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# Growth and relative living standards - testing 

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#### Abstract

The effect of business tax and regulation on growth, together with potential effects of government spending on education and $R \& D$, is embodied in a model of a small open economy with growth choices. The structural model is estimated on post-war panel data for 76 countries and the bootstrap is used to produce the model's sampling variation for the analysis of panel regressions of growth. Statistical rejection can occur at either the structural or the growth regression stage. The models featuring government spending on education and $R \& D$ are rejected while that with business taxation is accepted.


JEL: O41, O57, C52
Keywords: growth, living standards, business regulation, business taxation, public education, government R\&D, structural model, bootstrap testing

[^0]
## 1 Introduction:

In a series of contributions Parente and Prescott (1993, 1994,1999 and 2000) have argued that growth and accumulated living standards depend on the freedom of business to innovate and sell their innovations via free entry into business; this freedom could be withdrawn by regulations, taxes, or a variety of government restrictions designed to protect existing producers - 'barriers to riches' as they term it. Over the past decade a variety of indices have been constructed to measure this freedom. Here we use one compiled by the World Bank on the costs of entering and exiting from business. This index we combine with a measure of general taxation to create what we call for simplicity a 'business tax rate'. Our aim in this paper is to explore in a deliberately simplified model whether the Barriers to Riches theory, which we will call 'Incentivism', can explain postwar growth across the world ( 76 countries, 1970-2000, in data from the Penn Tables).

Plainly there has been a wide range of work, in the form of both history and case studies, devoted to explaining growth and the proximate factors causing it (see for example Easterly, 2001, and Snowdon, 2007, chapter 2, for a useful survey). Parente and Prescott themselves calibrate general equilibrium models and see whether these can replicate the facts of relative living standards and growth episodes since 1820. They reject in particular models in which capital accumulation (including and excluding intangibles) and education account for these when there are no barriers to the transmission of best world available practice; the problem is that investment and education rates are too similar across countries to generate the necessary differences in performance. The model that succeeds is one where the transmission of best practice is blocked differentially across countries by the government protection of insiders. However here we aim to carry out a more limited and complementary study in terms of econometrics on post-war panel data and attempt to set this Incentivist theory up in a way that is testable on this data.

Ranged up against this thesis are a variety of theories that growth depends on active government intervention to promote particular sorts of activity: two we focus on here are education and R\&D. We will measure the extent of government intervention to promote these by respectively government spending as a percent of GDP on all levels of education
and the government percentage share of total country R\&D spending. These theories we will call 'Activism'. According to Incentivism growth is triggered by people choosing to be entrepreneurial in response to incentives; these activities could take the form of acquiring skills via education or by doing 'R\&D', but if so these would just be some of the forms their activity could take but that activity would be defined by its focus on exploiting business opportunities. According to Activism it is these latter specific activities that generate higher productivity and therefore government can raise growth by subsidising them.

For our tests of these theories we abandon the widely-used method of regressing, usually in panel data, growth on a selection of exogenous 'growth drivers' and checking whether a particular driver is statistically significant ${ }^{1}$. We argue that this method is flawed by potential data-mining, by likely bias and by lack of identification. If one writes down a model of endogenous growth (as we will shortly do) one finds that it is complex and non-linear so that it does not have a linear reduced form; thus the linear 'reduced forms' written down for testing are no more than guesses at the variables, either exogenous or predetermined, that might be included among the determinants of growth. Even if their inclusion is correct, the omitted variables will in general include powers or other combinations of these included variables; hence the error terms will be correlated with the regressors and there will be bias whose size and direction cannot be estimated reliably.

Partly because authors have been conscious of these problems, they have included various menus of 'control' or 'nuisance' variables in these regressions. The trouble comes in the criteria for choosing them as many can be suggested in the absence of a tightly specified underlying structural model being applied. The statistical significance of the key variable under test will in general be much affected by this choice; hence the vulnerability of the method to data-mining.

[^1]The problem of identification arises because we do not know what model is generating these 'reduced forms'; many different models could give rise to some relationships between the chosen regressors and growth. For example if the regressors are correlated (due to transmission within the model) with the true causal mechanisms whatever they are one could obtain significant regression coefficients on the chosen regressors which in fact come from a quite different set of causes.

For such reasons the large literature above may not be regarded as entirely persuasive evidence. Those for example who think $\mathrm{R} \& \mathrm{D}$ is the major factor determining growth will not be impressed by regressions showing that tax rates are correlated with growth. Vice versa with regressions highlighting R\&D those favouring the tax explanation are unwilling to be persuaded.

In this paper we take a new approach to testing, one that has been used in time-series macroeconomics with some promising results. This approach we will term Popperian because it is an attempt to take as far as possible the principles of rejection put forward by Popper (1934). In these the idea is to set out the model to be tested and to use all possible implications of it for the data to reject it. For this purpose the model must be specified quite unambiguously and must be identified - that is it must not be able to be confused with another model. Secondly the model must be estimated on the data and must not be rejected at this stage. Thirdly, if there are other relationships it might imply or be consistent with - in this context these will be growth regressions - such regressions must be consistent with the model being true. This approach is new and therefore we set out carefully below the steps to be followed in this paper.

We start by insisting on a clearcut 'null hypothesis' by which we mean a hypothesis treated as true for purposes of testing (by 'null' has often strictly been meant the 'zero' hypothesis of no relationships at all because this is the one that is taken to be rejected in much statistical testing; however we adopt the definition of 'working' or 'initially believed' hypothesis here because in our approach it will be this, not the zero hypothesis that is to be rejected). This null hypothesis is the micro-founded (structural) theory of endogenous growth in this case that we are going to test. This theory will be set up to permit three
variants, viz. Incentivist or Activist, and in the latter case either stressing education or stressing R\&D.

This theory being specified is then estimated on the data in its structural form. It may be rejected at this stage, if the critical parameters cannot lie within the range required at some statistical confidence level. Here since the data we have is panel data there are practical limitations to how far we can apply the restrictions implied by the structural model. For example expectations of future variables cannot in practice be generated by solving each country's model forwards at each point of time as we would with time-series data for one country. Instead what we do is to estimate two key structural equations; these are the production function and the labour supply function, both necessary for our purpose in explaining output supplied. We fit these to the levels of variables in the data. Our model's specification is to variable levels, so it is appropriate to estimate it in these terms, so that it can be tested for these implications.

The theory as constructed and estimated in this way then, assuming it cannot rejected at this stage, can be reused to test its further implications for linear 'reduced forms' of interest. Here these forms are the growth regressions with which we began. We may ask whether each model can explain the growth regressions implied by it that we observe. To do this we note that each structural model as constructed above a) implies the exclusion of all variables other than those it identifies as causal (a zero restriction: these should therefore not appear in the reduced form) b) together with the panel data, implies certain errors - the 'structural error terms' in each structural equation. These latter errors can be regarded as the effects of non-systematic factors omitted from the model that may affect output (productivity) and labour supply (leisure preference). Clearly each model will partition the data differently into the part explained by the drivers it identifies as the identified causes and the part allocated to these errors. This difference of partitioning is what distinguishes one model from another.

To explore whether a model can explain the growth regressions, we bootstrap the random elements in the error processes together with the random elements in the exogenous variables to create the sampling variation of the data as implied by the model. This
sampling variation permits us to derive statistical confidence limits for the parameters of the growth regressions under the null hypothesis of the model. In turn this allows us to reject the model at this last stage.

In what follows we go into the exact methods in detail but this account has, we hope, outlined our method and the reasons for it.

## 2 A model of endogenous growth for a small open economy:

We begin (essentially as in Gillman and Kejak (2005) and following the same essential approach as Lucas (1988)) from a standard intertemporal utility function and a perfectly competitive firm sector with a Cobb-Douglas production function, from which households derive wages for their labour supply as workers and dividends for their capital; under constant returns to scale dividends and wages add up to total GDP. We assume that each household owns a corresponding firm for which it works (at competitive wage rates because it could always decide to work elsewhere) and also may undertake entrepreneurial activity to innovate its methods, so raising its productivity. This is the model's mechanism of growth; notice that it is essentially the same as the diversion of people's time to education or to R\&D, each of which also would raise productivity. However each household must buy its consumption and investment goods from other firms. Government taxes both in order to make transfer payments back to households (for redistributive purposes) and there is no government spending. The economy is open but is 'small' in the strictest sense; that is, it can borrow on world markets at the world real interest rate and its goods prices are also set on world markets. Each economy in our world of 76 countries will face the same world market; following Parente and Prescott each country has a different level of total factor productivity and the choices of its citizens determine how fast they raise this, by diverting their time from normal work to productivity-enhancing activities. In doing this they can draw on the world stock of available knowledge and borrow world capital to implement their resulting higher productivity output.

