CARDIFF BUSINESS SCHOOL
WORKING PAPER SERIES

CARDIFF UNIVERSITY

PRIFYSGOL CAERDYÐ

**Cardiff Economics
Working Papers**

Kateryna Onishchenko

*Can a pure real business cycle model explain the real
exchange rate: the case of Ukraine*

E2011/17

Can a pure real business cycle model explain the real exchange rate: the case of Ukraine[1]

Kateryna Onishchenko

PhD Doctoral Candidate

Department of Economics

Cardiff Business School

United Kingdom **E-mail: OnishchenkoK@cardiff.ac.uk**

Abstract: Real exchange rate (RER) is an important instrument for restoring sustainable economic growth in the small open economy with large export share. RER of Ukrainian currency can be explained within the real business cycle (RBC) framework without any forms of nominal rigidities. Fitting Ukrainian quarterly data for the period of 1996:Q1-2009:Q3 into the small open economy real business cycle model and testing it by method of indirect inference shows that RER can be reproduced by RBC framework. The generated pseudo-samples for RER by method of bootstrapping allow to obtain the distribution of the best fit ARIMA(2,1,4) parameters and to show with the Wald statistics that those parameters lie within 95% confidence intervals of those estimated for bootstrapped pseudo Q parameters.

Key words: sustainable economic growth, business cycle, real exchange rates, small open economy, indirect inference, ARIMA

JEL classification: E31, E32, E37, F31, F37

---

1. Introduction

The relationship between output level and real exchange rate is an important and controversial issue, therefore, has been thoroughly investigated by economists. As soon as real exchange rate is considered to be the tool for sustainable improvement of economic performance, the correlation between real exchange rate and output has been in the focus of attention. It is argued that in the open economy framework, the real depreciation improves the competitiveness of domestic goods on the international market, thus improving the terms of trade and raises employment and output. Therefore, according to the traditional view real devaluation has expansionary effect on the economy (Copeland, 2008).

Contrary to the traditional view, real devaluation may have contractionary effect via the redistribution effect between income groups and thus affecting the aggregate demand. Apart from that, in case of low import and export elasticities the output can also decline after real devaluation. As an example of the supply side contractionary effects of real depreciation may be relative increase of import prices for inputs of the production, increase of the domestic interest rates and wages that has an adverse supply side effect and reduces output.

In order to trace the linkage between the real exchange rate and the key macroeconomic fundamentals, I built a micro-founded general equilibrium open economy model that is based on optimising decisions of rational agents. In the small open economy real business cycle model money is irrelevant. Current account and the uncovered interest rate parity conditions that are derived from the consumer's maximisation problem links the medium size economy to the rest of the world. The foreign interest rate and foreign consumption are taken as given.

Small open economy real business cycle model allows to establish the link between the real exchange rate and key macroeconomic fundamentals and is calibrated for the Ukrainian data for the period 1996:Q1-2009:Q3. During this period Ukrainian economy has passed through at least three stages of extreme turbulence. The first shock occurred when the Soviet Union dissolved in 1991; Ukraine gained its political and economic independence and experienced few years of economic downturn, hyperinflation and unemployment. The National Bank of Ukraine (NBU) aimed its policy at keeping stable nominal exchange rate of Ukrainian currency to ensure export revenues, keep balance of payment positive and guarantee budget solvency. In mid-October 2008 as a consequence

of global financial crisis and rocketing inflation in Ukrainian economy NBU could not sustain its fixed exchange rate policy and rapid devaluation began with the hryvnia dropping 38.4 % from UAH 4.85 for USD 1 on 23 September 2008 to UAH 7.88 for USD 1 on 19 December 2008. Such unsterilized interventions lead to the monetary base fluctuations, amplified inflation, and balance of payment imbalances and obviously could not be sustainable in the long-run. Besides, fixing nominal exchange rate in relation to dollar constantly kept hryvnia over or undervalued and ignored the *real costs* of such exchange rate regime. As a result, fixed exchange rate policy imposed substantial restrictions on the monetary policy making NBU to conduct loose monetary policy and instead strict reserve requirements to the banks were used as alternative that created an additional pressure in the banking sector. Inconsistency of the Ukrainian Central Bank policy was revealed with the global financial crisis that in its turn caused currency and banking crisis in Ukraine, amplified inflation and forced NBU to nominally devalue hryvnia and change the general directions of the exchange rate policy towards more flexibility.

Fitting Ukrainian data into the small open economy framework demonstrates the 'cyclical pattern' of the real exchange rate after 1% sustainable rise in productivity within twelve periods, or in other words reproduces 'business cycle'. In the small open economy RBC model increase in productivity within twelve periods by 1% raises permanent income and also stimulates a stream of investments in capital stock. Labour and capital markets adjust relatively slow as compared to financial and currency markets. Therefore, on impact in the short-term it leads to real 2% appreciation of the real exchange rate according to the simulation results based on Ukrainian data. So the real exchange rate rises to reduce demand to the available supply. Uncovered Real Interest Parity (URIP) is violated and must be restored by a rise in Q relative to its expected future value. This rise is made possible by the expectation that Q will fall back steadily, so enabling URIP to be established consistently with a higher real interest rate. As real interest rates fall with the arrival on stream of sufficient capital and so output, Q also moves back to equilibrium. The new equilibrium however represents a real depreciation (by around 15% in my case) on the previous steady state since output is now higher and must be sold on world markets at a lower price.

In order to show that the model is consistent with the real data 'method of indirect inference' is used. It allows to capture non-linear interdependencies of the model parameters that enhance high explanatory power of the results. The sampling variability of parameters is generated within the model by the method of bootstrapping from the model's estimated residuals. The 95% confidence

limits around the real exchange rate ARIMA regression parameters are found. As it is indicated by Wald statistics that is in my case is equal 90.6% all ARIMA parameters jointly lie within 95% confidence intervals.

The rest of the paper is organised as follows: part 2 discusses the existing real exchange rate literature, part 3 analytically derives the small open economy model, part 4 describes calibration and testing techniques and reports econometric results obtained by method of indirect inference, part 5 concludes.

2. Literature review

The large body of the literature analyzed the link between the real exchange rate and productivity especially after continuous strengthening of the UD Dollar in 1990s at the background of productivity growth. The conventional view that real exchange rate depreciates in a response to a positive productivity shock has not been empirically approved and, therefore, has been argued in the literature.

Corsetti at al. (2004) employs standard two-country endowment international business cycle model and reconciles it in accord with empirical evidence on the lack of consumption risk sharing. Aarle, Jong, and Sosoian (2006) estimate a small macro-economic model of Ukraine using quarterly data for the period 1995–2004 in order to find out the most optimal exchange rate policy scenario. Mainly they question the sustainability of hryvnia peg after one-shot revaluation of 5% against the dollar in April 2005. They claim that in the period 2000-2005 stabilizing exchange rate of Ukrainian currency was a key ingredient of stabilizing macroeconomic policy that helped to curb inflation but further support of the rigid exchange rate is highly unfavourable due to the resulting non-sterilized interventions that fuel the domestic money supply and lead to high levels of inflation later on. Egert (2005) investigates the equilibrium exchange rates of three South-eastern European countries (Bulgaria, Croatia and Romania), of two CIS economies (Russia and Ukraine) and of Turkey. At the end of 2003, the Bulgarian lev, the Romanian lei and the Turkish lira became increasingly overvalued, while the real exchange rates in Croatia, Ukraine and also in Russia are found to be relatively fairly valued.

