Primary auction of slots at European Airports

Thomas Pertuiset and Georgina Santos

Thomas Pertuiset
Transport Studies Unit
Oxford University
Oxford, OX1 3QY
Tel: + 44 1865 275 991
Thomas.Pertuiset@polytechnique.org

Georgina Santos*
School of Planning and Geography, Cardiff University and
Transport Studies Unit, Oxford University
Glamorgan Building
King Edward VII Avenue
Cardiff ,CF10 3WA
Tel: +44 2920 874462
SantosG@Cardiff.ac.uk

*Corresponding author
Primary auction of slots at European Airports

Thomas Pertuiset and Georgina Santos

Key words


JEL codes

R41, D44

Abstract

We use the Vickrey-Clarke-Groves auction mechanism to propose a system of primary auctions of slots at congested European airports. The system would ensure allocative efficiency and would be incentive-compatible, flexible, understandable, implementable and transparent. Only 10% of slots would be auctioned per year. The current slot coordination mechanism used in Europe, based on historic use of slots, would thus be phased out and disappear within a decade.

Introduction

The air transport industry is of crucial importance for the economy. It links both people and businesses. Worldwide, since the early 1970s air passengers have increased ten-fold and air freight has increased fourteen-fold (International Air Transport Association, IATA, 2011, p.1). Airport capacity, however, has not kept pace with the growth in airport traffic and demand for air travel (Czerny, 2010) and as
a consequence, delays at airports are very common around the world. In Europe, for example, almost 18% of all Intra-European flights leaving from major airports departed more than 15 minutes later than their scheduled departure time in 2009 (Eurocontrol, 2010, p.18). Although weather is the most important and common reason for delays, the second reason is traffic exceeding airport capacity (Brueckner, 2002a, p.1357).

One obvious solution to reduce delays at airports is to invest in new runways, but ‘the long gestation period of such projects means that the benefits lie far in the future’ (Brueckner, 2002b, p.141). Despite the plans to increase capacity at several European airports, in order to meet projected demand growth, immediate action could be taken that would increase the efficiency of the system in the short-run.

According to basic theory of externalities (see for example, Baumol and Oates, 1988) the two main approaches to reduce the level of externality (in this case, delays) are command-and-control policies (where typically a cap on quantity is set) and incentive-based policies (where economic agents can make choices).

A slot coordination system can be seen as a command-and-control type of policy because it imposes a quantity control and in principle, trading is not allowed. It reduces congestion because it lessens the ‘clustering and randomness of arrivals and departures’ (Forsyth and Niemeir, 2008, p.63).

Congested airports in the EU are subject to a slot coordination process. Regulation (EC) N° 793/2004 (European Parliament and Council of the European Union, 2004), which amends Council Regulation (EEC) N° 95/93 (Council of the European Communities, 1993), requires member states to appoint an independent entity in charge of slot allocation at an airport, if it experiences excess demand for slots. Thus,
all airports in Europe can be classified as non-coordinated airports, schedule facilitated airports and fully coordinated airports.

Non-coordinated airports are airports that have no excess demand and where slot coordination is not needed. Schedules-facilitated airports are airports ‘where there is potential for congestion at some periods of the day, week or scheduling period’ (IATA, 2005, p.7) and where schedules are facilitated by a coordinator. Fully-coordinated airports are airports ‘where … congestion is at such high level that … the demand for facilities exceeds availability during the relevant period’ and ‘attempts to resolve problems through voluntary schedule changes have failed’ (IATA, 2005, p.11). All airlines wishing to land or take off at such airports during the periods for which they are fully coordinated need to have a slot allocated by a coordinator.

Unsurprisingly, slot coordination is not an efficient solution from an economic point of view, as airlines that value slots at peak times and would be prepared to pay for them, are not necessarily given the opportunity to do so.

The process of slot allocation in the EU is described in article 8(1) of Council Regulation (EEC) N° 95/93 (Council of the European Communities, 1993). Basically, ‘a slot that has been operated by an air carrier as cleared by the coordinator shall entitle that air carrier to claim the same slot in the next equivalent scheduling period’, which means that airlines are typically able to keep their slots.\(^1\) This set of rules is usually known as ‘grandfather rights’.

