Some aspects of gear tribology

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Abstract: The article briefly reviews some developments in gear tribology research against the background of three articles from the Institution's archives that have particularly influenced the authors' own contributions to the subject. The articles considered relate to three different aspects of gearing namely elastohydrodynamic lubrication, tooth contact phenomena, and the lubrication and wear of gears having a three-dimensional geometry.

Keywords: gears, lubrication, scuffing, pitting, Wildhaber–Novikov

1 INTRODUCTION

Three articles [**1–3**] from the Institution's archives have been selected and reprinted in this anniversary issue of *Journal of Mechanical Engineering Science* (*JMES*) as representative of the many important publications that have influenced subsequent research on gear tooth contact tribology. The following three sections give a brief introduction to each of the three articles together with some examples of the work that has been inspired by or has built on the ideas elucidated in them.

2 DOWSON AND HIGGINSON (1959)

It is appropriate that the first of these, the landmark 1959 article of Dowson and Higginson [**1**] on a numerical solution of the elastohydrodynamic lubrication (EHL) problem, appeared in Volume 1, Part 1 of *JMES* (reproduced on page 54 of this issue). Using the rudimentary computers of the time, these authors found the first detailed solution for pressure and film thickness in a heavily loaded lubricated line contact in which the two crucially important effects of pressure-dependent viscosity of the lubricant and elastic deformation of the surfaces were taken into account. Dowson and Higginson continued their effort to refine their solutions to reveal greater detail, in particular the first calculated existence of the 'pressure spike' at the exit of the contact (which had not appeared in their first solutions), and to take account of thermal effects. Their 1966 book [**4**] included a formula for minimum film thickness under isothermal conditions, and the practical relevance of EHL to gears was acknowledged by their inclusion of a short chapter entitled 'A Note on Gear Lubrication'. It had been known for many years previous to this work that gears benefited from effective hydrodynamic lubrication. For example, it was reported that when the main propulsion gears of the transatlantic liner 'Queen Mary' were examined after 11 years' operation '…no wear could be detected on the gear teeth' [**5**]. In spite of such overwhelming evidence for effective oil-film lubrication, it was not until the appearance of Dowson and Higginson's EHL theory that a detailed physical explanation for the successful operation of gear tooth contacts could be provided.

Developments since Dowson and Higginson's article have been concerned with refinements such as the study of 'point' contacts, the inclusion of non-smooth features in the surfaces like bumps, dents, regular waviness and, more recently, real, or measured, roughness. Early analyses of EHL were based upon the assumption of a Newtonian lubricant and an exponential (Barus) dependence of viscosity on pressure. These assumptions are generally adequate for the estimation of film thickness because this is determined under the relatively low-pressure conditions in the entry region of the overall contact, but they lead to gross overestimates in calculations of the viscous traction. To predict traction and frictional heating, it is necessary to adopt a non-Newtonian, shear-thinning model for the lubricant.

Initial attempts to model rough surface EHL were based on idealized rough surface features such as

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single irregularities [**6**] and sinusoidal waviness [**7–9**]. Later, the study of real (measured) roughness was carried out by Kweh *et al.* [**10**]. This model was limited to the time-independent case of simple sliding in which a smooth surface slides against a stationary rough surface. To model 'moving roughness', the transient effects due to interaction of asperities must be considered in a time-dependent solution. Chang and Webster [**11**] simulated the interactions of sinusoidal roughness on both surfaces in a transient analysis. Venner and Lubrecht [**12**] investigated indentation features and waviness in a transient analysis and showed the effects of the slide–roll ratio on the pressure and film thickness profile. Chang and Zhao [**13**] continued to study the differences between Newtonian and non-Newtonian lubricant models in transient micro-EHL and showed that the non-Newtonian effect was significant, particularly with roughness of relatively short wavelength. A study of micro-EHL under point contact conditions using measured roughness was carried out by Xu and Sadeghi [**14**] and Zhu and Ai [**15**]. In these studies, however, the amplitude of the roughness was relatively small compared to the thickness of the films generated.

The study of micro-EHL leads naturally to 'mixed' lubrication, where part of the load is carried by a hydrodynamic film and part of it is supported by 'dry' or boundary-lubricated contact between asperity features. The incorporation of these effects into a consistent mass-conserving physical model represents a particularly difficult challenge. In recent years, attempts have been made to address the problem using different numerical approaches. Chang [**16**] presented a deterministic model for line contacts that incorporates asperity contact and transient hydrodynamic behaviour. Jiang *et al.* [**17**] and Zhao *et al.* [**18**] solved the point contact problem by partitioning the overall contact area into subregions corresponding to solid contact or hydrodynamic separation. Hu and Zhu [**19**] developed a unified approach for full film, mixed lubrication, and boundary lubrication by using a simplification of the Reynolds equation for thin film conditions. The ability to model conditions in which the film was relatively thin compared to roughness was aided by the introduction of a radically new approach to solve the EHL problem developed by Hughes *et al.* [**20**] and Elcoate *et al.* [**21**]. The essential feature of the new technique is the 'differential deflection' approach to the elastic deformation aspect of the problem, which allows close coupling of the hydrodynamic and elasticity equations. In this way, the primary variables of film thickness and pressure can be treated as simultaneously active throughout the solution process. This leads to a highly robust and rapidly convergent numerical scheme that can cope with severe conditions of thin films/high roughness. Using the new technique, Elcoate *et al.* [**21**] were able to model the behaviour of real gear lubrication

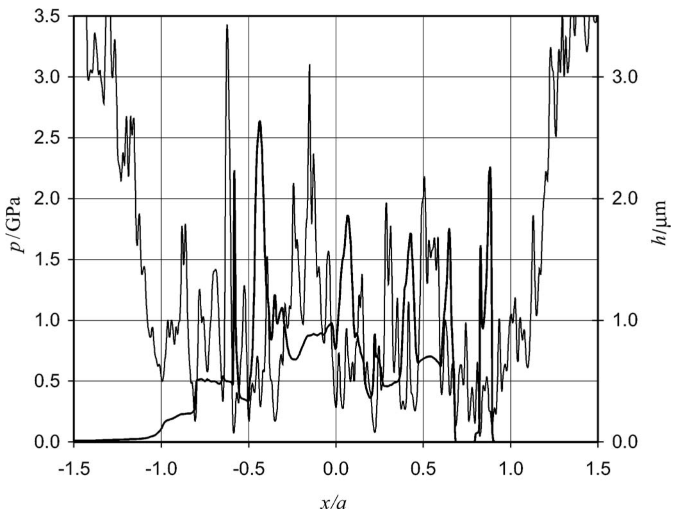

Fig. 1 Pressure (heavy curve) and film thickness (light curve) profiles at one particular time-step in a transient EHL analysis of two rough surfaces

conditions in which surface roughness was at least an order of magnitude greater than the minimum film thickness. Figure 1 shows the pressure and film thickness profiles at one particular time-step in a transient EHL analysis of two rough surfaces whose profiles have been taken from gear teeth used in a micropitting test. In this case, the equivalent smooth surface solution gives a maximum pressure of 1.0 GPa. The pressures developed at micro asperity contacts are considerably higher and are found to rise to the levels of 2–3 GPa. Later, the technique was extended to point contacts by Holmes *et al.* [**22**] and to incorporate mixed lubrication behaviour [**23**].

3 MERRITT (1962)

EHL theory could explain the *successful* operation of gears but it could not be relied upon to predict when *failure* occurred as a result of the numerous destructive forces at work in practical gear tooth contacts. The state of understanding of gear tooth contact phenomena in the early 1960s was admirably reviewed (in the second article reprinted here as Appendix 2) by Merritt [**2**]. The article was in the form of an Institution 'Nominated Lecture' and, as was the custom in those days, it attracted a considerable amount of in-depth written discussion from leading gear practitioners and researchers including W. A. Tuplin, A. W. Crook, B. A. Shotter, J. F. Archard, and H. J. Watson. At the time the article was published, scuffing (Fig. 2) was seen as particularly troublesome because it happened unexpectedly, often after only a few hours of operation. Scuffing still occurs from time to time in some heavily loaded, high-speed applications, particularly where high tooth surface temperatures are allowed to develop due to inadequate design of the cooling

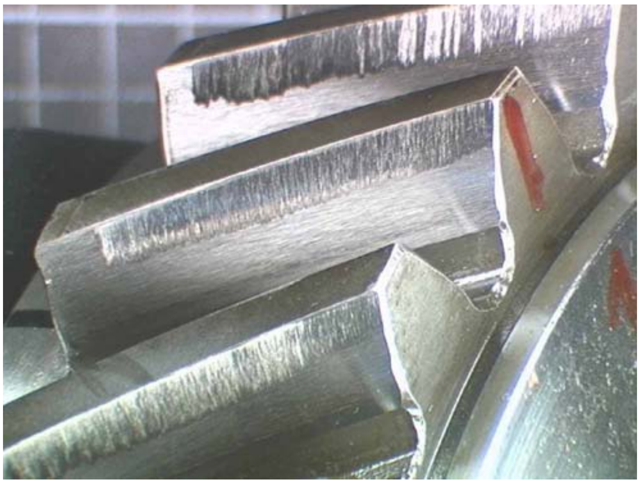

Fig. 2 Scuffing at the tips of helical gear teeth. Photograph courtesy of design unit, Newcastle University

arrangements. In the majority of transmissions, the problem has largely been overcome by the use of chemically active 'extreme pressure' oil additives and by careful attention to tooth geometry and alignment (tip relief and crowning) of the gears so as to avoid highly stressed and damaging edge contacts. It cannot be claimed that a 'cure' for scuffing has been discovered, however, and its exact physical mechanism, in particular the way in which EHL can totally break down under certain conditions, remains unexplained. In his article, Merritt drew attention to the influence of surface finish both in relation to scuffing and pitting. He made some particularly interesting comments about pitting behaviour as observed in disc tests and in actual gears. He noted that pitting (rolling contact fatigue) in gears occurred at nominal contact stresses lower than those which produced similar results in discs. He attributed this to 'macro errors' of tooth alignment and profile; but he went on to consider 'micro errors' (which may be interpreted as roughness effects), which can convert a nominally uniform band of contact into a series of small pressure areas. This, he said, produced effects analogous to the leakage effects seen in short journal bearings in which oil can escape laterally from the pressure areas. These comments (made almost half a century ago) remain highly relevant to current ideas on the lubrication of rough contacts and the attempts to model what has become known as 'micro-EHL'. In the case of scuffing, Merritt's particular idea of transverse leakage in a rough contact was in fact the basis of an EHL failure model proposed by the present authors [**24**]. This model predicted severe thinning of the lubricant film close to the edge of an elliptical contact due to sideways leakage of the lubricant in the transverse valley features present between rough surfaces even when in contact at a heavy load. In corresponding scuffing tests, using transverse-ground discs, it was found that scuffing invariably occurred at this location as predicted.

Among the contributions to the discussion on Merritt's article was an item from Shotter [**25**] who showed the results of pitting tests on Wildhaber–Novikov (W–N) gears. The sliding and entraining velocities at the tooth contact are roughly perpendicular because of the peculiar geometry of W–N gears (see below). Shotter noted that although the start of a pit pointed down the tooth flank (the sliding direction) its subsequent growth was along the tooth (the entraining, or rolling direction). These facts suggested that pitting was initiated by sliding (friction) but that subsequent propagation was assisted by hydraulic pressurization of the crack due to the rolling motion as had been suggested in an early article by Way [**26**]. Shotter continued to point out that real gear tooth surfaces did not have the perfect geometry assumed in the EHL models (at this time the models were based on the assumption of perfectly smooth surfaces). Due to this, the stresses that produced pitting were to be found at the contacts between roughness asperities rather than the nominal Hertzian contact because of the general curvature of the teeth. Shotter considered that these asperity contact effects were of importance in scuffing as well as pitting – a view that has been reinforced by the subsequent research in the field.

It is of interest to note that these ideas of surface roughness and crack pressurization effects being responsible for the initiation of surface distress are highly relevant to present day research on 'micropitting', which is currently identified as the main factor limiting gear performance and competitiveness [**27**]. It is a serious surface distress problem directly associated with roughness effects in gears finished by grinding. It causes serious wear and can develop into large-scale damage in the form of cracks that jeopardize tooth integrity. It is characterized by local plastic deformation and the formation of small pits some 10–30 μm in diameter and 5–10 μm deep that grow from surface initiated cracks at 10–30° inclination (Fig. 3). Micropitting may be classified as a form of erosive wear leading to serious loss of tooth profile accuracy and a consequent, unacceptable, increase in noise from the gears. A particularly serious feature of micropitting in hardened steels is subsurface crack branching that leads to large-scale pitting and eventual tooth fracture and total gear failures [**28**]. The problem of micropitting has a severe economic impact on industry and, to enhance national competitive advantage, is currently being investigated by several leading gear trade organizations (BGA in UK, FVA in Germany, and AGMA in USA). Key features of micropitting in gears are:

(a) damage occurs on the prominent parts of asperity features;

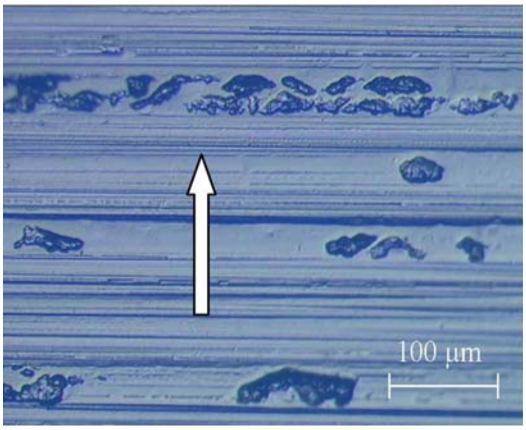

Fig. 3 Optical image of replica taken from a damaged gear tooth showing characteristic micropits occurring at the high points of surface texture. Arrow indicates rolling/sliding direction. Photograph courtesy of QinetiQ

(b) damage occurs across the whole face width of the gear;
(c) damage takes place in areas of the tooth profile that are subject to high sliding during the meshing cycle;
(d) micropitting predominantly occurs at the root of the teeth and does so without damage to the corresponding contacting part of the meshing tooth;
(e) shallow cracks grow in the opposite direction to that of the surface traction due to sliding (i.e. towards the pitch point on the driver and away from the pitch point on the driven wheel).

To achieve a fundamental understanding of the causes of the clearly observed and characteristic features of the damage produced as described above, it is necessary to establish the detailed nature of the tooth loading in terms of the transient asperity pressure, surface traction and subsurface stress, and temperature fields.

To model the conditions under which micropitting occurs, the theoretical treatment must include:

(a) low Λ ratios (ratio of calculated smooth-surface oil-film thickness to composite roughness), typically 0.1 or less;
(b) the transient effects brought about by two rough surfaces moving relative to each other;
(c) non-Newtonian lubricant behaviour caused by high shear rates in the film at the parts of the meshing cycle subject to high sliding;
(d) thermal effects brought about by high sliding, general temperature increase in the film and contacting bodies, and potentially catastrophic localized 'flash' temperatures at the site of asperity collisions.

Surface pressure distributions obtained from the foregoing transient micro-EHL analyses can be used to calculate elastic stress distributions in the highly stressed surface layer of the gear surfaces involved. Fatigue and accumulated damage may then be predicted on the basis of the available fatigue theories. In work being carried out by the authors, the stress components are evaluated at each time-step from a transient micro-EHL solution. This task is made more manageable by the use of the discrete convolution fast Fourier transform technique described by Liu *et al.* [**29**]. The time-varying stress distribution obtained in this way is related to a position fixed in the body concerned so that a 'history' of stress variation is obtained at a grid of points in a volume of material as it passes through the contact region as illustrated schematically in Fig. 4. Three different multi-axial fatigue criteria based on a critical plane approach (Findley [**30**], Matake [**31**], and Dang Van *et al.* [**32**]) were applied to the test volume and the results compared. In addition, a varying amplitude multi-axial fatigue theory based on shear strain cycles was also applied (Fatemi and Socie [**33**]). The cycle counts were obtained using the rainflow counting method [**34**] and the accumulated damage in a single pass through the contact area was calculated (i.e. the accumulated damage per gear meshing cycle). In comparing the results obtained, the various fatigue models were found to identify the same asperity features as being those most prone to fatigue. Results were obtained using a gear tooth profile from previously reported FZG tests [**35**]. The profile was run against itself in a transient EHL simulation and the time-varying stress field obtained as outlined above. A local friction coefficient of 0.1 was assumed where 'direct contact' events occurred in the simulation. Figure 5 shows a comparison of the fatigue parameter (FP) contours predicted by the three different critical plane models. The operating conditions correspond to a maximum equivalent Hertzian pressure of 10 GPa, an entraining speed of 25 m/s, and a slide–roll ratio of 0.5. The lubricant properties assumed are those of Mobiljet 2, a gas-turbine engine oil at a bulk temperature of 100 °C. It is clear that failure

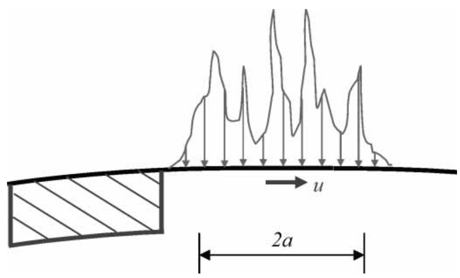

Fig. 4 Solid surface to which transient EHL pressure and shear stress (not shown) distributions are applied showing block of material that passes under the contact region

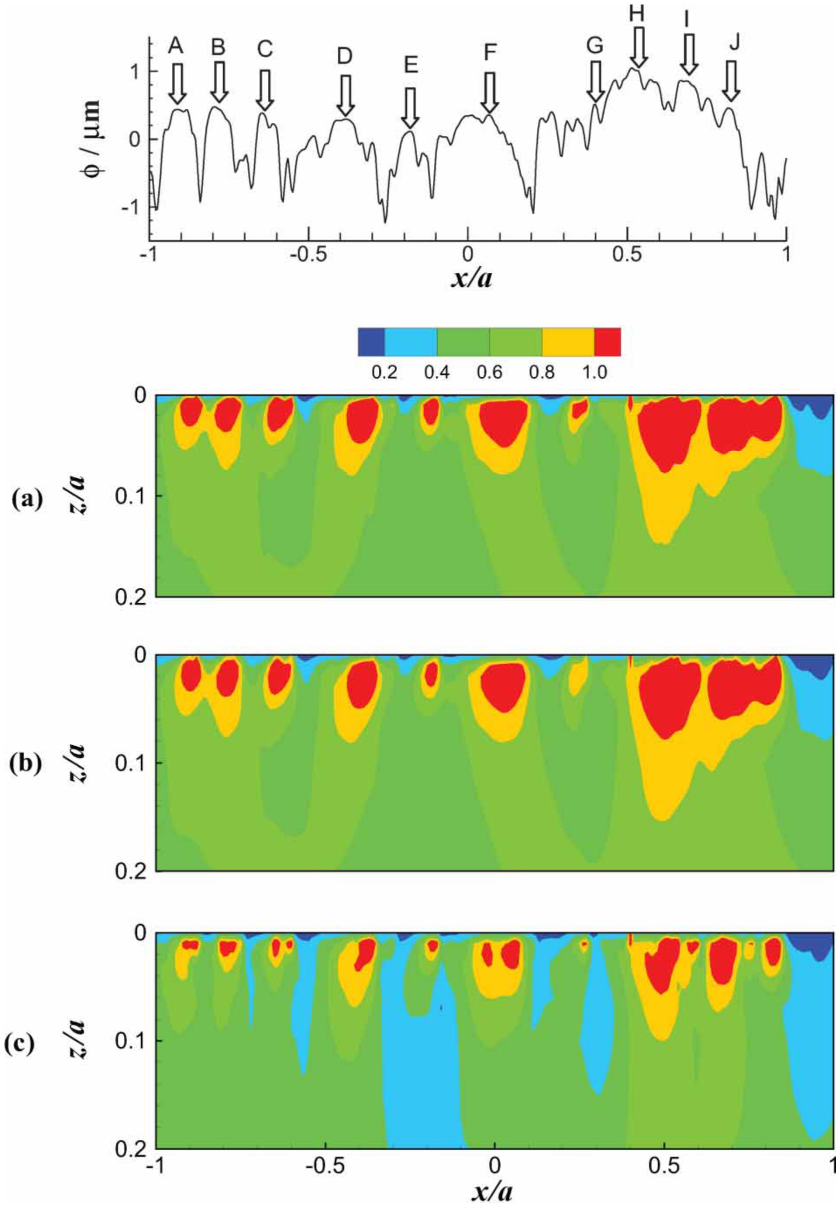

Fig. 5 Contours of FP for fatigue at 10^7 cycles determined by using (a) Findley, (b) Matake, and (c) Dang Van criteria. The upper graph shows the surface profile

zones are concentrated close to the surface of certain asperity features (arrowed in the figure). The results predicted by the Findley and Matake models are very similar, whereas the Dang Van model gives slightly lower predicted fatigue.

