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The optimality of optimal punishments in Cournot supergames

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Abstract

The result of Colombo and Labrecciosa (2006) that optimal punishments are inferior to Nash-reversion trigger strategies with decreasing marginal costs is due to the output when a firm deviates from the punishment path being allowed to become negative.

Keywords: Optimal punishments, trigger strategies, collusion, cartels.

JEL Classification: C73, D43, L13.

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1. Introduction

Recently, Colombo and Labrecciosa (2006) claimed to show that the optimal punishments of Abreu (1986) for sustaining collusion in a Cournot supergame were not optimal. They claimed that the optimal punishments were inferior to the Nash-reversion trigger strategies of Friedman (1971) when marginal cost decreased sufficiently with output. However, their analysis ignores their own assumption, A3, and the fact that the outputs of the firms must be non-negative. In particular, it turns out that the output when a firm deviates from the punishment path is actually negative for the parameter values where they obtain their result. This letter derives the optimal punishments and critical discount factor when the output of a firm that deviates from the punishment path is not allowed to be negative. Then, the critical discount factor with optimal punishments is compared to the critical discount factor with Nash-reversion trigger strategies, and it is shown that the critical discount factor with optimal punishments is always lower than the critical discount factor with Nash-reversion trigger strategies. Therefore, optimal punishments of Abreu (1986) are not sub-optimal with decreasing marginal cost.

2. The Model

The model and notation are the same as in Colombo and Labrecciosa (2006): two identical firms compete as Cournot duopolists in an infinitely repeated game with a common discount factor: $\delta \in (0,1)$. Demand for the homogeneous product is linear: $p = 1 - q_1 - q_2$, where q_i is the output of the *i*th firm with $i = 1,2$; and total costs are: $c(q_i) = \alpha q_i + \beta q_i^2 / 2$, so marginal cost is linear in output: $c'(q_i) = \alpha + \beta q_i$; where $\alpha \in (0,1)$ and $\beta \in (-2,0)$, which implies decreasing marginal cost. As in Colombo and Labrecciosa (2006), assumption A3, outputs are assumed to be non-negative.

As in Colombo and Labrecciosa (2006), the output and profits of each firm when the firms collude and maximise industry profits are:

$$q_C = \frac{1-\alpha}{4+\beta} \qquad \pi_C = \frac{(1-\alpha)^2}{2(4+\beta)} \tag{1}$$

The output and profits when a firm deviates from collusion and maximises its profits given that its competitor produces q_C are:

$$q_D = \frac{(1-\alpha)(3+\beta)}{(2+\beta)(4+\beta)} \qquad \pi_D = \frac{(1-\alpha)^2(3+\beta)^2}{2(2+\beta)(4+\beta)^2} \tag{2}$$

Collusion can be sustained using the Nash-reversion trigger strategies of Friedman (1971) if the discount factor is sufficiently large. Firms collude by producing output q_C thereby earning profits π_C as long as no firm has deviated. If a firm deviates from producing output q_C then both firms will produce the Cournot-Nash equilibrium output forever thereafter. As in Colombo and Labrecciosa (2006), the symmetric Cournot-Nash equilibrium output and profits of each firm are:

$$q_N = \frac{1-\alpha}{3+\beta} \qquad \pi_N = \frac{(1-\alpha)^2(2+\beta)}{2(3+\beta)^2} \tag{3}$$

For $\beta \in (-2,-1)$, the Hahn stability condition is not satisfied and the symmetric Nash equilibrium is not unique as there are two other Nash equilibria where one firm produces its monopoly output and the other produces an output of zero. However, as in Colombo and Labrecciosa (2006), we will only consider reversion to the symmetric Cournot-Nash equilibrium. Therefore, the critical discount factor with Nash-reversion trigger strategies is:

$$\delta_T^* = \frac{\pi_D - \pi_C}{\pi_D - \pi_N} = \frac{(3+\beta)^2}{2(3+\beta)^2 - 1} \tag{4}$$

This critical discount factor decreases from $\delta_T^* = 1$ when $\beta = -2$ to $\delta_T^* = 9/17$ when $\beta = 0$.