We go on to show that this economy for our purposes (examining its growth behaviour) can be summarised in three equations: the production function reduced to a function of productivity and labour supply, a labour supply function of labour/consumption taxation, and a productivity function of the accumulated tax rate on entrepreneurial activity.

## 3 Derivation of the 3-equation model:

The representative household's utility function seen from period 0 is:

$$
\begin{equation*}
U_{t}=E_{0} \sum_{t=0} \beta^{t}\left(\ln c_{t}+\alpha_{t} l \ln x_{t}\right) \tag{1}
\end{equation*}
$$

where $\alpha_{t}$ is a stationary preference error process, subject to

$$
\begin{equation*}
\left(1+\tau_{t}\right) c_{t}+k_{t}-(1-\delta) k_{t-1}+b_{t}=y_{t}+\left(1+r_{t-1}\right) b_{t-1}+\Gamma_{t}-\pi_{t} z_{t} \tag{2}
\end{equation*}
$$

where:
$\tau$ is the tax rate on consumption - this is assumed to be the sole general tax (so that dividends and wages are taxed indirectly through consumption);
$\pi$ is tax levied on entrepreneurial activity;
consumption ( $c$ ), capital stock ( $k$ ), foreign bonds (b), leisure $(x)$, entrepreneurial activity ( $z$ ) and government transfers ( $\Gamma$ ) are all expressed per capita;
$\delta$ is depreciation and $r$ is the real rate of interest on foreign bonds. Goods are bought by some system of organised barter and so we ignore the role of money in this economy; $y_{t}=A_{t} k_{t}^{\gamma} X_{t}^{\zeta}\left(1-x_{t}-z_{t}\right)^{1-\gamma-\zeta}$ is the Cobb-Douglas production function of the household (and firm combined). $X$ represents exogenous other production factors - such as 'land'/natural resources - assumed to be owned by households.

The household's first order conditions familiarly yield (where $\lambda$ are the Lagrangean parameters):

$$
\lambda_{0}=\frac{1}{c_{0}\left(1+\tau_{0}\right)} ; \lambda_{1}=\frac{\beta}{c_{1}\left(1+\tau_{1}\right)}
$$

(from the first derivatives of the Lagrangean with respect to current and future consumption)
$E_{0} \lambda_{1}[1-\delta]=\lambda_{0}\left[1-\frac{\gamma y_{0}}{k_{0}}\right]$ (from the first derivative with respect to capital, $k_{t}$ )
$E_{0} \lambda_{1}\left[1+r_{0}\right]=\lambda_{0}$ (from the first derivative with respect to foreign bonds, $b_{t}$ )
$\lambda_{0}\left[\frac{(1-\gamma) y_{0}}{1-x_{0}-z_{0}}\right]=\frac{\alpha_{0} l}{x_{0}}$ (from the first derivative with respect to leisure).
At this stage we treat entrepreneurial activity, $z$, as fixed. But we will return to it once we have introduced productivity determination below.

From these conditions (letting time zero be generalised to any $t$ ) we can derive the consumption condition:

$$
\begin{equation*}
c_{t}=\frac{1}{\beta\left(1+\tau_{t}\right)} E_{t} \frac{c_{t+1}\left(1+\tau_{t+1}\right)}{1+r_{t+1}} \tag{3}
\end{equation*}
$$

the condition relating the marginal product of capital (which we also denote by the shadow real dividend rate, $d_{t}$ ) to world real interest rates plus depreciation:

$$
\begin{equation*}
r_{t}+\delta=\frac{\gamma y_{t}}{k_{t}}=d_{t} \tag{4}
\end{equation*}
$$

and the condition relating labour supply to the marginal product of labour (which we also denote by the shadow real wage, $w_{t}$ ):

$$
\begin{equation*}
\text { a) } w_{t}=\frac{(1-\gamma-\zeta) y_{t}}{\left(1-x_{t}-z_{t}\right)} ; \text { b) } x_{t}=\frac{\alpha_{t} l c_{t}\left(1+\tau_{t}\right)}{w_{t}} \tag{5}
\end{equation*}
$$

Using the marginal productivity of capital condition, we can replace capital in the production function by terms in the shadow dividend (determined in 5).

$$
\begin{equation*}
y_{t}=A_{t}^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{d_{t}}\right)^{\frac{\gamma}{1-\gamma}} X_{t}^{\frac{\zeta}{1-\gamma}}\left(1-x_{t}-z_{t}\right)^{\frac{1-\gamma-\zeta}{1-\gamma}} \tag{6}
\end{equation*}
$$

What this means is that the household can obtain whatever capital it needs to produce its desired output at a fixed price on world markets; thus it is only limited in the output
it can produce by the supply of labour offered at the going shadow wage.
We now turn to the determination of productivity growth and the marginal condition determining $z$.

In this model representative households choose how much to invest and work within their available production technology. This technology is assumed here to improve through households' innovation by finding out about better processes. We assume that there is some innovative or entrepreneurial activity a household can undertake which involves spending the time denoted as $z$ above. In endogenous growth models one key channel of growth is via labour being withdrawn from 'normal' work and being used for an activity that raises productivity. Here we think of it as 'innovation', as in Klette and Kortum (2004); in Lucas' models (Lucas (1988)) it would be 'education'; in models stressing R\&D, as in Aghion and Howitt (1998), it would be research activity. Notice that in all three ways that productivity growth might be enhanced, the maximisation issue is exactly the same: the household must divert an appropriate amount of time away from standard work into this growth-enhancing activity. It decides how much time to devote to $z$ by maximising its expected welfare as above.

We write the growth of productivity as:

$$
\begin{equation*}
\frac{A_{t+1}}{A_{t}}=a_{0}+a_{1} z_{t}+u_{t} \tag{7}
\end{equation*}
$$

where $u_{t}$ is an error process, and the parameter $a_{1}$ is the effect of the entrepreneurial activity on productivity growth.

Going back therefore to the household's optimising decision, its first order condition for $z_{t}$ at time 0 is given by $^{2}$

$$
0=E_{0} \sum_{t=1}^{\infty} a_{1} \frac{A_{0}}{A_{1}} \beta^{t} \frac{y_{t}}{\left(1+\tau_{t}\right) c_{t}}-\lambda_{0}\left(w_{0}+\pi_{0}\right)
$$

[^2]from which we can obtain
$$
z_{0}=\frac{\left\{E_{0} \sum_{t=1}^{\infty} \beta^{t}\left(\frac{1}{\left(1+\tau_{t}\right)}\right) \frac{y_{t}}{c_{t}}\right\}}{\lambda_{0}\left(w_{0}+\pi_{0}\right)}-\frac{\left(a_{0}+u_{0}\right)}{a_{1}}
$$

We now compare $a_{1} z_{0}$ with (7) and find that

$$
\begin{equation*}
\frac{A_{1}}{A_{0}}=\frac{a_{1}\left\{E_{0} \sum_{t=1}^{\infty} \beta^{t}\left(\frac{1}{\left(1+\tau_{t}\right)}\right) \frac{y_{t}}{c_{t}}\right\}}{\lambda_{0}\left(w_{0}+\pi_{0}\right)} \tag{7a}
\end{equation*}
$$

What this is telling us is that entrepreneurs make allowance for the productivity growth already coming from other sources when they decide on optimal effort; they exactly offset these in their decision, so that it is purely entrepreneurs that determine productivity growth. To evaluate this equation we note that our tax rates are a random walk and that (see appendix) $\frac{c_{t}}{y_{t}}$ is non-stationary. We approximate the latter as a random walk. Omitting second order (variance and covariance) terms then the numerator of (7a) is given by $\frac{\beta}{1-\beta}\left(\frac{1}{\left(1+\tau_{0}\right)}\right) \frac{y_{0}}{c_{0}}$; then using (5) for $w_{0}$ and substituting for $\lambda_{0}$, we obtain

$$
\begin{align*}
\frac{A_{1}}{A_{0}} & =\left\{\frac{a_{1} \beta}{1-\beta} \frac{1}{\left(1+\tau_{0}\right)} \frac{y_{0}}{c_{0}}\right\} /\left\{\frac{1}{c_{0}\left(1+\tau_{0}\right)}\left(\frac{\alpha_{0} l c_{0}\left(1+\tau_{0}\right)}{x_{0}}+\pi_{0}\right)\right\} \\
& =\left\{\frac{a_{1} \beta}{1-\beta} \frac{y_{0}}{c_{0}} \frac{x_{0}}{\alpha_{0} l}\right\} /\left\{\left(1+\tau_{0}\right)+\pi_{0} \frac{x_{0}}{\alpha_{0} l c_{0}}\right\} \tag{7b}
\end{align*}
$$