Thin hard coatings have the potential to improve the surface durability of gears, and several successful applications have been reported [**36–38**]. Diamond-like carbon coatings show promise because of their high hardness and low friction behaviour, and their deposition methods and wear properties have therefore been studied extensively in recent years [**39–41**]. Naik *et al.* [**42**] carried out experiments on two coatings of this type by using both disc and gear rigs and reported promising results. Alanou *et al.* [**43**] performed experiments in a disc machine at high sliding speeds and reported a significant improvement in scuffing resistance although poor adherence was found for one particular combination of coating and substrate. More recently, Amaro *et al.* [**44**] reported good adhesion, low friction, excellent scuffing, and wear protection with a hard carbon/chromium coating.

A surface treatment that has recently been shown to be highly effective in increasing the surface fatigue (pitting) lives of gears is the application of a hard coating of the metal-containing diamond-like carbon (Me-DLC) type designed specifically for aerospace gearing applications. Krantz *et al.* [**45**] carried out accelerated surface fatigue experiments on both

Me-DLC-coated and uncoated hardened steel gears at a maximum Hertzian contact pressure of 1.7 GPa and demonstrated an approximately 4-fold increase in pitting life compared to that obtained with uncoated gears made from the same batch of steel. Some of the coated gears were operated at a maximum contact pressure of 1.9 GPa and showed good durability even at this relatively high loading.

Recent work [**46**] was carried out to investigate the *scuffing* resistance of the Me-DLC coating as used by Krantz *et al.* under conditions of high sliding and high Hertzian contact pressure representative of extreme gearing applications. The experiments were performed using axially ground carburized and hardened steel discs lubricated with a gas-turbine engine oil at a lubricant temperature of 100 °C. In addition to improving the scuffing resistance of test discs, the hard coating also gave a striking reduction in friction when applied to discs that had been ground. Figure 6 shows measured friction for four different surface conditions. Curve A shows the relatively high friction of ground/uncoated surfaces and curve B is the measured variation when the hard coating was applied. In both cases, the friction coefficient reduces with load, an effect which is attributed to running in and partial smoothing of the rough ground surface. Curve C shows the corresponding variation for superfinished/uncoated discs. In this case, friction tends to increase slightly with load; this is probably due to slight roughening of the very finely polished surfaces as a result of running. Although hard-coating of the superfinished discs (curve D) gave the lowest friction at a relatively light load, the behaviour deteriorated at heavier loads. This was found to be due to the coating becoming partly detached leading to a significant increase in the effective roughness of the surfaces.

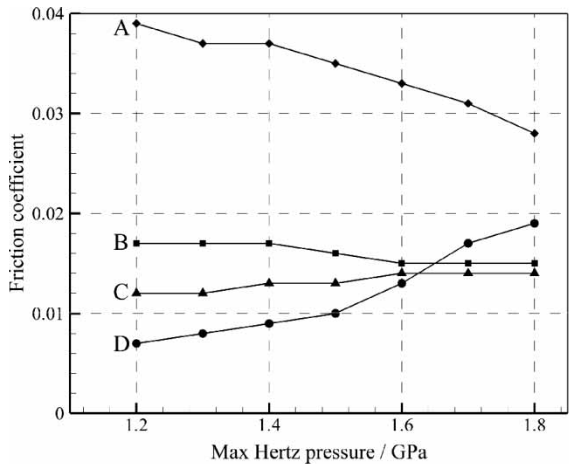

Fig. 6 Variation of friction coefficient with load during stages preceding scuffing. (A) ground versus ground; (B) ground/coated versus ground/coated; (C) superfinished versus superfinished; (D) superfinished/coated versus superfinished/coated

4 FRENCH (1965)

As mentioned above, Shotter's example of pitting failure occurred on the teeth of W–N gears. Unlike conventional involute gears, the teeth of W–N gears (Fig. 7) are not conjugate in the transverse plane. In the version of W–N gears used by Shotter ('Circarc'), the teeth were of convex circular arc form on the pinion and the teeth on the wheel were of concave circular arc form, the radius of curvature being slightly larger than the radius of the teeth on the pinion. To provide a constant velocity ratio between the gears, they must be of helical form. The elastic contact between the teeth under load is elliptical in shape, the long axis of the ellipse lying roughly along the teeth (i.e. the axial direction). When the gears rotate the contact moves across the gears in the axial direction but remains at the same height on the teeth. Entrainment (rolling motion) occurs in this axial direction, and this is accompanied by what may be described as conventional sliding in the direction up and down the teeth as in involute gears. In the gears described by Shotter (and later adopted by Westland helicopters in the Lynx aircraft), the teeth on the pinion were 'all addendum' and the mating teeth on the wheel 'all dedendum' so there was no position of zero sliding in the direction up/down the teeth. Such gears are described as having a recess action.

W–N gears acquired their name because they were patented by Ernest Wildhaber in USA in 1926 [**47**] and independently re-invented by Novikov in Soviet Russia in 1956 [**48**]. In spite of what were thought to be the theoretical advantages of W–N gears (reduced contact pressure and superior film formation), they were

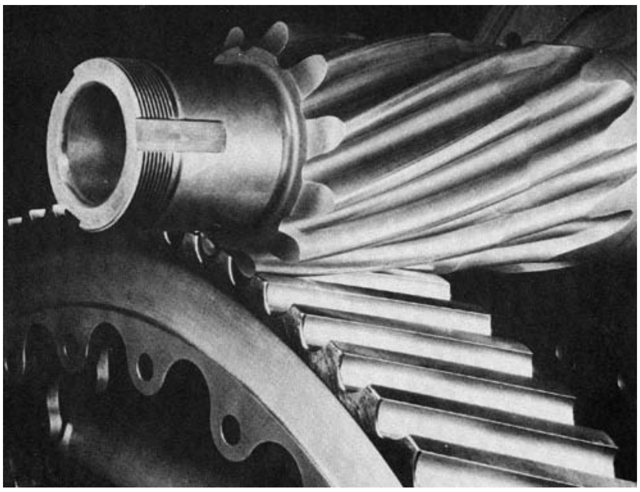

Fig. 7 Wildhaber–Novikov gears. The pinion (with convex teeth) is above and the wheel (with concave teeth) is below

not generally adopted in the West with the notable exceptions of the 'Circarc' and Westland Lynx designs mentioned above. Analysis of the geometry and kinematics of these gears had not progressed significantly, but the publication by French [**3**], which is the third article reproduced in this anniversary volume (as Appendix 3), stimulated the present authors to begin an investigation of the geometry of W–N gears which could be used as a firm basis for their design. French carried out an approximate analysis of the geometry of circular arc W–N gears and predicted that the elastic contact area was probably banana-shaped rather than

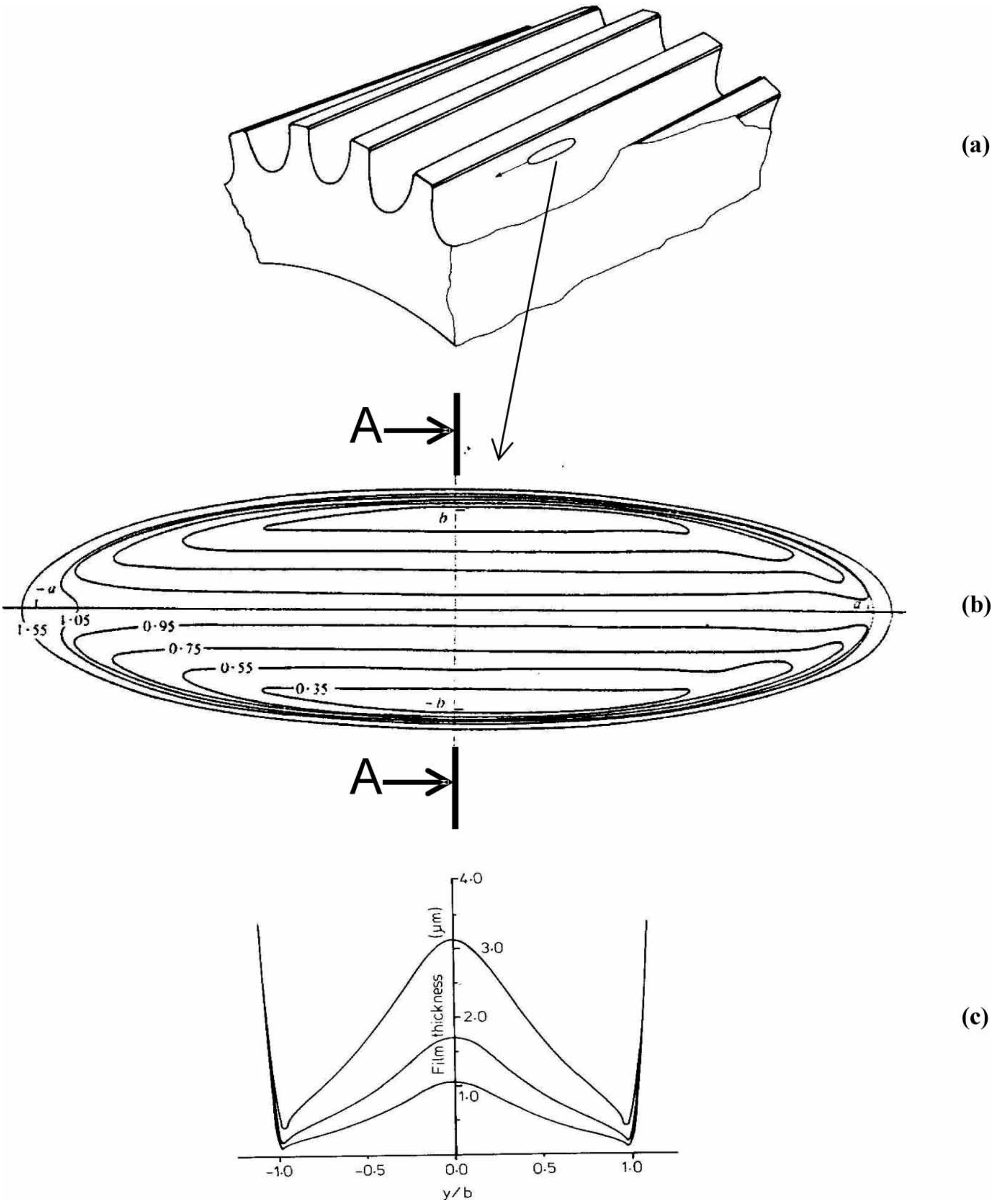

Fig. 8 EHL of a Wildhaber–Novikov gear tooth contact. (a) Illustration of wheel tooth showing orientation of Hertzian contact and its motion relative to the tooth; (b) contours of film thickness obtained from EHL analysis; and (c) section on A–A of film showing severe thinning at the transverse edges with increasing load

the ellipse previously assumed. The consequence of this suggestion was that an elongated contact would not spread along the teeth but would curve towards the tooth edges, thus limiting the conformity which could, in theory, be achieved by reducing the helix angle of the teeth. French had assumed in his analysis that the degree of mismatch between the radii of the contacting teeth in the transverse plane was small. In practical designs, being considered in the UK, however, significant mismatch was being proposed to reduce the sensitivity of the gears to variations in centres-distance setting due to deflection under load and thermal expansion. It was therefore decided to pursue an exact analysis of the geometry of W–N gears free from any assumptions about the degree of tooth conformity. This work was carried out with the help of Dr Alan Dyson, a retired Shell employee who had previously published a general theory of three-dimensional gears based on classic differential geometry techniques [**49**].

A detailed analysis [**50**] was developed and used to reveal the exact geometry and kinematics of the tooth contact from which the entraining velocity and its orientation relative to the contact could be obtained. This analysis was subsequently extended [**51**] to determine a number of properties of the gears, which are of direct relevance to their design and which could be used as a basis of a design optimization process. The contacts in gears of this type are relatively heavily loaded with maximum Hertzian contact pressures of up to 2 GPa. A special point contact EHL solver [**52**] was developed based on the inverse hydrodynamic techniques as used by Dowson and Higginson in their pioneering line contact solution. The elastic contact area in these gears is elliptical in shape with lubricant entrainment in the direction of the major axis. Due to the elongated shape of the contact side-leakage of the lubricant in the entry region of the contact was a dominant effect. This caused considerable thinning of the film at the edges of the nominal contact region as shown in Fig. 8. Thus, in spite of the relatively high entrainment speed at the contacts in W–N gears (which had often been cited as a principal film-forming advantage of the system), the oil-film generation in this type of gear was unfortunately not as favourable as had been expected.

Longitudinally entrained elliptical contacts also occur in worm gear designs. Worm gears are widely used in transmission arrangements in which a compact, high reduction, relatively low-speed drive is required. Due to the high degree of sliding present between the teeth of worm gears, it is an accepted practice to adopt a hard/soft combination of materials to prevent scuffing, the usual configuration being a hard steel worm and a softer worm wheel of bronze. An inevitable consequence of this arrangement is that the bronze gear teeth are subject to a rate of wear which is much higher than that which might be expected in conventional gearing. But in view of the other

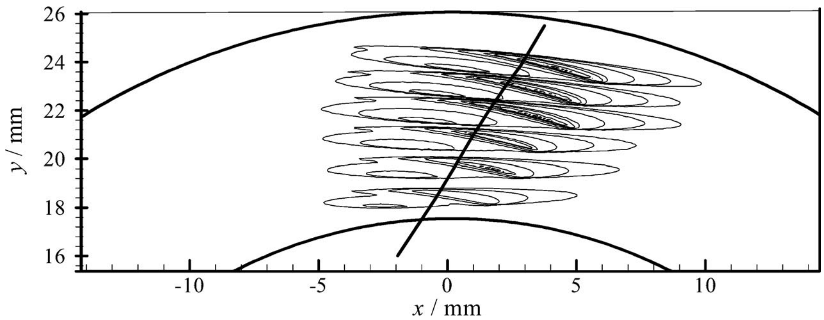

Fig. 9 Contours of wear rate/μm/s calculated at a series of meshing positions on a worm wheel tooth surface. Solid curve indicates path of nominal contact point over the meshing cycle

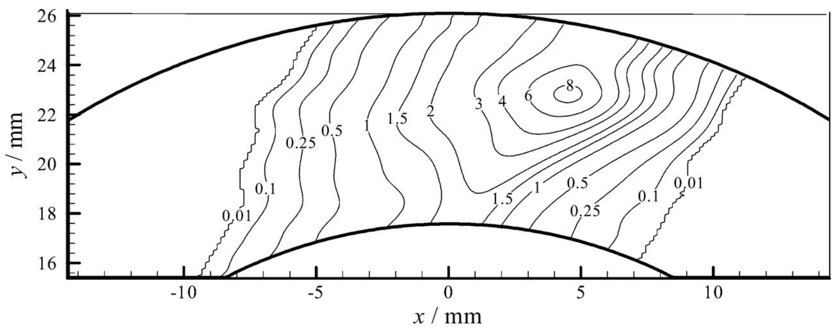

Fig. 10 Contours of wear/μm showing calculated worm wheel tooth wear over a single meshing cycle

advantages of worms this situation is largely tolerated, and indeed the process of 'bedding in' of new gears, in which an initially higher rate of wear occurs, is seen as beneficial in terms of the subsequent operation and life of the gears. In a recent project, the detailed geometry of wear in worm gears under lubricated conditions has been predicted on the basis of full EHL analysis combined with an empirical wear law.

The starting point for an EHL analysis of worm gears is the geometry and kinematics of the tooth surfaces close to their contact. The contact area over which the load is carried tends to be a long thin ellipse that is distorted into a banana shape by the enveloping nature of the surfaces. Such a configuration could yield relatively thick lubricant films if lubricant entrainment was in the direction of the minor axis, i.e. across the contact area. Unfortunately, this is not the case as the sweeping action of the worm means that the entrainment is effectively along the major axis of the contact ellipse. The authors have previously examined the film forming capability of a large number of worm gear designs [**53**, **54**] based on a full thermal EHL analysis, and this study identified locations of particularly thin films within the contact area caused by the relatively unfavourable kinematic action of the gears.

The EHL film forming analysis was extended to include a calculation of the wear rate at each point on the wheel tooth surface [**55**, **56**]. The following Archard-type wear law was adopted, which takes account of the local values of both contact pressure and film thickness in relation to the roughness of the surfaces

$$\text{Wear rate } = \frac{k}{H}\, p\, u_{\mathrm{s}} \left(\frac{R_{\mathrm{a}}}{h}\right)^{n}$$

where k is a wear constant, H is the hardness, p is the local contact pressure, u_s is the sliding speed, R_a is the roughness average, and h is the local film thickness. The local wear pattern is then integrated over the meshing cycle to obtain the tooth wear per wheel rotation. The wear is not uniform and modifies the effective contact area. This effect of wear on wheel tooth geometry is taken into account by recalculating the EHL film and pressure distributions at the end of a series of wear stages (or steps) and the calculated change in wheel tooth shape is incorporated into the subsequent calculations. In this way, the development of the pattern of wear and its effect on contact and lubrication performance can be studied. Figure 9 shows calculated contours of wear rate on the wheel tooth at six different meshing positions. The material removed by wear over the whole tooth during a complete meshing cycle is then obtained by a process of summation over the meshing cycle. A typical result of this process is shown in Fig. 10, where the contours plotted are the depth of material removed per meshing cycle.

5 CONCLUSION

Each of the three articles that were selected for reprinting in this anniversary issue had a marked influence on the development of fundamental research on gear lubrication, design and prevention of wear, and surface failure. It is hoped that the present authors' brief review of some subsequent developments demonstrates that significant progress towards a better understanding of gear tribology has been made, at the same time showing that further research is required, particularly on the surface fatigue phenomenon of micro-pitting.

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APPENDIX 1 – THIS PAPER IS REPRODUCED ON PG 54 OF THIS ISSUE.

A NUMERICAL SOLUTION TO THE ELASTO-HYDRODYNAMIC PROBLEM

By D. Dowson* and G. R. Higginson,*

This paper presents a solution to the problem of hydrodynamic lubrication of highly loaded elastic cylinders under isothermal conditions. A numerical method is developed which enables a pressure curve to be found which satisfies the elastic and hydrodynamic requirements of the system.

Results are presented which take account of elastic distortion and the dependence of viscosity upon pressure for centre-line pressures of 5, 10, 20, and 30 ton/in². In addition solutions are given for both rigid and elastic solids when the centre-line film thickness is the same in each case.

INTRODUCTION

SINCE REYNOLDS PRESENTED the hydrodynamic theory of lubrication to explain Tower's experimental results, the idea that a fluid film can completely separate bearing surfaces and allow loads to be transmitted with correspondingly low friction forces has been widely accepted. In recent years there has been a steady growth in the evidence which suggests that a fluid film might also separate the surfaces of machine elements which, when loaded, must necessarily be subjected to high local stresses (**1**)†. Examples are gears and rolling-contact bearings. Since high pressures will be generated in any fluid film which separates the surfaces of such machine elements, an analysis of the problem must clearly include the influence of pressure on the bounding solids and the lubricant itself. The solids will suffer elastic deformation and their surfaces will present an oil film shape which varies with the load. Also the viscosity of lubricating oils is a function of pressure, and the hydrodynamic equations governing the generation of pressure must include this effect. These factors are normally ignored in the application of hydrodynamic theory to bearings, since the pressures generated are relatively small. Several attempts have been made to solve the so-called elasto-hydrodynamic problem, in which these additional effects are included. Notable contributors to the literature include Blok (**2**), Grubin (**3**), Petrusevich (**4**), Poritsky (**5**), and Weber and Saalfeld (**6**). However, none of the theories so far advanced appears to be completely satisfactory.

Even the simplified problem of steady rolling and/or sliding of two circular cylinders separated by a lubricating film, which is the subject of this paper, awaits a complete solution. The solution must satisfy simultaneously the equations governing the generation of pressure in the oil and the elastic deformation of the solids. The main obstacle appears to be that a straightforward iteration of pressure, deformation, film-shape, pressure does not converge, even for the special case of constant viscosity of the oil. An ingenious device used by Weber and Saalfeld produces a converging solution for relatively light loading, but unfortunately the method is not applicable to cases of heavy loading, which would seem to be the important range in most applications.

When the loading is high, the film thickness will be small compared even with the local elastic displacements. When this is so, two important consequences arise. The first is that, except near the edges of the thin film zone, the pressure must be close to the Hertzian pressure of dry contact. This is because the variations in film thickness along the zone are everywhere small compared with the displacements, except near inlet and outlet, and so the pressure deforming the cylinders must be much the same as if no oil were present. Plainly this conclusion is independent of the properties of the oil. The second effect concerns side leakage which is examined in a later section.

Notation

e_x, e_y, e_z	Direct strains.
e_{xy}, e_{yz}, e_{zx}	Shear strains.
E	Young's modulus.
G	Shear modulus.
h	Oil film thickness.
h_o	Oil film thickness at the point of maximum pressure.
h_c	Oil film thickness on the line of centres.

