An e-Retailing Supply Chain Subject to Inventory Inaccuracies[1]

Yacine Rekik[a], Aris Syntetos[b], Zied Jemai[c]

[a] *EMLYON Business School, DISP, 23 av. Guy de Collongue, 69134 Ecully, France*
[b] *Cardiff Business School, Cardiff University, Cardiff CF10 3EU, UK*
[c] *Ecole Centrale Paris, Laboratoire Génie Industriel, Grande Voie des Vignes 92295 Chatenay Malabry, France*

Abstract

One of the implicit assumptions considered in the majority of investigations performed in the area of inventory management is that the physical flow of products in an inventory system is free from defects. The same is hypothesized for the associated information flow. However, various factors may create a difference between the actual physical and information system (IS) flows and perturb their synchronized evolution. The implications of such a discrepancy are particularly prevalent in contemporary supply chains, where sales commitments are based on IS records only. In this paper we model and analyze the impact of inventory inaccuracies on supply chain performance. We first provide an overview of potential errors that may occur within an inventory system and we then propose a general framework to model the impact of inaccuracy errors. Potential errors in both the physical and/or the IS record inventories are considered and optimal analytical solutions are provided for both centralized and decentralized (coordinated and uncoordinated) supply chains for three plausible scenarios: inaccuracy errors are ignored; errors are estimated; the utilization of a Radio Frequency Identification (RFID) technology enables the reduction of the relevant errors. The performance improvements enabled by the availability of error related information and the RFID technology are assessed and managerial insights are provided. The paper concludes with the implications of our work for supply chain design as well as with an agenda for further research in this area.

Keywords: inventory inaccuracies, supply chain management, RFID technology, newsvendor problem, internet ordering, random yield.

[1] All the Appendices to this paper are separately presented as supplementary material in an Electronic Companion.

1. Introduction

1.1 The inventory Inaccuracy Issue

One of the implicit assumptions considered in the majority of investigations performed in the area of inventory management is that the physical flow of products in an inventory system is free from defects. The same is commonly hypothesized for the associated information flow resulting in an expected match between physical and information system (IS) inventories. However, there is a plethora of factors that may generate a deviation between the inventory level displayed in the information system (i.e., what is available according to the IS) and the physical inventory (i.e. what is actually available). Such deviations are referred to as inventory inaccuracies and may deeply affect the operational and financial performance of firms (Schrady (1970)). Atali et al. (2009) discussed three different kinds of streams that may result in inventory inaccuracies. First, some streams result in **permanent inventory shrinkage** (such as theft and damage). Some others may result in **temporary discrepancies** and stock may be recovered by physical inventory audits and returned to the stock-base (such as misplacements). The final group of the relevant streams affects only the inventory records and leaves the physical inventory unchanged (an example relates to the consequences of scanning errors). **Transaction errors** are unintentional errors occurring during inventory transactions. Such errors arise while counting the inventory, receiving an order or checking out at the cash register.

Iglehart and Morey (1972) proposed an analytical tool to aid the task of controlling inventory errors; the objective of this study was to select the type and frequency of counts and to modify the predetermined inventory policy by adding a buffer that compensates for errors so as to minimize the total cost (inventory holding + inspection costs) per unit time. Sandoh and Shimamoto (2001) also built a model that determines the optimal frequency of periodic reviews within a supermarket, resulting from the trade off between the cost related to such counting exercises and the cost of investigating the causes of inventory deviations. More recently, the work conducted by Kök and Shang (2007) aimed at finding effective inventory replenishment and inspection policies that minimize inventory and inspection costs over a finite horizon. Record inaccuracies could also be the consequence of having an unreliable supply system. In fact, when the production process has a low yield or the supply process is unreliable, the physical available inventory may not be known until an inspection is performed and as a consequence it may be different from the inventory in the information system (Yano and Lee (1995), Inderfurth (2004), Kang and Gershwin (2005) and Rekik et al. (2007b)).

1.2 The Internet Retailing Context

From the above discussion it is evident that the very source of an inaccuracy may be either an error related to the physical inventory and/or one that affects the IS records only. In addition to the errors sources discussed above, the internet retailing context may also suffer from the existence of two databases: one associated with the e-retail website and another with the ERP system: a bad synchronization between the two databases constitute an additional source of inventory inaccuracy. Errors will always raise significant problems in traditional/in-store retail supply chains but their implications may become even more evident in electronic/internet retailing supply chains. In the later case not only ordering but also sales decisions are being taken based solely on the inventory level displayed in the IS. As such, a sales commitment may be made for example but not be respected when delivering the products (if the physical inventory level is lower than that indicated in the information system). In fact, under an electronic context, final customers make their order in front of a screen or by phone. Demand satisfaction is achieved based on the inventory levels in the Information System (and not based on the physical available stock) unlike the in-store case.

To the best of our knowledge, all inventory management investigations (except the ones conducted by Sahin and Dallery (2009) and Rekik (2011) which are discussed later on our paper) where the inventory system is subject to inaccuracies, have been performed in the context of a 'traditional' retail channel. Although we have used the e-Retailing case to illustrate the context of our inventory framework, it is clear that any other upstream supply chain actor (such as a wholesaler, distributor etc) is relying solely upon the IS records in order to satisfy demand. We will continue to be referring to the e-Retailing case for the purpose of our analysis but the reader should bear in mind that all our findings apply equally to any other upstream supply chain stage.
Despite the abundance of empirical evidence on the importance of the inventory inaccuracy issue in traditional/in-store retail supply chains (Raman et al. (2001)), no evidence has been put forward on the significance of this issue in internet businesses. The impact of inaccuracies in such a context may be studied thanks to 'review and opinion' websites which reflect the experience of e-customers with e-Retailers. Examples of such customers' feedback are available on platforms such as www.complaints.com, www.pissedconsumer.com or www.e-customer-satisfaction.com. To assess the presence of the inaccuracy issue among the e-customers' complaints, we have conducted a search with the keyword 'stock' or 'inventory' on these platforms resulting in a very great number of testimonies describing exactly the e-Retailing inaccuracy issue studied in our paper.

1.3 The RFID Technology

Dealing with inventory inaccuracy issues is not a straightforward exercise. Companies may of course just ignore the presence of such errors continuing their operations as if no errors have occurred. Alternatively, and should some information be available on the behavior of these errors, companies may attempt to estimate them for the purpose of improving their performance. Finally, various papers have studied the benefits of tackling the inventory inaccuracy issue through the adoption of the Radio Frequency IDentification (RFID) technology. RFID technologies offer several contributions to supply chain through their advanced properties such as unique identification of products, easiness of communication and real-time information (Michael and McCathie (2005), Saygin et al. (2007), Hozak and Collier (2008), Heim et al. (2009)). RFID can improve the traceability of products and the visibility throughout the entire supply chain, and may also render more reliable various operational processes such as tracking, shipping, checkout and counting, which lead to improved inventory flows and more accurate information (Chow et al. (2006), Tajima (2007), Bendoly et al. (2007), (Ti-Jun et al., 2014), (Piramuthu et al., 2014)). Miragliotta et al. (2009) provided an in-depth literature review and a classification of the main implications related to RFID applications. Most of the research/case study work in this area considers the evaluation of the hard (direct) benefits of RFID like its impact on inbound and outbound logistics (see, for example, Atkinson (2005), Vijayaraman and Osyk (2006), Altay and Taylor (2007), Gaukler and Hausman (2008) and Giusto et al. (2010)). Some investigations provide a compatative study between the barcode and the RFID identification technologies ((Chan et al., 2012)). The contribution of RFID in other areas, such as the reduction of the Bullwhip Effect and the more 'robust' application of inventory replenishment policies is also increasingly being recognised and evaluated (Sarac et al. (2010)). Similarly, several authors have been interested in RFID technologies to reduce the effects of inventory inaccuracy errors. Some of these studies will be discussed in the next section but the interested readers are referred to Lee and Ozer (2007) for a comprehensive review on that issue.

1.4 Contribution and Organization of the paper

The aim of our paper is to model and study in detail the implications of the presence of inventory inaccuracies in a supply chain. We do so by introducing an e-Retailing based modeling framework, a special case of which is the traditional in-store retail supply chain. We consider the possibility of errors being present in both the physical and/or the IS inventory levels and we focus on both centralized and decentralized (coordinated and uncoordinated) supply chains. For each scenario we assess: i) the impact of the errors being ignored; ii) the benefit of deploying better inventory strategies taking into account the error estimation; iii) the impact of the RFID technology to cope with inaccuracies. Such contributions constitute collectively a considerable extension of the current state of knowledge in this area. In particular, there have been only two studies concerned with inventory inaccuracies in an electronic context. In the first one developed by Sahin and Dallery (2009), a centralized supply chain was assumed coupled with the assumption that the physical inventory levels are free from defects. The second study developed by Rekik (2011) extended the framework provided by Sahin and Dallery (2009) by including the errors on the IS level but was only concerned with the case of a centralized supply chain.

Under the decentralized supply chain configuration, all previous work dealing with the inventory inaccuracy issue (Gaukler et al. (2007); Heese (2007); Rekik et al. (2007a); Çamdereli and Swaminathan (2009)) study the retail supply chain structure where the inventory level shown in the IS system does not play any role in satisfying the customers' demand. In these investigations, the inaccuracy issue is only impacting the physical inventory.

In addition to the above issues, and given the increased rates of using RFID it is natural to consider it as a possibility towards the resolution of inaccuracy related issues. The linkage between RFID and inventory accuracy is an area that may certainly benefit from further investigations and for this particular case we derive the conditions under which such a technology is cost effective and we evaluate the way its cost may be shared between supply chain actors.
The remainder of our paper is organized in three main parts as follows: the first part (Section 2) describes and formulates mathematically the framework under study. The second part of the paper aims to provide the optimal strategies for three supply chain scenarios: a centralized one (Section 3.1); a decentralized uncoordinated one (Section 3.2) and a decentralized coordinated supply chain (Section 3.3). For each supply chain structure, the inventory optimization contribution concerns three possible approaches: the first one considers the case of inaccuracy errors being estimated (and the relevant information being of course utilized for more effective inventory management decision making (Approach 1)); the second deals with the case according to which errors are being ignored (Approach 2); the last one examines the inventory implications of using the RFID approach (Approach 3). In the third and last part of the paper, we first provide managerial insights in Section 4. We do so by contrasting Approaches 1 and 2 & 1 and 3 for all three supply chain structures considered in our paper. The former comparison enables insights to be gained into the benefits of information on errors; the later highlights the implications of the RFID deployment. Our paper ends with the concluding remarks of our work along with the natural next steps of research in this area (Section 5).

2. The Framework under study

2.1 The Problem Setting

For the remainder of our paper, we consider the supply chain of an internet retailing channel. This supply chain is composed of two actors, the e-Retailer and the manufacturer managing a single seasonal product characterized by a unique selling season (newsvendor).We consider the following sequence of events: Let Q be the quantity that the e-Retailer orders from the manufacturer. When the necessary quantity has been produced, the manufacturer will deliver it to the e-Retailer. The e-Retailer will receive goods, update the information system by scanning products and store them in the warehouse. Because of errors, Q_{PH}, i.e. what is physically available in inventory for a product at the beginning of the selling period, may not be equal to Q_{IS}, i.e. what the information system shows as being available. Just before the beginning of the selling period, the e-Retailer will receive a cumulative online order from the final customers. He will compare the total quantity requested by the customers with the IS inventory record to accept or decline orders. If the cumulative order is less than Q_{IS}, the e-Retailer will accept all the orders. If not, he will only accept orders summing up to the IS inventory. Later on, products will be shipped from the e-Retailer's warehouse and delivered to the customers. All the orders that the e-Retailer has committed himself to should in principle be satisfied. However, this may not always be the case due to inventory inaccuracies when the commitment quantity is higher than Q_{PH}. Based on the sequence of events described above, one could consider three penalties that should be taken into account when deciding the quantity to order:

- An overage penalty which is paid by the e-Retailer when a product remains in its warehouse at the end of the selling season.
- A first (type 1) underage penalty which is incurred when, based on the IS system, the e-Retailer is not able to satisfy a customer demand.
- A second (type 2) underage penalty which is incurred when the e-Retailer is not able to respect his commitment.

The problem formulation discussed above implies a single lump-sum arrival of demand (i.e. a cumulative demand) which is representative of many e-Retailing settings procceding with an aggreagate and a collective end-of-period shipping.

