# Models of Energy in the United Kingdom

by

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“It is the journey that matters not the arrival.”- T. S. Eliot

To my Grandmother

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## Abstract

In this thesis, I examine the impact of energy price shocks in the United Kingdom using a New-Keynsian Dynamic Stochastic General Equilibrium (DSGE) model and a classic Real Business Cycle (RBC) model. The models are augmented with real rigidities and driven by exogenous shocks. Chapter 1 examines a DSGE model with New-Keynesian Philips Curve with three outputs of energy (petrol and utility), and non-energy output, using filtered data (1981:Q1-2014:Q4) of the UK. Chapter 2 examines a two-sector (RBC) model of energy intensive output and non-energy intensive output, using unfiltered data (1990:Q1-2014:Q4) of the UK. The models are econometrically estimated using indirect inference test that includes Monte Carlo simulation.

I show how the study can be quantitatively applied by evaluating the effects of different shocks on output, relative prices and interest rate. I also show how energy price shocks affect output, asset prices and aggregate consumption in a classic RBC model. By decomposition, the changes in these variables caused by each of the structural shocks showed that a fall in output during the financial crisis period 2008:Q2 to 2009:Q4 was driven by energy price shocks and sector-specific productivity shocks. Conversely, in the DSGE model with NKPC, the changes in these variables caused by each of the structural shocks showed that a fall in output during the financial crisis period 2008:Q2 to 2009:Q4 was driven by domestic demand shocks (consumption preference, government spending and capital adjustment cost), oil prices shock and world demand shock.

I found why the energy price shock reduces GDP in the models: In NKPC model with stationary shocks this is only a temporary terms of trade shock and so GDP only falls briefly, such that, the UK can borrow against such a temporary fall. In the RBC two-sector model, I found, it must be that the terms of trade rise permanently when world energy price increase as it is non-stationary and there is no other way to balance the current account than to reduce absorption due to lack of substitute for energy inputs. Finally, I found that the RBC two-sector model with non-stationary shocks performs better than NKPC model with stationary shocks. The performance can be credited to using unfiltered-data on the RBC model. This thesis show how estimated models can create additional input to the policymaker's choice of models through the economic shocks' effects of the macroeconomic variables.

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## 1.1 Introductory Chapter

Changes in energy prices (crude oil and gas) over the past decade is a concern for economists because of its high volatilty. This has resulted in the neeed for a new methodology to study the validity of the macroeconomic models and their assumptions. Soaring oil prices caused inflationary pressures, slowed economic growth, and created global disparities. Historically, energy prices increase the price of other goods at the same time because oil is used for the movement of most goods, as well as raw materials for extracting minerals infrastructural construction. As energy prices increase, the price of food and many other items also rise. This increases the expectations of inflation. The study of energy prices in a macroeconomic model will likely make the improvement, in the negative effects expected to be found in the study, by giving households, firms and monetary authorities a window to plan for energy price changes. This is because the public sector (central bank) and the private sector researchers, now see the price of oil as one of the main variables for macroeconomic study and in assessing macroeconomic risks. Energy prices have also directly affected other macroeconomics variables, such as exchange rates, foreign demand of goods and foreign exports prices which this study will emphasize. A continual increase in energy prices will result in a higher terms-of-trade shock in a net-energy-importing economy, like the United Kingdom. This will result in a persistent fall of the real exchange rate (Chaudhuri and Daniel 1998) that will put pressure on prices through cheaper imports.

Conversely, there has been a downward trend in crude oil prices in recent months, since December 2014. The decline in the oil prices has positive significant impacts in reducing costs in energy intensive sectors such as transportation and manufacturing. Declining oil prices are also favourable to economies that are importers and net-importers of oil, such as the United Kingdom, China, India and Japan. However, it is bad news for oil dependent economies, such as Nigeria, Venezuela, and Kuwait. The second quarter of 2015 UK CPI report showed that inflation in the energy intensive sectors fell by 1.8%. Empirical studies show that energy prices are non-stationary with high volatility as is evident in the past decade. A good example is that it took only five months, from July 2014 – December 2014, for the price of oil to fall from about $100 a barrel to $52 a barrel. Oil prices also fell from about $150 a barrel in 2008Q1 to under $40 a barrel in 2009Q1. Conversely, oil prices quickly reversed course, climbed steadily and reached more than $75 a barrel in 2009. Empirical studies shows that high oil prices were sufficient to explain the recent financial crisis, of 2008-2009. The inability of macroeconomic models to predict the crisis is one of the major reasons that economic models are under scrutiny. However, the study of Le, Meenagh, Minford and Ou (2013) suggested that an economist using a DSGE model for evaluation should take such weakness as a positive because dynamic linear models mirror the actual situation of an economy that includes recessions and booms.

In the next chapters, I will review related literature with regards to macroeconomics dynamic general equilibrium model (DSGE) evaluation methodology. I will also discuss some problems that are facing the model and how modellers were able to come up with solutions. I then present an evaluation of two different small-open economy DSGE models. The first model is a DSGE model with a New Keynesian Philips curve (NKPC) that incorporates oil and a gas (energy) producing firms together with non-energy producing firms. The study is carried out on stationary data of the United Kingdom. In the next chapter, I evaluate a two-sector model, with an energy-intensive sector and energy extensive (non-energy) sector, of the United Kingdom using nonstationary data.

## 1.2 Literature Review

### 1.2.1 Volatility of Energy prices

Volatile energy prices have brought concerns about slower economic activity and increased domestic prices to world economies. Oil as a feedstock and transportation fuel is the most significant commodity in terms of economic effects. Historical data shows oil-price spikes precede economic recessions. This occurred in the 1973, 1981, 1991 and 2008 recessions. Hamilton (2009) holds the opinion, many authors agree, that the high oil prices are partly the cause of the 2008 recession. There has been a long history of relating recessions to oil price shocks as well as monetary policy shocks. This is because most recessions are influenced by rising oil prices and by a tightening of monetary policy (Hoover and Perez 1994, Barsky and Kilian 2002, Killian and Vigfusson 2014).

Figure 1 World growth in oil price, UK growth in GDP, UK Inflation rate and UK Interest rate

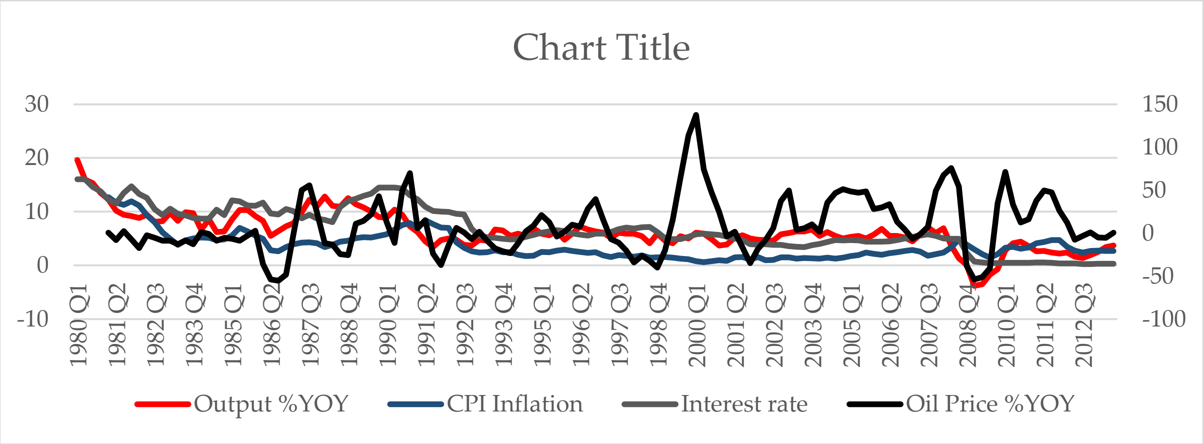

Figure 1 shows growth in UK output (percentage change year on year, %YOY), CPI inflation, interest rate and world growth in oil price (percentage change year on year, %YOY) from 1980: Q1 to 2013: Q1. This covers the great moderation period where the UK had the classic boom and bust of the late 1980's and early 1990's and extends beyond the 2008-2009 financial crisis. As oil prices rise, central banks are expected to tighten monetary policy. Borrowing rate is also expected to increase since investors demand higher interest rates, with an expectation of higher inflation. However, I did not find empirical evidence of Bank of England, like the Federal Reserve, responding to rising energy prices in the past. In the past thirty years, many studies have tried to examine the effects that oil shocks have had on the macroeconomy. Studies have established that oil shocks appear to have significant impacts on the economy. Similar studies, on oil shock, (Bernanke et al., (1997), Killian (2002), Hamilton (2009)) found that these shocks seem to have a lesser effect on output, interest rates and inflation during the great moderation period.

A structural break evidence shows data from 1986 with the estimates of the peak output impact decreasing from between 1 and 1.5% of GDP down to between 0.3 and 0.5%. The data from 2008: Q1, show that as the oil price increased, the output of the UK economy declined. As inflation increased and with interest rates high, it would be possible to conclude that the economy was heading for stagflation. However, the Bank of England was quick to respond to the situation by changing its monetary policy. As Killian and Vigfusson (2014) stated, that, most recessions are preceded both by higher energy prices and by a contraction of monetary policy and/or of financial markets, it is evident during the period of the recent financial crisis of 2008 as seen on figure 1.

### 1.2.2 DSGE models as standard tools of economic research

In macroeconomics, RBC/DSGE models have now become a standard research tool. These models highlight the dependency of existing choices on expected potential outcomes. Their use has spread from academic groups to the policymaking community. However, the general public is not very familiar with these models. DSGE models are now playing a key role in the formulation of monetary and fiscal policies at many of the world's central banks. Fundamentally, DSGE models are proposed to be constructed from microeconomic foundations that may incorporate simple (ah-hoc) fiscal and monetary rules. DSGE models have been used to explain a variety of macroeconomic problems. They have also been used to analyse the effects of fiscal and monetary policies and business cycle fluctuations.

The introduction of three revolutionary ideas by Kydland and Prescott (1982) changed macroeconomic research. The ideas from their seminal paper include: (i) The studying of business cycles using dynamic general equilibrium models based on the previous work by Lucas and Prescott (1971). These models describe economic agents that function in competitive markets which can form rational expectations about the future. (ii) The second key idea was the possibility of combining the business cycle and theory of growth by maintaining that real business cycle models are consistent with the empirical regularities of long-run growth. (iii) The third key idea was that it is possible to go far beyond the qualitative analysis of model properties using stylized facts that ruled theoretical work on macroeconomics until 1982. Hence, since then, researchers have now thought of how to take DSGE models to data. In order to capture important properties of the data, these models often also combine several nominal and real frictions such as rigid wages and prices, habit formation in labour choices and consumption, and adjustment costs in capital and capital utilisation. It also suggests that it is possible to calibrate models with parameters generated from microeconomic studies and long-run properties of the economy. These calibrated models can then be used to produce simulated data that can be matched with actual data.

DSGE-based models have also come to be widely used as laboratories for policy analysis[1] in general and, especially, for the discussion of the best fiscal and monetary policy. These policy implications echoed the fact that DSGE models represented an important step in realizing the challenge put out by Robert Lucas (Lucas (1980)) when he suggested that 'one of the functions of theoretical economics is to offer a well specified, artificial economic system that can serve as laboratories where policies that are costly to investigate in real life economies can be tested out at an affordable cost.'

The fluctuation of the DSGE model due to shock processes is a concern for modellers. The persistence of estimated shocks and the close mirroring of the path of

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[1] "DSGE models have become a workhorse for studying various aggregate economic phenomena." Chang, Doh and Schorfheide (2006).

one observable variable is a concern. One cannot tell whether these shocks depict aggregate uncertainty, or if it is a misspecification. An outstanding specification of the law of motion will remove the model misspecification, particularly for general time-series models such as vector-autoregressive models (VARs). Empirical results show that relaxing the restrictions of exogenous shocks exhibit AR(1) improves the fit of a DSGE model. Smets and Wouters (2007) use an ARMA mark-up shock to improve the model fit. Del Negro and Schorfheide (2009) allowed the exogenous government spending shock to follow a higher-order autoregressive process. Le, Minford and Wickens (2009) stated that one of the ways that a model is taken seriously is through the shock selection. They suggested how researchers should select shocks for a DSGE model when taking the model to data by assuming measurement errors[2] which the model's shock can account for in the model.

Several authors, including Hamilton (1996 and 2003) and Killian and Vigfusson(2014), stated that modelling the relationship of real output is important in explaining the role of oil price shocks. They mentioned that linear dynamic stochastic general equilibrium (DSGE) models assign low explanatory power to oil price fluctuations. This criticism can be overlooked because, since the financial crisis, several attempts have been made to incorporate oil into DSGE models.

Millard (2011) estimated an energy model in the United Kingdom using the Bayesian method. However, he found that energy price shocks (oil prices and gas prices)

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[2] Measurement errors means strictly that a variable is mis-measured, it is not different from the prediction of the equation.

have little effect on the variability of output and inflation. Other foreign shocks such as world demand shock made little contribution to output variability. Nonetheless, he found that the effects of higher world energy prices depends on the responses of monetary policy to increasing energy prices. The rate of self-sufficiency in energy also makes a great difference through the impacts on consumption and the real asset prices. His findings are consistent with Harrison et al., (2011). Other authors used the United Kingdom data in the estimation of their DSGE models, such as Harrison et al., (2010) and Faccini *et al.,* (2011). The model of inflation, used in the models estimated by these authors, is built around the 'New Keynesian Phillips Curve' (NKPC), which implies that inflation depends on lagged inflation, expected future inflation and the real marginal cost. In these models, real marginal cost will also be equivalent to real unit labour costs, although, as shown by Faccini *et al.,* (2011) and Kamber and Millard (2010), since energy and labour are complementary inputs to production, the real marginal cost is affected by changes in energy prices. Therefore, movements in energy prices will be significant for inflation. Since consumers are also users of energy, any shift in energy prices will have a direct impact on CPI inflation which will not be affected by the NKPC. The effects, from Figure 1, on CPI inflation can be seen from 2007: Q3 to 2008: Q3 as oil prices rise in 2007: Q3 to 2008: Q2.

Kim and Loungani (1992) and Finn (1995) study the significance of energy price shocks using closed economy real business cycle (RBC) models, with an emphasis on the United States. They find that energy shocks can provide little significance in explaining the real macroeconomic aggregate fluctuations in the economy. Conversely, the study of De Miguel, Manzano and Martín-Moreno (2003, 2005) finds that where they proposed a small open economy RBC model, the oil price shocks are highly significant in explaining aggregate fluctuations. Their results show that oil shocks can explain a significant percentage of GDP fluctuations in many southern European countries. Their models also replicate the cyclical path of the periods of oil crisis in the European economies. The rise in the relative price of oil had a negative impact on welfare, mostly in the southern European countries, which historical data relates to a lax monetary policy in oil crisis periods.

