

**Cardiff Economics  
Working Papers**

David Meenagh, Patrick Minford and David Peel

*Simulating Stock Returns under switching regimes — a new  
test of market efficiency*

E2006/13

Cardiff Business School  
Cardiff University  
Colum Drive  
Cardiff CF10 3EU  
United Kingdom  
t: +44 (0)29 2087 4000  
f: +44 (0)29 2087 4419  
[www.cardiff.ac.uk/carbs](http://www.cardiff.ac.uk/carbs)

ISSN 1749-6101  
February 2006

# Simulating Stock Returns under switching regimes- a new test of market efficiency.\*

David Meenagh<sup>a,†</sup>, Patrick Minford<sup>a,c</sup> and David Peel<sup>b</sup>

<sup>a</sup>Cardiff University, <sup>b</sup>Lancaster University, <sup>c</sup>CEPR

## Abstract

A model of profits switches between four regimes with fixed probabilities; the rationally expected profits stream implies the stock market value. This efficient market model is not rejected by UK post-war time-series behaviour of either profits or the FTSE index.

*Keywords:* regime switching, stock returns, efficient markets, rational expectations

*JEL Classification:* C15, C5, G14

---

\*We thank Laurence Copeland and workshop participants at Cardiff Business School

†Corresponding Author: David Meenagh, Cardiff University, Cardiff Business School, Aberconway Building, Colum Drive, Cardiff, CF10 3EU, UK; Tel: +44 29 20875198; Fax: +44 29 20874419; Email: MeenaghD@cf.ac.uk

# 1 Introduction

Tests of market efficiency based on various forms of regression are beset with problems of interpretation — Minford and Peel (2002, ch. 14) — including the possibility of variable risk-premia, peso problems and rational bubbles. Allowing for peso problems can make an important contribution to explaining apparent ex-post inefficiencies in asset markets— for example Rietz (1988) who specifies the Mehra and Prescott (1985) model to include a low-probability, depression-like, third state in order to provide some explanation of the equity-premium puzzle. Thus Markov switching models have become increasingly popular in analysis of asset prices as they are able to generate a wide range of coefficients for skewness and kurtosis and serial correlation in mean and variance even when based on a very small number of underlying states. (see e.g. Timmermann (2000) and Guidolin and Timmermann (2005, 2005(a)), who find that four separate regimes are required to capture the joint distribution of stock and bond returns.)

In this paper we build on this switching structure. Using quarterly data for the UK over the period 1963q2 to 2002q2 we create a regime switching model with four regimes (where the regimes represent high, normal and low growth, as well as a crash) with the probability of each regime constant over time, which generates a profits series. The rational expectation of the future profits is used to create the present discounted value which gives the implied stock market series, here the UK FTSE. There is a constant probability of each future regime and the variance around future returns is fixed, as are any risk-premium terms. We have an efficient market world of rational agents by construction. As our profits regime is a latent process it cannot be observed. However we require that the profits series produced by this latent process must be consistent with the actual profits data for the sample period, in just the same way that the FTSE series produced by it must be consistent with the actual FTSE data for the same period. Thus our model has to pass a double test: that it can generate not merely the FTSE but also the profits processes. We use stochastic simulations to check on the model's capacity to do so. At the end of the analysis we ask whether we can reject the hypothesis that the latent efficient market model is at work.

## 2 The Working Hypothesis

We can think of an economy (the UK in this case) where the capital stock generates the profits or dividends that are valued as equities (the FTSE). The profits are mainly driven by productivity shocks since variations in labour inputs mainly change wages while changes in the capital stock mainly dilute the equity base. So we shall assume that the fundamental, profits per share, can be identified with productivity as for example in a real business cycle (RBC) model. RBC models take the behaviour of productivity as exogenous, usually modelling it via some sort of univariate time-series. The empirical success of these models in matching the macroeconomic facts remains controversial; however we would argue that RBC models remain capable of supplying a good account of macroeconomic growth and fluctuations (see Rebello (2005)).