2.2 Approaches to Inventory Management Subject to Inaccuracy Problems

First, let us define the role of the RFID technology in such inventory systems where a non-agreement may exist between the ordered quantity, the PH and the IS inventories. The RFID technology may be linked to the inventory inaccuracy issue in the following important ways: **i)** The RFID may prevent or reduce the magnitude of some sources of inventory inaccuracy; **ii)** When errors are not reduced or eliminated, the visibility provided by this technology permits the alignment between the IS and the physical inventory levels.
In the case where the RFID technology is not deployed, one should distinguish between two situations depending on whether the e-Retailer is aware or not of the existence of errors. The case where the inventory manager is aware of errors occurring in the inventory system will be referred to as **Approach 1**. With regards to Approach 1, the average and the standard deviation of the errors are known thanks to statistical studies that the inventory manager may perform. In fact being aware that errors are occurring, he could perform periodic inspections to estimate and to improve his knowledge about the errors occurring in the warehouse. The publication of Pergamalis (2002) provides an excellent methodology permitting to measure the inaccuracy parameters. Another way to estimate errors is through the use of a Bayesian updating mechanism as illustrated in the investigation of DeHoratius et al. (2008). The case where the inventory manager is unaware or simply ignores errors will be referred to as **Approach 2**
In this paper, the scenario in which RFID is deployed will be referred to as **Approach 3**. Under Approach 3, we assume that the costs associated with the implementation of this technology consists of the RFID tags attached to each item individually, at a certain price t per unit. The fixed costs of the investment necessary to implement the technology are deliberately not part of our inventory

models. Such fixed costs may be included by a Return On Investment or a Net Present Value analysis. Under Approach 3, we assume that the RFID technology provides visibility to the inventory manager, i.e. permits to align the IS and the PH levels without having a preventive impact on errors. The case where the RFID technology prevents or eliminates the errors is indirectly studied in this paper since Approach 1 with lower errors' characteristics (average and variability) could model the RFID enabled approach where the technology decreases errors without eliminating them.
In contrast to Approach 1, Approaches 2 and 3 are easier to model and optimize. In fact, in Approach 2, the inventory manager acts as if there were no errors so, his replenishment policy coincides simply with the error free replenishment policy, i.e. the optimal newsvendor policy. Approach 3 is also a basic inventory problem with modified cost parameters where the RFID tag cost is included.

Due to constraints related to the length of the manuscript, in the next section we focus on the analysis associated with Approach 1. The optimal inventory policies pertaining to Approaches 2 & 3, which are adapted from the classical inventory literature, are provided in the Electronic Companion of this paper. Three scenarios are considered for the above discussed analysis:

1. **The Centralized scenario** (C) where we assume that there is a single decision-maker who is concerned with maximizing the entire chain's profit;
2. **The Decentralized Uncoordinated scenario** (DU) where we consider two decision-makers, the manufacturer and the e-Retailer, and each optimizes his own expected profit function;
3. **The Decentralized Coordinated scenario** (DC) where the manufacturer and the e-Retailer cooperate in order to render the total expected profit closer to the expected profit associated with the Centralized scenario.

2.3 Modeling of Errors and Notations

In a general setting the IS inventory, Q_{IS} (PH inventory, Q_{PH}) can be expressed as a function of the ordered quantity Q and either an additive, e_{IS} (e_{PH}) or multiplicative, γ_{IS} (γ_{PH}) random variable characterizing errors. The two cases are outlined below:

- The additive case: Q_{IS} and Q_{PH} are respectively given by $Q_{IS} = Q + e_{IS}$ and $Q_{PH} = Q + e_{PH}$
- The multiplicative case: Q_{IS} and Q_{PH} are respectively given by $Q_{IS} = \gamma_{IS}Q$ and $Q_{PH} = \gamma_{PH}Q$

In the first case, errors are independent of the ordered quantity. For example, they may arise as an outcome of administrative malfunctions (a wrong recording for instance of 7 units rather than 9 in the ordering process). In this case, the error realization does not depend on the ordered quantity. In the multiplicative case, the error realization depends on the ordered quantity. Factors such as theft can probably be modelled in this way since the higher the ordered quantity is, the higher will be the quantity potentially stolen. In contrast, it would be reasonable to assume that errors made by human beings, like transaction errors, may be represented by an additive structure. Similarly, misplacement errors made within the store or the storage warehouse can also be modelled by the additive setting. The probability that a customer takes a product, tries it, decides not to buy it and then places it in the wrong shelf is independent of the quantity or the batch initially available in the right shelf. The error setting considered in this paper is the additive one (assuming independence between e_{IS} and e_{PH}). Our choice is motivated by the following arguments:

- As mentioned above, the additive setting is representative of many sources of inventory inaccuracies particularly the transaction errors which impact exclusively the IS stock.

- From an empirical and practical point of view, the multiplicative link between errors and the actual stock level is not easy to show and validate. This is particularly true when errors accumulate over periods and are only discovered when an inspection is being made: it is not straightforward to link the gap between the IS and the PH stock levels with the inventory movements due to an accumulation of (many) errors whose traces are 'lost'.

- The distinction between the additive and the multiplicative modeling of errors could be analogically motivated by the distinction between the additive and the multiplicative modeling of the demand elasticity in the field of the coordinated pricing and inventory decisions. The literature in this field which has been reviewed by Yano and Gilbert (2004) and Huang et al. (2013), shows that there exist significant differences between the optimal pricing and inventory decisions when demand is additively or multiplicatively modeled (Petruzzi and Dada (1999)). Similarly to the inaccuracy formulation discussed above, the demand variance for the additive formulation is independent of price but the coefficient of demand variation is increasing in price. In contrast to the additive model, the coefficient of variation of demand in the multiplicative model is independent of the price but its variance is decreasing in price (Huang et al. (2013)).

The notations used for the remainder of our paper are as follows:

- D the random variable representing the demand;
- f (F): the PDF (CDF respectively) characterizing D;
- e_{IS} (e_{PH}): the random variable representing IS (PH) errors;
- μ_{IS} (μ_{PH}) the average of e_{IS} (e_{PH});
- σ_{IS} (σ_{PH}) the standard deviation of e_{IS} (e_{PH});
- $D_m = D - e_{IS}$: a random variable combining the demand and the IS error;
- f_m (F_m): the PDF (CDF respectively) characterizing the variable D_m;
- μ_m : the mean of the random variable D_m;

- $e = e_{IS} - e_{PH}$: a random variable that represents the difference between the IS and the PH errors;
- g (G): the PDF (CDF respectively) characterizing the variable e;
- r: the unit selling price;
- c: the unit production cost ($c \leq r$);
- w_{ij} the unit wholesale price paid by the e-Retailer under Approach j ($j = 1, 2, 3$) and scenario i ($i = DU, DC$) - Scenario C is not applicable since the supply chain is centralized, i.e.; there is not a wholesale price;
- b_{DCj}: the unit buy-back cost used under the Decentralized Coordinated scenario under Approach j ($j = 1, 2, 3$);
- s: the unit salvage cost ($s \leq c$);
- P: the unit cost paid for a non satisfied commitment;
- Q^*_{ij}: the optimal ordering quantity of Approach j ($j = 1, 2, 3$) under scenario i ($i = C, DU, DC$);
- $\pi_{ij}(.)$: the expected profit function of Approach j ($j = 1, 2, 3$) under scenario i ($i = C, DU, DC$);
- π^*_{ij}: the optimal expected profit of Approach j ($j = 1, 2, 3$) under scenario i ($i = C, DU, DC$).

3. The Optimal Inventory Policy under Approach 1

3.1 Analysis of the Centralized (C) Case

In the centralized scenario, both the e-Retailer and the manufacturer are part of the same organization and managed by the same institutional entity. As a consequence, we can ignore the wholesale transaction and the relevant price since these are not relevant. We note that the results provided in this section improve the ones provided in Rekik (2011). We deliberately present the centralized case in this paper for the sake of completeness and due to the fact that the results associated with the decentralized scenarios are to be contrasted to the centralized ones in the managerial insights section of this paper.

If D denotes the customers' demand, the sequence of events described in Section 2.1 enables us to deduce that the retailer's commitment is $C = Min(Q_{IS}, D)$. The sales achieved then are $Sales = Min(C, Q_{PH})$. For a given vector (D, Q_{PH}, Q_{IS}), the profit achieved by the inventory manager under the centralized case is given by:

$$\begin{aligned} Profit = rMin\left[Min(Q_{IS}, D), Q_{PH}\right] + s\left[Q_{PH} - Min(Q_{IS}, D)\right]^+ \\ -cQ_{PH} - P\left[Min(Q_{IS}, D) - Q_{PH}\right]^+ \end{aligned} \quad (1)$$

Elementary algebra and simplification enable us to write the achieved profit as expressed in the following result:

Result 1. *For a given vector (D, Q_{PH}, Q_{IS}), the profit achieved under the* *centralized case is as the following:*

$$\begin{aligned} Profit = (r-c)D - (c-s)\left[Q_{IS} - D\right]^{+} - (r-c)\left[D - Q_{IS}\right]^{+} \\ -(r-c+P)\left\{(Q_{IS} - Q_{PH}) - Min\left[(Q_{IS} - D)^{+}, (Q_{IS} - Q_{PH}\right]\right\} \\ +(c-s)Min\left(\left[Q_{IS} - D\right]^{+}, (Q_{IS} - Q_{PH})\right) \end{aligned} \quad (2)$$

where

- $(r-c)D$ *corresponds to the expected sales revenue.*
- $-(c-s)\left[Q_{IS} - D\right]^{+} + (c-s)Min\left(\left[Q_{IS} - D\right]^{+}, (Q_{IS} - Q_{PH})\right)^{+}$ *corresponds to the penalty related to an overstocking situation (**overage penalty**).*
- $-(r-c)\left[D - Q_{IS}\right]^{+}$ *corresponds to the penalty incurred if a demand is not satisfied when answering customers' requests (**type 1 underage penalty**).*
- $-(r-c+P)\left\{(Q_{IS} - Q_{PH}) - Min\left[(Q_{IS} - D)^{+}, (Q_{IS} - Q_{PH})\right]\right\}$ *corresponds to the penalty incurred when a commitment is made and then not respected (**type 2 underage penalty**).*

Proof. The proof follows directly from the application of some elementary algebra on Equation (1). □

As previously mentioned, we consider an additive error structure. Considering the variables $D_m = D - e_{IS}$ and $e = e_{IS} - e_{PH}$, the profit function achieved under the Centralized scenario could be expressed as follows:

$$\begin{aligned} Profit = (r-c)D - (c-s)(Q - D_m)^{+} - (r-c)(D_m - Q)^{+} \\ -(r-c+P)\left\{e - Min\left[(Q - D_m)^{+}, e\right]\right\} + (c-s)Min\left[(Q - D_m)^{+}, e\right] \end{aligned} \quad (3)$$

The following result states the expression of the expected profit, $\pi_{C1}(Q)$, achieved under the Centralized scenario by the inventory manager for a given ordering quantity Q:

Result 2. *The Expected profit for the Centralized scenario is given by:*

$$\begin{aligned} \pi_{C1}(Q) = (r-c)\mu - (c-s)\int_{D_m=-\infty}^{Q} (Q - D_m) f_m(D_m) dD_m \\ -(r-c)\int_{D_m=Q}^{+\infty} (D_m - Q) f_m(D_m) dD_m - (r-s+P)E\left[A\right] + (c-s)E[e] \end{aligned} \quad (4)$$

where

$$E\left[A\right] = \int_{e=0}^{+\infty} \left[e\left[1 - F_m(Q)\right] + \int_{D_m=Q-e}^{Q} \left[e - (Q - D_m)\right] f_m(D_m) dD_m \right] g(e) de$$

Proof. By defining $A = \{e - Min\left[(Q - D_m)^{+}, e\right]\}$ and by observing that $A = 0$ if $e < 0$, the proof of this result is a straightforward exercise after applying expectations and simplifying the profit function expressed in Equation 3. □

Under Approach 1, the inventory manager is aware of the errors occurring in the inventory system. Further an accurate estimation procedure is assumed to be in place, so that the parameters (mean and variance) associated with the distribution of e_{IS} and e_{PH} are known. Based on this information, the optimal ordering decision under Approach 1 may be derived considering the following theorem.

