The Manchester School Vol 00 No. 00 00–00 Month 2016
doi: 10.1111/manc.12155

THE STORM BEFORE THE CALM? ADVERSE EFFECTS OF TACKLING ORGANIZED CRIME *

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Policies targeted at high-crime neighbourhoods may have unintended consequences in the presence of organized crime. Whilst they reduce the incentive to commit crime at the margin, those who still choose to join the criminal organization have relatively high criminal propensities. Large organizations take advantage of this, substituting away from membership size towards increased individual criminal activity. Aggregate crime may rise. However, as more would-be recruits move into the formal labour market, falling revenue causes a reversal of this effect. Thereafter, the policy reduces both size and individual activity simultaneously.

1 Introduction

Over recent years, numerous innovative policies have been suggested to increase the opportunity cost of engaging in crime. Under normal circumstances, these discourage illegal activity and cause crime levels to fall. In the presence of organized crime, however, the outcome is less certain. Whilst they appear to be very successful in some cases, the same policies can lead to protracted escalations of criminal activity in others. I present a new framework that captures both scenarios. The subsequent analysis leads to a novel prediction: policies backfire only when organizations have sufficiently large membership bases. Over time, as the effects of a policy are felt more keenly and the size of the organization diminishes, this implies that the crime rate may get worse before it gets better; the storm before the calm.

Examples of backfiring policies abound. During the 'War on Drugs' in the 1980s, arrests for heroin and cocaine trafficking in the U.S. rose dramatically (Lee, 1993). Successful conviction of traffickers increased (from 85 per cent in 1985 to 92 per cent in 1989) and they were incarcerated for longer periods of time (up from 61 months to 76 months on average). Over the same period, the availability of both drugs increased, whilst their prices

* Manuscript received 29.10.14; final version received 4.3.16.
† I would like to thanks Leonardo Felli, Andrea Galeotti, Gilat Levy, Giacomo De Luca, Giovanni Mastrobuoni, Ákos Valentinyi, Alwyn Young, the editor and the two anonymous referees for their valuable insights. Equally useful have been comments by seminar participants at Cardiff, EEA-ESEM, KBTU, LSE, Stockholm and the Workshop on Organized Crime at York. Time spent at the Study Center in Gersenzee is also acknowledged.

remained stable. Various attempts have been made to explain this outcome based on the unique features of the drugs market. Consumers may have changed their purchasing behavior, buying larger quantities less often (Lee, 1993). The market structure itself might have changed, depending upon whether distributors or retailers are targeted (Poret, 2002). Competition could have increased, lowering the price (Mansour *et al.*, 2006; Poret and Téjédo, 2006). If backfiring policies were limited to this case, we might have little more to say. However, other examples exist.

In 1990, the Philippines customs authority clamped down on common forms of duty avoidance (Yang, 2008). Evidence suggests that not only did other forms of duty avoidance increase, but that the problem of avoidance may have got worse. Whilst the government raised an additional \$24.6 million from additional inspections, it is estimated that displacement towards other forms of avoidance cost an additional \$33.3 million in lost duties over a fifteen month period.

In response to the London riots of 2011, the Metropolitan Police developed a strategy of arresting known gang members in several neighbourhoods in London. One year later, a survey asked residents of these neighbourhoods to assess its impact (Centre for Social Justice, 2012). They stated that gang violence had increased. Despite the arrests, those who remained at liberty became more violent. A more recent report suggests that gang violence has subsequently declined (Home Office, 2013). This indicates that the backfiring effects may be temporary. Whilst riot-related offences may have declined, there is also evidence that activity may have been displaced towards other types of crime (Bell *et al.*, 2014).

In order to explain these phenomena, I develop a simple dynamic model of organized crime. An infinitely-lived criminal organization generates profit each period by recruiting individuals from its territory to engage in criminal activity on its behalf. Individuals vary in their willingness to engage in crime, which is private information, reflecting different moral values or honesty. Each individual consequently suffer different levels of disutility from working for the organization.

In the baseline model, all individuals are offered identical contracts, specifying a wage and a level of criminal activity for the current period. If they accept, they become members of the organization. Otherwise, they seek work in the formal labour market, representing the opportunity cost of crime. The contract acts as a simple screening device. Those most willing to commit crime prefer to join the organization. Those least willing enter the formal labour market.

I then subject the organization to a new policy that gradually improves individuals' formal labour market opportunities. For example, the neighbourhoods children could be offered intensive pre-school classes (as in the Perry Preschool Project in Michigan (Heckman *et al.*, 2010)) or its students could be given financial rewards for strong exam performances (as in the

Paper Project in Chicago (Fryer, 2011)). Alternatively, considering the cost of engaging crime more broadly, the policy could represent the gradual increase in mandatory minimum sentences across a broad range of criminal activities. As a result of the policy increasing the opportunity cost of engaging in crime, the marginal cost to the organization of hiring an additional recruit increases. It optimizes by offering a contract that fewer individuals would be willing to accept. Its size unambiguously falls. The impact on the criminal activity it requires of its members, however, is less clear cut. On the one hand, fewer members mean that increases in individual activity translate into a smaller increase in revenue (a revenue effect). On the other, those who are still prepared to join the organization are, on average, more willing to commit crime. The compensation that the organization needs to provide for greater activity has also fallen (a cost effect). Since both the marginal revenue product and marginal cost of individual activity have declined, the overall effect on its profitability is ambiguous.

When the organization is large, size and individual activity are substitutes. It recruits individuals with very low willingness to commit crime. As its size declines, the organization substantially increases the amount of activity it requires of each member (the cost effect dominates the revenue effect). The policy may appear to have backfired, as aggregate crime may increase or each individual crime becomes more intense.

As its size continues to fall, however, size and individual activity become complements. The organization begins to lose individuals who are relatively willing to commit crime. Whilst it still loses revenue through fewer members, it is no longer able to compensate by increasing individual activity (the revenue effect dominates the cost effect). The policy now proves highly effective, as both size and individual activity decline. Aggregate crime falls rapidly.

Of course, assuming identical contracts is not entirely plausible. The results are, however, broadly robust under two forms of heterogeneity. I first allow the organization to offer a menu of contracts that cause each member to reveal their willingness to commit crime. This gives rise to a hierarchical structure in which some individuals engage in higher levels of criminal activity in exchange for higher wages. When labour market conditions improve, the organization again chooses to reduce its size. This strengthens the degree of revenue complementarity between size and activity at the individual level. Initially, most members find that the organization asks them to engage in greater levels of activity as before. We still have a storm. As size falls, however, members increasingly find that their activity is reduced (although this happens at different points in time for each individual). Eventually, we have a calm. Secondly, I allow for overlapping generations of criminals. Once again, the intuition of the baseline model holds.

The remainder of the paper proceeds as follows. The next section relates the contribution to the literature. Section 3 outlines a simple model of organized crime. Section 4 discusses the equilibrium, highlighting the link between organization size and whether size and activity are complements or substitutes. The following two sections identify the impact of a gradual improvement in formal labour market conditions on the optimal size and activity. Section 7 briefly discusses the impact of introducing two different forms of contract heterogeneity. Section 8 discusses some extensions to the basic model and concludes. All proofs are provided in the appendices.

2 Related Literature

The economic analysis of crime began with the advent of the rational offender framework (Becker, 1968). In contrast to the established theories of the day, Becker suggested that individuals decide how much crime to commit by comparing the costs and benefits. This allowed the standard tools of microeconomics to be applied to combating crime (c.f. Polinsky and Shavell, 2000). As in the current work, the focus has been on raising the expected cost to an individual of committing crime, either through harsher punishment or, more broadly, by improving labour market opportunities.

Building on Becker's work, three approaches have been employed to investigate a more complicated environment with organized crime. Those considering its origins view a criminal organization as a pseudo-state (for example Skaperdas and Syropoulos, 1997; Anderson and Bandiera, 2005; Dixit, 2007). In the presence of weak law enforcement, organized criminal groups set up to supply private protection (often sparked by an increase in the value of assets needing protection, see Buonanno *et al.*, 2015). This literature emphasizes the localized nature of a lot of organized crime. Within their territory, they develop a monopoly over violence and, by extension, all illegal activity. In fact, many argue that this is the defining characteristic of organized crime (c.f. Gambetta, 1996). My model borrows from this, assuming that individuals within the organization's territory can only commit crime by becoming members.

Those interested in information transmission and learning within criminal groups have tended to view them as networks (Calvo-Armengol and Zenou, 2004; Baccara and Bar-Isaac, 2008; Ballester *et al.*, 2010). Vis-à-vis individual crime, this literature suggests that organized crime may be able to take advantage of network externalities. Individuals can share knowledge (Baccara and Bar-Isaac, 2008), and the asymmetric information problems inherent to trading in illegal markets can be overcome (Cook *et al.*, 2007). These gains are offset by individual competition between criminals, creating diminishing returns to crime within the organization's territory (Ballester *et al.*, 2010). In Section 7.1, I motivate the existence of a storm before the calm when contracts cause individuals to reveal their willingness to commit crime with reference to these network and congestion effects.

Finally, investigations of established criminal organizations tend to view them as profit-maximizing firms. I adopt this approach. Similar models have been used to investigate various aspects of organized crime's behavior. For example, Kugler *et al*. (2005) consider the impact of punishment on organized crime's willingness to bribe officials. Like me, they find that policy could backfire. As the official severity of punishment increases, the organization has a greater incentive to corrupt police officers. In turn, this lowers the actual expected punishment, leading to more crime. Whilst the organization faces a similar profit maximization problem to the current work (it employs risk neutral members to commit crime in exchange for a wage that compensates them for opportunity cost and expected punishment), they assume that each member only commits one crime. Moreover, they assume all individuals are identical. My results derive directly from relaxing these assumptions. Allowing the organization to vary both the intensive and extensive margins of employment opens up a new trade off that can cause policy to backfire.

Garoupa (2007) extends the optimal law enforcement literature to incorporate the presence of a profit-maximizing criminal organization. He finds that, if smaller organizations make sharing of information easier, then the most severe punishments may not be optimal. Instead, it is better to encourage the organization to employ larger numbers of individuals in order to increase the probability of mistakes being made. As with my model, the size of the organization can vary due to individual members suffering different levels of disutility when committing crime. However, once again, the intensive margin of activity is not modelled. Whilst Garoupa finds that severe policy can lead to higher crime, my results suggest that this may be a temporary phenomenon.

In line with much of this literature, my model assumes that individuals can either commit crime within the organization or work in the formal labour market. Of course, it may also be possible for individuals to commit crime by themselves (as in Chang *et al*., 2005). This tends to increase total crime, as individuals behave competitively, whereas a criminal organization acts like a monopolist (Garoupa, 2000). As in my model, individuals' decisions are shown to depend upon the type of contract on offer. If all members receive the same payoff (as in my baseline case) then those who gain the most from committing crime prefer to work alone. Increasing the severity of punishment tends to reduce crime, by causing these individuals to seek the protection of the criminal organization. If payoffs are commensurate to the revenue that each individual generates (as in my Section 7.1), then those with the highest willingness to commit crime become the first to join up in equilibrium.