Here we suggest that the recognition of several regimes for productivity growth could be a helpful generalisation of the time-series process governing it. Thus for example one might identify periods of poor productivity growth — during with poorly-adapted institutions (e.g. union power in the UK during the 1970s); periods of rapid productivity associated with surges of innovation (the industrial revolution, the computer revolution etc); and periods of normal growth, when innovation is being digested undisturbed by wars or dysfunctional institutions. Finally we included periods of ‘crash’ when profits drop off sharply and reduce the value of equities dramatically. The point has been made by Jorion and Goetzmann (1999) that from time to time around the world stock markets may suffer extreme loss or indeed total extinction because of a drastic interruption of profits from extreme negative events such as war or revolution. This possibility is present in even the most stable societies since such stability cannot ultimately be taken for granted.

Each of these productivity regimes we represent by an ARIMA (1,1, 0) process with drift. The unit root represents the idea that productivity changes are in principle irreversible; the serial correlation the idea that once a change occurs it will be followed by further similar changes. The drift represents the mean growth of the regime. Each period the economy chooses one of the regimes with fixed probabilities — but the lagged effects

of the previous regimes still are present. Thus if a war occurs, even when it is over its effects persist until they gradually disappear from the economy; meanwhile other shocks overlay them.<sup>1</sup> In our study we are unable to take our empirical work back beyond the actual profits series which we treat as the observable effect of productivity.

### 3 Our Test Procedure

We take the four regimes described above and assign the same iid normal-error to each. Each period the rational expectation of the future profits level is calculated and used to create the present discounted value (with a constant discount factor) which is the assumed FTSE. Notice that the conditional variance of profits around the future expectation is constant at all times since there is a constant probability of each future regime and hence of all future possible innovations; hence the variance surrounding future returns is equally fixed and with it any risk-premium terms attaching to FTSE valuation. Thus our set-up embodies all the standard assumptions of an efficient market world of rational agents.

At the next stage we search to find the best calibration of this model to the profits and FTSE data. Using the powerful method of grid search we choose the combination of parameters for the 4 ARIMA processes that minimises the distance between a linear combination of the moments of the simulated FTSE and the actual FTSE and those of the simulated profits process from those of the actual process.<sup>2</sup>

Using the best set of parameters the composite process is then simulated stochastically in 50000 runs of 150 periods. With these stochastic simulations we then carry out the tests described at the start of this paper. We wish to know whether the profits and FTSE data can each be regarded as a sample drawing from this model. We look first at the profits and FTSE sample moments; do they lie outside the 95% confidence limits of the null hypothesis distributions (found from the 50000 generated samples)? Secondly, we

---

<sup>1</sup>To calculate the order of magnitude for the probability of a crash we used share price index data for 38 countries.

<sup>2</sup>The method we use here is similar to Simulated Method of Moments, though we do not vary the weights on the moments in the critical value.

look at the profits ARIMA process and the FTSE GARCH process describing respectively the profits and the FTSE's dynamics; do the parameters of these processes lie outside the 95% confidence limits obtained from the 50000 generated samples?

#### *Model details*

The four regimes are each assumed to have their own mean ( $c_1, c_2, c_3, c_4$ ) and equal standard deviation ( $\sigma_\eta$ ). The low growth, high growth and crash regimes have the probability of occurrence  $\pi_1, \pi_2, \pi_3, \pi_4$ . For each period we choose the regime at random and the corresponding growth ( $c_i$ ), and then choose a random number  $\eta \sim N(0, \sigma_\eta)$ . This is then used to calculate our generated FTSE.

For our test procedure we estimate on the actual data the first four moments and also the best available parsimonious time-series descriptions of the two data series, DFT and DPROF: for the first an ARCH-ARMA(1,0), and for the second an ARMA(1,0). Thus:

$$\Delta \log FTSE_t = \beta_1 \Delta \log FTSE_{t-1} + \epsilon_t \quad (1)$$

with a ARCH(1) representation of the residuals of this regression<sup>3</sup>, and

$$\Delta \ln(PROF_t) = \alpha_1 + \alpha_2 \Delta \ln(PROF_{t-1}) + \varepsilon_t^1 \quad (2)$$

The estimated coefficients from this regression are then used as the normal growth rate (regime 2) and the serial correlation ( $\rho_{all}$ ). We then estimate the moments and the corresponding equations on the generated two series, to obtain the 95% confidence limits for both moments and equation parameters. The working hypothesis is rejected if any of these lie outside the 95% interval.

---

<sup>3</sup>We initially estimated  $\Delta \log FTSE_t = \beta_1 + \beta_2 \Delta \log FTSE_{t-1} + \epsilon_t$  with a GARCH(1,1) representation, but found that  $\beta_1$  and the GARCH parameter were not statistically significant.

