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Two Orthogonal Continents? Testing a Two-country DSGE Model of the US and the EU Using Indirect Inference*

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Abstract

We examine a two country model of the EU and the US. Each has a small sector of the labour and product markets in which there is wage/price rigidity, but otherwise enjoys flexible wages and prices with a one quarter information lag. Using a VAR to represent the data, we find the model as a whole is rejected. However it is accepted for real variables, output and the real exchange rate, suggesting mis-specification lies in monetary relationships. The model highlights a lack of spillovers between the US and the EU.

JEL Classification: C12, C32, C52, E1

Keywords: Bootstrap, Open economy model, DSGE, VAR, New Keynesian, New Classical, indirect inference, Wald statistic

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‘...if the conclusions (of a theory) have been falsified, then their falsification also falsifies the theory from which they were logically deduced. It should be noticed that a positive decision can only temporarily support the theory, for subsequent negative decisions may always overthrow it.’ Popper, The Logic of Scientific Discovery (p.10).

‘...my recollection is that Bob Lucas and Ed Prescott were initially very enthusiastic about rational expectations econometrics. After all, it simply involved imposing on ourselves the same high standards we had criticized the Keynesians for failing to live up to. But after about five years of doing likelihood ratio tests on rational expectations models, I recall Bob Lucas and Ed Prescott both telling me that those tests were rejecting too many good models.’ Tom Sargent, interviewed by Evans and Honkapohja (p.6)

In this paper we propose a two-country DSGE model for the US and EU. Our model is based on existing, well-known, but separate and either calibrated or estimated, models for each economy. The aim is to test this two-country model using indirect inference on data from the mid-1970s.

As the two quotes above indicate, evaluating RBC models using classical methods such as model likelihood ratio tests tends to lead to their rejection. Consequently, early RBC model-builders such as Prescott adopted a different way of evaluating their models. This was based on an informal comparison of key “facts” (usually moments and cross-moments) concerning observed data with the model’s simulated properties derived from a calibrated version of the model. Presumably, the implication is that a “good model” is one that is capable of explaining such key facts. Although it is increasingly common to incorporate prior knowledge of a DSGE model’s parameters by a mixture of calibration and Bayesian estimation, the problem remains of how to evaluate the model *on the basis of the key facts*, but using classical statistical procedures.

A second purpose of this paper is to suggest a way of doing this. We use a modified version of the method of indirect inference discussed in Le, Minford and Wickens (2008) and Meenagh, Minford and Wickens (2009). Model evaluation by indirect inference involves comparing the estimates of an auxiliary model derived from observed data and data generated by simulating the calibrated or estimated model. A common choice of auxiliary model is a VAR. A formal test is usually conducted using a Wald statistic derived from the difference between the two sets of parameters of the auxiliary model. The distribution of the Wald statistic may be based either on asymptotic distribution theory or numerical approximation using a bootstrap. Our modification of this procedure is to carry out the Wald test using only a sub-set of the auxiliary model’s parameters. We refer to the resulting test as a directed Wald test.

Our results provide further support for the pessimism of Lucas and Prescott that testing a DSGE model using all of the auxiliary model’s parameters is likely to lead to its rejection as on this criterion we reject both the two individual country models and our two-country model. Nevertheless, based on certain key facts, we establish that there are versions of these models that are able to pass these more limited tests. This provides a basis for ranking models that are rejected on a conventional Wald test.

We explain the test procedure in more detail in Section 1. In Section 2 we describe our two-country model. Our results are presented in Section 3. Some model implications are drawn in Section 4 and Section 5 concludes.

1 Model evaluation by indirect inference

Indirect inference has been widely used in the estimation of structural models, see Smith (1993), Gregory and Smith (1991, 1993), Gourieroux et al. (1993), Gourieroux and Montfort (1995) and Canova (2005). Here we make a different use of indirect inference as our aim is to evaluate an already estimated or calibrated structural model. The common element is the use of an auxiliary model. Before considering model evaluation by indirect inference, we discuss estimation by indirect inference.

1.1 Estimation

Estimation by indirect inference chooses the parameters of the macroeconomic model so that when this model is simulated it generates estimates of the auxiliary model similar to those obtained from the observed data. The optimal choice of parameters for the macroeconomic model are those that minimize the distance between a given function of the two sets of estimated coefficients of the auxiliary model. Common choices of this function are (i) the actual coefficients, (ii) the scores, and (iii) the impulse response functions. In effect, estimation by indirect inference gives the optimal calibration.

Suppose that y_t is an $m \times 1$ vector of observed data, $t = 1, ..., T$, $x_t(\theta)$ is an $m \times 1$ vector of simulated time series generated from the structural macroeconomic model, θ is a $k \times 1$ vector of the parameters of the macroeconomic model and $x_t(\theta)$ and y_t are assumed to be stationary and ergodic. The auxiliary model is $f[y_t, \alpha]$. We assume that there exists a particular value of θ given by θ_0 such that $\{x_t(\theta_0)\}_{s=1}^{S}$ and $\{y_t\}_{t=1}^{T}$ share the same distribution, i.e.

$$f[x_t(\theta_0), a] = f[y_t, \alpha]$$

where α is the vector of parameters of the auxiliary model.

The likelihood function for the auxiliary model defined for the observed data $\{y_t\}_{t=1}^{T}$ is

$$\mathcal{L}_T(y_t; \alpha) = \Sigma_{t=1}^{T} \log f[y_t, \alpha]$$

The maximum likelihood estimator of α is then

$$a_T = \arg\max_{\alpha} \mathcal{L}_T(y_t; \alpha)$$

The corresponding likelihood function based on the simulated data $\{x_t(\theta)\}_{s=1}^{S}$ is

$$\mathcal{L}_S[x_t(\theta); \alpha] = \Sigma_{t=1}^{S} \log f[x_t(\theta), \alpha]$$

with

$$a_S(\theta) = \arg\max_{a} \mathcal{L}_S[x_t(\theta); \alpha]$$

The simulated quasi maximum likelihood estimator (SQMLE) of θ is

$$\theta_{T,S} = \arg\max_{\theta} \mathcal{L}_T[y_t; \alpha_S(\theta)]$$

This is the value of θ that produces a value of α that maximises the likelihood function using the observed data. We suppose that the observed and the simulated data are such that this value of α satisfies

$$plim\ a_T = plim\ a_S(\theta) = \alpha$$

hence the assumption that $x_t(\theta)$ and y_t are stationary and ergodic, see Canova (2005). It can then be shown that

$$\begin{aligned} T^{1/2}(a_S(\theta) - \alpha) &\rightarrow N[0, \ \ (\theta)] \\ (\theta) &= E[-\frac{\partial^2 \mathcal{L}[\alpha(\theta)]}{\partial \alpha^2}]^{-1} E[\frac{\partial \mathcal{L}[\alpha(\theta)]}{\partial \alpha} \frac{\partial \mathcal{L}[\alpha(\theta)]}{\partial \alpha}'] E[-\frac{\partial^2 \mathcal{L}[\alpha(\theta)]}{\partial \alpha^2}]^{-1} \end{aligned}$$

The covariance matrix can be obtained either analytically or by bootstrapping the simulations.

The extended method of simulated moments estimator (EMSME) is obtained as follows. Consider the continuous $p \times 1$ vector of functions $g(a_T)$ and $g(\alpha_S(\theta))$ which could, for example, be moments or scores, and let $G_T(a_T) = \frac{1}{T}\Sigma_{t=1}^{T} g(a_T)$ and $G_S(\alpha_S(\theta)) = \frac{1}{S}\Sigma_{s=1}^{S} g(\alpha_S(\theta))$. We require that $a_T \rightarrow \alpha_S$ in probability and that $G_T(a_T) \rightarrow G_S(\alpha_S(\theta))$ in probability for each θ. The EMSME is

$$\theta_{T,S} = \arg\min_{\theta} [G_T(a_T) - G_S(\alpha_S(\theta))]' W(\theta) [G(a_T) - G_S(\alpha_S(\theta))]$$

1.2 Model evaluation

The parameters of the macroeconomic model and their distributions are taken as given—either estimated or calibrated. The aim is to compare the performance of the auxiliary model based on observed data with its performance based on simulations of the macroeconomic model derived from the given distributions of the parameters. The test statistic is based on the distributions of these functions of the parameters of the auxiliary model, or of a function of these parameters.

We choose the auxiliary model to be a VAR. This captures two features of a structural model: the variance-covariance relations among the variables is reflected through the covariance matrix of the VAR disturbances, and the dynamic behaviour of the structural model is reflected in the dynamics and impulse response functions of the VAR. Tests of the structural model are therefore based on the VAR characteristics, or equivalently, functions of the VAR coefficients.

Non-rejection of the null hypothesis is taken to indicate that the macroeconomic model is not significantly different from that of the observed data. Rejection is taken to imply that the macroeconomic model is incorrectly specified. Comparison of the impulse response functions of the observed and simulated data should reveal in what respects the macroeconomic model fails to capture the auxiliary model.

A Wald test statistic is obtained as follows. We assume that there exists a particular value of θ given by θ_0 such that $\{x_t(\theta_0)\}_{s=1}^{S}$ and $\{y_t\}_{t=1}^{T}$ share the same distribution, where $S = cT$ and $c \geq 1$. If $\widehat{\theta}$ is the estimated or calibrated value of θ then the null hypothesis can be expressed as $H_0 : \widehat{\theta} \to \theta_0$. Consider again the continuous $p \times 1$ vector of functions $g(a_T)$, $g(\alpha_S(\theta))$, $G_T(a_T) = \frac{1}{T}\Sigma_{t=1}^{T} g(a_T)$ and $G_S(\alpha_S(\theta)) = \frac{1}{S}\Sigma_{s=1}^{S} g(\alpha_S(\theta))$. The functions $g(.)$ may be impulse response functions. Given an auxiliary model and a function of its parameters, our test statistic for evaluating the macroeconomic model is based on the distribution of $G_T(a_T) - G_S(\alpha_S(\widehat{\theta}))$. The resulting Wald statistic is

$$[G_T(a_T) - G_S(\alpha_S(\widehat{\theta}))]' W(\widehat{\theta}) [G_T(a_T) - G_S(\alpha_S(\widehat{\theta}))]$$

where the estimate of the optimal weighting matrix is

$$W(\widehat{\theta}) = \{[\frac{\partial G(\alpha(\widehat{\theta}))}{\partial \alpha}] \quad (\widehat{\theta})[\frac{\partial G(\alpha(\widehat{\theta}))}{\partial \alpha}]'\}^{-1}$$

Alternatively, the distribution of $G_T(a_T) - G_S(\alpha_S(\widehat{\theta}))$ and the Wald statistic can be obtained using the bootstrap. We take the following steps in our implementation of the Wald test by bootstrapping:

Step 1: Determine the errors of the economic model conditional on the observed data and $\widehat{\theta}$.

Solve the DSGE macroeconomic model for the structural errors ε_t given $\widehat{\theta}$ and the observed data. The number of independent structural errors is taken to be less than or equal to the number of endogenous variables. The errors are not assumed to be Normal.

Step 2: Construct the empirical distribution of the structural errors

On the null hypothesis the $\{\varepsilon_t\}_{t=1}^{T}$ errors are omitted variables. Their empirical distribution is assumed to be given by these structural errors. The simulated disturbances are drawn from these errors. In some DSGE models the structural errors are assumed to be generated by autoregressive processes.

Step 3: Compute the Wald statistic

The test is here based on a comparison of the VAR coefficient vector itself rather than a multi-valued function of it such as the IRFs. Thus

$$g(a_T) - g(\alpha_S(\theta)) = a_T - \alpha_S(\theta)$$

also therefore

$$G_T(a_T) - G_S(\alpha_S(\widehat{\theta})) = a_T - \alpha_S(\widehat{\theta})$$

The distribution of $a_T - \alpha_S(\widehat{\theta})$ and its covariance matrix $W(\widehat{\theta})^{-1}$ are estimated by bootstrapping $\alpha_S(\widehat{\theta})$. This proceeds by drawing N bootstrap samples of the structural model, and estimating the auxiliary VAR on each, thus obtaining N $a_S(\widehat{\theta})$. This set of vectors represents the sampling variation implied by the structural model, enabling its mean, covariance matrix and confidence bounds to be calculated directly. N is generally set to 1000. We can now compute the properties of the model and compare them with those of the data; in particular we examine the model's ability to encompass the variances of the data. Assuming the model can do so, we go on to compute the bootstrap Wald statistic $[a_T - \alpha_S(\widehat{\theta})]' W(\widehat{\theta})[a_T - \alpha_S(\widehat{\theta})]$. This is expressed as the percentile in the distribution where the data value occurs.

A conventional Wald statistic like this provides a very stringent test of a model as it is based on all of the parameters of the auxiliary model whether they are "key facts" or not. If we interpret Lucas and Prescott's observation that a likelihood ratio test rejects too many good DSGE models as implying that a model may capture key features of the data yet be rejected because it fails to capture less important features, then this suggests that we base a test of the model only on the key features of the data. By "data" we mean certain key parameters of the auxiliary model. This results in a Wald test defined over a restricted parameter space. We call this test a directed Wald statistic.

To illustrate this argument consider Figure 1 which shows the joint sampling distribution of just two parameters of the auxiliary model based on simulated data from the structural model. The upper graph assumes that the estimates of the parameters are uncorrelated and the lower graph assumes that they are correlated. The position of the estimates of the two parameters based on the observed data is shown on each graph. The Wald test takes these values as given. Assuming a non-zero correlation between the parameter estimates results in the point estimate based on observed data lying well in the tail of the joint distribution implying that the Wald test would be significant. But assuming a zero correlation

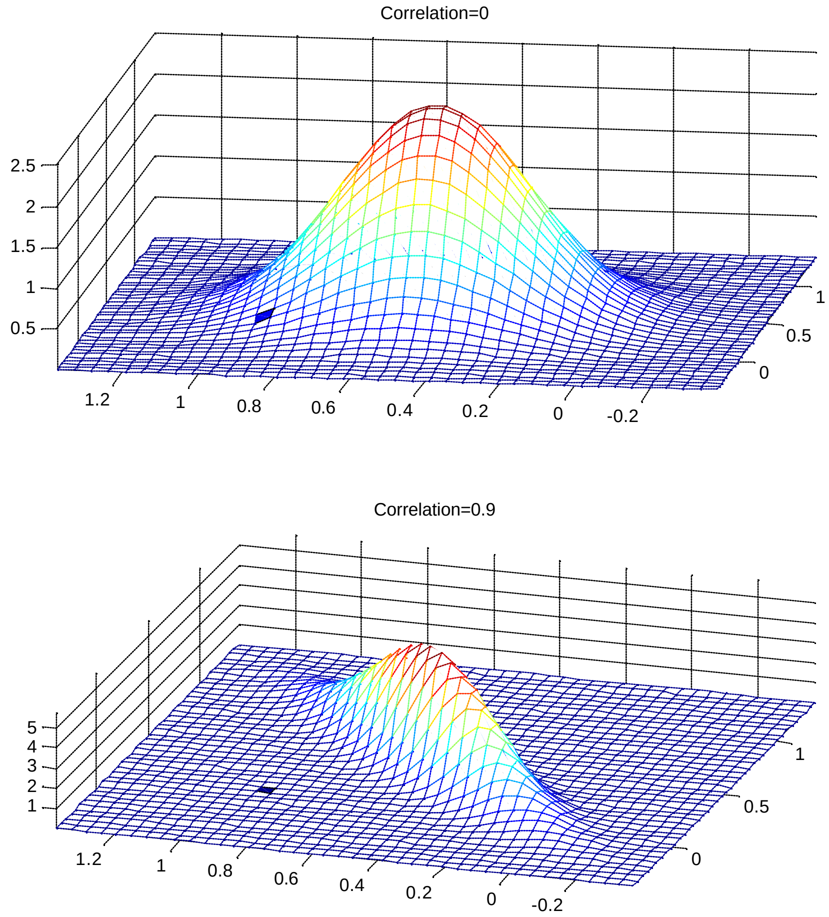

Figure 1: Bivariate Normal Distributions (0.1, 0.9 shaded) with correlation of 0 and 0.9.

results in the joint distribution covering the point estimate, suggesting that the Wald statistic is much less likely to be significant.

We calculate a variety of Wald statistics. First, we obtain the full Wald test based on the joint distribution of the VAR coefficients implied by their full covariance matrix, as in the second panel of Figure 1): this checks whether the VAR data-based coefficients lie within the DSGE model's implied joint distribution and is a test of the DSGE model's specification in absolute terms. Second, we use the Wald statistic computed from the DSGE model's distribution over the VAR coefficients on the assumption of zero covariances—as in the first panel of Figure 1. We call this the 'Average t-test' Wald statistic because by treating the VAR coefficients as orthogonal to each other, in effect, it averages each coefficient's t-value (which in turn just reflects the marginal distribution on that coefficient, taking the others as given). This test is not based on the model's true joint distribution, as Figure 1 illustrates clearly. Nonetheless, we use it as a way to assess the models' closeness on average to the data description. Third, we use what we have called the Directed Wald statistic in which we focus on particular features of the model—that is particular variables and groups of variables, and particular shocks or groups of shocks. For example, we can ask how well the model can reproduce the behaviour of US and EU output by running a VAR on just these variables and computing the Full Wald of the model for just this VAR. For a shock we can calculate the joint distribution of the IRFs of that shock on variables which it non-negligibly affects; comparing this with the data-generated IRFs gives us a Wald statistic for just this shock. Thus even if a model is overall mis-specified we can say whether it is well-specified enough to deal with some aspect of economic behaviour.

In addition to carrying out these Wald tests, we look at the usual battery of diagnostic statistics for these models, such as the ability to match (i.e. embrace within 95% condfidence limits) data variances, cross-correlations, and VAR-based IRFs. We attach particular importance to the ability to match data variances, arguing that a failure in this dimension is essentially terminal; for this reason we usually include these in the Wald test. In this respect our test procedure is quite traditional; but where it differs from much earlier work is that it considers *joint* distributions of all the key features under consideration.

2 The two-country model

We construct our world model from individual country models for the US and the EU. Together these two blocs make up some three quarters of world GDP at current exchange rates. Clearly leaving out the the Rest of the World (ROW), which is a quarter of the world economy, simplifies matters. In particular, it treats the effect of the ROW's GDP movements, and hence imports from the EU and US minus any movements in imports from them, as error terms in the US and EU market-clearing conditions which we allow to be correlated. Thus, we are exogenising the rest of the world's trade balances with the EU and US as a 'shock' like other shocks to the US and EU, such as productivity and consumption. We could think of these 'ROW shocks' as crudely consisting of a 'ROW demand' shock to exports to the ROW minus an 'outsourcing' shock to imports from the ROW.

