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Export Taxes under Bertrand Duopoly

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Abstract

This article analyses export taxes in a Bertrand duopoly with product differentiation, where a home and a foreign firm both export to a third-country market. It is shown that the maximum-revenue export tax always exceeds the optimum-welfare export tax. In a Nash equilibrium in export taxes, the country with the low cost firm imposes the largest export tax. The results under Bertrand duopoly are compared with those under Cournot duopoly. It is shown that the absolute value of the export subsidy or tax under Cournot duopoly exceeds the export tax under Bertrand duopoly.

Keywords: Trade Policy, Imperfect Competition, Oligopoly.

JEL Classification: F12, F13, L13.

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1. Introduction

The extensive literature on international trade and trade policy under imperfect competition has concentrated upon the Cournot duopoly model and paid relatively little attention to the Bertrand duopoly model with a few exceptions such as Eaton and Grossman (1986) and Cheng (1988). Recently, Clarke and Collie (2003) analysed the welfare effects of free trade in the Bertrand duopoly model with product differentiation. In contrast to the results of Brander and Krugman (1983) under Cournot duopoly, they showed that there were always gains from trade under Bertrand duopoly. They used the same model to analyse import tariffs in Clarke and Collie (2006), where they showed that the optimum-welfare tariff may exceed the maximum-revenue tariff under Bertrand duopoly, as Collie (1991) showed for Cournot duopoly, in contrast to the result of Johnson (1951-52) under perfect competition. They also showed that the optimum-welfare tariff and the maximum-revenue tariff were lower under Bertrand duopoly than under Cournot duopoly. Similarly, Cheng (1988) shows that the optimal import tariff and production subsidy are lower under Bertrand duopoly than under Cournot duopoly. This suggests that a general principle might be that the incentive to intervene in international trade is greater under Cournot duopoly than under Bertrand duopoly.

Regarding trade policy for export industries, Brander and Spencer (1985) showed that an export subsidy was optimal under Cournot duopoly whereas Eaton and Grossman (1986) showed that an export tax was optimal under Bertarnd duopoly. In the Brander and Spencer (1985) model, de Meza (1986) showed that the country with the lowest cost firm will give the largest export subsidy in the Nash equilibrium in export subsidies. This result was generalised by Collie and de Meza (2003) to allow for the possibility of export taxes and/or export subsidies in the Nash equilibrium under Cournot duopoly. They showed that the absolute value of export subsidy or tax of the country with the lowest cost firm will be larger than the absolute value of the export subsidy or tax of the other country. This suggests another general principle might be that the country with the lowest cost firm will have the greatest incentive to intervene in international trade compared to the other country.

This note will analyse export taxes under Bertrand duopoly using the same model as Clarke and Collie (2003 and 2006) in order to answer a number of questions. Firstly, how do the maximum-revenue and optimum-welfare export taxes under Bertrand duopoly compare with those under Cournot duopoly? Secondly, is it possible for the optimum-welfare export tax to exceed the maximum-revenue export tax as Clarke and Collie (2006) showed may happen with import tariffs? Thirdly, in the Nash equilibrium in export taxes as in Eaton and Grossman (1986), will the country with the lowest cost firm impose the largest export tax in line with the proposition of Collie and de Meza (2003) under Cournot duopoly?

The Bertrand duopoly model is presented in section two then the maximum-revenue and optimum-welfare export taxes are derived and compared in section three. The Nash equilibrium in export taxes is derived in section four and the conclusions are given in section five. For comparison, the Cournot duopoly model with product differentiation is analysed in the appendix.

2. The Model

As in Clarke and Collie (2003 and 2006), assume that there are two countries, a home and a foreign country and that each country has one firm that produces a differentiated good, with the home firm labelled as firm one and the foreign firm labelled as firm two.[1] The two firms both export to a third-country market where they compete in a Bertrand duopoly. Markets are assumed to be segmented. The home firm has constant marginal cost c_1, sets

[1] A similar model is used by Kikuchi (1998) to analyse optimum-welfare export taxes and subsidies under Bertrand and Cournot duopoly. He assumes symmetric demand functions and does not consider the maximum-revenue export tax or the Nash equilibrium in export taxes.

