FISCAL POLICY AT THE ZERO LOWER BOUND AND IN MACRO-FINANCE MODELS AT 'NORMAL TIMES'

By

Lorant Kaszab

A Thesis Submitted in Fulfilment of the Requirements for the Degree of Doctor of Philosophy of Cardiff University

Economics Section,
Cardiff Business School,
Cardiff University

Supervisor: Dr. Panayiotis Pourpourides

July 2014

DECLARATION

This work has not previously been accepted in substance for any degree and is not concurrently submitted in candidature for any degree.

Signed ....(candidate)

Date ....

STATEMENT 1

This thesis is being submitted in partial fulfillment of the requirements for the degree of ....(insert MCh, Md, MPhil, PhD etc, as appropriate)

Signed ....(candidate)

Date ....

STATEMENT 2

This thesis is the result of my own independent work/investigation, except where otherwise stated. Other sources are acknowledged by footnotes giving explicit references.

Signed ....(candidate)

Date ....

STATEMENT 3

I hereby give consent for my thesis, if accepted, to be available for photocopying and for inter-library loan, and for the title and summary to be made available to outside organisations.

Signed .... (candidate)

Date ....

Abstract

The main topic of the dissertation is the effect of fiscal policy in dynamic-stochastic general equilibrium (DSGE) models. Fiscal policy has received considerable attention after the introduction of the stimulus package of U.S. government in 2009. In addition, we allow for the fact that the zero lower bound on the short-term nominal interest rate was binding since 2008 in the United States. In Chapter 1 we study a deterministic labour tax cut at the zero lower bound and contribute to the literature by showing that a labour-tax cut increases GDP when non-Ricardian households and wage rigidity are included in the model. In chapter 2 and 3 (both co-authored with Ales Marsal) we depart from the assumption of zero lower bound. In particular, Chapter 2 discusses the effect of fiscal policy on the yields of non-defaultable zero-coupon government bonds employing a New-Keynesian model with only price rigidity and Ricardian consumers. There are several empirical papers supporting the view that there is positive relationship between indicators of fiscal stance (like budget deficit) and yields of different maturities. We show that income taxation raises long-term nominal bond yields implying higher inflation-risks than with lump-sum taxation. Income taxation also generates a rise in inflation-risks with smaller risk-aversion coefficient than the one needed in a model with time-varying inflation-target.

In chapter 3 we extend the model used in chapter 2 with costly firm entry and find that the model is able to account for high bond and equity premia without compromising the fit of the model to key macroeconomic regularities. The model is also successful in achieving inflation risks even with low coefficient on the output gap in the Taylor rule. The downside of this extension is that it yields a counterfactually low volatility of equity.

Acknowledgements

Contents

List of Figures

List of Tables

Introduction

In response to the financial crises and the recession started in 2008 the United States issued a massive $780 billion fiscal stimulus package called American Reinvestment and Recovery Act of February 2009 which provided tax credits of up to $800 per year to working households. At the onset of the crises banks were reluctant to lend to each other due to confidence problems. To ease credit conditions the Federal Reserve lowered the rate of its marginal lending facility to a level close to zero at the end of 2008 i.e. the zero lower bound on the nominal interest rate has become binding. In the peculiar environment characterised by low and fixed interest rates Eggertsson (2011) and Christiano et al. (2011) argued that a labour-tax hike is stimulating the economy in a basic New Keynesian model containing only price-rigidity. Their argument about the depressing effects of a labour tax-cut is the following. Lower taxes on labour causes an outward shift in labour supply decreasing wages and the marginal cost which leads to a fall in inflation through the New Keynesian Phillips curve. Falling prices (deflation) are associated with higher real interest rates (through the Fisher relationship) when nominal interest rate is stucked at zero level. Higher real interest rates discourage consumption today relative to future and results in lower aggregate demand. Therefore, aggregate demand and labour demand are *upward*-sloping at the zero-lower bound as demonstrated by Eggertsson (2010, 2011). All these previous models assume, however, that households in the model are Ricardian that is, they correctly anticipate the future tax-burden of the tax-cut in the present and, therefore they do not interpret the tax-

cut as an increase in their disposable income. Instead, Ricardians save up the amount of the tax-cut and do not consume it. Another way of telling this story is that Ricardians behave according to the Permanent Income Hypothesis and care only about their lifetime income that is not influenced by a tax-cut in the present.

However, there is some empirical evidence for non-Ricardian behaviour in the U.S. For instance, Parker (1999) finds that households respond to a temporary cut in payroll taxes by increasing their consumption expenditures. Hence, we enrich the previous model with non-Ricardian consumers along the lines of Gali et al. (2007) and show that the tax-cut boosts the disposable income of non-Ricardians who increase their consumption expenditures generating a demand/Keynesian effect. Due to sticky prices a large fraction of firms which are not able to revise their prices raises their labour demand and production in response to the positive demand shock (the tax-cut). The share of non-Ricardian households is chosen to be 30 percent in line with lower end of the estimates on the US economy.

The zero lower bound on the nominal interest rate in the model becomes binding due to a large negative demand shock (a discount factor shock that induces agents to save). With flexible wages the effects of the negative demand shock is a large fall in the real wage which is attenuated in the paper by introducing wage stickiness. As Christiano (2010) points out wage rigidity diminishes the relevance of the labour supply channel in the propagation of the labour tax-cut shock and there is more scope for the tax-cut to take positive effect on the economy through the labour-demand/aggregate demand channel whose importance is magnified by non-Ricardians. Therefore, the incorporation of non-Ricardian consumers and wage rigidity into the model of Christiano et al. (2011) helps to make the case for a labour-tax cut policy and the result remains true across different ways of financing this policy.

In particular, we consider two ways of financing the tax-cut. First, the tax-cut is covered by lump-sum taxes paid only by Ricardians such that government budget is balanced in each period. This is perhaps the simplest possible fiscal scenario that can be constructed and is useful for delivering closed-form solutions and for analysing certain properties of the model. Second, we depart from balanced budget and, instead, assume that the tax-cut is financed by government debt which is paid back through labour tax revenue collected from both types of households in the same proportion (uniform taxation) either in the short- or long-run. When debt is retired in the long-run non-Ricardians enjoy the increase in their disposable income by consuming more in the present. Whereas Ricardians consume less in anticipation of the future tax-burden (negative wealth effect). Alternatively we can say that non-Ricardians receive a transfer from Ricardians in the present and they pay back the transfer to Ricardians in the future. When debt is settled in the short-run the tax-cut is offset by a tax-hike in the near future so that the policy has no effects on output. In sum, the benign effects of the tax-cut are associated with governments operating with long-run bonds which usually constitute large part of the debt obligations of modern states.

Without giving a comprehensive overview of the empirical literature we note that papers by Romer and Romer (2010) and Mertens and Ravn (2012) using different types of identification of tax-changes in Vector Autoregressions (VARs) found that personal income tax cuts have stimulative effects on the economy.

Chapter 2 joins a growing body of work using a dynamic-stochastic general equilibrium (DSGE henceforth) model to study the interaction between asset prices and the macroeconomy. Rudebusch (2010) identifies three different directions of term-structure modeling. The most recent strand of macro-finance models (used in chapter 2 and 3) are fully structural and micro-founded DSGE models in the sense that

they are the result of optimal decisions of forward-looking agents with rational expectations. In fact, this literature studies the bond/equity-pricing implications of standard macroeconomic DSGE models which typically preclude the possibility of riskless arbitrage. DSGE-based term-structure modeling started to really evolve with the seminal paper of Jermann (1998) which applied a basic RBC model with capital adjustment costs and fixed labour supply and made the assumption of log-normal pricing kernel to derive a second-order approximation to bond and equity yields.

The second strand of literature employs the canonical finance linear (or affine) arbitrage-free model to give a statistical description of the yields through a couple of unobservable (or latent) factors with an arbitrage-free condition. While being popular with finance practitioners due to their superb empirical performance these models fail to offer sufficient insight into the economic nature of the underlying latent factors (Rudebusch (2010)). Perhaps the most celebrated arbitrage-free canonical finance model is the one with the Nelson-Siegel factor structure.

The third direction of the literature extends the canonical finance term-structure model with a few macroeconomic variables. For instance, Diebold et al. (2006) combines the dynamic Nelson-Siegel finance model of Diebold and Li (2006) with a vector autoregression (VAR) representation of the macroeconomy. Using Kalman filter they extract three latent factors (level, slope and curvature as usual) from a set to 17 yields on US Treasuries and associate these factors with observable macroeconomic variables. In particular, they find the level factor to be highly correlated with inflation and the slope factor co-moves strongly with output but the curvature factor occurs to be unrelated to key macroeconomic variables.

Our approach of modeling macroeconomy and asset prices in a common DSGE framework is greatly supported by Cochrane (2008, pp.

300): *"Clearly, there is much to do in the integration of asset pricing and macroeconomics. It's tempting to throw up one's hands and go back to factor fishing, or partial equilibrium economic models. They are only steps on the way. We will not be able to say we understand the economics of asset prices until we have a complete model that generates artificial time series that look like those in the data."*

Chapter 2 explores the interaction between fiscal policy and the term structure of interest rates in normal times when monetary policy is governed by a Taylor-rule and, thus, we abstract from zero-lower bound issues (see Swanson and Williams (2012) who argue that the ongoing zero lower bound experience in the US did not have large impact on long-term Treasury yields). Orszag and Gale (2004) provides empirical evidence on the positive relationship between fiscal deficits and yields on government bonds with various maturities. Production economies with either time separable preferences or habits face serious challenge in matching the empirical mean and standard deviation of the nominal term premium on long-term bonds without compromising the model's fit to key macroeconomic moments (see Rudebusch and Swanson (2008)). In a recent paper Rudebusch and Swanson (2012) make use of a New Keynesian model with only price rigidity and Epstein-Zin preferences to demonstrate that their model can account for high and variable nominal term premium without violating the fit of the model to key macroeconomic moments.

For time separable preferences the elasticity of intertemporal substitution (EIS) is the inverse of relative risk-aversion. By making households more risk-averse the EIS has to drop. However, Epstein-Zin preferences make it possible to disentangle intertemporal elasticity of substitution from relative risk-aversion so that consumers can be made sufficiently risk-averse without decreasing the EIS too much. In DSGE models forward-looking households wish to have a stable consumption path (consumption-smoothing motive). With habits consumers care

about uncertainty in consumption growth in the short-term but consumers with Epstein-Zin preferences are concerned about consumption growth over medium and long horizons as well as short horizons. It is important to emphasize that habits—even long-memory habits by Campbell and Cochrane (1999)—fail to account for the mean and variability of nominal term premium in a production economy as shown by Rudebusch and Swanson (2008). The New Keynesian model with habits has achieved partial success in terms of fitting the data. For instance, Hördahl et al. (2008) used a basic New Keynesian model with habits and managed to generate high real bond term premium (not nominal) and an upward-sloping real term structure at the cost of violating the fit of the model to some key macroeconomic moments like the standard deviation of hours worked as pointed out by Rudebusch and Swanson (2008). Therefore, we devote our attention to models with Epstein-Zin preferences which are able to facilitate the match of the model to bond market data without worsening the macroeconomic-fit.

Investors of long-term nominal bonds expect a term premium (or risk premium) to get compensated for the consumption and inflation-risks over the lifetime of the bond. In a seminal paper Piazzesi and Schneider (2006) and later Rudebusch and Swanson (2012) argue that temporary productivity shocks are the main source of inflation risks because they generate a negative comovement between consumption growth and inflation. A negative technology shock makes consumption fall due to its strong wealth effect and increases marginal cost and inflation (through the New-Keynesian Phillips curve) eroding real yields. Therefore, assets with low real payoffs in bad times (low productivity) are considered to be risky.

Chapter 2 contributes to the literature by showing that government spending shocks financed by distortionary income taxes produce the negative pattern between consumption growth and inflation similar

to productivity shocks. To illuminate this let us consider a positive innovation to government spending that is financed by issuing long-term bonds which are retired through raising income tax revenue. In a purely Ricardian world (as in chapter 2 and 3) consumers associate higher government purchases with a strong negative wealth effect that induces them to reduce consumption and leisure. The fall in leisure is equivalent to a rise in hours worked for a given time frame. Higher taxes in the future imply higher marginal cost and inflation. Also this chapter shows that fiscal policy engineers an increase in long-term inflation risks of similar magnitude to the version of the Rudebusch and Swanson (2012) model with a time-varying inflation target. We also inferred by comparing the models with fiscal policy and long-run inflation risks that the risk-aversion coefficient needed to achieve the empirical mean of the bond term-premium is lower in the former case.

The model used in chapter 2 performs well in terms of matching bond yields and selected macroeconomic moments. However, it does not capture the high mean and volatility of equities. The first contribution of chapter 3 is about the success of the entry model in matching the high empirical mean of the equity premium as well as the bond-premium without compromising the fit of the model to macro-moments. To do so chapter 3 extends the New Keynesian model of chapter 2 with costly firm entry. Originally firm entry has been introduced to match the countercyclical markup and procyclical profit in U.S. data (see Rotemberg and Woodford (1999)). To our knowledge this is the first paper in the literature which attempts to explain both bond and equity market data in a production economy with costly firm entry and variable labour supply.

In the entry model we make use of a new firm is associated with a new variety (or product) as in Bilbiie et al. (2007, 2012). Also this entry model features the love of variety effect whereby a new variety leads to a reduction in the overall price index. Bernard et al. (2010)

provide empirical evidence on the significant share of product creation and destruction in overall production using data on US manufacturing firms. In particular, they report that product creation by both existing and new firms make up for 46.6 of output in a 5-year period while the value from product destruction at existing or exiting firms constitutes 44 percent of output.

Firm entry in the model is subject to time-varying entry cost and an endogenous lag in production as in Bilbiie et al. (2007) who consider two different types of entry costs which are either specified in effective labour units (version 1) or in consumption units (version 2). We found version 2 of the model to match both macro and finance data better in several dimensions.

The success of the entry model in reproducing the empirical mean of the equity premium is related to the fact that there is a strong positive comovement between consumption and dividends (the yield on equity) in the model. In our simple model dividends correspond to profits. Similar to the bond premium the main driver of the equity premium are temporary technology shocks. In bad times (mostly negative innovations to technology) consumption as well as the payoff to equity (dividends) are low and, thus, equity (or a share of equity) is considered to be a risky asset.

Another contribution of chapter 3—which is related to our second finding (see below)—is the ability of model version 1 in achieving inflation risks when the coefficient on the output-gap is small in the Taylor rule. Papers which estimate the Taylor rule using US data usually produce two markedly different estimates of the coefficient on the output-gap depending on the sample period. For instance, Clarida et al. (1998) provide an estimate of 0.07 for the output-gap on US data over 1979-1994. While Clarida et al. (1999) infer a higher value of 0.93 by using a shorter interval of 1983-1996. The latter time interval can be tagged as a subperiod of what is often called as the period of

Great Moderation (from mid-1980s to 2007). The implications of a small/large output gap coefficient on the inflation risks component of long-term yields is explored in Chapter 3 using the Rudebusch and Swanson (2012) model with/without costly firm-entry.

A baseline New Keynesian model like the Rudebusch and Swanson (2012) model without entry implies a trade-off between stabilising fluctuations in inflation and the output-gap. In particular, a central bank which puts higher weight on the output-gap in the Taylor-rule can achieve lower unconditional standard deviation of the output-gap at the cost of generating higher unconditional standard deviation of inflation (see Clarida et al. (1999)). A large coefficient on the output-gap in the Taylor-rule mutes real risks i.e. reduces the standard deviation of the output-gap and magnifies inflation risks in the Rudebusch and Swanson (2012) model without entry.

Our second contribution in chapter 3 is pointing out that the New Keynesian model with entry features the inflation-output-gap volatility trade-off only for values of the output-gap coefficient lower than 0.5. With an output-gap coefficient higher than 0.5 it is no longer possible to mitigate fluctuations in the output-gap by placing higher coefficient on it in the Taylor-rule. Therefore, the entry model exhibits substantial real risks rather than inflation risks in case of a high output-gap coefficient. This finding is very interesting from policy point of view as it implies a constraint on the ability of monetary policy in stabilising the volatility of output-gap.

As a third contribution we point out that the second version of the entry model produces the property of fiscal policy already seen in chapter 2 i.e. fiscal shocks financed by distortionary income taxes create additional risks. However, fiscal policy is the source of consumption risks in the entry model in contrast to chapter 2 where fiscal shocks lead to a rise in inflation-risks.

As a fourth contribution we recognise that the entry model requires

a risk-aversion coefficient lower than that of Rudebusch and Swanson (2012) in order to match the mean of the nominal term premium. Still the risk-aversion coefficient (75) employed to help match the empirical mean of nominal term premium is considered to be high relative to the estimates of 5-10 by Vissing-Jorgensen and Attanasio (2003). Several suggestions were put forward explaining why a DSGE model needs to feature high risk-aversion in order to account for the high risk-premiums. One of them is Barillas et al. (2009) who identify the reason for the need of a high risk-aversion coefficient is that the agents have perfect knowledge of the model equations and parameters and face low degree of uncertainty about the economic environment.

Chapter 1

Rule-of-Thumb Consumers and Labour Tax Cut Policy at the Zero Lower Bound

1.1 Introduction

Following the enaction of the American Recovery and Reinvestment Act (ARRA) of 2009 which is a \$787 billion fiscal package containing labour tax cuts[1] and various forms of government purchases there has been discussion on the sign and magnitude of fiscal multipliers. On one hand some influential papers using New Keynesian type of models featuring price rigidity concluded that an increase in non-productive government spending can be very effective in stimulating the economy under the zero lower bound which is binding in the US since the end of 2008 (see, e.g., Christiano et al. (2011), Eggertsson (2010a,b) and Woodford (2010)). On the other hand these papers found that labour tax cuts can be contractionary when the zero lower bound on the nominal interest rate is binding.

This paper contributes to the literature on fiscal policy at the zero lower bound by showing that the incorporation of rule-of-thumb (or non-Ricardian) consumers into the baseline new-Keynesian model can render labour tax cut policy expansionary when nominal interest rate

---

[1] In December 2011 President Obama announced that the payroll tax cut is extended until end of 2012.

is zero. Non-Ricardian households behave in a Keynesian fashion i.e. they are willing to raise their consumption expenditure in response to a rise in their disposable income following the tax-cut[2]. Whereas Ricardians recognise that the tax-cut in the present is covered by taxes in the future and therefore their whole lifetime income on which they base their consumption decision is not affected. In this paper we reduce the employees' part of the labour taxes similar to what is prescribed by the US stimulus package of 2009.

First, let us discuss what the baseline log-linear new-Keynesian model with only price rigidity and perfectly competitive labour market with flexible wages delivers in the absence of non-Ricardian consumers in response to a labour tax cut. As in Christiano et al. (2011) the zero lower bound on the nominal interest rate becomes binding due to a discount factor shock (i.e. a negative demand shock) which leads to deflation and a fall in output, consumption and the marginal cost. In a similar model Eggertsson (2011) shows that cutting the labour tax rate has effects similar to the discount factor shock i.e. a random fraction of firms that can change their product price will lower the price because they face a reduction in their production costs. While other firms who cannot reset their price due to price stickiness will produce less and also decrease their demand for labour. When nominal interest rate is zero, the deflationary effect of the labour tax cut is coupled with a rise in the real interest rate that depresses consumption. Also, Ricardian consumers associate the current tax cut with future rises in taxes and decrease their consumption to save up (Ricardian equivalence is valid). Thus, with only Ricardian consumers in the model the tax cut cannot be stimulative.

However, the labour tax cut happens to be expansionary if we incorporate non-Ricardian consumers and wage rigidity into the model.

---

[2]Rule-of-thumb households are excluded from the financial market. Hence, they have no consumption-savings tradeoff (lack of Euler equation) and their decision problem is restricted to the optimal choice between consumption and leisure. The inclusion of rule-of-thumb households into DSGE models is a trivial way of generating incomplete asset markets.

Following the tax-cut Non-Ricardian households consume the rise in their disposable income generating a demand effect. Due to the higher consumption demand of non-Ricardian households firms which cannot alter their price as a result of price rigidity will demand more labour to be able to produce more. In the absence of an imperfectly competitive labour market with nominal wage inertia[3] the discount factor shock and the labour tax cut would lead to an enormous decline in the marginal cost (which equals to the real wage due to the constant returns-to-scale production function in the absence of productivity shocks and physical capital). But the introduction of wage rigidity into the model attenuates the reaction of real wage to the discount factor shock so that the real disposable income of non-Ricardians can rise following the tax cut. Hence, in our setting the effects of the negative demand shock is less severe if there is a simultaneous fall in the labour tax during the zero lower bound period. Our finding is completely in contrast to Christiano et al. (2011) who argue in favour of a labour tax rise at the zero lower bound using a middle-sized DSGE model without rule-of-thumb agents.

In this paper we consider two different ways of financing the labour tax-cut. First, we maintain the simplest fiscal scenario whereby government budget is balanced in each period by lump-sum taxes paid only by Ricardians. Secondly, we depart from balanced budget and introduce endogenous government debt which is retired through labour income taxes levied uniformly on both types of households and collected either in the short- or long-run. In the first case labour tax-cut stimulates the economy by construction. In the second case tax-cut policy boosts economic activity only when debt is paid back in the long-run so that non-Ricardians can enjoy higher disposable income due to the tax-cut in the present. An alternative interpretation is

---

[3]To motivate imperfectly competitive labor markets households (independently of whether they are Ricardian or non-Ricardian origin) become members of unions which set wages for them. It is costly for the unions to change wages because of wage adjustment costs. Hence, we have nominal wage rigidity. See details in the main text.

that non-Ricardians receive a transfer from Ricardians in the present, which is followed by a transfer from non-Ricardians to Ricardians in the future.

Again it needs to be emphasized that the tax-cut is stimulative in this paper because we reduce the employees' part of the labour taxes, which leads to an increase in non-Ricardians' income and a rise in labour demand. In general, it matters a lot whether we cut the employer's or the employee's part of the labour taxes (Bils and Klenow (2008)). In the latter case an average labour tax cut acts like a traditional stimulus tax-cut working through the labour demand while the labour supply is of reduced importance due to wage-setting frictions in the model (Christiano (2010)). However, in the previous case the payroll tax cut directly affects the marginal cost and, as we argue below, acts like a further deflationary factor on the economy besides the negative demand shock. Therefore, in this paper, it is the employee's part of the average labour tax which is reduced.

Our findings are based on deterministic labour tax cut experiments conducted using the shooting algorithm of Christiano et al. (2011) who studied small and middle-sized new-Keynesian models in log-linear form without non-Ricardian consumers. Thus, in our experiments the discount factor shock and the fiscal action (the labour tax cut) are on for a deterministic period of time. This modelling strategy received considerable attention in the recent literature. Here we touch upon two issues. First, Carlstrom et al. (2012) assert that inflation and output impulse responses of a negative demand shock might exhibit unorthodox behaviour—they rise instead of falling—depending on the number of periods for which the interest rate is fixed when there are state variables like price indexation in the log-linear model. However, we do not encounter such a problem for our calibrated value of the

length of the shock[4]. Second, a number of papers[5] raise concerns about the accuracy of the first-order perturbation in modeling the zero-lower bound. However, Christiano and Eichenbaum (2012) present evidence that first-order perturbation remains to be a fairly good approximation to the non-linear model.

This work is closely related to several papers in the literature. One of them is Coenen et al. (2012) who simulate various middle-sized DSGE models including rule-of-thumb households. We differ from that paper for at least two reasons. First, the zero lower bound period in our paper is generated endogenously as a result of a negative demand shock instead of arbitrarily fixing the interest rate for a given time period as Coenen et al. (2012) and Cogan et al. (2010) did. Siemsen and Watzka (2013) describe why it is misleading to model the ZLB in a way that Coenen et al. (2012) did. Even with an increase in government spending output, consumption and inflation is below their steady-state values due to the negative demand shock that makes the zero lower bound binding in Christiano et al. (2011) and, thus, real activity and inflation remain depressed even after the zero lower bound period. Whereas the same variables in Coenen et al. (2012) are above their steady-state even during the zero lower bound in the absence of the negative demand shock and therefore, monetary policy has to tighten outside the zero lower bound period as the Taylor rule, which engineers a sharp increase in real interest rates, is in operation. In contrast, economic activity in Christiano et al. (2011) still remain weak in the aftermath of zero interest rates and therefore monetary policy can be slack (real interest rates are low). Second, we employ simpler models than Coenen et al. (2012) so that we can provide intuition on what model features are needed in order for the labour

---

[4]Depending on the size and length of the discount factor and the fiscal shock (tax cut) the model endogenously generates the date at which the zero lower bound starts and ceases to bind.

[5]See, e.g., Braun and Körber (2011) and Mertens and Ravn (2011). Especially, Braun and Körber (2011) argue using a non-linear model that the ignorance of price adjustment cost in the aggregate feasibility constraint distorts the size and even sign of fiscal multipliers obtained from the log-linear model in which the price adjustment cost is zero.

tax cut policy to be expansionary.

Our paper is also closely related to Bilbiie et al. (2012b) who have shown that cuts in lump-sum taxes stimulate output and raises welfare in an economy featuring two types of households (savers and borrowers), price rigidity and which is constrained by the zero lower bound on the nominal interest rate. This paper is also related to the literature on models containing rule-of-thumb consumers like Bilbiie (2008) and Gali et al. (2007). The model used in this paper is closest to Ascari et al. (2011), Furlanetto (2011), Furlanetto and Seneca (2009) who enrich the model of Gali et al. (2007) with wage-setting frictions.

There is a growing empirical literature which founds labour tax cuts being stimulative. In a well-known study using a narrative approach Romer and Romer (2010) found that tax increases are contractionary. Also, Mertens and Ravn (2012) found using a new narrative account of federal tax liability changes to proxy tax shocks that the short run effects of a tax decrease on output are positive and large. Hall (2009) reviews several empirical studies arguing that households do respond with an increase in their consumption expenditures to a temporary cut in labour tax. Thus, there is enough empirical evidence in support of the positive effects of a labour tax cut.

The rest of the paper is organised as follows. Section 1 describes the agents in the model and their assumed behaviour. Section 2 contains the calibration. In Section 3 we conduct experiments in various models to investigate into the effects of the labour tax cut. The last section concludes.

1.2 The model

1.2.1 Households

Ricardians

There are two types of households: Ricardians and non-Ricardians. Ricardian households are able to smooth their consumption using state-contingent assets (risk-free bonds) while non-Ricardians cannot. The share of Ricardian and non-Ricardian households in the economy is $1 - \lambda$ and λ, respectively. The instantaneous utility function of type $i \in \{o, r\}$ household which can be Ricardian (optimiser (OPT), o) or non-Ricardian (rule-of-thumb (ROT), r), is given by:

$$U_t^i = \frac{(C_t^i - h_i \bar{C}_{t-1}^i)^{1-\sigma} - 1}{1 - \sigma} - \frac{(N_t^i)^{1+\varphi}}{1 + \varphi} \tag{1.1}$$

where C_t^i ($\bar{C}_t^i$) denotes the time-t consumption (aggregate consumption) of type $i \in \{o, r\}$ household and parameter $h_i > 0$ governs the degree of habit formation in consumption. σ is the inverse of the elasticity of intertemporal substitution (EIS) which measures the willingness of households' substituting consumption across time. Further σ stands for relative risk-aversion. The second term on the RHS of equation (1.1) denotes the disutility of household i from working. The Frisch elasticity of labour supply ($1/\varphi$) governs the sensitivity of hours with respect to changes in the pre-tax real wage assuming that the wealth of the household remains unchanged through an appropriate lump-sum transfer. The specification in equation (1.1) follows Constantinides (1990) and recently used by Furlanetto and Seneca (2012) who argue that habit formation is external in the sense that household chooses its own current consumption irrespective of its own past consumption while it is internal to the extent that household of type i relates its current consumption to aggregate consumption of the same class of households. When $h_i = 0$ there is no habit formation. N_t^i

is hours worked by household of type i. Habit formation has some solid psycological foundation according to Cochrane who argues that those *"who got used to an accustomed standard of living might got hurt in case of a fall in consumption after a few years of good times even though the same level of consumption might have seemed very pleasant if it arrived after years of bad times."* Cochrane (2008 pp. 276). Also importantly, Fuhrer (2000) used habits to explain the hump-shaped impulse responses obtained from VAR models in response to a monetary policy shock.

First, we discuss the problem of Ricardian households. They maximise their lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta_t U_t^i, \tag{1.2}$$

where E_0 is the expectation operator representing expectations conditional on period-0 information and β is the discount factor. This maximisation of the optimiser household is subject to a sequence of budget constraints[6]:

$$P_t C_t^o + R_t^{-1} E_t \{B_{t+1}^o\} = (1 - \tau_t^o) W_t N_t^o + D_t^o + B_t^o - P_t T_t^o - F_t - P_t S^o$$

where P_t is the aggregate price level, W_t is the nominal wage and N_t^o is hours worked by OPT. Thus, $W_t N_t^o$ is the labour income received by the optimiser household. T_t^o are lump-sum taxes (or transfers, if negative) paid by the Ricardian household (hence, the superscript o). τ_t^o is a tax rate on labour ($W_t N_t^o$). Profit income is denoted as D_t^o. Further, B_{t+1}^o is the amount of risk-free bonds and R_t is the gross nominal interest rate. Following Gali et al. (2007) and Rossi (2012) we assume, without loss of generality, that the steady-state lump sum taxes (S^o) are chosen in a way that steady-state consumption of ROT and OPT households equal in steady-state. Hence, S^o is a steady-

---

[6]For the rest of the paper, a variable without a time subscript denotes steady-state value.

state lump-sum tax used to facilitate the equality of the steady-state consumptions of ROT and OPT households. F_t stands for a nominal union membership fee (see later on it below). For an alternative approach when steady-state consumptions are not equal see Natvik (2008).

In summary, the optimiser household maximises its lifetime utility with respect to its budget constraint.

The OPT household first-order conditions (FOCs) with respect to consumption (C_t^o) and bonds (B_{t+1}^o) are:

$$\frac{\partial U_t^i}{\partial C_t^i} = (C_t^i - h_i \bar{C}_{t-1}^i)^{-\sigma} = \lambda_t, \text{ with } i = o, \qquad (1.3)$$

$$\beta_{t+1} E_t \left(\lambda_{t+1} \frac{R_{t+1}}{\pi_{t+1}} \right) = \lambda_t, \qquad (1.4)$$

where λ_t is the marginal utility of consumption. In all the above equations that contain expectations we ignore covariance terms.

The linearised[7] version of equation (1.4) is the intertemporal Euler equation:

$$c_t^o = \frac{h_o}{1 + h_o} c_{t-1}^o + \frac{1}{1 + h_o} E_t c_{t+1}^o - \frac{1 - h_o}{1 + h_o} \frac{\beta}{\sigma} [dR_t - E_t \pi_{t+1} - dr_t], \quad (1.5)$$

where $c_t^o \equiv \log(C_t^o/C)$, $\pi_t \equiv \log(P_t/P_{t-1})$ is the time-t rate of inflation, $dR_t \equiv R_t - R$, i.e. the deviation of nominal interest rate from its steady-state value. dr_t can be interpreted as the discount factor shock[8]. Notice that $h_o = 0$ delivers the usual Euler equation without habit formation.

---

[7]The fact that Eggertsson (2010a) log-linearise while Christiano (2010) linearise the same model does not affect the main conclusions. Here we follow the latter strategy.

[8]Following the appendix of Christiano (2010) the time varying discount factor is made equal to the inverse of the real interest rate (R_t^{real}) :

$$\beta_t = \frac{1}{1 + R_t^{\text{real}}}$$

which can be linearised as:

$$\beta \hat{\beta}_t = -\frac{1}{(1+r)^2} dr_t,$$

where $\hat{\beta}_t \equiv (\beta_t - \beta)/\beta$ and $dr_t \equiv R_t^{\text{real}} - R^{\text{real}}$.

The labour supply of OPT household is determined by the union's problem (discussed below).

Non-Ricardians

Non-Ricardian households cannot invest into bonds. In other words, they are excluded from financial markets. Hence, this is the case of limited asset market participation. Therefore, ROT do not make consumption-savings decision (i.e. the lack of consumption Euler equation). ROT households' consumption depends on their disposable income—i.e. the labour income after taxation, $(1 - \tau_t^r)W_t N_t^r$—which is reflected by their budget constraint:

$$\int_0^1 P_t(i) C_t^r(i) di = (1 - \tau_t^r) W_t N_t^r - P_t S^r, \tag{1.6}$$

where $C_t^r(i)$ and N_t^r are, respectively, the consumption of product i and hours worked by rule-of-thumb households. The steady-state lump-sum tax, S^r, ensures that the steady-state consumption of each types of households coincide.

ROT agents exploit relative price differences in the construction of their consumption basket and, in optimum, they obtain:

$$P_t C_t^r = \int_0^1 P_t(i) C_t^r(i) di.$$

Thus, a ROT household maximises its utility (equation (1.2) with $i = r$) with respect to its budget constraint (equation (1.6)).

The budget constraint of ROT households in equation (1.6) can be expressed in linear form as:

$$\hat{c}_t^r = w_t + n_t^r - \chi \hat{\tau}_t^r, \tag{1.7}$$

It follows by using the steady-state condition $\beta = 1/(1 + R^{\text{real}}) = 1/(1 + R)$ that:

$$\hat{\beta}_t = -\beta dr_t.$$

where $\hat{\tau}_t^r \equiv \tau_t^r - \tau^r$, $\chi \equiv 1/(1 - \tau^r)$.

ROT households delegate their labour supply decision to unions.

1.2.2 Firms

The intermediary goods are produced by monopolistically competitive firms of which a randomly selected $1 - \xi^p$ fraction is able to set an optimal price each period as in Calvo (1983) while the remaining ξ^p fraction keep their price fixed. Intermediary good j, denoted as $Y(j)$, is produced by a one-to-one production function:

$$Y_t(j) = N_t(j), \tag{1.8}$$

where $N_t(j)$ is an aggregator of different labour varieties:

$$N_t(j) = \left(\int_0^1 [N_t(j,z)]^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dz \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}},$$

where $N_t(j,z)$ stands for quantity of variety z labour employed by firm j. The one-to-one (constant return-to-scale) production function in equation (1.8) implies that the average (or economy-wide) marginal cost equals to the economy-wide real wage in the absence of technology shocks.

There is a competitive firm which bundles intermediate goods into a single final good through the Kimball (1995) aggregator:

$$\int_0^1 \mathcal{G}(X_t(j)) dj = 1, \tag{1.9}$$

where $X_t(j) \equiv Y_t(j)/Y_t$ is the relative demand and $\mathcal{G}$ is a function with properties $\mathcal{G}(1) = 1$, $\mathcal{G}' > 0$ and $\mathcal{G}'' < 0$. With Kimball specification the elasticity of demand is increasing in the price ($P_t(j)$) set by an individual firm or, equivalently, decreasing in its relative output ($X_t(j)$). After linearisation it turns out that Kimball demand reduces the slope of the Phillips curve (see more below). The Kimball aggre-

gator is also present in popular middle-sized DSGE models like the Smets-Wouters (2007) model.[9]

Proposition 1 *The profit maximisation problem of the perfectly competitive goods bundler (final good producer) gives way to the relative demand for the product of firm j:*

$$X_t(j) = \tilde{\mathcal{G}}\left(\frac{P_t(j)Y_t}{\mu_{f,t}}\right) \tag{1.10}$$

where $\tilde{\mathcal{G}} \equiv \mathcal{G}'^{-1}(.)$ and $\mu_{f,t}$ is multiplier on the constraint (equation (1.9)) in the Lagrangian representation of the final goods producer's maximisation problem:

$$\mu_{f,t} = P_t Y_t \left(\int_0^1 \mathcal{G}'\left(X_t(j)\right)X_t(j)dj\right)^{-1}$$

where the price deflator ($P_t Y_t$) is defined as

$$P_t Y_t = \int_0^1 P_t(j)Y_t(j)dj.$$

Proof. The final good producer problem is given by:

$$\max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(j)Y_t(j)$$

$$\text{subject to}$$

$$1 = \int_0^1 \mathcal{G}\left(\frac{Y_t(j)}{Y_t}\right)dj.$$

Let us denote the Lagrange multiplier as $\mu_{f,t}$ associated with the constraint of this problem.

The first-order conditions with respect to Y_t and $Y_t(j)$ are given, respectively, by:

$$P_t = \frac{\mu_{f,t}}{Y_t}\int_0^1 \mathcal{G}'\left(\frac{Y_t(j)}{Y_t}\right)\frac{Y_t(j)}{Y_t}dj \tag{1.11}$$

---

[9]There are several alternative ways to introduce strategic complementarity into price-setting. In chapter 2, for instance, we employ the model of Rudebusch and Swanson (2012) who use firm-specific capital. Firm-specific labor is another possibility (see e.g. Chapter 3 in Woodford (2003)).

$$P_t(j) = \mu_{f,t} \mathcal{G}'\left(\frac{Y_t(j)}{Y_t}\right)\frac{1}{Y_t}. \tag{1.12}$$

The combination of the previous first-order conditions leads to

$$Y_t(j) = Y_t \mathcal{G}'^{-1}\left[\frac{P_t(i)}{P_t}\int_0^1 \mathcal{G}'\left(\frac{Y_t(j)}{Y_t}\right)\frac{Y_t(j)}{Y_t}dj\right]$$

which is the same as the equation in proposition 1. ∎

Let us define the price elasticity of demand by

$$\Xi(X_t(j)) \equiv -\frac{\mathcal{G}'(X_t(j))}{\mathcal{G}''(X_t(j))X_t(j)}.$$

In the special case when

$$\mathcal{G}(X_t(j)) = [X_t(j)]^{\frac{\varepsilon_p-1}{\varepsilon_p}},$$

equation (1.9) boils down to the usual Dixit-Stiglitz aggregator which implies constant elasticity of substitution: $\Xi(X_t(j)) = \varepsilon_p$ for all $X_t(j)$ (Woodford (2003)). Also in steady-state $X = 1$ and $\Xi(1) \equiv -\frac{\mathcal{G}'(1)}{\mathcal{G}''(1)1} = \varepsilon_p$.

In the standard Dixit-Stiglitz case the demand function can be written as

$$X_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon_p},$$

where the price index is defined as:

$$P_t \equiv \left(\int_0^1 [P_t(j)]^{1-\varepsilon_p}\,dj\right)^{1/(1-\varepsilon_p)}.$$

In the general Kimball case the own price elasticity of the elasticity of demand can be defined as

$$\epsilon(X_t(j)) \equiv \frac{\partial\Xi(X_t(j))}{\partial P_t(j)}\frac{P_t(j)}{\Xi(X_t(j))} = 1 + \varepsilon_p + \varepsilon_p\frac{\mathcal{G}'''(X_t(j))}{\mathcal{G}''(X_t(j))}, \tag{1.13}$$

where in steady-state $\epsilon(1) = \epsilon > 0$ i.e. the elasticity declines if the firm sells more or, equivalently, elasticity is increasing in the price

(Furlanetto and Seneca (2009)).

Intermediary firm z that last reset its price at time $T = 0$ maximises its present and discounted future profits with the probability of not resetting its price:

$$\max_{P_t^*} \sum_{T=0}^{\infty} (\xi^p \beta)^T \Lambda_{t,t+T} \left[P_t^*(j) Y_{t+T}(j) - TC \left(Y_{t+T}(j) \right) \right], \qquad (1.14)$$

where P_t^* is the optimal reset price at time$-t$, ξ^p is the probability of not resetting the price, TC stands for the total cost of production and $\Lambda_{t,t+T}$ is the stochastic discount factor defined as:

$$\Lambda_{t,t+T} \equiv \left(\frac{C_{t+T}^o - h_o C_{t+T-1}^o}{C_{t+T+1}^o - h_o C_{t+T}^o} \right)^{\sigma} \frac{P_t}{P_{t+T}}.$$

This firm's maximisation problem is subject to the production function in equation (1.8) and to the demand function of good z in equation (1.10).

Proposition 2 *The New Keynesian price Phillips curve in case of Kimball demand can be written as:*

$$\pi_t = \beta E_t \pi_{t+1} + \kappa s_t \ \text{ with } \ \kappa \equiv A \frac{(1 - \xi^p)(1 - \beta \xi^p)}{\xi^p} \ \text{ and } \ A \equiv \frac{1}{1 + \mathcal{I} \frac{\bar{\epsilon}}{1 - \varepsilon_p}}$$
$$(1.15)$$

where we can see that the slope of the Phillips curve (κ) is influenced beyond the Calvo parameter (ξ^p) by the Kimball curvature parameter (ϵ) and the elasticity of demand (ε_p). The indicator variable $\mathcal{I}$ is equal to one when there is strategic complementarity in price setting (due to Kimball demand). The case of $\mathcal{I} = 0$ delivers the standard Phillips curve without Kimball demand.

Proof. Based on firm's problem in equation (1.14) the first-order

condition with respect to P_t^* is given by:

$$0 = \sum_{T=0}^{\infty} (\xi^p \beta)^T E_t \Lambda_{t,t+T} \left[\begin{array}{c} Y_{t+T} \mathcal{G}'^{-1}\left(\frac{P_t^*}{P_{t+T}} \tau_{t+T}^g\right) + P_t^* Y_{t+T} \mathcal{G}'(x_{t+T}^g) \frac{1}{P_{t+T}} \tau_{t+T}^g \\ - S_{t|t+T}(j) Y_{t+T} \mathcal{G}'(x_{t+T}^g) \frac{1}{P_{t+T}} \tau_{t+T}^g \end{array} \right]$$

where $x_{t+T}^g \equiv \mathcal{G}'^{-1}(z_{t+T}^g)$, $z_{t+T}^g \equiv \frac{P_t^*}{P_{t-1}} \frac{P_{t-1}}{P_{t+T}} \tau_{t+T}^g$ with $\tau_t^g \equiv \int_0^1 \mathcal{G}'\left(\frac{Y_t(j)}{Y_t}\right) \frac{Y_t(j)}{Y_t} dj$ and $S_{t|t+T}(j)$ is the time $t + T$ nominal marginal cost of firm j that last changed its price at time t. It is worth noting that in this simple economy the average nominal marginal cost is not firm-specific $(S_{t|t+T}(j) = S_{t|t+T})$ and is equal to the average real wage because of the constant-returns-to-scale assumption and the absence of productivity shocks.