Most of the academic literature studying the exchange rate stabilising policy are concentrating on the inflation curbing and medium run effects mainly ignoring long run effects of actually keeping the exchange rate at disequilibrium level. This can create highly unfavourable dynamics of sustainable economic growth, foreign direct investment/portfolio capital inflows and fiscal balances. RBC optimizing framework helps to establish theoretically justified relationship between exchange rate and the rest of the economy and look at the long run effects and steady state.

The attempts in the literature to explain the volatility and persistence of the real exchange rates have been major exercise. As it can be observed from the existing literature most of the model that attempt to explain the exchange rate puzzles use contrivances such as sticky prices and arbitrary set trade costs rather than optimizing framework.

Martinez-Garcia and Søndergaard (2008) re-examine the ability of sticky-price models to generate volatile and persistent real exchange rates. They use a DSGE framework with pricing-to-market akin to those in Chari et al. (2002) and Steinsson (2008) to illustrate the link between real exchange rate dynamics and what the model assumes about physical capital. Adjustment costs via intertemporal consumption margin, explain the real exchange rate volatility. Such a model combined with monetary policy shocks has the potential to replicate the observed real exchange rate volatility. But with real shocks, the same model produces real exchange rates that are far less volatile than in the data.

Moore and Roche (2010) paper succeeds in explaining the forward bias puzzle and exchange rate disconnect puzzle using the model of exchange economy with habit persistence. Nevertheless, the limitation of their approach is that different model is required to assess the impact of Campbell and Cochrane (2000) habits on other business cycle properties.

De Grauwe and Grimaldi (2003a; 2006) find that the exchange rate is mostly disconnected from the fundamentals; therefore, the agents use the simple forecasting rules based on the ex post evaluation of the relative profitability of these rules besides the introduction of the transaction costs in the goods market will explain the volatility of the exchange rates. These simple forecasting rules are applied to produce two types of equilibria, a fundamental and a bubble one producing crashes of bubbles at unpredictable moments (De Grauwe and Grimaldi, 2003b).

Minford at al. (2008) shows that the real business cycle framework produces very good story for the real exchange rates of the UK economy. Using UK data they show that a productivity simulation is capable of explaining initial real appreciation with subsequent depreciation to a lower steady state.

The obtained results advocate in favour that real business cycle models can explain the real exchange rate movements.

The literature mentioned above inspires for further research that will shed light on the situation that currently prevails in the Ukrainian economy. In particular, to understand what drives the real exchange rate of Ukrainian hryvnia in the last couple decades and whether real appreciation happened due to the macroeconomic factors or follows the trend that cannot be explained with the optimizing real business cycle framework.

3. Small open economy Real Business Cycle model

The theoretical model is developed by Minford et al. (2008) who calibrate the small open economy with UK data where the home economy is populated by identical infinitely lived agents who produce a single good as output and use it both for consumption and investment. The small domestic economy coexists with the rest of the world economy which is considered to be a large economy, therefore, its income and interest rate is unaffected by the developments of the domestic economy. The rationale behind such choice is obvious since Ukrainian economy by its size is a small open economy when the world price level is taken as given.

Consumer problem

The consumer maximizes expected lifetime utility in the stochastic environment subject to the budget constraint

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t U(C_t,L_t)\right\} \tag{3.1}$$

where β is the discount factor, C_t is consumption in period 't', L_t is the amount of leisure time consumed in period 't'.

The form of the utility function is time-separable that is common in the literature for example McCallum and Nelson (1999). Specifically, the utility function has the form:

$$U(C_t,1-N_t)=\theta_0(1-\rho_0)^{-1}\gamma_t C_t^{(1-\rho_0)}+(1-\theta_0)(1-\rho_2)^{-1}\xi_t(1-N_t)^{(1-\rho_2)} \tag{3.2}$$

where $0<\theta_0<1$, and $\rho_0,\rho_2>0$ are the substitution parameters and γ_t,ξ_t are preferences errors.

The representative agent's budget constraint is:

$$C_t + \frac{b_{t+1}}{1+r_t} + \frac{Q_t b_{t+1}^f}{(1+r_t^f)} + p_t S_t^p = (1-\tau)v_t N_t - T_t + b_t + Q_t b_t^f + (p_t + d_t)S_{t-1}^p \qquad (3.3)$$

where p_t denotes the real present value of shares because the consumer in this economy is the

owner of the firm, $v_t = w_t p_t^d$ is the real consumer wage ($v_t = \dfrac{W_t}{P_t}$) that is equal to the producer real

wage multiplied by the domestic goods price level (w_t is the producer real wage, is the wage

relative to the domestic goods price level; τ_t is labour income tax that is a stochastic process; T_t is

a lump-sum transfer and $Q_t = \dfrac{p_t^f}{p_t}$ is the real exchange rate.

In a stochastic environment the representative consumer maximizes the expected discounted stream

of utility subject to a budget constraint. The first order conditions with respect to $C_t, N_t b_t, b_t^f, S_t^p$ 1

(utility maximization problem and all derivations are provided in Appendix 1) derive present value

of the shares is the discounted stream of all future dividends

$$p_t = \sum_{i=1}^{\infty} \frac{d_{t+i}}{(1+r_t)^i} \qquad (3.4)$$

Where dividends d are surplus corporate cash flow:

$$d_t \bar{S}_t = Y_t - N_t^S w_t - K_t(r_t + \delta) \qquad (3.5)$$

$$d_t = \frac{Y_t - N_t^S w_t - K_t(r_t + \delta)}{\bar{S}_t} \qquad (3.6)$$

And market clearing for shares, S_{t+1}^p:

$$S_{t+1}^p = \bar{S}_t \qquad (3.7)$$

The uncovered interest parity condition from utility maximization problem is derived:

$$\left(\frac{1+r_t}{1+r_t^f}\right) = E_t \frac{Q_{t+1}}{Q_t} \qquad (3.8)$$

That in logs yields:

$$r_t = r_t^f + \log E_t \frac{Q_{t+1}}{Q_t} \qquad (3.9)$$

The **uncovered interest rate parity** postulates that if investors are risk-neutral and require zero risk-premium then the forward rate can be effectively replaced with the expected future spot rate. This makes it testable in case of presence of market expectations about the future exchange rates. Among other restrictions imposed by UIRP are no transaction cost, equal default risk over foreign and domestic currency denominated assets, perfect capital flow and no simultaneity induced by monetary authorities. For the purpose of current paper I construct the UIRP condition in real terms, which is more plausible and relevant.

The government

The government budget constraint is:

$$G_t + b_t = \tau_t v_t N_t + \frac{b_{t+1}}{1+r_t} \tag{3.10}$$

In order to finance its expenditures the government collects taxes from labour income and issues debt.

The representative firm

The representative firm buys labour and capital from households and in a stochastic environment maximizes present discounted stream of cash flows, subject to the constant-returns-to-scale production technology:

$$MaxV = E_t \sum_{i=0}^{\infty} d_{it}^i (Y_t - K_t(r_t + \delta) - w_t N_t) \tag{3.11}$$

The capital evolves according to the rule:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{3.12}$$

The constant-return-to-scale production function is:

$$Y_t = Z_{0t}(N_t + Z_{1t})^\alpha (K_t + Z_{2t})^{1-\alpha} \tag{3.13}$$

Where $0 < \alpha < 1$, Y_t is aggregate output per capita, K_t is capital carried over from previous period (t-1), and $Z_{it}(i = 0;1;2)$ reflect the state of technology.