We now linearise this as

$$
\frac{A_{1}}{A_{0}}=\phi_{0}-\phi_{1}\left(\tau_{0}+\pi_{0}^{\prime}\right)+\text { error }_{0}
$$

where $\pi_{0}^{\prime}=\pi_{0} \frac{x_{0}}{\alpha_{0} c_{0}}$ is the tax on entrepreneurs normalised by the ratio of preferenceadjusted leisure to consumption; and since $\frac{A_{t+1}}{A_{t}}=\Delta \ln A_{t+1}+1$, gathering constants as $\phi_{0}^{\prime}$ and letting $u_{t}^{\prime}=$ error $_{t}$ we obtain

$$
\Delta \ln A_{t+1}=\phi_{0}^{\prime}-\phi_{1}\left(\tau_{t}+\pi_{t}^{\prime}\right)+u_{t}^{\prime}
$$

What we see is that the 'tax rate on entrepreneurs' consists of both the general tax
rate and the particular imposts levied on business activity as such. These would include corporation tax for example if it is not rebated to the shareholder as an imputed tax already paid on dividends. Here we pay especial attention to the levies on entry and exit from business as measured by international bodies.

### 3.1 Generalising the analysis to different sorts of endogenous productivity growth:

The analysis above treats every household as being a potential entrepreneur in its choice of $z$; hence the relevant tax rate on $z$ is the business $\operatorname{tax}, \tau+\pi^{\prime}$. We now introduce the idea that different households face different opportunities for raising their productivity; for some education may be the way while for others (no doubt typically embodied in large firms) investment in $\mathrm{R} \& D$. In this way we propose to test the Activist theories that government subsidies to education and $\mathrm{R} \& \mathrm{D}$ are the essential drivers of productivity growth. We suppose that there are proportions $v_{\pi}, v_{e}$ and $v_{\rho}$ of these households respectively in the population; they add up to unity. Each type of household maximises exactly as above with the only change being that:

1. for each the relevant special tax rate corresponding to the tax on entrepreneurship alters to that special for its particular growth activity.
2. the total of household behaviour is obtained by adding together each of these groups and weighting it by its proportion in the population.

Thus now

$$
\Delta \ln A_{t+1}=\phi_{0}^{\prime}-v_{\pi} \phi_{\pi}\left(\tau_{t}+\pi_{t}^{\prime}\right)-v_{e} \phi_{e}\left(\tau_{t}+e_{t}^{\prime}\right)-v_{\rho} \phi_{\rho}\left(\tau_{t}+\rho_{t}^{\prime}\right)+u_{t}^{\prime}
$$

where the coefficients $\phi$ denote the response of each household type and $\pi_{t}^{\prime}, e_{t}^{\prime}, \rho_{t}^{\prime}$ represent the respective tax rates (or subsidies, negative tax rates).

We may also note that when we are dealing with macro aggregates, they are all weighted averages of these various types.

### 3.2 Completing the model:

To complete the model, we require:
(1) the government budget constraint which brings together the revenues it raises from households and the transfer it pays over; the government too can borrow from abroad via foreign bonds but for simplicity we assume it does not as it has no impact on the model's workings.

$$
\tau_{t} c_{t}+\sum\left(\pi_{t}^{\prime} z_{\pi t}+e_{t}^{\prime} z_{e t}+\rho_{t}^{\prime} z_{\rho t}\right)=\Gamma_{t}
$$

where the $z$ are subscripted by their relevant household type.
(2) goods market clearing in which households buy consumption and investment goods (gross investment $=k_{t+1}-(1-\delta) k_{t}$ ) from firms who may supply them either from their own output or from net imports $(m)$ purchaseable on the world market at going (exogenous) world prices. If firms have excess output they export it onto the world market at these prices. We set world prices at unity, ignoring terms of trade changes as an exogenous variable with no impact on the model's workings.

$$
y_{t}+m_{t}=c_{t}+k_{t+1}-(1-\delta) k_{t}
$$

It can easily be verified that the balance of payments constraint is implied (via Walras' Law) by the household and government budget constraints, the constraint that firms have no surplus profits (all earnings are distributed via wages and dividends) and goods market clearing. ${ }^{3}$

[^3]
### 3.3 Solution of the model:

The model is most conveniently analysed in loglinear form. We have from (3):

$$
\begin{equation*}
\ln c_{t}=-\ln \left(1+\tau_{t}\right)+E_{t} \ln c_{t+1}+E_{t} \ln \left(1+\tau_{t+1}\right)-E_{t} \ln \left(1+r_{t+1}\right)+\text { constant } \tag{8}
\end{equation*}
$$

Here we have made use of the fact that when $x$ is lognormally distributed $\ln E x=$ $E \ln x+0.5 v a r \ln x$. We assume throughout that our errors are lognormal and have a constant variance, so that the variance and covariance terms are included in the constant term above. To loglinearise $x_{t}$ we proceed by linearising (5b) as:

$$
\frac{\Delta x_{t}}{\bar{x}}=\overline{\left(\frac{\alpha l c}{w x}\right)} \Delta \tau_{t}+\left(\frac{1+\tau_{t}}{x_{t}}\right) \Delta\left(\frac{\alpha_{t} l c_{t}}{w_{t}}\right) \text { which we can approximate, adding a constant of }
$$ integration, as:

$$
\begin{equation*}
\ln x_{t}=\overline{\left(\frac{\alpha l c}{w x}\right)} \tau_{t}+\ln c_{t}-\ln w_{t}+\ln \alpha_{t}+\text { constant } \tag{9}
\end{equation*}
$$

Using (5a) above to substitute out wages (and assuming $\ln x_{t} \approx \ln \left(1-x_{t}-z_{t}\right)$ because leisure and working time are approximately equally divided and assuming entrepreneurial time is very small relative to the other two) yields:

$$
\begin{equation*}
\ln x_{t}=l^{*} \tau_{t}+0.5\left\{\ln c_{t}-\ln y_{t}\right\}+0.5 \ln \alpha_{t}+\text { constant } \tag{10}
\end{equation*}
$$

where $l^{*}=0.5\left[\overline{\left\{\frac{\alpha l c}{w x}\right.}\right]$.
It can be shown that $\left\{\ln c_{t}-\ln y_{t}\right\}$ is a non-stationary process (for the formal derivation see the appendix); the reason lies in the permanent income hypothesis, that consumption equals permanent income from home output plus interest on foreign assets. The stock of foreign assets then follows a random walk because consumers use foreign assets as a way of smoothing any fluctuations of home income around permanent income. It follows that we can replace this term (plus the stationary preference error) with a non-stationary error process, which will plainly be correlated with the other errors in the model and may also be autocorrelated.

In order to solve the model and eliminate expected future terms it is necessary to make assumptions about the behaviour of the exogenous variables. We assume that world real interest rates, $r$, are stationary and autoregressive of order 1: $r_{t}=(1-\lambda) r^{*}+\lambda r_{t-1}+\epsilon_{t}$. We assume that all the policy variables, essentially the tax rates, are random walks, which is frequently found empirically since tax changes are generally the result of policy change which is by construction unexpected.