At the same time, Article 8(4) specifies that slots can be ‘freely exchanged between air carriers or transferred by an air carrier from one route, or type of service, to another, by mutual agreement or as a result of a total or partial takeover or

---

\(^1\) The exception to that is detailed in article 10(3), which specifies that the airline will not be entitled to keep those slots unless it can demonstrate that they have been operated for at least 80% of the time during the period for which they were allocated.
unilaterally’, as long as the exchange is agreed by the coordinator. Although money payments are not legislated, a grey market, with secondary trading and monetary exchange has developed at London Heathrow (National Economic Research Associates, NERA, 2004, p.53).

If regulation 95/93, amended by regulation 894/2002, were amended to allow airlines to trade slots for money throughout the EU, the grey market would cease to exist and a new proper market would emerge. Indeed, although secondary trading has not been formalized yet, in April 2008 the European Commission issued a ‘clarification’ of the Slot Regulation (Commission of the European Communities, 2008), which endorsed the UK model of slot trading. Furthermore, in November 2012 a draft European Parliament Legislative Resolution was approved to allow market-based mechanisms in slot trading and a strengthened slot allocation process (European Parliament, 2012; Library of the European Parliament, 2012).

Slot trading, as proposed, would be a natural transition from slot coordination, and would increase economic efficiency in the sense that the slots would go to those airlines that value them the most. A step further yet, would be to auction slots in the first place. The slots at schedule facilitated and slot coordinated airports, currently allocated on the basis of historic use, commonly known as ‘grandfather rights’, could be initially auctioned. Needless to say, airlines will typically oppose the idea of auctioning (Sentance, 2003). Clearly, auctioning would improve allocation efficiency and would ensure that slots were used more effectively (Button, 2008, p.292).

Auctioning has a number of advantages over grandfathering: it reduces barriers to entry, increases regulation stringency, prevents the possibility of wind-fall profits, and generates revenues that can be recycled for environmental purposes and/or airport expansion/improvements, amongst other uses.
In this paper we propose the Vickrey-Clarke-Groves auction mechanism for slot allocation at European airports. It is important that any mechanism for primary auctions is efficient from an economic point of view and from the airlines’ point of view. Using the Vickrey-Clarke-Groves mechanism for primary auction would ensure so.

1. Description of the auction mechanism

The aim here is to develop an auction mechanism to allocate slots that will satisfy certain constraints.

First, we want the auction mechanism to be allocatively efficient, i.e. to maximise the value of the allocation, and to be incentive-compatible. A mechanism is incentive-compatible if it is structured such that each bidder finds in its interest to report his valuation honestly. We also want the auction mechanism to be flexible enough, so that airline carriers (especially hub carriers) can develop a strategy to schedule departures and arrivals. Finally, we want the auction mechanism to be understandable, quite easily implementable and transparent.

To allow airlines (especially the hub ones) to have a scheduling strategy, an interesting idea is to sell slots by set. That is why we chose a “generalised Vickrey-Clarke-Groves mechanism” for multiple non-identical objects, which yields efficiency. It is based on the auction mechanism developed by Vickrey (1961) for one good, and then extended by Clarke (1971) and Groves (1973) for multiple goods. The mechanism that we use is a light and adapted version of the generalisation of the Vickrey-Clarke-Groves (VCG) mechanism developed by Dasgupta and Maskin (2000) and by Ausubel and Milgrom (2002, 2005). Basically, the result of such an
auction will be a partition of the set of the auctioned goods across bidders, which maximises the income of the seller.

One idea developed by NERA (2004) would be to auction only 10% of slots per year, all slots being allocated in a rolling programme of ten-yearly auctions. To this 10% of slots all the slots in the pool\(^2\) would be added, which should not be significant if secondary trading was allowed too. But even if only 10% of slots were auctioned every year, given the quantity of slots involved, which at some airports can be 1,500 per day, for practical reasons we propose to split the day in different periods and to have as many auctions as periods. The periods must be neither too short (so that carriers can cluster departures and arrivals if they wish to), nor too long (so that the number of combinations is small enough to allow airline carriers to evaluate almost all the combinations of slots and to solve the maximisation program of the auction within a reasonable time). Therefore, we propose to split the day in periods of one hour at peak times and of two hours at off-peak times. This would of course need to be thought out and defined more accurately by and for each airport.

In the following paragraphs, we present the model we will use to describe the bidders (the airline carriers) and the set of goods (the slots). Then we present the program that the seller (the airport) will solve, the prices that the bidders will pay to get their set of goods, and finally, the efficiency properties of that auction mechanism.

