Spatial localisation of closeness and betweenness measures: a self-contradictory but useful form of network analysis

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Closeness and betweenness are forms of spatial network analysis grounded in a long standing tradition of measuring accessibility and flow potential. More recently, these measures have been enhanced by the concept of spatial localisation, producing effective models for the prediction of pedestrian and vehicle driver behaviour.

A contradiction arises where the distance metric used to define locality does not match the distance metric used to define shortest paths for closeness and betweenness. A typical case is the use of angular shortest paths within a Euclidean buffer as a pedestrian flow model. Such a model assumes that people make a mode choice based on distance, but a route choice based on least angular change – even when this results in an excessively long "problem route", which conflicts with their criterion for mode choice.

This study examines the prevalence of problem routes and the magnitude of their effect on some pedestrian and vehicle models. We show that while in a weighted analysis, pathological cases could invalidate an entire model, in the models presented the effect of this contradiction is minor. We do this by comparing model predictions to real flow data, using four strategies for handling problem routes: ignore, discard, reroute and strict locality. Strict locality is justified on grounds of bounded rationality. We find all strategies to give broadly similar results, although the reroute and strict strategies give marginally better simulation accuracy. We also present a discussion of the characteristics of each strategy, and findings on computational efficiency.

We conclude that it is prudent in any computation of localised closeness and betweenness to consider the impact of problem routes; however they do not necessarily invalidate these forms of analysis, which remain useful.

Keywords: spatial network analysis; closeness; betweenness; localisation; shortest angular paths; sDNA

1. Introduction

Closeness and betweenness are measures of network accessibility and flow potential based on the telecommunications work of Shimbel (1953) and Freeman (1977). Each measures a form of centrality based on shortest paths through the network; with closeness measuring potential for "to-movement" while betweenness measuring "through-movement". Modern implementations of these algorithms on spatial networks use a variety of distance metrics to define the shortest paths. Metrics may, for example, be Euclidean (minimizing the number of metres travelled along the network), angular (minimizing the cumulative angle turned along each route), topological (minimizing the number of nodes) or indeed based on collected data such as average journey times on individual links.

In a global computation of closeness and betweenness, such shortest path metrics are used to determine routes between all points on the network. If spatial locality of measures is required however – to specify a scale of analysis, maximum trip length, or size of catchment for example – shortest paths serve a second purpose of defining the locality of any point on the network. Such a locality, the set of all points 'within 500 metres', 'within ten minutes', and so on, constitutes a buffer of a specified *radius*.

The subtlety of spatial localization in these algorithms is that the locality of each point may be based on a different distance metric to that used to calculate shortest paths for closeness and betweenness. For example, in constructing a pedestrian movement model, we may decide that pedestrians will walk to any destinations within 1km of their origin by the simplest route possible. Hence the agent's *mode choice* (whether to walk, drive, not travel at all, etc) would be based on a Euclidean shortest path; but the "simplest route possible" is often not the shortest route, and in some cases will exceed

1km in length. In cases where this happens, we term the route a “problem route” as it leads to a contradiction in the analysis: why would a pedestrian, having already decided not to walk more than 1km, then choose a route which is longer than this? The question is rhetorical – while in this particular example we can envision reasons why this might occur in practice, such reasons will not apply to all analyses all of the time.

To state the problem more generally, then: if we compute closeness or betweenness for a subset of network defined in one manner, but with shortest paths defined in another, then the choice of shortest paths may violate the initial definition of the network subset. It is not unreasonable to treat this type of contradictory analysis as an accepted technique in GIS, and it is proper to investigate its intrinsic contradiction in at least one “use case” (to borrow a term from software engineering). We do this to see how the problems arising can be measured and mitigated, to defend the validity of the technique in at least one context and finally to pave the way for a similar defence in other domains of application. The “use case” in this paper relates to four models of pedestrians and vehicles in urban networks, in which human behaviour is approximated by angular shortest paths within a Euclidean network buffer.

The contribution of the paper is thus to demonstrate the existence of a contradiction in localized forms of closeness and betweenness, and to investigate its implications. We begin by investigating how often problem routes occur. In order to evaluate the impact of problem routes, it is necessary to define and compare different strategies to handle them, and compare the outcomes with the results arising from a baseline strategy of ignoring the problem. Therefore we discuss and experiment with techniques for handling problem routes, comparing our simulation results with real flows of pedestrians and vehicles, to see whether there is any difference in the predictive power of simulations using each technique. This is all, of course, within the context of our angular route-Euclidean buffer-transportation modelling use case; however, the techniques used both to analyse and mitigate problem routes can be generalized to other mismatched distance metrics in other domains of application.

The remainder of the paper is structured as follows. Section 2 explains the background, including a brief survey of current uses of spatially localized closeness and betweenness, a discussion of why we would want to use this contradictory analysis in the first place, and the implications of that contradiction. Section 3 presents four alternative methods for handling the contradiction in practice, and section 4 gives the results of empirical testing of these methods, both in terms of real world accuracy and computational efficiency. Section 5 concludes.