### 1.2.3 Methodologies of Evaluating DSGE Models

Minford (2006) outlines the methods of evaluating a DSGE model. One way is to treat the structural model is as a true model that follows the econometric method where the researcher asks the question, *how false is it?* Another way is to treat the DSGE model as a false model and then ask the question *how true is the model?* This method is the calibration method. The main difference in the two methods is the null hypothesis questions put forward by Canova (1994).

The econometric approach goes back over seventy years ago to the procedure of Haavelmo (1944). The evolution of this problem arises from the *stochastic singularity* issue, when written in state-space form, where the number of exogenous shocks in the model is less than the number of observable variables. This not been an issue recently since Smets and Wouters (2003) developed a model with ten structural shocks. The model can be estimated by the Kalman filter Algorithm for shock decomposition of the likelihood. Sargent and Hansen (2004) gave a detailed procedure for this evaluation. One of the shortcomings of this approach is the misspecification that comes with a standard DSGE model. The estimated parameters of the model show no consistency which makes the economic study irrelevant. There is also a case of partial identification that faces structural models due to little information about the model's structural parameters.

There are four groups of the calibration method, as classified by Canova (2005), namely: (1) $R^2 - type$ approach, (2) sampling variability of the actual data, (3) sampling variability of the simulated data and (4) sampling variability of both actual data and simulated data.

The $R^2 - type$ approach, measures goodness of fit ($R^2 - type$). The Watson (1993) method was developed to assess the statistical logic that the DSGE model is not true through an approximation of the stochastic process. This method depends on the number of shocks that are added to the model to measure the autocovariance from the implied shocks to match the autocovariance of the actual data. The procedure is to make the model as close to the actual data as possible. However, this method ignores non-linearity and the variance in conditional second and higher moments. There is also a reported shortcoming of the model due to lack of information provided when the need for re-specification of the model arises.

Christiano and Eichenbaum (1992), Rebelo (1993), among a few other authors responded to criticism of the calibration technique that structural parameters are assumed to be known with certainty by developing an evaluation method with uncertainty. They used conventional econometric methods to estimate a vector of structural parameters to fit their DSGE model with Hansen (1982) *Generalized Method of Moments* (GMM) and J-statistic. They developed a testing method to evaluate if the testing method comes from variability of sampling or from misspecification of the DSGE model. However, the use of GMM and the J-statistic requires stationary data time-series that need some kind of filter or differentiation for this condition to hold.

Diebold, Ohanian and Berkowitz (1998) develop a re-sampling method to extend the Watson (1993) method. They construct measures of fit based on the sample variance of the model data through long series simulations generated by the *Cholesky factor bootstrap* algorithm. The authors reported that the real macroeconomic data, interest rate and exchange rate, display non-linear behaviour that cannot fit the resampling method.

Calibration as testing provides a way to judge the distance between the statistics of a simulated DSGE model, $\hat{S}_x(\theta, y_t)$ and the actual model $\hat{S}_{y,T}$, where $\hat{S}_{y,T} \overset{P}{\rightarrow} S_y$. A measure of fit can be attained by randomization of the stochastic process of a DSGE model $y_t$. One can use a Monte Carlo technique to estimate the distance between the simulated and the actual models. The sequence of residuals is also drawn from the hypothetical distribution to calculate the simulated distribution while ordering the sequence numerically. They then check if the actual model falls within the simulated distribution or count the number of replications which gives the calibration test (Gregory and Smith, 1991). If the model shows a poor approximation of the data, that is not good enough. The simulated distribution will be far away from the simulated distribution (Minford, 2006). Gregory and Smith (1993), Oderlind (1994) and Colgey and Nason (1994) have also used this evaluation method on their, respective, DSGE models. Canova (1994, 1995) augmented the stated method with uncertainty of parameters, which caused criticism among DSGE modellers. A simulated quasi-maximum likelihood was developed by Smith (1993) as an estimation procedure on a non-linearized DSGE model that encompasses its own measure of fit. The parameters are chosen for the density of the simulated data to fit the density of the actual data. A VAR with identically independently distributed (i.i.d.) errors is selected to estimate the true conditional density due to its computational advantages.

Canova and De Nicolo (1995) evaluate a DSGE model by a resampling method based on the variability of a combination of actual and simulated data. A simple bootstrap technique is used to obtain the empirical distribution of the parameters. The evaluation method of variability of actual and simulated data is the method that was employed following the work of Le, Minford and Wickens (2009), and Le, Meenagh, Minford and Wickens (2010, 2011, 2012) to estimate their DSGE models of stationary and non-stationary data, respectively. A clear quantitative approach is outlined in the subsequent chapters.

### 1.2.4 Identification in a DSGE Model

An economic model can be exactly identified, over-identified or under-identified (not identified). It is exactly identified if and only if all of its coefficients can be derived exclusively from the solution of its reduced-form equation. It is over-identified if there is more than one set of structural parameters that can be estimated from the reduced-form solution. It is not identified (or under-identified) if it is not likely to estimate all of the structural parameters from the solution of the reduced-form equation. This includes situations where it may be possible to derive a subset of structural parameters from the solution of the reduced-form equation. Which of these situations prevails is determined prior to estimation. These principles also apply to DSGE models. However, there will be an extra feature that results from the necessity to account for the conditional expectations of future endogenous variables that initially include solving the model to take out the expected variables. If the DSGE model is over-identified, the solution is, in effect, a restricted reduced form; if the DSGE model is exactly identified then it is identical to an unrestricted reduced form; and if the DSGE model is under-identified then it is not possible to derive all of the structural parameters from the unrestricted reduced form.

Identification in a DSGE model is less transparent in a log-linear model as compared to the identification in a linear simultaneous equation model. The early literature on the DSGE has paid little attention to identification. Recent authors have found that objective functions are less informative with regards to structural parameters such as Philips curve coefficients or monetary policy rule parameters. The lack of transparency is seen in the system matrices of a given state-space[3] representation that are complicated nonlinear functions of DSGE model parameters that the most unrealistic DSGE model can only be evaluated numerically. Canova and Sala (2009) stated identification problems in New Keynesian DSGE models that were not globally identifiable but locally identifiable, for many values as a simple example. Furthermore, the work of Le, Minford and Wickens (2013) proposed a clear understanding of identification from its basics that goes back to Working (1927)[4].

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[3] State-Space Representation: Following log-linearized equilibrium conditions, the solutions to the rational expectations difference equations follows a state-space representation form of:

$$y_t = \Psi_0(\theta) + \Psi_1(\theta)t + \Psi_s(\theta)s_t$$

$$s_t = \Phi_1(\theta)s_{t-1} + \Phi_\epsilon(\theta)\epsilon_t$$

where $y_t$ is a vector of observed endogenous variables, e.g. GDP or Inflation; $s_t$ contains unobserved exogenous shock processes and unobserved endogenous state variables in the model.

[4] Le et al., (2013) prescribed the idea to rewrite Working (1927) model in terms of shocks as:

$$y_t = \beta_{11}\pi_t + \varepsilon_{d,t}$$

$$y_t = \beta_{21}\pi_t + \varepsilon_{s,t}$$

where $\beta s$ are constants, $\pi$ is price, $y$ is the quantity outputs. Given that, the above equations make the structural equations while the $\beta s$ make the structural parameters. With directly observed exogenous shocks, the model is identified because no linear combination is confused with either equation, and the shocks are different.

Assuming the supply equation is:

$$y_t = \beta_{21}\pi_t + \beta_{22}\varepsilon_{d,t} + \beta_{23}\varepsilon_{s,t}$$

This will make the linear combination not distinctive with either equation. The substitution of the true supply equation will give a linear combination of:

$$y_t = [\theta\beta_{11} + (1-\theta)\beta_{21}]\pi_t + [\theta + (1-\theta)\beta_{22}]\varepsilon_{d,t} + (1-\theta)\beta_{23}\varepsilon_{s,t}$$

which obtains the same reduced-form as:

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \frac{1}{\beta_{11} - \beta_{21}} \begin{bmatrix} -(1-\beta_{22}) & \beta_{23} \\ -(\beta_{21} - \beta_{11}\beta_{22}) & \beta_{11}\beta_{23} \end{bmatrix} \begin{bmatrix} \varepsilon_{d,t} \\ \varepsilon_{s,t} \end{bmatrix}$$

In principle, DSGE models may have very few or no exogenous variables. The exogenous errors in a DSGE model do not come from the model's inaccuracy, but are rather omitted exogenous variables to allow for instrumental effects in the model's feature. This is what makes shocks significant in a near perfect (DSGE) model since they are the only exogenous variables. Exogenous variables will be treated as errors since they will be directly observed from the data. The treatment of shocks is completely different given the mass of data that provides potential paths for exogenous variables. Identification will be investigated with knowledge of exogenous variables. The reduced-form solution of a DSGE model can be assumed as a function the exogenous variables to examine identification. Given the model parameters and data, the model shocks are extracted from the model and data and the exogenous shocks are a function of the model parameters.

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Hence, what the equation states is similar to Working (1927) when one does not impose a restriction, exclude the demand shock and the supply equation is not identified. If the supply equation is to be changed, the indirectly observed exogenous supply error must be also be modified as opposed to the Working (1927) technique.

Assuming the true model above, a linear combination of the two equations and substituted true supply equation will obtain the following supply equation:

$$y_t = \beta'_{21}\pi_t + \beta'_{22}\varepsilon_{d,t} + \beta'_{23}\hat{\varepsilon}_{s,t}$$

where $\beta'_{21} = [\theta\beta_{11} + (1-\theta)\beta_{21}], \beta'_{22} = [\ \theta + (1-\theta)\beta_{22}], \beta'_{23} = (1-\theta)\beta_{23}$

and $\hat{\varepsilon}_{s,t} = \dfrac{1}{\beta'_{23}}\{\beta_{23}\varepsilon_{s,t} - [\theta(\beta_{11} - \beta_{21})]\pi_t - \theta(1-\beta_{22})\varepsilon_{d,t}\}$

The reduced form equation of the model is given as:

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \frac{1}{\beta_{11} - \beta'_{21}} \begin{bmatrix} -(1-\beta'_{22}) & \beta'_{23} \\ -(\beta'_{21} - \beta_{11}\beta'_{22}) & \beta_{11}\beta'_{23} \end{bmatrix} \begin{bmatrix} \varepsilon_{d,t} \\ \hat{\varepsilon}_{s,t} \end{bmatrix}$$

In a case where a linear combination cannot be distinguished with the true supply equation, one can verify that this falls back to:

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \frac{1}{\beta_{11} - \beta_{21}} \begin{bmatrix} -(1-\beta_{22}) & \beta_{23} \\ -(\beta_{21} - \beta_{11}\beta_{22}) & \beta_{11}\beta_{23} \end{bmatrix} \begin{bmatrix} \varepsilon_{d,t} \\ \varepsilon_{s,t} \end{bmatrix}$$

One can see clearly the same reduced form despite being generated from different exogenous shocks and a different set of structural parameters, hence not identified.

### 1.2.5 Overcoming Identification

The suggestion of overcoming the lack of identification is for econometricians to use inferential procedures that are robust to a potential lack of identification when taking a model and data as given. Dreze (1974) opined that collecting richer data or resorting to more restrictive theory should be considered by econometricians worried with inference about parameters that are not identified. Lubik and Schorfheide (2004, 2007) demonstrated how restrictive theory leads to identification while there is a disagreement between authors if the application of such restrictions is correctly imposed in empirical studies.

Iskrev (2010) and Komunjer and Ng (2009) contributed to the issue of identification by developing 'necessary and sufficient conditions for identification' of DSGE model parameters. These conditions compare to the rank and order conditions that exist for simultaneous equation models but focus on a linear DSGE model with Gaussian innovations that will be cast into the state-space form. Iskrev (2010) developed a condition for identification based on the direct relationship of the parameter vector $\theta$ and the first and second population moments $\mathrm{m}_T(\theta)$ of a sequence observations $Y_{1:T} = [y_1, \dots, Y_t]'$. He stated that a sufficient and necessary condition for a global identification is $m_T(\tilde{\theta}) = m_T(\theta)$ for each pair $(\theta, \tilde{\theta})$. However, if the condition is in an open neighbourhood of $\theta$ only, then one can say $\theta$ is locally identifiable. Given a linear state-space form, the identification condition is necessary for normally distributed structural shocks $\epsilon_t$ and the initial state $s_0$. If $m_T(\theta)$ can be continuously differentiated, then $\theta$ is, again, locally identifiable as long as the Jacobian matrix $\partial m_T(\theta)/\partial\theta'$ has a full column rank. However, as the parameters of a linearized DSGE model are non-linear, there is need for the rank condition to be verified for a large number of empirically significant parameter values. As stated, the example above is not globally identifiable but locally identifiable for local values of $\theta$, but the latter fails if $\theta_1 = 0$. The procedure by Iskrev (2010) can be applied in DYNARE to help the one in detecting identification issues in all distinctive cases where such issues are not easily solved analytically. It is of note that all parameters of Smets and Wouters (2007) pass the rank condition that included multi-collinearity and pairwise correlation analysis. There is a suggestion of a possible weak identification but no problem was highlighted in their model.

Komunjer and Ng (2009) contributed by extending the above condition, of Iskrev (2010) from a finite number of second moments loaded in $m_T(\theta)$, to infinite-dimensional auto-covariance sequence. This issue faced some difficulties, however, since state-space representation has identification issues. The solutions to such issues are available in software packages such as DYNARE and available to empirical macroeconomists. This is a sign of the evolution that the DSGE model literature has made in the past decade.

### 1.2.6 Optimal Route of Identification with DSGE models

I review this literature explicitly because it is the route I follow in my model evaluation. The explanation of the method will give the reader a good knowledge of how efficient my methodology is. However, I will not be repeating this in further chapters.

Le, Minford and Wickens (2013) developed the idea of identification with DSGE models by finding an alternative set of parameters and complementary shocks. In this way, it is possible to obtain the same reduced form equation for the true model and its true shocks. In order to find a reduced form for alternative sets, one takes the alternative parameters and generates the shocks that would enable it to fit the data sample. This provides the alternative structural representation of the model that is consistent with the data sample. The procedure is repeated many times to avoid a data shortage that will be used to for reduced form estimation of both the alternative and true models. An indirect inference hypothesis test is carried out on the two parameters sets to see if they are the same on all samples. A 95% confidence will reject 5% of the time if that is the case. If a parameter set is found with no difference, the model is not identified. If otherwise, the model is identified. This will include raising the power of the test.