The individual models for these two countries have been evaluated by Le et al. (2008) for the US and Meenagh et al. (2009) for the EU and have been found to be empirically relatively successful. These two models are derivatives of the Smets and Wouters models for the US (Smets and Wouters, henceforth SW, 2007) and the EU (SW, 2003); a very similar type of model to the SW model was first assembled by Christiano et al. (2005).

SW's DSGE model of the EU and the US is in most ways an RBC model but with additional characteristics that make it 'New Keynesian'. First there are Calvo wage- and price-setting contracts under imperfect competition in labour and product markets, together with lagged indexation. Second, there is an interest-rate setting rule with an inflation target to set inflation. Third, there is habit formation in consumption. Fourth, in addition to the usual adjustment costs of the capital stock, there is variable capacity utilisation with a marginal cost of variation. What follows is an account of the SW model of the EU and how Meenagh et al. altered it in the light of their tests. It is followed by a brief account of the very similar US model of SW and how Le et al. altered that.

2.1 SW's EU model:

SW added ten exogenous shocks to their EU model. Eight—technical progress, preferences and cost-push shocks—were assumed to follow independent AR(1) processes. The whole model was then estimated using Bayesian procedures on quarterly data for the period 1970q1–1999q2 for seven euro-area macroeconomic variables: GDP, consumption, investment, employment, the GDP deflator, real wages and the nominal interest rate. It was assumed that capital and the rental rate of capital are not observed. By using Bayesian methods it was possible to combine key calibrated parameters with sample information. Rather than evaluate the DSGE model based only on its sample moment statistics, impulse response functions were also used. The moments and the impulse response functions for the estimated DSGE model were based on the median of ten thousand simulations of the estimated model.

Meenagh et al. applied the indirect inference testing procedure described above to this model. The same data as SW for the period 1970–1999 were used, and the same detrended series were obtained by taking deviations of all variables from a mean or a linear trend. Meenagh et al. began by estimating a VAR(1) using observed data for five main variables: inflation (quarterly rate), interest rate (rate per quarter), output, investment and consumption. Data for the capital stock, equity returns, and capacity utilisation were constructed variables using the model's identities. Real wages and employment were omitted from the VAR. All variables were expressed as percentage deviations from trend. Apart from constants, the VAR had 25 coefficients[1].

[1] In this study to achieve stationary data suitable for this analysis we follow SW's practice in their EU paper of detrending each series with a linear trend and constant. This achieves stationarity for all our data, EU and US, according to standard ADF tests. Meenagh et al. checked whether the EU study was robust to the widely-used alternative Hodrick-Prescott filter and found little difference to the results, though the filter does take a lot more of the variation out of the data. Thus the linear detrending method used here has the advantage of suppressing as little of the data as possible. However, filtering data at all is a concern and we are working on methods that can use the raw nonstationary data.

We use separate linear time trends to detrend each variable on the grounds that in a short sample such as ours here a number of separate trends are present, affecting each variable differently. Thus there are output productivity, labour supply, and inflation target trends, as well as trend developments not explicitly modelled such as terms of trade and net

As a benchmark against which to test the SW New Keynesian model (SWNK), a "New Classical" version of the model was formulated (SWNC) in which the Calvo wage and price equations were replaced by the assumption of complete price and wage flexibility. Further, in the household labour supply equation, a simple one-quarter information lag was imposed on the formation of expected inflation.

The structural model has six behavioural errors: consumption, investment, productivity, interest rates (monetary policy), wage- and price-setting. These can be estimated as equation residuals based on the given parameters of the structural model, the observed data and the expected variables in it[2]—see Figure 2. We refer to these residuals as the 'actual errors'. There is one exogenous process, 'government spending', which is the residual in the goods market-clearing equation (or 'GDP identity') and therefore includes net trade as discussed earlier. The first error is that of the Euler equation and has a standard error of 0.5(%), roughly half as much again as assumed by SW (see Canzoneri et al. (2007) on the peculiarities of actual Euler equation errors). The second error is that for investment which has a standard error of 1.2%, i.e. around ten times that assumed by SW. The AR coefficients (ρs) of these structural residuals are very different from those assumed by SW: there is hardly any persistence in the the residuals for consumption and investment, unlike the high persistence assumed by SW. In contrast, the residuals of the inflation and Taylor Rule equations are persistent and not zero, as assumed. Table 1 shows the comparison between SW's assumed shocks and those shown in the graphs below. These differences turned out to be an important factor in the tests that Meenagh et al. carried out. The good test performance of the orginal model came from the errors used by SW which were based largely on their priors; once the actual errors were substituted, the original model was rejected; it only achieved acceptance once the model was altered towards a predominantly New Classical form with only a small degree of stickiness.

Error Variances	Cons	Inv	Inflation	Wage	Gov	Prod	Taylor Rule
Data var	0.26	1.52	0.0007	0.278	0.141	0.091	0.227
SW var	0.088	0.017	0.026	0.081	0.108	0.375	0.017
Ratio	2.9	89	0.03	3.4	1.3	0.24	13.4
ρ							
Data	−0.101	0.063	0.154	−0.038	0.751	0.940	0.565
SW	0.886	0.917	0	0	0.956	0.828	0

Table 1: Variances of innovations and AR Coefficients (rhos) of shocks (data-generated v. SW assumed shocks) for the EU model

For this 'mixed NC' model, and using the actual errors, Meenagh et al., through bootstrap methods, found that the properties of the errors were the key element in the success or failure of tests of both the SWNK and SWNC models. The more the error properties conformed to New Keynesian priors, with dominant demand shocks, the better the SWNK model performed and the worse was the SWNC model. In contrast, the more the errors conformed to New Classical priors, the better the SWNC performed and the worse was the SWNK model. When the error properties were derived from observed data, both models had difficulty fitting the data, though the SWNC model was the closest to doing so. What is the explanation for these results?

In the SWNK model, because capacity utilisation is flexible, demand shocks (consumption/investment/money) dominate output and—via the Phillips Curve—inflation, then—via the Taylor Rule—interest rates. Supply shocks (productivity, labour supply, wages/inflation mark-ups) play a minor role as 'cost-

foreign assets. Of course the same ad hoc approach is taken with other filters that 'take out trend' according to the series' own movements.

Another issue arises in the choice of a VAR(1). Keeping the order of the VAR as low as possible reduces the complexity of the dynamics to be matched by the model, much as reducing the number of variables in the VAR does. Clearly it is possible to raise the order and the number, and so increase the challenge for the DSGE model. However, it appears from our work that the challenge from what we have chosen is quite enough.

[2] For equations in which expectations do not enter these residuals are simply backed out. Where expectations enter the residuals must be estimated. For this we followed a procedure of robust estimation of the structural residuals along the lines suggested by McCallum (1976) and Wickens (1982) under which the expectations on the right hand side of each equation are generated by an instrumental variable regression that is implied by the model. The instruments chosen are the lagged values of the endogenous variables. Thus, in effect, the generated expectations used in deriving the residuals are the predictions of the data-estimated VAR.

It should also be noted that we excluded the first 20 error observations from the sample because of extreme values; we also smoothed two extreme error values in Q. Thus our sample for both bootstraps and data estimation was 98 quarters, i.e. 1975(1)-1999(2).

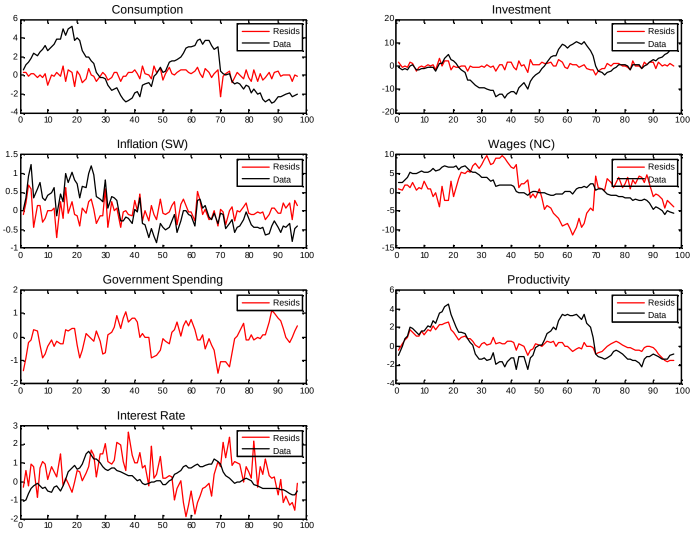

Figure 2: Single Equation Errors from SW EU Model

push' inflation shocks as they do not directly affect output. Persistent demand shocks raise 'Q' persistently and produce an 'investment boom' which, via demand effects, reinforces itself. Thus the model acts as a 'multiplier/accelerator' of demand shocks. Demand shocks therefore dominate the model, both for real and nominal variables. Moreover, in order to obtain good model perfomance for real and nominal data, these demand shocks need to be of sufficient size and persistence.

In the SWNC model an inelastic labour supply causes output variation to be dominated by supply shocks (productivity and labour supply) and investment/consumption to react to output in a standard RBC manner. These reactions, together with demand shocks, create market-clearing movements in real interest rates and—via the Taylor Rule—in inflation. Supply shocks are prime movers of all variables in the SWNC model, while demand shocks add to the variability of nominal variables. In order to mimic real variability and persistence suitably sized and persistent supply shocks are needed, but to mimic the limited variability in inflation and interest rates only a limited variance in demand shocks is required; and to mimic their persistence the supply shocks must be sufficiently autocorrelated.

The observed demand shocks have too little persistence to capture the variability of real variables in the SWNK model, but they generate too much variability in nominal variables in the SWNC model. The observed supply shocks matter little for the SWNK but are about right in size and persistence for the real variables in the SWNC. The implication is that the flexibility of prices and wages may lie somewhere between New Keynesian and the New Classical models. For example, adding a degree of price and wage stickiness to the SWNC model would bring down the variance of nominal variables, and boost that of real variables in the model.

A natural way to look at this is to assume that wage and price setters find themselves supplying labour and intermediate output partly in a competitive market with price/wage flexibility, and partly in a market with imperfect competition. We can assume that the size of each sector depends on the facts of competition and does not vary in our sample. The degree of imperfect competition could differ between labour and product markets.[3]

[3] Formally, we model this as follows. We assume that firms producing intermediate goods have a production function that combines in a fixed proportion labour in imperfect competition ('unionised') with labour from competitive markets-thus the labour used by intermediate firms becomes $n_t = n_{1t} + n_{2t} =$ $\left\{\left[\int_0^1 (n_{1it})^{\frac{1}{1+\lambda_{w,t}}} di\right]^{1+\lambda_{w,t}} + \left[\int_0^1 (n_{2it}) di\right]\right\}$ where n_{1it} is the unionised, n_{2it} the competitive labour provided by the ith household at t; we can think of n_t as representing the activities of an intermediary 'labour bundler'. Note that $n_{1t} = v_w n_t$, $n_{2t} = (1 - v_w) n_t$ so that $W_t = v_w W_{1t} + (1 - v_w) W_{2t}$. Each household's utility includes the two sorts of labour in the same way, that is $U_{it} = ... - \frac{n_{1it}^{1+\sigma_n} \epsilon_{1nt}}{1+\sigma_n} - \frac{n_{2it}^{1+\sigma_n} \epsilon_{2nt}}{1+\sigma_n} ...$ W_{1t} is now set according to the Calvo wage-setting equation, while W_{2t} is set equal to current expected marginal monetary disutility of work; in the latter case a 1-quarter information lag is assumed for current inflation but for convenience this is ignored in the usual way as unimportant in the

For the exercise they undertook Meenagh et al. (2009) initially assumed that it was the same in each market and given by a single free parameter, v. This implies that the price and wage equations will be a weighted average of the SWNK and SWNC equations, with the weights respectively of v and $(1-v)$. They also assumed that the monetary authority uses this parameter to weight its New Keynesian and New Classical Taylor Rules (different rules work best for SWNC than do so for SWNK). They chose the value of v for which the combined model was closest to matching the data variances while also passing the Wald test—an informal use of indirect inference. The optimal value turned out to be 0.06, implying quite a small NK sector of only 6% of the economy, but it was sufficient to bring the overall economy's properties close to the dynamic facts. They allowed the weight to be further varied around this to generate an optimum performance: in labour markets ($v_w = 0.08$), product markets ($v_p = 0.06$), and monetary policy ($v_m = 0.04$). We now consider how good a fit this gave.

The resulting model could replicate the variances in the data with all the data variances lying within the model's 95% bounds. The model therefore satisfied the necessary basic conditions to be taken seriously: it produced behaviour of the right size for both real and nominal variables and the structural errors were generated from the model using the observed data. All versions of the EU model were rejected on the full Wald statistic (100 in all cases), indicating that there are remaining dynamic specification faults in this model as far as the EU is concerned. However if one uses the Average t-test Wald statistic (assuming a diagonal variance-covariance matrix) for the VAR parameters then the SW weighted version is reasonably close on average to these parameters with a statistic of 90.8 while also as we have seen being the only model consistent with the data variances.

2.2 SW's US model

In the exercise carried out on SW's US model by Le et al. parallel results to their EU model are obtained. The key difference between the US and EU models lies in the much larger cost of capacity variation in the former. This has the effect of causing capital to provide a heavier drag on the ability of output to fluctuate in response to demand. Consequently demand shocks dominate output far less in the US model and productivity shocks are an important source of output variation. Otherwise the behaviour of the various forms of the SW model for the US, for both NK, NC versions follow the same general pattern as for the EU model.

There is a much longer sample period available for US data than for the EU—essentially the whole post-war period starting in the mid-1950s. SW achieved stationarity for this data by log differencing some series but leaving others, such as interest rates, in level form. This, of course, means that when combined in a data set where some relationships such as the IS curve relate output to interest rates, there is a problem as one cannot relate output when made stationary through first log differencing to interest rates made stationary in levels. Furthermore, it is well-known that differencing removes important information about the relative levels of variables from the data. When experimenting with the data in this stationarised form Le et al. found that their model produced very poor data matching under their indirect inference procedures—possibly for these reasons. They decided, therefore, to treat the data in the same way as SW treated their EU data, and simply de-trend each series by a linear time trend after which all time-series were found to be stationary.

As with the EU model, Le et al. found that with the errors assumed by SW, the US SWNK model was borderline accepted by the data. However, when the actual arrors were used in the bootstrapping process, both the SWNK and SWNC models failed to match the data variances in important ways: SWNK greatly underpredicted the data's interest rate variance while SWNC greatly overpredicted the data's inflation variance.

We can see below the actual errors for SWNK and their comparison with the errors SW assumed.

Calvo setting over the whole future horizon.

These wages are then passed to the labour bundler who offers a labour unit as above at this weighted average wage. Firms then buy these labour units off the manager for use in the firm.

Similarly, retail output is now made up in a fixed proportion of intermediate goods in an imperfectly competitive market and intermediate goods sold competitively. Retail output is therefore $y_t = y_{1t} + y_{2t} = \left\{ \left[\int_0^1 y_{j1t}^{\frac{1}{1+\lambda_{p,t}}} dj \right]^{1+\lambda_{p,t}} + \left[\int_0^1 y_{j2t} dj \right] \right\}$. The intermediary firm prices y_{1t} according to the Calvo mark-up equation on marginal costs, and y_{2t} at marginal costs.

Note that $y_{1t} = v_p y_t$, $y_{2t} = (1 - v_p) y_t$ so that $P_t = v_p P_{1t} + (1 - v_p) P_{2t}$. The retailer combines these goods as above in a bundle which it sells at this weighted average price.

Notice that apart from these equations the first-order conditions of households and firms will be unaffected by what markets they are operating in.

There are seven behavioural residuals: consumption, investment, productivity, monetary policy, wage- and price-setting, and one exogenous process, government spending, which enters the goods market clearing condition. These are shown in Figure 3.

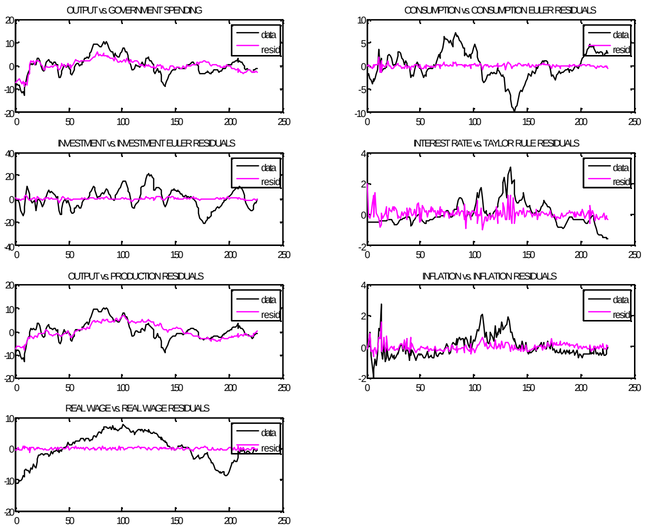

Figure 3: Single Equation Errors from SWNK US model

Five of these residuals were assumed to follow AR(1) and price- and wage- residuals follow ARMA(1,1) processes. The standard deviations of the estimated errors are in all cases larger than those assumed by SW, in the case of investment and the price mark-up nearly twice as large (see Table 2). Furthermore, the actual preference, investment and monetary shocks exhibit markedly less persistence than SW assumed. A vector bootstrap was used to preserve any dependence between the structural residuals.