2. Background

Closeness (Shimbel 1953) and betweenness (Freeman 1977) are based on a long history of spatial network analysis which can be traced back to concepts appearing in sociology (Bavelas 1948), geography (Haggett and Chorley 1969) and transportation (Garrison and Marble 1962; Ford and Fulkerson 1962; Kansky 1963). The spatial localization of closeness and betweenness is a more recent development, and several studies have used it to predict movement of pedestrian and vehicles in cities. Turner (2007) reports high correlation (R=0.91) of betweenness with real vehicle flows in the Barnsbury district of London. Hillier and Iida (2005) and Chiaradia et al (2014) report correlations of 0.64 to 0.91 in a more extensive series of vehicle and pedestrian models tested in the four districts of the current study: Barnsbury, Clerkenwell, South Kensington and Brompton. These micro foundations have supported macro epidemiological work such as Sarkar et al (2013) in which angular betweenness localized by a Euclidean buffer – again used as a predictor of pedestrian movement - was shown to correlate significantly to self-perceived health, hospital anxiety and depression in a longitudinal population study, after controlling for other factors. The same team went on to use these and related measures on an unprecedented scale to provide morphometric analysis of the UK BioBank medical dataset (Sarkar, Gallacher, and Webster 2014). Finally, further work by the current author awaits publication, including models of vehicle flow on inner city and city region scale (R=0.89 and 0.90 with measured flows respectively), city cyclist flows (R=0.78) and pedestrian flows (R=0.81). The GIS software used to create such models is publicly available (Cooper, Chiaradia, and Webster 2011; MIT 2011). Note that the "use case" in this paper (angular geodesics in a Euclidean buffer) is the same approach used by Turner (2007), Chiaradia et al (2014) and Sarkar et al (2013; 2014).

The means by which human beings choose paths is a long standing topic in spatial network analysis (Montello 2005). The idea of the shortest angular path – that is to say, selecting a route between an origin and destination which minimizes angular change – is widespread, either as a sole criterion (Golledge 1995; Hillier and Iida 2005; Turner 2007; Cooper, Fone, and Chiaradia 2014) or in conjunction with others (Dalton 2003). There are two main reasons for this. Firstly, the intersection of human perception with incomplete information makes fewest-turn routes a good strategy. We tend to simplify models of both the environment (Montello 1991; Klippel 2003) and our routes through it (Wiener et al. 2008) in order to handle the challenge (Kim 2001; Butler et al. 1993; Dogu and Erkip 2000) of navigating complex environments. Routes with fewer turns are usually easier to remember, placing less cognitive burden on the navigator, and are thus preferred by agents unfamiliar with a given area. (A related literature focuses on the selection of simplest rather than shortest paths, which usually implies minimizing the number of instructions needed to describe a path and hence the number of turns (Richter 2009; Mark 1986; Duckham and Kulik 2003; Haque, Kulik, and Klippel 2007)). Secondly, we suggest that in the case of vehicle traffic, routes with fewer turns tend to be faster; such routes tend to be designed as through roads with higher priority. Thus, shortest angular routes tend to represent real world route selection to some extent, and make an especially good proxy if data on road priority, congestion and so on are unavailable.

Although angular analysis of networks is popular, however, it does not of itself dictate a method for scale selection. A scale – whether a maximum trip length, a catchment size, the scale of administrative planning, etc – is required for most spatial network analysis, and that scale is rarely defined in angular units. To restrict analysis to the appropriate scale we define network buffers of a certain *radius,* as it is usually easiest to measure and interpret radius in terms of everyday (Euclidean) distance as measured in metres, miles or kilometres along the network. This is the case with the sDNA software used in this study (Cooper, Chiaradia, and Webster 2011) although other forms of measurement are used as well, for example 'steps' between axial lines (Hillier and Iida 2005), although the latter are not so easily interpreted.

An analysis that uses angular geodesics within a Euclidean buffer might proceed, therefore, as follows:

1. For each link L1 in the network
    a. Find all other links within a **<u>1000 metre radius</u>** as measured within the network (this defining a buffer)
    b. For each link L2 discovered in (a)
        i. Calculate shortest **<u>angular path</u>** SP_A from L2 to L1 (the whole network is available for the path to use)

ii. Incorporate summary measure of SP_A into accessibility statistics for L1 (thus computing some variant of closeness)

iii. Incorporate summary measure of SP_A into each link on SP_A (thus computing some variant of betweenness)

This is the form of analysis which, as explained in the introduction, is contradictory. In the current case, the buffer and shortest paths have real physical analogs: respectively, mode choice (walking, driving etc) and route choice. If a pedestrian is only willing to walk 1km, why would they proceed to choose a path that is longer than this? Conversely if the pedestrian truly navigates by choice of angular shortest paths, why would they consider travel to a destination which can only be reached (within comfortable walking distance) by taking a shortest Euclidean path instead? While there may be reasons for such behaviour in this instance, these are rhetorical questions which serve to illustrate the contradiction in a technique more generally applicable than pedestrian route finding.

The most logical resolution would be to insist that we use the same method for network buffering as geodesic computation: in the current context of transportation modelling, to insist that the same criteria are used for both mode choice and route choice. Intuitively it doesn't make sense that people always prefer angular shortest paths; indeed it is likely that using a suitable hybrid of angular and Euclidean distance metrics would make for a better model all round, and also remove the contradiction. But, in defence of current practice, a number of points should be raised: (i) that Euclidean radii are simpler to use and triangulate with real world experience and other research (such as government data on journey lengths); (ii) that Euclidean radii produce results which are far easier to comprehend and communicate; (iii) that it is often desirable to compare different rules for geodesic computation, in which case we may want to use a common means of defining a network buffer to produce a neutral comparison; (iv) that pure angular or pure Euclidean models are easier to explain and understand than hybrids of the two; and (v) that combining angular geodesic computation with a Euclidean radius has provided some of the best direct demand models of pedestrian and vehicle flow to date, where no information is available to the modeller other than the shape of the network itself (Hillier and Iida 2005; Chiaradia, Wedderburn, and Cooper 2014). In sum, mixing Euclidean buffers with angular geodesics, while contradictory, is useful, so we had better find a way to deal with the contradiction.

Note that while - by definition - all angular shortest paths which differ from their Euclidean counterparts will be longer, most of these do not concern us, as they will still be shorter than the specified radius for the analysis. Thus, we define a “problem route” to be an angular shortest path computed within a Euclidean network buffer, which exceeds the radius of that buffer. These are highly prevalent in continuous space analyses computed by the sDNA software, as this type of analysis includes partial links right up to the edge of the network buffer, rather than discarding links which cross it (Chiaradia, Cooper, and Webster 2012). Any angular geodesic which (1) touches the buffer edge, and (2) differs from its Euclidean counterpart will therefore become a problem route.