\(^2\) The pool contains the slots that were not requested by (allocated to) any carrier, plus all the slots that were returned by carriers, plus all new slots, plus the slots that were not used and were therefore lost by carriers.
2. Description of the subsets of goods

For the period \( t \), a seller has a quantity \( S_t \) of indivisible heterogeneous goods, denoted by \( s = 1, \ldots, S_t \), to sell in set to \( N \) bidders, denoted by \( i = 1, \ldots, N \).

There are \( R_t = 2^{S_t} \) possible subsets of goods and each combination is valued and ranked by all the bidders.

For the bidder \( i \), let \( \mathcal{Z}_{i,t} \) be the set of all the possible combinations of the \( S_t \) goods, where each combination is denoted by its rank \( r_i \).

For instance, if \( r_i \geq r_i' \), it means that the combination \( z_{i,t}^i \) is preferred to the combination \( z_{i,t}^{i'} \).

Each combination \( z_{i,t}^i \) is a vector of dichotomous elements:

\[
\forall s \in S_t, \quad z_{i,t}^i(s) \in \{0, 1\}
\]

If \( z_{i,t}^i(s) = 1 \), then the bidder bids to get the good \( s \) in the set; if \( z_{i,t}^i(s) = 0 \), then the bidder does not want the good \( s \) in its set.

To each combination \( z_{i,t}^i \), the bidder \( i \) associates a true value \( v_{i,t}^i \) (which is private information) and submits a corresponding bid \( b_{i,t}^i \). We make two assumptions about the value and the bid. First, we assume that if the combination

\footnote{All notation is summarised in the Appendix.}
$z_i^{r_i,t}$ is preferred to the combination $z_i^{r_i',t}$, then $z_i^{r_i,t} \succeq z_i^{r_i',t}$. Second, the bidder will never bid more than his true value, so that $z_i^{r_i,t} \succeq z_i^{r_i',t}$.

The goal of the auction is to allocate the goods among the bidders efficiently while respecting the capacity constraint, which is that only one bidder can get good $s$.

At the end of the auction, the seller tells each bidder which subset of goods he gets. Let $M_i = \{h_i^{r_i,t} \mid r_i \in R\}$ be the set of combinations allocated to bidder $i$, where $h_i^{r_i,t}$ is a dichotomous element, which is equal to 1 if $i$ gets his subset of rank $r_i$, and else equal to 0:

$R \sum_{r_i \in R} h_i^{r_i,t} \in \Omega_i \forall$ 

Each bidder gets one of the possible subsets of goods at most:

$R \sum_{r_i \in R} h_i^{r_i,t} = 1 \forall$ 

Each good $s$ can only be allocated to one bidder at most. Hence, the final allocation $H = \{h_i^{r_i,t} \mid i \in N\}$ must satisfy the following capacity constraint:

$N \sum_{i \in N} h_i^{r_i,t} \leq 1 \forall s$
3. Goods allocation

At the beginning of the auction, bidders simultaneously make sealed bids to the seller; and they send him a collection \( \left\{ \frac{j}{i} \right\} \) of bids for every possible slot combination. We make the assumption that a bid represents a real commitment of resources by the bidder and that he is not permitted to withdraw his bid. Then, the allocation of goods (slots) between bidders (airlines) is obtained by maximising the objective function, which is the sum of the bids for the allocated goods.

The maximisation program is built under the constraints above (i.e. that each bidder gets one subset of goods at most and that each good \( s \) can only be allocated to one bidder at most). Finally, the allocation \( H \) resulting from the maximisation program is announced.

Each bidder gets a slot combination and pays a price for it. This price is not exactly equal to his bid for that combination, and the way in which it is computed is presented in the next section.

The final allocation is the result of the following program:

\[
\max \sum_{i}^{N} \sum_{h}^{R} \frac{j}{i} \cdot \frac{j}{h}
\]

where:

\[
H \left\{ \frac{j}{i} \right\} \subseteq \left\{ \frac{j}{h} \right\}
\]

subject to:

\[
. \quad \frac{j}{i} \in \left\{ \frac{j}{r} \right\} \quad \text{(bidder } i \text{ gets subset of rank } r_i \text{ if } 1)
\]
(each bidder gets one subset at most)

(capacity constraint, each slot is allocated to one bidder at most)

4. Price

The price that bidder $j$ pays is defined as the difference between the sum of the resulting value of all the other bids (apart from his) after the auction has taken place (and the objective function has been maximised) and the sum of the resulting value (after maximisation of the objective function) of all bids if bidder $j$ did not join the auction.

