

# A new approach to modeling the magnetomechanical effect

D. C. Jiles and L. Li

Citation: J. Appl. Phys. 95, 7058 (2004); doi: 10.1063/1.1687200

View online: http://dx.doi.org/10.1063/1.1687200

View Table of Contents: http://jap.aip.org/resource/1/JAPIAU/v95/i11

Published by the American Institute of Physics.

#### **Related Articles**

Electronic origin of the negligible magnetostriction of an electric steel Fe1-xSix alloy: A density-functional study J. Appl. Phys. 111, 063911 (2012)

Modeling plastic deformation effect on magnetization in ferromagnetic materials J. Appl. Phys. 111, 063909 (2012)

Magnetic and calorimetric studies of magnetocaloric effect in La0.7-xPrxCa0.3MnO3 J. Appl. Phys. 111, 07D726 (2012)

Converse magnetoelectric effect dependence with CoFeB composition in ferromagnetic/piezoelectric composites J. Appl. Phys. 111, 07C725 (2012)

A model-assisted technique for characterization of in-plane magnetic anisotropy J. Appl. Phys. 111, 07E344 (2012)

## Additional information on J. Appl. Phys.

Journal Homepage: http://jap.aip.org/

Journal Information: http://jap.aip.org/about/about\_the\_journal Top downloads: http://jap.aip.org/features/most\_downloaded

Information for Authors: http://jap.aip.org/authors

### **ADVERTISEMENT**



# A new approach to modeling the magnetomechanical effect

#### D C Jiles

JOURNAL OF APPLIED PHYSICS

Center for Nondestructive Evaluation and Materials and Engineering Physics Program, Ames Laboratory, Iowa State University, Ames, Iowa 50011

#### L. Li

Materials and Engineering Physics Program, Ames Laboratory, Iowa State University, Ames, Iowa 50011

(Presented on 6 January 2004)

This paper reports on results taken to validate the extension to the theory of the magnetomechanical effect reported recently. This theory is based on a "law of approach" but the underlying equations have been generalized to include linear and nonlinear terms which are analogous to those in the well-known Rayleigh law of magnetization. It is shown that the generalized theory can be applied to materials with negative magnetostriction, such as nickel, and that the stress dependent model parameters can be determined from experimental measurements. It has been found that the results show improved agreement with experimental observation compared with the more restricted previous exposition of the model. © 2004 American Institute of Physics.

[DOI: 10.1063/1.1687200]

#### INTRODUCTION

Development of an accurate model description of the magnetomechanical effect becomes increasingly important in the application of stress sensors using magnetostrictive materials and applications of magnetic measurements to evaluation of stress in materials. One of the major challenges today is to provide reliable models for nonlinear and hysteretic effects in materials. The fundamental theory of the magnetomechanical effect is based on the "law of approach." This has recently been refined by including a linear term in the model equation, which relates to reversible changes in magnetization with stress, as described in a previous paper.<sup>2</sup> Although measurement of magnetomechanical effects in materials with positive magnetostriction had been performed before, there are few experimental data on materials with negative magnetostriction such as nickel that were taken to examine the relationship between the magnetic induction and stress under constant magnetic field.

Selection of model parameters for different materials with positive or negative magnetostriction can be difficult.<sup>3</sup> In order to verify the validity of the generalized model, a series of experiments have been performed on nickel samples which have different magnetostrictive properties. Both experimental and theoretical investigations on nickel samples have been carried out as part of this study in order to determine the relationship between model parameters and material properties. The experimental data have been used to evaluate the generality of two aspects of the new model theory, namely the model parameter determination and magnetomechanical effect simulation.