A full explicit solution in terms of the forcing processes requires dynamic programming. However as noted earlier we treat the log of the consumption/income ratio in (10) as a random walk error process and include it with the error due to work preferences, also likely to be a random walk like productivity. Thus our model for estimation becomes:

$$
\begin{equation*}
\Delta \ln A_{t+1}=\phi_{0}^{\prime}-v_{\pi} \phi_{\pi}\left(\tau_{t}+\pi_{t}^{\prime}\right)-v_{e} \phi_{e}\left(\tau_{t}+e_{t}^{\prime}\right)-v_{\rho} \phi_{\rho}\left(\tau_{t}+\rho_{t}^{\prime}\right)+u_{t}^{\prime} \tag{7}
\end{equation*}
$$

We integrate this into levels to become $\ln A_{t}=-\sum_{i=1960}^{t}\left\{v_{\pi} \phi_{\pi}\left(\tau_{i}+\pi_{i}^{\prime}\right)+v_{e} \phi_{e}\left(\tau_{i}+e_{i}^{\prime}\right)+\right.$ $\left.v_{\rho} \phi_{\rho}\left(\tau_{i}+\rho_{i}^{\prime}\right)\right\}+\phi_{0}^{\prime} t+\zeta_{t}$ where $\zeta_{t}=\frac{u_{t}^{\prime}}{1-L}$. We then substitute this for $\ln A_{t}$ in equation 6 , which becomes our first equation to be estimated

$$
\begin{equation*}
\ln \left(1-x_{t}\right)=-l^{*} \tau_{t}+v_{t} \tag{10}
\end{equation*}
$$

where as noted above we have treated $\ln (1-x) \approx-\ln x$
(6) in loglinearised form is now:

$$
\begin{align*}
\ln y_{t}= & \left(\frac{-1}{1-\gamma}\right) \sum_{i=1960}^{t}\left\{v_{\pi} \phi_{\pi}\left(\tau_{i}+\pi_{i}^{\prime}\right)+v_{e} \phi_{e}\left(\tau_{i}+e_{i}^{\prime}\right)+v_{\rho} \phi_{\rho}\left(\tau_{i}+\rho_{i}^{\prime}\right)\right\} \\
& +\psi \ln \left(1-x_{t}\right)+(1-\psi) \ln X_{t}+c+\phi_{0}^{\prime} t+\xi_{t}+\zeta_{t} \tag{6}
\end{align*}
$$

(where we have neglected the direct effect of $z_{t}$ on output for convenience as very small; where $\psi=\frac{1-\gamma-\zeta}{1-\gamma}$ and where $-\frac{\gamma}{1-\gamma} \ln r_{t}=\xi_{t}$ is the effect of world real interest rates this is assumed to be picked up by the time effects in the panel estimation process (as is $\left.\phi_{0}^{\prime} t\right)$ while $(1-\psi) \ln X_{t}$ is assumed to be picked up by the country and time effects).

Thus (10) and (6) are our two equations of the model to be taken to the data.

These equations have been chosen for tractability in the context of our panel set-up and data set. Forward-looking terms for example must be substituted out because we are in practice unable to solve each country model separately over the sample period. Other variables, such as wages, we have no data for. We have substituted out consumption in order to simplify the model to be estimated to its essentials. Thus one of our two equations (6) contains the production function which is essentially structural, an 'engineering relationship' with capital solved out in terms of its first-order condition; productivity in it is a function of the business or other taxes. The other equation is (10) the labour supply equation derived from first order conditions as partially solved out. The resulting two error terms include what the data and model imply are the omitted effects of the exogenous errors. These effects do not include the direct effects of interest in the model, of tax rates on productivity growth and on labour supply; these are explicitly included in the model.

Thus the two equations constitute a 'structural model' in the sense that they jointly exactly replicate the data country by country in a way entirely constrained by the model and its solution method; and that they can be solved to give the paths of output and labour supply.

### 3.4 Empirical work - Incentivism: the model with business tax alone

We begin by considering a model in which business tax alone, out of the three we have identified, is operative. Later we consider the addition of the other taxes. In this model the production function error should be purged of all effects of business taxation since this is explicitly entered in the estimated equation. Hence the model attributes a causal role to business taxation in creating growth.

The procedure we follow to test the model we have set out is that of bootstrapping. The idea is that we treat the model - in the form of the two equations set out above as the true or 'null' hypothesis. We estimate this model on the available post-war annual data, for 76 countries from 1970-2000. The resulting 2 structural errors for each country-
period are thus the implied 'true errors' under the model. These errors and the tax rates have time-series properties which we assume differ country by country; we estimate a time-series process for each country error and tax process, which in turn implies a set of 4 random errors (structural and tax) for each country over the period 1970-2000. Our bootstrapping procedure is then to draw the whole vector of random errors as a 76country bloc repeatedly with replacement for a 30-year sample period (we draw them as a vector to retain any patterns of simultaneous correlation); input them into the country time-series processes to generate a resulting set of 30-year errors; input these in turn into the model to generate a 30 -year sample of data for the endogenous variables. Such a 30-year sample of data is one pseudo-sample. We generate 1000 of these pseudo-samples. The idea is that these 1000 pseudo-samples represent the sampling variation that would occur according to the model.

We then investigate whether data descriptions that would emerge from the model are rejected by the data. We do this by estimating the descriptive form on the actual data and also on the pseudo-samples; if the estimate generated on the actual data lie within the $95 \%$ confidence limits given by the pseudo-samples, then we say that the model is not rejected by the data and vice versa.

The results for the model equations are as follows (with fixed country and time effects on each equation). We estimate equation (6) as:


We estimate the structural labour supply equation, (10) as:
$\ln \left(1-x_{t}\right)=c_{2}+0.0128 \ln \left(1-\tau_{t}\right)$

| Number of obs | $=2280$ |
| :--- | :--- |
| $\mathrm{~F}(103,2024)$ | $=278.53$ |
| $R^{2}$ | $=0.9308$ |
| $\bar{R}^{2}$ | $=0.9275$ |
| Root MSE | $=0.0442$ |

The error term from this equation is a combination of labour supply preferences and the $\log$ of the consumption/income ratio.

Notice that the business tax term in the production function has the right sign and is highly significant, while the general tax rate in the labour supply function also has the right sign, though at a low level of significance. Hence we may say that this Incentivist theory is not rejected at the structural model level.

Bootstrapping this model as described above generates 1000 pseudo-samples. With it we then investigate a data description for growth. In it growth depends on the (general plus entrepreneurial) tax rate and the rate of change of the general tax rate (the latter because growth in output not caused by productivity depends of the growth in labour supply which in turn depends on the rate of change of the general tax rate).

### 3.4.1 Growth rate and taxation - descriptions of data with model-generated $\mathbf{9 5 \%}$ confidence bands

We now turn to our test of this above model against the growth regressions. We proceed as follows. First we regress the data for growth on a set of potential regressors with a view to capturing the best (linear reduced form) description of the data. We consider four sets of regressors suggested by the theory: the level of the different special taxes; the rate of change of personal tax, $\Delta \tau_{t}$; country dummies; and time dummies.

In these growth regressions $\Delta \tau_{t}$ was insignificant though of the right sign. This term picks up the temporary effect on growth of the change in the personal tax rate (which affects labour supply); this effect however is very poorly determined, which is perhaps not

Table 1: Regression of Growth on Business Tax and the Rate of Change of Personal Tax With Fixed Time and Country Effects

| With fixed country and time effects |  |  | Number of obs | $=1748$ |
| :---: | :---: | :---: | :---: | :---: |
| $\Delta \operatorname{lny}_{t}=\alpha_{1}\left(\tau_{t}+\pi_{t}^{\prime}\right)+\alpha_{2} \Delta \tau_{t}$ |  |  |  |  |
|  | Actual | Growth regression |  | $=3.87$ |
|  |  | standard errors | $R^{2}$ | $=0.1903$ |
| $\alpha_{1}$ | -0.043 | 0.027 | $\bar{R}^{2}$ | $=0.1411$ |
|  | -0.039 | 0.043 | Root MSE | $=0.0506$ |

surprising as it works through labour supply and we know from other work that labour supply effects depend on expected tax and other variables. Here we are unable to pick up expectations effects (which could introduce a lead or a lag in the tax variable). We therefore decided to look also at an equation with solely the business tax effect whose level should directly determine growth on a permanent basis; we would expect this effect to come through powerfully in the data description and indeed it seems to do so.

The resulting equation is:

Table 2: Regression of Growth on Business Tax With Fixed Time and Country Effects

| With fixed country and time effects |  |  | Number of obs | $=1748$ |
| :---: | :---: | :---: | :---: | :---: |
| $\Delta \operatorname{lny}_{t}=\alpha_{1}\left(\tau_{t}+\pi_{t}^{\prime}\right)$ |  |  | F(99,1649) | $=3.90$ |
|  | Actual | Growth regression | $R^{2}$ | $=0.1899$ |
|  |  | standard errors | $\bar{R}^{2}$ | $=0.1412$ |
|  | -0.050 | 0.026 | Root MSE | $=0.0506$ |

Using panel data with fixed effects may not be the most efficient model to run. Estimating the model with random effects will give a more efficient estimator (the reason for this is that the estimator saves degrees of freedom by not using the fixed country dummies but instead using the regression with fixed country dummies with a weight,
to correct the regression with time dummies only). The results for the random effects estimator are shown in Table 3.