Theorem 1. *Under Condition 1, the expected profit function is concave and there exists a unique optimal ordering quantity Q^*_{C1} that maximizes the expected function $\pi_{C1}(Q)$. Q^*_{C1} solves the following equation:*

$$(r-s)F_m(Q^*_{C1})-(r-c)+(r-s+P)\int_{e=0}^{+\infty} g(e)\left[F_m(Q^*_{C1}-e)-F_m(Q^*_{C1})\right]de = 0 \tag{5}$$

where

- *Condition 1: the unit cost paid if a commitment is non satisfied is such that: $P \leq (r-s)\frac{G(0)}{1-G(0)}$*

The optimal expected profit, $\pi^*_{C1} = \pi_{C1}(Q^*_{C1})$, is given by:

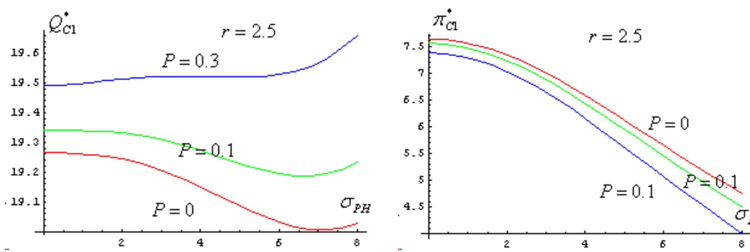
$$\pi^*_{C1} = (r-s)\int_{D_m=-\infty}^{Q^*_{C1}} D_m f_m(D_m)dD_m - (c-s)E[e]$$
$$-(r-s+P)\int_{e=0}^{+\infty}\left[e - e.F(Q^*_{C1}-e) + \int_{D_m=Q^*_{C1}-e}^{Q^*_{C1}} D_m f(D_m)dD_m\right] g(e)de \tag{6}$$

Proof. Cf. Appendix A. Please note that all the appendices of the paper are provided in the Electronic Companion. □

The reader could remark that an improved concavity condition on the optimal ordering strategy is provided in our paper compared with Rekik (2011) where two optimality conditions were proposed (one of them was concerned with the distribution of the random variable D_m).

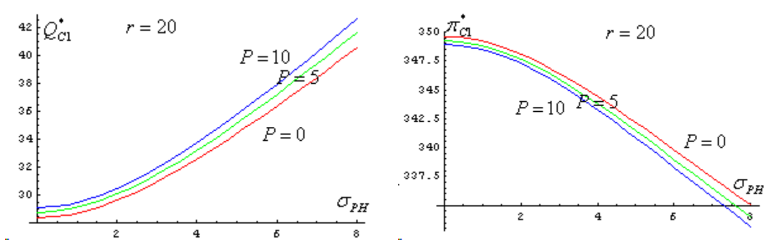
It could also be noticed that results associated with the Retailing context (where the IS level does not play a role in the demand satisfaction processes since the customers are physically present in the store and their demands are conrfonted to the PH level) could be derived from our results of the e-Retailing context by setting $IS = PH$ and $P = 0$ (no comittement in the Retailing context). For instance Equation 5 would apply to the Retailing context if we remove from it the intergral part which is specific to the e-Retailing case.

For the remainder of our paper we consider, for demonstration purpose, a numerical example the parameter values of which have been motivated by de Kok et al. (2008), Atali et al. (2009) and Kök and Shang (2007). The e-Retailer faces a normally distributed demand with a mean $\mu = 20$ and a standard deviation $\sigma = 4$. IS and PH errors are normally distributed with means equal to zero. The unit production cost is set to $c = 2$ and the unit salvage cost is set to $s = 1$. We set $\sigma_{IS} = 3$ and study the evolution of the optimal strategy (optimal ordered quantity and expected profit) as a function of σ_{PH} for different values of the cost parameters r and P. As in the study conducted by de Kok et al. (2008), we consider two particular situations: i) a product with a low margin where $r = 2.5$ and ii) an expensive product with a higher margin where $r = 20$. The single parameter set discussed above is not constraining in terms of the results we offer since additional analysis (not presented here) leads to the same insights.



(a) Optimal ordering quantity (b) Optimal expected profit

Figure 1: Behavior A of the optimal strategy under the centralised case: products with a low margin



(a) Optimal ordering quantity (b) Optimal expected profit

Figure 2: Behavior B of the optimal strategy under the centralised case: products with a high margin

The results illustrated in Figures 1 and 2 demonstrate an intuitively appealing behavior of the expected profit which is always decreasing with the PH error variability as well as with the commitment cost P. With regards to the optimal ordering quantity, there are 2 possible behaviors depending on the margin values:

- For products with a high margin, when the unit shortage penalty (i.e. the sum of the margin loss and the commitment cost, $(r - c) + P$) is more important than the overstock penalty, which is equal to $(c - s)$, Q^*_{C1} is an increasing function of the variability of the PH error. This is a natural consequence of the dominance of the unit shortage penalties that we aim to decrease in the case of high margin products.

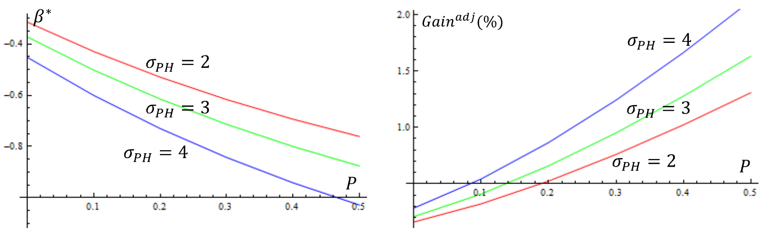
- For products with a low margin and for configurations according to which the sum $(r - c) + P$ is not as important in comparison with the overstock penalty $(c - s)$, Q^*_{C1} decreases with σ_{PH} in a first instance and then the slope of the function changes to become an increasing one on σ_{PH}. In this case, the initial behavior of Q^*_{C1} may be attributed to the dominance of overstocking costs; however, there is a threshold value above which the shortage penalties become dominant resulting in a necessary increase of the ordered quantity. As shown in Figure 1 (a), this value depends on P.

In addition to the concavity condition improvement, we extend in the following the results provided by Rekik (2011) from a managerial point of view by discussing: *i*) A policy adjusting the IS level before performing a commitment; *ii*) The profit loss illustration when the e-retail context is optimized using existing results pertaining to the retail context.

Remark 1.

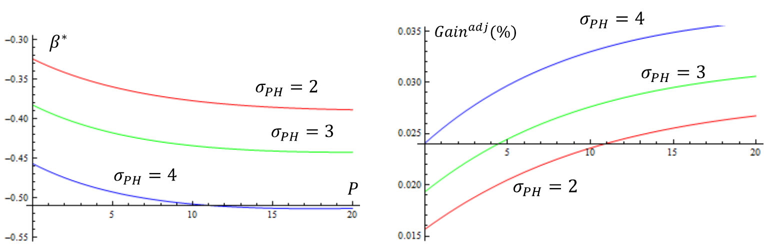
We assumed in our analytical study that the commitment, $C = Min(Q_{IS}, D)$, is decided by contrasting the demand D with the quantity observed in the Information System Q_{IS} since the PH level is not known. Once the optimal ordering quantity Q^* is decided and delivered, and before performing a commitment, the e-Retailer might adjust the IS record in anticipation of a discrepancy between the IS and the PH levels. Being aware that errors are additively modeled, the inventory manager could provide a commitment after changing the IS level from Q_{IS} to an adjusted one equal to $Q^{adj}_{IS} = Q_{IS} + \beta$, where β is an adjustment parameter that could be positive or negative.
By rewriting the Expected profit provided in Eq. 1 where the Q is set equal to the optimal ordering quantity given in Eq. 5 and where the Commitment is replaced by the adjusted one, $C^{adj} = Min(D, Q_{IS} + \beta)$, the e-Retailer obtains an adjusted profit function that could be numerically maximized by finding the best adjustment parameter β^*. Figures 3 and 4 provide the behavior of β^* as well as the relative gain resulting from the adjustment (calculated as a %) with the unit commitment penalty P for different values of the variability of the PH error. One could remark that:

- The magnitude of the adjustment is relatively low when compared with the optimal ordering quantity (cf. Figures 3 (a) and 4 (a)). In fact the inaccuracy issue appears to have been taken into account before ordering. Consequently, the relative percentage gain provided in Figures 3 (b) and 4 (b) also appears not to be particularly marked.
- When an adjustment is being made it should be correctly calculated to obtain a positive impact. A non-optimal adjustment will provide worse results than the non-adjustment scenario.



(a) Optimal ordering quantity (b) Optimal expected profit

Figure 3: Behavior A of the optimal adjustment strategy under the centralised case: products with a low margin



(a) Optimal ordering quantity (b) Optimal expected profit

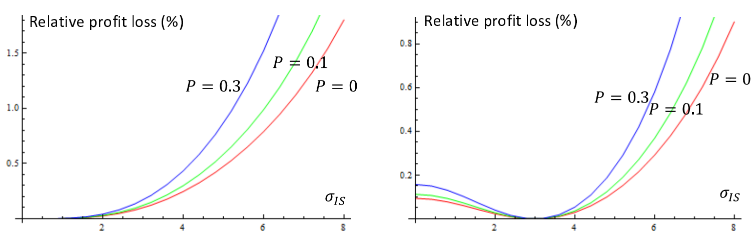
Figure 4: Behavior B of the optimal adjustment strategy under the centralised case: products with a high margin

- As intuitively expected, the absolute value of the adjustment parameter increases with the commitment penalty P and with the variability of the errors. The commitment adjustment aims to tackle the cases where a commitment is not satisfied.
- A less intuitive outcome of the numerical analysis is that the adjustment parameter is always negative meaning that the inventory manager has to underestimate his Q_{IS} level before performing the commitment. Such result could be explained as follows: the adjustment will impact the overage type 2 penalty (i.e. the penalty associated with a commitment non satisfaction) only in the configuration where $Q_{PH} \leq Q_{IS} \leq D$. In such a configuration adjusting positively (negatively) Q_{IS} will decrease the type 1 (type 2 respectively) overage cost. By remarking that the unit type 2 overage cost $(r - c + P)$ is higher than the unit type 1 overage cost $(r - c)$, the adjustment should be made on the negative side so that the commitment non satisfaction may be well avoided.

By noting that the adjustment strategy applied by the e-Retailer does not impact the optimal ordering quantity (and consequentially does not change the material transfer from the manufacturer side) and after remarking that the adjustment gain is relatively low, we assume for the remainder of the paper that the e-Retailer does not intentionally change his commitment by adjusting his IS quantity.

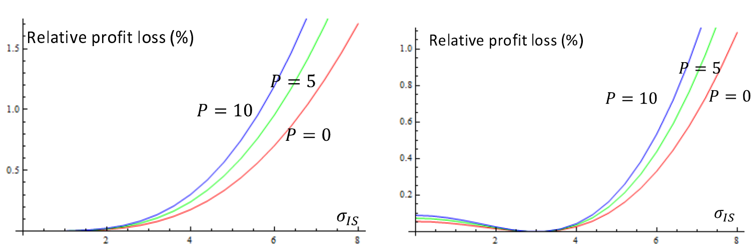
Remark 2.

We end our study of the centralized case by a comparative analysis of our contribution with the performance of the inventory system if it is managed by existing results in the inaccuracy literature. As mentioned in the introduction section, almost all past investigations are concerned with the retail context where the inaccuracy on the IS level is not integrated in the optimal decision. In such situation, the inventory manager could assume that: 1) case 1: the IS errors are simply set equal to zero, i.e., $Q_{IS} = Q$ or 2) case 2: the IS errors are assumed to be equal to the PH errors, i.e., $Q_{IS} = Q_{PH}$. Figures 5 and 6 illustrate the relative profit losses when our optimal inventory policy (provided in Theorem 1) is not applied under the low and the high margin settings (respectively).



(a) Relative loss under case 1 (b) Relative loss under case 2

Figure 5: Relative profit losses (resulted from not employing the optimal policy) under the centralized case: products with a low margin



(a) Relative loss under case 1 (b) Relative loss under case 2

Figure 6: Relative profit losses (resulted from not employing the optimal policy) under the centralized case: products with a high margin

It could be noticed that managing the inventory system under case 1 is more penalizing than case 2 because IS errors are totally ignored. Case 2, known in the literature as the random yield, improves the performance but remains a suboptimal policy when Q_{IS} is different from Q_{PH}.

3.2 Analysis of the Decentralized Uncoordinated (DU) Case

Under the Decentralized Uncoordinated scenario, we assume that the manufacturer and the e-Retailer are two independently owned and managed firms, where each party is trying to maximize its own expected profit. We analyze in this section the case where the two supply chain actors do not coordinate their decisions and for this purpose, we consider the wholesale contract: the manufacturer chooses the unit wholesale price w_{DU1} and the e-Retailer, after observing w_{DU1} chooses the order quantity Q_{DU1}.