The MS. of this paper was received at the Institution on 18th November 1958.

* *Lecturer, Department of Mechanical Engineering, Leeds University. Student of the Institution.*

† *A numerical list of references is given in the Appendix.*

APPENDIX 2

Nominated Lecture

GEAR-TOOTH CONTACT PHENOMENA

By H. E. Merritt, M.B.E., D.Sc. (Eng.) (*Member*)*

Current methods of assessing gear-tooth load carrying capacity remain largely empirical; so also is lubrication technique. There is need for more research and more endeavour to apply it. The lecture states the quantities, definitions and notation employed in the analysis of contact conditions; outlines the phenomena of oil-film formation and friction; gives experimental values obtained by disc machines; and develops a modified treatment of temperature calculation. Phenomena encountered in practice are summarized, and the conclusion is reached that scuffing begins not within the oil film but by dislocation of the bounding surfaces. The emphasis throughout is on the problems which still await solution.

THE DESIGN OF COMPONENTS for a predictable working life requires analysis of the destructive forces at work, and knowledge of the ability of chosen materials to withstand those forces. Gears made of steel left in a machinable condition, which represent the majority of heavy industrial gear units and marine reduction gearing, are controlled in major dimensions by the resistance of the teeth to contact stresses. But the nature of the destructive effects is not yet well understood; and 'wear rating formulae' are related, more or less directly, to permissible Hertzian contact stresses and cover only resistance to pitting. Formulae such as those given in B.S. 436:1940 represent codes of practice based on satisfactory experience, and contain implicit margins of safety which are thought to be adequate, but are not precisely known.

Tooth surface damage, in the form of scuffing or scoring, is more troublesome, because it can happen unexpectedly, and no satisfactory forecast of the conditions which will produce it has yet been devised. Scuffing and its variants, which begin with surface welding, are related to the conditions of lubrication, and any criterion of scuffing should contain a factor based on the properties of the lubricant. The results obtained by various types of machine, devised in the hope of permitting comparative measurement of the lubricating ability of different oils, merely show that no index of 'film strength' has been isolated. Lubricants themselves have been developed empirically, and orders of merit change with operating or test conditions.

The MS. of this lecture was received at the Institution on 4th August 1961. For a report of the meeting at which this lecture was presented see p. 164.

* *Consulting Engineer, Claverdon, Warwick.*

More basic study of the mechanics of oil-film formation and disruption, and of the destructive effects which arise, is needed. The purpose of this lecture is to present a synoptic view of the phenomena, and an indication of the directions which further research could profitably take, in the hope that more centres of investigation can be induced to join in the work.

RELATIVE SURFACE MOTION

Gear teeth are particular examples of the general case of line contact between moving surfaces differing in curvature. The approach to contact phenomena therefore begins with analysis of the relative surface motion. This is easy for involute spur and helical gears, more difficult for gears of sophisticated geometry such as worm gears and hypoids. But all cases can be reduced to one of the simplified representations shown in Fig. 1.

In Fig. 1*a*, a cylinder 1 is in line contact with a plane 2 and is moved over it with a combination of rotation and translation in a direction at right angles to its axis. The essential quantities are the speeds of motion of the line of contact over the respective surfaces. These speeds are termed the 'rolling velocities' (which being accompanied by sliding are not to be confused with 'pure rolling' in which sliding is absent). Thus the rolling velocity for the cylinder (V_{r1}) is its peripheral velocity, and that for the plane (V_{r2}) is the velocity of translation of the cylinder. The velocity of sliding in this case is the algebraic difference of the rolling velocities.

Fig. 1*b* represents the general case: in this, the cylinder is

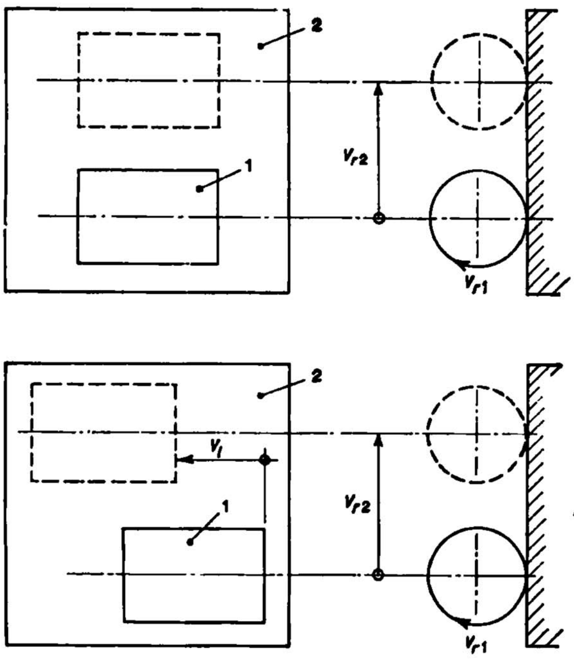


Fig. 1. Types of relative surface motion

additionally given simultaneous translation in the direction of its axis. If the rolling velocities of Fig. 1*a* are unchanged in Fig. 1*b*, the amount of sliding is increased by axial movement of the cylinder. A small proportion of axial movement corresponds to the contact conditions of helical gears, an intermediate proportion to hypoids, and a high proportion to worm gears of large ratio.

The principal concept, to be enlarged upon later, is that the algebraic sum of the rolling velocities acts to entrain an oil film, and the sliding velocity to generate heat in it.

QUANTITIES INVOLVED

Fig. 2 shows the profiles of a pair of spur-gear teeth, in contact at a point C, the projection of the line of contact between them. In the analysis of the loading and mechanical stresses, the quantities involved are as follows:

F_c is the load acting normal to the profiles per unit length of contact line.

R_1, R_2 are the radii of curvature of the profiles at the point of contact.

R_r is the relative radius of curvature, and

$$R_r = R_1.R_2/(R_1 + R_2) \quad . \quad . \quad . \quad (1)$$

E_1, E_2 are the respective moduli of elasticity.

E_m is the 'harmonic mean modulus', where

$$E_m = 2\,E_1.E_2/(E_1 + E_2) \quad . \quad . \quad . \quad (2)$$

S_c is a measure of the severity of line-contact loading, and

$$S_c = F_c/R_r \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

It is not in itself a measure of mechanical stress.

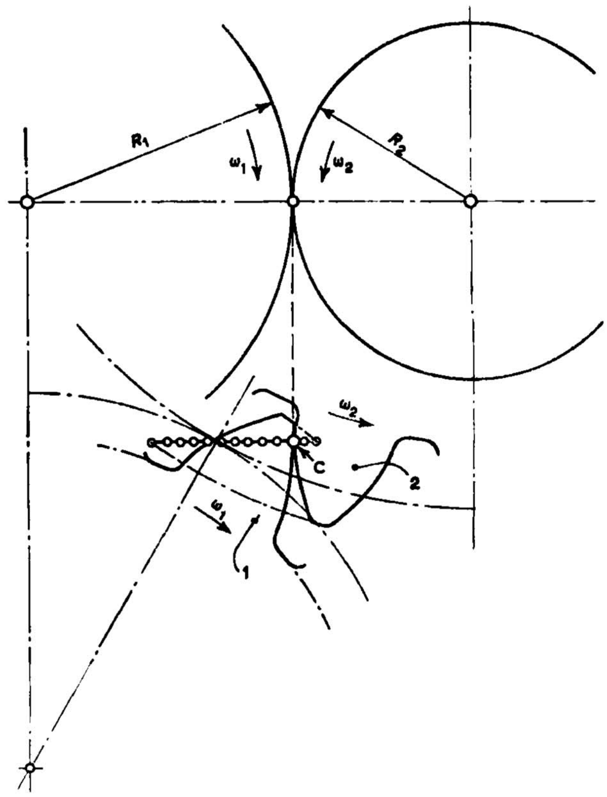


Fig. 2. Involute spur-tooth profiles in contact

S_{max} is the maximum Hertzian compressive stress, given by

$$S_{max} = 0{\cdot}418\,(F_c.E_m/R_r)^{\frac{1}{2}} \quad . \quad . \quad (4)$$

It can be derived from the value of S_c from

$$S_{max} = 0{\cdot}418\,(S_c.E_m)^{\frac{1}{2}} \quad . \quad . \quad . \quad (5)$$

and for a steel–steel combination,

$$S_{max} = 2250\,S_c^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (6)$$

while for a steel–bronze combination,

$$S_{max} = 1870\,S_c^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (7)$$

If the intensity of loading is expressed by Lloyds' 'K-factor', a useful approximate relationship is

$$S_c = 6{\cdot}3K \quad . \quad . \quad . \quad . \quad . \quad (8)$$

b is the width of the band of contact due to elastic deformation, and

$$b = 3{\cdot}04\,(F_c.R_r/E_m)^{\frac{1}{2}} = 3{\cdot}04\,R_r(S_c/E_m)^{\frac{1}{2}} \quad (9)$$

and for a steel–steel combination

$$b = 5{\cdot}55\,R_r.S_c^{\frac{1}{2}} \times 10^{-4} \quad . \quad . \quad . \quad (10)$$

while for a steel–bronze combination,

$$b = 6{\cdot}66\,R_r.S_c^{\frac{1}{2}} \times 10^{-4} \quad . \quad . \quad . \quad (11)$$

In the foregoing relationships, units are in pounds and inches. Particular values in any given case are derived from the design data, and from the position of a chosen point of contact where appropriate.

In the analysis of frictional and thermal problems, it is convenient to work in foot-pound-second units. Dimensions already referred to are then converted into feet. In addition,

ω_1, ω_2 are the angular velocities of the gears in Fig. 2, or of the discs in Fig. 3, in rad/sec.

V_{r1}, V_{r2} are the rolling velocities of the surfaces marked 1 and 2 respectively in Figs. 1, 2 and 3, and for spur gears and discs,

$$V_{r1} = R_1.\omega_1 \quad . \quad . \quad . \quad . \quad (12a)$$

$$V_{r2} = R_2.\omega_2 \quad . \quad . \quad . \quad . \quad (12b)$$

V_s is the sliding velocity (or relative surface velocity); for spur gears,

$$V_s = V_{r1} - V_{r2} \quad . \quad . \quad . \quad . \quad (13a)$$

and it is convenient to regard as 'surface 1' that which has the higher rolling velocity. In the general case (Fig. 1*b*) having a component V_l of relative surface velocity in the direction of the line of contact,

$$V_s = [(V_{r1} - V_{r2})^2 + V_l^2]^{\frac{1}{2}} \quad . \quad . \quad (13b)$$

V_e is an introduced quantity, the 'entraining velocity' and is

$$V_e = V_{r1} + V_{r2} \quad . \quad . \quad . \quad . \quad (14)$$

Q_{s1}, Q_{s2} here denote the commonly used relation, the 'slide/roll ratio' or 'specific sliding' for the respective surfaces, and

$$Q_{s1} = V_s/V_{r1} \quad . \quad . \quad . \quad . \quad (15a)$$

$$Q_{s2} = V_s/V_{r2} \quad . \quad . \quad . \quad . \quad (15b)$$

Q denotes a useful concept, the 'sliding/entraining ratio', represented by

$$Q_{se} = V_s/V_e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16a)$$

$$= (Q_{sr1} \cdot Q_{sr2})/(Q_{sr2} - Q_{sr1}) \quad . \quad (16b)$$

but in the following text, the ratio will be written V_s/V_e which is more explicit.

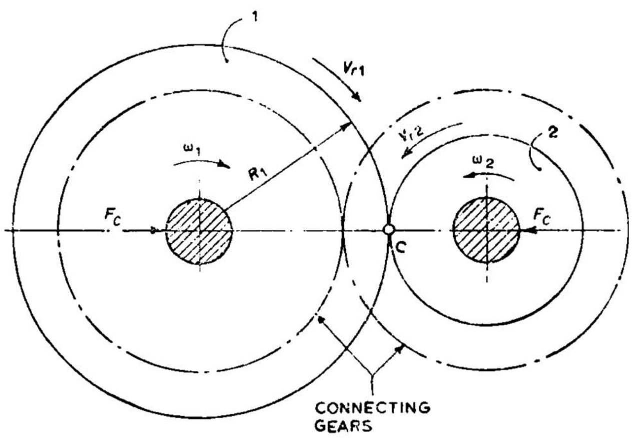


Fig. 3. Diagrammatic representation of a disc machine

DISC MACHINES

The contact conditions between the involute profiles of Fig. 2 at the point C can be simulated by arranging two discs, of radii R_1 and R_2, as in Fig. 3, and rotating them at the speeds of the gears, ω_1 and ω_2 respectively, in contact under the same intensity of loading. To simulate the conditions at any other point of contact, it would strictly be necessary to change the diameters of the discs, while keeping the speeds constant. It is more convenient, for general investigation, to use discs of fixed, arbitrarily chosen diameters, and adjust their respective speeds in order to reproduce the desired rolling velocities; the stress conditions can be selected by adjusting the load. But pairs of discs of different relative curvature produce different conditions of oil-film formation and friction.

A disc machine conveniently keeps the line of contact fixed in space, and makes the relative motion of the surfaces more easily recognizable. If the discs are connected by a pair of gears, their respective rolling velocities both act towards the line of contact, and are regarded as positive. The sliding velocity is the algebraic difference, but its direction depends upon the surface referred to. These are the conditions of engagement of spur gears. By measuring the torque required to drive the discs under load, the coefficient of friction can be deduced, after correcting for bearing friction.

Connecting the discs by a chain drive gives rolling velocities in opposite directions, a condition rarely encountered in gear drives, but used in some experiments in order to create an artificially severe condition. The condition when one disc is kept stationary does not exist between gear teeth (zero rolling velocity at the base circle of an involute also implies zero relative radius of curvature) but this, and also opposed rolling velocities, occur at some phases of contact between a cam and a tappet. The extreme conditions are, at one end of the scale, pure rolling, and at the other, pure sliding (i.e. equal and opposite rolling velocities). The pure rolling of discs, however, does not completely simulate the conditions of nominally pure rolling at the pitch point of spur gears, because some contact phenomena at this point are associated with the rapid rate of reversal of sliding velocity, not provided by constant-velocity discs. Discs driven at cyclically varying speeds can be and have been used, but make torque-measurement more difficult; and determination of the coefficient of friction is indispensable in the study of contact conditions.

SOME EXPERIMENTAL OBSERVATIONS

The lecturer, in association with Mr H. J. Watson and Mr H. Pearson, began experimenting with disc machines in the David Brown research department *circa* 1928, and reported some of the results in a paper (1)* to this Institution in 1935. Some early observations will here be recounted, however, in order to show why the phenomena of friction and lubrication were found so puzzling.

This early work was carried out using a bronze-hardened steel combination, chain-connected to give opposite

* *A numerical list of references is given in the Appendix.*

directions of the rolling velocities and severe conditions of lubrication. The coefficient of friction was found (a) to be but little influenced by the applied load, and (b) to diminish continually with increase of sliding velocity at any ratio V_s/V_e. These observations were not consistent with the classical theory of hydrodynamic lubrication. It was therefore believed that the lubrication was primarily of the boundary type, with an element of hydrodynamic lubrication and load-carrying in the convergent entry.

Measurement of the coefficient of friction of mineral oils showed it to depend chiefly upon the viscosity grade (or relative viscosity at a reference temperature) rather than upon the viscosity at the temperature prevailing during the tests; it also differed somewhat according to the nature of the base oil for the same viscosity grade. Castor oil gave lower values than mineral oil, and the lowest values of all were obtained with glycerol. It was thought that these differences supported the view that boundary lubrication prevailed.

But to confuse the picture, comparative tests of castor and mineral oil showed that the superiority of castor oil was substantial only when the contact conditions were favourable (i.e. ratio V_s/V_e small) and largely disappeared under unfavourable conditions (V_s/V_e large).

Three other observations might have led to earlier understanding. The first was that, in a test during which a chain-connected bronze-steel combination was run at $S_c = 600$ ($S_{max} = 45\,000$ lb/in^2) for 10 million revolutions of the bronze disc, not the slightest wear was measurable. The second was that a steel-steel combination seized instantly, under negligibly small load, when the discs were chain-connected to give them equal and opposite rolling velocities. The third was that, in measuring the coefficient of friction at break-away from a statically loaded condition, the torque necessary to produce motion increased with the interval of time between applying the load and applying the torque. It was not understood why a period of between 1 and 2 sec should have been required to expel the oil from a band of contact 0·025 in. wide under a pressure of 45 000 lb/in^2.

The significance of these observations escaped us because in the 1930's we did not know about the enormous increase in viscosity of lubricants under typical gear-tooth pressures, or that this could act to form a relatively thick oil film under the action of the entraining motion of the surfaces. This, when appreciated, explained the absence of wear in the first of the observations mentioned above, the immediate seizure in the second (when $V_e = 0$), and the time lag in the third.

The magnitude of the variation of viscosity with pressure and temperature will be seen from Fig. 4, for which the author is indebted to Dr Alastair Cameron. The viscosity of the comparatively light oil (about 20 cS at 210°F) to which the curves refer increases to 10^5 cS at a pressure of 120 000 lb/in^2 at the same temperature; increase of temperature to 425°F diminishes the viscosity to 10^3 cS or one-hundredth of its value at 210°F. These pressure-temperature conditions are exceeded in actual gear contacts.

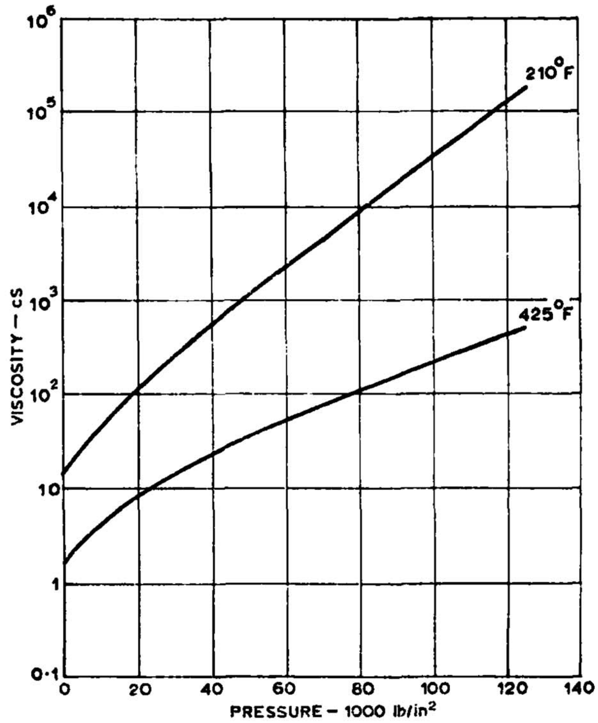


Fig. 4. Pressure–temperature–viscosity relations

SOME MEASURED FRICTIONAL VALUES

Fig. 5 shows in full line the coefficients of friction obtained with a chain-connected bronze–steel combination lubricated by a typical mineral worm-gear oil, at $S_c = 600$. The values are those given in Fig. 18 in the author's 1935 paper (1), but replotted in terms of the entraining velocity using as scales log V_e and log μ. For the several values of V_s/V_e, the points lie substantially on lines of constant slope, represented by

$$\mu \propto V_e^{-n}$$

where the exponential n is approximately 0·45.

Increase in the ratio V_s/V_e caused a progressive reduction in the coefficient of friction at any value of V_e.

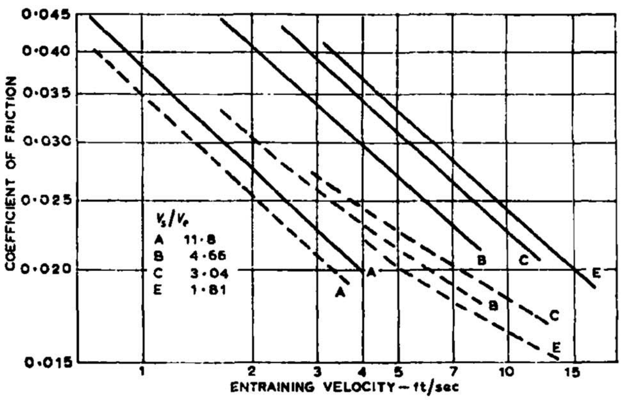


Fig. 5. Coefficients of friction, for mineral oil and castor oil: bronze–steel, chain-connected

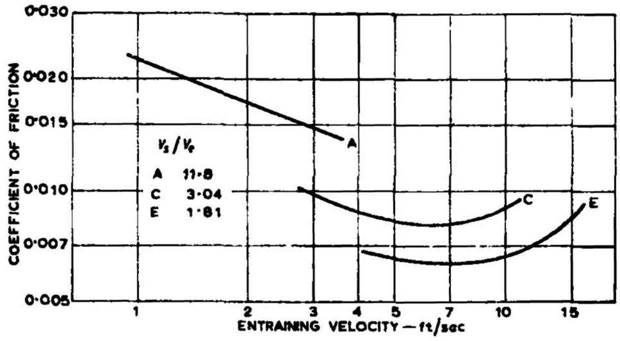


Fig. 6. Coefficients of friction, glycerol

On the same diagram, the values for castor oil are shown in dotted line. At the smaller values of V_s/V_e, the coefficients of friction are substantially lower than for the mineral oil, but at the highest ratio used, the difference had largely disappeared.