The first order conditions for the firm's problem with respect to K_t and N_t are the following:

$$K_t = \frac{(1-\alpha)Y_t}{r_t + \delta} - Z_t^2 \tag{3.14}$$

$$N_t = \frac{\alpha Y_t}{w_t} - Z_t^1 \qquad (3.15)$$

where r_t and w_t are the rental rates of capital and labour respectively.

The foreign sector

The foreign sector contains the import and export equations for the home economy, and the foreign bonds evolution equation that reflects the current account position.

The import equation for the domestic economy is as follows:

$$\log C_t^f = \log IM_t = \sigma \log(1-w) + \log C_t - \sigma \log Q_t + \sigma \log \varsigma_t \qquad (3.16)$$

The export equation for the domestic economy that corresponds to the existing import equation for the foreign economy is:

$$\log EX_t = \sigma^F \log(1-w^F) + \log C_t^F + \sigma^F \log Q_t + \sigma^F \log \varsigma_t^F \qquad (3.17)$$

Foreign bonds evolve over time to the balance of payment:

$$\frac{Q_t b_{t+1}^f}{(1+r_t^f)} = Q_t b_t^f + p_t^d EX_t - Q_t IM_t \qquad (3.18)$$

The goods market clearing condition:

$$Y_t = C_t + I_t + G_t + EX_t - IM_t. \qquad (3.19)$$

4. Calibration and testing techniques

4.1. Estimation and testing techniques: method of indirect inference

The paper is aimed at evaluation the *Real Business Cycle* ability to account for the real exchange rate behaviour, using the Ukrainian data. There is some evidence in the literature (Moore and Roche, 2010; Chari et al., 2002; Steinsson, 2008; Martinez-Garcia and Søndergaard, 2008) that the behaviour of the exchange rates is modelled using different forms of nominal rigidities like transaction costs, price rigidities, consumption externalities with habit persistence, etc.

In this paper I employ the real business cycle small open economy model *without any forms of nominal rigidities*. Calibration of the model for Ukrainian data shows that, first, a deterministic productivity growth shock creates a real appreciation on impact, reverting in steady state to a small

real depreciation, in the process producing a business cycle; and second, that the RBC alone can reproduce the univariate properties of the real exchange rate.

Following Minford et al. (2008) the model is tested by the *method of indirect inference*, bootstrapping the errors to generate 95% confidence limits for a time-series representation of the real exchange rate, as well as for various key data moments.

The data pattern for the real exchange rate is estimated by the best fitting ARIMA process based on the AIC to the real exchange rate. Augmented Dickey Fuller test and Phillips-Perron test check for stationarity of the data.

The method of indirect inference will test the model against the data to find out if the suggested theoretical model is consistent with the facts (Meenagh, Minford and Theodoridis, 2008). I fit the model to the Ukrainian data, and derive from it the behaviour of the seven shocks; there are shock to productivity, interest rate shock, shock to labour demand, shock to capital accumulation equation, producer real wage shock, export and import shocks; then generate the sampling variability within the model by the method of bootstrapping the random components of these processes as a vector to preserve any contemporary correlations. This allows us to generate a large number of pseudo-samples of Q (Minford et al., 2008).

After that I run an ARIMA for Q on all these pseudo-samples to generate the distribution of the ARIMA parameters. At the final stage of my data analysis I compare the estimated parameters for Q with this distribution, using a Wald statistic: whether I can reject the RBC model at the 95% level of confidence on the basis of the complete set of ARIMA parameters; we would do this if the ARIMA parameters lay outside the 95% confidence limits generated by the bootstrap process.

4.2. Data

The statistics of the Ukrainian Economy and Ukrainian foreign trade is obtained from State Statistics Committee, Budget Committee of Verkhovna Rada, Institute for Economic Research and Policy consulting and National Bank of Ukraine (IERPC). The quarterly historical data is for the period 1996:Q1-2009:Q3. The earlier periods' data is also available but the rationality behind using exactly this data range rests upon the introduction of hryvnia in 1996, relatively stabilized economy and general economic recovery after few years of hyperinflation. This will give more plausible results.

The model is estimated with the quarterly data that for the period of investigation contains in total 55 observations[2].

The real sector of the economy data set contains data series for Gross Domestic Product, Industrial production, sectors' shares in industrial output, etc.

Prices include inflation of the consumer price index and industrial producer prices.

Social statistics contains data sets for wages, households' monetary income, households' monetary expenditure, and employment in industry, and variety of other indicators.

From the monetary statistics I use the interest rate data series. In particular, interest rate statistics contain the variety of indicators for NBU refinancing rate, real NBU refinancing rate, real interbank interest rates, and real interest rates on credits and deposits.

From the available foreign trade indicators I use the data series for the foreign trade total balance of payment statistics, foreign trade with the rest of the world balance of payment statistics, Ukraine's merchandise exports and imports statistics.

Foreign sector statistics is obtained from IMF, World Bank and Fed.

The real exchange rates are calculated on the basis of relative GDP inflator indices. The following formula computes real exchange rate:

$$\ln(Q) = \ln(P^F) - \ln(P) \tag{4.1}$$

Where Q is the real exchange rate, P^F is the foreign price level; as soon as Ukraine has the dollar peg and in the model I use UAH/USD exchange rate, the best proxy for P^F is US GDP Implicit Price Deflator (US Department of Commerce) seasonally adjusted with Index 2005=100. P is the domestic consumer price index with Index 2005=100. Table below summarizes the data.

Table 4.1. *Data summary*

Source	Indicator	Description
NBU	**r**	Nominal interest rate: Interbank Interest Rate overnight, available from 1998, 1996-1998 period interest rate in proxied by NBU official interest rate
NBU, IERPC, own calculations	**Q**	Real Exchange Rate (ln(PF)- ln(P), PF=US price index, US GDP implicit seasonally adjusted price deflator, P=Ukrainian consumer price index [Index 2005=100)]
IERCP	**Y**	Gross Domestic Product
IERCP	**N**	Labour Supply (Employment Economically active population - thousands)
IERCP	**K**	Gross fixed capital accumulation
IERCP	**w**	Producer Real Wage (Average Earnings/PPI, PPI=Producer price index)
IERCP	**IM**	Imports (Imports: Total Trade in Goods & Services)

IERCP	**EX**	Exports (Trade in Goods & Services: Total exports)
IERCP	**C**	Consumption (Household final consumption expenditure)
IERCP	**I**	Investment [Changes in inventories including alignment adjustment+Gross Fixed Capital Formation]
IMF, World Bank	**b_F**	Foreign Bonds (calculated based on accumulated change in the current account divided by consumer price index: Year on Year growth)
IMF, World Bank	**r_F**	Foreign Interest rate (weighted average of EU(19%), US(60%), JP(21%), then made real using PF)
Own calculations	**Z**	Solow Residual (Y/(N^0.7*K^0.3))
IERCP	**tau**	VAT Tax paid by consumer from BoP as a % of GDP
IMF, World Bank, OECD	**C_F**	Foreign Consumption (World exports in goods and services (volume) s.a., in billions of 2000 US dollars))
IERCP	**G**	Government Expenditure
IERCP	**v**	Consumer Real Wage (Average Earnings/CPI, CPI=Consumer price index)
Own calculations	**T**	Lump sum taxes, residual (T=G-tau*v*N)

NBU - National Bank of Ukraine

IERCP - Institute for Economic Research and Policy Consulting

4.3. Best fitting ARIMA for real exchange rate.

In time series analysis, an autoregressive integrated moving average (ARIMA) model is a generalisation of an autoregressive moving average (ARMA) model. These models are fitted to time series data either to better understand the data or to predict future points in the series. They are applied in some cases where data show evidence of non-stationarity, where an initial differencing step (corresponding to the "integrated" part of the model) can be applied to remove the non-stationarity (see Wikipedia).

