amplitude and mean of the output MD (Figure 5.13) • Decreases natural frequency, increases damping ratio of responses (Table 5.1).	• Decreases the response and recovery time of both UO and IA, therefore has a significant negative impact on resilience (Figure 6.4).	5.4.2.1 5.5.3 6.2.1
Forrester's	Discontinuous Multi-valued	Shipment capacity	Frequency-dependent output (Figure 5.15) • Decreases the amplitude and phase of the output SS (Figure 5.16) • Decreases natural frequency but decreases damping ratio of responses (Table 5.1).	• Improves the response and recovery time of inventory but makes unfilled orders responses much worse (Figure 6.7). • Negatively affects resilience but its impact is less harmful than the one caused by low manufacturing capacity.	5.4.2.2 5.5.3 6.2.2
APIOBPCS	Discontinuous Single-valued	Manufact. capacity (Non-negativity)	• Decreases the amplitude and increases mean of the output ORATE (Figure 5.24) • Causes limit cycles on responses depending on parameters choice (Figure 5.31) • Decreases natural frequency but may increase or decrease damping ratio of responses depending on parameters (Figure 5.32).	• May improve or worsen the resilience performance depending on the parameters choice (Figure 6.5)	5.7.1.1 5.8.3 6.2.1
APIOBPCS	Discontinuous Multi-valued	Shipment capacity	• Frequency-dependent output (Figure 5.25) • Causes no effect in the system characteristic equation (Equation 5.95). Hence, no impact on inventory and order rate responses	• Impact on shipment rate (Equation 5.97), which becomes different from demand. Hence resilience is worsened (Figure 5.33)	5.7.1.2 5.8.3 6.2.2

Table 8.1: Summary answer to research question RQ:2b)

Different types of nonlinearities that commonly appear in supply chain systems and their impact on the system responses and resilience performance have been studied. Table 8.1 summarises the research findings and indicates the sections where each issue is addressed. For instance in the Forrester model, a continuous and single-valued nonlinearity was employed to represent a variable delay in the system of filling orders (DF). This type of nonlinearity causes an abrupt and sharp change in the responses of the inventory and shipment processes. This change is triggered by the values of AI and especially by the ratio ${}^{ID}/_{IA}$ when it is different from 1.

Capacity limitations are normally represented by CLIP functions, which cap the output to a minimum or maximum value. Hence these nonlinearities are normally discontinuous. In both models, the manufacturing capacities, represented by a maximum value of AL in Forrester's and a minimum of zero (non-negative order rate) in APIOBPCS, are single-valued nonlinearities. Both of these nonlinearities decrease the amplitude of the output responses but because one nonlinearity caps to a maximum value and the other caps to a minimum value, they produce different impacts on the mean of the output.

In the Forrester model, as the manufacturing capacity or the input amplitude increases, the system's natural frequency falls and the amplitude ratio becomes larger. This means that the system becomes slower and the supply chain resilience performance is negatively affected. On the other hand, in the APIOBPCS model as the input amplitude increases, natural frequency decreases but the effect on the damping ratio depends on the control parameters. Hence, the resilience performance may improve or degrade if order rates reach zero. Therefore, the effects of this capacity constraint on system responses are more controllable by the supply designer.

Regarding the shipment capacities, both nonlinearities in Forrester's and API-

OBPCS are multi-valued since the capacity constraint is not a fixed value but a dynamic one. They both depend on the input frequency. For Forrester's model, this nonlinearity decrease the amplitude and phase of the output SS as input amplitude increases. Moreover it changes the system's natural frequency and damping ratio, which consequently degrades resilience. For APIOBPCS, the nonlinearity in the shipment procedure does not cause any changes in the system's characteristic equation. The only usage of this nonlinearity is to represent the shipments when backlog occurs. Hence, when inventories are negative, the resilience performance is negatively affected.

8.3 Contribution to practice and industrial relevance

The resilience assessment framework developed in this study has been shown to be useful for supply chain analysts in designing supply chains that are more resilient to nonlinear system dynamics. Given the fact that the resilience performance trades-off with production, inventory and even transportation on-costs and this performance is very sensitive to changes in lead-times, companies may consider adjusting the control parameters to the resilience 'mode' only when resilience is needed or in times of high uncertainties. Hence, supply chain managers can prioritise change programmes that will deliver resilience or cost benefits.

Regarding capacity management, this study has shown that if supply chains want to invest in additional capacities in order to become more resilient, they should definitely invest in the manufacturing processes. Manufacturing constraints can considerably limit the recovery of resilience-related output responses. For this reason, flexibility in the manufacturing system is recommended although idle capacity costs may arise in times of low demand. Warehousing and shipment capacities are not as important as the manufacturing capacity for the resilience performance, but they should also be carefully considered. It has been shown that the supply chain designers do not have to be concerned about shipment constraints, which are normally determined by the available stock, when demand has medium to high frequencies and low amplitudes. However, if backlogs occur, shipment capacity should be able to accommodate the large quantity of goods that needs to be shipped when they are finally produced. Hence, transportation costs increase with the need for increased spare capacities or the hiring of third party logistics providers with associated premium freight rates.

This work brings awareness of the complex task of supply chain design in satisfying potentially conflicting desired performances. In particular, supply chains with high uncertainty in lead-times, such as may be found in the electronics sector (Berry *et al.*, 1998), need to trade-off robustness and cost-effectiveness against resilience. Companies with certain but long lead-times, such as some in the construction sector (Berry *et al.*, 1998), may have some difficulties in being resilient because of long lead-times but can design their system to respond quickly to changes in demand at the expense of increased operational costs. Those companies in an enviable position of having short and consistent lead-times, for instance the grocery industry (Fernie *et al.*, 2000), can more easily design a resilient system with only some compromise in increased costs.

To sum up, this study highlights several practical implications a supply chain designer has to consider before developing an inventory and production control system that is more resilient. Furthermore, the methodology for analysing nonlinearities in a real-world system suggested in this research can be used by supply chain designers to gain more insights into nonlinear systems without going through a time-consuming simulation process.

8.3.1 Recommendations for enhancing supply chain resilience

In summary, based on the research findings, but with the need to take due consideration of the cost-resilience trade-offs inherent in a practical setting, the following recommendations are suggested for supply chains seeking to be more resilient to disruptions caused by nonlinear system dynamics.

1. Follow a chase strategy or demand matching approach. In order words, supply chain managers should adjust capacity to match the demand pattern. This can be accomplished through hires and layoffs, overtime, extra shifts, outsourcing or subcontracting.

2. Similar to the previous recommendation, another suggestion is to keep spare production capacity. This study demonstrated that if production levels achieve maximum capacity, the resilience performance is deeply degraded.

3. Adjust inventory and work in process control parameters in order to decrease inventory recovery time. Instead of having a smooth inventory recovery, managers are recommended to accelerate the inventory recovery process by decreasing its time constants set in the production planning and inventory control system. However, the decrease of such parameters should be carefully considered and monitored in order to avoid system instability and chaotic behaviour.

4. Increase inventory redundancy by increasing target inventories. However, the new inventory target should be carefully designed in order to avoid unnecessary redundancies since that resilience may achieve a maximum level for particular input's amplitudes and frequencies.

5. Decrease lead-times whenever possible. This study demonstrated that the shorter the lead-time, the more resilient the supply chain is.

6. Keep spare transportation capacity. This can be achieved by guaranteeing fleet capacity (either holding it or subcontracting it). Moreover, in line with the previous recommendation, avoiding transport delays is also crucial.

8.4 Limitations and future research opportunities

This research is limited to the dynamics of single-echelon supply chain systems. Further research, motivated by the analytical research and due consideration of the literature review, could include:

1. Exploring different inventory policies in order to associate an inventory cost with supply chain resilience. In this way, it will be possible to recommend appropriate inventory policies for different supply chain requirements.

2. Exploring resilience from a dyadic to a multi-echelon supply chain perspective in order to determine alternative collaborative strategies, including altruistic behaviour, in sharing capacity across the supply chain.

3. Extending the developed resilience performance measure to MTO supply chain models and verifying how lead-time errors, response and recovery can be minimised.

4. Taking the latter two points together, extending the model to multiple echelons and investigating resilience of MTS-MTO decoupled supply chains.

5. Extending the model for stochastic disturbances and evaluating the impact of multi-event disruptions.

6. Investigating the effect of other contingency and mitigation strategies as highlighted in Figure 2.4. For instance, demand management, early sensing, multiple suppliers and vulnerability management strategies have not been explored by the literature, as illustrated in Table 2.2.

7. A combination of two or more sources of risk can be also investigated. For instance, combining supply chain dynamics with uncertain supply, demand, mismanagement of processes and unstable environmental, political and economic environments (Figure 2.4 and Table 2.2).

Moreover, this research lacks the use of real-word data to conduct statistical analyses which are advocated in the framework of Figure 7.3. This is because the ITAE index used would not be suitable to evaluate resilience. In order to use this statistical technique further research on measuring supply chain resilience for other demand patterns is suggested.

Regarding methodological constraints, this research has focused on the analysis of each nonlinearity individually. This means that at no point has more than one nonlinearity been considered active. However, simulations with all nonlinearities activated have been conducted and the results confirmed that the insights gained with the mathematical analysis of individual nonlinearities are consistent with the numerical results.

8.5 Summary

This chapter has brought the thesis to a close, by highlighting the overall findings and the contributions made to the supply chain theory, methodology and industrial practice. The limitations of this research due to the methods adopted and time constraints have also been discussed, along with future research opportunities.