The remainder of this paper examines the prevalence of problem routes, on pedestrian and short vehicle trip scales, in the Clerkenwell district of London. Four different methods for handling problem routes in spatial network analysis are proposed, and each method is tested by computing a pedestrian and vehicle flow model which is tested for accuracy against real-world data from three other districts (Barnsbury, South Kensington and Brompton) in addition to Clerkenwell.

3. Methodology

In order evaluate the magnitude of impact of problem routes in a transportation model, it is necessary to define and compare different strategies to handle them, and compare the outcomes with the results arising from a baseline strategy of ignoring the problem. Having established the magnitude of the problem, these strategies will also be useful in its mitigation. This section introduces four methods for handling problem routes (all with a publicly available implementation in the sDNA software), and explores some of their direct consequences.

The methods are:

(1) Ignore the problem. Analyse the geodesics as planned, even though they may not reflect practical route choices.

(2) Discard the route. Drop the geodesics from the analysis. If we consider whole links only, this is similar to computing the network buffer by measuring Euclidean distance along angular geodesics - in practice it is simpler to compute a Euclidean buffer and then discard the angular geodesics which do not fit. (If we wish to consider partial links, a different algorithm is required).

(3) Reroute them. Force the problem routes to take a Euclidean shortest path rather than an angular one, thus guaranteeing they will not exceed the buffer radius.

(4) Use only the portion of network within the radius for computation of geodesics; that is to say, choose the shortest angular path but only using network links which fall inside the buffer. We term this "strict network cutting" (in contrast to

relaxed network cutting, which allows use of network outside of the buffer even though we are only analysing the network inside it).

Figure 1 shows a real world example of a network radius, an angular problem route and its Euclidean and strict angular counterparts. These geodesics are the outcomes of methods 1, 3 and 4 respectively (method 2 produces no geodesic at all).

In the context of pedestrian and vehicle modelling, it is hard to see a behavioural justification for ignoring a route which is far too long (method 1), except to say that beyond a certain tolerance of excess length, such routes are unlikely to occur. However, it should be noted that if studying closeness as a function of buffer radius, methods 2-4 can introduce discontinuities in that function as progressively more preferable routes are 'revealed' by a growing buffer, or a growing threshold for discarding or rerouting. When ignoring routes, on the other hand, closeness remains a continuous function of buffer radius, so long as partial links are only partially counted as per Chiaradia et al. (2012).

For a behavioural justification of methods 2 and 3, we must make a distinction between mode choice (whether and how to travel) and route choice (which route to take). Method 2 is then easily justified - we maintain that people maintain a consistent criterion for mode choice by discarding excessively long routes. Method 3 is be justified by allowing inconsistent criteria for route choice in certain situations. Where the usual method of route selection results in an excessively long route, we assume that pedestrians or drivers change their route choice to match their mode choice, and use the shortest Euclidean route instead. Hence method 3 can be contrasted with method 2: method 2 is a revision of mode choice while method 3 is a revision of route choice.

Method 4 is perhaps the most interesting. By refusing to consider routes that stray (more than a certain distance) beyond the radius, we can consider it a strategy for navigating complex environments by wilfully discarding information. The heuristic used is to assume that places (more than a certain distance) further away than the desired destination will not be useful in navigation. It is thus a model of bounded rationality. This is consistent with the literature that shows information to be discarded when navigating complex environments (Wiener et al. 2008; Kim 2001; Montello 1991; Klippel 2003). Conveniently, the same process of discarding information helps the computer complete the analysis more quickly as well.

From a mathematical perspective, method 4 also ‘feels’ nicer, for two reasons. First, all geodesics are treated in exactly the same way, rather than introducing a bifurcation in behaviour based on the length of each geodesic. Second, the method preserves strict locality of analysis: changing network configuration outside of the radius of a particular origin cannot possibly affect geodesics from that origin. The routes thus produced are still angular shortest paths, albeit on a restricted portion of network – which is nice if we believe that angular geodesics are a good proxy for human behaviour. Thus this method may be desirable for some types of theoretical network analysis.

Finally, note that method 4 does not guarantee the removal of all problem routes; there will still be routes present which stay entirely within a network buffer but still have a length which exceeds the buffer radius. The ‘strict’ route of Figure 1 is one example of this. Fortunately these routes are likely to be considerably shorter than those derived from a non-strict network cutting (notwithstanding some pathological cases unlikely to occur in reality); still, it is possible to combine method 4 with either of 1, 2 or 3 in order to ‘mop up’ the problem routes which remain.

Methods 2 and 3 would perhaps be behaviourally unrealistic if applied to all routes which strictly exceed the radius, so we define a *threshold of interest*: a ratio by which the problem route must exceed the radius before any action is taken. That such a threshold is defined by ratio, rather than fixed distance, implies that agents plan a route in its entirety regardless of scale, and have limited cognitive capacity for perceiving differences between routes, which is not enhanced for longer routes. These assumptions are both already implicit in angular network analysis (Montello 1991; Klippel 2003; Wiener et al. 2008). For the purposes of this study the ratio is fixed at 120%. This ratio is intended to correct only the "long tail" of the distribution shown in Figure 3: it captures the longest 60% of problem routes, the estimated density of which is under 20% of the modal peak. Hence for routes which exceed the radius by less than the specified ratio, we apply method 1 – ignore. This parameter could be modified according to the characteristics of the study area, and experimentation with different thresholds of interest would be a possible avenue for future work.