Two recent contributions discuss substitution between members and individual activity, within the context of a utility-maximizing gang (Poutvaara and Priks, 2009, 2011). In their analysis, the gang's leader enjoys being head of a large, violent group. Following a change in police tactics (2009) or unemployment (2011), they show that the gang leader may reduce membership in favour of more violent activity. The relative price of gang size changes, and the leader maximizes utility by substituting towards violence. The intuition in my contribution is similar. However, I identify two effects (revenue and cost) which jointly determine the relationship between members and their individual effort. This enables me to discuss profit complements and substitutes in the same framework, and also to discern a link between the size of the organization and how it views the two inputs. These insights generate a new prediction about the reaction to policy. Whilst membership always falls, individual (and, potentially, aggregate) criminal activity will first increase, then stabilize, before rapidly declining.

Of course, trading off between the intensive margin (individual activity) and extensive margin (number of workers) is not new to labour economics. As such, the mechanisms discussed herein could equally apply to the formal labour market. In this more general context, there is evidence of substitution from the literature on work sharing (beginning with Calmfors, 1985 and Booth and Schiantarelli, 1987). These contributions assume that a firm's output depends upon total hours worked (individual hours multiplied by the number of workers). My approach nests this assumption. Empirical estimates for the elasticity of substitution between size and individual hours range between -0.1 and -1.7 (for a survey, see Freeman, 2000). The upper estimates are consistent with the view that aggregate activity increases when the number of workers falls. Criminal organizations are, however, different to firms operating in the formal economy. Whilst they share an incentive to maximize profit, the labour markets that they participate in are very different. Recruiting criminals is fraught with additional risks. Recruits could be undercover police officers, or could be targeted to become police informers. They could equally be spies for other organizations. To reduce these risks, organizations tend to recruit for well-defined pools of individuals: for example families; ethnicities; or geographic neighbourhoods (c.f. Jankowski, 1991; Polo, 1997; Paoli, 2003). Not only are individuals in these pools likely already known to the organization, but so are their families. This makes it is easier to credibly threaten recruits to prevent them from going against the interests of the group (Baccara and Bar-Isaac, 2008). As such, criminal organizations tend to act as the monopsonist employer of criminals within their labour pool. Nevertheless, the estimates of the elasticity of substitution between size and hours do suggest the possibility that criminal organizations too may substitute, and that this could inadvertently lead to greater amounts of crime.

3 A Model of Organized Crime

An infinitely-lived criminal organization recruits members from a neighbourhood with a population of unit mass, who are risk-neutral and supply their labour competitively. Every period, t, it offers an identical contract to everyone in the neighbourhood, comprising of a wage, g_t and a level of individual criminal activity a_t. Although this contract is unrealistically simple, it serves to illustrate the intuition underpinning the results. More complicated contract structures are then considered in Section 7.

Individuals vary in their willingness to engage in crime, denoted by $\sigma \sim \text{Exp}(\lambda)$, reflecting differences in moral values or honesty. An individual's effort cost of criminal activity a_t is given by a_t/σ. Those with higher σ suffer less disutility from committing crime. Willingness to commit crime is not observed by the organization (there is adverse selection).

Working for a criminal organization may bring an individual to the attention of the police. With instantaneous probability $p(a_t)$, a member of the organization is caught and suffers punishment $-f(a_t)$. Both the probability of being caught and the punishment are assumed to be increasing and weakly convex in the amount of criminal activity the member engages in. Since individuals are risk neutral, call the expected punishment $\phi(a_t) \equiv p(a_t)f(a_t)$. Following convention, individuals are still assumed to receive the benefit of their crime—the wage from the organization—irrespective of whether they are punished (Becker, 1968; Garoupa, 1997, 2000).

The expected payoff to an individual, with willingness to commit crime σ, from accepting the organization's period t contract is thus:

$$g_t - \frac{a_t}{\sigma} - \phi(a_t) \tag{1}$$

Alternatively, the individual could choose to seek work in the formal labour market. There they receive an expected wage, w_t, which is independent of their willingness to commit crime. The assumption that all individuals have identical formal labour market opportunities is common to many papers in this literature (c.f. Chang *et al.*, 2005; Garoupa, 2000, 2007). It may reflect the fact that many workers from crime-ridden neighbourhoods perform low-skilled jobs, where variation in wages is small (Levitt and Venkatesh, 2000) or simply that formal labour market work and criminal activity require different skills (Ballester *et al.*, 2010; Carvalho and Soares, 2016). Labour market opportunities are also likely to be unrelated to individuals' honesty or moral values, reflected in their willingness to engage in crime, which is the only source of heterogeneity in my model. In the following sections, I introduce dynamics by considering a gradual improvement in formal labour market opportunities. As such, I will assume that $w_t > w_{t-1}$ for all t.

Every period, the organization offers a new contract which individuals are free to accept or reject. This implicitly assumes that individuals are free to leave the organized crime whenever they like or, at least, when the organization no longer chooses to employ them. Of course this need not be the case, as organizations like the Sicilian Mafia are long-term commitments

(Gambetta, 1996; Paoli, 2003). In Section 7.2, I will relax this assumption, and discuss its impact upon the results. If contracts can be made contingent on w_t, however, the analysis of the next few sections holds. The organization recruits M_t members in order to generate revenue $R(M_t, a_t)$.[1] Revenue is subject to positive but diminishing marginal returns to both inputs. For simplicity, I also assume that it has constant returns to scale. This is useful insofar as it helps ensure that there is only one storm before the calm. Without it, the introduction of the policy could generate a series of false starts. Individual activity would eventually start to decline, only to rise again briefly, before continuing to fall. This would complicate the mathematical analysis, without adding much to our economic understanding of the problem. However, similar results can be derived without it.

Size and activity are revenue complements. I assume that: (i) the marginal revenue product of size, MRP_{Mt}, is increasing in a_t ($\partial \text{MRP}_{Mt}/\partial a_t > 0$); and (ii) it diminishes more rapidly as M_t increases ($\partial^2 \text{MRP}_{Mt}/\partial M_t \partial a_t < 0$). Both are satisfied by, for example, a CRTS Cobb-Douglas revenue function. With more active members, each increase in organization size leads to a larger rise in aggregate criminal activity, suggestive of a greater increase in revenue. However, with larger increases in aggregate activity, we would also expect to see a more rapidly diminishing marginal product. The first assumption is necessary for the results. Without it, as we shall see, the organization would never view size and activity as complements. The second simplifies the analysis, again guaranteeing that the storm before the calm is unique.

The organization chooses its contract in every period to maximize profit, given by its revenue minus its total wage bill:[2]

[1]The black box nature of revenue (as opposed to production) is purely for notational ease. One can think about it as an indirect revenue function: the one resulting from the optimal allocation of inputs across the wide range of activities the gang engages in. Kugler *et al.* (2005) consider a more structured approach, decomposing revenue into the number of crimes committed, and the booty collected from each crime.

[2]Technically, the organization's profit function constitutes an infinite-horizon Bellman equation as:

$$V(w_t, \phi) = \max_{g_t, a_t} [R(M_t, a_t) - g_t M_t + \beta V(w_{t+1}, \phi)]$$

where $\beta \in (0, 1)$ is the organization's discount factor. However, as there is no moral hazard and individuals supply labour competitively, the organization does not face any intertemporal trade-offs. Its profit maximization problem can be rewritten as:

$$V(w_t, \phi) = \max_{g_t, a_t} [R(M_t, a_t) - g_t M_t] + \beta V(w_{t+1}, \phi)$$

Solving the dynamic problem thus reduces to solving a sequence of static problems in each period t. Note that, in this case, it does not matter whether the increase in wages was expected or not. The organization simply adjusts its contract in each period to take account of current conditions.

$$\Pi(w_t,\phi)=\max_{g_t,a_t}\,[R(M_t,a_t)-g_tM_t]. \tag{2}$$

Summarizing, the timing is as follows. Each period, a new formal labour market wage is realized, and becomes common knowledge. The criminal organization then announces its contract. Next, individuals choose whether to join the organization or to work in the formal sector. Crime then takes place and wages are paid. Finally, members of the organization may be arrested and punished.

4 Equilibrium

This framework yields a sequence of (semi-separating) perfect Bayesian equilibria. Proceeding by backwards induction, first consider the choice of the neighbourhood's individuals. Given the contract on offer, (g_t, a_t), an individual will join the organization if and only if:

$$\begin{aligned} & g_t-\frac{a_t}{\sigma}-\phi(a_t)\geq w_t \\ & \Longleftrightarrow \sigma\geq\hat{\sigma}(g_t,a_t,w_t,\phi)\equiv\frac{a_t}{g_t-w_t-\phi(a_t)} \end{aligned} \tag{3}$$

The payoff from joining the organization is increasing in an individual's willingness to commit crime (the left-hand side of (3)). All members of the organization receive the same wage, engage in the same amount of criminal activity, and face the same expected punishment. However, those with higher willingness to commit crime suffer less disutility from criminal activity. In the formal labour market, on the other hand, everyone receives w_t. So there exists a unique *marginal individual*, with willingness to commit crime $\hat{\sigma}_t\equiv\hat{\sigma}(g_t,a_t,w_t,\phi)$ defined above, such that only those with willingness to commit crime exceeding that of the marginal individual join. Whilst the marginal individual is indifferent between either form of employment, all other members of the organization receive a positive surplus. Individuals with willingness to commit crime below $\hat{\sigma}_t$ strictly prefer working in the formal labour market. Since crime is always prohibitively costly for those with willingness to commit crime close to zero, $\hat{\sigma}_t>0$ in every period. The contract acts as a very simple screening device. The required level of individual activity provides a hurdle which only those with sufficiently high willingness to commit crime are willing to overcome.

We now turn to the organization's choice of optimal contract. It proves helpful to begin by rephrasing the profit maximization problem slightly. For a given criminal wage, g_t, individual activity, a_t, and formal labour market wage, w_t, the expected size of the organization is:

$$M(g_t, a_t, w_t, \phi) = e^{-\lambda\hat{\sigma}(g_t, a_t, w_t, \phi)}. \tag{4}$$

Size has a one-to-one relationship with g_t. Rather than choosing the contract, (4) suggests a different approach: the organization chooses its size, M_t, and individual activity level, a_t. Knowing how individuals respond in equilibrium, it then rearranges (4) to identify the willingness to commit crime it needs to make indifferent between joining and the formal labour market:

$$\hat{\sigma}(M_t) = -\frac{\ln M_t}{\lambda}$$

The organization then computes the wage needed to achieve this indifference, given its chosen individual activity level, the resulting expected punishment and w_t. In particular, it needs to pay:

$$\begin{aligned} & g(M_t, a_t, w_t, \phi) - \frac{a_t}{\hat{\sigma}(M_t)} - \phi(a) = w_t \\ \iff & g(M_t, a_t, w_t, \phi) = w_t + \phi(a_t) + \frac{a_t}{\hat{\sigma}(M_t)} \end{aligned} \tag{5}$$

The wage it offers just compensates the marginal individual for the opportunity cost of joining (w_t), the expected punishment for their crimes ($\phi(a_t)$) and the disutility they suffer from the activity that the organization requires of them. Infra-marginal members receive a positive surplus from joining, given by:

$$g(M_t, a_t, w_t, \phi) - \frac{a_t}{\sigma} - \phi(a_t) - w_t = \frac{a_t}{\hat{\sigma}(M_t)} - \frac{a_t}{\sigma} \geq 0$$

Members with higher willingness to commit crime always receive a greater surplus from the contract on offer than members with lower willingness to commit crime.