The previous equation can be further manipulated algebraically after multiplying through with P_t^* and substituting for $\frac{P_t^*}{P_{t+T}} \tau_{t+T}^g$ the combination of equations (1.11) and (1.12):

$$\sum_{T=0}^{\infty} (\xi^p \beta)^T E_t \left\{ \Lambda_{t,t+T} Y_{t+T}(j) \left[P_t^* + (P_t^* - S_{t|t+T}) \frac{1}{\mathcal{G}'^{-1}(z_{t+T}^g)} \frac{\mathcal{G}'(x_{t+T}^g)}{\mathcal{G}''(x_{t+T}^g)} \right] \right\}. \tag{1.16}$$

The preceding equation can be rewritten as

$$0 = \sum_{T=0}^{\infty} (\xi^p \beta)^T E_t \left\{ \begin{array}{c} \Lambda_{t,t+T} Y_{t+T}(j) \Gamma_{t+T} \times \\ \left[P_t^* + \frac{\mathcal{G}'^{-1}(z_{t+T}^g) \mathcal{G}''(x_{t+T}^g) P_t^*}{\mathcal{G}'(x_{t+T}^g)} - S_{t|t+T} \right] \end{array} \right\},$$

where $\Gamma_{t+T} \equiv \frac{1}{\mathcal{G}'^{-1}(z_{t+T}^g)} \frac{\mathcal{G}'(x_{t+T}^g)}{\mathcal{G}''(x_{t+T}^g)}$. After using $P_t^*/P_{t+T} = \mathcal{G}'(x_{t+T}^g)/\tau_{t+T}^g$ the former equation takes the form of

$$0 = \sum_{T=0}^{\infty} (\xi^p \beta)^T E_t \left\{ \begin{array}{c} \Lambda_{t,t+T} Y_{t+T}(j) \Gamma_{t+T} \times \\ \left[P_t^* + \frac{\mathcal{G}'^{-1}(z_{t+T}^g) \mathcal{G}''(x_{t+T}^g) P_{t+T}}{\tau_{t+T}^g} - S_{t|t+T} \right] \end{array} \right\}.$$

Nominal quantities in the squared bracket can also be expressed in real terms dividing them by P_{t+T}. Further, let us multiply and divide

the first term in the squared bracket by P_{t-1}:

$$0 = \sum_{T=0}^{\infty} (\xi^p \beta)^T E_t \left\{ \Lambda_{t,t+T} Y_{t+T}(j) \Gamma_{t+T} \left[\begin{array}{c} -\dfrac{P_t^*}{P_{t-1}} \dfrac{P_{t-1}}{P_{t+T}} \\ -\dfrac{\mathcal{G}'^{-1}(z_{t+T}^g)\mathcal{G}''(x_{t+T}^g)}{\tau_{t+T}^g} + \dfrac{S_{t|t+T}}{P_{t+T}} \end{array} \right] \right\}.$$

We make three observations about the previous expression. First, the term $\frac{P_t^*}{P_{t-1}}$ does not depend on T and therefore can be separated from the terms which are summed up with respect to T. Second, we can apply geometric summation to $\frac{P_t^*}{P_{t-1}}$ terms which are also incorporated implicitly in z_{t+T}^g and x_{t+T}^g. As a third point note that it is enough to loglinearise the expression in the squared bracket in the previous equation:

$$\frac{1}{1-\xi^p\beta}\left(2 + \frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right)(p_t^* - p_{t-1})$$

$$= \sum_{T=0}^{\infty} (\xi^p\beta)^T E_t \left[\left(2 + \frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right)(p_{t+T} - p_{t-1}) + Ss_{t+T}^{\text{real}}\right] \quad (1.17)$$

where $2 + \frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)} = 1 + \frac{\mathcal{G}''(x^g)}{\tau^g}\frac{1}{\mathcal{G}''(x^g)}\tau^g + \frac{\mathcal{G}'^{-1}(z^g)}{\mathcal{G}''(x^g)}\mathcal{G}'''(x^g)$, $\mathcal{G}'^{-1}(z^g) = 1$ and $\tau^g = \mathcal{G}'(1)$ is true in steady-state and $p_t^* \equiv \log(P_t^*)$, $\pi_t \equiv \log(P_t/P_{t-1}) = p_t - p_{t-1}$ and $s_{t+T}^{\text{real}} \equiv \log(S_{t+T}^{\text{real}}/S)$ with $S_{t+T}^{\text{real}} \equiv S_{t+T}/P_{t+T}$.

The RHS of the previous equation can be written as:

$$\sum_{T=0}^{\infty} (\xi^p\beta)^T E_t \left[\left(2 + \frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right)(p_{t+T} - p_t + p_t - p_{t-1}) + Ss_{t+T}^{\text{real}}\right]$$

$$= Ss_t^{\text{real}} + \left(2 + \frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right)\pi_t \quad (1.18)$$

$$+ \sum_{T=1}^{\infty} (\xi^p\beta)^T E_t \left[\left(2 + \frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right)(p_{t+T} - p_{t-1}) + Ss_{t+T}^{\text{real}}\right] \quad (1.19)$$

Let us forward equation (1.17) one period ahead to obtain:

$$\frac{1}{1-\xi^p\beta}\left(2+\frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right)E_t(p_{t+1}^* - p_t)$$

$$= \sum_{T=0}^{\infty}(\xi^p\beta)^T E_t\left[\left(2+\frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right)(p_{t+1+T} - p_t) + Ss_{t+1+T}^{\text{real}}\right]$$

$$= \sum_{T=1}^{\infty}(\xi^p\beta)^{T-1} E_t\left[\left(2+\frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right)(p_{t+T} - p_t) + Ss_{t+T}^{\text{real}}\right]$$

or alternatively

$$\frac{\xi^p\beta}{1-\xi^p\beta}\left(2+\frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right)E_t(p_{t+1}^* - p_t)$$

$$= \sum_{T=1}^{\infty}(\xi^p\beta)^T E_t\left[\left(2+\frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right)(p_{t+T} - p_t) + Ss_{t+T}^{\text{real}}\right].$$

After substituting the expression from LHS of the former equation into the third term in the second line of equation (1.18) we obtain:

$$\frac{1}{1-\xi^p\beta}\left(2+\frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right)(p_t^* - p_{t-1})$$

$$= Ss_t^{\text{real}} + \frac{\xi^p\beta}{1-\xi^p\beta}\left(2+\frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right)\pi_t$$

$$+ \frac{\xi^p\beta}{1-\xi^p\beta}\left(2+\frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right)E_t(p_{t+1}^* - p_t). \qquad (1.20)$$

The evolution of the aggregate price level in the Calvo model with Kimball demand is given by:

$$P_t \equiv (1-\xi^p)P_t^*\mathcal{G}'^{-1}(z_{t+T}^g) + \xi^p P_{t-1}\mathcal{G}'^{-1}(z_{t+T}^g)$$

which can be written after dividing with P_{t-1} as:

$$\frac{P_t}{P_{t-1}} = (1-\xi^p)\frac{P_t^*}{P_{t-1}}\mathcal{G}'^{-1}(z_{t+T}^g) + \xi^p\mathcal{G}'^{-1}(z_{t+T}^g).$$

The former equation can be loglinearised to yield:

$$p_t^* - p_{t-1} = \frac{1}{1 - \xi^p}\pi_t. \tag{1.21}$$

After substituting in the loglinear form of price index from equation (1.21) into the equation (1.20) we obtain the New Keynesian Phillips curve:

$$\frac{1}{1 - \xi^p\beta}\left(2 + \frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right)\frac{1}{1 - \xi^p}\pi_t$$

$$= Ss_t^{\text{real}} + \frac{\xi^p\beta}{1 - \xi^p\beta}\left(2 + \frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right)\pi_t$$

$$+ \frac{\xi^p\beta}{1 - \xi^p\beta}\left(2 + \frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right)\frac{1}{1 - \xi^p}E_t\pi_{t+1}$$

or

$$\left[\frac{1}{1 - \xi^p\beta}\frac{1}{1 - \xi^p} - \frac{\xi^p\beta}{1 - \xi^p\beta}\right]\pi_t$$

$$= \frac{S}{\left(2 + \frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right)}s_t^{\text{real}} + \frac{\xi^p\beta}{1 - \xi^p\beta}\frac{1}{1 - \xi^p}E_t\pi_{t+1} \tag{1.22}$$

The steady-state of the marginal cost can be inferred from equation (1.16):

$$1 + [1 - S]\frac{1}{\mathcal{G}'^{-1}(\mathcal{G}'(1))}\frac{\mathcal{G}'(1)}{\mathcal{G}''(1)} = 0$$

or

$$1 + \frac{\mathcal{G}''(1)}{\mathcal{G}'(1)} = \frac{\frac{\mathcal{G}'(1)}{\mathcal{G}''(1)} + 1}{\frac{\mathcal{G}'(1)}{\mathcal{G}''(1)}} = \frac{\varepsilon_p(1) - 1}{\varepsilon_p(1)} = S \tag{1.23}$$

where $P^*/P = 1$ and $\mathcal{G}'^{-1}(z^g) = 1$ hold in steady-state and $\Xi(1) = \varepsilon_p = -\frac{\mathcal{G}'(1)}{\mathcal{G}''(1)}$ is the elasticity of demand. The steady-state markup is given by $1/S$. Using the steady-state of the marginal cost from equation (1.23) we can rewrite the New Keynesian Phillips curve (equation

(1.22)) in its usual form:

$$
\begin{aligned}
\pi_t &= \frac{(1-\xi^p)(1-\beta\xi^p)}{\xi^p} \frac{\left[1+\frac{\mathcal{G}''(1)}{\mathcal{G}'(1)}\right]}{\left[2+\frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right]} s_t^{\text{real}} + \beta E_t \pi_{t+1} \\
&= \frac{(1-\xi^p)(1-\beta\xi^p)}{\xi^p} A s_t^{\text{real}} + \beta E_t \pi_{t+1} \text{ where } A \equiv \frac{\left[1+\frac{\mathcal{G}''(1)}{\mathcal{G}'(1)}\right]}{\left[2+\frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right]}
\end{aligned}
$$

Finally let us express A in terms of the Kimball curvature parameter (ϵ) and the elasticity of demand (ε_p):

$$
A \equiv \frac{\left[1+\frac{\mathcal{G}''(1)}{\mathcal{G}'(1)}\right]}{\left[2+\frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}\right]} = \frac{1-\frac{1}{\varepsilon_p}}{2+\frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)}}
$$

from which we express for

$$
\frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)} = \frac{1-\frac{1}{\varepsilon_p}}{A} - 2
$$

and substitute it back into the definition of the curvature (see equation (1.13)):

$$
\begin{aligned}
\bar{\epsilon} &= \epsilon(1) = 1 + \varepsilon_p + \varepsilon_p \frac{\mathcal{G}'''(1)}{\mathcal{G}''(1)} \\
&= 1 + \varepsilon_p + \varepsilon_p \left[\frac{1-\frac{1}{\varepsilon_p}}{A} - 2\right] = 1 + \varepsilon_p \left[\frac{1-\frac{1}{\varepsilon_p}}{A} - 1\right].
\end{aligned}
$$

From the previous equation we express A as a function of the Kimball curvature parameter (ϵ) and the elasticity of demand (ε_p):

$$
A = \frac{\varepsilon_p - 1}{\bar{\epsilon} + \varepsilon_p - 1} = \frac{1}{\frac{\bar{\epsilon}}{\varepsilon_p - 1} + 1}
$$

which is the same as the one in proposition 2. ∎

The following observations can be made about the new-Keynesian price Phillips curve and the slope of the Phillips curve derived in

proposition 2. Recall that the slope is given by

$$\kappa \equiv \frac{(1 - \xi^p)(1 - \beta\xi^p)}{\xi^p} \frac{1}{1 + \mathcal{I}\frac{\bar{\epsilon}}{1 - \varepsilon_p}}, \tag{1.24}$$

where $\mathcal{I}$ is an indicator variable that can take on the value of one or zero. When $\mathcal{I} = 1$ the model contains real rigidity in the form of Kimball (1995) demand. When $\mathcal{I} = 0$ the loglinear Phillips curve is the standard New Keynesian one without real rigidity. In Experiment **one** (see below) which utilises the above model without wage stickiness we found that real rigidity is necessary because it helps to avoid a non-uniqueness problem (for more on this see footnote (18)). Now we proceed to discuss the determination of labour supply.

1.2.3 Unions

To introduce wage stickiness into the model one usually assumes that households have monopoly power in determining their wage as in Erceg et al. (2001) who presume that each household can engage in perfect consumption smoothing. However, the presence of ROT households who cannot engage in intertemporal trade precludes the possibility of consumption smoothing. To motive a wage-setting decision we suppose following Gali et al. (2007) and Furlanetto and Seneca (2009) that there is a continuum of unions (on the unit interval), $z \in [0, 1]$, each representing a continuum of workers of which a fraction (λ) are members of rule-of-thumb and the remaining $(1 - \lambda)$ fraction consists of optimising households. Each union employs one particular type of labour (independently of the type households they originate from) that is different from the type of labour offered by other unions.

Each period the union maximises the weighted current and discounted future utility of its members:

$$E_t \sum_{T=0}^{\infty} \beta^T \left[\lambda U_{t+T}^r + (1 - \lambda)U_{t+T}^o\right]$$

subject to the labour demand function for labour of type z:

$$N_t(z) = \left(\frac{W_t(z)}{W_t}\right)^{-\varepsilon_w} N_t$$

where $W_t(z)$ is the nominal wage set by the union z, ε_w is the elasticity of labour demand and W_t is an aggregate of the wages set by unions:

$$W_t \equiv \left(\int_0^1 [W_t(z)]^{1-\varepsilon_w}\right)^{\frac{1}{1-\varepsilon_w}}.$$

Adjusting wages is costly as in Rotemberg (1982) who originally applied it to model price adjustment. In particular, there is a wage adjustment cost which is a quadratic function of the change in the nominal wage and proportional to the aggregate wage bill. The presence of this wage adjustment cost is justified by the fact that unions have to negotiate wages each period and this activity consumes real resources. The larger is the increase in nominal wage achieved by the union the higher is the effort associated with it. Each union members incurs an equal share of the wage adjustment cost. Thus, the nominal membership fee, F paid by a generic union member z at time t is given by:

$$F_t(z) = \frac{\phi_w}{2}\left(\frac{W_t(z)}{W_{t-1}(z)} - 1\right)^2 W_t N_t$$

where ϕ_w governs the size of the adjustment costs. In the special case of $\phi_w = 0$ the labour market features flexible wages.

The optimality condition from the union's problem can be derived by taking the FOC with respect to the wage, $\tilde{W}_t$:

$$\begin{aligned}
0 = {} & \left(\lambda\frac{\partial U_t^r}{\partial C_t^r} + (1-\lambda)\frac{\partial U_t^o}{\partial C_t^o}\right)(1-\tau_t^l)\tilde{W}_t[(\varepsilon_w - 1) + \phi_w(\Pi_t^w - 1)\Pi_t^w] \\
& - \varepsilon_w N_t^\varphi - \beta\left[\left(\lambda\frac{\partial U_{t+1}^r}{\partial C_{t+1}^r} + (1-\lambda)\frac{\partial U_{t+1}^o}{\partial C_{t+1}^o}\right)\right] \times \\
& \phi_w(\Pi_{t+1}^w - 1)\Pi_{t+1}^w \frac{W_{t+1}}{P_{t+1}}\frac{N_{t+1}}{N_t},
\end{aligned} \tag{1.25}$$

where $\Pi_t^w \equiv W_t/W_{t-1}$ is the wage inflation, $\tilde{W}_t \equiv W_t/P_t$ is the real wage and $\frac{\partial U_t^i}{\partial C_t^i}$ is defined by equation (1.3) for $i \in \{o, r\}$. The consumption also differs between the two types of consumers. When making a decision on labour demand the firm does not distinguish between different workers of type z. Thus, in the aggregate, $N_t^r = N_t^o = N_t$ holds i.e. they work the same amount of hours. The linearisation of equation (1.25) yields what we call the new-Keynesian wage Phillips curve:

$$\pi_t^w = \beta E_t \pi_{t+1}^w - \kappa^w \left[w_t - mrs_t - \chi\hat{\tau}_t \right], \tag{1.26}$$

where $\pi_t^w \equiv \log(\Pi_t^w/\Pi^w)$, $w_t \equiv \log(\tilde{W}_t/\tilde{W})$, $\hat{\tau}_t \equiv \tau_t - \tau$, $\kappa^w \equiv \frac{\varepsilon_w - 1}{\phi_w}$[10] and the linearised expression for the marginal rate of substitution is[11]:

$$mrs_t = \chi_r(c_t^r - h_r c_{t-1}^r) + \chi_o(c_t^o - h_o c_{t-1}^o) + \varphi n_t, \tag{1.27}$$

where

$$\chi_r \equiv \sigma \frac{\lambda}{1 - h_r} \frac{(1 - h_o)^{-\sigma}}{\lambda(1 - h_r)^{-\sigma} + (1 - \lambda)(1 - h_o)^{-\sigma}},$$

$$\chi_o \equiv \sigma \frac{1 - \lambda}{1 - h_o} \frac{(1 - h_r)^{-\sigma}}{\lambda(1 - h_r)^{-\sigma} + (1 - \lambda)(1 - h_o)^{-\sigma}}.$$

Note that in case of $h_o = h_r = 0$ equation (1.27) boils down to the case of CRRA utility without habits. Without loss of generality we postulate, following Furlanetto and Seneca (2012), that $h_r = h_o = h$ implying $\chi_r \equiv \lambda/(1 - h)$ and $\chi_o \equiv (1 - \lambda)/(1 - h)$. The connection between the wage inflation (π_t^w), price inflation (π_t) and the real wage (w_t) can be expressed, in linear form, as:

$$\pi_t^w = w_t - w_{t-1} + \pi_t. \tag{1.28}$$

---

[10]In calculating the value of κ^w we use $\frac{(1-\xi^w)(1-\beta\xi^w)}{\xi^w} \frac{1}{1+\varphi\frac{\varepsilon_w}{\varepsilon_w-1}}$ which results in case of Calvo wage setting and equivalent to $\frac{\varepsilon_w-1}{\phi_w}$ that we obtain under Rotemberg wage setting.

[11]Note that we assume a tax policy that equates steady-state consumptions across household types (i.e., $C^r = C^o$).

1.2.4 Fiscal and Monetary Policy

Fiscal policy

Similarly to Christiano (2010) and Christiano et al. (2011) we consider a deterministic experiment: the tax rate is cut by the same amount in each period for the entire duration of the shock.

We operate with two markedly different fiscal scenarios in this paper. The first one assumes a uniform tax cut (lowering labour taxes for both types of households by the same proportion) that is financed by lump-sum taxes levied on Ricardian agents. Hence, non-Ricardians do not pay taxes. In this case the government budget is balanced in each period. This is the simplest possible fiscal scenario that can be built into the model. Therefore, our setup is different from Gali et al. (2007) where non-Ricardians pay lump-sum taxes.

In the second fiscal scenario we depart from balanced budget and assume that a uniform tax cut is financed by government debt that is paid back through labour and profit income taxes that are levied on both types of households[12]. With this latter arrangement we relax the strong assumption that non-Ricardians do not bear the burden of the tax-cut. In the experiments below we assume in contrast to Gali et al. (2007) and in accordance to Rossi (2012) that the steady-state level of debt is not zero. The government budget constraint which implicitly describes the evolution of debt (B) reads as:

$$B_t + \tau_t W_t N_t = R_{t-1} B_{t-1} + P_t G_t$$

which gives way after linearisation to

$$b_t + \frac{\tau W N}{Y}(w_t + n_t + \frac{1}{\tau}\hat{\tau}_t) = \frac{1}{\beta}b_{t-1} + \gamma_b dR_{t-1} - \gamma_b \frac{1}{\beta}\pi_t + g_t,$$

where $b_t \equiv (B_t - B)/Y$, $dr_t \equiv r_t - r$, $y_t \equiv (Y_t - Y)/Y$, $g_t \equiv (G_t - G)/Y$ and γ_b is the government debt-to-GDP ratio. $\hat{\tau}_t$ and dR_t are

---

[12]Only Ricardians are entitled to profit income as they are the owner of the firms.

defined above. For the rest of the paper we set $g_t = 0$, $\forall t$.

Rossi (2012) proposes the following government revenue rule based on Leeper (1991):

$$\tau_t W_t N_t = \delta_0 + \delta_1 \frac{\tau Y}{B}(B_t - B) + \delta_2 \tau(Y_t - Y) + \varepsilon^\tau, \qquad (1.29)$$

where $\delta_0 > 0$. As in Leeper (1991) and Rossi (2012) there is no restriction on the values of δ_1 and δ_2. One usually refers to $\delta_2 > 0$ ($\delta_2 < 0$) as procyclical (countercyclical) fiscal policy. Here we simply set $\delta_2 = 0$ so that public debt does not fluctuate along the business cycle. We depart from the specification in equation (1.29) in the sense that we consider the deviation of real government debt from its steady-state value relative to the steady-state of GDP i.e. $b_t \equiv (B_t - B)/Y$. Exogenous shocks to the tax revenue are captured by ε^τ.

The latter revenue rule can be linearised to yield:

$$\hat{\tau}_t = \delta_1 \frac{Y}{WN} b_t - \tau(w_t + n_t) + \{d\varepsilon^\tau\}_{t=\text{zlb start}}^{T=\text{zlb end}}$$

where X is the steady-state value of variable X and $d\varepsilon^\tau \equiv \varepsilon_t^\tau - \varepsilon^\tau = -0.1$ is the deterministic 'tax cut shock' (a ten percentage points reduction) that is on for the duration of the zero-lower bound period. The abbreviations 'zlb start' and 'zlb end' stand for the start and the end date of the zero lower bound period, respectively. We investigate into the robustness of our findings with setting different values for δ_1.

Monetary Policy

Monetary policy is described by the rule in Christiano et al. (2011):

$$R_t = \max(Z_t, 0) \qquad (1.30)$$

where

$$Z_t = (1/\beta)(\Pi_t)^{\phi_1(1-\rho_R)}(Y_t/Y)^{\phi_2(1-\rho_R)}[\beta R_{t-1}]^{\rho_R} - 1, \qquad (1.31)$$

where Z_t is the shadow nominal interest rate which can take on negative values as well. As usual, we assume that $\phi_1 > 1$, $\phi_2 \in [0,1)$ and $0 < \rho_R < 1$. ϕ_1 controls how strongly monetary policy reacts to changes in inflation while ϕ_2 governs the strength of the response of nominal interest to changes in output gap[13]. The main implication of the rule in equation (1.30) is that whenever the nominal interest rate becomes negative, the monetary policy set it equal to zero, otherwise it is set by the Taylor rule specified in equation (1.31). The parameter ρ_R measures how quickly monetary policy reacts to changes in inflation and output gap. Furthermore, inflation in steady-state is assumed to be zero which implies that steady-state net nominal interest rate is $1/\beta$.

The monetary policy rule above can be written, in linear form, as:

$$dR_t = \begin{cases} dZ_t, & dZ_t \geq -\left(\frac{1}{\beta}-1\right), & \text{'zero bound not binding'} \\ -\left(\frac{1}{\beta}-1\right), & \text{otherwise,} & \text{'zero bound binding'} \end{cases},$$

$$dZ_t = \rho_R dR_{t-1} + (1-\rho_R)\frac{1}{\beta}\left[\phi_1\pi_t + \phi_2 y_t\right].$$

Hence, the ZLB on the nominal interest binds when $dR_t = -\left(\frac{1}{\beta}-1\right)$. Otherwise, we set $dR_t = dZ_t$.

1.2.5   Aggregation, Market Clearing and Equilibrium

The aggregate consumption is a composite of those of the two types of households:

$$C_t = \lambda C_t^r + (1-\lambda)C_t^o. \tag{1.32}$$

The aggregate dividend payments are determined by $D_t = (1-\lambda)D_t^o$. The presence of unions implies that both types of households work the same number of hours and, thus, $N_t^r = N_t^o = N_t$ for all t.

---

[13]Precisely, the term Y_t/Y does not stand for the output gap as the definition of the output gap contains the deviation of the actual GDP from its flexible price level equivalent. Here we simply use the deviation of output from its steady-state value.

It follows that equation (1.32) can be written in linear form as:

$$c_t = \lambda c_t^r + (1 - \lambda)c_t^o, \tag{1.33}$$

which is obtained by setting steady-state consumption and hours worked of each type equal in steady-state ($C^r = C^o$) using a lump-sum tax appearing in the budget constraint of Ricardian households.

The goods market clearing is

$$Y_t = C_t + G_t,$$

which can be expressed in linear form as

$$y_t = \gamma_c c_t + g_t, \tag{1.34}$$

where for the rest of the paper we set $g_t \equiv (G_t - G)/Y = 0$ and $\gamma_c \equiv C/Y$ is calculated as $\gamma_c = 1 - \gamma_g$ with $\gamma_g \equiv G/Y$. After having outlayed the building blocks we are ready to define equilibrium of this model.

Definition 3 *The equilibrium is characterised by a sequence of endogenous quantities*

$$\{N_t, C_t^o, C_t^r, C_t, Y_t, B_t\}_{t=0}^\infty,$$

price sequences

$$\{\Pi_t, \Pi_t^w, W_t, S_t, R_t, Z_t, \tau_t\}_{t=0}^\infty,$$

and a given set of exogenous deterministic shocks

$$\{\beta_t, \epsilon_t^\tau\}_{t=0}^\infty$$

and initial values for debt that satisfy equilibrium conditions of the household, firms, unions, government and monetary authority such that markets clear, the transversality conditions for the endogenous states are imposed and the aggregate resource constraint is satisfied.

1.3   Calibration

1.3.1   Households

The discount factor, β, is equal to 0.99 implying a real annual interest rate of 4%. The elasticity of intertemporal substitution (EIS), σ, is set to one implying log utility which is a usual choice in the literature. Following Christiano et al. (2010) the parameter governing the disutility of labour, φ is chosen to be one (i.e. Frisch elasticity of labour supply, $1/\varphi$, is also one) which is more conservative than the value of 0.2 used by Gali et al. (2007). Recently, Christiano et al. (2010) argued that unitary Frisch elasticity is the most reasonable choice—our baseline parametrisation—which is in line with both macro and micro evidence. Also, similarly to Christiano (2010) we use $\varepsilon_p = \varepsilon_w = 6$. For habit formation parameter, h, Furlanetto and Seneca (2011) set a high value of 0.85 while Smets and Wouters (2007) employing a model with various frictions estimate a value of 0.6. Therefore, we consider a value in the middle range and set $h = 0.7$. The steady-state government spending-GDP ratio, γ_g, is set to 0.2 mimicking the post-war US evidence. This implies a steady-state consumption-income ratio, γ_c of 0.8. Furlanetto and Seneca (2009) calibrates the share of rule-of-thumb consumers (λ) to be between 29% and 35% after reviewing a couple of econometric studies. Based on this we set $\lambda = 0.3$ which we think is more plausible empirically than the 0.5 used by Gali et al. (2007).

1.3.2   Fiscal and Monetary Policy

The steady-state quarterly debt-output ratio (B/Y) is 2.4 assuming that the yearly debt-output ratio is 60% as in Rossi (2012). The steady-state labour tax rate (τ) in the model with balanced budget is chosen to be 30% as in Christiano (2010) while it is 26.91% in the model with endogenous debt and pinned down by the discount factor,

the debt-output ratio and government spending-to-output ratio. The size of the discount factor shock, r_t, is set to -0.01 which is close to the mode estimate (-0.0104) by Denes and Eggertsson (2010) using a model that contains only price rigidity and specific labour market. The duration of the negative demand shock is 10 periods[14] which is in accordance with the modal estimate of Denes and Eggertsson (2009). The inflation coefficient in the Taylor rule, ϕ_1, is 1.5. Following Christiano (2010) and Christiano et al. (2011) there is neither interest-rate smoothing ($\rho_R = 0$) nor response to output gap in the Taylor rule ($\phi_2 = 0$).

1.3.3 Firms

The mean posterior estimates of Smets and Wouters (2007) for the Calvo parameters, $\xi^p = 0.66$ ($\xi^w = 0.7$) imply an average price (wage) stickiness of around two (three) quarters. The reduced form estimates (see for references Furlanetto and Seneca (2009)) on the new-Keynesian price Phillips curve imply $\kappa = 0.03$. Without real rigidity such a value of κ would imply a very long-period of price inertia ($\xi^p = 0.85$). In our baseline calibration without real rigidity (i.e. $\mathcal{I} = 0$) $\xi^p = 0.66$ implies $\kappa = .1786$. When $\mathcal{I} = 1$ the calibration of $\kappa = 0.03$ is achieved by setting an appropriate value for ϵ. The implied value of ϵ is 24.77 which is in the range of empirical estimates listed in Furlanetto and Seneca (2009).

---

[14]Eggertsson (2010a) and Denes and Eggertsson (2009) consider a stochastic experiment with a persistence estimate of $\mu = 0.9030$ for the discount factor shock process. This μ is easily translated into our deterministic experiment knowing that the average duration of this AR(1) is $1/(1 - \rho)$ which is roughly 10. For a similar argument see Appendix C of Carlstrom, Fuerst and Paustian (2012).

1.4 Experiments

1.4.1 Experiment 1—only price rigidity

Our zero lower bound experiments are in the spirit of Christiano (2010) and Christiano et al. (2011)[15] who assumed that the discount factor shock and the corresponding fiscal policy shock is on for a deterministic period of time. A discount factor shock (alternatively, savings or negative demand shock) hits the economy in period one. The model is in deterministic steady-state until $t = 1$. At $t = 1$ the discount rate drops from its steady-state value of 0.01 (per quarter) to $r = -0.01$ and remains low for $T = 10$ quarters[16]. From quarter 11 $(T + 1)$ on the discount rate is back to its steady-state value. Note that all deterministic experiments below assume that the discount factor shock is on for ten periods. The deterministic simulations are executed using a standard shooting algorithm to handle the ZLB problem. The details of this algorithm are available in the appendix of Christiano (2010)[17]. Briefly, the algorithm can be described as follows. Let t_1 and t_2 denote the guess values for dates of the start and the end of the zero lower bound period, respectively such that $1 \leq t_1 \leq t_2 \leq T$. Then taxes are decreased for period $t \in [t_1, t_2]$. Next check whether the zero lower bound binds for $t \in [t_1, t_2]$. If not, revise the guess for t_1 and t_2.

The steady-state level of labour tax in the model with balanced government budget is 30 per cent ($\tau = 0.3$). In the no policy response simulation the labour tax rate is at its steady-state level for the entire simulation. In the alternative simulation (denoted with dashed line)

---

[15]Note that section 2 and 3 of Christiano et al. (2011) consider a stochastic experiment similar to those in Eggertsson (2010a) and Woodford (2010) while section 4 and 5 consider deterministic experiments that are accomplished by using a standard shooting algorithm. In case of only price rigidity (or only wage riditiy) the system can be re-written using the Eggertsson and Woodford (2003) type of methodology applicable if the system contains no state variable. The latter is not true any more in case of the inclusion of both price and wage stickiness when one of the variables (potentially the real wage) becomes an endogenous state. Hence, we make use of the shooting algorithm of Christiano (2010).

[16]For comparison, Christiano (2010) considered a shock of similar size although a somewhat longer period ($T = 15$).

[17]In particular, we made use of some of the codes of Christiano (2010) and Christiano et al. (2011). The codes are available from Lawrence Christiano's website.

the labour tax rate is decreased (in contrast to Christiano (2010) and Christiano et al. (2011) who considered a rise in the tax rate) to 20 per cent for the time period in which the zero lower bound on the nominal interest rate is binding. The shooting algorithm determines endogenously the date at which the zero lower bound becomes binding and the date at which the zero lower bound ceases to bind. Thus we have at least two regimes. One of them is with fixed interest rate (the zero lower bound period) and the other one is with a Taylor rule. In general there can be many regimes with either fixed interest rate or with a Taylor rule operating. It is well known that there is indeterminacy in the new Keynesian model when interest rate is fixed (see, e.g., Woodford (2011)). However, inclusion of the Taylor rule in one of the regimes guarantees determinacy in the other regimes characterised by fixed interest rate (see, among others, Carlstrom et al. (2012)). We conduct four experiments and elements of the models used in each experiment are listed in Table (1.1).

Figure 1.1 shows experiment **one** featuring a model that includes two types of households and **only** price rigidity. In experiment one we assume that there is **no wage rigidity** in the economy (i.e. the wage Phillips curve in equation (1.26) is removed—equivalently ξ^w is set close to zero). Therefore, wages are flexible in experiment one[18]. In the absence of tax policy the ZLB ends in period 6 while the presence of tax policy makes the ZLB bind for 9 periods. *One can observe that each of the variables except for nominal and real interest rates decline due to the negative demand shock in each experiment and, therefore, the question is always whether the tax-cut is able to mitigate the negative effects of the demand shock or not.*

---

[18]In this experiment we found numerically that there are two solutions to the shooting problem (hence no unique solution). Also we realised that the drop in output and inflation is extremely large in this simplest variant of model (without capital, habits and wage rigidity) containing two types of households. To avoid the non-uniqueness problem and to reduce the extreme negative impact of the shock we introduce strategic complementarity into price setting in the way discussed above. As Ascari et al. (2011) argues the non-uniqueness problem is mitigated by the inclusion of wage rigidity into the baseline model. Thus, in the models containing wage rigidity we do not encounter such non-uniqueness problem.

Figure 1.1: This is experiment 1 in the text and in Table (1.1). The + signs indicate the date at which the zero lower bound on the nominal interest rate becomes binding and circles appear on the date at which the zero lower bound ceases to bind. ss means steady-state. There are two shocks in this experiment: a strong negative demand shock with size that equals to -0.01 in each period for ten quarters, which leads to huge fall in all variables. Also there is a labour tax cut of size -0.1 in each quarter during the zero lower bound period which does not necessarily last as long as the negative demand shock. Consumption (both ROT and OPT), hours, output and real wage rate are expressed as percentage deviation from their steady-state values (on the graphs it is indicated as "% deviation from ss") while price inflation, wage inflation, shadow interest rate, nominal and real interest rate is expressed in annual percentage rate (APR).
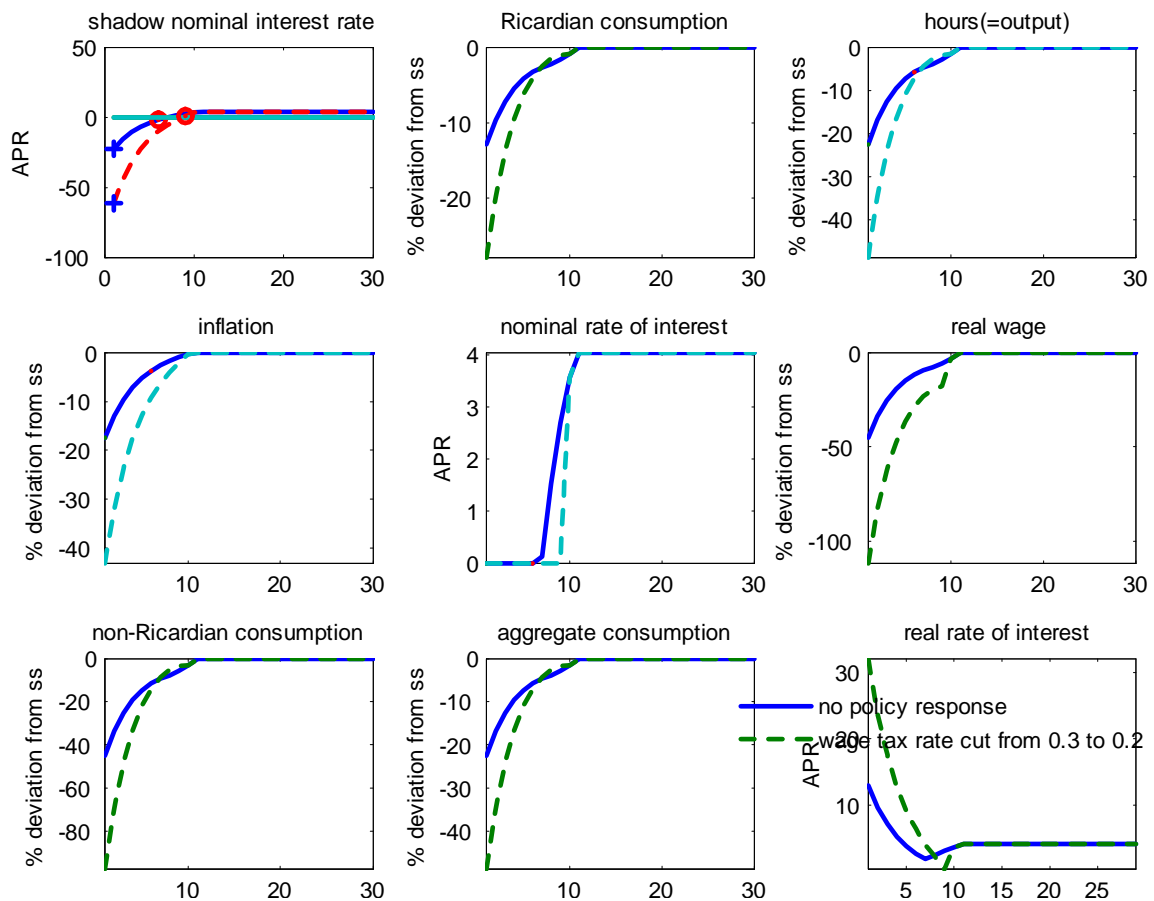
Figure 1.2: Labour demand and supply at the zero lower bound using the stochastic two-state version of the model used in experiment 1.
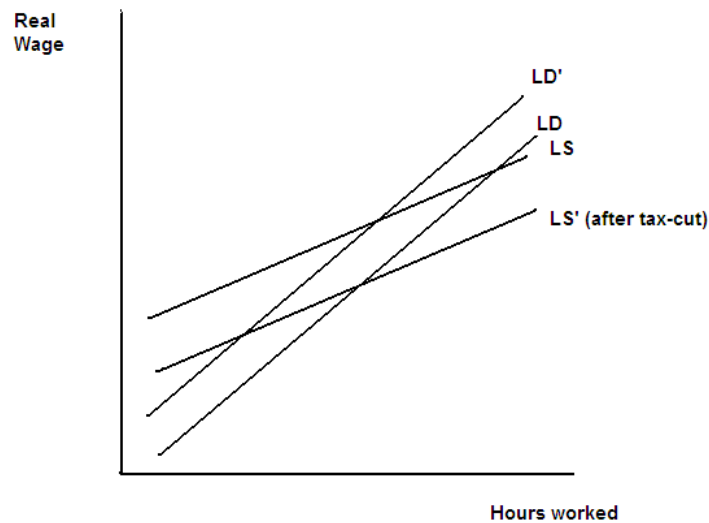
In experiment one consumption of non-Ricardians falls even more in case of the decrease in labour taxes and, hence, labour tax cut does not alleviate the negative consequences of the demand shock (huge deflation and fall in output). Also note that the drop in real wage—which equals to the marginal cost due to constant return-to-scale assumption—is considerable without and with the labour tax-cut. When zero lower bound ceases to bind the Taylor rule becomes operational and monetary policy reacts to positive inflation resulting from expansionary fiscal policy (i.e. the labour tax cut) by raising the nominal interest rate. Thus, there is a large upward movement in nominal interest rate following the zero lower bound period (see Figure 1.1). Similarly, real interest rate jumps during the zero lower bound period because nominal interest rate is fixed and there is huge deflation due to the negative demand shock. Real interest rate rises even more with a tax-cut that is associated with more pronounced fall in real wages and, hence, bigger drop in inflation through the new-Keynesian Phillips curve.

To provide intuition for experiment 1 let us study the labour market of model. The effects of the tax-cut is depicted on Figure 1.2.

Proposition 4 *Both labour demand and supply are upward-sloping at the zero lower bound such that the labour demand is steeper than the labour supply.*

Proof. Let us examine the stochastic case of the model used in experiment 1 as in Eggertsson (2011, 2010), Christiano et al. (2010) and Woodford (2011). The model in experiment 1 does not contain state variables and is therefore easy to implement. The analytical solution also provides lots of intuition.

Another reason why one can obtain analytical solution is that the simplifying assumption of the first fiscal scenario when non-Ricardians do not pay taxes i.e. neither lump-sum nor distortionary taxes (yet they enjoy the tax cut) making the government budget constraint redundant. More precisely, the government budget constraint is implicitly included in the aggregate resource constraint which combines the latter with Ricardian and non-Ricardian households' budget constraint. There is monopolistic competition on the labour market as in the main text and labour incomes are pooled by unions which set wages for each type of labour. It is useful to describe the system in the short-run and in the long-run. In the short-run (hence, the subscript s) an endogenous variable x_t is denoted as $x_t = x_s = \{w_s,\ n_s,\ y_s,\ c_s^o,\ c_s^r,\ \pi_s\}$. In the long-run we are back to the steady-state such that $x_t = 0$[19]. Here we consider the short-run with a binding zero lower bound. [See not-for-publication appendix of the paper version of this chapter for a characterisation of the model in the short-run with positive interest rates.]

The maximum number of periods for which the zero lower bound is binding is equal to the length of the shock (T). At time $t < T$ the economy is at the zero-lower bound and stays there, at the next period $t + 1$, with probability μ or going back to the steady-state with probability[20] $1 - \mu$. Therefore the exit from the zero lower bound pe-

---

[19]Remember that x is defined as percent deviation from steady-state.

[20]$1 - \mu$ can also be interpreted as the transition probability of switching from the zero lower

riod is stochastic and happens at $t^{exit} \le T$. The exogenous discount factor shock that makes the zero lower bound binding is $r_t^* = r_s^* < 0$ for $t \le T$ and $r_t^* = 0$ in the steady-state for $t > T$. [The discount factor shock is also zero when interest rate is positive.] The second exogenous shock is the tax cut that is lowered during the zero lower bound period $\hat{\tau}_t = \hat{\tau}_s$ for $t < t^{exit}$. Otherwise taxes do not deviate from the steady-state i.e. $\hat{\tau}_t = 0$ for $t \ge t^{exit}$. In the next labour demand and supply are derived. Then we argue that equilibrium at the zero lower bound is well-defined when labour demand is steeper than labour supply.

The aggregation of Ricardian and non-Ricardian intratemporal conditions results in:

$$w_s = \left(\frac{\sigma}{\gamma_c} + \varphi \right) n_s - \chi \hat{\tau}_s \tag{1.35}$$

where we used the aggregate resource constraint $(y_t = \gamma_c c_t)$ and the production function $(y_t = n_t)$.

The budget constraint of non-Ricardians is reproduced here:

$$c_s^r = w_s + n_s^r - \chi \hat{\tau}_s^r, \tag{1.36}$$

The Euler equation of Ricardians can be written as:

$$\sigma c_s^o - \sigma \mu c_s^o = -(0 - \mu \pi_s - r_s^*)$$

where the nominal interest rate is zero when the zero lower bound is binding. The previous can also be expressed as:

$$(1 - \mu) \sigma c_s^o = \mu \pi_s + r_s^* \tag{1.37}$$

Taxation is uniform i.e. Ricardians and non-Ricardians pay the same

---

bound state to the steady-state.

tax-rate:

$$\hat{\tau}_s = \lambda\hat{\tau}_s^r + (1-\lambda)\hat{\tau}_s^o$$
$$\hat{\tau}_s^r = \hat{\tau}_s^o = \hat{\tau}_s$$

The New Keynesian price Phillips curve can be written as:

$$(1-\beta\mu)\pi_s = \kappa w_s$$
$$\pi_s = \frac{\kappa w_s}{1-\beta\mu} \qquad (1.38)$$

Let us start with the aggregate resource constraint that equals to the consumption aggregator:

$$y_s = \gamma_c c_s = \gamma_c[\lambda c_s^r + (1-\lambda)c_s^o] = \gamma_c\left[\lambda(w_s + n_s^r - \chi\hat{\tau}_s) + (1-\lambda)\frac{(\mu\pi_s + r_s^*)}{(1-\mu)\sigma}\right],$$

where we substituted in for c_t^r and c_s^o from equations (1.36) and (1.37), respectively.

