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Taking Multi-Sector Dynamic General Equilibrium Models to the Data*

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Abstract

We estimate and compare two models, the Generalized Taylor Economy (GTE) and the Multiple Calvo model (MC), that have been built to model the distributions of contract lengths observed in the data. We compare the performances of these models to those of the standard models such as the Calvo and its popular variant, using the ad hoc device of indexation. The estimations are made with Bayesian techniques for the US data. The results indicate that the data strongly favour the GTE.

Keywords: DSGE models, Calvo, Taylor, price-setting.

JEL: E32, E52, E58.

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1 Introduction

The estimation of Dynamic Stochastic General Equilibrium (DSGE) models has increasingly attracted the attention of economists studying monetary policy, especially since the pioneering work of Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2005). However, the existing literature tends to focus on models that ignore the heterogeneity of price-spell durations that we observe in the data. This paper focusses on estimating and comparing the models that have been developed to account for the heterogeneity in price-spell duration as found in the microdata on prices and how they perform relative to the standard models. In particular, we take a common framework in the form of a DSGE, we calibrate the alternative models using the micro-data on prices and then estimate them using Bayesian methods. One advantage of the Bayesian methodology in this context is that we can use the posterior model probability to rank the different approaches. We aim to discover if the approaches that incorporate a realistic degree of heterogeneity are better (more likely to be true) than traditional models, and which is the best overall.

Two approaches have emerged that take the micro data on pricing seriously. The Generalized Taylor Economy (GTE) and the Multiple Calvo Economy (MC) were built to model heterogeneity in the length of price-spells. The GTE is set out in Dixon and Kara (2010a) and employed in Dixon and Kara (2010b) and Kara (2010)[1]. In the GTE, there are many sectors, each

[1] The idea of having several sectors with different Taylor contract lengths originates in

with a Taylor-style contract of a specific duration (we can think of the sectors in the GTE as *duration* sectors). The MC is developed in Carvalho (2006). The MC differs from the GTE in that the model assumes that within each CPI sector there is a Calvo-style contract, rather than a Taylor contract, resulting in a range of durations for each product or CPI category. Both of these approaches are cross-sectional: they describe the way firms (or more precisely price-setters) behave. These two approaches differ in how they divide up the economy into sectors: in the MC approach, firms (products) are partitioned into product groups; in the GTE approach, the sectors are defined by the duration of price-spells. Whilst both approaches model heterogeneity of price-spell durations, the pricing behavior is very different: in the Taylor based approach, firms know the exact duration of a price-spell when the price is set, whereas in the Calvo based approach they only know the distribution of possible price-spell durations. This difference affects the extent to which firms are forward-looking when they set their prices. In the GTE, firms are more myopic when they set their prices, since they only take into account things that happen during the spell. Calvo firms, however, have to look into the infinite future, since there is a positive probability of any duration occurring.

We calibrate the share of each sector in both models using the Klenow and Kryvtsov (2008) dataset derived from US CPI data covering 1988 to 2005. The data report monthly frequencies in disaggregated CPI categories and

Taylor (1993), and has also been used in Coenen, Christoffel and Levin (2007).

can be used directly in the MC model. In order to construct the GTE distribution, we need to make an additional assumption about the distribution of durations in each sector which gives rise to the observed frequency. As in Dixon and Kara (2010b), we adopt the assumption that the distribution within each CPI sector is Calvo. We then add up for each price-spell length across all of the sectors. This approach ensures that the two models we seek to compare have *exactly the same* distributions of price-spells in aggregate.

We then proceed to estimate the models with Bayesian techniques, as in Smets and Wouters (2003), using three key time-series: inflation, output and an interest rate. As in Smets and Wouters (2007), the number of structural shocks is the same as the number of observables used in the estimation. Specifically, there are three types of shocks: productivity shocks, monetary policy shocks and mark-up shocks. We also estimate and compare the performances of these two models (GTE and MC) to those of the standard models, notably the Calvo model with indexation, as in Smets and Wouters (2007) and Christiano et al. (2005).

The findings reported in the paper indicate that the data strongly favour the GTE. An impulse response analysis suggests that the main difference between the GTE and the other models is that inflation in the GTE adjusts more sluggishly in response to productivity shocks than in the other models. We also calculate an estimated variance decomposition for each of the models, which shows how much of the variation in each variable is attributable to each of the three shocks. The GTE suggests that productivity shocks and mark-up shocks are equally important in explaining the variance of inflation and that these shocks almost entirely account for the variations in inflation. In contrast, in the other models mark-up shocks dominate productivity shocks and explain the majority of the fluctuations in inflation, which is a common finding in the literature (see, for example, Smets and Wouters (2007)).

Before describing the models, it is useful to review briefly the literature on this topic. This paper is closely related to the paper by Coenen et al. (2007). Coenen et al. (2007) consider a multi-sector model with Taylor-style contracts. However, it is important to recognize the limitation of studies like Coenen et al. (2007), since the authors consider a model that has price-spells of up to 4 periods. Clearly, generating a more realistic case requires going beyond the cases these papers consider. This issue is important, as Kara (2010) shows, because the assumptions on contract structure significantly affect policy conclusions. To see this effect, consider a utility-based objective function for a central bank by following the procedure described in Woodford (2003). The loss function of a central bank in a multi-sector model depends on the variances of the output gap and on the cross-sectional price dispersion. Ignoring the heterogeneity in price-spells underestimates the degree of price dispersion in the economy. Reduced price dispersion would make it less important to control price stability and that increases the relative weight of the output gap term in the loss function. The same arguments apply to the case studied by Carvalho and Dam (2008), who extend the Coenen et al. (2007) approach by considering price-spells of up to 8 periods.

This paper is also closely related to papers by Rabanal and Rubio-Ramirez (2005) and Laforte (2007). These papers also compare alternative pricing models by using the Bayesian approach. Rabanal and Rubio-Ramirez (2005) estimate and compare the Calvo model and its extension with indexation and with wage rigidity, as in Erceg, Henderson and Levin (2000). Laforte (2007) compares and estimate the Calvo model, the sticky information model, as in Mankiw and Reis (2002), and the Generalized Calvo model (GC), as in Wolman (1999).[2]

The rest of the paper is organized as follows. In Section 2, we outline a generic macroeconomic framework which allows us to explore the different models. In Section 3, we explain the data and the priors. In Section 4, we report our estimates and compare the models three dimensions: fit of the data, impulse responses and variance decompositions. In Section 5 we conclude.