Abreu (1986) characterised the optimal symmetric punishments for sustaining collusion in a symmetric Cournot supergame. Firms collude by each producing output q_C thereby earning profits π_C in the collusion path but if a firm deviates from the collusion path then all firms shift to the punishment path where each firm produces q_{OP} thereby earning profits of π_{OP} for one period followed by a return to the collusive path. If a firm deviates from the punishment path then the punishment path will start again with the firms producing q_{OP} thereby earning profits of π_{OP}. The optimal punishments and critical discount factor can be derived by solving two simultaneous equations derived from the requirement that deviations from the collusive path and the punishment path are unprofitable for the firms.

The flaw in the analysis of Colombo and Labrecciosa (2006) is that they claim that the profits of the firm when it deviates from the punishment phase are positive and decreasing in β for $\beta < \sqrt{2} - 3 \approx -1 \cdot 586$. As a result, they claim that optimal punishments are inferior to Nash-reversion trigger strategies for $\beta \in (-1.8453, -1.7753)$, but output when a firm deviates from the punishment path turns out to be negative for $\beta \in \left(-2, \sqrt{2} - 3\right)$. The output and profits of the firm when it deviates on the punishment path in their analysis can be obtained by substituting their equation (12) into their equation (11), which yields:

$$q_{DOP}^* = \frac{(1-\alpha)\left(7 + 6\beta + \beta^2\right)}{2(2+\beta)(3+\beta)(4+\beta)} \qquad \pi_{DOP}^* = \frac{(1-\alpha)^2\left(7 + 6\beta + \beta^2\right)^2}{2(2+\beta)(3+\beta)^2(4+\beta)^2} \tag{5}$$

Since $\beta \in (-2, 0)$ and $\alpha \in (0, 1)$ all the terms are positive except $7 + 6\beta + \beta^2$, which is negative for $\beta < \sqrt{2} - 3$, therefore, although profits are positive, the output of the firm that deviates from the punishment path is negative for $\beta < \sqrt{2} - 3$. Obviously, the output of the firm when it deviates from the punishment path cannot be negative, so there will obviously be a corner solution where the firm produces zero output, $q_{DOP} = 0$, and makes zero profits, $\pi_{DOP} = 0$ for $\beta \in \left(-2, \sqrt{2} - 3\right)$. The firm will be forced down to its security level of zero in the punishment path. From Abreu (1986), the condition for sustaining cooperation in the punishment path is that deviation is not profitable:

$$\pi_{DOP} - \pi_{OP} \leq \delta\left(\pi_C - \pi_{OP}\right) \tag{6}$$

When $\beta \in \left(-2, \sqrt{2} - 3\right)$, the profits from deviating in the punishment path are zero, $\pi_{DOP} = 0$, and the profits with the optimal punishment are obtained by solving (6) when it holds with equality:

$$\pi_{OP} = \frac{-\delta}{1-\delta}\pi_C = \frac{\delta(1-\alpha)^2}{2(1-\delta)(4+\beta)^2} \tag{7}$$

From Abreu (1986), the condition for sustaining collusion is that deviation from the collusive path is not profitable:

$$\pi_D - \pi_C \leq \delta\left(\pi_C - \pi_{OP}\right) \tag{8}$$

Substituting (7) into (8) and solving when it holds with equality yields the critical discount factor with optimal punishments:

$$\delta_{OP}^* = \frac{1}{(3+\beta)^2} \qquad \forall \beta \in \left(-2, \sqrt{2} - 3\right) \tag{9}$$

This critical discount factor decreases from $\delta_{OP}^* = 1$ when $\beta = -2$ to $\delta_{OP}^* = \frac{1}{2}$ when $\beta = \sqrt{2} - 3$. Substituting the critical discount factor with optimal punishments into (7) yields the profits in the punishment path:

$$\pi_{OP} = -\frac{(1-\alpha)^2}{2(2+\beta)(4+\beta)^2} < 0 \qquad (10)$$

Profits in the punishment path are negative so one should check that the corresponding price will be positive in the punishment path. The corresponding output of each firm and price in the punishment path can easily be obtained, but the expressions are very messy. However, it can be shown that the price in the punishment path is positive provided α exceeds a critical value, $\alpha > \alpha^*$, which is less than one but which tends to one as $\beta \to -2$.