Let $G_i$ be the result of the following problem, which is the one in which bidder $j$ does not join the auction:

$$\max_{G_i} \sum_{i=1}^{N} R_i \forall i$$

where:

subject to:

$$R_i \in \mathbb{N}, \forall i$$
(each bidder gets one subset at most)

(bidder \( j \) does not join the auction)

(capacity constraint)

Hence, price \( p_{j,t} \) paid by bidder \( j \) to get his subset of goods is equal to:

**Proposition 1**: The price \( p_{j,t} \) paid for the allocated subset is always lower than the bid submitted. Formally,
By construction, $G_t$ lives in a space of constraints that is included in the space of constraints for maximising the objective function and thus for determining $H_t$ (see Section 3). By definition of $H_t$, the objective function computed with $G_t$ is necessarily lower than the objective function computed with $H_t$. Therefore, the equation above is necessarily positive, which means:

$$
\frac{R}{\mathcal{P}} \sum_{j \in A} \sum_{j'} \Phi_j \cdot r_{j'} - \sum_{j \in A} \frac{R}{\mathcal{P}} \sum_{i \in \mathcal{I}} \sum_{i'} \Phi_i \cdot r_{i'}
$$

5. Dominant strategy of a bidder

Let us start by recalling some definitions from game theory. A strategy is dominant if it maximises a player’s utility pay-off regardless of the opposition’s choice of strategy. A dominant strategy equilibrium is a choice of strategies by each player (bidder) such that each strategy dominates every other strategy available to that player. Trivially, a dominant strategy equilibrium is a Nash equilibrium.\(^4\)

\(^4\) A Nash equilibrium is a set of choices for which each player’s choice is optimal, given the choices of the other players.
We define the surplus of the auction for bidder \( j \), denoted \( \Pi_j(H_t) \), as the difference between his true value for his allocated subset of goods and the price he actually pays to the seller:

\[
\Pi_j(H_t) = \sum_{i \in H_t} t_i - \sum_{i \in J \setminus H_t} t_i
\]

If we assume that each bidder is risk-neutral, this surplus is the utility pay-off of the bidder \( j \).

Let \( B_j \) be a set of bids for \( t_R j \in U \), which is different from \( v_j \in U \), the bidder’s set of true values for the ranked combinations. Let \( H_j \) be the resulting allocation of the auction when bidder \( j \) bids \( B_j \) and all the other bidders bid a set of bids, which reflect their true values \( V_k \).

Let \( H_k \) be the resulting allocation of the auction when bidder \( j \) bids his true value \( V_k \) for all the different combinations and the other bidders bid the same set \( B_k \).
Proposition 2: *Ceteris paribus*, the surplus of bidder $j$ is always greater when he bids his true value,

\[
\text{Proof:}
\]

When bidder $j$ bids $B^j$, the maximised value of the surplus if he does not join the auction is the same as when he bids $V^j$, because the bids of the other bidders are the same in the two cases. Thus,

\[
\text{As } H^1 \text{ and } H^0 \text{ live in the same space of constraints for the maximisation of the objective function (in the case of the set of bids } V^j \text{), by definition of } H^0, \text{ the result is positive:}
\]

To sum up, for each bidder, the strategy of bidding his true value dominates all other strategies. Consequently, if each bidder bids his true value, the result of the auction is a dominant strategy equilibrium.

6. Efficiency of the VCG auction mechanism
An auction is said to be allocatively efficient if there exists an equilibrium such that every other allocation has a valuation that never exceeds the equilibrium valuation.

In our case, an allocation $H_t$ is efficient if it is the solution to the following program:

$$\max_{\prod_{i} R_i} \sum_{i} v_i$$

subject to:

- $H_i^{t} \subseteq \prod_{i} R_i$ \forall i \in N$ (one bidder = one set)

- $\sum_{i} H_i^{t} \subseteq \prod_{i} R_i$ \forall i \in N$ (constraint of capacity)

An auction mechanism is truth-revealing if the bidders have incentives to reveal their true value during the auction. An auction mechanism is incentive-compatible, if the best strategy for each bidder is to bid his true value. Therefore, if an auction mechanism is incentive-compatible, it is truth-revealing.