Overall, this research has explored the existing resilience-related literature to establish clearly elucidated performance criteria that encapsulate the attributes of resilience. This thesis has addressed the impact of system dynamics and control policies on supply chain resilience in order to propose a design framework for companies willing to be more resilient to these sources of risk. Moreover, nonlinearities have not been disregarded since capacity constraints play an important role in supply chains' ability to respond to and recover from disruptions. Hence, the impact of some of the most common nonlinearities present in supply chains on resilience has been investigated.

References

Aastrup, J. and Halldórsson, A., 2008. Epistemological role of case studies in logistics: A critical realist perspective. *International Journal of Physical Distribution & Logistics Management*, 38 (10), 746–763.

Adams, G.R. and Schavaneveldt, J.D., 1991. *Understanding Research Methods*. 2nd London: Longman.

Aggelogiannaki, E., Doganis, P. and Sarimveis, H., 2008. An adaptive model predictive control configuration for production-inventory systems. *International Journal of Production Economics*, 114 (1), 165–178.

Akkermans, H. and Dellaert, N., 2005. The rediscovery of industrial dynamics: the contribution of system dynamics to supply chain management in a dynamic and fragmented world. *System Dynamics Review*, 21, 173–186.

Allen, T.T., 2011. *Introduction to Discrete Event Simulation and Agent-based Modeling*. New York, USA: Springer.

Alsop, R. and Armstrong, D., 2010. Resilient supply chain in time of uncertainty. *The economist*, May, http://www.m2mevolution.com/conference/miami-13/presentations/oracle_supply_chain_web.pdf [Accessed 7 FEB 2013], 1–12.

Arbnor, I. and Bjerke, B., 1997. *Methodology for Creating Business Knowledge*. Newbury Park, CA.: Sage.

Arlbjørn, J. and Halldórsson, A., 2002. Logistics Knowledge Creation: Reflections on Content, Processes and Context. *International Journal of Physical Distribution & Logistics Management*, 31 (1), 22–40.

Asbjørnslett, B.E., 2008. Assessing the Vulnerability of Supply Chains. *In*: G.A. Zsidisin and B. Ritchie, eds. *Supply Chain Risk: A Handbook of Assessment, Management and Performance*. New York, NY: Springer.

Ashton, D., 1998. Systematic risk and empirical research. *Journal of Business Finance and Accounting*, 25 (9/10), 1325–1356.

Atherton, D.P., 1975. *Nonlinear Control Engineering*. 1st New York: International Offices.

Bailey, K.D., 1994. *Methods of social research*. 4th New York: The Free Press.

Barry, J., 2004. Supply chain risk in an uncertain global supply chain environment. *International Journal of Physical Distribution & Logistics Management*, 34 (9), 695–697.

Bechtel, C. and Jayaram, J., 1997. Supply chain management: a strategic perpective. *International Journal of Logistics Management*, 8 (1), 15–34.

Benton, T., 1977. *The philosophical foundation of the three sociologies*. London: Routledge.

Benton, T. and Craib, I., 2001. *Philosophy of social science*. 1st New York: Palgrave.

Berry, D., Evans, G.N. and Naim, M.M., 1998. Pipeline Information Survey: a UK Perspective. *Omega*, 26 (1), 115–131.

Berry, D. and Naim, M.M., 1996. Quantifying the Relative Improvements of Redesign Strategies in a Personal Computer Supply Chain. *International Journal of Production Economics*, 46-47 (181-196), 131–153.

Berry, D. and Towill, D.R., 1992. Material Flow in Electronic Product Based Supply Chains. *International journal of Logistics Management*, 3 (2), 77–94.

Bhamra, R., Dani, S. and Burnard, K., 2011. Resilience: the concept, a literature review and future directions. *International Journal of Production Research*, 49 (18), 5375–5393.

Bhaskar, R., 1975. *A Realist Theory of Science*. 1st London: Verso.

Bhatti, R.S., Kumar, P. and Kumar, D., 2012. Evolution of system dynamics in supply chain management. *International Journal Indian Culture and Business Management*, 5 (1), 1–14.

Bicheno, J., Holweg, M. and Niessmann, J., 2001. Constraint batch sizing in a lean environment. *International Journal of Production Economics*, 73 (1), 41–49.

Blanche, M.T., Durrheim, K. and Painter, D., 2007. *Research in Practice: Applied Methods for the Social Sciences*. 2nd Cape Town, South Africa: University of Cape Town Press.

Blumenfeld, D.E., Burns, L.D. and Diltz, J.D., 1985. Analyzing trade-offs between transportation, inventory and production costs on freight networks. *Transportation Research-B*, 5 (5), 361–380.

Boin, A., Kelle, P. and Clay Whybark, D., 2010. Resilient supply chains for extreme situations: Outlining a new field of study. *International Journal of Production Economics*, 126 (1), 1–6.

Bowen, J.T. and Sparks, B.A., 1998. Hospitality marketing research: A content analysis and implications for future research. *International Journal of Hospitality Management*, 17 (2), 125–144.

Bryman, A. and Bell, E., 2007. *Business Research Methods*. 2nd New York: Oxford University Press.

Burbidge, J.L., 1961. The new approach to production. *Production Engineer*, 40 (12), 769–84.

Burgess, K., Singh, P.J. and Koroglu, R., 2006. Supply chain management: a structured literature review and implications for future research. *International Journal of Operations & Production Management*, 26 (7), 703–729.

Burns, J.F. and Sivazlian, B.D., 1978. Dynamic analysis of multi-echelon Supply systems. *Computer & Industrial Engineering*, 2, 181–193.

Burrell, G. and Morgan, G., 1979. *Sociological Paradigms and Organisational Analysis*. London: Heinemann Books.

Callioni, G., de Montgros, X., Slagmulder, R., Wassenhove, L.N.V. and Wright, L., 2005. Inventory-Driven Costs. *Harvard Business Review*, March, 1–8.

Cameron, S. and Price, D., 2009. *Business Research Methods*. London: Chartered Institute of Personnel and Development.

Cannella, S., Ciancimino, E. and Márquez, A.C., 2008. Capacity constrained supply chains: a simulation study. *International Journal of Simulation and Process Modelling*, 4 (2), 139–147.

Carvalho, H., 2011. Resilience Index: Proposal and Application in the Automotive Supply Chain. *Proceedings of the 18th EUROMA Conference, Cambridge, UK, 3-6 July*.

Carvalho, H. and Cruz-Machado, V., 2011. Integrating Lean, Agile, Resilience and Green Paradigms in Supply Chain Management. May, http://www.intechopen.com/books/supply-chainmanagement/integrating-lean-agile-resilience-and-green-paradigms-in-supply-chain-manageme [Accessed 21 MAR 2014].

Cavinato, J.L., 1992. Total cost value model for supply chain competitiveness. *Journal of Business Logistics*, 13 (2), 258–291.

Celly, K.S. and Frazier, G.L., 1996. Outcome-based and behavior-based coordination efforts in channel relationships. *Journal of Marketing Research*, 33 (2), 200–210.

Chen, F., Drezner, Z., Ryan, J.K. and Simchi-Levi, D., 2000. Quantifying the Bullwhip Effect in a Simple Supply Chain: The Impact of Forecasting, Lead Times, and Information. *Management Science*, 46 (3), 436–443.

Chow, K.V. and Denning, K.C., 1994. On variance and lower partial movement betas: the equivalence of systematic risk measures. *Journal of Business Finance and Accounting*, 21 (2), 231–241.

Christopher, M. and Peck, H., 2004. Building the Resilient Supply Chain. *International Journal of Logistics Management*, 15 (2), 1–14.

Christopher, M. and Rutherford, C., 2004. Creating Supply Chain Resilience through Agile Six Sigma. *CriticalEYE Publications*, June-August, http://martin-christopher.info/wp-content/uploads/2009/12/critical_eye.pdf [Accessed 1 AUG 2010].

Christopher, M.L., 1992. *Logistics and Supply Chain Management*. London: Pitman.

Colicchia, C., Creazza, A. and Dallari, F., 2010a. Supply chain risk management: insights from an explorative study in the Italian industry. *Logistics Research Network Annual Conference proceedings, Leeds, UK, 8-10 September*.

Colicchia, C., Dallari, F. and Melacini, M., 2010b. Increasing supply chain resilience in a global sourcing context. *Production Planning & Control: The Management of Operations*, 21 (7), 680–694.

Collins English Dictionaries, 1995. *Dictionary and Thesaurus*. 2nd Glasgow: HarperCollins.

Comte, A., 1865. *A general view of positivism*. London, UK: Trübner and Co.

Cook, P.A., 1986. *Nonlinear Dynamical Systems*. Exeter, UK: Prentice.

Cooper, M.C., Lambert, D.M. and Pagh, J.D., 1997. Supply Chain Management: More Than a New Name for Logistics. *The International Journal of Logistics Management*, 8 (1), 1–14.

Coyle, R.G., 1977. *Management System Dynamics*. Chichester: Wiley.

Craighead, C.W., Blackhurst, J., Rungtusanatham, M.J. and Handfield, R.B., 2007. The Severity of Supply Chain Disruptions: Design Characteristics and Mitigation Capabilities. *Decision Sciences*, 38 (1), 131–155.

Cross, R., Grinfeld, M. and Lamba, H., 2009. Hysteresis and Economics. *IEEE Control Systems*, 29 (1), 30–43.

Cuypers, J.G.M., 1973. World Dynamics: Two Simplified versions of Forrester's Model. *Automatica*, 9, 399–401.

Cuypers, J.G.M. and Rademaker, O., 1974. An analysis of Forrester's World Dynamics Model. *Automatica*, 195-201, 195–201.