4. Results

4.1 Baseline occurrence of problem routes

Figure 2a shows the distribution of problem routes over origins in Clerkenwell, for network radii of varying size. At a 300m radius, problem routes are infrequent – likely a reflection of routes on the same scale as a city block, where the shortest angular and Euclidean paths will match. As radius increases, the incidence of problem routes appears to converge to a truncated, unimodal, positively skewed distribution over origins, with an average of approximately 20% of the routes from each origin being problematic. Figure 2a, however, overemphasizes the problem, as many of these routes exceed their radius by only a small amount. Figure 2b shows the distribution over origins of average excess length per trip; for all radii this appears to tail off as a truncated exponential with the most frequent value being small. Still, around 20% of origins have an average route length that exceeds 105% of the radius (note furthermore that this figure includes routes to destinations much closer than the buffer edge). The phenomenon is slightly less pronounced for larger radii, which proportionately allow more scope for deviation from the Euclidean shortest path to obtain a better angular route.

Note that while these figures apply to the average trip length within each buffer, in some types of weighted analysis the likelihood of pathological cases increases enormously. Weighted analysis can be used to consider access to specific facilities, for example shops and bus stops (Sarkar et al. 2013), green space (Gong et al. 2014) or alcohol outlets (Fone et al. 2012). Although these analyses usually conducted by Euclidean planar or network buffering, an obvious extension to this approach is to consider the actual route people will take, rather than the shortest path. This can be approximated with a weighted angular closeness measure, or spatial gravity model based on angular shortest paths; betweenness has also been extended with weighted measures (Karimi 2012; Karimi et al. 2013; Chiaradia, Cooper, and Webster 2012; MIT 2011; Space Syntax 2014). In cases such as these, it is possible for a specific facility location to fall on a particularly problematic location: in the tail of the distribution shown in Figure 2b, or on the endpoints of the route shown in Figure 1. This would assign a much higher weight to problem routes than in the current study. If measuring accessibility by Euclidean length of angular geodesics (e.g. Chiaradia, Cooper, and Webster 2012), these will be artificially long and accessibility will be underrepresented; if on the other hand mean angular distance is used as a measure of closeness (e.g. ibid; Turner 2007) then accessibility will be overrepresented due to the existence of low-angularity routes which will not be used in practice. If measuring betweenness, then flows to or from the facility will be overrepresented on the problem route itself and underrepresented on the route genuinely used. In either case, weighted analysis is much more prone to invalidation by problem routes.

4.2 Effect of mitigation measures

Figure 3 shows the effect of introducing strict network cutting on a 600m radius: as stated in the introduction, the worst outlying angular problem routes are redistributed to lengths either less than or not far exceeding the radius. Figure 4 shows a similar effect for other radii, compared to the baseline distributions in figure 2. Interestingly, after applying strict routing it is the larger radii that exhibit more problem routes – a reversal of the baseline trend. However all radii exhibit less excess length overall, so strict network cutting has improved things all round, albeit much more so for smaller radii. It makes sense that larger buffers, containing more links, allow more scope for deviation without straying from the network buffer.

The effects of strategies 2 and 3 are not graphed as they are easily described with reference to figure 3. Strategy 2 (discard problem routes) simply removes all data points above the threshold of interest, while strategy 3 (reroute to Euclidean shortest path) moves the same data points to the region just below the network radius of 600m.

4.3 Pedestrian and vehicle flow models

Betweenness-based models of pedestrian and vehicle flow were constructed using the above strategies, and their differences compared with one another. Figure 5 shows the spatial distribution of differences between the predictions of each model; remember that these models are the outcome of combining all geodesics less than a certain length between origin/destination pairs in the analysis. Compared to the baseline strategy of ignoring problem routes, strategy 4 (strict routing) predicts less traffic on long straight through-roads, which are presumably not permitted paths for a large number of geodesics as they do not fall within the relevant network radius for that geodesic. Strategy 3 (reroute) appears to have the opposite effect and emphasizes such links in the cases where they also form good Euclidean paths – seemingly favouring long straight roads at the expense of shorter straight roads. Strategy 2 discards either kind of geodesic, resulting in less traffic in the model overall.

4.4 Comparison with measured pedestrian and vehicle flows

Betweenness from models constructed using the four strategies was tested for its ability to predict real flows of vehicles and pedestrians. This was conducted over four separate data sets collected in central London: Barnsbury, Clerkenwell, South Kensington and Brompton (the same data set used in Hillier and Iida 2005; Turner 2007; Chiaradia et al 2014). Each data set consists of about 50 measured flows (exact counts are given in Table 1), and is placed in a spatial model based on the Ordnance Survey Integrated Transport Network, which buffers the entire study area by over 2km in each direction. As with any betweenness calculation, a key question is how to choose the appropriate radius. Here we sidestep that question by analysing a wide range of radii and graphing the model's correlation to actual measured flows for each radius.

Table 1 shows the best correlations between predicted and measured flows of pedestrians and vehicles for each strategy and each model; Figure 6 graphs the same correlations for varying radii. Looking at the best performing radius for each model, strategy 3 (reroute) shows the best correlation in all four vehicle models, and the best mean improvement in AIC for pedestrian models. The differences amount to a few percent in correlation, and are more dramatic in the South Kensington model where baseline correlations are low. Strategy 4 (strict network cutting) takes second place in all vehicle models, and equal first/second place in three of four pedestrian models. Strategy 2 (discard) does not perform so well, and on average underperforms strategy 1 (ignore) for pedestrian models. Strategies 2 and 1 usually exhibit the worst correlation with real flows, though as shown in Figure 6 they do perform equally well (or better) at lower radii. The greater variation in optimal strategy for pedestrian models could be a reflection of a less accurate spatial model (the vehicle network is used for the map in either case).

As to the most effective radius size, it is hard to discern clear trends. Strategy 3 (reroute) causes an upwards bias in the most effective radius in seven of nine cases where a clear peak appears in correlation. Strategy 1 (ignore) causes a downward bias in six of nine cases, while strategies 2 and 4 (discard and strict) fall in between. Overall the differences are not great, but the greatest downward bias appearing in strategy 1 is consistent with the picture that strategy 1 fails to correctly handle unrealistically long routes where they occur; the model thus attempts to compensate for this by using a shorter maximum trip length.