2 The Model

The framework is based on Dixon and Kara (2010a) and Dixon and Kara (2010b), and is able to encompass all of the main price setting frameworks. When we divide the economy into sectors based on the duration of price-spells, each duration-sector with a Taylor-style contract we have a Gener-

[2] The GC generalises the Calvo model to allow the reset probability to vary with the age of the contract. Thus, in this model the hazard rate is duration dependent, rather than constant, as in the Calvo model.

alized Taylor Economy (GTE). Alternatively, we can divide the economy into sectors based on product or CPI categories, and assume a Calvo-style contract resulting in a Multiple Calvo economy (MC). The exposition here aims to outline the basic building blocks of the model. We first describe the structure of the contracts in the economy, the price-setting process under different assumptions and then monetary policy. In fact, we are able to write the equations for the GTE in a way which allows us to re-interpret them as the appropriate equations in the MC, Calvo and Calvo-with-Indexation models.

2.1 Structure of the Economy

As in a standard DSGE model, in the model economy, there is a continuum of firms $f \in [0,1]$. Corresponding to the continuum of firms f, there is a unit interval of household-unions ($h \in [0,1]$). Each firm is then matched with a firm-specific union ($f = h$)[3]. The unit interval is divided into N sectors, indexed by $i = 1...N$. The share of each sector is given by α_i with $\sum_{i=1}^{N} \alpha_i = 1$. Within each sector i, there is a Taylor process. Thus, there are i equally sized cohorts $j = 1...i$ of unions and firms. Each cohort sets the price which lasts for i periods: one cohort moves each period. The share of each cohort j within the sector i is given by $\lambda_{ij} = \frac{1}{i}$. The longest contracts in the economy are N periods.

[3]This assumption means that there is a firm- specific labour market. The implications of this assumption on inflation dynamics are well known (see, for example, Edge (2002), Dixon and Kara (2010a) and Woodford (2003)).

A typical firm produces a single differentiated good and operates a technology that transforms labour into output subject to productivity shocks. The final consumption good is a constant elasticity of substitution (CES) aggregate over the differentiated intermediate goods. Given the assumption of CES technology, the demand for a firm's output (y_{ft}) depends on the general price level (p_t), its own price (p_{ft}) and the output level (y_t) : $y_{ft} = \theta(p_t - p_{ft}) + y_t$, where θ measures the elasticity of substitution between goods. Thus, the sole communality within a sector is the length of the price contract. The other elements of the model are standard New Keynesian: the representative household derives utility from consumption and leisure and the central bank conducts monetary policy according to a Taylor rule.

2.2 Log-linearized Economy

In this section, we simply present the log-linearised economy. Note that we render nominal variables such as reset price and the price level as stationary by re-expressing them in terms of log-deviations from the aggregate price level. For example, $\bar{x}_{it}$ and $\bar{p}_{it}$ denote the logarithmic deviation of reset price and the price level in sector i from the aggregate price level, respectively.

The linearized reset price for sector i is given by

$$\bar{x}_{it} = \sum_{j=1}^{T_i} \sum_{k=j}^{T_i} \lambda_{ij+k} \left(\pi_{t+j} - a\pi_{t+j-1}\right) + \lambda_{ij} \sum_{j=1}^{T_i} \gamma \tilde{y}_{t+j-1} + \tau_t \qquad (1)$$

with

$$\gamma = \frac{(\eta_{cc} + \eta_{_{LL)}}}{(1 + \theta\eta_{_{LL}})} \tag{2}$$

Where $\tilde{y}_t = y_t - y_t^*$ is the gap between actual output, y_t and the flexible-price equilibrium output level y_t^*, π_t is the aggregate inflation rate and θ is the elasticity of substitution of consumption goods. τ_t denotes mark-up shocks. $\eta_{cc} = \frac{-U_{cc}C}{U_c}$ is the parameter governing risk aversion, $\eta_{ll} = \frac{-V_{ll}L}{V_l}$ is the inverse of the labour elasticity. In the GTE, $T_i = i$.

In each sector i, relative prices are related to the reset price i through a relation of the form

$$\sum_{j=1}^{T_i} \lambda_{ij}\bar{p}_{it-j-1} = \sum_{j=1}^{T_i} \lambda_{ij} \left(\tilde{x}_{it-j-1} - \sum_{k=0}^{j-2} (\pi_{t+k} - a\pi_{t+k-1}) \right) \tag{3}$$

where $\lambda_{ij} = \frac{1}{T_i}$ and $0 < a \leq 1$ measures the degree of indexation to the past inflation rate. The reset prices will, in general, differ across sectors, since they take the average over a different time horizon. With indexation, the initial price at the start of the contract is adjusted to take into account of future indexation over the lifetime of the contract.

The two equations (1 and 3) can also represent the MC. Here the sectors are not defined by the duration of price-spells, but rather by CPI category. The proportion of prices changing in sector i is ω_i. To obtain the MC, the reset price in sector i at time t (x_{it}), the summation is in equation (1)made with $T_i = \infty$ and $\lambda_{ij} = \omega_i(1 - \omega_i)^{j-1} : j = 1...\infty$ and with no $a = 0$. When $i = 1$, the model reduces to the standard Calvo model with a single economy wide reset price. Assuming further that $0 < a \leq 1$ extends the Calvo model to the case in which the prices are indexed to past inflation.

By using the fact that the linearized price level in the economy is the weighted average of the ongoing prices in the economy, we obtain the following :

$$\sum_{i=1}^{N} \bar{p}_{it} = 0$$

The Euler equation in terms of output gap is given by

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \eta_{cc}^{-1} (r_t - E_t \pi_{t+1} - rr_t^*) \tag{4}$$

where r_t is the nominal interest rate. $rr_t^* = r_t^* - E_t \pi_{t+1}^* = \eta_{cc} \left(E_t y_{t+1}^* - y_t^* \right)$, π_t^* and y_t^* denote the nominal interest rate, the inflation rate and the output level when prices are flexible, respectively.