Recalling the low frictional values given by glycerol, Mr H. J. Watson kindly repeated the experiment, and the results are shown in Fig. 6. Extremely low values of μ are recorded but the shape of the curves is singular, probably associated with a reduction in the rate of diminution of viscosity with increase of temperature as the temperature increases, compared with the behaviour of mineral oils.

Fig. 7 shows, at curves A, B and C, mean values of a large number of determinations (also due to Watson) of the coefficient of friction between gear-connected discs, of case-hardened steel; the lubricant was OM 88 (23 cS at 70°C).

In a given pair of gears running at constant speed, the sum of the rolling velocities does not vary greatly during the cycle of tooth engagement. The sliding velocity changes direction and therefore varies from point to point of contact, but Fig. 7 indicates that the effect of this variation on the coefficient of friction is not large. This supports the convenient assumption of a constant coefficient of friction when calculating tooth efficiency. Other experimental data lead

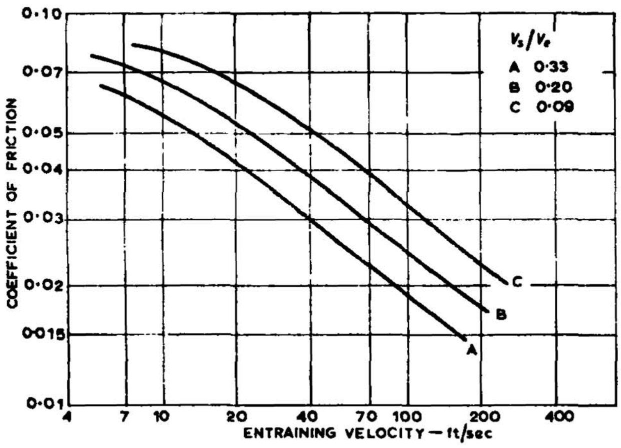


Fig. 7. Coefficients of friction, OM 88 steel–steel, gear-connected

to the following approximate expression for the coefficient of friction, of straight mineral oils:

$$\mu = 1{\cdot}6\,(\eta_0 . V_e . R_r)^{-0\cdot5} \quad . \quad . \quad . \quad (17)$$

where η_0 is the viscosity in centistokes at 140°F. For spur gears, $V_e = 2\ V \sin\psi$ where V is the pitch-line velocity and ψ is the pressure angle, and $R_r = \frac{1}{2}\ d.D/(d+D) \sin\psi$ where d and D are the pitch diameters. At small values of V_e, the value of μ will approach an upper limit corresponding to the coefficient of static friction.

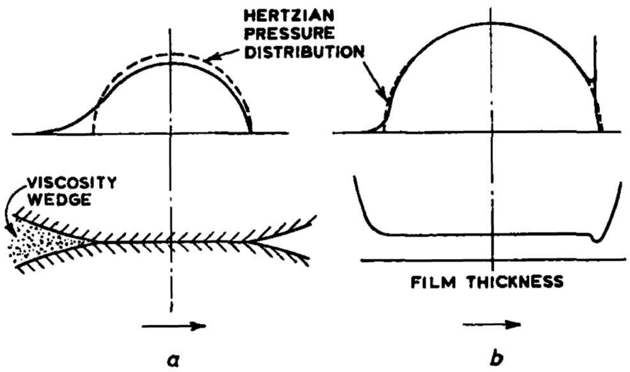
MECHANISM OF OIL-FILM FORMATION

Fig. 8*a* reproduces the hypothetical curve advanced by the author in 1935. It was believed (as mentioned earlier) that a hydrodynamic pressure rise was built up in the convergent entry, but that a boundary film existed between the elastically flattened surfaces over the Hertzian pressure zone. But Grubin (2) perceived that the convergent entry was capable of injecting a full oil film into the pressure zone, and accurately predicted the thickness of the film so formed.

Descriptively, the lubricant is dragged by its viscosity and the motion of the surfaces into the convergent entry. This produces an increase in pressure as the point of the entering 'wedge' of oil is approached. The increase in pressure causes an increase in viscosity, which further increases the entraining effect, and so on, until a pressure is reached sufficient to cause flow into the Hertzian pressure zone.

Petrusevich (3) obtained a solution of the hydrodynamic and elastic equations involved in the deformation of the surface by pressure changes, and derived the pressure distribution and expressions for the thickness of the oil film and for the coefficient of friction based on exponential pressure–viscosity and temperature–viscosity relationships. Dowson and Higginson (4) have pursued the analysis of pressure distribution and film thickness, for pure rolling, and a typical example is shown in Fig. 8*b*.

It would be a reasonable guess that the film, once established, would remain of constant thickness across the Hertzian pressure zone. An unexpected result, from Petrusevich's analysis, is the existence of a marked pressure



a Hypothesized, 1935.
b After Petrusevich, and Dowson and Higginson.

Fig. 8. Pressure distribution and film thickness

peak, of extremely short duration, at the point of exit, associated with a local constriction of the film.

The thickness of the film has been measured by Crook (5) using capacitance methods between discs, and by MacConochie and Cameron (6) between running gear teeth, using the principle of voltage drop across the film.

FRICTIONAL PHENOMENA

Most of the work lost in friction is expended in shearing the oil film, and the frictional resistance is therefore proportional to the width of the film, its mean viscosity and the rate of shear.

Considering first the case of gear-connected discs, and on the reasonable assumption that by the time a disc has completed a revolution its surface has returned to the bulk temperature, the formation of the oil film is probably not appreciably affected by temperature changes in the convergent entry, and the thickness of the film, mainly determined by the pressure–viscosity characteristic of the lubricant, increases with the entraining velocity. In the case of chain-connected discs, having opposite directions of rolling velocity, the slower-moving surface, which forms one boundary of the convergent entry, has been preheated by passing through the contact zone. It can be expected that it will give up some of its surface heat to the entering oil, diminishing its viscosity rise in the convergent entry and reducing the thickness of the film entrained.

In both instances, the film is sheared by the sliding velocity, and therefore heated, in its passage through the contact zone. This diminishes the mean viscosity which would otherwise result from the Hertzian pressure distribution. Deferring more detailed discussion of temperature changes, one can make the broad generalization that a steep pressure–viscosity characteristic results, other things being equal, in a thicker oil film having in consequence a reduced rate of shear, whilst a steep temperature–viscosity characteristic reduces the mean viscosity in the oil film.

These considerations can be related in a broad way to the frictional effects shown in the preceding curves of test results. In the case of Fig. 5, the coefficient of friction diminishes slowly with increase of V_e for a constant value of V_s, indicating the effect of the pressure–viscosity characteristic at entry. On the other hand, for a given value of V_e it diminishes rapidly with increase in V_s, showing the effect of the temperature–viscosity characteristic. The reduction in mean viscosity due to temperature rise must be very great, if the frictional resistance diminishes in spite of a greater sliding velocity, or rate of shear. This is borne out by Fig. 4.

The uniform slope of the curves for mineral oils in Figs. 5 and 7 does not reflect a simple physical law, but only a particular relation between the pressure–viscosity and temperature–viscosity characteristics of the mineral oils used. Other types of mineral oil give different slopes as shown by Misharin's results (7); whilst Figs. 5 and 6 show the variable slope of castor oil and the very peculiar results given by glycerol.

The pressure–viscosity effect, which promotes film thickness, 'tends to be greater for substances with large and complicated molecules' (8). This explains the lower friction associated with the larger molecules of the heavier fractions of mineral oils, and the significance of viscosity grade rather than viscosity at the operating temperature. In practical terms, a hot heavy oil is better than a cold light oil. Considerations of molecular structure doubtless explain, also, the lower friction of paraffinic than of asphaltic base oils, and some observed cases in which over-refining led to increased friction. With mixed fluids, each component plays its part; it has been found that a blend of a light fraction with bright stock gives less friction than a straight distillate of the same viscosity. The presence of an extreme-pressure additive can conceivably change the pressure–viscosity characteristic and therefore have a physical as well as a chemical effect.

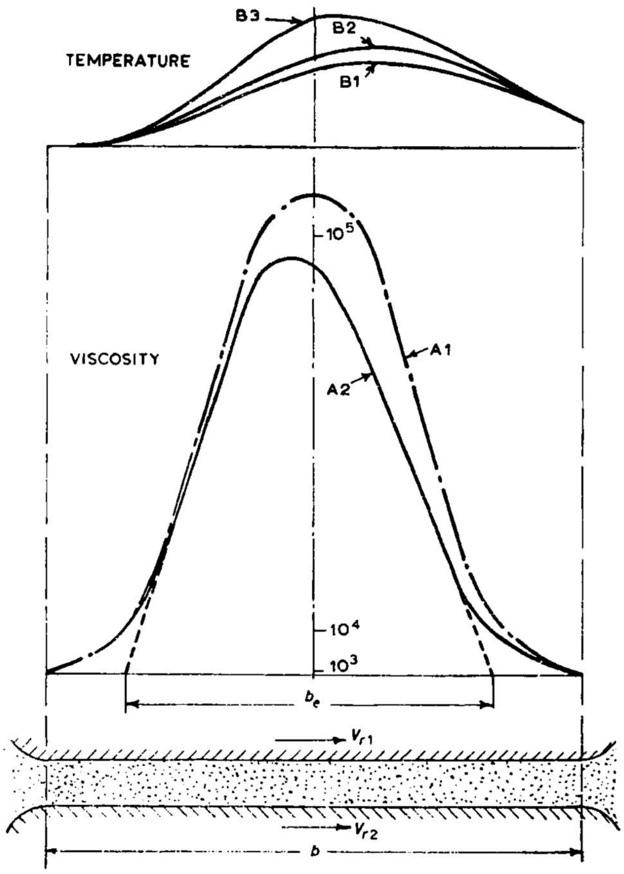
The instantaneous seizure when $V_e = 0$, mentioned earlier, is sufficient evidence that a boundary film will not carry gear-tooth loading.

THERMAL PHENOMENA AND TANGENTIAL STRESSES

Experimental results suggest that the surface cracks which precede pitting result from a combination, in uncertain proportions, of thermal stresses and tangential stresses due to friction. Scuffing and its variants, characterized by surface welding, are clearly related to surface temperature. The rippling of case-hardened surfaces, surprising though it seems, can be more easily visualized as occurring under the applied tangential stresses if these are associated with high transient skin temperatures. The following picture of what takes place at and between the surfaces in the band of contact, under good conditions of lubrication, will indicate that local temperatures and shear stresses are higher than would be deduced from simple assumptions.

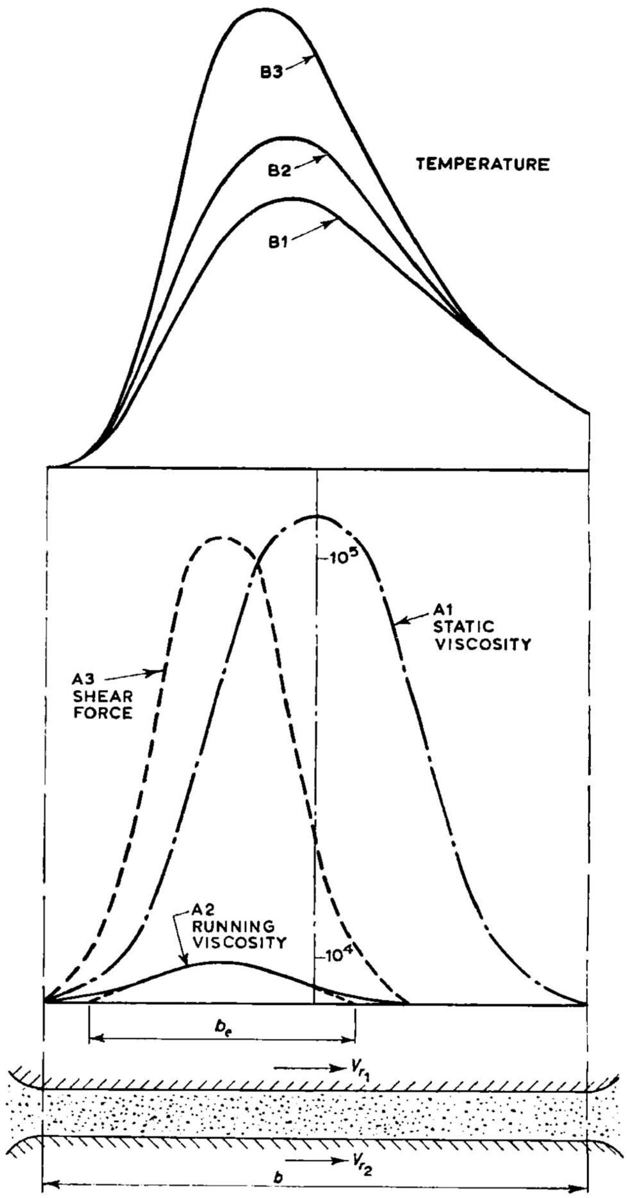
Fig. 9 represents the condition when the sliding velocity is very small in comparison with the entraining velocity. The dotted curve A1 indicates the variation in viscosity of the lubricant from point to point along the band of contact, under the influence of the Hertzian pressure alone, drawn to a linear scale, for a lubricant such as that to which Fig. 4 refers, with a maximum pressure of 120 000 lb/in^2. Curve A2 represents the estimated variation in mean viscosity of the oil film from section to section along the transit, under the influence of sliding/entraining conditions which cause an average temperature rise of 10°F across a section of the oil film (perpendicular to the direction of motion) in the region of maximum surface pressure.

To a scale dependent upon the film thickness and the sliding velocity, the ordinate to the viscosity curve at any point represents the shear stress between the surfaces, and also the rate of heat generation, at that point. The effect of the variable rate of heat generation as transit proceeds is to produce curves of temperature rise represented by curves B1, B2 and B3. Deferring for a moment any consideration of the significance of these three curves, the general result is that the maximum temperatures are reached more than halfway through the transit and the influence of



(V_s/V_e small)

Fig. 9. Temperature and viscosity variations across the band of contact



(V_s/V_e substantial)

Fig. 10. Temperature and viscosity variations across the band of contact

temperature rise on viscosity produces an unsymmetrical curve of viscosity distribution.

Curves B1 and B2 relate to the surfaces 1 and 2 respectively. The work of friction is expended in shearing the oil film and in generating heat in it. This heat flows laterally through the oil film into the metal surfaces, but if the conductivity of the oil film is not infinite, heat flow across it must be accompanied by a temperature gradient. Any element on surface 2, which has the lower velocity, is exposed to heat flow for a longer period and reaches a higher temperature; but because of the temperature gradients in the oil film, the maximum temperature within the film is greater than that on either surface, and is represented by curve B3.

Fig. 10 repeats the exercise, but for a greater proportion of sliding to entraining velocity assumed to produce an average temperature rise of 100°F in the mean temperature of the oil film in the region of maximum surface pressure.

Here, curve A1 represents, as before, the viscosity due to the Hertzian pressure only, and curve A2 the viscosity as influenced by temperature. The work of shearing the oil film is now concentrated over the first half of the transit and the peak temperatures are reached near the middle of the transit, where the pressures are also greatest. The shear force is proportional to the product of the rate of shear and the mean viscosity, and since the sliding velocity is greater in Fig. 10 than in Fig. 9, curve A3 represents curve A2 scaled up for the speed effect, so that areas under curve A2 in Fig. 9 and curve A3 in Fig. 10 correspond to the total friction in each case.

It is to be understood that because of the complex heat-generation and heat-flow mechanism, particularly across transverse sections of the oil film, the curves of Fig. 9 and Fig. 10 are indicative only, but the lecturer believes that they convey the character of the viscosity and temperature distribution. The higher local temperatures which can occur in heavily loaded gearing will still further accentuate the unsymmetrical form of the curve of mean viscosity distribution. The heat generation and the shear stress in the oil film and at the surfaces thus tend to be concentrated over the early portion of the transit, and will have peak values much greater than if they were taken as following the curve of Hertzian pressure distribution.

APPROXIMATE QUANTITATIVE TREATMENT

The following argument is presented as a first approach to the complex problem posed by Figs. 9 and 10. In the pursuit of scuffing criteria with the aid of tests on discs, it will be necessary to examine what correlation exists between the incidence of scuffing and the maximum temperatures reached. For reasons given later, it is not to be expected that maximum temperature will be the sole criterion, for any given lubricant. Other and still more complex phenomena appear to enter into the scuffing problem, but estimates of temperature may give a clue to their nature. For this purpose it is desirable that in the course of such tests the film thickness and the coefficient of friction be measured.

In the treatment which follows, foot-pound-second units are used throughout. Linear dimensions of the discs and of the oil film must be converted into feet, and F_c into pounds per foot of length of contact line.

Mean rate of heat generation within the oil film

Given, as experimental data, F_c, V_s and μ and the thickness h of the oil film, together with the calculated value of the width b of the band of contact from expression (9), the rate of heat generation per foot of length of contact line is

$$Q = \mu . F_c . V_s / J \quad . \quad . \quad . \quad . \quad (18)$$

The mean rate of heat generation within the oil film is then

$$H = Q/b.h = \mu . F_c . V_s / J.b.h \quad . \quad . \quad (19)$$

where J is Joule's equivalent.

Distribution of heat generated

It is for consideration in what proportions the heat generated flows to the respective surfaces. The mechanism by which heat generated by internal shear of the oil film flows through the film and into the surfaces is obviously extremely complex, and awaits investigation. It can be expected, however, that because of the violent internal motion of the film, its thermal resistance will be much less than if it were at rest. On the other hand, it would be unwise to assume that its thermal resistance is zero or negligible.

In discussing this subject with Dr R. T. Spurr and Mr T. P. Newcomb, they suggested that experimental evidence was needed, and Mr H. J. Watson devised and carried out such an experiment. Data relating to this experiment are given later, but it may here be stated that the ratio of heat distribution was not the same as that calculated on the assumption of zero thermal resistance: for whereas in this particular case the ratio calculated on that assumption was 1·732, the measured ratio was only about 1·4. For present purposes this ratio will be denoted by r, representing the ratio of the rate of heat flow to surface 1 to that to surface 2.

Maximum surface temperature

Surfaces 1 and 2 in Figs. 9 and 10 move across band sources of heat, of variable intensity, and the times of transit of given points on the moving surfaces will be t_{s1} and t_{s2} respectively, where $t_{s1} = b/V_{r1}$ and $t_{s2} = b/V_{r2}$.

If, as seems probable, the temperature gradients of the surfaces in the direction of motion are small in comparison with those in a direction at right angles to the surfaces, the temperature rise can be treated as that due to a stationary surface source of heat, of intensity varying with time, analogous to the case of a friction clutch.

Then the temperature rise above the bulk temperature (described by Blok (**9**) as 'temperature flash') is given, in general, by

$$\theta = B . q_m . t^{\frac{1}{2}} / m \quad . \quad . \quad . \quad (20)$$

where q_m is the mean rate of heat-flow per unit of area during the interval t_s, and m is a thermal constant for the material, represented by $m = (k . \rho . c)^{\frac{1}{2}}$, and k, ρ and c are the thermal conductivity, density and specific heat respectively. (The value of m is 0·65 for steel and 1·09 for bronze.) The coefficient B is a function of the form of the heat-flow–time curve, and it conveniently appears that, provided that the curve is fairly steep-fronted, the value of B does not vary greatly with the geometrical form of the curve. Thus, from the work of Newcomb (**10**) and Blok (**11**), $B = 1·128$ for a uniform rate of heat flow, 1·06 for linear variation from a maximum to zero, 1·17 for a parabolic and 1·11 for a semi-elliptical heat-flow–time curve.

In Figs. 9 and 10, the curves of heat generation have been extended by dotted lines to give steep-fronted curves, the effect of the disregarded areas being negligible. The base of the curve as terminated by the dotted extensions may be regarded as the 'virtual band-width' b_v equal to $x . b$ where x is a fraction.

