Rogue seasonality in supply chains-An investigation and a measurement approach

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Abstract

Purpose – Shukla et al (2012) proposed a signature and index to detect and measure rogue seasonality in supply chains, but which however, were not effectively validated. The authors have sought to investigate rogue seasonality using control theory and realistic multi echelon systems and rigorously validate these measures, so as to enable their application in practice. The paper aims to discuss these issues.

Design/methodology/approach – Frequency domain analysis of single echelon and simulated four echelon Beer game system outputs are used in the investigation, with the simulation incorporating realistic features such as non-linearities from backlogs and batching, hybrid make to order-make to stock ordering system and the shipment variable. Lead time, demand process parameters, ordering parameters and batch size are varied in the simulation to rigorously assess the validity of the index.

Findings –The signature based on the cluster profiles of variables, specifically whether the variables cluster together with or away from exogenous demand, was validated. However, a threshold for the proportion of variables that could be clustered with exogenous demand and the system still being classified as exhibiting rogue seasonality, would require to be specified. The index, which is derived by quantifying the cluster profile relationships, was found to be a valid and robust indicator of the intensity of rogue seasonality, and which did not need any adjustments of the kind discussed for the signature. The greater effectiveness of the frequency domain in comparison to time for deriving the signature and index was demonstrated.

Practical implications – This work enables speedy assessment of rogue seasonality in supply chains which in turn ensures appropriate and timely action to minimize its adverse consequences.

Originality/value – Detailed and specific investigation on rogue seasonality using control theory and Beer game simulation and rigorous validation of the signature and index using these methods.

Key words Control systems, Seasonality, Supply chain management, Simulation, Beer game

Paper type Research paper

1. Introduction

Supply chains are prone to disturbances not only from external sources but internal ones as well, such as from the use of inappropriate control systems and/or information to match supply with demand. These disturbances typically present themselves as the Bullwhip effect, where order variability increases from downstream to upstream echelons (Lee et al., 1997; Sterman, 1989), and/or as rogue seasonality, where system variables such as order and inventory show cyclicality in their profiles that is not present in exogenous demand (Forrester, 1961; McCullen and Towill, 2002). While extensive research has been done on the Bullwhip effect (Miragliotta, 2006; Geary et al., 2006), research on rogue seasonality has been minimal with only three studies identified in the literature on this subject. This lack of interest is despite the fact that rogue seasonality is commonly observed in practice (Kaipia et al., 2006; Thornhill and Naim, 2006; Torres and Moran, 2006) and significantly affects performance (Metters, 1997). The study of rogue seasonality is therefore important and forms the focus of this work.

One way to manage rogue seasonality could be through the "sense and respond" approach (Haeckel, 1999), which involves use of information to sense the context being studied, and then if required, initiating an appropriate corrective action/s. Appropriate information could include time series of orders, inventory, work in process and other system variables; sensing could mean assessing the presence and intensity of rogue seasonality; and corrective action could be to reduce intensity given that a higher rogue seasonality intensity is associated with a greater system inefficiency (Metters, 1997). Such an approach is discussed in Shukla et al (2012) with the authors also proposing a signature and index based on the cluster profiles of variables to sense rogue seasonality. However, the study by Shukla et al (2012) had significant research gaps such as:

1) A lack of analytical justification for the signature and index with sole reliance on simulated and empirical data.

2) Non rigorous testing of the signature and index. Only linear systems, and of the simplistic make to order (MTO) and make to stock (MTS) kind were simulated and assessed.

The validity of the signature and index is therefore insufficiently established and requires further investigation, which this study proposes to do. First, control theory, which is an established approach for studying system behaviour (Ortega and Lin, 2004; Sarimveisa et al., 2008) is used to investigate the dynamics associated with rogue seasonality and to justify the rationale of the signature and index. Rogue seasonality has not been rigorously studied using control theory methods in the past and this is one of the novelties of this work. Next, the dynamics of rogue seasonality is investigated using the Beer game system simulation (Sterman, 1989; O'Donnell et al., 2006), which incorporates real world characteristics such as batching in ordering and shipping, backlogs, hybrid MTO-MTS for ordering and shipment dynamics.

While frequency response analysis is used to understand the rogue seasonality characteristics, cluster profiles and index values are derived to validate the signature and index. To the best of our knowledge, such a comprehensive investigation of rogue seasonality has not been attempted in previous studies. This paper, therefore, not only improves our understanding of rogue seasonality generation, a subject of limited research interest in the past, it also establishes the validity of the signature and index through rigorous testing with realistic contexts.

The rest of the paper is structured as follows: Previous studies on rogue seasonality together with definitions of the signature and index are covered in the next section. Control theory analysis of a single echelon system from a rogue seasonality perspective is discussed in Section 3, while in Section 4 generation of rogue seasonality in the Beer game system, its characteristics and the effectiveness of the signature and index to detect/measure it are covered. We conclude in Section 5.

2. Relevant studies on rogue seasonality

Rogue seasonality is characterised by a cyclic pattern in order and other supply chain variables, which is generated endogenously from the inventory and production control system used, i.e. that pattern is not present in exogenous demand (Forrester, 1961; McCullen and Towill, 2002; Kim and Springer, 2008). The cyclic pattern could be of the order of months (Forrester, 1961;Thornhill and Naim, 2006), or for sectors with faster operating dynamics such as fast moving consumer goods and high technology, of the order of days and weeks (Fok et al., 2007; Neale and Willems, 2009). Rogue seasonality can be observed in numerous examples in the literature (Kaipia et al. ,2006; Torres and Moran, 2006; Thornhill and Naim, 2006). Cyclic variations from rogue seasonality could be misinterpreted as being of exogenous origin and be unnecessarily managed through either production ramp-up and ramp-down and/or increase in stock levels causing an increase in operating costs. This adverse impact on costs for a single echelon system is estimated to be around 10-20% (Metters, 1997), with that for realistic multiple echelon systems being significantly higher, given the propensity of the cyclicality to be transmitted to other echelons.

Surprisingly, rogue seasonality has received only a little academic interest. Few studies have exclusively focussed on rogue seasonality and most have considered it together with the Bullwhip effect (Forrester, 1961; Miragliotta, 2006; McCullen and Towill, 2002). A few studies, although also on the Bullwhip effect, have indirectly investigated rogue seasonality or endogenous amplification at certain frequency channels, in view of their analyses being in the frequency domain (Dejonckheere et al., 2003; Jaksic and Rusjan, 2008). However, the focus of these Bullwhip effect in the frequency domain (BEFD) studies is on the order variable, as its amplification alone defines the Bullwhip effect. Also, their analysis covers the entire frequency range so that output in the frequency domain could be equivalent, and hence comparable to that in the time (reference) domain. Rogue seasonality on the other hand is characterized by many system variables showing cyclicality in their profiles, and which therefore, requires the analysis of multiple variables (not just the order variable). The analysis also requires to be focused on select (amplification range) frequencies rather than the entire frequency range given that specific cycles are seen to be dominant in rogue seasonality presentations. Such a rogue seasonality focused analysis is not evident in previous BEFD studies.

Exclusive focus on rogue seasonality is seen in only three previous studies. Kim and Springer (2008) used an analytical system dynamics approach with a dyadic structure to determine the conditions under which rogue seasonality could be generated in a supply chain. Though useful, the practical utility of such top down approaches is limited. This is because appropriate policies developed under simplistic dyadic and other assumptions may not be so for most real world supply chains, which have multiple information and material flows, dynamic uncertainties and differing member objectives/constraints (Lawrie, 2003; Baader and Montanus, 2008). Also, real world decision making is characterized by behavioural biases and irrationality (Loch and Wu, 2007; Bendoly and Cotteleer, 2008), while optimal/appropriate policies are generally established under rational settings. Top down approaches therefore need to be complemented with the bottoms-up sense/detect and respond based approach (Haeckel, 1999; Craighead et al., 2007) which involves use of system information to detect anomalies (problematic supply chains from a rogue seasonality perspective in this case), and then initiating an appropriate corrective action. Such an approach was discussed by Thornhill and Naim (2006), who were able to discriminate exogenous from endogenous (rogue) cyclicality for a steel supply network (using monthly time series data of system variables and spectra principal component analysis technique). However, the intensity of rogue seasonality, which could indicate the extent of its negative impact on supply chain performance and is therefore more important to know, was not discussed (rogue seasonality is present in most supply chains as per Kim and Springer (2008) and therefore knowledge of its presence is not critical). Shukla et al (2012) partially addressed this shortcoming by proposing a signature to detect the presence and an index to indicate the intensity of rogue seasonality. While the signature was defined on the basis of (cluster) profiles of variables associated with the presence/absence of rogue seasonality, the index definition was based on the numerical values of the profiles and profile relationships (*exact definitions of the signature and index are discussed in the next section*). Though the relevance of signature and index for sensing rogue seasonality could be highlighted, Shukla et al (2012)'s study had some major weaknesses.