The e-Retailer's Decision: The e-Retailer's decision is the same as the one in the Centralized scenario with the exception that the e-Retailer now pays a wholesale price w_{DU1} to the manufacturer whose unit production cost is still c. The expected profit for the e-Retailer is as follows:

$$\pi_{DU1}^{e-retailer}(Q, w_{DU1}) = (r - w_{DU1})\mu - (w_{DU1} - s)\int_{D_m=-\infty}^{Q}(Q - D_m)f_m(D_m)dD_m$$
$$-(r - w_{DU1})\int_{D_m=Q}^{+\infty}(D_m - Q)f_m(D_m)dD_m - (r - s + P)E\left[A\right] - (w_{DU1} - s)E[e] \quad (7)$$

where:

$$E\left[A\right] = \int_{e=0}^{+\infty}\left[e\left[1 - F_m(Q)\right] + \int_{D_m=Q-e}^{Q}\left[e - (Q - D_m)\right]f_m(D_m)dD_m\right]g(e)de$$

As shown in the analysis conducted for the centralized case, under Condition 1 introduced in Theorem 1, the e-Retailer's profit function is concave and the optimal ordering quantity is given by:

$$(r - s)F_m(Q_{DU1}^{*}) - (r - w_{DU1})$$
$$+(r - s + P)\int_{e=0}^{+\infty}g(e)\left[F_m(Q_{DU1}^{*} - e) - F_m(Q_{DU1}^{*})\right]de = 0 \quad (8)$$

The Manufacturer's Decision: The manufacturer is concerned with the wholesale price w_{DU1} as a decision variable. The e-Retailer's order may be anticipated (known in advance) for any wholesale price. Consequently, the function $Q_{DU1}(w_{DU1})$ is deterministic as far as the manufacturer is concerned. The manufacturer's decision then is to choose the wholesale price w_{DU1} that maximizes his expected profit $\pi_{DU1}^{Manufacturer}(w_{DU1})$ which is as follows:

$$\pi_{DU1}^{Manufacturer}(w_{DU1}) = (w_{DU1} - c)Q_{DU1}(w_{DU1}) \quad (9)$$

Theorem 2. *Assuming that condition 1, introduced in the previous section, and Condition 2 (discussed below) hold:*

1. *The optimum is reached for* Q^*_{DU1}*, such that:*

$$1-a[F_m(Q^*_{DU1})+Qf_m(Q^*_{DU1})]-d\int_{e=0}^{+\infty}[F_m(Q^*_{DU1}-e)+Q^*_{DU1}f_m(Q^*_{DU1}-e)]g(e)de=0$$

2. *The corresponding optimum wholesale price is:*

$$w^*_{DU1}=c+(r-c)\{aQ^*_{DU1}f_m(Q^*_{DU1})+d\int_{e=0}^{+\infty}Q^*_{DU1}f_m(Q^*_{DU1}-e)g(e)de\}$$

3. *The optimal expected profit of the manufacturer is:*

$$\pi^{manufacturer^*}_{DU1}=(r-c)Q^{*^2}_{DU1}\{af_m(Q^*_{DU1})+d\int_{e=0}^{+\infty}f_m(Q^*_{DU1}-e)g(e)de\}$$

4. *The optimal expected profit of the e-Retailer is:*

$$\pi^{e-retailer^*}_{DU1}=\pi^{e-retailer}_{DU1}(Q^*_{DU1},w^*_{DU1})$$

Where:

- $a=\frac{(r-s)-(r-s+P)(1-G(0))}{r-c}$ *and* $d=\frac{r-s+P}{r-c}$
- *Condition 1:* $p\leq(r-s)\frac{G(0)}{1-G(0)}$
- *Condition 2: The random variable* D_m *is IGFR*[2] *and is such that* $f_m/f_m^{'}$ *is an increasing function.*

Proof. Cf. Appendix B. □

Remark 3. *We note that the IGFR condition is a well known one for wholesale type contracts and is not restrictive in the sense that most common demand distributions confirm it (Larivière and Porteus (2001)). Common demand distributions, in particular the normal one, verify also the* $f_m/f_m^{'}$ *increase condition.*

For the same numerical example introduced in section 3.1 ($\mu=20$, $\sigma=4$, $c=2$, $s=1$ and $\sigma_{IS}=3$), Figures 7 and 8 illustrate the behaviors of the ordering strategy (optimal ordering quantity and optimal wholesale cost) as a function of σ_{PH} for different values of P and r. As in the centralized case, two possible behaviors of Q^*_{DU1} could be observed:

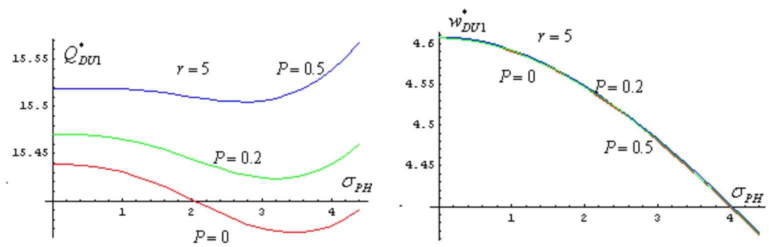
- Behavior A where the selling price is set to a low level resulting in a low margin for the e-Retailer (the selling price is $r=5$ and the proposed wholesale cost is around 4.5 as illustrated in Figure 7 (b))[3] . With a low

[2]Increasing General Failure Rate. A distribution is IGFR if its General Failure Rate defined by the function $g(x)=x\frac{1-F(x)}{f(x)}$ is weakly increasing for all x such that $F(x)<1$.

[3]This is a different r value than that utilized in the previous section in order to illustrate the insights of our analysis in the best possible way. Please also note that the very graphical presentation (e.g. scales used, origin of axes, etc) is 'optimised' by MATHEMATICA.

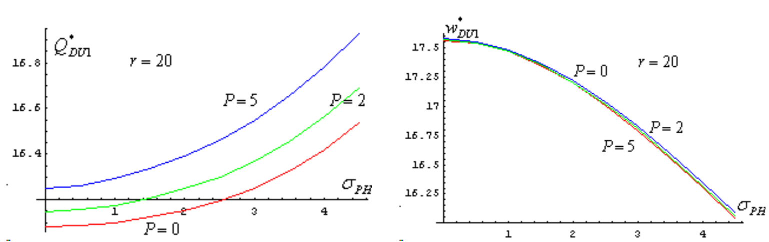
margin, the overage penalty is more important than the underage ones (types 1 & 2) and as a consequence we can observe a decrease of the ordering quantity with σ_{PH} in the first part of the respective curves. For higher σ_{PH}, errors are more impacting costs (especially the shortage and commitment ones) and as a consequence, we can observe an increasing Q^*_{DU1} to tackle the inaccuracy impact.

- Behavior B where the selling price is set to a higher level resulting in a higher margin for the e-Retailer (the selling price is $r = 20$ and the proposed wholesale cost is around 17 as illustrated in Figure 8 (b)). In such cost configuration, the overage penalty is less important than the underage ones and as consequence we always observe an increasing Q^*_{DU1} with σ_{PH}.



(a) Optimal ordering quantity (b) Optimal wholesale cost

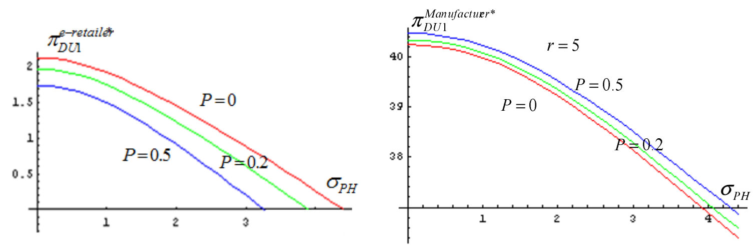
Figure 7: Behavior A of the optimal quantity and wholesale cost under the DU case: products with a low margin



(a) Optimal ordering quantity (b) Optimal wholesale cost

Figure 8: Behavior B of the optimal quantity and wholesale cost under the DU case: products with a high margin

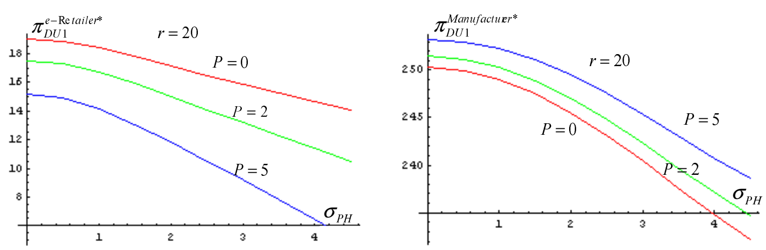
As intuitively expected in both behaviors, the ordering quantity increases with the commitment cost P but we remark that the proposed wholesale price (provided by the manufacturer) is insensitive to changes of P value. Such remark enables us to explain the following observations on expected profits: in contrast to the e-Retailer, the manufacturer obtains a higher profit when P increases (as illustrated in Figures 9 and 10). In fact, when P increases, the e-Retailer orders a higher quantity while the wholesale price stays unchanged which increases (decreases) the expected profit of the Manufacturer (e-Retailer).



(a) Optimal Expected Profit for the e-Retailer

(b) Optimal Expected Profit for the Manufacturer

Figure 9: Behavior A of the optimal expected profit under the DU case: products with a low margin



(a) Optimal Expected Profit for the e-Retailer

(b) Optimal Expected Profit for the Manufacturer

Figure 10: Behavior B of the optimal expected profit under the DU case: products with a high margin

3.3 Analysis of the Decentralized Coordinated (DC) Case

For the classical newsvendor problem, many solutions have been proposed to improve the supply chain performance. In a classical buy-back contract, the e-Retailer pays a wholesale price w_{DCj} $(j = 1, 2, 3)$ per unit ordered but can return the excess order quantity at a partial refund b_{DCj} $(j = 1, 2, 3)$ at the end of the selling season (Pasternack (1985)). As discussed by Cachon (2003), we assume that the e-Retailer salvages the units and the manufacturer credits him for the units subject to the buy-back agreement. Under Approach 1, we consider a modified buy back contract according to which the manufacturer buys back only the quantities that have not been subject to IS errors. Quantities that can be bought-back by the manufacturer correspond to the case where the PH inventory is higher than the IS record and when the IS is lower than D. That is, when overage stock is due to errors in the IS level, the relevant excess quantity is not bought-back by the manufacturer. By such contract configuration, we assume that the manufacturer is not responsible of errors affecting the IS of the e-Retailer

The coordination solution of our problem necessitates three decision variables to be determined: the wholesale price w_{DC1}, the buy-back payment b_{DC1} and the ordering quantity Q_{DC1}. It is obvious that there are many possible combinations with respect to who is determining which decision variables. The determination of all three variables by either the manufacturer or the e-Retailer alone constitutes an extreme case. Lariviere (1998) presents a quantity-forcing contract that eliminates the e-Retailer's choice. If such a contract allows coordinating performances for the supply chain, the profits' share is generally not impartial. In practice, the determination of either one of, or both, w_{DC1} and b_{DC1} by the e-Retailer is not reflective of real-world practices. Indeed, the e-Retailer tends to maximize b_{DC1} or to minimize w_{DC1} in order to optimize his objective function that will most probably yield to inefficient supply chain solutions and/or non-appealing solutions to the manufacturer. A more realistic case is when the manufacturer determines w_{DC1} and b_{DC1} and the e-Retailer determines Q^*_{DC1}. In certain cases, the market may impose certain values on some of these variables (such as the type of competition, etc.). In a newsvendor setting, the imposed variable is generally, the wholesale price w_{DC1}.

The e-Retailer's Decision: The e-Retailer decision is the same as the one under a wholesaling contract with two exceptions: *i)* The salvage cost s is replaced by $s + b_{DU1}$; *ii)* Since the manufacturer does not buy-back overage quantities resulting from IS errors, the e-Retailer's profit is decreased by b_{DU1} multiplied by the quantity that is not bought-back in such a situation.

There are two scenarios according to which quantities are not bought-back by the manufacturer: *i)* $Q_{PH} \geq D \geq Q_{IS}$: the quantity which is not bought-back is $D - Q_{IS}$; and *ii)* $D \geq Q_{PH} \geq Q_{IS}$: the quantity which is not bought-back is $Q_{PH} - Q_{IS}$. The expected quantity that is not bought-back by the manufacturer is written as follows:

$$E(B) = \int_{e=-\infty}^{0} [-e\{1 - F_m(Q-e)\} + \int_{x=Q}^{Q-e} (x-Q) f_m(x)dx] g(e)de \qquad (10)$$

Consequently, the e-Retailer's expected profit is given by:

$$\begin{aligned}\pi_{DC1}^{e-retailer}(Q, w_{DC1}, b_{DC1}) = (r - w_{DC1})\mu_D + (w_{DC1} - s - b_{DC1})E[e] - b_{DC1}E[B] \\ -(w_{DC1} - s - b_{DC1}) \int_{D_m=-\infty}^{Q} (Q - D_m) f_m(D_m) dD_m \\ -(r - w_{DC1}) \int_{D_m=Q}^{+\infty} (D_m - Q) f_m(D_m) dD_m - (r - s - b_{DC1} + P)E[A] \quad (11)\end{aligned}$$

For a given (w_{DC1}, b_{DC1}), as in the centralized case, Condition 1 is sufficient to verify the concavity of the e-Retailer expected profit. The optimal ordered quantity should satisfy:

$$\begin{aligned}(r - w_{DC1}) - [-P + (r - s + P)G(0)]F_m(Q^*_{DC1})\\ -(r - s - b + P)\int_{e=0}^{+\infty} g(e)\left[F_m(Q^*_{DC1} - e)\right]de + b\int_{e=-\infty}^{0} F(Q^*_{DC1} - e)g(e)de = 0\end{aligned}$$

The Manufacturer's Decision:
When the e-Retailer orders the quantity Q, the expected profit of the manufacturer is as follows:

$$\begin{aligned}\pi_{DC1}^{manufacturer}(Q, w_{DC1}, b_{DC1}) &= (w_{DC1} - c)Q - b_{DC1}\int_{D_m=0}^{Q}(Q - D_m)f_m(D_m)dD_m\\ &- b_{DC1}E(A) - b_{DC1}E[e] + b_{DC1}E(B) \qquad (12)\end{aligned}$$

To determine the set (w_{DC1}, b_{DC1}) enabling the coordination, we assume that w_{DC1} is fixed and we derive the value of b_{DC1} permitting to have an optimal ordering quantity for the manufacturer (resulting from the optimization of Equation 12) equal to the e-Retailer's one (Equation 11).