Datta, P.P., Christopher, M. and Allen, P., 2007. Agent-based modelling of complex production/distribution systems to improve resilience. *International Journal of Logistics: Research and Applications*, 10 (3), 187–203.

de Souza, R., Zice, S. and Chaoyang, L., 2000. Supply chain dynamics and optimization. *Integrated Manufacturing Systems*, 11 (5), 348–364.

Dejonckheere, J., Disney, S.M., Lambrecht, M.R. and Towill, D.R., 2003. Measuring and avoiding the bullwhip effect: A control theoretic approach. *European Journal of Operational Research*, 147 (3), 567–590.

Dejonckheere, J., Disney, S.M., Lambrecht, M.R. and Towill, D.R., 2004. The impact of information enrichment on the Bullwhip effect in supply chains: A control engineering perspective. *European Journal of Operational Research*, 153 (3), 727–750.

Denyer, D. and Tranfield, D., 2006. Using qualitative research synthesis to build an actionable knowledge base. *Management Decision*, 44 (2), 213–227.

Disney, S.M. and Grubbström, R.W., 2004. Economic consequences of a production and inventory control policy. *International Journal of Production Research*, 42 (17), 3419–3431.

Disney, S.M., Naim, M.M. and Towill, D.R., 2000. Genetic algorithm optimisation of a class of inventory control systems. *International Journal of Production Economics*, 68 (3), 259–278.

Disney, S.M., Potter, A. and Gardner, B., 2003. The impact of vendor managed inventory on transport operations. *Transportation Research Part E*, 39 (5), 363–380.

Disney, S.M. and Towill, D.R., 2002. A discrete transfer function model to determine the dynamic stability of a vendor managed inventory supply chain. *International Journal of Production Research*, 40 (1), 179–204.

Disney, S.M. and Towill, D.R., 2003a. On the bullwhip and inventory variance produced by an ordering policy. *Omega*, 31 (3), 157–167.

Disney, S.M. and Towill, D.R., 2003b. Vendor-managed inventory and bullwhip reduction in a two-level supply chain. *International Journal of Operations & Production Management*, 23 (6), 625–651.

Disney, S.M. and Towill, D.R., 2005. Eliminating drift in inventory and order based production control systems. *International Journal of Production Economics*, 93-94 (1), 331–344.

Disney, S.M., Towill, D.R. and Warburton, R.D.H., 2006. On the equivalence of control theoretic, differential, and difference equation approaches to modeling supply chains. *International Journal of Production Economics*, 101 (1), 194–208.

Dorf, R.C. and Bishop, R.H., 1998. *Modern Control Systems*. Addison Wesley Longmann, Inc.

Duberley, J. and Johnson, P., 2005. *Understanding Management Reseach: an introduction to epistemology*. London: Sage.

Dunn, S.C., Seaker, R. and Waller, M.A., 1994. Latent variables in Business Logistics Research: scale development and validation. *Journal of business logistics*, 15 (2), 145–172.

Durkheim, E., 1964. *The rules of sociological methods*. Glencoe: The Free Press.

Easterby-Smith, M., Thorpe, R. and Lowe, A., 1991. *Management research*. London: Sage.

Edghill, J. and Towill, D.R., 1990. Assessing manufacturing system performance: frequency response revisited. *Engineering Costs and Production Economics*, 19, 319–326.

Edghill, l., Olsmats, C. and Towill, D.R., 1988. Industrial Case Study on the Dynamics and Sensitivity of a Closecoupled Production-distribution System. *International Journal of Production Research*, 26 (10), 1681–1693.

Elkington, J., 1997. *Cannibals with forks: the triple bottom line of 21st century business*. Oxford: Capstone.

Elliott, S., 2001. *Signal Processing for Active Control*. London: Academic Press.

Eshlaghy, A.T. and Razavi, M., 2011. Modeling and Simulating Supply Chain Management. *Applied Mathematical Sciences*, 5 (17), 817–828.

Evans, G.N. and Naim, M.M., 1994. The dynamics of capacity constrained supply chain. *Proceedings of International System Dynamics Conference, Stirling, Scotland*, 28–39.

Falasca, M., Zobel, C.W. and Cook, D., 2008. A decision support framework to assess supply chain resilience. *Proceedings of the 5th International ISCRAM Conference, Washington, USA, May.*

Fernie, J., Pfab, F. and Marchant, C., 2000. Retail Grocery Logistics in the UK. *The International Journal of Logistics Management*, 11 (2), 83–90.

Fiksel, J., 2003. Designing Resilient, Sustainable Systems. *Environmental Science and Technology*, 37 (23), 5330–5339.

Fiksel, J., 2006a. A Framework for Sustainable Materials Management. *Journal of Materials*, August.

Fiksel, J., 2006b. Sustainability and Resilience: Toward a Systems Approach. *Sustainability: Science, Practice and Policy*, 2 (2), 14–21.

Fiksel, J., Low, J. and Thomas, J., 2004. Linking Sustainability to Shareholder Value. *Environmental Management*, June.

Fishman, G.S., 2001. *Discrete-Event Simulation: Modeling, Programming, and Analysis*. New York USA: Springer.

Forrester, J.W., 1958. Industrial dynamics: a major breakthrough for decision makers. *Harvard Business Review*, 36 (4), 37–66.

Forrester, J.W., 1961. *Industrial Dynamic*. Boston, MA: MIT Press.

Forrester, J.W., 1968. Industrial Dynamics - after the first decade. *Management Science*, 14 (7), 398–415.

Forrester, J.W., 1971. *World Dynamics*. Cambridge: Wright-Allen.

Frankel, R., Naslund, D. and Bolumole, Y., 2005. The “white space” of logistics research: a look at the role of methods usage. *Journal of Business Logistics*, 26 (2), 185–209.

Fransoo, J.C. and Wouters, M.J.F., 2000. Measuring the bullwhip effect in the supply chain. *Supply Chain Management: An International Journal*, 5 (2), 78–89.

Gaalman, G. and Disney, S.M., 2009. On bullwhip in a family of order-up-to policies with ARMA(2,2) demand and arbitrary lead-times. *International Journal of Production Economics*, 121 (2), 454–463.

Gammelgaard, B., 2004. Schools in logistics research? A methodological framework for analysis of the discipline. *International Journal of Physical Distribution & Logistics Management*, 34 (6), 479–491.

Georgiadis, P., Vlachos, D. and Iakovou, E., 2005. A system dynamics modeling framework for the strategic supply chain management of food chains. *Journal of Food Engineering*, 70 (3), 351–364.

Giannakis, M. and Croom, S.R., 2004. Towards the development of a Supply Chain Management Paradigm: a conceptual framework. *Journal of supply chain management*, 40 (2), 27–37.

Göcke, M., 2002. Various Concepts of Hysteresis Applied in Economics. *Journal of Economic Surveys*, 16 (2), 167–188.

Golicic, S.L., Davis, D.F. and McCarthy, T.M., 2005. A Balanced Approach to Research in Supply Chain Management. *In*: H. Kotzab, S. Seuring, M. Müller and G. Reiner, eds. *Research Methodologies in Supply Chain Management*. Germany: Physica-Verlag Heiedlberg.

Goranson, T., 1999. *The Agile Virtual Enterprise.*. Westport, CT: Quorum Books.

Gosling, J., Naim, M.M. and Towill, D.R., 2012. A supply chain flexibility framework for engineer-to-order systems. *Production Planning & Control*, 24 (7), 552–556.

Graham, D. and McRuer, D., 1961. *Analysis of nonlinear control theory*. 1st New York: John Wiley & Sons, Inc.

Grö*ß*ler, A. and Schieritz, N., 2005. Of Stocks, Flows, Agents and Rules - "Strategic" Simulations in Supply Chain Research. *In*: H. Kotzab, S. Seuring, M. Müller and G. Reiner, eds. *Research Methodologies in Supply Chain Management*. Germany: Physica-Verlag Heiedlberg.

Grübbstrom, R. and Wang, Z., 2000. Introducing capacity limitations into multi-level, multi-stage production-inventory systems applying the input-output: Laplace transform approach. *International Journal of Production Research*, 38 (17), 4227–4234.

Grubbström, R.W. and Huynh, T.T., 2006. Analysis of standard ordering policies within the framework of MRP theory. *International Journal of Production Research*, 44 (18-19), 3759–3773.

Grünwald, H.J. and Fortuin, L., 1992. Many steps towards zero inventory. *European Journal of Operational Research*, 59 (3), 359–369.

Gunasekaran, A., Patel, C. and Tirtiroglu, E., 2001. Performance measures and metrics in a supply chain environment. *International Journal of Operations & Production Management*, 21 (1/2), 71–87.

Hamdouch, Y., 2011. Multi-period supply chain network equilibrium with capacity constraints and purchasing strategies. *Transportation Research Part C: Emerging Technologies*, 19 (5), 803 – 820.

Harré, R., 1970. *The Principles of scientific thinking*. London: MacMillan.

Hennet, J.C., 2009. Supply chain dynamics. *In*: Y. Merkuryev, G. Merkuryeva, M.A. Piera and A. Guasch, eds. *Simulation-Based Case Studies in Logistics: Education and Applied Research*. London: Springer.

Hesse, M., 1966. *Models and Analogies in sciences*. South Bend: University of Notre Dame Press.

Hicks, M.J., 2004. *Problem Solving and Decision Making: Hard, Soft and Creative Approaches*. 2nd London, UK: Thomson Learning.