# MODEL THEORIES OF MAGNETOMECHANICAL EFFECTS

As shown previously, the effect of stress on the magnetization can be approximated as an effective field described by

$$H_{\sigma} = \frac{3}{2} \frac{\sigma}{\mu_0} \left( \frac{d\lambda}{dM} \right) (\cos^2 \theta - \upsilon \sin^2 \theta), \tag{1}$$

where  $\sigma$  is the stress which is negative for compression and positive for tension,  $\lambda$  is the magnetostriction, M is the magnetization of the material,  $\mu_0$  is the permeability of free space,  $\theta$  is the angle between the axis of the applied stress  $\sigma$  and the axis of the magnetic field H, and  $\nu$  is Poisson's ratio. Then the total effective field  $H_{\rm eff}$  including the stress contribution is

$$H_{\text{eff}} = H + \alpha M + H_{\sigma}, \tag{2}$$

where  $\alpha$  is a dimensionless mean field parameter representing interdomain coupling. In the isotropic limit, supposing that applied stress and magnetic field are applied coaxially, the stress dependence of the anhysteretic magnetization curve can be determined based on the generalized Langevin function as

$$M_{\rm an} = M_S \left[ \coth \left( \frac{H + \alpha M + H_{\sigma}}{a} \right) - \frac{a}{H + \alpha M + H_{\sigma}} \right],$$
 (3)

where  $a = k_B T/\mu_0 m$  in which  $k_B$  is Boltzmann's constant, T is the temperature, and m is the magnetic moment of a typical pseudodomain. The choice of a function for the anhysteretic magnetization must depend on the details of the particular material chosen. The model using the above anhysteretic magnetization equation, which applies to isotropic materials, works satisfactorily for soft magnetic materials such as iron and nickel. However other analytic anhysteretic functions are possible.<sup>4</sup>

Based on the irreversible, reversible, and anhysteretic components of magnetization, the differential equation for the total magnetization is

$$\frac{dM}{d\sigma} = \frac{1}{\varepsilon^2} (\sigma \pm \eta E) (1 - c) (M_{\text{an}} - M_{\text{irr}}) + c \frac{dM_{\text{an}}}{d\sigma}, \quad (4)$$

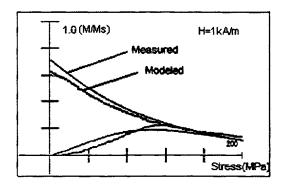


FIG. 1. Magnetic induction B vs stress curve for as-received nickel sample at a field of 1 kA/m along the initial magnetization curve. The nickel sample was subjected to 200 MPa first and then reduced to 0 MPa.  $M_S$  = 530 000 A/m, a = 6500 A/m, k = 2700 A/m,  $\alpha$  = 0.036, k = 0.1.

where E is the relevant elastic modulus, c is the reversibility coefficient,  $M_{irr}$  presents the irreversible component of magnetization,  $\varepsilon$  is the rate of approach parameter which has been defined previously, and  $\eta$  is a coefficient which represents the reversible change in the magnetization with the action of a stress. In this work, Eq. (3) has been used for modeling of anhysteretic magnetization and Eq. (4) has been used for modeling the stress dependence of magnetization.

#### **EXPERIMENTAL PROCEDURES**

Hysteresis loop and magnetization versus stress measurements were made on samples under various applied stresses within the elastic limit using a servohydraulic Instron mechanical testing system. The samples used for these measurements were nickel rods 140 mm in length and 8 mm diameter. All the measurements were conducted within the elastic limit in order to ensure a completely reversible mechanical process. An initial measurement was carried out to obtain the stress-strain curve for the material and for the purpose of obtaining preliminary hysteresis data under both tensile and compressive loading conditions. During the measurement the sample was magnetized using a solenoid. The magnetic field H was measured using a Hall sensor mounted on the sample surface. The output of a search coil wound on the sample was integrated to obtain the hysteresis loop. Magnetostriction was measured using strain gauges mounted on

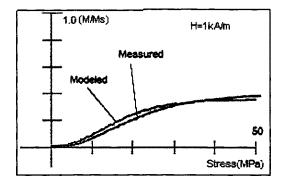


FIG. 2. Magnetic induction B vs stress curve for annealed nickel sample at a field of 1 kA/m along the initial magnetization curve. The nickel sample was subjected to 50 MPa first and then reduced to 0 MPa.  $M_S$  = 530 000 A/m, a = 400 A/m, k = 600 A/m,  $\alpha$  = 0.0009, c = 0.4.

