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The Rise of the English Economy 1300-1900: A Lasting Response to Demographic Shocks

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The Rise of the English Economy 1300-1900: A Lasting Response to Demographic Shocks

James Foreman-Peck[1] and Peng Zhou[2]

Abstract

We construct a Dynamic Stochastic General Equilibrium model of the interaction between demography and the economy for six centuries of English history. At the core of the four overlapping generations, rational expectations model is household choice about target number and quality of children, as well as female age at first marriage. The parameters are formally estimated rather than calibrated. Data on births, deaths, population and the real wage, and data moments can be closely matched by the estimated model. We show that the marriage age rises to reach that typical of the Western European Marriage Pattern at the end of the high mortality epoch of the 14th century. Higher marriage age lowers costs of child quality so that human capital gradually accumulates over the generations. But it does so more slowly than that of population initially, so that there is a negative correlation between population and wage. Ultimately the growth of human capital catches up with that of population and triggers a break out from the Malthusian equilibrium at the end of the 18th century. Without the contribution of late female age to human capital, human capital would have been about 20% of what it actually was around 1800, and real wages would only have attained about half their actual value.

Key Words: Economic development, Demography, DGSE model, English economy

JEL Classification: O11, J11, N13

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The Rise of the English Economy 1300-1900: A Lasting Response to Demographic Shocks

Whether the fourteenth century in England was a Malthusian crisis of over-population, as Postan and Titow (1959) contended, or simply an exogenous series of famines and plagues of extraordinary intensity and frequency (Hatcher and Bailey, 2001 pp55-65), it is clear that there was a massive decline in population, perhaps until the middle of the fifteenth century. Sustained high mortality rates are apparent from two longitudinal studies of late medieval monasteries (Hatcher, 1986; Bailey, 1996). In the Durham area, tenant numbers imply that population fell to 45 percent of pre-Black Death levels by the end of fourteenth century, and tithe evidence indicates a similar collapse of output (Dodds, 2004). Clark's (2007) calculations from wage data reach a broadly comparable conclusion for aggregate English population.

This appalling shock is associated with the emergence of the Western European Marriage Pattern (De Moor and Zanden, 2010). The pattern might therefore be interpreted as a means of avoiding over-population, breaking with the pervasive Malthusian regime (Ashraf and Galor, 2011), or maintaining the high wages that emerged in this period of labour scarcity.

The contention of this paper is that these changes set in motion a process that eventually culminated in the First Industrial Revolution[3]. Later female age at first marriage not only constrained population growth when mortality rates fell but also created more opportunities for female learning[4]. Through their mothers, increases in human capital were thus passed on to children (Foreman-Peck, 2011). Slow human capital accumulation gradually raised productivity over the centuries. In the key role of females and 'companionate' English marital relations (Stone, 1977) our model has affinities with that of Diebolt and Perrin (2013a, 2013b), but they focus on female empowerment and returns to education. Lagerlof (2003) also has explored long run demographic experience and industrial revolutions in a model quite similar to our own but does not include wages and nor does he explicitly measure the contribution of

[3] Alternative models include the idea that a greater population, accelerating the rate of technical progress, can break with the Malthusian regime (Galor and Weil, 2000), but this is not supported by the evidence from English data before the nineteenth century (Crafts and Mills, 2007). Another possible source of technological acceleration out of a Malthusian regime is natural selection in favour of higher child quality (Galor and Moav, 2002), a process in some respects similar to that proposed here but which depends not on selection but on parental preferences.

[4] The fourteenth century saw the first English book written by a female (Julian of Norwich, b 1342) and the first autobiographer (Margery Kempe, c1373). As a more practical use of female literacy, in 1448 Margaret Paston wrote to her husband requesting crossbows to defend their manor on the grounds that there was no room to use long bows (The Paston Letters, 1983 pp13-14, OUP).

human capital accumulation[5]. Our Dynamic Stochastic General Equilibrium (DSGE) model of this process for the English economy and population provides a closer fit to historical experience than can be obtained with other recent modelling styles. The stochastic approach allows more rigorous model testing than does deterministic calibration, by comparing second moments (as well as means) of the observable series. Also it permits distinguishing the consequences of different shocks to elicit more detailed consequences of the model[6].

Calibrated deterministic models tend to characterise human capital accumulation as an investment and focus exclusively on the contribution of falling death rates to raising the return to human capital (Boucekkine et al, 2003; Cervellati and Sunde, 2005). An empirical drawback of these interpretations is the apparent timing of human capital accumulation, considerably earlier than the Industrial Revolution growth spurt (Clarke, 2005 fig 7; Ahearn et al, 2009; Baten and van Zanden, 2008). A second disadvantage is that the English demographic pattern is distinguished from those of France and Sweden in the dominant role played by a rising birth rate (Wrigley and Schofield, 1981 p247).

DSGE modelling techniques in the recent years have been widely used by central banks to explain economic behaviour since the Second World War and draw policy implications both for the US (Yun, 1996; Chari et al., 2000; Christiano et al., 2005; Smets and Wouters, 2007) and for the Euro zone (Smets and Wouters, 2005). To our knowledge, however, the present exercise is the first attempt to explain such a long history with a DGSE model. A fundamental advantage of this approach over quasi-Malthusian VARs is that the VAR models require all explained variables to be observable (Nicolini, 2007; Moller and Sharp, 2008; Crafts and Mills, 2009), whereas unobservable endogenous variables (in particular, human capital) can also be derived and simulated in our DSGE model. Moreover, the DSGE model can clearly identify the structural shocks and their implications, whereas identification is always problematic with VARs.

Another novelty is the four-generation overlapping generation structure which allows us to explore the link between demographic and productivity shocks, incorporating human capital (quality) and fertility (quantity) into agents' decision. We supplement the late female marriage age effects with externalities from human capital formation and explain the onset of the Industrial Revolution by distant historical demographic shocks.

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[5] The Lagerlof model is restricted to two generations with a life expectation of 50 years compared with our four, and sixty years.

[6] Unlike Lagerlof (2003) we model multiple mortality rates and shocks.

In the following section 1 we discuss the data. We then specify a simplified and static version of the model in section 2 so as to elucidate the process we regard as critical. Section 3 presents the DSGE model and the shock structure, followed by the results in section 4. Section 5 is a concluding discussion.

1. The Data

We use Wrigley and Schofield's (1981) mortality and fertility rates which are established by family reconstitution of 404 Anglican parish registers of baptism, marriage and burial. English parish registration began in 1541. Wrigley and Schofield also calculated population by 'Back Projection' with aggregates from these parish registers. This involved back dating and revising the age structure of the 1871 population census at five year intervals by taking into account flows of previously occurring 'events'. Age at death was not stated in the register before 1813 so deaths were allocated to age groups by a model mortality schedule.

Wrigley and Schofield made other adjustments, to account for migration for instance. Migration from parishes of birth can bias estimates of mean marriage age and life expectancy even when demographic characteristics of migrants and non-migrants are similar (Ruggles, 1992). Only those married in their place of birth are included in the family reconstitution. Late marriage is more likely to take place after migration and so to be systematically excluded from the data. Early deaths are over-represented in reconstitutions because those who live longer and have time to migrate will die elsewhere—and be omitted.

For real wages we utilise Clark's (2013) series that brings together the wage and price data collections begun by Thorold Rogers (1866) in the nineteenth century and continued by other researchers, as well as Clark. Clark computes real wages back to 1200, when they were apparently higher than in 1800. In 1450 they were higher than at both these dates, an outcome conventionally attributed to labour scarcity. Clark's indices are generally reckoned to replace the earlier Phelps-Brown-Hopkins (1956) series.

The negative correlation between population and wages is clear until the end of the 18^{th} century, after which both population and wages start to grow together (Crafts and Mills, 2009). That is to say, the timing of take-off in wages lagged behind that in population. Population accumulated faster than human capital after the 14^{th} century. This created a Malthusian epoch before the 18^{th} century, because human capital growth was not high enough to compensate for the diminishing marginal product of labour due to faster population growth. Only towards the end of the 18^{th} century did the human capital growth rate eventually converge to the balanced growth path, breaking into the Solow epoch.

Unlike the demographic data and wages, other series on the economy—human capital in particular—are not available continuously (e.g. Baten and van Zanden, 2008; A'Hearn et al, 2009). Nonetheless, indications of their levels and trends at various dates can be gleaned from fragmentary data—as has Clark (2005) with the following Figure 1. Simple inspection of the figure suggests that the rise in male and female literacy was proceeding gradually and steadily long before the standard date for the industrial revolution. Similar results are obtained for numeracy by examining age heaping (A'Hearn et al, 2009).

Figure 1 The Long Rise of Human Capital (Literacy) in England

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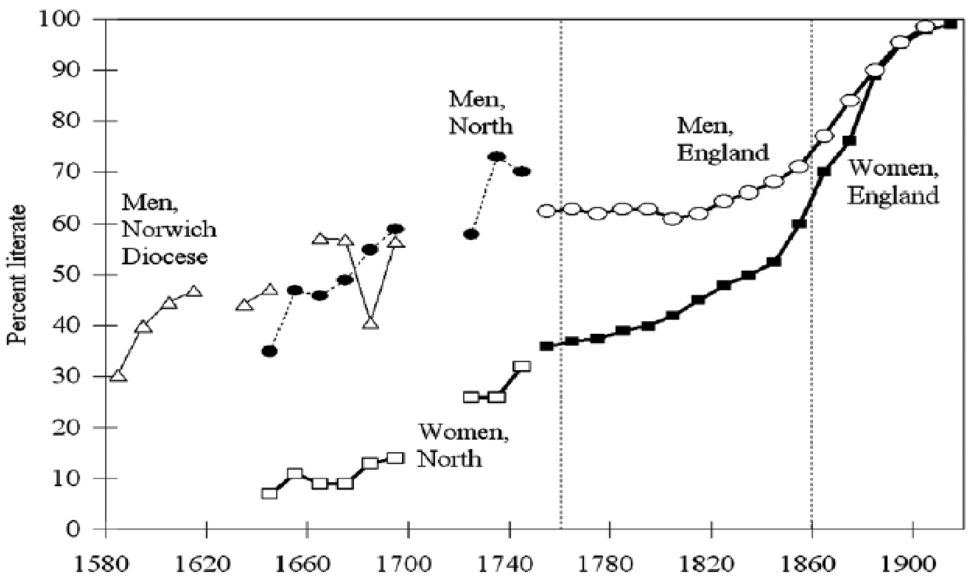


FIG. 7.—Average literacy in England, 1580–1920. Sources: 1750s–1920s, Schofield (1973), men and women who sign marriage registers; the North, 1630s–1740s, Houston (1982), witnesses who sign court depositions; Norwich Diocese, 1580s–1690s, Cressy (1977), witnesses who sign ecclesiastical court declarations.

Source: Clark (2005) Figure 7.

The two data sources used in this paper—demographic data from Wrigley and Schofield (1981) and wage data (builders, coal miners and agricultural workers) from Clark (2013)—are graphed and summarised in Appendix I. They are the fundamental basis for quantitatively evaluating the performance of our DSGE model.