An question of note is whether loss of information from the models has a consistent effect on their performance. Both strategy 2 (discard) and 4 (strict) lose information, either by discarding geodesics, or discarding parts of the network before computing a geodesic. However as shown in the discussion above, strategy 2 tends to perform similarly or worse than ignoring the problem, while strategy 4 consistently performs better. Thus it is not the loss of information, but the choice of which information is discarded, that determines model performance.

4.5 Computational efficiency

sDNA is used in large scale analyses such as Sarkar et al (2014), and ongoing work deals with networks on a national scale. The computational burden of large scale network analysis therefore formed part of the motivation for this study, in particular a concern for the inefficiency of the "strict" network cutting strategy which must recompute shortest paths for each scale individually. This section provides a brief discussion of CPU time requirements for each strategy.

The network analysis algorithm can be divided into four phases of significant computational expense:

(1) Computation of the network radius and Euclidean backlinks (from which shortest paths are constructed) surrounding each origin. This consists of running Dijkstra's (1959) algorithm for each origin; thus the time complexity of this phase is , where *N* is the number of links in the network, and is the maximum number of links in the largest network buffer necessary for the analysis. (This is the largest radius suitably increased to allow for the longer permitted geodesics, i.e. , as defined in section 2. This analysis also assumes the amortized number of nodes V in a subset of a spatial network is in practice is proportional to the number of links in that subset).

(2) Computation of the analytical path distance metrics and backlinks surrounding each origin. For a strict network cutting this must be performed once per origin per radius hence has time complexity , where r is the number of radii and is the largest number of links in the average radius (not increased to allow for longer geodesics). For a relaxed network cutting, a single set of distance metrics and backlinks is common to all radii, although as in (1) this must be computed for the largest network buffer, hence has complexity .

(3) Tracing of the backlinks to compute betweenness and other geodesic-based measures. This is done once per origin, radius and geodesic, so has complexity , where *L* is the largest number of links in a geodesic. This is in

practice the most costly operation affecting the scalability of closeness and betweenness algorithms.

(4) Incrementing of per-link betweenness accumulators. This has the same complexity as (3).

All of the above phases are slow in part due to nonlinear memory access which will not fully exploit the caching capabilities of modern CPUs. Even though graph edges may be linked directly through a pointer-based structure, the pattern of access prevents predictive prefetching of memory contents and thus stalls pipelines (e.g. Intel Corporation 2012, 2.23–2.25, although hyper threading mitigates this to some extent). Phase 4 includes an added cost of managing concurrency, as multiple threads must increment shared accumulators for betweenness. Table 2 shows benchmarking results for a continuous space angular analysis of Clerkenwell (12431 links). The key findings are as follows:

- As expected, the Discard strategy always takes less time than either Ignore or Reroute, as fewer geodesics are processed in phases 3 and 4.
- Reroute takes less time than Ignore, despite the additional work associated with rerouting. This could be because rerouted geodesics are always shorter and hence on average traverse fewer links, resulting in reduced cost in phase 4 (the L term is reduced). This interpretation is consistent with the very high relative cost of the Ignore strategy at low radii, for which shortest angular paths may exceed the radius by a greater proportion.
- The Strict strategy, for small radii and small numbers of radii, is the most efficient, despite the additional work necessary in phase 2. The cost of recomputing distance metrics and backlinks for each network buffer is

outweighed by the fact that the largest buffer is smaller, we need not compute backlinks beyond its limits, and the average shortest path length is also smaller when confined to a strict network buffer.

- For larger numbers of large radii, the Strict strategy is no longer as efficient.
- If the global radius (whole system) is analysed as part of the model, the time taken to do this dominates the causes the times for Ignore, Discard and Reroute strategies to converge towards the time for Ignore. This is because at global radius there are no problem routes, so the geodesics traced for the global measures match those that would be chosen by the Ignore strategy.

It is possible to envision a more efficient variant on stages 3 and 4 of the algorithm above, whereby all geodesics from an origin are traversed recursively in a treelike manner, giving a theoretical time complexity of . As it appears from Table 2 that this is the bottleneck in computation, an improvement in runtime may result for large analyses. This would come, however, at the expense of less flexibility in algorithm design, undesirable for software which is still rapidly evolving to meet research requirements.

Overall, there is no great variation in the order of magnitude of runtimes between any of the strategies, except for Ignore (at small radii) and possibly Strict (which may be exhibited when testing multiple radii larger than those shown here). In scaling up to country sized networks, it is likely that the trends in this section will be greatly exacerbated; however the key finding is that the number of large radii computed is as influential as well as the size of the radii on choice of the most efficient strategy.

5. Conclusions

This study has explored the contradictions that arise from computing localized forms of closeness and betweenness in which the distance metric used to define a locality differs from the distance metric used to shortest paths through it. This has been done through the lens of what we have termed ‘problem routes’, where angular shortest paths exceed the length of the Euclidean network radius they serve. Such routes are shown to be proportionately greater in number for large radii, but greater in excess length for smaller radii. Four strategies for handling these routes were explored. The differences in accuracy of model predictions were minor, but significant, showing that problem routes do have a significant effect in the models studied.

In practical terms, the *reroute* and *strict* strategies gave the best correlation with observed pedestrian and vehicle flows, though it should be noted that this is not the only consideration in choosing a strategy; different models may have different requirements. In terms of computation time, the *strict* strategy is the most efficient for small radii and small numbers of radii; for exceptionally large analyses the *discard* strategy is the cheapest, closely followed by *reroute*. If mathematical elegance is of importance, the *strict* strategy treats all geodesics in the same way, and preserves a strict locality of analysis, while the *ignore* strategy preserves continuity in closeness as a function of radius.