Firstly, the rogue seasonality signature was defined by visualizing the variable plots and cluster profiles and which therefore, is not effectively underpinned by theory. One way to fill this gap is through the use of control theory, which has previously been used to study the dynamics associated with the Bullwhip effect (Dejonckheere et al., 2003; Jaksic and Rusjan, 2008) but not specifically rogue seasonality, as discussed earlier. The second weakness is that the validation of the signature and index was based on the output from simplistic system simulations. Only linear systems were simulated, i.e. systems which did not have non-linearities from backlogs and batching (in ordering and shipping) which are important omissions (Riddalls and Bennett, 2001; Potter and Disney, 2006). Also, only extreme cases of MTO and MTS were considered in ordering, while hybrid MTO-MTS, which includes both actual customer demand and stock replenishment in the ordering decision and is more commonly observed in practice (Anderson et al., 2005) was not considered. Finally, the shipment variable was not included in the dynamics despite being routinely encountered in practice and known to affect the dynamics of variables (Shukla et al, 2009). These missing features need to be incorporated in the system for it to serve as an effective method of investigating rogue seasonality and for validating the related signature and index. One potential system which could be used is the Beer game system, which has previously been used to study the dynamic behaviour of production-distribution systems (Van Landeghem and Vanmaele, 2002; O'Donnell et al., 2006), but not specifically rogue seasonality.

3. Control theory based analysis

Control theory, with its roots in engineering systems provides a strong theoretical basis to simulation, and has extensively been used to study production-inventory dynamics (Ortega and Lin, 2004; Sarimveisa et al., 2008). Importantly, it facilitates analysis in the frequency domain (Disney and Towill, 2002), the preferred domain for data with cyclicality (Chatfield, 2004), and is therefore particularly suited for rogue seasonality investigations. Table 1 below gives the variables, parameters and abbreviations used.

Take in Table 1

3.1 System considered

Initially, a single echelon system is considered as it facilitates easier understanding of the dynamics besides being computationally tractable. However, in the next section the multi echelon Beer game system is discussed. The Automatic pipeline, inventory and order based production control system or APIOBPCS (John et al., 1994) is used as the ordering system, and this is because it mimics the heuristics used by humans to replenish inventory (Sterman, 1989) and has been used in several previous studies (Disney and Towill, 2003; Zhou et al., 2006; White and Censlive, 2013). The APIOBPCS ordering policy may be described as: "the order placed is equal to the average sales rate plus a fraction ($1/T_i$) of the inventory error plus a fraction ($1/T_w$) of the work-in-process (WIP) error", where T_i is termed the "time to adjust inventory" and T_w the "time to adjust WIP". The average sales rate is calculated using exponential smoothing, and is dependent on a parameter T_a related to the exponential smoothing parameter α. While T_p is the average delay/lead time between order placement and delivery/production output and is therefore a physical parameter, T_i , T_w and T_a are decision parameters and which are set according to performance criteria such as the minimisation of order variance, inventory availability and the speed of response to changes in demand. Another important factor is the order of delay, which reflects the distribution of delivery/production output around the average delay. In this study, order of delay infinity is used as it is more relevant in practice. It is characterized by delivery/production output exiting the system in the same sequence as order entry into the system earlier, and after the average lead time (or delay).

Three variants of the generic APIOBPCS control system are considered: make to order (MTO), make to stock (MTS) and hybrid MTO-MTS. MTO and MTS are considered because these are extreme contexts in terms of the intensity of rogue seasonality generated and are sometimes observed (Buxey, 1995) while hybrid MTO-MTS is considered as it is more commonly observed in practice than MTO or MTS (Anderson Jr et al., 2005). Actual customer demand and replenishment of stock and work in process (orders in pipeline) are all considered in the hybrid MTO-MTS ordering decision. The parameters used for MTO and MTS are based on Naim et al (2007) as: MTO**:** $T_a = 0$, $T_i = T_w = \infty$; MTS: $T_a = T_w = \infty$, $T_i = T_p$. For MTO-MTS, those proposed by John et al (1994) are used as: T_a=$2T_p$, T_i= T_p and T_w =$2T_p$. John et al.'s parameters are based on 'hard' engineering systems and optimize the dynamics from both customer service and demand amplification considerations. These parameters are therefore referred to as Optimal parameters in this study.

The variables considered in the model are consumption/sales rate (CONS), forecast of average consumption / sales rate (AVCONS), order rate (ORATE), work in process level (WIP), desired work in process level (DWIP), production completion rate or rate of goods receipts into inventory (COMRATE), actual inventory level (AINV), error between desired and actual inventory level (EINV) and error between desired and actual work in process level (EWIP). The block diagram representation and related equations for a single echelon APIOBPCS are given in figure 1.

Take in Figure 1

The next stage involves solving the equations for each system variable, which are usually determined in relation to the input (exogenous demand or CONS in this case) and referred to as the transfer function of that variable. Transfer functions are mathematical functions of decision and system parameters which could be analysed to assess the dynamic characteristics of variables. However, frequency domain analysis requires the transfer functions to be converted to that domain first.

3.2 Analytical analysis in the frequency domain

The transfer functions of the variables are converted to the frequency domain by substituting '$j\omega$' in place of 's' and determining their absolute value. The resulting functions called the frequency response functions (or FR) of those variables (same as Amplitude ratio or Amp R for the purpose of this study) give the amplitude of sinusoidal output to sinusoidal input at each frequency (ω) across the frequency range. In a linear system such as APIOBPCS, a sinusoidal input yields a sinusoidal output at the same frequency but a different amplitude (and phase) (Dejonckheere et al., 2003; Jaksic and Rusjan, 2008) and the relationship between the amplitudes is known through FR. This, together with the fact that every input time series consists of and can be broken down into its constituent sinusoids of different frequencies each with different amplitudes means that the FR function could provide information about the frequencies that would be amplified (corresponding to FR > 1), attenuated (corresponding to FR < 1) and for those for which there will be no change (corresponding to FR =1) by the system. This would be independent of the nature of exogenous demand. In other words, insights about endogenous generation of specific cyclicality (in variables) or rogue seasonality could be obtained by this approach, which was therefore explored.

FR functions are mathematical functions of frequency (ω) and the physical and decision parameters used, which for our context are T_p, T_a, T_i and T_w. The FR functions for different variables are not compared directly but instead, their values at critical points across the frequency range are computed so as to identify common characteristics, which could be related to the signature of rogue seasonality. The critical points include FR values at zero frequency (ω_0), very high frequency (ω_∞), maximum FR value and frequency corresponding to the maximum FR value ($\omega_{max\ FR}$) and the frequency corresponding to crossover from amplification to attenuation i.e. where FR value is equal to one ($\omega_{crossover}$). All of these were derived by applying basic calculus and algebra on the FR functions using the Matlab® symbolic toolbox.

The FR plots and values at critical points of variables in each of MTO, MTS and hybrid MTO-MTS systems are given in Table 2. It can be seen that the variables for each system are fewer here than in figure 1. This is because in Table 2 only the unique variables (in FR function terms) for each system are included. This means that for each system, the FR function of the excluded variables are equivalent to the FR function of at least one of the included variables.

Take in Table 2

An inspection of Table 2 reveals the following:

MTO: The maximum FR values of variables in this system are seen to be either one or T_p. A maximum value of one means, for that particular variable, the amplitude at any frequency will at most be equal to the amplitude of that frequency in exogenous demand. This means no amplification of any frequency and therefore, no rogue seasonality generation in the case of that variable. Other variables, which have a maximum FR value of T_p, have this value for almost the entire frequency range as seen in the plot for *EWIP/CONS*. Given that profiles are compared after amplitude scaling, this would mean maximum T_p gets reduced to maximum 1, so that the outcome from a rogue seasonality generation perspective is the same as that for the previous variable. Hence, with none of the system variables amplifying any frequency in CONS, this means that no rogue seasonality is generated by this system. An examination of the FR profiles of variables shows them to be either constant at one or at T_p (adjusted to one after amplitude scaling) for almost the entire frequency range. This means that the frequency characteristics of the variables would not be significantly different from those of exogenous demand in case of this system.

Hybrid MTO-MTS (optimal parameters): The FR profiles of all the variables initially increase reach a maxima and then decrease with frequency (ω). For three of the four unique variables, the FR values in the frequency range from ω_0 to $\omega_{crossover}$ are greater than one indicating that these frequencies would be amplified. In case of the fourth variable (*AINV/CONS*), the amplification range frequencies are immediately beyond ω_0 and till $\omega_{crossover}$. Examining the frequencies associated with maximum amplification ($\omega_{max\ FR}$) for the variables, these appear to be quite close to each other at $1.014/T_p$, $1.014/T_p$, $0.937/T_p$ and $0.983/T_p$. This implies that not only is rogue seasonality generated by this system, it is also characterized by all the system variables having cyclic profiles of a similar periodicity. This would also cause the profiles of these variables to be different from CONS or exogenous demand.