For the first 90 years of the fourteenth century high mortality rates are assumed. Thereafter a constant steady state mortality is imposed for the entire period The period 1300-1900 then divides into three phases. The break between Phases I and II is mainly due to the assumed demographic change, while that between Phases II (a Malthusian Epoch) and III (a Solow Epoch) is mainly attributable to technological change generated by human capital accumulation (leading to the sustainable growth of real wage rates) in the later period. We are going to show that the switches between phases are endogenously generated by the model, not imposed. Moreover, the timing of the phases can also be matched by the model.

2. The Static Deterministic Model

Before proceeding with the full DSGE model, it is helpful to show the key proposition of this paper with an illustrative simple static model. The model demonstrates how a major demographic shock affects human capital and economic growth, especially through women's marriage age. Although the link to the Western European Marriage Pattern is novel, the transmission process itself has been widely discussed.

So DeTray (1973) assumes household investment in human capital is reinforced by the experience and education of the mother. This experience might be expected to determine her children's education and health in a society where the family is the provider of these services (cf. Marshall, 1961 p469). Nordblom (2003) assumes 'child quality' is a function of 'within the family education' and parental education, as well as state schooling. Historical evidence of the transmission process is that children admitted to the Orphan House in Charleston, South Carolina, from the 1790s to 1840 were more likely to be literate if their mothers were literate (Murray, 2004). Moreover, David Mitch (1992 chapter 4) finds parental literacy a major cause of adult literacy in Victorian Britain.

There is much present day evidence for the relationship between later marriage age and greater education. Behrman et al. (1999) substantiate the higher productivity of more educated women and Gaiha and Kulkarni (2005) find an impact of mothers' age at marriage and education on a measure of child quality in India.

The present model therefore postulates that the representative household maximises utility by choosing the *target* number of children q, quality of children n and consumption z, subject to the budget constraint.

$$\max_{n,q,z} U\left(n,q,z\right) = \left[\alpha \cdot n^{\frac{s-1}{s}} + \beta \cdot q^{\frac{s-1}{s}} + \gamma \cdot z^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}}, \text{ subject to:}$$

(S1) General Budget Constraint: $\underbrace{\qquad\qquad}_{\equiv W} \qquad \cdot n + \pi_q \cdot qn + \pi_z \cdot z$;

(S2) Age at Marriage: $A = a_0 + a_1 n + a_2 m$;

(S3) Price of Consumption: $\pi_z \equiv p_z + w \cdot t_z$ (set to 1 as numeraire);

(S4) Price of Child Quality: $\pi_q \equiv \left(p_q + w \cdot t_q\right)\frac{1}{1-m} \approx b_0 + b_1 A + b_2 A^2 + b_3 m$;

(S5) Price of Child Quantity: $\pi_n \equiv \left(p_n + w \cdot t_n\right)\frac{1}{1-m} \approx c_0 + c_1 m$.

The budget constraint (S1) combines time costs and monetary costs of the three activities, so the broadly defined income W includes both time endowment $w \cdot t$ and goods endowment v (which covers charity and Poor Law provision). Similarly, the general prices (π's) are defined in terms of monetary value, so the time spent in each activity is translated into monetary cost.

An increase in wages shifts the budget constraint (S1) outwards, allowing higher target numbers of children, but also raises their 'price' (S5), reducing projected family size. Lower mortality (m) reduces the 'prices' of child quality and numbers (S4 and S5), encouraging more of both. Premature deaths (m) probabilistically affect the costs of raising a surviving child.

A Malthusian preventative check relates female age at first marriage (A) negatively to the number of births (b). The higher the marriage age, the shorter is the period available for giving birth. Assuming a family target n (surviving children), more births will be needed the higher is m, the mortality rate: $b = \frac{n}{1-m}$. Hence a negative mortality shock (a lower death rate) induces a higher marriage age so as to reduce the numbers of births and achieve the previous target family size (S2).

The price of child quality is negatively related to A (S4). The higher is female age at marriage the greater is the accumulated human capital of the mother and therefore the lower the costs of educating children. A higher marriage age increases the opportunity for female learning, and raises female general educational experience. This reduces the price of child quality, since informal home education by mothers was the main channel for accumulating children's human capital.

2.1. Simulation Results

The parameters of the static model are calibrated to illustrate the change between Phase I and Phase II; the emergence of the Western European Marriage Pattern by the end of the fifteenth century. The simulation shows that a drop in child mortality (to say age 15) will raise both women's age at (first) marriage and human capital accumulation, among other effects. One limitation of this static model is that it cannot show the consequences of higher human capital for wages and other variables, because we do not model production here. Moreover, no dynamic story can be told and people are assumed to have perfect foresight in decision making. We need a fully stochastic and dynamic system to address these issues, which are covered in the DSGE model below.

Some parameter values are calibrated to match the stylised facts on age at marriage (around 25—Hajnal, 1965)) and average number of children (2 to 3—Flinn, 1981 p33) in the historical period under study. Other parameters are chosen because they are very similar to those estimated in the later DSGE model. The focus is on the qualitative rather than the quantitative changes.

Table 1 Calibrated Parameters of the Static Model

Parameter	Meaning	Calibration
α	utility weight for n	0.3
β	utility weight for q	0.3
γ	utility weight for z	$1-\alpha-\beta$
s	elasticity of substitution	0.1
a_0	constant term on A	30
a_1	coefficient of n on A	-1
a_2	coefficient of m on A	-0.05
b_0	baseline price of q	1
b_1	coefficient of A on π_q	-0.05
b_2	coefficient of A^2 on π_q	0.0005
b_3	coefficient of m on π_q	0.02
c_0	baseline price of n	1
c_1	coefficient of m on π_n	0.02
W	Wealth	10
m	mortality rate	50%; 20%

The simulation results are summarised in the following table (as the elasticity of substitution rises, quality becomes a closer substitute for quantity).

Table 2 Simulation Results of the Static Model

	Variable	m=50%	m=20%	% change	Direction
b	number of births	3.7631	3.3028	-12.23%	drop
n	number of child	1.8815	2.6422	40.43%	rise
q	quality of child	2.0304	2.8944	42.55%	rise
z	consumption	2.2363	3.0165	34.89%	rise
A	age at marriage	25.6185	26.3578	2.89%	rise
u	utility level	2.0167	2.8217	39.92%	rise
π_q	price of q	1.0472	0.4295	-58.99%	drop
π_n	price of n	2	1.4	-30.00%	drop

As the mortality rate drops from 50% to 20% to simulate the end of the fourteenth and fifteenth century demographic shocks, both π_n and π_q drop relative to π_z (because m does not influence π_z), so both n and q rise. Consumption (z) also rises due to the "income effect", but this positive change is partly offset by the "substitution effect". Hence the change is quantitatively smaller than those in n and q.

Although the target number of children (n) rises, the actual number of birth b drops, because of a lower mortality rate. This is why women's age at first marriage (A) rises, providing another reason for a fall in the price of child quality (π_q). Therefore the decline in price of q is greater than the fall in the price of n. The consequent higher quality of children in the current period means higher human capital for the labour force in the future, which will generate stronger economic growth, a feature that we add to the dynamic model below.

Although savings are not modelled, in the interests of minimising complexity, the process of acquiring female human capital might also be supplemented by savings for the separate household necessary to attain the target child quality. The age at marriage is indirectly influenced not only by the number of births but the target child quality.

The model is presented as one of individual choice, but should be interpreted as the outcome of collective optimisation with customary behaviour. The utility function then becomes a 'culture function', representing the shared values of the group. In such a framework later age at marriage may be enforced by customary economic requirements for household formation. Higher wages bring about more children by reducing marriage age because less time is required to save for the separate household. But the appropriate or customary costs of a separate household are determined by the culturally chosen target number of children and perhaps their quality. Lower mortality then increases the customary costs of the separate household because it increases general consumption and child quality (which requires space and that children not be put to work early).

3. The Dynamic Stochastic General Equilibrium Model

There are four novel features of this DSGE model to improve historical realism and the 'fit' to the data:

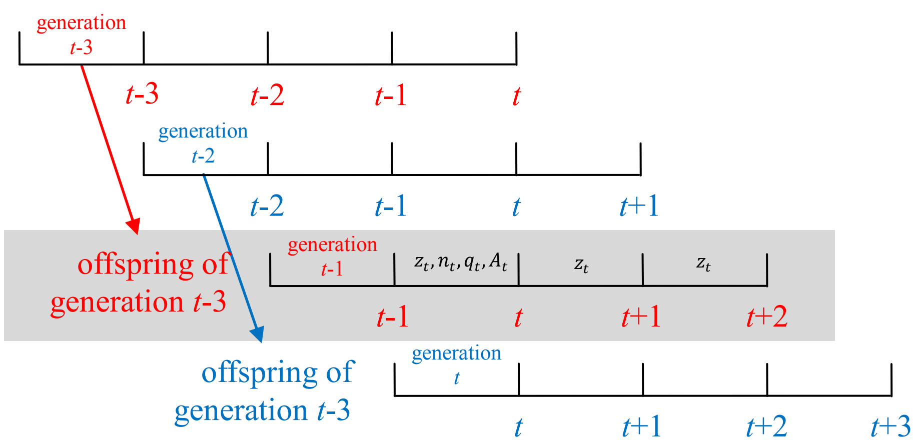
- Four overlapping (half) generations interacting in each period (a heterogeneous agent model).

- The formation of human capital is mainly carried out by family education, i.e. the parents' (mainly the mothers') human capital and rates of financial return are irrelevant.
- Human capital accumulation is also affected by an externality; society's human capital influences children's accumulation of human capital.
- Women's marriage age is a key transmission channel for economic growth from the mortality shocks: lower death rates → higher marriage age → lower costs of children quality → higher future human capital → higher economic growth.

3.1. The Model

We assume that a representative agent in a particular generation may live up to 4 periods (with each period around 15 years). There periods are childhood (0-15 years), youth (16-30 years), reproduction (31-45 years) and senior (46-60 years). Each period has a mortality rate (mi). The generation born in period $t-1$ only make decisions in period t, if they survive the childhood period. The representative agent of a generation decides on the female age at marriage (A_t) ('companionate' marriage), the target number of children (n_t), the target quality of children relative to the parents (q_t), and the lifetime consumption flows (z_t). Note that the time subscript denotes when the variable is determined, so the variables with subscript t may be realised in different periods. For example, A_t occurs in period t; n_t and q_t take effect in period $t+1$; while z_t is relevant throughout periods t to $t+3$. Figure 2 shows this overlapping generation structure.