The solution for y_t^* is given by

$$y_t^* = \frac{(1 + \eta_{ll})}{(\eta_{ll} + \eta_{cc})} a_t \tag{5}$$

We assume that the central bank follows a Taylor style rule under which the short term interest rate is adjusted to respond to the lagged interest rate, the inflation rate and the output gap:

$$r_t = \phi_r r_{t-1} + (1 - \phi_r) \left(\phi_\pi \pi_t + \phi_y \tilde{y}_t \right) + \xi_t$$

where ξ_t is a monetary policy shock and follows a white noise process with zero mean and a finite variance and $\phi-$coefficients denote the coefficients on the targeting variables.

3 Data

We estimate the models with Bayesian estimation techniques using three key macro-economic series at quarterly frequency[4]. Specifically, the macro-economic series are the log difference of real GDP, the log difference of the GDP deflator and the federal funds rate. Our sample covers the period from January 1984 to December 2004.[5] The reason for this choice is that this sample period is the most appropriate sample for the Klenow and Kryvtsov (2008) dataset, which covers the period from 1988 to 2005. We did not want to use data that included the great inflation, when pricing behavior might have been different.

3.1 Prior distribution of the parameters

Bayesian estimation methodology requires to specify prior distributions for the parameters we would like to estimate. These disributions are typically

[4] Appendix B provides a description of the Bayesian estimation methodology.

[5] We obtain these series from the Smets and Wouters (2007) dataset, which is available at http://www.e-aer.org/data/june07/20041254_data.zip. GDP is taken from the US Department of Commerce-Bureau of Economic Analysis databank. Real GDP is expressed in billions of chained 1996 dollars and expressed per capita by dividing it by the population over 16. Inflation is the first difference of the GDP price deflator. The interest rate is the federal funds rate. See Smets and Wouters (2007) for a more detailed description of the data.

centered around standard calibrated values of the parameters. Table 1 reports our assumptions on the priors of the parameters. We assume that the shocks are assumed to follow $AR(1)$ process. The persistence of the $AR(1)$ process is assumed to follow a beta distribution with mean 0.5 and standard deviation 0.2.We assume that the standard errors of the shocks follow an inverse-gamma distribution. Monetary policy is assumed to follow a Taylor rule. The coefficient on inflation (ϕ_π) is assumed to follow a normal distribution with mean 1.5 and a standard error of 0.125. The coefficients on the output gap$\left(\phi_y\right)$ follows a normal distribution with mean 0.125 and standard deviation of 0.05. The mean of $\phi_\pi = 1.5$ and of $\phi_y = 0.125$ are Taylor's original estimates. The lagged interest rate (ϕ_r) is assumed to follow a normal distribution of 0.75 with a standard error of 0.1. The prior on η_{cc} follow a normal distribition with mean 4 and a standard error of 0.5. The prior on θ is assumed to follow a inverse-gamma distribution with mean 8 and a standard error of 3.5. The parameter $\eta_{_{LL}}$, which denotes the inverse of the labour elasticity, is fixed in the estimation. We set $\eta_{_{LL}} = 4.5$, which is n standard assumption in the literature (see Dixon and Kara (2010a) and references therein). These assumptions are common across the models and in line with those made in much of the literature (see, for example, Levin, Onatski, Williams and Williams (2005), Reis (2008) and Smets and Wouters (2007), among others). In the IC model, following Smets and Wouters (2007), we assume that the prior distribution for the indexation parameter is a beta distribution with mean 0.5.

The share of each sector in the GTE and in the MC is calibrated according to the micro data. To do so, we use the Klenow and Kryvtsov (2008) dataset. The data are derived from the US Consumer Price Index data collected by the Bureau of Labor statistics. The period covered is 1988-2005, and 330 categories account for about 70% of the CPI. The dataset provides the frequency of price change per month for each category. We interpret these frequencies as Calvo reset probabilities. We then convert the monthly numbers to quarterly numbers and use them to calibrate a Multiple Calvo model with 330 separate sectors. To calibrate the share of each duration in the GTE, following Dixon and Kara (2010b), we generate the distribution of completed durations for that category using the formula put forward by Dixon and Kara (2006). We then sum all sectors using the category weights.[6] The distribution in months is plotted in Figure 1. Whilst there are many flexible prices with short spells, there is a long tail of price-spells lasting many quarters. However, the most common contract duration is one month. The mean price-spell is around one year. In the Calvo model and its variant with indexation, we set $\omega = 0.4$, so that the average price-spell in these models is the same as that in the other models. However, notice that with indexation, prices change every period, so that although the "contract" or price-plan lasts for 12 months, prices change every period.

[6]For computational purposes, the distribution is truncated at $N = 20$, with the 20-period contracts absorbing the weights from the longer contracts.

4 Results

This section presents our results. Firstly, we present our posterior estimates for each of the models. Secondly, we report the estimates of the marginal likelihood for each of the models. Thirdly, we report the estimated impulse response functions for inflation and output to the three shocks for each model. Finally, we report a variance decomposition analysis for each of the models. Note, in our method, we treat the price-data distributions as calibrated parameters, they are not "priors" to be updated. This reflects the fact that there is so much hard evidence about prices embodied in the pricing microdata. In this we differ from Carvalho and Dam (2010) who use the microdata as to form a prior.

We compare 4 different models within our common framework. From Klenow and Kryvstov (2008) we calibrate the GTE ($i = 1...20$) and the 330 sector MC model. We also have the Calvo model with and without indexation.

4.1 Posterior estimates of the parameters

Table 1 reports the means of the posterior distributions of the parameters obtained by the Metropolis-Hastings algorithm[7].

[7] The posterior distributions reported in Table 1 have been generated by $20,000$ draws, from a Metropolis Hastings sampler. The first 20% of draws are discarded. In estimating each model, a step size is chosen to ensure a rejection rate of 70%. Various statistical convergence tests show that the Markov chains have converged. An appendix that documents these tests is available from the authors upon request.