The reduced form of a DSGE model can be in several forms. The aim of the reduced form is to show the data characteristics that are generated by the structural model. Identification will fail if the alternative structural model can generate data that has the same feature. The test determines whether the alternative False model can generate the data feature that is generated by the True model. It does this by, finding via simulation, the distribution of the data feature parameters for the False model compared with what it is for the True model. If the distributions are not dissimilar according to the test, the model is not identified. The test is whether the false parameters can be considered as true according to the Indirect Inference Test. How exactly one measure, the data features do not matter for the test's validity, provided one measure it in the same way for both True and False models. The only effect on the test would be on the power of the test that is reduced by a very inaccurate degree. VAR representations are used for the tests that show a high power against False models.

They presented a prototype New Keynesian model similar to Clarida, Gali and Gertler (1999). The model has three equations: Model (1)

$$\pi_t = \omega \mathrm{E}_t \pi_{t+1} + (1-\omega)\pi_{t-1} + \lambda y_t + \varepsilon_t^{\pi} \tag{1}$$

$$y_t = \mathrm{E}_t y_{t+1} - \frac{1}{\sigma}(r_t - \mathrm{E}_t \pi_{t+1}) + \varepsilon_t^{y} \tag{2}$$

$$r_t = \rho r_{t-1} + (1-\rho)\big(\gamma \pi_t + \eta y_t + \psi(y_t - y_{t-1})\big) + \varepsilon_t^{r} \tag{3}$$

The first representation of the model (1) is the New-Keynesian Philips curve. Assuming $\omega = 0$, then one can assume a backward-looking Philips curve and if $\omega = 1$ then it is a forward-looking Philips curve. The next equation is the demand equation followed by an interest rate rule with a smoothed interest rate by the parameter $\rho$. The Philips curve at the heart of the model is a subject of complex econometric arguments on whether it should be forward looking or backward looking[5]. The model also includes a problem of specification of error processes with regards to serial correlation. The arguments also includes identification issues that Le, Minford and Wickens (2005) provided a methodology for its solution.

The shocks follow AR(1) process:

$$\varepsilon_t^j = \rho_j \varepsilon_{t-1}^j + u_t^j \qquad (j = \pi, y, r)$$

A less complex version of the model is: (model 2)

$$\pi_t = \omega \mathrm{E}_t \pi_{t+1} + \lambda y_t + e_{\pi,t}, \quad \omega < 1 \tag{4}$$

$$y_t = \mathrm{E}_t y_{t+1} - \frac{1}{\sigma}(r_t - \mathrm{E}_t \pi_{t+1}) + e_{y,t} \tag{5}$$

$$r_t = \gamma \pi_t + \eta y_t + e_{r,t} \tag{6}$$

$$e_{j,t} = \rho e_{j,t-1} + \varepsilon_{j,t} \qquad (j = \pi, y, r) \tag{7}$$

where the model possesses five structural parameters and three autoregressive parameters. Thus, rewriting the model with a lag operator, $\mathrm{E}_t \pi_{t+1} = L^{-1} \pi_t$ gives:

$$\begin{bmatrix} e_{\pi,t} \\ e_{y,t} \\ e_{r,t} \end{bmatrix} = \begin{bmatrix} \pi_t \\ y_t \\ r_t \end{bmatrix} \begin{bmatrix} 1 - \omega L^{-1} & -\lambda & 0 \\ -\frac{1}{\sigma} L^{-1} & 1 - L^{-1} & \frac{1}{\sigma} \\ -\gamma & -\eta & 1 \end{bmatrix} \tag{8}$$

The solution of the model is, therefore:

$$q_t = A e_t \tag{9}$$

where $q'_t = [\pi_t y_t r_t]$, $e'_t = [e_{\pi,t}, e_{y,t}, e_{r,t}]$. The $A$ matrix is restricted with 9 elements and includes only 5 structural parameters while $\rho_j$ is generated from the shock processes. This implies that the model is over-identified. Assuming $\rho_j = 0$ for all $j$, then there will be another solution: model (3)

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[5] The papers of Gali et al., (2005), and Rudd and Whelan (2005) were based on these arguments. The Journal of Monetary Economics (Volumes 52, 6, 2005)

$$\begin{bmatrix} \pi_t \\ y_t \\ r_t \end{bmatrix} = \frac{1}{1 + \frac{\eta + \gamma\lambda}{\sigma}} \begin{bmatrix} 1 + \frac{\eta}{\sigma} & \lambda & -\frac{\lambda}{\sigma} \\ -\frac{1}{\sigma}\gamma & 1 & -\frac{1}{\sigma} \\ -\gamma & \eta + \gamma\lambda & 1 \end{bmatrix} \begin{bmatrix} e_{\pi,t} \\ e_{y,t} \\ e_{r,t} \end{bmatrix} \tag{10}$$

The solution shows the significance of shock dynamics in identification with the disappearance the parameter $\omega$, hence, not identified and the other parameters are termed as over-identified. Thus, without shock dynamics, the variables with future expectations will not appear in the model since their values will be zero which makes their coefficients disappear from the structural and reduced form equations. The solution of the model is similar to the model (2), less complex model. It includes two backward roots from the interest rate smoothing parameter and Philips curve indexation lag:

$$\begin{aligned} \Delta L = {} & \left(\frac{\sigma\rho+\psi}{\sigma}\right)(1-\omega)L^2 - \left[\left(\frac{\sigma\rho+\psi}{\sigma}\right) + (1-\omega)\left(1+\rho+\frac{\eta+\psi}{\sigma}\right)\right]L + \\ & \left(1+\rho+\left(\frac{\eta+\psi}{\sigma}\right)+\omega\left(\frac{\sigma\rho+\psi}{\sigma}\right)+\frac{\rho\lambda+(1-\rho)\gamma\lambda}{\sigma}+1-\omega\right) - \left[\frac{\lambda}{\sigma}+\omega\left(1+\frac{\eta}{\sigma}\right)\right]L^{-1} + \\ & \omega L^{-2} \end{aligned} \tag{11}$$

The solution will have two backward roots and two forward roots inside one full circle, given parameter values. The restricted model has seven structural parameters, with $\rho_j$ directly estimated from shocks, and is over-identified. The unrestricted model has 24 parameters with 6 coming from lagged endogenous variables and 18 coefficients from the errors $e_{j,t}$. Le et al., (2013) stated that an analytical identification can be carried out with smaller models, like this 3-equations model, but may be impractical with larger models, like the log-linearized form model of Smets and Wouters (2003 and 2007). They found that the Smets and Wouters model using the numerical approach to be over-identified. The impracticality of larger models is what motivated them to propose indirect inference on structural parameters as a numerical procedure[6] of resolving identification. The numerical approach is a way of resolving identification since the authors have taken that route[7]. Canova and Sala (2009) resolved identification based on properties of data implied impulse responses using maximum likelihood.

The route of overcoming identification by Le et al., (2013) reconciles with the numerical methodology of Canova and Sala (2009) on three points:

(i) The disappearance parameters may likely occur but not as often in DSGE models due to the lag parameters both in the model and in the shock processes.

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[6] The numerical procedure is as follows:
a) Generate a large number of samples of large size, by Monte Carlo sampling, from the true DSGE model that is being tested.
b) The sample implied VAR distribution is computed for a high order VAR on the maximum number of variables.
c) Carry out a Wald test to check whether there are DSGE models in the region of the true model that are not-rejected; if not then regard the DSGE model as identified.

[7] Furthermore, Le, et al., (2013) argued that the choice of model features to estimate is significant for a numerical approach to weak identification.
The procedure is to choose a VAR to describe the data, and the VAR coefficients as the important data properties; and then use indirect inference as the base of the estimation procedure. They maintain this allows one to check the identification of DSGE models rather accurately.

With errors having a univariate AR coefficient, this can easily be transformed into a VARMA(3,2):

$$q_t = \left(\sum \rho_j\right) q_{t-1} - \left(\sum \rho_j \rho_k\right) q_{t-2} + \left(\prod \rho_j\right) q_{t-3} + A \begin{bmatrix} (1-\rho_y L)(1-\rho_r L)\epsilon_{\pi,t} \\ (1-\rho_\pi L)(1-\rho_r L)\epsilon_{y,t} \\ (1-\rho_y L)(1-\rho_\pi L)\epsilon_{r,t} \end{bmatrix}$$

By substituting the solutions of the expected variables into model (2) and rearranging, the equation can be written as:

$$\begin{bmatrix} I & -A \\ 0 & I \end{bmatrix} \begin{bmatrix} q_t \\ e_t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} q_{t-1} \\ e_{t-1} \end{bmatrix}$$

(ii) The impulse responses of the model may not hold as much evidence for identification as a full set of VAR parameters.

(iii) The likelihood used by Canova and Sala (2009) appears to be not as well-determined as the Wald statistic used by Le et al., (2013).

### 1.2.7 Non-stationarity of observed energy shocks

Another point of note in this study is the non-stationary behaviour of oil prices which is related to exchange rates. The filtering of observed data is a standard practice before estimating a DSGE model to confirm that the data is stationary that will obviously produce a stationary residual of the structural model (Le, et al., 2012). Given that world prices are exogenous, and the world price of oil is non-stationary, a misrepresentation of this data will be difficult to uncover. A typical example is how the generally-accepted Hodrick-Prescott (HP) filter changes the lag structure of the data, creating cycles without the certainty of its occurrence. It was found by Christiano and den Haan (1996) that the use of HP filter causes persistent serial correlation in residuals, thereby, making the results of the study disappointing.

Most of the researchers that studied US data over the post-Bretton Woods period found evidence that there is a cointegration relationship between the real exchange rate and real oil prices. There is an agreement among researchers[8] that study the impact of real oil price behaviour to the non-stationary behaviour of the real exchange rate. The oil price tends to be the dominant source of persistent shocks and

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[8] See Amano and Van Norden (1988a) and (1988b), Chaudhuri and Daniel (1998) for evidence.

the nonstationarity of real exchange rates. Chaudhuri (2000) revealed that a significant relationship exists between real oil prices and real prices of primary commodities. His study showed that the nonstationary behaviour of real commodity prices is due to the nonstationary pattern of real oil prices. Evidently, this effect differs depending on the type of output produced. He emphasized that the results are the same even if oil is not being used directly in the production of output. He also noted that the oil price change may affect the prices of value-added output through the effect of the changes in oil prices on real exchange rates.

In conclusion, one can see that despite the DSGE models becoming significant in real business cycle economic analysis, it is important for the model to be identified. Identification is significant for both the model calibration as well as the statistical analysis. This is one of the areas that has been neglected until Canova and Sala (2009), Minford et al., (2009) made emphasis on. It is also imperative to note that world energy prices are nonstationary and therefore, to see the real effects of energy prices its data should be unfiltered.

# Chapter 1 Evaluation of a DSGE model of energy in the United Kingdom using stationary data

## 2.1 Introduction

The model that I propose closely follows the work of Millard (2011)[9] who augmented and estimated a model of the United Kingdom using a Bayesian estimation method. However, using the Bayesian approach includes a vague prior knowledge or even non-existence of it. The question of objectivity arises because different study use different priors[10]. The Bayesian method also involves high-dimensional integrals. Nevertheless, Bayesian inference that assumes proper priors does not necessitate identification as a condition, so long as the prior and posterior distribution have a total probability mass of one. The requirement in inference is that the curvature in the likelihood functions should be flat. However, challenges arise when a more sensitive inference occurs following a prior distribution choice. Secondly, a lack of identification ends up complicating the estimation of the model from the posterior draws. Variability is generated from the variability of the stochastic process. In a Bayesian framework, variability arises from model parameters uncertainty.

My aim is to use a completely different methodology to estimate this DSGE model. I will be using the indirect inference test method to estimate this model on United Kingdom stationary data. This is a procedure of variability of actual and simulated

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[9] The model was originally developed by Harrison et al., (2011) that studied the impact of permanent energy price increases on the UK economy using a calibrated DSGE model.

[10] This is evident in the estimation of this model, from Harrison and Oomen (2010) to Harrison, et al., (2011) to Millard (2011) since all used different priors.

data that follows the work Le, Minford and Wickens (2009). Unlike Bayesian estimation, my evaluation requires the observed data of the endogenous variables in the functional form in order to estimate the model residuals. I use similar observed data that was used by Millard (2011) but because of the evaluation approach, I used twice as much data as he employed. He also hard-coded[11] parameters estimated from the shock processes of the five foreign shocks as he estimated the model, which I did not. Lastly, an aggregation for consumer inflation is introduced, equation (49)[12]. This is an approach that will also focus on the effects of changes in all the output firms'[13] factors of production[14] on inflation that can be used to study how a central bank should react to changes in the prices of energy in order to attain its inflation target. I will estimate a macroeconomic model that can be used to quantitatively evaluate the impact of exogenous shocks, which includes energy prices among many others, on monetary policy as well as how inflation and output can respond to such shocks. Moreover, estimating the model showed how the shocks evolved in the long-run and the effects of the changes in output and inflation.

This is a single sector model with three different types of value-added goods. The study will look at the effects that the oil price shocks, among other shocks, will have on the price changes of goods, changes in output and monetary policy. This will be

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[11] Following Harrison and Oomen (2010), and Harrison, et al., (2011)

[12] Recommended by Professor Minford

[13] It is assumed in the model that there are three producers in the economy, given value-added produced which is sold according to sector specifics: Non-energy output producers, petrol producers and utility producers.

[14] The factors of production are capital, labour, imported intermediates and energy input. This is similar to Rotemberg and Woodford (1996) that included oil as a production input, although it represents a small portion of the total marginal cost and their result showed that oil had a huge impact on output.

analysed, in this study, by looking at the variation in output, inflation and interest rate in the UK economy during the crisis period. The study of Millard (2011) did not show the difference between the shocks that may have caused the oil price to increase. However, they showed that the response to oil prices in the UK was expected to be sensitive to changes in wage stickiness as well as the reaction of the policy-makers.