Figure 2 Generation Structure of the DSGE model



[Household] Where $\mathbf{E}_t$ indicates expectations at time t, the representative agent born in period $t-1$ faces the maximization problem in period t:

$$\max_{A_t,n_t,q_t,z_t} \mathbf{E}_t\left[U\left(n_t, q_t, \frac{z_t}{z_{t-1}}\right)\right] \equiv \mathbf{E}_t\left[\alpha\cdot\left(\frac{n_t}{2}\right)^{\frac{s-1}{s}} + \beta\cdot\left(q_t\right)^{\frac{s-1}{s}} + \gamma\cdot\left(\frac{z_t}{z_{t-1}}\right)^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}}$$, subject to:

(H1) Budget Constraint: $w_t + w_{t+1} + w_{t+2} = \frac{1}{2}\left(\pi_{n,t+1}\cdot n_t + \pi_{q,t+1}\cdot q_t n_t\right) + 3z_t$;

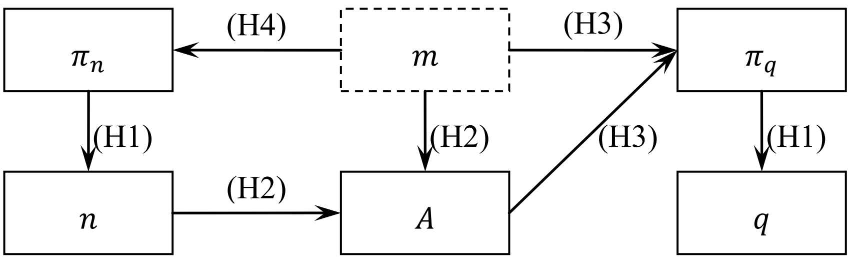
(H2) Marriage Age Equation: $A_t = a_0 - a_1 \frac{n_t}{\left(1-m1_{t+1}\right)\left(1-m2_{t+2}\right)}$;

(H3) Price of Children Quality Equation: $\pi_{q,t} = \frac{b_0}{\left(1-m1_t\right)\left(1-m2_{t+1}\right)}\left(\frac{b_1}{A_{t-1}}\right)^{\eta_A} w_t$;

(H4) Price of Children Quantity Equation: $\pi_{n,t} = \frac{c_0}{\left(1-m1_t\right)\left(1-m2_{t+1}\right)} w_t$.

The utility function has three new features in a dynamic setting: (i) The target number of children n is *relative* to the baseline number 2, which keeps the whole society sustainable in the long run. (ii) The target quality of children q is the ratio of parents' to children's human capital levels. (iii) Consumption z_t/z_{t-1} is again a relative magnitude, a ratio rather than an absolute level. This "habit persistence", as described in the macro-economic literature, in consumption is necessary, since the other two utility inputs (n and q) are stationary and the third utility input must also be stationary.

The following chart summarises the topology of the household's constraints.



Constraint (H1) describes the lifetime earnings equal to the lifetime expenditures (by 'earnings' we also mean earnings in kind from domestic production consumed in the household). Expenditure on children is shared by both husband and wife, so it is halved for the representative agent. Furthermore, the inter-temporal margin (or Euler equation) implies that consumption must be perfectly smoothed across the three periods because there is no mature capital market. Therefore, the total consumption for the lifetime is $3z_t$ for the representative agent.

Constraint (H2) imposes a particular restriction on female's marriage age A_t, which negatively depends on the total planned births, taking premature deaths in 'generations' 1 and 2 ($m1, m2$) into consideration[7]. a_0 is the age women usually stop planning for, or expecting, new children (assumed around 35[8]), and a_1 is the average gap between births (assumed 2 to 3 years).

Constraint (H3) states that the *relative* price of educating each surviving child depends positively on mortality rates, and negatively on mother's marriage age (which is an indicator of the level of the mother's human capital). The parameter b_0 can be interpreted as the average proportion of income spent on q, while b_1 is the lower bound of marriage age. $-\eta_A$ measures the elasticity of marriage age effects on human capital accumulation.

Constraint (H4) states that the *relative* price of feeding and clothing each surviving child increases with mortality rates, where c_0 is the average proportion of income spent on n. All these constraints are dynamic counterparts of the static model in the previous section.

In addition to the quantities and prices involved in the individual maximisation problem, some aggregate variables can also be defined in terms of existing variables. The law of motion for population (P_t: population stock at time t) is:

$$P_t = P_{t-1} - D_t + B_t .$$

Total deaths in period t (D_t: death flow in period t) are a sum of premature deaths and natural deaths:

$$D_t = m1_t \cdot B_t + m2_t \cdot G1_{t-1} + m3_t \cdot G2_{t-1} + G3_{t-1} .$$

Here, Gi_{t-1} denotes the population of the generation born in period $t - i$ still living at time $t - 1$ (at the beginning of period t):

$$\begin{aligned} G1_t &= (1 - m1_t) B_t \\ G2_t &= (1 - m1_{t-1})(1 - m2_t) B_{t-1} \\ G3_t &= (1 - m1_{t-2})(1 - m2_{t-1})(1 - m3_t) B_{t-2} \end{aligned}$$

Total births in period t (B_t: birth flow in period t) can be defined as:

$$B_t = \frac{(1-\mu) G2_{t-1}}{2} \frac{n_{t-1}}{(1 - m1_t)(1 - m2_{t+1})} .$$

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[7] Family targeting plans are assumed to be base on survival of children to (approximately) age 30, perhaps to support parents in old age.

[8] This rather early assumed stopping age compensates for not taking into account increasing birth intervals with parity (Flinn, 1981 p33).

The parameter μ here captures the proportion of each generation without any children. This proportion includes both never-married people and married people with no offspring.

Based on B_t and D_t, we can define birth rate (b_t) and death rate (d_t) to link with the observable historical data. In particular, the birth rate is the birth flow during period t divided by the population stock at the beginning of period t (or at time $t-1$). The death rate is the death flow during period t divided by the sum of the population at $t-1$ and the birth flow during period t, because the death rate takes into account the premature deaths of those in their childhood period.

$$b_t \equiv \frac{B_t}{P_{t-1}}$$

$$d_t \equiv \frac{D_t}{P_{t-1}+B_t}$$

Lastly, the labour force of period t is the population stock (P_{t-1}) excluding the generations in childhood period at the beginning of period t (or at time $t-1$).

[Production] Where Y is output, H is human capital, X is productivity, F is a fixed factor, especially land and e^{x_t} a random productivity shock, the representative production unit's (farm's) problem is:

$$\max \Pi_t = Y_t - w_t P_{t-1} - \text{fixed costs}, \text{ subject to:}$$

(F1) Production Function:

$$Y_t = X_t \left(H_{t-1}P_{t-1}\right)^{\theta} \bar{F}^{1-\theta};$$

(F2) Aggregate Human Capital:

$$H_t = Q_t \frac{G1_t}{P_t} + Q_{t-1}\frac{G2_t}{P_t} + Q_{t-2}\frac{G3_t}{P_t};$$

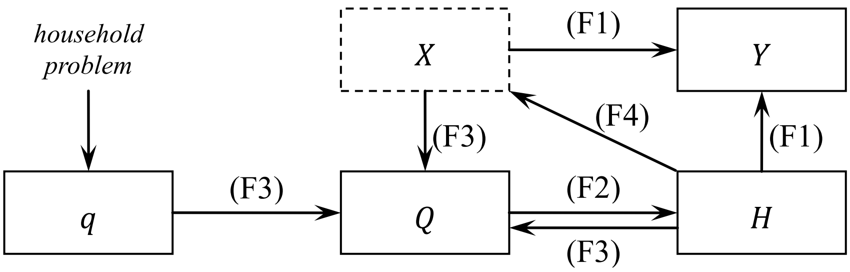
(F3) Generational Human Capital:

$$Q_t = X_t H_t^{\varepsilon}\left(Q_{t-2}q_{t-1}\right)^{1-\varepsilon};$$

(F4) Total Factor Productivity Function:

$$X_t = \left(\frac{H_{t-1}}{H_{t-2}}\right)^{\eta_H} \bar{X}e^{x_t}.$$

The following chart summarises the topology of the farm's constraints.



Constraint (F1) describes the production technology with X_t being the total factor productivity. h_{t-1} is the average human capital of the labour force influencing period t, by which the 'raw' labour force (P_{t-1}) is augmented. $\bar{F}$, representing factors that cannot be reproduced such as land, for simplicity is normalised to unity.

Constraint (F2) defines the aggregate human capital (H_t), which is a weighted average of the generational human capital in the labour force.

Constraint (F3) describes how each generation's human capital level (Q_t) is formed. Apart from the parents' influence ($Q_{t-2}q_{t-1}$: the target quality of children formed by "family education"), the generation born in period t is also influenced by the average human capital of existing generations H_t (the parameter ϵ is interpreted as "social education" externality). Total factor productivity also affects the formation of Q_t, on the grounds that a society with higher productivity tends to be more effective in the transmission of knowledge.

Constraint (F4) describes how total factor productivity (X_t) fluctuates. The baseline total factor productivity is $\bar{X} = 1$ in steady state, while the growth rate of aggregate human capital in the last period (H_{t-1}/H_{t-2}) is assumed to shift the steady state higher or lower than 1. Stronger growth in human capital leads to greaterer total factor productivity in next period. This is a feedback effect from human capital to productivity, as opposed to the direction described in constraint (F3).

Finally, to complete the system of equilibrium conditions, a competitive labour market is postulated, so that the marginal product of labour equals the marginal cost:

$$MP_t \equiv \frac{\partial Y_t}{\partial P_{t-1}} = \theta \frac{Y_t}{P_{t-1}} = w_t \equiv MC_t \, .$$

3.2. Stationarisation

The above system is non-stationary because of growth in human capital and population. But standard numerical methods for solving this dynamic equation system require that we stationarise many of the original conditions. U_t, n_t, q_t, A_t are stationary by definition, so for them no change is necessary. The non-stationary endogenous variables can be categorised into five groups in terms of their balanced growth path rates, or of their deflators. Where a hat "^" indicates a stationarised variable;

Deflated by P_{t-1}: $\hat{B}_t \equiv \frac{B_t}{P_{t-1}} \equiv b_t$, $\hat{D}_t \equiv \frac{D_t}{P_{t-1}}$, $\hat{Gi}_{t-1} \equiv \frac{Gi_{t-1}}{P_{t-1}}$, $\hat{P}_t \equiv \frac{P_t}{P_{t-1}}$

Deflated by H_{t-1}: $\hat{H}_t \equiv \frac{H_t}{H_{t-1}}$, $\hat{Q}_t \equiv \frac{Q_t}{H_{t-1}}$

Deflated by $H_{t-1}^{\theta}P_{t-1}^{\theta}$: $\hat{Y}_t \equiv \frac{Y_t}{H_{t-1}^{\theta}P_{t-1}^{\theta}}$

Deflated by $H_{t-1}^{\theta}P_{t-1}^{\theta-1}$: $\hat{w}_t \equiv \frac{w_t}{H_{t-1}^{\theta}P_{t-1}^{\theta-1}}$, $\hat{\pi}_{n,t} \equiv \frac{\pi_{n,t}}{H_{t-1}^{\theta}P_{t-1}^{\theta-1}}$, $\hat{\pi}_{q,t} \equiv \frac{\pi_{q,t}}{H_{t-1}^{\theta}P_{t-1}^{\theta-1}}$, $\hat{z}_t \equiv \frac{z_t}{H_{t-1}^{\theta}P_{t-1}^{\theta-1}}$

Inflated by $H_{t-1}^{\theta}P_{t-1}^{\theta-1}$: $\hat{\lambda}_t = \lambda_t H_{t-1}^{\theta}P_{t-1}^{\theta-1}$

3.3. Shock Structure

There are four structural shocks—one productivity shock and three mortality shocks. Runs of poor harvests and livestock disease constitute a negative productivity shock. Epidemic diseases such as bubonic plague, typhus and smallpox were mortality shocks. We assume the exogenous productivity process is AR(1): $x_t = \rho_x \cdot x_{t-1} + ex_t$, where ex_t is Gaussian white noise with standard deviation equal to σ_x.

