# Data formats for numerical relativity

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This document proposes data formats to exchange numerical relativity results, in particular gravitational waveforms. The primary goal is to further the interaction between gravitational-wave source modeling groups and the gravitational-wave data-analysis community. We present a simple and extendible format which is applicable to various kinds of gravitational wave sources including binaries of compact objects and systems undergoing gravitational collapse, but is nevertheless sufficiently general to be useful for other purposes.

#### I. INTRODUCTION

Numerical relativity (NR) has made enormous progress within the last few years. Following the initial breakthroughs of 2005 [1–3], a number of numerical codes are available to perform sufficiently accurate simulations of the inspiral, merger, and ringdown phases of generic black-hole-binary systems (for overviews see e.g. [4–8]). Similarly, significant progress has been made in the numerical simulation of the inspiral, coalescence, and post-merger dynamics of binaries involving neutron stars, and of stellar gravitational collapse (see e.g. [9–11] for recent overviews). All these processes are among the most promising sources of gravitational radiation, and gravitational wave observations have been a key motivation driving numerical relativity.

The exploration of the parameter space of gravitational-wave sources is a large-scale effort that involves many NR research groups. A standard data format is needed to share their results with other research communities and for collaborative projects. The first example of such an application is the use of NR waveforms in gravitational-wave data analysis codes, but numerical results are also used to produce analytical template banks, in particular for the case of black-hole binaries (see e.g., [12, 13]), and in systematic comparisons between NR groups [14].

The aim of this document is to suggest such formats for data exchange between numerical relativists and the "numerical relativity data user community", in particular gravitational wave data analysts. It is clear that there are

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still outstanding conceptual and numerical issues remaining in numerical simulations; the goal of this document is not to resolve them, but to spell out the technical details of the waveform data in a way that is both precise and sufficiently flexible to adapt to future research. A primary aim is that NR waveforms can be incorporated seamlessly within the data-analysis software currently being developed within the LIGO/Virgo Collaboration (LVC). The relevant software development is being carried out as part of the LSC Algorithms Library<sup>1</sup> which contains core routines for gravitational-wave data analysis written in ANSI C99, and is distributed under the GNU General Public License.

While it is, in principle, straightforward to extend the data format to all kinds of gravitational wave sources or NR problems, we first focus on black-hole-binaries, and discuss in particular applications to the NINJA project [15, 16]. We also specify an extension of the data format for neutron star binaries and stellar collapse.

The key ideas of the data format are as follows:

- Simulation data (e.g. time series for different gravitational wave strain multipoles) are distributed together with a metadata file that describes the data set (including information on authors, codes used, physical parameters, links to publications, etc.) and contains the file names of the actual simulation data.
- The metadata file is a simple text file that is easy to read and edit for humans, and contains key = value pairs,
  organized in sections.
- In this paper we define the basic syntax of the metadata file, together with a collection of keys that would typically be included, and the specific set of keys required for submissions to the NINJA project.
- We also define formats for the gravitational wave strain data to be processed with LVC software tools. Formats for other type of data, say the time evolution of black hole spins, can straightforwardly be defined in analogy.

The first version of this document was posted on the arXiv in September 2007 [17], and included a simple data format for representing the gravitational-wave strain as three-column data sets of  $\{t, h_+, h_x\}$ , with the time t being given in equally spaced intervals, together with a simple metadata format that specified only black hole (BH) parameters (mass ratio and spin), the authors and the numerical code. This format has been used for data exchange in a number of subsequent research projects, notably the first NINJA project. The present format addresses some of the shortcomings noted during the first NINJA project, and the requirements of the second NINJA project, which involves extremely long hybrid post-Newtonian-plus-numerical-relativity waveforms, and a larger number of waveforms, which have to be processed automatically. In order to represent very long waveforms we here define a second, more economical, data format for gravitational waveforms. Also, we have revised the metadata format as necessary to allow representing all relevant scientific information, and to facilitate the future use of waveforms from sources that involve matter. The current version of this document describes both the old and new formats, described respectively in Sections IV A and IV B. Note that citations to this document prior to 2011 are to version one.

The remainder of this document is structured as follows: section II describes our conventions for decomposing the gravitational wave data in terms of spherical harmonics, section III specifies the format for metadata, and section IV specifies data formats for waveform data. We conclude with section V discussing the specific incarnations of data format specifications for the different phases of the NINJA project.

# II. MULTIPOLE EXPANSION OF GRAVITATIONAL WAVES

The output of a numerical-relativity code is the full spacetime of a black-hole-binary system. On the other hand, what is required for gravitational-wave data-analysis purposes is the strain h(t), as measured by a detector located far away from the source. The quantity of interest is therefore the gravitational-wave metric perturbation  $h_{ab}$  in the wave zone, where a and b are space-time indices. We always work in the Transverse Traceless (TT) gauge so that all information about the metric perturbation is contained in the TT tensor  $h_{ij}$ , where i and j are spatial indices. The wave falls off as 1/r where r is the distance from the source:

$$h_{ij} = A_{ij} \frac{M}{r} + \mathcal{O}\left(r^{-2}\right). \tag{II.1}$$

Here  $A_{ij}$  is a transverse traceless tensor and M is the total mass of the system; this approximation is, naturally, only valid far away from the source.

Available from http://www.lsc-group.phys.uwm.edu/daswg/projects/lal.html.

There are different methods for extracting  $h_{ij}$  from a numerical evolution. One common method is to use the complex Weyl tensor component  $\Psi_4$  which is related to the second time derivative of  $h_{ij}$ . Another method is to use the Zerilli function which approximates the spacetime in the wave-zone as a perturbation of a Schwarzschild spacetime. For our purposes, it is not important how the wave is extracted, as different NR groups are free to use methods they find appropriate. The starting point of our analysis is the multipole moments of  $h_{ij}$  and it is important to describe explicitly our conventions for the multipole decomposition. The corresponding values of  $\Psi_4$  or the Zerilli function can be described analogously, in particular also regarding the data formats described later. Since the strain and  $\Psi_4$  or the Zerilli functions are related by numerical operations and other potential ambiguities, it is often useful to have these original data available for consistency checks with the strain waveforms.

Let (x, y, z, t) be a Cartesian coordinate system in the wave zone, sufficiently far away from the source. Let  $\vec{e}_x$ ,  $\vec{e}_y$  and  $\vec{e}_z$  denote the coordinate basis vectors. Given this coordinate system, we define standard spherical coordinates  $(r, \iota, \phi)$  where  $\iota$  is the inclination angle from the z-axis and  $\phi$  is the phase angle. At this point, we have not specified anything about the source. In fact, the source could be a binary system, a star undergoing gravitational collapse or anything else that could be of interest for gravitational wave source modeling. In later sections we will specialize to particular GW sources and suggest possibilities for some of the various choices that have to be made. However, as far as possible, these choices are eventually to be made by the individual source modeling group.

We break up  $h_{ij}$  into modes in this coordinate system. In the wave zone, the wave will be propagating in the direction of the radial unit vector

$$\vec{e}_r = \vec{e}_x \sin \iota \cos \phi + \vec{e}_y \sin \iota \sin \phi + \vec{e}_z \cos \iota$$
 (II.2a)

A natural set of orthogonal basis vectors from which to build the transverse-traceless basis tensors is

$$\vec{e}_{\iota} = \vec{e}_x \cos \iota \cos \phi + \vec{e}_y \cos \iota \sin \phi - \vec{e}_z \sin \iota, \qquad (II.2b)$$

$$\vec{e}_{\phi} = -\vec{e}_x \sin \phi + \vec{e}_y \cos \phi. \tag{II.2c}$$

In the transverse traceless gauge,  $h_{ij}$  has two independent polarizations

$$\overrightarrow{h} = \sum_{i,j} h_{ij} \, \overrightarrow{e}_i \otimes \overrightarrow{e}_j = h_+ \overrightarrow{e}_+ + h_\times \overrightarrow{e}_\times , \qquad (II.3)$$

where  $\overleftrightarrow{e}_+$  and  $\overleftrightarrow{e}_\times$  are the usual basis tensors for transverse-traceless tensors in the wave frame

$$\stackrel{\leftrightarrow}{e}_{+} = \vec{e}_{\iota} \otimes \vec{e}_{\iota} - \vec{e}_{\phi} \otimes \vec{e}_{\phi}, \quad \text{and} \quad \stackrel{\leftrightarrow}{e}_{\times} = \vec{e}_{\iota} \otimes \vec{e}_{\phi} + \vec{e}_{\phi} \otimes \vec{e}_{\iota}.$$
 (II.4)

It is convenient to use the combination  $h_+ - ih_{\times}$ , which is related to  $\Psi_4$  by two time derivatives<sup>2</sup>

$$\Psi_4 = \ddot{h}_+ - i\ddot{h}_\times \,. \tag{II.5}$$

It can be shown that  $h_+ - ih_{\times}$  can be decomposed into modes using spin-weighted spherical harmonics  ${}^{-s}Y_{lm}$  of weight -2:

$$h_{+} - ih_{\times} = \frac{M}{r} \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} H_{\ell m}(t)^{-2} Y_{\ell m}(\iota, \phi) .$$
 (II.6)

The expansion parameters  $H_{lm}$  are complex functions of the retarded time t-r and, if we fix r to be the radius of the sphere at which we extract waves, then  $H_{lm}$  are functions of t only.