	Prior Distribution			Posterior Distribution			
	Distribution	Mean	SD	Mean			
				GTE	MC	Calvo	IC
θ	Invgamma	8	3.5	7.98	7.59	7.83	7.58
η_{cc}	Normal	4	0.5	4.56	4.41	4.57	4.72
ϕ_r	Beta	0.5	0.1	0.78	0.78	0.78	0.78
ϕ_π	Normal	1.5	0.25	1.64	1.87	1.72	1.58
ϕ_y	Normal	0.125	0.05	0.13	0.10	0.11	0.12
ρ_z	Beta	0.5	0.2	0.99	0.99	0.99	0.99
ρ_τ	Beta	0.5	0.2	0.86	0.85	0.68	0.50
ρ_r	Beta	0.5	0.2	0.65	0.57	0.59	0.61
σ_z	Invgamma	0.6	0.6	1.32	1.25	1.22	1.26
σ_τ	Invgamma	0.6	0.6	0.21	0.13	0.16	0.25
σ_r	Invgamma	0.1	0.2	0.13	0.13	0.13	0.13
a	Beta	0.5	0.15	-	-	-	0.51

Table 1: Prior and posterior distribution of parameters and shock processes (Note: SD stands for standard deviation)

The parameter estimates are surprisingly similar across the different models, with the major exception of the persistence of the mark-up shocks ρ_τ. In all four models the productivity shocks are nearly a unit root process. The monetary policy shock is less persistent compared to the productivity shock and the persistence parameter is around 0.6. The mark-up shocks are highly persistent in the GTE and in the MC: the persistence parameter is around 0.85. In the case of the Calvo and IC, the mark-up shocks are not as persistent as in the GTE and MC. In the Calvo model, the persistence parameter of the mark-up shock is around 0.7, whereas in the IC model, it is around 0.5.

The reason why the mark-up shocks are less persistent in the IC model seems to be related to the presence of indexation in that model. We estimate the mean degree of indexation to be 0.51. This estimate is higher than that of Smets and Wouters (2007). Smets and Wouters (2007) estimate the parameter to be 0.24. However, Smets and Wouters (2007) estimate that mark-up shocks are highly persistent, with an AR(1) coefficient of 0.9. It appears that the presence of indexation reduces the need for persistent mark-up shocks and there is a trade-off between the degree of indexation and the persistence of mark-up shocks: the more persistent the mark-up shocks the lower the indexation or vice versa. Indeed, Rabanal and Rubio-Ramirez (2005) assume that the mark-up shocks follow a white-noise process and estimate a higher degree of indexation at around 0.67. It should also be noted that it appears that the data is not informative on the indexation parameter, as indicated by the fact that the posterior and prior distributions are quite similar. This is not surprising, as there is little micro-evidence of this type of indexation occurring. The assumption of indexation implies that all prices change each period, whereas, as discussed above, the micro-evidence suggests that they remain unchanged for several months.

The mean of the standard error of the productivity shock in each model is around 1.3. In contrast, the standard deviations of the monetary policy and mark-up shocks are relatively low. The standard deviation of the mark-up shocks in the MC and the Calvo is around 0.15, whereas in the GTE and in the IC, it is slightly larger, at around 0.2. The standard deviation of the monetary policy shock in each model is 0.13.

Turning to the estimates of the behavioural parameters (η_{cc}, θ), the means of the posterior distributions for both parameters in each model are similar to those of the prior distributions. The posterior mean of θ is around 8, while the posterior mean of η_{cc} is around 4.5. The estimates are in line with the typical calibration of these parameters and with the estimates reported by Rabanal and Rubio-Ramirez (2005).[8] The estimate of η_{cc} implies an elasticity of intertemporal substitution $\eta_{cc}^{-1} \approx 0.2$.

Finally, we focus on the coefficients on the targeting variables in the monetary policy rule. The table indicates that there is little difference between the estimates. All of the models suggest a strong reaction to inflation by policy makers. There is a significant degree of interest rate smoothing. The mean of the coefficients on the lagged interest rate is as high as 0.8. The coefficient on the output gap is small at around 0.1. Perhaps the most notable difference here is that the MC and the Calvo models suggest a slightly stronger reaction to inflation than the GTE and the IC. The MC suggests that the coefficient on inflation is around 1.9; whereas, according to the GTE, it is around 1.6. The estimates of the coefficient on the output gap and on the interest rate smoothing parameter are similar to those reported by Clarida, Gali and Gertler (2000), Smets and Wouters (2007) and Rabanal and Rubio-Ramirez (2005).

[8] Rotemberg and Woodford (1998) obtain a similar estimate by using a different estimation method. Rotemberg and Woodford (1998) estimate the Calvo model by minimizing the distance between model-based and VAR impulse responses.

4.2 Model Comparison

We now turn to our main question: which model do the data favour? Bayesians typically present posterior odds and Bayes factors to compare models, which can be used to calculate posterior model probabilities. Before presenting our results, let us briefly describe these concepts, for those who are unfamiliar with them (see Kass and Raftery (1995) and Schorfheide (2008) for a more detailed description). We denote models by M_i for $i = 1, ..m$. The posterior model probability of model i is given by

$$p\left(M_i \mid y\right) = \frac{p\left(y \mid M_i\right) p\left(M_i\right)}{\sum_j p\left(y \mid M_j\right) p\left(M_j\right)} \tag{6}$$

where $p\left(M_i \mid y\right)$ is the posterior model probability, $p\left(y \mid M_i\right)$ is the marginal likelihood and $p\left(M_i\right)$ is the prior model probability Note that $\sum_{i=1}^{m} p\left(M_i \mid y\right) = 1$. Consider for example the case in which there are only two models, then the posterior odds ratio $\left(P_{ij}^{O}\right)$ is given by

$$P_{ij}^{O} = \frac{p\left(M_i \mid y\right)}{p\left(M_j \mid y\right)} = \frac{p\left(y \mid M_i\right) p\left(M_i\right)}{p\left(y \mid M_j\right) p\left(M_j\right)} \tag{7}$$

By using the fact that $p\left(M_1 \mid y\right) + p\left(M_2 \mid y\right) = 1$ and $P_{12}^{O} = \frac{p(M_1|y)}{p(M_2|y)}$, we can express $p\left(M_1 \mid y\right)$ as

$$p\left(M_1 \mid y\right) = \frac{P_{12}^{O}}{1 + P_{12}^{O}}. \tag{8}$$

$p\left(M_2 \mid y\right)$ is given by$1 - p\left(M_1 \mid y\right)$. The Bayes factor $\left(B_{ij}\right)$ is given by

$\frac{p(y|M_i)}{p(y|M_j)}$. Thus, to put it differently, posterior odds are given by

$$\text{posterior odds=Bayes factor*prior odds}$$

When there are more than two models to compare, then we choose one of the models as a reference model and calculate Bayes factors relative to that model.