MTS: The FR profiles of variables are similar to those in the hybrid MTO-MTS system (optimal parameters) case with amplification in the ω_0 to $\omega_{crossover}$ frequency range and a similar frequency (corresponding to maximum amplification) for all variables. This implies that rogue seasonality is generated in this system as in the case of hybrid MTO-MTS, and with the same characteristic presentation. However, a key difference between the two cases is in the maximum FR values of variables.

The maximum FR values of variables for MTS at 2.307, 2.307 and 2.141 (after normalization), are significantly more than those for hybrid MTO-MTS (optimal parameters), which are 1.688, 1.688, 1.62 and 1.863 (the last two after normalization). This implies that the intensity of rogue seasonality generated in an MTS system is higher than that in a hybrid MTO-MTS (optimal parameters) system.

Overall, it is seen that generation/presence of rogue seasonality, such as in the case of the hybrid MTO-MTS (optimal parameters) and MTS systems, is associated with system variables having similar cyclic profiles, and which therefore, are significantly dissimilar from exogenous demand in profile terms. On the other hand in the case of rogue seasonality not being generated, such as in the MTO system, the variable profiles are not significantly different from exogenous demand. This is exactly the signature for rogue seasonality that was proposed by Shukla et al. (2012): rogue seasonality is considered present when system variables are clustered together and away from exogenous demand, and not if the variables are clustered together with the exogenous demand. The rogue seasonality signature has thus been validated in a generic sense.

However, an important issue which is still unresolved is how to discriminate two systems with rogue seasonality. For example, the signature will similarly indicate generation/presence of rogue seasonality in both hybrid MTO-MTS (optimal parameters) and MTS cases. What is therefore needed is an indicator of rogue seasonality intensity, and such an indicator called the index was proposed in Shukla et al. (2012).

$$\textit{Rogue Seasonality Index} = \left(\frac{\textit{Minimum dissimilarity between CONS and the other variables}}{\textit{Average dissimilarity between all variables except CONS}} \right)$$

The index is based on the logic of the signature, but with numerical values being used to represent the variable profiles and profile relationships. CONS in the definition refers to exogenous demand, and dissimilarity is measured in terms of the Euclidean distance between the variables in the time or frequency domain. In order to assess the effectiveness of the index, single echelons of each of MTO, hybrid MTO-MTS (optimal parameters) and MTS systems were simulated and the time series profiles of variables generated. Gaussian exogenous demand and lead time of one week ($T_p = 7$) were considered with one year of data (250 daily data points) being simulated. The variable profiles were normalized, transformed to the frequency domain (using Fourier transform or FT) and amplitudes of all frequencies (after FT) used to derive the index. The index values for MTO, hybrid MTO-MTS (optimal parameters) and MTS systems were computed to be 0, 2.46 and 7.04 respectively which accurately reflect the relative rogue seasonality intensities of these systems as per the FR analysis. The index value is 0 for the system in which no rogue seasonality is generated (MTO), while for systems with rogue seasonality, the index values are large and significantly greater than 0. Also, the index value for the MTS system, where relatively a higher intensity of rogue seasonality is generated, is seen to be greater than that for the hybrid MTO-MTS (optimal parameters) system.

The FR based approach used here has some similarities to the seminal work of Dejonckheere et al (2003) and therefore it is important to delineate the respective contributions. While Dejonckheere et al (2003) highlighted the effectiveness of the FR based approach in assessing the effectiveness of alternative replenishment policies (including APIOBPCS), their focus was on the Bullwhip effect and hence only on the order variable. Also, the FR profiles were analysed at a high level, with select parameter values and spanning the entire frequency range (to establish equivalence between the time and frequency domains). The FR based analysis in this study on the other hand, covers all system variables (not just the order variable) in view of their relevance in rogue seasonality presentation, is generic in terms of parameter values (within the MTO, MTS and hybrid MTO-MTS configurations), is detailed in terms of FR profile assessments and is focused only on the amplification range frequencies and not the entire frequency range. Therefore, even though a similar FR based approach as discussed in Dejonckheere et al (2003) is used here, the nature of investigation is different and is also more in-depth.

Overall, the frequency response approach strengthened the theoretical basis for the signature and index of rogue seasonality. In the next section, the Beer game is simulated and the simulation data used to investigate the rogue seasonality generation characteristics and to validate the signature and index.

4. Beer game simulation based analysis

The Beer game (Sterman, 1989) is a four echelon production-distribution supply chain consisting of Retailer (R), Wholesaler (W), Distributor (D) and Factory (F), where orders flow upstream from the Retailer through to the Factory and products are shipped downstream in response. There are delays in order transmission, shipping and production; and backlogs are generated in case of demand being greater than available stock, which require to be fulfilled subsequently. The game is triggered by exogenous demand at the Retailer, with the role of players at each echelon being to act as inventory managers and decide on the order (quantity) to be placed in each time period.

The Beer game was considered for the simulation as: a) it is a realistic multi echelon system, b) it allows inclusion of real world features such as backlogs, batching in ordering and shipping and the shipment variable, c) it has been used in previous studies to study the dynamics associated with the Bullwhip effect (O'Donnell et al., 2006; Hwarng and Xie, 2008), but not specifically rogue seasonality. The simulation here includes different exogenous demand processes, lead times, ordering policies and batch sizes as a part of rogue seasonality analysis, and is presented in three stages. First, the frequency response (FR) functions of variables derived from actual simulated data are compared with those derived theoretically for a corresponding linear four echelon system. This is done to verify the Beer game simulation, as well as to assess the impact of backlogs and batching on system behaviour and related impact, if any, on the logic of the signature and index. Next, the time series data of variables from select Beer game simulations are used to test the validity of the signature and index. Finally, the index is computed for different configurations of the Beer game system in the time and frequency domains, and its effectiveness discussed.

4.1 Simulation of the Beer game

Simulation of the Beer game involved translating its structure and decision making into difference equations which are given in Table 3 below.

Table 3

The initial demand is assumed to be the same as in Sterman (1989) at 4 units to enable effective comparison of results. However, while Sterman considered a step jump in demand from 4 to 8 units, we have assumed demand to be autoregressive (AR), as the same is observed for many products (Chopra and Meindl, 2004). The initial demand is considered in Equation 2.1a, while the first order autoregressive demand process at the Retailer is considered in Equation 2.1b. Equation 2.1c models the delay (lead time or LT_{order}) in order transmission between echelons, while the delays in shipping between echelons and in factory production are considered in Equations 2.2a and 2.2b. Inventory and backlogs are modeled separately in equations 2.6 and 2.7 respectively. Shipments are included in the model to take account of non-linearities associated with the backlog situation. Shipments are set to zero when there is no inventory available, and when inventory does become available, the echelon ships what is ordered plus any backlog that has been accumulated (equations 2.3, 2.4, 2.5). Equation 2.8 captures forecasting based on exponential smoothing while the work in process is calculated in equations 2.10a and 2.10b. The work in process computation for all except the factory echelon includes three terms: orders that have been placed but not transmitted to the upstream echelon, shipments made by the upstream echelon but not yet received and backlogs. In case of the factory, the orders placed are received after a fixed production delay; therefore, work in process only includes previous orders in the computation. The human decision maker in the physical version of the game is replaced by the APIOBPCS ordering process, which is captured in equation 2.14. Finally, batching constraints are applied on orders and shipments by rounding them to the nearest multiple of the batch size as given in equations 2.15 and 2.16. Overall, a significant increase in complexity in comparison to the single echelon analysis (refer to figure 1) is evident, which is on account of multiple echelons and inclusion of backlogs and shipping in the dynamics. One year data (250 daily data points) was simulated in Excel as described earlier. The rogue seasonality analysis including signature and index computations were undertaken using Matlab.

As per this formulation, the dynamics of variables in the Beer game system depends on the autoregressive demand parameter (ρ), delays or lead times in order transmission, shipping and factory production (LT order, shipping, factory production) APIOBPCS ordering parameters (T_i, T_a, T_w) and batch size (b*).*These were varied as follows:

- Autoregressive parameter in demand process: A change in the parameter (ρ) value in the AR(1) demand process changes the low frequency (amplification range) content in the demand profile, which in turn changes the intensity of rogue seasonality generated in the system variables, as per the frequency response analysis in the previous section. ρ values of 0.1 and 0.2 were considered, with the rogue seasonality intensity in the latter case expected to be higher (than in the former) because of its relatively higher low frequency content (Gottman, 1981). ρ values were chosen very close to each other, in order to assess the sensitivity of the index in capturing small differences in intensities.
- Delays (Lead times): LT 2,2,3 was considered as in the original game, although to assess the sensitivity of the results, LT 3,3,4 was also applied.
- APIOBPCS ordering parameters: Two variants of the hybrid MTO-MTS process are considered, the objective being to provide different rogue seasonality contexts with which to test the signature and index. One variant referred to as optimal parameters, which is based on parameters proposed by John et al (1994), was discussed earlier. The other variant does not use any pipeline feedback ($T_w = \infty$), with the other parameter values being the same as in the optimal case. Because such a parameter setting results in greater order amplification as seen in Sterman (1989), the same is referred to as un-optimal parameters. While the choice of optimal and un-optimal parameters is based on the Bullwhip effect, it was expected that these would generate different rogue seasonality characteristics on the basis of the findings in Kim and Springer (2008).
- Batch sizes (b) are considered in relation to the average demand, which is assumed to be 4 units per day. Batch sizes of 50% and 100% of average demand per day, which means batch sizes of 2 and 4, are considered. The no batching option is also considered by keeping the b value at one.