Error Variances	Government Spending*	Pref	Inv	Mon	Prod	Price Mark-up	Wage Mark-up
SW stdev	0.53	0.23	0.45	0.24	0.45	0.14	0.24
Data stdev	0.673	0.371	0.704	0.344	0.553	0.239	0.311
SW AR(1)	0.97	0.22	0.71	0.15	0.95	0.89	0.96
SW MA(1)						−0.69	−0.84
Estimated AR(1)	0.944	−0.064	0.530	−0.062	0.971	0.925	0.915
Estimated MA(1)						−0.709	−0.848

*this includes a response to current productivity

Table 2: Standard deviations of innovations and coefficients of shocks (data-generated vs. SW's assumed) for the US model

Le et al. then considered a weighted model and found that the optimal values were $v_w = 0.1$ (the NK share for wages) and $v_p = 0.2$ (the NK share for prices). That is, only 10% of labour markets and only 20% of product markets are imperfectly competitive. Therefore, the model only requires a small amount of nominal rigidity in order to match the data. They found that the Taylor rule became:

$$R_t = 0.6R_{t-1} + (1-0.6)\{2.3\pi_t + 0.08y_t\} + 0.22\,(y_t - y_{t-1}) + \varepsilon_t.$$

This is somewhat more aggressive towards inflation than either the NK rule

$$R_t = 0.81R_{t-1} + (1-0.81)\left\{2.04\pi_t + 0.08(y_t - y_t^P\right\} + 0.22\left[\left(y_t - y_t^P\right) - \left(y_{t-1} - y_{t-1}^P\right)\right] + \varepsilon_t^r$$

or NC rules (the NC is the same as NK except that it sets 'potential output' to a constant). Notice that if one substitutes out for the interest rate from a simple money demand function with an exogenous money supply growth process, then one obtains a 'Taylor Rule' that has the form

$$\Delta R_t = \frac{1}{\beta}\{\pi_t + \gamma \Delta y_t) + v_t$$

where β is the semi-log interest rate (γ the income) elasticity of money demand, and v_t is a combination of the money supply growth process and the change in the money demand error. This is fairly close to the rules adopted in these models where the lagged term in interest rates is large and the term in the output gap is small compared with the term in the rate of change of output.

The difference between the combined NK and NC model and the individual models is its ability to reproduce the variances in the data. All the data variances lie within the model's 95% bounds. While as in the EU case the model is rejected absolutely (with a Full Wald of 100), the Average t-test Wald (inclduing the data variances) was a marginally acceptable 99.0.

	Y	R	π	C	I
Actual	18.035	0.654	0.446	10.413	72.098
Lower	9.698	0.292	0.439	7.305	61.415
Upper	61.85	0.765	0.891	72.017	301.772
Mean	26.851	0.470	0.631	26.767	146.509

Table 3: Variance of Data and Bootstraps for Weighted Model with estimated rhos

2.3 Putting the two models together into a world model

Given that these two models performed reasonably well empirically for their two economies, in spite of the evidence of overall mis-specification, we decided to combine them. To do so we allowed households in each economy to buy goods produced by the other in order to satisfy their consumption needs. Thus consumption is a CES function of home and foreign goods for each country's households. We assume that there is arbitrage between each country's goods prices at home and abroad so that in each country the relative price of home to foreign goods is the same, the real exchange rate.

The consumption, C_t in the SW model's utility function we make a composite per capita consumption, made up of agents' consumption of domestic goods, C_t^d and their consumption of imported goods, C_t^f. We treat the consumption bundle as the numeraire so that all prices are expressed relative to the general price level, P_t. The composite consumption utility index can be represented as an Armington (1969) aggregator of the form

$$C_t = \left[\omega \left(C_t^d\right)^{-\varrho} + (1-\omega)\varsigma_t \left(C_t^f\right)^{-\varrho}\right]^{\left(\frac{-1}{\varrho}\right)} \tag{1}$$

where ω is the weight of home goods in the consumption function, σ, the elasticity of substitution is equal to $\frac{1}{1+\varrho}$ and ς_t is a preference error.

The consumer maximises this composite utility index, given that an amount $\widetilde{C_t}$ has been chosen for total expenditure, with respect to its components, C_t^d and C_t^f subject to $\widetilde{C_t} = p_t^d C_t^d + Q_t C_t^f$, where p_t^d is the domestic price level relative to the general price level and Q_t [4] is the foreign price level in domestic currency relative to the general price level (the real exchange rate). The resulting expression for the home demand for foreign goods is

$$\frac{C_t^f}{C_t} = [(1-\omega)\varsigma_t]^{\sigma}(Q_t)^{-\sigma} \tag{2}$$

[4] We form the Lagrangean $L = \left[\omega \left(C_t^d\right)^{-\varrho} + (1-\omega)\left(C_t^f\right)^{-\varrho}\right]^{\left(\frac{-1}{\varrho}\right)} + \mu(\widetilde{C_t} - \frac{P_t^d}{P_t}C_t^d - \frac{P_t^f}{P_t}C_t^f)$. Thus $\frac{\partial L}{\partial \widetilde{C_t}} = \mu$; also at its maximum with the constraint binding $L = \widetilde{C_t}$ so that $\frac{\partial L}{\partial \widetilde{C_t}} = 1$. Thus $\mu = 1$ — the change in the utility index from a one unit rise in consumption is unity. Substituting this into the first order condition $0 = \frac{\partial L}{\partial C_t^f}$. yields equation (2) . $0 = \frac{\partial L}{\partial C_t^d}$. gives the equivalent equation: $\frac{C_t^d}{C_t} = \omega^{\sigma}(p_t^d)^{-\sigma}$ where $p_t^d = \frac{P_t^d}{P_t}$. Divide (1) through by C_t to obtain $1 = \left[\omega\left(\frac{C_t^d}{C_t}\right)^{-\varrho} + (1-\omega)\left(\frac{C_t^f}{C_t}\right)^{-\varrho}\right]^{\left(\frac{-1}{\varrho}\right)}$; substituting into this for $\frac{C_t^f}{C_t}$ and $\frac{C_t^d}{C_t}$ from the previous two equations gives us equation (4).

We also note that:

$$1 = \omega^{\sigma} \left(p_t^d\right)^{\sigma\varrho} + \left[(1-\omega)\varsigma_t\right]^{\sigma} Q_t^{\sigma\varrho} \tag{3}$$

Hence we can obtain the logarithmic approximation:

$$\log p_t^d = -\left(\frac{1-\omega}{\omega}\right)^{\sigma} \log\left(Q_t\right) - \frac{1}{\varrho}\left(\frac{1-\omega}{\omega}\right)^{\sigma} \log\varsigma_t + constant \tag{4}$$

We also allowed households to buy foreign bonds; this gives us uncovered interest parity as an extra first order condition. We choose to express this in real terms, so that the real interest rate differential is equal to the expected change in the real exchange rate. Financial markets are otherwise not integrated and are incomplete[5].

We note here that the first order condition can be written $E_t \frac{u'_{c,t+1}(1+R_{ft})(S_{t+1}/S_t)}{P_{t+1}} = E_t \frac{u'_{c,t+1}(1+R_t)}{P_{t+1}}$; where u'_c is the marginal utility of consumption, R is the nominal interest rate, P is the price level and S is the nominal exchange rate (home per foreign currency, so that a rise is a depreciation). Taking natural logs, assuming that error distributions are lognormal, and invoking the result that when $\ln x_t$ is distributed with a random error ϵ_t, $lnE_t x_{t+1} = E_t \ln x_{t+1} + 0.5var\epsilon$, we obtain $\ln(1+R_{ft}) + E_t \ln S_{t+1} - \ln S_t = \ln(1+R_t) + c_0$ where c_0 is the term with the implied error variances and covariances with marginal utility. Being constant under our assumption that the model has fixed parameters and errors are homoscedastic, we then ignore this term as not affecting the model's responses to shocks. Finally, we use the approximation that for a small fraction $\ln(1+x) \simeq x$ to obtain uncovered interest rate parity:

$R_{ft} + E_t \ln S_{t+1} - \ln S_t = R_t$

To turn this into real uncovered interest parity we add and subtract expected inflation,

$(R_{ft} - E_t \ln P_{ft+1} + \ln P_{ft}) + (E_t \ln S_{t+1} + E_t \ln P_{ft+1} - E_t \ln P_{t+1}) - (\ln S_t + \ln P_{ft} - \ln P_t) = (R_t - E_t \ln P_{t+1} + \ln P_t)$ whence

$r_{ft} + E_t \ln Q_{t+1} - \ln Q_t = r_t$

To test the combined model using actual errors (which are similar to the ones we showed above for the two economies separately) against the data we use a VAR in eight variables designed to represent the key open economy behaviour: output, inflation and interest rates for each economy, the real exchange rate (note that a rise in this is a real appreciation of the european currency versus the dollar) and the bilateral US-EU trade balance, defined as the difference between log US-to-EU exports and log EU-to-US exports. We use the period for which data is available for both economies, namely from 1975–1999. We focus in what follows on the combination of the two weighted models. (As with the individual economies so with the combined, the alternative models, SWNK and SWNC, failed to match the data variances.)

It turns out that the combined weighted model performs reasonably well also as one would expect from the models' individual success. The model's predicted variance bounds embrace the data variances. While as with both the individual economy models the Full Wald statistic rejects at 100, the model's Average t-test Wald statistic (including data variances) is 82.7, indicating substantial closeness to the data—remembering that this is average closeness of each feature taken individually, much as is done is the current literature comparing models with data moments etc. The components in this are shown in the Table below; at its foot are the data variances and their 95% bounds.

[5] However, as noted by Chari et al (2002), assuming complete asset markets imposes the condition that the real exchange rate equals the ratio of the two continents' marginal utilities of consumption at all times. This implies that the the expected log change in the real exchange rate equals the expected log change in this ratio, ie the the real interest differential- the real UIP condition again. Thus the conditions are in practice similar: under complete markets the real exchange rate exactly moves with relative consumption whereas under incomplete it is only expected to do so, so that random shocks can drive them apart.

	Actual	Lower	Upper	State
$A^{Y^{US}}_{Y^{US}}$	0.848156	0.734829	1.154908	IN
$A^{\pi^{US}}_{Y^{US}}$	0.007809	-0.02349	0.189169	IN
$A^{R^{US}}_{Y^{US}}$	0.022051	-0.03326	0.127268	IN
$A^{NE}_{Y^{US}}$	0.012121	-0.82331	1.022503	IN
$A^{Y^{EU}}_{Y^{US}}$	-0.01314	-0.19684	0.179005	IN
$A^{\pi^{EU}}_{Y^{US}}$	0.006146	-0.12742	0.102156	IN
$A^{R^{EU}}_{Y^{US}}$	-0.01151	-0.14166	0.069589	IN
$A^{RXR}_{Y^{US}}$	0.038521	-0.3344	0.587617	IN
$A^{Y^{US}}_{\pi^{US}}$	0.36066	-0.77054	0.109068	OUT
$A^{\pi^{US}}_{\pi^{US}}$	0.52233	0.414022	0.830549	IN
$A^{R^{US}}_{Y^{\pi^{US}}}$	0.291259	0.002895	0.312914	IN
$A^{NE}_{\pi^{US}}$	-1.53716	-0.61581	3.010255	OUT
$A^{Y^{EU}}_{\pi^{US}}$	-0.07586	-0.35787	0.421549	IN
$A^{\pi^{EU}}_{\pi^{US}}$	0.376145	-0.23157	0.178844	OUT
$A^{R^{EU}}_{\pi^{US}}$	0.063915	-0.19015	0.166679	IN
$A^{RXR}_{\pi^{US}}$	0.82309	-0.25858	1.4899	IN
$A^{Y^{US}}_{R^{US}}$	-0.36486	-1.40835	0.284688	IN
$A^{\pi^{US}}_{R^{US}}$	0.159178	-0.82883	0.053359	OUT
$A^{R^{US}}_{R^{US}}$	0.799615	0.015509	0.702261	OUT
$A^{NE}_{R^{US}}$	0.859896	-5.91636	1.220997	IN
$A^{Y^{EU}}_{R^{US}}$	0.075664	-0.87739	0.617746	IN
$A^{\pi^{EU}}_{R^{US}}$	-0.00657	-0.56982	0.337031	IN
$A^{R^{EU}}_{R^{US}}$	0.01617	-0.45617	0.371871	IN
$A^{RXR}_{R^{US}}$	$-4.20E-05$	-3.33276	0.11577	IN
$A^{Y^{US}}_{NE}$	-0.014	-0.19792	0.24851	IN
$A^{\pi^{US}}_{NE}$	0.002316	-0.07708	0.155404	IN
$A^{R^{US}}_{NE}$	-0.00233	-0.08023	0.093029	IN
A^{NE}_{NE}	0.91884	-0.04945	1.899053	IN
$A^{Y^{EU}}_{NE}$	-0.0045	-0.22315	0.17572	IN
$A^{\pi^{EU}}_{NE}$	0.001812	-0.174	0.09997	IN
$A^{R^{EU}}_{NE}$	-0.00059	-0.16908	0.066184	IN
A^{RXR}_{NE}	0.043954	-0.38027	0.567782	IN
$A^{Y^{US}}_{Y^{EU}}$	0.136033	-0.3435	0.312359	IN
$A^{\pi^{US}}_{Y^{EU}}$	0.016644	-0.27198	0.058838	IN
$A^{R^{US}}_{Y^{EU}}$	-0.03228	-0.18539	0.045186	IN
$A^{NE}_{Y^{EU}}$	0.747852	-1.57235	1.10588	IN
$A^{Y^{EU}}_{Y^{EU}}$	0.981749	0.522227	1.067224	IN
$A^{\pi^{EU}}_{Y^{EU}}$	0.046921	-0.23584	0.116394	IN
$A^{R^{EU}}_{Y^{EU}}$	0.04625	-0.20801	0.105403	IN
$A^{RXR}_{Y^{EU}}$	-0.55428	-0.75171	0.520743	IN

Table 4: VAR coefficients and variances for the SW EU-US weighted model

	Actual	Lower	Upper	State
$A^{Y^{US}}_{\pi^{EU}}$	-0.19749	-1.46488	0.66343	IN
$A^{\pi^{US}}_{\pi^{EU}}$	0.234097	-1.36936	-0.20396	OUT
$A^{R^{US}}_{\pi^{EU}}$	0.00023	-1.0349	-0.19345	OUT
$A^{NE}_{\pi^{EU}}$	0.262419	-8.32418	1.148038	IN
$A^{Y^{EU}}_{\pi^{EU}}$	0.19797	-0.16902	1.761419	IN
$A^{\pi^{EU}}_{\pi^{EU}}$	0.363756	0.10701	1.286825	IN
$A^{R^{EU}}_{\pi^{EU}}$	-0.05942	-0.23094	0.745215	IN
$A^{RXR}_{\pi^{EU}}$	-0.8632	-4.77715	-0.22719	IN
$A^{Y^{US}}_{R^{EU}}$	-0.23979	-0.65119	1.62167	IN
$A^{\pi^{US}}_{R^{EU}}$	-0.19154	0.300122	1.516317	OUT
$A^{R^{US}}_{R^{EU}}$	0.158096	0.226208	1.149287	OUT
$A^{NE}_{R^{EU}}$	-2.64786	-0.56199	9.468748	OUT
$A^{Y^{EU}}_{R^{EU}}$	-0.50621	-2.01799	0.122446	IN
$A^{\pi^{EU}}_{R^{EU}}$	0.034226	-0.7663	0.407617	IN
$A^{R^{EU}}_{R^{EU}}$	0.860364	-0.23734	0.784417	OUT
$A^{RXR}_{R^{EU}}$	0.326105	0.710531	5.647189	OUT
$A^{Y^{US}}_{RXR}$	0.032076	-0.47781	0.446239	IN
$A^{\pi^{US}}_{RXR}$	-0.01252	-0.23182	0.235984	IN
$A^{R^{US}}_{RXR}$	-0.00368	-0.11584	0.241804	IN
A^{NE}_{RXR}	-0.29155	-1.84215	2.255991	IN
$A^{Y^{EU}}_{RXR}$	-0.01729	-0.36227	0.451164	IN
$A^{\pi^{EU}}_{RXR}$	0.017608	-0.17549	0.363608	IN
$A^{R^{EU}}_{RXR}$	-0.00042	-0.1415	0.338608	IN
A^{RXR}_{RXR}	0.920969	-0.1275	1.867483	IN
$\sigma^2_{Y^{US}}$	5.945734	3.493806	36.88094	IN
$\sigma^2_{\pi^{US}}$	0.385286	0.301302	0.788716	IN
$\sigma^2_{R^{US}}$	0.806059	0.230181	0.824717	IN
σ^2_{NE}	351.4456	48.61269	530.9087	IN
$\sigma^2_{Y^{EU}}$	3.608468	1.49129	11.46202	IN
$\sigma^2_{\pi^{EU}}$	0.245893	0.221595	0.855852	IN
$\sigma^2_{R^{EU}}$	0.363569	0.197476	0.751455	IN
σ^2_{RXR}	43.54279	9.932144	95.91408	IN

Table 5: VAR coefficients and variances for the SW EU-US weighted model (cont.)

What is striking is how little spill-over there is between the US and EU economies. A variance decomposition shows that while both economies' shocks affect the real exchange rate and the trade balance, only the home shocks affect home macro variables, much in the way they did when each works as a closed economy. We can also see that both models give a preponderant effect to productivity and labour supply shocks, with no demand shocks contributing much of the variation in either inflation or output in the US. Only in the EU does the monetary shock contribute an important share of output and inflation variation.

Shock↓\Variable⟶	Y^{US}	π^{US}	R^{US}	NE	Y^{EU}	π^{EU}	R^{EU}	RXR
$Prod^{EU}$	0.003	0.015	0.024	10.445	23.895	7.397	19.670	10.078
$Cons^{EU}$	0.000	0.000	0.000	0.006	3.057	8.975	7.906	0.234
Res^{EU}	0.000	0.000	0.001	0.162	2.533	0.446	0.916	0.151
Inv^{EU}	0.000	0.003	0.005	1.479	5.512	2.500	6.706	1.489
Mon^{EU}	0.003	0.011	0.018	6.894	16.732	61.357	26.844	6.774
$Price^{EU}$	0.000	0.000	0.000	0.050	0.037	4.962	2.481	0.055
$LabSup^{EU}$	0.009	0.029	0.047	20.891	48.233	14.361	35.470	19.844
$Wage^{EU}$	0.000	0.000	0.000	0.000	0.000	0.002	0.004	0.000
Res^{US}	1.031	1.535	2.809	1.328	0.000	0.000	0.000	1.547
$Cons^{US}$	0.612	2.229	2.695	0.145	0.000	0.000	0.000	0.658
Inv^{US}	2.441	1.878	3.994	1.871	0.000	0.000	0.000	1.921
Mon^{US}	0.310	5.877	0.416	0.594	0.000	0.000	0.000	0.612
$Prod^{US}$	31.489	28.209	29.697	17.531	0.000	0.000	0.001	19.357
$Price^{US}$	0.792	1.336	0.726	0.334	0.000	0.000	0.000	0.311
$Wage^{US}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$LabSup^{US}$	63.308	58.876	59.566	38.271	0.001	0.001	0.002	36.967
$Total$	100	100	100	100	100	100	100	100

Table 6: Variance Decomposition for US-EU weighted Model

3 Evaluating dimensions of the model's performance

As we have just seen from the variance decomposition this model says that the domestic performance of the EU and US economies is entirely determined by domestic shocks in each case and that these shocks are predominantly supply shocks. Only in the EU is there an important role for a monetary demand shock. All these shocks do however impact on the real exchange rate (and trade balance) of course, as the 'buffer' between the two economies.