The explicit expression for the spin-weighted spherical harmonics in terms of the Wigner d-functions is

$${}^{-s}Y_{lm} = (-1)^s \sqrt{\frac{2\ell+1}{4\pi}} d_{m,s}^{\ell}(\iota) e^{im\phi},$$
 (II.7)

<sup>&</sup>lt;sup>2</sup> We define  $\Psi_4$  as  $\Psi_4 := C_{abcd}\bar{m}^a n^b \bar{m}^c n^d$  where  $C_{abcd}$  is the Weyl tensor and  $a,b\dots$  denote abstract spacetime indices. If we denote the unit timelike normal to the spatial slice as  $e^a_{\hat{t}}$  and the promotions of  $\{\vec{e}_r, \vec{e}_\iota, \vec{e}_\phi\}$  to the full spacetime as  $\{e^a_{\hat{r}}, e^a_{\hat{\ell}}, e^a_{\hat{\ell}}\}$ , then the null tetrad adapted to the constant r spheres is  $\{\ell^a, n^a, m^a, \bar{m}^a\}$  where  $\ell^a = (e^a_{\hat{t}} + e^a_{\hat{r}})/\sqrt{2}$ ,  $n^a = (e^a_{\hat{t}} - e^a_{\hat{r}})/\sqrt{2}$ ,  $m^a = (e^a_{\hat{t}} + ie^a_{\hat{\phi}})/\sqrt{2}$ , and  $\bar{m}^a$  is the complex conjugate of  $m^a$ .

where

$$d_{m,s}^{\ell}(\iota) = \sum_{k=k_1}^{k_2} \frac{(-1)^k [(\ell+m)!(\ell-m)!(\ell+s)!(\ell-s)!]^{1/2}}{(\ell+m-k)!(\ell-s-k)!k!(k+s-m)!} \times \left(\cos\left(\frac{\iota}{2}\right)\right)^{2\ell+m-s-2k} \left(\sin\left(\frac{\iota}{2}\right)\right)^{2k+s-m}$$
(II.8)

with  $k_1 = \max(0, m - s)$  and  $k_2 = \min(\ell + m, \ell - s)$ . For reference,

$$^{-2}Y_{22} = \sqrt{\frac{5}{64\pi}} (1 + \cos\iota)^2 e^{2i\phi}, \qquad (II.9)$$

$$^{-2}Y_{21} = \sqrt{\frac{5}{16\pi}} \sin \iota (1 + \cos \iota) e^{i\phi}, \qquad (II.10)$$

$$^{-2}Y_{20} = \sqrt{\frac{15}{32\pi}}\sin^2\iota\,, (II.11)$$

$$^{-2}Y_{2-1} = \sqrt{\frac{5}{16\pi}} \sin \iota (1 - \cos \iota) e^{-i\phi}, \qquad (II.12)$$

$$^{-2}Y_{2-2} = \sqrt{\frac{5}{64\pi}} (1 - \cos\iota)^2 e^{-2i\phi} \,. \tag{II.13}$$

The mode expansion coefficients  $H_{lm}$  are given by

$$H_{\ell m} = \frac{1}{M} \oint {}^{-2}Y_{lm}^{\star}(\iota,\phi)(rh_{+} - irh_{\times}) d\Omega.$$
 (II.14)

If  $\Psi_4$  is used for wave extraction, then  $H_{lm}$  is given by two time integrals of the corresponding mode of  $\Psi_4$ . In this case, it is important that the information provided contains details about how the integration constants are chosen. We define  $h_+^{(\ell m)}$  and  $h_\times^{(\ell m)}$  as the real and imaginary parts of the  $H_{lm}$  according to

$$rh_{+}^{(\ell m)}(t) - irh_{\times}^{(\ell m)}(t) := M H_{\ell m}(t).$$
 (II.15)

It is these modes  $rh_{+,\times}^{(\ell m)}$  of  $rh_+$  and  $rh_\times$  that we suggest to be provided as functions of time in units of M for vacuum spacetimes and in units of  $M=1M_{\odot}=4.92549059\times 10^{-6}\,\mathrm{s}$  for spacetimes involving matter (using the values of  $M_{\odot}$  and fundamental constants listed in section E.3 of [18]).

## A. Application to Waveforms extracted in the Quadrupole Approximation

Many simulation codes for non-vacuum spacetimes still operate in Newtonian gravity and dynamics or employ post-Newtonian or conformally-flat approximations to general relativity. Such codes generally follow the quadrupole approximation for estimating gravitational waveforms. In this approximation, the transverse-traceless gravitational wave field is related to the second time derivative of the reduced mass quadrupole tensor  $\ddot{I}_{ij}$  as

$$rh_{ij}^{TT,\text{quad}} = \frac{2G}{c^4} \Pi_{ijkl} \ddot{I}_{kl} , \qquad (II.16)$$

where  $\Pi_{ijkl}$  is the necessary projection operator to transform the right-hand side into the transverse-traceless gauge. Here and in the following we assume that  $\ddot{I}_{ij}$  is given in standard cgs units.  $\ddot{I}_{ij}$  is a common wave extraction output of simulation codes. In order to express  $H_{2m}$  in terms of  $\ddot{I}_{ij}$ , one first expresses  $h_{+}(\iota,\phi)$  and  $h_{\times}(\iota,\phi)$  in terms of  $\ddot{I}_{kl}$ , then convolves these with  $^{-2}Y_{lm}^{*}$  (cf. eq. II.14). The result is

$$H_{20}^{\text{quad}} = \sqrt{\frac{32\pi}{15}} \frac{G}{c^4} \left( \ddot{I}_{zz} - \frac{1}{2} (\ddot{I}_{xx} + \ddot{I}_{yy}) \right) , \qquad (\text{II}.17)$$

$$H_{2\pm 1}^{\text{quad}} = \sqrt{\frac{16\pi}{5}} \frac{G}{c^4} \left( \mp \ddot{I}_{xz} + i\ddot{I}_{yz} \right) ,$$
 (II.18)

$$H_{2\pm 2}^{\text{quad}} = \sqrt{\frac{4\pi}{5}} \frac{G}{c^4} \left( \ddot{I}_{xx} - \ddot{I}_{yy} \mp 2i\ddot{I}_{xy} \right) .$$
 (II.19)

#### III. METADATA FORMAT

Simulation data are distributed together with a metadata file that describes the data set and contains the file names of the actual simulation data. The metadata file is a simple text file that is easy to read and edit for humans, and contains key = value pairs, organized in sections, adopting the format that is very commonly used for software configuration files. The metadata file aims in particular to allow automated production of waveform tables such as the simulation tables in the NINJA and Samurai papers [14–16], and to provide all the scientific information that is required to process the data sets.

This metadata file contains (at least) two sections, one for the simulation metadata, and the other listing the filenames which correspond to the various  $(\ell, m)$  modes of the waveform. A separate metadata file is required for each simulation.

The metadata format proposed here is based on that used for the first NINJA project, and described in detail in Version 1 of this document [17], which included a minimum essential set of information related to a given numerical simulation—the NR group that produced the data, their contact information, and the initial spins of the two black holes, their mass ratio, and the initial frequency of the waveform.

During the course of the first NINJA project, and discussions related to follow-up projects, it became clear that the metadata format needs to be extended to include more information on the source of the waveforms, their physical properties, their accuracy, the methods used to combine them with PN/EOB results to produce hybrid waveforms, and the availability of these waveforms for use by others in scientific projects. A more precisely defined metadata format is also required to allow the automated construction of waveform catalogs.

Examples of the metadata format are given at the end of this document. Further details, and modifications more recent than the production of this document, can be found on the NINJA project website, www.ninja-project.org.

#### A. Syntax for sections, entries and comments

A metadata file consists of sections headed by

#### [sectionname]

and entries of the form

key = value

The key strings, which we will also refer to as tags, are not allowed to contain spaces, but spaces can be used in the value field, where they are interpreted as field separators or text depending on the context implied by key.

Values default to those set in the [metadata] section, and values set in any other section override the values set in the [metadata] section, and are defined only locally in that section.

Keys that point to data files (such as mode names, i.e "2,-2", see below) are ignored in the [metadata] section. All keys that can appear in a [metadata] section can also appear in other sections.

The comment character is "#"—all text to the right of a comment character is interpreted as a comment.

## B. Format of 'names'

Names of files (metadata files, simulation data, documentation files, etc.) and -name keys, such as simulation-name should *only* contain alphanumeric characters plus "-", ".".

The suggested naming convention for binary black hole metadata files is "name.bbh".

#### C. Format of email addresses

To indicate the full name of authors and waveform submitters, the format Joan Smith <joan.smith@example.edu> should be used consistently, i.e. the email address field should contain the real name.