The first row of Table 2 presents the log-marginal likelihood of each model, the second row of the table reports Bayes factors, where we assume that the GTE is the reference model, and, finally, the third row of the table gives posterior model probabilities.

	GTE	MC	Calvo	IC
Log Marginal Likelihood ($lnp\,(y \mid M_i)$)	-43.46	-62.93	-63.12	-62.91
Bayes Factors relative to the GTE	e^{0}	$e^{19.47}$	$e^{19.66}$	$e^{19.45}$
Posterior Model Probability (%)	1.00	0.00	0.00	0.00

Table 2: Statistical measures to compare models

We first compare the models according to Bayes factors. The use of Bayes Factors to compare models was first suggested by Jeffreys (1935) (cf. Kass and Raftery (1995)). Jeffreys (1961) suggests the following rule of thumb for interpreting Bayes factors:

Bayes Factors (B_{ij})	
1 to 3.2	Not worth more than a bare mention
3.2 to 10	Substantial
10 to 100	Strong
>100	Decisive

Table 3: Jeffrey's guidelines for interpreting Bayes factors

The data provide "decisive" evidence for the GTE. Surprisingly, introducing heterogeneity to the Calvo model does not improve its empirical performance. The Bayes factor between the MC and the Calvo is only $e^{0.19}$, which, according to Jeffreys's guidelines, means that there is evidence for the MC but it is "not worth more than a bare mention". This is also true for the IC. Adding indexation to the Calvo model does not significantly improve its ability to explain the data. The latter result is in line with the findings reported in Coenen et al. (2007) and Smets and Wouters (2007).

Kass and Raftery (1995) suggest alternative guidelines for interpreting Bayes factors, which are reported in Table 4. Kass and Raftery (1995) propose to consider twice the natural logarithm of the Bayes factor. The Kass and Raftery (1995) guideline is useful as it is on the same scale as the likelihood statistics.

$2lnB_{ij}$	B_{ij}	
0 to 2	1 to 3	Not worth more than a bare mention
2 to 6	3 to 20	Positive
6 to 10	20 to 150	Strong
>10	>150	Very Strong

Table 4: The Kass and Raftery (1995) guidelines for interpreting Bayes factors

The conclusions, however, do not change if we consider the Kass and Raftery (1995) guidelines, rather than Jeffreys's guidelines. The third row of Table 2 reports the posterior model probabilities, under the assumption that the models have equal prior probabilities. The probability that the GTE is the correct model, among the models considered, is one.

We also estimate a Carvalho and Dam (2008) (CD) style GTE. Carvalho and Dam (2008) consider a multi-sector economy that has price-spells of up to 8 periods. To achieve this, we truncate the KK-distribution plotted in Figure 1 at $N = 8$, with the 8-period contracts absorbing the weights from the longer contracts.[9] The main advantage of this approach is that the CD-GTE is computationally easier to estimate than the KK-GTE. However, this simplification comes at a cost. The CD-GTE performs worse than the KK-GTE. The marginal likelihood for the CD-GTE is -48.3. The Bayes factor between the KK-GTE and CD-GTE is $e^{4.8}$. According to the Jeffreys guidelines, there is again decisive evidence for the KK-GTE. This is also almost the case with the Kass and Raftery (1995) guidelines. In this case, the evidence for the KK-GTE is strong. Clearly, there is a trade-off to be made in terms of how many sectors you have in the GTE, and the optimal choice will depend on the particular application. In our case, since we wanted to have exactly the same distribution for the GTE and the MC, we needed

[9]However, their estimates are not dissimilar to the numbers we use to estimate the CD-GTE. The sectoral weights we use to estimate the CD-GTE are as follows: $\alpha_1 = 0.30$, $\alpha_2 = 0.12$, $\alpha_3 = 0.10$, $\alpha_4 = 0.08$, $\alpha_5 = 0.07$, $\alpha_6 = 0.05, \alpha_7 = 0.04$ and $\alpha_8 = 0.22$. We also used the estimates reported in Carvalho and Dam (2008) to estimate the model, and the results do not change significantly.

$N = 20$ to capture the long tail in the Calvo distributions. Note that the CD methodology differs from ours in that we treat the price-data distributions as calibrated parameters, whereas Carvalho and Dam (2010) use the microdata to form a prior to estimate the share of each duration.

4.3 Impulse Responses

In order to understand why the GTE explains the data better than the other models, we have studied the impulses responses of output and inflation in each model to each of the three shocks. Figure 2 reports the mean estimated impulse response functions of inflation and Figure 3 the corresponding responses for output.

A key difference among the models arises when it comes to the effects of productivity shocks. As Figure 2 shows, the inflation response to a productivity shock in the GTE is very different from the responses in the other models. Inflation in the GTE has a hump, peaking at the 20th quarter, whereas in the MC and in the Calvo models, the maximum effect of a productivity shock is on impact and the responses are less persistent compared to that in the GTE. The IC model also has a hump-shaped response but the peak response is rapid compared to the GTE. Productivity shocks are highly persistent: under all model specifications the posterior is 0.99. A positive productivity shock gives a long and lasting negative effect on prices (in all models, there is still a clear effect even after 40 quarters). Why should the GTE behave so differently? The key concept here is that on average the firms resetting prices are less forward looking in the Taylor framework: they know exactly how long their price-spells will last and when they reset their prices they only look forward as far as their price is going to last. This makes the price-setters on average more myopic (less forward looking[10]) than in the different Calvo frameworks, where they are uncertain as to the duration of the price-spell and therefore all have to take into account the distant future. This means that reset prices respond less in the GTE on impact, so that the general price-level responds more sluggishly in impact. The full impact of the productivity shock takes time to feed through and eventually peaks at a little under 20 quarters. Note that the peak effect is less than the peak effect in the GTE is less than the peak in the other models: this is because even with an autoreggressive coefficient of 0.99, after 20 quarters almost 20% of the shock has died away.