Higuchi, T. and Troutt, M.D., 2004. Dynamic simulation of the supply chain for a short life cycle product. *Computers & Operations Research*, 31 (7), 1097–1114.

Holliday, C. and Pepper, J., 2001. Walking the Talk: The Business Case for Sustainable Development. *World Business Council for Sustainable Development.*

Holling, C.S., 1973. Resilience and stability of ecological systems. *Annual review of Ecology and Systematics*, 4, 1–23.

Holling, C., 1959. The components of predation as revealed by a study of small mammal predation of the European Pine Sawfly. *The Canadian Entomologist*, 91 (5), 293–320.

Holweg, M. and Bicheno, J., 2000. The reverse amplification effect in supply chains. *Global Logistics for the New Millennium - 5th International Symposium on Logistics*, 12-15 July, Tokyo, Japan, 549–554.

Hong-Minh, S.M., Disney, S.M. and Naim, M.M., 2000. The dynamics of emergency transhipment supply chains. *International Journal of Physical Distribution and Logistics Management*, 30 (9), 788–815.

Hsia, T.C., 1972. On the simplification of linear systems. *IEEE Transactions on Automatic Control*, June, 372,374.

Hwarng, H.B. and Xie, N., 2008. Understanding Supply Chain Dynamics: A Chaos Perspective. *European Journal of Operational Research*, 184 (3), 1163–1178.

Jeong, S., Oh, Y. and Kim, S., 2000. Robust Control of multi-echelon production-distribution systems with limited decision policy (II). *KSME International Journal*, 14 (4), 380–392.

Johansson, M., 2003. *Piecewise Linear Control Systems.* Berlin, Germany: Springer.

John, S., Naim, M.M. and Towill, D., 1994. Dynamic analysis of a WIP compensated decision support system. *International Journal of Manufacturing System Design*, 1 (4), 283–297.

Juntunen, J. and Juga, J., 2009. Controlling the bullwhip with transport capacity constraints. *International Journal of Services and Standards*, 5 (2), 160–171.

Jüttner, U., 2011. Supply chain resilience in the global financial crisis: an empirical study. *Supply Chain Management: An International Journal*, 16 (4), 246–259.

Jüttner, U., Peck, H. and Christopher, M., 2003. Supply chain risk management: outlining an agenda for future research. *International Journal of Logistics: Research and Applications Vol. 6, No. 4, 2003*, 6 (4), 197–210.

Kaplan, H.B., 1990. Toward an understanding of resilience: a critical review of definitions and models. *In*: M.D. Glantz and J.L. Johnson, eds. *Resilience and devepment: positive life adaptation*. Kluwer Academic.

Kaplan, S. and Garrick, B.J., 1981. On the quantitative definition of risk. *Risk Analysis, Vol. I , No. I , 1981*, 1 (1), 11–27.

Kearney, A.T., 2003. Supply chains in a vulnerable, volatile world. *Executive Agenda*, 6 (3), 5–16.

Keat, R. and Urry, J., 1975. *Social theory as sciences*. London: Routledge.

Kogut, B. and Kulatilaka, N., 1994. Operating flexibility, global manufacturing, and the option value of a multinational network. *Management Science*, 10, 123–139.

Kolk, W.R. and Lerman, R.A., 1992. *Nonlinear System Dynamics*. New York: Van Nostrand Reinhold.

Kouvelis, P., 1998. Global sourcing strategies under exchange rate uncertainty. *In*: S. Tayur, R. Ganeshan and M. Magazine, eds. *Quantitative Models for Supply Chain Management*. Springer.

Kovács, G. and Spens, K.M., 2007. Humanitarian logistics in disaster relief operations. *International Journal of Physical Distribution & Logistics Management*, 37 (2), 99–114.

Kovács, G. and Tatham, P., 2009. Responding to disruptions in the supply network - from dormant to action. *Journal of Business Logistics*, 30 (2), 215–229.

Kovács, G. and Spens, K.M., 2005. Abductive reasoning in logistics research. *International Journal of Physical Distribution and Logistics Management*, 35 (2), 132–144.

Kumar, S. and Nigmatullin, A., 2011. A system dynamics analysis of food supply chains - Case study with non-perishable products. *Simulation Modelling Practice and Theory*, 19 (10), 2151–2168.

Kumar, S. and Yamaoka, T., 2007. System dynamics study of the Japanese automotive industry closed loop supply chain. *Journal of Manufacturing Technology Management*, 18 (2), 115–138.

Kuo, B.C. and Golnaraghi, F., 2003. *Automatic control systems.* Vol. 1. John Wiley & Sons.

Lagoudis, I.N., Lalwani, C.S., Naim, M.M. and King, J., 2002. Defining a conceptual model for high-speed vessels. *International Journal of Transport Management*, 1 (2), 69–78.

Lalwani, C.S., Disney, S.M. and Towill, D.R., 2006. Controllable, observable and stable state space representations of a generalized order-up-to policy. *International Journal of Production Economics*, 101 (1), 172–184.

Lane, D.C. and Sterman, J.D., 2011. Jay Wright Forrester. *In*: S. Gass and A. Assad, eds. *Profiles in Operations Research: Pioneers and Innovators.* New York: Springer.

Lang, D. and de Peretti, C., 2009. A strong hysteretic model of Okun's Law: theory and a preliminary investigation. *International Review of Applied Economics*, 23 (4), 445–462.

Larsen, E.R., Morecroft, J.D.W. and Thomsen, J.S., 1999. Complex behaviour in a production-distribution model. *European Journal of Operational Research*, 119, 61–74.

Larson, P.D. and Halldórsson, A., 2004. Logistics versus supply chain management: An international survey. *International Journal of Logistics Research and Applications*, 7 (1), 17–31.

Lassar, W.M. and Kerr, J.L., 1996. Strategy and control in supplier-distributor relationships: an agency perspective. *Strategic Management Journal*, 17 (8), 613–632.

Laugesen, J. and Mosekilde, E., 2006. Border-collision bifurcation in a dynamic management game. *Computers & Operations Research*, 33, 464–478.

Lee, H.L., 2003. The Triple-A Supply Chain. *Harvard Business Review*, 82 (10), 102–112.

Lee, H.L., Padmanabhan, V. and Whang, S., 1997a. The bullwhip effect in supply chains. *Sloan Management Review*, 38 (3), 93–102.

Lee, H.T. and Wu, J.C., 2006. A study on inventory replenishment policies in a two-echelon supply chain system. *Computers & Industrial Engineering*, 51, 257–263.

Lee, H.L., Padmanabhan, V. and Whang, S., 1997b. Information Distortion in a Supply Chain: The Bullwhip Effect. *Management Science*, 43 (4), 546–558.

Leigh, J.R., 2004. *Control Theory*. 2nd London: The Institution of Electrical Engineers.

Lodree Jr., E.J. and Taskin, S., 2007. An insurance risk management framework for disaster relief and supply chain disruption inventory planning. *Journal of the Operational Research Society*, 59 (5), 1–11.

Lummus, R.R., Duclos, L.K. and Vokurka, R.J., 2003. Supply Chain Flexibility: Building a New Model. *Global Journal of Flexible Systems Management*, 4 (4), 1–13.

Lummus, R.R., Krumwiede, D.W. and Vokurka, R.J., 2001. The relationship of logistics to supply chain management: developing a common industry definition. *Industrial Management and Data Systems*, 101 (8), 426–432.

Mahadevan, B., 2007. *Operation Management: Theory and Practice*. 4th Delhi, India: Dorling Kindersley.

Mangan, J., Lalwani, C. and Butcher, T., 2008. *Global Logistics and Supply Chain Management*. Chichester: John Wiley & Sons.

Mangan, J., Lalwani, C. and Gardner, B., 2004. Combining quantitative and qualitative methodologies in logistics research. *International Journal of Physical Distribuition & Logistics*, 34 (7), 565–78.

March, J.G. and Shapira, Z., 1987. Managerial perspectives on risk and risk taking. *Management Science*, 33 (11), 1404–1418.

Marques, V.L., Naim, M.M. and Disney, S.M., 2010. The impact of capacitated logistics operations on supply chain dynamics. *Proceedings of the 16th International Working Seminar on Production Economics, Innsbruck, 1-5 March, Austria,* Pre-Prints, Vol. 3., pp. 365-376.

Mason, R. and Lalwani, C., 2006. Transport integration tools for supply chain management. *International Journal of Logistics: Research and Applications*, 9 (1), 57–74.

Mason-Jones, R., 1998. 'The holistic strategy of information enrichment through the supply chain dynamics'. Thesis (PhD). University of Wales, Cardiff.

Mason-Jones, R. and Towill, D., 1998. Shrinking the supply chain uncertainty circle. *Control*, 24 (7), 17–23.

Mason-Jones, R., Naim, M.M. and Towill, D.R., 1997. The impact of pipeline control on Supply Chain Dynamics. *The International Journal of Logistics Management*, 8 (2), 47–62.

Matsubara, M., 1965. On the equivalent dead time. *IEEE Transactions on Automatic Control*, 464–466.

May, T., 1997. *Social Research: issues, methods and process*. 2nd Buckingham, UK: Open University Press.

McManus, S., Seville, E., Brunsdon, D. and Vargo, J., 2007. Resilience Management: A Framework for Assessing and Improving the Resilience of Organisations. *Christchurch, New Zealand: Resilient Organisations (www.resorgs.org.nz).*

Mentzer, J. and Kahn, K., 1995. A framework of logistics research. *Journal of Business Logistics*, 16 (1), 231–50.