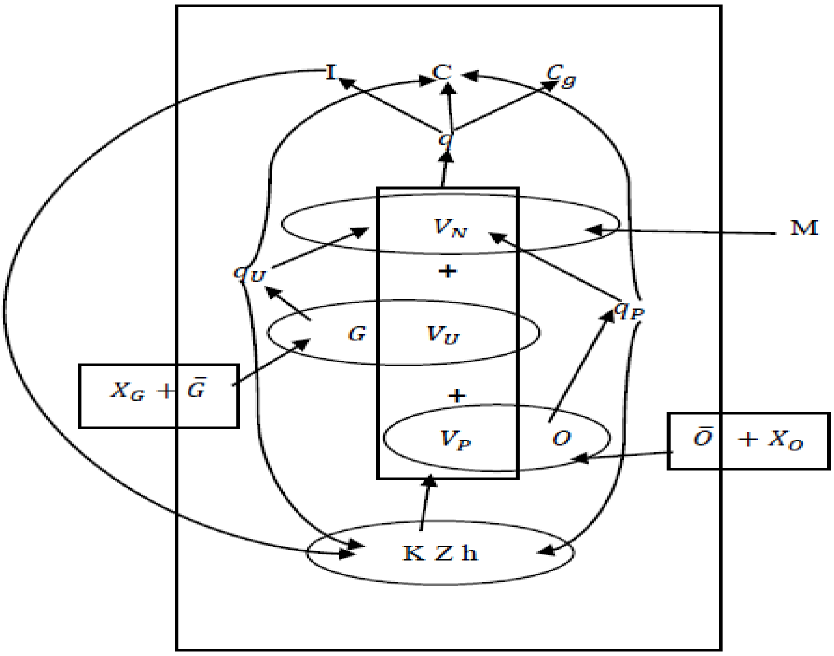

Figure 2[15] Model diagram

The UK economy, in this study, is characterized as a small open economy and also a primary producer of crude oil and gas (energy). This assumption may not be a reality since the production of oil and gas in the UK is in decline according to Webb (2013). The UK is a currently a net importer of oil and will continue for the next 20 years by about seventy-five percent. The continuous decline of energy resource

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[15] Harrison, et al., (2011)

extraction is likely to particularly effect domestic consumption and the exchange rate since energy prices will be changing permanently. As a result, it will have implications on the UK monetary policy.

Figure 2 shows how investment accumulates into the capital stock. It shows how the capital (K), capital utilisation rate ( z ) and labour hours ( h ) are pooled to produce value added (V). This is considered to be GDP in the model. Value added is distributed to the three producing firms: the non-energy goods sector ($V_n$ ); the utilities sector ($V_u$); and petrol sector ($V_p$ ). Value added is used with other inputs to produce other types of goods. The petrol sector uses value added ($V_p$ ) and oil (*O*) to produce petrol ($q_p$). The amount of crude oil used in UK petrol production is the total of the UK's endowment of oil ($\bar{O}$ ) and net trade in oil with the rest of the world ($X_O$). The utilities sector also uses value added ($V_u$) and gas ($\bar{G}$ ) to produce the utilities output ($q_u$ ) and the amount of gas combined in production comes from the endowment ($\bar{G}$ ) and net trade with the rest of the world ($X_g$ ). The energy output (including petrol and utilities) is combined with value added ($V_n$) and intermediate imports (*M*) to produce the final output (*q*) of non-energy (Gross GDP less energy). This final non-energy output is traded to households for consumption (C), for investment (*I*), to government ($C_g$ ) and to the rest of the world as exports (*X*).

## 2.3 The Log-Linearized model

### 2.3.1 The Household

The model prescribes households to consume the three final goods as they supply differentiated labour to all three firms. Households are also assumed to own the capital stock and make decisions about capital accumulation and utilisation. Proceeds from the sale of oil and gas on world markets are distributed lump sum to consumers. Also, it is assumed that the capital utilisation decision depends on the price of energy, following Finn (2000).

The consumption Euler equation:

$$\hat{c}_t = \frac{\psi_{hab}(1-\sigma_c)}{1+\psi_{hab}(1-\sigma_c)}\hat{c}_{t-1}\frac{1}{1+\psi_{hab}(1-\sigma_c)} + E_t\hat{c}_{t+1}\left(-\frac{\sigma_c}{1+\psi_{hab}(1-\sigma_c)}\right)\begin{pmatrix} i_t - E_t\pi_{c,t+1} \\ -\left(\frac{1}{\beta}-1\right) + \varepsilon_{b,t} \end{pmatrix} \tag{12}$$

$$\widehat{w}_{k,t} = \phi_z\hat{z}_t \tag{13}$$

The equation for capital accumulation shows lagged capital due to the assumption of capital adjustment costs:

$$\left(i_t - E_t\pi_{c,t+1} - \left(\frac{1}{\beta}-1\right) + \varepsilon_b\right) = \left(\frac{\varepsilon_k}{1-\delta+\chi_z} + (1+\varepsilon_k)\right)\chi_k\hat{k}_{t-1} - \tag{14}$$

$$\left(\frac{(1+\varepsilon_k)}{1-\delta+\chi_z} + 1\right)\chi_k\hat{k}_t + \frac{\chi_k}{1-\delta+\chi_z}E_t\hat{k}_{t+1} - \chi_k\varepsilon_k\hat{k}_{t-2} + \frac{\chi_k}{1-\delta+\chi_z}E_t\widehat{w}_{k,t+1} + \varepsilon_{inv,t}$$

Aggregate consumption is composed of consumption of non-energy, petrol and utilities.

Consumption of 'energy' will be given by:

$$\hat{c}_{E,t} = (1-\psi_P)\hat{c}_{U,t} + \psi_P\hat{c}_{P,t} \tag{15}$$

Hence, aggregate consumption is:

$$\hat{c}_t = (1-\psi_e)\hat{c}_{n,t} + \psi_e \hat{c}_{e,t} \tag{16}$$

Relative prices are given by:

$$\hat{P}_{U,t} = -\frac{1}{\sigma_e}\hat{c}_{n,t} + \left(\frac{1}{\sigma_e} - \frac{1}{\sigma_p}\right)\hat{c}_{E,t} + \frac{1}{\sigma_p}\hat{c}_{U,t} \tag{17}$$

and $$\hat{P}_{U,t} - \hat{P}_{P,t} = -\frac{1}{\sigma_p}\hat{c}_{U,t} + \frac{1}{\sigma_p}\hat{c}_{P,t} \tag{18}$$

The households assume to have an option of holding either foreign or domestic bonds, as trade in foreign bonds incurs quadratic costs. This results in the UIP condition:

$$E_t\hat{s}_{t+1} - \hat{s}_t = -\left(i_t - \left(\frac{1}{\beta} - 1\right)\right) + \chi_{bf} b_{f,t} + \varepsilon_{rf,t} \tag{19}$$

The model assumes household to be a monopoly supplier of differentiated labor. Therefore, households will set real wage as a mark-up over the marginal rate of substitution between consumption and leisure that is the percentage deviation denoted by *mrs*. This is subject to nominal wage stickiness and partial indexation of wages to inflation. Hence, wage inflation will be given by:

$$\dot{W}_t = \frac{\xi_w}{1+\beta\xi_w}\dot{W}_{t-1} + \frac{\beta}{1+\beta\xi_w}E_t\dot{W}_{t+1} - \left(\frac{\psi_w(1-\beta(1-\psi_w))}{\left(1+\frac{\sigma_w}{\sigma_h}\right)(1-\psi_w)(1+\beta\xi_w)}\right)(\hat{w}_t - mrs_t) + \varepsilon_{w,t} \tag{20}$$

where $$mrs = \frac{1}{\sigma_h}\hat{h}_t + \frac{1}{\sigma_c}(\hat{c}_t + \psi_{hab}(\sigma_c - 1)\hat{c}_{t-1}) \tag{21}$$

and real $$\hat{w}_t = \dot{W}_t + \hat{w}_{t-1} - \pi_{c,t} \tag{22}$$

wages

### 2.3.1 The firm

Production is assumed to be divided into three sectors of non-energy producing firm and energy producing firm:

#### 2.3.1.1 Non-energy producing firm

$$\hat{q}_t = \left(1-\alpha_q\right)\hat{B}_t + \alpha_q\hat{e}_t + \varepsilon_{a,t} \tag{23}$$

where
$$\hat{B}_t = (1-\alpha_B)\hat{V}_{n,t} + \alpha_B\hat{M}_{n,t} \tag{24}$$

and
$$\hat{e}_t = \hat{I}_{p,t} = \hat{I}_{u,t} \tag{25}$$

where $q$ denotes output of non-energy, and $\varepsilon_a$ represents the productivity shock. $B$ denotes bundle of value-added, $V_{n,}$, and intermediate imported goods, $M_{n,}$; $e$ denotes energy input in this sector, which will be given by (25). The cost minimization shows the demand curve for:

Value-added
$$\hat{V}_{n,t}=\hat{\mu}_t - \hat{p}_{vc,t} + \frac{1}{\sigma_q}\hat{q}_t + \frac{\sigma_q-1}{\sigma_q}\hat{B}_t + \frac{\sigma_q-1}{\sigma_q}\varepsilon_{a,t} \tag{26}$$

imports
$$\hat{M}_{n,t}=\hat{\mu}_t - \hat{p}_{m,t} + \frac{1}{\sigma_q}\hat{q}_t + \frac{\sigma_q-1}{\sigma_q}\hat{B}_t + \frac{\sigma_q-1}{\sigma_q}\varepsilon_{a,t} \tag{27}$$

energy
$$\hat{e}_t = \sigma_q\hat{\mu}_t + \hat{q}_t - \sigma_q\left(\psi_n\hat{p}_{p,t} + (1-\psi_n)\hat{p}_U\right) + (\sigma_q-1)\varepsilon_{a,t} \tag{28}$$

where $\mu$ is real marginal cost and $p_{vc}$ is the 'competitive' price of value-added (the marginal cost of producing it). Firms in the non-energy sector are also subject to nominal rigidities in their price-setting. In particular, each period they are only allowed to set their price optimally with a probability of $1-\chi p$. If they cannot change their price optimally, they partially index their price to lagged inflation.

The resulting NKPC is:

$$\pi_t = \frac{\beta}{(1+\beta\varepsilon)} E_t\pi_{t+1} + \frac{\varepsilon}{(1+\beta\varepsilon)}\pi_{t-1} + \frac{(1-\chi_p)(1-\beta\chi_p)}{(1+\beta\varepsilon)\chi_p}\hat{\mu}_t + \varepsilon_{\mu,t} \qquad (29)$$

**2.3.1.2 Value-added:**

The producers of value-added use capital to produce value-added, V: The equation (30) represents output.

$$\hat{V}_t = (1-\alpha_v)\hat{h}_t + \alpha_v(\hat{k}_{t-1} + z_t) \qquad (30)$$

*z* denotes that the efficient use of capital in production depends on the intensity of capital utilization. It is assumed that value-added producers need to borrow the money to finance a proportion, $\omega_{wc}$ of their wage bill. This assumption has been used by many others, such as Fuerst (1992) and Christiano and Eichenbaum (1992, 1995), and implies a 'cost channel' of monetary transmission.

Cost minimization by value-added producers implies the following demand curves for capital and labor:

$$\hat{h}_t = \hat{V}_t + \sigma_V\left(\hat{p}_{vc,t} - \hat{w}_t - \omega_{wc}\left(i_t - \left(\frac{1}{\beta} - 1\right) + \varepsilon_{b,t}\right)\right) \qquad (31)$$

$$\hat{k}_{t-1} + \hat{z}_t = \hat{V}_t + \sigma_V(\hat{p}_{vc,t} - \hat{w}_{k,t}) \qquad (32)$$

**2.3.1.3 Petrol producers**

Petrol, $\hat{q}_p$ is produced using inputs of crude oil, $\hat{I}_o$ and value-added $\hat{V}_p$. A simple Leontieff production function is assumed:

$$\hat{q}_{p,t} = \hat{I}_{o,t} = \hat{V}_{p,t} \qquad (33)$$

$$\pi_{pb,t} = \frac{\beta}{\left(1 + \beta\varepsilon_{pp}\right)} E_t \pi_{pb,t+1} + \frac{\varepsilon}{\left(1 + \beta\varepsilon_{pp}\right)} \pi_{pb,t-1} + \frac{\left(1 - \chi_p\right)\left(1 - \beta\chi_p\right)}{\left(1 + \beta\varepsilon_{pp}\right)\chi_{pp}} \hat{\mu}_{pt} \tag{34}$$

$$\hat{\mu}_{p,t} = \psi_{qp}\hat{p}_{vc,t} + \left(1 - \psi_{qp}\right)\hat{p}_{o,t} - \hat{p}_{pb,t} \tag{35}$$

$$\pi_{pb,t} = \pi_t + \hat{p}_{pb,t} - \hat{p}_{pb,t-1} \tag{36}$$

#### 2.3.1.4 Utilities producers

$$\hat{q}_{u,t} = \hat{I}_{g,t} = \hat{V}_{u,t} \tag{37}$$

$$\pi_{u,t} = \frac{\beta}{(1 + \beta\varepsilon_u)} E_t \pi_{u,t+1} + \frac{\varepsilon}{(1 + \beta\varepsilon_u)} \pi_{u,t-1} + \frac{\left(1 - \chi_p\right)\left(1 - \beta\chi_p\right)}{(1 + \beta\varepsilon_u)\chi_u} \hat{\mu}_{u,t} \tag{38}$$

$$\hat{\mu}_{u,t} = \psi_u \hat{p}_{vc,t} + (1 - \psi_u)\hat{p}_{g,t} - \hat{p}_{u,t} \tag{39}$$

$$\pi_{u,t} = \pi_t + \hat{p}_{u,t} - \hat{p}_{u,t-1} \tag{40}$$

### 2.3.2 Monetary and fiscal policy

$$i_t - \left(\frac{1}{\beta} - 1\right) = \theta_{rg}\left(i_{t-1} - \left(\frac{1}{\beta} - 1\right)\right) + (1 - \theta_{rg})(\theta_{pdot}\pi_{c,t} + \theta_y \hat{V}_t) + \varepsilon_{i,t} \tag{41}$$

The government's budget constraint is:

$$G_t = \psi_d p_{p,t} q_{p,t} + T_t \tag{42}$$

### 2.3.3 Foreign sector

World oil prices: $\hat{p}_{o,t} = \varepsilon_{p_o,t} - \hat{s}_t$ (43)

World gas prices: $\hat{p}_{g,t} = \varepsilon_{p_g,t} - \hat{s}_t$ (44)

NKPC for UK import prices

$$\pi_{m,t} = \frac{l_{pm}}{(1+\beta l_{pm})} \pi_{m,t-1} + \frac{\beta}{(1+\beta l_{pm})} E_t \pi_{m,t+1} + \frac{(1-\xi_{pm})(1-\beta\xi_{pm})}{(1+\beta l_{pm})\xi_{pm}} (\varepsilon_{p_{mf},t} - \hat{s}_t - \hat{p}_{mt}) \tag{45}$$

$$\pi_{m,t} = \pi_t + \hat{p}_{m,t} - \hat{p}_{m,t-1} \tag{46}$$

World demand:

$$\hat{x}_{n,t} = \psi_z \hat{x}_{n,t-1}(1-\psi_z)(\varepsilon_{yf} - \eta_x \hat{s}_t) \tag{47}$$