An ARIMA(p,d,q) process expresses this polynomial factorisation property, and is given by:

$$(1-\sum_{i=1}^{p}\phi_i L^i)(1-L)^d X_t = (1+\sum_{i=1}^{q}\theta_i L^i)\varepsilon_t \qquad (4.2)$$

This is a particular case of an ARMA(p+d,q) process having the auto-regressive polynomial with some roots in the unity. For this reason every ARIMA model with $d>0$ is not wide sense stationary.

Pattern of real exchange rate is expressed in ARIMA with *d=1* and has constant deterministic trend (i.e. a non-zero average).

Non-stationarity tests results are provided in the table below. Augmented Dickey Fuller test and Phillips-Perron test reject the null hypothesis of no-unit root at 1% level of significance. First differencing allows for stationarity of the series at high significance level. Therefore, it is rational to apply d=1 in the best fitting ARIMA.

Table 4.2. *Results of Augmented Dickey Fuller test and Phillips-Perron test for non-stationarity of real exchange rate, Q*

Unit Root Tests		
	Levels	First Differences
ADF Test Statistics	-1.593658	-3.934591
	(0.4783)	(0.0036)
PP Test Statistics	-1.640389	-3.866941
	(0.4550)	(0.0044)

Having established non-stationarity of the Q series and having proved that first differencing approach helps to make the series stationary I found that ARIMA(2,1,4) is the best fitting configuration. Q series is rather persistent in the moving average (MA) term exhausting its persistency in the 4th lag; and relatively less persistent in its autoregressive (AR) term that exhausts its persistency in the 2nd lag. The best fit ARIMA analysis is based on the AIC presented in the Table 4.3.

Table 4.3. *Best fitting ARIMA results for real exchange rate.*

	C	AR(1)	AR(2)	AR(3)	AR(4)	MA(1)	MA(2)	MA(3)	MA(4)	$\overline{R}^2$	AIC
ARIMA(1,1,0)	-0.034	0.398								0.143	-2.292
ARIMA(1,1,1)	-0.033	-0.092				0.533				0.131	-2.261
ARIMA(1,1,2)	-0.036	0.762				-0.318	-0.218			0.130	-2.243
ARIMA(1,1,3)	-0.036	0.713				-0.313	-0.273	0.222		0.141	-2.238
ARIMA(1,1,4)	-0.035	0.662				-0.267	-0.262	0.227	0.063	0.125	-2.204
ARIMA(2,1,0)	-0.035	0.408	-0.035							0.124	-2.241
ARIMA(2,1,1)	-0.035	-0.455	0.306			0.976				0.193	-2.306

Model										
ARIMA(2,1,2)	-0.037	-0.055	0.570			0.520	-0.468			0.192	-2.288
ARIMA(2,1,3)	-0.038	-0.077	0.601			0.570	-0.485	-0.088		0.171	-2.246
ARIMA(2,1,4)*	**-0.033**	**-0.601**	**0.264**			**1.277**	**-0.045**	**0.053**	**0.382**	**0.311**	**-2.414**
ARIMA(3,1,0)	-0.035	0.418	-0.111	0.188						0.139	-2.221
ARIMA(3,1,1)	-0.036	0.726	-0.239	0.203		-0.321				0.125	-2.187
ARIMA(3,1,2)	-0.038	0.107	0.601	-0.100		0.402	-0.581			0.178	-2.233
ARIMA(3,1,3)	-0.038	1.235	0.152	-0.533		-0.988	-0.665	0.881		0.280	-2.349
ARIMA(3,1,4)	-0.037	0.888	0.586	-0.661		-0.644	-0.978	0.800	0.208	0.314	-2.381
ARIMA(4,1,0)	-0.036	0.410	-0.108	0.158	0.065					0.124	-2.165
ARIMA(4,1,1)	-0.038	0.688	-0.224	0.181	0.031	-0.283				0.107	-2.130
ARIMA(4,1,2)	-0.037	0.007	0.496	-0.054	0.139	0.489	-0.507			0.183	-2.202
ARIMA(4,1,3)	-0.034	0.662	0.039	-0.542	0.277	-0.229	-0.372	0.850		0.241	-2.259
ARIMA(4,1,4)	-0.036	0.039	-0.378	-0.122	0.530	0.492	0.498	0.502	-0.445	0.235	-2.236

*Best fitting ARIMA

Table 4.4 reports the values and description of parameters of the model used in the open economy RBC literature. These coefficients are applied in the basic run of the model.

Table 4.4. *Basic run model parameters.*

Coefficient	Description of the coefficient	Value – single equation
α	Effective labour share in goods production	0.70
β	Discount factor	0.97
δ	Physical capital depreciation rate	0.0125
ρ_0	Coefficient of relative risk aversion	1.20
θ_0	Time separable utility function substitution parameter	0.50
γ_G	Share in the goods sector	0.05
ρ_2	Substitution elasticity between consumption and leisure	1.00
ω	Home bias	0.70
ρ	Discount rate	-0.50
ω^F	Home bias	0.70
h	Leisure preference parameter	0.80
ρ_3	Substitution elasticity between consumption and leisure	-0.50
σ	Substitution elasticity between home and foreign goods	2.00
σ_1	Substitution elasticity between home and foreign goods	2.00

4.4. Testing the model to the Ukrainian data by method of indirect inference

Meenagh, Minford and Theodoridis (2008) describes method of indirect inference based using a Wald statistic, whether the parameters of a time-series representation estimated on the actual data lie within some confidence interval of the model-implied distribution.[3]

At the first stage using the calibrated above coefficients I run the basic model and derive for this fitted into the model data the behaviour for the productivity and preference shocks. The null hypothesis is that the model holds meaning that by default I assume that my theoretical model is the true model describing Ukrainian economy. Therefore, extracted shocks are the shocks that are jointly implied by the model and the data. As soon as these shocks represent the stochastic part of the model I need to check extracted shocks for stationarity. If the shock is I(0) then I can estimate AR(1) process; if the shock is non-stationary I estimate ARIMA(1,1,0) process, i.e. AR(1) for first differencing. Table below provides with the ADF and PP tests for non-stationarity.