In view of the stochastic nature of demand, thirty independent replications were generated for each of the above simulation variants based on common random numbers.

4.2 Analysis in the frequency domain

The Beer game system involves high order non-linear differential equations and therefore is not analytically tractable (Sterman, 1989). One alternative could be to consider a similar four echelon system as the Beer game in terms of structure, delays and ordering policy, but which did not have backlogs and batching. The frequency response functions (FR) of variables for this linear system could be derived analytically by coupling together the single echelon versions discussed in the previous section, and then compared with the same derived from the Beer game simulation data. This would not only be useful in verifying the simulation but could also help in assessing the impact of backlogs and batching on the logic of the signature and index. The Beer game and corresponding linear system (and related outputs) were therefore compared for different ρ values, delays, ordering policies and batch sizes as per the earlier discussion.

Since the FR functions of variables for only optimal parameters (in a hybrid MTO-MTS system) were determined earlier, the first requirement is to derive the same for un-optimal parameters. Therefore, as in Table 2 but considering un-optimal parameters, the FR profiles and function values at critical points were determined for *ORATE/CONS*, *COMRATE/CONS*, *WIP/CONS* and *AINV/CONS* (not shown here to conserve space). Comparison of the FR profiles of variables for the un-optimal parameter case with that for optimal parameters showed them to be similar, and therefore similarly indicative of rogue seasonality being generated. However, the FR values at critical points for the two cases are different: the $\omega_{max.FR}$ values of the above four variables based on un-optimal parameter values at $1.311/T_p$, $1.311/T_p$, $1.298/T_p$ *and* $1.305/T_p$ respectively, when compared with those for optimal parameters given in Table 2, can be seen to be significantly different. More importantly, the maximum FR values of variables for un-optimal parameters, which were determined as 3.335, 3.335 $3.09*T_p$ and $3.01*T_p$, are significantly greater than those for optimal parameters (refer Table 2), indicating a greater intensity of rogue seasonality being generated in the former case.

Next, the FR functions for a four echelon linear system are derived by coupling together the single echelon functions (assuming ORATE of the downstream echelon to be equivalent to the CONS of the upstream echelon). FR functions are derived for each of the four variables, for each of the four echelons as a function of exogenous demand or CONS. These are then compared with the FR functions of the corresponding variable at the corresponding echelon derived from the simulation data. One such comparison is shown in figure 2, where the lead times for both the Beer game simulation and the linear four echelon system are LT 3, 3, 4 and hybrid MTO-MTS (un-optimal parameters) is used in ordering.

Take in Figure 2

The choice of lead times (higher) and parameters (un-optimal) for ordering was driven by the need to have more backlogs in the Beer game system, so that it could be an extreme case for comparison with the linear system. For a similar reason, batch size of 4 was also considered in the simulation though a separate analysis without batching was also done. The fill rates for the no batching and batch size 4 options with LT 3, 3, 4 and un-optimal parameters were observed to be 0.81 and 0.98 (average of 30 replications) respectively at the Distributor echelon; the corresponding maximum backlogs were observed to be 23 and 20 units. In the figure Amplitude ratio is used in place of FR for the sake of clarity, although for the purposes of this study, they are essentially the same.

Examining the last row in the figure, the analytically derived Amplitude ratio profiles for each of the four variables in each of the four echelons (some profiles are merged and hence not separately identifiable) all show peak amplification at a frequency of 0.03/day. This is in conformance with the FR analysis discussed above assuming a 6 day lead time for each echelon (3 days for order information + 3 days for shipping), and that the frequency characteristics are maintained from single to multiple echelons in linear systems. Although the factory echelon has a lead time of 4 days, the dynamics is transmitted through other echelons having lead times of 6 days, and therefore variables for the factory echelon also show similar peak amplification. Next, we compare the FR profiles derived from simulated data (plotted in the first two rows) with the analytically derived ones. No significant difference between the two is evident. Also, in both sets of simulation data based plots, all variables in all echelons are seen to have a similar cyclicality. Outputs for simulations with a different lead time (LT 2, 2, 3), ordering policy (optimal parameters), batch size (2 units) and demand parameter (ρ 0.2) showed similar characteristics. The good correlation between analytical and simulated outputs therefore serves to verify the simulation model. Also, the characteristic presentation of rogue seasonality even in systems with non-linearities from backlogs and batching provides support to the logic for signature and index. Finally, it is important to explain the difference between the maximum Amplitude ratios derived analytically to those derived from the Beer game simulation output as evident in the figure. In the analytical derivation, which is for a linear system, only previous orders are included under work in process (WIP), while in the Beer game simulation backlogs are also included under WIP. The ORATE in the latter case is therefore relatively smaller which has a concomitant impact on the other variables.

4.3 Signature and index for the Beer game system

While the logic of the signature and index is established in the previous section, it is important to derive them now (from the simulation data) and assess their validity. Three contrasting systems from a rogue seasonality perspective were therefore considered: Pass on orders (or MTO) with lead times of 2, 2, 3, hybrid MTO-MTS (optimal parameters) with lead times of 2, 2, 3 and hybrid MTO-MTS (un-optimal parameters) with lead times of 3,3,4. These systems were excited with an AR process of ρ 0.1. The number of variables is large (35) akin to what one would encounter in practice when analyzing multi echelon systems. The time series and spectra profiles of variables for the three simulations are given in figures 3a and 3b below. The time series profiles are all normalised (i.e. mean centred and amplitude scaled), while the spectra profiles are scaled with respect to the largest spectral peak to enable better visualization of the frequency characteristics. The spectra profiles are seen to stop at 0.5 on the frequency scale in view of the Nyquist sampling theorem (Chatfield, 1994).

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Take in Figures 3a and 3b

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Examination of the variable profiles show no rogue seasonality being generated in the pass on orders (MTO) case, with the variable profiles similar to each other and to the market demand in both time series and spectra plots. On the other hand for hybrid MTO-MTS (optimal parameters), rogue seasonality generation in the system is evident from the plots. Cyclicality is seen to be generated in the variable profiles in lower echelons and then propagated upstream as per the Forrester effect (Forrester, 1961). Many variables share this cyclicality, and whose frequency, which is more clearly evident in the spectra profiles of the variables, is seen to be around 0.04/day. This is equivalent to the frequency for peak amplification for a single echelon of this system (with lead time of 4 days) as per the earlier analytical FR analysis, thereby indicating only an insignificant impact of backlogs on the rogue seasonality dynamics. Finally, in case of hybrid MTO-MTS (un-optimal parameters), the variables profiles in both time series and spectra representations show a similar generation and propagation of cyclicality as in the case of optimal parameters. However, the cycles in this case appear to be more consistent, as seen in the time series plots and which is also evident in the spectra plots as higher peak values. This indicates that rogue seasonality of greater intensity is generated in this system in comparison to MTO-MTS (optimal parameters). The cyclicality of most variables appears to be of frequency 0.03/day as evident from the peak spectra values in figure 3b. This again is in conformance with the FR analysis findings for a single echelon hybrid MTO-MTS (un-optimal parameters) system and suggests minimal impact of backlogs on the dynamics.

Given that the presentation/ non-presentation of rogue seasonality does not appear to be significantly impacted by backlogs and batching (for the chosen levels), and whose nature is similar to that seen for single echelon systems in Section 3, implies that the signature and index for indicating the presence and intensity of rogue seasonality should also be similarly valid. In order to test this, the time series data of variables were transformed into their spectra representations and used to develop the cluster profiles and index. These are presented in figure 4 below.

Take in Figure 4

In the figure, exogenous demand (CONS) is seen to be clustered with the other variables in the pass on orders (MTO) case, which as per the signature accurately indicates no rogue seasonality being generated. Similarly, the signature also correctly indicates generation of rogue seasonality in the hybrid MTO-MTS optimal and un-optimal parameter cases, with CONS being clustered separately from the other variables which are clustered together. However, unlike the single echelon contexts discussed in Section 3, separation of CONS from the other variables is not complete with some variables pertaining to the Retailer seen to be similar to CONS. This is a realistic possibility in any context involving a large number of variables, where, despite the presence of rogue seasonality, some variables could be similar to CONS and be clustered with it. To allow this possibility, the signature needs to be redefined in the following way: rogue seasonality is considered present if CONS is clustered separately from most (not necessarily all) of the variables, and these variables are clustered together. This would require 'most' to be specified, which could be in terms of proportion of total number of system variables. It would depend on the application context and the sensitivity of indication required.