The organization's period t profit maximization problem can thus be rewritten as:

$$\max_{M_t, a_t} [R(M_t, a_t) - g(M_t, a_t, w_t, \phi) M_t] \tag{6}$$

where $g(M_t, a_t, w_t, \phi)$ is defined by (5). Any unconstrained solution must satisfy the following:

Proposition 1: (First-order conditions):

The profit maximization problem (6) has a solution, $(M^*, a^*) > 0$, given by:

$$\begin{aligned}\mathrm{MRP}_M(M_t^*, a_t^*) &= g(M_t^*, a_t^*, w_t, \phi) + M_t^* \frac{\partial g_t}{\partial M_t^*} \\ &= g(M_t^*, a_t^*, w_t, \phi) + \frac{a_t^*}{\lambda \hat{\sigma}(M_t^*)^2}\end{aligned} \tag{7}$$

$$\begin{aligned}\mathrm{MRP}_a(M_t^*, a_t^*) &= M_t^* \frac{\partial g_t}{\partial a_t^*} \\ &= M_t^* \phi'(a_t^*) + \frac{M_t^*}{\hat{\sigma}(M_t^*)}\end{aligned} \tag{8}$$

Equation 7 describes the first-order condition for size. Given individual activity, a_t^*, new members increase the organization's revenue by $\mathrm{MRP}_M(M_t^*, a_t^*)$. However, they must be paid $g(M_t^*, a_t^*, w_t, \phi)$. Moreover, attracting new members involves recruiting those with lower willingness to commit crime than the current marginal individual. In order to compensate for the new recruits' higher cost of activity, the organization must increase its wage (in (5) the willingness to commit crime of the new marginal individual is lower). This involves offering higher compensation to the infra-marginal individuals too. The marginal cost of members exceeds $g(M_t^*, a_t^*, w_t, \phi)$.

Equation 8 gives the first-order condition for activity. Increasing the level of individual activity enables the organization to generate more revenue. Each member commits more crime, and total revenue increases by $\mathrm{MRP}_a(M_t^*, a_t^*)$. However, in order to ensure that no member chooses to switch to working in the formal labour market, the organization must compensate them for the higher effort cost that they incur. The member requiring the greatest payment is the marginal individual. From (5) the organization must raise its wage by $1/\hat{\sigma}(M_t^*)$. However, all M^* members receive this pay rise. The marginal cost of activity is thus $M_t^*/\hat{\sigma}(M_t^*)$. Each member also takes on a greater risk of a more severe punishment, $\phi'(a_t^*)$. Once again, the organization must increase the wage it offers to compensate them for this, increasing the total wage bill by $M_t^* \phi'(a_t^*)$.

The profit-maximizing level of individual activity and organization size can be described as the point of intersection between two **restricted demand curves**, $\tilde{M}(a_t)$ and $\tilde{a}(M_t)$. Each curve gives the optimal choice of one input, for any given quantity of the other. They are implicitly defined directly from the first-order conditions, as follows:

$$\mathrm{MRP}_M\left[\tilde{M}(a_t), a_t\right] = g\left[\tilde{M}(a_t), a_t, w_t, \phi\right] + \frac{a_t}{\lambda \hat{\sigma}\left[\tilde{M}(a_t)\right]^2} \tag{9}$$

$$\mathrm{MRP}_a[M_t, \tilde{a}(M_t)] = M_t \phi'[\tilde{a}(M_t)] + \frac{M_t}{\hat{\sigma}(M_t)} \tag{10}$$

For each a_t, the solution to equation 9 states the organization's profit-maximizing size. Equation 10 has a similar interpretation for activity. Profits are maximized when both equations are satisfied, as size maximizes profit given individual activity and activity maximizes profit given the organization's size. The restricted demand curves provide a very intuitive way to assess the endogenous effects of changes in the policy environment on the organization's optimal choice of inputs. Also, by substituting $\tilde{a}(M_t)$ for a_t in (9), we can express the organization's profit-maximization problem in terms of a single input, M_t. Understanding the shape of these curves in more detail is hence our next task.

Consider how an increase in size impacts upon the marginal profitability of activity, given by (8), at the profit-maximizing combination of inputs:

$$\frac{\partial}{\partial M_t}\left(\frac{\partial \Pi_t}{\partial a_t}\right)=\frac{\mathrm{MRP}_{at}}{M_t}\left\{\underbrace{\eta(M_t,a_t)}_{\text{Revenue Effect}}-\underbrace{[1+\varepsilon(M_t,a_t)]}_{\text{Cost Effect}}\right\} \tag{11}$$

Whether individual activity becomes more profitable depends upon the sign of the term inside the parentheses.[3] The first element, $\eta(M_t,a_t)=(M_t/\mathrm{MRP}_{at})(\partial\mathrm{MRP}_{at}/\partial M_t)$, is an elasticity and states the percentage increase in the marginal revenue product of activity following a one per-cent rise in size. It represents a **revenue effect**. With more members, a small rise in each member's individual activity leads to larger growth in aggregate crime and hence in the organization's revenue. Size and activity are revenue complements. The second term is the percentage increase in the marginal cost of activity. It represents a **cost effect**. Given constant wages, a one per-cent increase in size causes a one per-cent increase in the organization's wage bill. When membership expands, however, the new marginal individual has a lower willingness to commit crime. $\varepsilon(M_t,a_t)=(M_t/g_t)(\partial g_t/\partial M_t)$ represents the percentage increase in wages required to compensate them for one percent increase individual activity. The marginal cost of individual activity also increases in size.

Activity only becomes more profitable following an increase in size if the revenue effect dominates the cost effect. As alluded to in the introduction, this has important implications for how the organization responds to a change in the policy environment. Fortunately, we can easily distinguish between the two cases:

Proposition 2: (Complements vs. substitutes):

There exists a unique $\overline{M}\geq 0$ such that the revenue effect dominates the cost effect if and only if $M_t<\overline{M}$.

[3]These effects are, of course, entirely symmetric. (11) also describes $\partial/\partial a_t(\partial\Pi_t/\partial M_t)$.

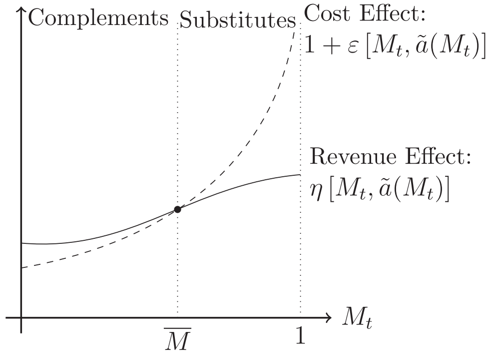

FIG. 1. Size and activity can be complements or substitutes

Figure 1 illustrates. The cost effect is increasing in size, even when endogenous changes in activity are taken into account. As size increases, the willingness to commit crime of the marginal individual, $\hat{\sigma}_t$, declines. This raises the marginal cost to the organization of increasing individual activity. Proposition 2 shows that the revenue effect only can only intersect with the cost effect once. The figure has two regions. Small organizations only recruit individuals with very high willingness to commit crime. Any increase in activity only requires a small rise in wages. The cost effect is small, and is dominated by the revenue effect. Size and activity are profit **complements**. Larger organizations, on the other hand, are forced to recruit individuals who are much less willing to commit crime. Any increase in activity necessitates a much larger rise in wages in order to maintain the indifference of the marginal individual. The cost effect is very high. Moreover, the revenue effect is small, due to diminishing marginal product. If size is large enough, it is dominated by the cost effect. In this region, size and activity are profit

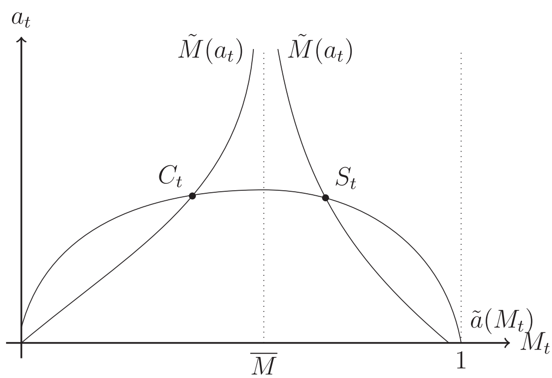

FIG. 2. Profit-maximizing size and activity level

substitutes. Where the two curves intersect, the revenue and cost effects exactly cancel each other out. An increase in activity has no effect on the optimal size of the organization and vice-versa.

Figure 2 displays the restricted demand functions, given Proposition 2. For each M_t on the horizontal axis, $\tilde{a}(M_t)$ gives the profit-maximizing level of individual activity. Similarly, for each a_t on the vertical axis, $\tilde{M}(a_t)$ gives the profit-maximizing organization size. Profits are maximized where the two curves intersect. The space represented in the diagram can be partitioned into two regions. To the left of $\overline{M}$, size and activity are complements. The revenue effect dominates. Both restricted demand curves are upward sloping.[4] As the organization's size increases, the inputs' complementarity weakens ($\partial^2\Pi_t/\partial M_t\partial a_t$ approaches zero). From Fig 1, the difference between the revenue and cost effects declines. An increase in size causes a smaller endogenous increase in activity ($\tilde{a}(M_t)$ becomes less steep). An increase in activity causes a smaller endogenous increase in size ($\tilde{M}(a_t)$ becomes steeper). When $M_t=\overline{M}$, $\partial^2\Pi_t/\partial M_t\partial a_t=0$, so the restricted demand for activity achieves a maximum, whereas the restricted demand for size asymptotes towards infinity.

In the complements region, profit is maximized at point C_t. At C_t, size maximizes profit given the level of individual activity (we are on the $\tilde{M}(a_t)$-curve) and activity maximizes profit given size (we are also on the $\tilde{a}(M_t)$-curve). Since the restricted demand for activity becomes less steep as size increases, whereas the restricted demand for size becomes steeper, the curves can only intersect once. C_t is unique.

In the region to the right of $\overline{M}$, size and activity are substitutes. The cost effect dominates, and the restricted demand curves are downward

[4]The slope of the $\tilde{a}(M_t)$ curve is:

$$\frac{\partial \tilde{a}_t}{\partial M_t} = -\frac{\frac{\partial^2\Pi_t}{\partial M_t\partial a_t}}{\frac{\partial^2\Pi_t}{\partial a_t^2}}$$

by the Implicit Function Theorem. In the complements region, $\partial^2\Pi_t/\partial M_t\partial a_t > 0$, so $\partial\tilde{a}_t/\partial M_t > 0$. Similarly:

$$\frac{\partial \tilde{M}_t}{\partial a_t} = -\frac{\frac{\partial^2\Pi_t}{\partial M_t\partial a_t}}{\frac{\partial^2\Pi_t}{\partial M_t^2}}$$

As Fig 2 displays M_t on the horizontal axis and a_t on the vertical axis, the slope of the $\tilde{M}(a_t)$ is:

$$\left(\frac{\partial \tilde{M}_t}{\partial a_t}\right)^{-1} = -\frac{\frac{\partial^2\Pi_t}{\partial M_t^2}}{\frac{\partial^2\Pi_t}{\partial M_t\partial a_t}}$$

Once again, in the complements region, $\left(\partial\tilde{M}_t/\partial a_t\right)^{-1} > 0$

sloping. As the size of the organization increases, the difference between the cost and revenue effects gets larger (see Fig 1). The inputs become stronger substitutes. An increase in size causes a larger endogenous decline in individual activity ($\tilde{a}(M_t)$ becomes steeper), and vice versa ($\tilde{M}(a_t)$ becomes less steep).