# D. Documentation and References

Any tag can be extended by the following tags as tag-subtag to add documentation or references:

documentation = eccentricity\_writeup.tex # file included with the submission.

#### E. Units

Black hole waveforms are given in units of M, where  $M = m_1 + m_2$  is the total initial black hole mass (for spinning BHs, this mass includes the spin contribution). Black hole separations are measured in coordinate distance, measured in M.

For simulations involving matter fields, we use the convention of time in units of  $M_{\odot} = 4.92549095 \times 10^{-6}$  s, and strain  $rh_{\{+,\times\}}$  in units of  $M_{\odot} = 1.47662504 \times 10^{5}$  cm. Frequencies in metadata are in units of  $1/M_{\odot} = 203.025447$  kHz. These geometrized solar units are compatible with LAL values of  $M_{\odot}$  and fundamental constants listed in section E.3 of [18]. Use of these units can be flagged with metadata mass-scale = 1.

#### F. Gravitational wave start frequency

For use in gravitational wave data analysis, it is essential to indicate the start frequency of the waveform. Since different spheroidal harmonic modes differ in frequency, we choose to specify the start frequency of the l=m=2 harmonic with the key freq-start-22. For black hole binaries, where all results scale with the total mass M, this is best done in units of M, and frequency  $f_0/M$  is specified. For a given value of the mass, this gives the physical start frequency of the waveform, and this will need to be less than the lower cut-off frequency relevant for a particular detector. For example,  $f_0=40\,\mathrm{Hz}$  is an appropriate value for the initial LIGO detectors, and  $f_0=10\,\mathrm{Hz}$  is appropriate for advanced detectors.

## G. Accuracy measures and error bars

Any number-valued tag also implies the tags tag-error-relative and tag-error-range to quote relative and absolute errors. We use the following syntax:

```
value-error-relative = 0.01 means tag has a relative error of 1%, while value-error-range = 3 9 means 3 \le \text{value} \le 9.
```

Useful accuracy measures on time series data are more difficult to specify, and depend on the application. For waveform data, mismatch values can be defined. For the NINJA project, we have specified three tags that relate to the accuracy of the waveforms—uncertainty-in-number-of-cyles, relative-amplitude-uncertainty, and mismatch. Details of how these uncertainties are defined (in particular for what follows the tag "mismatch") can be specified in comments fields or decided upon for specific projects. These three quantities are relatively easy to calculate, and they can be easily extended to more sophisticated accuracy measures in the future.

## H. Vector valued tags

For vector-valued tags we use the syntax

```
value = \{1, 2, 3\}.
```

Relative and absolute errors can be specified as

```
value-error-relative = {.01, .01, .01}
or
value-error-range = {.5, 1.5, 2.5} {4, 4, 4}.
```

#### I. Black-hole binaries

# 1. Parameterization of parameter space

A numerical-relativity simulation of black-hole binary coalescence has many parameters that need to be specified, and several of them may not be directly relevant to the data-analysis problem. We need to determine which parameters of the numerical simulation are useful for the users of the data. For our purposes, a single numerical waveform is defined by at least seven parameters: the mass ratio  $q = M_1/M_2$  and the three components of the individual spins  $\vec{S}_1$  and  $\vec{S}_2$ . The choice of precisely how  $M_1$ ,  $M_2$  and the spins are calculated is left up to the individual numerical relativity groups. Note that for systems with matter, as discussed below, it makes more sense to specify all masses in physical units, e.g. in solar masses, instead of using the mass ratio.

The mass ratio q is assumed to be in the form  $q = m_1/m_2$ . It is essential to adhere to this definition for spinning binaries, because this is the only way for an automated script to determine which spin belongs to which black hole. In addition, for simplicity we suggest to adhere to the convention  $m_1 \ge m_2$ , i.e., the first BH is the heavier one.

#### 2. Coordinate system

In the default coordinate system, all binaries start out in the xy-plane, consistent with the convention of Table 1 in the first NINJA paper [15].

For aligned-spin systems, it is recommended that the initial momenta be tangent to the xy-plane. For precessing systems this will not necessarily be convenient, and BH momenta or velocity vectors should be specified. To fix the rotation in the xy-plane, the key initial-separation-angle specifies the direction of the separation vector from the first to the second BH in degrees, e.g., if this direction points along the x-axis then

```
initial-separation-angle = 0 and if this direction points along the negative y-axis then initial-separation-angle = 270 # or initial-separation-angle = -90. Specifying a value for initial-separation-angle implies initial-bh-position1z = 0
```

and sets initial-bh-position1x, initial-bh-position1y, initial-bh-position2x, initial-bh-position2y accordingly, assuming a center of mass frame and using the value for mass-ratio.

In order to override these definitions of the coordinates, specify the keys

initial-bh-position2z = 0

```
initial-bh-position1x, initial-bh-position1y, initial-bh-position1z.
```

In order to specify the initial motion of the BHs, define the momenta or velocities,

```
initial-bh-momentum1x, initial-bh-momentum1y, initial-bh-momentum1z,
initial-bh-velocity1x, initial-bh-velocity1y, initial-bh-velocity1z,
```

and analogously for the second BH. If one component of the momenta or velocities is specified, the other components default to zero.

#### J. Binaries involving neutron stars

Double-neutron-star or mixed black-hole/neutron-star binaries have many characteristics in common with binary-black-hole systems. When the bodies are far apart, the dynamics are also determined by total mass, mass ratio, and individual spins, as in the specification above.

However, the properties of matter set a physical scale which fixes, for example, the radius of a neutron star relative to the length scale of the mass. Waveforms from such systems cannot be scaled to different masses, and the masses of all objects have to be specified explicitly using fixed units of  $M_{\odot}$  as discussed in Sec. III E. Thus, unlike in the simpler BBH case, the mass ratio is not used to parameterize configurations. Note that the convention  $q \leq 1$  in the neutron star simulation community is opposite to the BBH data format convention.

In the simplest case the equation of state (EOS) for a simulated neutron star may be a polytrope or ideal fluid characterized by two parameters of adiabatic index and compactness; more generally a cold EOS is an arbitrary tabled or parameterized function of pressure vs. density, and even more generally details of heating and composition may also be described (and influence system dynamics). Simple metadata descriptions are therefore more challenging, and full information will most likely need to be given in comments or simulation-details. If useful characteristic parameters can be agreed on (for example compactness, tidal deformability, tidal disruption frequency estimates), these can be included in the metadata.

Since systems involving neutron stars (except for extreme mass ratio cases) are low mass systems, hybrid waveform construction will be required for most realistic detector injections.

## K. Stellar Collapse and Single Compact-Star Spacetimes

Simulations of stellar collapse, core-collapse supernovae, and single compact stars (neutron stars, strange stars, etc.) have less global parameters, but the physics involved in their modeling is generally more complex, is frequently done under some symmetry assumptions and involves complicated physics, such as nuclear equations of state, neutrino transport, realistic initial stellar models, etc. Depending on application, this modeling complexity is reflected in a more complicated and extensive set of metadata information needed to characterize a given waveform. An example is given in section VD.

A general convention to follow is that the system spin should be aligned with the positive z-axis.

# IV. DATA FORMATS FOR WAVEFORMS

In the following subsections we outline two data formats for waveforms, represented as multipole components of the complex wave strain h(t).

The first format directly represents the strain h(t), and the second instead specifies the phase and amplitude of the wave, which can then be used to reconstruct the wave strain. The second format has the advantage that the phase and amplitude functions factor out the oscillations of h(t) with the gravitational wave frequency, and can thus be accurately represented with fewer data points.

Data in both formats are written as 3-column ASCII plain text files, which can optionally be compressed with gzip (i.e., file readers for such data are supposed to support gzip). Note that time steps need not be equidistant, which was required in the first version of the data format [17].

For a given simulation, numerical groups decide the maximum value of  $\ell = \ell_{\text{max}}$  to which they will provide the waveform. For every  $\ell \leq \ell_{\text{max}}$ , waveforms must be provided for all values of  $m = -\ell, \ldots, \ell$ , irrespective of any symmetries that may be present in the simulation. If there are certain modes which vanish, or have a small amplitude which cannot be accurately determined, then such data can either be set to zero, or equivalently left out. Thus, any modes which will not be provided will be assumed to have zero amplitude, and providing an  $(\ell, m)$ -mode without the corresponding  $(\ell, -m)$ -mode will in general lead to incorrect results.

It is natural to use the total mass M of the binary as the unit for the time and strain columns. However, there are subtleties in the choice of M, and in the definition of black hole masses in a dynamical spacetime, e.g. one could alternatively use approximations to the ADM or Bondi mass of the spacetime. Similar ambiguities exist for other relevant quantities, such as the black hole spins. While for many applications these subtleties have little or no relevance, it is important that all definitions can be reproduced, and therefore the metadata file should include links to sufficient documentation on the details for that simulation.

For simplicity, we identify M with an estimate of the total initial black hole mass, and clearly highlight deviations from this convention.

## A. Waveforms represented as a complex time series

In this format, the data for a single mode  $rh_{+,\times}^{(\ell m)}$  is written as a plain text file in three columns for the time t,  $rh_{+}^{(\ell m)}$  and  $rh_{\times}^{(\ell m)}$  respectively.