The previous one can be expressed as:

$$y_t = \gamma_c\left[\lambda(w_s + n_s - \chi\hat{\tau}_s) + (1-\lambda)\frac{\left(\mu\frac{\kappa w_s}{(1-\beta\mu)} + r_s^*\right)}{(1-\mu)\sigma}\right]$$

where we substituted in from equation (1.38) for π_s.

Multiplying out terms in the previous equation leads to

$$y_t = \gamma_c\lambda w_s + \gamma_c\lambda n_s - \gamma_c\lambda\chi\hat{\tau}_s + \frac{\gamma_c(1-\lambda)\mu\kappa}{(1-\beta\mu)(1-\mu)\sigma}w_s + \frac{(1-\lambda)\gamma_c}{(1-\mu)\sigma}r_s^*.$$

After collecting some terms the preceding one results in:

$$n_s(1-\gamma_c\lambda) = \frac{\gamma_c\lambda(1-\beta\mu)(1-\mu)\sigma + \gamma_c(1-\lambda)\mu\kappa}{(1-\beta\mu)(1-\mu)\sigma}w_s - \gamma_c\lambda\chi\hat{\tau}_s + \frac{(1-\lambda)\gamma_c}{(1-\mu)\sigma}r_s^*$$

And the resulting labour demand is a function of the real wage, the

tax shock and the discount factor shock:

$$n_s^{ld} = \frac{\gamma_c\lambda(1-\beta\mu)(1-\mu)\sigma + \gamma_c(1-\lambda)\mu\kappa}{\Gamma(1-\beta\mu)(1-\mu)\sigma}w_s - \frac{\gamma_c\lambda\chi}{\Gamma}\hat{\tau}_s + \frac{(1-\lambda)\gamma_c}{\Gamma(1-\mu)\sigma}r_s^*$$

(1.39)

where $\Gamma \equiv 1 - \gamma_c\lambda$. Two observations can be made about the preceding equation. First, the slope of the labour demand in equation (1.39) is positive ($\Gamma > 0$ for our baseline calibration) similar to Eggertsson (2011, 2010) who used a model without non-Ricardian households. In figure 1.2 labour demand is steeper than labour supply as in Eggertsson (2011) and the tax cut has contractionary effects that is, a rightward shift of the labour supply leads to a fall in real wages and hours worked. It needs to be stressed that changes in the tax-rate ($\hat{\tau}_s$) directly affect aggregate labour demand through the budget constraint of non-Ricardian households. The fact that the tax cut raises disposable income of rule-of-thumbers and also leads to higher labour demand is of reduced significance in experiment 1 while it is the key to generate a demand effect in experiment 2 with rigid wages (see discussion following this proof and also experiment 2).

Second, by choosing an appropriate value for κ ceteris paribus we make sure that labour demand is steeper than labour supply so that the condition for a determinate equilibrium is satisfied as in Eggertsson (2011). It turns out that κ has to be sufficiently low in order for the labour demand to be steeper than the labour supply[21]. The introduction of Kimball demand (see the firm's problem in the main text) helps to achieve low κ without departing from the baseline calibration of the other parameters of the model. In fact, Kimball demand (and other types of real rigidities in general see Woodford (2003) chapter 3) is a useful tool to reduce κ instead of assuming unrealistically long average duration of price rigidity (high ξ^p) that could also reduce κ. Next we determine the equilibrium amount of hours worked as a function of the shocks and argue that conditions for a binding zero lower

---

[21]We plot wages on the vertical and hours worked on the horizontal axis.

bound are satisfied.

Substituting from aggregate labour supply equation (1.35) into equation (1.39) we derive an equilibrium expression for n as a function of the shocks (r^* and $\hat{\tau}$):

$$n_s^{ld} = \frac{\gamma_c\lambda(1-\beta\mu)(1-\mu)\sigma + \gamma_c(1-\lambda)\mu\kappa}{\Gamma(1-\beta\mu)(1-\mu)\sigma}(\sigma c_s + \varphi n_s^{ls} + \chi\hat{\tau}_s)$$
$$-\frac{\gamma_c\lambda\chi}{\Gamma}\hat{\tau}_s + \frac{(1-\lambda)\gamma_c}{\Gamma(1-\mu)\sigma}r_s^*$$

and after using the aggregate resource constraint (equation 1.34) and the production function ($y_t = n_t$) we obtain:

$$n_s^{ld} = \frac{\gamma_c\lambda(1-\beta\mu)(1-\mu)\sigma + \gamma_c(1-\lambda)\mu\kappa}{\Gamma(1-\beta\mu)(1-\mu)\sigma}\left[\left(\frac{\sigma}{\gamma_c}+\varphi\right)n_s^{ls} + \chi\hat{\tau}_s\right]$$
$$-\frac{\gamma_c\lambda\chi}{\Gamma}\hat{\tau}_s + \frac{(1-\lambda)\gamma_c}{\Gamma(1-\mu)\sigma}r_s^*$$

In equilibrium labour supply equals labour demand ($n_s^{ld} = n_s^{ls}$) so that it follows

$$n_s\left[1 - \frac{[\gamma_c\lambda(1-\beta\mu)(1-\mu)\sigma + \gamma_c(1-\lambda)\mu\kappa]\left(\frac{\sigma}{\gamma_c}+\varphi\right)}{\Gamma(1-\beta\mu)(1-\mu)\sigma}\right]$$
$$= \left[\frac{[\gamma_c\lambda(1-\beta\mu)(1-\mu)\sigma + \gamma_c(1-\lambda)\mu\kappa]\chi}{\Gamma(1-\beta\mu)(1-\mu)\sigma} - \frac{\gamma_c\lambda\chi}{\Gamma}\right]\hat{\tau}_s$$
$$+\frac{(1-\lambda)\gamma_c}{\Gamma(1-\mu)\sigma}r_s^*$$

which can also be written as

$$n_s = \frac{\Phi}{\Xi}\hat{\tau}_s + \frac{(1-\lambda)\gamma_c(1-\beta\mu)}{\Xi}r_s^*$$

where $\Phi \equiv [\lambda(1-\beta\mu)(1-\mu)\sigma + (1-\lambda)\mu\kappa - \lambda(1-\beta\mu)(1-\mu)\sigma]\gamma_c\chi > 0$ and $\Xi \equiv \Gamma(1-\beta\mu)(1-\mu)\sigma - [\gamma_c\lambda(1-\beta\mu)(1-\mu)\sigma + \gamma_c(1-\lambda)\mu\kappa]\left(\frac{\sigma}{\gamma_c}+\varphi\right) > 0$. For reasonable parameter values Φ and Ξ are positive constants and the deflationary shock ($r^* < 0$) as well as the tax cut ($\hat{\tau} < 0$) are both decreasing labour so that the zero-lower bound is binding. It is useful

to note that $\Xi > 0$ corresponds to condition C2 in Eggertsson (2011). Assuming that γ_c and λ are fixed expressions Φ and Ξ are positive for high values of the IES or Frisch labour supply elasticities (i.e. when σ or φ is low, respectively).

Furthermore, our baseline calibration ensures that the shock is large enough for the zero lower bound on the nominal interest rate to start binding so that another condition which is equivalent to C1 in Eggertsson (2011) is also respected by our model solution (for further details see our not-for-publication appendix). A unique bounded equilibrium at the zero lower bound respects both C1 and C2. It can be shown that C2 is enough to hold for a well-defined equilibrium at positive interest rates (see Eggertsson (2011) and the not-for-publication appendix of the paper version of this chapter).

Our baseline calibration of the model containing Kimball demand that keeps κ low and helps to maintain $\Phi, \Xi > 0$ is consistent with $\mu < 0.7$ which implies a duration much shorter than with $\mu = 0.903$ estimated by Denes and Eggertsson (2011) using a model that contains only Ricardian households. Therefore, the model with two types of households satisfy the condition of binding zero lower bound ($\Phi, \Xi > 0$) only when the zero lower bound period is not too long with a maximum duration of $1/(1-0.7) = 3.33$ quarters (on average) *and* there is strategic complementarity in price-setting ensuring a low value of κ.

∎

The labour supply shifts to the right from LS to LS' due to a decrease in the tax rate (see again figure 1.2) leading to a drop in wages and hours worked along labour demand LD. The leftward shift of labour demand is associated with a drop in the consumption of Ricardians after the tax-cut. There are at least two reasons why consumption of Ricardians declines to higher extent in case of a tax cut (see figure 1.1). First, the sharp increase in real interest rates make them delay their consumption expenditures. Second, the tax cut in

the present is associated with future tax-hikes (a large negative wealth effect) to satisfy the budget constraint of the government and in experiments 1-3 it is only the Ricardians who bear the burden of the tax cut. As a result labour demand falls so much (from LD to LD') due to lower consumption demand of Ricardians (firms produce less) that equilibrium hours worked finally drops even more with tax cut relative to the case of no policy intervention. Also importantly, with flexible wages the tax cut $(-\chi\hat{\tau}_t > 0)$ is not strong enough to counteract the decline in real wages and hours worked due to the negative demand shock and the tax cut so that the disposable income of non-Ricardians' cannot rise[22]. In sum the tax cut magnifies the deflationary effects described by Eggertsson (2011): *price deflation and the contraction in hours worked are more severe with the tax cut even after the inclusion of non-Ricardian households in the absence of wage rigidity.*

1.4.2 Experiment 2—price and wage rigidity

Figure 1.3 shows an experiment similar to the first one but this time we introduce wage stickiness into the model (experiment **two**). The discount rate is set to -0.01 per quarter. The ZLB binds for period 1 to 6 with or without policy. Wages are set by unions and assumed to remain fixed for about 3 quarters. The wage tax cut increases the disposable income of ROT households who consume it. *Again, the rise in the consumption of ROT households (and similarly for the other variables) in response to a labour tax cut should be read as the consumption of ROT households (and also other variables) fall less in case of the tax-cut than without the policy (see Figure 1.3).*

Real wage does not fall dramatically due to the presence of wage stickiness in sharp contrast to the previous experiment (the absence of wage rigidity). But, still, the tax cut remains deflationary (labour supply shifts slightly more to the right than labour demand) and real

---

[22]Based on the budget constraint of non-Ricardians (see equation (1.7)) c_t^r drops due to the fall in n_t^r and w_t which is not neutralised by $-\chi\hat{\tau}_t^r > 0$.

Figure 1.3: This is experiment 2. The model used in the experiment 1 is extended with wage rigidity. The + signs indicate the date at which the zero lower bound on the nominal interest rate becomes binding and circles appear on the date at which the zero lower bound ceases to bind. ss means steady-state. There are two shocks in this experiment: a strong negative demand shock with size that equals to -0.01 in each period for ten quarters leading to a huge fall in all variables. Also there is a labour tax cut of size -0.1 in each quarter during the zero lower bound period which does not necessarily last as long as the negative demand shock.
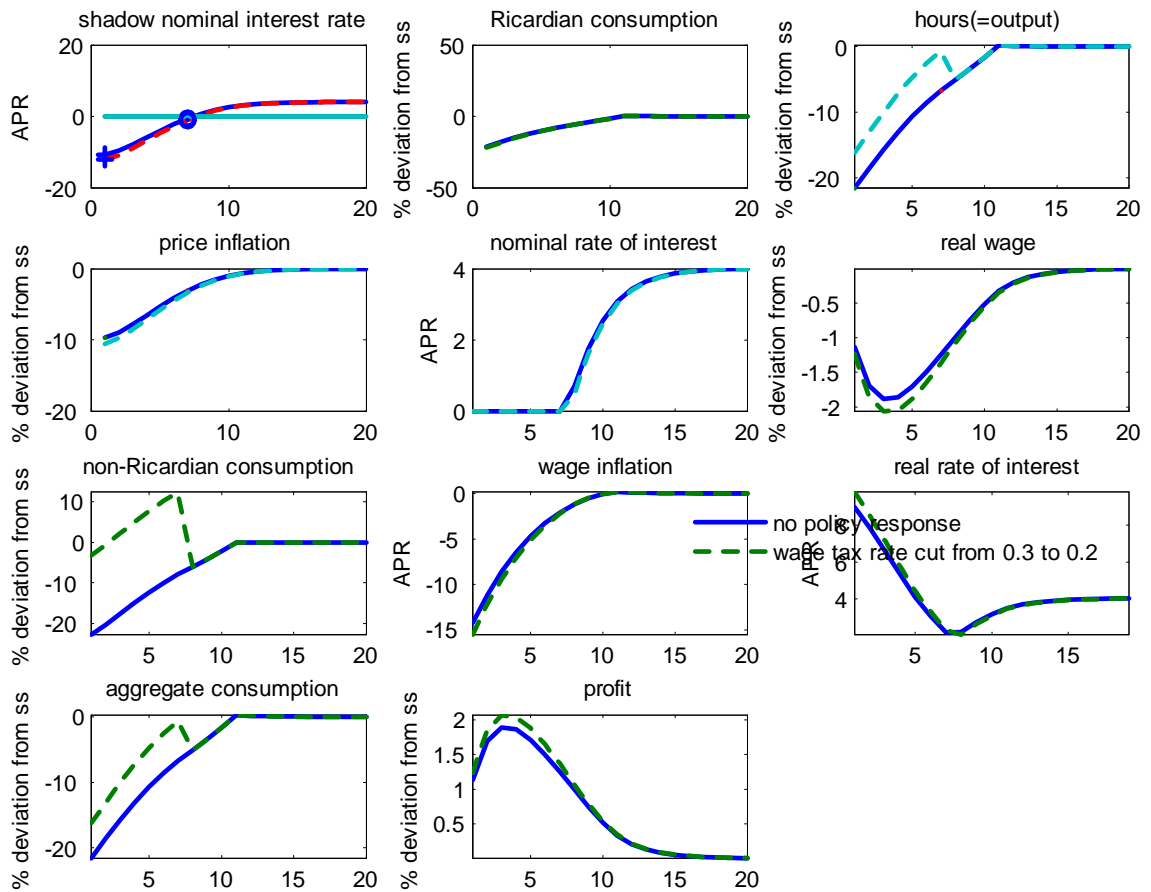
wage in the case of tax policy falls more than without policy. Observing the graph we can also see that the wage deflation is higher than the price deflation implying a fall in the real wage rate. With perfect wage-stickiness (ξ^w is close to one)—which is not the case here but serves as a useful abstraction (see e.g. the argument of Christiano (2010))—the labour supply would remain inact. In the next we analyse the indirect reaction of labour demand to the tax cut.

The higher consumption demand of non-Ricardian agents induces many of the firms which cannot charge a higher price due to price stickiness to increase their production. To produce more firms demand more labour i.e. the labour demand shifts out. As it is well-known in sticky-price models a rise in aggregate demand—due to the higher consumption expenditures of ROT households—leads to a fall in the markup, which induces an outward shift in the labour demand. Below we discuss the reaction of labour supply following the tax cut.

On the one hand, the labour tax cut raises the pre-tax real wage creating an incentive for the union to increase the labour supply (substitution effect). On the other hand, the labour tax cut has a strong negative wealth effect: Ricardians know that the present tax cut will be offset by higher lump-sum taxes in the future and, therefore, they decrease their demand for consumption and leisure. As the time frame is normalised to one, the fall in leisure implies spending more time working i.e. Ricardians supply more labour. Due to unions non-Ricardians work the same number of hours as Ricardians. Thus, both Ricardians and non-Ricardians satisfy higher labour demand by working more. On figure 1.3 the pre-tax real wage falls more for the tax-cut scenario relative to the case of no policy change. Also we observe that hours worked increases i.e. it decreases less with tax cut. It follows that the labour supply must have increased more than the labour demand.

It also needs to be emphasized that rigid wages imply that labour

supply (or wage schedule, WS) curve is flatter with rigid relative to flexible wages (Ascari et al. 2011) and the outward shift of labour demand in response to higher consumption expenditures of non-Ricardians is associated with more movement in hours worked (drop) rather than real wage (rise) which is rather apparent on Figure 1.4.

The profit on figure 1.3 rises either with or without a tax-cut. It is easy to show algebraically that this is always the case. In first-order loglinear terms profit can be written as $profit_t = y_t - w_t - n_t$. Because of the constant returns to scale assumption $y_t = n_t$ and, therefore $profit_t = -w_t$. The real wage always drops due to the negative demand shock so it follows that profit has to increase. Profit income that accrues only to Ricardians has an important role in the model. It helps Ricardian consumers who are the owner of the firms to insulate themselves from the negative wealth effects of the future tax increases and also from the rise in the real interest rate that discourages them from further consumption in the present. Thus, profit income enables Ricardians to avoid larger cuts in consumption due to the future tax burden and the higher real rates.

1.4.3 Experiment 3—price rigidity, wage rigidity and consumption habits

Experiment **three** that is shown on Figure 1.5 makes use of the model in the previous simulation but now it includes external habit formation in consumption as well. Due to the lagged consumption term habit formation injects some endogenous persistence into the model and leads to hump-shaped impulse responses in consumption and hours. Habit formation is a well-known feature of middle-sized DSGE models like the one of Smets and Wouters (2007) and is found useful in matching the empirical VAR evidence. Also habit formation is usually regarded to have some solid psychological foundation. The presence of habits mitigates the effects of the negative demand shock. This can be ex-

Figure 1.4: Comparison of labour markets under positive and zero nominal interest rate. WS-sticky stands for the wage schedule under sticky wages while WS-flexible means the wage schedule under flexible wages. Source: the left-hand-side figure is a reproduction of Ascari et al. (2011 page 12) while the right-hand-side one is based on figure 4 of Eggertsson (2010b page 15)
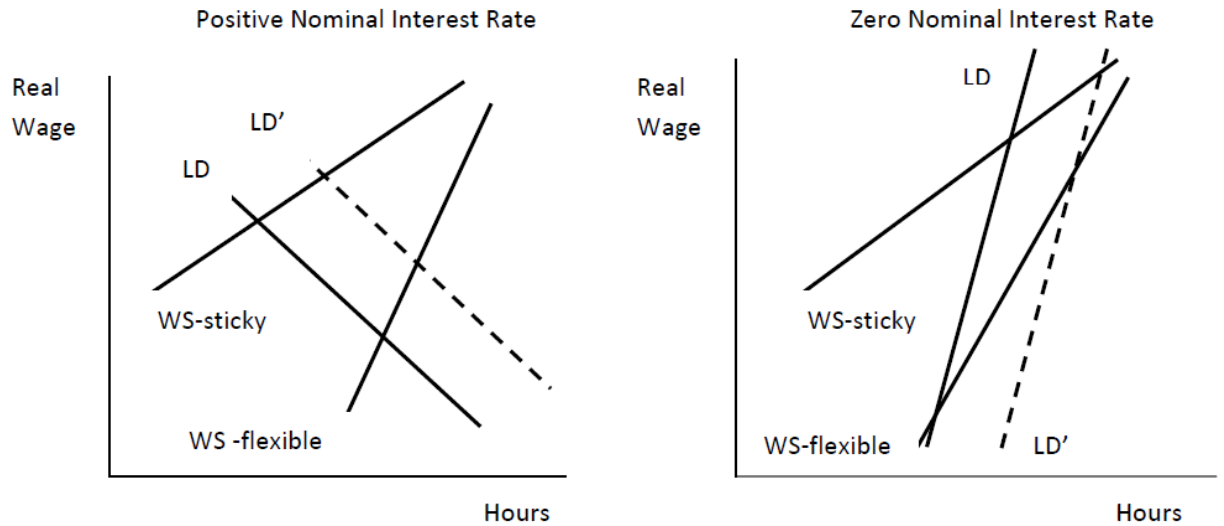
plained as follows. As argued above it is the rise in the real interest that makes people delay their consumption expenditure. The introduction of habits reduces the sensitivity of consumption to changes in the real interest (this can be quickly verified by looking at equation (1.5) where the coefficient on the interest is smaller in case of habits $[\frac{1-h_o}{1+h_o}\frac{\beta}{\sigma}]$ than it is for the standard CRRA case $[\beta/\sigma]$). The ZLB binds from period 1 to 8 (9) without (with) policy. Still output (hours) declines less when labour tax cut policy is applied.

1.4.4 Experiment 4—price rigidity, wage rigidity, consumption habits and government debt

In experiment **four** we assume more realistically that the tax cut is financed by government debt which is retired through an increase in labour tax revenue either in short- or long-run. We allow for inherited debt from the past: that is, the steady-state debt-to-output ratio is positive and interest payments on current debt affects the evolution of future debt. In previous experiments it was only the Ricardians

Figure 1.5: This is called Experiment 3 in the text. Here we used the model in Experiment 2 extended with external habit formation in consumption.
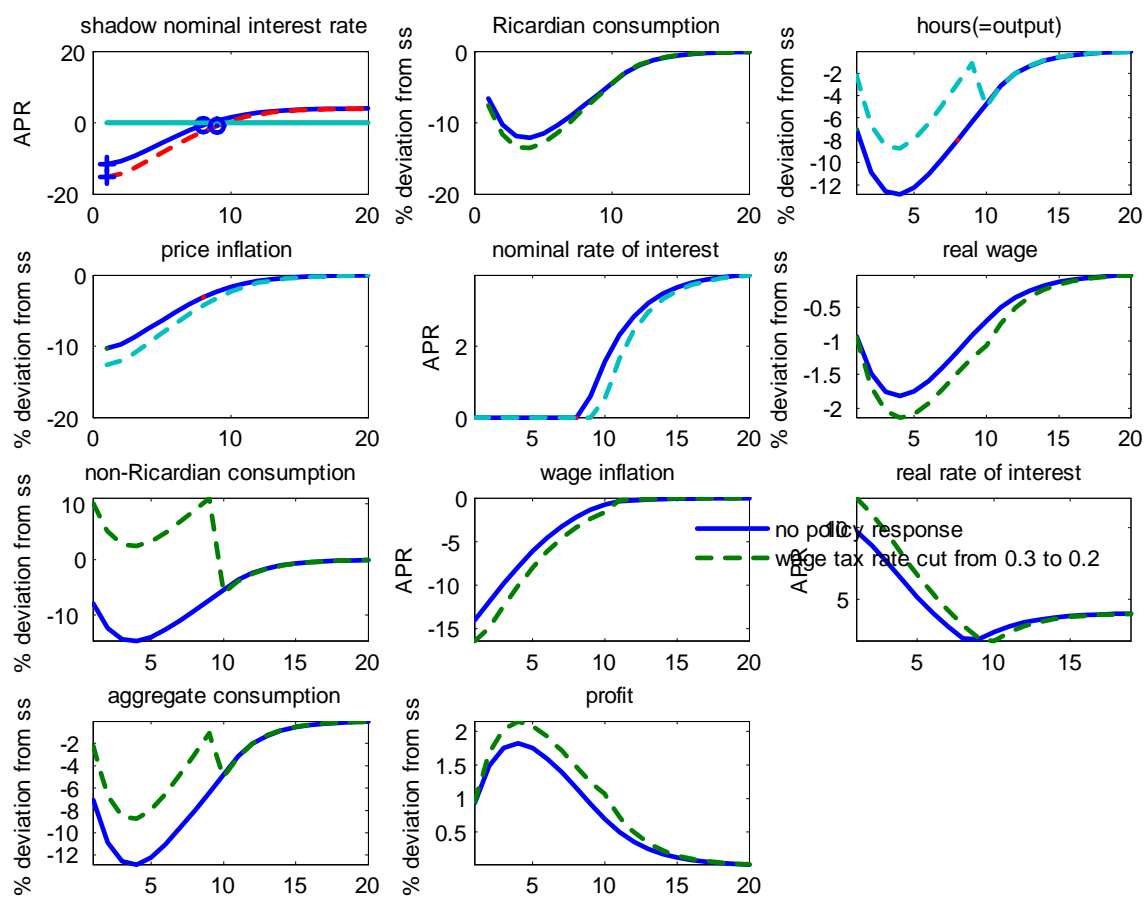
Figure 1.6: This is experiment 4a in the text. Here we set $\delta_1 = 0.02$.
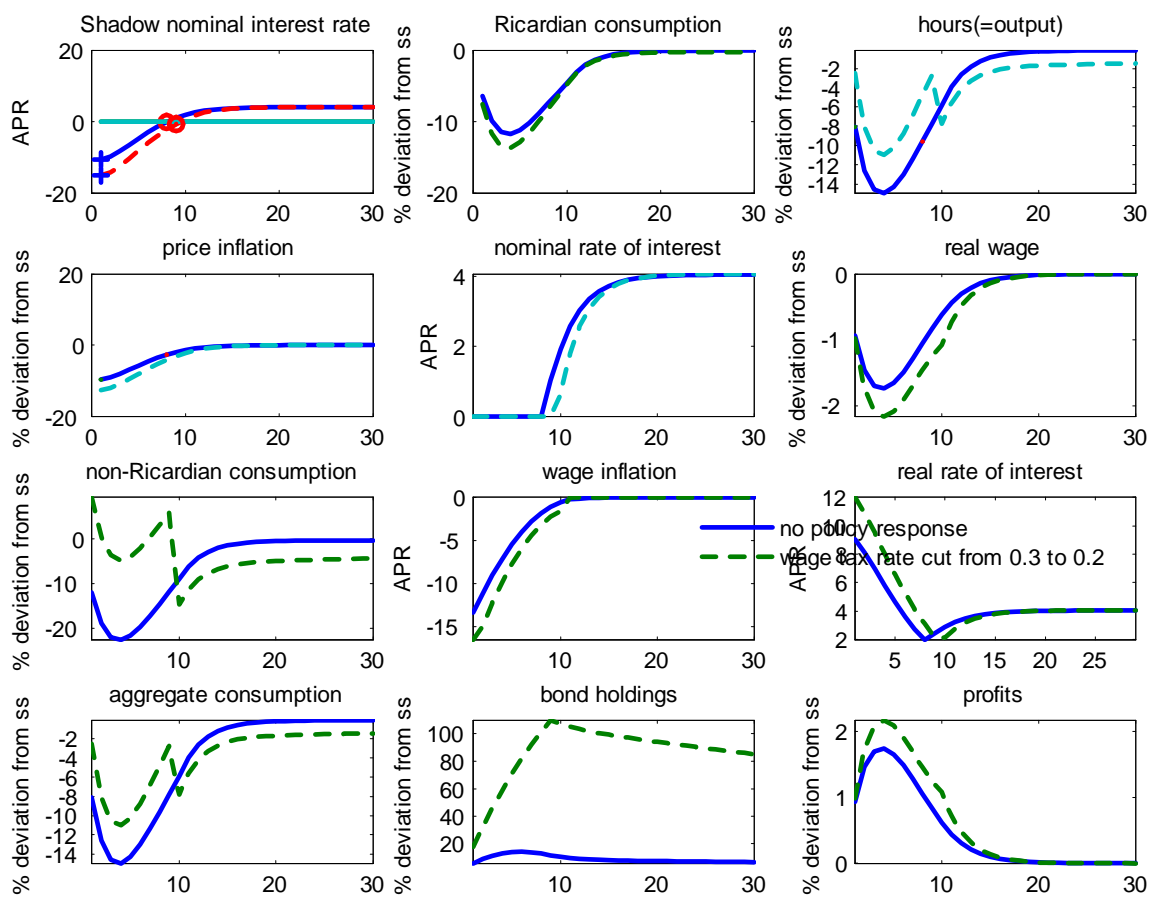
Table 1.1: Details of the models used in experiment 1-4

	Features of the model used
Experiment 1	price rigidity
Experiment 2	price and wage rigidity
Experiment 3	price and wage rigidity, consumption habits
Experiment 4a	price and wage rigidity, consumption habits and government debt with $\delta_1 = 0.02$
Experiment 4b	see Experiment 4a with $\delta_1 = 0.9$
	Robustness checks
Experiment 4c	see Experiment 4a with $\sigma = 1.5$ (lower IES)
Experiment 4d	see Experiment 4a with $\varphi = 1.5$ (lower Frisch)
Experiment 4e	see Experiment 4a with $\lambda = 0.2$ (lower share of non-Ricardians)
Experiment 4f	see Experiment 4a with $\xi^p = 0.6$ (lower price rigidity)
Experiment 4g	see Experiment 4a with $\xi^w = 0.66$ (lower wage rigidity)
Experiment 4h	see Experiment 4a with (wage inflation in the Taylor rule)

All experiments contain both Ricardian and non-Ricardian households. Experiment 1, 2 and 3 assumes a uniform tax cut that is financed by lump-sum taxes levied on Ricardian agents. Experiment 4 assumes that the uniform tax cut is financed by government debt that is retired through labour tax revenue collected from both types of households either in the short-run (δ_1 is close to one) or in the long-run (δ_1 is close to zero).

who bear the burden of the tax cut. However, now, it is also the non-Ricardians who have to take part in settling the bill. Taxation is uniform: both types of households pay the same tax rate.

The parameter choice for δ_1 in the fiscal rule (equation 1.29) turns out to be crucial for the outcome. When δ_1 is low the tax-cut is mainly financed with debt which paid back in the distant future (see, for instance, the impulse responses with $\delta_1 = 0.02$ on Figure (1.6) where bond holdings refer to real debt). This case lends support for tax-cut policy and is in accordance to the findings of Bilbiie et al. (2012) who argues in favour of a (lump-sum) tax-cut[23] as follows. When debt repayment happens far in the future a uniform tax cut is pure redistribution (transfer) from Ricardians to non-Ricardians in the present

---

[23]In this chapter we found that intuition in case of labour-tax cut that is very similar to the lump-sum tax-cut discussed by Bilbiie et al. (2012) who also assume consolidation of the debt in form of higher lump-sum taxes. In this chapter, however, consolidation of debt is carried out through increases in labour taxes which distort consumption-leisure tradeoff, discourage from work and depress output even more beyond the zero lower bound period.

while it is a transfer from non-Ricardians to Ricardians in the future. The evolution of Ricardian consumption is totally consistent with our story. Ricardians pay attention to changes in their whole lifetime income (the present discounted value of income) which remains unaltered with temporary changes in taxes (see the straight and dashed lines coincide on Figure (1.6)).

Thus, the outcome of the policy in case of endogenous debt with low δ_1 is very similar to the first fiscal scenario in which only Ricardians pay for the tax-cut through lump-sum taxes. In particular, Ricardians do not react to the policy while non-Ricardians enjoy the tax-decrease as it is not offset in the present by a rise in the labour tax. Also note that this tax cut policy leads to better outcome only in the zero-lower bound period which coincides with the period of sharp accumulation in debt. As soon as the Taylor rule is operative again (from period 11) this type of tax cut policy is strictly worse mainly because of its negative effects on the consumption of non-Ricardians. Indeed, from period 11 the tax-cut is over and non-Ricardians also have to take part in settling the debt accumulated in the zero lower bound period. As a result non-Ricardian consumption with fiscal policy beyond the zero lower bound period is below the one without fiscal policy.

On the contrary when δ_1 is closer to one (see Figure (1.7) for $\delta_1 = 0.9$) the tax cut in the present is counteracted by a tax rise and there is no rationale for such a policy. This is also confirmed by looking at the evolution of bonds on the same Figure. In the first ten periods bond holdings are positive but from period eleven they are the same as in the case of no policy. This is markedly different from the previous experiment where the late repayment ensured that bond holdings are positive even beyond period ten in case of a tax-cut relative to no policy intervention. Values of δ_1 that equals to one or above are to be avoided as they would render the path of debt explosive.

It deserves explanation why we observe a run-up of the debt in the

Figure 1.7: This is experiment 4b in the text. Here we set $\delta_1 = 0.9$.
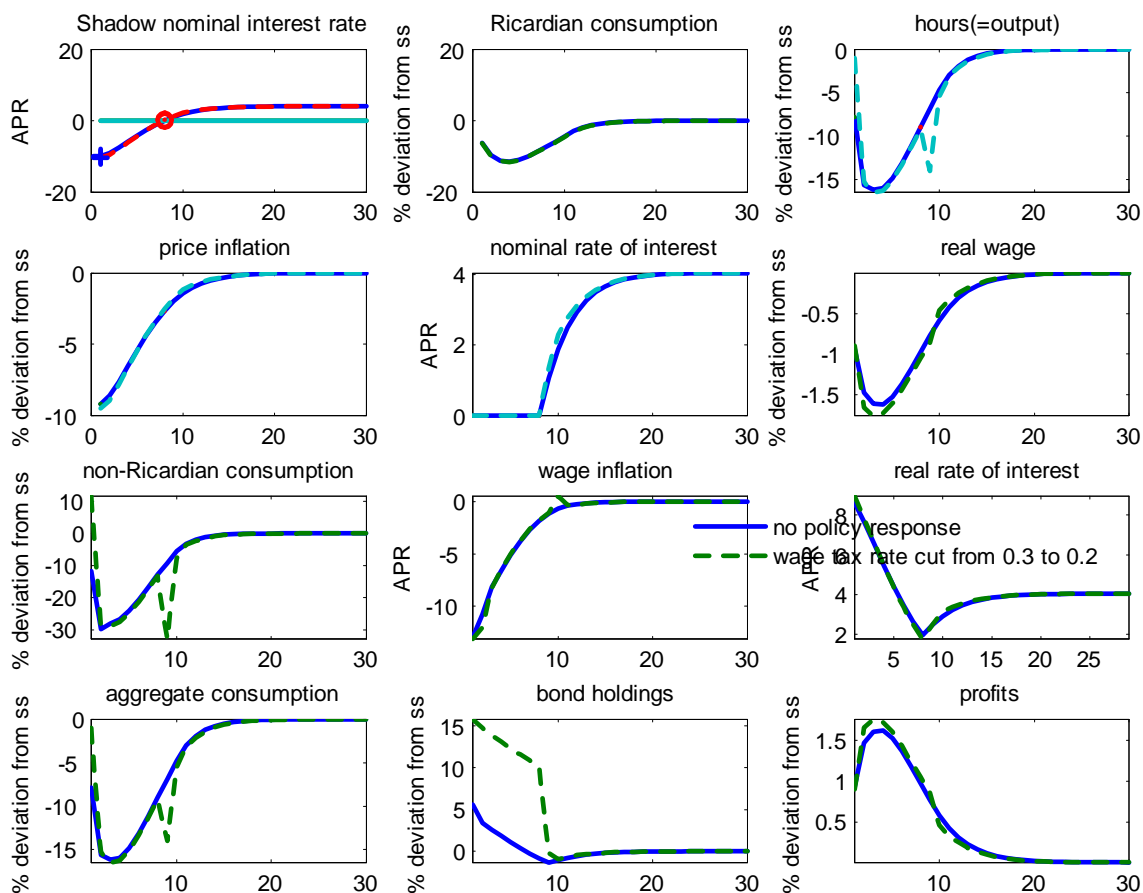
zero lower bound period even without fiscal policy (see straight lines for bond holdings on Figures 1.6 and 1.7). All the figures plot the evolution of real debt. In particular, increases in debt during the zero lower bound period (in the absence of fiscal policy) reflects the fact that the real value of debt increases due to deflation.

1.4.5   Robustness checks—using the model in Experiment 4

It is necessary to test the robustness of the tax-cut policy in case of government debt that is settled far in the future (i.e. δ_1 is close to zero). Several estimated middle-sized DSGE models like Smets and Wouters (2007) consider a value of lower than one for the IES and Frisch elasticities. Figure (1.8) and (1.9) show experiments with either IES or Frisch elasticity chosen to be 2/3 (ceteris paribus). Based on the

figures we conclude that the main result is not sensitive to the choice of the values of the EIS or the Frisch elasticity. In fact, the tax-cut seems to be even more desirable when the EIS and the Frisch elasticity are below one. Figure (1.10) is the case when we reduce the share of non-Ricardian households to 20% in the economy (baseline calibration is 30% which is not considered to be high according to several estimates, see the section on calibration) and the positive effect of the tax-cut disappears. Figure (1.11) demonstrates that our result is robust if we use a shorter average duration of price rigidity. Our result is also very sensitive to the average duration of wage stickiness: see figure (1.12) where the average duration of wage contract is 3 quarters instead of the baseline choice of 3.33 quarters on average. Finally, we report an experiment where we replace price inflation with wage inflation in the Taylor-rule (see figure (1.13)). With wage inflation in the Taylor rule tax-cut policy is still somewhat better than the absence of it.

In sum, we have seen that the result is very sensitive to the share of non-Ricardian households, the duration of wage stickiness and which variable is included in the Taylor rule. However, it needs to be stressed that these experiments were conducted using unitary values for the consumption and leisure curvatures parameters (Frisch and IES, respectively). As previously pointed out several empirical studies estimated these values to be slightly or well below one. Hall (1988) review the early literature on estimates of EIS and settles with a value of around 0.1. Vissing-Jorgensen (2002) uses data from the U.S. Consumer Expenditure Survey over 1980-1996 and estimates EIS to be around 0.3-0.4 for stockholders and 0.8-1 for bondholders. Importantly, Vissing-Jorgensen (2002) utilises time-separable utility function without habits to estimate EIS. However, Vissing-Jorgensen and Attanasio (2003) conclude that the EIS might be higher than one for stockholders estimating a structural model that is based on Epstein-Zin preferences which help to separate EIS from risk-aversion. Also

note that in chapter two and three we found that a model containing Epstein-Zin preferences match the macro and finance data best when IES and Frisch elasticity are below one.

An influential labour market literature estimated $1/\varphi$ to be small using household level data (see, e.g., Pistaferri (2003)). First, $1/\varphi$ can be treated as the elasticity of hours worked to changes in the real wage holding household wealth fixed (the intensive margin) and individual labour supply elasticities are found to be small based on micro data. It is also true that data shows little variation in either hours worked or real wage along the business cycle. However, data shows substantial movement in employment (extensive margin) in response to changes in the wage. Therefore, Christiano et al. (2010) argue that it is more reasonable to treat N_t as the number of people working in a particular household and reinterpret $1/\varphi$ as *"the elasticity with which different members of the households enter or leave employment in response to shocks"* (Christiano et al. (2010) pp. 18). This suggests that φ is meant to capture the aversion to work by different members of the household and, thus a small φ means that several members of the households are close to be indifferent between working and not working and a small change in the wage triggers a large labour supply response. Based on this Christiano et al. (2010) argue that it makes sense to set a value of around one or larger than one (their Bayesian estimate) for $1/\varphi$.

Based on the previous arguments we set IES to be 0.5 and Frisch elasticity to 1.67 (for higher value of the Frisch elasticity the numerical approximation of the policy function is found to be inaccurate). Then the tax-cut policy is still better even with a share of non-Ricardian consumers of 25% which is lower than the baseline calibration of 30%.