Similarly, there are three exogenous premature mortality rates:

$$m1_t = \overline{m1} \cdot \exp(mm1_t), \text{ where } mm1_t = \rho_1 \cdot mm1_{t-1} + emm1_t;$$
$$m2_t = \overline{m2} \cdot \exp(mm2_t), \text{ where } mm2_t = \rho_2 \cdot mm2_{t-1} + emm2_t;$$
$$m3_t = \overline{m3} \cdot \exp(mm3_t), \text{ where } mm3_t = \rho_3 \cdot mm3_{t-1} + emm3_t.$$

Here, $\overline{mi}$'s are the steady state mortality rates and $emmi_t$'s are Gaussian white noises with standard deviations equal to σ_i. Following Wrigley and Schofield (1981 p714, levels 3 and 7), we set the steady states of mortality rates to match the historical life expectancy pattern over the 600 years:

Table 3 Calibration of Steady States of Mortality Rates

	Initial State	**Steady State**
$\overline{m1}$	0.50	0.30
$\overline{m2}$	0.29	0.10
$\overline{m3}$	0.49	0.21
Implied Life Expectancy	**23.54**	**35.36**

In addition, the changes of the three mortality rates are positively correlated ($\rho_{ij} > 0$), since they are driven by similar factors, such as progress in healthcare, wars and plagues. However, the productivity shock is assumed to be uncorrelated with mortality shocks, so $\rho_{ix} = 0$.

4. Results

The model is solved, simulated and estimated using the methods described in Appendix III.

4.1. Parameter Estimation

Most parameters in Table 4 are estimated within their theoretical boundaries to minimise the distance between the simulated and the historical mean values and standard deviations of the four data series. Some parameters are set according to such historical information as is available ('Calibrated' in Table 4).

Table 4 Calibrated and Estimated Parameters of the DSGE model

Parameter	Meaning	Value	Notes
a_0	Average stopping birth age	**35**	**Calibrated**
a_1	Gaps between births	2.523	Estimated
b_0	average proportion of income spent on q	0.100	Estimated
b_1	Lower bound of marriage age	**16**	**Calibrated**
c_0	average proportion of income spent on n	0.188	Estimated
α	Utility weight for children quantity	0.175	Estimated
β	Utility weight for children quality	0.316	Estimated
γ	Utility weight for consumption	0.509	Estimated
s	Elasticity of substitution	0.100	Estimated
μ	Proportion of never-married	0.120	Estimated
θ	Income share of labour	0.500	Estimated
ϵ	Externality in human capital formation	0.116	Estimated
η_A	– Elasticity of marriage age effect on π_q	1.457	Estimated
η_H	Elasticity of human capital effect on X	0.734	Estimated
ρ_1	AR coefficient of shock mm1	0.498	Estimated
ρ_2	AR coefficient of shock mm2	0.000	Estimated
ρ_3	AR coefficient of shock mm3	0.010	Estimated
ρ_x	AR coefficient of shock x	0.000	Estimated
σ_1	Standard deviation of shock emm1	0.010	Estimated
σ_2	Standard deviation of shock emm2	0.444	Estimated
σ_3	Standard deviation of shock emm3	0.438	Estimated
σ_x	Standard deviation of shock ex	0.112	Estimated
ρ_{12}	Correlation coefficient between emm1 and 2	**0.5**	**Calibrated**
ρ_{13}	Correlation coefficient between emm1 and 3	**0.5**	**Calibrated**
ρ_{23}	Correlation coefficient between emm2 and 3	**0.5**	**Calibrated**

4.2. Matching the Moments

Using the estimated/calibrated values of the parameters above, the DSGE model can simulate the first moments (means or steady states) and second moments (standard deviations) of the endogenous variables (Table 5). The simulated moments are contrasted with the observed moments. Note that the simulated periods (600 years, 40 periods) are longer than the observable periods of demographic variables (330 years, 22 periods).

From the high mortality simulated initial state, target family size increases in the simulated steady state from the end of the 14th century. Child quality rises by a larger proportion because the price of quality falls further than does the price of child numbers. Age of female first marriage goes up from 19 to 25. Steady state population, wage and income growth increase in response to the low mortality regime.

Table 5 Simulated Moments versus Data Moments[9]

	Simulated Initial State	Simulated Steady State	**Sample Mean**	Simulated SD	**Sample SD**
n	2.233	2.497		0.149	
q	1.291	1.443		0.079	
$\hat{z}$	0.443	0.560		0.043	
π_n	0.303	0.181		0.023	
π_q	0.124	0.050		0.007	
A	19.13	25.00	**25**	0.598	
$\hat{P}$	0.991	1.048	**1.100**	0.076	**0.080**
b	0.951	0.598	**0.536**	0.070	**0.067**
d	0.492	0.344	**0.272**	0.031	**0.026**
$\hat{H}$	1.202	1.299		0.098	
g_w	1.101	1.113	**1.113**	0.165	**0.166**
g_y	1.092	1.167		0.159	

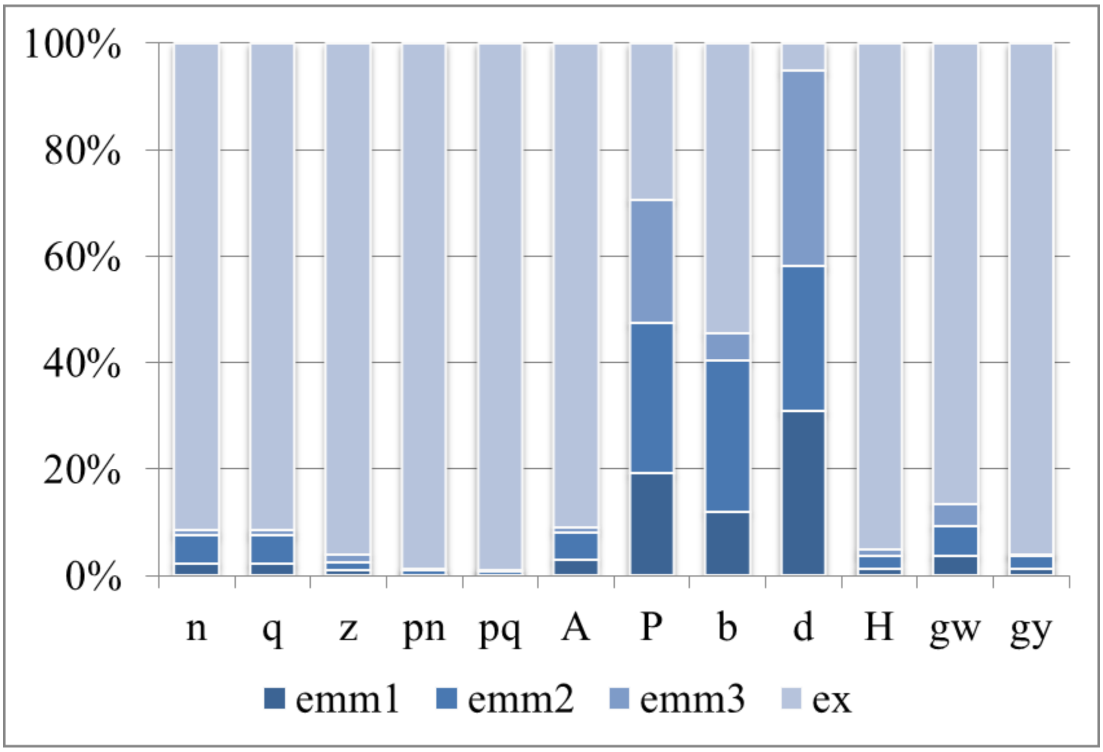
The model has very accurately matched the second moments (up to two decimal places), but there are slight discrepancies between the simulated steady states and sample means (Table 5). However, steady states (the simulated first moments) are not equivalent to sample means (the sample first moments) in the first place. The sample means measure the average levels of the variables in real history, which were never in a steady state, because of continual shocks. In contrast, the steady states are defined as the long-run levels when there are no shocks. Arguably, the comparison of second moments makes better sense.

[9] Note that $\hat{P}$, $\hat{H}$, g_w and g_y are gross growth rates. For example, a 0.991 growth rate of population means population shrinks 0.9%, and a 1.101 growth rate of wage means wage grows 10.1%.

4.3. Variance Decomposition

The variation of demographic and economic variables can be attributed to the four structural shocks: the three mortality shocks and the productivity shock, which are estimated to match the observed history below in 4.5.1. According to the variance decomposition implied from the model simulation (Figure 3), productivity shocks contribute the most to fluctuations in economic variables such as consumption, human capital, wage and output. Productivity shocks also dominate the fluctuations in individual decisions on n, q and A through the budget constraint. In contrast, the demographic variables, population, birth rate and death rate are mainly explained by the demographic shocks[10].

Figure 3 Variance Decomposition of Key Variables



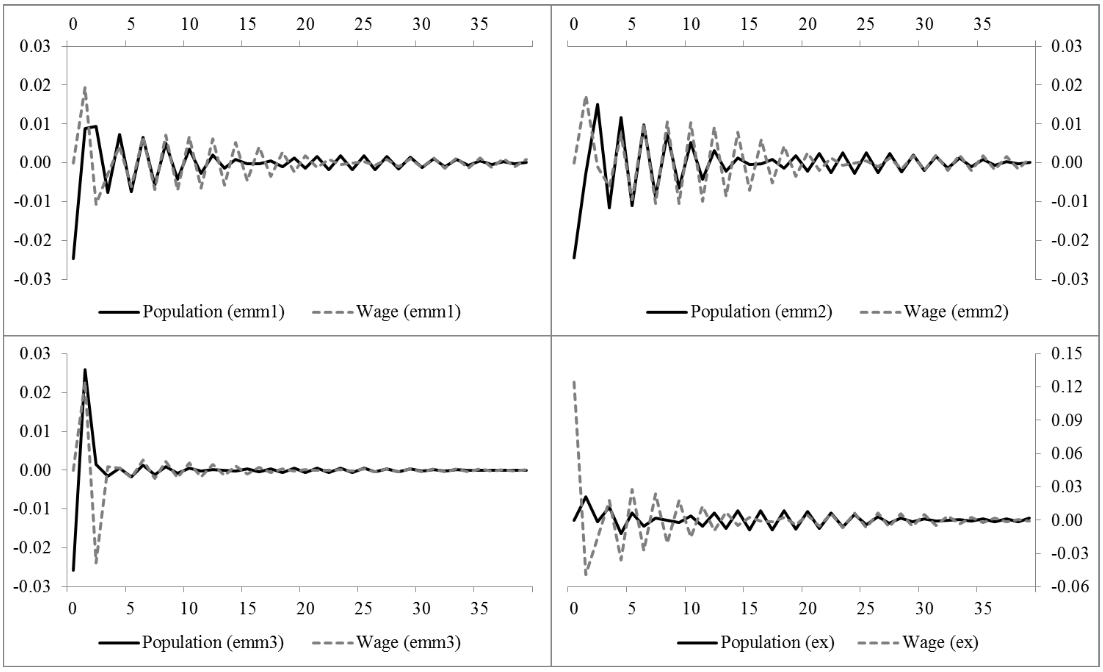
4.4. Impulse Response Functions

A key implication of this stochastic model is the response of endogenous variables to shocks in transition. Impulse response functions describe the time path of an endogenous variable after a one-off shock, keeping all the other shocks equal to zero. They represent the dynamic effect of a specific shock over time. Each point in the impulse response function denotes the difference from its steady state.