Mentzer, J.T., DeWitt, W., Keebler, J.S., Min, S., Nix, N.W., Smith, C.D. and Zacharia, Z.G., 2001. Defining supply chain management. *Journal of Business Logistics*, 22 (2), 1–25.

Meredith, J., Raturi, A., Gyampah, K. and Kaplan, B., 1989. Alternative research paradigms in operations. *Journal of Operations Management*, 8 (4), 297–326.

Metz, P.J., 1998. Demystifying Supply Chain Management. *Supply Chain Management Review*, Winter, 46–55.

Mingers, J., 2000. The contribution of critical realism as an underpinning philosophy for OR/MS and systems. *Journal of Operational Research Society*, 51 (11), 1256–1270.

Mitra, K., Gudi, R.D., Patwardhan, S.C. and Sardar, G., 2009. Towards resilient supply chains: Uncertainty analysis using fuzzy mathematical programming. *Computer Aided Chemical Engineering*, 87 (7), 967–981.

Mohapatra, P.K.J., 1980. Nonlinearity in system dynamics models. *Dynamica*, 6 (1), 36–52.

Mosekilde, E. and Laugesen, J.L., 2007. Nonlinear dynamic phenomena in the beer model. *Systems Dynamics Review*, 23 (2/3), 229–252.

Naim, M.M. and Towill, D.R., 1995. What's in the pipeline?. *Second International Symposium on Logistics, Nottingham, UK, 11-12th July*, 135–142.

Naim, M.M., Wikner, J., Towill, D.R. and Marques, V.L., 2012. Mitigating the bullwhip effect by exploiting archetypes of complex production and inventory control systems. *Proceedings of the 17th International Working Seminar on Production Economics, Igls/Innsbruck, 20-25 February, Austria*, 1, 373–382.

Naim, M.M., Disney, S.M. and Towill, D.R., 2004. Supply chain dynamics. *In*: S. New and R. Westbrook, eds. *Understanding Supply Chains: Concepts, Critiques, and Futures*. Oxford University Press.

Naim, M.M. and Towill, D.R., 1994. Establishing a Framework for Effective Materials Logistics Management. *International journal of Logistics Management*, 5 (1), 81–88.

Näslund, D., 2002. Logistics needs qualitative research: especially action research. *International Journal of Physical Distribution & Logistics*, 32 (5), 321–38.

Newman, W.R., Hanna, M. and Maffei, M.J., 1993. Dealing with the uncertainties of manufacturing: flexibility, buffers and integration. *International Journal of Operations and Production Management*, 13 (1), 19–34.

Nise, N.S., 2000. *Control systems engineering*. 3rd New York: John Wiley & Sons.

Odame, K. and Hasler, P.E., 2008. Harmonic distortion analysis via perturbation methods. *51st Midwest Symposium on Circuits and Systems*, 554–557.

Odame, K. and Hasler, P.E., 2010. Nonlinear Circuit Analysis via Perturbation Methods and Hardware Prototyping. *VLSI Design*, 2010, 1–10.

Oliver, R.K. and Webber, M.D., 1982. Supply Chain Management: Logistics Catches Up with Strategy. *In*: M. Christopher, ed. *Logistics: The Strategic Issues*. Chapman & Hall, 63–75.

Pagell, M. and Krause, D.R., 1999. A Multiple-Method Study of Environmental Uncertainty and Manufacturing Flexibility. *Journal of Operations Management*, 17 (3), 307–326.

Patton, M.Q., 2002. *Qualitative research and evaluation methods*. 3rd London: Sage.

Peck, H., 2005. Drivers of supply chain vulnerability: an integrated framework. *International Journal of Physical Distribution and Logistics Management*, 35 (4), 210–232.

Peirce, C.S., 1931. Kinds of reasoning. *In*: C. Hartshorne and P. Weiss, eds. *Collected papers of Charles Sanders Peirce: Principles of Philosophy*. Cambridge, MA: Harvard University Press, p. 28.

Pettit, T.J., Fiksel, J. and Croxton, K.L., 2010. Ensuring supply chain resilience: development of a conceptual framework. *Journal of Business Logistics*, 31 (1), 1–21.

Poles, R., 2013. System Dynamics modelling of a production and inventory system for remanufacturing to evaluate system improvement strategies. *International Journal of Production Economics*, 144 (1), 189–199.

Ponomarov, S.Y. and Holcomb, M.C., 2009. Understanding the concept of supply chain resilience. *International Journal of Logistics Management*, 20 (1), 124–143.

Potter, A. and Lalwani, C., 2008. Investigating the impact of demand amplification on freight transport. *Transportation Research Part E*, 44 (5), 835–846.

Pytel, A. and Kiusalaas, J., 2003. *Mechanics of Materials*. Stamford, USA: Cengage Learning.

Raj, T.S. and Lakshminarayanan, S., 2008. Performance Assessment/Enhancement Methodology for Supply Chains. *Industrial and Engineering Chemistry Research*, 47 (3), 748–759.

Ratick, S., Mecham, B. and Aoyama, Y., 2008. Locating backup facilities to enhance supply chain disaster resilience. *Growth and Change*, 39 (4), 642–666.

Ratnatunga, A.K. and Sharp, J.A., 1976. Linearization and order reduction in system dynamics models. *Dynamica*, 2 (3), 87–94.

Rice, J.B. and Caniato, F., 2003. Building a secure and resilient supply network. *Supply Chain Management Review*, 7 (5), 22–30.

Richardson, G.P., 2008. The Core of System Dynamics. *In*: R. Meyers, ed. *Encyclopedia of Complexity and Systems Science*. Springer, 8967–8974.

Richardson, G.P., 2011. Reflections on the foundations of system dynamics. *System Dynamics Review*, 27 (3), 219–243.

Rose, A., 2004. Economic resilience to disasters: toward a consistent and comprehensive formulation. *In*: D. Paton and D. Johnston, eds. *Disaster resilience: an integrated approach*. Springfield, 226–248.

Rugh, W.J., 2002. *Nonlinear System Theory: The Volterra/Wiener Approach*. The Johns Hopkins University Press.

Sacha, A. and Datta, S., 2005. Review of supply chain management and logistics research. *International Journal of Physical Distribution and Logistics Management*, 35 (9), 664–705.

Sanchez Rodrigues, V., Stantchev, D., Potter, A., Naim, M. and Whiteing, A., 2008. Establishing a transport operation focused uncertainty model for the supply chain. *International Journal of Physical Distribution and Logistics Management*, 38 (5), 388–411.

Saunders, M.N.K., Lewis, P. and Thornhill, A., 2009. *Research Methods for Business Students*. 5th Harlow, England: Pearson Education Limited.

Sayer, A., 2000. *Realism and Social Science*. 1st London: Sage.

Schmidt, A.J. and Singh, M., 2012. A quantitative analysis of disruption risk in a multi-echelon supply chain. *International Journal of Production Economics*, 139 (1), 22–32.

Sheffi, Y., 2001. Supply Chain Management under the Threat of International Terrorism. *International Journal of Logistics Management*, 12 (2), 1–11.

Sheffi, Y., 2005a. Preparing for the big one. *IEE Manufacturing Engineer*, Oct./Nov., 12–15.

Sheffi, Y. and Rice, J.B., 2005. A supply chain view of the resilient enterprise. *MIT Sloan Management Review*, 47 (1), 41–48.

Sheffi, Y., 2005b. Building a Resilient Supply Chain. *Harvard Business Review*, 1 (8), 1–5.

Shinners, S.M., 1998. *Modern Control System Theory and Design*. New York, USA: John Wiley & Sons.

Shukla, V., Naim, M.M. and Yaseen, E.A., 2009. 'Bullwhip' and 'backlash' in supply pipelines. *International Journal of Production Research*, 47 (23), 6477–6497.

Simon, H.A., 1952. On the application of servomechanism theory in the study of production control. *Econometrica*, 20, 247–268.

Skipper, J.B. and Hanna, J.B., 2009. Minimizing supply chain disruption risk through enhanced flexibility. *International Journal of Physical Distribution and Logistics Management*, 39 (5), 404–427.

Slack, N., Chambers, S. and Johnst, R., 2010. *Operations and Process Management*. 6th Essex: Pearson Prentice Hall.

Spekman, R.E. and Davis, E.W., 2004. Risky business: expanding the discussion on risk and the extended enterprise. *International Journal of Physical Distribution & Logistics Management*, 34 (5), 414–433.

Spens, K.M. and Kovács, G., 2006. A content analysis of research approaches in logistics research. *International Journal of Physical Distribution and Logistics Management*, 36 (5), 374–390.

Srivastava, M., Srivastava, M.C. and Bhatnagar, S., 2009. *Control Systems*. Nagar, New Delhi: Tata Macgraw-Hill.

Stalk, G. and Hout, T., 1990. *Competing against Time*. New York, NY: The Free Press.

Sterman, J.D., 1989. Modelling managerial behaviour: misperceptions of feedback in a dynamic decision making experiment. *Management Science*, 35 (3), 321–339.

Stevens, G.C., 1989. Integrating the Supply Chain. *International Journal of Physical Distribution and Logistics Management*, 19 (8), 3–8.

Stewart, G., 1995. Supply chain performance benchmarking study reveals keys to supply chain excellence. *Logistics Information Management*, 8 (2), 38–44.

Stewart, M., Reid, G. and Mangham, C., 1997. Fostering Children's Resilience. *Journal of Pediatric Nursing*, 12 (1), 21–31.

Suri, R., 1998. *Quick Response Manufacturing: A Companywide Approach to Reducing Lead Times*. Productivity Press.