Figure 1.8: This is experiment 4c in the text. The elasticity of intertemporal substitution (EIS) is 2/3 here (i.e. $\sigma = 1.5$)
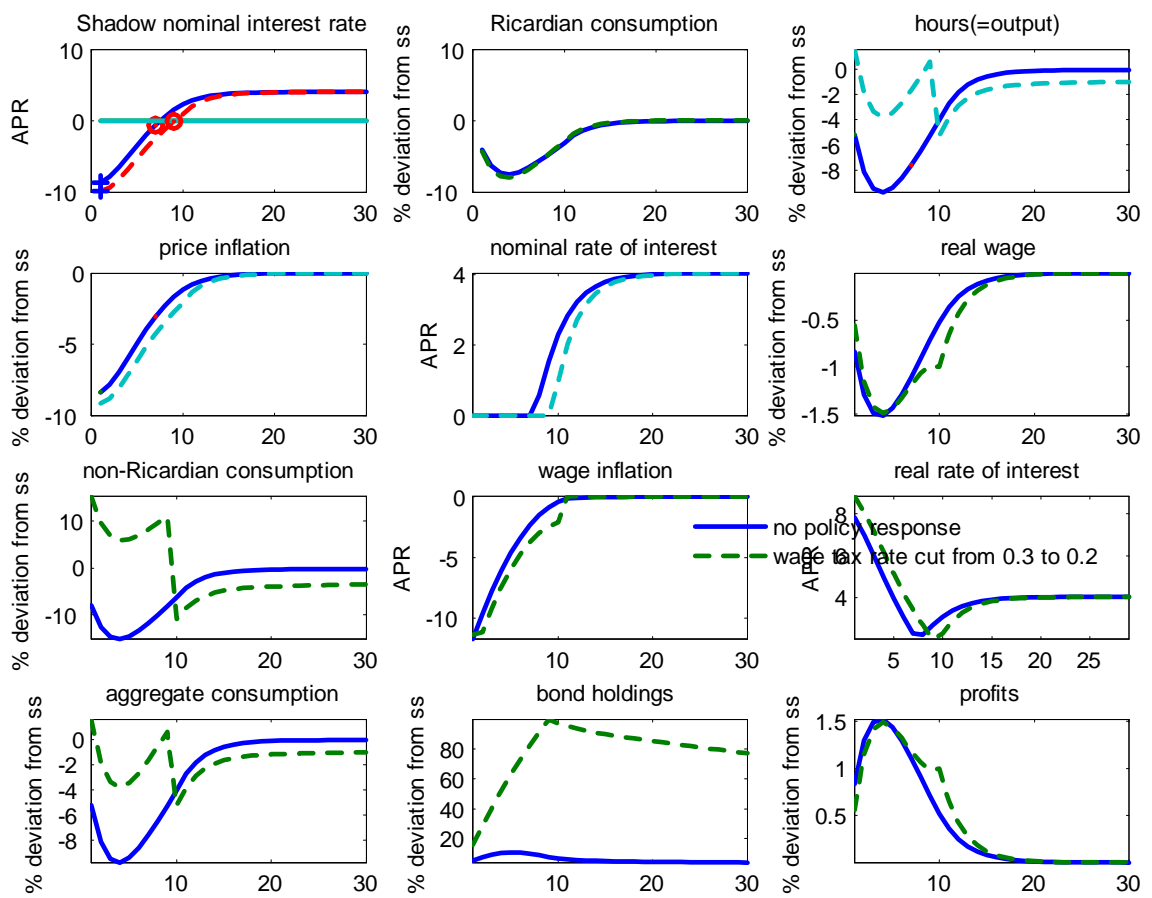
Figure 1.9: This is experiment 4d in the text. The Frisch elasticity of labor supply is 2/3 here (i.e. $\varphi = 1.5$)
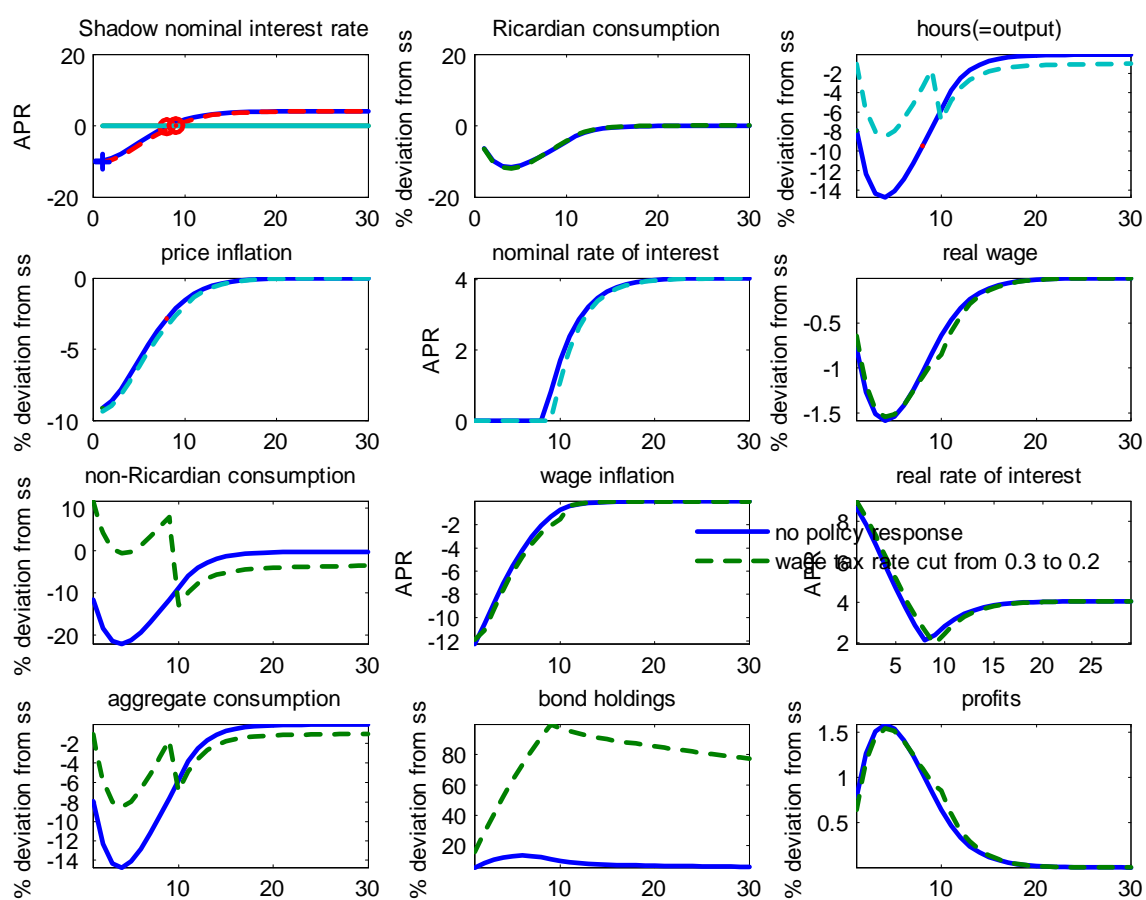
Figure 1.10: This is experiment 4e. The share of non-Ricardian households ($\lambda = 0.2$) is lower.
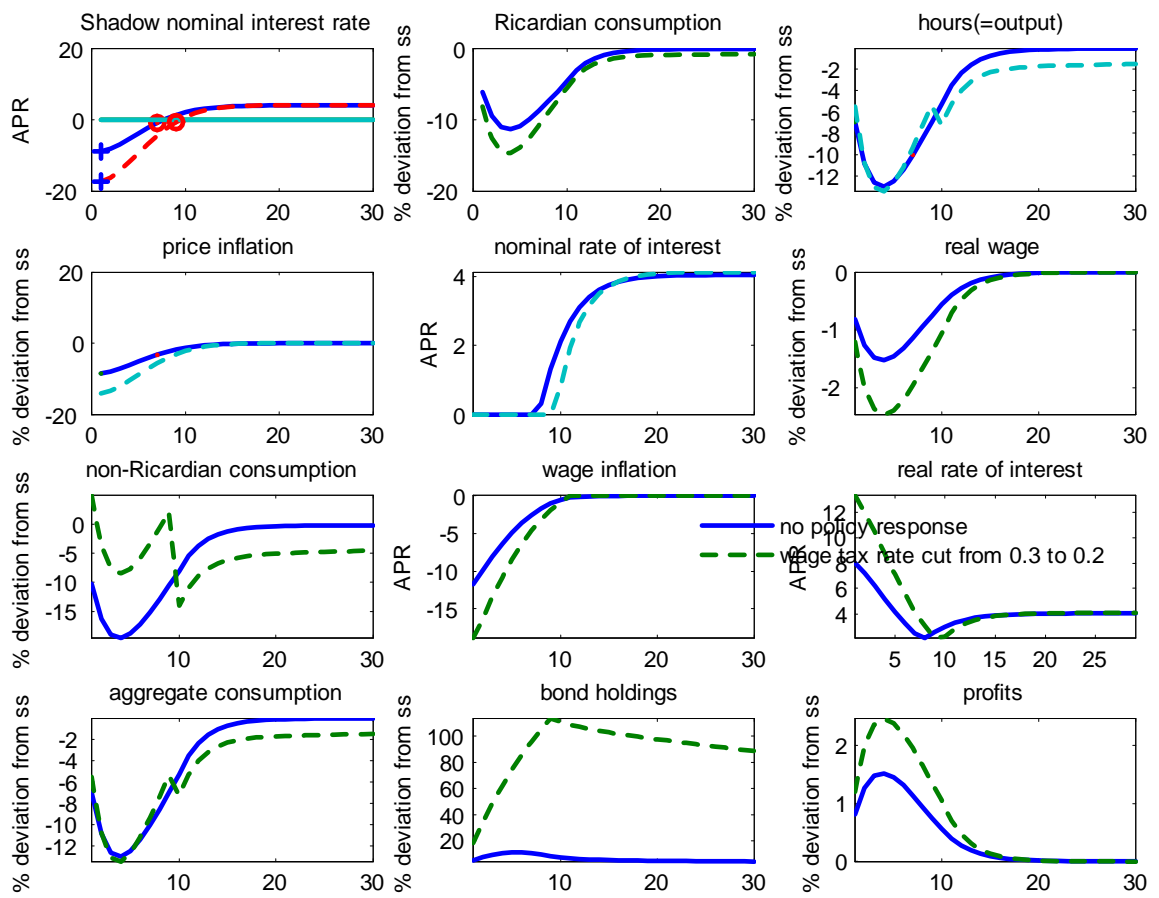
Figure 1.11: This is experiment 4f. Price rigidity lasts for 2 quarters (on average) instead of the baseline of 3 quarters (on average).
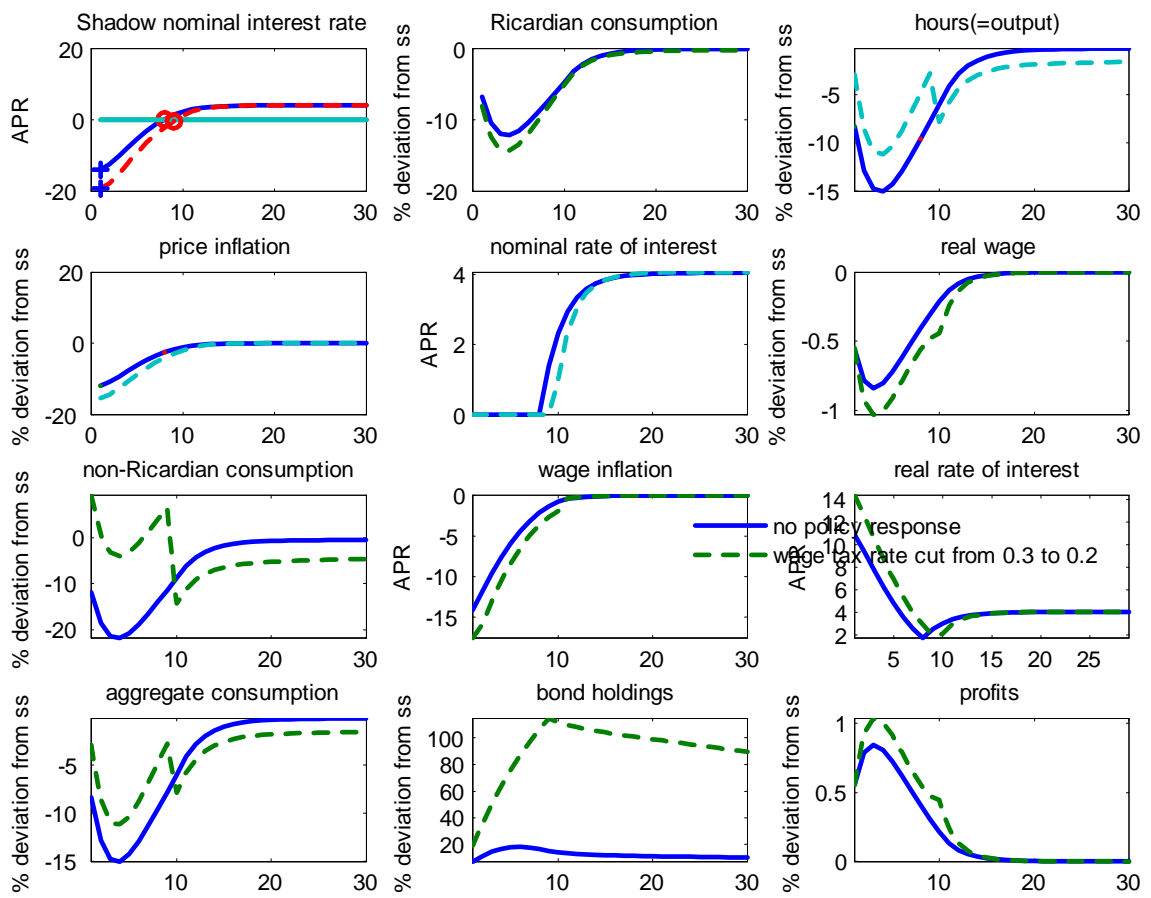
Figure 1.12: This is experiment 4g. The duration of wage rigidity is reduced to 3 quarters (on average) instead of the baseline 3.33 quarters (on average).
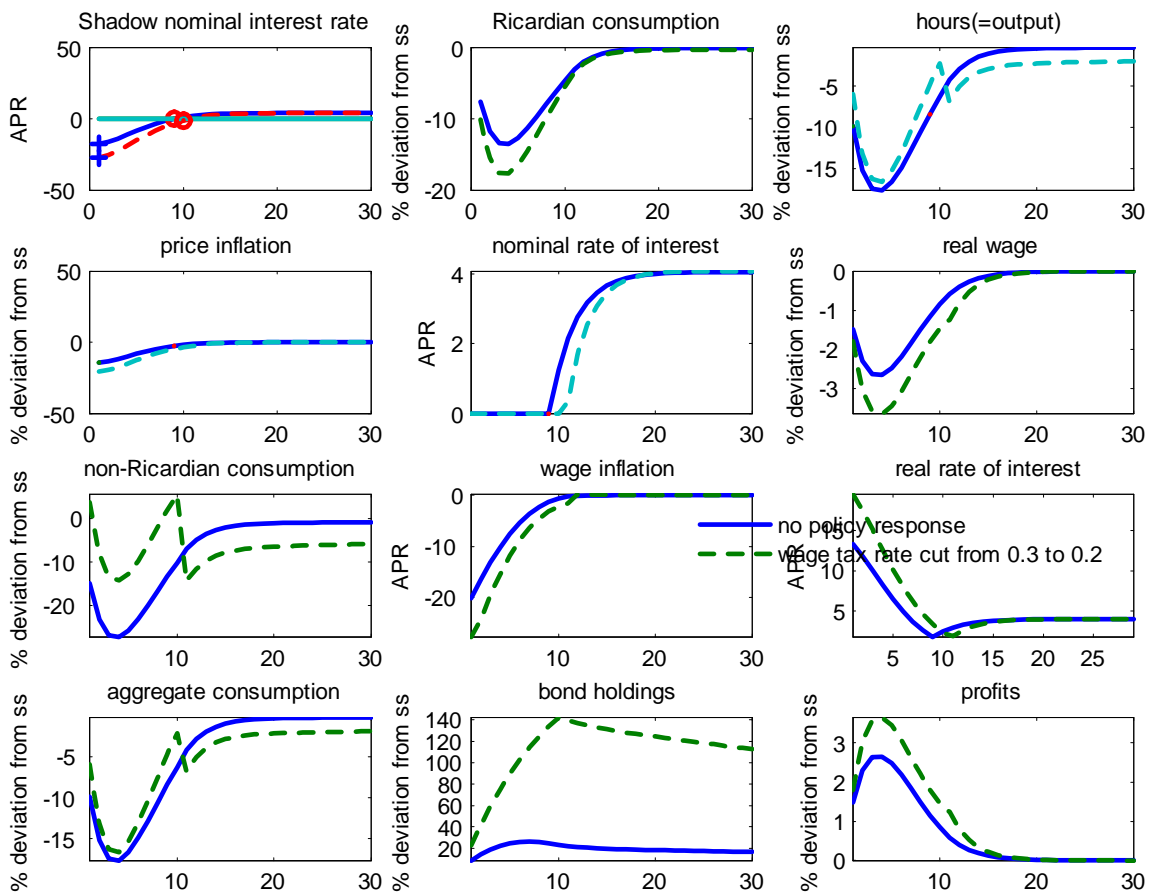
Figure 1.13: This is experiment 4h. Here price inflation is replaced for wage inflation in the Taylor rule.
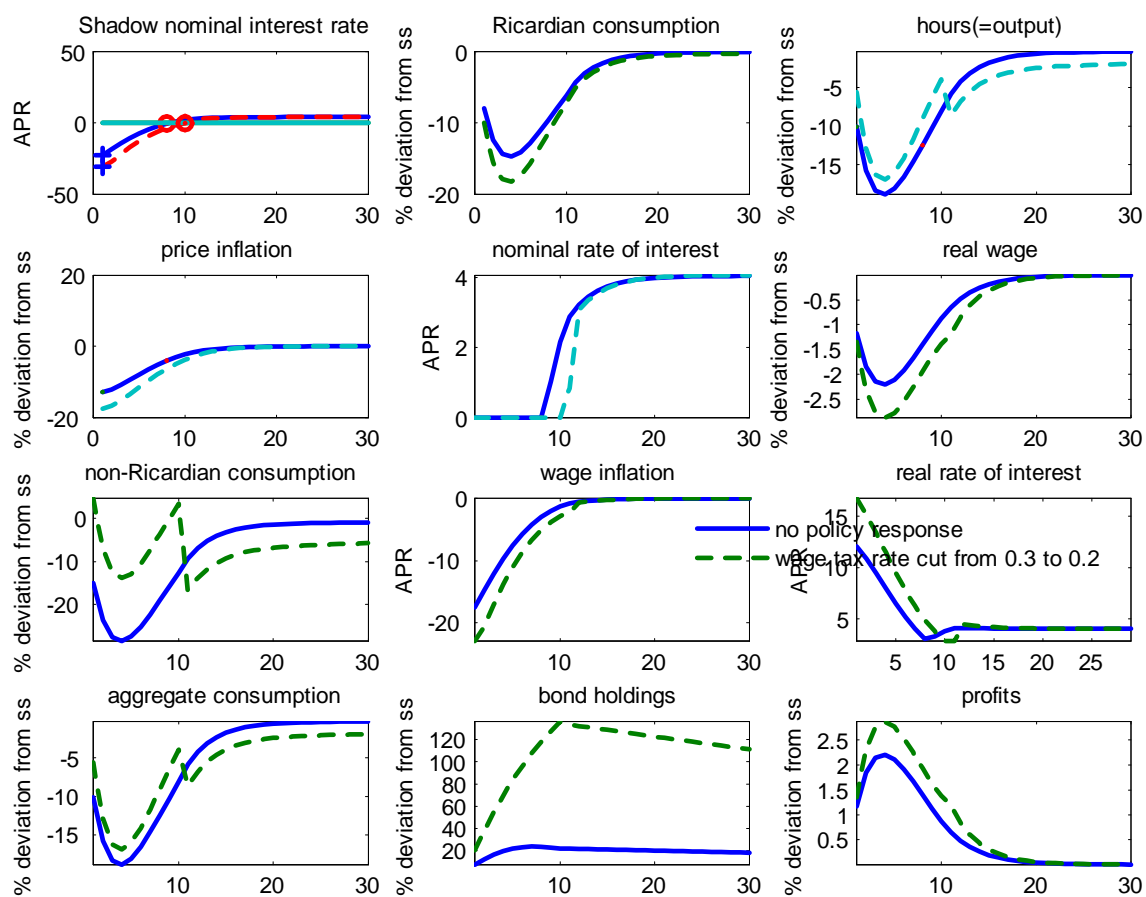
1.5 Conclusion

After augmenting the baseline new-Keynesian model containing price and wage rigidity with rule-of-thumb (or non-Ricardian) households we argued that a labour tax cut can partly offset the fall in output and deflation caused by a negative demand shock that made the zero lower bound on the nominal interest rate binding. Importantly, we assumed that we cut the labour tax rate that is levied upon the households and not upon the firms. Under such an arrangement the labour tax cut acts like a traditional fiscal stimulus that raises aggregate demand. We found that the tax-cut policy is stimulative if it is financed by lump-sum taxes levied completely on Ricardian agents. We also explored a more realistic scenario when tax-cut is covered by long-term government debt which is settled by both types of households in the form of taxes on labour income. Still the tax-cut is found to have positive effects on output because non-Ricardians who doesn't take into the consideration the future tax burden enjoy the increase in their disposable income by spending more. Finally, rule-of-thumb consumers can be thought of as a shortcut of modeling agents with borrowing constraint. Based on the logic of the model with rule-of-thumb households our finding should remain valid in a model with savers and borrowers who face borrowing constraints.

Chapter 2

Fiscal Policy and the Nominal Term Premium

with Ales Marsal[1]

2.1 Introduction

Long-term nominal bonds deliver term-premium in order to compensate for future inflation and consumption risks the bond-holder has to bear. Rudebusch and Swanson (2012) make use of a basic New Keynesian model and finds that the term premium can be large and volatile. They assume the simplest fiscal scenario where government spending is financed with lump-sum taxes and deficits are not allowed. Further they show that long-run inflation risks like uncertainty about the inflation target substantially increase the term-premium. Van Binsbergen et al. (2012) estimate a simplified version of the model in Rudebusch and Swanson (2012) with Bayesian methods and points toward further investigation of the model with a fiscal structure added. There are several empirical papers like Engen and Hubbard (2004), Laubach (2009) and Canzoneri et al. (2002) who find a positive connection between deficit and long-term bond yields using various econometric methods. Barth et al. (1991) is an early paper surveying fourty-two studies on

---

[1]Ales Marsal is based at the Institute of Economic Studies, Faculty of Social Sciences, Charles University in Prague.

the effects of fiscal policy on the yield curve. He finds positive effects in eighteen cases, six of them are mixed and the rest of them (nineteen cases) exhibits not significant or negative effects. The reduced form econometric models differ mainly by the variables they use to make inference. In particular, they explore the effect of current (or projected) deficit on current (or forward) interest rates. For instance, Elmendorf (1993) finds that 1% increase in projected deficit-to-GNP ratio raises 5-year bond yields by 43 basis points. Also Canzoneri et al. (2002) predicts a 1 percent increase in the future deficit/GDP ratio to elevate long-term yields by 53-60 basis points. Orszag and Gale (2004) find a 1% rise in the expected deficit-to-GDP ratio to lift 10-year bond yields by 30-36 basis points. They also provide a summary of the more recent literature and report thirteen positive, five mixed and one case with no effects. Similar to Orszag and Gale (2004) Engen and Hubbard (2004) estimate the effect of 1 percent increase in the predicted deficit-to-GDP ratio to drive up long forward rates by 18-24 basis points. Laubach (2009) reports that a 1 percent jump in expected deficit-to-GDP lifts long-term forward rates by 24-40 basis points.

This chapter proposes a simple alternative way to generate long-run inflation risks. In particular, we introduce distortionary income taxation into the model of Rudebusch and Swanson (2012) and find that the term premium on nominal bonds is higher than with lump-sum taxes. Thus, fiscal policy provides another good reason why long-term nominal bonds are risky. Further we show that the model with income taxation is able to generate the mean level of the empirical nominal term premium with a risk-aversion coefficient that is lower than the one needed in case of the model with long-run inflation risks. We consider only Ricardian fiscal policy where agents anticipate current rises in government spending to be covered by present or future taxes causing a negative wealth effect that make households reduce their

consumption in the present in order to save up for the tax-burden. Even if Ricardian fiscal policy occupies a prominent place in modern macroeconomic theory it has to be noted that it is not necessarily consistent with some of the VAR evidence predicting a positive response of consumption to positive government spending shocks (see, for instance, Gali et al. (2007)).

Rudebusch and Swanson (2012) show that a basic New Keynesian model approximated to the third order in the sense of Taylor series is able to generate large and volatile term premium that is in line with US data. The term-premium is constant to the second order while it becomes time-varying to the third-order. The only asset in their model is a default-free government bond. These assets are risky as their real payoff covaries positively with consumption. In particular, shocks that result in low consumption, high inflation and, thus, low real yields are important sources of the term premia. There are no liquidity risks in their model.

The most important feature of Rudebusch and Swanson (2012) model are Epstein-Zin preferences. Earlier papers in the literature considered preferences with consumption habits (see, for instance, de Paoli et al. (2006) and the literature review in Rudebusch and Swanson (2008)). With habits households are mainly concerned about sudden changes in consumption. However, with Epstein-Zin preferences they are unhappy with changes in consumption over medium and long horizons as well as short horizons. The household can offset short-run changes in consumption by modifying its labour supply and savings (also known as precautionary savings) but its ability to smooth consumption at medium- and long-run is much more limited.

Unlike previous papers of endowment economies with either long-memory habits (see Campbell and Cochrane (1999)) or Epstein-Zin preferences (like Wachter (2006)) Rudebusch and Swanson (2012) is a production economy. In endowment economies it is easy to gen-

erate high term-premium with consumption habits as consumption risk cannot be insured away by varying labour supply. In production economies the effects of a negative consumption shock can be mitigated by increasing labour supply so that term-premium on risky assets is considerably lower.

This paper departs from Rudebusch and Swanson (2012) who assume that government spending is financed by lump-sum taxes in each period. In their model there is no deficit allowed and, hence, no role for government debt. Below we show that allowing for government debt change properties of the term-premium to a small extent if deficit is financed by lump-sum taxes. As an alternative of lump-sum taxes we introduce distortionary taxation into the Rudebusch and Swanson (2012) model following Linnemann (2006) who postulates that government collect revenue by levying the same tax rate on labour and profit income. When government debt is paid back through income taxes the mean term-premium is substantially higher than with lump-sum taxes and similar in magnitude to what can be obtained using the long-run inflation risk version of Rudebusch and Swanson (2012). Also the fit of the model with our fiscal extension and the one with long-run inflation risks relative to US data are quite similar using the baseline calibration of Rudebusch and Swanson (2012).

The paper is organised as follows. Section 2 describes the model. Section 3 is on calibration and solution method. Section 3 provides details about the main results. The last section concludes.

2.2 The model

2.2.1 The household's problem with Epstein Zin preferences

The household maximises the continuation value of its utility (V):

$$V_t = \begin{cases} U(C_t, L_t) + \beta \left[E_t V_{t+1}^{1-\alpha} \right]^{\frac{1}{1-\alpha}} & \text{if } U(C_t, L_t) \geq 0 \\ U(C_t, L_t) - \beta \left[E_t (-V_{t+1})^{1-\alpha} \right]^{\frac{1}{1-\alpha}} & \text{if } U(C_t, L_t) < 0 \end{cases} \tag{2.1}$$

with respect to its flow budget constraint. β is the discount factor. Utility (U) at period t is derived from consumption (C_t) and leisure $(1 - L_t)$. E_t denotes expectations conditional on information available at time t. As the time frame is normalised to one leisure time $(1 - L_t)$ is what we are left with after spending some time working (L_t). The recursive functional form in equation (2.1) is called Epstein-Zin preferences and is the same as the one used by Rudebusch and Swanson (2012). To be consistent with balanced growth Rudebusch and Swanson (2012) imposes the following functional form on U:

$$U(C_t, L_t) = \frac{C_t^{1-\varphi}}{1 - \varphi} + \chi_0 Z_t^{1-\varphi} \frac{(1 - L_t)^{1-\chi}}{1 - \chi}, \quad \varphi, \chi > 0.$$

where Z_t is an aggregate productivity trend and φ, χ, $\chi_0 > 0$. The intertemporal elasticity of substitution (IES) is $1/\varphi$ and the Frisch labour supply elasticity is given by $(1 - \bar{L})/\chi\bar{L}$ where $\bar{L}$ is the steady-state of hours worked. As we restrict our attention (see details on calibration below) to $\varphi > 1$ and $\chi > 1$ we employ the second row of equation (2.1).

Rudebusch and Swanson (2012) derives the following relationship between coefficient of relative risk-aversion $(CRRA)$ and the curvature parameter α in the recursive utility (2.1):

$$CRRA = \frac{\varphi}{1 + \frac{\varphi}{\chi}\frac{1-\bar{L}}{\bar{L}}} + \alpha \frac{1 - \varphi}{1 + \frac{1-\varphi}{1-\chi}\frac{1-\bar{L}}{\bar{L}}}.$$

The budget constraint of the household can be written as:

$$p_{s^t} b_{s^t} + P_{s^t} C_{s^t} = (1 - \tau_t^i)(W_{s^t} L_{s^t} + D_{s^t}) + p_{s^t} b_{s^t_{t-1}}$$

where assets b_{s^t} with price p_{s^t} are available in each period t and state of the world s^t. The same tax-rate (τ_t^i) is levied on labour $(W_{s^t} L_{s^t})$ and profit (D_{s^t}) income. With the choice of $\tau_t^i = 0$ we obtain the model in RS.

The household sets up the following Lagrangian and chooses state-

contingent paths for consumption (C_{s^t}), labour (L_{s^t}) and asset holdings (b_{s^t}).

$$\max_{C_t, L_t, b_t} \mathcal{L} \equiv V_{s^0} - \sum_{t=0}^{\infty} \sum_{s^t} \mu_{s^t} \left\{ \begin{array}{c} V_{s^t} - U(C_{s^t}, L_{s^t}) \\ + \beta \left[\sum_{s^{t+1}} \pi_{s^{t+1}|s^t} (V_{s^{t+1}})^{1-\alpha} \right]^{\frac{1}{1-\alpha}} \end{array} \right\}$$

$$- \sum_{t=0}^{\infty} \sum_{s^t} \lambda_{s^t} \left[p_{s^t} b_{s^t} + P_{s^t} C_{s^t} - (1 - \tau_t^i)(W_{s^t} L_{s^t} + D_{s^t}) - p_{s^t} b_{s_{t-1}^t} \right]$$

where $\pi_{s^{t+1}|s^t}$ stands for the probability of realising state s^{t+1} at time $t+1$ conditional on being in state s^t at time t. The price per unit of consumption is P_{s^t}.

The first-order conditions associated with this problem are given by:

$$\frac{\partial \mathcal{L}}{\partial c_{s^t}} : \mu_{s^t} U_1(C_{s^t}, L_{s^t}) = P_{s^t} \lambda_{s^t},$$

$$\frac{\partial \mathcal{L}}{\partial L_{s^t}} : -\mu_{s^t} U_2(C_{s^t}, L_{s^t}) = (1 - \tau_t^i) W_{s^t} \lambda_{s^t},$$

$$\frac{\partial \mathcal{L}}{\partial b_{s^t}} : \lambda_{s^t} p_{s^t} = \sum_{s^{t+1} \supseteq s^t} \lambda_{s^{t+1}} p_{s^{t+1}},$$

$$\frac{\partial \mathcal{L}}{\partial V_{s^t}} : \mu_{s^t} = \beta \pi_{s^{t+1}|s^t} \mu_{s_{t-1}^t} \left[\sum_{\tilde{s}^t \supseteq s_{t-1}^t} \pi_{\tilde{s}^t|s_{t-1}^t} (V_{\tilde{s}^t})^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} V_{s^t}^{-\alpha}; \mu_{s^0} = 1,$$

where U_1 and U_2 denote the partial derivaties of U with respect to its first and second arguments, respectively. Letting $1 + r_{s^{t+1}} \equiv p_{s^{t+1}}/p_{s^t}$ denote the gross rate of return on assets and defining the discounted Lagrange multipliers as $\tilde{\lambda}_{s^t} \equiv \beta^{-t} \pi_{s^t|s^0}^{-1} \lambda_{s^t}$ and $\tilde{\mu}_{s^t} \equiv \beta^{-t} \pi_{s^t|s^0}^{-1} \mu_{s^t}$ we arrive at (after making some substitutions):

$$\frac{\partial \mathcal{L}}{\partial c_{s^t}} : \tilde{\mu}_{s^t} U_1(C_{s^t}, L_{s^t}) = P_{s^t} \tilde{\lambda}_{s^t}, \tag{2.2}$$

$$\frac{\partial \mathcal{L}}{\partial L_{s^t}} : -\tilde{\mu}_{s^t} U_2(C_{s^t}, L_{s^t}) = (1 - \tau_t^i) W_{s^t} \tilde{\lambda}_{s^t}, \tag{2.3}$$

$$\frac{\partial \mathcal{L}}{\partial b_{s^t}} : \tilde{\lambda}_{s^t} = \beta E_{s^t} \tilde{\lambda}_{s^{t+1}} (1 + r_{s^{t+1}}), \tag{2.4}$$

$$\frac{\partial \mathcal{L}}{\partial V_{s^t}} : \tilde{\mu}_{s^t} = \tilde{\mu}_{s^{t-1}} \left[E_{s^t_{t-1}} (V_{\tilde{s}^t})^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} V_{s^t}^{-\alpha}; \tilde{\mu}_{s^0} = 1 \qquad (2.5)$$

where $E_{s^t_t}$ denotes the expected value of being in state s^t. We can combine equation (2.2) and (2.3) to obtain the intratemporal condition:

$$-\frac{U_2(C_{s^t}, L_{s^t})}{U_1(C_{s^t}, L_{s^t})} = \frac{(1-L_t)^{-\chi}}{C_t^{-\varphi}} = (1-\tau_t^i)\frac{W_{s^t}}{P_{s^t}} \qquad (2.6)$$

Similarly the substitution of equation (2.2) and (2.4) into equation (2.5) results in the intertemporal condition:

$$U_1(C_{s^t}, L_{s^t}) = \beta E_{s^t} \left[E_{s^t_{t-1}} (V_{\tilde{s}^t})^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} V_{s^t}^{-\alpha} U_1(C_{s^{t+1}}, L_{s^{t+1}})(1+R_{s^{t+1}}^{net})P_{s^t}/P_{s^{t+1}}$$

$$(2.7)$$

where $R_{s^{t+1}} \equiv (1+R_{s^{t+1}}^{net}) = B_t/B_{t-1}$ is the gross return on a short-term nominal bonds. Equation (2.7) is equivalent to the equation in row 13 (the Euler equation to price bonds) in Table (2.1).

2.2.2   The intermediary firm's problem

Intermediary firm i produces output $(Y_t(i))$ using the technology:

$$Y_t(i) = A_t[K_t(i)]^{1-\eta}[Z_t L_t(i)]^\eta \qquad (2.8)$$

which after substituting for $Y_t(i)$ the demand for product i ($Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\theta}{\theta}} Y_t$) and aggregating across firms gives way to:

$$Y_t = \Delta_t^{-1} A_t[K_t]^{1-\eta}[Z_t L_t]^\eta, \quad 0 < \eta < 1, \qquad (2.9)$$

where $K_t = Z_t \bar{K}$ is the aggregate capital stock ($\bar{K}$ is fixed), η is the share of labour in production, $\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\theta}{\theta}} di$ is price dispersion (which can also be defined recursively, see Table (2.1) below), A_t is a stationary aggregate productivity shock:

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A,$$

where ε_t^A is an independently and identically distributed (iid) stochastic technology shock with mean zero and variance σ_A^2.

Intermediary firms which maximise their profits face price-setting frictions of Calvo style. With Calvo frictions a $1 - \xi$ fraction of firms can set its price optimally in each period. We follow Rudebusch and Swanson (2012) in laying out the intermediary firm's problem. Intermediary firm i chooses a state-contingent plan for prices that maximises its current and future profits:

$$E_t \left\{ \sum_{T=0}^{\infty} (\xi \beta)^T \mathcal{K}_{t,t+T} \left[P_t(i) Y_{t+T}(i) - W_{t+T} L_{t+T}(i) \right] \right\} \qquad (2.10)$$

where $\mathcal{K}_{t,t+T}$ is the representative household's (nominal) stochastic discount factor (or pricing kernel defined below in equation (2.17)). The first term in the squared bracket in equation (2.10) is the revenue of the firm while the latter one is the cost of labour.

There is a perfectly competitive sector that purchases the continuum of intermediary goods and turns them into a single final good using a CES aggregator:

$$Y_t = \left[\int_0^1 [Y_t(i)]^{\frac{1}{1+\theta}} di \right]^{1+\theta} .$$

Each intermediary firm i faces a downward-sloping demand curve:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\theta}{\theta}} Y_t \qquad (2.11)$$

where the economy-wide price index P_t is a CES aggregator of the individual prices:

$$P_t \equiv \left[\int_0^1 [P_t(i)]^{-\frac{1}{\theta}} di \right]^{-\theta} .$$

After taking the derivative of the profit equation (2.10) with respect to the optimal price $P_t(i)$ we obtain the standard optimality condition

in case of sticky prices:

$$P_t(i) = \frac{(1+\theta)\sum_{T=0}^{\infty}(\xi\beta)^T\mathcal{K}_{t,t+T}MC_{t,t+T}(i)Y_{t,t+T}(i)}{\sum_{T=0}^{\infty}(\xi\beta)^T\mathcal{K}_{t,t+T}Y_{t,t+T}(i)} \tag{2.12}$$

where $1+\theta$ is the gross markup and the (nominal) marginal cost of firm i can be written as:

$$MC_t(i) = \frac{W_t L_t(i)}{\eta Y_t(i)} \tag{2.13}$$

After making use of the aggregate production function (equation (2.9)) and aggregating equation (2.13) across firms we arrive at the formula of marginal cost in Table (2.1).

Also the optimality condition in equation (2.12) can alternatively be expressed in recursive (and aggregate) form as the equations in rows three, four and five in Table (2.1) and in the next proposition.

Lemma 5 *It can be shown that the optimality condition in equation (2.12) can be rewritten as*

$$\tilde{P}_t^{1+\frac{(1+\theta)(1-\eta)}{\theta\eta}} \equiv \left(\frac{P_t(i)}{P_t}\right)^{1+\frac{(1+\theta)(1-\eta)}{\theta\eta}}$$

$$= \frac{(1+\theta)\sum_{T=0}^{\infty}(\xi\beta)^T\mathcal{K}_{t,t+T}^{\text{real}}MC_{t,t+T}^{\text{real}}\pi_{t,t+T}^{\frac{1+\theta}{\theta\eta}}Y_{t+T}}{\sum_{T=0}^{\infty}(\xi\beta)^T\mathcal{K}_{t,t+T}^{\text{real}}\pi_{t,t+T}^{\frac{1}{\theta}}Y_{t+T}} \tag{2.14}$$

where the (real) stochastic discount factor is

$$\mathcal{K}_{t,t+T}^{\text{real}} \equiv \left(\frac{C_{t+1}}{C_t}\right)^{-\varphi}\left[\frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}}\right]^{-\alpha}$$

and the (real) marginal cost is

$$MC_{t,t+T}^{\text{real}} \equiv \frac{W_{t+T}/P_{t+T}}{\eta}\left(\frac{Y_{t+T}}{\bar{K}}\right)^{\frac{1-\eta}{\eta}}A_{t+T}^{-\frac{1}{\eta}}.$$

Proof. One can decompose the nominal marginal cost of an individual

firm in the following way:

$$MC_t(i) \;=\; \frac{W_t/P_t}{\eta} P_t \left(\frac{Y_t(i)}{\bar{K}}\right)^{\frac{1-\eta}{\eta}} A_t^{-\frac{1}{\eta}} \tag{2.15}$$

$$=\; \left(\frac{P_t(i)}{P_t}\right)^{-\frac{(1+\theta)(1-\eta)}{\theta\eta}} \frac{W_t/P_t}{\eta} P_t \left(\frac{Y_t}{\bar{K}}\right)^{\frac{1-\eta}{\eta}} A_t^{-\frac{1}{\eta}}$$

$$=\; \left(\frac{P_t(i)}{P_t}\right)^{-\frac{(1+\theta)(1-\eta)}{\theta\eta}} P_t MC_t^{\text{real}}$$

where the second line made use of the demand function in equation (2.11).

The previous equation can be substituted for $MC_t(i)$ in equation (2.12) to obtain:

$$E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}_{t,t+T} P_t(i) Y_{t+T}(i)$$

$$=\; (1+\theta) E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}_{t,t+T} \left(\frac{P_t(i)}{P_t} \frac{P_t}{P_{t+T}}\right)^{-\frac{(1+\theta)(1-\eta)}{\theta\eta}} \times$$

$$MC_{t,t+T}^{\text{real}} P_{t+T} Y_{t+T}(i) \tag{2.16}$$

where

$$\mathcal{K}_t \equiv \left(\frac{C_{t+1}}{C_t}\right)^{-\varphi} \left[\frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}}\right]^{-\alpha} \frac{1}{\pi_{t,t+T}} = \mathcal{K}_{t,t+T}^{\text{real}} \frac{1}{\pi_{t,t+T}}. \tag{2.17}$$

Then using the demand for an individual product (equation (2.11)) we can express equation (2.16) as:

$$E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}_{t,t+T}^{\text{real}} \frac{1}{\pi_{t,t+T}} P_t(i) \left(\frac{P_t(i)}{P_{t,t+T}}\right)^{-\frac{1+\theta}{\theta}} Y_{t+T}$$

$$=\; (1+\theta) E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}_{t,t+T}^{\text{real}} \frac{1}{\pi_{t,t+T}} \left(\frac{P_t(i)}{P_t} \frac{P_t}{P_{t+T}}\right)^{-\frac{(1+\theta)(1-\eta)}{\theta\eta}} \times$$

$$MC_{t,t+T}^{\text{real}} P_{t+T} \left(\frac{P_t(i)}{P_{t,t+T}}\right)^{-\frac{1+\theta}{\theta}} Y_{t+T}.$$

Next we multiply both sides of the previous equation by $P_t(i)^{\frac{1+\theta}{\theta}}$ and $P_t^{-\frac{1+\theta}{\theta}}$ and derive:

$$
E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}_{t,t+T}^{\text{real}} \frac{1}{\pi_{t,t+T}} P_t(i) \left(\frac{P_t}{P_{t,t+T}} \right)^{-\frac{1+\theta}{\theta}} Y_{t+T}
$$

$$
= (1+\theta) E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}_{t,t+T}^{\text{real}} \left(\frac{P_t(i)}{P_t} \frac{P_t}{P_{t+T}} \right)^{-\frac{(1+\theta)(1-\eta)}{\theta\eta}} \times
$$

$$
MC_{t,t+T}^{\text{real}} [\pi_{t,t+T}^{-1} P_{t+T}] \left(\frac{P_t}{P_{t,t+T}} \right)^{-\frac{1+\theta}{\theta}} Y_{t+T}.
$$

We can make use of the identity

$$
[\pi_{t,t+T}^{-1} P_{t+T}] = [P_t]
$$

to rewrite the previous expression as:

$$
E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}_{t,t+T}^{\text{real}} \pi_{t,t+T}^{-1} P_t(i) \left(\pi_{t,t+T} \right)^{\frac{1+\theta}{\theta}} Y_{t+T}
$$

$$
= (1+\theta) E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}_{t,t+T}^{\text{real}} \left(\frac{P_t(i)}{P_t} \frac{P_t}{P_{t+T}} \right)^{-\frac{(1+\theta)(1-\eta)}{\theta\eta}} \times
$$

$$
MC_{t,t+T}^{\text{real}} [P_t] \left(\frac{P_t}{P_{t+T}} \right)^{-\frac{1+\theta}{\theta}} Y_{t+T}.
$$

Regarding $\frac{P_t}{P_{t+T}}$ terms in the previous equation the following algebraic manipulation can be carried out on the RHS:

$$
\left(\frac{P_t}{P_{t+T}} \right)^{-\frac{(1+\theta)(1-\eta)}{\theta\eta}} \left(\frac{P_t}{P_{t+T}} \right)^{-\frac{1+\theta}{\theta}}
$$

$$
= \left(\frac{P_t}{P_{t+T}} \right)^{\frac{(1+\theta)(\eta)}{\theta\eta}} \left(\frac{P_t}{P_{t+T}} \right)^{-\frac{(1+\theta)}{\theta\eta}} \left(\frac{P_t}{P_{t+T}} \right)^{-\frac{1+\theta}{\theta}}
$$

$$
= \left(\frac{P_t}{P_{t+T}} \right)^{-\frac{(1+\theta)}{\theta\eta}} = \pi_{t,t+T}^{\frac{(1+\theta)}{\theta\eta}}
$$

and the inflation term on the LHS can be written as:

$$\pi_{t,t+T}^{-1}\left(\pi_{t,t+T}\right)^{\frac{1+\theta}{\theta}} = \pi_{t,t+T}^{-\frac{\theta}{\theta}}\left(\pi_{t,t+T}\right)^{\frac{1+\theta}{\theta}} = \pi_{t,t+T}^{\frac{1}{\theta}}.$$

Using the previous result the optimality condition can be transformed as:

$$E_t\sum_{T=0}^{\infty}(\xi\beta)^T\mathcal{K}_{t,t+T}^{\text{real}}\pi_{t,t+T}^{\frac{1}{\theta}}Y_{t+T}P_t(i)$$

$$= (1+\theta)E_t\sum_{T=0}^{\infty}(\xi\beta)^T\mathcal{K}_{t,t+T}^{\text{real}}\left(\frac{P_t(i)}{P_t}\right)^{-\frac{(1+\theta)(1-\eta)}{\theta\eta}}MC_{t,t+T}^{\text{real}}[P_t]\pi_{t,t+T}^{\frac{(1+\theta)}{\theta\eta}}Y_{t+T}$$

which is the same as the expression in the Lemma. ■

The preceding lemma has shown that the optimality condition of the firm in equation (2.12) can be rewritten as equation (2.14).