This can allow us to evaluate the model's ability to replicate certain aspects of the data. First of all, we can limit the number of variables we examine and estimate a VAR for that group of variables alone, and compute the Wald statistic for its VAR coefficients. Then we can also ask whether the model's IRFs for a particular shock are rejected by the data. Thus we evaluate the IRFs for say the productivity shock on output and inflation by grouping the average IRFs for the US productivity shock on US output and inflation, and those for the EU productivity shock on EU output and inflation; these make 4 average IRFs. We then find the joint distribution of these average IRFs to compute the Wald statistic for the data-observed average IRFs.

We call these 'Directed Wald' statistics. The idea is that they give us a guide to where the model is mis-specified—or, looked at another way, to what uses in analysis we can safely put the model.

What we find—Table 7—is that the model passes these Wald tests at the 95% level for outputs and the real exchange rate, but fails them when interest rates or inflation are added. This pinpoints the DSGE model's failure in respect of nominal relationships (echoing the fact that of the VAR coefficients lying outside their 95% bounds, many involve inflation and interest rate effects). Thus the model, which indeed has properties essentially like a real business cycle model, can replicate the behaviour of real variables well but fails on nominal variables. This suggests rather clearly that monetary relationships in the model require attention—perhaps not surprisingly given the turbulence in monetary policy during this era.

If we turn to shocks—Table 8—we find that US and EU productivity and labour supply shocks all pass their individual Directed Wald tests quite comfortably when interest rate effects are excluded; but when interest rates are included only productivity shocks do. Thus we can see here again that while the shocks do well for most variables they fail on interest rates. The US monetary shock only contributes non-trivially to the variance of US inflation, whose IRF to it comes comfortably within its 95% bounds (see Annex). The EU monetary shock explains a big part of the variation in EU inflation, whose IRF to it lies within its 95% bounds. What all this shows is that taken individually the shock effects are well-modelled on the whole, apart mostly from their effects on interest rates; but as we know from the tests on variables, taken together they cannot account for nominal behaviour. Apart from failing on interest rates, they fail when combined even when interest rates are excluded, because of the general failure to pick up inflation effects.

Finally, we note that the model fits the data variances as a group at the 95% level (Table 9), provided

NE is excluded. Although the model can match the NE variance singly, it cannot match it jointly, no doubt because it is hugely variable. We do not attach much importance to this variable as it is the difference between the natural logs of US exports to the EU and EU exports to the US; this trade is actually quite small, specialised, and volatile. We can omit it from the VAR and the results are essentially unchanged.

Summarising these tests of particular capacities of the model, we can say that it fits the data variability, it is capable of replicating output and real exchange rate behaviour, but that it fails on nominal variables. It captures the effects of important shocks individually in both economies on output and inflation, but not generally on interest rates. Thus this model can be rigorously tested econometrically for certain properties and it passes some of these limited tests and gives us some clues about the location of mis-specifications.

Variable combinations	Direct Wald
Y^{EU}, Y^{US}	80.7
π^{EU}, π^{US}	99.9
R^{EU}, R^{US}	96.1
Y^{EU}, Y^{US}, RXR	94.2
$Y^{EU}, Y^{US}, RXR, R^{EU}, R^{US}$	100
$Y^{EU}, Y^{US}, RXR, R^{EU}, R^{US}, \pi^{EU}, \pi^{US}$	100
$RXR\ (AR(2))$	81.3

Table 7: Directed Wald Statistics by variable combinations

Shocks	Variables	Direct Wald
Mon^{EU}	$Y^{EU}\quad R^{EU}\quad \pi^{EU}\quad RXR$	84.1
$Prod^{EU}$	$Y^{EU}\quad \pi^{EU}\quad RXR\ (R^{EU} incl)$	47.1 (86.5)
$LabSup^{EU}$	$Y^{EU}\quad \pi^{EU}\quad RXR\ (R^{EU} incl)$	66.1 (96.1)
$Prod^{US}$	$Y^{US}\quad \pi^{US}\quad RXR\ (R^{US} incl)$	77.5 (87.7)
$LabSup^{US}$	$Y^{US}\quad \pi^{US}\quad RXR\ (R^{US} incl)$	94.9 (99.7)
$Prod^{BOTH}$	*without (with) interest rates*	64.1 (97.3)
$LabSup^{BOTH}$	*without (with) interest rates*	93.0 (100)
$(Prod, LabSup)^{BOTH}$	*without (with) interest rates*	100 (100)

Table 8: Directed Wald Statistics by shocks

Variances of data	Direct Wald
$\sigma^2_{Y^{EU}}\quad \sigma^2_{Y^{US}}\quad \sigma^2_{\pi^{EU}}\quad \sigma^2_{\pi^{US}}$ $\sigma^2_{R^{EU}}\quad \sigma^2_{R^{US}}\quad \sigma^2_{RXR}$	94.3

Table 9: Directed Wald Statistic for Variances of the Data

4 What does this model tell us about the world economy?

4.1 The nature of the world economy

This model suggests the world is bi-polar: the US and the EU are essentially independent blocs with little mutual spill-over. This can be illustrated by deterministic productivity shocks for each bloc shown below.

This bi-polarity implies that the exchange rate acts as a buffer between the two blocs enabling each to achieve its own market-clearing real interest rates under its own policy preferences. Hence its wild swings as uncorrelated shocks come from each direction. This can be seen in the IRFs below for the exchange rate reaction. Notice that it is pro-cyclical in the sense that positive demand and supply shocks in one economy cause its real interest rates to rise and its real exchange rate to appreciate. This is what we see for example in the data-based IRFs (coming from the data VAR as indentified by the model) for a US productivity shock. Thus the data too can be seen through the lens of this model as supporting this interpretation.

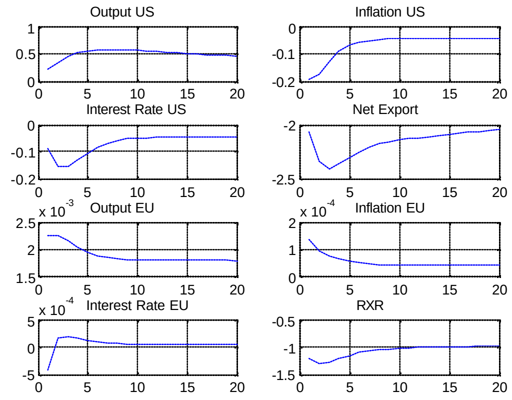

Figure 4: US productivity shock

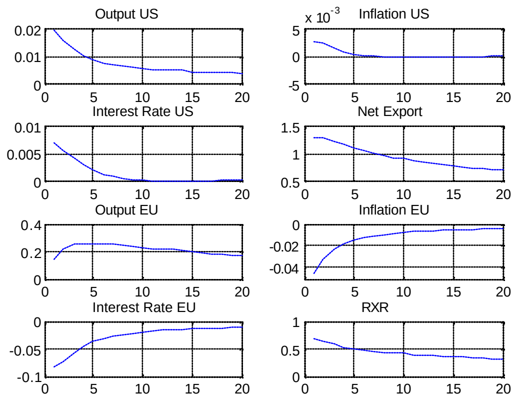

Figure 5: EU productivity shock

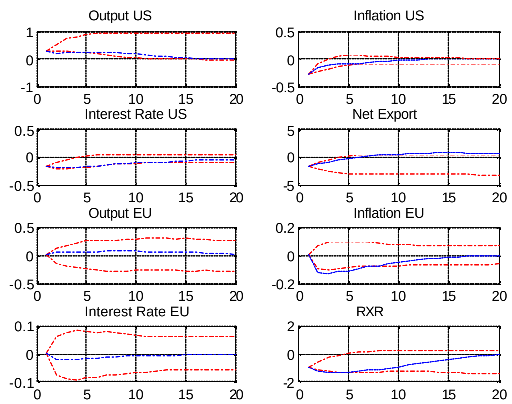

Figure 6: VAR IRFs for US productivity shock with model bounds

The trade balance similarly reacts to these shocks, essentially mirroring relative outputs and the real exchange rate swings. Again we see this in the data-based IRF for US productivity shown below.

4.2 The banking crisis shock

Since 1992 the world has experienced shocks to the US (9/11 and the internet stock crash of 2000/1 in the US for example) and to the EU (low productivity growth from 1992 in the big three EU nations accompanied by rapid productivity and demand growth in Spain, Portugal and Ireland). But since these shocks to the two blocs have been uncorrelated the world economy has grown fairly uninterruptedly. Common shocks like the 1998 Asian financial crisis have had modest effects in the US and the EU. However the recent banking crash was a common shock, even if originating in the US (politically driven) sub-prime mortgage bonanza; it was propagated across the whole world via capital market reactions to bank collapses in the form of rising risk premia on lending to banks and hence on lending by banks to non-banks. What then does this model have to say about the effects of such a common shock, interpreted here as a rise in risk-premia to consumers and investors demanding capital? The model suggests that the depressing effect on output and inflation is rapid and large in both continents, induces a powerful response from monetary policy such that interest rate falls more than offset the rise in risk-premia, and that recovery in output begins after only 2 quarters in the US and 3 in the EU, thereafter proceeding quickly in the US and rather more slowly in the EU. Of course fiscal policy is absent here but in a rough way the simulation does appear to have fitted experience in the banking crisis since the collapse of Lehman in September 2008. For a more detailed discussion of the application of this model to the current crisis see Minford (2009).

How well does the model fit such data as we have on this shock? No such shock with such persistence occurred in the sample period. Nevertheless we can evaluate the IRFs from an innovation like this shock, albeit with past average persistence. Thus we show the VAR shock that would be implied by this shock to the structural model. We then plot the resulting VAR response in the data (as implied by the data-based VAR) and the 95% bounds implied by the model. These lie mostly within their 95% bounds, indicating that the model is consistent with past data on these shocks.

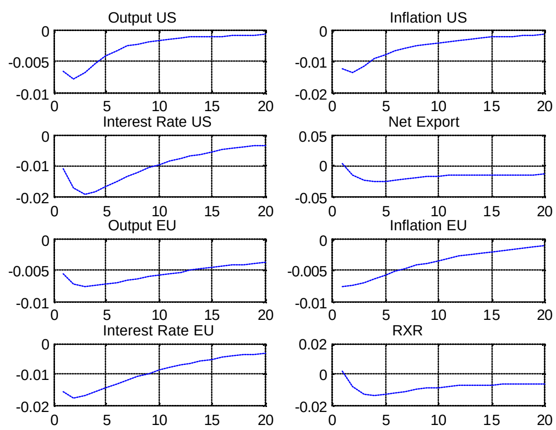

Figure 7: Deterministic credit crunch shock to both economies (AR(1) coefficient of 0.95)

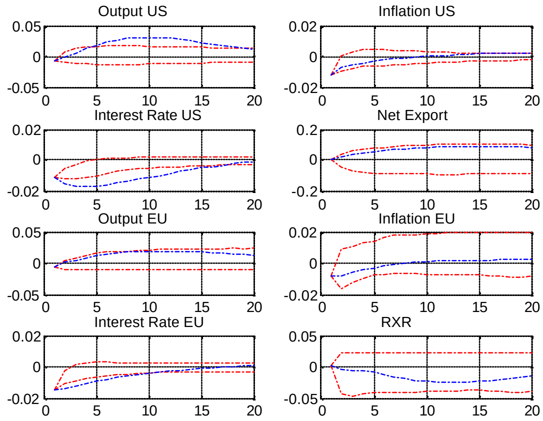

Figure 8: Credit crunch shock on both economies (AR(1) coefficient of 0.95)

5 Conclusion

We have investigated a two-country model of the world economy consisting of the US and the EU. As noted, by leaving out the Rest of the World (ROW), which is a quarter of the world economy, the model in effect treats the ROW's GDP movements, and hence net imports from the EU and US, as error terms in the US and EU market-clearing conditions. But through our bootstrapping procedure, any correlation in these errors with other shocks to the EU and US is picked up. Thus we are exogenising the rest of the world's trade balances with the EU and US as a 'shock' like other shocks to the US and EU such as productivity and consumption. We could think of these shocks as crudely consisting of an 'ROW demand' shock to exports to the ROW minus an 'outsourcing' shock to imports from the ROW. Of these two shocks the ROW demand shock is to some extent endogenous to the EU and US model, as plainly the ROW will respond to EU/US shocks. However, as noted this endogeneity may be at least partly picked up by the correlation between EU/US shocks and this ROW demand shock. We conjecture that the model may therefore not be sensitive to leaving out the ROW explicitly—but clearly it is something we should investigate in future work.

Subject to this caveat we have found that we can explain the dynamic behaviour of key US and EU macro variables, of their bilateral exchange rate and of their bilateral trade balance better by this model than available DSGE alternatives. The model exhibits rather limited price/wage rigidity, and largely therefore behaves like a real business cycle model driven predominantly by supply shocks. Because the EU and US trade little with each other—or indeed with anyone much, relative to their GDP—they each behave much like a closed economy. Their bilateral exchange rate and trade balance therefore fluctuate wildly in response to their relative shocks. However when there is a large common shock as in the current banking crisis the two economies' spill-overs somewhat amplify the direct effects of the shock on each other.

DSGE models such as this have come in for criticism in the current crisis because in them money demand and supply are not explicitly modelled but are embedded implicitly through an interest rate response function. However one can think of the current crisis as creating direct shocks to the risk-premia entering the Euler and other first-order conditions via the cost of borrowing. The impulse responses to such shocks are in line with the VAR evidence we have.

We must enter the caveat that even though the model performs relatively well and absolutely well in a number of dimensions, the full model is in an absolute sense strongly rejected by the data on a full Wald statistic. This echoes the Sargent remark at the top of this paper. We can however be rather precise about where the model fails: when we look only at the real variables, outputs and the real exchange rate, the model passes the Wald test. Thus this DSGE model is mis-specified in a way that leads us to focus on monetary relationships, since it fails to replicate the behaviour of nominal variables. This should perhaps not surprise us given thet monetary policy regimes have experienced considerable turbulence since 1975, the start of our sample.

One final caveat is in order which is generic to work of this sort. We have obtained stationarity by extracting linear time trends for each series. It is well known to be hard to distinguish trend-stationarity from non-stationarity with drift. We have found our results to be generally insensitive to filtering the data by the widely-used Hodrick-Prescott filter. Nevertheless this too may not be an appropriate way of dealing with non-stationary data and we are investigating other ways of working with non-stationary data. This too is a matter for further work.

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Appendix A Listing of weighted model

A.1 Home Country (USA)

Consumption Euler equation

$$\begin{aligned} c_t^{US} &= \frac{\frac{0.71}{1.0043}}{1+\frac{0.71}{1.0043}} c_{t-1}^{US} + \frac{1}{1+\frac{0.71}{1.0043}} E_t c_{t+1}^{US} + \frac{(1.3952-1)\,0.83192}{\left(1+\frac{0.71}{1.0043}\right)1.3952}\left(l_t^{US} - E_t l_{t+1}^{US}\right) \\ &\quad - \left(\frac{1-\frac{0.71}{1.0043}}{\left(1+\frac{0.71}{1.0043}\right)1.3952}\right)\left(r_t^{US} - E_t \pi_{t+1}^{US}\right) + eb_t^{US} \end{aligned} \tag{1}$$

Investment Euler equation

$$inn_t^{US} = \frac{1}{1+0.99(1.0043)} inn_{t-1}^{US} + \frac{0.99(1.0043)}{1+0.99(1.0043)} E_t inn_{t+1}^{US} + \frac{1}{(1+0.99(1.0043))\,(1.0043^2)\,5.74} qq_t^{US} + einn_t^{US} \tag{2}$$

Tobin Q equation

$$qq_t^{US} = \frac{1-0.025}{1-0.025+0.032649} E_t qq_{t+1}^{US} + \frac{0.032649}{1-0.025+0.032649} E_t rk_{t+1}^{US} - \left(r_t - E_t \pi_{t+1}^{US}\right) + \frac{1}{\frac{1-\frac{0.71}{1.0043}}{\left(1+\frac{0.71}{1.0043}\right)1.3952}} eb_t^{US} \tag{3}$$

Capital accumulation equation

$$k_t^{US} = \left(1-\frac{0.025}{1.0043}\right) k_{t-1}^{US} + \frac{0.025}{1.0043} inn_t^{US} + \left(1-\frac{0.025}{1.0043}\right)\left(1+1.0043\,(0.99)\right)\left(1.0043^2\right)(5.74)\left(enn_t^{US}\right) \tag{4}$$

Marginal Product of Labour

$$0.19 rk_t^{US} + (1-0.19)\, w_t^{US} = ea_t^{US} \tag{5}$$

Price Setting equation

$$\begin{aligned} p_t^{US} &= \phi_p^{US} \left\{ \left[\begin{array}{c} \frac{0.99x1.0043}{1+0.99x1.0043\times 0.24} E_t \pi_{t+1}^{US} + \frac{0.24}{1+0.99\times 1.0043x0.24} \pi_{t-1}^{US} - \left(\frac{1}{1+0.99x1.0043\times 0.24}\right) \\ \left(\frac{(1-0.99(1.0043)(0.66))(1-0.66)}{0.66((1.6-1)(10)+1)}\right)\left(0.19 rk_t^{US} + (1-0.19)\, w_t^{US} - ea_t^{US}\right) \end{array} \right] + ep_t^{US} \right\} \\ &\quad + \left(1-\phi_p^{US}\right)\left\{\frac{1}{0.19}\left[ea_t^{US} - (1-0.19)\, w_t^{US}\right]\right\} \end{aligned} \tag{6}$$