In the case investigated here, other differences such as the choice of an appropriate radius, and accuracy in the spatial model, are likely to have a far greater influence on the correctness of predictions than the differences between problem route handling strategies. In a weighted model of accessibility to specific facilities, on the other hand, there is potential for pathological cases to have an overriding influence, resulting in an over-estimated, unvalidated accessibility index (and miscomputed flows) to be used in transportation planning.

They key message of this paper therefore, is that using different distance metrics to specify spatial localization and actual routes analysed, can lead to contradicted closeness and betweenness measures. It is essential to handle such contradicted routes in order to provide accurate localizations of these measures, and the four approaches demonstrated in this paper provide feasible and effective ways to mitigate the problem. But so long as due consideration is given, the contradiction we have discussed should not cause undue concern to practitioners; it is a useful contradiction to maintain.

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Table 1. Best correlations (r^2) with measured flow data per strategy, relative Akaike information criterion for each strategy, and number of data points per model.

	n	r^2 for best performing radius				AIC relative to Strategy 1, "ignore"		
		Strategy 1 Ignore	Strategy 2 Discard	Strategy 3 Reroute	Strategy 4 Strict	Strategy 2 Discard	Strategy 3 Reroute	Strategy 4 Strict
Pedestrian								
Barnsbury	109	0.48	0.46	0.48	0.46	7.8	2.7	7.8
Brompton	87	0.60	0.60	0.60	0.60	0.0	0.0	0.0
Clerkenwell	56	0.65	0.64	0.63	0.65	1.5	3.9	-0.2
South Kensington	69	0.27	0.29	0.36	0.32	-4.1	-18.5	-11.1
mean	***80***	***0.50***	***0.50***	***0.52***	***0.51***	***1.3***	***-3.0***	***-0.9***
Vehicle								
Barnsbury	82	0.63	0.65	0.67	0.66	-8.0	-18.5	-12.1
Brompton	62	0.58	0.58	0.62	0.60	-0.8	-14.6	-7.2
Clerkenwell	43	0.83	0.83	0.85	0.85	-0.4	-13.1	-11.0
South Kensington	48	0.25	0.26	0.37	0.28	-1.6	-16.8	-3.8
mean	***59***	***0.57***	***0.58***	***0.63***	***0.60***	***-2.7***	***-15.8***	***-8.5***

Table 2. Benchmarking data for the four problem route strategies. Computer is Intel Core i3-2120 @3.30GHz, 32 GB ram, 2 cores/4 hyper threads.

Radii	**600m**	**2000m**	**600, 1200, 1500, 2000, 2500m**	**600, 1200, 1500, 2000, 2500m and global**
Ignore Time (s)	374.6	398.0	703.1	962.7
Discard Time (s)	93.0	168.9	502.1	953.0
Reroute Time (s)	94.2	174.2	519.7	978.5
Strict Time (s)	92.7	152.1	526.5	1047.7

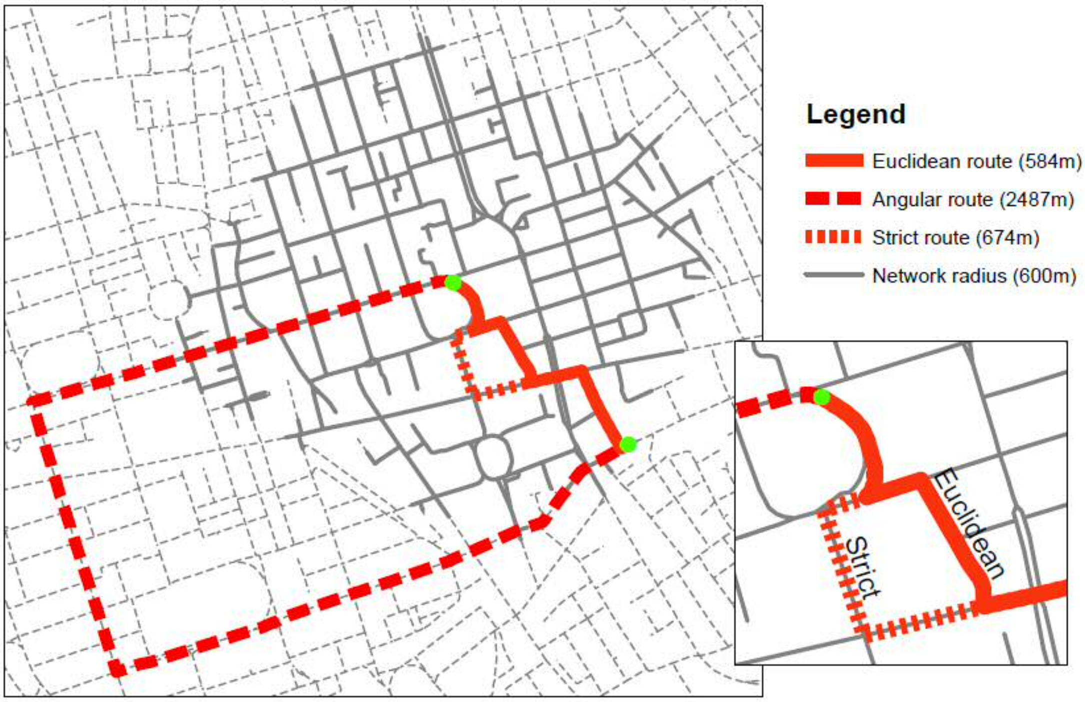

Figure 1. Illustration of a Euclidean network buffer, and Angular, Euclidean and Strict Angular shortest paths between an origin and destination. Inset shows detail of where the Strict and Euclidean paths diverge, to avoid two 'backward' turnings. Note continuous space network buffer which includes partial links falling on the radial boundary. OS ITN mapping ©Crown Copyright/database right 2013. An Ordnance Survey/EDINA supplied service.