Proposition 6 *Here we demonstrate that equation (2.14) can be expressed recursively as follows (so that we can input them into Dynare):*

$$\tilde{P}_t^{1+\frac{(1+\theta)(1+\eta)}{\theta\eta}} = \frac{Zn_t}{Zd_t}$$

where

$$Zn_t = (1+\theta)MC_t^{\text{real}}Y_t + \xi\beta\mathcal{K}_t^{\text{real}}\pi_{t+1}^{\frac{1+\theta}{\theta\eta}}Zn_{t+1} \qquad (2.18)$$

and

$$Zd_t = Y_t + \xi\beta\mathcal{K}_t^{\text{real}}\pi_{t+1}^{\frac{1}{\theta}}Zd_{t+1}. \qquad (2.19)$$

Proof. The nominator of the fraction in equation (2.14) can be tagged as Zn:

$$Zn_t \equiv (1+\theta)E_t\sum_{T=0}^{\infty}(\xi\beta)^T\mathcal{K}_{t,t+T}^{\text{real}}MC_{t,t+T}^{\text{real}}\pi_{t,t+T}^{\frac{1+\theta}{\theta\eta}}Y_{t+T}$$

which can also be written as:

$$Zn_t = (1+\theta)MC_t^{\text{real}}Y_t + (1+\theta)E_t\sum_{T=1}^{\infty}(\xi\beta)^T\mathcal{K}_{t+T}MC_{t+T}^{\text{real}}\pi_{t,t+T}^{\frac{1+\theta}{\theta\eta}}Y_{t+T}.$$

$$(2.20)$$

After iterating the definition of Zn one period ahead we obtain:

$$
\begin{aligned}
Zn_{t+1} &= (1+\theta)E_{t+1}\sum_{T=0}^{\infty}(\xi\beta)^{T}\mathcal{K}_{t+1+T}^{\text{real}}MC_{t+1+T}^{\text{real}}\pi_{t+1,t+1+T}^{\frac{1+\theta}{\theta\eta}}Y_{t+T+1} \\
&= (1+\theta)E_{t+1}\sum_{T=1}^{\infty}(\xi\beta)^{T-1}\mathcal{K}_{t+T}^{\text{real}}MC_{t+T}^{\text{real}}\pi_{t+1,t+T}^{\frac{1+\theta}{\theta\eta}}Y_{t+T} \\
&= (1+\theta)E_{t+1}\sum_{T=1}^{\infty}(\xi\beta)^{T-1}\mathcal{K}_{t+T}^{\text{real}}MC_{t+T}^{\text{real}}\pi_{t,t+T}^{\frac{1+\theta}{\theta\eta}}\pi_{t+1}^{-\frac{1+\theta}{\theta\eta}}Y_{t+T}(2.21)
\end{aligned}
$$

where the last line made use of

$$
\pi_{t+1,t+T} \equiv \frac{P_{t+T}}{P_t}\frac{P_t}{P_{t+1}} = \pi_{t,t+T}\pi_{t+1}^{-1}.
$$

Finally let us multiply equation (2.21) by $\xi\beta\pi_{t+1}^{\frac{1+\theta}{\theta\eta}}$:

$$
\xi\beta\pi_{t+1}^{\frac{1+\theta}{\theta\eta}}Zn_{t+1} = (1+\theta)E_{t+1}\sum_{T=1}^{\infty}(\xi\beta)^{T}\mathcal{K}_{t+T}^{\text{real}}MC_{t+T}^{\text{real}}\pi_{t,t+T}^{\frac{1+\theta}{\theta\eta}}Y_{t+T} \quad (2.22)
$$

and recognise that the resulting expression is the the second term on the RHS of equation (2.20).

Hence the combination of equations (2.20) and (2.22) give way to equation (2.18) in the proposition. Similar derivation can be used to obtain equation (2.19). ∎

The average real marginal cost is defined as follows:

$$
MC_t^{\text{real}} = \frac{W_t/P_t}{MPL_t} = \frac{\frac{C_t^{\varphi}(1-L_t)^{-\chi}}{1-\tau_t^i}}{MPL_t}. \quad (2.23)
$$

where MPL_t denotes the marginal product of labour and can be obtained from equation (2.15). For the real wage the intratemporal condition is substituted in from equation (2.6). Equation (2.23) can be loglinearised as

$$
\widehat{mc}_t = \varphi\hat{c}_t + \frac{\bar{L}}{(1-\bar{L})}\chi\hat{l}_t + d\tau_t^i - \widehat{mpl}_t \quad (2.24)
$$

where $\hat{c}_t \equiv \log(C_t/\bar{C})$, $\hat{l}_t \equiv \log(L_t/\bar{L})$, $d\tau_t^i \equiv \tau_t^i - \tau^i$ and $\widehat{mpl}_t \equiv \log(MPL_t/\overline{MPL})$. Variables with an upper bar mean steady-state.

Based on a first-order Taylor-series approximation of equation (2.12) and aggregate version of equation (2.13) one can derive the New-Keynesian Phillips curve that establishes log-linear connection between inflation rate ($\hat{\pi}_t$) and the real marginal cost $(\widehat{mc}_t)^2$:

$$\hat{\pi}_t = \tilde{\beta} E_t \hat{\pi}_{t+1} + \kappa \widehat{mc}_t. \tag{2.25}$$

where $\hat{\pi}_t \equiv \log(\pi_t/\bar{\pi})$, $\widehat{mc}_t \equiv \log(MC_t^{\text{real}}/\overline{MC}^{\text{real}})$. $\bar{\pi}$ and $\overline{MC}^{\text{real}}$ stand for steady-state inflation and real marginal cost, respectively. The parameter $\kappa > 0$ is an inverse function of ξ, η and ε which is the elasticity of substitution among intermediary goods. Variables without a time index denote steady-states. $\tilde{\beta}$ stands for the discount factor that is corrected by the growth rate (γ) of the productivity trend (Z_t) i.e. $\beta\gamma^{-\varphi}$. The positive connection between income tax rate and inflation is described below by making use of the Phillips curve in equation (2.25) where loglinear marginal cost is defined by equation (2.24).

Intermediary products are bundled into a final product through a Dixit-Stiglitz aggregator. Bundlers are perfectly competitive firms.

2.2.3 Monetary policy

The New-Keynesian model is closed by a monetary policy rule (so called Taylor rule):

$$R_t = \rho R_{t-1} + (1-\rho)[R + \log\bar{\Pi}_t + g_\pi(\log\bar{\Pi}_t - \log\Pi_t^*) + g_y(Y_t - Y_t^*)/Y_t^*] + \varepsilon_t^i \tag{2.26}$$

where R_t is the policy rate, $\bar{\Pi}_t$ is a four-quarter moving average of inflation and Y_t^* is the trend level of output yZ_t (where y denotes the steady-state level of Y_t/Z_t). Π_t^* is the target rate of inflation, ε_t^i is an

---

[2]Here we use the log-linear version of the Phillips curve for illustration purposes the model is solved using the Phillips curve in its non-linear form.

iid shock with mean zero and variance σ_i^2. In the baseline version of the Rudebusch and Swanson (2012) model without long-run inflation risks the inflation target is constant ($\Pi_t^* = \Pi^*$ for all t).

The four-quarter moving average of inflation ($\bar{\Pi}_t$) can be approximated by a geometric moving average of inflation:

$$\log \bar{\Pi}_t = \theta_\pi \log \bar{\Pi}_t + (1 - \theta_\pi) \log \Pi_t, \qquad (2.27)$$

where the choice of $\theta_\pi = 0.7$ ensures that the geometric average in equation (2.27) has an effective duration of about four quarters.

In the long-run inflation risk version of Rudebusch and Swanson (2012) model they make the inflation target stochastic:

$$\Pi_t^* = \rho_{\pi^*}^* \Pi_{t-1}^* + \vartheta_{\pi^*}(\bar{\Pi}_t - \Pi_t^*) + \varepsilon_t^{\pi^*}, \quad \vartheta_{\pi^*} > 0, \, \varepsilon_t^{\pi^*} > 0, \qquad (2.28)$$

where $\varepsilon_t^{\pi^*}$ is an iid inflation target shock with mean zero and variance $\sigma_{\pi^*}^2$. Note that equation (2.28) can also be described as a learning process in the sense that the inflation target (Π_t^*) is updated in proportion to the deviation of the actual average inflation from the target one ($\bar{\Pi}_t - \Pi_t^*$). The specification in equation (2.28) follows Gürkaynak et al. (2005).

Note that in this paper we set $\vartheta_{\pi^*} = \rho_{\pi^*} = \varepsilon_t^{\pi^*} = 0$ when considering different sorts of fiscal scenarios.

2.2.4   Fiscal Policy

The government spending follows the process:

$$\log(g_t/g) = \rho_G \log(g_{t-1}/g) + \varepsilon_t^G, \quad 0 < \rho_G < 1,$$

where g is the steady-state level of $g_t \equiv G_t/Z_t$, and ε_t^G is an iid shock with mean zero and variance σ_G^2.

Rudebusch and Swanson (2012) assume that government spending is financed through lump-sum taxes in each period i.e. government

budget is balanced. Instead, we can allow for deficit that is retired through lump-sum taxes:

$$b_t + t_t = \frac{\gamma^{-1} r_{t-1} b_{t-1}}{\pi_t} + g_t \qquad (2.29)$$

where b_t, t_t, $\tilde{R}_t$ and π_t stand for the de-trended government debt, lump-sum taxes, short-term nominal interest rate and inflation, respectively. All quantities are expressed as real except for the nominal interest rate (r_t). $r_{t-1} b_{t-1}$ are interest-payments on the previous period debt. If one imposes the restriction of $b_t = b_{t-1} = 0$ for all t expression (2.29) boils down to the case of balanced budget ($g_t = t_t$ for all t).

The tax rule in case of lump-sum taxes is given by:

$$t_t = \psi \gamma^{-1} b_{t-1} \qquad (2.30)$$

where $\psi \in (0,2)$ ensures that fiscal policy is passive in the sense of Leeper (1991). When ψ is set to be close to zero debt is paid back in the very long-run. By contrast a coefficient of ψ close to two roughly mimics the case of balanced budget.

An alternative way to retire government debt is through income tax revenue ($\tau_t y_t$):

$$b_t + \tau_t^i y_t = \frac{\gamma^{-1} r_{t-1} b_{t-1}}{\pi_t} + g_t. \qquad (2.31)$$

where τ_t^i is the income tax rate. y_t is the de-trended level of output that equals to the sum of profit and labour income which are taxed at the same rate.

The tax revenue rule for the latter case is given by:

$$\tau_t^i y_t = \psi \gamma^{-1} b_{t-1}. \qquad (2.32)$$

To observe the role of steady-state debt we linearise equation (2.31)

to the first-order:

$$\hat{b}_t + d\tau_t^i + \tau^i \hat{y}_t = \gamma^{-1}(\gamma_b dr_{t-1} + r\hat{b}_{t-1} - \gamma_b r\hat{\pi}_t) + \hat{g}_t \qquad (2.33)$$

where $\hat{b}_t \equiv (b_t - b)/y$, $d\tau_t^i \equiv \tau_t^i - \tau^i$, $dr_t \equiv \tilde{R}_t - \tilde{R}$, $\hat{y}_t \equiv (y_t - y)/y$, $\hat{g}_t \equiv (g_t - g)/y$ and γ_b is the government debt-to-GDP ratio. Variables without a time index denote steady-state values. Note that the deviations of debt and government spending from their respective steady-states is defined relative to the steady-state output, which is standard in the literature (see, e.g., Linnemann (2006)). When steady-state debt is zero i.e. $\gamma_b = 0$ real interest rate $(dr_{t-1} - r\pi_t)$ does not have a direct effect on taxes $(d\tau_t^i)$. More intuition is provided below.

Finally we note that goods and labour markets clear in equilibrium and the transversality condition regarding the bond-holdings is satisfied.

The equations of the model are summarised in Table 2.1 and follows close the Appendix of Rudebusch and Swanson (2012).

2.2.5   Pricing Real and Nominal Assets

Real bonds

We closely follow Ferman (2011) on the pricing of inflation-protected (or real) bonds.

Under no-arbitrage the Euler equation for real bonds can be written as:

$$B_{\tau,t}^r = E_t[M_{t+1} B_{\tau-1,t+1}^r] \qquad (2.34)$$

where $B_{\tau,t}^r$ is the price of a real bond of maturity τ.

Therefore, bond prices with maturity ranging from $\tau = 1$ to $\tau = 40$

Table 2.1: Summary of the model

Pricing kernel	$K_t^{\text{real}} \equiv \left(\frac{C_{t+1}}{C_t}\right)^{-\varphi} \left[\frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}}\right]^{-\alpha}$
Intratemporal condition	$(1-\tau_t^i) W_t/P_t = C_t^\varphi (1-L_t)^{-\chi}$
Optimal-Aggregate price ratio ($\tilde{P}_t \equiv \frac{P_t^*}{P_t}$)	$\tilde{P}_t^{1+\frac{(1+\theta)(1+\eta)}{\theta\eta}} = \frac{Zn_t}{Zd_t}$
Definition of Zn	$Zn_t = (1+\theta) MC_t^{\text{real}} Y_t + \xi\beta K_t^{\text{real}} \pi_{t+1}^{\frac{1+\theta}{\theta\eta}} Zn_{t+1}$
Definition of Zd	$Zd_t = Y_t + \xi\beta K_t^{\text{real}} \pi_{t+1}^{\frac{1}{\theta}} Zd_{t+1}$
Price index in case of Calvo pricing	$\Pi_t^{-\frac{1}{\theta}} = (1-\xi)(\tilde{P}_t \Pi_t)^{-\frac{1}{\theta}} + \xi$
Definition of Real Marginal Cost	$MC_t^{\text{real}} = \frac{W_t/P_t}{\eta} \left(\frac{Y_t}{\bar{K}}\right)^{\frac{1-\eta}{\eta}} A_t^{-\frac{1}{\eta}}$
Production function	$Y_t = \Delta_t^{-1} A_t \bar{K}^{1-\eta} L_t^\eta$
Price dispersion	$\Delta_t^{\frac{1}{\eta}} = (1-\xi)\tilde{P}_t^{\frac{-(1+\theta)}{\theta\eta}} + \xi\Pi_t^{\frac{1+\theta}{\theta\eta}} \Delta_{t-1}^{\frac{1}{\eta}}$
Aggregate accounting	$Y_t = C_t + \delta\bar{K} + G_t$
Taylor rule	see equation (2.26)
Definition of $\bar{\Pi}$ in the Taylor rule	see equation (2.27)
Euler equation to price bonds	$1 = \beta K_t E_t \frac{R_t}{1+\Pi_{t+1}}$
Recursive utility (the second row of equation 2.1)	$V_t = \frac{C_t^{1-\varphi}}{1-\varphi} + \chi_0 Z_t^{1-\varphi} \frac{(1-L_t)^{1-\chi}}{1-\chi} + \beta V k_t$
The definition of Ve	$Ve_t = \left(\frac{V_{t+1}}{\bar{V}}\right)^{1-\alpha}$
The definition of Vk	$Vk_t = \bar{V} Ve_t^{\frac{1}{1-\alpha}}$
Government Budget Constraint	see equation (2.29) or (2.31)
Fiscal Policy Rule	see equation (2.30) or (2.32)

are constructed recursively using a chain of 40 Euler equations:

$$B_{1,t}^r = E_t[M_{t+1}]$$
$$B_{2,t}^r = E_t[M_{t+1}B_{1,t+1}^r]$$
$$B_{3,t}^r = E_t[M_{t+1}B_{2,t+1}^r]$$
$$\vdots$$
$$B_{40,t}^r = E_t[M_{t+1}B_{39,t+1}^r]$$

where we assumed that $B_{0,t+1}^r = 1$. In order to convert bond prices into yields let us take the log of equation (2.34), denote the τ-period yield-to-maturity as $R_{\tau,t}^r = \log(1+R_t^{r,net}) \equiv -\frac{1}{\tau}\log B_{\tau,t}^r$ and we arrive at:

$$R_{\tau,t}^r = E_t m_{t+1} + R_{\tau-1,t}^r$$

which can be written after a second-order Taylor approximation and forward iteration as:

$$R^r_{\tau,t} = -\frac{1}{\tau} \sum_{j=1}^{\tau} E_t[m_{t+j}] - \frac{1}{2\tau} Var_t[\sum_{j=1}^{\tau} m_{t+j}] + O(\epsilon^3)$$

where $m_{t+1} \equiv \log(M_{t+1})$ and $O(\epsilon^3)$ denotes terms of order three or higher.

Ferman (2011) shows that the Real Term Premium (RTP) can be expressed as:

$$
\begin{aligned}
RTP &\equiv R_{\tau,t} - \frac{1}{\tau} \sum_{j=1}^{\tau} E_t[R^r_{\tau,t+j-1}] \\
&\simeq -\frac{1}{\tau} \sum_{j=1}^{\tau-1} \sum_{k=j+1}^{\tau} Cov_t(m_{t+j}, m_{t+k}) \\
&\quad -\frac{1}{2\tau} \sum_{j=1}^{\tau} Var_t[E_{t+j-1}[m_{t+j}]]
\end{aligned}
\tag{2.35}
$$

where the first line indicates that real term premium is measured as the difference between the long-term real yield ($R_{\tau,t}$) and the one implied by the Expectations Hypothesis of the term structure ($\frac{1}{\tau} \sum_{j=0}^{\tau-1} E_t[R^r_{t+j}]$). $R_{\tau,t}$ can be interpreted as the *risk-adjusted* average of future short-term interest rates while $\frac{1}{\tau} \sum_{j=0}^{\tau-1} E_t[R^r_{t+j}]$ is simply the average of future short-term interest rates (see more intuition below). This difference is able to capture changes in risk-premiums when approximated to the third-order. In practice, yields consistent with Expectations Hypothesis are computed in a recursive fashion similar to the way we calculated risk-adjusted bond prices above i.e. a set of 40 equations (assuming that the highest maturity is 40 quarters).

Based on the second line in equation (2.35) RTP depends mainly on the autocovariance structure of the real stochastic discount factor (m_t). The second term which is a convexity term is quantitatively negligible.

Nominal bonds

Under no-arbitrage the Euler equation for real bonds can be written as:

$$B_{\tau,t} = E_t[M_{t+1}B_{\tau-1,t+1}]$$

where $B_{\tau,t}$ is the price of a nominal bond of maturity τ. Let us denote the τ-period yield-to-maturity as $R_{\tau,t} \equiv \log(1 + R_t^{net}) = -\frac{1}{\tau}\log B_{\tau,t}$
. Nominal bond prices and yields can be computed in a recursive fashion similar to real bond prices and yields in previous section but this time the nominal stochastic discount factor has to be used in order to to build risk-adjusted nominal bond prices and yields. Also nominal yields consistent with the Expectations Hypothesis are simply a set of recursive equations ranging all maturities and are regarded as the forward iteration of the short-term nominal interest rate.

Ferman (2011) decomposes nominal term premium in the following way:

$$
\begin{aligned}
NTP_{\tau,t} \simeq\ & RTP_{\tau,t} +\\
& Convexity^{\pi}_{\tau,t} +\\
& \left(
\begin{array}{c}
\frac{1}{\tau}cov_t[\hat{m}_{t,t+\tau}, \hat{\pi}_{t,t+\tau}]\\
-\frac{1}{\tau}\sum_{j=0}^{\tau-1} E_t\left\{cov_{t+j}[\hat{m}_{t+j,t+j+1}, \hat{\pi}_{t+j,t+j+1}]\right\}
\end{array}
\right) \quad (2.36)
\end{aligned}
$$

where a variable with a hat means percentage deviation from steady-state. The first term is the τ-period real term premium (RTP) which is defined as $RTP_{\tau,t} \equiv R_{\tau,t} - \frac{1}{\tau}\sum_{j=0}^{\tau-1} E_t[R_{t+j}]$ where $R_{\tau,t}$ is the yield-to-maturity on a τ-period real bond (or inflation-indexed bond/TIPS) and R_t is the real short-rate. As the previous section has shown RTP can also be described by the autocorrelation structure of the real stochastic discount factor (SDF). The second component of $NTP_{\tau,t}$ is an inflation convexity term that is of small size and, thus, is of minor importance.

The third-term in the parenthesis is the compensation for inflation

risk and has two parts. The first part shows that covariance of inflation from period t until maturity with the SDF. The latter term is positive when nominal bonds loose real value in times of low consumption growth i.e. the SDF is high (recession). The second part measures the one-period-ahead inflation co-variability risk that is relatively low in such short horizon and, therefore, less important.

The first part of the inflation risk-premium in equation (2.36) can be further rewritten using the definition of correlation as:

$$\frac{1}{\tau}cov_t[\hat{m}_{t,t+\tau}, \hat{\pi}_{t,t+\tau}] = \frac{1}{\tau}corr_t[\hat{m}_{t,t+\tau}, \hat{\pi}_{t,t+\tau}]std_t(\hat{m}_{t,t+\tau})std_t(\hat{\pi}_{t,t+\tau}).$$

Thus, the covariance between inflation and the SDF is governed by three elements: the correlation between inflation and the SDF, the standard deviation of the SDF (real uncertainty) and the standard deviation of inflation (nominal uncertainty). Again, the higher is the correlation between inflation and the stochastic discount factor (or alternatively the more negative is the correlation between inflation and consumption growth) the higher are inflation risks. Also when either real or nominal uncertainty is higher inflation risks are higher.

2.3 Calibration and solution method

Calibration can be found in Table (2.2) that follow the baseline parameter values of Rudebusch and Swanson (2012). The coefficient in case of the lump-sum tax rule (ψ) is set to 0.01 which is the choice of Corsetti et al. (2012). Unlike Linnemann (2006) who linearises his model to the first-order we make use of the tax-rule in its non-linear form so that ψ is needed to pin down the steady-state tax rate which is given by $\tau = ((1/\beta)(b/y)(1/\gamma) + g/y)/(1 + \gamma/\psi)$. The quarterly steady-state debt-to-GDP ratio ($b/4y$) of sixty per cent is based on Rossi (2012). As a baseline we set $\psi = 0.12$ so that the steady-state tax rate is 0.2767 which is slightly higher than that of Linnemann

$(2006)^3$. The whole model is approximated to the third order using Dynare (Adjemian et al. 2011) in Matlab when calculating all the moments in table (2.3) and (2.4). To speed up calculations for figure (2.6) we used second-order approximation as unconditional means—unlike standard deviations—are quite similar in magnitude for second- and third-order approximations. Yearly data on US Treasury yields with maturities ranging from one- to thirty-year for 1961-2014 are taken from Gürkaynak et al. (2007). Data on inflation-indexed bonds (or Treasury Inflation Protected Securities, TIPS) is available from Gürkaynak et al. (2008) for period 1999-2014. The US TIPS market opened in 1997 only.

<div align="center">Table 2.2: Calibration</div>

γ	1.0025	φ	2	ρ_i	0.73	ρ_A	.95
$\tilde{\beta}$	0.99	χ	3	g_π	0.53	ρ_G	.95
δ	0.02	CRRA	75	g_y	0.93	σ_A^2	0.005^2
l	1/3	η	2/3	Π^*	1	σ_G^2	0.004^2
K/Y	10	θ	0.2	ρ_{π^*}	0.99	σ_i^2	0.003^2
G/Y	0.17	ξ	0.75	$\sigma_{\pi^*}^2$	$.0005^2$	γ_b	2.4
ε	6			ϑ_{π^*}	0.01	ψ	0.12

where G/Y is the government spending-GDP ratio, K/Y is the share of fixed capital in GDP, δ is the depreciation rate of fixed capital. The rest of the parameters are explained above.

2.4 Results

The mean and standard deviation of the empirical US nominal and real yield curves for various maturities and time intervals are plotted on Figure (2.1). Time periods 1961-2014 and 1985-2007 are associated with the highest levels of the average nominal yield curve. Although it is also apparent that the data ranging from 1961 to 2014 including several crises is associated with higher standard deviations than the one based on the period of Great Moderation (1985-2007). The

---

[3]The choice of $\psi = 0.02$—used by Corsetti et al. (2012) in case of the model with lump-sum taxes—implies a steady-state tax rate of less than 1 percent that is empirically implausible.

Figure 2.1: Empirical yield curves based on US data.
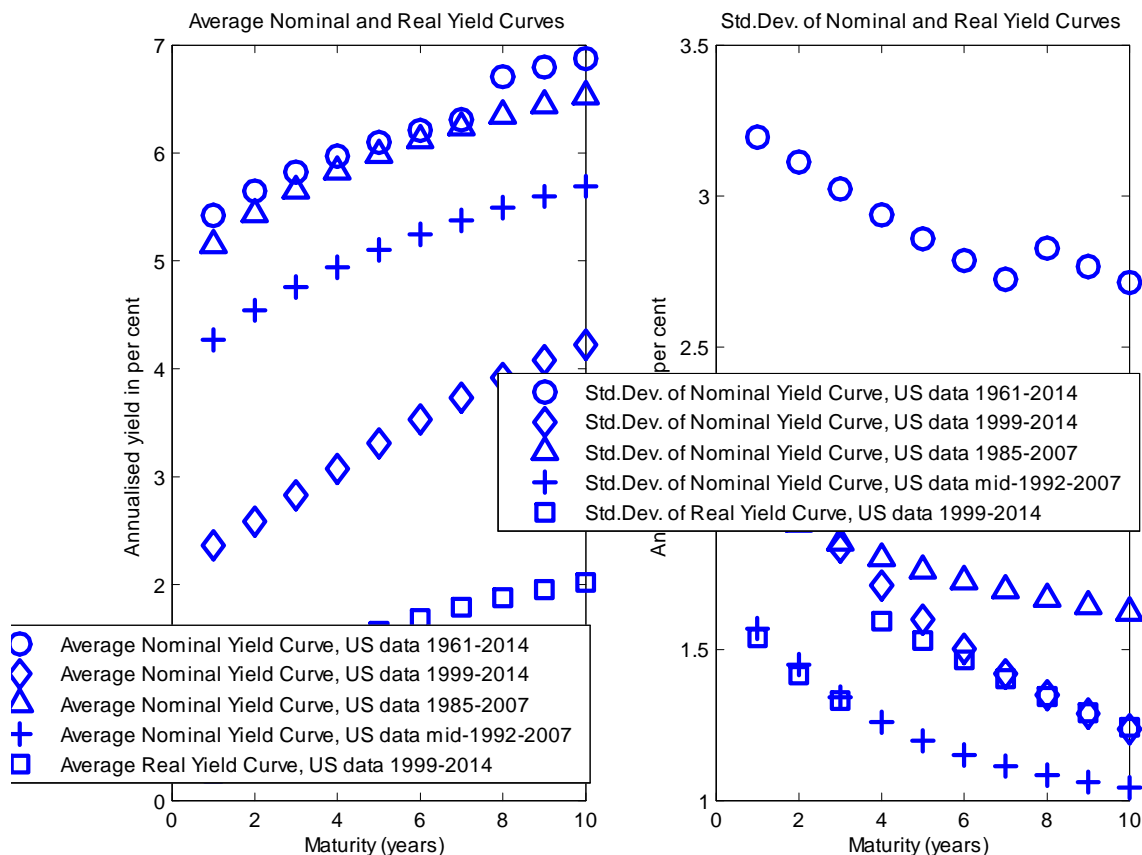
difference between the average nominal and real yield curve is called break-even inflation rate which contains investors' expectation of future inflation and also inflation risks[4] (Hördahl, 2008). Based on the previous graph, break-even inflation rate is lowest for the period 2003-2014. The means and standard deviations of the yield curves implied by the model are depicted on Figure (2.2) and (2.3), respectively. Similar to the data, average nominal and real yield curves from the model are upward-sloping and standard deviations exhibit a negative slope. Other features of the model graphs are discussed below.

Binsbergen et al. (2012) and Piazzesi and Schneider (2006) show that a DSGE model successfully replicates several properties of the

---

[4]However, D'Amico et al. (2010) recently argued that a correct estimate inflation expectations (and risks) based on TIPS yields should correct for the liquidty premium content of TIPS. Since start liquidy on the US TIPS market was quite low compared to the vast turnorver of nominal Treasuries.

term-structure when consumption growth and inflation are negatively correlated contemporaneously and forecast each other with a negative sign. The Rudebusch and Swanson (2012) model features temporary productivity shocks in order to generate the previous patterns. In particular, a temporary negative shock to productivity leads to a fall in consumption and an increase in inflation due to the rise in marginal cost. As a result, nominal bonds carry positive term premia as depressed consumption is associated with a time of low real bond yields due to higher inflation. The negative relationship between consumption and inflation in case of a positive productivity shock is illustrated on Figure (2.4).

The negative correlation between consumption growth and inflation is prerequisite for a positive nominal term premium and, therefore, it is of major interest whether there is empirical evidence in support of it. In a comment to Piazzesi and Schneider (2006) Benigno (2006) investigated whether this negative correlation can be recovered from US data. In particular, he found that for the whole sample (1952-2004) and for the subsample of 1952-1984 the correlation between inflation and consumption growth is significant at the one percent significance level and is equal to -0.35 and -0.44, respectively. However, for the subsamples of 1984-2004, 1987-2004 and 1995-2004 the correlation is less negative and not significant.

Ferman (2011) who argues that nominal term premium mainly captures inflation risks through the negative correlation between consumption growth and inflation if real term premium which is can be described by the autocorrelation of the real stochastic discount factor is close to zero. de Paoli et al. (2010) investigate into the determinants of nominal term premium using a second-order approximation and find that it is the covariance between consumption growth and inflation that matters if real term-premium is low. Further, Andreasen (2012) proves that inflation risk-premium can be well approximated by

Figure 2.2: Average nominal and real yield curves from the model
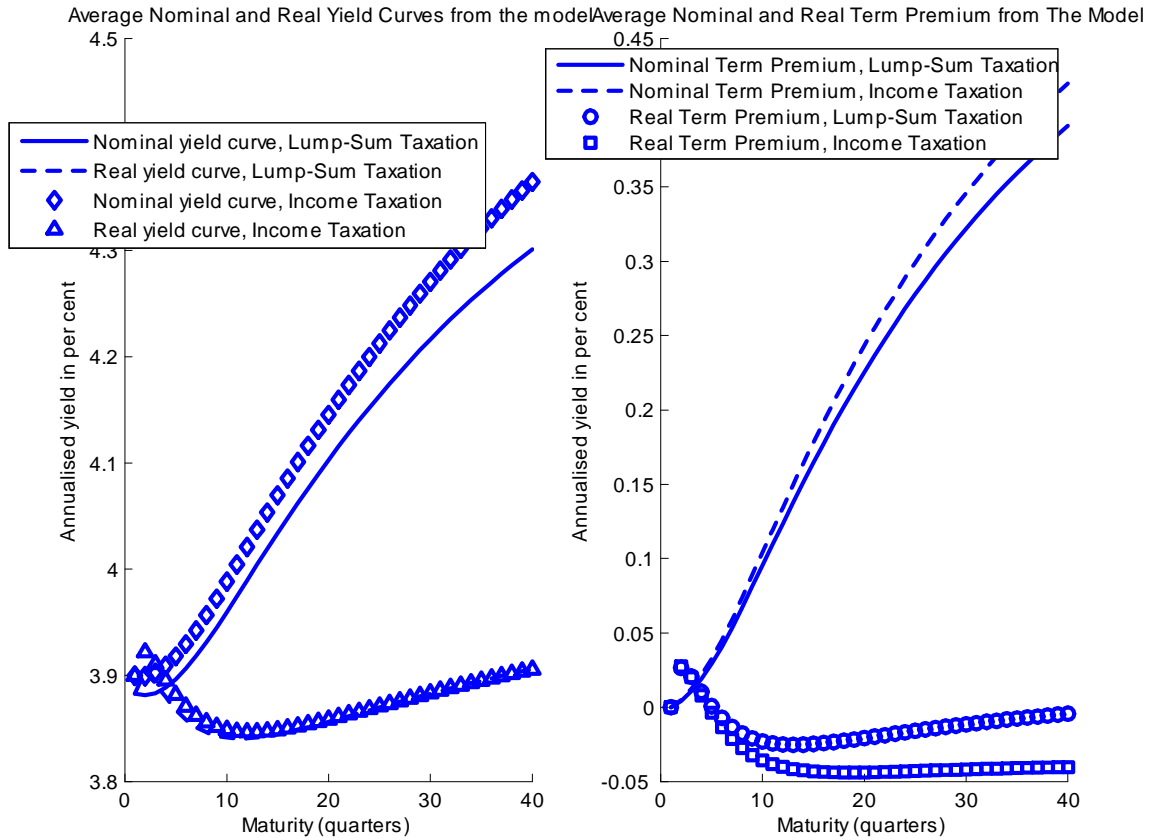
the difference between nominal term premium and real term premium in case of a third-order approximation. Our graphs are based on a third-order approximation of the model and, thus, the area between nominal term premium and real term premium for each maturity is an indicator of positive inflation risks which are even higher for income taxation. In chapter 3 we illuminate the role of the size of the coefficient on the output gap in driving inflation risks.

Figure (2.2) and (2.3) show that income taxation is associated with higher nominal term premium and also higher standard deviation of the nominal yield curve for all maturities. Our fiscal extension with income taxation has the implication similar to that of productivity shocks. The inverse connection between consumption and inflation in case of positive government spending shock is depicted on Figure (2.5). To shed light on the mechanism let us study what happens after

Figure 2.3: Standard deviations of the yield curves from the model
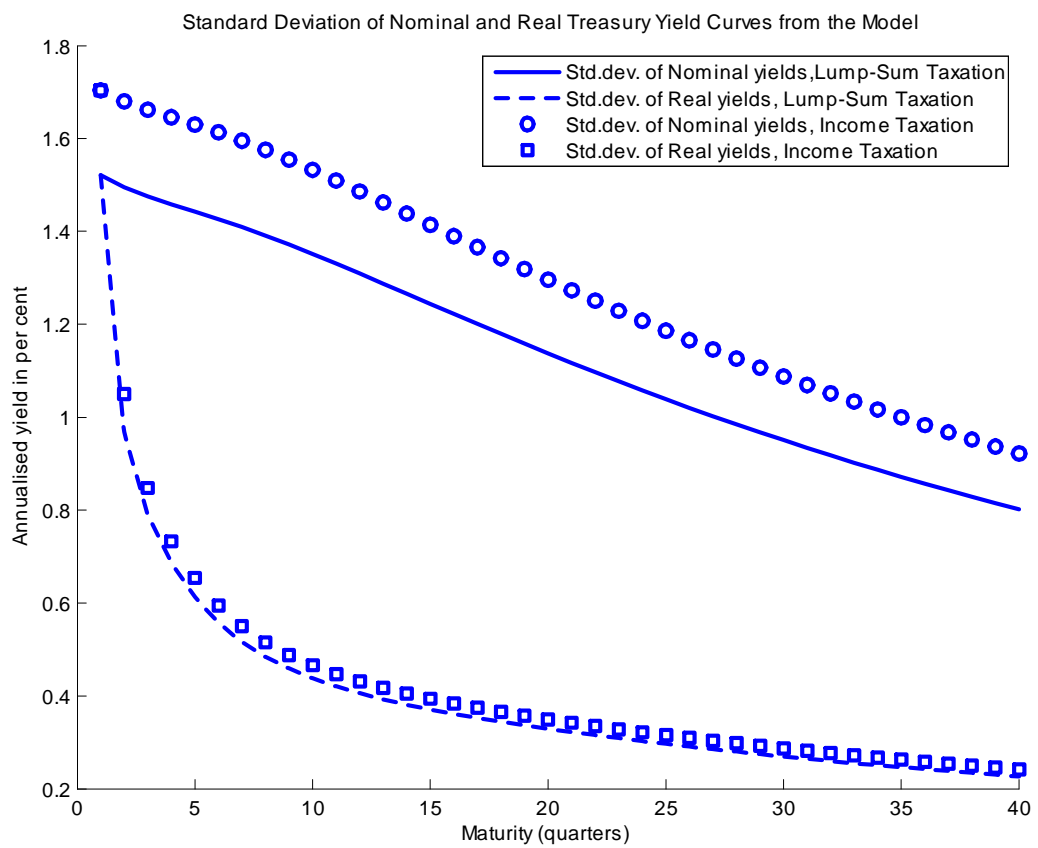
a positive innovation to government spending that needs to be financed sooner (ψ is close to two) or later (ψ is close to zero) with income taxes. Higher taxes on income imply less hours worked and lower output because households substitute away from labour to leisure. Also higher income taxes means higher real marginal cost (see equation (2.24)) and higher inflation through the New Keynesian Phillips curve (see equation (2.25)). The effect of the previous channel is magnified by positive steady-state debt (see $\gamma_b > 0$ in equation (2.33)) establishing direct connection between taxes and real interest rate which surely rises after a stimulative shock according to the logic of the Taylor rule to curb inflation expectations (Linnemann, 2006). It would be of interest to test the significance of the size of the steady-state debt-to-GDP ratio. However, it is impractical to carry out an experiment with zero steady-state debt-to-GDP ratio because this would result in a steady-state tax rate that is extremely low (around 1.8 percent).

Figure (2.6) shows that there is positive linear relationship between the risk-aversion coefficient and mean of the nominal term premium. The straight line reproduces the baseline model of Rudebusch and Swanson (2012) without debt and long-run risks. The introduction of debt into the baseline model with lump-sum taxes raises the term-premium (see dashed line). The baseline model with long-run risks is depicted with dots. The model with debt and income taxes are demonstrated for two different values of the coefficient in the policy rule, ψ (see the circles and diamonds). One can see that the extension of the Rudebusch and Swanson (2012) model with debt and distortionary taxes generates a term-premium close to the one obtained with long-run risks for both values of ψ. When taxation is distortionary the curve is steeper than with lump-sum taxes implying higher inflation/real risks in case of income taxation.

Table (2.3) summarises means and standard deviations of selected macro and finance variables using the baseline calibration of Rude-

Figure 2.4: Impulse responses of selected variables to a positive technology shock. All of them are expressed in percentage deviation from steady-state. Interest rates, yields and inflation are annualised.
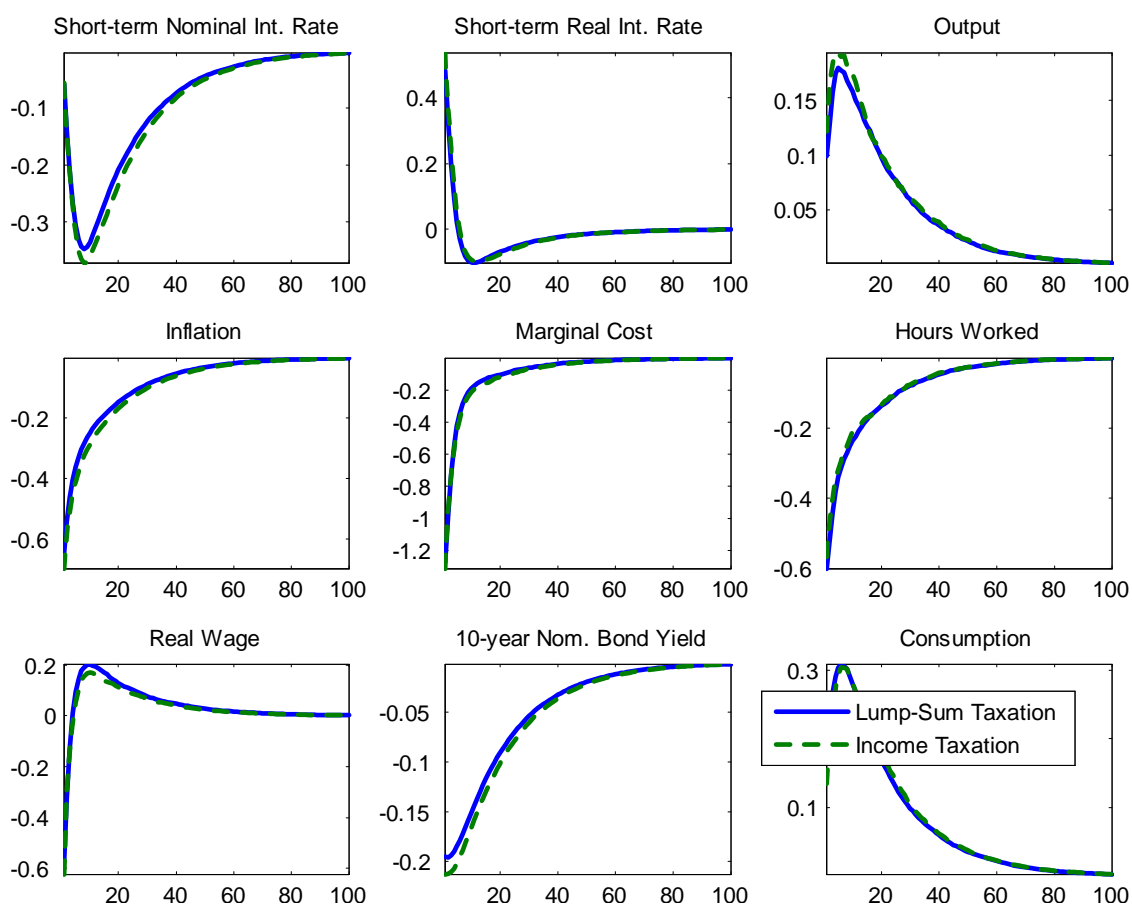
Figure 2.5: Impulse responses of selected variables to a positive government spending shock. All of them are expressed in percentage deviation from steady-state. Interest rates, yields and inflation are annualised.
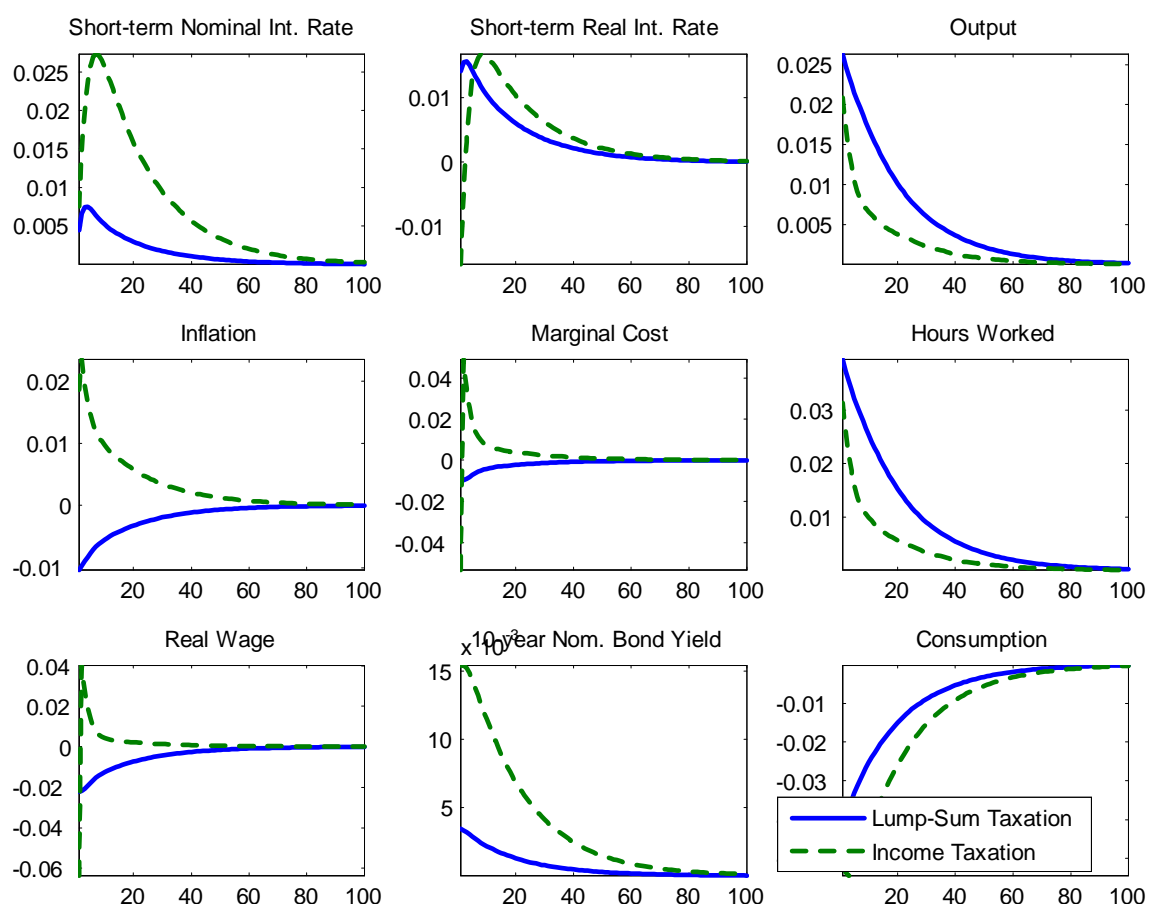
Figure 2.6: The relationship between the coefficient of relative risk-aversion (CRRA) and the mean of the nominal term premium using different versions of the Rudebusch and Swanson (2012) (=RS) model.
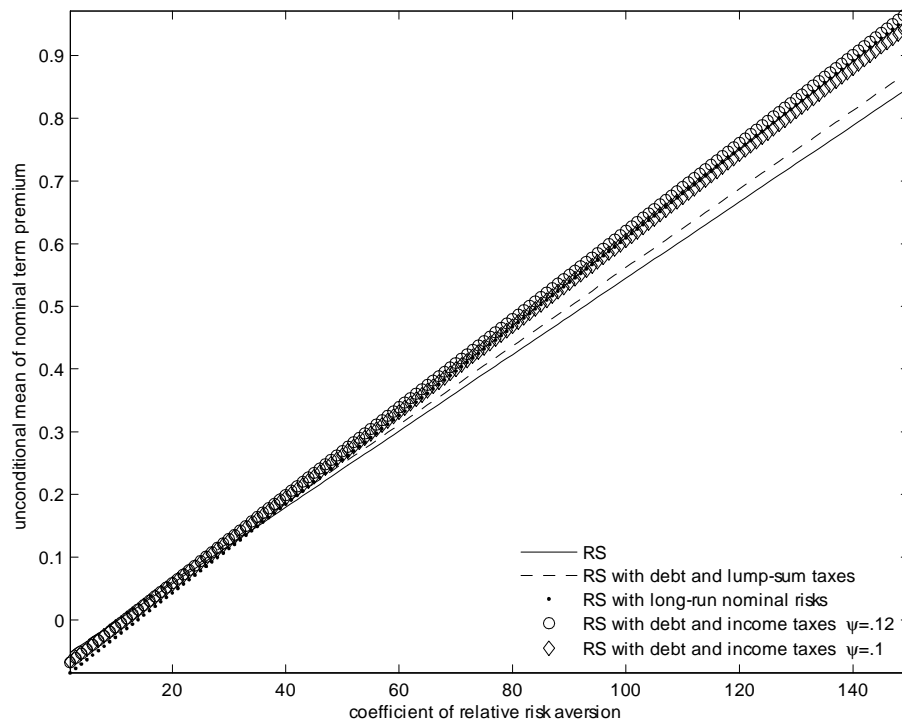
busch and Swanson (2012). Nominal term premium on a long-term bond, say a 10 year-bond, is computed as the return on the risky 10-year bond minus the return on a bond that is rolled over 10 years. The yield on the latter strategy is often called as risk-neutral yield which is consistent with the expectations hypothesis of the term structure. The nominal term premium cannot be observed empirically. Therefore, we also calculate alternative measures of the term premium like the mean and standard deviation of slope of the term structure and excess holding period returns that are observable.