## 4 Results

We investigated this model initially in a variety of versions with less than four regimes. However, they all in various ways failed to match jointly the properties of the FTSE and the profits data. Turning to the model of four regimes described above, we found that the combination of parameters from the search algorithm that minimised the critical value were

$$\begin{aligned} \rho_1 &= -0.2 & \rho_2 &= -0.10868384 & \rho_3 &= 0.3 & \rho_{crash} &= 0.9 \\ c_1 &= -0.063978 & c_2 &= 0.006414 & c_3 &= 0.1 & c_4 &= -0.3 \\ \pi_1 &= 0.07828 & \pi_2 &= 0.91172 & \pi_3 &= 0.01 & \pi_4 &= 0.0002 \\ \sigma_\eta &= 0.003267 \end{aligned}$$

Note that here for the crash regime alone the  $\rho$  parameter is set at  $\rho_{crash} = 0.9$ , an important element in the process of matching the moments. We had to raise the  $\rho$  parameter for a crash because the variance of the FTSE is large compared to the variance of profits, so that a larger  $\rho$  was required to match the variance of the FTSE. Also, the  $\rho$ 's for the other regimes are calculated so that the average  $\rho$  is equal to the estimated value ( $\alpha_2$  from Equation (2)). The results for this model are in Table 1: the model matches all the properties of both the FTSE and the profits series, in the sense that at a 95% confidence interval it cannot be rejected by our chosen descriptive measures of these two data series.

## 5 Conclusions

We have shown here that the hypothesis of efficiency, if constructed to incorporate the possibility of extreme events, can mimic the behaviour of the FTSE. It remains to be seen if the same is true of alternative hypotheses, such as behavioural finance.

	Lower 2.5% Limit	Upper 2.5% Limit	Actual
<b>FTSE</b>			
Mean	-0.009916	0.013217	0.004356
Variance	0.002323	0.012068	0.007137
Skewness	-1.278085	1.356789	-0.579837
Kurtosis	2.503174	41.085846	5.715775
<b>FTSE Regression</b>			
$\beta_1$	-0.011192	0.014199	0.008181
ARCH constant	0.002103	0.005427	0.003327
ARCH	0.000000	0.456564	0.455670
<b><math>\Delta \log(\text{Profits})</math> Regression</b>			
$\beta_1$	-0.011207	0.014974	0.006414
$\beta_2$	-0.272949	0.046430	-0.111544
<b><math>\Delta \log(\text{Profits})</math> Regression Residuals</b>			
Mean	$-4.09e^{-18}$	$4.08e^{-18}$	$-4.08e^{-19}$
Variance	0.002679	0.004964	0.002714
Skewness	-0.650569	0.477425	-0.390942
Kurtosis	2.309765	4.920717	3.991456

Table 1: Results for the markov switching model with a crash



## References

- [1] Guidolin, Massimo, and Allan Timmermann, 2005, Optimal Portfolio Choice Under Regime Switching, Skew and Kurtosis Preferences, *Federal Reserve Bank of St. Louis Working Paper 2005-006A*
- [2] Guidolin, Massimo, and Allan Timmermann, 2005(a), Strategic Asset Allocation and Consumption Decisions under Multivariate Regime Switching, *Federal Reserve Bank of St. Louis Working Paper 2005-002B*
- Gultekin, Mustafa N. and N. Bulent Gultekin, 1983, Stock Market Seasonality: International Evidence, *Journal of Financial Economics*, 12, 469–481
- [3] Jorion, Philippe, and William N. Goetzmann, 1999, Global Stock Markets in the Twentieth Century, *The Journal of Finance* 54, 953–980
- [4] Mehra, Rajnish and Edward C. Prescott, 1985, The equity premium: A puzzle, *Journal of Monetary Economics*, 15, 145–161.
- [5] Minford, Patrick and David Peel, 2002, *Advanced Macroeconomics: A Primer*, Edward Elgar
- [6] Rietz, Thomas A., 1988, The Equity Risk Premium: A Solution, *Journal of Monetary Economics* 22, 117–131
- [7] Rebello, S.(2005) Real Business Cycle Models:Past, Present, and Future.
- [8] Timmermann, Allan, 2000, Moments of Markov Switching Models, *Journal of Econometrics* 96, 75–111