The total rate of heat generation per unit of length of contact band is Q given by expression (18). Taking into account the heat-distribution ratio r, the rates of heat flow to the respective surfaces per unit of length of contact band are

$$Q_1 = Q . r/(r+1) \quad . \quad . \quad . \quad (21a)$$

$$Q_2 = Q/(r+1) \quad . \quad . \quad . \quad (21b)$$

Regarding these quantities of heat as flowing into the virtual band width $b_v = x . b$, the mean rates of heat flow per unit of area will be, for the respective surfaces,

$$q_{m1} = Q . r/(r+1)x . b \quad . \quad . \quad (22a)$$

$$q_{m2} = Q/(r+1)x . b \quad . \quad . \quad . \quad (22b)$$

The times of transit are also to be reckoned over the virtual band width, i.e.,

$$t_{s1} = b_v/V_{r1} = x . b/V_{r1} \quad . \quad . \quad (23a)$$

$$t_{s2} = b_v/V_{r2} = x . b/V_{r2} \quad . \quad . \quad (23b)$$

Inserting these quantities in expression (20) gives

$$\theta_1 = B.\frac{Q.r}{(r+1)x.b}\cdot\left(\frac{x.b}{V_{r1}}\right)^{\frac{1}{2}}\cdot\frac{1}{m_1}$$

or

$$\theta_1 = \frac{B.Q.r}{(r+1)(x.b)^{\frac{1}{2}}.V_{r1}^{\frac{1}{2}}.m_1} \quad . \quad . \quad (24a)$$

and similarly

$$\theta_2 = \frac{B.Q}{(r+1)(x.b)^{\frac{1}{2}}.V_{r2}^{\frac{1}{2}}.m_2} \quad . \quad . \quad (24b)$$

If the oil film offered no resistance to heat flow, the temperatures of points on the respective surfaces facing each other across the oil film would be equal, θ_1 would be equal to θ_2, and equating expressions (24*a*) and (24*b*) leads to

$$r = V_{r1}^{\frac{1}{2}}.m_1/V_{r2}^{\frac{1}{2}}.m_2 \quad . \quad . \quad . \quad (25)$$

or for similar materials, $r = (V_{r1}/V_{r2})^{\frac{1}{2}}$.

If the actual value of r is less than this, the temperature flash of surface 2 is greater than that of surface 1; and surface 2 corresponds to the dedendum portion of the teeth of a gear.

Application: Watson's experiment

In this experiment, identical steel discs, 3-in. diameter, were run in a David Brown disc machine, and were arranged for measurement of the temperatures of the inner surfaces of the rims of the discs, which were flanged. The discs were run under load until steady temperatures were reached, and the coefficient of friction was measured. The discs were then separated, running continued, and temperatures measured during cooling. In order to avoid extraneous heat losses or transfer, by the oil bath or oil stream ordinarily used in disc tests, the surfaces of the discs were initially coated with a thin film of very heavy lubricant, which adhered to the surfaces throughout the test, its efficacy being shown by the constancy of the coefficient of friction.

The data and results of the tests were as follows:

Applied load	315 lb	
Contact face	0·1875 in. = $1{\cdot}5625\times10^{-4}$ ft	
Loading	1680 lb/in. = 20 400 lb/ft	$= F_c$
Surface speeds (1)	5·57 ft/sec	$= V_{r1}$
(2)	1·85 ft/sec	$= V_{r2}$
Sliding velocity	3·72 ft/sec	$= V_s$
Relative radius of curvature	0·75 in. (from equation (1))	$= R_r$
Contact load criterion	2240 lb/in./in. (from equation (3))	$= S_c$
Contact band width	0·0197 in. = $1{\cdot}64\times10^{-3}$ ft	$= b$
Coefficient of friction	0·0358	$= \mu$
Ambient temperature	65°F	
Terminal rim temperature, (1)	193°F	
Terminal rim temperature, (2)	176°F	
Temperature rise, bulk (1)	128°F	
Temperature rise, bulk (2)	111°F	
Ratio of slopes of cooling curves	1·20/1	
Ratio of rates of heat loss at terminal temperature	1·20×128/111 = 1·39/1	$= r$

Derived results were as follows:

Rate of heat generation	3·42 Btu/sec/ft from equation (18)	$= Q$

It will be assumed that the curve of mean viscosity across the transit resembles that shown in Fig. 10, and that $b_v = 0{\cdot}5b$ or $x = 0{\cdot}5$. Taking the curve to approximate to a parabola, the value of B is 1·17; and inserting the data in expressions (24*a*) and (24*b*), including $m = 0{\cdot}65$,

$$\theta_1 = 53°\text{F}$$
$$\theta_2 = 65°\text{F}$$

and the maximum flash temperatures of the surfaces are therefore

$$T_1 = 246°\text{F}$$
$$T_2 = 241°\text{F}$$

It should be mentioned that in comparison with high-duty case-hardened gears, the severity of loading on the discs in this experiment as measured by the product $S_c\times V_s$ is very modest. On the other hand, the bulk rim temperatures were relatively high because of the absence of oil cooling, and the difference between that of disc 1 and that of disc 2 exceeded the difference between the calculated flash-temperature rises.

TEMPERATURE GRADIENTS WITHIN THE OIL FILM

This aspect of the subject requires further discussion, if only to emphasize its difficulties.

Fig. 11 illustrates a concept of what occurs within the oil film, which subsequent comment will show to be over-simplified. The diagram represents an element of oil film separating the surfaces which, where they are in contact with the oil-film element, have temperatures of T_1 and T_2; these are local and not the maximum temperatures. The curve abc is a hypothetical curve of temperature distribution across the oil film, and on the assumptions of a uniform rate of heat generation per unit of volume of the oil film, and uniform thermal conductivity, curve abc will be a parabola having its vertex at b, lying in the local plane of zero heat flow ZZ. All the heat generated in the thickness h_1 of the film, lying below the portion ab of the curve, will flow to surface 1, and all that in thickness h_2 will flow to surface 2. The displacement of the plane ZZ from a position midway between the surfaces is consistent with the concept that surface 2 receives less heat but reaches a higher temperature than surface 1.

It is not, however, consistent with the results deduced from Watson's experiment, in which the calculated value of T_2 was less than T_1. It must therefore be concluded

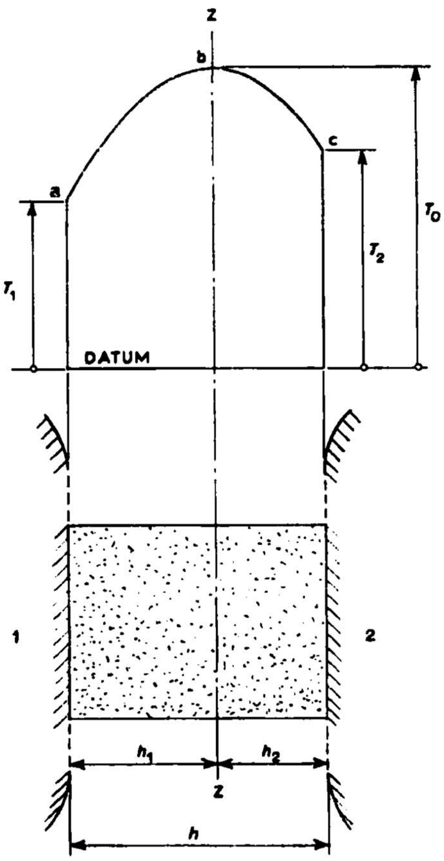


Fig. 11. Temperature distribution across the oil film

that the opening assumptions were invalid, and that some asymmetry in the mechanism of heat generation or conduction in the oil film exists. Further work is clearly needed.

INFLUENCE OF SURFACE FINISH

All the preceding discussion assumes geometrically perfect surfaces.

With an oil film a few thousandths of an inch in width, and less than 100 μ-in. thick, initial surface irregularities can be expected to change the situation. Where disc machines are concerned, the technique of securing optimum surface finish is difficult. Variations in the degree of surface finish lead to non-repeatability of friction measurements, and Shotter (**12**) has reported the results of many experiments on the effects of initial surface finish and running-in.

In the case of actual gears, macro-errors of tooth alignment and profile obviously result in localized contact pressures, and pitting occurs under nominal stresses lower than those which produce a similar result in discs. Micro-errors can convert a nominally uniform band of contact into a series of small pressure areas, with effects analogous to those in short journal bearings in that oil can escape laterally from the pressure areas. It has been stated that gear teeth having fine-pitched axial irregularities scuff more readily than if similar irregularities exist in the profiles. The correlation of tests on discs and gears is therefore difficult.

TYPES OF SURFACE DAMAGE

These fall broadly into two groups, one in which failure begins within the surface, the other by surface welding.

In the first group, pitting is the type most commonly encountered, in gears which are not skin-hardened, and usually begins with cracks at the surface, although some cracks may start from sub-surface defects. In skin-hardened gears, case-cracking or exfoliation may occur under high surface loading, and may begin at the junction of the case and the core if the case depth or core hardness is inadequate to resist sub-surface shear stress. A variant in case-hardened gears may be termed 'pitch-line fissure', and takes the form of irregular break-up of the case in an area close to the pitch line, attributable to the reversal of tangential stress in that region. An unusual type of failure, 'fine flaking', is probably a member of the first group. Plastic flow occurs at the surfaces of most gears, including heavily loaded case-hardened gears, and in the presence of an unsatisfactory lubricant may appear as 'rippling', the surfaces remaining polished.

In the second group, local welding produces scuffing, or scoring if more severe; and ridging-and-grooving along the pitch line, implying both welding and plastic flow. 'Smooth abrasion', or wear not attributable to the presence of foreign abrasive matter in the lubricant, is sometimes encountered, and is probably due to an undefined type of intermittent local scuffing, since it can be prevented or arrested by using a heavier oil. Gears after long service often show a pronounced step along the pitch line, the addendum standing proud and remaining smooth, while the dedendum is worn and pitted. Each of these types of failure must have its particular mechanism.

PITTING

This has been much discussed, and present thinking need be reviewed but briefly.

In the United States of America, rating for surface durability (in effect, pitting) is commonly based on permissible Hertzian compressive stresses, increasing with the square of the hardness of the material. British standard wear rating formulae are more conservative as hardness increases, but are indirectly based on Hertzian stresses.

Results given by Chesters (**13**) of experiments with rolling discs accord with the square law; but other experiments indicate that the question is not simple. Way, often quoted, found that pitting did not occur with unlubricated discs, but only in the presence of lubricant, and that pitting increased with reduction of viscosity; he therefore suggested that a factor in pitting was hydrostatic pressure in lubricant trapped in surface cracks when sealed by contact. It remains to be shown why the cracks are first formed, if not present before running commences.

Meldahl and others have shown by disc tests, and observation of many gears in service confirms, that pitting occurs more readily if sliding is present, particularly in the surface having the larger slide/roll ratio (the dedendum of gear teeth). It does not appear, from published work, that any comprehensive programme of pitting tests on discs has been carried out, to determine the separate effects, on given materials, of the lubricant, entraining velocity, sliding velocity and load, and to seek a possible correlation with friction and temperatures. Timms (14), however, has reported experiments on gears, in which the pitting resistance increased with speed. Since the coefficient of friction diminishes with increase of speed, other things remaining unchanged, but surface temperature increases with speed, it seems that tangential stresses are more significant than thermal stresses where pitting is concerned.

SCUFFING

Before one attempts to speculate about the mechanism of scuffing, some facts from experiment and experience of gears may be cited.

(1) In scuffing tests on gears, it is difficult to obtain repeatable results. Scuffing may be induced or accelerated by reducing the rate of oil flow.

(2) It has been reported that teeth which have been hand-dressed after scuffing do not scuff thereafter. Inadequate profile modification promotes scuffing; the hand-dressing mentioned may have acted to correct this.

(3) Scuffing can be inhibited by various processes of pretreatment of the tooth surfaces, e.g. phosphate treatment or copper plating.

(4) Extreme-pressure additives inhibit scuffing, without marked change in the viscosity grade of the oil. Different types of e.p. oil have different anti-scuffing characteristics.

(5) Occasionally, in favourable circumstances, scuffing can be seen to progress steadily from the tip of the driving pinion inwards, and correspondingly over the driven profiles.

(6) In an improvised two-ball machine, the lecturer found that cracks, substantially at right angles to the direction of sliding, and similar in appearance to thermal cracks sometimes seen in plain thrust faces, were visible at loads just below that at which scuffing occurred. The edges of the cracks showed temper colours, which faded out before the next crack was reached.

(7) The scuffing resistance of glycerol was found by Watson to be only about one-tenth of that of the reference mineral oil, in spite of much lower coefficients of friction.

(8) Watson (15) also reported that some material combinations scuff at lower loads than bronze–steel, and other combinations will not run at all.

Taken together, these observations run counter to the common belief that, when scuffing occurs, the primary failure takes place within the oil film itself. The whole question is seen to be much more complex, particularly in the light of item (8) above.

It is conceivable that some fluids may be unstable, and dissociate under the high temperatures which occur: this may be the reason for the low scuffing resistance of glycerol. But the success of surface pretreatment, or the effect of e.p. additives, in preventing scuffing, means that by some action which preserves the character of the surface, an oil which would otherwise have been accompanied by scuffing is made to work satisfactorily.

An additional reservation is necessary, in that some cases of scuffing in practice may result from inadequate direction of the necessary quantity of lubricant to the right place. Even if seemingly well-directed, too fine an oil spray may be blown away by the air currents generated by the tooth mesh, may fail to cool the tooth surfaces adequately, and by the consequent increase in the body temperature of the gears will diminish the thickness of the entrained oil film. This topic could be enlarged upon if space permitted.

The broad conclusion which can be drawn from the facts given above is that with possible reservations about non-stable fluids, scuffing is the consequence of disruption of the geometry of the oil film; but that this may have varying origins. Thus:

(1) Progressive scuffing starting at the tip of an unmodified profile may be attributed to a discontinuity of the surfaces between which an oil film should be formed. This discontinuity progresses over the profiles, with the extending boundaries of the scuffed area.

(2) Thermal or other surface cracking can cause a local break in the surfaces which bound the oil film; but if not too drastic, the oil film can re-form when the crack has passed. (The severity of thermal and shear stresses under high V_s/V_e conditions is evident from Fig. 10.)

(3) Under pressure or surface temperatures so high that scuffing occurs, even in the presence of e.p. additives, the surfaces may yield locally under plastic flow and destroy the regularity of contour necessary for the maintenance of an oil film, so diminishing film thickness that temperatures mount further and the surfaces melt.

(4) The behaviour of recalcitrant material combinations opens up a new line of speculation and a new field for investigation. Observation of cases of smooth abrasion leads to the conclusion that under similar conditions of contact velocities and loading, it occurs more readily with normalized or temper-hardened nickel and nickel–chromium steels than with straight carbon steels of equal hardness, and must be counteracted by using a heavier lubricant.

In chain-connected disc tests (Watson (15)) the thinner oil films which then occur act to accentuate the effect of materials, and some combinations will not run. Inasmuch as material combinations which run successfully under the same conditions have frictional characteristics consistent with the behaviour of relatively thick oil films, some additional factors must come into play. The usual belief is that polar affinity provides the explanation. But the tests on castor oil, for which the effects of polar affinity should be more marked than for mineral oil, showed little reduction of friction under adverse contact conditions. More significantly, Mr Harry Pearson, in a recent conversation, mentioned that when a bronze disc was heated and re-cooled between friction measurements, the coefficient of friction was increased. The behaviour of good bearing metals is commonly attributed to the presence in them of hard nodules in a softer matrix, and although this seems difficult to prove, it is supported by the results recently reported to this Institution by Wells. Practical experience also shows that sliding surfaces can be too smooth.

Other assumptions made in deriving Hertzian stresses and oil-film geometry are that the surfaces are perfectly elastic and isotropic. It would be rash to assume that these assumptions remain valid over the small area of a contact band, and it would seem reasonable to surmise that under the pressures and temperature changes which exist at a gear-tooth or disc contact, local surface deformations occur which may either aid or hinder the establishment of a freely flowing oil film. If this is so, the simple concept of a parallel oil film must be replaced by one of a vast number of very small asperities.

LUBRICANTS AND GEAR DESIGN

What has been said explains the difficulty of evaluating the lubricating properties of oils, and the variability of relative ratings given by different types of testing machine.

For most gear applications, satisfactory lubricants are available, and the type of lubricant which will ultimately be used is not taken into consideration in initial design. But difficult applications remain, chiefly those in which great variation in operating temperature or speed is involved, or in which a common lubricant has a diversity of duties to perform. It goes almost without saying that the results of mechanical tests of a lubricant, carried out under one set of conditions of materials, load and surface motion, will no longer be valid if the conditions are greatly changed. Lubricants for particularly severe types of duty must be, and in fact are, rated or compared by testing with actual gears.

But if, in the general run of gear drives, the resistance to pitting is influenced by the characteristics of the lubricant, increased precision in design will have to take the lubricant into account. There is, therefore, an obvious need for more work in this field, with ultimate repercussions on gear rating formulae.

The ability to predict, and therefore to avoid, the onset of scuffing would also be an invaluable addition to the gear designer's armoury.

In both cases, *ad hoc* tests may give useful, but limited, data. The situation really requires more fundamental research, and a comprehensive programme of experimental work. Whilst recognizing that disc machines have their limitations, they are relatively simple and inexpensive to build and to run, and an increase in their number would do much to contribute urgently needed knowledge, particularly if coupled with techniques of film-thickness measurement.

ACKNOWLEDGEMENTS

The author desires particularly to acknowledge his indebtedness to Mr T. P. Newcomb, of Ferodo Limited, for his stimulating instruction and comments on thermal phenomena, to Mr H. J. Watson, of David Brown Industries Limited, for his willing co-operation in experimental work and the supply of test data, and to Dr Alastair Cameron of the City and Guilds College for much helpful comment.

APPENDIX

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Discussion

Mr Ewen M'Ewen, M.Sc. (Eng.) (*Member of Council*), in opening the discussion said that gear technology had exercised a fascination on very many men from the great days of Sang, that improbable Scots professor at the University of Istanbul, onwards: and he was delighted to see a good turnout of what might be described as the 'Gear Technologists Club' for this nominated lecture by their old friend, colleague, and, might he call him, past-president of the 'Club', Dr H. E. Merritt.

The most important part of the work which had been undertaken by him, and by some of the other members of the 'Club', had been the way in which gear technology had progressed from being an exercise in geometry, of a kind which he feared the modern geometer would scorn, although nonetheless fascinating, to the practical application.

In this lecture, Dr Merritt had again provided the groundwork for the practical use of much of the theoretical analysis which had been done by Blok, Petrusevich and others on scuffing, temperature flash and lubrication.

The lecture contained a wealth of information and material, some of it controversial, and at first reading he had been tempted to make a list of points which he wished either to agree with, or disagree with, in the discussion. However, when he had been asked to open the discussion of this very important lecture he had decided that the detailed points were better dealt with either in personal conversation with the lecturer or in writing, and in any case unless the temper of the meeting had departed radically from what they normally had on a gearing occasion, somebody else was bound to raise all his detailed points anyway: having in the past suffered himself from having some small point which had seemed important to him included, swept up and swallowed in the opening of the discussion, he was more than ever convinced of the necessity for a broader approach.

One of the most important aspects of the lecture was the bringing together of the very little that was known about scuffing, either in theory or in the test laboratory, and the obvious need widely to extend this knowledge in practical cases. Much of what was known about gearing was wrong because, of course, of the gross oversimplifications in their theoretical analyses in which they normally considered the gear teeth as being infinitely rigid, perfectly mounted and free from error. It was true that in some of the work that had been published in the last twenty or thirty years by the lecturer and by such pioneers as Dr Harry Walker some consideration had been given to the flexibility of the gear tooth in the transverse plane, and also to the wind up over a wide face with a helical pinion, but the bulk of this work had been on the macro effects especially as related to the fatigue or breakage strength of the tooth and theoretical attacks on the scuffing problem and other forms of surface failure were still premised on perfect accuracy.

As Dr Merritt had pointed out, the oil film thicknesses being referred to were of the same order of magnitude as the surface roughnesses and smaller than some of the combinations of deflection and error, so that one must not be surprised if the temperature flash predictions led one astray from time to time in practice.

It might be wrong, but he was unrepentantly of the opinion that boundary conditions did exist owing to these asperities in at least some cases and that this was the probable reason for the fact that certain combinations of materials were good and others were bad, from the point of view of resistance to scuffing.

He would like to comment a little in more detail, although still broadly, on one or two aspects of the problems raised by the lecture.

First, on the subject of the lubricant and its point of application, it could be seen from the work reported, summarized and commented upon here how hydrodynamic oil films would occur within the limits of the asperities already referred to, but he thought the most important aspect of the lubricant was apt to be overlooked—which was, in fact, that one must cool the tooth sufficiently between one passage to the contact zone and the next to prevent a continuous build-up in temperature to an excessive figure.

Experience showed that in many cases the addition of further lubricant to the unit, or the spraying of a larger quantity of lubricant to the point of mesh, resulted not in better but in deteriorated performance. He recalled, for example, an extremely difficult case of this kind, being a high-speed 6-in. centres increasing gear of 3/1 ratio, 1500 rev/min input, and the combination of 3½ per cent nickel–chromium–molybdenum steel in the 65 ton condition and a 45 carbon steel in the 45 ton condition, where scuffing problems failed to yield to profile modifications, to increases in the quantity of lubricant supplied and to variations in the nature of the lubricant, until it was noticed that if the gear were run in the reverse direction they did not have the trouble. The reason for this was, of course, that they were then applying very large quantities of lubricant to the teeth as they emerged from the point of mesh and were producing the necessary cooling. He regarded this cooling as being an important point which was too frequently overlooked.