[10] In another practice, we verified that the effects of mortality shocks become significantly weaker as the steady state levels of mortality rates decline.

Figure 4 shows the impulse response functions of population growth rate and wage growth rate after the four shocks. There are three noteworthy aspects of the impulse response functions. Firstly, the **OSCILLATING** feature is mainly due to the heterogeneous generation structure in the model, and is also apparent in the data (Figure 13) but the data is generated by compounded shocks rather than a one-off shock. Secondly, the effects of mortality shocks (especially $emm1$ and $emm2$) are highly **PERSISTENT**, bearing in mind that each period is 15 years. Thirdly, there is a **REVIVING** feature in the impulse response function of population growth around 15 periods and the impulse response function of wage growth around 30 periods. Since the simulation starts in 1300, the 15th period (c. 1525) is the beginning of Phase II (when the population begins to take off) and the 30th period (c. 1750) is about the beginning of Phase III, i.e. the Industrial Revolution (when the real wage begins to take off). Therefore, the timing match here enables the model to endogenously explain the division of historical phases, purely based on the demographic shocks.

Figure 4 Impulse Responses of Population and Wage Growth Rates



We can see that the Malthusian negative correlation between wage and population is only exhibited during Phase II (16th-18th century), but not for Phase I or III. Note that these impulse response functions describe the reactions of population and wage after a one-off mortality shock, so they simulate an aspect of history without other complications of reality.

4.5. Other Implications

4.5.1. Simulated History based on Mortality Shocks

The previous subsection describes the theoretical features of the economy, but a historian will want to know what actually happened in real history according to the model. We therefore simulate the 600-year demographic and economic history by compounding the impulse response functions of the productivity and the mortality shocks. The occurrences of demographic shocks are chosen to minimise the gap between the simulated and the observed series in the sample. In Figures 5-8 the grey broken lines are the simulated series based on the compounded mortality shocks. Most of the fluctuations in population and wage growth in the data (black solid lines) can be well matched by mortality shocks only.

Figure 9 shows an increase in the marriage age (A) in the first half of the 15th century. The initial state was obtained under the high mortality rates (50%, 29%, 49%)—Wrigley and Schofield's level 3. The eventual state of age 25 was obtained under the 'normal' uniform mortality rates (30%, 10%, 21%)—Wrigley and Schofield's level 7.

In the transition the postulated fall in mortality to a low m does not guarantee a high A, because n is high at the same time (surviving children are less costly so demand for them expands). Recall the constraint (H2):

$$A_t = a_0 - a_1 \frac{n_t}{(1-m1_{t+1})(1-m2_{t+2})}$$

In the steady state, there are two consequences of the permanent decline of mortality rates (m). There is a "wealth effect", as human capital accumulation more than offsets diminishing returns, raising wages. But this takes a considerable time to work through. There is also a steady state "price effect" which lowers generalised child prices and the price of child quality, as well as an additional channel through a higher marriage age and a lower price of child quality. The wealth effect tends to increase n, q and z, while the price effect reinforces this in the steady state.

In the transition, diminishing returns dominate the wealth effect so wages fall because human capital accumulates slowly. This reduces n, q and z but raises the marriage age, The price effect in transition tends to offset these wealth consequences. Falling wages lower child price and the price of child quality reducing the marriage age.

Both observed and simulated population growth turn upwards after the first quarter of the eighteenth century (Figure 5), whereas real wage growth begins to rise permanently much later (Figure 8). The intensity and frequency of mortality crises diminish with the success of western European quarantine regulations from the early eighteenth century (Chesnais, 1992 p141). The demographic consequences, more surviving children, are realised more rapidly than the effects operating through human capital accumulation. This is because greater child quality must take longer to be translated into higher wages than it takes the stronger demand for children's numbers to trigger an increase in population.

Figure 5 Simulated and Observed Population Growth Rate (15-Year)

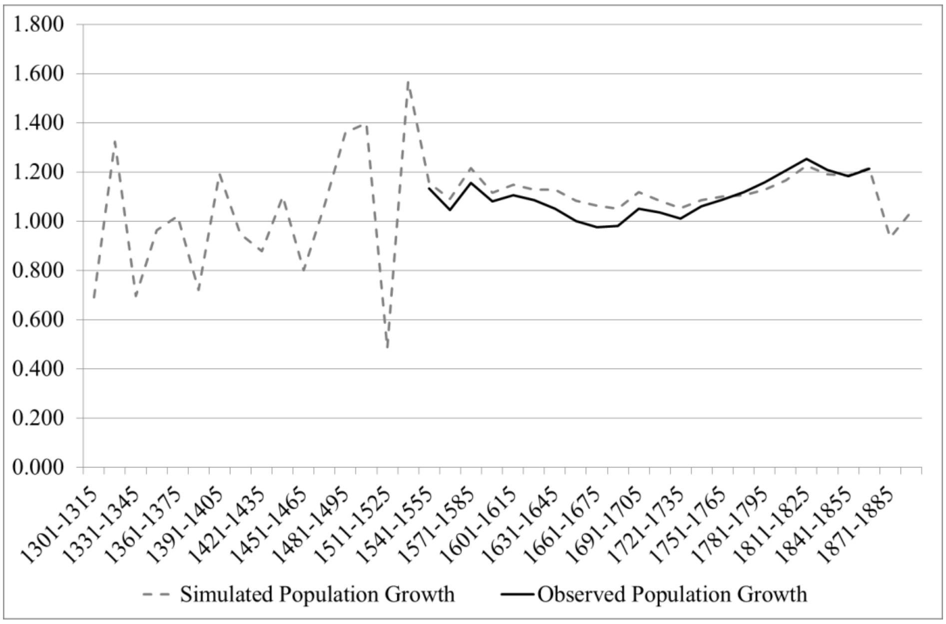


Figure 6 Simulated and Observed Birth Rate (15-Year)

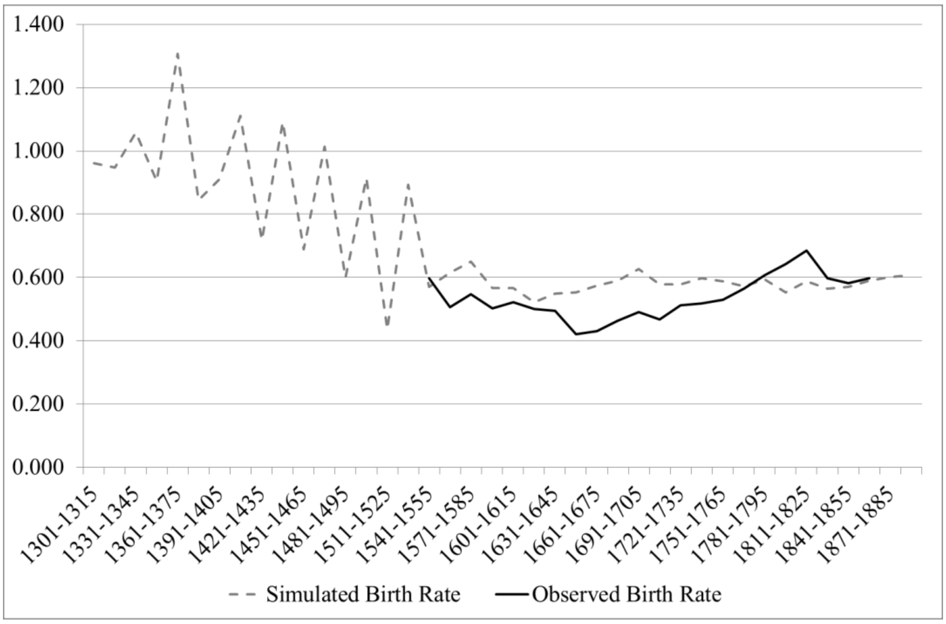


Figure 7 Simulated and Observed Death Rate (15-Year)

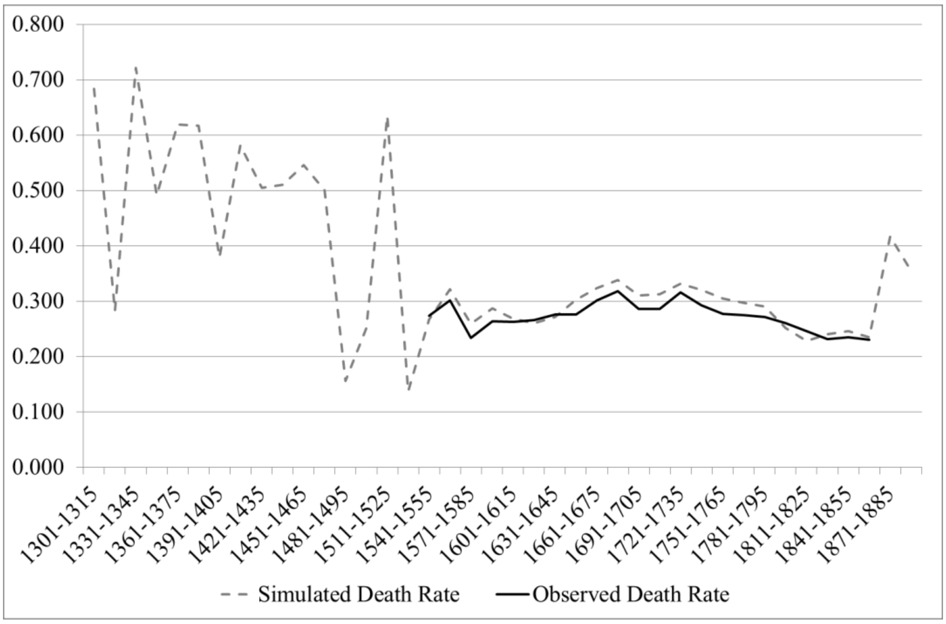


Figure 8 Simulated and Observed Wage Growth Rate (15-Year)

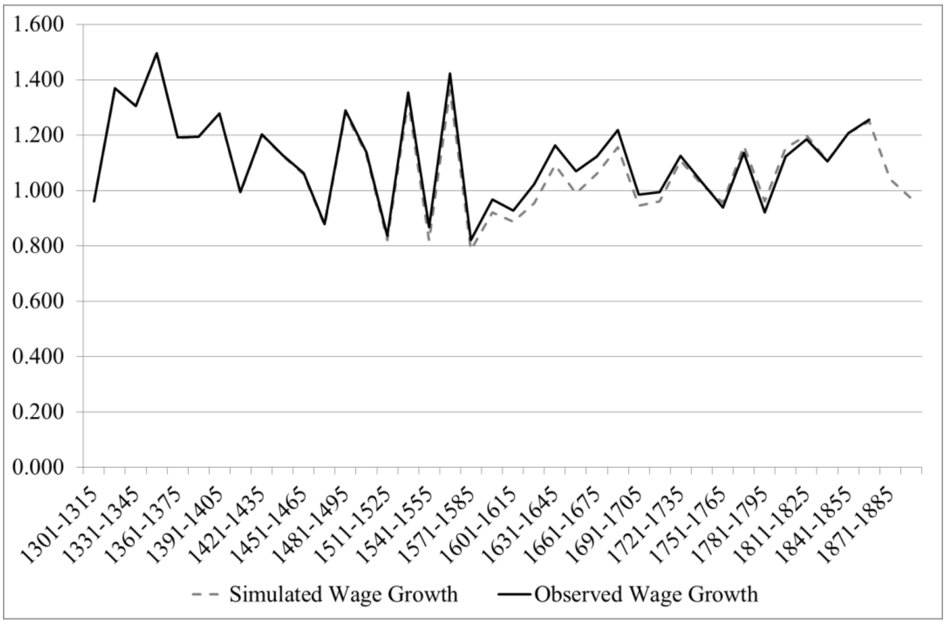


Figure 9 Simulated Female Age at Marriage

4.5.2. Importance of Marriage Age

The importance of the marriage age can be seen by shutting down the effect of marriage age on the price of child quality. This is achieved by setting $\eta_A = 0$ in (H3). The simulation below in Table 6 compares the scenarios with and without this mechanism. All four variables have steady state values lower without the human capital effect.