Table 3: Regression of Growth on Business Tax with Random Effects

| With random effects |  |  | Number of obs <br> Wald $\chi^{2}(1)$ | $\begin{aligned} & =1748 \\ & =9.91 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\Delta \ln _{t}=\alpha_{1}\left(\tau_{t}+\pi_{t}^{\prime}\right)$ |  |  |  |  |
| Actual |  | Growth regression | $R^{2}$ within | $=0.0035$ |
|  |  | standard errors | $R^{2}$ between | $=0.0615$ |
|  | -0.043 | 0.026 | $R^{2}$ overall | $=0.0088$ |

To test whether we should use the fixed or random effects model we run a Hausman test, the results from this test are shown in Table 4.

Table 4: Hausman Test

$$
\Delta \ln _{t}=\alpha_{1}\left(\tau_{t}+\pi_{t}^{\prime}\right)
$$

|  | Fixed | Random | Difference | Standard error | $\chi^{2}(1)$ | P-value |
| :---: | :---: | :---: | :--- | :--- | :--- | :--- |
| $\alpha_{1}$ | -0.050 | -0.043 | -0.007 | 0.022 | 0.10 | 0.751 |

From Table 4 we find that we can use either fixed or random effects in the actual data sample without serious risk of inconsistency. However it can be seen that the fixed effects regression, which we can be sure is free of inconsistency, gives effects essentially no different from the random effects regression.

We now turn to the bootstrapping exercise where we wish to establish the sampling distributions of the growth regression coefficients according to our model. For this exercise it is essential that the estimator used is consistent in all the potential data samples; otherwise the distribution of 'potentially estimated' coefficients will be wrongly measured. Hence in what follows we use the fixed effects estimator throughout the bootstrapping
process, since it is known definitely to be consistent in all samples; thus each sample estimate will give us a 'central' value for the coefficients.

We now report how our chosen growth regression - with the business tax rate only compares with our basic model. We take the growth regression and run it on our bootstrap data for each model. As noted earlier, this allows us to find the $95 \%$ confidence interval implied by the model. In addition it gives the overall 'M-metric', that is the percentile in the bootstrap distribution of all parameters ${ }^{4}$ jointly where the actual data regression lies; the higher the percentile, the further into the tail the actual regression lies. As it happens, in this case with only one parameter of interest the M-metric is directly related to the distribution of this one parameter.

Table 5: Bootstrap Results for Model with Estimated Tax Effects
$\Delta \operatorname{lny}_{t}=\alpha_{1}\left(\tau_{t}+\pi_{t}^{\prime}\right)-$ With fixed country and time effects

| $95 \%$ interval for basic model | Actual | Lower | Upper | M-metric |
| :--- | :---: | :--- | :---: | :---: |
| $a_{1}$ | -0.050 | -0.05680 | 0.0112 | $90.4 \%$ |

What we see is that the model is accepted at the $95 \%$ level.
We also reestimated the model imposing an increased coefficient on business tax and retrieving the implied new errors. We used two cases, one in which we set the coefficient to -0.02 and another in which we set the coefficient to -0.04 . The results for the -0.02 case are shown in Table 6 and the -0.04 case in Table 7.

[^4]Table 6: Bootstrap Results for Model with Tax Effects and Coefficient on Business Tax set to -0.02

Coefficient on Business Tax Set to -0.02
$\Delta \operatorname{lny}_{t}=\alpha_{1}\left(\tau_{t}+\pi_{t}^{\prime}\right)-$ With fixed country and time effects

|  | Actual | Lower | Upper | M-metric |
| :---: | :---: | :---: | :---: | :---: |
| $a_{1}$ | -0.050 | -0.0580 | 0.0083 | $87.2 \%$ |

Table 7: Bootstrap Results for Model with Tax Effects and Coefficient on Business Tax set to -0.04

| Coefficient on Business Tax Set to -0.04 |  |  |  |  |
| :--- | :--- | :--- | :---: | :---: |
| $\Delta \operatorname{lny}_{t}=\alpha_{1}\left(\tau_{t}+\pi_{t}^{\prime}\right)-$ With fixed country and time effects |  |  |  |  |
| Actual |  |  |  |  |
| Lower | Upper | M-metric |  |  |
| $a_{1}$ | -0.050 | -0.0729 | -0.0034 |  |

What is interesting about this is that there is an improvement in the model's performance vis-a-vis the data description as the model's business tax effect is raised. Thus if it is raised in absolute size by two standard errors to -0.02 (from the estimated -0.017) the M-metric falls from $90.4 \%$ to $87.2 \%$. This parameter imposition on the production function is not rejected by the F-test (3.06); hence we can happily accept the higher effect from both the structural and the simulation viewpoints.

The improvement in the simulation test continues from here on up. If the parameter is raised further to -0.04 the M-metric improves to $47.5 \%$. However of course this higher coefficient is massively outside the two standard error range on the production function estimate; thus it fails to fit the structural model quite badly (the F-test value is a massive 225). Hence the data estimation of the model itself combined with the data description tells us that a business tax parameter of around -0.02 is the most compatible with the data.

### 3.4.2 A discussion of the empirical results on the Incentivist Model (business tax rate alone)

We may start by discussing the 'conventional' way of testing the model using the standard reduced form approach. Thus we note that the model implication - viz that the level of business tax and the rate of change of general tax both affect growth - meets a mixed reception. The business tax effect alone is fairly significant against the usual zero alternative; the general tax effect is not. We concluded from this that the data description should not include the general tax effect as it does not contribute to explaining growth. We might also have concluded that there was reduced form evidence of a business tax effect. However as we have argued above this is not a persuasive test for two reasons. First, the error terms in the reduced form will include omitted nonlinear effects of tax on growth that can bias the reduced form coefficient. Second, other models in which tax plays no part could also generate this reduced form result.

So we reviewed next the evidence from the bootstrapping method, where instead of the confidence intervals generated by the 'reduced form' we look at those produced by bootstrapping the structural model. We found here that the model was accepted by the data description. In fact a model with a higher business tax coefficient of -0.02 is also accepted and with a lower M-metric by the growth regression (and still compatible with the production function as checked by an F-test).

What is also striking is the insight afforded by the bootstrapping procedure into the biases in the linear reduced form coefficients under the null hypothesis. Thus we suggest from the combination of the structural estimates and from simulating the model for a shock to the business tax rate that growth (in steady state) increases by some $0.2 \%$ for every 0.1 (ie 10 percentage point) fall in the business tax rate under the model. However the reduced form coefficients give a value for this business tax effect that is up to two and a half times as big. This indicates a possible bias in these linear reduced form coefficients.

A further point of interest is that there is some tension between what the structural model will tolerate in estimation and what gives the best results when simulated results are compared with the linear reduced form. The simulated method of moments would
give all the weight to the latter comparison; here we treat the structural estimation results as a separate Popperian hurdle or source of potential rejection, on a par with the growth regressions hurdle.

### 3.5 Adding other potential sources of growth into the model testing the Activist Models

From here on we expand the model to include, besides or instead of the business tax, tax/subsidies to education and R\&D. Each of these we think of as constructed of the general tax rate and the direct tax or (usually) subsidy on this element. We measure the subsidy to education as the share of GDP spent by government on education at all levels; that to $R \& D$ by the government expenditure on $R \& D$ as a percentage of total GDP. As these are subsidies they are counted as negative tax rates and referred to as 'taxes' symmetrically with business taxes; hence the sign of their effect in our regressions should be negative. Our procedure is the same as before except that now we replace the (cumulant of the) business tax rate in the production function with the cumulant of these other tax rates. As our estimate of the effect of these direct tax rates we use the freely estimated coefficient on each entered on its own with the general tax rate; the only exception is education discussed below. In the growth regressions we enter the general tax rate and the direct tax/subsidy separately, to allow the effect of the direct element to be freely determined.