Table 4.5. *Check for stationarity of the productivity and preference shocks.*

Shocks	ADF Test Statistics		PP Test Statistics	
	Levels	First Differences	Levels	First Differences
ε_t^r	-1.8707	-4.2599	-5.3369	-10.7341
Prob	0.3431	0.0014	0.0000	0.0000
$\varepsilon_t^{N^d}$	2.2764	-9.8184	1.6242	-10.1283
Prob	0.9999	0.0000	0.9994	0.0000
ε_t^Y	1.4807	1.1223	-1.6968	-7.0954
Prob	0.9991	0.9972	0.4272	0.0000
ε_t^K	-3.3992	-2.9304	-0.5102	-6.2332
Prob	0.0157	0.0494	0.8807	0.0000
ε_t^w	1.7293	-9.5678	1.6079	-9.4802

Prob	0.9996	0.0000	0.9994	0.0000
ε_t^{EX}	-0.7562	-2.3485	-1.0029	-5.0548
Prob	0.8231	0.1613	0.7461	0.0001
ε_t^{IM}	0.4100	-1.8482	0.2272	-6.1070
Prob	0.9817	0.3536	0.9720	0.0000

As it is shown all of the seven shocks are non-stationary and represent I(1) processes so I estimate ARIMA(1,1,0) processes so the residual of those ARIMA(1,1,0) processes will allow to generate the sampling variability within the model by the method of bootstrapping the random components (or the residuals) of these processes as a vector to preserve any contemporary correlations. Appendix 2 contains the estimated ARIMA(1,1,0) shocks process with descriptive statistics of the random component. In my model 1000 pseudo samples for Q is generated. On each of the pseudo series of Q best fit ARIMA(2,1,4) is run in order to obtain the distribution of the parameters and to test with the Wald-statistics test if the ARIMA parameters estimated in the previous section of the historical data lie within confidence intervals of those estimated for bootstrapped pseudo Q.

If the ARIMA parameters for the historical data lie outside the 95% confidence limits generated by the bootstrap process then it is valid to reject the employed RBC model at the 95% level of confidence.

Appendix 3 demonstrates some of the bootstraps of Q against the historical Q. It's worth noticing that fixed exchange rate policy within the last few years affected the estimated historical Q series used in the model. Therefore, historical Q follows more flat pattern as compared to the bootstrapped Q.[4]

The Wald test is a parametric statistical test can be applied whenever a relationship within or between data items can be expressed as a statistical model with parameters to be estimated from a sample (in our case those parameters are estimated with best fitting ARIMA(2,1,4) from generated pseudo-samples), the Wald test can be used to test the true value of the parameter based on the sample estimate.

The following formula of the Wald test is applied:

16

$$t_{Wald} = (\Phi - \overline{\Phi})' \sum{}_{(\Phi\Phi)}{}^{-1} (\Phi - \overline{\Phi})^5 \qquad\qquad (4.3)$$

Where Φ is a matrix of the joint ARIMA parameters; $\overline{\Phi}$ is a matrix that contains mean values of the parameters generated by sampling distribution; $\sum_{(\Phi\Phi)}$ variance-covariance matrix of the model parameters.

Table 4.6 summarises the results of the Wald test and confidence limits for best fitting ARIMA.

Table 4.6. *Confidence limits for best fitting ARIMA(2,1,4) and Wald test results.*

|	*AR(1)*	*AR(2)*	*MA(1)*	*MA(2)*	*MA(3)*	*MA(4)*
Actual coefficients	-0.6010	0.2640	1.2770	-0.0450	0.0530	0.3820
Lower	-1.4093	-0.9388	-1.5850	-1.0500	-0.7757	-0.7371
Upper	1.4200	0.7144	1.5808	1.5012	0.6379	0.8958
Mean coefficients	-0.0280	-0.2198	-0.0013	0.2379	-0.0433	-0.0426
Wald statistics	90.6%					

Wald test statistics is equal to 90.6% and clearly shows that actual data coefficients jointly lie within 95% confidence intervals. Besides, as is it demonstrated in Table 4.6 actual coefficients lie between lower and upper values of the coefficients of the pseudo-generated best fitting ARIMA parameters. The obtained result advocates in favour of the hypothesis that real exchange rate can be explained with real business cycle framework.

4.7. Data pattern for real exchange rate after sustained one-off rise in productivity.

A sustained one-off rise in productivity allows to qualitatively explaining large cyclical swings in Q. Figure 4.1 shows the model simulation of a rise of the productivity level by 1% for 12 periods. Basically it proves that that RBC alone can reproduce the univariate properties of the real exchange rate. The process behind the behaviour of the real exchange rate after 1% permanent productivity rise can be described as follows. Increase in productivity raises permanent income and also stimulates a stream of investments to raise the capital stock in line. Output, though, cannot be

increased without extra capital and labour. Labour and capital markets adjust relatively slow as compared to financial and currency markets. Therefore, on impact in the short-term it leads to real 2% appreciation of the real exchange rate according to the simulation results based on Ukrainian data. So the real exchange rate rises to reduce demand to the available supply.

Figure 4.1. *Real exchange rate after 1% permanent rise in productivity.*
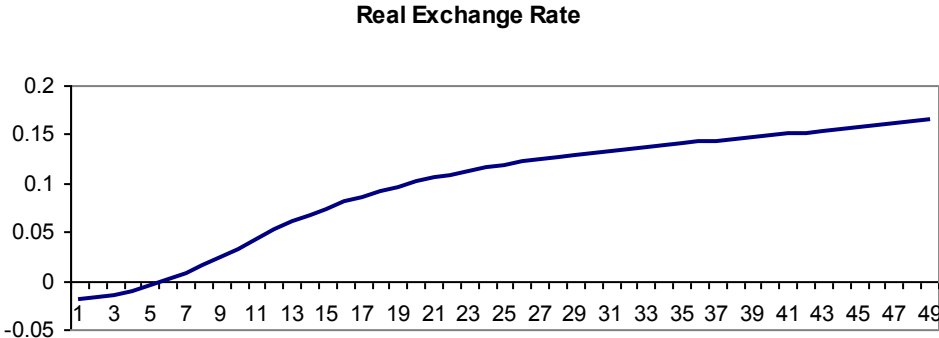
Real Exchange Rate

Uncovered Real Interest Parity (URIP) is violated and must be restored by a rise in Q relative to its expected future value. This rise is made possible by the expectation that Q will fall back steadily, so enabling URIP to be established consistently with a higher real interest rate. As real interest rates fall with the arrival on stream of sufficient capital and so output, Q also moves back to equilibrium in the 5th quarter. The new equilibrium however represents a real depreciation (by around 15% in our case) on the previous steady state since output is now higher and must be sold on world markets at a lower price (adopted from Minford et al., 2008).

The experiment is based on the Ukrainian data. The rest of the data patterns for other variables is provided in Appendix 4.

It is worth mentioning that other variables of the model also tell very good story. Output increases steadily to the new steady state level that is accompanied by slow arrival of labour and capital. Net exports falls below its' original level while the exchange rate appreciates. Later when the exchange rate depreciates to a new steady state level net exports grow demonstrating that competitiveness has now improved. The interest rate increases by one percent but then steadily converges to a new steady state level, which is above the original one. Obviously productivity rise is associated with wage, consumptions and government spending permanent increase.

5. Conclusions

All of the ARIMA parameters for the real exchange rate lie within 95% confidence intervals and the parameters jointly lie within 95% confidence interval as it is indicated be the Wald statistics. Therefore, real exchange rate of Ukrainian hryvnia during the period 1996-2009 can be explained within sustainable real business cycle framework. RBC small open economy model tells very good story about the behaviour of the real exchange rate and also the interdependencies between multiple time series of the model.