### 2.3.4 Market clearing conditions:

$$\hat{p}_{c,t} + \hat{c}_t = \tfrac{c_n}{p_c c}\hat{c}_{n,t} + \tfrac{p_u c_u}{p_c c}\hat{p}_{U,t} + \hat{c}_{U,t} + (1 - \tfrac{c_n}{p_c c} - \tfrac{p_u c_u}{p_c c})(\hat{p}_{p,t} + \hat{c}_{p,t}) \tag{48}$$

$$\pi_{c,t} = \tfrac{c_n}{p_c c}\hat{\pi}_t + \tfrac{p_u c_u}{p_c c}\pi_{u,t} + \left(1 - \tfrac{c_n}{p_c c} - \tfrac{p_u c_u}{p_c c}\right)\pi_{pb,t} \tag{49}$$

$$V_t = \frac{V_n}{V_t}\hat{V}_{n,t} + \frac{V_u}{V_t}\hat{V}_{u,t} + \left(1 + \frac{V_n}{V_t} - \frac{V_u}{V_t}\right)\hat{V}_{p,t} \tag{50}$$

$$\hat{q}_{p,t} = \tfrac{c_p}{q_p}\hat{c}_{p,t} + (1 - \tfrac{c_p}{q_p})\,\hat{I}_{p,t} \tag{51}$$

$$\hat{I}_{O,t} = -\frac{X_o}{I_o}\hat{X}_{O,t} \tag{52}$$

$$\hat{I}_{G,t} = -\frac{X_g}{I_g}\hat{X}_{G,t} \tag{53}$$

$$\hat{q}_{u,t} = \tfrac{c_U}{q_U}\hat{c}_{U,t} + (1 - \tfrac{c_U}{q_U})\,\hat{I}_{U,t} \tag{54}$$

$$\hat{q}_t = \tfrac{c_n}{q}\hat{c}_{n,t} + \tfrac{k}{q}\hat{k}_t - \tfrac{(1-\delta)k}{q}\hat{k}_{t-1} + \tfrac{\chi_z k}{q}\hat{z}_t + \tfrac{x_n}{q}\hat{x}_{n,t} + \varepsilon_{g,t} \tag{55}$$

$$b_{f,t} = \tfrac{1}{\beta}b_{f,t-1} + \tfrac{x_n}{q}\hat{x}_{n,t} + \tfrac{X_g}{q}(\hat{p}_{g,t} + \hat{X}_{g,t}) + \tfrac{X_o}{q}(\hat{p}_{o,t} + \hat{X}_{o,t}) - \tfrac{M_n}{q}(\hat{p}_{m,t} + \hat{M}_{n,t}) \tag{56}$$

### 2.3.5 The exogenous shock processes

Shock processes follow AR(1)

$$\varepsilon_{a,t} = \rho_a \varepsilon_{a,t-1} + \eta_{a,t} \tag{57}$$

$$\varepsilon_{b,t} = \rho_b \varepsilon_{b,t-1} + \eta_{b,t} \tag{58}$$

$$\varepsilon_{g,t} = \rho_g \varepsilon_{g,t-1} + \eta_{g,t} \tag{59}$$

$$\varepsilon_{i,t} = \rho_i \varepsilon_{i,t-1} + \eta_{i,t} \tag{60}$$

$$\varepsilon_{\mu,t} = \rho_\mu \varepsilon_{\mu,t-1} + \eta_{\mu,t} \tag{61}$$

$$\varepsilon_{inv,t} = \rho_{inv} \varepsilon_{inv,t-1} + \eta_{inv,t} \tag{62}$$

$$\varepsilon_{w,t} = \rho_w \varepsilon_{w,t-1} + \eta_{w,t} \tag{63}$$

$$\varepsilon_{yf,t} = \rho_{yf} \varepsilon_{yf,t-1} + \eta_{yf,t} \tag{64}$$

$$\varepsilon_{pmf,t} = \rho_{pmf} \varepsilon_{pmf,t-1} + \eta_{pmf,t} \tag{65}$$

$$\varepsilon_{po,t} = \rho_{po} \varepsilon_{po,t-1} + \eta_{po,t} \tag{66}$$

$$\varepsilon_{pg,t} = \rho_{pg} \varepsilon_{pg,t-1} + \eta_{pg,t} \tag{67}$$

$$\varepsilon_{rf,t} = \rho_{rf} \varepsilon_{rf,t-1} + \eta_{rf,t} \tag{68}$$

where $\eta's$ are all assumed to be i.i.d. normal processes.

Following the log-linearized model, there are 48 endogenous variables and twelve exogenous shocks have been added to the model which follow AR(1) process. These shocks are assumed to be temporary shocks in the economy. I divided the shocks into two: domestic shocks and foreign shocks. Domestic shocks include: productivity, monetary, consumption preference, capital adjustment cost, government exogenous spending, wage mark-up and price mark-up. While the foreign shocks are: foreign real interest rate, foreign demand, foreign exports price as well as oil price and gas price shocks.

## 2.4 Data

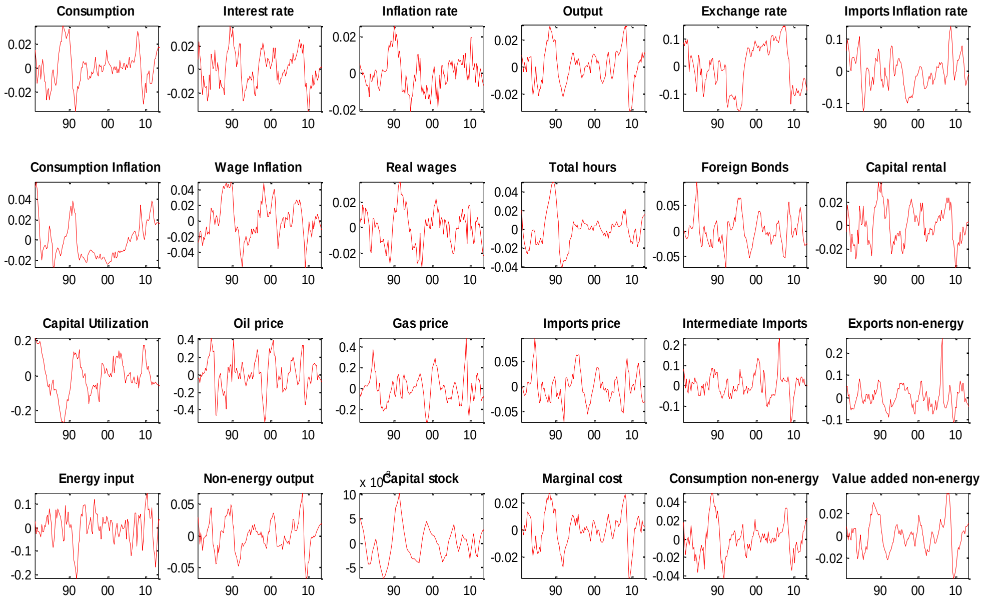
In this section the data sources and construction are presented. The data for endogenous variables and exogenous forcing processes covers the period from 1981 Q1 to 2013 Q1. This period takes in the great moderation era of the UK and includes the 2008 financial crisis. Twenty-six variables were used in total for the estimation, with all variables being expressed in real terms. All variables are per capita and this is calculated by dividing through a UK working-age population, before taking natural logs and then detrended using the Hodrick-Prescott (HP) filter setting - the smoothing parameter $\lambda = 1600$ except where the spatial econometrics toolbox has been used to detrend interest rate, inflation rate and, capital rental rate.

The ONS quarterly series (UKMGSL.Q) has been used when considering population. To calculate the aggregate consumption, the methodology of Harrison and Oomen (2010) was used, where the final consumption expenditure of households and NPISHs (ABJR.Q + HAYO.Q) has been used (ZAVO08) when considering consumption of energy. The consumption deflator is derived as (ABJQ.Q + HAYE.Q)/(ABJM.Q + HAYO.Q). For output I have used GDP at basic prices (ABMM.Q) and the output gap (XOGAP.R) has been used as a proxy for marginal cost. The interest rate used is the three-month Treasury bill rate series from Bank of England (BoE) database (IUQAAJNB). For total hours of employment, I have used the ONS series of (YBUS.Q). To calculate real wages, the UK wages (XPEWF.B) from ONS series have been divided by the total hours worked (YBUS.Q) and then divided through by the consumption deflator. Wage inflation is represented by wages and salaries YOY changes.

Inflation is CPI year-on-year, YOY henceforth, (XCPI.YR). The inflation on consumption is final consumption expenditure YOY (UKES&NMZR). For non-energy gross output the data of BoE similar to Millard (2011) is used, the volume of the final output of the private non-oil and gas extraction sector (QNOCP.Q/PYNODEF.Q). For exchange rate, the Quarterly Average Effective exchange rate index XUQABK67 from BoE is used. Capital stock is constructed using gross fixed capital formation. The foreign bonds are represented by (UKNIJJ10). For the capital rental rate, the official bank rate (IUQABEDR) from BoE is used, while the capital utilization rate is represented by (XCAPU.R). The energy input data is a combination of gas sale to energy generators, gas sale to refinery, gas sale to iron and steel industry and finally gas sale to other sectors (SGASOIF+SGASISF+SGASPWF+RUFUELF). This is achieved without double counting.

For world data I have used the series of world imports prices (Q76.X.F) and followed the BEQM described in Harrison et al., (2005) to construct intermediate imports while I used the UK total imports price YOY as imports inflation (KH3K. R). Non-energy exports are data on trade in goods, less oil and eratics (UKBPBLQ). Finally, for oil and gas prices the world prices of each (WDXWPOB.A and WDXGASJ.A) were collected and then converted to pounds using the exchange rate series of US Dollar to British Pound (UKAUSSQ).

Figure 3 Filtered data of the UK



The estimated[16] persistence and volatility of the shocks, following AR(1) process are:

$\varepsilon_{a,t} = 0.6453\varepsilon_{a,t-1} + \eta_{a,t}$, $\sigma_{a,t}$=0.0106

$\varepsilon_{b,t} = 0.8809\varepsilon_{b,t-1} + \eta_{b,t}$, $\sigma_{b,t}$=0.0150

$\varepsilon_{g,t} = 0.7811\varepsilon_{g,t-1} + \eta_{g,t}$, $\sigma_{g,t}$=0.0111

$\varepsilon_{i,t} = 0.7631\varepsilon_{i,t-1} + \eta_{i,t}$, $\sigma_{i,t}$=0.0097

$\varepsilon_{\mu,t} = 0.3473\varepsilon_{\mu,t-1} + \eta_{\mu,t}$, $\sigma_{\mu,t}$=0.2021

$\varepsilon_{inv,t} = 0.6542\varepsilon_{inv,t-1} + \eta_{inv,t}$, $\sigma_{inv,t}$=0.0041

$\varepsilon_{pg,t} = 0.8701\varepsilon_{pg,t-1} + \eta_{pg,t}$, $\sigma_{pg,t}$=0.0744

$\varepsilon_{pmf,t} = 0.8755\varepsilon_{pmf,t-1} + \eta_{pmf,t}$, $\sigma_{pmf,t}$=0.0382

$\varepsilon_{po,t} = 0.7944\varepsilon_{po,t-1} + \eta_{po,t}$, $\sigma_{po,t}$=0.1265

$\varepsilon_{rf,t} = 0.8369\varepsilon_{rf,t-1} + \eta_{rf,t}$, $\sigma_{rf,t}$=0.0155

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[16] Details of the estimation is provided in the methodology.

$\varepsilon_{w,t} = 0.2891\varepsilon_{w,t-1} + \eta_{w,t}$, $\sigma_{w,t}$=0.0042

$\varepsilon_{yf,t} = 0.7840\varepsilon_{yf,t-1} + \eta_{yf,t}$, $\sigma_{yf,t}$=0.0430

One can see that the filtered data World oil prices have shown high persistence and volatility.

## 2.5 Calibration

The calibrated parameters are taken from Millard (2011). The paramters are split into two groups, with the first group of parameters being the set that are important in deriving the model's steady state, derived by taking average ratios, with little or no influence on the dynamics properties. These parameters are set to match steady-state values in Harrison et al., (2011), except elasticity of demand for differentiated labour that is in the second category of parameters. When I estimate the model, these sets of parameters are fixed, hence, the name: fixed parameters shown in figure 1 below.

Table 1 Fixed parameters

| | **Value** | **Parameter Description** |
|---|---|---|
| $\beta$ | 0.9925 | Discount factor |
| $\chi_{bf}$ | 0.001 | Cost of adjusting portfolio of foreign bonds |
| $\delta$ | 0.013 | Depreciation rate |
| $\chi_z$ | 0.0206 | Scales the effect of capital |
| $\sigma_e$ | 0.4 | Elasticity of substitution between non-energy and energy in consumption |
| $\sigma_p$ | 0.1 | Elasticity of substitution between petrol and utilities in energy consumption |
| $\sigma_v$ | 0.5 | Elasticity of substitution between labour and capital in value-added |
| $\sigma_q$ | 0.15 | Elasticity of substitution between energy and everything else in non-energy |
| $\psi_e$ | 0.0526 | Share of energy in consumption |
| $\psi_p$ | 0.5913 | Share of petrol in energy consumption |
| $\alpha_q$ | 0.0528 | Cost share of energy in non-energy output |
| $\alpha_B$ | 0.3154 | Cost share of imports in 'bundle' |
| $\alpha_v$ | 0.1701 | Cost share of capital in value-added |
| $\psi_n$ | 0.3096 | Cost share of petrol in energy output |
| $\psi_{qp}$ | 0.1844 | Cost share of value-added in petrol output |
| $\psi_u$ | 0.4834 | Cost share of value-added in utilities output |
| $\psi_d$ | 0.617 | Share of duty in petrol prices |
| $\frac{c_n}{p_c c}$ | 0.9474 | Share of non-energy consumption in total consumption |
| $\frac{p_u c_u}{p_c c}$ | 0.0215 | Share of utility consumption in total consumption |
| $\frac{V_n}{V}$ | 0.9815 | Share of value-added used as input in non-energy goods |

| | | |
|---|---|---|
| $\frac{V_u}{V}$ | 0.0145 | Share of value-added used as input in utilities |
| $\frac{c_p}{q_p}$ | 0.4202 | Share of petrol output going to consumption |
| $\frac{c_u}{q_u}$ | 0.4054 | Share of utilities output going to consumption |
| $\frac{X_o}{I_o}$ | 0.4551 | Ratio of oil exports to oil inputs |
| $\frac{X_g}{I_g}$ | - 0.0792 | Ratio of gas exports to gas inputs |
| $\frac{c_n}{q}$ | 0.5801 | Share of private consumption in non-energy output |
| $^{k}/_{q}$ | 4.7202 | Ratio of capital to non-energy output |
| $^{k}/_{q}$ | 4.7202 | Ratio of capital to non-energy output |
| $\frac{x_n}{q}$ | 0.2552 | Share of exports in non-energy output |
| $\frac{M_n}{q}$ | 0.2581 | Ratio of imports of non-energy goods to output of non-energy goods |
| $\frac{X_O}{q}$ | 0.0035 | Ratio of oil exports to output of non-energy goods |
| $\frac{X_g}{q}$ | - 0.0007 | Ratio of gas exports to output of non-energy goods |

The second set of parameters are priors used in Millard (2011). The prior for the parameter on inflation in Taylor's rule is in line with Taylor's original paper. This is the set that we will estimate in the study using indirect inference testing. This set of parameters as estimated parameters is shown in table 2. The value of the capital adjustment cost is set at 201 is justified from equation (14). It shows how capital costs gives incentives for households to change the capital stock slowly (Harrison and Oomen (2010)). This means that a higher adjustment cost parameter, $\chi_z$, will decrease the change elasticity in capital stock with regards to interest rate, shadow price of capital and the capital rental rate.