### 3.5.1 The Education tax/subsidy:

We obtain a nonsensical positive and significant effect of the cumulated education tax rate in the production function when we allow it to be freely estimated, with the cumulated business tax rate (and insignificant education tax effect without the cumulated business tax rate). Hence the presence of an education tax/subsidy effect is rejected at the structural level.
(a) without business tax

$$
\begin{align*}
& \ln y_{t}=c_{1}+0.792 \ln \left(1-x_{t}\right)+ 0.011 \sum_{i=1960}^{t} \tau_{i}+0.002 \sum_{i=1960}^{t} e_{i}^{\prime} \\
&(0.081)(0.005) \\
& \text { Number of obs }=2280 \\
& \mathrm{~F}(107,2172)=814.09 \\
& R^{2}=0.9757 \\
& \bar{R}^{2}=0.9745 \\
& \text { Root MSE }=0.1651 \\
& \begin{aligned}
\text { (b) with business tax } & \\
\ln y_{t}=c_{1}+0.38 \ln \left(1-x_{t}\right) & -0.019 \sum_{i=1960}^{t}\left(\tau_{i}+\pi_{i}^{\prime}\right)+0.024 \sum_{i=1960}^{t}\left(\tau_{i}\right)+0.055 \sum_{i=1960}^{t}\left(e_{i}^{\prime}\right) \\
(0.085) & (0.0015) \\
\text { Number of obs } & =2280 \\
\mathrm{~F}(105,2174) & =864.07 \\
R^{2} & =0.9773 \\
\bar{R}^{2} & =0.9761 \\
\text { Root MSE } & =0.1596
\end{aligned}
\end{align*}
$$

However, for completeness we did go on to consider whether the education tax could have a marginal effect at some (imposed calibrated) level in the growth regressions test. So our procedure was to impose on the production function the same coefficient as for the business tax rate: -0.02 . The table below shows that the education tax cannot predict any of the growth regressions, either on its own or when combined with the business tax.

The table can be understood as follows. In the first Table we show the model with the education tax/subsidy alone. Then the table shows the relevant growth regressions in the actual data in the first line of each pair, with the standard errors of the coefficients in brackets. In the second line of each pair we show the $95 \%$ confidence limits of the same coefficients derived from running the same regression on the bootstrapped data from the model; the final column shows the M-Metric which indicates what level of confidence we would need to accept the model overall. In the second Table we show the equivalent for the model with both education and business tax operative.

Hence we can say that any role for the education tax in the model is strongly rejected at

Table 8: Bootstrap Results for Model With Education Tax
$\Delta \operatorname{lny}_{t}=\alpha_{0}\left(\tau_{t}+\pi_{t}^{\prime}\right)+\alpha_{1} \tau_{t}+\alpha_{2}\left(e_{t}^{\prime}\right)+$ country $/$ time dummies (standard errors)

- Model has education tax alone

|  | $\tau_{t}+\pi_{t}^{\prime}$ | $\tau_{t}$ | $e_{t}^{\prime}$ | M-metric |
| :--- | :---: | :---: | :---: | :--- |
| Growth regression |  | $-0.0329(0.0270)$ | $0.3934(0.1663)$ |  |
| Bootstrap 95\% limits | $-0.0621,0.0101$ | $-0.1873,0.1673$ | $100 \%$ |  |

Table 9: Bootstrap Results for Model With Education Tax and Business Tax $\Delta \operatorname{lny}_{t}=\alpha_{0}\left(\tau_{t}+\pi_{t}^{\prime}\right)+\alpha_{1} \tau_{t}+\alpha_{2}\left(e_{t}^{\prime}\right)+$ country $/$ time dummies (standard errors)

- Model has education with bus. tax

|  | $\tau_{t}+\pi_{t}^{\prime}$ | $\tau_{t}$ | $e_{t}^{\prime}$ | M-metric |
| :--- | :--- | :---: | :---: | :--- |
| Growth regression | $-0.0481(0.0105)$ | $0.0152(0.0294)$ | $0.3934(0.1663)$ |  |
| Bootstrap 95\% limits | $-0.0302,0.0003$ | $-0.0529,0.0134$ | $-0.1490,0.1543$ | $100 \%$ |

both the structural and the growth regression levels. We therefore eliminate the education tax/subsidy from any further consideration.

### 3.5.2 Government subsidy to R\&D:

When we turn to the Government R\&D tax/subsidy (GOVRD) we find a different picture. This (cumulated) is now significant and of the right sign at the structural level. The coefficient is -0.003 with a standard error of 0.001 on its own though when entered with the cumulated business tax it drops to insignificance. Thus at the structural level it is rejected if we assume that the business tax rate is operating. We would only accept it if we could reject the business tax rate in favour of the $R \& D$ model alone. We proceed therefore to test the R\&D model on its own on the growth regressions.

Table of structural equation with GOVRD on its own and with bus tax.
(a) without business tax

$$
\begin{gather*}
\ln y_{t}=c_{1}+0.768 \ln \left(1-x_{t}\right)+0.009 \sum_{i=1960}^{t}\left(\tau_{i}\right)-0.003 \sum_{i=1960}^{t}\left(\rho_{i}^{\prime}\right) \\
(0.081) \tag{0.001}
\end{gather*}
$$

| Number of obs | $=2280$ |
| :--- | :--- |
| $\mathrm{~F}(105,2174)$ | $=815.71$ |
| $R^{2}$ | $=0.9757$ |
| $\bar{R}^{2}$ | $=0.9745$ |
| Root MSE | $=0.1649$ |

(b) with business tax

$$
\begin{aligned}
& \ln y_{t}=c_{1}+0.36 \ln \left(1-x_{t}\right)-0.019 \sum_{i=1960}^{t}\left(\tau_{i}+\pi_{i}^{\prime}\right)+0.019 \sum_{i=1960}^{t}\left(\tau_{i}\right)+0.0005 \sum_{i=1960}^{t}\left(\rho_{i}^{\prime}\right) \\
& \quad(0.085) \quad(0.0016) \\
& \text { Number of obs }=2280 \\
& \mathrm{~F}(105,2174)=862.9 \\
& R^{2}=0.9772 \\
& \bar{R}^{2}=0.9761 \\
& \text { Root MSE }=0.1597
\end{aligned}
$$

If we now test the Government $\mathrm{R} \& \mathrm{D}$ Model on the growth regressions we find that it is rejected. The Table below shows first the Model with the R\&D tax/subsidy on its own, and it is rejected by the growth regression. Secondly we show the Model with both tax rates; again it is rejected by the growth regression with both.

Table 10: Coefficient on GOVRD -0.003(structural production function estimate)
$\Delta \operatorname{lny}_{t}=\alpha_{0}\left(\tau_{t}+\pi_{t}^{\prime}\right)+\alpha_{1} \tau_{t}+\alpha_{2}\left(\rho_{t}^{\prime}\right)+$ country $/$ time dummies (standard errors)

- Model has R\&D tax alone

|  | $\tau_{t}+\pi_{t}^{\prime}$ | $\tau_{t}$ | $\rho_{t}^{\prime}$ | M-metric |
| :--- | :---: | :---: | :---: | :--- |
| Growth regression | $-0.0674(0.0270)$ | $-0.0269(0.0110)$ |  |  |
| Bootstrap 95\% limits | $-0.0621,0.0002$ | $-0.0144,0.0058$ | $100 \%$ |  |

Table 11: Coefficient on GOVRD -0.003(structural production function estimate)
$\Delta \operatorname{lny}_{t}=\alpha_{0}\left(\tau_{t}+\pi_{t}^{\prime}\right)+\alpha_{1} \tau_{t}+\alpha_{2}\left(\rho_{t}^{\prime}\right)+$ country/time dummies (standard errors)

- Model has R\&D with bus. tax

|  | $\tau_{t}+\pi_{t}^{\prime}$ | $\tau_{t}$ | $\rho_{t}^{\prime}$ | M-metric |
| :--- | :---: | :--- | :--- | :--- |
| Growth regression | $-0.0025(0.0126)$ | $-0.0649(0.0312)$ | $-0.0269(0.0110)$ |  |
| Bootstrap 95\% limits | $-0.0312,-0.0018$ | $-0.0525,0.0063$ | $-0.0113,0.0074$ | $99.9 \%$ |

These results indicate that the R\&D tax/subsidy is rejected on the growth regressions whether business tax is included in the structural model or not and also whether or not it is included in the growth regression. However notice that in the growth regression the negative coefficient on $R \& D$ is higher in absolute value than the bootstrap range, indicating that a structural coefficient that is more negative would do better. Accordingly we subtracted two standard errors from the -0.003 estimated and reran the bootstraps; the results are shown in the next table.