*Proposition 3*: The mechanism described above is incentive-compatible and is allocatively efficient.
Proof:

As concluded in Proposition 2, the mechanism is incentive-compatible. Moreover, as the best strategy for each bidder is to submit the set of bids \( B \), the program of maximisation presented in the above becomes exactly the program which gives the social optimum. Therefore, that auction mechanism is allocatively efficient.

7. **Strengths and weaknesses of the mechanism**

The advantages and disadvantages of the VCG auction are well-known and have been carefully described in Green and Laffont (1979), Holmstrom (1979), Milgrom (1981), Rothkopf *et al* (1990), McMillan (1994), Klemperer (1998), Williams (1999), and Ausubel and Milgrom (2002, 2005). Here we only present the ones that directly affect our model.

The first important advantage of the VCG mechanism that affects our model is the dominant-strategy property, which reduces the costs of the auction by making it easier for bidders to determine their optimal bidding strategies and by eliminating bidders’ incentives to spend resources learning about competitors’ values or strategies. Such spending is pure waste from a social perspective, since it is not needed to identify the efficient allocation, yet it can be encouraged by auction formats in which each bidder’s best strategy depends on its opponents’ likely actions.

The dominant strategy property also has the advantage of adding reliability to the efficiency prediction, because it means that the conclusion is not sensitive to assumptions about what bidders may know about each others’ values and strategies. This is a distinctive advantage of the VCG mechanism (Green and Laffont, 1979; Holmstrom, 1979).
Another advantage of the VCG mechanism is its scope of application. Propositions 1, 2 and 3 above do not impose any restrictions on the bidders’ rankings of possible slot combinations. The basic rules of the auction mechanism presented in this paper can be further adapted if the auctioner (the airports) or the European Commission wish to impose some additional constraints. For example, the Commission may wish to limit the concentration at European airports by setting a cap on the proportion of slots that airlines hold at a particular airport. One can add a constraint such as for example, $\sum \frac{s_i}{m}$, where $m$ is a certain number set by the administration, or any other linear constraint, without affecting the preceding theory or arguments in any essential way.

Apart from the complexity and the cost of determining valuations (discussed below), the VCG mechanism has several possible weaknesses. The most important disadvantage is that the revenues it yields can be very low or zero, even when the items being sold are quite valuable. A classic example is that of a hypothetical auction of two spectrum licenses to three bidders (Ausubel and Milgrom, 2005). Bidder 1 wants only the package of two licenses, for which he is willing to pay $2 billion, while bidders 2 and 3 are each willing to pay $2 billion for a single license. A quick calculation establishes that the Vickrey outcome allocates the licenses to bidders 2 and 3, each at a price of zero (!). This defect of the VCG mechanism is, by itself, decisive for most practical applications.

The second and third disadvantages of the Vickrey design are its vulnerability to shill bidding (a bidder’s use of multiple identities in the auction) and collusion, even by losing bidders. However, in economic environments where goods are substitutes for all bidders, Ausubel and Milgrom (2002, 2005) show that these first three
weaknesses never occur. Besides, in our case, if the slots can be later traded, there will never be shill bidding and the risk of collusion can be reasonably prevented.\(^5\)

Finally, another potentially significant issue is the cost of determining valuations. When there are many combinations, as is the case of slots at airports, it can be prohibitively expensive for the airline to assign a value to each combination. Having said that, the airline could decide to evaluate only a few from the whole possible number of combinations. That is why the splitting of the day and the setting of a cap on slots holding can help to lower the implementation costs by lowering and improving the number of valuable slot combinations. In any case, and as in any market, there will be transaction costs greater than zero, which may diminish part of the economic benefits of trading.

8. Conclusions

The VCG auction mechanism applied to primary allocation of slots at airports as presented above would be allocatively-efficient, truth-revealing and it would also allow airlines to develop their scheduling strategies. Under the assumption that the bids of each bidder were revealed at the same time as the results from the auction, it would be transparent because the maximisation program and all its parameters (the bids, the available slots, and the constraints) would be shared information across bidders, so that they would be able to check the results of the auction \textit{a posteriori}. This mechanism would also be fairly easy to understand for airlines, because all they would need to produce would be a list of bids for each subset of goods – even if the production of that list would probably be long and complicated.