Theorem 3. *Under Approach 1, for a given w_{DC1} the channel is coordinated for (b^*_{DC1}, Q^*_{DC1}) solving the following two-variable equation system:*

$$b^*_{DC1} = \frac{w_{DC1} - c}{\int_{e=-\infty}^{+\infty} F_m(Q^*_{DC1} - e)g(e)de}$$

$$\begin{aligned}(r - w_{DC1}) - [-P + (r - s + P)G(0)]F_m(Q^*_{DC1})\\ -(r - s - b^*_{DC1} + P)\int_{e=0}^{+\infty} g(e)\left[F_m(Q^*_{DC1} - e)\right]de\\ +b^*_{DC1}\int_{e=-\infty}^{0} F(Q^*_{DC1} - e)g(e)de = 0\end{aligned}$$

Proof. cf. Appendix C. □

At this point we should remark that the optimal ordering quantity under the decentralized coordinated scenario is equal to the one under the Centralized Scenario $Q^*_{DC1} = Q^*_{C1}$ where :

$$(r - s)F_m(Q^*_{C1}) - (r - c) + (r - s + P)\int_{e=0}^{+\infty} g(e)\left[F_m(Q^*_{C1} - e) - F_m(Q^*_{C1})\right]de = 0$$

For a given w_{DC1}, the manufacturer is able to coordinate the supply chain by offering a buy-back unit cost equal to:

$$b^*_{_{DC1}} = \frac{w_{DC1} - c}{\int_{e=-\infty}^{+\infty} F_m(Q^*_{C1} - e)g(e)de} \tag{13}$$

In other words, when the pricing rules defined by the 2-tuple (w_{DC1}, b^*_{DC1}) are established by the manufacturer, the ordering quantity decided by the e-Retailer coincides with the ordering quantity of the Centralized scenario $Q^*_{DC1} = Q^*_{C1}$ and the optimal expected profit of the whole supply chain is also the one achieved under the centralized case.

4. Managerial Insights: Information and RFID impacts

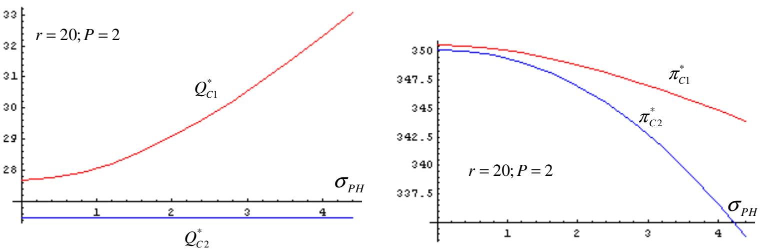
The aim of this section is twofold: *i*) To analyze the impact of errors on the inventory system and in particular to assess how the performance could be improved by taking into account errors when establishing the ordering decision. That is, we provide a comparison between approaches 1 & 2; and *ii*) To analyze the impact of the RFID technology on the inventory system. To do so, we will study the comparative performance of approaches 1 & 3 and we will derive conditions on the RFID tag cost that render the RFID deployment cost effective. Please recall that results related to Approaches 2 and 3 are presented in the Electronic Companion of the paper. It is also important to recall that we assume in this paper that the RFID deployment leads to a visibility on errors permitting ta align the PH and the IS levels. The case where RFID prevents errors could also analysed by modeling the RFID approach using Approach 1 results by considering lower errors parameters. For the numerical analysis, we consider the same parameters values used in the previous sections ($\mu = 20$; $\sigma = 4$; $c = 2$; $s = 1$; $\sigma_{IS} = 3$) but we limit the presentation to products with a high margin ($r = 20$) only. The insights derived below are the same for products with a low margin as well.

4.1 Comparison of Approaches 1 & 2: Benefit of the Information on Errors

To analyze the potential effects of an ordering strategy that takes into account information on errors (their probability distributions), let us compare numerically the optimal expected profits for each supply chain actor under approaches 1 & 2.

4.1.1 The Centralized (C) Case

Under the centralized scenario, it is straightforward to observe (as illustrated in Figure 11 (b)) and to prove that the benefit of estimating errors is always present since Q^*_{C1} optimizes the expected profit function $\pi_{C1}(.)$. Q^*_{C2} which is used under Approach 2 is a sub-optimal solution for the inventory manager under the centralized case. To tackle the inaccuracy issue, Q^*_{C1} should be higher than Q^*_{C2} as illustrated in Figure 11 (a).



(a) Q^*_{C1} vs Q^*_{C2} (b) π^*_{C1} vs π^*_{C2}

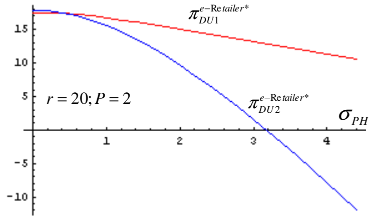
Figure 11: Implications of estimating errors under the C scenario

4.1.2 The Decentralized Uncoordinated (DU) Case

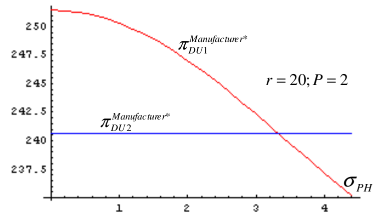
Under the Decentralized Uncoordinated scenario, the situation is different (cf. Figure 12) since the manufacturer may in fact face some negative consequences when estimating and taking into account error distributions. When errors are estimated and taken into account in the ordering strategy, the e-Retailer tends to order a higher quantity than the error free model (cf. figure 12 (d)). When the ordered quantity is higher, the wholesale price provided by the manufacturer is lower (as illustrated in figure 12 (c)). As a consequence, estimating errors leads the manufacturer to decrease his unit margin and the expected profit is not improved for the manufacturer even if the ordered quantity is higher.

4.1.3 The Decentralized Coordinated (DC) Case

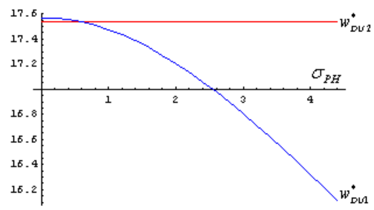
Under the Decentralized Coordinated scenario, the power of each supply chain actor is controlled as stated in Result 5 (provided in the Electronic Companion of the paper) by the value of ϵ. For a given value of ϵ, the wholesale price is deduced for both approaches and the buy-back cost is calculated such that the channel is coordinated. With regards to the comparison of Approaches 1 & 2, the situation is totally different from what it was the case for the previous scenario since we may observe (cf. Figures 13 and 14) that estimating errors offers improvements to the manufacturer and not to the e-Retailer regardless of how power is assigned in a supply chain (i.e. regardless of whether the manufacturer or the e-Retailer has more power). Such a result may be explained in terms of the design of the modified buy-back contract under Approach 1. In fact, recall that we assumed the manufacturer to buy back only quantities that have not been subject to IS errors. Obtaining information about error distributions permits to change the ordering vector $(Q_{DC1}, w_{DC1}, b_{DC1})$ and in particular to increase the ordering quantity which results in a higher risk that the e-Retailer finishes the selling period with more unsold inventory that won't be bought-back by the manufacturer.



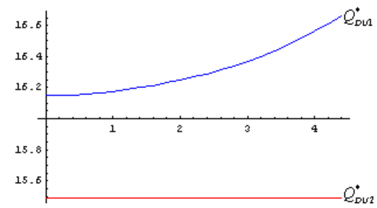
(a) Benefit achieved by the e-Retailer



(b) Benefit achieved by the Manufacturer

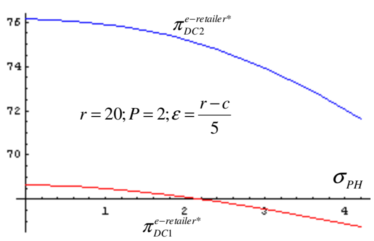


(c) Evolution of the wholesale cost

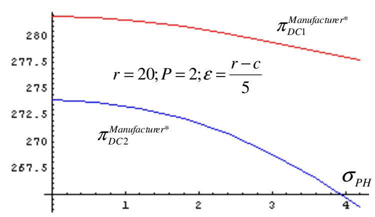


(d) Evolution of the ordered quantity

Figure 12: Implications of estimating errors under the DU scenario



(a) Benefit achieved by the e-Retailer



(b) Benefit achieved by the Manufacturer

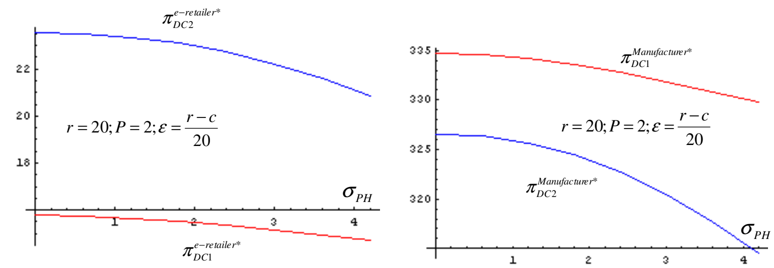
Figure 14: Implications of estimating errors under the DC Scenario: power is assigned to the e-Retailer $\epsilon = \frac{r-c}{5}$

4.2 Comparison of Approaches 1 & 3: the Impact of the RFID Deployment

The aim of this section is to provide the conditions under which the RFID deployment is cost effective. We do so by comparing approaches 1 & 3 under the three supply chain structures considered in our work.

4.2.1 The Centralized (C) Case

Under the centralized scenario, we are able to provide an analytical expression of the critical RFID tag cost over which RFID deployment is not recom-



(a) Benefit achieved by the e-Retailer

(b) Benefit achieved by the Manufacturer

Figure 13: Implications of estimating errors under the DC Scenario: power is assigned to the Manufacturer $\epsilon = \frac{r-c}{20}$

mended. This is stated in the following theorem:

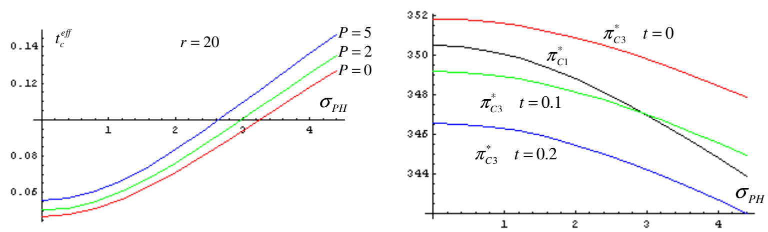
Theorem 4. *We can identify a critical tag cost t_c such that for $t \geq t_c$ the implementation of the RFID technology yields a negative benefit under the centralized scenario. t_c solves:*

$$\int_{F_m^{-1}[\frac{r-c-t_c}{r-s}]}^{F_m^{-1}[\frac{r-c}{r-s}]} D_m f_m(D_m) dD_m = \frac{r-s+P}{r-s}(1-G(0)) \tag{14}$$

Proof. Let's define $Q_{RY} = F_m^{-1}[\frac{r-c}{r-s}]$ which corresponds to the optimal quantity ordered under a centralized random yield problem with an additive error setting (cf. Rekik et al. (2007b)). First, it is straightforward to verify that $Q_{C3}^* \leq Q_{RY} \leq Q_{C1}^*$ by comparing the expression of Q_{RY} with the ones of Q_{C1}^* and Q_{C3}^* provided by equations (5) and (8) (Appendix D in the Electronic Companion of the paper), respectively.