Table 2 Parameters to be estimated

| | **Description** | **Initial value** |
|---|---|---|
| $\theta_y$ | Taylor Rule Coefficient on output | 0.125 |
| $\varepsilon_p$ | Degree of indexation: non-energy sector | 0.5 |
| $\chi_p$ | Probability of not being able to change price: non-energy sector | 0.5 |
| $\varepsilon_{pm}$ | Degree of Indexation: importers | 0.5 |
| $\xi_{pm}$ | Probability of not able to change price: importers | 0.5 |
| $\eta_x$ | Elasticity of demand for exports | 1.5 |
| $\psi_z$ | Degree of persistence in export demand | 0.5 |
| $\psi_{hab}$ | Degree of habit persistence in consumption | 0.5 |
| $\sigma_c$ | Intertemporal elasticity of substitution | 0.66 |
| $\varepsilon_k$ | Degree of persistence in investment adjustment costs | 0.5 |
| $\psi_w$ | Probability of being able to change wages | 0.5 |
| $\xi_w$ | Degree of wage indexation | 0.5 |
| $\sigma_h$ | Frisch elasticity of labour supply | 0.43 |
| $\theta_{rg}$ | Degree of Taylor-rule interest-rate smoothing | 0.5 |
| $\theta_{pdot}$ | Taylor rule coefficient on inflation | 1.5 |
| $\chi_z$ | Scale of capital adjustment cost | 201 |
| $\psi_{wc}$ | Share of wage bill paid financed by borrowing | 0.5 |
| $\chi_u$ | Probability not being able to change price: utility | 0.5 |
| $\chi_{pp}$ | Probability not being able to change price: petrol | 0.5 |
| $\varepsilon_u$ | Degree of indexation: utilities sector | 0.5 |
| $\varepsilon_{pp}$ | Degree of indexation: petrol sector | 0.5 |
| $\phi_z$ | Inverse elasticity of capital utilisation costs | 0.56 |
| $\sigma_w$ | Elasticity of demand for differentiated labour | 3.8906 |

## 2.6. Methodology

In this section, this model is applied to the UK stationary data. In standard practice, there are conventional tools used to understand how a simulated DSGE model works. Tools such as Variance decomposition and Impulse response functions are explored in this study. The VAR-impulse response functions[17] will be added to assess the fit of the estimated model. I will also be accounting for the crisis period with the model's shock decomposition. This follows the model estimation method used with the powerful simulated annealing algorithm[18]. I adopt the approach of sampling variability of the simulated data to match the actual data using indirect inference testing. This is in contrast to indirect inference estimation.

### 2.6.1 Model evaluation by indirect inference

Indirect inference test method of model evaluation offers a classical econometrics inferential structure for assessing calibrated models Le, Meenagh, Minford and Wickens (2012). This method is used to judge partially or fully estimated models while maintaining the fundamental ideas utilized in the evaluation of early RBC models of comparing data generated moments from the model simulation by the actual data. Instead of using moments to compare with no distributions, this method provides a simple model (auxiliary model) that includes the conditional mean of the

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[17] Christiano, et al., (2005) evaluated their model of the US exclusively on the fit to the structural shock

[18] I use a Simulated Annealing algorithm due to Ingber (1996). This mimics the feature of the steel cooling process, with a degree of reheating at randomly chosen moments in the cooling process which ensures that the defects are minimised globally.

distribution which one can compare the features of the model estimated from actual and simulated data. The indirect inference test methodology, although different, has similar features in the widely used *indirect estimation* method. The primary feature of this similarity is utilization of the auxiliary model in addition to the structural macroeconomic model. The estimation by indirect inference chooses the parameters of the DSGE model in a way that the simulated model generates estimates of the auxiliary model that is similar to those obtained from the data.

An account of inferential problem is as follows: using Canova (2005) notations designed for indirect inference estimation, where $y_t$ is defined as $m \times 1$ vector observed data $(t = 1, \dots, T)$ and $x_t(\theta)$ is a $m \times 1$ vector of simulated (time series) data with the number of observations $S$ which is generated from the structural model, $\theta$ is a $k \times 1$ vector of the model's structural parameters. The assumption here is that $y_t$ and $x_t(\theta)$ are stationary and ergodic. Then set $S = T$ with the requirement of the actual dal data sample being regarded as the expected imitation from the population of the samples that have been bootstrapped by the data. The auxiliary model is assumed as $f(y_t, \alpha)$, with $\alpha$ as the vector of descriptors. From the given null hypothesis $H_0\colon \theta = \theta_0$, the auxiliary model then becomes $f[x\_t\ (\theta), (\theta\_0\ )]$= as $f[y_t, \alpha]$. The test of the null hypothesis is by a q × 1 vector of a continuous function $g(\alpha)$. Therefore, under the null hypothesis, one is going to have $g(\alpha) = g(\alpha(\theta_0))$. The estimator for $\alpha$ using the actual data is $a_T$ while the estimator for $(\theta_0)$ based on simulated data is $a_S(\theta_0)$. This gives us $g(a_T)$

and $g(a_S(\theta_0))$. We then get the mean of the bootstraps as: $\overline{g(\alpha(\theta_0))} = \frac{1}{N}\sum_{k=1}^{N} g_k(\alpha(\theta_0))$. From here, we get the Wald statistic ($WS$) by using the bootstrapped distribution of $g(a_S)$- $\overline{g(a_S(\theta_0))}$. This is then defined as:

$$WS = g(a_T)\text{-}\ \overline{g(a_S(\theta_0))}'W(\theta_0)^{-1}(\theta_0)g(a_T)\text{-}\ \overline{g(a_S(\theta_0))} \tag{69}$$

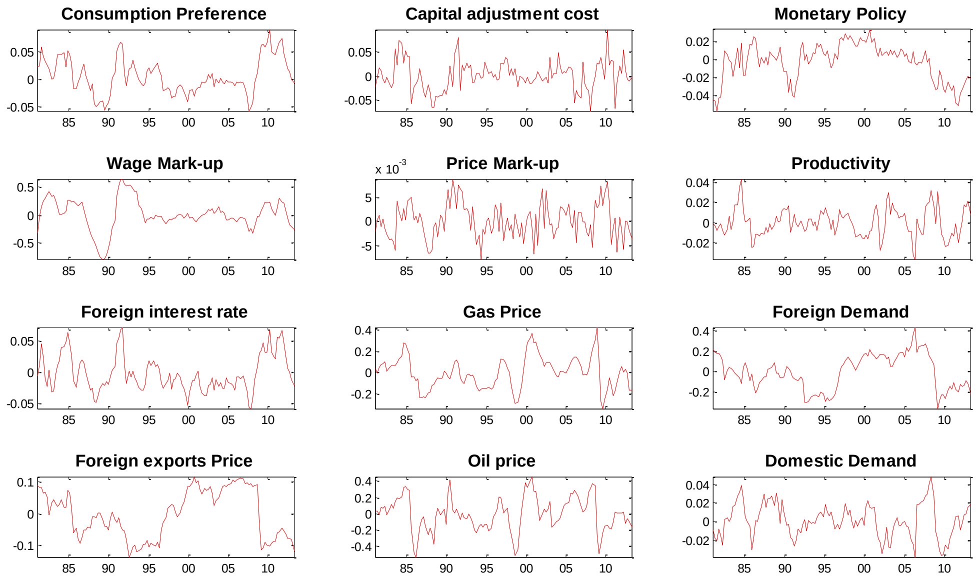
where $W(\theta_0)^{-1}$ is the variance-covariance of the bootstrapped distribution of $g(a_T)$- $\overline{g(a_S(\theta_0))}$. Furthermore, $W(\theta_0)$ is obtained from the asymptotic distribution of $g(a_T)$- $\overline{g(a_S(\theta_0))}$ and then the asymptotic distribution of the Wald statistic would then be chi-squared. Unlike the above, with an indirect inference test one will obtain an empirical distribution of the Wald statistic bootstrap using a bootstrap method through defining $g(\alpha)$ as a vector consisting of the VAR coefficients and the variances of the data or the disturbances of the VAR model.

Following the work of Meenagh, Minford and Wickens (2012), I will show how the Wald test by bootstrap is conducted:

*Step 1: Estimating the errors of the structural model based on observed data and* $\theta_0$.

The number of exogenous shocks must be equal to or less than the endogenous variables in the DSGE model. The structural residuals $\varepsilon_t$ are estimated from the DSGE model $x_t(\theta_0)$, given the stated values of $\theta_0$ and the actual observed data. There is an assumption the errors will be normally distributed and will follow AR(1) process. If a structural equation contains no expectation, the residuals may be backed out of the equation and the observed data. If the equation includes some expectations on some variables then there will be estimation for the expected variables. In this case, I carry this out using McCallum (1976) and Wickens (1982) a robust instrumental variables method with lagged endogenous observed data as the instruments. This is more or less an auxiliary model VAR.

Figure 4 Estimated structural residuals



### *Step 2: Deriving the simulated data*

In this model, like many DSGE models, the structural shocks are assumed to be autoregressive processes rather than being serially independent. OLS is used to estimate the innovations from the residuals[19]. The innovations are repeatedly drawn by time vector to preserve any simultaneity between the shocks, and then solving the model by *dynare*. I then go on to obtain $N$ bootstrapped simulations by repeating the drawing of the sample independently. $N=1000$.

### *Step 3: Compute the Wald Statistic*

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[19] The coefficients of the residuals from the OLS estimation are the model's persistence.

The auxiliary model is then estimated, a VAR(1), on the bootstrap sample and the actual data to obtain the estimates[20], of the distribution of the observed data and the VAR coefficients, $a_T$ and $a_S$ of the vector $\alpha$. I am able to obtain the covariance matrix $W(\theta_0)$ of the distribution $(a_S)$- $\overline{(a_S(\theta_0))}$ through estimating the auxiliary VAR on the 1000 bootstrapped simulations of $a_S(\theta_0)$ while the covariance of the simulated variables from the bootstrap samples were obtained. This shows the variations in the data sampling as implied by the model from the result set of $a_k$ vectors ( $k = 1, \dots . N$), thus the estimate of $W(\theta_0)$ will be:

$$\frac{1}{N}\sum_{k=1}^{N}(a_k - \overline{a_k})'(a_k - \overline{a_k}) \tag{70}$$

where $\overline{a_k} = \frac{1}{N}\sum_{k=1}^{N} a_k$. From here, the Wald statistic is calculated for the data sample and then the bootstrap distribution of the Wald from the 1000 samples of the bootstrap is estimated

### 2.6.2 Assessing the fit of the estimated model

The indirect inference test is based on the significant comparison of the actual data with the simulated data from the structural model that comes through an auxiliary model. The test is based on the VAR coefficients and the data variances of the variables in the VAR.

$$\begin{bmatrix} y_t \\ \pi_t \\ r_t \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{21} & \beta_{31} \\ \beta_{12} & \beta_{22} & \beta_{32} \\ \beta_{13} & \beta_{23} & \beta_{33} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ r_{t-1} \end{bmatrix} + \Omega_t$$

---

[20] Actual and simulated data variances have been included in the estimates to determine the model's dynamics and volatility.

A combination of output ($y$), Inflation rate ($\pi$) and real interest rate ($r$) were chosen as the auxiliary model of VAR, for the evaluation to fit the model although other combinations were used, this set was used in the estimation as the variables in the VAR auxiliary model. The descriptors provide a strong argument for the structural model to match.

Using the method of indirect inference testing to test and estimate the model, VAR(1) is used as the auxiliary model. A VAR(1) $\alpha$ contains 12 elements, that is 9 VAR coefficients and 3 variances of the actual data used. Increasing the VAR order will increase the VAR coefficients. VAR(2)[21] will generate 18 VAR coefficients which will make 21 elements in total, making it difficult to match the data. VAR(1) was chosen and it proves to be effective.

The model was tested using the calibrated parameters and the test shows rejection. I show the Wald statistic bootstrap distribution and where the Wald statistic data lies. I also show the joint distribution's Mahalanobis distance, which was normalized to a t-statistic, as well as the Wald p-value. In Table 3, the VAR coefficients of the joint distribution's variables chosen using the calibrated parameters show the Wald statistic bootstrap distribution and where the Wald statistic data lies. The joint

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[21] Le, et al., (2012) produced the result of a VAR(2) and showed how difficult it could be to find a favourable result in the testing.

distribution's Mahalanobis Distance[22], normalized to a t-statistic as well as the Wald p-value is also shown.