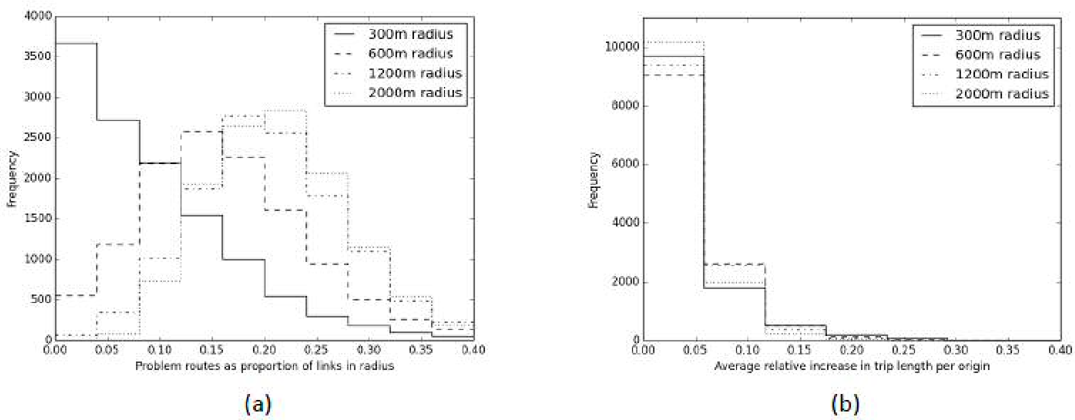

Figure 2. Distribution of problem routes over all origins in the Clerkenwell district of London (a) by number of problem routes as a proportion of links in radius, (b) by average increase (relative to radius) in trip length per origin.

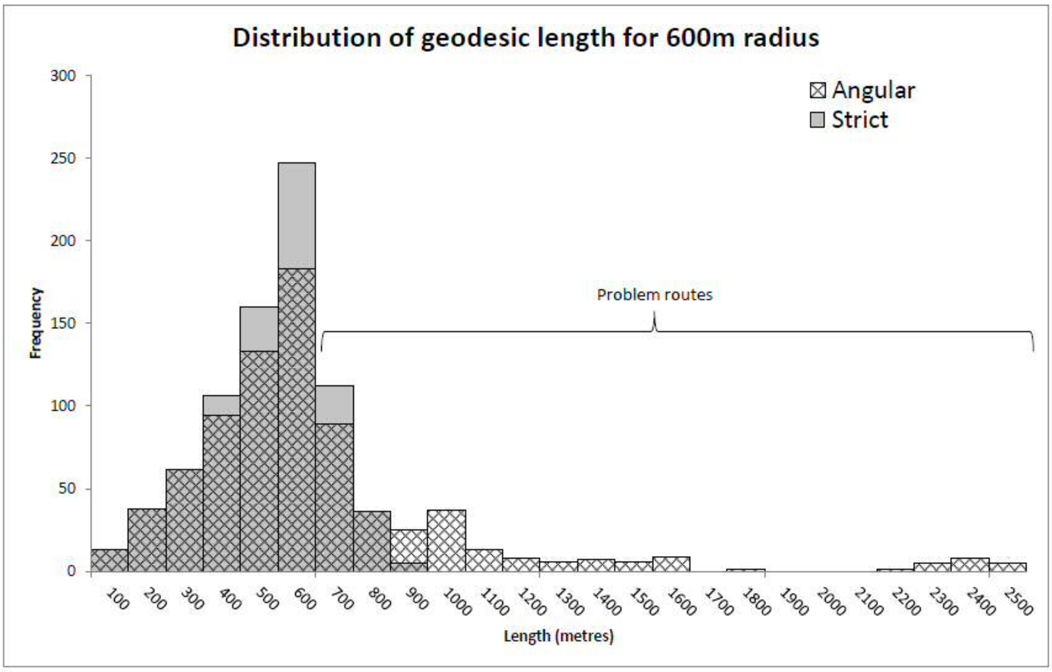

Figure 3. Distribution of geodesic length over geodesics for a 600m Euclidean network buffer surrounding three random origins in Clerkenwell; showing the effect of introducing strict network cutting on geodesic distribution.

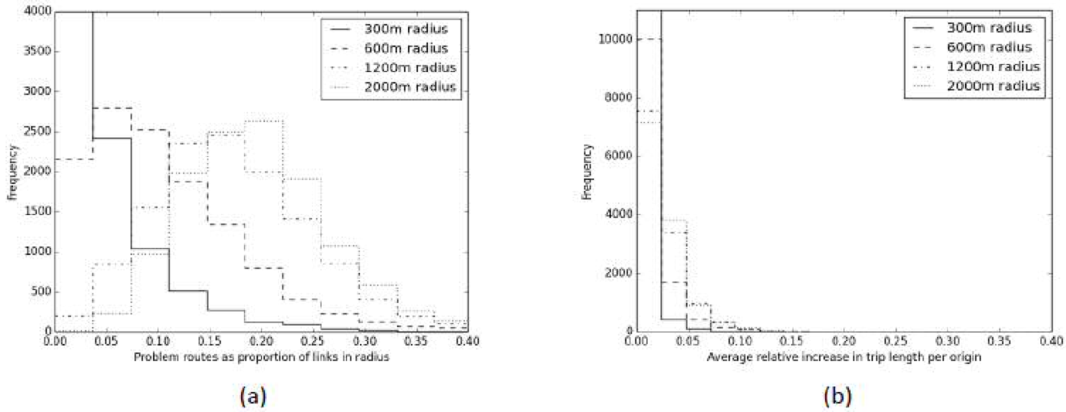

Figure 4. Distribution of problem routes over origins in the Clerkenwell district of London, after introducing strict network cutting, (a) by number of problem routes as a proportion of links in radius, (b) by average increase (relative to radius) in trip length per origin.