If t is such that $t \geq t_c$, we deduce that $\int_{F_m^{-1}[\frac{r-c-t}{r-s}]}^{F_m^{-1}[\frac{r-c}{r-s}]} D_m f_m(D_m) dD_m \geq \frac{r-s+P}{r-s}(1-G(0))$ which could also written as $\int_{Q_{C3}^*}^{Q_{RY}} D_m f_m(D_m) dD_m \geq \frac{r-s+P}{r-s}(1-G(0))$. Using the fact that $Q_{RY} \leq Q_{C1}^*$, we deduce that $\int_{Q_{C3}^*}^{Q_{C1}^*} D_m f_m(D_m) dD_m \geq \frac{r-s+P}{r-s}(1-G(0))$ which can easily be used to verify that $\pi_{C1}(Q_{C1}^*)$ (Eq. 6) is higher than $\pi_{C3}(Q_{C3}^*)$ (Eq. (8) of Appendix D in the Electronic Companion of the paper). $\square$

It is important to note that the analytical expression of t_c provided in Theorem 4 corresponds to the maximum value of the RFID tag cost over which the RFID technology is not beneficial from an economical point of view. The effective RFID critical tag cost, t_c^{eff} under which RFID is cost effective solves the equation $\pi_{C1}^* = \pi_{C3}^*$ as illustrated in Figure 15 (b). Solving numerically $\pi_{C1}^* = \pi_{C3}^*$ permits us to deduce the results presented in Figure 15 (a) where the behavior of t_c^{eff} is illustrated. Figure 15 (a) illustrates the behavior of the critical tag cost t_c as a function of the PH error variability and P.



(a) Evolution of t_c^{eff} with σ_{PH} and P

(b) Approach 1 vs Approach 3

Figure 15: Implications of deploying the RFID technology under the C scenario

We notice that t_c^{eff} increases with both σ_{PH} and P: that is, when errors result in higher penalties to the performance of the inventory system, the RFID deployment is easier to justify from an economical point of view.

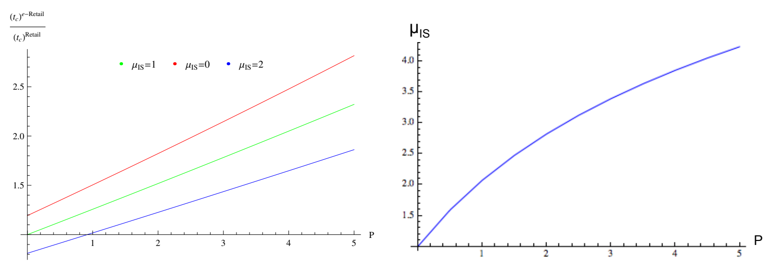
We extend in the following the managerial insight under the centralized scenario by comparing the attrictiveness of the RFID technology in the Retailing and e-Retailing context. The critical RFID tag cost above which RFID is not cost effective in the Retailing context could be derived from Theorem 4 by setting the right hand side of equation equal to $\frac{1}{2}$.

A comparative study between the Retailing and the e-Retailing context permits to derive interesting managerial insights concerning the degree of the RFID attractiveness:

- Figure 16 (a) illustrates the ratio between the critical tag cost in the retailing and e-Retailing context. A ratio higher than one means RFID is cost effective up to a critical tag price, which is higher in the e-Retailing context than the retailing one; that is, RFID is easier to economically justify in the e-Retailing context than the Retailing one.

- According to figure 16 (a), RFID is always easier to justify in the e-Retailing context if the IS error average is lower than the PH one; this is independent of the value taken by the commitment unit cost P. If the IS average stock is underestimated, both the u_1 and h penalties are higher in the e-Retailing context as compared to the Retailing one and consequently the need for RFID to provide visibility is more important.

- In the case where the IS average is higher than the PH one, it exists a couple (P, μ_{PH}) permitting to delimit the attractiveness of RFID (cf. Figure 16 (b)). Compared to the retailer, for small $IS - PH$ inaccuracies and small P values, the e-Retailer could profit from the inaccuracies by committing more (so decreasing his u_1 penalty) and additionally profiting from a smaller end stock (which decreases his h penalty). In such a case the e-Retailer's interest to the RFID technology is lower than that of the retailer until the commitment penalties (u_2) become more important.



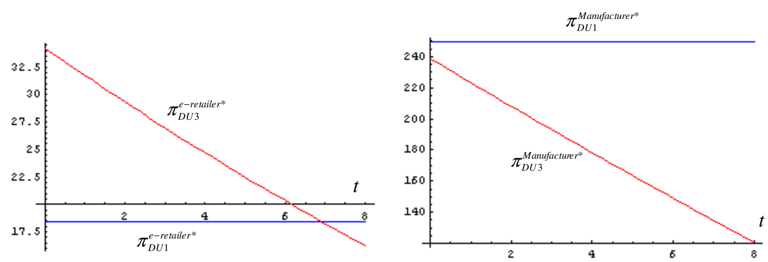
(a) Critical RFID tag cost: e-Retailing vs Retailing

(b) Conditions under which RFID is more attractive in the e-Retailing context

Figure 16: RFID attractivenss: e-Retail vs Retail

4.2.2 The Decentralized Uncoordinated (DU) Case

Comparing approaches 1 & 3 under the DU scenario (cf Figure (17)), permits us to deduce an interesting impact of the RFID deployment. The manufacturer would be better off without RFID since inaccuracy errors permit him to produce and sell more units to the e-Retailer. In addition, since RFID offers visibility to the e-Retailer the relevant benefit depends on the gap between IS and PH errors. In Figure (17), the benefit of the e-Retailer and manufacturer is presented as a function of the RFID tag cost, and for $r = 20$, $P = 0$, $\sigma_{PH} = 1$ and $\sigma_{IS} = 3$.



(a) The e-Retailer benefit

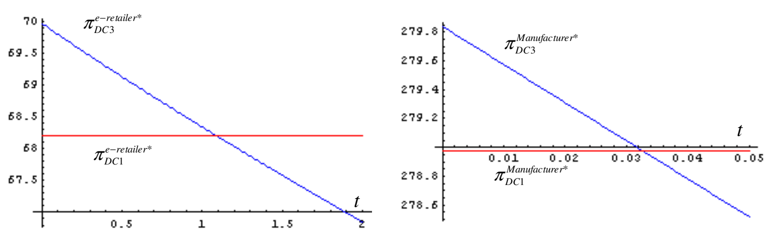
(b) The Manufacturer benefit

Figure 17: Implications of deploying the RFID technology under the DU scenario

As mentioned in the Appendix associated with the Decentralized RFID scenario without coordination (cf. Remark 4 in the Electronic Companion), the research and practical question associated with the RFID tag cost sharing between the two supply chain actors has no particular meaning under the 'take it or leave it' contract with the manufacturer being the Stackelberg leader. In fact the manufacturer is the one that fixes the wholesale price w and will include it in the additional RFID tag cost if he is charged with the technology cost. Our managerial findings are different from the result provided by (Choi, 2011) where the problem is modeled in a way that the RFID could influence the demand profile. In the presence of the RFID technology, the author assumes that the real demand is being met because shelf replenishment may be performed in a continuous and accurate way. Without RFID, the replenishment is done on a periodic basis and consequently the effective demand is shrunk by the shelf capacity. In addition to this difference concerning demand modeling, the author also considers risk analysis in his framework and shows that the manufacturer's level risk can be reduced when RFID is deployed.

4.2.3 The Decentralized Coordinated (DC) Case

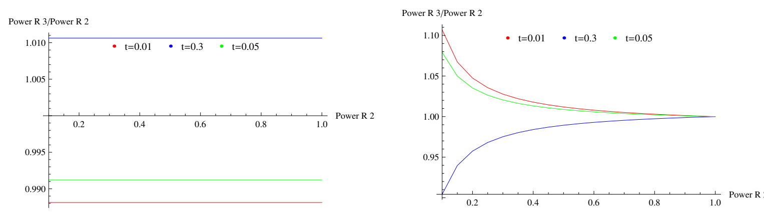
The implications of RFID are different under the DC scenario since by coordinating the channel, some conditions on the RFID tag cost should be respected so that the technology deployment is cost effective for both supply chain actors. In fact, critical RFID tag costs ensuring a positive benefit, could be deduced for each supply chain actor (cf Figure (18)). Such critical RFID costs are linked directly to the coordination parameters, especially, to the parameter ϵ which governs the power of each actor. The implication of using the RFID technology are presented in Figure 18 for $r = 20$, $P = 0$, $\sigma_{PH} = 1$, $\sigma_{IS} = 3$ and $\epsilon = \frac{r-c}{5}$. For a given power configuration of each SC actor in Approach 2, we can derive the unit RFID tag cost under which the technology deployment is cost effective and ensures the same power sharing under the RFID Approach (i.e. the sharing configuration of the total supply chain profit is the same with and without RFID).



(a) The e-Retailer benefit (b) The Manufacturer benefit

Figure 18: Implications of deploying the RFID technology under the DC scenario

We further extend the analysis by assuming that the RFID tag cost is fixed (exogenous variable fixed by the RFID tag market) and we derive the potential loss or gain in power for each SC actor to ensure that the technology deployment is cost effective. In other words, we assess who should make concessions in his power distribution and at which scale in order to enable the RFID deployment.



(a) The e-Retailer power impact

(b) The Manufacturer power impact

Figure 19: Impact of deploying the RFID technology on the power sharing configuration

Figure 19 (a) (Figure 19 (b)) illustrates for an initial e-Retailing power under Approach 2, Power R 2, what the e-Retailing power under Approach 3 should become from a e-Retailer's (Manufacturer) perspective. For example, let us assume that the e-Retailer power is 0.25 under approach 2 (which means that the e-Retailer has 25% of the centralized supply chain profit under DC2 and the manufacturer has the remaining 75%) and let's suppose that the RFID tag price is t=0.01 (red curves). According to Figure 19 (b) (Figure 19 (a)), the Manufacturer (e-Retailer respectively) would accept to deploy RFID if the new e-Retailer power under approach 3 becomes equal to 1.03 (0.98 respectively) the power under approach 2, i.e. 25.75% (24.50% respectively). In other words, the Manufacturer (e-Retailer) could decrease his power by 0.75% (0.5% respectively) in order to adopt the RFID technology if its cost is equal to t=0.01: both of them could make a sacrifice. The Manufacturer's sacrifice will stop and will become a reclaim for additional power for high values of the RFID tag cost (blue curve) when the technology becomes harmful for the entire supply chain (centralized profit under approach 3 is lower than the one under approach 2). It is interesting to notice (not intuitively expected) that the manufacturer's sacrifice decreases with the e-Retailer initial power while the e-Retailer's sacrifice is independent of it. The e-Retailer does not profit from the RFID technology to gain more power and the manufacturer becomes sparing of power sacrifice if his initial power without RFID is higher.

4.3 Summary Managerial Insights

The main managerial contributions of the paper could be summerized as follows:

- Compared to the well-studied Retailing context under inventory inaccuracies, our work in the e-Retailing context showed some conflicting managerial results on the behavior of the optimal ordering strategy. The ordered quantity in the centralized case is not monotonously behaving as shown in Figure 1. Such a behavior impacts the interaction between the e-Retailer and its manufacturer as well as the RFID adoption decision.

- Our results on the e-retail context extend previous retail-related ones since the latter could be derived by setting IS=PH and the commitment penalty $P = 0$.

- Managing the e-retail context subject to inventory inaccuracies with existing results (associated with the error free model, the random yield problem or results specifically developped for the Retailing context) sets the system in suboptimal situations with non-negligible loss in the profit function.

- The e-Retailer could apply an adjustment policy according to which the IS record is changed before providing a commitment. We showed how and when the adjustment could be done and we deduced that the adjustment should be well performed otherwise the system will be set in a suboptimal situation. The adjustment strategy we propose could be linked to recent studies around the superimposition of human judgement on statistically derived decisions in Operations Management, such as demand forecasts (Syntetos et al. (2009), (Syntetos et al., 2010)) or replenishment decisions (Syntetos et al. (2015)).

- We suggest that studying the additive error structure may be a more pragmatic approach since it is not straightforward to empirically identify multiplicative links between error realizations and actual stock levels.

- Regarding the RFID as a solution to tackle inaccuracies, we derived conditions under which its deployment is cost effective. We focused on the comparison of the degree of attractiveness of the technology under the retail and e-retail contexts. We intuitively expected to find RFID more attractive in the e-retail context (because of the additional shortage type 2 cost), but we showed that the errors in the IS could be beneficial in some situations and rendering the RFID less attractive as compared to the retail case.

- In contrast with existing managerial results and intuitive expectations, we showed that the RFID sharing cost between supply chain actors under a wholesale contract is not a relevant issue; we illustrated how the manufacturer could indirectly reflect in the wholesale price his part of the RFID cost.