The slope of the term structure is the difference between the yield on a long-term bond (say a 40-quarter bond) and the short-term bond $(R^{(40)} - R)$. The excess holding period return $(x^{(40)})$ is defined e.g. for a 40-quarter bond as: $p_t^{(39)}/p_{t-1}^{(40)} - R_{t-1}$ where the first term is the gross return to holding the 40-quarter bond for one period (p is the price of the bond) and the second term is the gross one-period risk-free rate.

RS (2012 pp. 119) consider the slope and the excess holding period return as "imperfect measures of the riskiness of the long-term bond because they can vary in response to shocks even if all investors in the model are risk-neutral."

Note that the mean value of the nominal term premium is very similar across the four versions of the baseline model. Regarding the standard deviations of finance variables it is the model with inflation risks that fits the data best. With the introduction of lump-sum taxes and endogenous debt (column three) we improve upon the performance of the baseline model. The macro and finance moments derived using the model with long-run inflation risks (column four) and the one with fiscal extension (column five) are roughly similar. The moments reported in column one (called RS) are directly comparable with those in Rudebusch and Swanson (2012).

We note that the standard deviations of macro and finance moments in column one (called RS) are slightly lower than those of Rudebusch and Swanson (2012, pp. 124, column 3 of table 2) for at least two reasons. First, they calculate theoretical moments while we obtain simulated moments. Second, they use a numerical precision of 90 digits available in software Mathematica while we have only sixteen digits available in Matlab.

Rudebusch and Swanson (2012) decrease their baseline values of IES and Frisch elasticity and increase the CRRA parameter in order to match US data[5]. A lower IES means that households dislike changes in consumption over time more than with a higher IES. A lower Frisch elasticity implies that households are less able to use their labour supply to offset negative shocks to consumption. As a conse-

---

[5]To be precise, Rudebusch and Swanson (2012) change not only the IES and Frisch parameters when they consider their best-fit experiments but also properties of the stochastic processes and duration of the price rigidity. Instead, here, we focus only on changes in the IES, Frisch and CRRA parameters whose effects are well documented in the literature (see, e.g., van Binsbergen et al. (2012)). de Paoli et al. (2006) describes how a shorter duration of price rigidity raises the term-premium. However, we do not intend to deviate from the price-stickiness of a duration of 4 quarters on average (this is the baseline calibration).

quence consumption stream is more volatile for relatively low values of IES and Frisch elasticity. Also a higher risk-aversion coefficient represents that households are more concerned about changes in future consumption flow and, therefore, require higher compensation in order to hold risky bonds whose real return co-moves positively with the consumption stream.

In Table (2.4) we lower the IES from 0.5 to 0.09 and the Frisch elasticity from 2/3 to 0.28 leaving other parameters at the level of the baseline calibration. The risk-aversion coefficient of 106 is chosen so that the model with fiscal policy can match a nominal term premium of 1.06 inferred from US data. Indeed after a reduction in the values of IES and Frisch elasticity the macro and finance variables exhibit higher standard deviations. Still some of the simulated moments like the standard deviation of the excess holding period return and the slope for a 10-year bond (denoted by $\text{SD}(x^{(40)})$ and $\text{SD}(R^{(40)} - R)$ respectively) are below the corresponding US data.

Also importantly, the model with fiscal policy produces a mean of the nominal term premium (1.06) that is higher than the one with inflation risks (0.72). The version of the model with inflation risks is able to generate the empirical mean of the term premium (1.06) only if the risk-aversion coefficient is 155. Thus, it follows that a particular level of the average nominal term premium can be achieved with lower CRRA using the fiscal extension compared to the inflation risk alternative. However, it is also true that the inflation risk model performs better in terms of matching standard deviations of US data. A shortcoming of the model with either long-run inflation risks or with fiscal policy is that the simulated standard deviation of real wage (2.77 and 2.35 for each version respectively) is in excess of US data (0.82). One might suggest to remedy this problem by introducing wage-rigidity. However, it is well-known since at least Rudebusch and Swanson (2008) that wage-stickiness reduces the standard deviation

of real wage at the cost of a high standard deviation of labour well above the corresponding US statistic.

2.5 Conclusion

Fiscal policy can be an important source of long-run nominal risks in the sense that nominal term premium on government bonds rises substantially when spending is financed through income taxes relative to lump-sum taxes. Also employing the model with income taxes we can match the empirical level of the nominal term premium with lower risk-aversion coefficient than the one needed in case of the model with long-run inflation risks. In a companion paper we augment the above model with physical capital as in de Paoli et al. (2006) and find our main message to be robust.

Table 2.3: Moments from variants of the Rudebusch and Swanson (=RS) (2012) model compared to US data

Unconditional Moment	US data, 1961-2007	RS	A	B*	C
SD(C)	0.83	1.42	1.46	1.35	1.49
SD(L)	1.71	1.5	1.54	1.44	1.46
SD(W)	0.82	1.32	1.32	1.35	1.21
SD(π)	2.52	1.64	1.64	2.11	1.86
SD(R)	2.71	1.6	1.61	2.01	1.76
SD(R^{real})	2.30	0.93	0.93	0.98	0.98
SD ($R^{(40)}$)	2.41	0.85	0.85	1.33	0.95
Mean($NTP^{(40)}$)	1.06	0.39	0.4	0.42	0.44
SD($NTP^{(40)}$)	0.54	0.04	0.04	0.1	0.05
Mean($R^{(40)} - R$)	1.43	0.43	0.45	0.48	0.49
SD($R^{(40)} - R$)	1.33	0.9	0.89	0.87	0.96
Mean($x^{(40)}$)	1.76	0.69	0.72	0.81	0.79
SD($x^{(40)}$)	23.43	7.81	7.98	10.14	8.88

where SD=unconditional standard deviation, $NTP^{(40)}$=nominal term premium on a 40-quarter bond, Mean=Unconditional Mean, $R^{(40)} - R$ is the slope and $x^{(40)}$ is the excess holding period return for a 40-quarter bond. Each version of the models listed above utilises the baseline calibration of Rudebusch and Swanson (2012) that does not fit macro and finance moments of US data (neither here nor in their paper). $\mathcal{RS}$=reproduction of the results of Rudebusch and Swanson (2012) without entry (Note that their results are very close to ours.)
$\mathcal{A}$: RS with debt and lump-sum taxes.
$\mathcal{B}$*: RS with debt and long-run inflation risks.
$\mathcal{C}$: RS with debt and income taxation.
*Note that the results in this column are obtained using the baseline calibration of RS while they provide results using their best-fit calibration (see column 3 of table 3 on pp. 136 in Rudebusch and Swanson (2012)). In this paper we deem it important to compare the performance of the long-run inflation risk version with other versions making use of the baseline calibration of RS.

Table 2.4: Moments from variants of the Rudebusch and Swanson (=RS) (2012) model compared to US data

Unconditional Moment	US data, 1961-2007	RS with long-run risks	RS with debt and income taxes
SD(C)	0.83	0.44	0.65
SD(L)	1.71	1.41	1.83
SD(W)	0.82	2.77	2.35
SD(π)	2.52	2.57	2.20
SD(R)	2.71	2.24	2.42
SD(R^{real})	2.30	1.55	1.42
SD ($R^{(40)}$)	2.41	1.92	1.51
Mean($NTP^{(40)}$)	1.06	0.72	1.06
SD($NTP^{(40)}$)	0.54	0.57	0.25
Mean($R^{(40)} - R$)	1.43	0.79	0.68
SD($R^{(40)} - R$)	1.33	1.13	1.25
Mean($x^{(40)}$)	1.76	1.41	2.01
SD($x^{(40)}$)	23.43	16.18	14.58

where SD=unconditional standard deviation, $NTP^{(40)}$=nominal term premium on a 40-quarter bond, Mean=Unconditional Mean, $R^{(40)} - R$ is the slope and $x^{(40)}$ is the excess holding period return for a 40-quarter bond.

In this table we used a risk-aversion coefficient ($CRRA$) of 106, a Frisch elasticity ($(1-\bar{L})/\chi\bar{L}$) of 0.28 and an IES coefficient ($1/\varphi$) of 0.09. The rest of the parameters follow the baseline calibration of Rudebusch and Swanson (2012).

Chapter 3

Explanining Bond and Equity Premium Puzzles with Epstein-Zin Preferences in a New Keynesian Model of Costly Firm Entry

with Ales Marsal

3.1 Introduction

Rudebusch and Swanson (2012; RS henceforth) produce high and volatile nominal bond term premium using a basic New Keynesian model with Epstein-Zin (1989) preferences. RS point toward further investigation of the model including other types of assets like equities. The empirical literature estimates the mean value of the equity premium to be around 6 per cent and a volatility of equity returns of around 15 per cent based on post-war US data (see the literature review in Donaldson and Mehra (2008)). Kim and Wright (2005) use an arbitrage-free three-factor model and report estimates of the mean and standard deviation of a 10-year bond term-premium of around one and 0.54 per cent, respectively. We contribute to the macro-finance literature by showing that the RS model extended with costly firm entry is able to generate a high mean value of bond and equity risk-

premia without compromising the model's fit to macro data. Using US data Clarida et al. (1998) estimate the Taylor-rule coefficient on the output gap to be 0.07 for the period 1979-1994 while Clarida et al. (2000) obtained an output gap coefficient estimate of 0.93 (both in quarterly terms) for 1984-1996 which is a subperiod of the Great Moderation (from mid1980s until the onset of the recent financial crises). We refer to the previous as low and the latter one as high estimate on the output gap. RS utilised the high output gap estimate in their simulations.

Standard New Keynesian models like the RS model without entry imply a trade-off between stabilising the standard deviation of inflation and the output gap, that is, a lower volatility of inflation can be achieved at the cost of higher standard deviation of the output gap (see also Clarida et al. (1999) and Woodford (2003)). Further, this trade-off means that the larger is the coefficient on the output gap the higher is the relative weight a central bank places on stabilising fluctuations in the output gap and, therefore, the lower is the unconditional standard deviation of the output gap and the higher is the unconditional standard deviation of inflation. The case of a high coefficient on the output gap in the standard New Keynesian model is associated with low standard deviation of the output gap and a relatively high standard deviation of inflation (nominal uncertainty) and high inflation risks. However, the RS model with entry implies the existence of this trade-off for output gap coefficients lower than 0.5. For values of the output gap coefficient higher than 0.5 it is no longer possible to engineer a decrease in the volatility of the output gap at the cost of higher volatility of inflation i.e. they move together. It follows that the RS with entry and with a high coefficient on the output gap generates real risks instead of inflation risks which is the implication of the RS model without entry.

As a second contribution we show that the RS model with entry

produces inflation risks when the coefficient on output gap in the Taylor rule is small unlike RS where only a high coefficient on the output gap guarantees the existence of inflation risk premia. Our third contribution is that the RS model with entry imply substantial increase in consumption risks in case of distortionary income taxation relative to lump-sum taxation even for risk-aversion coefficients below one hundred and, from this aspect, improves upon Kaszab and Marsal (2013) [chapter 2] who highlight the possibility of fiscal policy with distortionary income taxation in raising inflation risks when households are sufficiently risk-averse.

We introduce firm entry into the RS model along the lines of Bilbiie et al. (2007, 2012) where the mass of firms entering the industry in each period are subject to a time-varying sunk entry cost and a time-to-build lag in production. The RS model features Epstein-Zin preferences which are widely employed to increase risk-aversion of the consumer without decreasing intertemporal elasticity of substitution. Vissing-Jorgensen and Attanasio (2003) estimated risk-aversion to be around 5-10 for stockholders using US data over 1982-1996. This paper, however, maintains a high value of risk-aversion similar to RS to obtain a reasonable amount of nominal term premium on long-term default-free bonds. As a fourth contribution, we demonstrate that the entry model makes some progress by matching the estimated mean of the equity premium with a risk-aversion (75) smaller than that of RS (110). RS cite a number of papers in order to support the high risk-aversion coefficient. One of them is based on Barillas et al. (2009) who show that a model with Epstein-Zin preferences and high risk-aversion is "isomorphic to a model in which households have low risk aversion but a moderate degree of uncertainty about the economic environment." (RS pp. 123). Another interpretation can be derived from Malloy et al. (2009) who find that consumption of stockholders has higher standard deviation than consumption of nonstockholders.

Therefore, risk-aversion should be higher in a representative agent model like the RS model with/without entry than in a model which can distinguish between agents with different consumption smoothing behaviour. To put it differently, the DSGE models we use might understate the quantity of risks faced by households so that a higher risk-aversion is needed to match risk premiums in the data.

Firm entry has been incorporated into basic RBC model in order to reproduce the countercylical markup and procyclical profit found in the data (see, e.g. Rotemberg and Woodford (1999)). In fact, it is the strong procyclicality of the profit that justifies the high-premium on equities which bring low return in bad times i.e. when output (and consumption) is also low. RS is able to match a high nominal term premium on long-maturity bonds. Besides the high-equity premium the extension of the RS with entry in this paper exhibits a reasonable bond-premium as well because the negative covariance between consumption and inflation—a pre-requisite for the existence of a positive bond term premium—is also maintained. Investors expect long-term government bonds to pay an excess return (a term premium) in order to be compensated for consumption/inflation risks over the duration of the bond. Thus, a bond is considered to be risky when low consumption is coupled with high inflation that erodes the real payoff of the bond.

The representative consumer of Bilbiie et al. (2012) model has love for variety i.e. a new (monopolistically competitive) firm is associated with a new product. Also, consumers benefit from the appearance of new varieties through a reduction in the aggregate price index. Indeed, there is empirical evidence on the contribution of product creation and destruction to aggregate output. In a pioneering work Bernard et al. (2010) studied US manufacturing firms and showed that the value of new products account for 33.6% of the overall output over a business cycle horizon (5 years) while product destruction explaining

-30% loss in the value of output (both at existing firms). Hence the overall contribution of the extensive margin (product creation and destruction) is quite substantial.

The paper proceeds as follows. The second section describes the model. The third section explains why we need to employ translog preferences in the model. Then the parametrisation of the model is presented. Results follow with particular attention given to different specifications of the entry cost and finally we conclude.

3.2 The Model

3.2.1 Firm entry and profit maximisation

Our short description of the production sector borrows heavily from Bilbiie et al. (2007) who feature a two-sector RBC model with price rigidity. Labour is the only factor of production. In one sector labour is used to produce consumption goods. The other sector requires labour effort to set up new firms. We start with the description of the latter one.

There is a mass of firms. Firm ω employs labour $(l_t(\omega))$ in order to produce output $(y_t(\omega))$ using a constant-return-to-scale technology: $y_t(\omega) = Z_t l_t(\omega)$ where Z_t is a stationary productivity shock:

$$\log Z_t = \rho_Z \log Z_{t-1} + \varepsilon_t^Z,$$

where ε_t^Z is an independently and identically distributed (iid) stochastic technology disturbance with mean zero and variance σ_Z^2. The unit cost of production in units of consumption good C_t is w_t/Z_t where $w_t \equiv W_t/P_t$ is the real wage. There is also a mass of prospective entrants. Firms pay an entry cost of f_E effective labour units, equal to $w_t f_E/Z_t$. Each period firms correctly anticipate their future profits and the probability δ of the exit-inducing shock. The model features a time-to-build lag in the sense that firms entering at time t start to

produce one period later. Therefore, the number of firms producing at period t, N_t, is described by:

$$N_t = (1 - \delta)(N_{t-1} + N_{E,t-1}) \tag{3.1}$$

where N_E stands for new entrants and both new entrants and incumbents survive with probability $1 - \delta$.

The real profits of firm ω at time t (transferred back to households in the form of dividends, $d_t(\omega)$) can be expressed as:

$$d_t(\omega) = \rho_t(\omega)y_t^D(\omega) - w_t l_t(\omega) - pac_t(\omega)\rho_t(\omega)y_t^D(\omega)$$

where $\rho_t(\omega) \equiv p_t(\omega)/P_t$ is the real price of firm ω, $y_t^D(\omega)$ is the demand schedule coming from the cost-minimisation problem ($y_t^D(\omega) = (p_t(\omega)/P_t)^{-\theta}[C_t + G_t + PAC_t]$). Lower-case letters denote firm-specific variables while upper-case ones stand for the aggregate.

Adjusting prices is costly. Hence, nominal rigidity is introduced in the form of price adjustment costs that can be described with a quadratic function as in Rotemberg (1982):

$$PAC_t(\omega) = \frac{\phi_P}{2} \left[\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1 \right]^2$$

where ϕ_P measures how strong price adjustment costs are.

The real value of firm ω in units of consumption at time t, denoted as $v_t^{firm}(\omega)$ can be expressed as the sum of present and discounted future dividends:

$$v_t^{firm}(\omega) = E_t \sum_{j=0}^{\infty} \lambda_{t+j} d_{t+j}(\omega)$$

where λ_t is the marginal utility of consumption used to discount future profits. Firms face a death shock occuring with probability $\delta \in (0, 1)$ in each period.

Thus, at time t, firm ω chooses $p_t(\omega)$ to maximise $d_t(\omega)$ subject to

$y_t(\omega) = y_t^D(\omega)$ taking w_t, P_t, C_t, PAC_t, and Z_t as given. Equivalently, firm ω maximises the present and future discounted value of its profits:

$$\max_{p_t(\omega)} E_t \sum_{j=0}^{\infty} [\beta(1-\delta)]^j \lambda_{t+j} \left[\begin{array}{c} \rho_{t+j}(\omega)y_{t+j}^D(\omega) - w_t l_{t+j}(\omega) \\ -\frac{\phi_P}{2} \left(\frac{P_{t+j}(\omega)}{P_{t-1+j}(\omega)} - 1 \right)^2 \rho_{t+j}(\omega)y_{t+j} \end{array} \right]$$

where λ_t is the marginal utility of consumption, $\rho_{t+j}(\omega)y_{t+j}^D(\omega)$ is the revenue, $w_t l_{t+j}(\omega)$ is the cost of labour and the last term appears because of Rotemberg price adjustment costs.

Next we make use of the demand curve for the product of an individual firm ω $\left(y_{t+j}^D(\omega) = \left(\frac{p_{t+j}(\omega)}{P_{t+j}} \right)^{-\varepsilon} y_{t+j} \right)$, the production function $(y_t^D(\omega) = Z_t l_{t+j}(\omega))$ and the price ratio $\rho_{t+j}(\omega) = \frac{p_{t+j}(\omega)}{P_{t+j}}$:

$$E_t \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} \left[\begin{array}{c} \left(\frac{p_{t+j}(\omega)}{P_{t+j}} \right)^{1-\varepsilon} y_{t+j} - \psi_{t+j} \left(\frac{p_{t+j}(\omega)}{P_{t+j}} \right)^{-\varepsilon} y_{t+j} \\ -\frac{\phi_P}{2} \left(\frac{p_{t+j}(\omega)}{p_{t-1+j}(\omega)} - 1 \right)^2 \frac{p_{t+j}(\omega)}{P_{t+j}} y_{t+j} \end{array} \right]$$

where $\psi_t = w_t/Z_t$ is the economy-wide real marginal cost. Note that this is true for the marginal cost as long as we have a one-to-one production function as in this chapter (and similarly in chapter 1).

The first-order condition with respect to the price of an individual firm $(p_t(\omega))$ can be written as:

$$\begin{aligned} 0 = \ & (1-\varepsilon)\lambda_t \left(\frac{p_t(\omega)}{P_t} \right)^{-\varepsilon} \frac{y_t}{P_t} + \lambda_t \varepsilon \psi_t \left(\frac{p_t(\omega)}{P_t} \right)^{-\varepsilon-1} \frac{y_t}{P_t} \\ & -\lambda_t \phi_P \left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1 \right) \left(\frac{p_t(\omega)}{P_t} \right)^{1-\varepsilon} y_t \frac{1}{p_{t-1}(\omega)} \\ & -\lambda_t \frac{\phi_P}{2} \left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1 \right)^2 (1-\varepsilon) \left(\frac{p_t(\omega)}{P_t} \right)^{-\varepsilon} y_t \frac{1}{P_t} \\ & -\beta E_t \left\{ \lambda_{t+1} \phi_P \left(\frac{p_{t+1}(\omega)}{p_t(\omega)} - 1 \right) \frac{p_{t+j}(\omega)}{P_{t+j}} y_{t+1} \left(-\frac{p_{t+1}(\omega)}{p_t^2(\omega)} \right) \right\} \end{aligned}$$

After multiplying out each term with p_t and dividing by λ_t we obtain:

$$
\begin{aligned}
0 = {} & (1 - \varepsilon) \left(\frac{p_t(\omega)}{p_t} \right)^{-\varepsilon} y_t + \varepsilon \psi_t \left(\frac{p_t(\omega)}{P_t} \right)^{-\varepsilon - 1} y_t \\
& - \phi_P \left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1 \right) \left(\frac{p_t(\omega)}{P_t} \right)^{1-\varepsilon} y_t \frac{P_t}{p_{t-1}(\omega)} \\
& - \frac{\phi_P}{2} \left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1 \right)^2 (1 - \varepsilon) \left(\frac{p_t(\omega)}{P_t} \right)^{-\varepsilon} y_t \\
& - \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \phi_P \left(\frac{p_{t+1}(\omega)}{p_t(\omega)} - 1 \right) \frac{p_{t+1}(\omega)}{P_{t+1}} \frac{y_{t+1}^D}{y_t^D} \left(-\frac{p_{t+1}(\omega)}{p_t^2(\omega)} \right) P_t \right\},
\end{aligned}
$$

which can be further simplified after dividing each term by $\left(\frac{P_t(\omega)}{P_t} \right)^{-\varepsilon} y_t = y_t^D(\omega)$ and imposing symmetric equilibrium such that the producer-price inflation (PPI) is $\frac{p_t(\omega)}{p_{t-1}(\omega)} = \frac{P_t}{P_{t-1}} \equiv 1 + \pi_t$ and $y_t^D(\omega) = y_t^D$ for any firm ω:

$$
\begin{aligned}
0 = {} & (1 - \varepsilon) + \varepsilon \psi_t \frac{1}{\rho_t} \\
& - \phi_P (\pi_t)(1 + \pi_t) \\
& - \frac{\phi_P}{2} (\pi_t)^2 (1 - \varepsilon) \\
& + \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \phi_P (\pi_{t+1}) \frac{y_{t+1}^D}{y_t^D} (1 + \pi_{t+1}) \frac{\rho_{t+1}}{\rho_t} \right\}.
\end{aligned}
$$

The previous one can alternatively be written as:

$$
\begin{aligned}
\varepsilon \psi_t \frac{1}{\rho_t} = {} & (\varepsilon - 1) \left(1 - \frac{\phi_P}{2} (\pi_t)^2 \right) + \phi_P (\pi_t)(1 + \pi_t) \\
& - \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \phi_P (\pi_{t+1}) \frac{y_{t+1}^D}{y_t^D} (1 + \pi_{t+1}) \frac{\rho_{t+1}}{\rho_t} \right\}
\end{aligned}
$$

and we obtain the same formula as in Bilbiie et al. (2007) (it is equation 2 there):

$$
\rho_t = \mu_t \psi_t = \mu_t \frac{w_t}{Z_t} \tag{3.2}
$$

where

$$\mu_t \equiv \frac{\varepsilon}{(\varepsilon - 1)\left(1 - \frac{\phi_P}{2}(\pi_t)^2\right) + \phi_P(\pi_t)(1 + \pi_t) - \beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\Upsilon_{t+1}\right\}}$$

with $\Upsilon_{t+1} \equiv \phi_P(\pi_{t+1})\frac{y_{t+1}^D}{y_t^D}(1 + \pi_{t+1})\frac{\rho_{t+1}}{\rho_t}$ (3.3)

which is the definition of the time-varying price markup. Intuitively, equation (3.2) can be interpreted as follows: the firm sets the relative price of its product (ρ_t) as a markup (μ_t) above the marginal cost (w_t/Z_t). The markup is time-varying because of the presence of the Rotemberg price-setting frictions.

Next we can also make use of the aggregate production function

$$Y_t^C = \rho_t N_t y_t^D = \rho_t N_t Z_t l_t$$

to substitute for y_t^D in equation (3.3):

$$\mu_t \equiv \frac{\varepsilon}{(\varepsilon - 1)\left(1 - \frac{\phi_P}{2}(\pi_t)^2\right) + \phi_P(\pi_t)(1 + \pi_t) - \beta E_t\left\{\mathcal{K}_{t,t+1}\Upsilon_{t+1}\right\}}$$

$$\Upsilon_{t+1} \equiv \phi_P(\pi_{t+1})\frac{\frac{Y_{t+1}^C}{\rho_{t+1}N_{t+1}}}{\frac{Y_t^C}{\rho_t N_t}}(1 + \pi_{t+1})\frac{\rho_{t+1}}{\rho_t}$$

where we applied the notation of Table (3.1) for the ratio of marginal utilities ($\mathcal{K}_{t,t+1} \equiv \frac{\lambda_{t+1}}{\lambda_t}$).

3.2.2   The household's problem

The representative household maximises the continuation value of its utility (V):

$$V_t = \begin{cases} U(C_t, L_t) + \beta\left[E_t V_{t+1}^{1-\alpha}\right]^{\frac{1}{1-\alpha}} & \text{if } U(C_t, L_t) \geq 0 \\ U(C_t, L_t) - \beta\left[E_t(-V_{t+1})^{1-\alpha}\right]^{\frac{1}{1-\alpha}} & \text{if } U(C_t, L_t) < 0 \end{cases}$$ (3.4)

with respect to its flow budget constraint. $\beta \in (0, 1)$ is the subjective discount factor. Utility (U) at period t is derived from consumption

(C_t) and leisure $(1-L_t)$. As the time frame is normalised to one leisure time $(1-L_t)$ is what we are left with after spending some time working (L_t). The recursive functional form in equation (3.4) is called Epstein-Zin preferences and is the same as the one used by Rudebusch and Swanson (2012). The period utility U which is additively separable in consumption and labour is given by[1]:

$$U(C_t, L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi_0 \frac{L_t^{1+\varphi}}{1+\varphi} \tag{3.5}$$

where σ is the inverse of the intertemporal elasticity of substitution (IES), φ is the inverse of the Frisch elasticity of labour supply to wages and $\chi_0 > 0$. Note that in this paper we consider an IES<1 so that $U < 0$ and, thus, the second line of equation (3.4) is employed.

Swanson (2012) shows that the connection between coefficient of relative risk-aversion $(CRRA)$ and parameter α of the recursive utility in equation (3.4) is[2]:

$$CRRA \simeq \frac{\sigma}{1 + \frac{\sigma}{\varphi}} + \frac{\alpha(1-\sigma)}{1 + \frac{\sigma-1}{1+\varphi}}.$$

Households possess two types of assets: shares in a mutual fund of firms and government bonds. Let x_t denote the share in the mutual fund of firms entering period t. In each period the mutual fund pays the representative household a total profit (in units of currency) of all firms that produce in that period, $P_t N_t d_t$. In period t the representative household purchases x_{t+1} shares in a mutual fund of $N_{H,t} \equiv N_t + N_{E,t}$ firms where the first term refer to firms already operating at time t while the second term stands for the new entrants. Only $N_{t+1} = (1-\delta)N_{H,t}$ firms will produce and pay dividends at time $t + 1$. As the household does not know the share of firms induced to leave the market due to the exogenous exit shock δ at the end of

---

[1]Note that this felicity function is slightly different from the one of RS mainly because we abstract from deterministic growth in line with Bilbiie et al. (2007, 2012).

[2]Note that this formula applies only when the utility function in equation (3.5) is used.

period t, it finances the continuing operation of all preexisting firms and all new entrants during period t. The nominal price of a claim to the future profit stream of the mutual fund of $N_{H,t}$ firms at time t equals to $V_t^{firm} \equiv P_t v_t^{firm}$.

At time t the representative household holds nominal bonds and a share x_t in the mutual fund. It receives labour income ($W_t L_t$) interest income i_{t-1} on nominal bonds and dividend income (in nominal terms) on mutual fund share holdings ($D_t \equiv P_t d_t$) in nominal terms and the value of selling its initial share position (V_t^{firm}).

Therefore, the period budget constraint of the representative household (in units of currency) can be written as:

$$B_{N,t+1} + V_t^{firm} N_{H,t} x_{t+1} + P_t C_t = (1+i_{t-1}) B_{N,t} + (D_t + V_t^{firm}) N_t x_t + W_t L_t + T_t^L$$

where D_t stands for the nominal value of dividends ($D_t \equiv P_t d_t$), $1 + i_t$ is the gross nominal interest rate and T_t^L are lump-sum taxes in nominal terms.

The previous equation can be expressed in real terms by dividing both sides with the price level (P_t):

$$B_{t+1} + v_t^{firm} N_{H,t} x_{t+1} + C_t = (1 + r_t) B_t + (d_t + v_t^{firm}) N_t x_t + w_t L_t + t_t^L$$

where $B_{t+1} \equiv B_{N,t+1}/P_t$ and $1 + r_t \equiv (1 + i_{t-1})/(1 + \pi_t^C)$ is the consumption-based real interest rate on bond holdings between time $t - 1$ and t with consumer-price inflation (CPI) defined as $\pi_{t+1}^C \equiv P_{t+1}/P_t - 1$, and $t_t^L \equiv T_t^L/P_t$.

The first-order conditions derived from the households' optimisation problem are the same as those in chapter 2. The Euler equations for bond and share-holdings are repeated here:

$$1 = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1 + i_t}{1 + \pi_{t+1}^C} \right\}$$

$$v_t^{firm} = \beta(1-\delta)E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}(v_{t+1}^{firm} + d_{t+1})\right\}$$

The intratemporal condition says that labour effort is chosen optimally when the marginal disutility of labour equals to the marginal utility from consuming real wage received for one more hour worked:

$$\chi\frac{L_t^{\frac{1}{\varphi}}}{C_t^{-\sigma}} = (1-\tau^i)\frac{W_t}{P_t}$$

where χ is set such that hours worked makes up for one-third of the total time endowment and τ^i is a tax rate on labour income.

3.3 Equilibrium

In the symmetric equilibrium all firms make identical choices so that $p_t(\omega) = p_t$, $d_t(\omega) = d_t$, $y_t(\omega) = y_t$, $v_t^{firm}(\omega) = v_t^{firm}$, $l_t(\omega) = l_t$, $\mu_t(\omega) = \mu$ and $pac_t(\omega) = pac_t$.

The labour market clearing is given by:

$$L_t = N_t l_t + N_{E,t}\frac{f_{E,t}}{Z_t} \tag{3.6}$$

where the firm term on the RHS denotes the amount of labour used in production while the second term stands for the amount of labour employed to set up new firms. One can use equation (3.6) to back out $N_{E,t}$ in equation (3.1).

The aggregate output of the consumption basket (Y_t^C) is used for private (C_t) and public consumption (G_t) and to pay price adjustment costs:

$$\begin{aligned} Y_t^C &\equiv C_t + G_t + PAC_t \\ &= N_t\rho_t y_t \\ &= N_t\rho_t Z_t l_t \end{aligned}$$

The previous accounting identity says that total absorption (the first

line) equals to total production (second line). The last line made use of the production function.

3.3.1   Monetary and fiscal policy

The New-Keynesian model is closed by a monetary policy rule (so called Taylor rule):

$$R_t = \rho R_{t-1} + (1-\rho)[R + \log \bar{\Pi}_t + \phi_\pi (\log \bar{\Pi}_t - \log \Pi_t^*) + \phi_y (Y_t - Y_t^*)/Y_t^*] + \varepsilon_t^i$$

$$(3.7)$$

where $R_t \equiv 1 + i_t$ is the policy rate, $\bar{\Pi}_t$ is a four-quarter moving average of inflation and Y_t^* is the trend level of output yZ_t (where y denotes the steady-state level of Y_t/Z_t). As in RS we annualise $\log \bar{\Pi}$ and R so that the choice of $\phi_\pi = 0.53$ corresponds to roughly one-fourth of the empirical estimates using quarterly data (see, e.g., Clarida et al. (2000)).

Π_t^* is the target rate of inflation, ε_t^i is an iid shock with mean zero and variance σ_i^2. In the baseline version of the Rudebusch and Swanson (2012) model without long-run inflation risks the inflation target is constant ($\Pi_t^* = \Pi^*$ for all t).

The four-quarter moving average of inflation ($\bar{\Pi}_t$) can be approximated by a geometric moving average of inflation:

$$\log \bar{\Pi}_t = \theta_\pi \log \bar{\Pi}_t + (1 - \theta_\pi) \log \Pi_t, \qquad (3.8)$$

where the choice of $\theta_\pi = 0.7$ ensures that the geometric average in equation (3.8) has an effective duration of about four quarters. In this paper we do not consider long-run inflation risks as in one of the version of RS.

In Rudebusch and Swanson (2012) exogenous fiscal spending is assumed to be financed with lump-sum taxes. In the previous chapter we have shown that the way government spending is financed affects first and second moments calculated from the model to a large extent.

Therefore, as an alternative of lump-sum taxes we present results for the case when spending is financed by distortionary taxes levied on labour and profit income (for a formal description see chapter 2).

3.4 Comparison of the model with that of Rudebusch and Swanson (2012)

We summarised the equations of the model in Table (3.1) below. This is slightly different from the Table 5.1 of Bilbiie et al. (2007) because our extension contains Epstein-Zin preferences and government spending as well (as in Rudebusch and Swanson (2012)). Note that the Calvo-type of price stickiness used by RS is equivalent to the Rotemberg style price rigidity applied in Bilbiie et al. (2007) and in this paper.

The model used in this paper departs from Rudebusch and Swanson (2012) to the extent of i) the inclusion of firm entry, ii) the omission of fixed capital (and hence fixed investment)[3] and iii) using a lower estimate on the coefficient of output gap in the Taylor rule ($\phi_y = 0.125$ which is close to one of the estimate by Clarida et al. (1998) instead of $\phi_y = 0.93$ in Rudebusch and Swanson (2012)).

We shortly elaborate on points ii) and iii). First, we describe consequences of the omission of fixed capital (see point ii) above). In Woodford (2003) fixed/firm-specific capital is a way of introducing strategic complementarity into price-setting. A higher level of strategic complementarity manifest in a smaller coefficient on the marginal cost (or output gap) in the New Keynesian Phillips curve. A smaller coefficient on marginal cost in the Phillips curve is equivalent to lower level of price-rigidity in the model. However, alternatively, we can induce a lower level of price stickiness by reducing κ which is the parameter of price adjustment costs in the Phillips curve (see equation

---

[3]We plan to explore the role of physical capital with adjustment costs in the entry model in another paper.

called markup in Table 3.1 below).

Regarding iii) we motivate low coefficient on the output gap in the Taylor rule for four reasons. First, in the RS model with entry a coefficient of $\phi_y = 0.93$ leads to indeterminacy[4] when entry cost is specified in consumption units (discussed below). The highest output gap coefficient with which the model can be solved is around 0.6. Second, most estimated New Keynesian models place small coefficient on the output gap (see e.g. Smets and Wouters (2007)). Bilbiie et al. go even further asserting that a small coefficient on the output gap is consistent with the behaviour of Federal Reserve since that 1980s and places a coefficient of zero on the output gap. Third, a small positive output gap coefficient is also in line with the empirical evidence (see Clarida et al. (1998) and more in the calibration section) Fourth, it is argued below that the higher is the output gap coefficient the stronger is the negative covariance between consumption and inflation, which is a pre-requisite for achieving a high term premium on long-term bonds (see chapter 2 for more on this).

The behaviour of the RS model with entry and a zero coefficient on the output gap is contrasted with the case of a small, positive coefficient ($\phi_y = 0.125$) by looking at the impulses responses of a positive technology shock (see Figure 3.3). In both versions the markup responds positively on impact although it turns to negative (below zero) sooner when the output gap coefficient is zero.

Most importantly inflation falls more when ϕ_y is higher than zero stengthening the negative comovement between consumption and inflation and contributing more to the nominal term premium. The reason why inflation plummets to higher extent in case of a positive ϕ_y is due to the reaction of the real interest rate. As Figure 3.3 indicates real interest rate rises more with ($\phi_y > 0$) and depressing aggregate demand so much that it leads to huge deflation.

---

[4]We do not face this challenge when entry costs are defined in effective labour units.

Table 3.1: Summary of the Model

Pricing kernel	$\mathcal{K}_{t,t+1} \equiv \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left[\frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}}\right]^{-\alpha}$
Pricing	$\rho_t = \mu_t \frac{w_t}{Z_t}$
Markup	$\mu_t = \frac{\theta(N_t)}{(\theta(N_t)-1)\left[1-\frac{\phi_P}{2}\Pi_t^2\right]+\phi_P[(1+\Pi_t)\Pi_t-\beta(1-\delta)E_t[\mathcal{K}_{t,t+1}\Upsilon_{t+1}]]}$
Variety effect	$\rho_t = \exp\left(-\frac{1}{2}\frac{\tilde{N}-N_t}{\varsigma\tilde{N}N_t}\right)$
Profits	$d_t = \left[1-\frac{1}{\mu_t}-\frac{\phi_P}{2}\Pi_t^2\right]\frac{Y_t^C}{N_t}$
Free Entry	$v_t^{firm} = w_t\frac{f_{E,t}}{Z_t}$
Number of firms	$N_t = (1-\delta)(N_{t-1}+N_{E,t-1})$
Intratemporal Condition	$\chi C_t^\sigma N_t^{\frac{1}{\varphi}} = W_t$
Euler equation (shares)	$v_t^{firm} = \beta(1-\delta)E_t\left\{\mathcal{K}_{t,t+1}(v_{t+1}^{firm}+d_{t+1})\right\}$
Euler equation (bonds)	$1 = \beta\mathcal{K}_{t,t+1}\frac{R_t}{1+\Pi_{t+1}^C}$
Output of the consumption sector	$Y_t^C = \left[1-\frac{\phi_P}{2}\Pi_t^2\right]^{-1}(C_t+G_t)$
Aggregate accounting	$Y_t^C + N_{E,t}V_t^{firm} = W_tL_t + N_tD_t$
CPI inflation	$\frac{1+\Pi_t}{1+\Pi_t^C} = \frac{\rho_t}{\rho_{t-1}}$
Taylor Rule	The non-linear version of equation (3.7).
The definition of $\bar{\Pi}$	The non-linear version of equation 3.8

Plus three more equations describing $U(C_t, L_t) - \beta\left[E_t(-V_{t+1})^{1-\alpha}\right]^{\frac{1}{1-\alpha}}$.
For details see Chapter 2 and the appendix of Rudebusch and Swanson (2012).

Here we provide a brief description of some equations of interest in Table 3.1. The first equation is the pricing kernel used to value payoffs across time and states of nature. The second and third equations say that the optimal price ratio (or a value of a variety) equals to the marginal cost with a markup which is time-varying due to endogenous entry and price rigidity. Equation four is the variety effect in case of translog preferences (see more below). Equation five is the definition of profits. Equation six is the free-entry condition stating that the value of the firm (the present value of profit) equals to a sunk cost. The entry cost could fluctuate for exogenous reasons (Bilbiie et al. (2012) study deregulation as an exogenous fall in $f_{E,t}$) but we keep it fixed i.e. $f_{E,t} = f_E$. Equation seven describes the evolution of the number of firms at time t (N_t) as a function of firms in the previous period (N_{t-1}) and new entrants ($N_{E,t-1}$) allowing for the fact that

some of existing firms exit with probability δ. The interpretation of the rest of the equations is quite standard.

3.5 Translog preferences

Bilbiie et al. (2007) show that the basic New Keynesian model produces countercyclical markup in response to productivity shocks when i) variety effect operates through a CES aggregator ($\rho(N_t) = N_t^{1/(\theta-1)}$ and θ is the constant elasticity of substitution among goods), ii) aggregate labour supply is inelastic and iii) the coefficient on the output gap in the Taylor rule is zero.

However, we found that the markup is not counter-cyclical anymore under a CES aggregator if labour supply is endogenous and there is a positive coefficient in the Taylor-rule on the output gap. In order to maintain the countercyclical markup the model has to feature the competition effect which makes the elasticity of substitution among goods ($\theta(N)$) rise with the appearance of new entrants and, hence, new varieties (N). In the literature there are several ways to induce competition effect. In particular, Colciago and Etro (2010a) induce strategic interactions among firms through quantity competition a la Cournot while Colciago and Etro (2010b) consider competition in prices a la Bertrand. Furthermore, Colciago and Etro (2010b) compare the response of the markup to a temporary technology shock under Cournot, Bertrand and translog preferences and conclude that competition effect is the strongest under translog preferences as in Bilbiie et al. (2012). Therefore, the markup which is an inverse function of θ declines with an increase in the number of varieties. The variety effect ($\rho(N)$) and the markup ($\mu(N_t)$) under translog preferences can be written as:

$$\rho(N_t) = \exp\left(-\frac{1}{2}\frac{\tilde{N} - N_t}{\varsigma \tilde{N} N_t}\right), \ \tilde{N} \equiv Mass(\Omega)$$

$$\mu(N_t) = 1 + \frac{1}{\varsigma N_t}$$

where ς is chosen such that the steady-state number of firms under the CES and translog case is the same.