If we look at the effect of mark-up shocks on inflation, as in the case of productivity shocks, we see that inflation in the GTE adjusts more sluggishly compared to the other models, although the difference in responses in the case of mark-ups are not as great as in the case of productivity shocks. In the GTE, the effect of mark-up shocks dies out after approximately 20 quarters, whereas in the MC, it dies out after 12 quarters.

Turning to te response of inflation to monetary policy shocks, the responses in the Calvo model and in the IC model is considerably less persistent that those in the GTE and in the MC. In the IC model, the effect of the

[10] For a formalisatin of the concept of forward lookingness, see Dixon (2012).

shocks dies before 10 quarters. The responses of inflation to monetary policy shocks are similar across the three models. This result is in contrast to several other studies: Dixon and Kara (2010b), Dixon and Le Bihan (2012) where the GTE has a hump shaped response in contrast to the other specifications. Why is there not a hump shaped response of inflation in this model? The reason for this is that, as we will show when we look at the variance decomposition of shocks: monetary policy does not have an important role to play in determining inflation. Prices respond much more to productivity shocks because these are highly persistent. In Dixon and Kara (2010b), Dixon and Le Bihan (2012), the impulse responses were derived in a framework without any productivity or mark up shocks. In this case, monetary policy alone drives inflation and we get the hump shaped response of inflation with the GTE. Is it reasonable to find that monetary policy in our model plays so little a role in explaining inflation? We believe it is in this time period of the great moderation. The period 1988-2005 is the great moderation when a mixture of sound monetary policy and benign supply-side shocks kept inflation low. What the estimated models are telling us is that most of the variation in inflation we see in the data over this period was coming from real shocks. This is not to say that at a causal level monetary policy had little effect: one would expect sound monetary policy to have the feature that it did not contribute much to the variance in inflation becuase it is designed to reduce the variance.

We will now consider the effects of the three shocks on output. As Figure

3 shows, the responses are more similiar, except that in response to a mark-up shock the GTE is more persistent than those in the other models. There are slight differences in the responses of output to monetary and productivity shocks, but these are minor. Again, this is similar to what has been found in other models: the key differences between the models are found in the response of inflation.

Table 3 and 4 present the variance decompositions associated with the estimates presented above for the contribution of each shock to the total variance.

	Shock		
Model	Productivity	Mark-up	Monetary
GTE	47.81	48.43	3.75
MC	25.56	70.34	4.07
Calvo	41.71	52.54	5.75
IC	37.93	53.91	8.17

Table 5: Variance Decompostion of inflation (in percent)

	Shock		
Model	Productivity	Mark-up	Monetary
GTE	82.48	4.03	13.49
MC	81.45	9.29	9.26
Calvo	85.71	3.82	10.47
IC	86.13	3.91	9.96

Table 6: Variance Decompostion of output growth (in percent)

Table 4 reports the variance decompositions for output. As the table shows, there is no significant difference between the models: all of them suggest that the variance of output is almost entirely accounted for by productivity shocks. However, this is not the case when it comes to inflation. Table 3 reports the variance decompositions for inflation. The GTE suggests that both the mark-up and productivity shocks are equally important in explaining the variance of inflation. In the MC the mark-up shock is by far the most important. Specifically, the MC suggests that around 75% of the variance is attributable to the mark-up shocks. In the Calvo and IC, the mark-up shocks account for around 60% of the variations in inflation. The latter result is in line with the findings reported in Smets and Wouters (2007) and is unsurprising, since in these models the response of inflation to productivity shocks is muted compared to that in the GTE. Finally, in line with the findings reported in Smets and Wouters (2007) and in Christiano et al. (2005), monetary policy shocks are relatively unimportant for these two variables.

5 Conclusions

In Dixon and Kara (2010), we proposed the concept of the Generalized Taylor Economy (GTE), in which there can be many sectors with different price-spell durations, to model macroeconomic adjustment in a way that is consistent with microdata on prices. In this paper, we develop a common framework that enables us to estimate and compare the GTE and other alternatives: in particular the Multiple Calvo (MC) model, in which there are Calvo style contracts within each sector as in Carvalho (2006), the Calvo model and the widely used Calvo-with-indexation, as in Christiano et al. (2005) and Smets and Wouters (2007). We use Bayesian methods to estimate and compare these models. It should be emphasised that the GTE and MC model have exactly the same distribution of price-spell durations by construction. The indexed-Calvo model we know to be wrong: it implies that all prices adjust every period which flies in the face of the emprical evidence on prices. We include it as a useful reference point, since it has become the standard model used in the literature.

Our results indicate that the data strongly favours the GTE. A main difference between the GTE and its popular alternatives arises when it comes to how inflation responds to productivity shocks. In the GTE, inflation exhibits a delayed response to productivity with a hump shape peaking at the 20th quarter. In the other three models the adjustment is more rapid compared to that in the GTE. Moreover, inflation in the GTE adjusts more sluggishly in response to mark-up shocks compared to the other models. A variance decomposition analysis indicates that in the GTE, mark-up shocks and productivity shocks are equally important in explaining the variations in inflation, whereas the other models attribute most of this variation to mark-up shocks. The reason for the better performance of the GTE arises from the fact that the pricing decisions of firms are more myopic since they know how long their prices will last for (as in the Taylor framework) than in the other Calvo settings. This tends to make the response of inflation more sluggish and leads to a possible hump shape. We did not find a hump-shaped response of inflation to monetary policy for the GTE, unlike other papers. This reflects that fact that in our estimated model, monetary policy is not an important source of variation in inflation.

The implications of our results are that we can use the Bayesian framework to evaluate and compare different ways of modelling pricing behavior in macroeconomic models. Using Bayes factors, we find that the ex post model probability of the GTE is almost 1: the other three models are many times less likely relative to the GTE. The general framework we have adopted is simple and abstracts from factors that may be of interest to policy makers such as capital accumulation and an explicit credit channel. These factors might alter the relative performance of the models. However, we hope to have shown the promise of a model that uses empirical data to model the heterogeneity in price-spell durations and a possible method for comparison and evaluation.