Table 12: Coefficient on GOVRD changed to -0.005
$\Delta \operatorname{lny}_{t}=\alpha_{0}\left(\tau_{t}+\pi_{t}^{\prime}\right)+\alpha_{1} \tau_{t}+\alpha_{2}\left(\rho_{t}^{\prime}\right)+$ country /time dummies (standard errors)

- Model has R\&D tax alone

|  | $\tau_{t}+\pi_{t}^{\prime}$ | $\tau_{t}$ | $\rho_{t}^{\prime}$ | M-metric |
| :--- | :---: | :---: | :---: | :---: |
| Growth regression |  | $-0.0674(0.0270)$ | $-0.0269(0.0110)$ |  |
| Bootstrap $95 \%$ limits | $-0.0649,0.0051$ | $-0.0171,0.0050$ | $99.8 \%$ |  |

Table 13: Coefficient on GOVRD changed to -0.005
$\Delta \operatorname{lny}_{t}=\alpha_{0}\left(\tau_{t}+\pi_{t}^{\prime}\right)+\alpha_{1} \tau_{t}+\alpha_{2}\left(\rho_{t}^{\prime}\right)+$ country $/$ time dummies (standd errors)

- Model has R\&D with bus. tax

|  | $\tau_{t}+\pi_{t}^{\prime}$ | $\tau_{t}$ | $\rho_{t}^{\prime}$ | M-metric |
| :--- | :---: | :--- | :--- | :--- |
| Growth regression | $-0.0025(0.0126)$ | $-0.0649(0.0312)$ | $-0.0269(0.0110)$ |  |
| Bootstrap 95\% limits | $-0.0302,-0.0015$ | $-0.0538,0.0078$ | $-0.0126,0.0049$ | $100 \%$ |

What we see in this Table is that the GOVRD coefficient of -0.005 still fails to match up with the growth regression. We found that it needed to be put at -0.017 before the structural model with R\&D would be accepted (Table following). This is fourteen standard errors above the estimated coefficient and the F-test on this in the structural equation is 122 , implying massive rejection.

Table 14: Coefficient on GOVRD changed to -0.017
$\Delta \operatorname{lny}_{t}=\alpha_{0}\left(\tau_{t}+\pi_{t}^{\prime}\right)+\alpha_{1} \tau_{t}+\alpha_{2}\left(\rho_{t}^{\prime}\right)+$ country $/$ time dummies (standd errors)

- Model has R\&D tax alone

|  | $\tau_{t}+\pi_{t}^{\prime}$ | $\tau_{t}$ | $\rho_{t}^{\prime}$ | M-metric |
| :--- | :---: | :---: | :---: | :---: |
| Growth regression |  | $-0.0674(0.0270)$ | $-0.0269(0.0110)$ |  |
| Bootstrap 95\% limits | $-0.0612,0.0071$ | $-0.0280,-0.0081$ | $97.8 \%$ |  |

What this evidence is telling us is that the government share of R\&D does have a moderate effect on output in the production function estimate when business tax is not included but that this coefficient cannot explain the growth regressions with this factor
in it, with or without business tax. These growth regressions show a much larger effect of this GOVRD factor than can be explained by the only model with this factor accepted at the structural stage, viz one with the GOVRD factor on its own. Thus this model must be rejected. This implies that the apparent role of the GOVRD factor in the growth regression is the result of some other causal factor than the GOVRD factor itself - i.e. it is spurious.

Discussion of the empirical results: The theory we are investigating is in terms of the level of output, labour supply and productivity. These levels depend on the history of people's diversion of effort into productivity-raising activities. In the Incentivist Barriers to Riches theory people's incentive to do this depends on the barriers erected to it in the form of taxes and regulations. In the Activist theory people's incentive to acquire education and do R\&D depends on direct government encouragement of these activities. To test the two theories on our panel data we have first estimated a production function whose productivity level depends on the accumulated history of these factors; also a labour supply function of an ordinary sort from the households' first order condition. At this structural stage we are mirroring Parente and Prescott's emphasis on the need for the theory to explain the level of living standards. This first 'structural-stage' test is then complemented by the test on growth regressions - the second stage. Each theory implies that the growth regression should include only factors it identifies.

Our findings are that the model with only tax variables both fits at the structural level and predicts the parameters of the implied growth on tax variables. Plainly growth regressions can be done on other variables; but according to this theory any correlations with such variables is spurious, i.e. the result of reverse or joint causation - that is, the tax causal mechanism of growth also causes these other variables' movement either directly or via some link from growth to them. However this is not in this model but in some extended model. Thus for example we find that there is a partial correlation between growth and GOVRD, in addition to the one with business tax. According to the Incentivist model this is spurious.

When we turned to the Activist model focused on the government subsidy to R\&D,
we find that it fits at the structural stage. However, the growth regression it implies has a rather weak partial correlation with GOVRD which is inconsistent with the much stronger one in the data. Thus this model is rejected at the second stage; by implication the strong partial correlation with growth is spurious, in terms of the only model (the Incentivist) that survives both test stages.

How could this partial correlation come about in practice? It may well be that when growth is rapid it induces governments to participate directly in the process by paying for R\&D - in defence or strategic industries for example. Perhaps it sees opportunities for more tax revenue; or the politicians involved see opportunities for personal gain. There are many possible avenues in political economy for such processes.

Another way of looking at the matter is that the true relationship is in levels; differences of accumulated government GOVRD for example should show up in differences in living standards. If they do not, because there is no effect, then it is nevertheless possible for a relationship to show up in the growth rate of living standards and the levels of government R\&D, should there be some process that causes growth to affect the latter. Of course this has nothing to do with the model. This echoes the point emphasised by Parente and Prescott that a theory must explain the difference of living standards as well as the difference of growth rates. Our double-stage test should check out this capacity of the theory. Where a theory fails, it should reveal the existence of a spurious relationship coming from outside the causal model driving growth and living standards.

## 4 Conclusions

We have argued that regressions of growth on its supposed causes are not on their own persuasive evidence of these causes. Instead we proposed to test theories by a two-stage Popperian procedure in which rejection can occur at each stage. In the first stage the model as tightly specified must pass an estimation test in its structural form; in the second its bootstrapped implications must be consistent with the growth regressions it implies. We tested two main classes of growth theory: one was the Incentivist theory set
out by Parente and Prescott in Barriers to Riches, the other was the Activist theory that direct government intervention to stimulate particular activities - specifically education and R\&D - caused growth. We were able to reject the latter for education at both the structural and the bootstrap levels; and for R\&D at the bootstrap level, though not the structural. We accepted the Incentivist theory at both levels.

We suggest that these methods of testing are a useful way to proceed when there are many competing theories which are hard to distinguish in their reduced form. Further work would be interesting, for example both to test other theories of growth and to consider different measures of the relevant tax incentives. Finally we emphasise, as we began, that this sort of econometric testing can only complement and not replace the wideranging investigation of the historical evolution and particular case-studies of growth in terms of other methods.

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## 5 Appendix A: the non-stationarity of $\left\{\ln c_{t}-\ln y_{t}\right\}$

Start with the household budget constraint after substituting out tax and transfer terms via the government budget constraint and wage and dividends from the firm's first order conditions; this is line 4 of footnote 3 :

$$
\begin{equation*}
c_{t}+k_{t+1}-(1-\delta) k_{t}+b_{t+1}=y_{t}+\left(1+r_{t}\right) b_{t} \tag{A1}
\end{equation*}
$$

In expectational form the household's consumption plan must satisfy this constraint as follows after an infinite forward recursion in the value of future bonds:

$$
\begin{equation*}
\left(1+r_{t}\right) b_{t}=c_{t}-y_{t}^{\prime}+E_{t} \sum_{i=0}^{\infty}\left\{\prod_{j=1}^{i}\left(1+r_{t+j}\right\}^{-1}\left(c_{t+i}-y_{t+i}^{\prime}\right)\right. \tag{A2}
\end{equation*}
$$

where $y_{t}^{\prime}=y_{t}-\left[k_{t+1}-(1-\delta) k_{t}\right]$
Now note that from the household's first order condition
$E_{t}\left\{\prod_{j=1}^{i}\left(1+r_{t+j}\right\}^{-1} c_{t+i}=\beta^{i} c_{t}\right.$ since for example $c_{t}=\frac{1}{\beta} E_{t} \frac{c_{t+1}}{1+r_{t+1}}=\frac{1}{\beta^{2}} E_{t} \frac{c_{t+2}}{\left(1+r_{t+1}\right)\left(1+r_{t+2}\right)}$

It follows that

$$
\begin{equation*}
c_{t}=(1-\beta)\left\{\left(1+r_{t}\right) b_{t}+y_{t}^{\prime}+E_{t} \sum_{i=0}^{\infty}\left\{\prod_{j=1}^{i}\left(1+r_{t+j}\right\}^{-1} y_{t+i}^{\prime}\right)\right\} \tag{A4}
\end{equation*}
$$

The term inside the braces is the household's spendable wealth hence the whole RHS expression is permanent net income or

$$
\begin{equation*}
c_{t}=.(1-\beta)\left(1+r_{t}\right) b_{t}+\bar{y}_{t}^{\prime} \tag{A5}
\end{equation*}
$$