Labour supply

$$\begin{aligned} w_t^{US} &= \phi_w^{US} \left\{ \left[\begin{array}{c} \frac{0.99x1.0043}{1+0.99x1.0043} E_t w_{t+1} + \frac{1}{1+0.99x1.0043} w_{t-1} + \frac{0.99x1.0043}{1+0.99x1.0043} E_t \pi_{t+1} - \frac{1+0.99x1.0043\times 0.58}{1+0.99x1.0043} \pi_t \\ + \frac{0.58}{1+0.99x1.0043} \pi_{t-1} - \frac{1}{1+0.99x1.0043}\left(\frac{(1-0.99x1.0043x0.7)(1-0.7)}{(1+(10)(1.5-1))0.7}\right) \\ \left(w_t^{US} - 1.83 l_t^{US} - \left(\frac{1}{1-\frac{0.71}{1.0043}}\right)\left(c_t^{US} - \frac{0.71}{1.0043} c_{t-1}^{US}\right)\right) \end{array} \right] + ew_t^{US} \right\} \\ &\quad + \left(1-\phi_w^{US}\right)\left\{1.83 l_t^{US} + \left(\frac{1}{1-\frac{0.71}{1.0043}}\right)\left(c_t^{US} - \frac{0.71}{1.0043} c_{t-1}^{US}\right) - \left(\pi_t^{US} - E_{t-1}\pi_t^{US}\right) + ew_t^{USNC}\right\} \end{aligned} \tag{7}$$

Labour Demand

$$l_t^{US} = -w_t^{US} + \left(1+\frac{1-0.54}{0.54}\right) rk_t^{US} + k_{t-1}^{US} \tag{8}$$

Market clearing condition

$$y_t^{US} = 0.64 c_t^{US} + 0.17 inn_t^{US} + 0.19\frac{1-0.54}{0.54} rk_t^{US} + 0.016 im_t^{EU} - 0.02 im_t^{US} + eg_t^{US} \tag{9}$$

Production function

$$rk_t^{US} = \frac{1}{1.6\,(0.19)\,\frac{1-0.54}{0.54}}\left(y_t^{US} - 1.6\,(0.19)\,k_{t-1}^{US} - 1.6\,(1-0.19)\,l_t^{US} - 1.6 ea_t^{US}\right) \tag{10}$$

Taylor Rule

$$r_t^{US} = 0.81 r_{t-1}^{US} + (1-0.81)\left(2.03\pi_t^{US} + 0.08 y_t^{US}\right) + 0.22\left(y_t^{US} - y_{t-1}^{US}\right) + er_t^{US} \tag{11}$$

Imports

$$im_t^{US} = c_t^{US} - 0.8 s_t \tag{12}$$

Exchange Rate

$$s_t = r_t^{US} - E_t\pi_{t+1}^{US} - \left(r_t^{EU} - E_t\pi_{t+1}^{EU}\right) + E_t s_{t+1} \tag{13}$$

Consumption Decomposition

$$c_t^{US} = 0.7 c_t^{USd} + 0.3 im_t^{US} \tag{14}$$

A.2 Overseas (EU)

Market-clearing equation

$$c_t^{EU} = \frac{1}{(1-0.025\,(2.2)-0.18)}\left(y_t^{EU} - 0.025\,(2.2)\, i_t^{EU} - eg_t^{EU} - 0.001222 im_t^{US} + 0.001413 im_t^{EU}\right) \tag{15}$$

Investment Euler equation

$$i_t^{EU} = \frac{1}{1+0.99} i_{t-1}^{EU} + \frac{0.99}{1+0.99} E_t i_{t+1}^{EU} + \frac{\frac{1}{7.0}}{1+0.99} Q_t^{EU} + ei_t^{EU} \tag{16}$$

Tobin Q equation

$$Q_t^{EU} = -\left(R_t^{EU} - E_t\pi_{t+1}^{EU}\right) + \frac{1-0.025}{1-0.025+0.04} E_t Q_{t+1}^{EU} + \frac{0.04}{1-0.025+0.04} E_t rk_{t+1}^{EU} + eq_t^{EU} \tag{17}$$

Capital accumulation equation

$$k_t^{EU} = (1-0.025)\, k_{t-1}^{EU} + 0.025 i_{t-1}^{EU} \tag{18}$$

Taylor Rule equation

$$\begin{aligned} \pi_t^{EU} \quad = \quad & \phi_m^{EU}\left\{\pi_{t-1}^{EU} + \frac{1}{0.221}\left(R_t^{EU} - 0.931 R_{t-1}^{EU} - (1-0.931)\left(1.661\pi_{t-1}^{EU} + 0.143 y_t^{EU} - 0.173\left(y_t^{EU} - y_{t-1}^{EU}\right)\right)\right)\right\} \\ & + \left(1-\phi_m^{EU}\right)\left\{\frac{1}{1.5}\left(R_t^{EU} - E_t\pi_{t+1}^{EU}\right)\right\} + e\pi_t^{EU} \end{aligned} \tag{19}$$

Labour supply

$$\begin{aligned} w_t^{EU} \quad = \quad & \phi_w^{EU}\left\{\begin{array}{c} \frac{0.99}{1+0.99} E_t w_{t+1}^{EU} + \frac{1}{1+0.99} w_{t-1}^{EU} + \frac{0.99}{1+0.99} E_t \pi_{t+1}^{EU} - \frac{1+0.99(0.663)}{1+0.99}\pi_t^{EU} + \frac{0.663}{1+0.99}\pi_{t-1}^{EU} \\ -\left(\frac{1}{1+0.99}\right)\left(\frac{(1-0.99(0.758))(1-0.758)}{\left(1+\left(\frac{(1+0.596)1.188}{0.596}\right)\right)0.758}\right)\left(w_t^{EU} - 1.188 N_t^{EU} - \frac{1.608}{1-0.552}\left(c_t^{EU} - 0.552 c_{t-1}^{EU}\right)\right) + ewSW_t^{EU} \end{array}\right. \\ & + \left(1-\phi_w^{EU}\right)\left\{1.188 N_t^{EU} + \frac{1.608}{1-0.552}\left(c_t^{EU} - 0.552 c_{t-1}^{EU}\right) - \left(\pi_t^{EU} - E_{t-1}\pi_t^{EU}\right) + ewNC_t^{EU}\right\} \end{aligned}$$

Labour Demand

$$N_t^{EU} = -w_t^{EU} + (1+0.175) rk_t^{EU} + k_{t-1}^{EU} \tag{21}$$

Production function

$$y_t^{EU} = 1.487\,(0.3)\, 0.175 rk_t^{EU} + 1.487\,(0.3)\, k_{t-1}^{EU} + 1.487\,(1-0.3)\, N_t^{EU} + 1.487 ea_t^{EU} \tag{22}$$

Price-setting equation

$$
\begin{aligned}
rk_t^{EU} \quad = \quad & \phi_p^{EU} \left\{ \frac{1}{\left(\frac{1}{1+0.99(0.425)}\right)\left(\frac{(1-0.99(0.909))(1-0.909)}{0.909}\right)0.3} \left(\begin{array}{r} \pi_t^{EU} - \frac{0.99}{1+0.99(0.425)} E_t \pi_{t+1}^{EU} \\ -\left(\frac{1}{1+0.99(0.425)}\right)\left(\frac{(1-0.99(0.909))(1-0.909)}{0.909}\right)\pi_{t-1}^{EU} + eps_t^{EU} \end{array} \right) \right. \\
& \left. -\frac{1-0.3}{0.3} w_t^{EU} + \frac{1}{0.3} ea_t^{EU} \right\} \\
& + \left(1-\phi_p^{EU}\right)\left\{\frac{1}{0.3} ea_t^{EU} - \frac{1-0.3}{0.3} w_t^{EU}\right\} \qquad (
\end{aligned}
$$

Consumption Euler equation

$$R_t^{EU} = E_t \pi_{t+1}^{EU} + \frac{1}{\frac{1-0.552}{(1+0.552)1.608}} \left(\frac{0.552}{1+0.552} c_{t-1}^{EU} + \frac{1}{1+0.552} E_t c_{t+1}^{EU} - c_t^{EU} + eb_t^{EU} \right) \tag{24}$$

Imports

$$im_t^{EU} = c_t^{EU} + 0.8 s_t \tag{25}$$

Consumption Decomposition

$$c_t^{EU} = 0.7 c_t^{EUd} + 0.3 im_t^{EU} \tag{26}$$

Net Exports

$$nx_t = im_t^{EU} - im_t^{US} \tag{27}$$

Annex A Weighted Model Results

A.1 Randomly selected Bootstraps (Blue) versus Actual Data (Red)

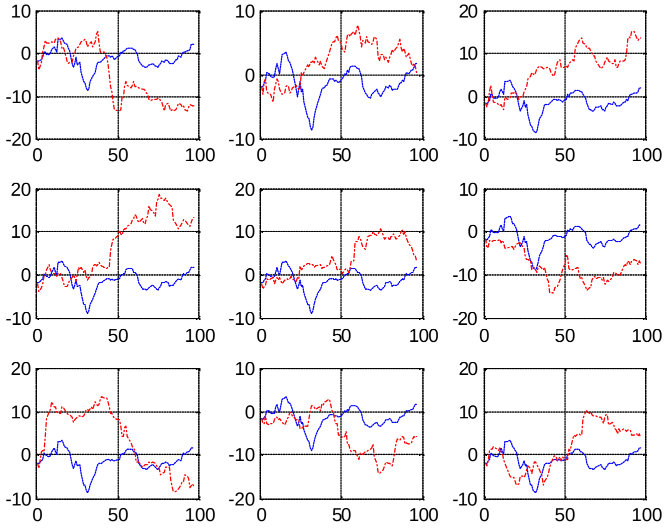

Figure 9: US Output

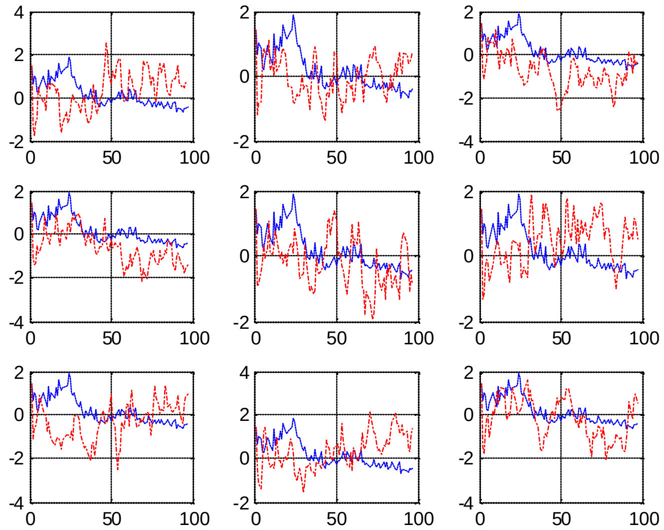

Figure 10: US Inflation

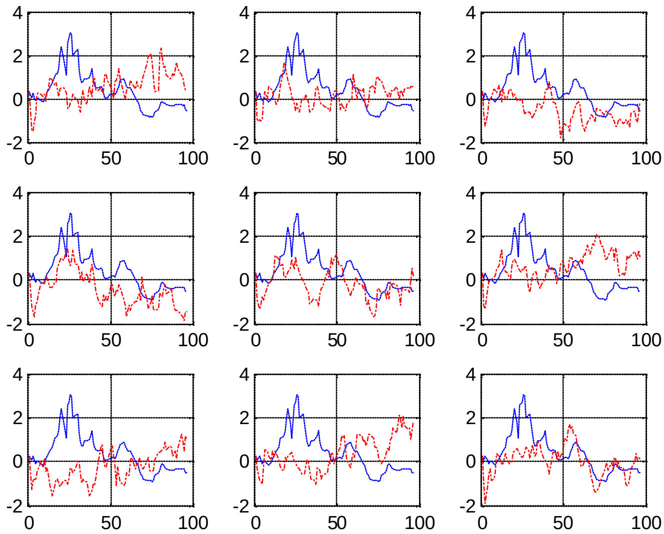

Figure 11: US Interest Rate

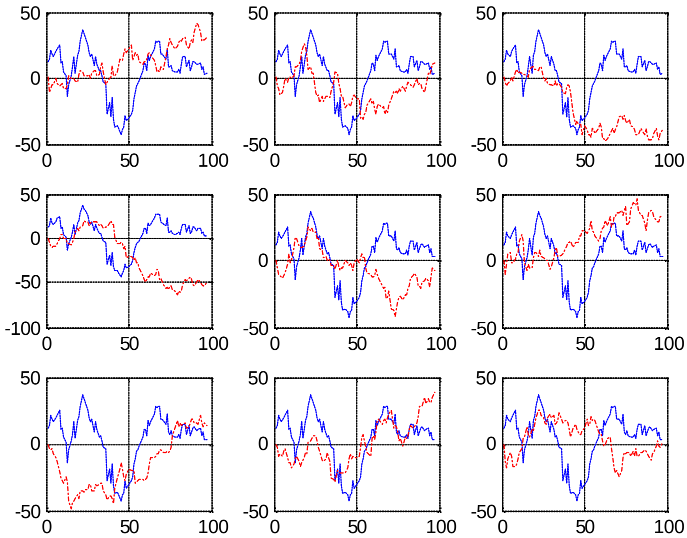

Figure 12: Net Exports

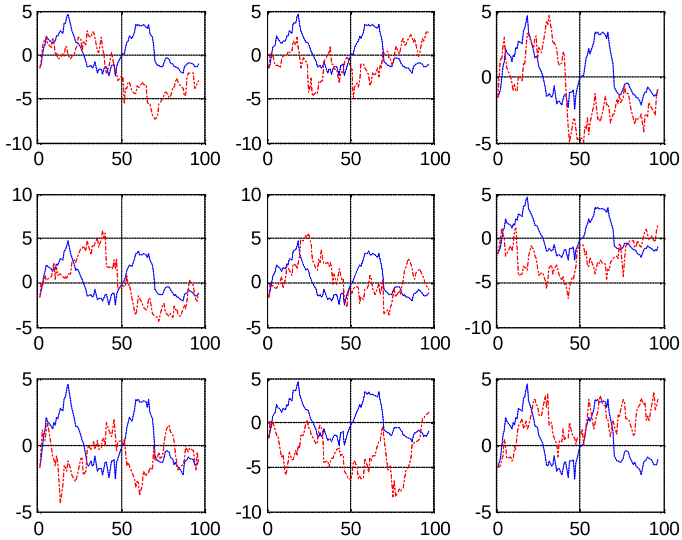

Figure 13: EU Output

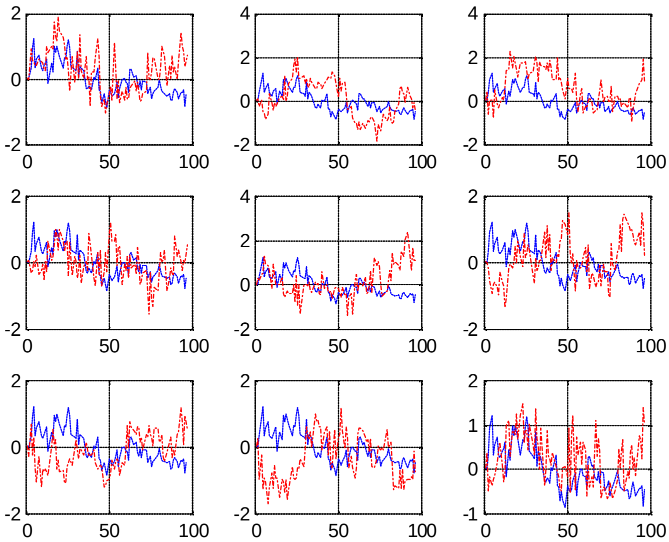

Figure 14: EU Inflation

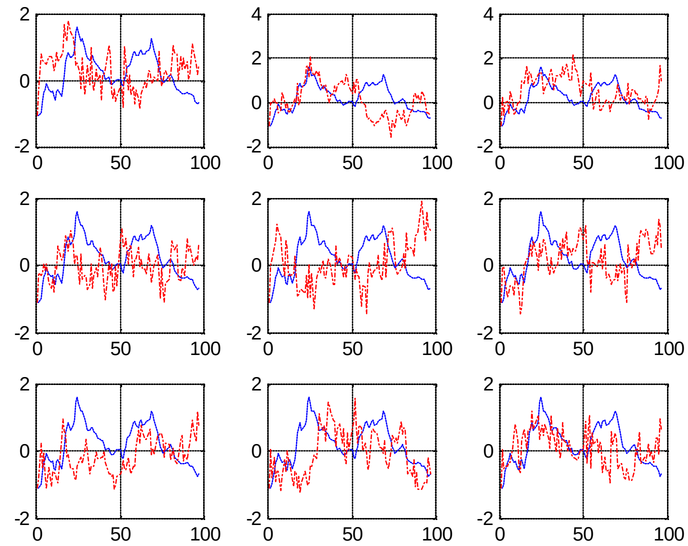

Figure 15: EU Interest Rate

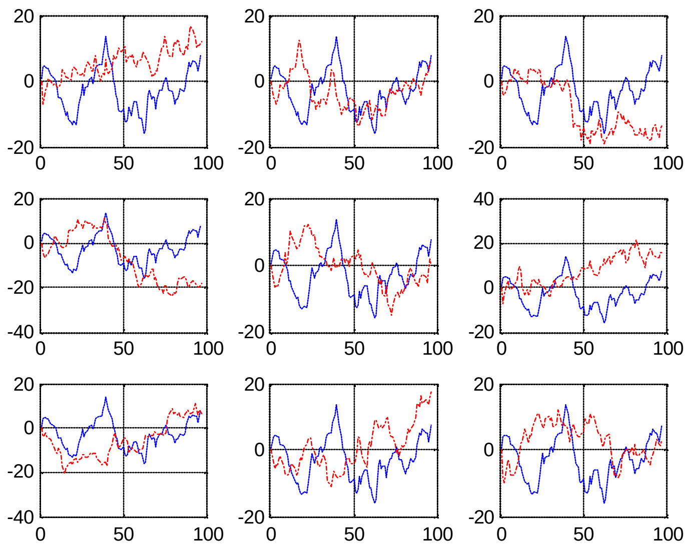

Figure 16: RXR

A.2 Analysis of IRFs of VAR (blue) versus 95% VAR bounds implied by the model

The red dotted lines denotes the upper and the lower bounds of the distribution of the IRF and the blue solid one is the IRF of the actual data.