In the substitutes region, there is also a profit-maximizing point, S_t, where the restricted demand curves intersect. Once again, S_t is unique, due to how changing size affects the slopes of both curves.

Considering the whole range of inputs on offer to the organization, we have two candidates for the profit-maximizing combination of inputs, C_t and S_t. Fortunately, one offers strictly higher profits than the other:

Proposition 3: (Profit maximization):

Suppose that C_t and S_t are both interior solutions to the organization's profit maximization problem in the complements and substitutes regions respectively. Then the organization's profits are higher at S_t than at C_t.

At either C_t or S_t, profit is positive. Making use of Euler's Theorem, profit at any point where the restricted demands for both inputs intercept can be calculated as:

$$\Pi^*(M_t^*, a_t^*) = \frac{M_t^* a_t^*}{\hat{\sigma}(M_t^*)}\left[1 + \frac{1}{\lambda\hat{\sigma}(M_t^*)}\right] + a_t^* M_t^* \phi'(a_t^*)$$

Using (10) to substitute for a_t^*, this can be shown to be increasing in the size of the organization. Since size at S_t is larger than at C_t, and the restricted demands intercept at both points, profit must be higher at S_t. Left to its own devices, the profit-maximizing criminal organization will operate in a region where size and individual activity are substitutes.

5 The Storm

We now begin to evaluate the impact of a steady increase in the wage that individuals would earn in the formal labour market, w_t. Consider first the effect of an increase in w_t on the profitability of size, holding individual activity constant. From (9):

$$\frac{\partial \tilde{M}_t}{\partial w_t} = \frac{1}{\frac{\partial^2 \Pi_t}{\partial M_t^2}} < 0$$

When w_t increases, the surplus each member enjoys from the organization declines. The marginal individual, who was indifferent between employment in either sector, now prefers the formal labour market. If the organization wishes to maintain its size, it must increase the wage it offers in order to restore this indifference. The marginal cost of size increases.

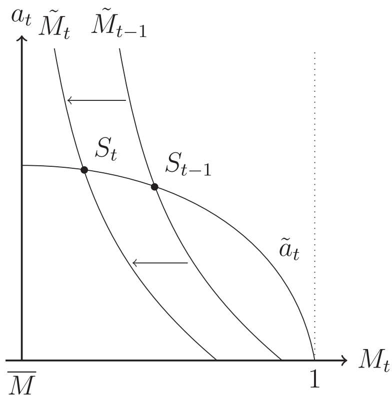

FIG. 3. Activity increases with substitutes

$\mathrm{MRP}_M(M_t^*, a_t^*)$, on the other hand, is unaffected. The increase in w_t makes size less profitable for any level of individual activity. The restricted demand for size shifts left.

Turning to the profitability of individual activity for any given size, from (10), we have:

$$\frac{\partial \tilde{a}_t}{\partial w_t}=0.$$

The increase in w_t has no impact upon the marginal cost of activity. For a given organization size, the marginal individual has the same willingness to commit crime, and hence faces the same effort cost. Varying activity whilst maintaining this individual's indifference thus necessitates the same change in wages. Similarly, $\mathrm{MRP}_a(M_t^*, a_t^*)$ has not changed. For any given organization size, the level of individual activity maximizes profit is unchanged. The restricted demand for activity is unaffected.

From Proposition 3, the organization initially operates in the substitutes region. Figure 3 shows the impact of the policy change: the $\tilde{M}(a_t)$ curve shifts left. Whilst it is clear that the policy is effective at reducing membership, it has had an unintended consequence. Due to revenue complementarity between inputs, the marginal revenue product of activity, MRP_{at}, declines. In (10), this provides the organization with an incentive to reduce activity. However, as it now recruits fewer members, the willingness of the new marginal individual to commit crime is higher. The remaining recruits require less compensation for greater individual activity. The marginal cost of activity has also declined. Since size and activity are substitutes, we know that the marginal cost has fallen to the greater extent. The organization chooses to increase its activity.

The rise in individual activity generates further endogenous effects. The surplus each member receives from being part of the organization declines due to a higher effort cost. For the marginal individual, this is sufficient to cause them to prefer the formal labour market. The marginal cost of size has increased again. At the same time, greater individual activity means that new members generate more revenue for the organization. However, in the substitutes region, this increase in MRP_{Mt} is not sufficient to maintain the profitability of size. The organization reduces its membership further.

The secondary reduction in membership leads to further endogenous increases in activity, which further impacts upon membership etc. Eventually, as in Fig 3, the profit-maximizing combination of inputs moves from S_{t-1} to S_t. Whilst the organization has fewer members, each member commits more crime. This is the storm.

Proposition 4: (The storm):

An increase in the formal labour market wage, w_t, reduces size, but increases individual activity in the interior of the substitutes region.

How bad can things get? On the one hand, the organization is smaller. This reduces aggregate crime. On the other, each member is more active, increasing it. If activity increases enough, the aggregate amount of crime could increase.

Corollary 1: (Aggregate crime):

There exists $\hat{M} > \overline{M}$ such that, if $M_t > \hat{M}$ an increase in the formal labour market wage, w_t, will raise aggregate crime, $M_t^* a_t^*$.

For large organization, the marginal individual has a very low willingness to commit crime. Small increases in activity require a very large increase in compensation, both due to the large number of members and the sensitivity of marginal individual. When policy reduces the size of the organization, the marginal cost of activity declines very rapidly relative to the marginal revenue product. The organization optimally increases individual activity more than proportionally. In effect, the restricted demand for activity is elastic with respect to size. This is consistent with the higher estimates of the elasticity of hours with respect to workers reported by Freeman, 2000. As size declines, total criminal activity increases.

As the policy reduces size further, the revenue effect becomes important (in Fig 1 the gap between the revenue and cost effects decline). Whilst individual activity continues to increase, it becomes less responsive to changes in M_t. Eventually, the restricted demand for activity becomes inelastic. At this point, further improvements in the formal labour market lower the total amount of criminal activity the organization commits (consistent with the lower estimates in Freeman, 2000). Nevertheless, individual criminal activity increases. This could represent a greater number of the same type of crime being committed, or a movement towards more serious crime.

As the policy environment continues to increase the opportunity cost of engaging in crime, the marginal cost of size grows. Since size declines and activity increases, the marginal revenue product of size also increases. However, in the substitutes region, size is bounded below by $\overline{M}$ and effort is bounded above by $\tilde{a}(\overline{M})$. So:

$$\mathrm{MRP}_M(M_t^*, a_t^*) \leq \mathrm{MRP}_M\left[\overline{M}, \tilde{a}(\overline{M})\right].$$

The marginal cost of size faces no such restriction. In fact, it is bounded below by w_t. As w_t increases, the compensation the organization must provide its members grows indefinitely. Eventually, the marginal cost of size increases above the marginal revenue product. At this point, the organization reaches a corner solution with $M_t = \overline{M}$, and the policy appears to become ineffective:

Proposition 5: (An impasse):

There exists $w^S > 0$ such that, when w_t increases above w^S size and individual activity become unresponsive to the changing policy environment.

w^S is defined by:

$$\mathrm{MRP}_M\left[\overline{M}, \tilde{a}(\overline{M})\right] = g\left[\overline{M}, \tilde{a}(\overline{M}), w^S, \phi\right] + \frac{\tilde{a}(\overline{M})}{\lambda \hat{\sigma}(\overline{M})^2}$$

$$\Longleftrightarrow w^S \equiv \mathrm{MRP}_M\left[\overline{M}, \tilde{a}(\overline{M})\right] - \phi\left[\tilde{a}(\overline{M})\right] - \frac{\tilde{a}(\overline{M})}{\hat{\sigma}(\overline{M})}\left[1 + \frac{1}{\lambda \hat{\sigma}(\overline{M})}\right], \tag{12}$$

the expected cost of engaging in crime at which $\overline{M}$ becomes the optimal organization size in the substitutes region. Since, when $w_t = w^S$, $\overline{M}$ is the (unconstrained) optimal size in the substitutes region, by Proposition 3, the profit it generates still strictly exceeds that of any combination of inputs in the complements region. It is therefore the unique profit-maximizing organization size. Profit-maximizing individual activity is given by $\tilde{a}(\overline{M})$.

As w_t rises above w^S, although profit declines rapidly, $\overline{M}$ still maximizes profits. It is optimal in the substitutes region, and offers greater profits than any combination of inputs in the complements region. Changing the policy environment appears to have run out of steam. Size and activity are unresponsive. Of course, this is not sustainable…

6 The Calm

As the formal labour market wage continues to increase, the organization appears unwilling to adjust its size and activity, but must nevertheless pay out higher wages to its members. Its profits decline rapidly. Whilst profits in the complements region also fall in the face of this increasing cost, the

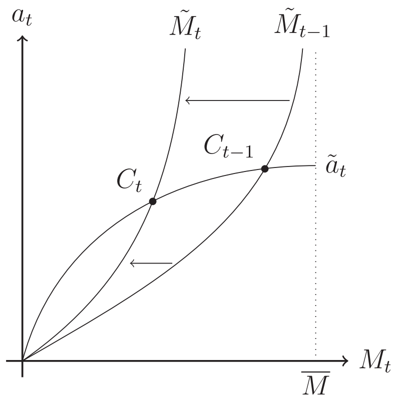

FIG. 4. Activity decreases with complements

greater flexibility afforded by an interior solution curtails the rate at which they shrink. Eventually, the organization finds it profitable to switch:

Proposition 6: (The calm):

There exists $w^C > w^S$ such that as the formal labour market wage increases above w^C, size and activity both decline.

Having moved production into the complements region, the impact of further increases in the w_t are shown in Fig 4:

As before, an increase in w_t exogenously increases the marginal cost of size. Without a corresponding exogenous increase in its marginal revenue product, the organization recruits fewer members. The $\tilde{M}_{t-1}$-curve shifts in to $\tilde{M}_t$. This has two effects. Firstly, the marginal revenue product of activity falls (the revenue effect). Secondly, those who are still recruited have relatively high willingness to commit crime. They consequently need little compensation for the criminal activity they engage in. The marginal cost of activity also falls (the cost effect). Since the organization is operating in the complements region, the revenue effect dominates, and the marginal profitability of activity falls. The decline in size causes an endogenous decrease in activity.

The endogenous effects that were so troublesome in the substitutes region now reinforce the impact of the policy. The decline in activity further reduces the marginal profitability of size. The organization recruits even fewer members. This secondary fall in size leads to further declines in individual activity. Eventually, the organization's profit-maximizing input combination moves from C_{t-1} to C_t, consisting of both fewer members and lower activity. Now, increases in the cost of engaging in crime lead to rapid declines in both the organization size, and the extent of its criminal activities. In contrast to its initial effect, the policy of improving labour market opportunities has become very effective indeed.