Svensson, G., 2000. A conceptual framework for the analysis of vulnerability in supply chains. *International Journal of Physical Distribution & Logistics Management*, 30 (9), 731–749.

Tang, C. and Tomlin, B., 2008. The power of flexibility for mitigating supply chain risks. *International Journal of Production Economics*, 116 (1), 12–27.

Tang, C.S., 2006. Robust strategies for mitigating supply chain disruptions. *International Journal of Logistics: Research and Applications*, 9 (1), 33–45.

Thomas, A.B., 2004. *Research Skills for management studies*. 1st London: Routledge.

Thomas, A. and Fritz, L., 2006. Disaster Relief, Inc.. *Harvard*, 84 (11), 114–122.

Tibben-Lembke, R.S., 1998. The impact of reverse logistics on the total cost of ownership. *Journal of Marketing Theory and Practice*, 6 (4), 51–60.

Tierney, K. and Bruneau, M., 2007. Conceptualizing and measuring resilience: a key to disaster loss reduction. *TR news*, May-June (250), 14–17.

Tomlin, B., 2006. On the Value of Mitigation and Contingency Strategies for Managing Supply Chain Disruption Risks. *Management Science*, 52 (5), 639–657.

Torres, O.C. and Maltz, A.B., 2010. Understanding the financial consequences of the bullwhip effect in a multi-echelon supply chain. *Journal of Business Logistics*, 31 (1), 23–41.

Towill, D.R., 1970. *Transfer Function Techniques for control engineers*. London: Iliffe Books Ltd.

Towill, D.R., 1982. Dynamic analysis of an inventory and order based production control system. *International Journal of Production Research*, 20 (6), 671–687.

Towill, D.R., 1991. Supply chain dynamics. *International Journal of Computer Integrated Manufacturing*, 4 (4), 197–208.

Towill, D.R., 1992a. Supply chain dynamics: the change engineering challenge of the 1990s. *Proceedings Institute of Mechanical Engineering Conference on Engineering Manufacture*, 206 (4), 233–245.

Towill, D.R., 1997a. Forridge - principles of good material flow. *Production Planning and Control*, 8 (7), 622–632.

Towill, D.R., Naim, M.M. and Wikner, J., 1992. Industrial dynamics simulation models in the design of supply chains. *International Journal of Physical Distribution & Logistics Management*, 22 (5), 3–13.

Towill, D.R., 1981. *Coefficient Plane Models for Control System Analysis and Design*. Herts, England: John Wiley & Sons.

Towill, D.R., 1992b. The practical use of control theory in manufacturing design and manufacturing management. *Journal of Systems Engineering*, 1 (2), 2–14.

Towill, D.R., 1996. Time compression and supply chain management - a guided tour. *Supply Chain Management: An International Journal*, 1 (1), 15–27.

Towill, D.R., 1997b. The seamless supply chain - the predator's strategic advantage. *International Journal of Technology Management*, 13 (37-56), 1.

van Hoek, R., Harrison, A. and Christopher, M., 2001. Measuring agile capabilities in the supply chain. *International Journal of Operations and Production Management*, 21 (1/2), 126–147.

Venetoulis, J., Chazan, D. and Gaudet, C., 2004. *Ecological footprint of nations*. Oakland: Redefining Progress.

Vukic, Z., Kuljaca, L., Donlagic, D. and Tesaknjak, S., 2003. *Nonlinear Control Systems*. New York: Marcel Dekker.

Wagner, S.M. and Neshat, N., 2010. Assessing the vulnerability of supply chains using graph theory. *International Journal of Production Economics*, 126 (1), 121–129.

Wang, X. and Disney, S.M., 2012. Stability Analysis Of Constrained Inventory Systems. *European Journal of Operational Research*, 223, 86–95.

Warburton, R. and Disney, S., 2007. Order and inventory variance amplification: The equivalence of discrete and continuous time analyses. *International Journal of Production Economics*, 110 (1-2), 128–137.

Weick, K.E., Sutcliffe, K.M. and Obstfeld, D., 1999. Organizing for high reliability: processes of collective mindfulness. *Research in Organizational Behavior*, 21, 13–81.

Weng, Z., 1999. Strategies for integrating lead time and customer-order decisions. *IIE Trans.*, 31, 161–171.

Wikner, J., Naim, M.M. and Towill, D.R., 1992. The system simplification approach in understanding the dynamic behaviour of a manufacturing supply chain. *Journal of Systems Engineering*, 2, 164–178.

Wikner, J., Towill, D.R. and Naim, M.M., 1991. Smoothing supply chain dynamics. *International Journal of Production Economics*, 22 (3), 231–248.

Wikner, J., Naim, M.M. and Rudberg, M., 2007. Exploiting the Order Book for Mass Customized Manufacturing Control Systems With Capacity Limitations. *IEEE Transactions on Engineering Management*, 54 (1), 145–155.

Wildavsky, A., 1988. *Searching for Safety*. New Brunswick, NJ: Transaction Books.

Williams, T., 2008. *Management Science in Practice*. West Sussex: John Wiley.

Wilson, B., 2001. *Soft Systems Methodology: Conceptual Model Building and Its Contribution*. Chichester, UK: Wiley.

Wilson, M., 2007. The Impact of Transportation Disruptions on Supply Chain Performance. *Transportation Research Part E: Logistics and Transportation Review*, 43 (295-320), 4.

Wolf, J., 2008. *The nature of supply chain management research*. Frankfurt: Gabler-Verlag.

Yates, J.F. and Stone, E.R., 1992. The risk construct. *In*: J. Yates, ed. *Risk Taking Behavior*. New York: Wiley, 1–25.

Zhou, L., Disney, S. and Towill, D., 2010. A pragmatic approach to the design of bullwhip controllers. *International Journal of Production Economics*, 128 (2), 556–568.

Zobel, C.W., 2011. Representing perceived tradeoffs in defining disaster resilience. *Decision Support Systems*, 50 (2), 394–403.

Zobel, C.W. and Khansa, L., 2011. Characterizing multi-event disaster resilience. *Computers & Operations Research*, doi:10.1016/j.cor.2011.09.024.

Zsidisin, G.A., 2003. A grounded definition of supply risk. *Journal of Purchasing and Supply Managemen*, 9 (5-6), 217–224.

Appendix A: Low order modelling

Let a high-order system be represented by a transfer function in the following form:

$$T(s) = \frac{1 + b_1 s + b_2 s^2 + \cdots + b_q s^q}{1 + a_1 s + a_2 s^2 + \cdots + a_n s^n} \tag{A-1}$$

The low order model will then be:

$$T_M(s) = \frac{1 + B_1 s + B_2 s^2 + \cdots + b_Q s^Q}{1 + A_1 s + A_2 s^2 + \cdots + A_N s^N} \tag{A-2}$$

so that $Q \leq q$ and N must be less than n.

Matsubara time delay theorem for low order modelling

This method initially involves choosing the poles nearest to the imaginary axis to determine $T_M(s)$. However, the Matsubara time delay theorem is also incorporated to compensate for inaccuracies in the low order model. This gives us the following model:

$$T_M(s) = e^{-\tau s}\left(\frac{1 + B_1 s + B_2 s^2 + \cdots + b_Q s^Q}{1 + A_1 s + A_2 s^2 + \cdots + A_N s^N}\right) \tag{A-3}$$

where τ is a time delay in the response which is determined by matching the system and model step responses according to the integral of error from time zero to infinity. In other words, the area between the input and output lines in the system, $T(s)$, should match the respective area in the low order model, $T_M(s)$ plus the area caused by this time delay.

The area, D, between the input and output lines given any transfer function is represented by the hatched area in Figure A-1a. This area can be found by calculating the integral of error from time zero to infinity as:

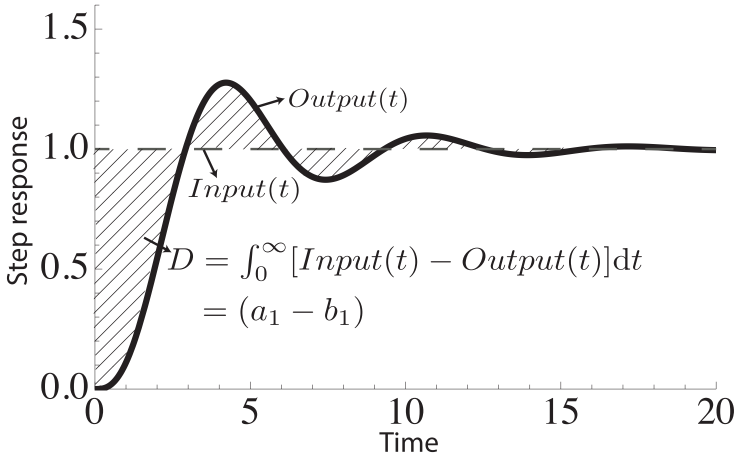
$$D = \int_0^\infty [input(t) - output(t)]\mathrm{d}t \tag{A-4}$$