Table 6 Key Variables with and without Marriage Age Mechanism

	With	**Without**
$\hat{P}$	1.048	1.033
$\hat{H}$	1.299	1.220
g_w	1.113	1.087
g_y	1.167	1.122

The following Table 7 simulates human capital, real wage and output under the assumption that the female marriage age does not contribute to human capital accumulation. Supposing the effects had begun by the end of the fourteenth century, by 1850 (i.e. about 30 periods after the steady states shift in mortality rates) the model implies that without the marriage age process, wages would have been slightly less than half what they actually were. Population would have been one third lower, human capital would have been 85% less than the actual level, and output would have been one third of what it actually was. That means, without marriage age mechanism, England would need to wait another $1.299^{24} = 1.220^{x} \rightarrow x \approx 31.6$ periods (or 475 years) to have the same human capital as actually achieved by 1850!

Table 7 Differences in Levels without Marriage Age Mechanism

	360-year difference Up to 1750	**390-year difference** Up to 1780	**450-year difference** Up to 1850
Population	69.89%	67.84%	63.90%
Human Capital	21.99%	19.38%	15.05%
Wage	56.09%	53.45%	48.54%
Output	39.20%	36.26%	31.02%

4.5.3. Preventative and Positive Checks

Simulation of DSGE model can shed light on the "preventative checks" and "positive checks", of Malthus and others by generating the correlations between endogenous variables. The following table 9 summarises the correlation matrix between marriage age, birth rate, death rate, real wage per effective unit of labour ($\hat{w}$) and wage growth rate (g_w). Both the "preventative check" (lower wage, lower births and higher marriage age) and "positive check" (lower wage, higher death rate, lower marriage age to replace deaths) are confirmed, though the "positive check" is very weak[11]. An *unexpected* positive mortality shock leads to a high death rate and a high marriage age when the system is still in transition—a positive relationship. By contrast the static simulation of section 2 is concerned with steady states, where the relationship is negative.

Table 8 Simulated Correlations

	A	b	d	$\hat{w}$	g_w
A	1				
b	-0.254	1			
d	0.012	-0.265	1		
$\hat{w}$	-0.838	0.156	-0.078	1	
g_w	-0.902	0.052	-0.152	0.675	1

5. Conclusion

The complexities of the interaction between demography and economy for several centuries of English history inevitably require simplification if the main elements of the process are to be understood. Our model is a judgement based upon evidence as to what mattered most for the economic development that culminated in the first industrial revolution. Understanding English long run economic development must involve

[11] The weakness of positive checks is supported by other researchers. Nicolini (2007) concludes that they disappeared during the seventeenth century; Moller and Sharp (2008) find insignificant positive checks and Crafts and Mills (2009) discover no evidence of a positive check during the period 1541–1800.

the interplay between data and modelling. Both are uncertain. Though based on a large number of parishes, the demographic data of necessity are processed on the basis of a number of assumptions that might be questioned, as might the real wage data[12]. So the precise patterns to be modelled are open to doubt. Nonetheless, the data do contain information and a model that matches the available data is likely to be a better explanation of economic development than one that does not. Similar considerations apply to the DSGE model; it may be in some respects simplistic or even mis-specified but it does capture more historically warranted characteristics of the interaction between the demography and the English economy than any others of which we are aware.

The model can show how the Western European Marriage Pattern emerged as a result of the high mortality of the fourteenth century. Whereas Malthus saw the consequence of this 'preventative check' as being simply population control, the model proposed here represents the change as more fundamental. It can demonstrate how this high female age at first marriage ultimately contributed to the break out from the Malthusian equilibrium- through a shift towards 'child quality' and family-based human capital accumulation.

Continuing mortality crises in the remainder of the fourteenth and fifteenth centuries diminished by the early sixteenth century so that a new phase of development begins about 1525 ('Phase II'). This period shows Malthusian inverse fluctuations between wage and population growth. Around 1750 the model shows the economy entering a third phase in which first population and then real wages grow secularly, the industrial revolution. Economic growth lags behind demographic growth. As the intensity and frequency of mortality crises diminish, more children survive and are planned immediately. The consequences of their higher quality, greater human capital, take longer to work through the economy.

The cyclical behaviour of the data series is matched by a similar pattern in the impulse response functions. This seems to be due to the multiple overlapping generations structure of the model. Each generation has its own smoothed impulse responses (a feature of rational behaviour), but they act in their own interest with different timing. There are four generations in each period, and their aggregation creates the final shape of the overall impulse responses. The interactions among generations might sometimes reinforce each other (peak/trough), sometimes staggering among each oth-

[12] Using a different price index, Allen (2007 Figure 19) places the take-off in real wages perhaps 30 years or more after Clark's (2005) series.

er (see-sawing) and sometimes cancelling out each other (reviving). Given the regular and persistent pattern, this behaviour must be rooted in the structure of the model.

The model findings on positive and preventative checks in pre-industrial England conform generally with those of other recent researchers. However, the DSGE approach draws attention to the difference between transition dynamics of these checks, and the steady state relations between the key variables. This matters when mortality rates and female age at first marriage are concerned. The steady state predicts a negative relationship but the transition can give rise to a positive relation as the correlation of Table 8 shows. This weak positive association stems from the price effect dominating the wealth effect of falling wages. Lower wages reduce the opportunity cost of children, and lower marriage age, as the target completed family size rises.

Mortality shocks, not productivity shocks, drive almost the entire English growth process from 1300 into the nineteenth century[13]. The most lasting of these in effect was the sustained period of high mortality in the fourteenth century. For this ultimately triggered the break out to the industrial revolution through the change in marriage pattern and induced human capital formation. Without the marriage/child quality process, real wages by 1850 would have been perhaps half, or less, of what they actually were.

[13] Compare Grantham's (1999 p210) contention that by the thirteenth century agricultural productivity was able to match that of the eighteenth century because the technical capacity already existed; the techniques had been available since the early Iron Age.

Appendix I: Graphs and Tables of Data

Demographic data are based on Wrigley and Schofield (1981) and wage data are based on Clark (2013).

Figure 10 Birth Flows and Death Flows (Annual)

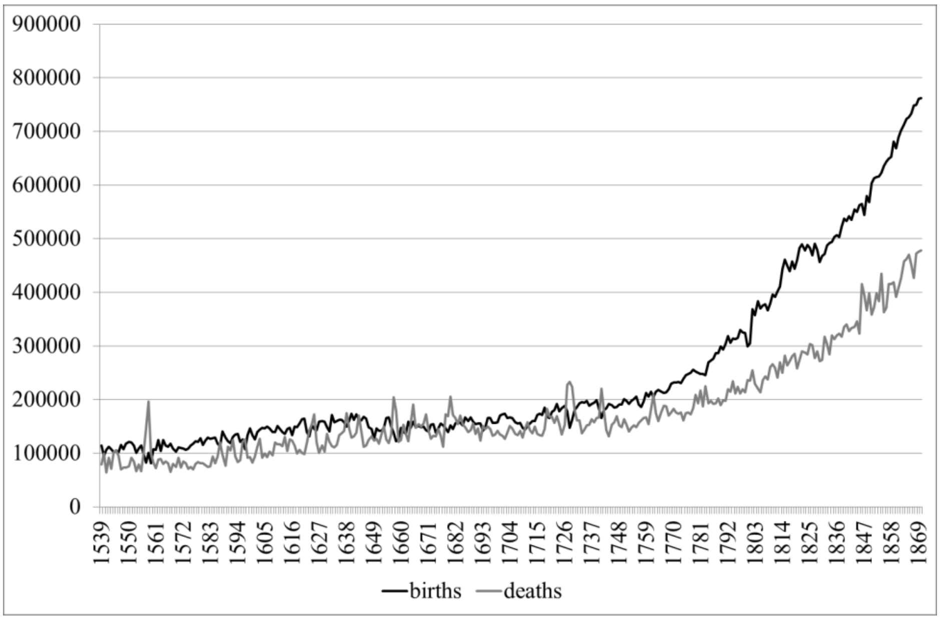


Figure 11 Birth Rate and Death Rate (Annual)

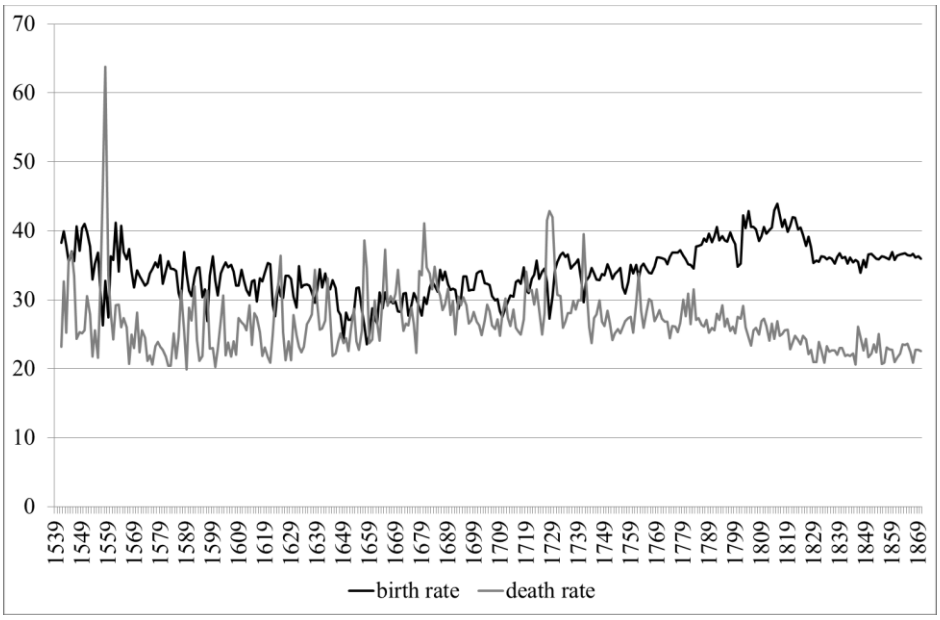


Figure 12 Population Growth Rate and Wage Growth Rate (Annual)