7 Heterogeneous Contracts

The baseline model presented in Section 3 was very simple. Every member was paid the same, and engaged in the same amount of crime. I now briefly discuss how the storm before the calm is affected under two forms of heterogeneous contract.

7.1 Separating Contracts

Many criminal organizations feature strict hierarchies. Different individuals engage in different levels of activity, and receive different wages (Levitt and Venkatesh, 2000; Carvalho and Soares, 2016). I now allow for this possibility, by considering the organization's profit maximization problem when it is able to offer a menu of contracts which cause members to reveal their willingness to commit crime.

Since every member will be employing a different level of activity, we must first modify the baseline model. Let $r(M, a)$ be the revenue generated by an individual when the organization size is M and their activity level is a. Even at the individual level, there are several reasons to think that M will play a role in determining the revenue that they can generate. Members may be able to take advantage of **network effects**. For example, by vouching for individuals and punishing those who cheat, organizations can increase the sharing of capital equipment between its members used in criminal activities (Cook *et al.*, 2007). Larger organizations may also stretch police resources, reducing the efforts individuals must go to avoid capture (Sah, 1991). Conversely, larger organizations can generate **congestion effects**. Whilst size and activity may be complementary at the aggregate level, an increase in membership reduces an individual's opportunities to generate revenue due to greater competition between criminals (Ballester *et al.*, 2010).

The organization's profit maximization problem is:[5]

$$\Pi(w_t,\phi)=\max_{[g_t(\sigma),a_t(\sigma)]_{\sigma=0}^{\infty}}\left(M_tE\{r[M_t,a_t(\sigma)]-g_t(\sigma)|\sigma\text{ joins}\}\right)$$

It chooses its contract schedule, $[g_t(\sigma),a_t(\sigma)]_{\sigma=0}^{\infty}$, to maximize the sum of all of its members revenues, minus the wage it must pay them.

As in the baseline case, it is straightforward to show that the size of the organization is synonymous with the identity of a marginal individual. Suppose that an individual with willingness to commit crime σ' is indifferent between accepting a contract $(g_t(\sigma'),a_t(\sigma'))$ and working in the formal labour market

$$g_t(\sigma')-\frac{a_t(\sigma')}{\sigma'}-\phi[a_t(\sigma')]=w_t$$

[5]Once again, the organization's problem is not subject to moral hazard and labour is supplied competitively. The full dynamic problem is equivalent to a sequence of static problems.

Any individual with a higher willingness to commit crime, $\sigma'' > \sigma'$, strictly prefers accepting $(g_t(\sigma'), a_t(\sigma'))$ to receiving w_t. They receive the same wage $(g_t(\sigma'))$, and suffer the same expected punishment $(\phi[a_t(\sigma')])$, but face a lower disutility of effort $(a_t(\sigma')/\sigma'')$:

$$g_t(\sigma') - \frac{a_t(\sigma')}{\sigma''} - \phi[a_t(\sigma')] > w_t$$

So all individuals with willingness to commit crime greater than σ' join the organization. If they accept a different contract, it must offer an even higher payoff. Since the individual with willingness to commit crime $\sigma = 0$ would never wish to join the organization,[6] there must exist a unique willingness to commit crime $\hat{\sigma}_t$ such that individuals join if and only if $\sigma \geq \hat{\sigma}_t$. The organization's size is once again $M_t = e^{-\lambda\hat{\sigma}_t}$.

For members to reveal their willingness to commit crime to the organization, the contract that the organization offers in each period t must satisfy the following incentive compatibility constraint:

$$\sigma = \arg\max_{s \geq 0} \left\{ g_t(s) - \frac{a_t(s)}{\sigma} - \phi[a_t(s)] \right\} \tag{13}$$

i.e. it must be implementable. Such a contract takes the following form:

Proposition 7: (Implementable contracts):

An implementable contract schedule, $[g_t(\sigma), a_t(\sigma)]_{\sigma \geq \hat{\sigma}_t}$, takes the following form. For every willingness to commit crime, σ, any organization size, $M_t = e^{-\lambda\hat{\sigma}_t}$, and any activity schedule, $[a_t(\sigma)]_{\sigma \geq \hat{\sigma}_t}$, the organization offers a wage:

$$g_t[\sigma; M_t, (a_t), w_t, \phi] = w_t + \frac{a_t(\sigma)}{\sigma} + \phi[a_t(\sigma)] + \int_{s=\hat{\sigma}_t}^{\sigma} \frac{a_t(s)}{s^2} ds$$

As in the baseline case, the wage offered must compensate each member for the opportunity cost (w_t), disutility of effort $(a_t(\sigma)/\sigma)$ and expected punishment $(\phi[a_t(\sigma)])$ associated with working for the organization. The final term is information rent, and causes each individual to weakly prefer revealing their willingness to commit crime to choosing any other contract in the schedule.

The organization's profit maximization problem is equivalent to:

$$\max_{M_t, [a_t(\sigma)]_{\sigma=\hat{\sigma}_t}^{\infty}} \left(\int_{\sigma=\hat{\sigma}_t}^{\infty} \{ r[M_t, a_t(\sigma)] - g_t[\sigma; M_t, (a_t), w_t, \phi] \} \lambda e^{-\lambda\sigma} d\sigma \right)$$

Solving yields the following familiar result:

[6]Unless, of course, $a_t(0) = 0$. In this case they do not generate revenue, but cost the organization at least w_t. The organization would never offer such a contract.

Proposition 8: (Complements vs. substitutes):

If $\frac{\partial}{\partial a_t}\left(\frac{\partial^2 r}{\partial M_t^2}\right) < 0$ and $\frac{\partial}{\partial M_t}\left(\frac{\partial^2 r}{\partial a_t^2}\right) < 0$, then for every σ there exists a $\overline{M}(\sigma)$ such that:

1. If $M_t > \overline{M}(\sigma)$ then a fall in M_t^* increases $a_t^*(\sigma)$. Size and activity are profit substitutes; and
2. If $M_t < \overline{M}(\sigma)$ then a fall in M_t^* decreases $a_t^*(\sigma)$. Size and activity are profit complements.

Each individual's optimal level of activity in period t satisfies:

$$\frac{\partial r(M_t^*, a_t^*)}{\partial a_t} - \phi'(a_t^*) - \frac{1}{\sigma}\left(1 + \frac{1}{\lambda\sigma}\right) \equiv 0 \tag{14}$$

Unlike the baseline case, the size of the organization has no direct impact upon the marginal cost of activity. Whereas before the willingness to commit crime of the marginal individual determined how much each member would be compensated, this contract enables the organization to pay individuals according to their own σ.

The individual's marginal revenue product of activity does still depend upon size, due to network and congestion effects. Under the conditions of the proposition, it can be proved that, for 'large enough' M_t, it declines as size increases, even taking into account optimal changes in activity. In this case, size and activity are substitutes. What constitutes 'large enough' depends on the individual's equilibrium activity, in turn determined by their willingness to commit crime. This yields a unique threshold for each member, $\overline{M}(\sigma)$ such that size and activity are profit substitutes if and only if $M_t > \overline{M}(\sigma)$.

Turning to the impact of policy, it is straightforward to show that an increase in w_t reduces the optimal size of the organization. As before, the marginal cost of hiring an additional individual is higher. Whilst the organization is still relatively large, it substitutes away from size towards individual activity. The majority of members find that they are required to commit more crime. If the organization is sufficiently large, aggregate crime increases.[7] However, as formal labour market opportunities continue to improve, this process is reversed. More and more members find

[7]Aggregate crime is $\tilde{A}_t(M_t) \equiv \int_{\sigma=\hat{\sigma}_t}^{\infty} \tilde{a}_t(M_t, \sigma)e^{-\lambda\sigma}d\sigma$. So:

$$\frac{\partial \tilde{A}_t}{\partial M_t} = \frac{\tilde{a}_t(M_t, \hat{\sigma}_t)}{\lambda M_t} + \int_{\sigma=\hat{\sigma}_t}^{\infty} \frac{\partial \tilde{a}_t}{\partial M_t} e^{-\lambda\sigma} d\sigma.$$

As $M_t \to 1$, $\partial\tilde{A}_t/\partial M_t \to \int_{\sigma=0}^{\infty} \partial\tilde{a}_t/\partial M_t e^{-\lambda\sigma}d\sigma < 0$ since size and activity are substitutes for all individuals in the limit. There thus exists a region $[\hat{M}, 1]$ such that if $M_t > \hat{M}$ then $\tilde{A}_t$ increases as M_t falls.

that the organization treats their activity as complementary to its size. Eventually, aggregate crime begins to fall. Once again, we get a storm before the calm.

7.2 Overlapping Generations of Members

An alternative source of heterogeneity may derive from different individuals joining at different times. If contracts can be offered contingent on w_t, then the model presented in Section 3 needs no adjustment. This extension considers a simple situation in which this is not true. Although the organization recruits in each period, the contract it offers in fixed for two periods. This gives rise to the possibility that old members prefer to seek work in the formal labour market, but are prevented from doing so by the organization. Moreover, new *recruits* will have a different contract to *existing members* who continue from the previous period.

Introducing overlapping generations of members necessitates further changes to the organization's profit maximization problem. In period t, M_{t-1} existing members remain from period $t-1$, each exerting effort a_{t-1} in exchange for a wage of g_{t-1}. The organization then recruits a further M_t members, who exert effort a_t in exchange for g_t. In its most general form, this enables it to generate revenue $R(M_{t-1}, a_{t-1}, M_t, a_t)$. Of course, I maintain the assumption that size and activity are revenue complements. The organization's profit maximization problem can be represented recursively as follows:

$$V(M_{t-1}, a_{t-1}, g_{t-1}, w_t, \phi)$$
$$= \max_{g_t, a_t} [R(M_{t-1}, a_{t-1}, M_t, a_t) - g_{t-1}M_{t-1} - g_tM_t + \beta V(M_t, a_t, g_t, w_{t+1}, \phi)].$$

In contrast to previous sections, the organization faces an intertemporal trade off, due to the interplay of existing and new members' activity within the revenue function.

Turning to the solution of the model, an individual with willingness to commit crime σ will join the organization if and only if:

$$(1+\beta)\left[g_t - \frac{a_t}{\sigma} - \phi(a_t)\right] \geq w_t + \beta w_{t+1}$$

$$\iff g_t - \frac{a_t}{\sigma} - \phi(a_t) \geq \frac{w_t + \beta w_{t+1}}{1+\beta} \tag{15}$$

If they join, they are tied into a contract with the organization for two periods. If they decide to seek work in the formal labour market, they receive an increasing wage profile over the two periods. In order to entice an individual to become a member, the organization must offer a contract that pays a new recruit more than they could earn in the formal labour market (since $[w_t + \beta w_{t+1}]/[1+\beta] > w_t$). In their second period of employment, however, this need not be true. Although they may prefer to leave the organization (since $[w_t+\beta w_{t+1}]/[1+\beta] < w_{t+1}$), the contract prevents them from doing so.

(15) has a very similar form to that of (3). Whilst the left hand side increases in σ, the right hand side is independent of it. As before, there thus exists a unique willingness to commit crime, $\hat{\sigma}(g_t, a_t, w_t, w_{t+1}, \phi)$, such that individuals join if and only if $\sigma \geq \hat{\sigma}_t$.