Figure (3.1) and (3.2) show simulated autocorrelations of the markup with lagged GDP for the case of zero and positive coefficient on the output gap, respectively. The figures also provide details on which shocks contribute most to the autocorrelation between markup and GDP.

We make the following observations. First, the autocorrelation between the markup and lags of the GDP is negative although the model cannot generate the shape observed in data[5]. Second, a positive coefficient on the output gap decreases the absolute value of the autocorrelations especially when only technology and fiscal shocks are considered. Third, monetary policy shocks facilitate matching the empirical markup-GDP autocorrelation especially in the case of $\phi_y = 0$ when technology shock causes this correlation to be excessively negative. To understand why monetary policy shocks mitigate the negative autocorrelation between markup and lagged GDP we can contrast impulse responses for technology and monetary policy shocks (Figure 3.3 and 3.4, respectively). Although for a positive technology shock markup increases on impact it switches to negative for most of the transition to the steady-state (Figure 3.3). For a negative technology shock the markup falls already on impact and remains negative for most of the transition path (not shown).

From the graphs we can also recognise that a monetary policy shock has much stronger effect on the markup (0.6% per cent on impact) compared to a technology shock with 0.15% even if the monetary policy shock is assumed to have zero persistence in line with the literature. Thus, in sum, highly persistent technology shocks are the main drivers

---

[5]This paper does not endeavour to match the exact shape of the empirical autocorrelation function. Our aim to maintain the negative correlation between markup and GDP is fulfilled.

Figure 3.1: Autocorrelation of the markup with lagged GDP when coefficient on the output gap is zero in the Taylor rule
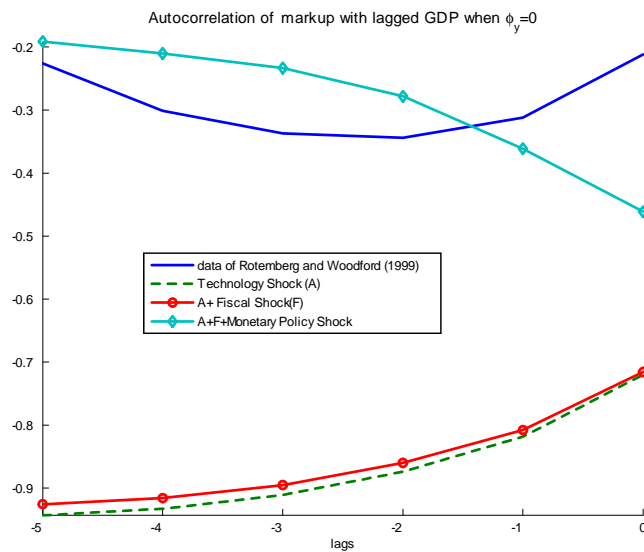
of the countercyclical markup and monetary policy shocks have the potential to limit the strength of technology shocks.

3.6 Parametrisation and solution method

Parameter values are collected in Table 3.3 which closely follows Rudebusch and Swanson (2012). The model is approximated to the third-order using Dynare (see Adjemian et al. (2011)). We follow Bilbiie et al. (2007) and assume that the entry cost is unity ($f_E = 1$). We haven't found proper guidance[6] on how to calibrate $\tilde{N}$ so we picked a high value of 10,000 varieties. In fact we don't experience a major change in results when we use either $\tilde{N} = 1000$ or $\tilde{N} = 10,000$. Parameter χ_0 is chosen such that steady-state hours worked is normalised to one-third of the total time endowment ($\bar{L} = 1/3$). Data on equity premium and the standard deviation of equity is taken from Beaubrun-Driant and Tripier (2005). In this chapter the implications of two different estimates of the output gap coefficient (either 0.07 or

---

[6]Bilbiie et al. (2007, 2012) use the first-order loglinear form of the entry model and they can rewrite the model in such a way that $\tilde{N}$ drops. However, when the non-linear model is maintained as in this paper $\tilde{N}$ can't be eliminated.

Figure 3.2: Autocorrelation of the markup with lagged value of the GDP when coefficient on the output gap is positive in the Taylor rule
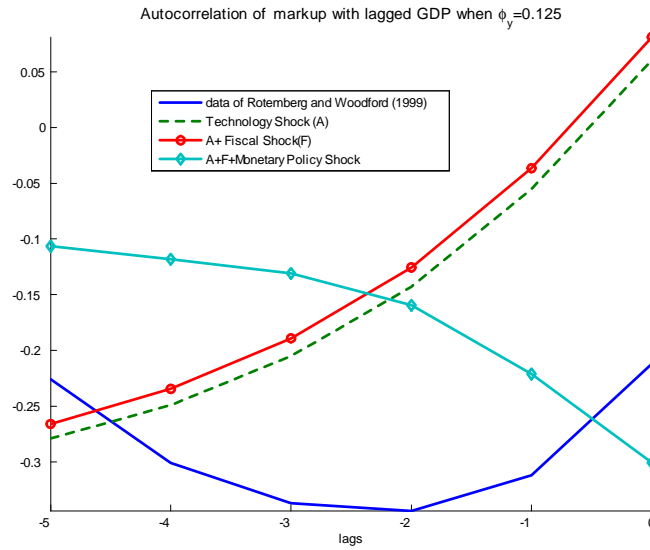
Figure 3.3: Impulse responses of selected variables to a positive (temporary) technology shock. All of them are expressed in percentage deviation from steady-state. Inflation, real and nominal interest rates and the return on equity are annualised.
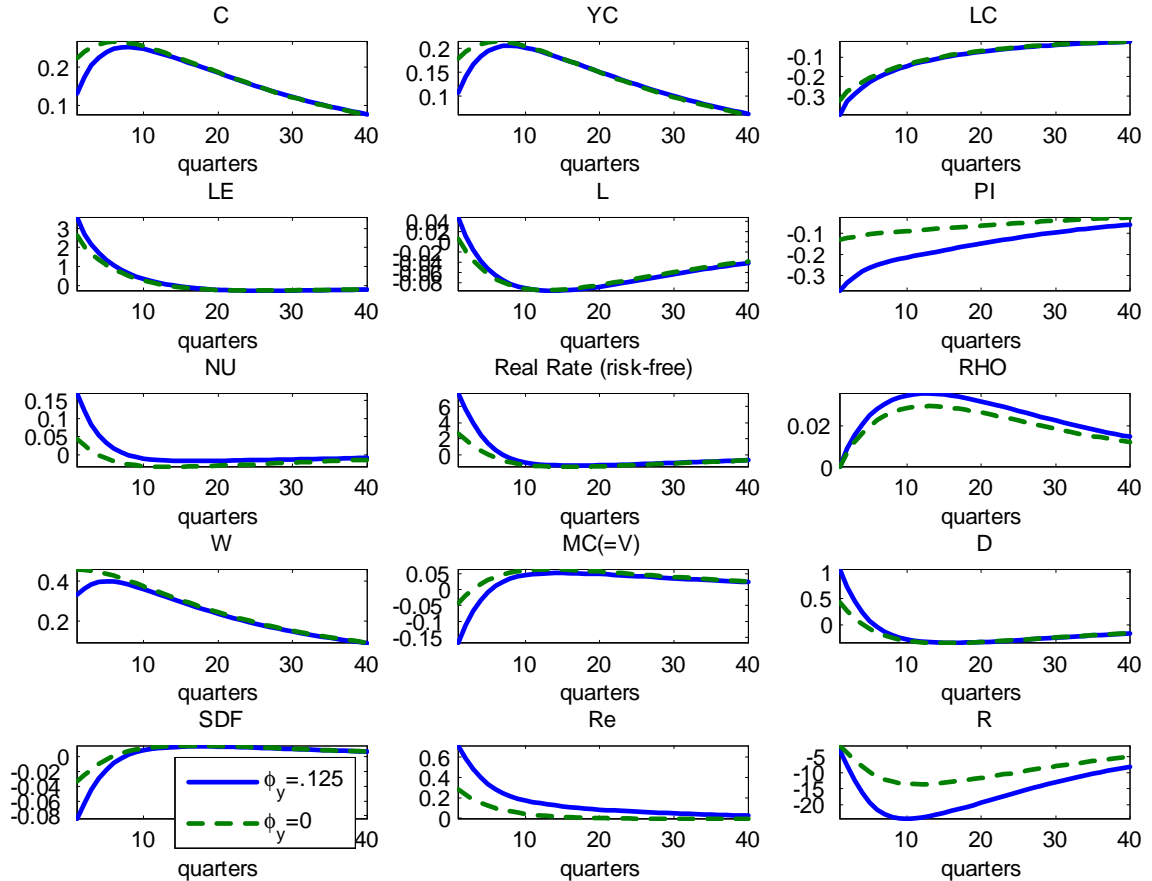
Figure 3.4: Impulse responses in case of contractionary monetary policy shock. All of them are expressed in percentage deviation from steady-state. Inflation, real and nominal interest rates and the return on equity are annualised.
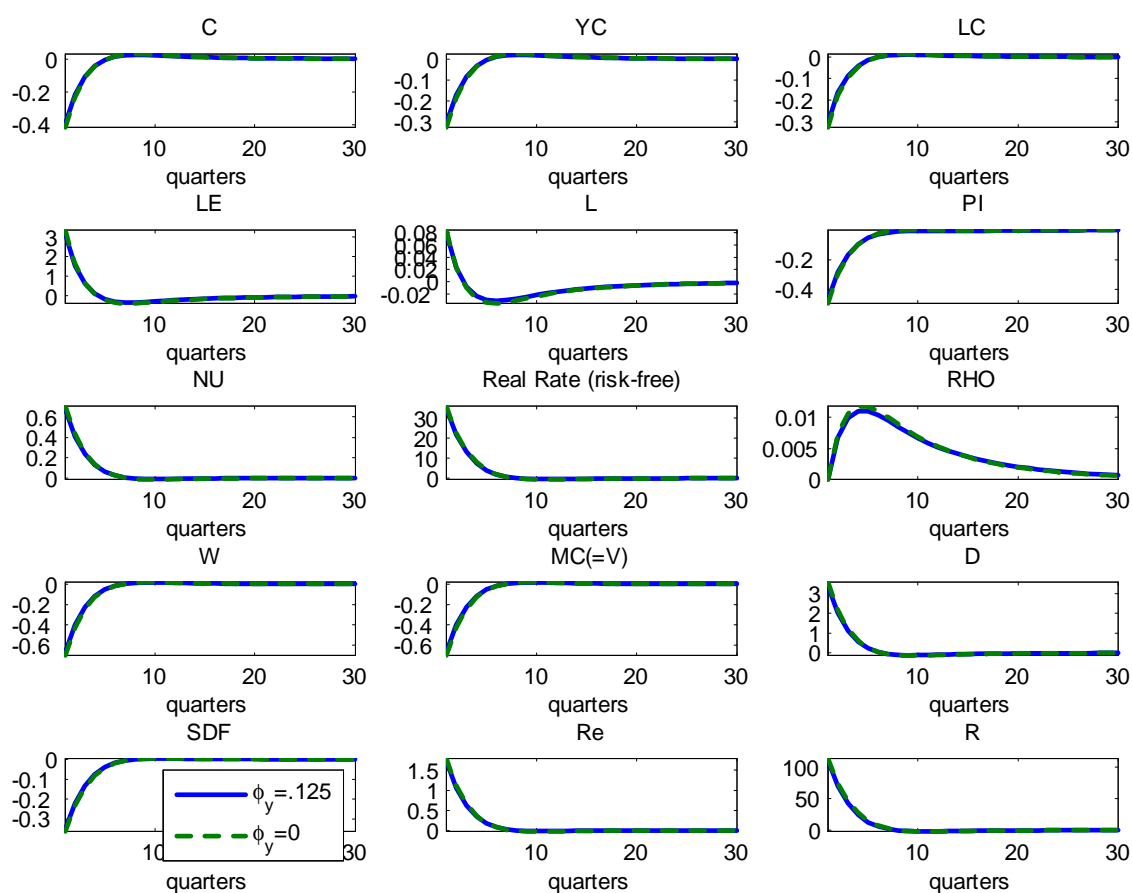
0.93) are explored in line with the estimates of Clarida et al. (1998, 2000) and Rudebusch (2002) (for more information see Table 3.2). The price-adjustment cost ($\phi_P = 77$) coincides with the one in Bilbiie et al. (2007) and is in line with most of the literature.

Table 3.2: Calibration

Taylor-rule estimates of Clarida et al. (1998, 2000) for the US

	ρ	ϕ_π	ϕ_y
Rule 1 (Clarida et al. 1998) for 1979-1994	0.92	1.79	0.07
Rule 2 (Clarida et al. 2000) for 1983-1996*	0.79	2.16	0.93

Clarida et al. (1998, 2000) estimated the following forward-looking Taylor rule: $i_t = \rho i_{t-1} + (1 - \rho)[\phi_\pi \bar{\pi}_{t+1} + \phi_y y_t]$. In Rudebusch and Swanson (2012) and also in our chapter 2 and 3 $\bar{\pi}_t$ is used instead of $\bar{\pi}_{t+1}$, although we found similar results for the case of $\bar{\pi}_{t+1}$. *Quite close to the values of Rudebusch and Swanson (2012) who utilised the estimate by Rudebusch (2002): $\rho = 0.73$, $\phi_\pi = 2.1$ and $\phi_y = 0.93$ [Remark: in Rudebusch and Swanson (2012) inflation is annualised in their Taylor rule as well as in our chapter 2 and 3 and, therefore, $\phi_\pi = 0.53$ is set].

Table 3.3: Calibration

σ	2	φ	3/2	ρ_i	0.73	ρ_Z	0.95
β	0.99	CRRA	75	ϕ_π	0.53	ρ_G	0.95
$\bar{L}$	1/3	θ	0.2	ϕ_y	0.93	ρ_M	0
G/Y	0.17	ϕ_P	77	Π^*	1	σ_Z^2	0.005^2
ε	6					σ_M^2	0.003^2
						σ_G^2	0.004^2

where G/Y is the government spending-to-GDP ratio. This table closely follows the calibration of Rudebusch and Swanson (2012).

3.7 Results

3.7.1 Entry cost in effective labour units

Inflation Risks

In this section we argue that the RS model with entry exhibits inflation risks when coefficient on the output gap in the Taylor-rule is

small. Before that we shortly summarise the empirical evidence on the inflation risk content of long-term nominal bonds. Early studies which don't include information from indexed bonds usually predict substantial inflation risk-premia based on no-arbitrage models. One such paper is by Buraschi and Jiltsov (2005) who find an average inflation risk premium of 70 basis points from 1960. Ang et al. (2008) estimate a term-structure model in which US inflation display regime-switching and report an inflation risk premium of around 115 basis points on average for bonds of five-year maturity over the period 1952-2004. Studies that incorporate information from indexed bonds reveal smaller estimates. For instance, Durham (2006) estimates a no-arbitrage model using US TIPS data and find a slightly positive inflation risk-premia for the sample starting at 2003. Before 2003 risk premia on 10-year inflation indexed bonds reflected liquidity risks rather than inflation risks. D'Amico et al. (2008) employ a model similar to Durham (2006) utilising data from 1990 onwards and report a positive and relatively stable 10-year inflation risk premium of about 50 basis points.

The case when entry cost is defined in terms of effective labour units is treated as the baseline in Bilbiie et al. (2012) and in our paper as well. We present a detailed decomposition of the nominal and real term premium in Table 3.4. In particular, nominal term premium is calculated as the difference between the yield on a 10-year nominal bond held by a risk-averse investor ($\text{yield}_{\text{10-year}}^{nom}$) and the yield on a nominal bond that is rolled over—in each quarter—for 10 years ($\text{yield}_{\text{10-year}}^{nom,\,eh}$). The latter can be interpreted as the yield expected by a risk-neutral investor and is consistent with the expectations hypothesis (eh) of the term structure. Similarly, real term premium is the difference between the same measures but for inflation-indexed (real) bonds i.e. $\text{RTP}=\text{yield}_{\text{10-year}}^{real}-\text{yield}_{\text{10-year}}^{real,\,eh}$.

We make the following observations. First, the correlation between

inflation and consumption growth is negative in the RS model with entry unlike the RS without entry where it is negative only when coefficient on output gap is high in the Taylor-rule. Second, nominal uncertainty ($\sigma(\hat{\pi})$) is much higher in the model with entry (5.43) than in the model without entry (1.63). Third, real uncertainty ($\sigma(\Delta\hat{c})$) is slightly higher in the model with entry. Fourth, nominal term premium is somewhat higher in the model without entry irrespective of the size of the coefficient on the Taylor rule. Fifth, RTP is low in the model without entry when output gap coefficient is high and there are positive inflation risks with the opposite is true in case of a high coefficient on the output gap. Sixth, exactly the inverse of observation five is true in the model with entry i.e. RTP is high in the model with entry.

Before we provide intuition regarding the previous observations, it is worth having a look at nominal and real yield curves as well as the inflation risk premium obtained from the entry model (see Figure 3.5). On the left-hand side of the graph we can see that the nominal yield curve is above the real yield curve mainly for bonds with maturities of at least 10 or 15 quarters depending on whether lump-sum or distortionary taxation is assumed. The difference between nominal and real term structure captures inflation risks. On the right hand side we observe that the nominal term premium is higher than real term premium and, thus, inflation risks emerge. The previous plot is generated based on the assumption that the coefficient on the output gap is low ($\phi_y = 0.125$). However, the RS model without entry implies positive inflation risks only when the coefficient on the output gap is high ($\phi_y = 0.93$). To show this we plot yield curves of the RS model without entry on Figure 3.6. On the left panel of the preceding figure inflations risks are positive for $\phi_y = 0.93$ while, on the right panel, they are negative for $\phi_y = 0.125$. Hördahl et al. (2008) employ a model similar to RS with low coefficient on the output gap and also

Figure 3.5: Yield curves, nominal and real term and inflation-risk premium
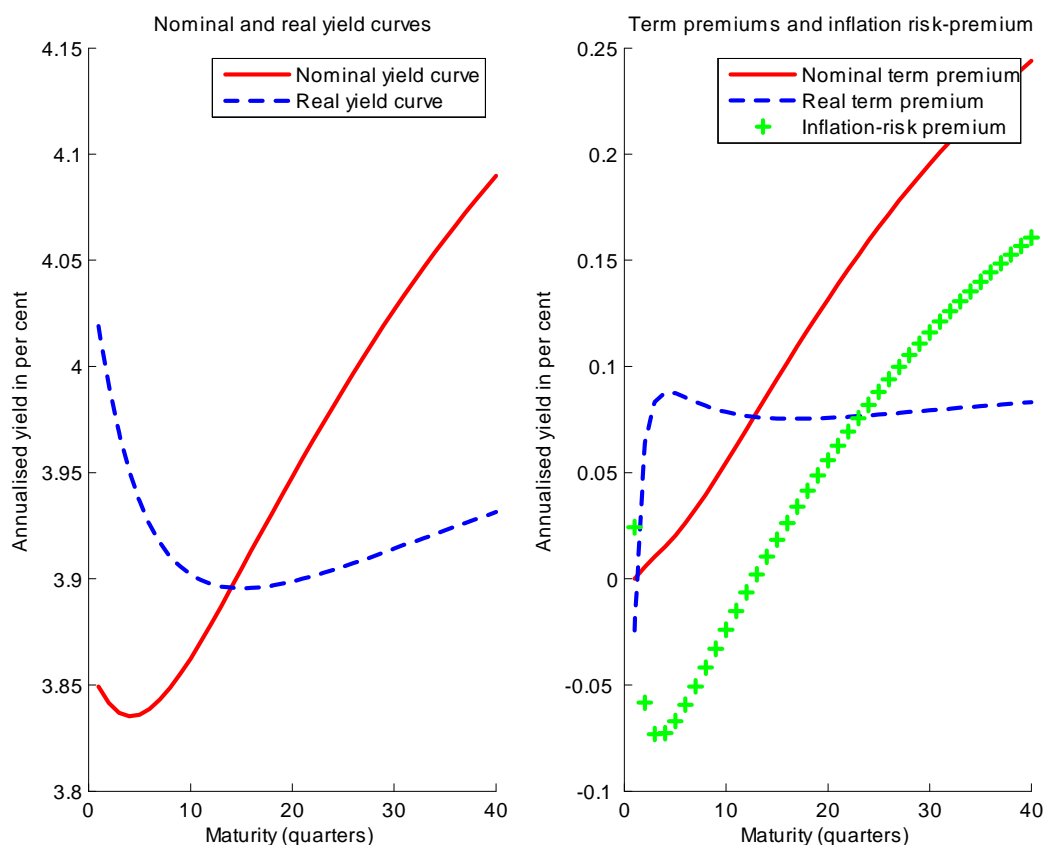
find that inflation risks are zero. It is well-known about the New Keynesian model that the higher is the coefficient on the output gap in the Taylor rule (ceteris paribus) the higher is the standard deviation of inflation (or other nominal variables in general) and the amount of inflation risks (see e.g. Clarida et al. (1999)). Hence, the size of the coefficient on the output gap appears to be a key determinant of inflation risks.

One can gain insight into the workings of the models with/without entry by inspecting the inflation and output gap volatility trade-off. In the RS model without entry the inflation-output gap volatility trade-off is the standard one: a higher coefficient on the output gap reduces real uncertainty (the standard deviation of the output gap) and raises nominal uncertainty (the standard deviation of inflation) (see, e.g., Clarida et al. (1999)). Rather surprisingly this trade-off becomes non-linear after adding entry to the New Keynesian model. For an output

Figure 3.6: Yield curves from the Rudebusch and Swanson (2012) model without entry and with different coefficients on the output gap.
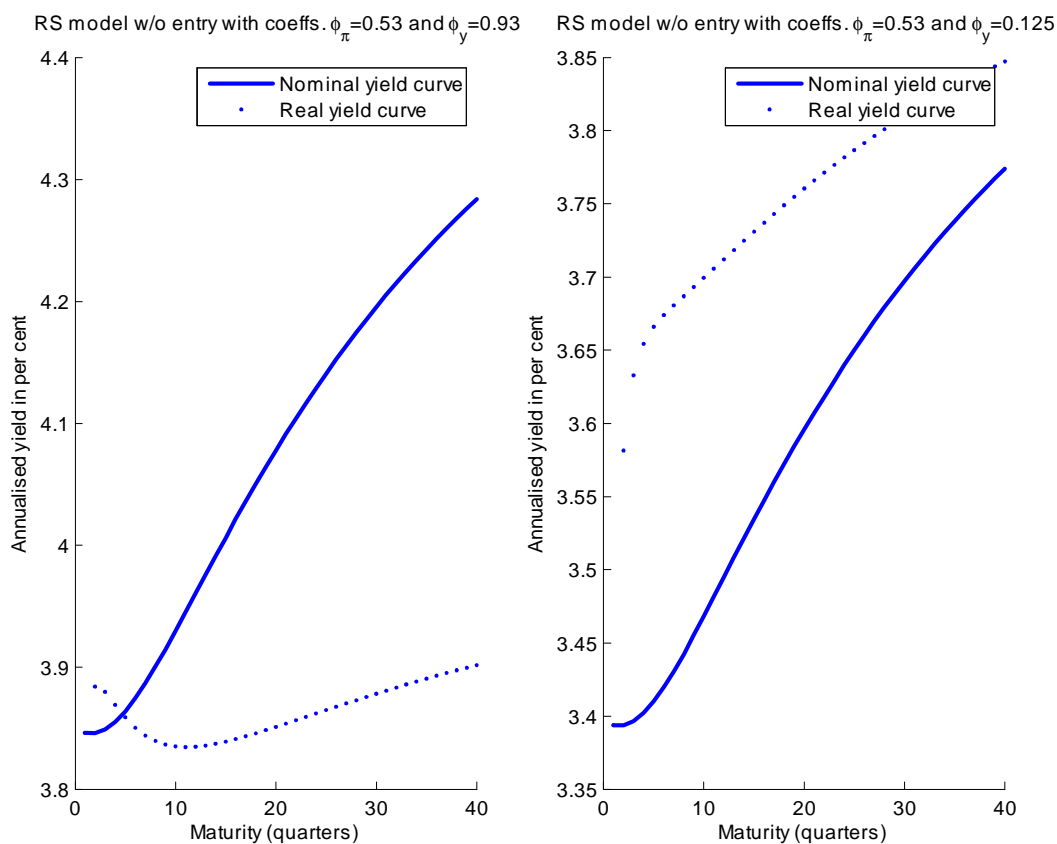
gap coefficient of lower than 0.5 the trade-off is maintained similar to the standard New Keynesian model. However, for output gap coefficients of 0.5 or above the trade-off between inflation and output gap volatility disappears. Figure (3.7) and (3.8) show that these findings are independent of the specification of the entry costs that can be either in effective labour units or in consumption units (see more on this below), respectively. The figures also indicate that the standard deviations for both inflation and output gap are higher for the entry model in general.

After comparing Figure 3.9 and 3.10 we recognise that the response of the short-term real interest rate on impact is much stronger in the model with entry compared to the one without entry implying that real interest rate increases to a large extent in order to counteract the sizeable real and nominal uncertainty when coefficient on the output gap is high ($\phi_Y = 0.93$). Figure 3.9 and 3.10 also reveal that differences in the short-term real rates are reflected by long-term real rates as well. Indeed, the 10-year real rate falls in the model without entry after a positive innovation to technology while 10-year real rate in the entry model jumps to the same shock. These differences are more pronounced the higher is the coefficient on the output gap.

Therefore, it follows that the RS model without entry implies low real and high nominal term premiums when the output gap coefficient is large. On the contrary the entry model exhibits substantial real term premium (real/consumption risks) for a high output gap coefficient and inflation risks exist only when coefficient on the output gap is low.

Unconditional moments

Table 3.5 collects several simulated macro and finance moments. The first column is taken from RS and contain moments calculated from US data for the period 1961-2007. The second column shows simulated

Figure 3.7: Trade-off between inflation and output-gap volatility in the Rudebusch and Swanson (2012) model with and w/o entry (entry costs are specified in effective labour units). The coefficient on the output-gap increases as we move from left to the right.
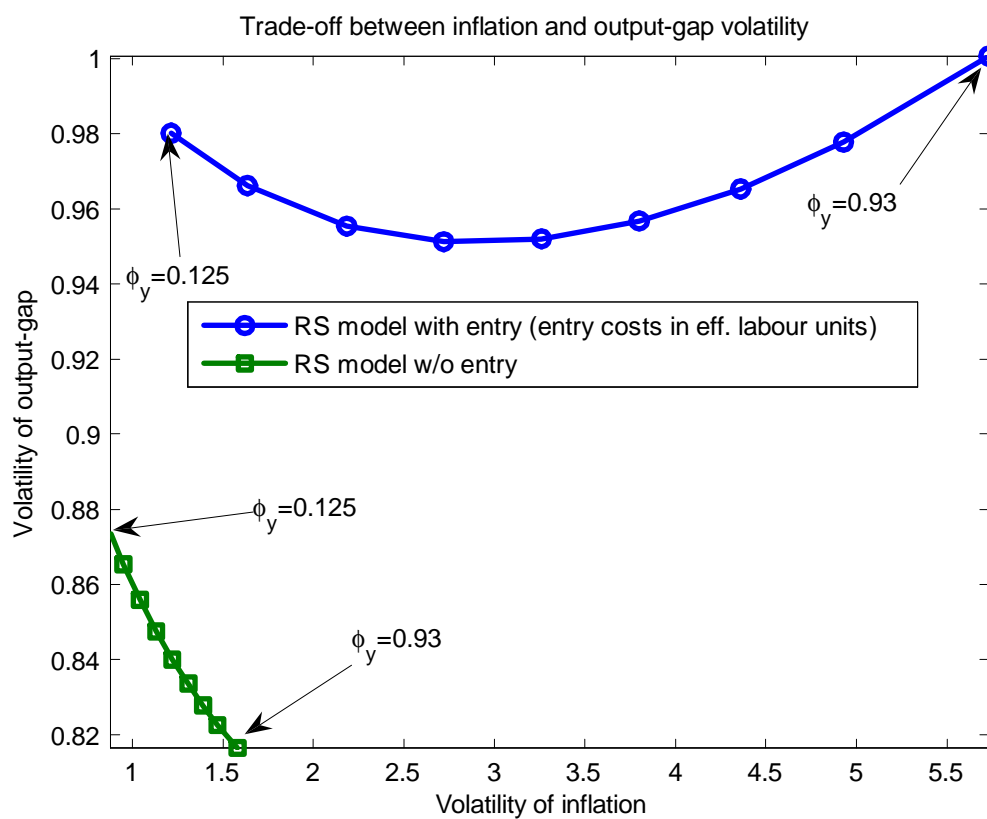
Figure 3.8: Trade-off between inflation and output-gap volatility in the Rudebusch and Swanson (2012) model with and w/o entry (entry costs are specified in consumption units). In this case we faced difficulties with simulations when the output-gap coefficient was higher than $\phi_y > 0.5$ so that we graph inflation-output-gap volatility pairs for $\phi_y \in [0.125, 0.5]$. The coefficient on the output-gap increases as we move from left to the right.
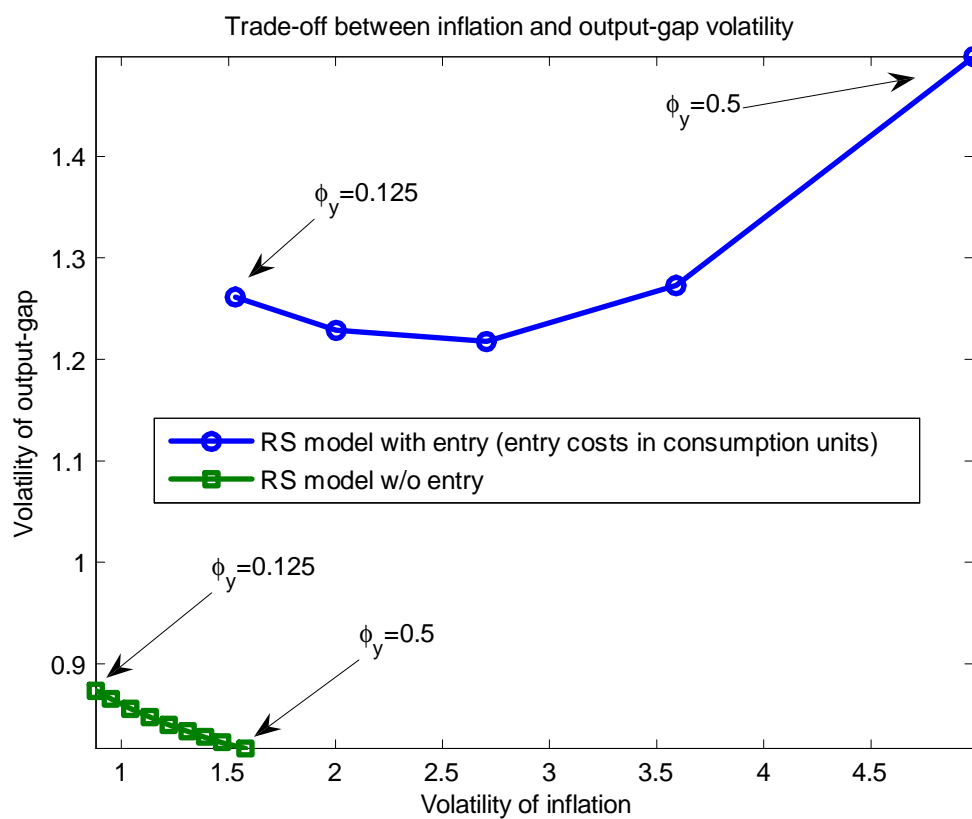
Figure 3.9: Impulse responses of selected variables to a positive (temporary) technology shock using the Rudebusch and Swanson (2012) model with entry. All of them are expressed in percentage deviation from steady-state. Inflation, real and nominal interest rates and the return on equity are annualised.
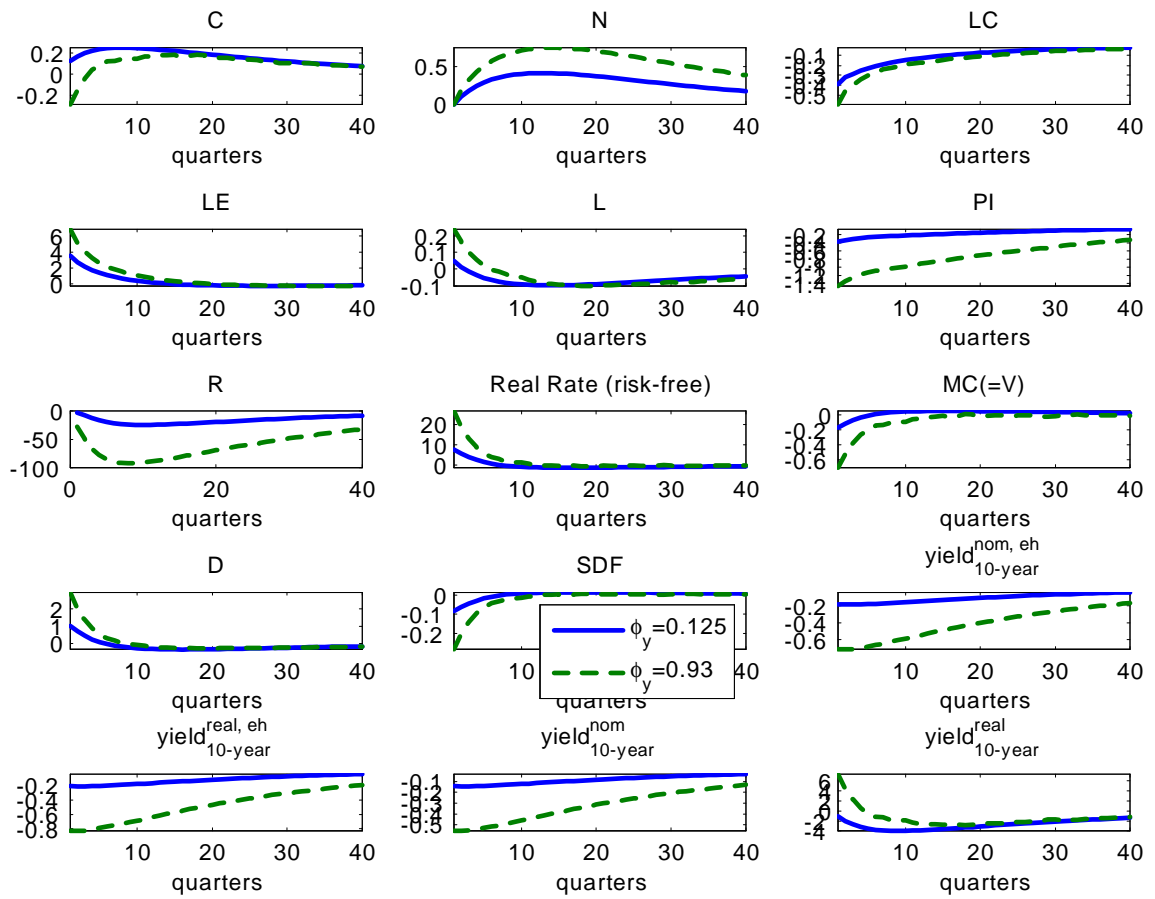
Figure 3.10: Impulse responses of selected variables to a positive (temporary) technology shock using the Rudebusch and Swanson (2012) model without entry. All of them are expressed in percentage deviation from steady-state. Inflation, real and nominal interest rates and the return on equity are annualised.
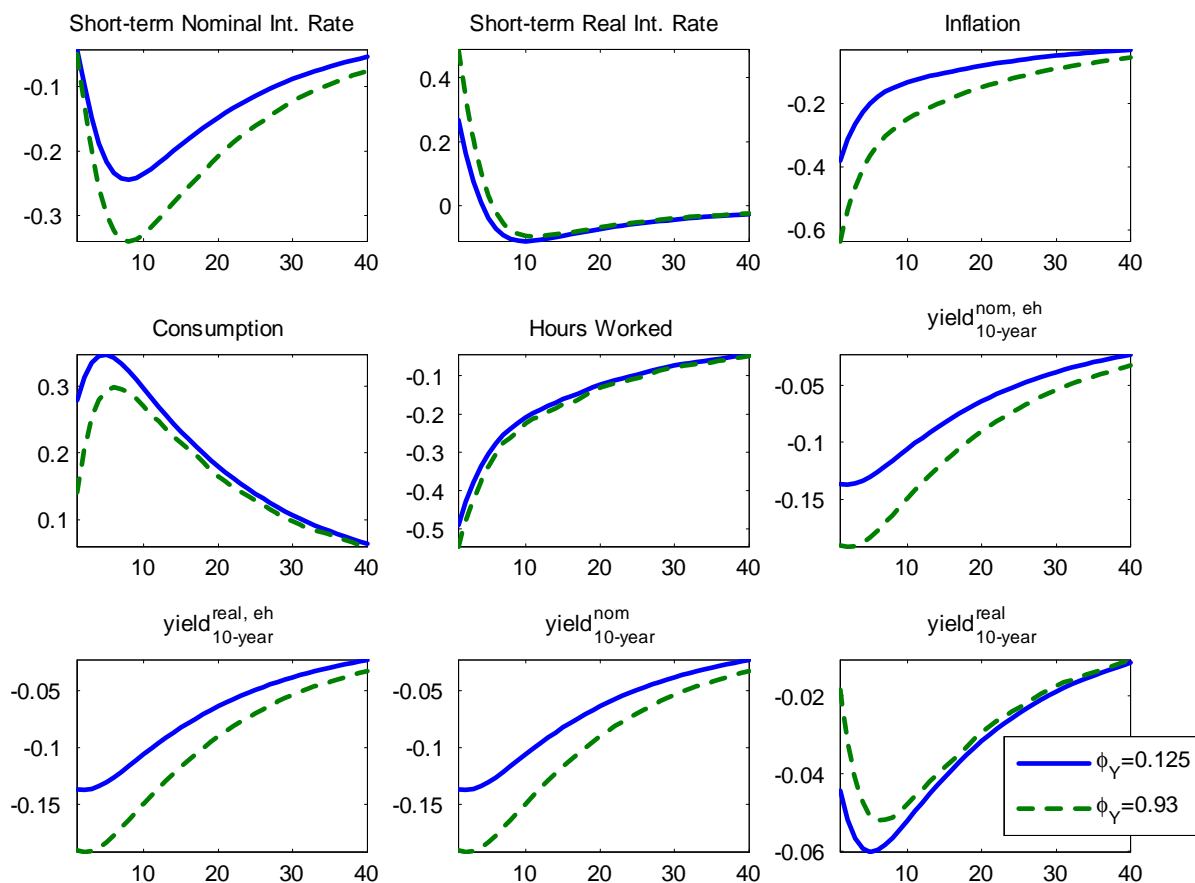
moments based on our reproduction of the RS model (our results and theirs are quite similar). The third column (denoted with A) provides simulated moments of the RS model with firm entry using temporary technology and fiscal shocks. All the columns except for E and F are based on the model in which government spending is financed by lump-sum taxes. However, column E and F present an alternative when spending is covered through income taxation as in the previous chapter.

Nominal term premium on a long-term bond, say a 10 year-bond, is computed as the return on the risky 10-year bond minus the return on a bond that is rolled over for 10 years. The yield on the latter strategy is often called as risk-neutral yield which is consistent with the expectations hypothesis of the term structure. Besides the mean and standard deviation of the nominal term premium we report its alternative measures like the slope of the term structure and the excess holding period return. The slope of the term structure is the difference between the yield on a long-term bond (say a 40-quarter bond) and the short-term bond $(R^{(40)} - R)$. The excess holding period return $(x^{(40)})$ is defined e.g. for a 40-quarter bond as: $p_t^{(39)}/p_{t-1}^{(40)} - R_{t-1}$ where the first term is the gross return to holding the 40-quarter bond for one period (p is the price of the bond) and the second term is the gross one-period risk-free rate. Equity premium is defined as the return on the profit (dividend) claim minus the return on the risk-free asset.

Consistent with Bilbiie et al. we also provide moments of real variables that are consistent with the change in the composition of goods of the consumption basket after the arrival of new varieties. In particular, a data-consistent variable $\tilde{X}$ is calculated as X/ρ where ρ is the price ratio that changes with the appearance of new varieties. Data-consistent variables can be found in column B, D and F. Generally, data-consistent standard deviations are higher than the baseline ones except for consumption.

The inclusion of the monetary policy shock (see columns C and D) facilitates the match of data for nominal and real interest rates. Also, we establish after comparing column A with columns C and D that the RS model with entry containing all three shocks outperforms the RS model without entry in achieving higher standard deviations for short-term nominal and real interest rates and a lower standard deviation for consumption. A shortcoming of the RS model with entry is the small standard deviation of labour compared to data. However, the introduction of fiscal policy with income taxation mitigates this problem to some extent (see columns E and F). On the negative side fiscal policy with income taxation further magnifies the standard deviation of real wage departing more from its empirical counterpart. It deserves some explanation why fiscal policy with income taxation is able to raise the standard deviation of labour and real wage.

Fiscal policy has substitution and wealth effects. The higher variability of hours worked is associated with a rise in government spending that is covered (partly) by higher labour taxes making people reduce (increase) their labour supply due to the substitution (wealth) effect. At the same time income taxation leads to higher standard deviation of the pre-tax real wage compared to the lump-sum taxes case and we depart more from its empirical counterpart. Thus, fiscal policy with income taxation improves (worsens) the fit in terms of labour (real wage).

3.7.2   Entry cost in consumption units

Unconditional moments

In the previous section entry cost is defined in units of effective labour that is equal to the firm's value ($v_t = w_t \frac{f_{E,t}}{A_t}$). Following Bilbiie et al. (2012) we can instead assume that all labour is utilised in the goods-producing sector and entry cost is defined in units of the consumption basket (entry cost has a different notation now: $f_{E,t}^C$). Still

we maintain the assumption that entry cost is constant ($f_{E,t}^C = f_E^C$ for all t). This modification imply some changes in the equations listed in Table 3.1 above. In particular, there is no need to differentiate output of the consumption sector from the whole GDP so that the accounting identity becomes $Y_t = N_t \rho_t y_t = w_t L_t + N_t d_t$ and there is no longer sectoral reallocation of labour between product creation and production of consumption goods. This also means that Y_t replaces Y_t^C in the definition of profits and there is no need for a separate equation defining Y_t^C. Remember that in the model of Bilbiie et al. the real price of investment (v_t) is time-varying and is equal to the entry cost in effective labour units ($w_t \frac{f_{E,t}}{A_t}$). However, in this case the consumption-based price of investment is constant and equal to one unit of consumption (after imposing the normalisation of $f_E^C = 1$ as in Bilbiie et al.)

Results obtained from applying a second-order approximation[7] are collected in Table 3.6. The structure of the columns is similar to that of the previous table. This version of the model definitely improves upon the baseline one not just in terms of higher nominal term premium but also producing higher standard deviations for both macro and finance moments. The standard deviation of the nominal term premium is zero because of the second-order approximation. However, it would turn to positive with an approximation to the third-order[8].