The index values, which are derived on the basis of the formula discussed earlier, are shown next to the respective cluster profiles in the figure. These are 0, 1.09 and 1.22 for MTO, hybrid MTO-MTS (optimal parameters) and hybrid MTO-MTS (un-optimal parameters) respectively. Relating these values to the actual intensity of rogue intensity generated in these systems as discussed earlier of 0, high and highest intensity shows the index to be an accurate indicator. More importantly unlike the signature, the index definition is robust and does not require any adjustment on account of certain variables (some Retailer variables in this case) being similar to CONS. This is because despite a reduction of the numerator value (in the index) from such variables, the remaining variables being aligned at a common amplification frequency reduces the denominator value sufficiently to still yield a high index value which accurately characterizes the rogue seasonality intensity. This is evident in the index values for hybrid MTO-MTS optimal and un-optimal parameter cases in the figure, in both of which one of the variables (Retailer shipping) is very similar to CONS.

While the analysis in the preceding section established the validity of the signature and index, it was based on a limited data set. We now assess the effectiveness of the index using data from various Beer game simulation contexts.

4.4 Consistency assessment of the index from detailed simulation of the Beer game system

Detailed simulation of the Beer game system was carried out by considering lead times of 2,2,3 and 3,3,4, hybrid MTO-MTS with optimal and un-optimal parameters, batch sizes of 1 (no batching) ,2 and 4 and AR(1) demand process with ρ values of 0.1 and 0.2. The rationale for these choices has been discussed earlier in this section. One year of daily data (250 days) was simulated with 30 replications (based on common random numbers) for each simulated case. Average fill rate and maximum backlog for echelons varied from 0.8 to 1 and 0 to 50 respectively for the different simulation cases.

The index values were derived in the time as well as frequency domains, with amplitudes of all frequencies after Fourier Transform (FT Total) used for the latter domain. Besides using all variables, the index was also computed from order and inventory variables only, as in Shukla et al. (2012). This is done because from a practical perspective, many organisations may choose to share information on only a few (rather than all) variables, and order and inventory are the most common variables on which information is shared amongst companies (Lee and Whang, 2000). The index values for each simulation case, and which are derived in alternative domains and with alternative number of variables, are given in Table 4 below.

Take in Table 4

Examining the index values based on FT Total in the table, these are seen to be significantly greater than 0 which is the value for a system with no rogue seasonality. Rogue seasonality is therefore indicated in all the systems considered, and appropriately so, based on the previous discussion. However, it is also important to assess if the index is consistent, i.e. its value accurately reflects the rogue seasonality intensity in the system. For this, the index values of simulation contexts with higher/lower expected rogue seasonality intensities are compared. For example, the index value for a system using hybrid MTO-MTS (un-optimal parameters) should be more than when optimal parameters are used, in view of the relatively greater intensity of rogue seasonality being generated in the former case. Similarly, a system excited with an AR (1) demand process of higher ρ value, which has relatively higher proportion of low frequency content, is expected to generate rogue seasonality of greater intensity. The index value in this case should be more than when a demand process with a lower ρ value is used. However, the important thing to note here is that, as in a real system, the other factors are not being held constant. For example, ρ values, batch sizes, lead times and backlogs are all being varied when optimal/un-optimal parameters are applied. The same is done when systems are excited alternately with ρ values of 0.1/ 0.2.

Consistency of the index for alternative domains is assessed separately. In the frequency domain, for assessing the consistency of the index in terms of ordering policy, the number of times the index value (based on FT Total) for a system with un-optimal parameters is greater than that with optimal parameters (for different ρ values, lead times and same batch sizes), is counted. This can be seen to be 16 out of a maximum 16. A similar count, but of cases where index value for ρ 0.2 is greater than ρ 0.1 (with same lead time, ordering and batch sizes) is seen to be 13 out of a maximum 16. The overall consistency is therefore around 90%, which although lower than the 98% seen in the case of Shukla et al (2102) for a linear three echelon system, is still quite high to justify confidence in the effectiveness of index as a measure. The index values based on time are seen to have a lower consistency of 17/32, and which are also less discriminating of the intensities.

Overall, the signature and index are seen to be valid indicators of the presence and intensity of rogue seasonality respectively in the Beer game system which included backlogs, batching in orders and shipping, shipping variable and used variants of hybrid MTO-MTS in ordering. While no adjustment was required in the definition of the index, a threshold for the proportion of variables clustered separately from CONS now needs to be specified for the signature.

5. Conclusions

Rogue seasonality or endogenous generation of cyclicality has a negative effect on supply chain performance; therefore, approaches to indicate its presence and intensity in a supply chain could be useful for timely initiation of mitigative action/s. Such indicators in the form of a signature and index of rogue seasonality were proposed by Shukla et al (2012), although these were not rigorously validated, with only simplistic linear make to order (MTO) and make to stock (MTS) simulated systems being used, and no use of analytical approaches in the investigation. This study sought to plug these gaps and establish the robustness of the signature and index, for which control theory and simulation of a multi echelon non-linear system called the Beer game were used. While a linear system was used in the control theory based analysis, the simulated Beer game systems included order backlogs, batching in ordering and shipping, variants of hybrid make to order-make to stock for ordering and the shipment variable. Each context was first analysed in terms of the nature of rogue seasonality generated whereafter data from each context was used to assess the effectiveness of the signature and index.

The signature based on the cluster profiles of variables, specifically whether the variables cluster together with or away from exogenous demand, was seen to be effective in indicating the presence/absence of rogue seasonality in all the cases. The definition of the signature, however, needs to be flexible to accommodate minor inconsistencies, such as a system with rogue seasonality still having a few variable profiles similar to exogenous demand and being clustered with it. This would need setting of appropriate thresholds, which could be in terms of the proportion of variables clustered, and be based on prior knowledge and/or sensitivity of indication required. The index, which is derived by quantifying the cluster profiles, was found to be similarly effective as the signature, though to indicate the intensity of rogue seasonality, with a higher value indicating a greater intensity. The index is robust and does not require adjustment of the kind required for the signature discussed above. Finally, the relative superiority of the frequency domain (amplitudes of all frequencies after Fourier Transform) for deriving the signature and index in comparison to time was also established.

Validation of the index and signature with realistic systems considered in the study has increased the confidence that these will be useful in practice. The process for exploiting these, described through a flowchart discussed in Shukla et al (2012), still remains valid and useful. The only change required is an additional pre-processing step of specifying a threshold (in terms of the proportion of variables clustered together or away from the exogenous demand) for the signature. The threshold could be established through a sample data analysis. The flow chart based approach could be used to discriminate supply chains on the basis of their index values, with corrective action taken against those with high values, if possible and necessary.

A significant ground on rogue seasonality has been covered in this study. However, a few questions still remain such as: a) What would be the nature of rogue seasonality generated in different network configurations and would the signature and index be valid for such contexts, b) How to identify the main source/contributor of rogue seasonality based on index values at select points in the chain/network.

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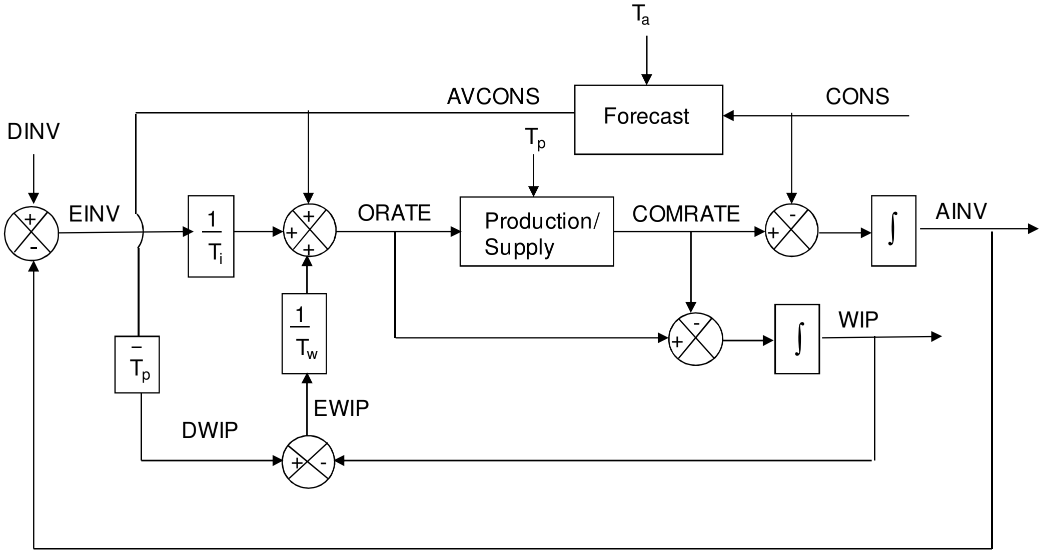
Table 1- Nomenclature