The strain multiplied by the distance will also be in units of the total mass M of the binary. There can be any number of comment lines at the top of the file; it should however be noted that this can put restrictions on the range of plotting tools that can be used. It is recommended that all information contained in such comments is also available in the metadata files.

For uniformly sampled waveforms, a rate of  $1 \times M/m$  (where m labels the harmonic) is believed to be sufficient for most data analysis purposes, and is recommended unless a more careful analysis suggests the use of a different value for a particular case. This time resolution is generally sufficient during the merger and ringdown, when the waveform frequency is highest; it is acceptable to use a coarser time sampling at lower frequencies.

Gravitational strain waveforms in the complex time series representation are referenced in a ht-data section of the metadata file, e.g. as

# [ht-data] 2,2 = hmod.r5.l3.l2.m2 2,1 = hmod.r5.l3.l2.m1 2,-1 = hmod.r5.l3.l2.m-1 2,-2 = hmod.r5.l3.l2.m-2.

For  $\Psi_4$  or Zerilli waveforms, the section headers should be [psi4t-data] and [zerillit-data]. An example data file looks like:

```
# numerical waveform from ....
# equal mass, non spinning, 5 orbits, 1=m=2
# time
             hplus
                         hcross
0.000000e+00 1.138725e-02 -8.319811e-04
2.000000e-01 1.138725e-02 -1.247969e-03
4.000000e-01 1.138726e-02 -1.663954e-03
6.000000e-01 1.138727e-02 -2.079936e-03
8.000000e-01 1.138728e-02 -2.495913e-03
1.000000e-00 1.138728e-02 -2.911884e-03
1.200000e+00 1.138729e-02 -3.327850e-03
1.400000e+00 1.138730e-02 -3.743807e-03
1.600000e+00 1.138731e-02 -4.159757e-03
1.800000e+00 1.138733e-02 -4.575696e-03
2.000000e+00 1.138734e-02 -4.991627e-03
2.200000e+00 1.138735e-02 -5.407545e-03
2.400000e+00 1.138737e-02 -5.823452e-03
2.600000e+00 1.138739e-02 -6.239345e-03
2.800000e+00 1.138740e-02 -6.655225e-03
3.000000e+00 1.138752e-02 -7.071059e-03
3.200000e+00 1.138754e-02 -7.486903e-03
3.400000e+00 1.138757e-02 -7.902739e-03
```

# B. Waveforms represented by phase and amplitude

NINJA waveforms need to be scaled across a wide range of masses, and injected without aliasing into the data stream. This presents two conflicting motivations. First, in order to cover the low frequency bands of advanced detectors, simulating low-mass systems, the waveforms need to be very long. On the other hand, in order to prevent aliasing—especially of the merger and ringdown of high-mass systems—the data needs to be sampled very finely. Naively, these two problems taken together would lead to enormous data sets, especially as NINJA extends to more interesting regimes of low-frequency sensitivities and low-mass systems. By changing the format of stored data, we can sample the waveform very coarsely during the long inspiral, and very finely during the highly dynamic merger and ringdown, then reconstitute the data as necessary.

We have chosen to represent the waveform through the wave phase and amplitude, because they are typically very simple functions of time, meaning that interpolation is quite accurate, so the waveforms can be represented with a relatively coarse time sampling. In this format, the data for the phase and amplitude of a single mode of  $rh_{+,\times}^{\ell m}$  is written as a plain text file in three columns for the time t, amplitude A, and phase  $\phi$ , respectively. The wave strain at a given time can be reconstructed by

$$rh_{+}^{\ell m} - irh_{\times}^{\ell m} = A_{\ell m}e^{i\phi_{\ell m}} . \tag{IV.1}$$

Because the strain is insensitive to offsets of  $2\pi$  in the phase, a stepwise function of integer multiples may be added to  $\phi_{\ell m}$  to make it somewhat continuous; the objective is for interpolation of the time series for  $\phi_{\ell m}$  to be accurate. With these choices of variables, the time sampling need not be uniform.

The only requirement is that the time sampling be fine enough at any given time that the wave strain can be reconstructed to be identical (within some desired accuracy) to the "original" waveform phase and amplitude data using only first-order interpolation between the data points provided. Applying higher-order interpolation to the provided data will presumably yield results with greater fidelity to the original hybrid waveform, and may be useful if it is expected that accurate derivatives will need to be taken of the waveform. However, the defining requirement is that the phase and amplitude can be accurately reconstructed using only first-order interpolation. The accuracy of the reconstruction is left to contributing NR groups. In general, two separate tolerances will need to be defined:  $Tol_A$  and  $Tol_{\phi}$ . Presumably, these will be chosen to be roughly the same as the estimated accuracy of the input data.

Within this requirement, NR groups may choose any method to sample the data points. A simple but robust algorithm for removing unnecessary time steps from the data is as follows. First, we assume that both the initial and final time steps should be included in the final data set. Next is a recursive stage in which each interval of the new coarser time series is checked to ensure that  $\phi_{\ell m}$  at the midpoint of that interval can be linearly interpolated to within the desired tolerance of that point in the original data set. If the interpolation is not sufficiently accurate, the midpoint is included in the coarse set, and the test continues on the two new intervals thus formed. Finally, another recursive stage checks each point of the input data set to ensure that both  $A_{\ell m}$  and  $\phi_{\ell m}$  can be correctly reproduced. If not, we include the midpoint of the coarse interval in which that data point is found, and continue and repeat the check.

This algorithm has been implemented in the NINJA code repository as MinimizeGrid. Depending on the length of the waveform, it reduces the size of the data set by anywhere from a few percent for short numerical data to 99% for very long hybrid waveforms. For the long waveforms needed in the second NINJA project (to allow injection into detector noise at low masses), this technique is crucial to make the data sets manageable.

Gravitational-strain waveforms in the phase-amplitude series representation are referenced in a ht-phiamp-data (alternatively psi4t-data and zerillit-data) section of the metadata file as, e.g.,

```
[ht-phiamp-data]
2,2 = hmod.r5.13.12.m2
2,1 = hmod.r5.13.12.m1
2,-1 = hmod.r5.13.12.m-1
2,-2 = hmod.r5.13.12.m-2.
```

#### V. APPLICATIONS

In this section we list the compulsory metadata fields for some concrete projects, in particular the first NINJA project, the NINJA waveform catalog—a publicly available catalog of BBH simulation metadata from simulations that have been performed in the NR community—, the second NINJA project, and the Matter NINJA proposal.

# A. First NINJA project

This is an example for the old format [17], used in the first NINJA project [15, 16]

```
[metadata]
simulation-details = initial separation 11M, QC parameters
nr-group = friendlynrgroup
email = sub.mitter@good-science.org
mass-ratio = 1.0
spin1x = 0.0
```

```
spin1y = 0.0
spin1z = 0.5
spin2x = 0.0
spin2y = 0.0
spin2z = 0.5
freqStart22 = 0.05

[ht-data]
2,2 = hmod.r5.13.12.m2
2,1 = hmod.r5.13.12.m1
2,-1 = hmod.r5.13.12.m-1
2,-2 = hmod.r5.13.12.m-2
```

#### B. NINJA waveform catalog

A waveform catalog is currently under construction at www.ninja-project.org, for use in the second and ongoing NINJA projects. For this catalog, the compulsory fields are give below, and examples of options are given in Sec. V C.

## [metadata] section

```
license = ninja2
documentation
                        # can be "none"
publication
simulation-bibtex-keys
simulation-name
simulation-uuid
authors-tag
submitter-email
authors-emails
nr-uuid,data-type = hybrid
code-bibtex-keys
initial-separation
initial-data-type
initial-data-bibtex-keys
quasicircular-bibtex-keys
eccentricity
mass-ratio
spin1x
spin1y
spin1z
spin2x
spin2y
spin2z
freq-start-22
number-of-cycles-22
extraction-radius
uncertainty-in-number-of-cycles
relative-amplitude-uncertainty
mismatch
[ht-data] section
```

Entries for the 2,2 and 2,-2 modes.