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6 Appendix A: The Model

6.1 Firms

A typical firm in the economy produces a differentiated good which requires labour as the only input, with a CRS technology represented by

$$Y_{ft} = A_t L_{ft} \tag{9}$$

where $a_t = \log\ A_t$ is a productivity shock. $f \in [0, 1]$ is firm specific index. Differentiated goods $Y_t(f)$ are combined to produce a final consumption good Y_t. The production function here is CES and corresponding unit cost function P_t

$$Y_t = \left[\int_0^1 Y_{ft}^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}, P_t = \left[\int_0^1 P_{ft}^{1-\theta} df\right]^{\frac{1}{1-\theta}} \tag{10}$$

The demand for the output of firm f is given by

$$Y_{ft} = \left(\frac{P_{ft}}{P_t}\right)^{-\theta} Y_t \tag{11}$$

The firm chooses $\{P_{ft,}Y_{ft}, L_{ft}\}$ to maximize profits subject to (9, 11), and this yields the following solutions for price, output and employment at the firm level given $\{Y_t, W_{ft}, P_t\}$.

$$P_{ft} = \frac{\theta}{\theta - 1} \frac{W_{ft}}{A_t} \tag{12}$$

$$Y_{ft} = \left(\frac{\theta}{\theta - 1}\right)^{-\theta} \left(\frac{W_{ft}}{A_t P_t}\right)^{-\theta} Y_t \tag{13}$$

$$L_{ft} = \left(\frac{\theta}{\theta - 1}\right)^{-\theta} \left(\frac{1}{A_t}\right) \left(\frac{W_{ft}}{A_t P_t}\right)^{-\theta} Y_t \tag{14}$$

Price is a markup over marginal cost, which depends on the wage rate (W_{ft}) and the sector specific productivity shocks.

6.2 Household-Unions

The representative household h has a utility function given by

$$U_h = E_t \left[\sum_{t=0}^{\infty} \beta^t \left[U(C_{ht}) + V\left(1 - H_{ht}\right) \right] \right] \tag{15}$$

where C_{ht}, H_{ht} are household $h's$ consumption and hours worked respectively, t is an index for time, $0 < \beta < 1$ is the discount factor, and $h \in [0, 1]$ is the household specific index.

The household's budget constraint is given by

$$P_t C_{ht} + \sum_{s^{t+1}} Q(s^{t+1} \mid s^t) B_h(s^{t+1}) \leq B_{ht} + W_{ht} H_{ht} + \Pi_{ht} - T_{ht} \tag{16}$$

where $B_h(s^{t+1})$ is a one-period nominal bond that costs $Q(s^{t+1} \mid s^t)$ at state s^t and pays off one dollar in the next period if s^{t+1} is realized. B_{ht} represents the value of the household's existing claims given the realized state of nature. W_{ht} is the nominal wage, Π_{ht} is the profits distributed by firms and $W_{ht}H_{ht}$ is labour income. Finally, T_t is a lump-sum tax.

The first order conditions derived from the consumer's problem are as follows:

$$u_{ct} = \beta R_t E_t \left(\frac{P_t}{P_{t+1}} u_{ct+1} \right) \tag{17}$$

$$\sum_{s_{t+1}} Q(s^{t+1} \mid s^t) = \beta E_t \frac{u_{ct+1} P_t}{u_{ct} P_{t+1}} = \frac{1}{R_t} \tag{18}$$

$$W_{it} = \frac{\theta}{\theta - 1} \frac{V_L \left(1 - H_{it}\right)}{\left[\frac{u_c(C_t)}{P_t}\right]} \tag{19}$$

Equation (17) is the Euler equation. Equation (18) gives the gross nominal interest rate. Equation (19) shows that the optimal wage in sector i (W_{it}) is a constant "mark-up" over the ratio of marginal utilities of leisure and marginal utility from consumption. Note that the index h is dropped in equations (17) and (19), which reflects our assumption of complete contingent claims markets for consumption and implies that consumption is identical across all households in every period ($C_{ht} = C_t$).

7 Appendix B: The Bayesian estimation methodology

The Bayesian estimation methodology involves the following steps:

- Step 1, the log-linearised model is solved to obtain a state equation in its predetermined variables.
- Step 2 prior distributions are specified for the parameters to be estimated. The distributions are centered around standard calibrated values of the parameters.
- Step 3 the likelihood function is derived using the Kalman filter.
- Step 4 involves combining this likelihood function with prior distributions over the parameters to form the posterior density function.
- Finally, Step 5 involves numerically deriving the posterior distributions of the parameters using a Monte Carlo Markov Chain (MCMC) algorithm. The MCMC method we use is Metropolis-Hastings.

An and Schorfheide (2007) provide a detailed description of the Bayesian methodology. All these calculations are performed by using Dynare (see Juillard (1996)).

Note that following An and Schorfheide (2007) and Smets and Wouters (2007), we assume that the number of observables equals the number of shocks to remove the singularity of the covariance matrix of the endogenous variables. If the number of shocks are less than the observables, then a stochastic singularity problem arises. In this case, the model suggests that certain combinations of the endogenous variables will be deterministic. If these relationships do not hold in the data, likelihood estimation will fail. An alternative approach to coping with stochastic singularity is to add measurement errors to the model (see for example Ireland (2004)).