For a unit step input, the Laplace transform of the about equation gives us

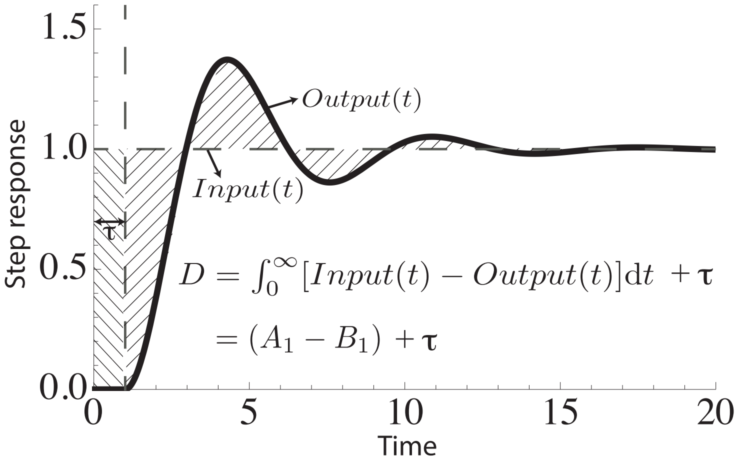
$$D = \frac{1}{s}\left[\frac{1}{s} - \frac{T(s)}{s}\right] \tag{A-5}$$

where T(s) is the transfer function of the high order system in Equation A-1. Replacing this general form of transfer function results:

$$D = \frac{1}{s^2}\left[\frac{(a_1 - b_1)s + (a_2 - b_2)s^2 + \cdots + b_q s^q}{1 + a_1 s + a_2 s^2 + \cdots + a_n s^n}\right] \tag{A-6}$$



(a) System $T(s)$, unit step response



(b) Model $T_{M\tau}(s)$, unit step response

Figure A-1: Low order modelling: Matsubara

Using the final value theorem, it is obtained that the area between the input and output lines, D, in the system is simply equal to $(a_1 - b_1)$. Analogously, the corresponding area in the low order model (see Figure A-1b) will be $(A_1 - B_1)$, which is normally smaller than the area, $(a_1 - b_1)$, in the system. Hence, adding the time delay proposed by Matsubara, the relation $(a_1 - b_1) = (A_1 - B_1) + \tau$ is obtained.

Initially, the low order model, $T_M(s)$, is determined by choosing the poles nearest to the imaginary axis. The number of poles chosen will determine the order of the model. In Figure A-2, the boundary in Forrester pipeline case for determining low order models of first, second and third order is illustrated. Note that it is irrelevant how far the poles are from each other and how dominant each pole is. This choice of poles is just an initial recommendation.

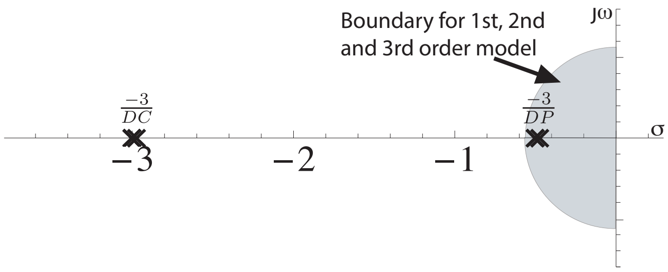


Figure A-2: Poles of Forrester's pipeline

Hsia method for low order modelling

The approximation method proposed by Hsia (1972) is based on selecting A_i and B_i, in such way that $T_M(s)$ has a frequency (ω) response very close to that of $T(s)$. In order words, the magnitude of the frequency function $T(i\omega)/T_M(i\omega)$ is required to deviate the least amount from unity for various frequencies. Hence, the following relation should be satisfied as closely as possible:

$$\frac{|T(i\omega)|^2}{|T_M(i\omega)|^2} = 1, \text{ for } 0 \leq \omega \leq \infty \tag{A-7}$$

The ration $T(s)/T_M(s)$ can also be written as:

$$\begin{aligned}\frac{T(s)}{T_M(s)} &= \frac{(1 + b_1 s + b_2 s^2 + \cdots + b_q s^q)}{(1 + a_1 s + a_2 s^2 + \cdots + a_n s^n)} \cdot \frac{(1 + A_1 s + A_2 s^2 + \cdots + A_N s^N)}{(1 + B_1 s + B_2 s^2 + \cdots + b_Q s^Q)} \\ &= \frac{(1 + m_1 s + m_2 s^2 + \cdots + m_u s^u)}{(1 + l_1 s + l_2 s^2 + \cdots + l_v s^v)}\end{aligned} \tag{A-8}$$

where $u = q + N$ and $v = n + Q$. Equation A-7 can be re-written as

$$\frac{|T(i\omega)|^2}{|T_M(i\omega)|^2} = \left.\frac{T(s)T(-s)}{T_M(s)T_M(-s)}\right|_{s=iw}$$
$$= \frac{(1 + m_1 s + m_2 s^2 + \cdots + m_u s^u)}{(1 + l_1 s + l_2 s^2 + \cdots + l_v s^v)} . \frac{(1 - m_1 s + m_2 s^2 + \cdots + (-1)^u m_u s^u)}{(1 - l_1 s + l_2 s^2 + \cdots + (-1)^v l_v s^v)} \tag{A-9}$$

This can be re-written in the form of

$$\frac{|T(i\omega)|^2}{|T_M(i\omega)|^2} = 1 + \left.\frac{(e_2 - f_2)s^2 + (e_4 - f_4)s^4 + \cdots + (e_{2u} - f_{2u})s^{2u}}{1 + f_2 s^2 + f_4 s^4 + \cdots + f_{2v} s^{2v}}\right|_{s=iw}, \text{ if u=v} \tag{A-10}$$

Then, to satisfy the condition of Equation A-7, $e_2 = f_2, e_4 = f_4, \cdots, e_{2u} = f_{2u}$ should hold true. However, if $u < v$, which it is in most practical cases, then we have that:

$$\frac{|T(i\omega)|^2}{|T_M(i\omega)|^2} = 1 + \left.\frac{(e_2-f_2)s^2+(e_4-f_4)s^4+\cdots+(e_{2u}-f_{2u})s^{2u}-f_{2(u+1)}s^{2(u+1)}-f_{2(u+2)}s^{2(u+2)}-\cdots-f_{2v}s^{2v}}{1+f_2s^2+f_4s^4+\cdots+f_{2v}s^{2v}}\right|_{s=iw} \tag{A-11}$$

This will imply an error. Given the conditions that $e_2 = f_2, e_4 = f_4, \cdots, e_{2u} = f_{2u}$ and Equation A-9, the unknown coefficients for determining $T_M(s)$ once T(s) is given ca be calculated by solving the system of non-linear equations.

Appendix B: The Forrester model

Equations used in the DYNAMO programme (factory echelon)

$$RR.KL = \begin{cases} RRI, \text{if } t \leq 0 \\ RRI + STEP, \text{if } t > 0 \end{cases} \tag{B-1}$$

$$UO.K = UO.J + (DT)(RR.JK - SS.JK) \tag{B-2}$$

$$IA.K = IA.J + (DT)(SR.JK - SS.JK) \tag{B-3}$$

$$ST.K = UO.K/DF.K \tag{B-4}$$

$$NI.K = IA.K/DT \tag{B-5}$$

$$SS.KL = CLIP(ST.K, NI.K, NI.K, ST.K) \tag{B-6}$$

$$DF.K = (ID.K/IA.K)(DU) + DH \tag{B-7}$$

$$ID.K = (AI)(RS.K) \tag{B-8}$$

$$RS.K = RS.J + (DT)(1/DR)(RR.JK - RS.J) \tag{B-9}$$

$$MW.K = RR.KL + (1/DI)(ID.K - IA.K + LD.K - LA.K + UO.K - UN.K) \tag{B-10}$$

$$MD.KL = CLIP(MW.K, AL, AL, MW.K) \tag{B-11}$$

$$LD.K = (RS.K)(DC + DP) \tag{B-12}$$

$$LA.K = CP.K + OP.K \tag{B-13}$$

$$UN.K = (RS.K)(DH + DU) \tag{B-14}$$

$$CP.K = CP.J + (DT)(MD.JK - MO.JK) \tag{B-15}$$

$$MO.KL = DELAY3(MD.KL, DC) \tag{B-16}$$

$$OP.K = OP.J + (DT)(MO.JK - SR.JK) \quad \text{(B-17)}$$

$$SR.KL = DELAY3(MO.KL, DP) \quad \text{(B-18)}$$

Variables used in the DYNAMO programme (factory echelon)

CP	clerical in-process orders	NI	negative inventory limit rate
DF	delay (variable) in filling orders	OP	orders in production
IA	inventory actual	RR	requisition (orders) received
ID	inventory desired	RS	requisition (orders) smoothed
LA	pipeline orders actual in transit	SR	shipment received inventory
LD	pipeline orders desired in transit	SS	shipment sent
MD	manufacturing rate decision	ST	shipping rate tried
MO	manufacturing orders	UN	unfilled orders normal
MW	manufacturing rate wanted	UO	unfilled orders

Constants used in the DYNAMO programme (factory echelon)

AI=4	constant for inventory
AL=1000(RRI)	constant specifying capacity limit *
DC=1	delay clerical
DH=1	delay due to minimum handling time
DI=4	delay in inventory/pipeline adjustment
DP=6	delay in production lead time
DR=8	delay in smoothing requisitions
DU=1	delay, average, in unfilled orders
DT=1	solution time interval
RRI=1000	initial value of demand *
STEP=100	requisition step change *

* Different values of AL were considered when evaluating the impact of manufacturing constraints.
**Author used standard unit step input in order to compare simulation and mathematical results.