To compare the sustainability of collusion, subtract the critical discount factor with optimal punishments (9) from the critical discount factor with Nash-reversion trigger strategies (4), which yields:

$$\delta_T^* - \delta_{OP}^* = \frac{(2+\beta)^2(4+\beta)^2}{(3+\beta)^2\left[2(3+\beta)^2 - 1\right]} > 0 \qquad (11)$$

Since $2(3+\beta)^2 - 1 > 0$ for $\beta \in (-2,0)$, the difference in the critical discount factors is positive so the optimal punishments of Abreu (1986) are superior to the Nash-reversion trigger strategies of Friedman (1971). This is shown in figure one and leads to the following proposition:

***Proposition**: For $\beta \in \left(-2, \sqrt{2}-3\right)$, the critical discount factor for Nash-reversion trigger strategies is larger than the critical discount factor for optimal punishments.*

This contradicts the result of Colombo and Labrecciosa (2006) who claimed that optimal punishments were not optimal but they ignored the non-negativity constraint for output when a firm deviates in the punishment path.

3. Conclusion

This note has shown that the result in Colombo and Labrecciosa (2006) that the optimal punishments of Abreu (1986) are inferior to Nash-reversion trigger strategies of Friedman (1971) with rapidly decreasing marginal costs was due to the output when a firm deviates from the punishment path being allowed to become negative. When outputs are required to be non-negative, the superiority of the optimal punishments over Nash-reversion trigger strategies is restored. There seems no reason to believe that the optimal punishments of Abreu (1986 and 1988) do not apply to the case of decreasing marginal costs.

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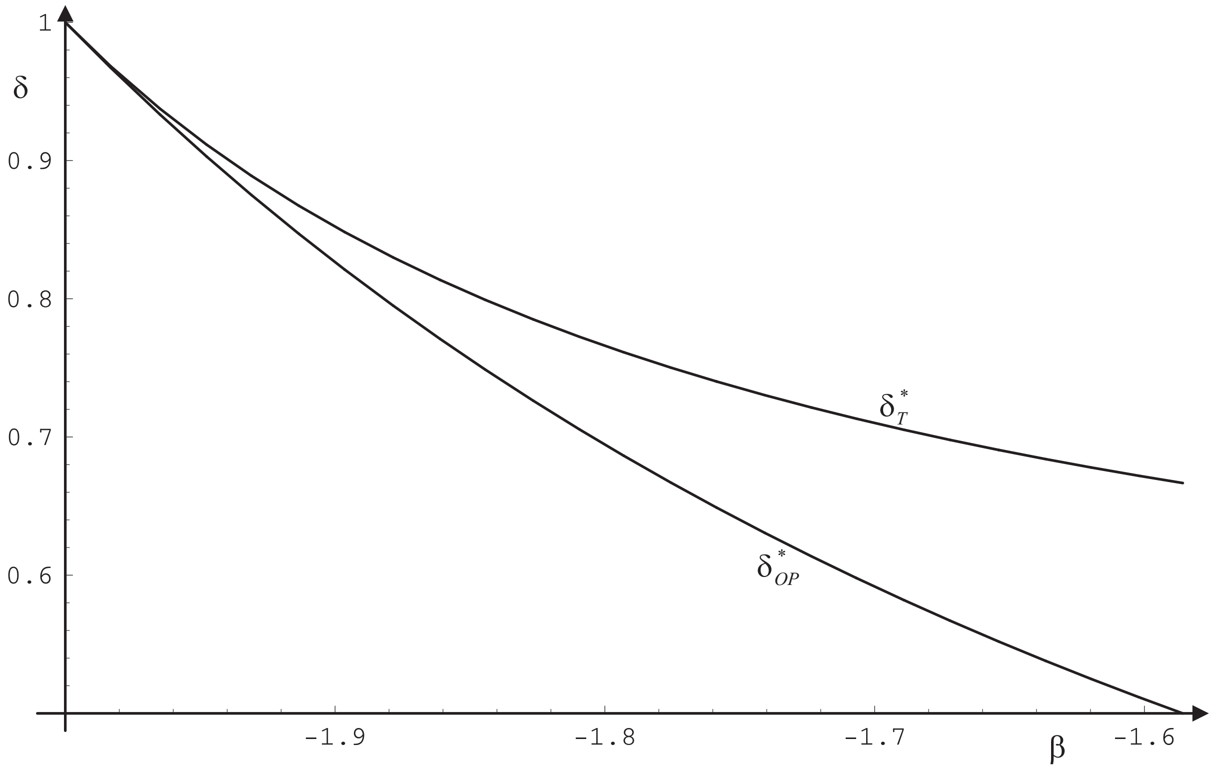

Figure 1: Critical Discount Factors