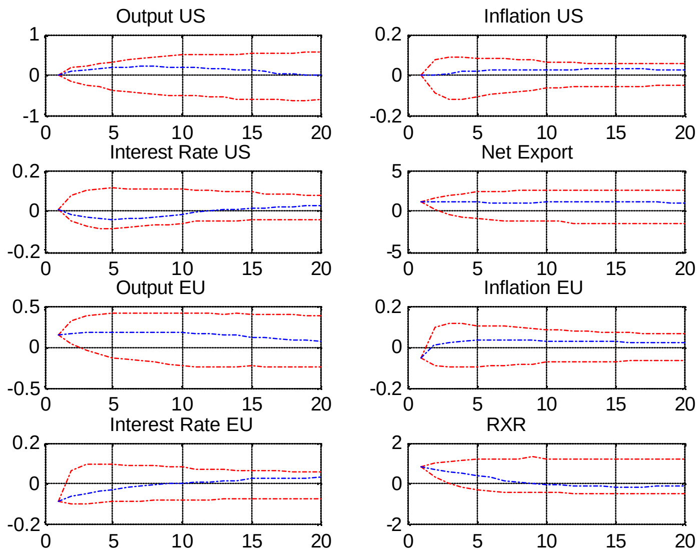

Figure 17: EU Productivity Shock

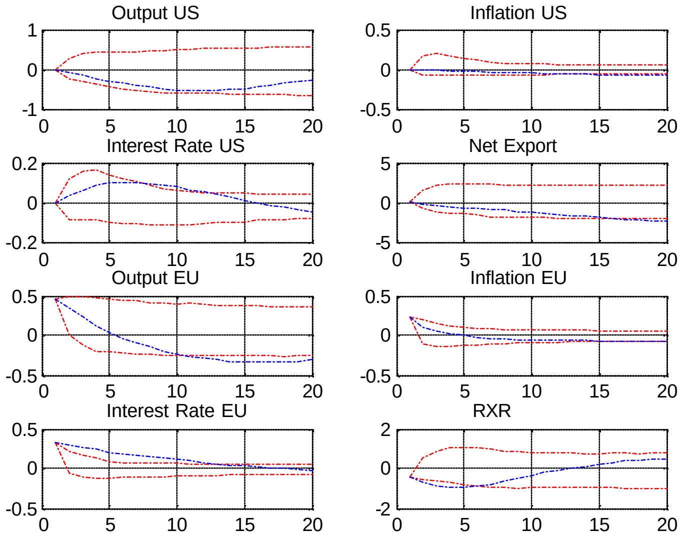

Figure 18: EU Euler Consumption Shock

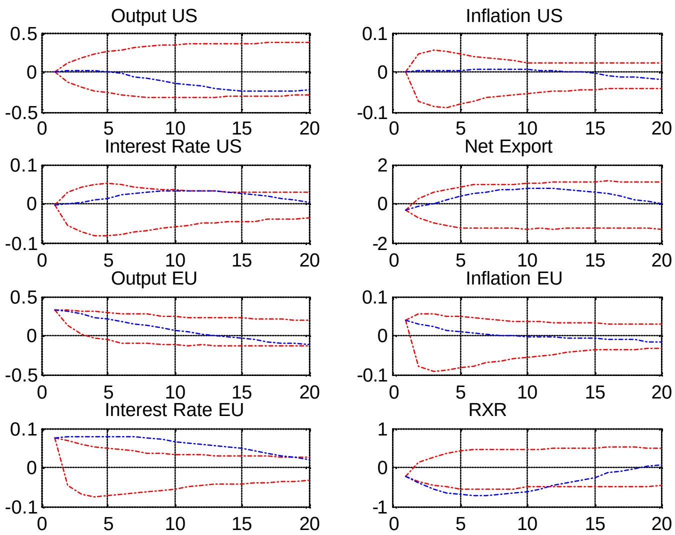

Figure 19: EU Residual (World) Shock

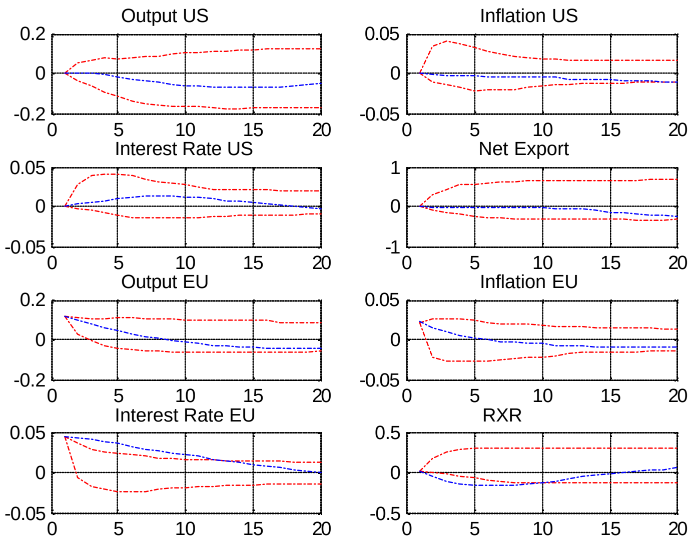

Figure 20: EU Investment Euler Shock

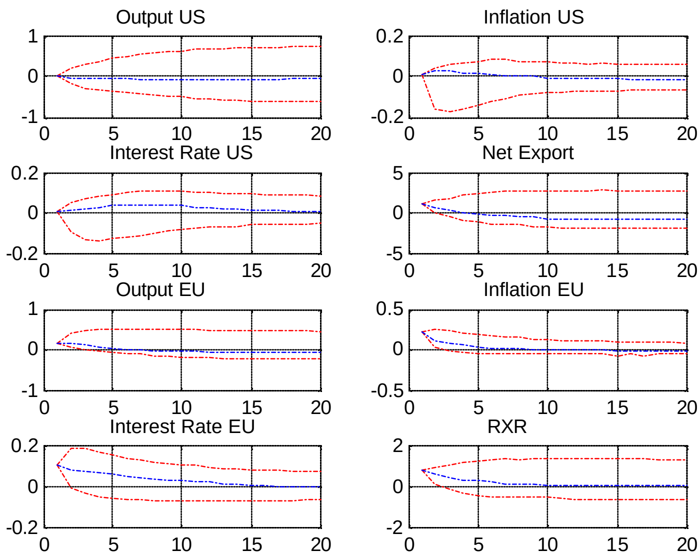

Figure 21: EU Monetary Shock

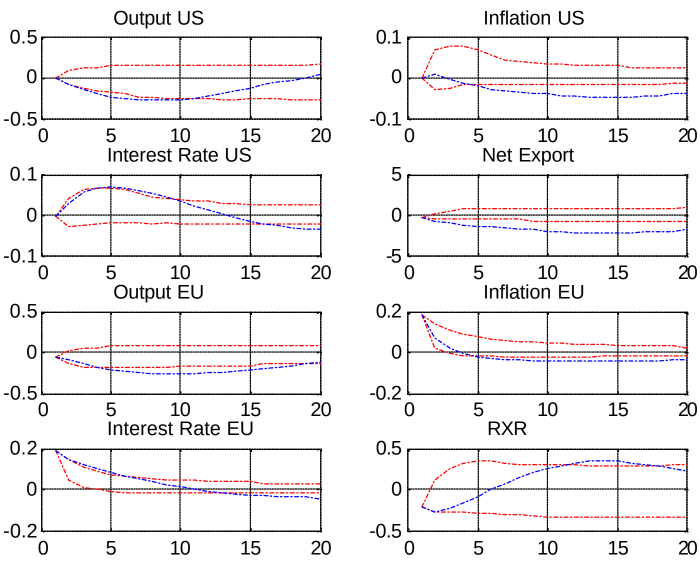

Figure 22: EU Price Mark-up Shock (NK)

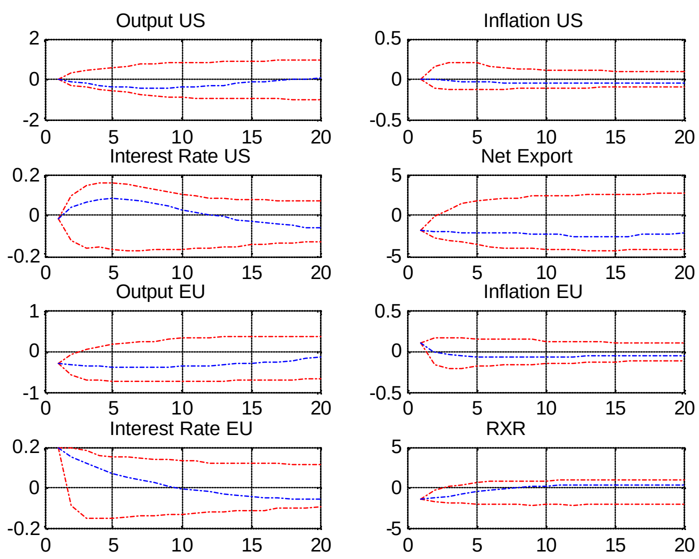

Figure 23: EU Labour Supply Shock (NC)

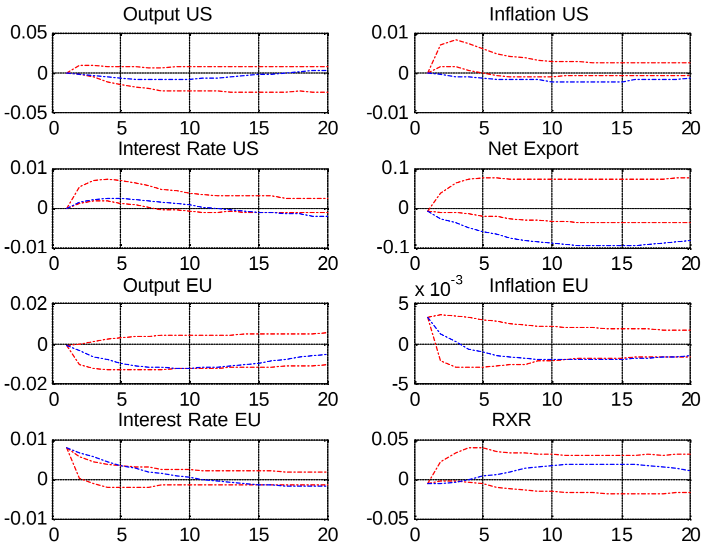

Figure 24: EU Wage Mark-up Shock (NK)

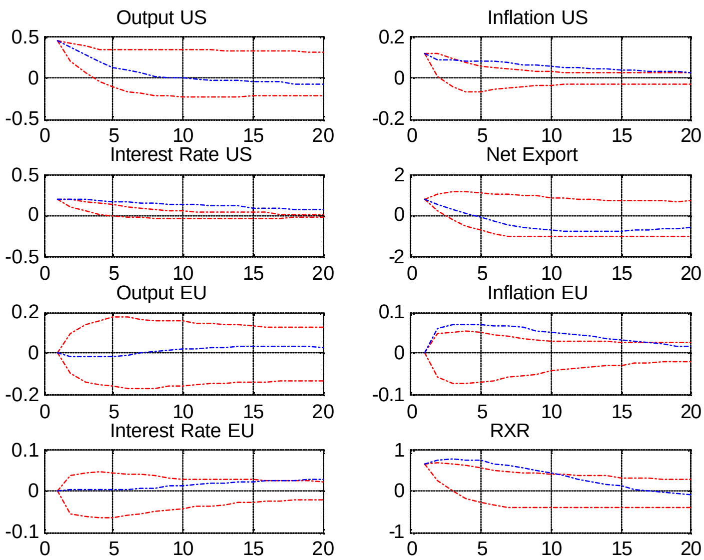

Figure 25: US Residual (World) Shock

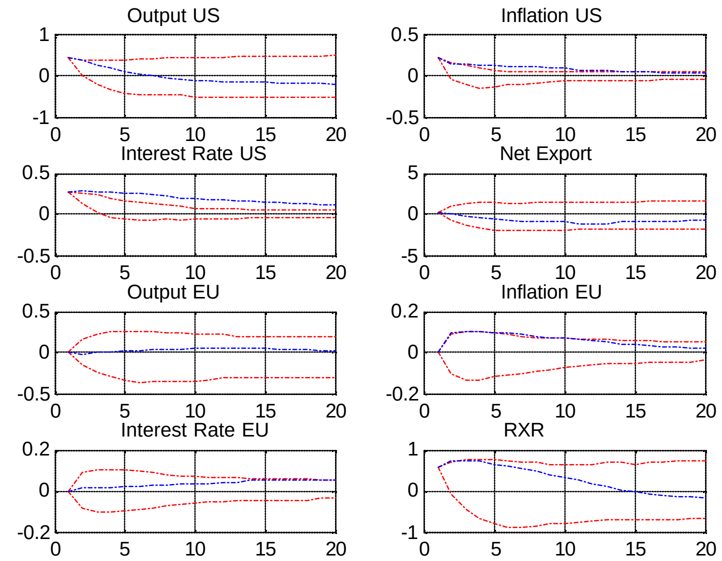

Figure 26: US Consumption Euler Shock

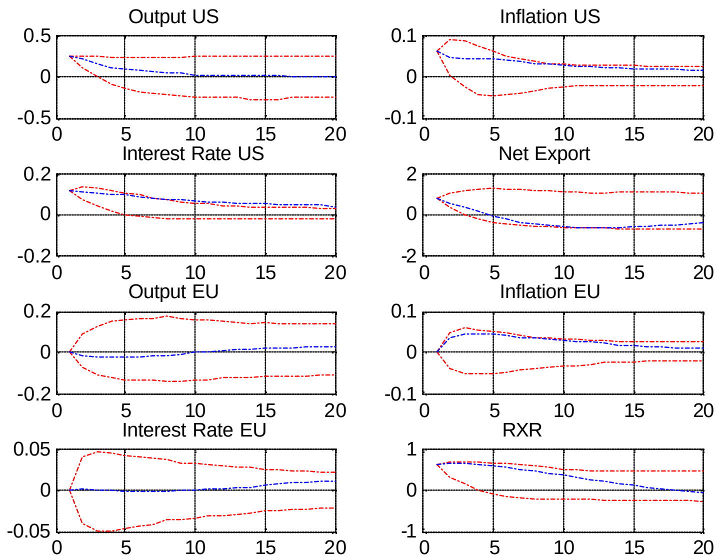

Figure 27: US Investment Euler Shock

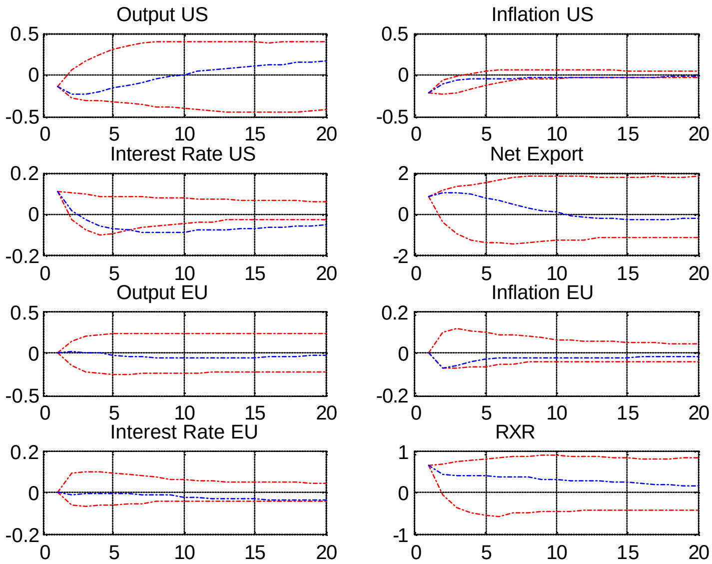

Figure 28: US Monetary Shock

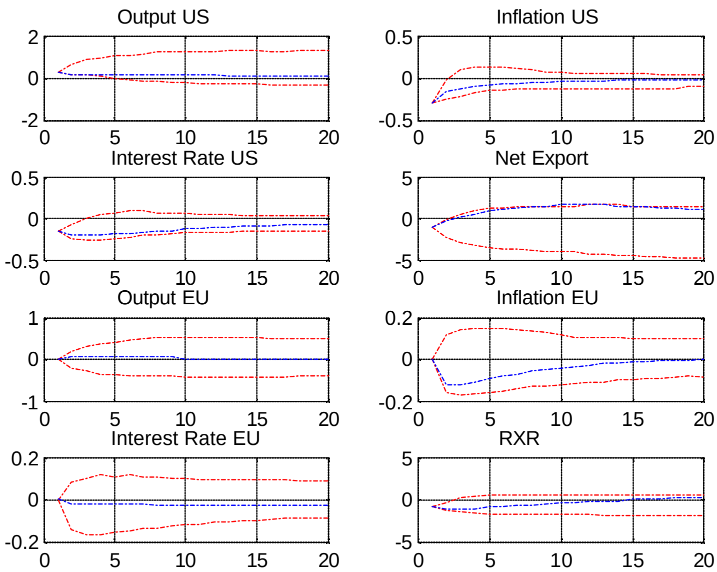

Figure 29: US Productivity Shock

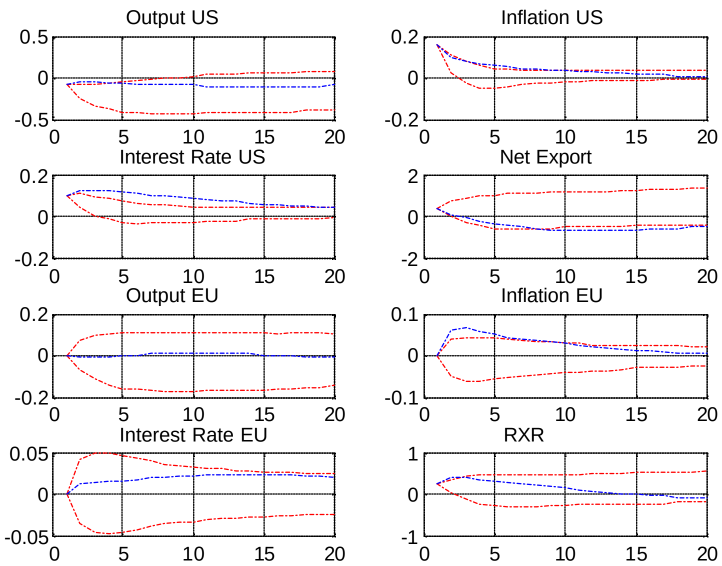

Figure 30: US Price Mark-up Shock (NK)