price p_1, and sells output y_1 while the foreign firm has constant marginal cost c_2, sets price p_2, and sells output y_2. The government in the home (foreign) country imposes a specific export tax of e_1 (e_2) per unit exported by the home (foreign) firm. It is assumed that there is a representative consumer in the third-country market with quasi-linear preferences that can be represented by a quadratic utility function, as in Vives (1985):

$$U(\mathbf{y},z)=\sum_{i=1}^{2}\alpha_i y_i-\frac{1}{2}\sum_{i=1}^{2}\beta_i y_i^2-\gamma y_1 y_2+z \qquad \alpha_i,\beta_i,\gamma>0; \quad \beta_i>\gamma \tag{1}$$

where z is consumption of a numeraire good and $0<\gamma^2/\beta_1\beta_2<1$ is a measure of the degree of product substitutability ranging from zero when the products are independent to one when they are perfect substitutes. It is straightforward to show that the utility function (1) yields the following inverse and direct demand functions:

$$\begin{aligned} p_i &= \alpha_i-\beta_i y_i-\gamma\, y_j \\ y_i &= \frac{1}{F}\left[\left(\alpha_i\beta_j-\alpha_j\gamma\right)-\beta_j\, p_i+\gamma\, p_j\right] \end{aligned} \qquad i,j=1,2 \quad i\neq j \tag{2}$$

where $F=\beta_1\beta_2-\gamma^2>0$. With segmented markets and constant marginal costs, the third-country export market can be analysed independently of the home and foreign markets. Then, the profit functions of the home firm and the foreign firm from exports to the third-country market are:

$$\pi_i=\left(p_i-c_i-e_i\right) y_i \qquad i=1,2 \tag{3}$$

Assuming an interior solution, where both firms export positive quantities to the third-country market, it is straightforward to solve for the Bertrand equilibrium prices:

$$p_i=c_i+e_i+\frac{1}{H}\left[G(\alpha_i-c_i-e_i)-\beta_i\gamma(\alpha_j-c_j-e_j)\right] \tag{4}$$

where $G=2\beta_1\beta_2-\gamma^2>0$ and $H=4\beta_1\beta_2-\gamma^2>0$. Substituting these prices into the demand functions (2) yields the Bertrand equilibrium exports of the home firm and the foreign firm:

$$y_i = \frac{\beta_j}{FH}\left[G\left(\alpha_i - c_i - e_i\right) - \beta_i\gamma\left(\alpha_j - c_j - e_j\right)\right] \tag{5}$$

As with (4) and (5), many of the results that follow will be functions of $\left(\alpha_1 - c_1\right)$ and $\left(\alpha_2 - c_2\right)$ so it is worth noting that α_i is the consumer's maximum willingness to pay for the product of the *i*th firm while c_i is the marginal cost of the *i*th firm. Also, from (4) and (5), it can be seen that an increase in the export tax of the home country results in both firms increasing their prices, a decrease in exports for the home firm, and an increase in exports for the foreign firm. These results can now be used to derive the maximum-revenue and the optimum-welfare export taxes.

3. Optimum-Welfare and Maximum-Revenue Export Taxes

This section analyses the trade policy of the home country and, for simplicity, it will be assumed that the foreign country pursues a policy of laissez-faire so $e_2 = 0$ throughout this section. Firstly, consider the maximum-revenue export tax of the home country. The maximum-revenue export tax is the export tax that maximises the export tax revenue collected by the government in the home country. The export tax is e_1 per unit of exports and the quantity of exports is y_1 so export tax revenue is $R_1 = e_1 y_1$. Assuming an interior solution where the market is supplied by exports from both firms, the first-order condition for the maximisation of export tax revenue by the government in the home country is:

$$\frac{\partial R_1}{\partial e_1} = y_1 + e_1 \frac{\partial y_1}{\partial e_1} = 0 \tag{6}$$

Thus, using (5), to solve for an explicit expression for the maximum-revenue export tax yields (where the superscript *B* is used to denote the Bertrand duopoly and *R* is used to denote the maximum-revenue export tax):

$$e_1^{BR} = \frac{FH}{\beta_2 G} y_1 = \frac{1}{2}\left[G(\alpha_1 - c_1) - \beta_1 \gamma(\alpha_2 - c_2)\right] > 0 \tag{7}$$

Clearly, the maximum-revenue export tax must be positive if there are any exports, $y_1 > 0$, which implies that the expression in square brackets must be positive if there are any exports. The maximum-revenue export tax is decreasing in the costs of the home firm and increasing in the costs of the foreign firm.