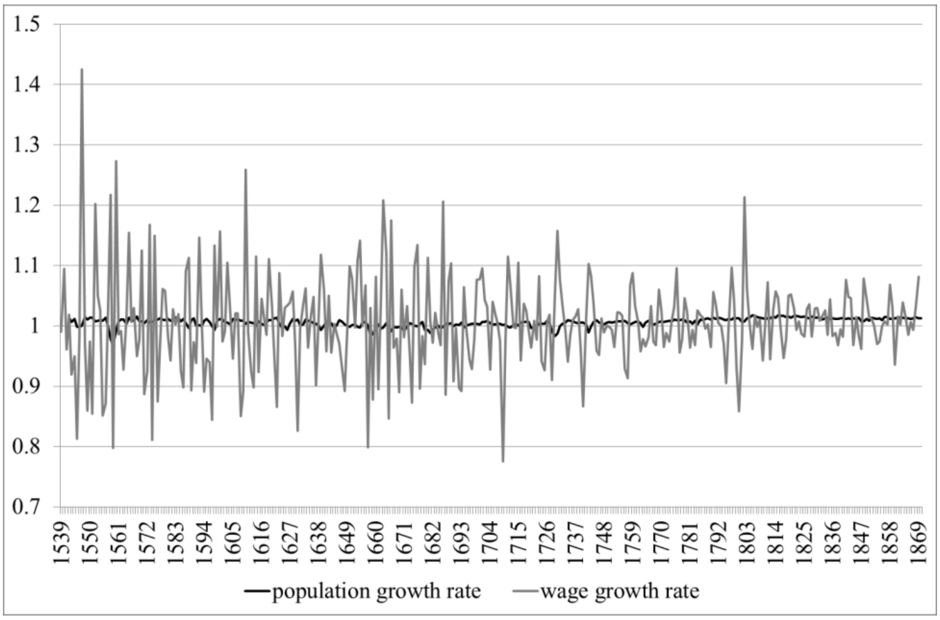


Figure 13 Historical Fluctuations of Key Variables (15-year)

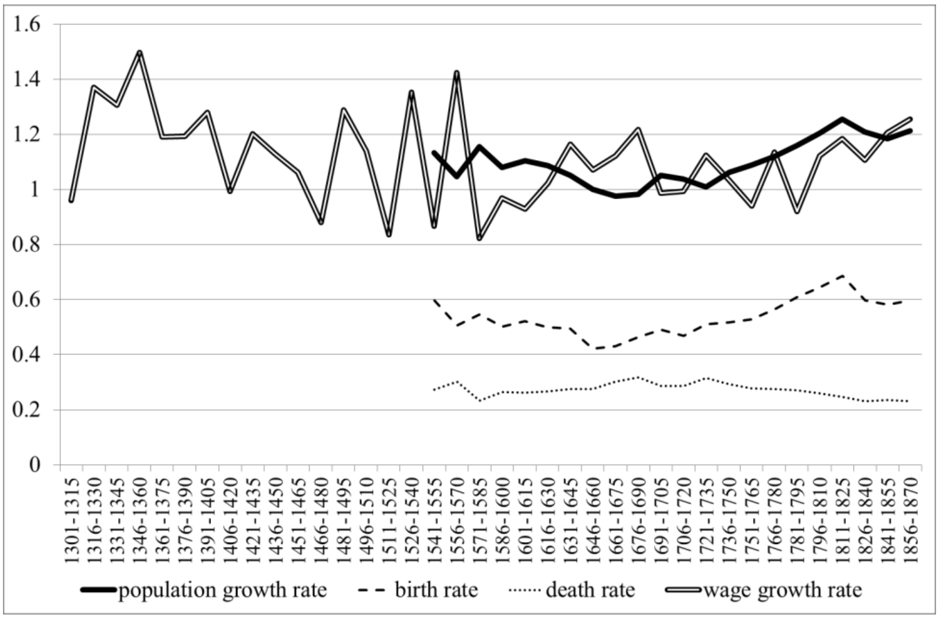


Table 9 Empirical Data in 15-year Frequency

Period	Year	Wrigley and Schofield			Clark
		$\widehat{P}_t$	b_t	d_t	g_w
1	1301-1315				0.961
2	1316-1330				1.370
3	1331-1345				1.306
4	1346-1360				1.497
5	1361-1375				1.191
6	1376-1390				1.193
7	1391-1405				1.279
8	1406-1420				0.994
9	1421-1435				1.203
10	1436-1450				1.128
11	1451-1465				1.063
12	1466-1480				0.879
13	1481-1495				1.288
14	1496-1510				1.139
15	1511-1525				0.836
16	1526-1540				1.354
17	1541-1555	1.133	0.598	0.274	0.868
18	1556-1570	1.046	0.507	0.301	1.424
19	1571-1585	1.156	0.547	0.234	0.822
20	1586-1600	1.081	0.503	0.264	0.968
21	1601-1615	1.105	0.522	0.263	0.928
22	1616-1630	1.087	0.500	0.266	1.022
23	1631-1645	1.051	0.496	0.276	1.164
24	1646-1660	1.000	0.422	0.277	1.071
25	1661-1675	0.977	0.431	0.302	1.123
26	1676-1690	0.982	0.464	0.318	1.218
27	1691-1705	1.051	0.491	0.286	0.986
28	1706-1720	1.037	0.468	0.286	0.994
29	1721-1735	1.010	0.511	0.316	1.124
30	1736-1750	1.061	0.518	0.293	1.033
31	1751-1765	1.088	0.529	0.277	0.940
32	1766-1780	1.119	0.565	0.275	1.136
33	1781-1795	1.159	0.608	0.271	0.921
34	1796-1810	1.205	0.643	0.260	1.123
35	1811-1825	1.254	0.686	0.246	1.185
36	1826-1840	1.209	0.598	0.231	1.106
37	1841-1855	1.184	0.582	0.235	1.208
38	1856-1870	1.213	0.597	0.231	1.256
	Mean	**1.100**	**0.536**	**0.272**	**1.113**
	S.D.	**0.080**	**0.067**	**0.026**	**0.166**

Appendix II: Linearised Equilibrium Conditions

$$U_t = \left[\alpha \cdot \left(\frac{n_t}{2} \right)^{\frac{s-1}{s}} + \beta \cdot \left(q_t \right)^{\frac{s-1}{s}} + \gamma \cdot \left(\frac{\hat{z}_t \hat{H}_{t-1}^{\theta} \hat{P}_{t-1}^{\theta-1}}{\hat{z}_{t-1}} \right)^{\frac{s-1}{s}} \right]^{\frac{s}{s-1}}$$

$$\alpha \left(\frac{2U_t}{n_t} \right)^{\frac{1}{s}} = \frac{1}{2} \hat{\lambda}_t \hat{H}_t^{\theta} \hat{P}_t^{\theta-1} \left[\hat{\pi}_{n,t+1} + \hat{\pi}_{q,t+1} q_t + \frac{\eta_A \hat{\pi}_{q,t+1}}{A_t} \frac{a_1}{\left(1-m1_{t+1}\right)\left(1-m2_{t+2}\right)} q_t n_t \right]$$

$$\beta \left(\frac{U_t}{q_t} \right)^{\frac{1}{s}} = \frac{1}{2} \hat{\lambda}_t \hat{H}_t^{\theta} \hat{P}_t^{\theta-1} \hat{\pi}_{q,t+1} n_t$$

$$\gamma \left(\frac{\hat{z}_{t-1} U_t}{\hat{z}_t} \right)^{\frac{1}{s}} = 3 \hat{\lambda}_t \hat{z}_{t-1} \left(\hat{H}_{t-1}^{\theta} \hat{P}_{t-1}^{\theta-1} \right)^{\frac{1-s}{s}}$$

$$\frac{\hat{w}_t}{\hat{H}_t^{\theta} \hat{P}_t^{\theta-1}} + \hat{w}_{t+1} + \hat{w}_{t+2} \hat{H}_{t+1}^{\theta} \hat{P}_{t+1}^{\theta-1} = \frac{1}{2} \left(\hat{\pi}_{n,t+1} n_t + \hat{\pi}_{q,t+1} q_t n_t \right) + \frac{3 \hat{z}_t}{\hat{H}_t^{\theta} \hat{P}_t^{\theta-1}}$$

$$A_t = a_0 - a_1 \frac{n_t}{\left(1-m1_{t+1}\right)\left(1-m2_{t+2}\right)}$$

$$\hat{\pi}_{q,t} = \frac{b_0}{\left(1-m1_t\right)\left(1-m2_{t+1}\right)} \left(\frac{b_1}{A_{t-1}} \right)^{\eta_A} \hat{w}_t$$

$$\hat{\pi}_{n,t} = \frac{c_0}{\left(1-m1_t\right)\left(1-m2_{t+1}\right)} \hat{w}_t$$

$$\hat{P}_t = 1 - \hat{D}_t + \hat{B}_t$$

$$\hat{D}_t = m1_t \hat{B}_t + m2_t \hat{G}1_{t-1} + m3_t \hat{G}2_{t-1} + \hat{G}3_{t-1}$$

$$\hat{G}1_t = \left(1-m1_t\right) \frac{\hat{B}_t}{\hat{P}_t}$$

$$\hat{G}2_t = \left(1-m1_{t-1}\right)\left(1-m2_t\right) \frac{\hat{B}_{t-1}}{\hat{P}_t \hat{P}_{t-1}}$$

$$\hat{G}3_t = \left(1-m1_{t-2}\right)\left(1-m2_{t-1}\right)\left(1-m3_t\right) \frac{\hat{B}_{t-2}}{\hat{P}_t \hat{P}_{t-1} \hat{P}_{t-2}}$$

$$b_t \equiv \hat{B}_t = \frac{\left(1-\mu\right) \hat{G}2_{t-1}}{2} \frac{n_{t-1}}{\left(1-m1_t\right)\left(1-m2_{t+1}\right)}$$

$$d_t = \frac{\hat{D}_t}{1 + \hat{B}_t}$$

$$\hat{Y}_t = \hat{H}_{t-1}^{\eta_H}\bar{X}e^{x_t}$$

$$\hat{H}_t = \hat{Q}_t\hat{G}1_t + \frac{\hat{Q}_{t-1}}{\hat{H}_{t-1}}\hat{G}2_t + \frac{\hat{Q}_{t-2}}{\hat{H}_{t-1}\hat{H}_{t-2}}\hat{G}3_t$$

$$\hat{Q}_t = \hat{H}_{t-1}^{\eta_H}\bar{X}e^{x_t}\left(\frac{\hat{Q}_{t-2}}{\hat{H}_{t-1}\hat{H}_{t-2}}q_{t-1}\right)^{1-\varepsilon}$$

$$\theta\hat{Y}_t = \hat{w}_t$$

$$g_{w_t} = \frac{\hat{w}_t}{\hat{w}_{t-1}}\hat{H}_{t-1}^{\theta}\hat{P}_{t-1}^{\theta-1}$$

$$g_{Y_t} = \frac{\hat{Y}_t}{\hat{Y}_{t-1}}\hat{H}_{t-1}^{\theta}\hat{P}_{t-1}^{\theta-1}$$

$$m1_t = \overline{m1}\cdot\exp\left(mm1_t\right), \text{ where } mm1_t = \rho_1\cdot mm1_{t-1} + emm1_t$$

$$m2_t = \overline{m2}\cdot\exp\left(mm2_t\right), \text{ where } mm2_t = \rho_2\cdot mm2_{t-1} + emm2_t$$

$$m3_t = \overline{m3}\cdot\exp\left(mm3_t\right), \text{ where } mm3_t = \rho_3\cdot mm3_{t-1} + emm3_t$$

$$x_t = \rho_x\cdot x_{t-1} + ex_t$$

NB: For simplicity, we omit the expectation operators $\mathbf{E}_t[\cdot]$, but any variable with a time subscript beyond period t should be treated as the expected value of that variable based on information set available in period t.