Still on the subject of the lubricant, Dr Merritt had referred to the fact that in practical experience a hot heavy oil was better than a cold light one. He would confirm that statement which had especial importance in a good deal of development testing where one, from time to time, misled oneself, because as a matter of convenience of carrying out a rig test one used a lighter oil at a lower temperature to 'simulate' a hotter, heavier oil. The fact that this simulation was invalid was fairly well-known, although he feared it was sometimes overlooked and it was interesting to have this practical result supported on theoretical grounds.

Dr Merritt had spent most of his time dealing with other forms of failure than pitting, and he agreed with him that pitting had had too much emphasis compared with other forms of surface defect.

There was one aspect of the resistance to pitting, however, which he personally would like to see explored further, in order that greater economy in the use of materials might be achieved. Dr Merritt had pointed out that in the United States of America rating for surface durability was commonly based on permissible Hertzian compressive stresses increasing with the square of the hardness of the material and that the British standard wear rating formulae were more conservative as hardness increased but were the same general form. Practical experience gave rise to an interesting and important question here, in that pitting resistance did increase approximately as the square of the hardness up to about 60 to 70 ton tensile per square inch and the pitting resistance of case-hardened materials of much greater hardness was up more or less where one would expect it to be, but hardly as the square of the hardness.

However, in the range 70 to 110 ton/in^2, or Rockwell C 35 to 50, the pitting resistance did not increase as would be predicted from formulae of this type. It would be interesting to know why, so that one could take advantage from the point of view of surface load-carrying capacity of steels with tensile strengths in this range.

Of course the formulae used in any manner of rating were really only for the one-off, or small quantities, and had to be fairly conservative, whereas large-quantity production gearing was usually the subject of intensive development which might effect an increase in load-carrying capacity of 100 per cent between the initial unsatisfactory samples and the final satisfactory ones.

Finally, he would support to the fullest Dr Merritt's plea for further work in the research laboratories of institutions and companies and for the freest possible publication of material arising therefrom. This was not a case where commercial advantage was to be obtained by keeping secret the results of investigations of this type, but rather could all gain by the sharing of the knowledge which resulted.

It seemed to him that the most important thing that could be done in this field was to improve the correlation between the practical application and the necessarily oversimplified theory which one had to use to guide one in one's original designs.

There was a deplorable fashion in some circles to regard engineering in general, and gear technology in particular, as applied science. Engineers must remember that in these cases they were practising an art in which they used a simplified scientific theory to guide them in their first approximations, and from which they proceeded on the basis of trial, error, experience, frustration and, all too occasional, triumph.

Professor W. A. Tuplin, D.Sc. (*Member*), said that the lecture was rather unusual in that it did not merely admit ignorance about the subject but proclaimed it. There were papers which did not precisely admit ignorance but the ignorance showed through. There were other papers in which ignorance was not admitted at all. That was the case with some papers on allied subjects, such as for example bending stress in gear teeth. So much was known about it that it was almost a pity that, provided that gears were properly designed, they never failed by bending. But that did not stop people from pursuing refinement of the bending stress factor and thus affecting to improve something that was good enough already.

This lecture was quite different. The author had referred to a number of more or less accepted concepts, and he had talked about rolling velocities, not to be confused with pure rolling. He himself doubted whether this sort of motion was properly called rolling at all, but that was what it had been called in the literature of gearing. To him, rolling implied absence of sliding.

The author had referred to S_c and K, giving S_c as equal to $6{\cdot}3K$. He ventured to suggest, however, that for conventionally designed gears the ratio ranged from about 6 for spur gears to 4 for helical gears. He had referred to the disc machine and to a line of contact 'in space'. He suggested that it might be preferable to define the conditions by saying that the line of contact was fixed relative to the base of the machine and therefore to the observer and to the common plane of the axes of the discs.

Disc machines had been used since 1880, and much work was still being done with them despite the fact that they did not represent gear tooth contact conditions because the sliding velocity of a pair of gear teeth changed sign in every cycle whereas the conventional disc machine gave constant sliding speed. Nevertheless, disc machines had become numerous and one sensed a fear of trying any other device that might closely simulate gear teeth in case its results should differ from those obtained from disc machines and thus undermine a great many research results. There was a fear of discovering something that one would rather not know.

No doubt many people had thought of making machines for simulating the action of gear teeth. They had done this at Sheffield University and, helped by the D.S.I.R., they had made a machine. There had been some resistance to the suggestion, but it had gone through, and the machine justified the resistance by giving results that did not agree with any of the currently fashionable ideas about contact stresses in gear teeth.

One of the graphs shown by the author (Fig. 5) gave variations of the coefficient of friction with speed and with slide/sweep ratio on the basis of entraining velocity. If the solid lines were replotted on the basis not of entraining velocity but of sweep velocity in respect of the large disc they compressed themselves very nearly into one straight line. He could not say the same for the dotted lines nor could he say it for the lines (Fig. 6) applying to lubrication by glycerol.

There was a reference in the lecture to plastic flow. At least in respect of what might be called ductile material, it must be appreciated that plastic flow of the surface layers of tooth flanks was normal. He was inclined to think that plastic flow occurred until the effective stress corresponded to the yield stress of the material, and further running produced work hardening that raised the fatigue limit above that yield stress. That was how gears got by if the loading was not too great. If it was, then the plastic flow occurred in such a way that the radius of relative curvature increased and the stress was reduced. That was why gears continued to work although the tooth profiles were different from their original form.

The author had referred to cracks in steel. At least one theory was that steel was largely built of cracks tied together by metal and that one steel was better than another only because it had finer and smaller cracks. One did not need to reject any theory about the origin of pitting merely because it demanded that the material be already cracked.

Scuffing had been mentioned. He suggested that that was not so difficult to understand if one accepted the theory that it started from metal-to-metal contact. The fact that scuffing tests showed very scattered results might be explained by assuming that scuffing started when a hard particle in the oil happened to come between two high spots that would otherwise just miss each other. The large element of chance thus involved might explain why scuffing loads could not be predicted with any useful degree of precision.

Early troubles with scuffing had resulted from attempts to lubricate gears with oil that had been primarily selected for some other purpose. In most cases use of an ordinary mineral oil of adequate viscosity had overcome the trouble.

Dr A. W. Crook (Aldermaston) said that he would like to express to Dr Merritt his appreciation of his review of the more speculative aspects of his subject. Dr Merritt had long been a notable advocate of scientific measurement and analysis in relation to the lubrication and failure of gears

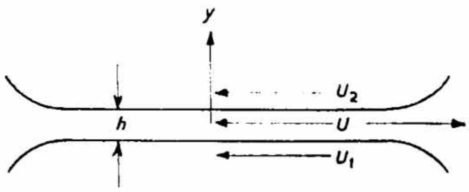


Fig. 12. Co-ordinates

and he had given a considerable sketch of the way in which the work could go forward.

Dr Merritt had posed the problem of the temperature distribution within the oil film and it was on that topic that he would like to contribute to the discussion.

For some time they had been making measurements of film thickness and of friction with disc machines and had based their interpretation upon the following equations: the hydrodynamic equation,

$$-\frac{\partial P}{\partial x} = \eta\frac{\partial^2 u}{\partial y^2} + \frac{\partial \eta}{\partial y}\frac{\partial u}{\partial y} \quad . \quad . \quad . \quad (26)$$

where P was pressure, η viscosity, u velocity taken positive in the negative direction of x and the co-ordinates were those of Fig. 12;

the equation of heat generation,

$$q = \eta\left(\frac{\partial u}{\partial y}\right)^2$$

where q was the rate of generation of heat per unit volume; the equation of heat flow,

$$\rho c u\frac{\partial \theta}{\partial x} + k\frac{\partial^2 \theta}{\partial y^2} = -q \quad . \quad . \quad . \quad (27)$$

where ρ, c and k were respectively the density, specific heat and thermal conductivity of the oil and θ was temperature; and lastly upon,

$$\eta_{P_2\theta_2} = \eta_{P_1\theta_1} \exp\left[\delta(P_2-P_1)-\gamma(\theta_2-\theta_1)\right]$$

which related the viscosity of the oil at any combination of temperature and pressure to that of another combination.

When frictional heat was to be considered it was essential that the hydrodynamic equation be taken in a form which gave to η freedom to be a function of y as well as of x; because of that the term $\frac{\partial \eta}{\partial y}\frac{\partial u}{\partial y}$ appeared in equation (26). The frictional heat would be dissipated by conduction to the surfaces of the discs and by transport with the oil in its passage through the conjunction (convection). The first term on the left of equation (27) referred to convection and the second to conduction.

It could be shown as Archard (**16**)* had suggested that, in general, the convection term might be neglected in comparison with the conduction term. The above set of equations was then analytically tractable (Crook (**17**)).

The analysis showed that the dimensionless quantity

$$\psi = \eta_x\gamma(u_2-u_1)^2/8k$$

where η_x was the viscosity at a point x on the surfaces of the discs (Fig. 12) was of central importance. For example θ_c, the temperature rise on the median plane of the oil film with respect to the temperature at the surfaces of the discs, was given by

$$\theta_c = \frac{1}{\gamma}\ln(1+\psi)$$

and the effective viscosity throughout the pressure zone ($\bar{\eta}_m$) was given by

$$\bar{\eta}_m = \frac{1}{2b}\int_0^{2b}\eta_x f(\psi)\,dx$$

* *A numerical list of references is given in the Appendix, p. 163.*

where $2b$ was the width of the Hertzian flat and $f(\psi)$ was a function of ψ (Crook (17)) which would not be specified here. The significance of the effective viscosity was that it was the constant viscosity throughout the pressure zone which would give rise to the observed friction. The analysis suggested that $\bar{\eta}_m$ and indeed the temperature distribution did not depend explicitly upon film thickness, that they did not depend upon the entraining velocity although, of course, they did depend upon the sliding velocity and it also suggested, in agreement with a suggestion of Dr Merritt, that the thermal conductivity of the oil was important. As the thermal conductivity entered the expressions along with the exponent representing the temperature dependence of viscosity it was just as important as that dependence.

The effective viscosity had also been deduced from simultaneous measurements of friction and film thickness. As experimentally determined it referred to the oil as it actually behaved under the conditions within the conjunction. A typical comparison of the variation of $\bar{\eta}_m$ with sliding speed as determined experimentally and as predicted by theory was given in Fig. 13. There was a sufficient agreement to establish the theory as a first approximate solution to the problem.

Unfortunately it was often impossible, because of missing information, to consider published measurements of friction with disc machines in relation to theory as fully as one would like to do so (e.g. Fig. 5 of Merritt's lecture).

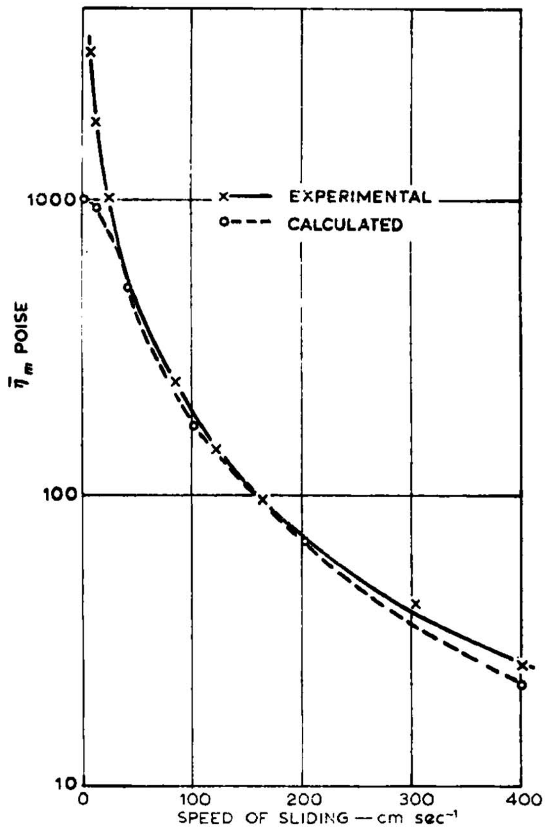


Fig. 13

The major difficulty was that seldom was the surface temperature of the discs noted. Consequently the viscosity of the oil at the surface temperature of the discs was unknown and it was this viscosity which was of dominant importance with respect to the thickness of the hydrodynamic film (Crook (18), Archard and Kirk (19), Crook (20)). With respect to film thickness it was just as important to record this viscosity as to record the speeds of revolution of the discs so he would like to ask Dr Merritt to couple with his advocacy of further work a plea that the surface temperature of the discs be measured and the viscosity of the oil at that temperature be given.

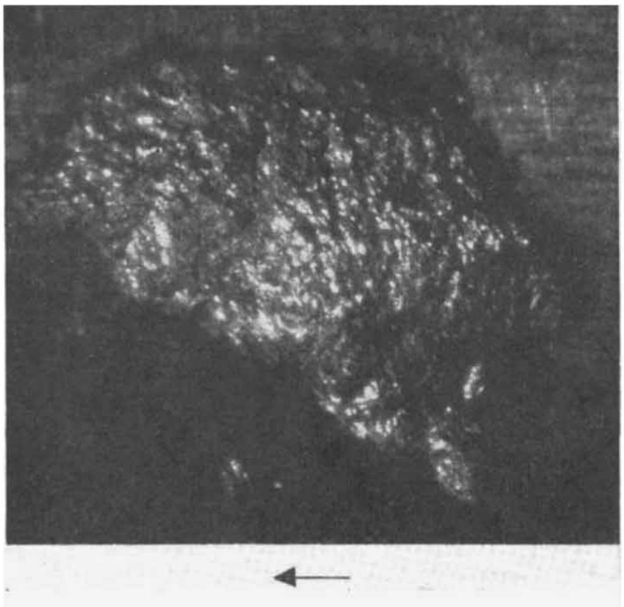
Mr W. J. Davies, B.Sc. (Eng.), Wh.Sch. (*Associate Member*), said that whilst Dr Merritt had emphasized the problems awaiting solution, it must be very satisfying after a lapse of 30 years to have data becoming available which shed material light upon some of the old results which at that time had appeared perplexing. The appreciation of pressure–viscosity variation and afterwards of elasto-hydrodynamic behaviour had had powerful eroding effects on the lack of understanding. Elasto-hydrodynamic appreciation altered the pressure distribution from the old Hertzian concept to, at first sight, little degree except for the realization of the vicious pressure spike at the outgoing edge of the contact. Incidentally, this seemed to suggest that fatigue-wise a point in the tooth surface received almost the equivalent of double the hitherto appreciated frequency.

One of the many minor mysteries of the behaviour of surface fatigue of gears related to the position of onset of such break-up. This was, in so many cases, just under the pitch line in the dedendum region. Dr Merritt's lecture indicated higher temperature on the dedendum surface rather than the addendum partner of the contact, which might contribute. Also the tangential force of friction was shown to be greatest with low sliding and it would therefore have its worst effect near the pitch line. Analytical work by Professor Weber contained in a Pametrada report (21) showed appreciable surface tension stress 2μ times maximum Hertzian stress occurring at the edge of tooth contact. For μ of 0·08, such a tension stress became an appreciable addition to thermal stresses and the general conditions of stress. The relevant contact edge in the just-below-pitch-line condition would correspond to the lower one—in other words the outgoing edge of the rolling contact zone, where one also had the spike. One wondered whether the combination of temperature, Petrusevich spike and tangential stress combined to cause the onset of surface cracking, leading to pitting in the popular region.

The opening descriptions of the fundamental components of surface motion had quite correctly included rolling and sliding in more than one plane, covering conditions of helical, hypoid and worm gearing. The subsequent work was wholly two-dimensional in motion because of the simplicity achieved by the conventional disc test, but it was, he thought, very relevant to the subject of sliding and entraining velocities to mention the outstanding and peculiar properties of the Wildhaber–Novikov type of gearing. For high surface capacity this was best designed with low helix angle. Contact in the transverse plane was of momentary nature and was accompanied by sliding in the transverse plane of a conventional order, but with this low helix angle the contact passed very speedily across the face width giving a very high order of sweep rolling or entraining velocity in this direction. The evidence was that this induced a very thick film to the great benefit of efficiency, and anti-scuffing. This was a case in which a high degree of rolling entirely unconnected with the sliding—which was virtually normal to it—conferred on the lubricant exceptional power to carry the load with less than usual danger of metallic contact.

Mr B. A. Shotter (Rugby) said that Dr Merritt had discussed pitting in a general way and suggested that tests should be conducted to try and separate the effects of entraining and sliding velocities. Those two velocities were conveniently separated in the Wildhaber–Novikov (W–N) gear-tooth system. In this case sliding motion occurred in the transverse plane but the rolling or entraining velocity was produced by the motion of the contact area along the gear teeth.

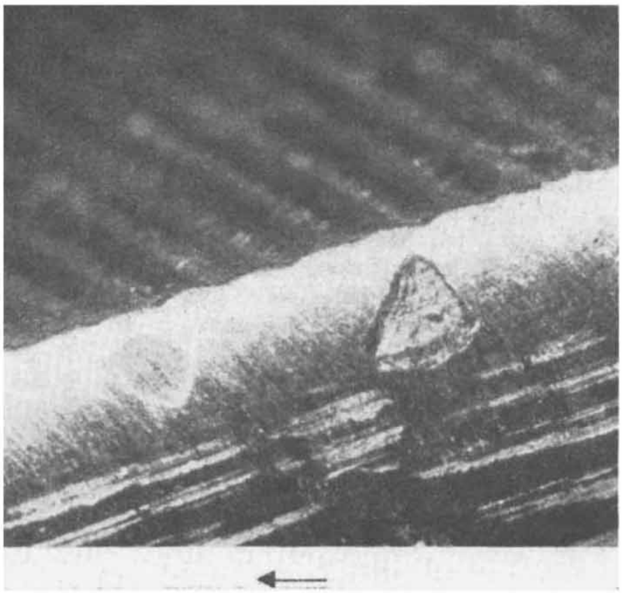
Fig. 14 showed a large pit on the flank of a W–N tooth. It was interesting to note that although the start of the pit pointed down the tooth flank its subsequent development had caused the crack to spread mainly in the rolling direction. These facts suggested that pitting was initiated by the sliding motion, though the subsequent propagation of the crack was probably assisted by hydraulic pressure from the rolling motion as suggested by S. Way in 1940. The small



×23

Arrow indicates the rolling direction.

Fig. 14. Large pit on a W–N tooth flank



×23

Arrow indicates the rolling direction.

Fig. 15. Small pit and pit-print on a W–N tooth flank

pit and the pit-print shown in Fig. 15 also showed the asymmetry mentioned previously but the fact that pits on opposed surfaces pointed in opposite directions was additional evidence that it was surface traction forces which were responsible for the initiation of pitting.

A lot of work had been undertaken to establish the basic theory of gear tooth lubrication but it must be remembered that actual gears were only approximations to the perfect geometry assumed by the theory. Because of this the stresses which produced pitting were to be found at the contact of asperities rather than the general stresses produced by the contact of geometrical bodies (Shotter (**22**)). This concept should also be considered in investigations into the cause of scuffing.

Mr A. D. Newman, B.Sc. (*Associate Member*), said that if he were a psychiatrist he might have been tempted to try a word association test on the audience. He was sure that if he said 'gears' a large number of people present would immediately respond 'Merritt'. That was quite an achievement on Dr Merritt's part, and it gave him very great pleasure, therefore, to be able to contribute to the discussion on the lecture.

He was pleased that Dr Merritt was on the side of the disc testers—kept in their right place, of course. This aspect of gear research had been under somewhat of a cloud lately, partly he thought because its results were sometimes used out of context and partly because there was a lack of usable data because vital factors were often not measured, or controlled, or reported. He was therefore pleased to hear Dr Merritt say that determination of the coefficient of friction was indispensable to the study of contact conditions between discs. He had found that the way in which the coefficient of friction changed in the process leading up to scuffing between discs was highly significant, and it often grieved him to see so much data published on disc testing which did not include friction values.

As Dr Merritt pointed out in connection with research into contact between discs, pairs of discs of different relative curvature produced different conditions of oil film formation and friction. At Pametrada some time ago a disc machine had been designed and built in which the centre-distance between the discs could be varied, as well as the load and slide-roll conditions. The object was to be able to determine the effect of varying relative curvature for the same sliding and rolling speeds, and consequently the same slide/roll ratio. No data from this machine had yet been published, but the results, particularly the effect on coefficient of friction, should be valuable.