#### C. NINJA 2

This is a suggested format for the Ninja2 black holes project, http://www.ninja-project.org/doku.php?id=ninja2:home. Slashes are used to indicate suggested options.

```
[metadata]
                        = ninja-catalog/ninja2/thirdparty/private/public
license
comments
                       = short.txt myfile.pdf
documentation
                        = arXiv:0901.4399/unpublished/inprint
publication
simulation-bibtex-keys = Aylott:2009ya
simulation-name = spp50
simulation-uuid = 373b1340-db3d-11de-aef4-0002a5d5c51b
authors-tag = SubCorrIts
submitter-email = Sub Mitter <submitting@author.edu>
authors-emails = Corr Espondent<corresponding@author.edu>, Its Me <somebody@else.edu>
data-type = NR/PN/hybrid
hybrid-nr-uuid = 373b1340-db3d-11de-aef4-0002a5d5c51ba # optional
hybrid-approximant = TaylorT1
                                                            # only for hybrids
hybrid-method = hybrid-doc.txt
                                                            # only for hybrids
{\tt simulation-relative-resolution} = 3 {\tt d} 18 {\tt f} {\tt d} {\tt a} 3 {\tt c} 58 {\tt d} - 4 {\tt a} 95 - 9 {\tt a} 87 - {\tt b} 1275 {\tt d} {\tt a} 258 {\tt e} 6
# resolution is 2 times better than for this simulation-uuid
code
                  = mycode
code-version
                 = 247
code-bibtex-keys =
initial-data-type = Bowen-York quasicircular
initial-data-uuid =
initial-data-bibtex-keys =
quasicircular-bibtex-keys =
initial-separation = 11
initial-separation-angle = 90 # only for aligned spins
                                # alternatively specify initial-bh-position1 etc.
# direction of the separation vector from the first to the second BH, in degrees
initial-bh-momentum1x = -0.0900993
# if momenta for second BH are not given, we are in the center of mass frame
initial-bh-momentum1y = -0.000709412
# alternatively, use initial-bh-velocity1x etc.
initial-bh-momentum1z = 0
eccentricity = 0.002 \ 0.001
eccentricity-error-range = 0 0.003
mass-ratio = 1.0
spin1x = 0.0
spin1y = 0.0
spin1z = 0.5
spin2x = 0.0
spin2y = 0.0
spin2z = 0.5
freq-start-22/freqStart22 = 0.05
```

```
number-of-cycles-22 = 19.3
extraction-radius = somenumber/infinity/extrapolated
extraction-techniques =
number-of-cycles-22-error-relative = 0.001
amplitude-error-relative = 0.02

[ht-data]
2,2 = hmod.r5.13.12.m2
2,1 = hmod.r5.13.12.m1
2,-1 = hmod.r5.13.12.m1
2,-2 = hmod.r5.13.12.m-1
```

## D. First Matter NINJA project

This is a suggested format for a project such as http://www.ninja-project.org/doku.php?id=matter:home. Slashes are used to indicate suggested options.

1. Binary Systems

```
[metadata]
simulation-type = BNS/NSBH
mass-scale = 1
documentation = short.txt
publication = arXiv:1001.1234/unpublished/inprint
submitter-email = Sub Mitter <submitting@author.edu>
authors-email = Cor Espondent<corresponding@author.edu>, Sub Mitter <submitting@author.edu>
#Use convention that body 1 is BH in NSBH
#Use convention that body 1 is larger NS in NSNS
mass1 = 1.4
mass2 = 1.4
# calculate mass-ratio from mass specification as needed
# to avoid issues of convention conflict
spin1x = 0.0
spin1y = 0.0
spin1z = 0.0
spin2x = 0.0
spin2y = 0.0
spin2z = 0.0
# simulation setup and physics
gravity-type = NR/CFC/ApproxGR/Newtonian
fluid-type = GRHD/GRMHD/HD/MHD/OtherType
symmetries = 3Dbitant/3Dnone/OtherSymmetry
eos-name = Gamma2Poly/APR/LS/PP-HB/OtherName
eos-reference = SPIRES bibtex code or ADS link
eos-details = Specify EOS parameters (string may be long)
eos-details-reference = SPIRES bibtex code or ADS link
neutrino-treatment = None/CoolingFunction/Leakage/MGFLD/OtherName
neutrino-treatment-reference = SPIRES bibtex code or ADS link
# waveform properties
inspiral = Y/N # suitable target for inspiral search?
```

```
freq-Start-22 = 1.50E-4 \# in 1/M_sun = 203.025447 \text{ kHz}
data-type = NR/PN/hybrid
# pn-method = pn-doc.txt
# hybrid-method = hybrid-doc.txt
[ht-data]
2,2 = example1_12_m2.dat
2,-2 = example1_12_m-2.dat.
                                        2. Single-star systems
[metadata]
simulation-type = CCSN/NSCollapse/NSOscillations/OtherType
mass-scale = 1
documentation = short.txt
publication = SPIRES or ADS link/arXiv:1001.1234/unpublished/inprint
submitter-email = Sub Mitter <submitting@author.edu>
authors-email = Cor Espondent <corresponding@author.edu>, Sub Mitter <submitting@author.edu>
mass = 2.8 #total system gravitational mass in solar masses (M_Sun)
mass-baryonic = 3.0 #total system gravitational mass in M_Sun
spin = 1.0 #total system spin in c=G=M_Sun=1
# simulation setup and physics
gravity-type = NR/CFC/ApproxGR/Newtonian
fluid-type = GRHD/GRMHD/HD/MHD/OtherType
symmetries = axi/3Doctant/3Dbitant/3Dquadrant/3Dnone
# rotation
rotation = describe how rotation is set up; define meaning of rotation parameters
rotation-parameters = specify rotation parameters (string may be long)
rotation-reference = SPIRES bibtex code or ADS link
# microphysics
eos-name = Hybrid/HShen/LS/OtherName
eos-reference = SPIRES bibtex code or ADS link
eos-details = Specify EOS parameters (string may be long)
eos-details-reference = SPIRES bibtex code or ADS link
neutrino-treatment = None/CoolingFunction/Leakage/MGFLD/OtherName
neutrino-treatment-reference = SPIRES bibtex code or ADS link
# describe waveform type
data-type = NR/Quadrupole/OtherName
inspiral = N
freq-Start-20 = 0.0
freq-Start-22 = 0.0
[ht-data]
2,0 = example1_20.dat
2,2 = example1_22.dat.
```

#### Acknowledgments

We are grateful to various numerical relativists for numerous discussions and suggestions. In particular, we would like to thank the following people for valuable inputs to this document: Peter Diener, Luis Lehner, Lee Lindblom, Carlos Lousto, Hiroyuki Nakano, Harald Pfeiffer, Luciano Rezzolla, and Erik Schnetter.

## Appendix: Post-Newtonian waveforms in the adiabatic approximation

There are two basic elements to obtaining a post-Newtonian waveform: (1) finding the orbital phase of the binary, and (2) using that phase to find the so-called "amplitude" of the waveform. Previous references have been incomplete, or predate recent errata involving spin terms [19, 20]. Here, we gather together the most complete and current formulas for phase and amplitude when spins are *non-precessing* (i.e. spins aligned or anti-aligned with the orbital angular momentum), using consistent notation.

## 1. Phasing

The orbital phase evolution  $\Phi(t)$  of the binary can be computed by considering energy conservation. The energy of the full system is accounted for in three parts, each computed as a function of the post-Newtonian expansion parameter

$$v := \left(M \frac{d\Phi}{dt}\right)^{1/3} . \tag{A.1}$$

The first part is the kinetic and gravitational binding energy of the binary—the orbital energy E(v). As the system evolves, it gives off energy to infinity in the form of gravitational waves accounted for as the flux  $\mathcal{F}(v)$  leaving the system. Finally, we must also account for the tide raised on each black hole by the other, and the flow of energy into the black holes due to the motion of these tides, given as a rate of change in the mass of the black holes  $\dot{M}(v)$ . Using this threefold accounting for the energy, we can express the conservation of energy as

$$\frac{dE}{dt} + \mathcal{F} + \dot{M} = 0 \ . \tag{A.2}$$

Now, because the expression for the orbital energy is written in terms of v, we can straightforwardly differentiate to find E'(v). With the chain rule, dE/dt = E'(v) dv/dt, we can rearrange this into a differential equation for v:

$$\frac{dv}{dt} = -\frac{\mathcal{F}(v) + \dot{M}(v)}{E'(v)} \ . \tag{A.3}$$

Given the expressions for  $\mathcal{F}(v)$ ,  $\dot{M}(v)$ , and E'(v), this equation can be integrated to find v(t). Then, using the definition of v, we see that

$$\frac{d\Phi}{dt} = \frac{v^3}{M} \,, \tag{A.4}$$

which can be integrated in turn to find  $\Phi(t)$ . We now exhibit the formulas for  $\mathcal{F}(v)$ ,  $\dot{M}(v)$ , and E'(v), and discuss various methods for integrating the balance equation (A.3).

Given the masses  $M_1$  and  $M_2$  and spin vectors  $\mathbf{S}_1$  and  $\mathbf{S}_2$ , we define the following parameters:

$$M := M_1 + M_2 \tag{A.5}$$

$$\eta := M_1 M_2 / M^2 ,$$
(A.6)

$$\delta := (M_1 - M_2)/M \tag{A.7}$$

$$\boldsymbol{\chi}_i := \mathbf{S}_i / M_i^2 \,, \tag{A.8}$$

$$\chi_s := (\chi_1 + \chi_2)/2 , \tag{A.9}$$

$$\chi_a := (\chi_1 - \chi_2)/2$$
 (A.10)

We also define the quantities  $\chi_s$  and  $\chi_a$  to be the components of the spin vectors perpendicular to the orbital plane, namely  $\chi_s := \chi_s \cdot \ell$  and  $\chi_a := \chi_a \cdot \ell$ , where  $\ell$  is the unit vector along the Newtonian angular momentum.