Initial conditions in the DYNAMO programme (factory echelon)

SS=RR	IA=AI.RR
MD=RR	CP=DC.RRI
RS=RRI	OP=DP.RRI
UO=RRI (DH+DU)	

Appendix C: The APIOBPCS model

Difference equations for simulating the Beer Game using the APIOBPCS ordering rule (single echelon)

$$CONS(t) = \begin{cases} 0, \text{if } t \leq 0 \\ 1, \text{if } t > 0 \end{cases} \tag{C-1}$$

$$ORATE(t - Tp) \tag{C-2}$$

$$MAXSHIP(t) = AINV(t-1) + INSHIP(t) \tag{C-3}$$

$$DSHIP(t) = BACKLOG(t-1) + CONSJ(t) \tag{C-4}$$

$$SHIP(t) = MIN[DSHIP(t), MAXSHIP(t)] \tag{C-5}$$

$$AINV(t) = AINV(t-1) + INSHIP(t) - SHIP(t) \tag{C-6}$$

$$BACKLOG(t) = BACKLOG(t-1) + CONS(t) - SHIP(t) \tag{C-7}$$

$$AVCON(t) = AVCON(t-1) + \frac{1}{1+Ta}\left(CONSJ(t) - AVCON(t-1)\right) \tag{C-8}$$

$$DWIP(t) = Tp \times AVCON(t) \tag{C-9}$$

$$WIP(t) = \sum_{i=1}^{Tp} ORATE(t - Tp - i) \tag{C-10}$$

$$EWIP(t) = DWIP(t) - WIP(t) \tag{C-11}$$

$$EINV(t) = DINV - AINV(t) + BACKLOG(t) \tag{C-12}$$

$$ORATE(t) = MAX\left[0, AVCON(t-1) + \frac{EINV(t-1)}{Ti} + \frac{EWIP(t-1)}{Tw}\right] \tag{C-13}$$

Variables used in the APIOBPCS model (single echelon)

CONS	consumption	AVCON	average consumption
INSHIP	shipments received	DWIP	desired work in process
MAXSHIP	maximum shipment	EWIP	error in work in process
DSHIP	desired shipment	WIP	work in process
SHIP	actual shipment	EINV	error in inventory
AINV	actual inventory	ORATE	order rate
BACKLOG	backlog		

Constants used in the APIOBPCS model (single echelon)

$T_a = 2T_p$*	time to average demand
$T_i = T_p$*	time to recover inventory
$T_w = 2T_p$*	time to recover work in process
$T_p = 3$**	lead-time
DINV=12***	desired inventory

* Recommended by John *et al.* (1994). When different values were used, it is indicated in the text.
** In this thesis, the nominal setting was for $T_p = 8$.
***Initial setting for the Beer Game. When different values were used, it is indicated in the text.

Initial conditions in the APIOBPCS model (single echelon)

CONS=0	BACKLOG=0
ORATE=0	AVCON=0
SHIP=0	WIP=0
AINV=0	EINV=0
EWIP=0	

Appendix D: Calculating ITAE of unfilled orders

In order to obtain the unfilled orders time equation through inverse Laplace Transform, it is necessary to substitute the physical parameters with actual values Appendix B into Equation 5.60. Inserting a unit step change in the customer's requisition (RR), the actual inventory will have the following response:

$$\begin{aligned} UO =& (2 + (10 + 7DI + 7DR)s + (10DI + 10DR + 7DIDR)s^2 + 10DIDRs^3 + \\ & AI(2 + (21 + 3DI + 2DR)s + (30 + 21DI + 14DR + 2DIDR)s^2 + 2(15DI \\ & + 10DR + 7DIDR)s^3 + 20DIDRs^4/((1 + 2s)(1 + 5s)(1 + AI + 2AIs) \\ & (1 + DIs)(1 + DRs)) \end{aligned} \tag{D-1}$$

where the pole ($s = 0$) represents the step input. Under the assumption that the poles in Equation D-1 differs from each other, simple partial fraction expansion can be applied. In the case of repeated poles the special case of the partial fraction expansion method has to be used. Hence, Equation D-1 can now be rewritten as:

$$UO = \frac{A}{(\frac{1}{5}+s)} + \frac{B}{(\frac{1}{2}+s)} + \frac{C}{(\frac{1+AI}{2AI}+s)} + \frac{D}{(\frac{1}{DI}+s)} + \frac{E}{(\frac{1}{DR}+s)} + \frac{F}{s} \tag{D-2}$$

From Equation D-2, it is found that the coefficient of the transient response is given by A, B, C, D and E, while the steady state of the system will be equal to F. By solving the partial fraction expansion, the coefficients can be determined as:

$$A = \frac{125(5(AI+DI)-(-5+DI)DR)}{3(5+3AI)(-5+DI)(-5+DR)} \tag{D-3}$$

$$B = \frac{4}{3} - \frac{8(5+AI)}{3(-2+DI)(-2+DR)} \tag{D-4}$$

$$C = \frac{2AI^2(5DI+AI(-10+4AI+13DI)-5(AI(-2+DI)+DI)DR)}{(5+3AI)(AI(-2+DI)+DI)(AI(-2+DR)+DR)} \tag{D-5}$$

$$D = \frac{(5+AI)DI^4}{(-5+DI)(-2+DI)(AI(-2+DI)+DI)(DI-DR)} \tag{D-6}$$

$$E = \frac{DR\left(5DR^3+AI(DI(-5+DR)(-2+DR)+DR(-10+7DR))\right)}{(-5+DR)(-2+DR)(-DI+DR)(AI(-2+DR)+DR)} \tag{D-7}$$

$$F = \frac{DR\left(5DR^3+AI(DI(-5+DR)(-2+DR)+DR(-10+7DR))\right)}{(-5+DR)(-2+DR)(-DI+DR)(AI(-2+DR)+DR)} \tag{D-8}$$

The time function of the unfilled orders can be obtained by undertaking the inverse Laplace transform:

$$UO(t) = A.e^{-\frac{t}{5}} + B.e^{-\frac{t}{2}} + C.e^{-\frac{(1+AI).t}{2AI}} + D.e^{-\frac{t}{DI}} + E.e^{-\frac{t}{DR}} + F \tag{D-9}$$

In order to calculate ITAE of the unfilled orders, the error between the target and actual unfilled orders are needed. Investigating Figures 5.8 and 5.9 again, it is found that the target unfilled orders in Forrester's model is fixed and equal to (DU+DH)=2. Hence, the error in the unfilled orders is the difference between (DU+DH) and UO. Hence ITAE of unfilled order can be estimated as:

$$ITAE_{(UO)} = 2 - \left(A.5^2 + B.2^2 + C.\left(\frac{2AI}{(1+AI)}\right)^2 + D.DI^2 + E.DR^2\right)$$

$$= \frac{-8+5DI^2+5DI(7+DR)+5DR(7+DR)+AI^2(45+7DR+DI(9+DI+DR))+AI\left(23+6DI^2+DR(52+5DR)+DI(52+6DR)\right)}{(1+AI)^2} \tag{D-10}$$

Appendix E: Finding limit cycle via describing functions

A general block diagram for a closed loop system containing a discontinuous nonlinear characteristic is shown in Figure E-1. The output of the nonlinearity, f(u), is fed into a linear element with transfer function G(s), generating a signal c which is then subtracted from an external reference input r, giving u=r-c as the input to the nonlinear element.

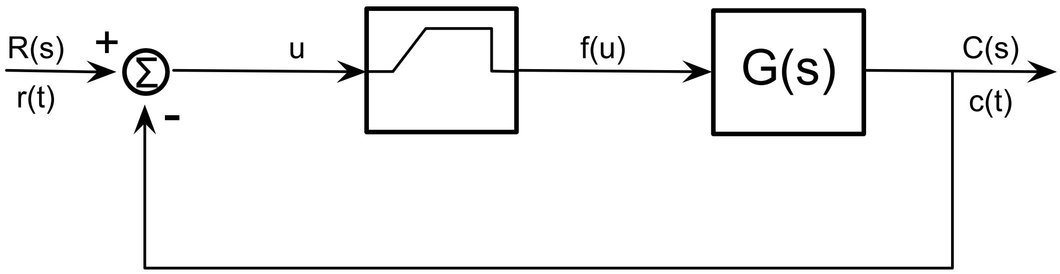


Figure E-1: General block diagram of a nonlinear feedback system

Considering the reference signal r to be constant, so that the system is autonomous, it is assumed that if a limit cycle occurs, it ca be adequately approximated by a sinusoidal oscillation, so that we can take:

$$u \approx A.sin(\omega t + \phi) + B \tag{E-1}$$

and correspondently

$$f(u) \approx N_A.A.sin(\omega t) + N_B.B \approx \mathrm{Re}\{N_A\}A.sin(\omega t) + \mathrm{Im}\{N_A\}A.cos(\omega t) + N_B.B \tag{E-2}$$

where N_A and N_B are the describing function components. Hence,

$$c \approx \mathrm{Re}\{G(i\omega).N_A\}A.sin(\omega t) + \mathrm{Im}\{G(i\omega).N_A\}A.cos(\omega t) + G(0).N_B.B \tag{E-3}$$

so that, setting

$$u + c = r \tag{E-4}$$

and separating oscillatory terms from constants, we get

$$B + G(0).N_B.B = r \tag{E-5}$$

$$A + \mathrm{Re}\{G(i\omega).N_A\}A + \mathrm{Im}\{G(i\omega).N_A\}A = 0 \tag{E-6}$$

From the Equation E-6,

$$\begin{aligned} A(1 + \mathrm{Re}\{G(i\omega).N_A\} + \mathrm{Im}\{G(i\omega).N_A\}) &= 0 \\ 1 + G(i\omega)N_A &= 0 \\ \frac{c}{u} = G(i\omega)N_A &= -1 \end{aligned} \tag{E-7}$$

Hence, oscillations will occur if the product of the open loop frequency response and the describing function is equal to minus one. Nyquist diagrams can be used to to plot the frequency response locus $G(i\omega)$ and $-{}^{1}/_{N_A}$ to find any intersection which corresponds the solution of Equation E-7.