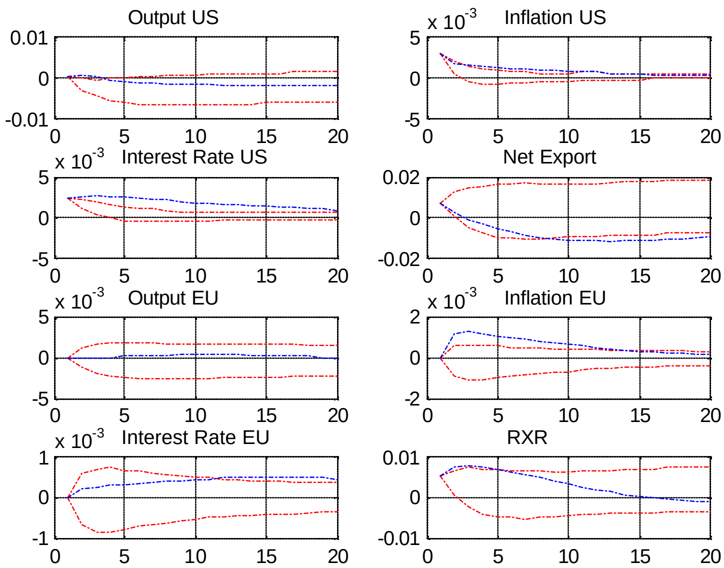

Figure 31: US Labour Supply Shock (NC)

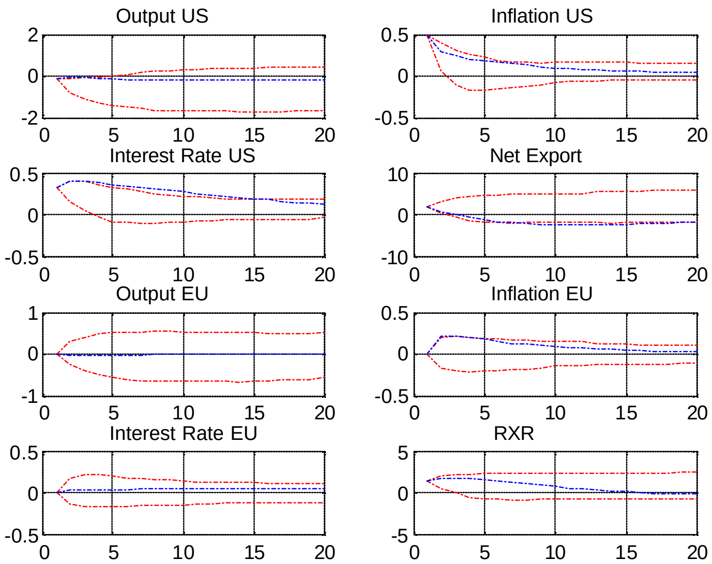

Figure 32: US Wage Mark-up Shock (NK)

A.3 VAR Parameters, Model Bootstrap Bounds and Wald statistic

	Actual Estimate	Lower Bound	Upper Bound	State
A_{YUS}^{YUS}	0.848156	0.734829	1.154908	IN
$A_{YUS}^{\pi US}$	0.007809	-0.02349	0.189169	IN
A_{YUS}^{RUS}	0.022051	-0.03326	0.127268	IN
A_{YUS}^{NXUS}	0.012121	-0.82331	1.022503	IN
A_{YUS}^{YEU}	-0.01314	-0.19684	0.179005	IN
$A_{YUS}^{\pi EU}$	0.006146	-0.12742	0.102156	IN
A_{YUS}^{REU}	-0.01151	-0.14166	0.069589	IN
A_{YUS}^{RXR}	0.038521	-0.3344	0.587617	IN
$A_{\pi US}^{YUS}$	0.36066	-0.77054	0.109068	OUT
$A_{\pi US}^{\pi US}$	0.52233	0.414022	0.830549	IN
$A_{\pi US}^{RUS}$	0.291259	0.002895	0.312914	IN
$A_{\pi US}^{NXUS}$	-1.53716	-0.61581	3.010255	OUT
$A_{\pi US}^{YEU}$	-0.07586	-0.35787	0.421549	IN
$A_{\pi US}^{\pi EU}$	0.376145	-0.23157	0.178844	OUT
$A_{\pi US}^{REU}$	0.063915	-0.19015	0.166679	IN
$A_{\pi US}^{RXR}$	0.82309	-0.25858	1.4899	IN
A_{RUS}^{YUS}	-0.36486	-1.40835	0.284688	IN
$A_{RUS}^{\pi US}$	0.159178	-0.82883	0.053359	OUT
A_{RUS}^{RUS}	0.799615	0.015509	0.702261	OUT
A_{RUS}^{NXUS}	0.859896	-5.91636	1.220997	IN
A_{RUS}^{YEU}	0.075664	-0.87739	0.617746	IN
$A_{RUS}^{\pi EU}$	-0.00657	-0.56982	0.337031	IN
A_{RUS}^{REU}	0.01617	-0.45617	0.371871	IN
A_{RUS}^{RXR}	$-4.20E-05$	-3.33276	0.11577	IN
A_{NXUS}^{YUS}	-0.014	-0.19792	0.24851	IN
$A_{NXUS}^{\pi US}$	0.002316	-0.07708	0.155404	IN
A_{NXUS}^{RUS}	-0.00233	-0.08023	0.093029	IN
A_{NXUS}^{NXUS}	0.91884	-0.04945	1.899053	IN
A_{NXUS}^{YEU}	-0.0045	-0.22315	0.17572	IN
$A_{NXUS}^{\pi EU}$	0.001812	-0.174	0.09997	IN
A_{NXUS}^{REU}	-0.00059	-0.16908	0.066184	IN
A_{NXUS}^{RXR}	0.043954	-0.38027	0.567782	IN

Table 10: VAR Parameters & Model Bootstrap Bounds (Weighted Model)

	Actual Estimate	Lower Bound	Upper Bound	State
A_{YEU}^{YUS}	0.136033	−0.3435	0.312359	IN
$A_{YEU}^{\pi US}$	0.016644	−0.27198	0.058838	IN
A_{YEU}^{RUS}	−0.03228	−0.18539	0.045186	IN
A_{YEU}^{NXUS}	0.747852	−1.57235	1.10588	IN
A_{YEU}^{YEU}	0.981749	0.522227	1.067224	IN
$A_{YEU}^{\pi EU}$	0.046921	−0.23584	0.116394	IN
A_{YEU}^{REU}	0.04625	−0.20801	0.105403	IN
A_{YEU}^{RXR}	−0.55428	−0.75171	0.520743	IN
$A_{\pi EU}^{YUS}$	−0.19749	−1.46488	0.66343	IN
$A_{\pi EU}^{\pi US}$	0.234097	−1.36936	−0.20396	OUT
$A_{\pi EU}^{RUS}$	0.00023	−1.0349	−0.19345	OUT
$A_{\pi EU}^{NXUS}$	0.262419	−8.32418	1.148038	IN
$A_{\pi EU}^{YEU}$	0.19797	−0.16902	1.761419	IN
$A_{\pi EU}^{\pi EU}$	0.363756	0.10701	1.286825	IN
$A_{\pi EU}^{REU}$	−0.05942	−0.23094	0.745215	IN
$A_{\pi EU}^{RXR}$	−0.8632	−4.77715	−0.22719	IN
A_{REU}^{YUS}	−0.23979	−0.65119	1.62167	IN
$A_{REU}^{\pi US}$	−0.19154	0.300122	1.516317	OUT
A_{REU}^{RUS}	0.158096	0.226208	1.149287	OUT
A_{REU}^{NXUS}	−2.64786	−0.56199	9.468748	OUT
A_{REU}^{YEU}	−0.50621	−2.01799	0.122446	IN
$A_{REU}^{\pi EU}$	0.034226	−0.7663	0.407617	IN
A_{REU}^{REU}	0.860364	−0.23734	0.784417	OUT
A_{REU}^{RXR}	0.326105	0.710531	5.647189	OUT
A_{RXR}^{YUS}	0.032076	−0.47781	0.446239	IN
$A_{RXR}^{\pi US}$	−0.01252	−0.23182	0.235984	IN
A_{RXR}^{RUS}	−0.00368	−0.11584	0.241804	IN
A_{RXR}^{NXUS}	−0.29155	−1.84215	2.255991	IN
A_{RXR}^{YEU}	−0.01729	−0.36227	0.451164	IN
$A_{RXR}^{\pi EU}$	0.017608	−0.17549	0.363608	IN
A_{RXR}^{REU}	−0.00042	−0.1415	0.338608	IN
A_{RXR}^{RXR}	0.920969	−0.1275	1.867483	IN
	Wald Statistic	100	Joint t-stat	99.0

*t-stat from bootstrap mean

Table 11: VAR Parameters & Model Bootstrap Bounds (Weighted Model) Continued

A.4 Results for Weighted Model

	Output^{US}	Inflation^{US}	Int. Rate^{US}	Net Exports	Output^{EU}	Inflation^{EU}	Int. Rate^{EU}	RX
Actual	5.945734	0.385286	0.806059	351.4456	3.608468	0.245893	0.363569	43.542
Lower	3.493806	0.301302	0.230181	48.61269	1.49129	0.221595	0.197476	9.9321
Upper	36.88094	0.788716	0.824717	530.9087	11.46202	0.855852	0.751455	95.914
Mean	13.68411	0.498109	0.458167	177.0114	4.512572	0.441191	0.38496	33.984

Table 12: Variance of Data and Bootstraps (Weighted Model)

Shock↓\Variable⟶	Y^{US}	π^{US}	R^{US}	NE	Y^{EU}	π^{EU}	R^{EU}	RXR
$Prod^{EU}$	0.003	0.015	0.024	10.445	23.895	7.397	19.670	10.078
$Cons^{EU}$	0.000	0.000	0.000	0.006	3.057	8.975	7.906	0.234
Res^{EU}	0.000	0.000	0.001	0.162	2.533	0.446	0.916	0.151
Inv^{EU}	0.000	0.003	0.005	1.479	5.512	2.500	6.706	1.489
Mon^{EU}	0.003	0.011	0.018	6.894	16.732	61.357	26.844	6.774
$Price^{EU}$	0.000	0.000	0.000	0.050	0.037	4.962	2.481	0.055
$LabSup^{EU}$	0.009	0.029	0.047	20.891	48.233	14.361	35.470	19.844
$Wage^{EU}$	0.000	0.000	0.000	0.000	0.000	0.002	0.004	0.000
Res^{US}	1.031	1.535	2.809	1.328	0.000	0.000	0.000	1.547
$Cons^{US}$	0.612	2.229	2.695	0.145	0.000	0.000	0.000	0.658
Inv^{US}	2.441	1.878	3.994	1.871	0.000	0.000	0.000	1.921
Mon^{US}	0.310	5.877	0.416	0.594	0.000	0.000	0.000	0.612
$Prod^{US}$	31.489	28.209	29.697	17.531	0.000	0.000	0.001	19.357
$Price^{US}$	0.792	1.336	0.726	0.334	0.000	0.000	0.000	0.311
$Wage^{US}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$LabSup^{US}$	63.308	58.876	59.566	38.271	0.001	0.001	0.002	36.967
$Total$	100	100	100	100	100	100	100	100

Table 13: Variance Decomposition (Weighted Model)

A.5 Deterministic Simulations

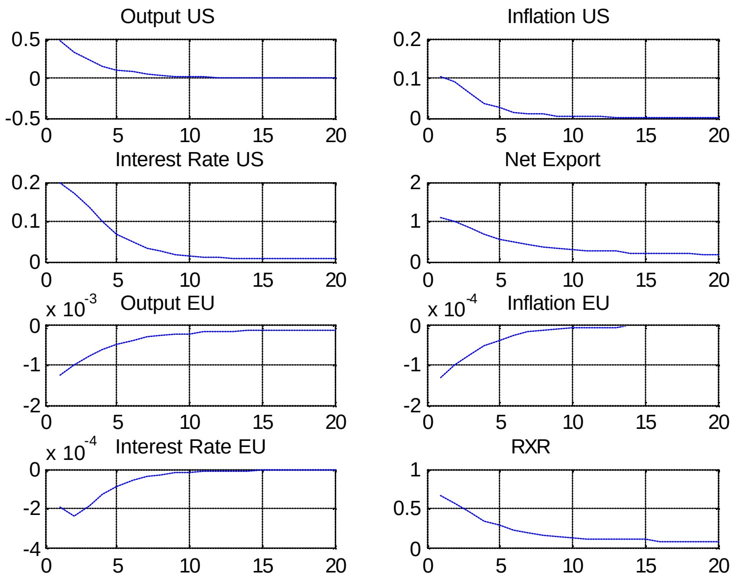

Figure 33: US Residual (World) Shock

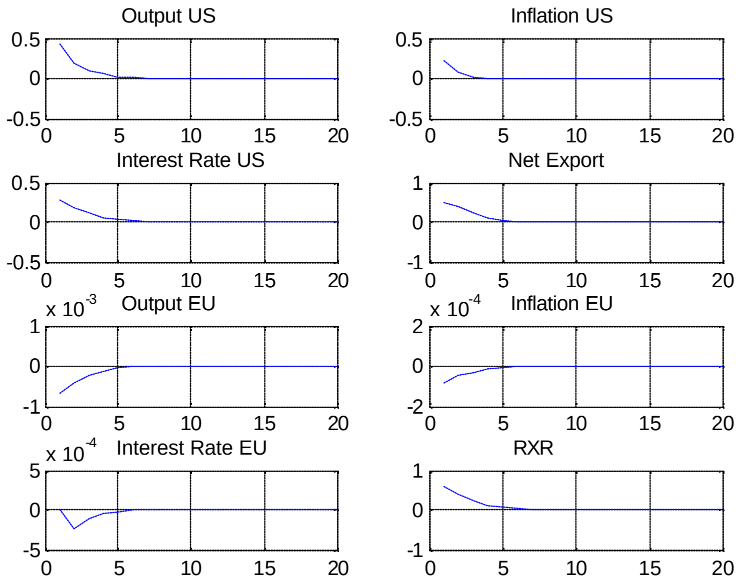

Figure 34: US Consumption Euler Shock

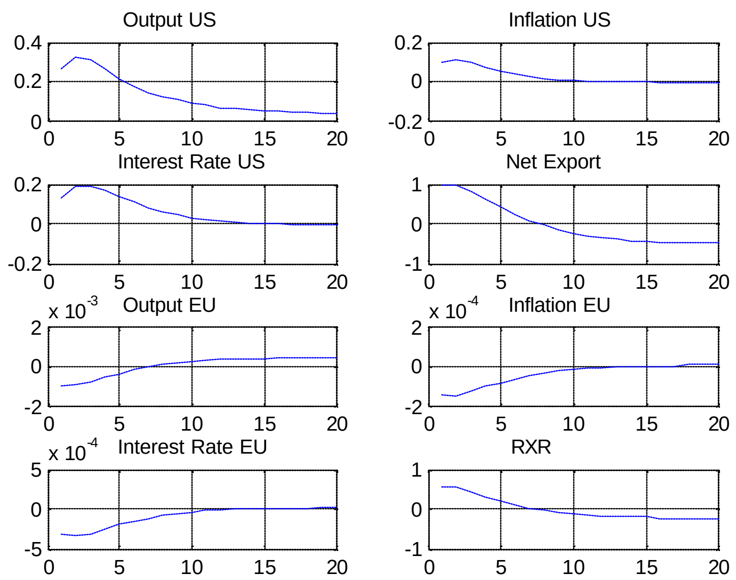

Figure 35: US Investment Euler Shock

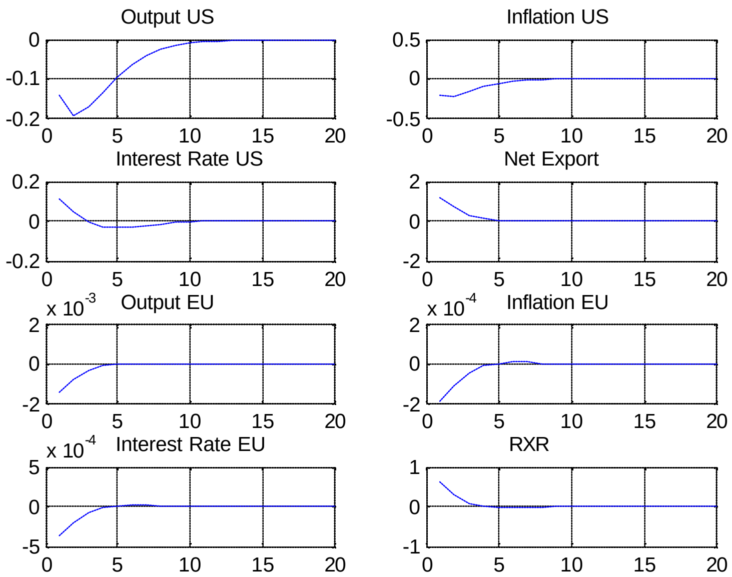

Figure 36: US Monetary Shock

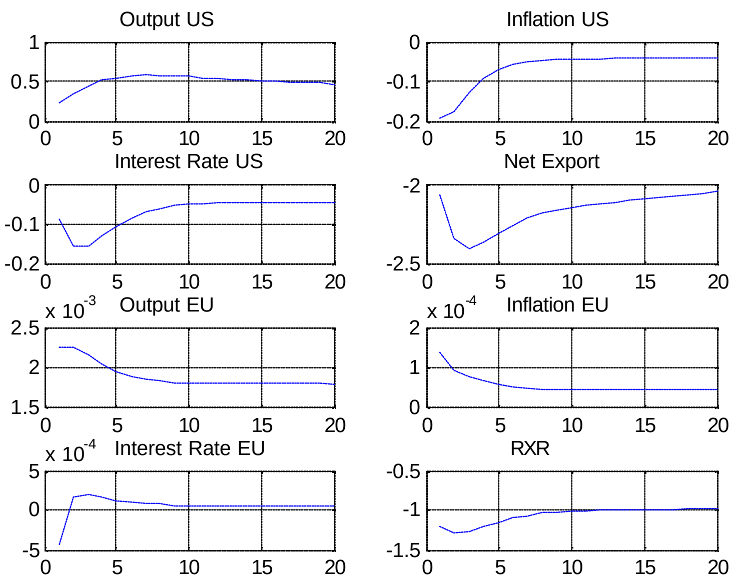

Figure 37: US Productivity Shock

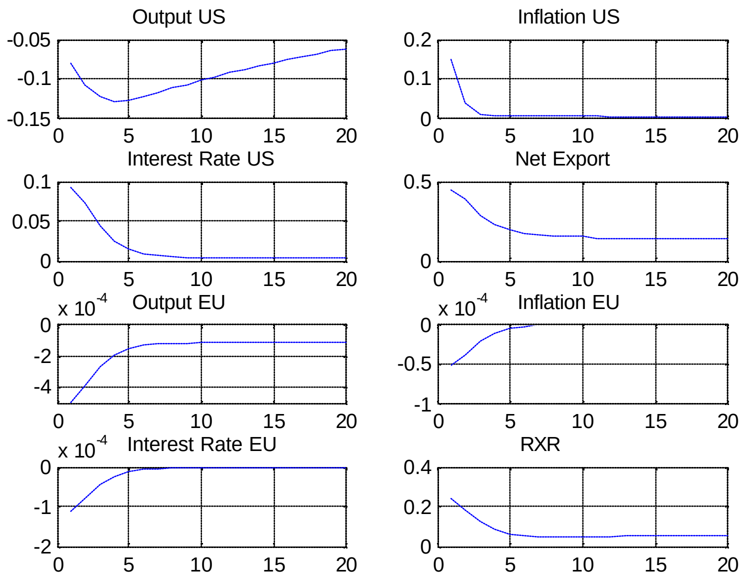

Figure 38: US Price Mark-up Shock (NK)