The orbital energy function can be written in terms of the PN expansion parameter v defined above as  $[19-25]^3$ 

$$E(v) = -\frac{M\eta v^2}{2} \left\{ 1 + v^2 \left( -\frac{3}{4} - \frac{\eta}{12} \right) + v^3 \left[ \frac{8 \delta \chi_a}{3} + \left( \frac{8}{3} - \frac{4\eta}{3} \right) \chi_s \right] \right.$$

$$\left. + v^4 \left[ -2\delta \chi_a \chi_s - \frac{\eta^2}{24} + (4\eta - 1)\chi_a^2 + \frac{19\eta}{8} - \chi_s^2 - \frac{27}{8} \right] \right.$$

$$\left. + v^5 \left[ \chi_a \left( 8 \delta - \frac{31\delta\eta}{9} \right) + \left( \frac{2\eta^2}{9} - \frac{121\eta}{9} + 8 \right) \chi_s \right] \right.$$

$$\left. + v^6 \left[ -\frac{35\eta^3}{5184} - \frac{155\eta^2}{96} + \left( \frac{34445}{576} - \frac{205\pi^2}{96} \right) \eta - \frac{675}{64} \right] \right\}.$$
(A.11)

We simply take the derivative of this formula with respect to v to find the energy function appearing in the phasing formula:

$$E'(v) = -M\eta v \left\{ 1 + v^2 \left( -\frac{3}{2} - \frac{\eta}{6} \right) + v^3 \left[ \frac{20\delta \chi_a}{3} + \left( \frac{20}{3} - \frac{10\eta}{3} \right) \chi_s \right] \right.$$

$$\left. + v^4 \left[ -6 \delta \chi_a \chi_s - \frac{\eta^2}{8} + (12\eta - 3)\chi_a^2 + \frac{57\eta}{8} - 3\chi_s^2 - \frac{81}{8} \right] \right.$$

$$\left. + v^5 \left[ \chi_a \left( 28\delta - \frac{217\delta\eta}{18} \right) + \left( \frac{7\eta^2}{9} - \frac{847\eta}{18} + 28 \right) \chi_s \right] \right.$$

$$\left. + v^6 \left[ -\frac{35\eta^3}{1296} - \frac{155\eta^2}{24} + \left( \frac{34445}{144} - \frac{205\pi^2}{24} \right) \eta - \frac{675}{16} \right] \right\} . \tag{A.12}$$

Similarly, the flux function can be written as [19–25]<sup>4</sup>

$$\mathcal{F}(v) = \frac{32}{5} v^{10} \eta^2 \left\{ 1 + v^2 \left( -\frac{1247}{336} - \frac{35}{12} \eta \right) + v^3 \left[ -\frac{11\delta \chi_a}{4} + \left( 3\eta - \frac{11}{4} \right) \chi_s + 4\pi \right] \right.$$

$$\left. + v^4 \left[ \frac{33\delta \chi_a \chi_s}{8} + \frac{65\eta^2}{18} + \left( \frac{33}{16} - 8\eta \right) \chi_a^2 + \left( \frac{33}{16} - \frac{\eta}{4} \right) \chi_s^2 + \frac{9271\eta}{504} - \frac{44711}{9072} \right] \right.$$

$$\left. + v^5 \left[ \left( \frac{701\delta \eta}{36} - \frac{59\delta}{16} \right) \chi_a + \left( -\frac{157\eta^2}{9} + \frac{227\eta}{9} - \frac{59}{16} \right) \chi_s - \frac{583\pi\eta}{24} - \frac{8191\pi}{672} \right] \right.$$

$$\left. + v^6 \left[ -\frac{1712}{105} \ln(4v) - \frac{1712\gamma}{105} - \frac{775\eta^3}{324} - \frac{94403\eta^2}{3024} + \left( \frac{41\pi^2}{48} - \frac{134543}{7776} \right) \eta + \frac{16\pi^2}{3} + \frac{6643739519}{69854400} \right] \right.$$

$$\left. + v^7 \left[ \frac{193385\pi\eta^2}{3024} + \frac{214745\pi\eta}{1728} - \frac{16285\pi}{504} \right] \right\} , \tag{A.13}$$

where  $\gamma$  is the Euler Gamma.

Alvi [26] derived an expression for the transfer of energy from the orbit to each black hole by means of tidal heating. The calculation involves computing the deformation of each hole's horizon due to the Newtonian field of the other, then using that expression in formulas for energy absorption due to tidal deformation. In particular, his expression is applicable in the comparable-mass case. By combining the rates of mass change for both black holes, we obtain the total rate of change:

$$\dot{M}(v) = \frac{32}{5}v^{10}\eta^2 \left\{ -\frac{v^5}{4} \left[ (1 - 3\eta)\chi_s(1 + 3\chi_s^2 + 9\chi_a^2) + (1 - \eta)\delta\chi_a(1 + 3\chi_a^2 + 9\chi_s^2) \right] \right\}. \tag{A.14}$$

<sup>&</sup>lt;sup>3</sup> The 1.5PN and 2.5PN spin terms were taken from Eq. (7.9) of [19]. The 2PN spin term and all nonspinning terms were taken from Eq. (C4) of [20]. Note that Eq. (C5) in the original published version of [20] is erroneous.

<sup>&</sup>lt;sup>4</sup> The 1.5PN and 2.5PN spin terms were taken from Eq. (C10) of [20], except that the term  $\eta \left\{-\frac{103}{48}(\chi_s^2 - \chi_a^2) + \frac{289}{48}[(\chi_s \cdot \ell)^2 - (\chi_a \cdot \ell)^2]\right\}$  is omitted. (The authors of [20] have confirmed that this term should not be present.) Also note that Eq. (C11) in the original published version of [20] is erroneous.

The coefficient above is the leading-order term in the flux, meaning that this term is comparable to a relative 2.5PN spin effect in the flux. A similar calculation has been carried out in the extreme-mass-ratio limit [27], and agrees with this formula in that limit. Note that higher-order spin terms were calculated in [26], but are not included here, as they are at relative 3.5PN order, which is higher than the relative 2.5PN order to which other spin terms are known. Except for its explicit presence in the balance equation (A.3), we always treat the mass as a constant. This leads to additional errors at the 3.5PN spin level, which we ignore.

Below we will define some of the standard variants of computing the post-Newtonian phase from the energy and flux functions, using the naming convention of [28].

## a. TaylorT1 phasing

The TaylorT1 approximant is computed by numerically integrating the ordinary differential equation for v(t) in Eq. (A.3), using the expressions for orbital energy, flux, and mass change given in Eqs. (A.12), (A.13), and (A.14). The phase is then computed using this result for v(t) in Eq. (A.4).

# b. TaylorT4 phasing

The TaylorT4 approximant is similar to the TaylorT1 approximant, except that the ratio of the polynomials on the right-hand side of Eq. (A.3) is first expanded as a Taylor series, and truncated at consistent PN order. Explicitly, the formula to be integrated is

$$\begin{split} \frac{dv}{dt} &= \frac{32}{5M} v^{9} \eta \left\{ 1 + v^{2} \left[ -\frac{11\eta}{4} - \frac{743}{336} \right] + v^{3} \left[ -\frac{113\delta\chi_{a}}{12} + \left( \frac{9\eta}{3} - \frac{113}{12} \right) \chi_{s} + 4\pi \right] \right. \\ &+ v^{4} \left[ \frac{81\delta\chi_{a}\chi_{s}}{8} + \frac{59\eta^{2}}{18} + \left( \frac{81}{16} - 20\eta \right) \chi_{a}^{2} + \left( \frac{81}{16} - \frac{\eta}{4} \right) \chi_{s}^{2} + \frac{13661\eta}{2016} + \frac{34103}{8144} \right] \\ &+ v^{5} \left[ -\frac{189\pi\eta}{8} - \frac{4159\pi}{672} + \left( \frac{3\eta}{4} - \frac{3}{4} \right) \delta\chi_{a}^{3} + \left( \frac{9\eta}{4} - \frac{9}{4} \right) \delta\chi_{a}\chi_{s}^{2} + \left( \frac{1165\eta}{24} - \frac{31571}{1008} \right) \delta\chi_{a} \right. \\ &+ \left( -\frac{79\eta^{2}}{3} + \frac{27\eta\chi_{a}^{2}}{4} - \frac{9\chi_{a}^{2}}{4} + \frac{5791\eta}{63} - \frac{31571}{1008} \right) \chi_{s} + \left( \frac{9\eta}{4} - \frac{3}{4} \right) \chi_{s}^{3} \right] \\ &+ v^{6} \left[ -\frac{1712\gamma}{105} - \frac{5605\eta^{3}}{2592} + \frac{541\eta^{2}}{896} + \left( \frac{451\pi^{2}}{48} - \frac{56198689}{217728} \right) \eta_{s} + \frac{16\pi^{2}}{3} + \frac{16447322263}{139708800} \right. \\ &- \frac{1712\ln(4v)}{105} + \left( \frac{1517\eta^{2}}{72} - \frac{23441\eta}{288} + \frac{128495}{22016} \right) \chi_{s}^{2} + \left( \frac{565\delta^{2}}{3} + \frac{89\eta^{2}}{3} - \frac{2435\eta}{224} + \frac{215}{224} \right) \chi_{a}^{2} \\ &+ \left( \left( \frac{128495\delta}{1008} - \frac{12733\delta\eta}{144} \right) \chi_{a} + \frac{40\pi\eta}{3} - \frac{80\pi}{3} \right) \chi_{s} - \frac{80\pi\delta\chi_{a}}{3} \right] \\ &+ v^{7} \left[ \frac{91495\pi\eta^{2}}{1512} + \frac{358675\pi\eta}{6048} - \frac{4415\pi}{4032} \right. \\ &+ \left( \left( \frac{3\eta^{2}}{24} + \frac{917\eta}{24} - \frac{505}{8} \right) \chi_{s}^{3} + \left( \frac{\delta\eta^{2}}{8} + \frac{742\delta\eta}{3} - \frac{505\delta}{8} \right) \chi_{a}^{3} \right. \\ &+ \left( \left( \frac{124\delta^{2}}{3} - \frac{339017\eta^{2}}{24} + \frac{7007\eta}{24} - \frac{523}{8} \right) \chi_{s} - 48\pi\eta + 12\pi \right) \chi_{a}^{2} \\ &+ \left( \left( \frac{2045\eta^{3}}{216} - \frac{398017\eta^{2}}{2016} + \frac{70779221\eta}{54432} - \frac{2529407\delta}{27216} \right) \chi_{s} + 24\pi\delta\chi_{a}\chi_{s} \right. \\ &+ \left( -\frac{415515\eta^{2}}{864} + \frac{845827\delta\eta}{6048} - \frac{2529407\delta}{27216} \right) \chi_{a} \right] \right\}. \end{split}$$