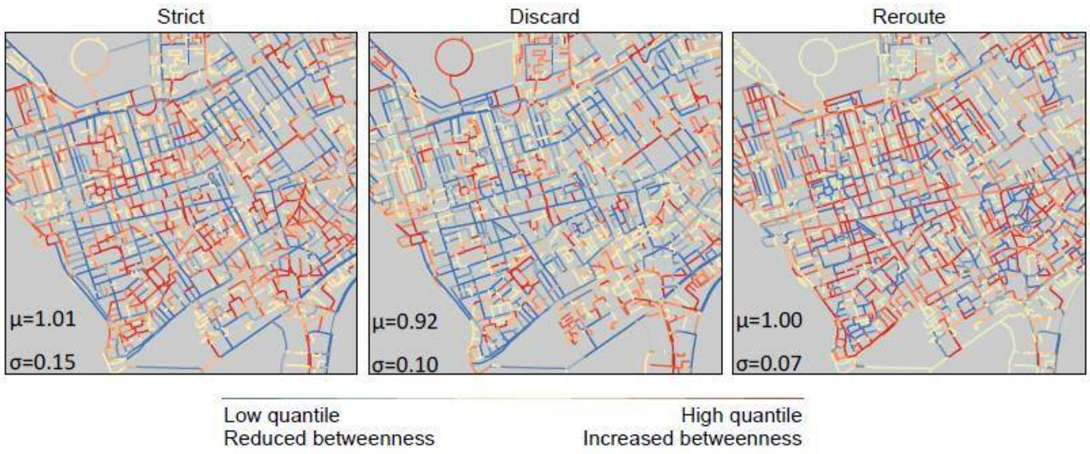

Figure 5. Plot of differences in betweenness (as used to predict pedestrian and vehicle flows), as a ratio, arising from use of discard, reroute or strict strategies compared to the baseline ignore strategy. Annotations show mean change in link betweenness (as a ratio) and standard deviation of that change for each model overall. OS ITN mapping ©Crown Copyright/database right 2013. An Ordnance Survey/EDINA supplied service.

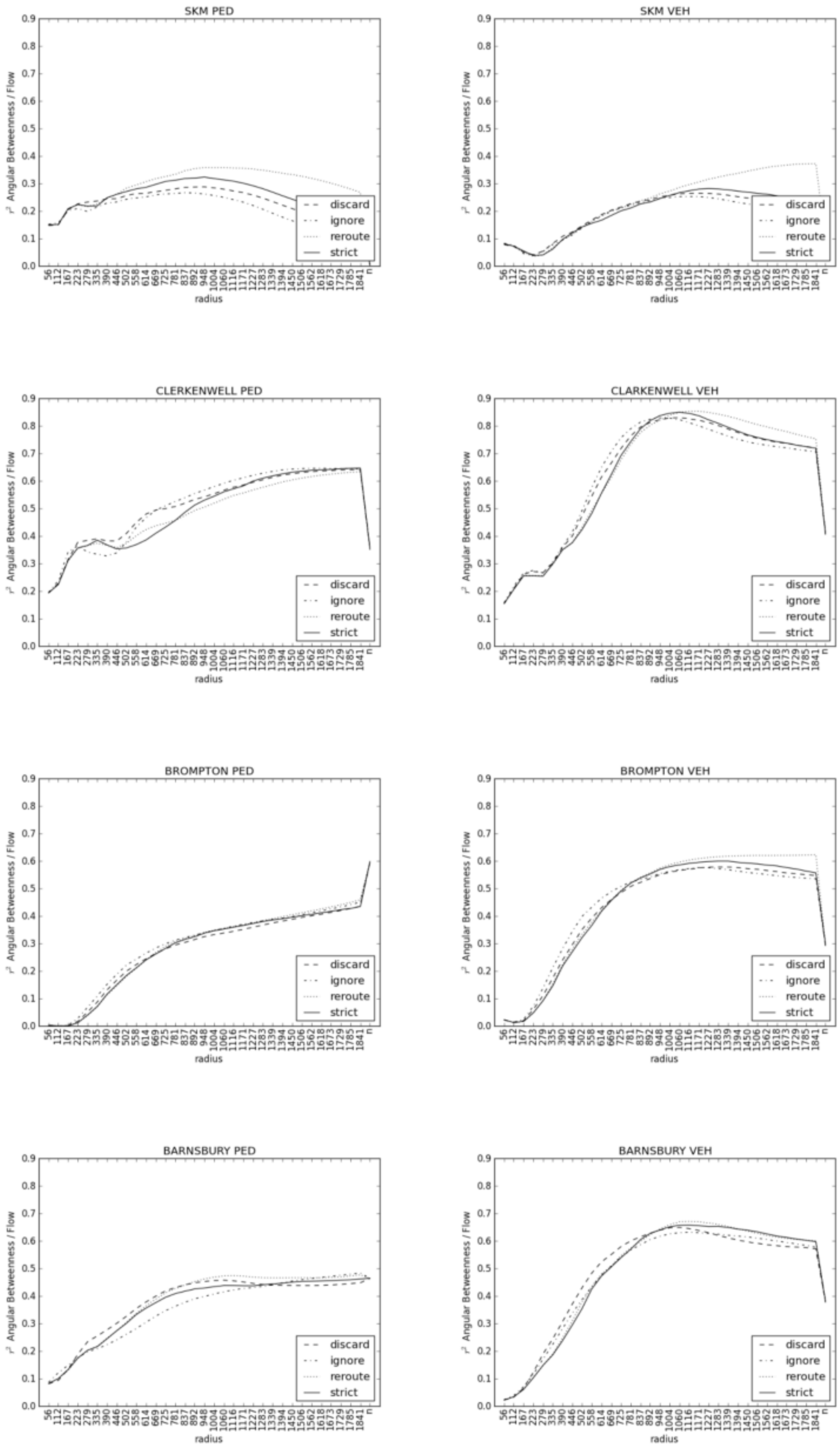

Figure 6. Effect of problem route handling strategy on accuracy of pedestrian and vehicle flow predictions, shown for four areas of London at multiple radii. VEH = vehicle, PED = pedestrian, SKM = South Kensington Museum district.