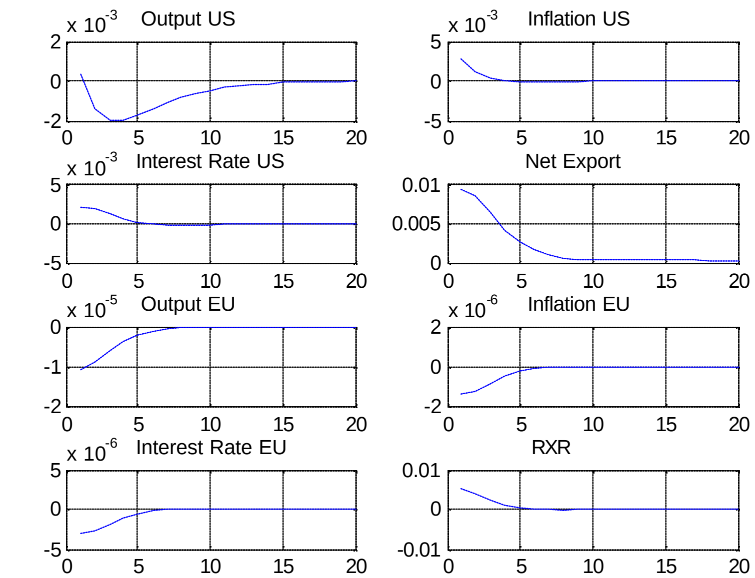

Figure 39: US Wage Mark-up Shock (NK)

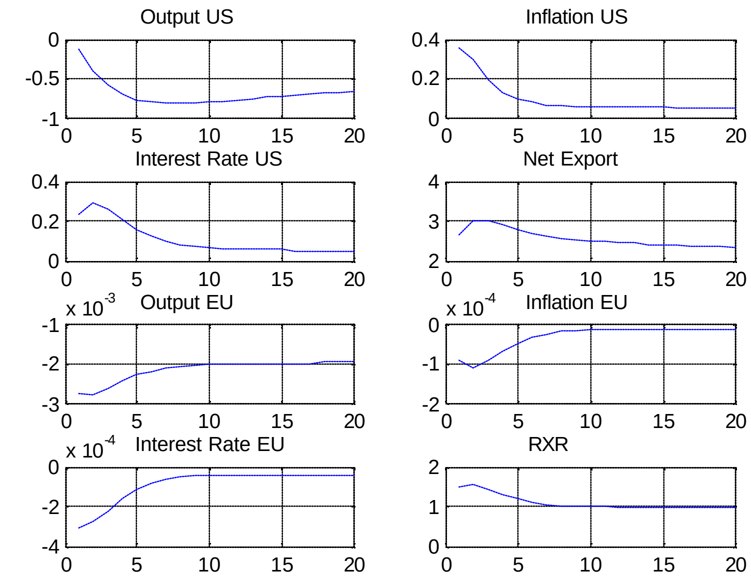

Figure 40: US Labour Supply Shock (NC)

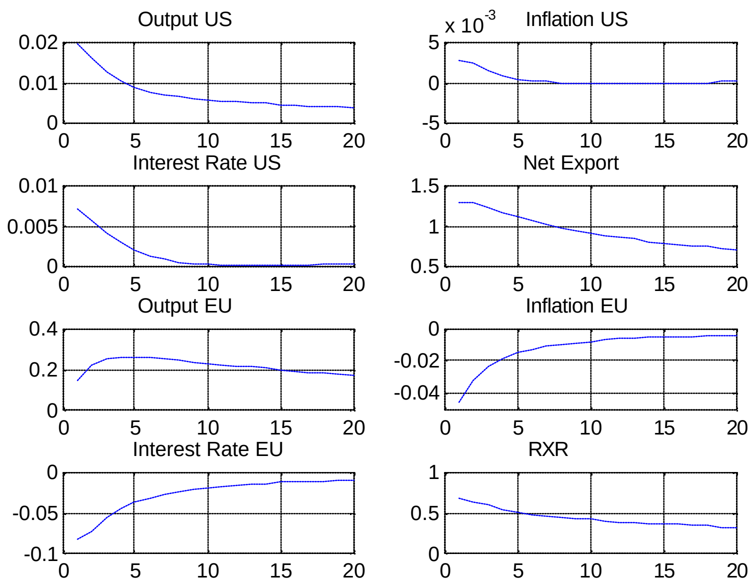

Figure 41: EU Productivity Shock

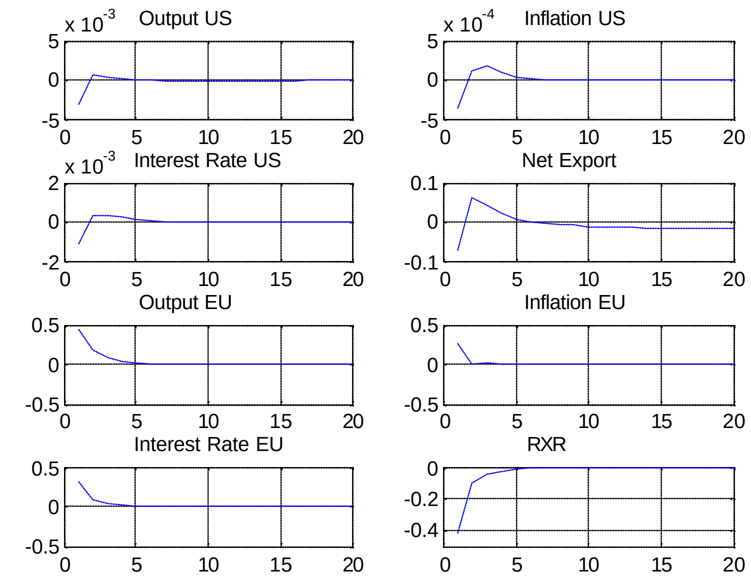

Figure 42: EU Consumption Euler Shock

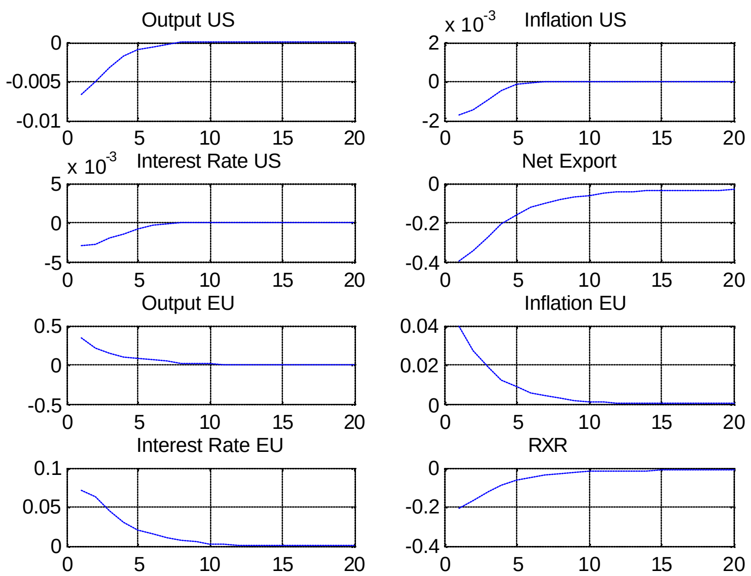

Figure 43: EU Residual (World) Shock

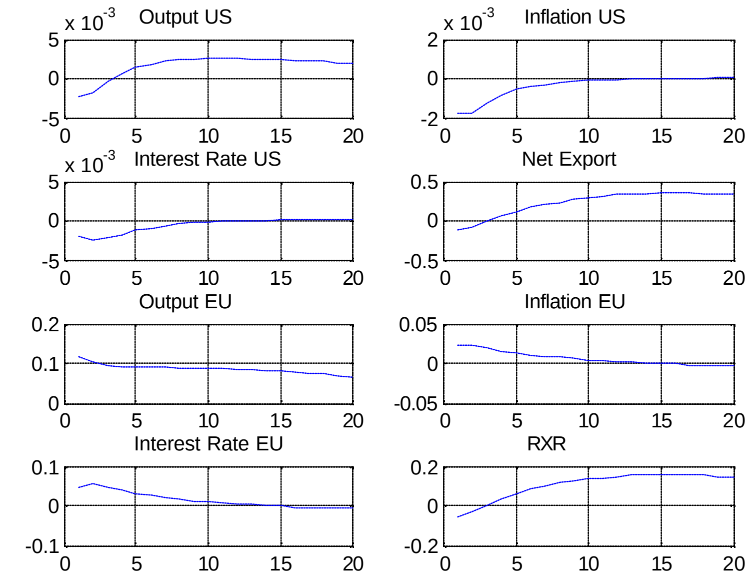

Figure 44: EU Investment Euler Shock

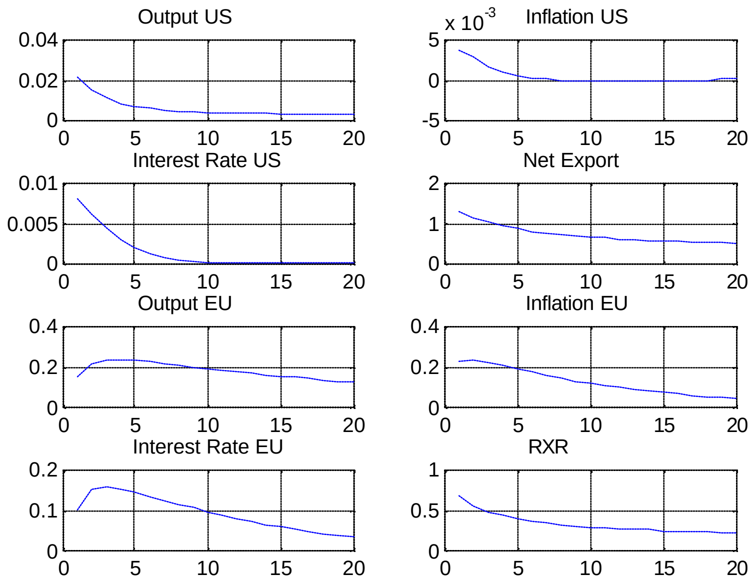

Figure 45: EU Monetary Shock

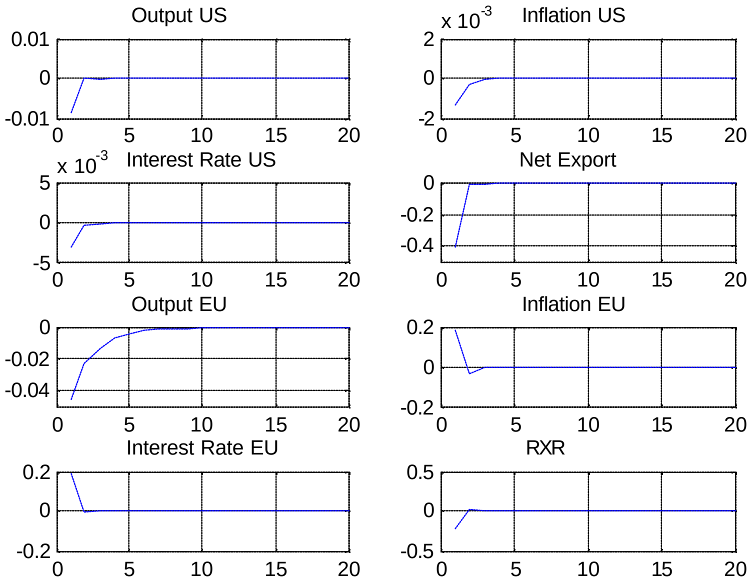

Figure 46: EU Price Mark-up Shock (NK)

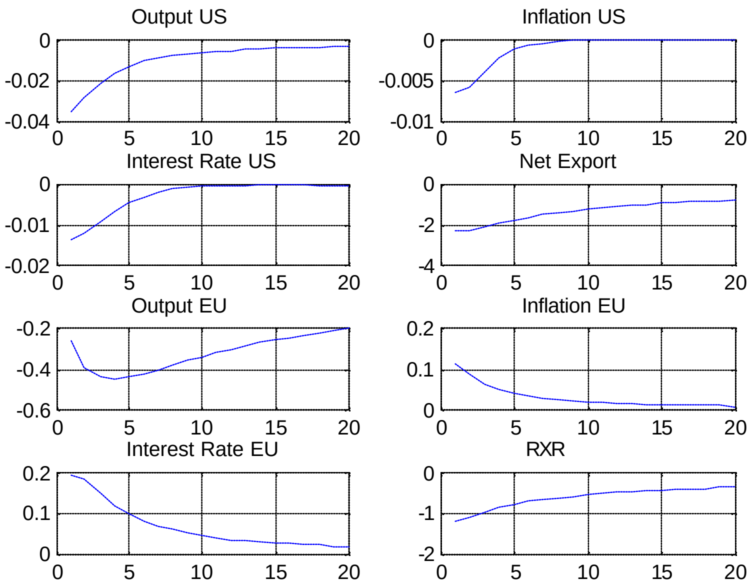

Figure 47: EU Labour Supply Shock (NC)

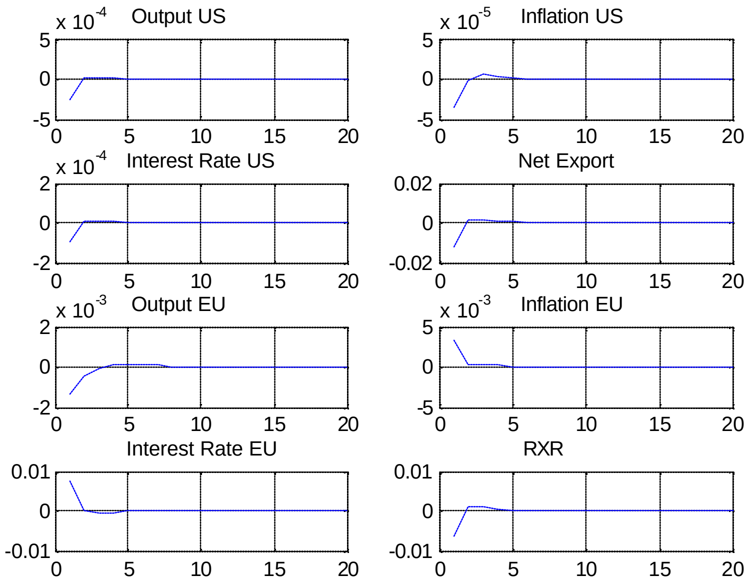

Figure 48: EU Wage Mark-up Shock (NC)

A.6 Cross-Correlations for Weighted Model

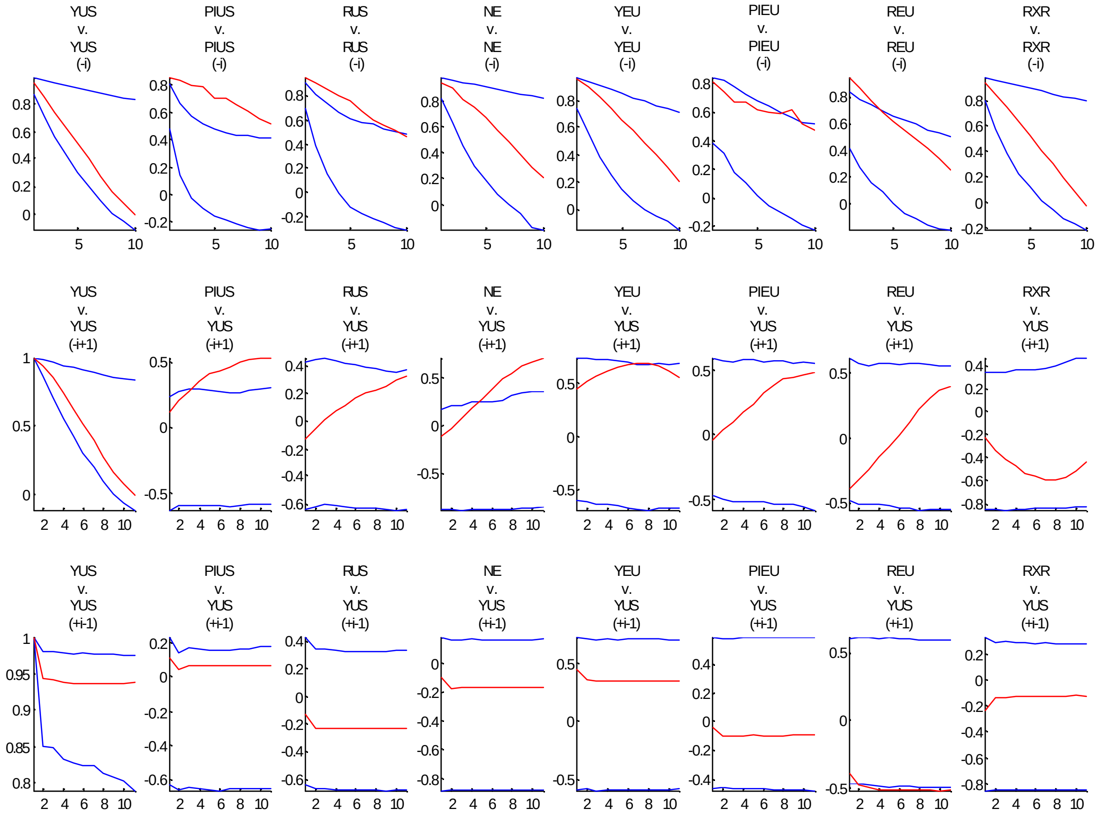

Figure 49: Cross-Correlations for Weighted Model