Variable/Parameter Abbreviation/	**Description**
MTO	Make to order. Ordering decision only based on customer orders
MTS	Make to stock. Ordering decision only based on replenishing stock to appropriate level
Hybrid MTO-MTS	Customer orders and stock replenishment are both included in the ordering decision
CONS	Consumption/Sales rate
AVCONS	Forecast of average Consumption/Sales rate
WIP	Work in process level
DWIP	Desired work in process level
EWIP	Error between desired and actual work in process level
AINV	Actual inventory level
DINV	Desired inventory level
EINV	Error between desired and actual inventory level
ORATE	Order rate
COMRATE	Production completion rate or rate of goods receipt into inventory
T_p	Actual delay or lead time
$\overline{T}_p$	Estimated delay or lead time
T_i	Time to adjust inventory
T_a	Time to average demand
α	Forecasting constant used in exponential smoothing forecast $\alpha = 1/(1+T_a)$
T_w	Time to adjust WIP
s	Laplace transform operator
FT	Fourier Transform
j	The imaginary number $\sqrt{-1}$
FR	Frequency Response
Amp R	Amplitude Ratio
f	Frequency (cycles per time period)
ω	Angular frequency (radians per time period)
ω_0	0 angular frequency
$\omega_{max\ FR}$	Angular frequency corresponding to maximum value of FR
$\omega_{crossover}$	Angular frequency corresponding to FR crossover from amplification to attenuation range
ω_∞	Very high angular frequency
AR	Autoregressive model
ρ	Autoregressive model parameter
LT x, y, z	Used in the context of Beer game to capture three lead times or delays in one term. x: order transmission lead time between two adjacent echelons; y: shipping lead time between two adjacent echelons; z: factory production lead time
LT_{order}	Order transmission lead time (or delay) between 2 adjacent echelons used in the context of the Beer game
$LT_{shipping}$	Shipping lead time (or delay) between 2 adjacent echelons used in the context of the Beer game
$LT_{factory\ production}$	Factory production lead time (or delay) used in the context of the Beer game
b	Batch size. Material to be ordered and shipped only in multiples of batch size.



$$\frac{AVCONS}{CONS} = \frac{1}{1+T_a s} \textbf{.... (1)} \qquad DWIP = \overline{T_p}\,(AVCONS) \textbf{....(2)} \qquad AINV = \frac{1}{s}(COMRATE - CONS) \textbf{....(3)}$$

$$\frac{COMRATE}{ORATE} = \mathrm{e}^{-\mathrm{Tps}} \approx \frac{(sT_p)^2 - 6(sT_p) + 12}{(sT_p)^2 + 6(sT_p) + 12} \; \textit{(Order infinity)}\textbf{... (4)}$$

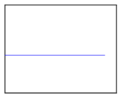
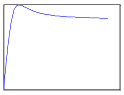
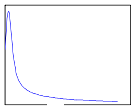
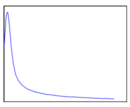
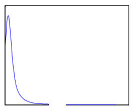
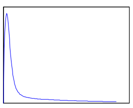
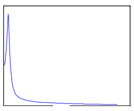
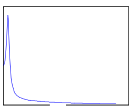
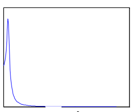
[2nd order Pade's approx.(Nise, 1995)]

$$WIP = \frac{1}{s}(ORATE - COMRATE) \textbf{....(5)} \qquad EINV = DINV - AINV \textbf{....(6)}$$

$$EWIP = DWIP - WIP \textbf{....(7)} \qquad ORATE = AVCONS + \frac{EWIP}{T_w} + \frac{EINV}{T_i} \textbf{....(8)}$$

Figure 1. Block diagram and system equations for a single echelon APIOBPCS system

Table 2 – Frequency response (FR) functions of variables for single echelon MTO (make to order), hybrid MTO-MTS and MTS (make to stock) systems (Delay order infinity for all)

	Variables (unique)	Profile of FR function*	FR at ω_0	$\omega_{max\ FR}$	Max FR value	$\omega_{crossover\ (or\ FR\ =\ 1)}$	FR at ω_∞
MTO	$\frac{COMRATE}{CONS}$		1	all	1	NA	1
	$\frac{EWIP}{CONS}$		0	∞	T_p	NA	T_p
Hybrid MTO-MTS with Optimal parameters (John, 1994)	$\frac{ORATE}{CONS}$		1	$\frac{1.014}{T_p}$	1.688	$\frac{2.135}{T_p}$	0
	$\frac{COMRATE}{CONS}$		1	$\frac{1.014}{T_p}$	1.688	$\frac{2.135}{T_p}$	0
	$\frac{WIP}{CONS}$		T_p	$\frac{0.937}{T_p}$	$1.62{*}T_p$	Complex function of T_p	0
	$\frac{AINV}{CONS}$		0	$\frac{0.983}{T_p}$	$1.863{*}T_p$	Complex function of T_p	0
MTS	$\frac{ORATE}{CONS}$		1	$\frac{1.306}{T_p}$	2.307	$\frac{1.906}{T_p}$	0
	$\frac{COMRATE}{CONS}$		1	$\frac{1.306}{T_p}$	2.307	$\frac{1.906}{T_p}$	0
	$\frac{WIP}{CONS}$		T_p	$\frac{1.293}{T_p}$	$2.141{*}T_p$	Complex function of T_p	0

*Plotted for $T_p = 5$

Table 3 - Difference equations used for the Beer game system simulation

J = Supply chain echelon with J = 1, 2, 3, 4 representing the Retailer (R), Wholesaler (W), Distributor (D) and Factory (F) respectively

$CONS^{J}$ (J = 1 to 4, t ≤ 24) = 4 ----- *Initial condition;* Rest of the equations are for t > 24 **(2.1a)**

$CONS^{J}$ (J = 1, t) = $4 + \varepsilon_t + \rho CONS^{J}(J{=}1)_{t-1}$ [AR (1) demand process with parameter ρ] **(2.1b)**

$CONS^{J}$ (J = 2 to 4, t) = Order $^{J-1}$ (t – LT $_{order}$) **(2.1c)**

Shipment Received ^{J} (J = 1 to 3, t) = Shipping $^{J+1}$ (t – LT $_{shipping}$) **(2.2a)** Shipment Received ^{J} (J = 4, t) = ORATE ^{J} (t – LT $_{factory\ production}$) **(2.2b)**

Maximum Possible Shipping ^{J} (J = 1 to 4, t) = AINV ^{J} (t-1) + Shipment Received ^{J} (t) **(2.3)**

Desired Shipping ^{J} (J = 1 to 4, t) = Backlog ^{J} (t-1) + CONS ^{J} (t) **(2.4)**

Shipping ^{J} (J = 1 to 4, t) = MIN [Desired Shipping ^{J} (t), Maximum Possible Shipping ^{J} (t)] **(2.5)**

AINV ^{J} (J = 1 to 4, t) = AINV ^{J} (t-1) + Shipment Received ^{J} (t) - Shipping ^{J} (t) **(2.6)**

Backlog ^{J} (J = 1 to 4, t) = Backlog ^{J} (t-1) + CONS ^{J} (t) - Shipping ^{J} (t) **(2.7)**

AVCONS ^{J} (J = 1 to 4, t) = AVCONS ^{J} (t-1) + α [CONS ^{J} (t) – AVCONS ^{J} (t –1)], where $\alpha = 1/(1+T_a/\Delta t)$; Δt is simulation time increment set at 1 **(2.8)**

DWIP ^{J} (J = 1 to 4, t) = T_p * AVCONS ^{J} (t) where T_p is the lead time between placing an order and receiving the material for J supply chain echelon; T_p (J = 1 to 3) = LT $_{order}$ + LT $_{shipping}$ – 1; T_p (J = 4) = LT $_{factory\ production}$ – 1 **(2.9)**

WIP ^{J} (J = 1 to 3, t) = $\sum_{1}^{LT_{Order}-1}$ ORATE ^{J} (t-i) + $\sum_{0}^{LT_{Shipping}-1}$ Shipping $^{J+1}$ (t-k) + Backlog $^{J+1}$ (t) **(2.10a)** WIP ^{J} (J = 4, t) = $\sum_{1}^{LT_{Order}-1}$ ORATE ^{J} (t-i) **(2.10b)**

EWIP ^{J} (J = 1 to 4, t) = DWIP ^{J} (t) – WIP ^{J} (t) **(2.11)** DINV ^{J} (J = 1 to 4, t) = 12 **(2.12)**

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EINV ^{J} (J = 1 to 4, t) = DINV ^{J} (t) – AINV ^{J} (t) + Backlog ^{J} (t) **(2.13)**

ORATE ^{J} (J = 1 to 4, t) = MAX [0, AVCONS ^{J} (t) + (EINV ^{J} (t) /T_i) + (EWIP ^{J} (t) /T_w)] **(2.14)**

ORATE ^{J} (J = 1 to 4, t) =b* Ceiling (ORATE ^{J}/b) **(2.15)** Shipping ^{J} (J = 1 to 4, t) = b*Floor (Shipping ^{J}/b) where b = batch size **(2.16)**