It also needs to be added that distortionary fiscal policy elevates real/consumption risks and not inflation risks in this version of the model. An analogous way of stating this is that the additional increase in the nominal term premium occurs due to a rise in the real term premium and not inflation risk premium. This is in stark contrast to RS where the additional increase in the nominal term premium is due

---

[7]Unfortunately, we obtain a non-trivial error in Dynare when taking a third-order approximation so we have to rely on second-order approximation in this case.

[8]This confirms Rudebusch and Swanson (2012) who argue that the nominal term-premium is time-varying only when the model (or, at least, the asset-pricing equations) is approximated to the third-order.

to higher inflation risks. It seems to be reasonable that the model with entry costs in effective labour units predicts higher inflation risks than the model where entry costs are in consumption units as the former one implies higher wage costs to finance new entrants following a positive productivity shock and also higher marginal cost and inflation through the New Keynesian Phillips curve.

Robustness Checks

In Table (3.7) we perform robustness checks using the model in which entry costs are expressed in terms of consumption units and government spending is covered by income taxes. All three shocks are employed. In column A and B we gauge how much our results change in the absence of price-rigidity i.e. setting $\phi_P = 0$ which is the case of fully flexible prices. In line with findings of previous literature (see, e.g., de Paoli et al. (2010)), nominal term premium has increased. However, the standard deviation of real interest rate became counterfactually low.

Some papers like Binsbergen et al. (2012) argue that it is relatively easier to generate high equity premium with lower EIS as in our paper rather than a higher one. In Column C and D we raised EIS in our model to two, ceteris paribus. Even if there is some improvement in terms of matching standard deviation of the equity return, the model undershoots in terms of the volatilities of finance variables relative to data. Most importantly, we find that the size of the equity premium is not affected by the higher EIS.

In column E and F we investigate into the case of a higher Frisch elasticity ($1/\varphi = 2$). On the negative side, the model overshoots the standard deviation of consumption, real wage and hours worked relative to data. On the positive side the finance moments are closer to the data. For instance, nominal term premium has risen to 75 basis points from 55 basis points.

In the last two columns (G and H) EIS and Frisch elasticity are simultaneously reduced while risk-aversion is increased—as a further attempt to match data (for a similar experiment see RS and also Chapter 2)—to values which can help match the empirical level of the nominal term premium. In particular, the EIS, the Frisch elasticity and risk-aversion are set to 0.3, 0.28 and 85 respectively. In the RS model without entry risk-aversion needs to be raised to 110 in order to arrive at a high mean value of the nominal term premium consistent with data. However, the RS model with entry allows us to produce the empirical mean of the nominal term premium with smaller upward movement in risk-aversion (to 85) relative to the baseline value (75).

3.8 Conclusion

This paper has shown that a slightly modified version of the Rudebusch and Swanson (2012, RS) model extended with firm entry can explain the means of bond and equity premium reasonably well without worsening the fit of the model to key macroeconomic variables. Also the model produces procyclical profits and countercyclical markups in line with estimates of Rotemberg and Woodford (1999). The empirical evidence in Clarida et al. (1998, 2000) reveals that the coefficient on the output gap can be either low or high depending on the choice of the sample period. The RS model without entry features a trade-off between the volatility of inflation and the output gap. RS relies on a high estimate of the output gap coefficient in the Taylor-rule to achieve low or even negative real risks and, thus, positive inflation risks. However, in the RS model with entry the trade-off between inflation and output gap volatility is present only for an output gap coefficient of 0.5 at most. For values of the output gap coefficient higher than 0.5 inflation and consumption risks mount simultaneously. We also show that inflation risks emerge in the entry model only when the coefficient on the output gap is relatively low.

A shortcoming of our model is that it cannot capture the enormous volatility of the stock return (16%). Therefore, future research should address the ways our model can increase the standard deviation of the return on equity without magnifying the volatility of macro variables extremely. This is quite a challenging exercise if we insist on small-size shocks as done in this paper. However, an extension of our model with capital definitely deserves further exploration.

3.9   Appendix—The steady-state number of firms

We generalise Bilbiie et al. (2012) for the case of IES<1 and positive government spending in the resource constraint. Of course, the steady-state is affected by these two pieces of modifications.

The steady-state number of firms in the case of entry costs in effective labour units can be obtained numerically from

$$
N \left[\frac{\delta[\mu(N) - 1] + (r + \delta)}{\delta[\mu(N) - 1]} \right]
$$

$$
= \frac{1 - \delta}{\delta} \frac{Z}{f_E} \left[\frac{1 - \tau}{\left(1 - \frac{G}{Y^C}\right)^\sigma \left(\frac{[1 - \beta(1 - \delta)] f_E \frac{\rho(N)}{\mu(N)}}{\beta(1 - \delta)\left(1 - \frac{1}{\mu(N)}\right)\frac{1}{N}} \right)^\sigma} \frac{\rho(N)}{\mu(N)} \frac{Z}{\chi} \mathcal{M} \right]^{1/\varphi}
$$

while in the case of entry costs in consumption units:

$$
1 = \frac{\mu}{\rho} \frac{\chi(1 - \frac{G}{Y})^\sigma (\frac{1}{\rho Z})^{\frac{1}{\varphi}} \left(\frac{\left(\frac{1}{\beta(1-\delta)} - 1\right)N}{1 - \frac{1}{\mu}} \right)^{\frac{\sigma\varphi + 1}{\varphi}}}{Z(1 - \tau)}
$$

where we have functional forms for $\rho(N)$ and $\mu(N)$. In the CES case $\mu(N) = \theta/(\theta - 1)$ and $\rho(N) = N^{\frac{1}{\theta - 1}}$ while in the translog case $\mu(N) = 1 + 1/(\varsigma N)$ and $\rho(N) = \exp(-\frac{1}{2}\frac{\tilde{N} - N}{\varsigma \tilde{N} N})$, $\tilde{N} \equiv Mass(\Omega)$ where ς is chosen such that the steady-state number of firms under the CES and translog case is the same. G/Y^C is the government spending-GDP ratio.

Table 3.4: Unconditional moments of the term-structure

Unconditional moments	Data	$\phi_Y = 0.93$	$\phi_Y = 0.125$
		RS model without entry	
$\text{corr}(\Delta\hat{c}, \hat{\pi})$	-0.35*	-0.20	0.11
$\sigma(\hat{\pi})$	2.52	1.63	0.92
$\sigma(\Delta\hat{c})$	1.96*	0.86	0.74
(a) $\text{yield}_{10\text{-year}}^{nom}$	6.94^	4.2989	3.7851
(b) $\text{yield}_{10\text{-year}}^{nom,\ eh}$	na	3.9078	3.4406
(c) $\text{yield}_{10\text{-year}}^{real}$	1.96^	3.9047	3.8527
(d) $\text{yield}_{10\text{-year}}^{real,\ eh}$	na	3.9078	3.4406
(e) NTP(=a-b)	1.06	0.39	0.34
(f) RTP(=c-d)	$\simeq 1.00°$	0.01	0.42
IRP(=e-f)	0.5°	0.38	-0.08
EQPR	6.2	0.85	0.71
		RS model with entry	
$\text{corr}(\Delta\hat{c}, \hat{\pi})$	-0.35*	-0.45	-0.55
$\sigma(\hat{\pi})$	2.52	5.43	1.38
$\sigma(\Delta\hat{c})$	1.96*	2.16	1.83
(g) $\text{yield}_{10\text{-year}}^{nom}$	6.94^	3.6551	4.0898
(h) $\text{yield}_{10\text{-year}}^{nom,\ eh}$	na	3.5221	3.4859
(j) $\text{yield}_{10\text{-year}}^{real}$	1.96^	3.9872	3.9314
(k) $\text{yield}_{10\text{-year}}^{real,\ eh}$	na	3.5254	3.8482
(l) NTP(=g-h)	1.06	0.13	0.24
(m) RTP(=j-k)	$\simeq 1.00°$	0.46	0.08
IRP(=l-m)	0.5°	-0.33	0.16
EQPR	6.2	11.43	10.51

where $\text{yield}_{10\text{-year}}^{nom}$ ($\text{yield}_{10\text{-year}}^{real}$) stands for the unconditional mean on a 10-year nominal (real) bond while $\text{yield}_{10\text{-year}}^{nom,\ eh}$ ($\text{yield}_{10\text{-year}}^{real,\ eh}$) denote yields for a 10-year nominal (real) bond which is consistent with the expectations hypothesis of the term structure. Corr, σ, NTP, RTP, IRP and EQPR are abbreviations of the unconditional correlation, standard deviation, nominal term premium, real term premium, inflation risk premium and equity premium. Data on $\sigma(\pi)$ and NTP is from Rudebusch and Swanson (2012).
*the source is Binsbergen et al. (2012) who used US NIPA data over 1953-2008.
^the source are the databases of Gürkaynak et al. (2007, 2008).
°inflation risk premium and RTP is the estimate by D'Amico et al. (2008). They note that most of the RTP is liquidity premium (especially in the early years of TIPS).
na=not identified from the data.

Table 3.5: Moments from variants of the Rudebusch and Swanson (=RS) (2012) model compared to US data

Unconditional Moment	US data, 1961-2007	$\mathcal{RS}$	A	B	C	D	E	F
SD(C)	0.83	1.42	0.93	0.87	1.01	0.97	1.15	1.09
SD(L)	1.71	1.5	0.40	0.63	0.43	0.66	0.59	0.83
SD(W)	0.82	1.32	1.33	1.33	1.47	1.49	1.54	1.52
SD(π)	2.52	1.64	1.44	-	1.62	-	1.59	-
SD(R)	2.71	1.6	1.43	-	2.01	-	2.09	-
SD(R^{real})	2.30	0.93	0.57	0.63	1.87	2.06	1.9	2.07
SD ($R^{(40)}$)	2.41	0.85	1.01	-	1.03	-	0.96	-
Mean($NTP^{(40)}$)	1.06	0.39	0.28	-	0.29	-	0.30	-
SD($NTP^{(40)}$)	0.54	0.04	0.02	-	0.02	-	0.02	-
Mean($R^{(40)} - R$)	1.43	0.43	0.27	-	0.30	-	0.37	-
SD($R^{(40)} - R$)	1.33	0.9	0.56	-	1.47	-	1.58	-
Mean($x^{(40)}$)	1.76	0.69	0.48	-	0.51	-	0.57	-
SD($x^{(40)}$)	23.43	7.81	8.94	-	9.14	-	8.85	-
Equity Premium	6.2	0.85	7.46	-	7.49	-	10.61	-
SD(Re)	15.98	1.20	1.46	1.29	2.63	2.68	2.75	2.79

where SD=standard deviation, $NTP^{(40)}$=nominal term premium on a 40-quarter bond, Mean=Unconditional Mean, $R^{(40)} - R$ is the slope and $x^{(40)}$ is the excess holding period return for a 40-quarter bond. Re is the return on equity. Each version of the models listed above utilises the baseline calibration of Rudebusch and Swanson (2012) that does not fit macro and finance moments of US data (neither here nor in their paper).

$\mathcal{RS}$=reproduction of the results of Rudebusch and Swanson (2012) without entry (Note that their results are very close to ours.)

A=Technology and fiscal shock (lump-sum taxation).

B=Technology and fiscal shock (lump-sum taxation), data-consistent real variables.

C=Technology, fiscal (lump-sum taxation) and monetary policy shocks.

D=Technology, fiscal (lump-sum taxation) and monetary policy shocks, data-consistent real variables.

E=Technology, fiscal (income taxation) and monetary policy shocks.

F=Technology, fiscal (income taxation) and monetary policy shocks, data-consistent real variables.

Table 3.6: Moments from variants of the Rudebusch and Swanson (=RS) (2012) model compared to US data

Unconditional Moment	US data, 1961-2007	$\mathcal{RS}$	A	B	C	D	E	F
SD(C)	0.83	1.42	1.12	0.98	1.33	1.08	1.58	1.41
SD(L)	1.71	1.5	0.83	1.28	0.92	1.50	0.92	1.31
SD(W)	0.82	1.32	1.18	1.09	1.87	1.75	1.89	1.72
SD(π)	2.52	1.64	1.58	-	1.90	-	2.03	-
SD(R)	2.71	1.6	1.53	-	2.21	-	2.61	-
SD(R^{real})	2.30	0.93	0.54	0.64	0.97	1.16	1.25	1.36
SD ($R^{(40)}$)	2.41	0.85	0.88	-	1.59	-	1.10	-
Mean($NTP^{(40)}$)	1.06	0.39	0.30	-	0.38	-	0.55	-
SD($NTP^{(40)}$)	0.54	0.04	0.00	-	0.00	-	0.00	-
Mean($R^{(40)} - R$)	1.43	0.43	0.47	-	0.57	-	0.70	-
SD($R^{(40)} - R$)	1.33	0.9	0.75	-	1.59	-	1.83	-
Mean($x^{(40)}$)	1.76	0.69	0.80	-	0.94	-	1.18	-
SD($x^{(40)}$)	23.43	7.81	7.95	-	9.20	-	11.78	-
Equity Premium	6.2	0.85	10.77	-	11.00	-	11.17	-
SD(Re)	15.98	1.20	1.63	1.94	1.88	3.66	2.44	4.10

where SD =standard deviation, $NTP^{(40)}$=nominal term premium on a 40-quarter bond, Mean=Unconditional Mean, $R^{(40)} - R$ is the slope and $x^{(40)}$ is the excess holding period return for a 40-quarter bond. Re is the return on equity. Each version of the models listed above utilises the baseline calibration of Rudebusch and Swanson (2012) that does not fit macro and finance moments of US data (neither here nor in their paper). Note that we are unable to carry out a third-order approximation when entry cost is specified in consumption as Dynare stops with a non-trivial error. Thus we have to resort to a second-order approximation when calculating moments in this table (except for column $\mathcal{RS}$).
$\mathcal{RS}$=reproduction of the results of Rudebusch and Swanson (2012) without entry (Note that their results are very close to ours.)
A=Technology and fiscal shock (lump-sum taxation).
B=Technology and fiscal shock (lump-sum taxation), data-consistent real variables.
C=Technology, fiscal (lump-sum taxation) and monetary policy shocks.
D=Technology, fiscal (lump-sum taxation) and monetary policy shocks, data-consistent real variables.
E=Technology, fiscal (income taxation) and monetary policy shocks.
F=Technology, fiscal (income taxation) and monetary policy shocks, data-consistent real variables.

Table 3.7: Moments from variants of the Rudebusch and Swanson (=RS) (2012) model compared to US data

Unconditional Moment	US data, 1961-2007	A	B	C	D	E	F	G	H
SD(C)	0.83	1.75	1.64	1.16	0.98	1.72	1.61	1.42	1.32
SD(L)	1.71	0.42	0.85	1.25	1.79	0.73	1.06	0.89	1.35
SD(W)	0.82	2.03	1.93	1.95	1.80	2.73	2.77	2.73	2.75
SD(π)	2.52	5.02	-	1.78	-	2.40	-	2.33	2.48
SD(R)	2.71	2.65	-	2.51	-	2.77	-	2.89	-
SD(R^{real})	2.30	0.71	0.81	1.18	1.32	1.21	1.37	1.23	1.37
SD ($R^{(40)}$)	2.41	1.19	-	1.08	-	1.30	-	1.51	-
Mean($NTP^{(40)}$)	1.06	0.67	-	0.51	-	0.75	-	1.06	-
SD($NTP^{(40)}$)	0.54	0.00	-	0.00	-	0.00	-	0.00	-
Mean($R^{(40)} - R$)	1.43	0.78	-	0.52	-	0.84	-	1.09	-
SD($R^{(40)} - R$)	1.33	1.61	-	1.73	-	1.81	-	1.78	-
Mean($x^{(40)}$)	1.76	1.23	-	0.91	-	1.44	-	1.93	-
SD($x^{(40)}$)	23.43	13.10	-	11.13	-	13.25	-	14.69	-
Equity Premium	6.2	10.57	-	11.11	-	11.47	-	11.87	-
SD(Re)	15.98	1.69	1.78	2.03	3.73	2.79	4.16	2.66	3.97

where SD =standard deviation, $NTP^{(40)}$=nominal term premium on a 40-quarter bond, Mean=Unconditional Mean, $R^{(40)} - R$ is the slope and $x^{(40)}$ is the excess holding period return for a 40-quarter bond. Re is the return on equity.
$\mathcal{RS}$=reproduction of the results of Rudebusch and Swanson (2012) without entry (Note that their results are very close to ours.)
Here we used the second version of the entry model where entry costs are defined in consumption units. All three types of shocks are employed and spending is financed by income taxation. Note that we are unable to carry out a third-order approximation when entry cost is specified in consumption as Dynare stops with a non-trivial error. Thus we have to resort to a second-order approximation when calculating moments in this table.
A=removing price rigidity ($\phi_P = 0$).
B=removing price rigidity ($\phi_P = 0$), data-consistent real variables.
C=higher elasticity of intertemporal substitution ($EIS = 1/\sigma = 2$ instead of the baseline 0.5).
D=higher elasticity of intertemporal substitution ($EIS = 1/\sigma = 2$ instead of the baseline 0.5), data-consistent real variables.
E=Higher Frisch elasticity of labour supply ($1/\varphi = 2$ instead of the baseline 2/3).
F=Higher Frisch elasticity of labour supply ($1/\varphi = 2$ instead of the baseline 2/3), data-consistent real variables.
G=Lowering EIS to 0.3 and Frisch elasticity to $1/\varphi = 0.28$.
H=Lowering EIS to 0.3 and Frisch elasticity to $1/\varphi = 0.28$, data-consistent real variables.

Conclusion and Policy Implications

The first chapter has shown that labour tax-cuts can stimulate output when short-term interest rate is close zero (the zero lower bound on the nominal interest rate is binding) as in the United States since end of 2008 if, on one hand, the economy contains non-Ricardian households who raise their consumption expenditure in response to the tax-cut generating a Keynesian-type demand effect. On the other hand, wage rigidity is also necessary for the tax-cut to have positive effects since it does reduce the sensitivity of real wage to the negative demand shock that makes the zero lower bound on the nominal interest rate binding.

Our model contains both Ricardian and non-Ricardian households. Ricardian households are forward-looking agents with perfect foresight exhibiting life-cycle consumption behaviour and knowing that the tax-cut at the present will be followed by tax-hikes in the future such that the budget constraint of the government is satisfied in present value terms. Therefore, Ricardians do not react to the tax-cut through a change in their consumption expenditure. Instead they save up the tax-cut in order to cover future tax-burden. However, non-Ricardians have a static horizon and can alternatively be interpreted as households with a borrowing constraint which is relaxed due to the tax-cut. As a result non-Ricardians spend the increase in their disposable income.

Our results are in stark contrast to Christiano et al. (2011) and Eggertsson (2011) who made the policy recommendation of increasing

the labour-tax rate to enhance economic growth during the zero lower bound using a model that contains only Ricardian agents. From policy point of view it is important to stress that chapter 1 allows for a uniform labour-tax cut i.e. the tax rate is reduced for both Ricardian and non-Ricardian households by the same extent. As a result the policy maker does not have to be able to distinguish between Ricardian and non-Ricardian households. The policy recommendation of our model is consistent with what was enacted as part of the American Recovery and Reinvestment package of 2009 which prescribed cuts in the labour taxes paid by employees as in our model.

In chapter 2 we investigate into the connection between fiscal policy and long-term yields on non-defaultable zero-coupon bonds in a fully Ricardian setup. Yields on long-term nominal bonds include term premium paid as a compensation to investors for consumption/inflation risks over the duration of the bond. Nominal bonds are risky in the sense that real payoff correlates positively with consumption. If a rise in future inflation which erodes bond prices due to heavier discounting of future nominal coupons coincides with times of low consumption (growth) then nominal bonds carry inflation risks (i.e. bond yields contain risk-premium) because bonds loose value at the time when household values consumption the most (Rudebusch and Swanson (2012)). Accordingly shocks like innovations to technology which induce negative comovement between inflation and consumption are most likely to generate risk-premium.

Orszag and Gale (2004) present empirical evidence on the positive relationship between government deficits and treasury yields of various maturities. We study the response of nominal bond yields to government spending shocks which are either financed by lump-sum or distortionary income taxes. We used the New Keynesian model of Rudebusch and Swanson (2012) where government spending shocks are financed by lump-sum taxes and government budget is balanced in

each period quite similar to the fiscal side of the Smets and Wouters (2007) model of the US economy. As an alternative we allowed for government debt—the possibility of deficit—that can be retired either in the short- or long-run through lump-sum or distortionary income taxes. We found that government spending has little impact on nominal yields if it is financed with lump-sum taxes either with or without debt.

However, distortionary income taxation implies negative correlation between inflation and consumption and therefore contributing to inflation risks. The ability of the model with income taxation in contributing to the risk-premium is similar to Rudebusch and Swanson (2012) model with long-run inflation risks where inflation target is updated in each period according to a moving average of inflation and also contains a stochastic element. The performance of the model with income taxes in terms of moments of key macroeconomic and finance variables is also very similar to one with long-run inflation risks when the baseline calibration of Rudebusch and Swanson (2012) is considered. However, the Rudebusch and Swanson (2012) model with either long-run inflation risks or with income taxation is unable to match finance data *when* we consider their baseline calibration.

Further research needs to be conducted to explore the empirical relevance of the relationship between tax system and bond yields. In particular, it would be interesting to see whether countries featuring more distortionary tax system or higher taxes on labour income are associated with higher bond yields.

In chapter 3 we introduce costly firm entry and an endogenous lag in production into the New Keynesian asset-pricing model of Rudebusch and Swanson (2012) used in chapter 2. As a first contribution we show that this extension explains both bond and equity premium puzzles without compromising the fit of the model to key macroeconomic moments. The inclusion of costly firm entry in RBC or New

Keynesian setups (see, e.g. Bilbiie et al. (2007, 2012)) proved to be successful in accounting for procyclical profits and countercyclical markups found in the data by Rotemberg and Woodford (1999).

In chapter 2 we argue that in a world featuring technology shocks there is a strong negative comovement between consumption growth and inflation leading to inflation risks that are compensated for in the form of bond risk premium. When firm entry is added to the basic model the positive correlation between consumption and profits (dividends) is magnified. In case of a negative technology shock both consumption and the return on equities (dividends) fall. Therefore, equity claims are considered to be risky as times of low consumption are coupled with reduced yields on equity and investors who are willing to hold them command a premium.

The standard New Keynesian model like the Rudebusch and Swanson (2012) model without entry implies a trade-off between stabilisation of the volatility of inflation and the output-gap (see also Clarida et al. (1999)). It is also true in the New Keynesian model without entry that the higher is the coefficient on the output-gap the more successful is monetary policy in driving down the volatility of the output-gap (or stabilising the output-gap) at the cost of increasing the volatility of inflation and, hence, a trade-off exists between volatility of inflation and the output-gap. This chapter also recognises that the size of Taylor-rule coefficient is crucial for the ability of the New Keynesian model with/without entry in generating inflation-risks. In particular, one can show that the Rudebusch and Swanson (2012) model without firm entry implies high inflation risks and zero consumption risks when the coefficient on the output-gap in the Taylor rule is large in line with the estimate Clarida et al. (2000) using US data of 1983-1996. It is generally true in a New Keynesian model that a large coefficient on the output-gap reduces real risks and magnifies inflation risks (see, e.g., Clarida et al. (1999)). On the contrary the Rudebusch and Swanson

(2012) model with costly firm entry exhibits inflation-risks when entry costs are expressed in effective labour units and the coefficient on the output-gap is low consistent with the estimate of Clarida et al. (1998) using US data over 1979-1994.

The second main finding of chapter 3 is that the trade-off between the standard deviation of the inflation and the output-gap is non-linear in the New Keynesian model with entry. In particular, we found that the trade-off between the volatility of inflation and the output-gap exists for an output-gap coefficient below 0.5 but disappears for an output gap coefficient of at least 0.5 or above. Therefore, the entry model implies that a coefficient on the output-gap higher than 0.5 does not help stabilise the volatility of the output-gap but, in fact, raises real uncertainty even further. It follows that the entry model is unable to produce high inflation risks with relatively high output-gap coefficient leading to large real term premium. The latter result is found to be robust irrespectively of the specification of the entry costs which can be either in terms of effective labour units (which is the baseline) or consumption units.

The limited trade-off between inflation and output gap volatility in the entry model is very interesting from policy point of view as it imposes a constraint on monetary policy in its ability of stabilising fluctuations in the output-gap.

The third main finding of chapter 3 is that fiscal policy in the form of income taxation also raises nominal term premium when entry cost is expressed in consumption units (instead of effective labour units). However, the increase in the nominal term premium is due to a rise in consumption (real) risks in contrast to chapter 2 where fiscal policy shocks are the source of inflation risks.

A fourth result of chapter 3 is that empirical mean of the nominal term premium is matched with a risk-aversion coefficient lower than in previous literature. However, the risk-aversion coefficient we maintain

is still considered to be well above the empirical estimates of about 5-10 (see, e.g. Vissing-Jorgensen and Attanasio (2003)) versus the value of 75 used in chapter two and three.

A shortcoming of the model used in chapter 3 is that it produces counterfactually low volatility of equity. Therefore, future research should address the ways of magnifying the variability of equity returns. In particular, the inclusion of physical capital in the entry model with adjustment costs might be a promising avenue to explain further stylised facts like the excess volatility of equity.

References

[1] Adjemian Stéphane , Houtan Bastani, Michel Juillard, Frédéric Karamé, Ferhat Mihoubi, George Perendia, Johannes Pfeifer, Marco Ratto and Sébastien Villemot (2011), "Dynare: Reference Manual, Version 4," Dynare Working Papers. CEPREMAP.

[2] Altig, Christiano, Lawrence, Eichenbaum, Martin and Jesper Linde (2012), "Firm-Specific Capital, Nominal Rigidities and the Business Cycle," Review of Economic Dynamics. 14:225-247.

[3] Andreasen Martin M. (2012), "An Estimated DSGE Model: Explaining Variation in Nominal Term Premia, and Inflation Risk Premia," European Economic Review. 56:1656-1674.

[4] Ang, Andrew, Bekaert Gert and Min Wei (2008), "The Term Structure of Real Rates and Expected Inflation, " The Journal of Finance. 63:797-849.

[5] Ascari, Guido, Colciago, Andrea and Lorenza Rossi (2011), "Limited Asset Market Participation, Sticky Wages and Monetary Policy," mimeo.

[6] Barillas Francisco, Hansen Lars Peter and Thomas J. Sargent (2009), "Doubts or Variability?" Journal of Economic Theory. 144(6):2388-2418.

[7] Barth James R., George Iden, Frank S. Russek, and Mark Wohar (1991), "The Effects of Federal Budget Deficits on Interest Rates and the Composition of Domestic Output," In The Great Fiscal

Experiment, edited by Rudolph G. Penner. Washington: Urban Institute Press. pp. 71-141.

[8] Beaubrun-Driant Kevin E. and Fabien Tripier (2005), "Asset Returns and Business Cycles in Models with Investment Adjustment Costs," Economics Letters. 86:141-146.

[9] Benigno, Pierpaolo (2006), "Comment on "Equilibrium Yield Curves by Monika Piazzesi and Martin Schneider,"" NBER Macroeconomics Annual.

[10] Bernard A. B., S. J. Redding and P. K. Schott (2010), "Multi-product Firms and Product Switching," American Economic Review. 93:1268–90.

[11] Bilbiie Florin O., Fabio Ghironi and Marc J. Melitz (2007), "Monetary Policy and Business Cycles with Endogenous Entry and Product Variety," NBER Macroeconomics Annual.

[12] Bilbiie Florin O., Fabio Ghironi and Marc J. Melitz (2012), "Endogenous Entry, Product Variety, and Business Cycles," Journal of Political Economy. 120(2):304-345.

[13] Bilbiie, Florin (2008), "Limited Asset Market Participation, Monetary Policy and (Inverted) Aggregate Demand Logic," Journal of Economic Theory. 140(1):162-196.

[14] Bilbiie, Florin, Tomasso Monacelli and Roberto Perotti (2012a), "Public Debt Redistribution with Borrowing Constraints." The Economic Journal. 123 (566):F64-F98.

[15] Bilbiie, Florin, Tomasso Monacelli and Roberto Perotti (2012b), "Tax Cuts vs. Spending Increases: Welfare at the Zero Lower Bound," mimeo.

[16] Bilbiie, Florin and Roland Straub (2004), "Asset Market Participation and Distortionary Taxation: Keynesian and Non-

Keynesian Effects of Fiscal Policy," Hungarian National Bank Working Papers.

[17] Bils, Mark and Peter J. Klenow (2004), "Some Evidence on the Importance of Sticky Prices," Journal of Political Economy. 112:947-985.

[18] Braun, R. Anton, Lena Mareen Körber and Yuichiro Waki (2012), "Some Unpleasant Properties of Log-Linearized Solutions When the Nominal Rate is Zero," February 15, unpublished manuscript, Federal Reserve Bank of Atlanta.

[19] Buraschi, Andrea and Alexei Jiltsov (2007), "Habit Formation and Macroeconomic Models of the Term Structure of Interest Rates," The Journal of Finance. 62(6):3009-3063.

[20] Calvo, Guillermo (1983), "Staggered Prices in a Utility-Maximizing Framework," Journal of Monetary Economics 12:383-398.

[21] Campbell John H. and John H. Cochrane (1999), "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behaviour," Journal of Political Economy. 107:205-251.

[22] Canzoneri, Matthew B., Robert E. Cumby, and Behzad Diba (2002), "Should the European Central Bank and the Federal Reserve be Concerned About Fiscal Policy?," in Rethinking Stabilization Policy. Proceedings of the Symposium sponsored of the Federal Reserve Bank of Kansas City, Jackson Hole. pp. 333-389

[23] Carlstrom, Charles, Timothy Fuerst, and Matthias Paustian (2012), "How Inflationary is an Extended Period of Low Interest Rates?," Federal Reserve Bank of Cleveland Working Paper.

[24] Chari V.V., Kehoe, Patrick Patrick J. and Elen McGrattan (2009), "New Keynesian Models: Not Yet Useful for Policy Analysis", American Economic Journal: Macroeconomics. 1:242-266.

[25] Chrisitano, Lawrence (2010), "Comment on Eggertsson, "What Fiscal Policy Is Effective at Zero Interest Rates?,"" NBER Macroeconomics Annual.

[26] Christiano, Lawrence, Martin Eichenbaum and Charles Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," Journal of Political Economy. 113(1):1-45.

[27] Christiano, Lawrence, Martin Eichenbaum and Sergio Rebelo (2011), "When is The Government Spending Multiplier Large?," Journal of Political Economy. 119:78-121.

[28] Christiano, Lawrence and Martin Eichenbaum (2012), "Notes on Linear Approximations, Equilibrium Multiplicity and E-learnability in the Analysis of the Zero Lower Bound," mimeo. March.

[29] Christiano, Lawrence, Trabandt Mathias and Karl Walentin (2010), "DSGE Models for Monetary Policy Analysis," The Handbook of Monetary Economics edited by Benjamin Friedman and Michael Woodford.

[30] Clarida, Richard, Gali Jordi and Mark Gertler (1998), "Monetary Policy Rules in Practice: Some International Evidence." European Economic Review 42:1033-1067.

[31] Clarida, Richard, Gali Jordi and Mark Gertler (1999), "The Science of Monetary Policy," Journal of Economic Literature. 37:1661-1707.

[32] Clarida, Richard, Gali Jordi and Mark Gertler (2000), "Monetary Policy Rules and Macroeconomic Stability,". The Quarterly Journal of Economics. 115:147-180.

[33] Cochrane John H. (2008), "Financial Markets and the Real Economy," Handbook of the Equity Risk Premium. Chapter 7.

[34] Coenen, Günter, Christopher J. Erceg, Charles Freedman, Davide Furceri, Michael Kumhof, René Lalonde, Douglas Laxton, Jesper Lindé, Annabelle Mourougane, Dirk Muir, Susanna Mursula, Carlos de Resende, John Roberts, Werner Roeger, Stephen Snudden, Mathias Trabandt, and Jan in't Veld (2012), "Effects of Fiscal Stimulus in Structural Models," American Economic Journal: Macroeconomics. 4(1):22-68.

[35] Cogan John F., Tobias Cwik, John B. Taylor and Volker Wieland (2010), "New Keynesian versus Old Keynesian Government Spending Multipliers," Journal of Economic Dynamics and Control. 34(3):281-295.

[36] Colciago, Andrea and Federico Etro (2010a), "Real Business Cycles with Cournot Competition and Endogenous Entry," Journal of Macroeconomics. 32:1101-1117.

[37] Colciago, Andrea and Federico Etro (2010b), "Endogenous Market Structures and the Business Cycle," The Economic Journal. 120:1201-1233.

[38] Colciago, Andrea (2011), "Rule-of-Thumb Consumers Meet Sticky Wages," Journal of Money Credit and Banking. 43(2–3):325-353.

[39] Constantinides, George (1990), "Habit Formation: A Resolution of the Equity Premium Puzzle," Journal of Political Economy. 98(3):519-543.

[40] Corsetti, Giancarlo, Meier Andre and Gernot Müller (2012), "Fiscal Stimulus with Spending Reversals," Review of Economics and Statistics. 94(4):878-895

[41] D'Amico, Stefania, Don H. Kim and Min Wei (2010), "Tips from TIPS: the Informational Content of Treasury Inflation-Protected

Security Prices," Board of Governors of the Federal Reserve System. Finance and Economics Discussion Series Working Paper.

[42] Denes, Matthew and G. Eggertsson (2009), "A Bayesian Approach To Estimating Tax and Spending Multipliers," New York Fed Staff Paper.

[43] De Paoli Bianca, Scott Alasdair and Olaf Weeken (2010), "Asset Pricing Implications of a New Keynesian Model," Journal of Economic Dynamics and Control. 34:2056-2073.

[44] Diebold, Francis X. and Canlin Li (2006), "Forecasting the Term Structure of Government Bond Yields," Journal of Econometrics. 130:337-364.

[45] Diebold, Francis X., Glenn D. Rudebusch and S. Boragan Aruoba (2006), "The Macroeconomy and the Yield Curve: a Dynamic Latent Factor Approach," Journal of Econometrics. 131:309-338.

[46] Donaldson, John and Rajnish Mehra (2008), "Risk-Based Explanations of the Equity Premium," Handbook of the Equity Premium. Chapter 2:37-99.

[47] Drautzburg Thorsten and Harald Uhlig (2011), "Fiscal Stimulus and Distortionary Taxation," NBER Working Papers 17111. National Bureau of Economic Research.

[48] Durham, Benson (2006), "The Estimate of the Inflation Risk Premium Using a Three-Factor Affine Term Structure Model," Board of Governors of the Federal Reserve System. Finance and Economics Discussion Series. 42.

[49] Eggertsson, Gauti (2011), "What Fiscal Policy is Effective at Zero Interest Rates?," NBER Macroeconomic Annual.

[50] Eggertsson, Gauti (2010), "The Paradox of Toil," New York Fed Staff Report no. 433. March.

[51] Eggertsson, Gauti and Michael Woodford (2003), "The Zero Bound on Interest Rates and Optimal Monetary Policy". Brookings Papers on Economic Activity.

[52] Elmendorf, Douglas W. (1993), "Actual Budget Deficit Expectations and Interest Rates," Harvard Institute of Economic Research Working Papers 1639.

[53] Engen, Eric M. and Glenn R. Hubbard (2004), "Federal Government Debt and Interest Rates," edited by Mark Gertler and Kenneth Rogoff. NBER Macroeconomics Annual 2004. Volume 19. MIT Press

[54] Erceg, Christopher J., Henderson, Dale W. and Andrew T. Levin (2000), "Optimal monetary policy with staggered wage and price contracts," Journal of Monetary Economics, 46(2):281-313.

[55] Fuhrer, Jeffrey C. (2000), "Habit Formation in Consumption and Its Implications for Monetary-Policy Models." American Economic Review. 90(3):367-390.

[56] Furlanetto, Francesco (2011), "Fiscal Stimulus and the Role of Wage Rigidity," Journal of Economic Dynamics and Control. 35:512–527.

[57] Furlanetto, Francesco and Martin Seneca (2009), "Fiscal Shocks and Real Rigidities," The B.E. Journal of Macroeconomics. 9(1):1-31.

[58] Gali, Jordi, Lopez-Salidó David and Javier Vallés (2007), "Understanding the Effect of Government Spending on Consumption," Journal of European Economic Association. 5(1):227-270.

[59] Gürkaynak, Refet, Sack Brian and Eric Swanson (2005), "The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models." American Economic Review. 95(1):425-436.

[60] Gürkaynak Refet, Sack Brian and Jonathan H. Wright (2007), "The U.S. Treasury yield curve: 1961 to the present," Journal of Monetary Economics. 54:2291-2304.

[61] Gürkaynak, Refet S., Sack Brian and Jonathan H. Wright (2008), "The TIPS yield curve and inflation compensation," Finance and Economics Discussion Series. Board of Governors of the Federal Reserve System.

[62] Hall, Robert (1988), "Intertemporal Substitution in Consumption," Journal of Political Economy. 96:339-57.

[63] Hall, Robert (2009), "By How Much Does GDP Rise If the Government Buys More Output?," Brookings Papers on Economic Activity.

[64] Hördahl, Peter (2008), "The Inflation-Risk Premium in the Term Structure of Interest Rates," BIS Quarterly Review. September.

[65] Hördahl, Peter, Tristani, Oreste and David Vestin (2008), "The Yield Curve and Macroeconomic Dynamics," The Economic Journal. 118:1937-1970.

[66] Jermann, Urban (1998), "Asset Pricing in a Production Economy," Journal of Monetary Economics. 41:257-275.

[67] Kim, Don and Jonathan H. Wright (2005), "An Arbitrage-Free Three-Factor Term Structure Model and the Recent Behaviour of Long-Term Yields and Distant-Horizon Forward Rates." Board of Governors of the Federal Reserve System. Finance and Economics Discussion Series. 33.

[68] Kimball, Miles (1995), "The Quantitative Analytics of the Basic Neomonetarist Model," Journal of Money, Credit and Banking 27(4):1241-1277.

[69] Laubach, Thomas (2009), "New Evidence on the Interest Rate Effects of Budget Deficits and Debt," Journal of the European Economic Association. 7(4):858-885.

[70] Leeper, Eric (1991), "Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies," Journal of Monetary Economics. 27:129-147.

[71] Linnemann, Ludger (2006), "Interest rate policy, debt and indeterminacy with distortionary taxation," Journal of Economic Dynamics and Control. 30:487-510.

[72] Nakamura, Emi and Ion Steinnsson (2008), "Five Facts About Prices: A Reevaluation of Menu Cost Models," Quarterly Journal of Economics. 123(4):1415-1464.

[73] Natvik, Gisle (2012), "Government Spending Shocks and Rule-of-Thumb Consumers with Steady-State Inequality," Scandinavian Journal of Economics. 114(4):1414–1436.

[74] Malloy, Christopher, Moskowitz Tobias and Anette Vissing-Jorgensen (2009), "Long-Run Stockholder Consumption Risk and Asset Returns," Journal of Finance. 64(6):2427-2479.

[75] Mertens, Karel and Morten Ravn (2012), "Empirical Evidence on the Aggregate Effects of Anticipated and Unanticipated U.S. Tax Policy Shocks." American Economic Journal: Economic Policy 4(2):145-81.

[76] Monacelli, T. and R. Perotti (2008), "Fiscal Policy, Wealth Effects, and Markups," NBER WP 14584, December.

[77] Orszag, Peter R. and William G. Gale (2004), "Budget Deficits, National Saving, and Interest Rates," Brookings Papers on Economic Activity.

[78] Parker, Jonathan (1999), "The Reaction of Household Consumption to Predictable Changes in Social Security Taxes, " American Economic Review. 89(4):959-973.

[79] Piazzesi, Monika and Martin Schneider (2006), "Equilibrium Yield Curves," NBER Macroeconomic Annual.

[80] Pistaferri, Luigi (2003), "Anticipated and Unanticipated Wage Changes, Wage Risk, and Intertemporal Labour Supply," Journal of Labour Economics. 21:729-754.

[81] Ravn, Morten, Stephanie Schmitt-Grohe and Martin Uribe (2006), "Deep habits," Review of Economic Studies. 73:195-218.

[82] Romer, David and Christina Romer (2010), "The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks," American Economic Review. 100:763-801.

[83] Rossi, Raffaele (2012), "Designing Monetary and Fiscal Policy Rules in a New Keynesian Model with Rule-of-Thumb Consumers," Macroeconomic Dynamics. 18:395-417.

[84] Rotemberg, Julio (1982), "Monopolistic Price Adjustment and Aggregate Output," Review of Economic Studies. 49:517-31.

[85] Rotemberg, Julio and Michael Woodford (1999), "The Cyclical Behaviour of Prices and Costs," In: Handbook of Macroeconomics edited by J. B. Taylor and and M. Woodford 1B:1051-1135.

[86] Rudebusch, Glenn (2002), "Term-Structure Evidence on Interest-Rate Smoothing and Monetary Policy Inertia," Journal of Monetary Economics. 49:1161-1187.

[87] Rudebusch, Glenn (2010), "Macro-Finance Models of Interest Rates and the Economy," Federal Reserve Bank of San Francisco Working Paper.

[88] Rudebusch, Glenn and Eric Swanson (2008), "Examining the Bond Premium Puzzle with a DSGE Model," Journal of Monetary Economics, 55:S111-S116.

[89] Rudebusch, Glenn and Eric Swanson (2012), "The Bond Premium in a DSGE Model With Long-Run Real and Nominal Risks," American Economics Journal of Macroeconomics. 4(1):105-143.

[90] Siemsen, Thomas and Sebastian Watzka (2013), "Fiscal Multipliers and the Choice of Zero Lower Bound Modeling," CESIFO Working Paper No. 4342.

[91] Smets, Frank and Rafael Wouters (2007), "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," The American Economic Review. 97(3):586-606.

[92] Swanson, Eric (2012), "Risk-Aversion and the Labour Margin in Dynamic Equilibrium Models," American Economic Review. 102(4):1663-1691.

[93] Swanson, Eric and John Williams (2012), "Measuring the Effects of the Zero Lower Bound on Medium- and Longer-Term Interest Rates," American Economic Review. Forthcoming.

[94] Van Binsbergen, Jules, Jesus Fernández-Villaverde, Ralph S. Koijen, and Juan F. Rubio-Ramírez (2012) "The Term Structure of Interest Rates in a DSGE Model with Recursive Preferences," Journal of Monetary Economics 59(7):634-648.

[95] Vissing-Jorgensen, Anette (2002), "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution," Journal of Political Economy. 110(4):825-853.

[96] Vissing-Jorgensen, Anette and Orazio P. Attanasio (2003), "Stock-Market Participation, Intertemporal Substitution and Risk-Aversion," American Economic Review Papers and Proceedings. 93(2):383-391.

[97] Woodford, Michael (2003), "Interest and Prices: Foundations of a Theory of Monetary Policy." Princeton University Press.

[98] Woodford, Michael (2011), "Simple Analytics of the Government Expenditure Multiplier," American Economic Journal: Macroeconomics. 3(1):1–35.