If the organization wishes to recruit $M_t = e^{-\lambda\hat{\sigma}_t}$ members in period t to exert effort a_t, it must offer the following wage:

$$g_t(M_t, a_t, w_t, w_{t+1}, \phi) = \frac{w_t+\beta w_{t+1}}{1+\beta} + \phi(a_t) + \frac{a_t}{\hat{\sigma}(M_t)}$$

The organization must compensate its members for the opportunity cost of joining, the expected punishment and the disutility of criminal activity. Offering this wage makes an individual with willingness to commit crime, $\hat{\sigma}_t$, indifferent between joining and working in the formal labour market, giving rise to the desired number of recruits.

By offering the above wage, the organization is free to choose both the number of recruits and the individual activity level that maximize its profits. The solution satisfies the following (by now familiar) first-order conditions:

$$\frac{\partial R_t}{\partial M_t^*} + \beta \frac{\partial R_{t+1}}{\partial M_t^*} = (1+\beta)\left[g_t(M_t^*, a_t^*, w_t, w_{t+1}, \phi) + \frac{a_t^*}{\lambda\hat{\sigma}(M_t^*)^2}\right] \tag{16}$$

$$\frac{\partial R_t}{\partial a_t^*} + \beta \frac{\partial R_{t+1}}{\partial a_t^*} = (1+\beta) M_t^* \left[\phi'(a_t^*) + \frac{1}{\lambda\hat{\sigma}(M_t^*)}\right] \tag{17}$$

(16) is equivalent to (7), and provides the profit maximizing condition for the number of recruits. Each new recruit generates revenue in both the current and next period. The marginal cost of recruiting them is the wage that they receive, plus the increase in wages offered to all inframarginal recruits required to entice the marginal individual to join.

(17) is equivalent to (8), and states the profit maximizing condition for individual activity. Again, an increase in activity amongst new recruits generates additional revenue in both the current and next period. This is offset by an increase in wages required to compensate for harsher expected punishment and additional disutility of criminal activity.

How does the profitability of individual activity vary with the number of recruits?

$$\frac{\partial}{\partial M_t}\left(\frac{\partial V_t}{\partial a_t}\right) = \underbrace{\frac{\partial^2 R_t}{\partial M_t \partial a_t} + \beta \frac{d^2 R_{t+1}}{dM_t da_t}}_{\text{Revenue Effect}} - \underbrace{\left[\phi'(a_t) + \frac{1}{\hat{\sigma}(M_t)}\left(1 + \frac{1}{\lambda\hat{\sigma}(M_t)}\right)\right]}_{\text{Cost Effect}} \gtreqless 0 \tag{18}$$

We once again have revenue and cost effects. An increase in the number of members the organization recruits means that greater individual activity translates into a larger increase in aggregate criminal activity: the marginal revenue product of activity increases. This is counteracted by the impact upon wages. More recruits require compensation for the harsher expected punishment and greater disutility associated with an increase in activity, raising the total wage bill. Moreover, increasing the number of recruits necessitates attracting individuals with relatively low willingness to commit crime. This further increases the marginal cost of activity.

Whether the revenue or cost effect dominates will, as in Section 4, determine whether the organization treats new recruits and individual activity as complements or substitutes. In turn, how these inputs are viewed will determine whether policy will backfire. Further consideration of (18) provides insight. When the organization is large, $\hat{\sigma}(M_t)$ is very small ($\hat{\sigma}(M_t) \rightarrow 0$ as $M_t \rightarrow 1$). The cost effect dominates, and the organization treats its inputs as substitutes. Intuitively, the organization recruits individuals with very low willingness to commit crime. The amount of compensation these individuals require in order to suffer the disutility of criminal activity makes activity unprofitable. If, on the other hand, the organization is small, it will treat recruits and activity as complements. It only attracts those with relatively high willingness to commit crime, who require little compensation for the disutility criminal activity creates.

As wages in the formal labour market improve, the organization optimally reduces its size. Initially, this will cause it to substitute away from recruits towards increased individual activity. The policy appears to backfire. Eventually, however, changing labour market conditions will reduce the optimal number of recruits to the point where the organization view them as complementary to activity. Thereafter, size and activity will both decline. Once again, we have a storm before the calm.

8 Discussion

Recent years have seen numerous innovative policies put forward to tackle high-crime neighbourhoods. These approaches tend to be based upon a rational offender argument. Improving formal labour market conditions increase the opportunity cost of engaging in crime. The offender weighs up these higher costs against the benefits they enjoy from successfully committing crime. The crime rate falls.

The presence of organized crime may confound this argument, at least initially. Whilst those on the margin do indeed move away from a life of crime, the criminal organization reacts by adjusting its recruitment policy. Those who still opt for a career in the organization are hardened criminals. They require relatively little compensation for engaging in criminal acts. With this in mind, the organization substitutes away from a large, inactive membership towards a small, prolific one. This may help to explain evidence suggesting that policy can backfire in the presence of organized crime.

All is not lost, however. As the size of the organization continues to fall, the endogenous effects that hampered the policy now reinforce it. With so few recruits, increasing each member's individual activity does little to increase the organization's revenue. Conversely, its costs continue to grow, as members must be compensated for their efforts. This counteracts the incentive to substitute. Eventually, the organization switches to a strategy whereby increasing the opportunity cost of engaging in crime reduces both its size and the amount of crime each of its members commits. Of course, several simplifications have been made to ensure that the model remains tractable. For example, although the model is dynamic, it does not allow for human capital accumulation (as in Mocan *et al.*, 2005). Prolonged membership of the organization could both increase an individual's willingness to commit crime and reduce their expected formal labour market earnings. This would introduce a further source of contract heterogeneity, as well as counteracting the effect of labour market policy. It is worthy of further study.

It is also assumed that changes in policy do not impact upon the marginal cost of individual activity. In many cases, this is accurate. Increases in a minimum wage or the number of vacancies in a neighbourhood simply raise the opportunity cost of engaging in crime. They do not affect the marginal cost of one more criminal act within the organization. Similarly, more police on the streets or tougher sentences across the board (even if different crimes warrant different punishments) will simply raise the expected cost of punishment—they act as a fixed cost associated with each crime. However, if more serious crimes saw their expected punishment rise more sharply, then crime would also become more costly at the intensive margin. This would generate an exogenous decline in individual criminal activity. Such policies may help mitigate the storm. Stretching the metaphor a bit far, perhaps, they are all-weather policies.

Appendix

Throughout the appendices, I will use subscript to denote derivative. For example, $R_M=\partial R/\partial M_t$ and $R_{Ma}=\partial^2 R/\partial M_t \partial a_t$.

Proof of Proposition 1

The proof proceeds in two steps. Firstly, I show that the solution to the profit maximization problem, if there is one, must be contained within a compact set. Appealing to Weierstrass's theorem immediately proves that a maximum exists. I then show that none of the boundaries of the compact set maximize profits, so the solution must be interior. Hence the first-order (and second-order) conditions must be satisfied when profits are maximized.

Lemma 1: A solution to the organization's profit maximization problem exists.

Proof: Clearly, if a solution exists, it must be the case that $M_t^* \in [0,1]$. a_t^*, on the other hand, appears to be unbounded. For each given M_t, consider how Π_t varies with a_t:

$$\frac{\partial \Pi_t}{\partial a_t} = R_a(M_t, a_t) - M_t \phi'(a_t) - \frac{M_t}{\hat{\sigma}(M_t)} \gtreqless 0$$

$$\frac{\partial^2 \Pi_t}{\partial a_t^2} = R_{aa}(M_t, a_t) - M_t \phi''(a_t) < 0$$

For each M_t define $\tilde{a}(M_t)$ to be the solution to $\partial \Pi_t / \partial a_t = 0$. Since $\partial^2 \Pi_t / \partial a_t^2 < 0$, $\tilde{a}(M_t)$ maximizes profits given M_t and, importantly, the organization would never choose a higher level of activity. Since $\tilde{a} : [0,1] \to \mathbb{R}$ is continuous in M_t and defined over a compact set, it must have a maximum by Weierstrass's theorem. Call this maximum $\hat{a}$. For any M_t, $\tilde{a}(M_t) \le \hat{a}+1$. So $a_t^* \le \hat{a}+1$.

Any solution to the organization's profit maximization problem is thus contained within a compact set: $(M_t^*, a_t^*) \in [0,1] \times [0, \hat{a}+1]$. Since profits are continuous, by Weierstrass's theorem a solution exists. This completes the proof of the lemma. ■

I now show that the profit maximization problem does not have a corner solution.

Lemma 2: The solution to the organization's profit maximization problem is either in the interior of $[0,1] \times [0, \hat{a}+1]$ or it involves the organization shutting down.

Proof: There are four boundaries of the compact set defined in Lemma 1 to check:

1. $M_t^* \in (0,1)$ and $a^* = 0$. We have:

$$\Pi(M_t^*, 0) = -M_t^* w_t < 0$$

The organization would be better off choosing $M_t^* = 0$. This boundary cannot maximize profits.

2. $M_t^* \in (0,1)$ and $a_t^* = \hat{a}+1$. By construction, the organization could increase profits by reducing activity to $\tilde{a}(M_t^*)$. This boundary cannot maximize profits.
3. $M_t^* = 0$ and $a_t^* \in (0, \hat{a}+1)$. This is equivalent to the organization temporarily shutting down in period t.
4. $M_t^* = 1$ and $a_t^* \in (0, \hat{a}+1)$. As $M_t \to 1$, $\hat{\sigma}(M_t) \to 0$. From (5), for any $a_t^* > 0$, $g(M_t, a_t^*, w_t, \phi) \to +\infty$ and so $\Pi(M_t, a_t^*) \to -\infty$. This cannot maximize profits.

Profit maximization involves either an interior solution, or the organization shuts down. This completes the proof of the lemma. ■

Finally, it is necessary to show that the organization does not wish to shut down. Making use of constant returns to scale, the profit generated at an interior maximum is:

$$\begin{aligned}
\Pi^* &= M_t^* R_M(M_t^*, a_t^*) + a_t^* R_a(M_t^*, a_t^*) - g(M_t^*, a_t^*, w_t, \phi) M_t^* \\
&= M_t^* \left[g(M_t^*, a_t^*, w_t, \phi) + \frac{a_t^*}{\lambda \hat{\sigma}_t^{*2}} \right] + a^* R_a(M_t^*, a_t^*) - g(M_t^*, a_t^*, w_t, \phi) M_t^* \\
&= \frac{a_t^* M_t^*}{\lambda \hat{\sigma}_t^{*2}} + a_t^* R_a(M_t^*, a_t^*) > 0,
\end{aligned}$$

where the second line comes from substituting (7) for $R_M(M^*, a^*)$. The organization makes positive profits. This completes the proof. ■

Proof of Proposition 2

Firstly, consider the first-order condition for a_t given by (8). For each M_t, this gives the a_t that maximizes profit. Once again, let us denote this restricted demand for a_t by $\tilde{a}(M_t)$, defined implicitly by:

$$R_a[M_t,\tilde{a}(M_t)]-\phi'[\tilde{a}(M_t)]-\frac{M_t}{\hat{\sigma}_t}\equiv 0.$$

We can now consider whether size and effort are complements in equilibrium purely as a function of M_t. Consider (11). For any M_t size and effort are complements if and only if:

$$\eta(M_t,\tilde{a}_t)\geq 1+\varepsilon(M_t,\tilde{a}_t) \tag{19}$$

$$\iff \eta(M_t,\tilde{a}_t)\geq 1+\frac{1}{\lambda\hat{\sigma}_t[1+\phi'(\tilde{a}_t)\hat{\sigma}_t]}. \tag{20}$$

The right-hand side is strictly increasing in M_t. Moreover, as $M_t\to 1$, $1+\varepsilon(M_t,\tilde{a}_t)\to\infty$. So, as $M_t\to 1$, size and activity are substitutes. By the definition of a limit, there must exist some $\overline{M}\geq 0$ such that if $M_t\geq\overline{M}$ then size and activity are substitutes.