To compare the maximum-revenue export tax under Bertrand duopoly with that under Cournot duopoly, subtract (7) from the corresponding equation under Cournot duopoly (A3) in the Appendix, which yields:

$$e_1^{CR} - e_1^{BR} = \frac{\gamma^3}{4\beta_2 G}(\alpha_2 - c_2) > 0 \tag{8}$$

This is clearly positive, which leads to the following proposition:

Proposition 1: *The maximum-revenue export tax is always lower under Bertrand duopoly than under Cournot duopoly.*

An export tax causes a larger reduction in exports under Bertrand duopoly than under Cournot duopoly so the maximum-revenue export tax is lower under Bertrand duopoly than under Cournot duopoly. Clarke and Collie (2006) obtain the same result for the maximum-revenue import tariff.

Now consider the optimum-welfare export tax of the home country. The optimum-welfare export tax is the export tax that maximises the welfare of the home country, where the welfare of the home country is given by the sum of the profits of the home firm and export tax revenue:

$$W_1 = \pi_1 + R_1 = (p_1 - c_1 + e_1)y_1 + e_1 y_1 = (p_1 - c_1)y_1 \tag{9}$$

Thus, assuming an interior solution where both firms export to the third-country market, the first-order condition for welfare maximisation by the government of the home country is:

$$\frac{\partial W_1}{\partial e_1} = \frac{\partial \pi_1}{\partial e_1} + \frac{\partial R}{\partial e_1} = \left(p_1 - c_1\right)\frac{\partial y_1}{\partial e_1} + y_1 \frac{\partial p_1}{\partial e_1} = 0 \tag{10}$$

The effect of the export tax on welfare can be decomposed in two different ways. Firstly, the export tax has an effect on the profits of the home firm and an effect on the tax revenue of the government, which is positive for low export taxes but will become negative for high export taxes. Secondly, it can be decomposed into the usual profit-shifting effect, which is negative, and the terms of trade effect, which is positive.

Using (4) and (5) to solve (10) for an explicit solution for the optimum-welfare export tax yields (where the superscript W is used to denote the optimum-welfare export tax):

$$e_1^{BW} = \frac{F\gamma^2}{\beta_2 G} y_1 = \frac{\gamma^2}{4\beta_1\beta_2 G}\left[G\left(\alpha_1 - c_1\right) - \beta_1\gamma\left(\alpha_2 - c_2\right)\right] > 0 \tag{11}$$

Clearly, as shown by Eaton and Grossman (1986), the optimum-welfare export tax is positive under Bertrand duopoly if there are any exports, $y_1 > 0$, which implies that the expression in square brackets is positive. As in Kikuchi (1998), the optimum-welfare export tax is decreasing in the marginal cost of the firm in the home country.

Brander and Spencer (1985) have shown that the optimum-welfare export tax is negative (an export subsidy) under Cournot duopoly. Therefore, to compare the two policies, the absolute value of the optimum-welfare export tax under Bertrand duopoly will be compared with the absolute value of the export subsidy under Cournot duopoly. Using (11) and the corresponding equation under Cournot duopoly (A4) from the Appendix yields:

$$\left|e_1^{CW}\right| - \left|e_1^{BW}\right| = \frac{\gamma^4}{4\beta\beta G}\left(\alpha_1 - c_1\right) > 0 \tag{12}$$

This is clearly positive, which leads to the following proposition:

Proposition 2: *The absolute value of the optimum-welfare export tax under Bertrand duopoly is lower than the absolute value of the export subsidy under Cournot duopoly.*

Although the optimal policy is an export tax under Bertrand duopoly and an export subsidy under Cournot duopoly, the comparison of the absolute values of the policies shows the export subsidy under Cournot duopoly is larger than the export tax under Bertrand duopoly. Clarke and Collie (2006) obtain the same result for the optimum-welfare import tariff except that it is positive under both Bertrand and Cournot duopoly. This suggests a general result that the incentive to intervene in international trade is greater under Cournot duopoly than under Bertrand duopoly.

To compare the optimum-welfare export tax with the maximum-revenue export tax under Bertrand duopoly, subtract (7) from (11), which yields:

$$e_1^{BR} - e_1^{BW} = \frac{1}{4\beta_1\beta_2}\left[G(\alpha_1 - c_1) - \beta_1\gamma(\alpha_2 - c_2)\right] > 0 \tag{13}$$

This is clearly positive, which leads to the following proposition:

Proposition 3: *The maximum-revenue export tax always exceeds the optimum-welfare export tax under Bertrand duopoly.*

The export tax always reduces the profits of the home firm so the first-order condition for welfare maximisation (10) implies that the export tax revenue must be increasing at the optimum-welfare export tax so it must be lower than maximum-revenue export tax. This result is in contrast to the result in Clarke and Collie (2006) that the optimum-welfare import tariff may exceed the maximum-revenue import tariff under Bertrand duopoly. The explanation for the difference is that an import tariff has a positive profit-shifting effect but an export tax has a negative profit-shifting effect.