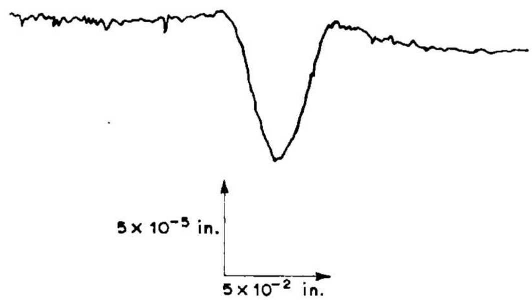
Dr Merritt had mentioned in his section on friction phenomena in disc tests that the presence of an extreme-pressure additive in an oil could conceivably change the pressure–viscosity characteristic of the oil, and thus have a physical as well as a chemical effect. This might be borne out by the friction measurements in disc tests where the load was gradually increased until scuffing occurred—the friction curve up to the scuffing point was different with an extreme-pressure additive in the oil from that obtained with the base oil alone. He thought that this effect might also occur with certain additives other than those having an extreme-pressure function, for example with anti-oxidants.

In his experience of marine gearing, the most troublesome difficulty was the combination of ridging and scuffing, often leading to flaking or pitting of the gear teeth, which appeared to be peculiarly associated with certain gear steels in combination. If he had understood Professor Tuplin's remarks correctly, the inference was that this was not a serious trouble. His own experience was that it sometimes got worse and worse. This sort of trouble had been described by Dr Darlington some time ago in his paper on the Nestor gearing (**23**) and was alluded to by Dr Merritt. He asked whether Dr Merritt had any views as to the association between this sort of trouble and the particular combinations of gear steels involved, and could he expand a little his views of the basic cause of the trouble?

He would like to underline Mr M'Ewen's point about gears in relation to scuffing and cooling. He did so by the example of a marine gear on low load at high speed, scuffing apparently because of an excess of cooling oil, so much indeed that the gearcase filled with oil. This proved a practical demonstration of Joule's law, and it indicated that unless one tried to understand the reality of this sort of situation one would not get anywhere.

One last thought: he wondered why, in helical gears, one did not involve the apparent effect of movement of the line of contact in the axial direction—a very high rate of movement—in lubricant considerations?

Dr J. F. Archard (Aldermaston) said that Dr Merritt's lecture emphasized the change in ideas about gear lubrication



$R = 0{\cdot}6$ in. (1·52 cm); $V = 280$ ft/min (140 cm/sec); $\mu \simeq 0{\cdot}04$; $W = 925$ lb (420 kg); temp. 17°C; electrical resistance $= 10^3 - 10^4$ ohm.

Fig. 16. Plastic flow of hardened steel under hydrodynamic conditions

which had occurred in recent years. His lecture implied that it was now accepted that the lubrication of successful gearing was primarily elasto-hydrodynamic.

In this connection it was of interest to explain that Kirk and himself (**19**) had recently explored the range of conditions over which elasto-hydrodynamic lubrication could operate. They had found that, even under the unfavourable conditions of a point contact geometry, elasto-hydrodynamic lubrication occurred with smooth surfaces down to speeds as low as 1 cm/sec (2 ft/min). At the higher speeds they had found that elasto-hydrodynamic lubrication persisted up to loads so large that the hardened steel specimens were plastically deformed. Fig. 16 showed a Talysurf record of the track on one of the crossed-cylinders used in this experiment. The measured value of the electrical resistance (more than 10^3 ohm) and the complete absence of any form of surface damage showed clearly that the lubrication was elasto-hydrodynamic. (Perhaps in these circumstances one was tempted to use a new piece of jargon and call it 'plasto-hydrodynamic'!)

It was relevant to consider the consequences of the change in ideas about lubrication outlined in the lecture. He would suggest that the most important consequence was that the role of boundary lubricants, the action of e.p. additives and, indeed, the initiation of many forms of failure must be considered in terms of local contacts through a primary elasto-hydrodynamic film. This concept of a primary elasto-hydrodynamic film with local penetration at asperity contacts also suggested that scuffing depended upon surface finish. Indeed in the crossed-cylinders experiments there was some evidence that this was so.

The behaviour of glycerol as a lubricant described in the lecture could be explained in terms of the dependence of viscosity upon temperature and pressure. The effects upon the viscosity of glycerol and oil of a change in temperature followed by a change in pressure were listed in Table 1. The results for glycerol were taken from Bridgman's (**24**) work and those for an oil from the A.S.M.E. Report (**25**). It would be seen that, for glycerol, the increase of viscosity with pressure was relatively small and that this increase was more than cancelled by the assumed increase in temperature. If one changed from an oil to glycerol, the

Table 1. Effect of pressure and temperature upon viscosity

(For simplicity only relative values are considered, the viscosity at normal pressure and temperature being taken as unity.)

Pressure	Temperature, °C	Viscosity	
		Glycerol	Typical oil
Atmospheric .	30	1	1
57 000 lb/in² .	30	8·6	> 10^3
57 000 lb/in² .	75	0·34	> 50

reduced pressure–viscosity coefficient would cause a reduction in the film thickness. However, the value of the friction would fall because this was dominated by the much larger fall in the effective viscosity of the lubricant in the pressure zone. It was possible that the poor behaviour of glycerol in scuffing tests was associated with the reduced thickness of the film.

Turning to the temperature gradients within the film one might note that the temperature distribution of Fig. 11 had been based upon the assumption of a uniform rate of heat generation throughout the film. This assumption was only a crude approximation (26) (16) since temperature gradients caused variations in viscosity across the film; in this direction the shear stress was approximately constant and the rate of heat generation was inversely proportional to the viscosity. Thus along any plane normal to the surfaces the rate of heat generation was greatest towards the centre of the film where the highest temperatures existed. As an extreme example one could consider all the heat being generated on a central plane, in which case the temperature distribution was triangular rather than parabolic. A rigorous analysis of the problem had been made by his colleague Dr Crook (17) and this confirmed that the generation of heat was far from uniform; in the symmetrical case most of the heat was generated in a band centred on the mid-plane between the surfaces.

Thus in the problem posed by Dr Merritt in connection with Fig. 11 the difference in temperature between the two surfaces assumed an unwarranted importance. Because uniform heat generation and a parabolic temperature distribution was assumed this temperature difference was the only factor which could cause an asymmetry in the distribution of heat generation. Moreover, the actual value (5 deg F) was hardly significant; it was small compared with the probable temperature differences (50 deg F to 100 deg F) which occurred within the film.

Mr H. J. Watson, B.Sc. (*Associate Member*), said that there was no doubt that the lecture was a major contribution to the knowledge of gear tooth contact, which was by no means a simple subject, although Dr Merritt had the knack of making it seem much simpler than it was.

As one who had watched while a small part of this work was being done he would like to comment briefly on one or two items which formed the subject of the tests. Fig. 17 showed a diagram of the modified disc machine described

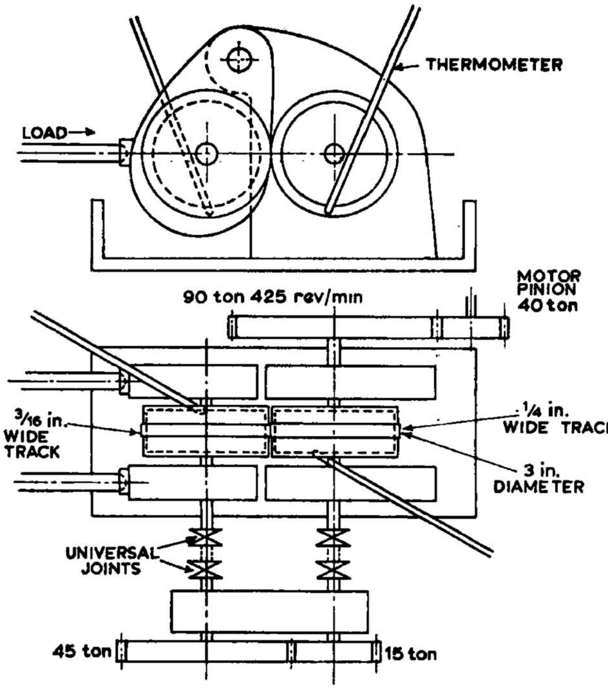


Fig. 17. Disc machine, 3-in. centre distance, fitted with hollow discs

in the lecture and used for making the measurements of heat flow in the experiments. It was very similar to that which had been shown already except that the discs used were hollow and thermometers were placed in the hollow rims which contained a quantity of oil. The heat was transferred from the surface to the oil and measured by the thermometers. The experiments had been carried out by Mr Pearson. Originally tests had been made with the hope of measuring surface temperature by means of a contact pyrometer or thermometer on the outside, but the work was not successful. Measurements had been taken in this case every minute, during heat and cooling.

Table 2 showed some of the results. The author had given the list of readings on the top line, which showed that the flash temperature was lower in disc number 2 than in number 1 and not as his calculations predicted, but they had two other examples using steel–steel discs where the flash temperatures were in accordance with the theory and another where they were equal. Then they had two bronze–bronze disc tests in which the calculated flash temperature did not agree with the theory. In the case of the steel–bronze specimens, the results were a little complex and more work was needed on them.

Might he comment on scuffing? When news of this lecture was first received hopes had been expressed that an advance would be obtained towards a practical solution of the scuffing problem, and there was no doubt that that advance had been made. The author had concluded that scuffing resulted from the disruption of the geometry of the oil film; this was probably true. He had mentioned that in some

Table 2. Results from hollow disc tests

Material combination		Applied load, lb	Coefficient of friction, μ	Measured rim temperatures		Ratio of cooling curve slopes	Temperature rise values		Max. flash temperatures	
Disc 1	Disc 2			T_1, °F	T_2, °F		θ_1, °F	θ_2, °F	$T_1+\theta_1$	$T_2+\theta_2$
Steel	Steel	315	0·036	193	176	1·2	53	66	246	242
Steel	Steel	150	0·060	164	153	1·2	51·5	65	215·5	218
Steel	Steel	315	0·050	240	220	1·2	74	94	314	314
Steel	Steel	315	0·060	236	224	1·2	87	117	323	341
Bronze	Bronze	315	0·030	168	158	1·25	23	28	191	186
Bronze	Bronze	440	0·033	210	197	1·3	33	40	243	237
Steel	Bronze	315	0·032	195	181	1·3	44·5	32	239·5	213
Bronze	Steel	315	0·029	170	166	1·6	25	43·5	195	209·5

cases it had been possible to detect the breakdown of the metallic surfaces prior to scuffing having occurred. On the other hand, the cracks described were rather similar to those which occurred sometimes on very heavily loaded worm threads where there were cracks normal to the direction of sliding. In many cases the worm continued to run satisfactorily if the surface was in compression and the crack edges did not rise; on the other hand should they rise the film was disrupted and scuffing was very drastic.

Professor Tuplin had mentioned the inclusion of foreign matter in gear units as a possible means of promoting scuffing, but he did not think that was always true. They had tried to produce scuffing on occasions by the introduction of foreign matter into hypoid gear units but without success. The most likely reason for scuffing was an inadequate supply of oil at the contact zone on loaded gear teeth and there was a very good example of scuffing produced by interruption in the supply of oil on a Pilger drive mill. On one side the teeth were in perfect condition but at 180° around the pitch circle they were completely melted. Between the two they had had all the signs of deficient oil supply including scuffing and excessive abrasion. Mr M'Ewen had mentioned that the effect of oil, apart from lubrication, was to cool the surfaces of gear teeth and that had been demonstrated.

There seemed to be considerable practical support for the theory that scuffing occurred when the oil film was thinner than a minimum critical value. If this could be calculated from factors which engineers used it would be a major contribution to the design of serviceable gears and a formula generally following the lines of predicting the minimum film thickness would be useful. Additional practical results were still needed before a reliable formula could be used in gear design but the lecture undoubtedly had given a new lead in that direction.

He would like to suggest a formula for critical oil film thickness generally following the pattern:

$$h \propto \frac{\eta V_e}{S_c E_m r_s \mu}$$

where h was the oil film thickness; η the viscosity at the actual point of contact under the pressure and temperature conditions which could be calculated from the author's proposed formula; V_e the entraining velocity; S_c the surface stress criterion which might preferably be in the form of load divided by relative radius of curvature; E_m a factor concerning the materials of the gears; r_s a surface roughness factor; μ the coefficient of friction, involving sliding velocity.

Mr R. Tourret, M.Sc. (Eng.) (*Associate Member*), said that Dr Merritt was to be congratulated on producing an excellent lecture designed to set people thinking. He rightly pointed out that much research remained to be done before one could understand the fundamentals underlying gear lubrication. He had recently attempted much the same task (27) but with perhaps more emphasis on detailing actual results and less on thinking! So, he knew how difficult the subject was, and especially how difficult it was to explain the observed facts!

He would briefly recapitulate some of his own views on one particular aspect, that of scuffing, since he wished to try to take his thinking one stage further.

He felt that scuffing was primarily related to temperature and he followed Professor Blok in viewing peak temperature —that was, ambient+bulk+flash temperature—as the critical parameter. However, it should be realized that many factors affected whether or not this temperature was attained. He emphasized that this critical parameter had no unique value. In other words, an oil had not got a unique 'film strength'.

Scuffing, he thought, occurred when the peak temperature reached a sufficiently high value for the lubricant film to break down and for the surface asperities to melt. If microseizures followed, then what was known as scuffing occurred. This sounded childishly simple but it was necessary to follow through and see what factors could affect scuffing as so defined, either by affecting the critical temperature or the chances of reaching it. Then one should see whether this concept would permit an explanation of observed facts such as those recorded by Dr Merritt.

Some of the factors which affected the value of the critical temperature were as follows: *Gear material factors:* component with the lowest melting point; when asperities melted, did they smooth out or seize? This depended on the mating surface; surface treatment (i.e., phosphating, plating, etc.), etc.

Lubricant factors: type of fluid (i.e., oil, water, etc.);

viscosity grade (i.e., molecule size); type of e.p. agent, if present.

Some of the factors which affected whether or not the critical temperature was reached were as follows: *Gear design factors:* sliding speed; rolling speed; heat dissipating characteristics; loading.

Gearbox design factors: rigidity (affected amount of local loading); heat dissipating characteristics; if oil was circulated, the amount of oil in circuit and the degree that it was cooled *en route*; if oil was not circulated, the size of sump and the amount of oil in it.

Gear manufacturing factors: surface finish (affected the number of asperities in contact and the height of the asperities would affect the likelihood of collision); direction of markings (some surface finishes had directional markings); macro-finish (affected local loading, e.g. effect of diurnal error); hardness of surface of teeth.

Gear material factors: rigidity (affected local loading).

Site of gearbox factors: ambient temperature; was gearbox cooled? vibration; did foundation flex (e.g. ship's hull)?

Lubricant factors: viscosity; thickness of oil film.

Could one now explain observed facts? He thought so. This mechanism—and the factors affecting it—were consistent with all the facts (1) to (8) listed by Dr Merritt.

He was sure that Dr Merritt wanted to promote discussion on this topic so he would say that he disagreed with him on two points so far as scuffing was concerned. First, Dr Merritt had referred to scuffing and friction in one breath as it were—as if these phenomena were related—or, more accurately, inversely related. A notable example was item (7) under the scuffing facts. It was his own opinion that these were quite independent phenomena, each proceeding on its own way according to its own set of rules. This being so, there was no cause for surprise at glycerol with its low friction giving a poor anti-scuffing performance.

Secondly, there were to be considered the bronze–steel discs which did not scuff and the steel–steel discs which did. While the increase in viscosity under pressure was admittedly large, it must be remembered that temperature also increased and this had the effect of decreasing the viscosity. To an order of magnitude, the two effects cancelled out. In any case, why should the oil protect the bronze–steel combination but not the steel–steel set? He preferred to think that this striking difference in behaviour was due largely to bronze asperities not seizing when operated with steel. The melting points were too far apart. Steel on steel was much more likely to seize since the melting temperatures of asperities on each disc were likely to be very similar. It was common knowledge that it was preferable to make pinion and wheel of a gear set of different materials.

Mr J. Braddyll (Barrow-in-Furness) said that he had always considered Dr Merritt to be *the* authority on gears. It was a real pleasure to be able to say that in the lecture he had presented that evening Dr Merritt had maintained the same high standard of lucidity, simplicity and clarity of expression that had always characterized his work.

He was particularly interested in the concepts put forward by Dr Merritt in relation to gear-tooth lubrication and the influence of sliding and rolling velocities on gear-tooth performance.

Some ten years previously when he had been specifically concerned with the scuffing problem associated with naval gears and at which time he had been working with Mr Arthur Fisher, to whom credit for much of the sense of what he was going to say should be given, they had conceived as a scuffing criterion that loading should be limited such that the intensity of loading F_c multiplied by the square of the sliding velocity V_s and divided by the 'rolling velocity' V_{r2} did not exceed a predetermined figure. Application of this criterion to the redesign of a particular set of naval gears had produced the desired result, i.e., the original gears would not run without scuffing except on an e.p. oil; the redesigned gears ran satisfactorily on O.M.100 oil. It should be stated, however, that the modifications had been considerable including a reduction in pitch, a reduction in proportions of working depth to pitch and also an increase in the working pressure angle. It was instructive to note that these modifications had not only improved the slide/roll ratio but had also, owing to the increase in pressure angle, increased the velocity of entrainment, as described by Dr Merritt. At this point he would like to make it clear that Mr Fisher and himself had never accepted the concept of rolling velocity as described by the lecturer. Whilst Dr Merritt said that the rolling velocity of a disc was its peripheral velocity, they said that a single disc spinning in space had no rolling velocity and that it was not until it ran in contact with another disc or surface that a rolling velocity could have any meaning. In spur and helical toothed gearing there was always a condition of a long arc, an addendum, working against a short arc which was that part of the mating dedendum which lay within the contact zone. In both approach and recess contact the amount of rolling which took place was the length of the short arc and the amount of sliding which took place was the difference in length between the long arc and the short arc. Using the terms Dr Merritt had set forth in his lecture, the sliding velocity $V_s = V_{r1} - V_{r2}$ as indicated, but the rolling velocity was properly the peripheral speed of the slower moving disc, i.e., V_{r2} in the lecturer's terms.

In gears the rolling velocity was the speed at which the point of contact was moving over the dedendum in contact either on approach or recess.

The interesting point which arose here was that broadly Dr Merritt had said that sliding was bad because it generated heat and entraining velocity was good because it generated an oil film. What they had said in their $\dfrac{F_c \times V_s^2}{V_{r2}}$ criterion was that sliding speed was bad because it generated heat and rolling speed was good because it indicated the speed at which (on the dedendum) new surface was brought into the contact zone. This was perhaps best illustrated by referring to discs; and if one considered one disc rotating and the other stopped, one could visualize a hot spot developing on the stationary disc at the point of contact. If the stationary disc was now given a rolling velocity, fresh cooled surface was continuously brought into the contact zone, heat dissipation would be improved and contact conditions upon this disc made less severe.

Following this idea forward and looking at the results of Watson's experiment, one's first reaction was that the final answer was wrong because T_2 was smaller than T_1, but this experiment was unrepresentative of gears for several reasons. The first was that one did not have a long arc working with a short arc because the discs were of equal size and the slide/roll ratio between them was obtained by rotating them at different speeds. Thus whilst it was invariably true in gearing that the short arc suffered most, in this experiment the conditions were worse on the faster-moving disc which in the kinematics of its motion represented positive specific sliding or addendum contact, i.e., the long arc. Another point of interest was that in his calculations the lecturer had indicated that the high-speed disc generated more heat on its own side of the oil film and further received more heat because it exposed a greater expanded area to the hot spot than the slower-moving disc. In practical gears there would also be a benefit because the higher peripheral speed would expose also a greater expanded area for cooling. Perhaps he ought not to argue that an experiment designed to prove a point should necessarily simulate gear-tooth contact; but it was important to recognize that it did not or one might misinterpret the results.

In what he had just been saying he had deliberately introduced the term specific sliding. This term had always confused him and he thought it needed clarification. In practical terms the area of positive specific sliding was the addendum or the long arc in contact and the area of negative specific sliding was the dedendum or the short arc in contact. It was often said that preferential pitting or wear took place in the area of negative specific sliding, but it had never surprised him that if one continuously worked a long arc against a short arc then it was the short arc which wore out first.

An interesting extension of this theme was this, that when one redesigned gears to lower the ratio of sliding to rolling, whether one decreased the pitch or increased the pressure angle then in involute gears, one was in practice always trying to make the length of the addendum more nearly equal to the length of its mating dedendum. Left to themselves, gears which wore tried to do this in a different way—they wore away both dedenda until their teeth approached cycloidal form. There was a lesson to be learned from this, particularly for low-speed gears.