- Under the same contract, we showed that it is not certain that the e-Retailer will have a manufacturer interested in deploying RFID because errors increase the orders and consequently the margin of the manufacturer. This managerial result depends on the way the RFID impacts the inventory system. In a set of investigations (Gaukler et al. (2007), Choi (2011), RFID presence is shown to improve the profile of the demand and consequently its deployment could be of interest to the manufacturer since it leads to an increase of the sales of the Retailer. In our model, the demand is not impacted by the RFID presence; however, the commitment and the sales are impacted if RFID enables the alignment between the IS and the PH levels. Without RFID the e-Retailer orders more to anticipate errors, which increases the manufacturer revenue. A risk analysis as performed by Choi (2011) would further enrich the managerial insights.

- Under the coordination scenario, RFID could become an interesting alternative to both supply chain actors to avoid the double marginalization effect.

- In addition to finding the threshold RFID tag cost under which the technology is cost effective under the coordination scenario, we also studied the impact on the supply chain power loss and/or gain for each actor if the RFID cost is assumed to be an exogenous variable fixed by the technology market.

5. Conclusions and Further Research

In this paper, we have been concerned with the implications of inventory inaccuracies on the performance of an electronic supply chain channel. An e-Retailing context was considered for demonstration purposes but our results apply to any other upstream supply chain actor (such as a supplier or a distributor) where orders are received electronically. An analytical framework has been developed for the purposes of our work, built on the premise that the inventory inaccuracy problem can be seen as an extended random yield problem. We have proved the connection under concern and studied analytically the effects of inaccuracy errors that may be present either in the physical inventory and/or the IS records. Both centralized and decentralized supply chains have been considered in conjunction with three possible approaches for handling the errors: i) errors are estimated and subsequently the information under concern is utilized for the purpose of improving performance; ii) the errors are ignored, and iii) errors are dealt with through the RFID employment. Comparisons of the first and the third approach with the naive approach where errors are ignored led to important insights with regards to the implications of error estimation and the RFID employment respectively.

In particular, and for a decentralized uncoordinated supply chain, error estimation may lead to a reduction of the manufacturer's profit margin as compared to the case where errors are just ignored. In contrast, the e-Retailer will almost always find such an estimation procedure beneficial. The picture is different for decentralized coordinated supply chains where error estimation offers improvements only to the manufacturer and not to the e-Retailer regardless of how the power between the supply chain players is distributed. For the centralized case, error estimation offers a clear benefit.

With regards to the RFID deployment, performance in the centralized scenario is subject to the RFID tag cost. For a decentralized uncoordinated supply chain, the manufacturer is found to be better off without RFID since inaccuracy errors permit him to produce and sell more units to the e-Retailer. Since RFID offers visibility to the e-Retailer, the relevant benefit depends on the gap between the information system and physical inventory levels. Finally, for the decentralized coordinated scenario, critical RFID tag costs may be deduced for each supply chain actor ensuring a positive benefit; such costs link directly to the power available to the supply chain members.

An important assumption upon which the first approach discussed above was developed is the fact that a mechanism is in place to accurately estimate inaccuracy errors. Although some work has been performed with regards to error estimation (see section 3.2), further contributions that link error estimation methodologies with the inventory inaccuracy problem would be very valuable additions to the current state of knowledge. In addition, it is important to note that our analytical developments have been conducted under the assumption that the inaccuracy errors may be adequately represented by an additive structure. Although such an error representation reflects many real-world cases, it would certainly be worthwhile extending the analysis conducted here to consider the case of multiplicative and mixed error settings as well. Further, our work has been developed upon a single period formulation and in the next steps of our research we plan to extend our proposed framework to the multi-period setting. Finally, and given the prevalence of electronic supply chains in modern business settings, further work into the modelling requirements and idiosyncrasies of such chains would appear to be merited.

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Supplement to "An e-Retailing Decentralized Supply Chain Subject to Inventory Inaccuracies"

6. Appendix A. Proof of Theorem 1

Starting from the definition of $A = \{e - Min\,[(Q - D_m)^+, e]\}$ and by using Leibniz Formula[4], the first derivative of $E\,[A]$ with respect to Q is given by:

$$\begin{aligned}\frac{dE[A]}{dQ} &= -\int_{e=0}^{+\infty}\int_{x_m=Q-e}^{Q} f_m(x_m)g(e)dx_m de \\ &= \int_{e=0}^{+\infty}\left[F_m(Q-e) - F_m(Q)\right]g(e)de\end{aligned}$$

The first derivative of the expected profit function $\pi_{C1}(Q)$ with respect to Q can also be derived:

$$\begin{aligned}\frac{d\pi_{C1}(Q)}{dQ} &= (r-c) - (r-s)F_m(Q) + (r-s+P)\int_{e=0}^{+\infty} g(e)\left[F_m(Q) - F_m(Q-e)\right]de \\ &= (r-c) - [r-s-(r-s+P)(1-G(0))]F_m(Q) \\ &- (r-s+P)\int_{e=0}^{+\infty} g(e)\left[F_m(Q-e)\right]de \\ &= (r-c) - [-P + (r-s+P)G(0)]F_m(Q) \\ &- (r-s+P)\int_{e=0}^{+\infty} g(e)\left[F_m(Q-e)\right]de\end{aligned}$$

The second derivative of the expected profit function is given as follows:

$$\frac{d^2\pi_{C1}(Q)}{d^2Q} = -[-P + (r-s+P)G(0)]f_m(Q) - (r-s+P)\int_{e=0}^{+\infty} g(e)\left[f_m(Q-e)\right]de$$

Under Condition 1, it is clear that the expected profit function is concave and is maximized for Q^*_{C1} verifying:

$$(r-s)F_m(Q^*_{C1}) - (r-c) + (r-s+P)\int_{e=0}^{+\infty} g(e)\left[F_m(Q^*_{C1}-e) - F_m(Q^*_{C1})\right]de = 0$$

7. Appendix B. Proof of Theorem 2

From Equation 9 (provided in the main paper), we derive the relationship between the wholesaling contract and the quantity subsequently ordered by the

[4] Leibniz Formula: $\frac{d}{dy}\int_{a_1(y)}^{a_2(y)} h(x,y)dx = \int_{a_1(y)}^{a_2(y)} \frac{\partial h(x,y)}{\partial y}dx + h\left(a_2(y),y\right)a_2^{'}(y) - h\left(a_1(y),y\right)a_1^{'}(y)$

e-retailer as:

$$w_{DU1}(Q) = r - (r-s)F_m(Q) - (r-s+P)\int_{e=0}^{+\infty}[F_m(Q-e) - F_m(Q)]g(e)de \quad (15)$$

By denoting k=G(0)(r-s+p)-p, the wholesale price is written as:

$$w_{DU1}(Q) = r - kF_m(Q) - (r-s+P)\int_{e=0}^{+\infty}[F_m(Q-e)]g(e)de \quad (16)$$

Such a relationship is deterministic for the manufacturer who has to chose the best wholesale price which maximizes his expected profit $\pi_{DU1}(Q) = (w_{DU1} - c)Q$. Using the methodology employed by Larivière and Porteus (2001), we will deduce a condition on the probability distribution function permitting to show the concavity of the manufacturer's profit function. For this purpose let us denote by $R(Q) = w_{DU1}(Q)Q$ the manufacturer's revenue function. The first derivative of $R(Q)$ is as follows:

$$R^{'}(Q) = w_{DU1}[1 - \frac{1}{v(Q)}] \quad (17)$$

where $v(Q) = \frac{-w_{DU1}}{Q.w^{'}_{DU1}(Q)}$ measures the price sensitivity. It is the percent decrease (increase) in the e-retailer's order from a percent increase (decrease) in the wholesale price at stocking level Q. The second derivative of the revenue function is as follows:

$$R^{''}(Q) = -k[Qf^{'}_m(Q) + 2f_m(Q)]$$

$$-(r-s+p)\int_{e=0}^{+\infty}[Qf^{'}_m(Q-e) + 2f_m(Q-e)]g(e)de$$

If $Q \leq \mu_m$, $R^{''}(Q)$ is clearly positive. Otherwise we need two additional assumptions to show that the first order conditions are sufficient to derive the optimal ordering quantity of the decentralized uncoordinated scenario. If k is positive, i.e. if condition 3 is satisfied, and if D_m is IGFR, we can use Larivière and Porteus (2001)'s results in addition to condition 2 defined in Theorem 2 to show that the revenue function is unimodal and concave on an interval $[a, \bar{Q})$ ($\bar{Q}$ corresponds to the least upper bound on the set of points such that $v(Q) \geq 1$). Within the interval $[\mu_m, \bar{Q})$, we use the fact that $(Qf^{'}_m(Q) + 2f_m(Q))$ is positive (IGFR assumption) and we show that $\int_{e=0}^{+\infty}[Qf^{'}_m(Q-e) + 2f_m(Q-e)]g(e)de$ is also positive. For this purpose we simply use condition 2: $\frac{f}{f'}$ is increasing and f_m is decreasing for all values higher than the average μ_m. The manufacturer's first order condition may be written as expressed in point 1 of Theorem 2. The proof of the other three points is a direct consequence of the first order condition.

8. Appendix C. Proof of Theorem 3

The first derivative of the manufacturer's expected profit is given as follows:

$$\begin{aligned}
\frac{\partial \pi_{DC1}^{manufacturer}(Q, w_{DC1}, b_{DC1})}{\partial Q} &= (w_{DC1} - c) - b_{DC1} F_m(D_m) dD_m \\
&- b_{DC1} \frac{\partial E(A)}{\partial Q} + b_{DC1} \frac{dE(B)}{dQ} \\
\frac{\partial E(A)}{\partial Q} &= -\int_{e=0}^{+\infty} [F_m(Q) - F_m(Q-e)] g(e) d \\
\frac{\partial E(B)}{\partial Q} &= \int_{e=-\infty}^{0} [F_m(Q) - F_m(Q-e)] g(e) de
\end{aligned}$$

As a consequence :

$$\frac{\partial \pi_{DC1}^{manufacturer}(Q, w_{DC1}, b_{DC1})}{\partial Q} = (w_{DC1} - c) - b_{DC1} \int_{e=-\infty}^{+\infty} F_m(Q-e) g(e) de$$

The second derivative of the manufacturer's expected profit could also be written as follows:

$$\frac{\partial^2 \pi_{DC1}^{manufacturer}(Q, w_{DC1}, b_{DC1})}{\partial^2 Q} = -b_{DC1} \int_{e=-\infty}^{+\infty} f_m(Q-e) g(e) de = -b_{DC1}(f_m \times g)$$

which is clearly negative.
The first condition of the manufacturer's expected profit permits us do derive the expression of the buy-back unit cost:

$$b_{DC1} = \frac{w_{DC1} - c}{\int_{e=-\infty}^{+\infty} F_m(Q-e) g(e) de}$$

For a given w_{DC1} , to coordinate the channel, the optimal ordering quantities associated with the optimization of the manufacturer and the e-retailer's decision should be the same.