Table 3 Summary of VAR results

| Variables used in testing: Output, inflation and interest rate | **Normalized T-statistic** | **Wald** | **p-value** |
|---|---|---|---|
| Dynamics | 9.4939 | 100% | 0.00 |
| Dynamics and Volatility | 13.5826 | 100% | 0.00 |
| Volatility | 9.7516 | 100% | 0.00 |

VAR Results

| | **95% lower** | **95% upper** | **Actual** | **IN/OUT** |
|---|---|---|---|---|
| $\boldsymbol{A_y^y}$ | 0.459416 | 0.773121 | 0.933917 | OUT |
| $\boldsymbol{A_y^\pi}$ | -0.656821 | 0.273008 | -0.054771 | IN |
| $\boldsymbol{A_y^r}$ | -0.512248 | 0.098587 | -0.062042 | IN |
| $\boldsymbol{A_\pi^y}$ | 0.022581 | 0.125566 | 0.107079 | IN |
| $\boldsymbol{A_\pi^\pi}$ | 0.666408 | 0.885087 | 0.810838 | IN |
| $\boldsymbol{A_\pi^r}$ | -0.034367 | 0.136235 | -0.093553 | OUT |
| $\boldsymbol{A_r^y}$ | -0.031974 | 0.087848 | 0.151025 | OUT |
| $\boldsymbol{A_r^\pi}$ | -0.086830 | 0.257084 | 0.190834 | IN |
| $\boldsymbol{A_r^r}$ | 0.768280 | 0.987982 | 0.735061 | OUT |
| $\boldsymbol{\sigma_y^2}$ | 0.000609 | 0.000986 | 0.000032 | OUT |
| $\boldsymbol{\sigma_\pi^2}$ | 0.000056 | 0.000095 | 0.000029 | OUT |
| $\boldsymbol{\sigma_r^2}$ | 0.000072 | 0.000131 | 0.000067 | OUT |

Following the estimation, using the simulated annealing algorithm, it found the best set of parameters, with a non-rejection of quite a few variables combinations. Above all, the auxiliary model used in the estimation, output-inflation-interest rate, fits the data. The results in table 4 gives the summary of the VAR results. The Wald statistic bootstrap distribution, the joint distribution's Mahalanobis Distance, normalized to a t-statistic and the p-value. One can conclude, with respect to the summary of the

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[22] The Mahalanobis Distance is the square root value of the Wald chi-squared distribution then into a normalised t-statistic by adjusting the mean and the size. The value is normalised by ensuring that the resulting t-statistic is 1.645 at the 95% point of the distribution, following Le and Meenagh (2013).

result, that the model is not rejected by the data. The VAR coefficients for the auxiliary model in Table 5 shows all the VAR coefficients of the bootstrapped model (dynamics), together with its variances (volatility) in the test. Here, one can see that the output and inflation variances are outside the 95% percentile but the data does not reject the model.

Table 4 Summary of VAR results

| Variables used in testing:<br>Output, inflation and interest rate | **Normalized T-statistic** | **Wald** | **p-value** |
|---|---|---|---|
| Dynamics | 0.7980 | 83.1% | 0.169 |
| Dynamics and Volatility | 1.498 | 94% | 0.060 |
| Volatility | 2.1861 | 97.4% | 0.026 |

Table 5 VAR results

| | **95% lower** | **95% upper** | **Actual** | **IN/OUT** |
|---|---|---|---|---|
| $\boldsymbol{A_y^y}$ | 0.721125 | 0.955407 | 0.933917[23] | IN |
| $\boldsymbol{A_y^\pi}$ | -0.159182 | 0.039341 | -0.054771 | IN |
| $\boldsymbol{A_y^r}$ | -0.089259 | 0.083968 | -0.062042 | IN |
| $\boldsymbol{A_\pi^y}$ | -0.059268 | 0.200526 | 0.107079 | IN |
| $\boldsymbol{A_\pi^\pi}$ | 0.744558 | 0.933653 | 0.810838 | IN |
| $\boldsymbol{A_\pi^r}$ | -0.167904 | 0.036061 | -0.093553 | IN |
| $\boldsymbol{A_r^y}$ | -0.025819 | 0.273290 | 0.151025 | IN |
| $\boldsymbol{A_r^\pi}$ | -0.079204 | 0.197448 | 0.190834 | IN |
| $\boldsymbol{A_r^r}$ | 0.701350 | 0.924074 | 0.735061 | IN |
| $\boldsymbol{\sigma_y^2}$ | 0.000034 | 0.000061 | 0.000032 | OUT |
| $\boldsymbol{\sigma_\pi^2}$ | 0.000039 | 0.000078 | 0.000029 | OUT |
| $\boldsymbol{\sigma_r^2}$ | 0.000059 | 0.000107 | 0.000067 | IN |

Table 6 shows the estimated structural parameters of the model. The value of the habit persistence parameter, 0.7, is consistent with the value reported by Boldrin, Christiano and Fisher (2001). They argued the ability of a standardized DSGE model accounts for the equity premium among other points. The Taylor rule coefficient of

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[23] Falls within 1 percent boundary.

output and inflation, elasticity of demand for exports and imports are consistent with a lot of authors' estimations, e.g. Christiano et al., (2005), Smets and Wouters (2007) and LMMW (2012). Looking at the persistence[24] and volatility[25] of the shocks,

Table 6 Estimated parameters

| **Parameter** | **Definition** | **Initial value** | **Estimated value** | **% change** |
|---|---|---|---|---|
| $\theta_y$ | Taylor rule Coefficient on output | 0.125 | 0.1291 | 3.3 |
| $\varepsilon_p$ | Degree of indexation: non-energy sector | 0.5 | 0.4055 | -18.9 |
| $\chi_p$ | Probability of not being able to change price: non-energy sector | 0.5 | 0.6474 | 29.5 |
| $\varepsilon_{pm}$ | Degree of Indexation: importers | 0.5 | 0.5145 | 2.9 |
| $\xi_{pm}$ | Probability of not able to change price: importers | 0.5 | 0.2109 | -57.8 |
| $\eta_x$ | Elasticity of demand for exports | 1.5 | 2.4545 | 63.6 |
| $\psi_z$ | Degree of persistence in export demand | 0.5 | 0.1844 | -63.1 |
| $\psi_{hab}$ | Degree of habit persistence in consumption | 0.5 | 0.6965 | 39.3 |
| $\sigma_c$ | Intertemporal elasticity of substitution | 0.66 | 0.6681 | 1.2 |
| $\varepsilon_k$ | Degree of persistence in investment adjustment costs | 0.5 | 0.9055 | 81.1 |
| $\psi_w$ | Probability of being able to change wages | 0.5 | 0.3809 | -23.8 |
| $\xi_w$ | Degree of wage indexation | 0.5 | 0.9678 | 93.6 |
| $\sigma_h$ | Frisch elasticity of labour supply | 0.43 | 0.0149 | -96.5 |
| $\theta_{rg}$ | Degree of Taylor-rule interest-rate smoothing | 0.5 | 0.4770 | -4.6 |
| $\theta_{pdot}$ | Taylor rule coefficient on inflation | 1.5 | 2.0637 | 37.6 |
| $\chi_z$ | Scale of capital adjustment cost | 201 | 18.5928 | -90.7 |
| $\psi_{wc}$ | Share of wage bill paid financed by borrowing | 0.5 | 0.0272 | -94.6 |
| $\chi_u$ | Prob. not being able to change price: utility | 0.5 | 0.0886 | -82.3 |
| $\chi_{pp}$ | Prob. not being able to change price: petrol | 0.5 | 0.6296 | 25.9 |
| $\varepsilon_u$ | Degree of indexation: utilities sector | 0.5 | 0.4476 | -10.5 |
| $\varepsilon_{pp}$ | Degree of indexation: petrol sector | 0.5 | 0.9363 | 87.3 |
| $\phi_z$ | Inverse elasticity of capital utilisation costs | 0.56 | 0.8453 | 50.9 |
| $\sigma_w$ | Elasticity of demand for differentiated labour | 3.8906 | 1.3617 | -65.0 |

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[24] Each shock persistence is given as the coefficient (rho), of that shock, generated from the data residual regressed on its lagged data. (Wickens,1982)

[25] The volatility is the standard error from the shock's innovation (Wickens,1982). This is also what is given to generate the impulse response functions of each shock using dynare.

**Error! Not a valid bookmark self-reference.**, with focus on foreign shocks, one can conclude there exists high persistence except energy price shocks. These shocks possess high volatility compared to all the shocks. Productivity shock has low persistence and low volatility which is only bettered by the mark-up shock of prices.

Table 7 Estimated parameters of structural shocks AR(1)

| Shock ($j$) | $\rho_j$ | $\sigma_j$ |
|---|---|---|
| Productivity shock | 0.6453 | 0.0106 |
| Consumption preference shock | 0.8796 | 0.0153 |
| Government spending shock | 0.7811 | 0.0111 |
| Monetary policy shock | 0.8363 | 0.0106 |
| Capital adjustment cost shock | 0.4545 | 0.0284 |
| Price mark-up shock | 0.5695 | 0.0037 |
| Gas price shock | 0.8701 | 0.0744 |
| Foreign export price shock | 0.9415 | 0.0256 |
| Oil price shock | 0.7944 | 0.1265 |
| Foreign interest rate shock | 0.8348 | 0.0160 |
| Persistence of wage mark-up shock | 0.9381 | 0.0322 |
| Persistence of foreign demand shock | 0.9083 | 0.0559 |

## 2.7 VAR impulse response functions (VAR-IRFs)

In assessing the fit of the calibrated model, I add the VAR-IRFs to compliment the analysis. Authors like Christiano et al., (2005) evaluated their model of the US exclusively on the fit to the structural shock IRFs. This follows Le, Meenagh and Minford (2012), where the model estimation base on passing the Wald test using VAR(1). The process generates 95 percent confidence limits for implied VAR responses that simply includes the data-based VAR responses to the structural shocks for the variables in the auxiliary model, output, inflation and interest rate. Here, I show the VAR IRFs of the twelve structural shocks. The red lines indicate 95 percent confidence intervals about the point estimates. Overall, the auxiliary model falls within the 95 percent boundary. Overall, the auxiliary model falls within the 95 percent boundary. The response is identified in a similar assumption of the real aggregate output, aggregate demand and real exchange rate evolve in this DSGE model. The behaviour of these endogenous variables displays the fit of the DSGE model. The VAR-IRFs here simply shows the fit of the model with the data. More analysis follows when I discuss the impulse responses of the model. See appendix 1.2.

## 2.8 A Stochastic Variance Decomposition[26]

Table 8 shows the significance of each shock in terms of how much each shock explains the variance in the endogenous variables. It is quite surprising that the productivity shock does not have effect on output. This is because the productivity shock affects gross non-energy output[27], with output (value-added) used as input. Hence, one can see productivity shock explains only 4% of its variability and just a little over 1% of the total gross output and output. Due to the feature of productivity shock, it explains most of the variables including investment at 0.5%, employment at about 2% except marginal cost which it contributes almost 10% to its variability. The monetary policy shock dominates as it contributes 20% to gross output and 9% of output. 16% of consumption is explained by this shock as it also contributes 41% to wage inflation and 49% to consumption inflation.

Domestic demand shock (a combination of preference shock, capital adjustment cost shock and government spending shock[28]) explains about 80% of the variance in interest rates. It also explains about 55% of the variations of capital stock and 53% of investment, 49% of inflation rates as well as 38% of consumption inflation. Demand shock contributes 20% to the variation, except exchange rate, and has effects on real wage rate as it contributes 25% to its variance. It also contributes 20% and 21% in

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[26] In this analysis, the shocks are classified as foreign or domestic. The domestic shocks are classified as productivity, monetary, domestic demand; which include consumption preference, capital adjustment cost and government spending (this is following Smets and Wouters (2007)), mark-up; includes price and wage mark-up. and finally the foreign shocks (world oil price, world gas price, foreign interest rate, foreign demand and world imports price) .

[27] Value-added are used as inputs for gross output.

[28] Following Smets and Wouters, 2007

explaining consumption and output, respectively. The mark-up shock (a combination of price and wage mark-up shock) explains about 42% of GDP, 51% of employment and 60% of the marginal rate of substitution (MRS).

However, it is the combined foreign shocks that explain 57% of GDP variation. These shocks explain that about 60% of the exchange rate variation is impacted by the foreign shocks with the foreign interest rate shock accounting for 32% and 46% of gross output of non-energy. The energy price shock that includes oil price and gas price shocks have little effect on the economic variables. Looking at the energy sector inflation, one can see the impact of the energy shocks as it explains 57% of the petrol price inflation, 75% of the oil price and 36% of the utility price inflation. Comparing with related literature, authors like Bjornland (2000)[29] as well as Jimenez-Rodriguez and Sanchez (2004)[30] finds the oil price shock explains 9% of the variability in the GDP in the UK.

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[29] Bjornland (2000) looked at variance decomposition for countries in the euro area that includes Germany, Norway and the United Kingdom.

[30] Jimenez-Rodriguez and Sanchez (2004) find empirical evidence for some OECD.

Table 8 Variance Decomposition of Domestic shocks

|  | Productivity | Monetary Policy | Consumption preference | Capital adjustment cost | Government spending | Wage mark-up | Price mark-up |
|---|---|---|---|---|---|---|---|
| Consumption | 1.39 | 16.23 | 17.69 | 0.49 | 1.30 | 4.08 | 0.82 |
| Output[31] | 1.69 | 9.29 | 12.26 | 3.40 | 6.52 | 38.10 | 3.79 |
| Gross Output[32] | 1.19 | 19.96 | 21.85 | 0.39 | 0.87 | 2.97 | 1.18 |
| Gross Output (non-energy) | 4.22 | 18.69 | 20.20 | 1.28 | 3.20 | 2.96 | 1.71 |
| Investment | 0.54 | 25.90 | 27.00 | 0.30 | 11.52 | 0.18 | 0.19 |
| Capital stock | 0.67 | 24.19 | 29.78 | 0.34 | 7.00 | 0.67 | 0.31 |
| Interest rate | 1.07 | 2.67 | 76.69 | 0.27 | 0.51 | 0.16 | 0.49 |
| Inflation rate | 1.35 | 48.88 | 38.70 | 0.14 | 0.13 | 0.66 | 1.15 |
| Consumer price inflation | 0.90 | 48.99 | 38.35 | 0.10 | 0.09 | 0.65 | 0.69 |
| Exchange rate | 0.63 | 16.68 | 18.05 | 0.30 | 1.63 | 1.93 | 0.27 |
| Real wage rate | 1.87 | 23.31 | 25.35 | 0.44 | 1.04 | 1.86 | 4.17 |
| Total hours | 1.77 | 2.58 | 4.97 | 3.50 | 5.21 | 49.05 | 2.81 |
| Utilization | 0.70 | 23.85 | 29.43 | 0.47 | 6.97 | 1.11 | 0.57 |
| Rental rate | 0.70 | 23.85 | 29.43 | 0.47 | 6.97 | 1.11 | 0.57 |
| Foreign bond | 0.92 | 4.74 | 7.52 | 0.13 | 0.28 | 8.00 | 0.81 |
| Gas price | 0.22 | 7.17 | 7.32 | 0.19 | 1.02 | 0.08 | 0.1 |
| Oil Price | 0.14 | 4.66 | 4.75 | 0.12 | 0.66 | 0.05 | 0.07 |
| Net exports | 1.28 | 13.41 | 15.72 | 0.20 | 0.48 | 6.17 | 1.56 |
| Imports price | 0.38 | 13.03 | 14.47 | 0.17 | 1.09 | 1.48 | 0.19 |
| Imports inflation | 0.68 | 24.64 | 17.76 | 0.29 | 1.38 | 0.86 | 0.33 |
| Petrol inflation | 0.19 | 17.25 | 12.91 | 0.03 | 0.24 | 0.34 | 0.14 |
| Utility inflation | 0.29 | 26.51 | 20.72 | 0.04 | 0.17 | 0.36 | 0.65 |
| Wage inflation | 0.81 | 41.03 | 46.09 | 1.45 | 0.54 | 1.26 | 2.89 |
| Imports | 0.37 | 19.41 | 20.09 | 0.68 | 1.84 | 0.1 | 0.99 |
| Bundle of inputs | 0.69 | 19.25 | 20.81 | 1.33 | 3.33 | 3.13 | 1.76 |
| Energy inputs | 0.69 | 20.20 | 21.84 | 1.30 | 3.19 | 2.60 | 1.82 |
| Consumption (non-energy) | 1.42 | 16.21 | 17.67 | 0.49 | 1.33 | 4.10 | 0.83 |
| Consumption-energy | 1.03 | 16.22 | 17.72 | 0.39 | 0.92 | 3.79 | 0.61 |
| Marginal cost | 9.55 | 18.22 | 27.17 | 1.67 | 0.22 | 0.03 | 17.46 |
| MRS | 1.68 | 8.26 | 17.79 | 2.26 | 0.21 | 58.26 | 2.07 |

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[31] Throughout this study, value-added is referred to as output which is assumed GDP

[32] Gross output is a combination of output from the three producing sectors given value-added. One can see this as gross GDP.