\(^5\) Oligopolistic dominance and tacit collusion are prohibited in the EU under article 81(1) of the EC Treaty and the abuse of collective dominance is controlled and condemned by article 82 (Official Journal of the European Communities, 2002).
NERA (2004, p.214) concludes that “auctions of 10% of slots, combined with secondary trading could, in theory, achieve the most efficient allocation of slots possible. But in practice, many of the auctions are likely to be so complex, both for auction organisers and for airlines bidding for slots, that it is probably unlikely that an efficient allocation of slots will emerge from this process”.

This paper proposes the use of the VCG mechanism which, although complex, is quite easily solvable by common softwares. It also ensures allocative efficiency, as long as the day is split in effective periods (for example, periods of 1 hour at peak times and of 2 hours at off-peak times) and both airlines and airports have incentives to take part. The system has all the desired characteristics, described at the beginning of the paper: it is allocatively-efficient, incentive-compatible, flexible, understandable, implementable and transparent.

Future lines of research include the estimation of transaction costs as well as different objectives of a primary auction, such as airport revenue maximisation instead of allocative efficiency. Further research should also look into secondary trading mechanisms, in line with the latest proposals from the European Commission. Even after an initial free allocation of slots, these could be later traded amongst airlines. Grandfather rights do not ensure ‘that slots are allocated to those who attach the highest value to them’ (Menaz and Matthews, 2008, p.35) and therefore ‘will typically not be particularly efficient’ (DotEcon, 2006, p.2). Secondary markets would remove ‘residual inefficiencies from the primary allocation process, responding to changing circumstances between primary allocations and allowing adjustments of slot
holdings for international coordination across congested airports’ (Maldoom, 2003, p.59).6

Acknowledgements

Support from the Economic and Social Research Council under award RES-000-22-0896 is gratefully acknowledged. The authors are grateful to Thomas Greve from Cambridge University, to Paul Milgrom from Stanford University and to two anonymous referees for suggestions on an earlier version of this paper. Any remaining errors are the authors’ own.

References


6 Having said that, when secondary trading was introduced at US slot constrained airports, an initial active trading period was followed by a quiet market with few transactions. Although ‘the small number of transactions is not itself proof that the secondary market failed, it is doubtful the result was an optimal use of airport capacity’ (Whalen et al, 2008, p.33).


### Appendix: Summary of the notations used for the auction

<table>
<thead>
<tr>
<th>Notation</th>
<th>Set</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>day</td>
<td>Period of the day (e.g. 1 hour)</td>
</tr>
<tr>
<td>$s$</td>
<td>$[1,S]$</td>
<td>Slot number $s$</td>
</tr>
<tr>
<td>$i$</td>
<td>$[1,N]$</td>
<td>Airline $i$</td>
</tr>
<tr>
<td>$r_i$</td>
<td>$[1,R_i]$</td>
<td>Rank given to a slot combination by airline $i$</td>
</tr>
<tr>
<td>$z_{r_i,t}^i$</td>
<td>$Z_t^i$</td>
<td>Slot combination of rank $r_i$ for carrier $i$</td>
</tr>
<tr>
<td>$\left( \begin{array}{c} i \ r_i,t \end{array} \right)_s$</td>
<td>${0,1}$</td>
<td>Dichotomous variable: 1 if carrier $i$ wants slot $s$ in the combination of rank $r_i$; 0 otherwise</td>
</tr>
<tr>
<td>$v_{r_i,t}^i$</td>
<td>$V_t^i$</td>
<td>Valuation of the slot combination of rank $r_i$ by $i$</td>
</tr>
<tr>
<td>$b_{r_i,t}^i$</td>
<td>$B_t^i$</td>
<td>Submitted bid for the slot combination of rank $r_i$ by $i$</td>
</tr>
<tr>
<td>$h_{r_i,t}^i$</td>
<td>${0,1}$</td>
<td>Dichotomous variable: 1 if carrier $i$ gets its slot combination of rank $r_i$; 0 otherwise</td>
</tr>
<tr>
<td>$p_{j,t}^i$</td>
<td>$[0, +\infty]$</td>
<td>Price paid by $i$ to get slot combination of rank $r_i$</td>
</tr>
</tbody>
</table>