9. Appendix D. Results associated with Approaches 2 & 3 which are derived from the classical literature

Approach 2 under the Centralized case: Errors are Ignored

Please recall that errors are not taken into account when establishing the ordering decisions under Approach 2. The e-retailer and the manufacturer's decisions are also independent of the error parameters. Under a newsvendor framework, they will establish their decisions based on the error free models (in

C, DU and DC scenarios), i.e., their decisions for each scenario correspond to the newsvendor ordering strategy independently of the presence of errors.
For the Centralized scenario, the ordered quantity under Approach 2 is simply equal to the optimal newsvendor quantity and is given by:

$$Q^*_{C2} = Q^*_{Newsvendor} = F^{-1}\left[\frac{r-c}{r-s}\right] \tag{18}$$

When ordering Q^*_{C2}, the profit achieved by the inventory manager is not the optimal profit of the basic newsvendor problem, but rather:

$$\pi^*_{C2} = \pi_{C1}(Q^*_{C2}) \tag{19}$$

Approach 3 under the Centralized case: the RFID Enabled Approach

Under Approach 3, the RFID technology leads to the removal of discrepancies between the IS and the PH inventories. We assume that the RFID technology only provides visibility (IS inventory level = Physical inventory level) but does not eliminate the inaccuracy errors. The inventory problem is also the same as the classical random yield problem with an additional cost pertaining to the RFID tag t embedded to each product. The random yield problem under an additive error setting is nothing but the classical newsvendor problem where the demand distribution D is replaced by the random variable D_m (cf Rekik et al. (2007b) for more details). As a consequence the analysis of Approach 3 under the different scenarios (C, DU and DC) is derived from newsvendor related results where the production cost c is replaced by $c+t$ and the demand distribution D is replaced by that of D_m.
For a given vector $(D, Q_{IS} = Q_{PH})$, the profit could be written as follows:

$$\begin{aligned} Profit &= (r-c-t)[D-Q_{IS}]^+ + (c+t-s)[Q_{IS}-D]^+ \\ &= (r-c-t)[D-(Q_{C3}+e_{IS})]^+ + (c+t-s)[(Q_{C3}+e_{IS})-D]^+ \\ &= (r-c-t)[D_m-Q_{C3}]^+ + (c+t-s)[Q_{C3}-D_m]^+ \end{aligned} \tag{20}$$

Under the Centralized scenario, the general form of the expected profit as a function of the model parameters is given by:

$$\begin{aligned} \pi_{C3}(Q_{C3}) &= (r-c-t)\mu - (r-c-t)\int_{D_m=Q_{C3}}^{+\infty}(D_m-Q_{C3})f_m(D_m)dD_m \\ &- (c+t-s)\int_{D_m=-\infty}^{Q_{C3}}(Q_{C3}-D_m)f_m(D_m)dD_m \end{aligned} \tag{21}$$

The expected profit function is concave and is maximized at the value of Q^*_{C3} such that:

$$F_m(Q^*_{C3}) = \frac{r-(c+t)}{r-s} \tag{22}$$

The optimal expected profit for Approach 3 in the Centralized scenario is given by:

$$\begin{aligned}\pi^*_{C3} &= \pi_{C3}(Q^*_{C3}) \\ &= (r-s)\int_{D_m=-\infty}^{Q^*_{C3}} D_m f_m(D_m)dD_m \end{aligned} \tag{23}$$

Approach 2 under the Decentralized Uncoordinated case: Errors are Ignored

Under Approach 2, errors are ignored and the optimal ordering strategy is the same as the one of the error free model, i.e. the newsvendor problem. The following result states the ordering quantity and the wholesale price, resulting from the optimization of the error free framework that is utilized under Approach 2:

Result 3. *Larivière and Porteus (2001)*
Under Approach 2, the first-order condition is sufficient and its solution is a unique global maximum for an IGFR demand distribution

1. *The optimum is reached for Q^*_{DU2}, such that:* $1-F(Q^*_{DU2})-Q^*_{DU2}f(Q^*_{DU2}) = \frac{c-s}{r-s}$
2. *The corresponding optimum wholesale price is* $w^*_{DU2} = c+(r-s)Q^*_{DU2}f(Q^*_{DU2})$

Proof. Cf Larivière and Porteus (2001) □

- The expect profit of the manufacturer is

$$\pi^{Manufacturer*}_{DU2} = (w^*_{DU2} - c)Q^*_{DU2} \tag{24}$$

- The expect profit of the e-retailer is obtained by considering the expected profit of Approach 1:

$$\pi^{e-retailer*}_{DU2} = \pi^{e-retailer}_{DU1}(Q^*_{DU2}, w^*_{DU2}) \tag{25}$$

Approach 3 under the Decentralized Uncoordinated case: the RFID Enabled Approach

The formulation and the optimization of Approach 3 is similar to that conducted in Larivière and Porteus (2001) with a unit production cost $c+t$ and by considering the demand D_m instead of D.

The e-retailer's Decision:. The expected profit function of the retailer is given by:

$$\begin{aligned}\pi^{e-retailer}_{DU3}(Q_{DU3}, w_{DU3}) &= (r-w_{DU3})\mu - (r-w_{DU3})\int_{D_m=Q_{DU3}}^{+\infty}(D_m - Q_{DU3})f_m(D_m)dD_m \\ &- (w_{DU3}-s)\int_{D_m=-\infty}^{Q_{DU3}}(Q_{DU3}-D_m)f_m(D_m)dD_m \end{aligned} \tag{26}$$

The optimal ordering quantity should also verify:

$$Q^*_{DU3}(w_{DU3}) = F_m^{-1}\left[\frac{r - w_{DU3}}{r - s}\right] \tag{27}$$

The Manufacturer's Decision:. The manufacturer treats the wholesale price as his decision variable. The manufacturer's expected profit is given by:

$$\pi_{DU3}^{Manufacturer}(w_{DU3}) = (w_{DU3} - (c + t))Q_{DU3}(w_{DU3}) \tag{28}$$

By using the inverse of $Q_{DU3}(w_{DU3})$ which is $w_{DU3}(Q_{DU3}) = (r-s)\left[1 - F_m(Q_{DU3})\right] + s$, the expected profit function of the manufacturer can be written as follows:

$$\pi_{DU3}^{Manufacturer}(Q_{DU3}) = \{\, (r - s)\left[1 - F_m(Q_{DU3})\right] - (c - s + t)\}\, Q_{DU3} \tag{29}$$

The result of Larivière and Porteus (2001) can be invoked directly, as the following result shows:

Result 4. *For Approach 3 under an IGFR condition on the random variable D_m:*

1. *The optimal ordering quantity is such that:*

$$1 - F_m(Q^*_{DU3}) - Q^*_{DU3} f_m(Q^*_{DU3}) = \frac{c - s + t}{r - s}$$

2. *The corresponding optimum wholesale price is:*

$$w^*_{DU3} = c + t + (r - s)Q^*_{DU3} f_m(Q^*_{DU3})$$

3. *The optimum expect profit of the manufacturer is:*

$$\pi_{DU3}^{Manufacturer^*} = (r - s)(Q^*_{DU3})^2 f_m(Q^*_{DU3})$$

4. *The optimum expect profit of the retailer is:*

$$\pi_{DU3}^{e-retailer^*} = (r - s)\int_{D_m=0}^{Q^*_{DU3}} D_m f_m(D_m) dD_m$$

Proof. cf. Larivière and Porteus (2001) by considering $c+t$ as a unit production cost. □

Remark 4. *As expected, even if we assumed that the manufacturer pays the tag price, he adjusts his wholesale price in order to include this additional cost. This is why the notion of sharing the tag price is not relevant under a wholesale contract. To eleaborate on this result, let's consider two settings according to which the manufacturer pays a fraction $\alpha_1 t$ ($\alpha_2 t$) and the e-retailer pays the rest $(1 - \alpha_1)t$ ($(1 - \alpha_2)t$). Using the same analysis as before, we can easily show that*

$[w^*_{DU3}]_{\alpha_2} - [w^*_{DU3}]_{\alpha_1} = (\alpha_2 - \alpha_1)t$. *As a consequence* $[Q^*_{DU3}]_{\alpha_2} = [Q^*_{DU3}]_{\alpha_1}$ *which assures that* $[\pi^{Manufacturer^*}_{DU3}]_{\alpha_2} = [\pi^{Manufacturer^*}_{DU3}]_{\alpha_1}$ *and* $[\pi^{e-retailer^*}_{DU3}]_{\alpha_2} = [\pi^{e-retailer^*}_{DU3}]_{\alpha_1}$.

Approach 2 under the Decentralized Coordinated case: Errors are Ignored

As in Approach 1, here we also use a buy-back contract which is completely determined by a 2-tuple (w_{DC2}, b_{DC2}), where w_{DC2} and b_{DC2} are the wholesale price and the buy-back price, respectively.

The e-retailer's Decision:. The expected profit function of the e-retailer is given by:

$$\begin{aligned}\pi^{e-retailer}_{DC2}(Q_{DC2}, w_{DC2}, b_{DC2}) &= (r - w_{DC2})\mu - (r - w_{DC2})\int_{x=Q_{DC2}}^{+\infty}(x - Q_{DC2})f(x)dx \\ &- (w_{DC2} - b_{DC2})\int_{x=0}^{Q_{DC2}}(Q_{DC2} - x)f(x)dx \qquad (30)\end{aligned}$$

By assuming $b_{DC2} < w_{DC2} < r$, the retailer's profit is strictly concave and the optimal ordering quantity Q^*_{DC2} satisfies

$$Q^*_{DC2}(w_{DC2}, b_{DC2}) = F^{-1}\left[\frac{r - w_{DC2}}{r - b_{DC2}}\right] \qquad (31)$$

The Manufacturer's Decision:. The expected profit of the manufacturer is as follows:

$$\begin{aligned}\pi^{Manufacturer}_{DC2}(Q_{DC2}, w_{DC2}, b_{DC2}) = (w_{DC2} - c)Q_{DC2}(w_{DC2}, b_{DC2}) \\ -(b_{DC2} - s)\int_0^{Q_{DC2}} F(x)dx\end{aligned}$$

The following result (from Pasternack (1985)) outlines the coordination conditions of the buy-back contract under Approach 2:

Result 5. *Suppose that the manufacturer offers a contract* $(w_{DC2}(\varepsilon), b_{DC2}(\varepsilon))$ *for* $\varepsilon \in (0, r - c)$ *where* $w_{DC2}(\varepsilon) = r - \varepsilon$ *and* $b_{DC2}(\varepsilon) = r - \varepsilon\frac{r-s}{r-c}$:

1. *The e-retailer orders the optimal solution of the Centralized Scenario and the system profit is also equal to the Centralized Scenario profits*
2. *The parameter* ε *governs the distribution of market power: a high* ε *implies a strong retailer.*

Proof. cf. Pasternack (1985). □

- The expected Manufacturer profit is given by:

$$\pi^{Manufacturer*}_{DC2} = \pi^{Manufacturer}_{DC2}(Q^*_{C2}, w^*_{DC2}, b^*_{DC2}) \qquad (32)$$

- The expected e-retailer profit is given by

$$\pi^{e-retailer*}_{DC2} = \pi^{e-retailer}_{DC1}(Q^*_{C2}, w^*_{DC2}, b^*_{DC2}) \qquad (33)$$

Approach 3 under the Decentralized Coordinated case: the RFID Enabled Approach

Similarly to previous analysis, under approach 3 we also assume a back contract according to which the manufacturer offers to buy-back all unsold units of the e-retailer at the price b_{DC3}. We interpret the 'operation' of the buy-back contract such that the manufacturer pays $(b_{DC3} - s)$ for each unsold unit, and the e-retailer salvages the item for s. Also, we assume as previously that the tag price is totally paid by the manufacturer.

The e-retailer's Decision:. The expected profit function of the e-retailer is given by:

$$\begin{aligned}\pi_{DC3}^{e-retailer}(Q_{DC3}, w_{DC3}, b_{DC3}) &= (r - w_{DC3})\mu \\ &- (r - w_{DC3})\int_{x=Q_{DC3}}^{+\infty}(x - Q_{DC3})f_m(x)dx \\ &- (w_{DC3} - b_{DC3})\int_{x=0}^{Q_{DC3}}(Q_{DC3} - x)f_m(x)dx \end{aligned} \tag{34}$$

By assuming $b_{DC3} < w_{DC3} < r$, the e-retailer's profit is strictly concave and the optimal ordering quantity Q^*_{DC3} satisfies:

$$Q^*_{DC3}(w_{DC3}, b_{DC3}) = F_m^{-1}\left[\frac{r - w_{DC3}}{r - b_{DC3}}\right] \tag{35}$$

The Manufacturer's Decision:. The expected profit function of the manufacturer is like the one considered by Pasternack (1985) with the exception that the unit production price is no longer c but $c + t$:

$$\begin{aligned}\pi_{DC3}^{Manufacturer}(Q_{DC3}, w_{DC3}, b_{DC3}) &= (w_{DC3} - (c + t))Q_{DC3}(w_{DC3}, b_{DC3}) \\ &- (b_{DC3} - s)\int_0^{Q_{DC3}} F_m(x)dx \end{aligned} \tag{36}$$

The following result (from Pasternack (1985)) outlines the coordination conditions of the buy back contract:

Result 6. *Suppose that the manufacturer offers a contract* $(w_{DC3}(\varepsilon), b_{DC3}(\varepsilon))$ *for* $\varepsilon \in (0, r - c - t)$ *where* $w_{DC3}(\varepsilon) = r - \varepsilon$ *and* $b_{DC3}(\varepsilon) = r - \varepsilon\frac{r-s}{r-(c+t)}$:

1. *The e-retailer orders the optimal quantity of the Centralized Scenario and the system profit is also equal to the Centralized Scenario profits*
2. *The e-retailer profit is increasing in* ε*. In particular* $\pi_{DC3}^{e-retailer^*}(w_{DC3}(\varepsilon), b_{DC3}(\varepsilon)) = \frac{\varepsilon}{r-(c+t)}\pi^*_{C3}$
3. *The manufacturer profit is decreasing in* ε*. In particular* $\pi_{DC3}^{Manufacturer^*}(w_{DC3}(\varepsilon), b_{DC3}(\varepsilon)) = (1 - \frac{\varepsilon}{r-(c+t)})\pi^*_{C3}$

Proof. cf Pasternack (1985) $\square$