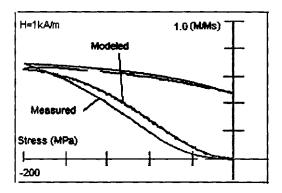


FIG. 3. Magnetic induction B vs stress curve for as-received nickel sample at a field of 1 kA/m along the initial magnetization curve. The nickel sample was subjected to  $-200 \,\mathrm{MPa}$  first and then reduced to  $0 \,\mathrm{MPa}$ .  $M_S = 530\,000 \,\mathrm{A/m}, \ a = 6500 \,\mathrm{A/m}, \ k = 2700 \,\mathrm{A/m}, \ \alpha = 0.036, \ c = 0.1$ .

the sample surface. The stress-strain curve of the sample was also measured to determine the mechanical properties such as the Young's modulus for use in the simulations.

#### **RESULTS AND DISCUSSION**

The experimental data include hysteresis parameters such as the coercivity, remanence, initial permeability, and maximum differential permeability. Stress-induced changes in magnetization were calculated using the improved model equation. The theoretical parameters were determined from measured hysteresis loops and magnetostriction curves under various applied stresses using the previously published inversion algorithm.<sup>6</sup> Hysteresis loops were modeled under zero stress levels using different sets of parameters for both asreceived and annealed nickel samples.

Good agreement was observed between the experimental and modeled hysteresis loops in the low field regime (that is at field strengths below the coercive field). But the modeled hysteresis curves also showed deviations from the experimental results in particular at the knee of the hysteresis loops. A possible explanation is that at the knee of the hysteresis loop the magnetization reversal processes involve mainly reversible rotation of domain magnetization towards the applied field, as indicated by the small hysteresis in high field regime of the experimental loops.<sup>7</sup>

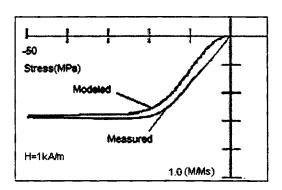


FIG. 4. Magnetic induction *B* vs stress curve for annealed nickel sample at a field of 1 kA/m along the initial magnetization curve. The nickel sample was subjected to -50 MPa first and then reduced to 0 MPa.  $M_S = 530\,000$  A/m, a = 400 A/m, k = 600 A/m,  $\alpha = 0.0009$ , c = 0.4.

Figures 1 and 2 show the measured and modeled comparison for the as-received and annealed nickel under tensile stress. For the initial part of the magnetization curve, the modeled results show good agreement with the experimental observations both for as-received and annealed nickel samples. But along the reverse part of the curve of annealed nickel the model calculations did not agree so well with the measured results.

Similar results for the compression test on the asreceived and annealed nickel samples were obtained following the same method in Figs. 3 and 4. It can be seen that better agreement between theory and experiment was obtained.

#### **CONCLUSIONS**

A modification of the "law of approach" for the magnetomechanical effect was achieved by taking into account both linear and nonlinear terms. In order to verify the generality of the improved model, a series of experiments were made on two different nickel samples. The new model equation gave results that were better agreement both qualitatively and quantitatively with observations. However, the model did not reproduce the experimental results accurately in all of the situations examined and further refinement of the model may still be needed.

#### **ACKNOWLEDGMENTS**

This work was supported by the NSF Industry/ University Cooperative Research Program at the Center for Nondestructive Evaluation, Iowa State University.

<sup>1</sup>D. C. Jiles, J. Phys. D **28**, 1537 (1995).

<sup>&</sup>lt;sup>2</sup>L. Li and D. C. Jiles, IEEE Trans. Magn. 39, 3037 (2003).

<sup>&</sup>lt;sup>3</sup>P. R. Wilson, J. N. Ross, and A. D. Brown, IEEE Trans. Magn. **37**, 989 (2001).

<sup>&</sup>lt;sup>4</sup>D. C. Jiles, S. J. Lee, J. Kenkel, and K. Metlov, Appl. Phys. Lett. **77**, 1029 (2000).

<sup>&</sup>lt;sup>5</sup>W. F. Brown, Phys. Rev. B **75**, 541 (1949).

<sup>&</sup>lt;sup>6</sup>D. C. Jiles, J. B. Thoelke, and M. K. Devine, IEEE Trans. Magn. **28**, 27 (1992).

<sup>&</sup>lt;sup>7</sup>C. C. H. Lo, S. J. Lee, L. Li, L. C. Kerdus, and D. C. Jiles, IEEE Trans. Magn. 38, 2418 (2002).