To summarise, this nonlinear dynamic system has 25 endogenous variables, denoted as $\mathbf{y}_t$, and 4 exogenous shocks, denoted as $\mathbf{u}_t$, where:

$$\mathbf{y}_t \equiv \begin{bmatrix} U_t; n_t; q_t; \hat{z}_t; \hat{\pi}_{nt}; \hat{\pi}_{qt}; A_t; \hat{\lambda}_t; \\ \hat{P}_t; \hat{B}\left(b_t\right); \hat{D}_t; d_t; \hat{G}1_t; \hat{G}2_t; \hat{G}3_t; \\ \hat{Y}_t; \hat{Q}_t; \hat{H}_t; \hat{w}_t; g_{wt}; g_{Yt}; mm1_t; mm2_t; mm3_t; x_t \end{bmatrix}, \text{ and}$$

$$\mathbf{u}_t \equiv \left[emm1_t; emm2_t; emm3_t; ex_t\right].$$

Appendix III: Solution, Simulation and Estimation Methods

The DSGE model can be summarised by a nonlinear system of equilibrium conditions described in Appendix II. After transformation, all the endogenous variables are stationary so that steady states exist. Denote the vector of stationarised endogenous variables as $\mathbf{y}_t$, the vector of exogenous shocks as $\mathbf{u}_t$, and the vector of structural parameter as $\boldsymbol{\theta}$. Hence, the *nonlinear dynamic* system can be written as:

$$\mathbf{E}_t\left[f\left(\mathbf{y}_{t-1},\mathbf{y}_t,\mathbf{y}_{t+1},\mathbf{u}_t;\boldsymbol{\theta}\right)\right]=0.$$

Solution

In steady state, we assume the shocks are all equal to zero, and the endogenous variables are equal to their steady state levels $\bar{\mathbf{y}}$. The system in steady state is:

$$f\left(\bar{\mathbf{y}};\boldsymbol{\theta}\right)=0.$$

Since the number of equations are equal to the number of unknowns ($\bar{\mathbf{y}}$), the nonlinear equation system can be numerically solved using the Newton algorithm. Once the steady state levels ($\bar{\mathbf{y}}$) are solved (in terms of $\boldsymbol{\theta}$), we can then log-linearise the original system around the steady state. This leads to a *linear dynamic* system:

$$\mathbf{AE}_t\left[\mathbf{y}_{t+1}\right]+\mathbf{By}_t+\mathbf{Cy}_{t-1}+\mathbf{Du}_t=0, \text{ where } \mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D} \text{ are functions of } \boldsymbol{\theta}.$$

The system can be solved using perturbation method as described in Schmitt-Grohe and Uribe (2004). Note that to be solvable, the linearised system of difference equation needs to satisfy the Blanchard-Khan condition (1980), i.e. the number of forward-looking variables is equal to the number of unstable eigenvalues. The solution here is a first order approximation to the true solution to the original nonlinear system, which is actually a VAR process with restrictions:

$$\mathbf{y}_t=\bar{\mathbf{y}}+\mathbf{g}_\mathrm{y}\left(\mathbf{y}_{t-1}-\bar{\mathbf{y}}\right)+\mathbf{g}_\mathbf{u}\mathbf{u}_t, \text{ where } \mathbf{g}_\mathrm{y} \text{ and } \mathbf{g}_\mathbf{u} \text{ are functions of } \boldsymbol{\theta}.$$

Simulation

The solution can be used to simulate the moments of the endogenous variables and to track them, either by theoretical derivation based on the probability distributions of $\mathbf{u}_t$ or by Monte Carlo simulation. In this paper, we adopt the former procedure, so:

$$\mathbf{Var}\left[\mathbf{y}_t\right]=\mathbf{g}_\mathbf{u}\mathbf{Var}\left[\mathbf{u}_t\right]\mathbf{g}_\mathbf{u}'.$$

Based on this above equation, we can obtain the conditional variance-covariance matrix of the endogenous variables. We can also conduct a variance decomposition to analyse the contribution of each shock to the variance of each endogenous variable.

The impulse response functions can be derived based on the solution by rewriting the VAR into a VMA process:

$$\left(\mathbf{I}-\mathbf{g}_{\mathrm{y}}L\right)\mathbf{y}_t=\left(\mathbf{I}-\mathbf{g}_{\mathrm{y}}\right)\bar{\mathbf{y}}+\mathbf{g}_{\mathbf{u}}\mathbf{u}_t\text{, where } L \text{ is lag operator;}$$

$$\mathbf{y}_t=\bar{\mathbf{y}}+\left(\mathbf{I}-\mathbf{g}_{\mathrm{y}}L\right)^{-1}\mathbf{g}_{\mathbf{u}}\mathbf{u}_t\text{, if it is invertible;}$$

$$\mathbf{y}_t=\bar{\mathbf{y}}+\mathbf{\Phi}(L)\mathbf{u}_t\text{, where } \mathbf{\Phi}(L) \text{ is a polynomial of order } L\text{;}$$

$$\mathbf{y}_t=\bar{\mathbf{y}}+\sum_{i=0}^{\infty}\mathbf{\Phi}_i\mathbf{u}_{t-i}\text{, where } \mathbf{\Phi}_i \text{ is a function of } \boldsymbol{\theta} \text{ and } \bar{\mathbf{y}}.$$

The impulse response functions of a particular variable in $\mathbf{y}_t$ with respect to a particular shock in $\mathbf{u}_t$ can be extracted from the matrices $\mathbf{\Phi}_i$:

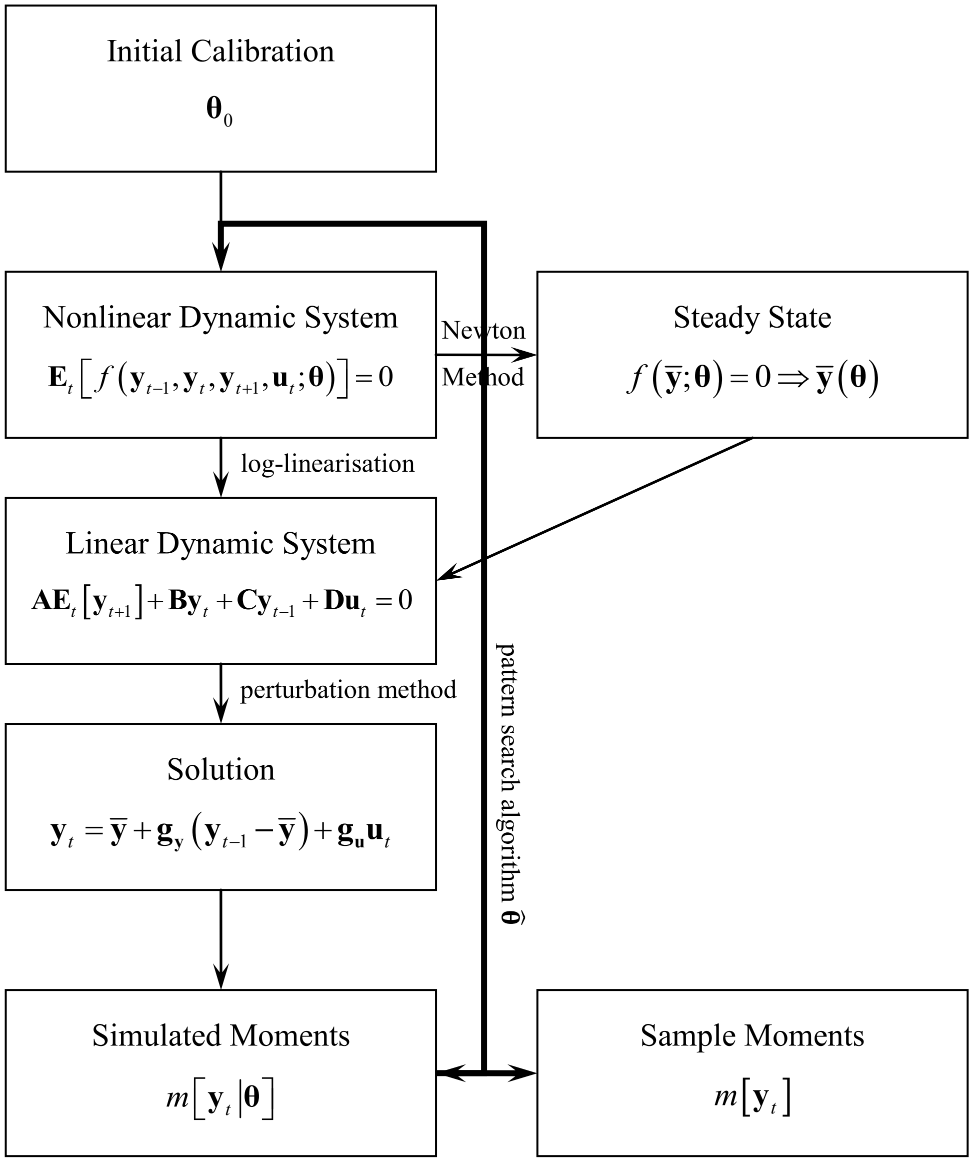
$$\mathbf{\Phi}_i=\frac{\partial \mathbf{y}_{t+i}}{\partial \mathbf{u}_t'}.$$

The solution and simulation procedures can be conveniently accomplished by the Dynare toolbox in Matlab or Octave environments.

Estimation

We assume the parameter value $\boldsymbol{\theta}$ is known in solution and simulation. However, we only know an approximate value of $\boldsymbol{\theta}_0$, its upper bound $\overline{\boldsymbol{\theta}}$ and lower bound $\underline{\boldsymbol{\theta}}$. This initial value $\boldsymbol{\theta}_0$ is chosen to ensure the model converges. A popular way of judging the performance of a model is to check its ability to match the moments observed in the data. We have historical data of four series, i.e. population growth, birth rate, death rate and wage growth. It is easy to work out the sample mean and the sample variance of the four series. In addition, we also know that the marriage age is around 25 years during the sample period. To maximise the ability of the model to match these sample moments, we implement an optimisation procedure based on a pattern search algorithm. The objective function is the gap—the squared difference—between model-simulated moments and the sample moments, while the choice variable is $\boldsymbol{\theta}$ and the constraints are the two bounds. The algorithm then searches the best $\widehat{\boldsymbol{\theta}}$ over the whole parameter space to minimise the objective function. For further references, see the Matlab manual on the global optimisation toolbox.

We use this global optimisation algorithm, because the objective function cannot be differentiated—the simulated moments depend on the solution of the model which in turn depends on the *numerical* solution of steady state $\bar{\mathbf{y}}$. The following flow charts illustrates the links among solution, simulation and estimation procedures.



This estimation procedure is termed as optimal calibration, indirect estimation or simulated method of moments in DSGE literature.

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