Now, consider how $\partial^2\Pi_t/\partial a_t\partial M_t$ varies with M_t, accounting for endogenous changes in a_t:

$$\frac{d}{dM_t}\left(\frac{\partial^2\Pi_t}{\partial a_t\partial M_t}\right)=\frac{\partial^3\Pi_t}{\partial a_t\partial M_t^2}+\frac{\partial^3\Pi_t}{\partial a_t^2\partial M_t}\frac{\partial\tilde{a}_t}{\partial M_t}$$

If a_t and M_t are complements, then $\partial^2\Pi_t/\partial a_t\partial M_t>0$. Conversely, if a_t and M_t are substitutes, then $\partial^2\Pi_t/\partial a_t\partial M_t<0$. Thus, at any point where the relationship between size and effort changes from complements to substitutes or vice versa, $\partial^2\Pi_t/\partial a_t\partial M_t=0$.

Now, at such a point, by the Implicit Function Theorem:

$$\frac{\partial\tilde{a}_t}{\partial M_t}=-\frac{\dfrac{\partial^2\Pi_t}{\partial a_t\partial M_t}}{\dfrac{\partial^2\Pi_t}{\partial a_t^2}}=0.$$

So:

$$\begin{aligned}\frac{d}{dM_t}\left(\frac{\partial^2\Pi_t}{\partial a_t\partial M_t}\right)&=\frac{\partial^3\Pi_t}{\partial a_t\partial M_t^2}\\&=R_{aMM}+\frac{\partial\hat{\sigma}_t}{\partial M_t}\left(\frac{1}{\hat{\sigma}_t^2}+\frac{2}{\lambda\hat{\sigma}_t^3}\right)\\&=R_{aMM}-\frac{1}{\lambda M_t\hat{\sigma}_t^2}-\frac{2}{\lambda^2 M_t\hat{\sigma}_t^3}.\end{aligned}$$

If $R_{aMM} < 0$ then, at any point where the relationship between size and activity changes, $d/dM_t\left(\partial^2\Pi_t/\partial a_t \partial M_t\right) < 0$. As the organization's size increases, it must be that they switch from being complements to substitutes. This implies that, once the organization's size is sufficiently large to make both inputs substitutes, they cannot become complements again. $\overline{M}$ is the only point at which the relationship changes. This completes the proof. ■

Proof of Proposition 3

The proof proceeds in two steps. Firstly, I show that equilibrium profit is always increasing in M_t^* in the complements region ($M_t < \overline{M}$). Second, applying a different technique, I show that it also increases in M_t^* in the substitutes region ($M_t \geq \overline{M}$).

Lemma 3: Profit is increasing in M_t in the complements region.

Proof: From the proof of Proposition 1, the profit generated by at any point of intersection between the restricted demand curves is:

$$\begin{aligned}\Pi^*(M_t, a_t) &= \frac{a_t M_t}{\lambda \hat{\sigma}_t^2} + a_t R_a(M_t, a_t)\\ &= \frac{a_t M_t}{\hat{\sigma}_t}\left[1 + \frac{1}{\lambda \hat{\sigma}_t}\right] + a_t M_t \phi'(a_t),\end{aligned}$$

where the second line comes from substituting for $R_a(M, a)$ using (8). We can assess what happens to profit as we move along the $\tilde{a}(M_t)$ curve, increasing M:

$$\Pi^*(M_t, \tilde{a}_t) = \frac{\tilde{a}_t M_t}{\hat{\sigma}_t}\left[1 + \frac{1}{\lambda \hat{\sigma}_t}\right] + \tilde{a}_t M_t \phi'(\tilde{a}_t)$$

So:

$$\begin{aligned}\frac{d\Pi^*(M_t, \tilde{a}_t)}{dM_t} &= \frac{\partial \Pi_t^*}{\partial M_t} + \frac{\partial \Pi_t^*}{\partial a_t}\frac{d\tilde{a}_t}{dM_t}\\ &= \left[\frac{\tilde{a}_t}{\hat{\sigma}_t}\left(1 + \frac{1}{\lambda \hat{\sigma}_t}\right) + \frac{\tilde{a}_t}{\lambda \hat{\sigma}_t^2}\left(1 + \frac{2}{\lambda \hat{\sigma}_t}\right) + \tilde{a}_t \phi'(\tilde{a}_t)\right]\\ &\quad - \left[\frac{M_t}{\hat{\sigma}_t}\left(1 + \frac{1}{\lambda \hat{\sigma}_t}\right) + M_t \phi'(\tilde{a}_t) + M_t \tilde{a}_t \phi''(\tilde{a}_t)\right] \frac{R_{Ma} - \phi'(\tilde{a}_t) - \frac{1}{\hat{\sigma}_t} - \frac{1}{\lambda \hat{\sigma}_t^2}}{R_{aa} - M_t \phi''(\tilde{a}_t)}\end{aligned}$$

Both expressions in square parentheses are clearly positive. In the complements region, $R_{Ma} - \phi'(\tilde{a}_t) - 1/\hat{\sigma}_t - 1/(\lambda \hat{\sigma}_t^2) > 0$, and so:

$$\frac{R_{Ma} - \phi'(\tilde{a}_t) - \frac{1}{\hat{\sigma}_t} - \frac{1}{\lambda \hat{\sigma}_t^2}}{R_{aa} - M_t \phi''(\tilde{a}_t)} < 0$$

We conclude that $d\Pi^*[M_t, \tilde{a}(M_t)]/dM_t > 0$ in the complements region. This completes the proof of the lemma. ■

Turning to the substitutes region:

Lemma 4: Equilibrium profits are increasing in M_t^* in the substitutes region.

Proof: The proof of this lemma pre-empts some results in the following section. M_t^* and a_t^* are determined simultaneously by the first-order conditions (7) and (8). Since everything (up to functional forms) is endogenous in (8), the only exogenous variable that can shift the equilibrium is w_t in (7). Applying the Implicit Function Theorem, and noting that the second-order conditions for a maximum require that $\Pi_{aa} < 0$ and $\Pi_{MM}\Pi_{aa} - \Pi_{Ma}^2 > 0$, we have that:

$$\frac{dM_t^*}{dw_t} = \frac{\Pi_{aa}}{\Pi_{MM}\Pi_{aa} - \Pi_{Ma}^2} < 0$$

Now, what happens to Π_t^* as w_t increases? From the Envelope Theorem:

$$\begin{aligned}\frac{d\Pi_t^*}{dw_t} &= -M_t^* \frac{\partial g_t}{\partial w_t} \\ &= -M_t^* < 0.\end{aligned}$$

So, as M_t^* falls, Π_t^* falls. Equivalently, equilibrium profits rise in the substitutes region as M_t^* rises. This completes the proof of the lemma. ■

Finally, note that profit is a continuous function of M_t. So as M_t increases from $M_t < \overline{M}$ to $M_t > \overline{M}$, profit increases. If S represents an interior solution in the substitutes region, it must yield higher profits than C. This completes the proof. ■

Proof of Proposition 4

Applying the Implicit Function Theorem to (7) and (8), we have that:

$$\frac{dM_t^*}{dw_t} = \frac{\Pi_{aa}}{\Pi_{MM}\Pi_{aa} - \Pi_{Ma}^2} < 0,$$

$$\frac{da_t^*}{dw_t} = -\frac{\Pi_{Ma}}{\Pi_{MM}\Pi_{aa} - \Pi_{Ma}^2}$$

Size unambiguously falls. Since size and activity are substitutes, $\Pi_{Ma} = R_{Ma} - \phi'(a_t^*) - 1/\hat{\sigma}_t - 1/(\lambda\hat{\sigma}_t^2) < 0$. So $da_t^*/dw_t > 0$. This completes the proof. ■

Proof of Corollary 1

Aggregate criminal activity is given by M^*a^*. Differentiating with respect to ϕ:

$$\begin{aligned}\frac{dM_t^* a_t^*}{dw_t} &= \frac{dM_t^*}{dw_t} a_t^* + M_t^* \frac{da_t^*}{dw_t} \\ &= \frac{\Pi_{aa} a_t^* - \Pi_{Ma} M_t^*}{\Pi_{MM}\Pi_{aa} - \Pi_{Ma}^2}\end{aligned}$$

So $\partial M_t^* a_t^* / \partial w_t > 0$ if and only if:

$$\Pi_{Ma} M_t^* - \Pi_{aa} a_t^* < 0$$

$$\Longleftrightarrow \left\{ M_t^* R_{Ma} - M_t^* \phi'(a_t^*) - \frac{M_t^*}{\hat{\sigma}_t}\left[1+\frac{1}{\lambda\hat{\sigma}(M_t)}\right]\right\} - \left[a_t^* R_{aa} - a_t^* M_t^* \phi'\prime(a_t^*)\right] < 0$$

Making using of Euler's Theorem, $M_t^* R_{M^* a^*} + a_t^* R_{aa} = 0$, so:

$$2R_{Ma} - \phi'(a_t^*) - \frac{1}{\hat{\sigma}_t}\left(1+\frac{1}{\lambda\hat{\sigma}_t}\right) + a_t^* \phi'\prime(a_t^*) < 0$$

$$\Longleftrightarrow R_{Ma} - \frac{1}{2}\left[\phi'(a_t^*) - \frac{1}{\hat{\sigma}_t}\left(1+\frac{1}{\lambda\hat{\sigma}_t}\right)\right] + \frac{1}{2} a_t^* \phi'\prime(a_t^*) < 0$$

We thus require:

$$\frac{R_a}{M_t^*}\left(\eta(M_t^*, a_t^*) - \frac{1}{2}\left\{1+\frac{1}{\lambda\hat{\sigma}_t[1+\phi'(a_t^*)\hat{\sigma}_t]}\right\}\right) + \frac{1}{2} a_t^* \phi'\prime(a_t^*) < 0$$

The first term in parentheses is the revenue effect. The second term is half the cost effect. Since the revenue effect declines in M and the cost effect increases towards infinity as M approaches 1, for large enough M this must be negative. Since $a_t^* \to 0$ as $M_t^* \to 1$, the final term tends to zero as M_t^* gets larger. For large enough M_t^*, the inequality holds. Conversely, if $M = \overline{M}$ the revenue and cost effect are equal, so the term above is positive. There therefore exists a unique $\hat{M} \in (\overline{M}, 1)$ such that for $M_t^* > \hat{M}$ aggregate crime increases. This completes the proof. ■