4. Nash Equilibrium Export Taxes

This section considers the Nash equilibrium in export taxes when both governments use export taxes and they are both maximising their own welfare. Since the objective is to analyse how the Nash equilibrium export taxes depend upon the relative costs of the firms, it will be assumed that the demand functions are symmetric so that the cost comparisons are meaningful; thus, $\beta_1 = \beta_2 = \beta$ and $\alpha_1 = \alpha_2 = \alpha$. The welfare of the home country is defined by (9) and the welfare of the foreign country is defined analogously. Therefore, the first-order conditions for the Nash equilibrium in export taxes are:

$$\frac{\partial W_i}{\partial e_i} = \left(p_i - c_i\right)\frac{\partial y_i}{\partial e_i} + y_i \frac{\partial p_i}{\partial e_i} = 0 \qquad i = 1,2 \tag{14}$$

Solving (14) using (4) and (5) yields the Nash equilibrium export taxes of the two countries (where the superscript *N* is used to denote the Nash equilibrium):

$$e_i^{BN} = \frac{\gamma^2 F}{\beta G} y_i = \frac{\gamma^2}{\Delta}\left[\beta\left(4\beta^2 - 3\gamma^2\right)\left(\alpha_i - c_i\right) - \gamma\left(\beta^2 - \gamma^2\right)\left(\alpha_j - c_j\right)\right] > 0 \qquad i \neq j \tag{15}$$

where $\Delta \equiv \beta\left(16\beta^4 - 12\beta^2\gamma^2 + \gamma^4\right) > 0$. The Nash equilibrium export taxes are positive if both firms export positive quantities to the third-country market, which implies that the expression in square brackets is positive. Using (15) to compare the Nash equilibrium export taxes of the two countries yields:

$$e_1^{BN} - e_2^{BN} = \frac{\gamma^2\left(\beta + \gamma\right)}{\beta\left(4\beta^2 + 2\beta\gamma - \gamma^2\right)}\left(c_2 - c_1\right) \tag{16}$$

If the home firm has the lowest costs, $c_1 < c_2$, then the government in the home country will impose the largest export tax, $e_1^{BN} > e_2^{BN}$ and vice versa. This leads to the following proposition:

Proposition 4*: The country with the lowest cost firm imposes the largest export tax in the Nash equilibrium in export taxes under Bertrand duopoly.*

This result is consistent with the proposition of Collie and de Meza (2003) that the country with the lowest cost firm would give the largest export subsidy or tax in absolute terms under Cournot duopoly. The country with the lowest cost firm intervenes most in international trade whether it uses an export tax under Bertrand duopoly or an export subsidy under Cournot duopoly.

5. Conclusions

Firstly this article has derived the maximum-revenue tariff and the optimum-welfare export tax in the Bertrand duopoly model with differentiated products and, for comparison, in the Cournot duopoly model with differentiated products. It was shown that the maximum-revenue export tax was lower under Bertrand duopoly than under Cournot duopoly. The absolute value of the optimum-welfare export tax under Bertrand duopoly was shown to be lower than the absolute value of the export subsidy under Cournot duopoly. These results are in line with the findings of Cheng (1988) and Clarke and Collie (2006) for import tariffs and support the general principle that the incentive for governments to intervene in international trade is greater under Cournot duopoly than under Bertrand duopoly.

Secondly, it was shown that the maximum-revenue export tax will always exceed the optimum-welfare export tax under both Bertrand and Cournot duopoly. This result is in line with the result of Johnson (1951-52) under perfect competition, but contrasts with the result of Collie (1991) and Clarke and Collie (2006) for import tariffs under both Cournot and Bertrand duopoly. Clarke and Collie (2006) showed that the optimum-welfare tariff may exceed the maximum-revenue tariff under both Bertrand and Cournot duopoly. The reason for the difference in the results for an export tax and an import tariff is that the profit-shifting effect of an import tariff is positive whereas the profit-shifting effect of an export tax is negative.

Thirdly, this note derived the Nash-equilibrium in export taxes under Bertrand duopoly and showed that the country with the lowest cost firm will impose the largest export tax in the Nash equilibrium. This extends to the case of Bertrand duopoly the proposition of Collie and de Meza (2003) that the country with the lowest cost firm will have the largest export tax or subsidy in absolute terms in the Nash equilibrium under Cournot duopoly. It also suggests a general principle that the country with lowest cost firm will have the greatest incentive to intervene in international trade.