Note that this expression does not include *all* of the spin-dependent terms at 3PN and 3.5PN, since the spin terms in the energy and flux functions are known only up to 2PN and 2.5PN, respectively. However, the 3PN and 3.5PN

terms shown here will still be present in this formula when the higher-order terms are included in the energy and flux formulas.

## c. TaylorT2 phasing

Expanding the inverse of Eq. (A.3) allows for the analytical integration of t(v). The result reads

$$\begin{split} t(v) &= t_0 - \frac{5M}{256\eta v^8} \Bigg\{ 1 + v^2 \left[ \frac{11\eta}{3} + \frac{743}{252} \right] + v^3 \left[ -\frac{32\pi}{5} + \frac{226\delta\chi_a}{15} + \left( \frac{226}{15} - \frac{152\eta}{15} \right) \chi_s \right] \\ &+ v^4 \left[ \frac{3058673}{508032} + \frac{5429\eta}{504} + \frac{617\eta^2}{72} - \frac{81}{4}\delta\chi_a\chi_s - \left( \frac{81}{8} - \frac{\eta}{2} \right) \chi_s^2 - \left( \frac{81}{8} - 40\eta \right) \chi_a^2 \right] \\ &+ v^5 \left[ -\frac{7729\pi}{252} - \frac{13\pi\eta}{3} + \left( \frac{147101}{756} - \frac{4906\eta}{27} - \frac{68\eta^2}{3} \right) \chi_s + \left( \frac{147101}{756} + \frac{26\eta}{3} \right) \delta\chi_a \right. \\ &+ \left. \left( 6 - 6\eta \right) \delta\chi_s^2 \chi_a + \left( 6 - 18\eta \right) \chi_s \chi_a^2 + \left( 2 - 6\eta \right) \chi_s^3 + \left( 2 - 2\eta \right) \delta\chi_a^3 \right] \\ &+ v^6 \left[ \frac{6848\gamma}{105} - \frac{10052469856691}{23471078400} + \frac{128\pi^2}{3} + \left( \frac{3147553127}{3048192} - \frac{451\pi^2}{12} \right) \eta - \frac{15211\eta^2}{1728} + \frac{25565\eta^3}{1296} \right. \\ &+ \frac{6848\ln(4v)}{105} - \left( \frac{584\pi}{3} - \frac{448\pi\eta}{3} \right) \chi_s - \frac{584\pi\delta\chi_a}{3} + \left( \frac{6845}{672} - \frac{43427\eta}{168} + \frac{245\eta^2}{3} \right) \chi_s^2 \\ &+ \left( \frac{6845}{672} - \frac{1541\eta}{12} + \frac{964\eta^2}{3} \right) \chi_a^2 + \left( \frac{6845}{336} - \frac{2077\eta}{6} \right) \delta\chi_s \chi_a \right] \\ &+ v^7 \left[ -\frac{15419335\pi}{127008} - \frac{75703\pi\eta}{756} + \frac{14809\pi\eta^2}{378} + \left( \frac{4074790483}{1524096} + \frac{30187\eta}{112} - \frac{115739\eta^2}{216} \right) \delta\chi_a \right. \\ &+ \left( \frac{4074790483}{1524096} - \frac{869712071\eta}{381024} - \frac{2237903\eta^2}{1512} + \frac{14341\eta^3}{54} \right) \chi_s + \left( 228\pi - 16\pi\eta \right) \chi_s^2 \\ &+ \left( 228\pi - 896\pi\eta \right) \chi_a^2 + 456\pi\delta\chi_s \chi_a - \left( \frac{3237}{14} - \frac{14929\eta}{84} + \frac{362\eta^2}{3} \right) \chi_s^3 \\ &- \left( \frac{3237}{14} - \frac{87455\eta}{84} + 34\eta^2 \right) \delta\chi_a^3 - \left( \frac{9711}{14} - \frac{39625\eta}{84} + 102\eta^2 \right) \delta\chi_s^2 \chi_a \\ &- \left( \frac{9711}{14} - \frac{267527\eta}{84} + \frac{3574\eta^2}{3} \right) \chi_s \lambda_a^2 \Bigg] \Bigg\} \, . \end{split}$$

The comment made below Eq. (A.15) about spin contributions at 3PN and 3.5PN order is valid for Eq. (A.16) and the following expansions as well.

The orbital phase  $\Phi$  can be integrated similarly to the time t. Eq. (A.4) and Eq. (A.3) yield

$$\frac{d\Phi}{dv} = \frac{v^3}{M} \frac{dt}{dv} = -\frac{v^3}{M} \frac{E'(v)}{\mathcal{F}(v) + \dot{M}(v)} , \qquad (A.17)$$

which, after re-expanding in a Taylor series, can be integrated analytically. The final result reads

$$\begin{split} \Phi(v) &= \Phi_0 - \frac{1}{32\eta \, v^5} \Bigg\{ 1 + v^2 \left[ \frac{3715}{1008} + \frac{55\eta}{12} \right] + v^3 \left[ -10\pi + \frac{565\delta\chi_a}{24} + \left( \frac{565}{24} - \frac{95\eta}{6} \right) \chi_s \right] \\ &+ v^4 \left[ \frac{15293365}{1016064} + \frac{27145\eta}{1008} + \frac{3085\eta^2}{144} - \frac{405}{8}\delta\chi_a\chi_s - \left( \frac{405}{16} - \frac{5\eta}{4} \right) \chi_s^2 - \left( \frac{405}{16} - 100\eta \right) \chi_a^2 \right] \\ &+ v^5 \ln v \left[ \frac{38645\pi}{672} - \frac{65\pi\eta}{8} - \left( \frac{735505}{2016} - \frac{12265\eta}{36} - \frac{85\eta^2}{2} \right) \chi_s - \left( \frac{735505}{2016} + \frac{65\eta}{4} \right) \delta\chi_a \right. \\ &- \left( \frac{45}{4} - \frac{45\eta}{4} \right) \delta\chi_s^2 \chi_a - \left( \frac{45}{4} - \frac{135\eta}{4} \right) \chi_s \chi_a^2 - \left( \frac{15}{4} - \frac{45\eta}{4} \right) \chi_s^3 - \left( \frac{15}{4} - \frac{15\eta}{4} \right) \delta\chi_a^3 \right] \end{split} \tag{A.18}$$

$$+ v^6 \left[ \frac{12348611926451}{18776862720} - \frac{1712\gamma}{21} - \frac{160\pi^2}{3} - \left( \frac{15737765635}{12192768} - \frac{2255\pi^2}{48} \right) \eta + \frac{76055\eta^2}{6912} - \frac{127825\eta^3}{5184} \right. \\ \left. - \frac{1712 \ln \left( 4v \right)}{21} + \left( \frac{730\pi}{3} - \frac{560\pi\eta}{3} \right) \chi_s + \frac{730\pi\delta\chi_a}{3} - \left( \frac{34225}{2688} - \frac{217135\eta}{672} + \frac{1225\eta^2}{12} \right) \chi_s^2 \right. \\ \left. - \left( \frac{34225}{2688} - \frac{7705\eta}{48} + \frac{1205\eta^2}{3} \right) \chi_a^2 - \left( \frac{34225}{1344} - \frac{10385\eta}{24} \right) \delta \chi_s \chi_a \right] \\ + v^7 \left[ \frac{77096675\pi}{2032128} + \frac{378515\pi\eta}{12096} - \frac{74045\pi\eta^2}{6048} - \left( \frac{20373952415}{24385536} + \frac{150935\eta}{1792} - \frac{578695\eta^2}{3456} \right) \delta \chi_a \\ - \left( \frac{20373952415}{24385536} - \frac{4348560355\eta}{6096384} - \frac{11189515\eta^2}{24192} + \frac{71705\eta^3}{864} \right) \chi_s - \left( \frac{285\pi}{4} - 5\pi\eta \right) \chi_s^2 \\ - \left( \frac{285\pi}{4} - 280\pi\eta \right) \chi_a^2 - \frac{285\pi}{2} \delta \chi_s \chi_a + \left( \frac{16185}{224} - \frac{74645\eta}{1344} + \frac{905\eta^2}{24} \right) \chi_s^3 \\ + \left( \frac{16185}{224} - \frac{437275\eta}{1344} + \frac{85\eta^2}{8} \right) \delta \chi_a^3 + \left( \frac{48555}{224} - \frac{198125\eta}{1344} + \frac{255\eta^2}{8} \right) \delta \chi_s^2 \chi_a \\ + \left( \frac{48555}{224} - \frac{1337635\eta}{1344} + \frac{8935\eta^2}{24} \right) \chi_s \chi_a^2 \right] \right\}.$$