Testing the real exchange rate in the context of dynamic general equilibrium small open economy model has important policy implications. Apart from partial equilibrium approaches of estimating different types of equilibrium exchange rates and the deviations of the existing RER from equilibrium, general equilibrium approach allows to link major external and internal macroeconomic fundamentals together. Basically, the analysis shows that real exchange rate is itself driven by fundamentals. Using historical data for the Ukrainian economy, the Real Business Cycle establishes the ability to account for the real exchange behaviour of Ukrainian currency within the last one and a half decades. I employed the real business cycle small open economy model *without any forms of nominal rigidities*. RBC alone can reproduce the univariate properties of the real exchange rate and 1% productivity shock on impact leads to real appreciation of the real exchange rate and as soon as capital and labour arrives it is established on the new equilibrium level below the original one so leads to the real depreciation due to the output growth that need to be sold on the international markets at a lower price.

This result implies that restoring sustained growth requires real currency depreciation. As it is suggested in the literature there is couple of ways to achieve this result. One option is to conduct deflationary policies, i.e. to reduce prices and wages by simultaneously that will increase the real exchange rate, thus making foreign goods price in terms of the domestic goods price more expensive for domestic consumer. That has the risk of persistent recession that can bring the country to fiscal non-sustainability so neither political system can accept such direction.

The second policy prospective that sounds more attractive is to accelerate structural reforms and corporate restructuring to increase productivity growth while keeping wage growth moderate. That

will restore higher growth rates with moderate currency depreciation but might incur some short run costs of resource reallocation.

Refining the official exchange rate role is viable. It has been more volatile within the last two years; this volatility and nominal devaluation of hryvnia was unavoidable and stipulated by the necessity to adjust to unfavourable external economic shocks and to rescue domestic economy from inflation and recession. Nevertheless, official exchange rate should be revised by the central bank authorities on the regular daily basis and to be set in accord with the economic conditions.

In the *long run*, assuming independence of the National Bank of Ukraine to be able to conduct effective monetary policy (adopted from Volosovych, 2002), inflations targeting rather than fixing exchange rate could also be a potential policy option.

References

Aarle, B., Jong, E. and Sosoian, R. (2006) 'Exchange Rate Management in Ukraine: Is there a Case for More Flexibility?' *Economic Systems*, Vol. 30, No. 3. pp. 282-305.

Campbell, J.Y. and Cochrane, J. H. (1999) 'Explaining the Poor Performance of Consumption-Based Asset Pricing Models', *The Journal of Finance*, Vol. LV, No. 6, pp. 2863-2878.

Chari, V. V., Kehoe, P.J. and McGrattan, E.R. (2002) *Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?* Federal Reserve Bank of Minneapolis.

Copeland, L. (2008) Exchange Rates and International Finance, Fifth ed, Prentice Hall.

Corsetti, G., Dedola, L. and Leduc, S. (2004), *International Risk Sharing and the Transmission of Productivity Shocks*, ECB Working Paper No. 308.

De Grauwe, P. and Grimaldi, M. (2003a) 'Exchange rates puzzles. A tale of switching attractors', paper presented at the EEA Meeting, Stockholm, Sweden.

De Grauwe, P. and Grimaldi, M. (2003b) 'Bubbling and Crashing Exchange Rates', Open Access publications from Katholieke Universiteit Leuven with number urn:hdl:123456789/119410.

De Grauwe, P. and Grimaldi, M. (2006) 'Exchange Rate Puzzles: A tale of Switching Attractors', *European Economic Review*, No. 50, pp. 1-33.

Egert, B. (2005) 'Equilibrium exchange rates in South Eastern Europe, Russia, Ukraine and Turkey: Healthy or (Dutch) diseased?' *Economic Systems*, Vol. 29, No. 2, pp. 205-241.

IERPC (Institute for Economic Research and Policy Consulting). Obtained through the Internet: http://www.ier.kiev.ua/English/data/data_eng.cgi.

Martinez-Garcia, E. and Sondergaard, J. (2008) *The real exchange rate in sticky price models: does investment matter?,* Globalization and Monetary Policy Institute Working Paper 17, Federal Reserve Bank of Dallas.

McCallum, B.T. and Nelson, E. (1999) 'An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis', *Journal of Money Credit and Banking*, Vol. 31, No. 3, pp. 296-316.

Meenagh, D., Minford, P. and Theodoridis, K. (2008) 'Testing a Model of the UK by the Method of Indirect Inference', *CEPR Discussion Paper No. DP6849*.

Meenagh, D., Minford, P., Nowell, E. and Sofat, P. (2008) *Can a pure Real Business Cycle Model explain the real exchange rate?* Cardiff Economics Working Papers No. E2005/2.

Minford, P., Meenagh, D., Nowell, E. and Sofat, P. (2008) *Can a Real Business Cycle Model without price and wage stickiness explain UK real exchange rate behaviour?* (previously published as 'Can a pure Real Business Cycle Model explain the real exchange rate?') Cardiff Economics Working Papers.

Moore, M. and Roche, M. (2010) 'Solving exchange rate puzzle with neither sticky prices nor trade costs', *Journal of International Money and Finance*, Vol. 29, No.6, pp. 1151-1170.

Steinsson, J. (2008) *The Dynamic Behaviour of the Real Exchange Rate in Sticky Price Models*, NBER Working Papers No. 13910.

US Department of Commerce, Bureau of Economic Analysis. Downloadable from Federal Reserve Bank of St. Louis: http://research.stlouisfed.org/fred2/series/GDPDEF?cid=21.

Volosovych, V. (2002) *The feasibility of monetary targeting in Ukraine*. In: von Cramon-Jadubal, S., Akimova, I. (Eds.), Fostering Sustainable Growth in Ukraine. Physica Verlag, Heidelberg.

Notes

1. Instead of maximizing with respect to capital K_{t+1} consumer decides on the demand for domestic shares S_t^p that derives the present value of shares being equal to the present value of discounted future dividends; where dividends are the surplus corporate cash flows. Equations (3.5)-(3.7) close the model for optimal choice of capital and, thus, optimize investment decision.

2. For the purpose of econometric exercise the data series is extrapolated to 100 observations that do not violate the key results of the paper.

3. See Appendix 5 for more details.

4. Alternative estimates of RER can be applied on the future stages of research.

5. Wald statistic $[(g(a_T) - g(\alpha_S)]'W[(g(a_T) - g(\alpha_S)]$ where $W = \sum_g^{-1}$ and $\sum_g$ is the covariance matrix of the (quasi) maximum likelihood estimates of $g(\alpha_S)$ which is obtained using a bootstrap simulation.