In steady state (at $T$ ) we have (where $g$ is the growth rate)

$$
\begin{align*}
c_{T} & =(1-\beta)\left\{\left(1+r^{*}\right) b_{T}+\sum_{i=0}^{\infty}\left\{\frac{1+g}{1+r^{*}}\right\}^{i}\left[1+\frac{\gamma \delta}{r^{*}+c}\right] y_{T}\right\} \\
& =(1-\beta)\left(1+r^{*}\right)\left\{b_{T}+\frac{1}{r^{*}-g}\left[1+\frac{\gamma \delta}{r^{*}+c}\right] y_{T}\right\} \\
& =(1-\beta)\left(1+r^{*}\right) b_{T}+\bar{y}_{T}^{\prime} \tag{A6}
\end{align*}
$$

in which all of $c_{T}, b_{T}, y_{T}$ will be growing at $g$.
Now consider the movement of $\frac{c_{t}}{y_{t}}$ which from A5) is:

$$
\begin{equation*}
\frac{c_{t}}{y_{t}}=.(1-\beta)\left(1+r_{t}\right) \frac{b_{t}}{y_{t}}+\frac{\bar{y}_{t}^{\prime}}{y_{t}} \tag{A7}
\end{equation*}
$$

Hence using the approximation that $\ln (x+y)=\frac{x}{x+y} \ln x+\frac{y}{x+y} \ln y$

$$
\begin{equation*}
\ln c_{t}-\ln y_{t}=.(\text { share of net income from abroad }) \ln \frac{b_{t}}{y_{t}}+\ln \frac{\bar{y}_{t}^{\prime}}{y_{t}} \tag{A8}
\end{equation*}
$$

From the balance of payments (footnote 3)

$$
\begin{equation*}
\frac{b_{t+1}}{y_{t+1}} \frac{y_{t+1}}{y_{t}}-\frac{b_{t}}{y_{t}}=r_{t} \frac{b_{t}}{y_{t}}-\frac{m_{t}}{y_{t}} \tag{A9}
\end{equation*}
$$

or

$$
\frac{b_{t+1}}{y_{t+1}}-\frac{b_{t}}{y_{t}}=\left(r_{t}-g_{t}\right) \frac{b_{t}}{y_{t}}-\frac{m_{t}}{y_{t}}
$$

We know that in steady state $\frac{b_{t}}{y_{t}}$ will tend to some steady level because of household behaviour. However until this has occurred it is driven by a difference equation of the form:

$$
\begin{equation*}
x_{t+1}=\left(1+q_{t}\right) x_{t}+\xi_{t} \tag{A10}
\end{equation*}
$$

where $q_{t}=r_{t}-g_{t}$ will vary from positive to negative and $\xi_{t}=-\frac{m_{t}}{y_{t}}$ will move randomly between steady states. Plainly $x_{t}=\frac{b_{t}}{y_{t}}$. will for at least some of the periods between steady states will be a randomly disturbed explosive (or unit root) difference equation and will
therefore be non-stationary (in other words it will end up at a new steady state randomly different from its initial value). So therefore will $\ln c_{t}-\ln y_{t}$ which contains its $\log$.

## 6 Appendix B: Data Description



## Data definitions and sources:

1. $y$ is GDP per capita from the Penn World Table;
2. General tax is public spending-to-GDP rate, from the Penn World Table
3. Business tax is general tax plus marginal costs levied by the country on firm closure \& firm set-up, from the World Ban Database;
4. Education tax is the negative value of government spending on education (primary, secondary, and tertiary) as the share of tot: GDP, from the World Bank Database;
5. Infrastructure tax is the negative indexed value of government spending on infrastructure (airports, electricity, telephones, road: as the share of total GDP, from the World Bank Database;
6. $R \& D$ tax is negative value of the government expenditure on $R \& D$ as the percentage of total GDP, from UNESCO yearbook;
7. Labour supply is workforce as the share of total population, from the World Bank Database.

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[^1]:    ${ }^{1}$ Leach (2003) and OECD (Leibfritz et al, 1997) provide useful surveys of this literature. Studies include Barro (1991), Koester and Kormendi (1989), Hansson and Henrekson (1994), Cashin (1995), Engen \& Skinner (1996), Leibfritz et al (op. cit.), Alesina et al. (2002), Bleaney, Gemmell \& Kneller (2000), Folster \& Henrekson (2000), Bassanini \& Scarpetta (2001), Benson and Johnson (1986), Chao and Grubel (1998), Easterly and Rebelo (1993), Grier and Tullock (1989), King and Rebelo (1990), Levine and Renelt (1992), Peden and Bradley (1989), Plosser (1992), Scully (1989, 1991, 1995), Slemrod (1995), Smith (2001), Vedder and Gallaway (1998).

[^2]:    ${ }^{2}$ This is obtained by differentiating the Lagrangean above with respect to $z_{0}$ remembering that (7) determines $A_{t}$. Thus we obtain
    $0=E_{0} \sum_{t=1}^{\infty} a_{1} \frac{A_{0}}{A_{1}} \lambda_{t} y_{t}-\lambda_{0}\left(w_{0}+\pi_{0}\right)$
    Note that $\frac{\partial y_{t}}{\partial z_{0}}=k_{t}^{\gamma} X_{t}^{\zeta}\left(1-x_{t}-z_{t}\right)^{1-\gamma-\zeta} \frac{\partial A_{t}}{\partial z_{0}}=\frac{y_{t}}{A_{t}} \frac{\partial A_{t}}{\partial z_{0}}(t \geq 1)$; while since $A_{t}=\frac{A_{t}}{A_{1}} \frac{A_{1}}{A_{0}} A_{0}$ and $\frac{A_{t}}{A_{1}}$ is independent of $z_{0}$ it follows that $\frac{\partial A_{t}}{\partial z_{0}}=\frac{A_{t}}{A_{1}} A_{0}\left\{\partial \frac{A_{1}}{A_{0}} / \partial z_{0}\right\}=\frac{A_{t}}{A_{1}} A_{0} a_{1}$. Hence finally $\frac{\partial y_{t}}{\partial z_{0}}=y_{t} \frac{A_{0}}{A_{1}} a_{1}=$ $y_{t} \frac{a_{1}}{a_{0}+a_{1} z_{0}+u_{0}}$

[^3]:    ${ }^{3}$ Thus taking the household budget constraint
    $\left(1+\tau_{t}\right) c_{t}+k_{t+1}-(1-\delta) k_{t}+b_{t+1}=y_{t}+\left(1+r_{t}\right) b_{t}+\Gamma_{t}-\sum\left(\pi_{t}^{\prime} z_{\pi t}+e_{t}^{\prime} z_{e t}+\rho_{t}^{\prime} z_{\rho t}\right)$
    we note that the tax terms cancel with the government transfer via the government's budget constraint so that
    $c_{t}+k_{t+1}-(1-\delta) k_{t}+b_{t+1}=y_{t}+\left(1+r_{t}\right) b_{t}$
    Now we use market clearing to substitute out for $y_{t}$ so that
    $c_{t}+k_{t+1}-(1-\delta) k_{t}+b_{t+1}=c_{t}+k_{t+1}-(1-\delta) k_{t}-m_{t}+\left(1+r_{t}\right) b_{t}$.
    Cancelling terms yields the balance of payments
    $b_{t+1}-b_{t}=r_{t} b_{t}-m_{t}$
    where net lending abroad (the capital account deficit) equals net interest from abroad minus net imports (the current account surplus).

[^4]:    ${ }^{4}$ In assessing whether the model is rejected or not we need to use the joint distribution of all the parameters in the description. The $95 \%$ confidence intervals shown by each parameter apply to that parameter taken on its own, that is holding the other parameters as given by their estimated values. For the model as a whole the question is whether the joint values of the estimated parameters lie within the ' $95 \%$ contour' of the joint distribution. The idea here is that the model generates a joint distribution of the descriptive ('reduced form') parameters around the mean of the bootstrap distribution. This is assumed to be symmetric and the Mahalanobis distance of each parameter combination from the bootstrap mean is computed. The bootstrap distribution over this distance value - the 'M-metric' can be used to compute their percentile values. The model as a whole is then rejected if the actual Mmetric estimated on the data exceeds say the 95 th percentile, this being the 95 th percentile 'contour line' on the joint distribution. Clearly such a rejection is related somehow to the rejection on the parameters individually; however, this relationship depends on the covariance matrix of these parameters which is a crucial ingredient of the joint distribution. Thus there is no simple link from the individual rejections to the overall rejection of the model.