Eq. (A.16) and Eq. (A.18) together define  $\Phi(t)$  implicitly.

## d. TaylorF2 phasing

Starting from the explicit expressions for time and orbital phase in the TaylorT2 approximant, it is possible to analytically construct the Fourier transform of the GW strain in the framework of the stationary phase approximation (SPA) [28–30]. Denoting the Fourier transform of Eq. (IV.1) by  $\tilde{A}_{\ell m}e^{i\psi_{\ell m}}$ , the phase in the frequency domain can be approximated by

$$\psi_{\ell m}(f) = 2\pi f \, t_f - m\Phi(t_f) - \frac{\pi}{4}.$$
 (A.19)

Here, f is the Fourier variable and  $t_f$  corresponds to the time when the instantaneous GW frequency coincides with f, i.e.,

$$\frac{d(m\Phi)}{dt}(t_f) = 2\pi f \qquad \Rightarrow \quad v(t_f) = \left(\frac{2\pi Mf}{m}\right)^{1/3}.$$
 (A.20)

The form of the Taylor series of  $\psi_{\ell m}$  obviously depends on the spherical harmonic mode's m. For the sake of brevity, only the expansion for m=2 is given below.

$$\begin{split} \psi_{\ell 2}(v) &= 2t_0v^3 - 2\Phi_0 - \frac{\pi}{4} + \frac{3}{128\eta\,v^5} \Bigg\{ 1 + v^2 \left[ \frac{3715}{756} + \frac{55\eta}{9} \right] + v^3 \left[ -16\pi + \left( \frac{113}{3} - \frac{76\eta}{3} \right) \chi_s + \frac{113\delta\chi_a}{3} \right] \\ &+ v^4 \left[ \frac{15293365}{508032} + \frac{27145\eta}{504} + \frac{3085\eta^2}{72} - \left( \frac{405}{8} - \frac{5\eta}{2} \right) \chi_s^2 - \left( \frac{405}{8} - 200\eta \right) \chi_a^2 - \frac{405}{4} \delta\chi_s\chi_a \right] \\ &+ v^5 \left( 1 + 3\ln v \right) \left[ \frac{38645\pi}{756} - \frac{65\pi\eta}{9} - \left( \frac{735505}{2268} - \frac{24530\eta}{81} - \frac{340\eta^2}{9} \right) \chi_s - \left( \frac{735505}{2268} + \frac{130\eta}{9} \right) \delta\chi_a \\ &- \left( 10 - 10\eta \right) \delta\chi_s^2\chi_a - \left( 10 - 30\eta \right) \chi_s\chi_a^2 - \left( \frac{10}{3} - 10\eta \right) \chi_s^3 - \left( \frac{10}{3} - \frac{10\eta}{3} \right) \delta\chi_a^3 \right] \\ &+ v^6 \left[ \frac{11583231236531}{4694215680} - \frac{6848\gamma}{21} - \frac{640\pi^2}{3} - \left( \frac{15737765635}{3048192} - \frac{2255\pi^2}{12} \right) \eta + \frac{76055\eta^2}{1728} - \frac{127825\eta^3}{1296} \right. \\ &- \frac{6848\ln(4v)}{21} + \left( \frac{2920\pi}{3} - \frac{2240\pi\eta}{3} \right) \chi_s + \frac{2920\pi}{3} \delta\chi_a - \left( \frac{34225}{672} - \frac{217135\eta}{168} + \frac{1225\eta^2}{3} \right) \chi_s^2 \\ &- \left( \frac{34225}{672} - \frac{7705\eta}{12} + \frac{4820\eta^2}{3} \right) \chi_a^2 - \left( \frac{34225}{336} - \frac{10385\eta}{6} \right) \delta\chi_s\chi_a \bigg] \end{split}$$

$$+ v^{7} \left[ \frac{77096675\pi}{254016} + \frac{378515\pi\eta}{1512} - \frac{74045\pi\eta^{2}}{756} - \left( \frac{20373952415}{3048192} + \frac{150935\eta}{224} - \frac{578695\eta^{2}}{432} \right) \delta\chi_{a} \right.$$

$$- \left( \frac{20373952415}{3048192} - \frac{4348560355\eta}{762048} - \frac{11189515\eta^{2}}{3024} + \frac{71705\eta^{3}}{108} \right) \chi_{s} - (570\pi - 40\pi\eta)\chi_{s}^{2}$$

$$- \left( 570\pi - 2240\pi\eta \right) \chi_{a}^{2} - 1140\pi\delta\chi_{s}\chi_{a} + \left( \frac{16185}{28} - \frac{74645\eta}{168} + \frac{905\eta^{2}}{3} \right) \chi_{s}^{3}$$

$$+ \left( \frac{16185}{28} - \frac{437275\eta}{168} + 85\eta^{2} \right) \delta\chi_{a}^{3} + \left( \frac{48555}{28} - \frac{198125\eta}{168} + 255\eta^{2} \right) \delta\chi_{s}^{2}\chi_{a}$$

$$+ \left( \frac{48555}{28} - \frac{1337635\eta}{168} + \frac{8935\eta^{2}}{3} \right) \chi_{s}\chi_{a}^{2} \right] \right\}. \tag{A.21}$$

According to Eq. (A.20), v should be understood as  $v = (M\pi f)^{1/3}$  in the equation above.

#### 2. Waveform amplitudes

Now, given the orbital phase  $\Phi$  and the related post-Newtonian expansion parameter v defined in Eq. (A.1), we can obtain the waveform observed at infinity. Currently, the most complete expressions for the nonspinning parts of the waveform are found in [31]. In particular, Eqs. (9.3) and (9.4) of that reference give the decomposition of  $h_+ - ih_\times$  into harmonics as requested in Eq. (II.6). Due to space considerations and the danger of transcription errors, we do not reproduce those equations here, but simply refer the reader to that paper. To these, we must add<sup>5</sup> the spin terms given most completely in [32]. There, the spin terms were not explicitly decomposed into harmonics, however, using Eq. (9.2) of [31], it is a simple matter to deduce them. Using Eqs. (F24) and (F25) of [32], and noting the overall sign error in Eq. (F25c), we obtain the only nonzero spin contributions to the harmonics:

$$H_{2,2} = -\frac{16}{3} \sqrt{\frac{\pi}{5}} v^5 \eta \left[ 2\delta \chi_a + 2(1-\eta)\chi_s + 3v\eta \left(\chi_a^2 - \chi_s^2\right) \right] e^{-2i\Phi} , \qquad (A.22)$$

$$H_{2,1} = 4i\sqrt{\frac{\pi}{5}}v^4\eta(\delta\chi_s + \chi_a)e^{-i\Phi} , \qquad (A.23)$$

$$H_{3,2} = \frac{32}{3} \sqrt{\frac{\pi}{7}} v^5 \eta^2 \chi_s e^{-2i\Phi} . \tag{A.24}$$

In all cases, modes with negative values of m can be obtained from

$$H_{\ell,-m} = (-1)^{\ell} \bar{H}_{\ell,m} .$$
 (A.25)

The appropriate SPA amplitude in Fourier space can easily be deduced from its time-domain description  $A_{\ell m}$  by

$$\tilde{A}_{\ell m} = A_{\ell m} \sqrt{\frac{2\pi}{m\ddot{\Phi}}} = A_{\ell m} \sqrt{\frac{2\pi M}{3mv^2 \dot{v}}} ,$$
 (A.26)

where  $\dot{v}$  can be taken for instance from Eq. (A.15) and all arguments should be replaced according to Eq. (A.20).

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<sup>&</sup>lt;sup>5</sup> Note that Refs. [31] and [32] share the notation set forth in [33], so that we can simply add the relevant terms. Also note that each of those references uses a different normalization for the variable *H* compared to the one used here.

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