Appendix 1. Small open economy model derivations (replicated from Meenagh et al., 2008)

Consumer utility maximization problem:

$$\underset{C_t, N_t b_t, b_t^f, s_t^p}{Max}\; L = E_0 \left\{ \begin{array}{l} \sum_{t=0}^{\infty} \beta^t \left(\theta_0 (1-\rho_0)^{-1} \gamma_t C_t^{(1-\rho_0)} + (1-\theta_0)(1-\rho_2)^{-1} \xi_t (1-N_t)^{(1-\rho_2)} \right) \\[2mm] - \lambda_t \left(C_t + \dfrac{b_{t+1}}{1+r_t} + \dfrac{Q_t b_{t+1}^f}{(1+r_t^f)} + p_t S_t^p - (1-\tau)v_t N_t + T_t - b_t - Q_t b_t^f - (p_t + d_t) S_{t-1}^p \right) \end{array} \right\}$$

The first order conditions with respect to $C_t, N_t b_t, b_t^f, S_t^p$:

$$C_t : \theta_0 \gamma_t c_t^{-\rho_0} = \lambda_t \tag{A.1}$$

$$N_t : (1-\theta_0)\xi_t(1-N_t)^{-\rho_2} = \lambda_t (1-\tau_t)v_t \tag{A.2}$$

$$b_t : \frac{\lambda_t}{1+r_t} = \beta E_t \lambda_{t+1} \tag{A.3}$$

$$b_t^f : \frac{\lambda_t Q_t}{1+r_t^f} = \beta E_t \lambda_{t+1} Q_{t+1} \tag{A.4}$$

$$S_t^p : \lambda_t p_t = \beta E_t \lambda_{t+1} (p_{t+1} + d_{t+1}) \tag{A.5}$$

Substituting A.3 into A.1 yields:

$$(1+r_t) = \left(\frac{1}{\beta}\right) E_t \left(\frac{\gamma_t}{\gamma_{t+1}}\right) \left(\frac{C_t}{C_{t+1}}\right)^{-\rho_0} \tag{A.6}$$

Substituting A.1 and A.3 into A.2 yields the following:

$$(1-N_t) = \left(\frac{\theta_0 C_t^{-\rho_0}(1-\tau_t)v_t}{(1-\theta_0)\xi_t} \right)^{\frac{-1}{\rho_2}} \tag{A.7}$$

Since real consumer wage $v_t = w_t p_t^d$ and logarithmic approximation of domestic price level

$$\log p_t^d = -\left(\frac{1-\omega}{\omega}\right)^\sigma \log(Q_t) - \frac{1}{\rho}\left(\frac{1-\omega}{\omega}\right)^\sigma \log \varsigma_t + const \quad \text{A.7 transforms into}$$

$$(1-N_t) = \left(\frac{\theta_0 C_t^{-\rho_0}\left[(1-\tau_t)\exp\left(\log w_t - \left(\frac{1-\omega}{\omega}\right)^\sigma \left(\log Q_t + \frac{1}{\rho}\log\varsigma_t\right)\right)\right]}{(1-\theta_0)\xi_t} \right)^{\frac{-1}{\rho_2}}$$

And finally substituting A.3 into A.5 yields

$$p_t = \left(\frac{p_{t+1} + d_{t+1}}{1+r_t}\right) \text{ that after forward iterations and arbitrage yields:}$$

$$p_t = \sum_{i=1}^{\infty} \frac{d_{t+i}}{(1+r_t)^i} \text{ that tells that present value of the shares is the discounted stream of all future}$$

dividends.

Substituting A.3 into A.4 derives the uncovered interest rate parity condition $\left(\dfrac{1+r_t}{1+r_t^f}\right) = E_t \dfrac{Q_{t+1}}{Q_t}$

That in logs yields:

$$r_t = r_t^f + \log E_t \frac{Q_{t+1}}{Q_t}$$

Variables listing

C_t is consumption in period 't'

L_t is the amount of leisure time consumed in period 't'

p_t is the real present value of shares because the consumer in this economy is the owner of the firm

v_t is the real consumer wage

w_t is the producer real wage

τ_t is labour income tax

T_t is a lump-sum transfer

Q_t is the real exchange rate

p_t^d is the domestic goods price level

r_t is the domestic interest rate

r_t^f is foreign interest rate

G_t is government expenditures

b_t is the domestic government bonds

Y_t is aggregate output per capita

K_t is physical capital carried over from previous period (t-1)

C_t^f is foreign consumption

IM_t is the domestic import

EX_t is the domestic export

w^F is foreign producer real wage

b_t^f is foreign bonds

$Z_{it}\,(i = 0;1;2)$ is the state of technology

Appendix 2. Estimates of the ARIMA(1,1,0) of the shock processes and descriptive statistics of the random component.

Interest rate shock process ε_t^r

Dependent Variable: D(ERR_R)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.009338	0.023207	-0.402358	0.6891
AR(1)	-0.014578	0.137041	-0.106379	0.9157

Productivity shock process ε_t^Y

Dependent Variable: D(ERR_Y)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.016715	0.029077	-0.574871	0.5679
AR(1)	0.057958	0.144133	0.402115	0.6893

Labour demand shock process $\varepsilon_t^{N_d}$

Dependent Variable: D(ERR_N)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.218303	0.051570	4.233175	0.0001
AR(1)	-0.318714	0.134311	-2.372962	0.0215

Capital accumulation shock process ε_t^K

Dependent Variable: D(ERR_K)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.012840	0.016663	0.770545	0.4446
AR(1)	0.137386	0.145539	0.943978	0.3497

Producer real wage shock process ε_t^w

Dependent Variable: D(ERR_W)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.221104	0.050415	4.385705	0.0001
AR(1)	-0.287197	0.134534	-2.134760	0.0376

***Exports shock process*** ε_t^{EX}

Dependent Variable: D(ERR_EX)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.065873	0.031275	2.106243	0.0401
AR(1)	0.352673	0.133014	2.651404	0.0107

Imports shock process ε_t^{IM}

Dependent Variable: D(ERR_IM)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.120068	0.029087	-4.127856	0.0001
AR(1)	0.131736	0.142702	0.923154	0.3603

Appendix 3. Comparing real exchange rate historical data with model generated pseudo samples
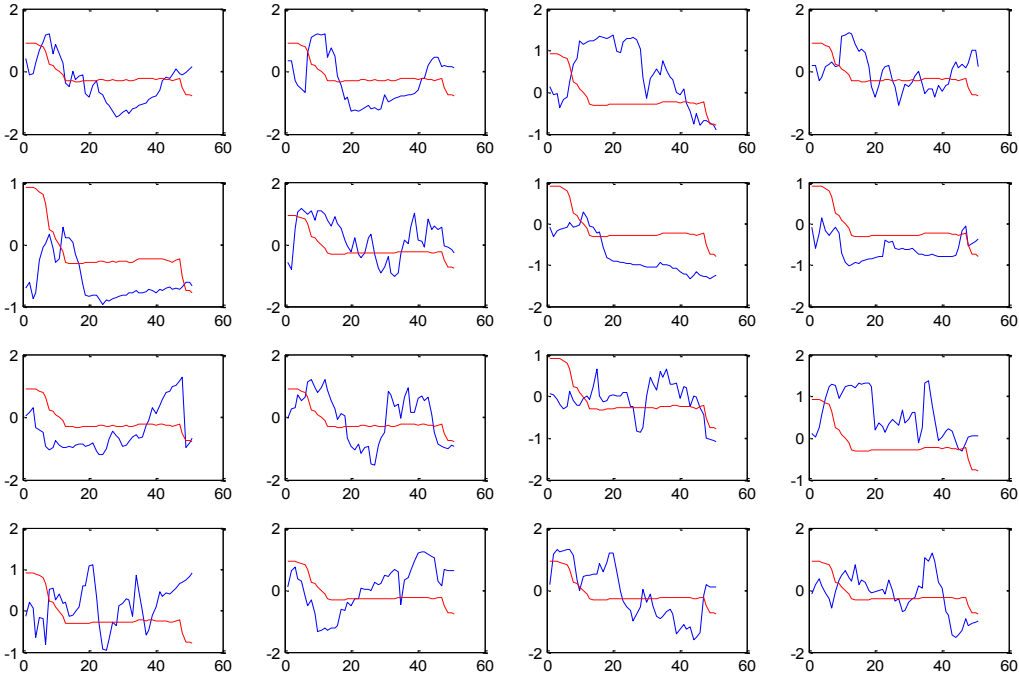
Note: Red line is actual RER, blue line is bootstrapped RER