He had already said that Dr Merritt had perpetrated the common misconception of rolling velocity but this was perhaps not a valid criticism of this lecture because he then proceeded to ignore the effect of rolling as such, using his so-called rolling velocities only to calculate the important entraining velocity. He believed that the conception of rolling as he himself had described it was important and certainly if one could contrive to make the lengths of mating arcs equal, then the scuffing problem would disappear. As an empirical formula their $\dfrac{F_c \times V_s^2}{V_{r2}}$ criterion was not too bad and he suspected that the results of their future researches might indicate that a modification to include also an entrainment factor might get them a little nearer to the truth. By such an empirical approach they might be able to design out of scuffing trouble but scuffing would be a puzzle for a long time. One often saw gears which did not wear and there was now theoretical backing for what practical people had always believed—that hydrodynamic lubricating conditions must, in such gears, exist whatever the theory of the day might say. What physical or chemical forces came into play and caused the hydrodynamic film to disrupt if the mating surfaces were incompatible was the real problem, and here he could only echo the lecturer and say 'If we want to solve this problem then we must work harder at it'.

Mr T. I. Fowle (*Associate Member*) said that Dr Merritt had mentioned that Blok's flash-temperature theory only applied to conditions of boundary lubrication, and since he considered that boundary conditions did not arise in gear-tooth contact, it followed that Blok's theory could not apply to gear teeth. It was quite true that, as originally conceived, the flash-temperature theory had been developed on the basis of direct contact between gear teeth. However, it was later realized that it must also apply with reasonable accuracy to contacts separated by a continuous film of oil provided that the film was very thin. This had been mentioned by Professor Blok in his paper to the 1958 Gearing Conference (**28**). Whether the oil film was of the boundary type, or elasto-hydrodynamic type, practice had shown that the flash-temperature theory was able to predict the onset of scuffing, in practical gears with reasonable accuracy. This was vouched for by Kelley (**29**) and by Wydler (**30**) in papers they had presented on the subject. In addition, of course, in the I.A.E. gear rig up to about 4000 rev/min over which range a coefficient of friction could be assumed constant, good correlation had also been shown (**31**). Finally, of course, Blok (**32**) himself had established a correlation between the theory and results in the special planetary gear rig used in his original work.

The point that the flash-temperature theory could apply to thin continuous film lubrication was important, not only to put the record straight, but because it could help to reconcile the controversy over whether boundary lubrication conditions could operate in gear teeth lubrication or not.

On another point, Dr Merritt had suggested, and Mr Newman had supported him, that e.p. oils might be effective because the e.p. additive significantly increased the viscosity of the oil under pressure. It was, however, extremely unlikely on general grounds that any oil-soluble additive in concentrations of a few per cent could have any significant effect on a bulk property like viscosity under pressure. Certainly, in one case examined no such effect had been detected. There was, however, a large body of evidence, including the work of Bowden, to cite only one of many authorities, showing that e.p. agents acted by reacting chemically with the surfaces concerned.

APPENDIX

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Author's Reply

Dr H. E. Merritt wrote in reply, expressing pleasure that the lecture had induced so much discussion. The contributions reflected the many diverse attitudes encountered whenever gear problems were discussed.

Those contributions which described experiences added to the general picture of the problem and reinforced the main purpose of the lecture, which was to stimulate further investigation and, by inference, a re-examination of the questionable and oversimplified notions on which much gear design practice was based.

Future progress must rely upon fundamental studies, having the character of those described by Dr Crook and Dr Archard. But much might be learnt from the study of troubles and failures in service, if more guides to diagnosis could be provided. Mr Watson went to the heart of the matter in suggesting that an immediate need was a means of calculating film thickness 'from factors which engineers use'. He urged that scientists and engineers should get together, to seek a way of establishing mutual understanding and communication.

APPENDIX 3

CONFORMITY OF CIRCULAR-ARC GEARS

By M. J. French*

The conformity of circular-arc profile gears of the Wildhaber-Novikov sort is examined. It is indicated that the contact area may be a banana shape rather than the ellipse hitherto assumed. Two consequences of this are that too small a difference between the profile radii may reduce the useful conformity, and that it is not possible to increase the torque capacity per unit face width indefinitely by reducing the helix angle.

INTRODUCTION

WILDHABER-NOVIKOV GEARS (1)† have helical teeth whose working profiles consist of circular arcs, generally convex on the pinion and concave on the wheel, of nearly equal radius. At zero load the contact between them is at one or more points. This note explores the contours of constant separation round a contact point, which may be expected to bear some relation to the form of the contact zone under load.

Notation

- a Pitch radius of pinion.
- c Subscript denoting a value at culmination.
- G Gear ratio (> 1).
- K Relative Gaussian curvature.
- R_H Longitudinal relative radius of curvature.
- R_T Transverse relative radius of curvature.
- r Mean radius of tooth profiles.
- s Tooth separation.
- t Time.
- z Distance parallel to gear axes.
- δ Distance between centres of tooth profiles.
- σ Pitch helical angle.
- σ', μ Angles explained in the text.
- ψ Transverse pressure angle. Dots denote differentiation with respect to t.

KINEMATICS

In Fig. 1a a transverse section of a pair of Wildhaber-Novikov gears is shown diagrammatically: A and B are respectively the centres of the pinion and wheel, and S and Q the centres of the circular tooth profiles. The pinion and wheel rotate steadily with angular velocities $G\omega$ and ω, and the pitch point P lies in AB such that AP $= a$ and PB $= Ga$.

In one configuration only, which will be called the culmination, will the profiles at this section be in contact under a small load, and values relating to the culmination will be denoted by a subscript c. At the culmination SQ will be equal to the difference of radii of the tooth profiles, and before and after culmination it will be less, since the profiles will have separated. If SQ is δ, then δ_c is the difference of radii of profiles and since at culmination δ is a maximum, $\dot{\delta}_c = 0$ and $\ddot{\delta}_c < 0$, where a dot denotes differentiation with respect to t, time. The condition $\dot{\delta}_c = 0$ will be met if, in the culminating position, SQ passes through P, and it is built into the individual teeth in the form of the helical angles: there is only one point in AB at which the helical angles of the wheel and pinion are equal, and that is the pitch point.

The angle between SP and the perpendicular to AB will be denoted by ψ and called the pressure angle. Suppose it is required to design a pair of gears and a, G, δ_c and ψ_c, the culminating pressure angle, have already been chosen. We draw the line through P at ψ_c as in Fig. 1a, and we still have to decide where in its length S shall fall in the culminating configuration: Q will then be δ_c behind S. There is one degree of freedom of choice, and it should be exercised to give the highest conformity.

Let t be the time since the section under consideration culminated, and z its distance from the section culminating at time t, i.e. the section where contact is occurring. Then if σ is the pitch helical angle,

$$z = G\omega t a \cot \sigma \quad . \quad . \quad . \quad . \quad (1)$$

By means of this relation between time and distance along the axis of the gears, kinematic properties of δ and ψ can be converted to geometrical properties of the contacting surfaces. The point of closest approach of the tooth profiles in any configuration occurs on the line of centres

The MS. of this paper was received at the Institution on 15th October 1964.

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† *References are given in Appendix 2.*

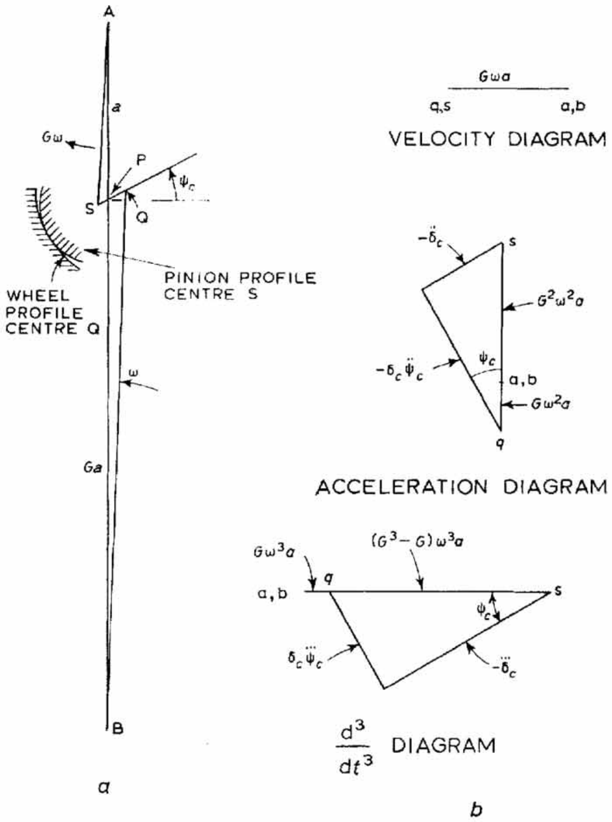


Fig. 1

SQ and the minimum separation (in the transverse section) is

$$\delta_c-\delta = \delta_c-(\delta_c+\dot{\delta}_c t+\tfrac{1}{2}\ddot{\delta}_c t^2+\cdots)$$
$$= -\tfrac{1}{2}\ddot{\delta}_c\frac{z^2\tan^2\sigma}{G^2\omega^2a^2}+\cdots \quad . \quad . \quad (2)$$

since $\dot{\delta}_c = 0$. Equation (2) provides a measure of the way the separation varies along the face width, and hence of the lengthwise conformity. The actual separation of the surfaces along SQ will be the projection of $(\delta_c-\delta)$ on the normal plane or

$$(\delta_c-\delta)\cos\sigma' = (\delta_c-\delta)(1+\cos^2\psi\tan^2\sigma)^{-1/2} \quad (3)$$

where σ' is the angle between the separation in the transverse plane and the normal plane.

In a practical case $\cos\sigma'$ may differ from unity by only about 1 per cent.

When z is small the distance along the tooth is $z\sec\sigma$ so that the relative radius of curvature along the tooth R_H is given by

$$(\delta_c-\delta)\cos\sigma' = \frac{1}{2}\frac{z^2\sec^2\sigma}{R_H} = -\tfrac{1}{2}\ddot{\delta}_c\frac{z^2\tan^2\sigma}{G^2\omega^2a^2}\cos\sigma'$$

or

$$R_H = -\frac{G^2\omega^2a^2}{\ddot{\delta}_c\sin^2\sigma\cos\sigma'} \quad . \quad . \quad . \quad . \quad (4)$$

Equation (4) suggests that the culminating position of S should be chosen to make $\ddot{\delta}_c$ small (but of course negative) and so R_H large. This is not a good idea, because the configuration so obtained is one in which $\dot{\psi}_c$ is large, the line of minimum curvature crosses the transverse section at a small angle, the Gaussian curvature is large in spite of a large R_H and the contact area, being skewed sharply across the teeth, is truncated at their tips.

To avoid skewing of the contact area, $\dot{\psi}_c$ must be small, and if δ_c is small, as it must be in a competitive design, S must be close to P at culmination. The condition $\dot{\psi}_c = 0$ will obtain if at culmination AS is parallel to QB, as in Fig. 1*a*. The velocity, acceleration and rate of change of acceleration diagrams are then as in Fig. 1*b*, in which the difference between AS and AP and the angle SAP have been neglected (in the example quoted later angle SAP is one minute roughly and AS exceeds AP by about one five-thousandth of a). The diagrams are particularly simple because of the conditions $\dot{\delta}_c = 0$, $\dot{\psi}_c = 0$ which cause centripetal, Coriolis and many higher terms to disappear. The diagrams give

$$\ddot{\psi}_c = -\frac{1}{\delta_c}(G^2+G)\omega^2a\cos\psi_c \quad \dddot{\psi}_c = \frac{1}{\delta_c}(G^3-G)\omega^3a\sin\psi_c$$
$$\ddot{\delta}_c = -(G^2+G)\omega^2a\sin\psi_c \quad \dddot{\delta}_c = -(G^3-G)\omega^3a\cos\psi_c \quad . \quad . \quad . \quad (5)$$

CONFORMITY

Equations (1), (2) and (5), with the condition $\dot{\psi}_c = 0$, yield

$$\left.\begin{aligned}\psi_c-\psi &= \frac{1}{2\delta_c}\frac{G+1}{Ga}z^2\tan^2\sigma\cos\psi_c+\text{higher terms}\\ \delta_c-\delta &= \frac{1}{2}\frac{G+1}{Ga}z^2\tan^2\sigma\sin\psi_c+\text{higher terms}\end{aligned}\right\}\quad(6)$$

By means of these equations we can examine the contours of constant separation s between the tooth surfaces in the neighbourhood of the point of contact C.

Let D be any point on the surface of the pinion tooth near to C, lying in a transverse section of co-ordinate z. The points S and Q will be regarded as coincident in what follows (Fig. 2). In the section of co-ordinate z let M be the point on the pinion surface such that MS makes an angle ψ_c with HS, the perpendicular to AB. Let θ be the angle subtended at S by MD. Then for co-ordinates of D

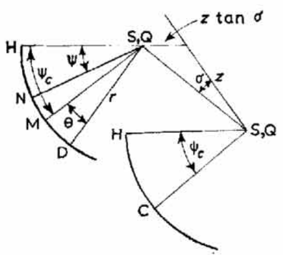


Fig. 2

we use z and $r\theta$, where r is the radius of the pinion profile. Let N be the point on the profile in the section z which is closest to the wheel tooth profile. Then the angle HSN is ψ and the angle MSN is $\psi_c - \psi$.

The separation at N in the transverse section is $\delta_c - \delta$, given by equations (5). The additional separation at D over that at N is roughly

$$\frac{1}{2R_T}(\text{arc ND})^2$$

where R_T is the relative radius of curvature of the profiles so that

$$R_T = \frac{r^2}{\delta_c} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where r is the mean radius of the profiles.

The arc ND is

$$r(\theta + \psi_c - \psi)$$

The total separation at D parallel to the common normal at C, the point of contact, is given approximately by

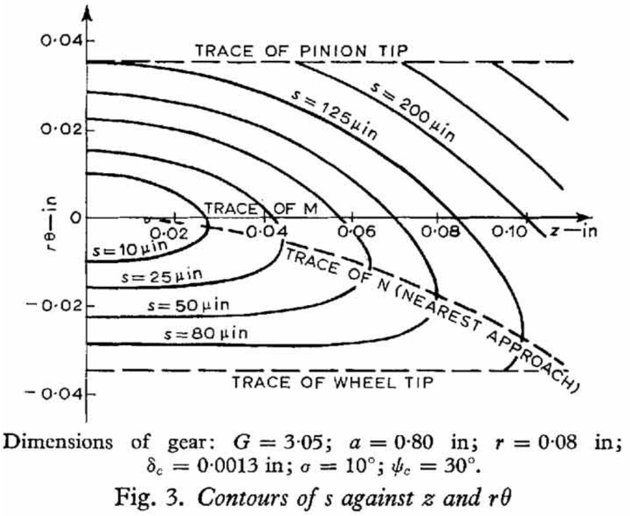
$$s = \frac{1}{2}\frac{G+1}{Ga} z^2 \tan^2\sigma \sin\psi_c \cos\sigma' + \frac{\delta_c}{2}\left(\theta + \frac{1}{2\delta_c}\frac{G+1}{Ga} z^2 \tan^2\sigma \cos\psi_c\right)^2 \cos\sigma' \quad (8)$$

Contours of constant s against z and $r\theta$ for an existing design (2) are given in Fig. 3. The trace of N in this plot is given by

$$\theta = -\frac{1}{2\delta_c}\frac{G+1}{Ga} z^2 \tan^2\sigma \cos\psi_c \quad . \quad (9)$$

and forms a camber line. The contours are ellipses sheared parallel to their minor axes so that their major axes conform to this camber line. Also plotted in Fig. 3 are the traces in the z, $r\theta$ plane of the wheel and pinion teeth tips, which under load may truncate the contact zone. The rotation of the radius to the pinion *tip* between the culminating (C) section and the z (D) section is

$$-\frac{z}{a}\tan\sigma$$



Dimensions of gear: $G = 3{\cdot}05$; $a = 0{\cdot}80$ in; $r = 0{\cdot}08$ in; $\delta_c = 0{\cdot}0013$ in; $\sigma = 10°$; $\psi_c = 30°$.

Fig. 3. *Contours of s against z and rθ*

so that the change in $r\theta$ ordinate is

$$-\frac{rz}{a}\tan\sigma$$

and the angle between the trace of the pinion tip and that of M is approximately

$$-\frac{r}{a}\sigma$$

Similarly the angle between the trace of the wheel tip and that of M is roughly

$$\frac{r\sigma}{Ga}$$

An examination of the higher order terms shows that in practical designs their effect is negligible, and the simple symmetrical form of equation (7) will generally suffice.

Small adjustments of the culminating position of S cause small changes in the effective value of R_H, the form of the trace of N and the point at which it is truncated by the wheel tip. The assessment of the value of such small adjustments (for example, the common design method of putting the culminating position of S at the pitch point (3)) depends upon elastic considerations which are beyond the scope of this paper. It appears certain, however, that no substantial improvement can be made on putting S at P at culmination.

It has been suggested (4) (5) that because R_H is roughly proportional to σ^{-2} (equation (4)) the torque capacity of circular arc gears is proportional to σ^{-2}. From equation (9), however, it can be seen that the value of z at which the camber-line runs off the tooth is roughly proportional to σ^{-1}. As this value effectively limits the length of the zone of contact and as it is necessary to increase the face width in proportion to σ^{-1} to preserve the necessary overlap, the capacity for fine helix angles will be nearly proportional to face width.

The value of z at camber-line run-off is also proportional to $\delta_c^{1/2}$, so that too small a value of δ_c reduces the length of the zone of contact. This implies that there is a finite optimum value of δ_c even if the difficulties of maintaining centre-distance do not exist (6).

PRODUCT OF PRINCIPAL RELATIVE RADII OF CURVATURE

It should be noted that while R_H is always a principal relative radius of curvature, R_T is not, the tangent in the transverse section making a small angle μ with the other plane of principal relative curvature, where

$$\sin\mu \simeq \sin\sigma \sin\psi_c \quad . \quad . \quad . \quad (10)$$

so that the inverse of the Gaussian relative curvature K, or product of principal relative radii of curvature, is approximately

$$R_H R_T \left\{1 + \sin^2\sigma \sin^2\psi_c \left(1 - \frac{R_T}{R_H}\right)\right\}^{-1} \quad (11)$$

For most practical cases a more than sufficiently accurate value is given by

$$\frac{1}{K}=\frac{(1+\cos^2\psi_c\tan^2\sigma)^{1/2}}{1+\sin^2\sigma\sin^2\psi_c}\frac{G}{G+1}\frac{r^2a}{\delta_c\sin^2\sigma\sin\psi_c} \quad (12)$$

This agrees substantially with expressions derived by Niemann (7) and Johnson (4), but not with that of Fedyakin and Chesnokov (3). Using the same approach as Fedyakin and Chesnokov, namely, the differential geometry of the first and second quadratic forms, the present writer obtains results in agreement with those given above.

In this application it is interesting to observe how much more elegant, economical and powerful this geometrical use of velocity and acceleration diagrams proves than the standard methods of differential geometry.

This being a purely geometrical problem, the identification of t with time is strictly irrelevant: it could be regarded simply as a geometrical parameter. Velocity and acceleration diagrams, however, are familiar and the use of the same principles for purely geometrical purposes, though obvious, is unfamiliar.

PROFILES CIRCULAR IN THE NORMAL SECTION

The effects of profiles circular in the normal section appear to be very slight, but favourable. The analysis proceeds in the same fashion, the elliptical traces of S and Q in the normal section being approximated by arcs of circles having three-point contact at the ends of the minor axes, as is commonly done with helical involute gears.

CONCLUSIONS

This approach to the rigid body geometry of contact of Wildhaber–Novikov gears suggests that the contact area should be banana-shaped rather than elliptical. As a result it is not possible to increase the torque capacity per unit face width indefinitely by reducing the helix angle. Another consequence is that reducing the difference in profile radii beyond a certain point will reduce the conformity of the teeth, apart from the difficulties of maintaining centre distance.

It is unlikely that any substantial improvement can be made by departing from the design condition in which the pinion centre of curvature coincides with the pitch point at culmination.

APPENDIX 1

EXACT EXPRESSIONS

Culmination at $\dot{\psi}_c = 0$.

$$\ddot{\delta}_c = -(G^2+G)\omega^2 a\sin\psi_c\left(1+\frac{\delta_c}{(G+1)a\sin\psi_c}\right)$$

$$\delta_c\dddot{\psi}_c = (G^3-G)\omega^3 a\sin\psi_c\left(1+\frac{\delta_c}{(G+1)a\sin\psi_c}\right)$$

Culmination with S at P

$$\dot{\psi}_c = \omega$$

$$\ddot{\delta}_c = -G(G+1)(G+2)\omega^2 a\cos\psi_c$$

$$\delta_c\ddot{\psi}_c = G(G+1)(G+2)\omega^3 a\sin\psi_c$$

$$\sin\mu = \frac{\sin\sigma\left(\sin\psi_c-\frac{r}{Ga}\tan\sigma\right)}{\left(1+\frac{r^2}{G^2a^2}\tan^2\sigma-\frac{2r}{Ga}\sin\psi_c\tan^2\sigma\right)}$$

Expressions not repeated are exact in the text.

APPENDIX 2

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