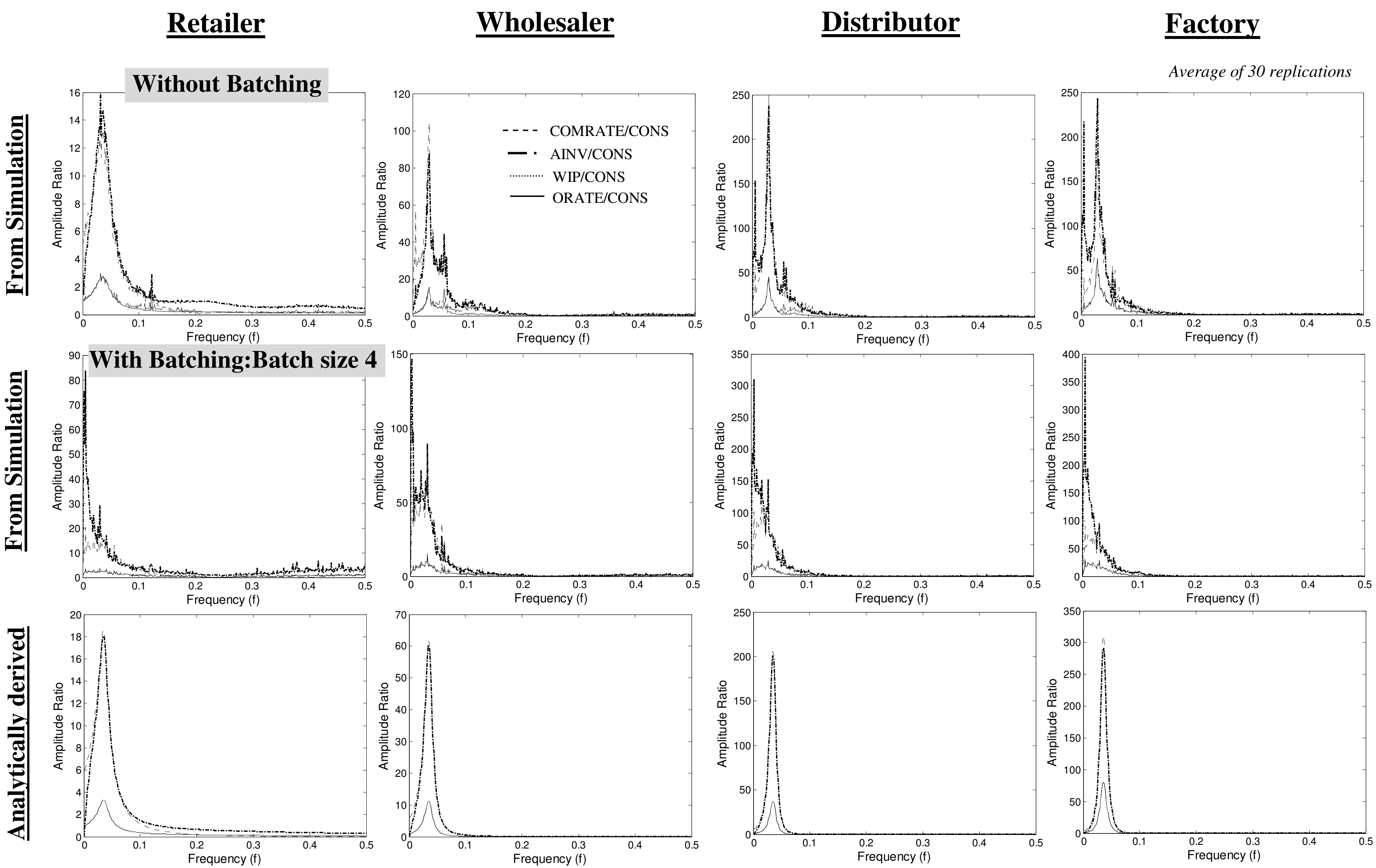

Figure 2. Amplitude ratio (Frequency response) of select variables for a simulated Beer game system with LT 3,3,4, Hybrid MTO-MTS (un-optimal parameters) as ordering policy and AR(1) with ρ 0.1 as demand process

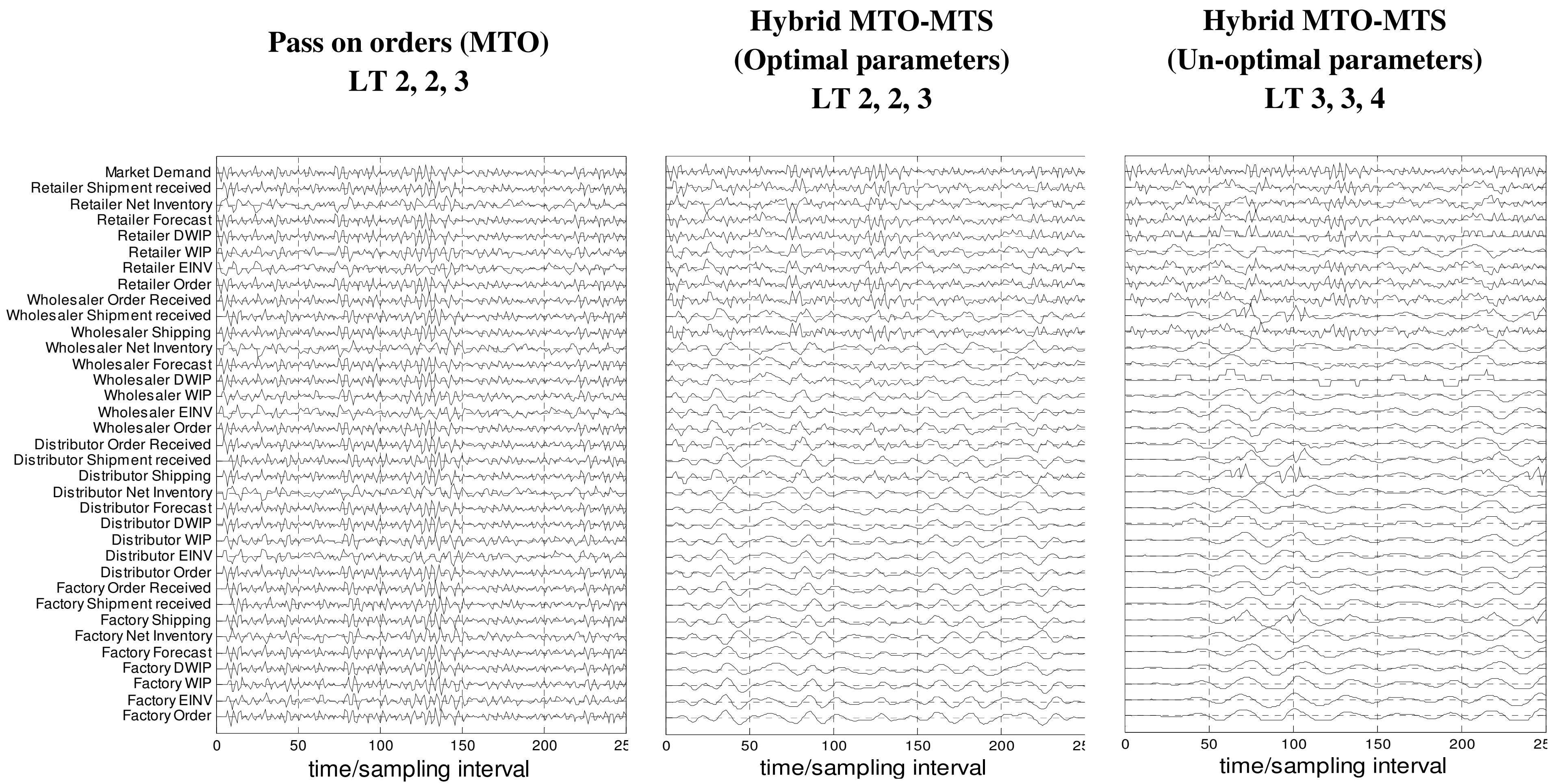

Figure 3a. Normalised time series profiles for a simulated Beer game system (*LT x,y,z*: order transmission, shipping and factory production lead times); *Optimal parameters*: As per John, 1994; *Un-optimal parameters*: No pipeline feedback (Sterman, 1989); Exogenous demand: AR(1) ρ 0.1