Appendix 4. Data pattern for key variables of the model after one-off sustainable rise in productivity
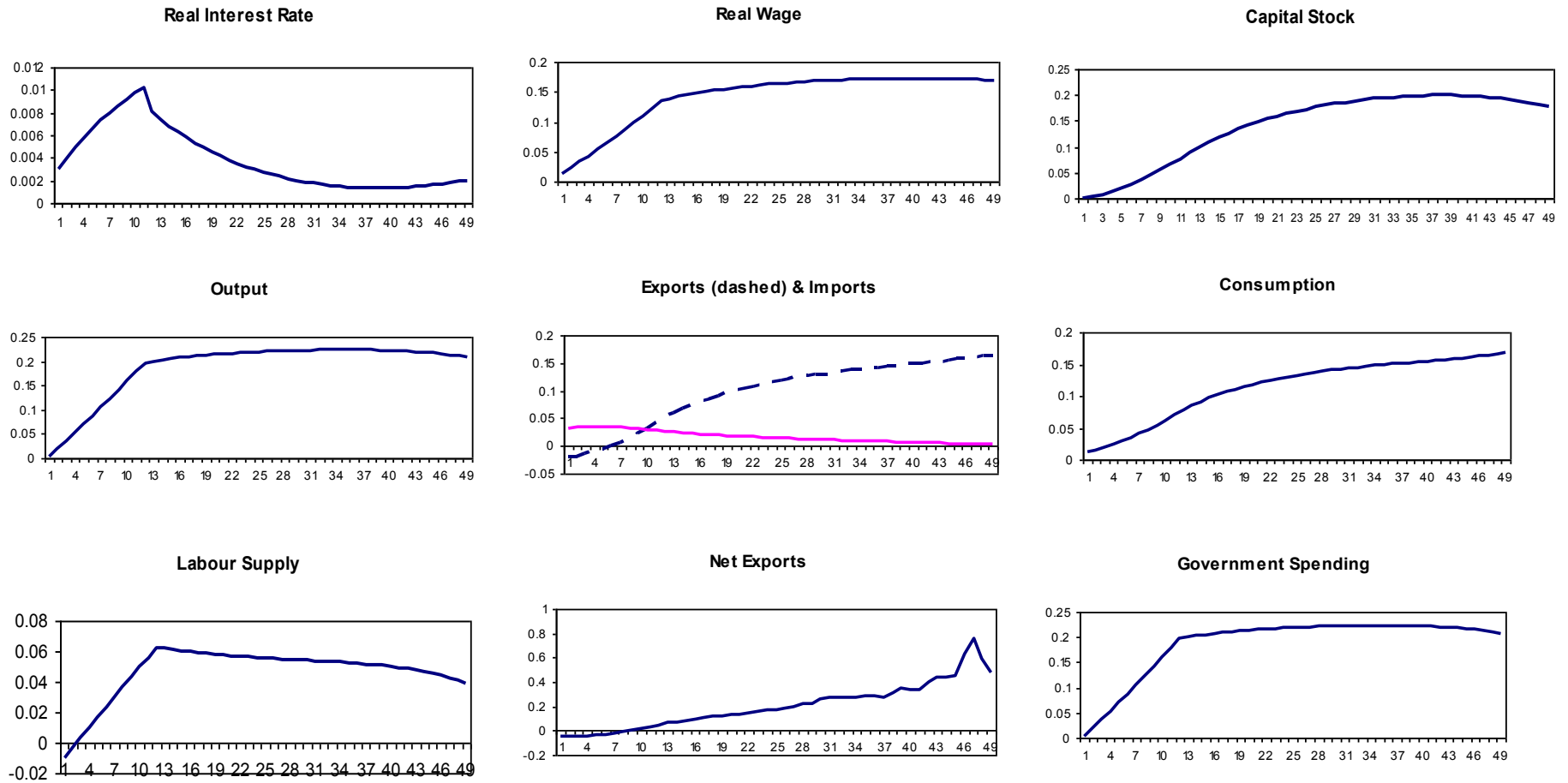
Appendix 5. Method of indirect inference.

Meenagh, Minford and Theodoridis (2008) explain the method of indirect inference. Let $x_t(\theta)$ be an $m \times 1$ vector of simulated time series dependent on the $k \times 1$ parameter vector θ and let y_t be the actual data. Assume that $x_t(\theta)$ is generated from a structural model. We assume that there exists a particular value of θ given by θ_0 such that $\{x_t(\theta_0)\}_{s=1}^T$ share the same distribution, where $S = cT$ and $c \geq 1$. Thus the null hypothesis is $H_0 : \theta = \theta_0$.

Let the likelihood function defined for $\{y_t\}_{t=1}^T$, which is based on the auxiliary model, be $L_T(y_t; a)$. The maximum likelihood estimator of a is then

$$a_T = \arg\max L_T(y_t; a)$$

The corresponding likelihood function based on the simulated data $\{x_t(\theta_0)\}_{s=1}^T$ is $L_T[x_t(\theta_0); \alpha]$. Let

$$\alpha_S = \arg\max L_T[x_t(\theta_0); \alpha]$$

Define the continuous $p \times 1$ vector of functions $g(a_T)$ and $g(\alpha_S)$ and let

$$G_T(a_T) = \frac{1}{T} \sum_{t=1}^T g(a_T) \text{ and } G_S(\alpha_S) = \frac{1}{S} \sum_{s=1}^S g(\alpha_S).$$

We require that $a_T \to \alpha_S$ in probability and that $G_T(a_T) \to G_S(\alpha_S)$ in probability for each θ. If $x_t(\theta)$ and y_t are stationary and ergodic then these hold a.s., see Canova (2005). It then follows that on the null hypothesis, $E[(g(a_T) - g(\alpha_S)] = 0$.

Thus, given an auxiliary model and a function of its parameters, we may base our test statistic for evaluating the structural model on the distribution of $(g(a_T) - g(\alpha_S))$ using the Wald statistic

$$[(g(a_T) - g(\alpha_S)]'W[(g(a_T) - g(\alpha_S)]$$

where $W = \sum_g^{-1}$ and $\sum_g$ is the covariance matrix of the (quasi) maximum likelihood estimates of $g(\alpha_S)$ which is obtained using a bootstrap simulation. The auxiliary model is a time-series model- here a univariate ARIMA- and the function $g(.)$ consists of the impulse response functions of the ARIMA. In what follows we specialize the function $g(.)$ to $(.)$; thus we base the test on a_T and α_S; the ARIMA parameters themselves. Notice that though Q and its bootstrap samples are I(1) processes, in the test they are stationarised through the ARIMA estimation.

Non-rejection of the null hypothesis is taken to indicate that the dynamic behaviour of the structural model is not significantly different from that of the actual data. Rejection is taken to imply that the structural model is incorrectly specified.

Comparison of the impulse response functions of the actual and simulated data should then reveal in what respects the structural model differs.