Appendix: Cournot Duopoly with Product Differentiation

The appendix generalises the well known results for Cournot duopoly in Brander and Spencer (1985) by introducing differentiated products to allow a comparison with the results under Bertrand duopoly. The firms face the inverse demand functions in (2) and set outputs to maximise profits in (3). Assuming an interior solution, it is straightforward to show that the Cournot equilibrium outputs are:

$$y_i = \frac{1}{H}\left[2\beta_j\left(\alpha_i - c_i - e_i\right) - \gamma\left(\alpha_j - c_j - e_j\right)\right] \qquad i,j = 1,2 \quad j \neq i \tag{A1}$$

Substituting the Cournot equilibrium outputs into the demand functions in (2) yields the Cournot equilibrium prices:

$$p_i = c_i + e_i + \frac{\beta_i}{H}\left[2\beta_j\left(\alpha_i - c_i - e_i\right) - \gamma\left(\alpha_j - c_j - e_j\right)\right] \tag{A2}$$

The maximum-revenue export tax under Cournot duopoly is obtained by solving (6) using (A1). Assuming an interior solution, this yields (the superscript *C* denotes the Cournot equilibrium and the *R* denotes maximum-revenue):

$$e_1^{CR} = \frac{H}{2\beta_2} y_1 = \frac{1}{4\beta_2}\left[2\beta_1\left(\alpha_1 - c_1\right) - \gamma\left(\alpha_2 - c_2\right)\right] > 0 \tag{A3}$$

The maximum-revenue export tax is positive if exports are positive, $y_1 > 0$, which implies that the expression in square brackets is positive. The maximum-revenue export tax is decreasing in the costs of the home firm and increasing in the costs of the foreign firm, as in the Bertrand case.

The optimum-welfare export tax under Cournot duopoly is obtained by solving (10) using (A1) and (A2). Assuming an interior solution, this yields:

$$e_1^{CW} = \frac{\gamma^2}{2\beta_2} y_1 = \frac{-\gamma^2}{4\beta_2 G}\left[2\beta_2\left(\alpha_1 - c_1\right) - \gamma\left(\alpha_2 - c_2\right)\right] < 0 \tag{A4}$$

The optimum-welfare export tax is negative (an export subsidy) as in Brander and Spencer (1985). The positive profit-shifting effect of the export subsidy outweighs the negative terms of trade effect under Cournot duopoly.

The maximum-revenue export tax, which is positive, is obviously algebraically larger than the optimum-welfare export tax, which is negative. Therefore, to compare the two policies, the difference in the absolute size of maximum-revenue export tax and the optimum-welfare export tax is, using (A3) and (A4):

$$\left|e_1^{CR}\right|-\left|e_1^{CW}\right|=\frac{F}{2\beta_2 G}\left[2\beta_2\left(\alpha_1-c_1\right)-\gamma\left(\alpha_2-c_2\right)\right]>0 \tag{A5}$$

This is clearly positive, which leads to the following proposition:

Proposition A1: *The maximum-revenue export tax exceeds the optimum-welfare export tax under Cournot duopoly both in terms of its algebraic and its absolute value.*

Now consider the Nash equilibrium in export taxes. Solving (14) for the Nash equilibrium export taxes yields:

$$e_i^{CN}=\frac{-\gamma^2}{2\beta}y_i=\frac{-\gamma^2}{\Delta}\left[\left(4\beta^2-\gamma^2\right)\left(\alpha_i-c_i\right)-2\beta\gamma\left(\alpha_j-c_j\right)\right]<0 \tag{A6}$$

The Nash equilibrium export taxes are negative (export subsidies) under Cournot duopoly as in Brander and Spencer (1985). Comparing the absolute sizes of the export taxes of the two countries yields:

$$\left|e_1^{CN}\right|-\left|e_2^{CN}\right|=\frac{\gamma^2}{4\beta^2-2\beta\gamma-\gamma^2}\left(c_2-c_1\right) \tag{A7}$$

If the home firm has the lowest costs, $c_1<c_2$, then the home government will give the largest export subsidy in the Nash equilibrium in export subsidies, $\left|e_1^{CN}\right|>\left|e_2^{CN}\right|$ and vice versa. This leads to the following proposition:

Proposition A2*: The country with the lowest cost firm gives the largest export subsidy in the Nash equilibrium in export subsidies under Cournot duopoly.*

As in de Meza (1986) and Collie and de Meza (2003), the export subsidy of the country with the lowest cost firms is larger than the export subsidy of the other country. The country with the low cost firm intervenes most in international trade whether it uses an export subsidy or an export tax.

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