Proof of Proposition 5

In the substitutes region:

$$R_{Mw}(M_t^*, a_t^*) = R_{MM}\frac{dM_t^*}{dw_t} + R_{Ma}\frac{da_t^*}{dw_t} > 0$$

From the previous proof $dM_t^*/dw_t < 0$ and $da_t^*/dw_t > 0$. As w_t increases, the marginal revenue product of size increases. Moreover, conditional on $M_t^* \geq \overline{M}$:

$$R_M(M^*, a^*) \leq R_M\left[\overline{M}, \tilde{a}(\overline{M})\right]$$

So, if:

$$w_t \geq w^S \equiv R_M\left[\overline{M}, \tilde{a}(\overline{M})\right] - \overline{M}\phi'[\tilde{a}(\overline{M})] - \frac{\tilde{a}(\overline{M})}{\hat{\sigma}(\overline{M})}\left[1+\frac{1}{\lambda\hat{\sigma}(\overline{M})}\right]$$

then $M^* = \overline{M}$ and $a^* = \tilde{a}(\overline{M})$. The organization has reached a corner solution. When $w_t = w^S$, $\overline{M}$ is the unconstrained profit-maximizing organization size. By Proposition

3, the organization makes strictly more profit than it would it were to switch into the complements region. As the wage continues to increase, the organization thus prefers to remain at $\left(\overline{M}, \tilde{a}\left(\overline{M}\right)\right)$. This completes the proof. ■

Proof of Proposition 6

The proof is presented in two stages. We first compare the profit from the optimal input combination under complements to that when size is $\overline{M}$, summarizing the result in the following lemma:

Lemma 5: There exists w^C such that, for all $w_t \geq w^C$ the organization makes more profit using an input combination in the complements region than it does using an input combination in the substitutes region.

Proof: The proof consists of two steps. Firstly, I show that, for high enough w_t, the profit from optimal input combination under complements exceeds that when size is $\overline{M}$. Secondly, I show that, as w_t increases, profit when size is $\overline{M}$ decline more rapidly than when size and activity are complements.

The profit when size is $\overline{M}$ is given by:

$$\Pi\left[\overline{M}, \tilde{a}\left(\overline{M}\right)\right] = R\left[\overline{M}, \tilde{a}\left(\overline{M}\right)\right] - \overline{M} w_t - \overline{M} \phi\left[\tilde{a}\left(\overline{M}\right)\right] - \frac{\overline{M} \tilde{a}\left(\overline{M}\right)}{\hat{\sigma}\left(\overline{M}\right)} + \beta \Pi_{t+1}$$

So, for:

$$w_t > \frac{R\left[\overline{M}, \tilde{a}\left(\overline{M}\right)\right]}{\overline{M}} - \phi\left[\tilde{a}\left(\overline{M}\right)\right] - \frac{\tilde{a}\left(\overline{M}\right)}{\hat{\sigma}\left(\overline{M}\right)}$$

we have that $\Pi\left[\overline{M}, \tilde{a}\left(\overline{M}\right)\right] < \beta \Pi_{t+1}$. Now, with complements, we always have an interior solution. So:

$$\Pi^*(M_t, a_t) = \frac{a_t M_t}{\hat{\sigma}_t} \left[1 + \frac{1}{\lambda \hat{\sigma}_t}\right] \geq 0$$

For high enough w_t the profit under complements exceeds that under substitutes. Moreover:

$$\frac{d\Pi\left[\overline{M}, \tilde{a}\left(\overline{M}\right)\right]}{dw_t} = -\overline{M},$$

$$\frac{d\Pi[M, \tilde{a}(M)]}{dw_t}\bigg|_{M<\overline{M}} = -M_t$$

Since, under complements, $M_t < \overline{M}$ we have that:

$$-\frac{d\Pi\left[\overline{M}, \tilde{a}\left(\overline{M}\right)\right]}{dw_t} > -\frac{d\Pi}{dw_t}\bigg|_{M<\overline{M}}$$

$\Pi\left[\overline{M}, \tilde{a}\left(\overline{M}\right)\right]$ and $\Pi[M, \tilde{a}(M)]|_{M<\overline{M}}$ can only intersect once as w_t increases. Call the formal labour market wage which equates the two profit functions w^C. For all w_t above w^C, $\Pi[M, \tilde{a}(M)]|_{M<\overline{M}} > \Pi\left[\overline{M}, \tilde{a}\left(\overline{M}\right)\right]$. ■

Following on from the proof of Proposition 4, size unambiguously falls. However, in the complements region, $\Pi_{Ma} > 0$, and so $da_t^*/dw_t < 0$. Activity also declines. This completes the proof. ■

Proof of Proposition 7

For (13) to be satisfied it must be the case that, for all σ:

$$\frac{\partial g_t}{\partial s}(\sigma) = \phi'[a_t(\sigma)]\frac{\partial a_t}{\partial s}(\sigma) + \frac{1}{\sigma}\frac{\partial a_t}{\partial s}(\sigma)$$

Integrating over $[\hat{\sigma}_t, \sigma]$ yields:

$$g_t(\sigma) - g_t(\hat{\sigma}_t) = \phi[a_t(\sigma)] - \phi[a_t(\hat{\sigma}_t)] + \int_{s=\hat{\sigma}_t}^{\infty} \frac{1}{s}\frac{\partial a_t}{\partial s} ds$$

Now, it must be the case that the marginal member's participation constraint binds, i.e. that:

$$g_t(\hat{\sigma}_t) - \phi[a_t(\hat{\sigma}_t)] - \frac{a(\hat{\sigma}_t)}{\hat{\sigma}_t} = w_t$$

If the organization pays less, the individual will not join. If they pay more, they could maintain the same organization size and activity profile, but increase profits by reducing the wage the marginal member receives. Substituting yields:

$$g_t(\sigma) = w_t + \phi[a_t(\sigma)] + \frac{a(\hat{\sigma}_t)}{\hat{\sigma}_t} + \int_{s=\hat{\sigma}_t}^{\infty} \frac{1}{s}\frac{\partial a_t}{\partial s} ds$$

Finally:

$$\frac{\partial}{\partial s}\left[\frac{a_t(s)}{s}\right] = \frac{1}{s}\frac{\partial a_t}{\partial s} + \frac{a(s)}{s^2}$$
$$\iff \frac{1}{s}\frac{\partial a_t}{\partial s} = \frac{\partial}{\partial s}\left[\frac{a_t(s)}{s}\right] - \frac{a(s)}{s^2}$$

Substituting this into $\int_{s=\hat{\sigma}_t}^{\infty} (1/s)(\partial a_t/\partial s) ds$ yields:

$$g_t(\sigma) = w_t + \phi[a_t(\sigma)] + \frac{a_t(\sigma)}{\sigma} + \int_{s=\hat{\sigma}_t}^{\infty} \frac{a_t(s)}{s^2} ds$$

as required. This completes the proof. ■

Proof of Proposition 8

The organization's profit function is:

$$\Pi[M_t, a_t(\sigma)] = \max_{M_t, a_t(\sigma)} \left(\int_{\sigma=\hat{\sigma}(M_t)}^{\infty} r[M_t, a_t(\sigma)]\lambda e^{-\lambda\sigma} d\sigma - \int_{\sigma=\hat{\sigma}(M_t)}^{\infty} \left\{ w_t + \phi[a_t(\sigma)] + \frac{a_t(\sigma)}{\sigma} + \int_{s=\hat{\sigma}_t}^{\sigma} \frac{a_t(s)}{s^2} ds \right\} \lambda e^{-\lambda\sigma} d\sigma \right)$$

Integrating the information rent by parts simplifies the expression:

$$\int_{\sigma=\hat{\sigma}(M_t)}^{\infty}\int_{s=\hat{\sigma}_t}^{\sigma}\frac{a_t(s)}{s^2}ds\lambda e^{-\lambda\sigma}d\sigma=\left[-\int_{s=\hat{\sigma}_t}^{\sigma}\frac{a_t(s)}{s^2}ds\times e^{-\lambda\sigma}\right]_{\sigma=\hat{\sigma}_t}^{\infty}+\int_{\sigma=\hat{\sigma}(M_t)}^{\infty}\frac{a_t(\sigma)}{\lambda\sigma^2}\lambda e^{-\lambda\sigma}d\sigma$$
$$=\int_{\sigma=\hat{\sigma}(M_t)}^{\infty}\frac{a_t(\sigma)}{\lambda\sigma^2}\lambda e^{-\lambda\sigma}d\sigma$$

Substituting into the organization's profit function:

$$\Pi[M_t,a_t(\sigma)]=\max_{M_t,a_t(\sigma)}\left(\int_{\sigma=\hat{\sigma}(M_t)}^{\infty}r[M_t,a_t(\sigma)]\lambda e^{-\lambda\sigma}d\sigma\right.$$
$$\left.-\int_{\sigma=\hat{\sigma}(M_t)}^{\infty}\left\{w_t+\phi[a_t(\sigma)]+\frac{a_t(\sigma)}{\sigma}+\frac{a_t(\sigma)}{\lambda\sigma^2}\right\}\lambda e^{-\lambda\sigma}d\sigma\right)$$

Now, for every $\sigma\geq\hat{\sigma}_t$, $a_t(\sigma)$ must maximize the profit that the member generates for the organization. Taking first-order conditions:

$$r_a[M_t,a_t(\sigma)]-\phi'[a_t(\sigma)]-\frac{1}{\sigma}\left(1+\frac{1}{\lambda\sigma}\right)\equiv 0$$

The associated second-order condition is:

$$r_{aa}[M_t,a_t(\sigma)]-\phi''[a_t(\sigma)]<0$$

consistent with a maximum.

How does $a_t(\sigma)$ vary with M_t? Once again, call $\tilde{a}_t(\sigma,M_t)$ the restricted demand for activity for a member with willingness to commit crime σ:

$$\frac{\partial\tilde{a}_t}{\partial M_t}=-\frac{r_{Ma}}{r_{aa}-\phi''(\tilde{a}_t)}$$

So $\partial\tilde{a}_t/\partial M_t>0$ if and only if $r_{Ma}>0$. Now:

$$\frac{dr_{Ma}(M_t,\tilde{a}_t)}{dM_t}=r_{MMa}+r_{Maa}\frac{\partial\tilde{a}_t}{\partial M_t}$$
$$=r_{MMa}-r_{Maa}\frac{r_{Ma}}{r_{aa}-\phi''(\tilde{a}_t)}$$

So, if $r_{MMa}<0$ and $r_{Maa}<0$, then $r_{Ma}(M_t,\tilde{a}_t)$ falls as M_t rises when $r_{Ma}>0$. If $r_{Ma}<0$ then an increase in M_t cannot make it positive.

Define $\overline{M}(\sigma)$ as follows:

$$\overline{M}(\sigma) = \underset{M \in [0,1]}{\arg\min} \{M : r_{Ma}[M, \tilde{a}(\sigma, M)] = 0\}$$

Then, for $M_t < \overline{M}(\sigma)$, $\partial \tilde{a}_t / \partial M_t > 0$. For $M_t > \overline{M}(\sigma)$, $\partial \tilde{a}_t / \partial M_t < 0$. This completes the proof. ■

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