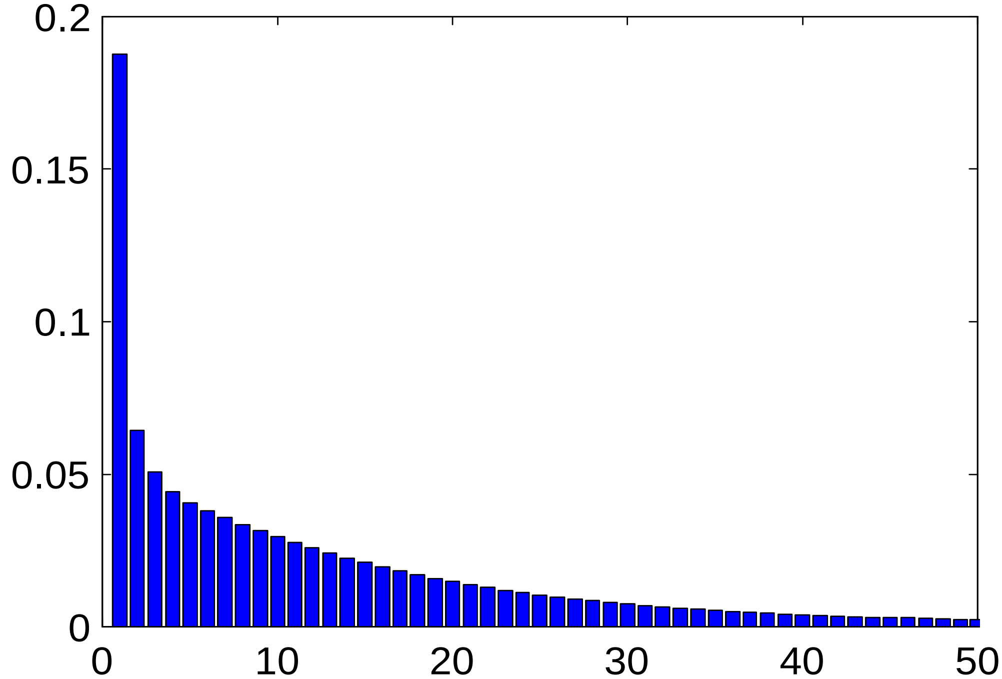

Figure 1: KK-distribution

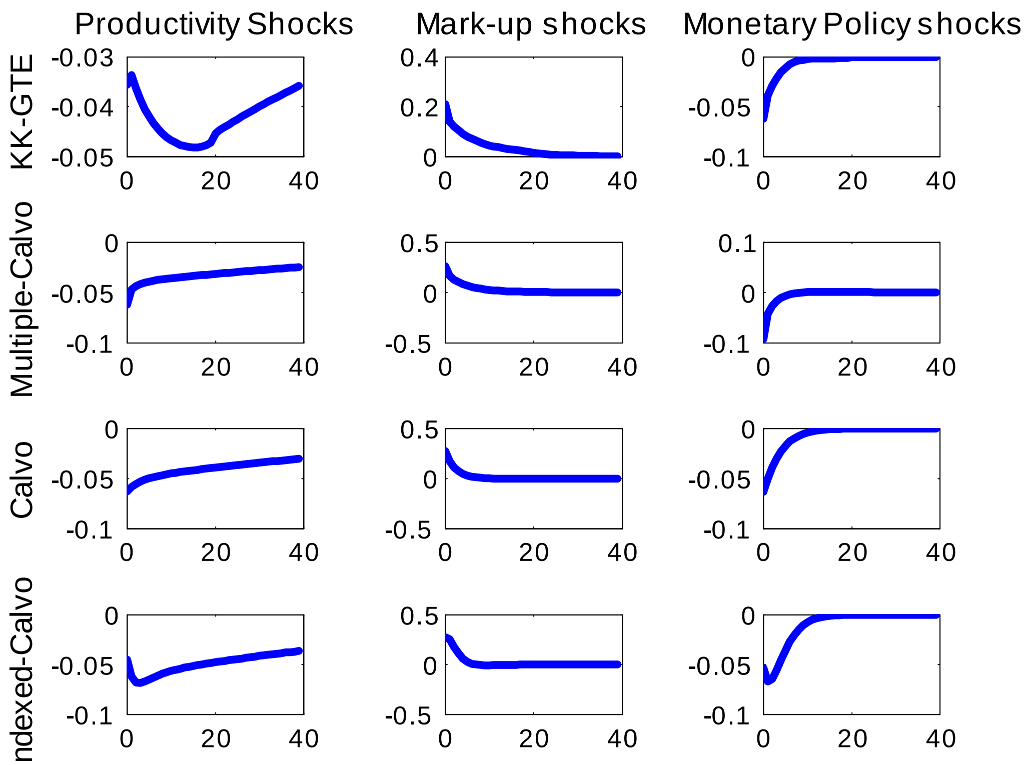

Figure 2: The estimated mean response impluse functions of inflation to the three shocks.

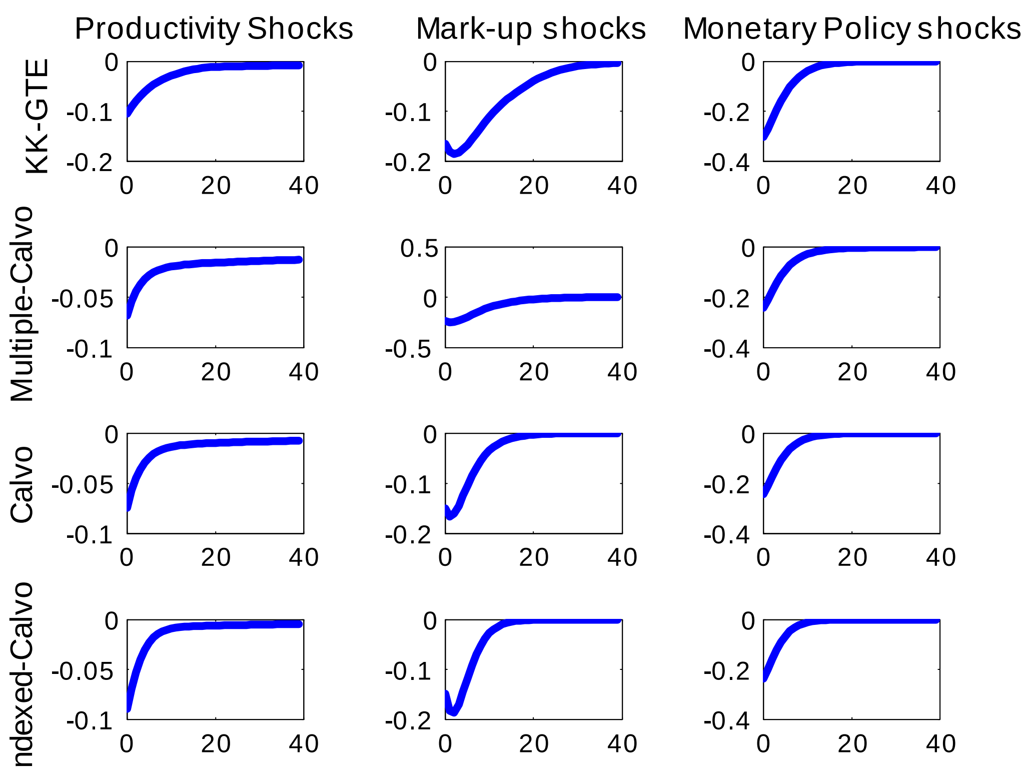

Figure 3: The estimated mean response impluse functions of the output gap to the three shocks.