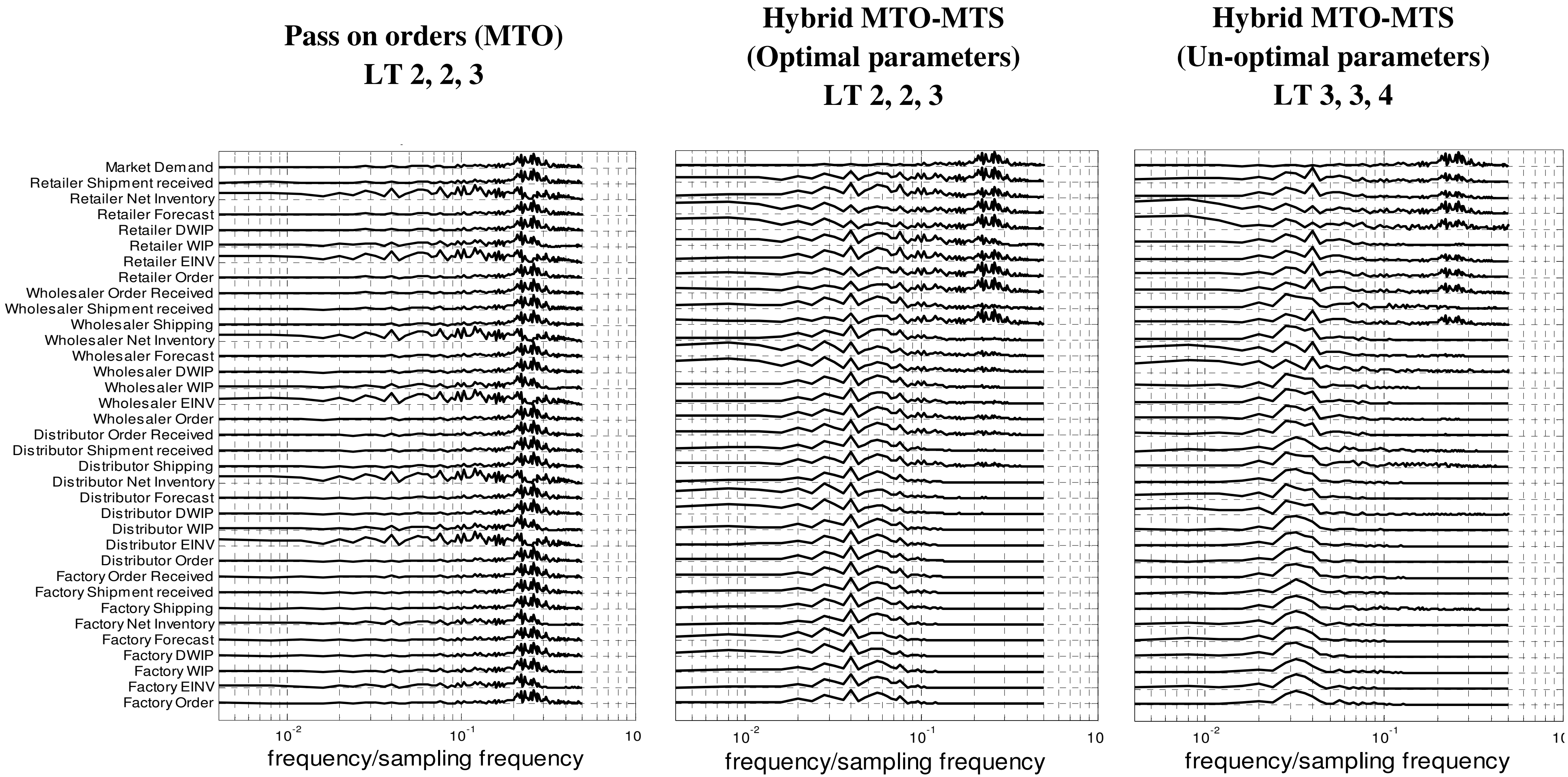

Figure 3b. Spectra profiles for a simulated Beer game system (*LT x,y,z*: order transmission, shipping and factory production lead times); *Optimal parameters*: As per John, 1994; *Un-optimal parameters*: No pipeline feedback (Sterman, 1989); Exogenous demand: AR(1) ρ 0.1

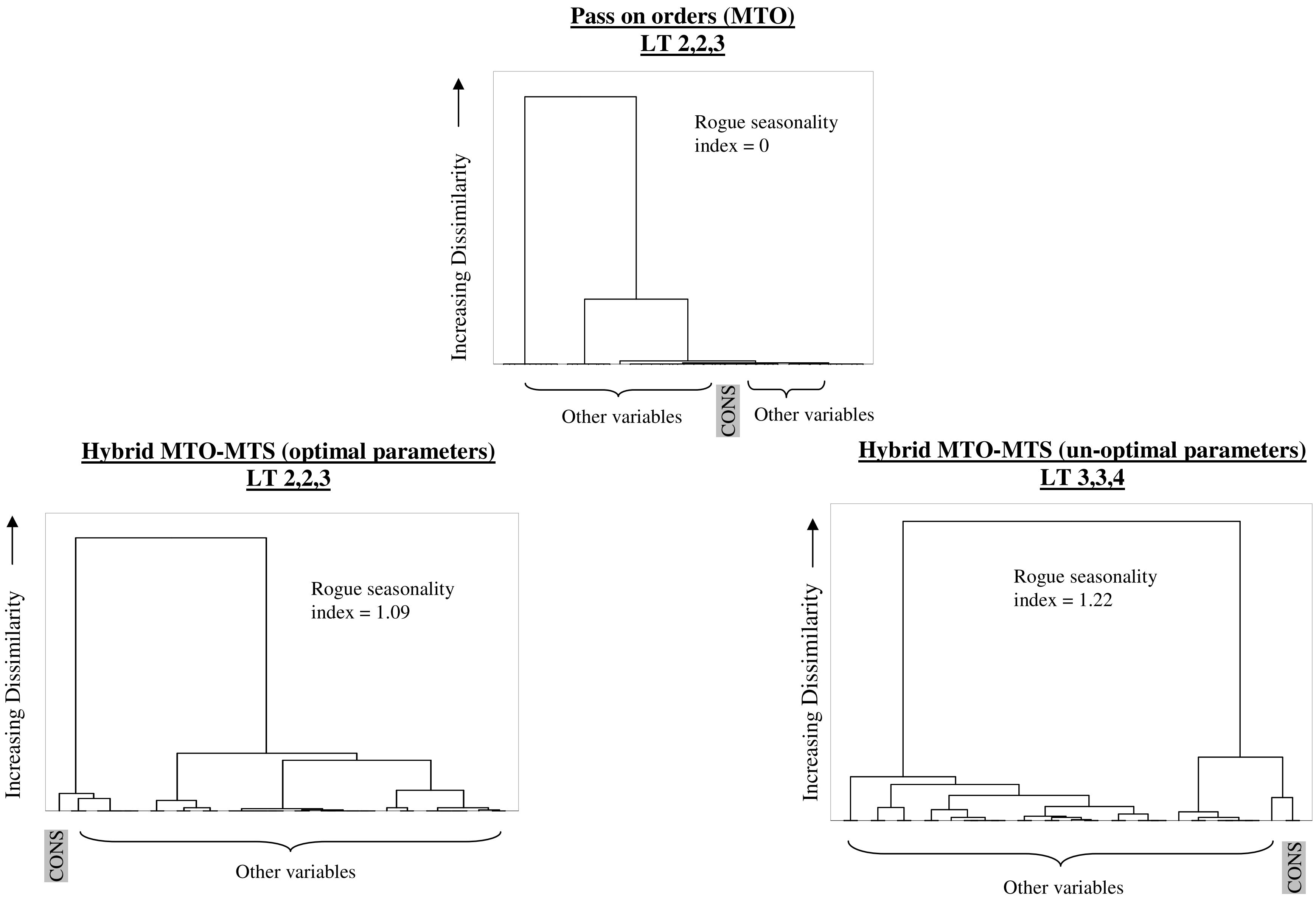

Figure 4. Rogue seasonality signature and index for different ordering practices in a simulated Beer game system (LT x,y,z: order transmission, shipping and factory production lead times ; Exogenous demand: AR (1) ρ 0.1; FT (Total) or amplitudes of all frequencies used in clustering and deriving index

Table 4 – Rogue seasonality index values for different simulated variants of the Beer game system

			Exogenous demand: AR(1) with ρ 0.2				Exogenous demand: AR(1) with ρ 0.1			
			All variables used		Only order and inventory variables		All variables used		Only order and inventory variables	
Lead Time*	Ordering**	Batching***	Time	FT Total	Time	FT Total	Time	FT Total	Time	FT Total
LT 334	Unoptimal	Batch 50%	0.88	1.41	0.83	1.62	0.77	1.19	0.84	1.43
		Batch 100%	0.88	1.29	0.90	1.38	0.91	1.19	1.04	1.41
	Optimal	Batch 50%	0.90	1.35	0.76	1.40	0.78	1.14	0.83	1.19
		Batch 100%	0.90	1.07	0.86	1.01	0.79	1.07	0.93	1.20
LT 223	Unoptimal	Batch 50%	0.82	1.44	0.77	1.60	0.72	1.10	0.78	1.55
		Batch 100%	0.83	1.27	0.85	1.31	0.73	0.95	0.91	1.30
	Optimal	Batch 50%	0.83	1.30	0.69	1.41	0.72	1.07	0.73	1.23
		Batch 100%	0.84	1.10	0.81	1.03	0.73	0.92	0.90	1.19

*LT xyz (x: order transmission, y: shipping, z:factory production lead time)

** Variants of hybrid MTO-MTS

*** Batch 50% (2 units) and Batch 100% (4 units) are in relation to the average demand per period (considered to be 4 units)

FT Total: Amplitudes of all frequencies after Fourier Transform (FT)