Table 9 Variance Decomposition of Foreign shocks

| | Foreign interest rate | Gas price | Oil price | Foreign exports price | Foreign demand |
|---|---|---|---|---|---|
| Consumption | 16.50 | 0.49 | 0.08 | 20.64 | 20.28 |
| Output | 15.67 | 0.84 | 0.45 | 1.40 | 6.60 |
| Gross Output | 14.13 | 1.92 | 0.63 | 17.32 | 17.61 |
| Gross Output (non-energy) | 11.76 | 0.58 | 0.24 | 19.04 | 16.11 |
| Investment | 31.84 | 0.06 | 0.03 | 1.13 | 1.30 |
| Capital stock | 27.52 | 0.17 | 0.04 | 5.81 | 3.51 |
| Interest rate | 17.54 | 0.15 | 0.08 | 0.14 | 0.24 |
| Inflation rate | 7.59 | 0.18 | 0.90 | 0.16 | 0.18 |
| Consumer price inflation | 9.72 | 0.25 | 0.10 | 0.10 | 0.06 |
| Exchange rate | 32.67 | 0.10 | 0.04 | 1.90 | 25.80 |
| Real wage rate | 9.72 | 1.26 | 0.51 | 16.18 | 14.28 |
| Total hours | 12.01 | 0.64 | 0.32 | 5.89 | 11.24 |
| Capital utilization | 27.23 | 0.20 | 0.07 | 5.01 | 4.40 |
| Capital rental rate | 27.23 | 0.20 | 0.07 | 5.01 | 4.40 |
| Foreign bond | 19.50 | 0.50 | 0.06 | 15.78 | 41.77 |
| Gas price | 59.97 | 2.01 | 0.02 | 13.01 | 8.89 |
| Oil Price | 0.03 | 1.31 | 73.99 | 8.44 | 5.77 |
| Net exports | 4.06 | 7.30 | 1.20 | 19.38 | 29.24 |
| Imports price | 20.66 | 0.05 | 0.01 | 28.19 | 20.27 |
| Imports inflation | 36.17 | 0.23 | 0.54 | 11.70 | 5.42 |
| Petrol price inflation | 10.60 | 0.11 | 57.01 | 0.18 | 1.02 |
| Utility price inflation | 13.91 | 35.09 | 1.08 | 0.71 | 0.48 |
| Wage inflation | 1.60 | 0.60 | 0.37 | 1.27 | 2.09 |
| Intermediate imports | 0.36 | 27.35 | 0.17 | 13.66 | 14.97 |
| Bundle of inputs | 12.15 | 0.46 | 0.23 | 20.26 | 16.62 |
| Energy inputs | 12.48 | 2.68 | 0.53 | 15.50 | 17.17 |
| Consumption (non-energy) | 16.39 | 0.45 | 0.07 | 20.85 | 20.19 |
| Consumption (energy) | 18.02 | 1.99 | 1.11 | 16.88 | 21.32 |
| Marginal cost | 1.43 | 0.23 | 2.34 | 21.09 | 0.58 |
| MRS | 1.02 | 0.9 | 0.2 | 5.83 | 1.53 |

## 2.9 Impulse response function of the structural model

Here, I evaluate the structural macroeconomic model with the given, sets of, parameters showing the impulse response functions (IRFs). The impulse responses come from positive shocks of each of the twelve exogenous shocks in the model that are assumed to follow AR(1) processes. The figures here is shown using the model's estimated parameters. In each figure, the x-axis refers to 'quarters' as the shocks are presumed to occur in the first quarter. *DYNARE* is used to generate the IRFs.

Figure 5 Consumption preference shock

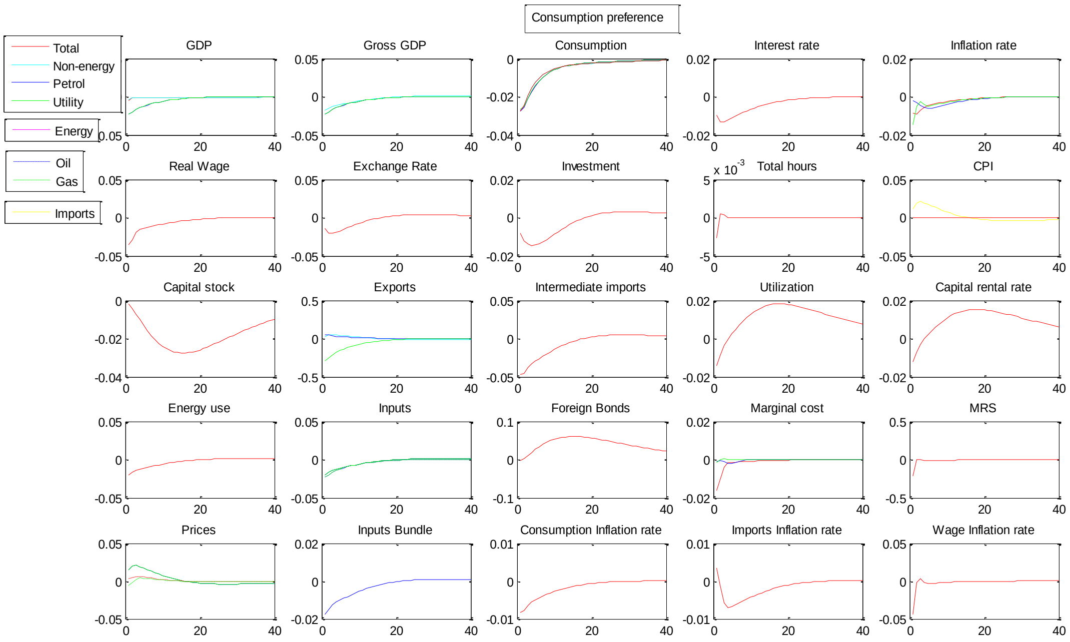

Figure 5 shows the effect a 15% consumption preference[33] shock has on the real macroeconomic aggregates. This shock is similar to an increase in risk premium such as credit control, and this will result in a fall in aggregate consumption, inflation, and output. To meet higher demand, the firm raises capacity utilization and

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[33] This is a shock that will increase the interest rate aimed at the consumers in relation to the policy rate.

employment as both are falling. The effect of falling consumption is also reflected in falling consumption inflation as consumer confidence is low. The response in falling real wages shows the willingness to work by households so that they can earn more to make purchases and also because of wage stickiness. However, the exchange rate rises as a response to the shock as demand falls in the United Kingdom relative to prices abroad. The movement in foreign bonds comes through in the foreign interest rate shock, hence foreign risk premium. The shock response to inflation and interest rates falling are as a result of flexible prices and central bank's Taylor rule.

Figure 6 Productivity shock

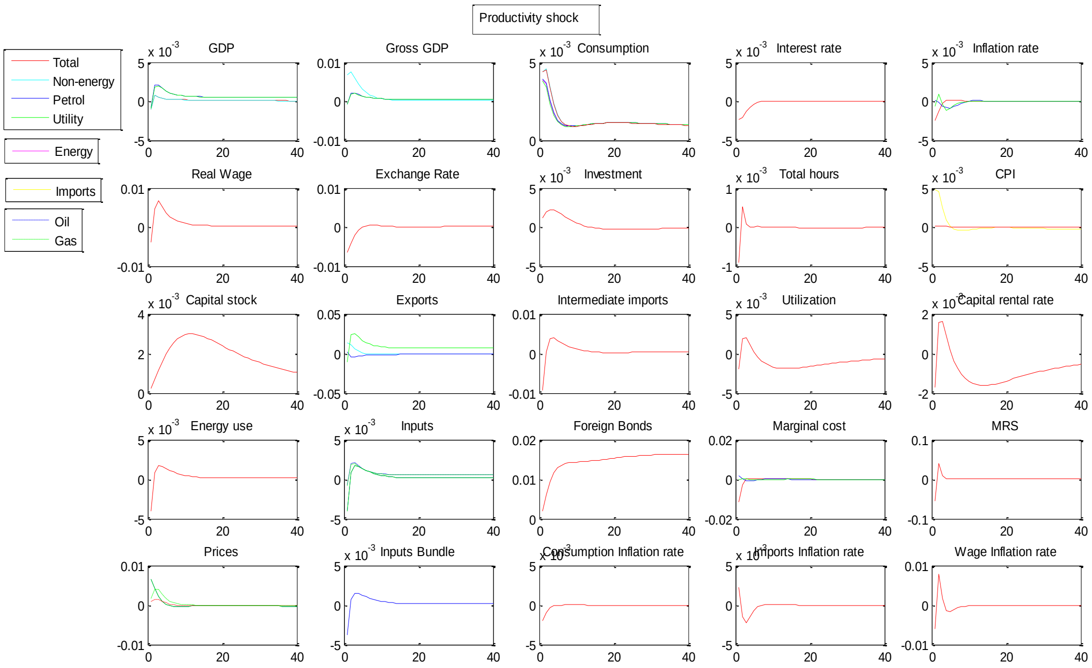


Figure 6 show the response to the model variables following a unit shock of productivity shock. The productivity shock affects gross non-energy output given value-added input (a fraction of total value-added that is proxy of output in this model) as it drives it to rise by almost 0.75%. As we can see, surprisingly, output fall that causes the fall in employment and capital utilization as aggregate demand does not correspond to output. This the makes marginal cost to increase for all firms as they respond by lowering prices to stimulate aggregate demand by reducing total hours and demand for capital. As a result of the shock, investment falls immediately but recovers within the year so that capital stock could be built up. The response by investment is due to the impact of capital adjustment costs. Assuming sticky prices, the demand for non-energy gross output will not respond much to the increase in productivity, which makes producers cut down on inputs, and this will include value-added. Also, assuming sticky wages, there will be a 'knock-on' effect on total hours of work. Consumption will rise while the shock makes households richer. Annual inflation and interest rates will fall and exchange rate depreciates as goods inside the United Kingdom will be produced at a lower cost compared to foreign goods.

Figure 7 shows the effects of a positive unit government spending shock. This shock leads to a fall in consumption which reflects 'crowding-out' effect (because increase in government spending is usually financed by higher lump-sum taxes from households). The overall effect is a positive one as firms demand more labour for 2 quarters and increase capacity utilization. The rental rate of capital rises as does real wage rate because of households' willingness to work more. Although the rise in output is much smaller than the increase in government spending, the increase in demand leads to a rise in inflation, though this is close to zero, and also gave a little push to the interest rates as the Bank of England moves to cut demand. Finally, the increase in exogenous government spending relative to foreign demand pushes the exchange rate that appreciates.

Figure 7 Government spending shock

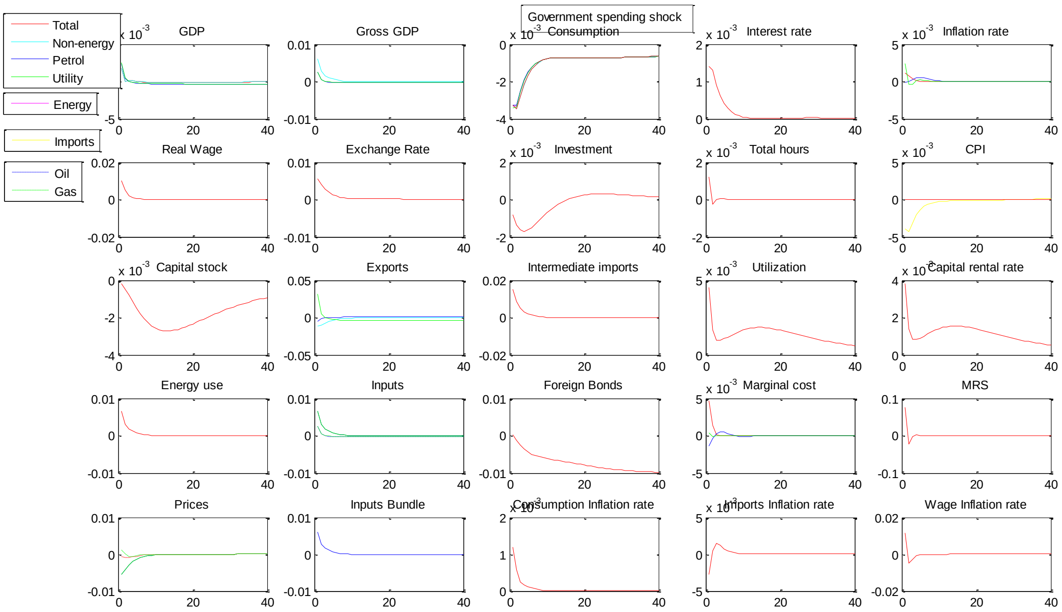


Figure 8 Monetary policy shock

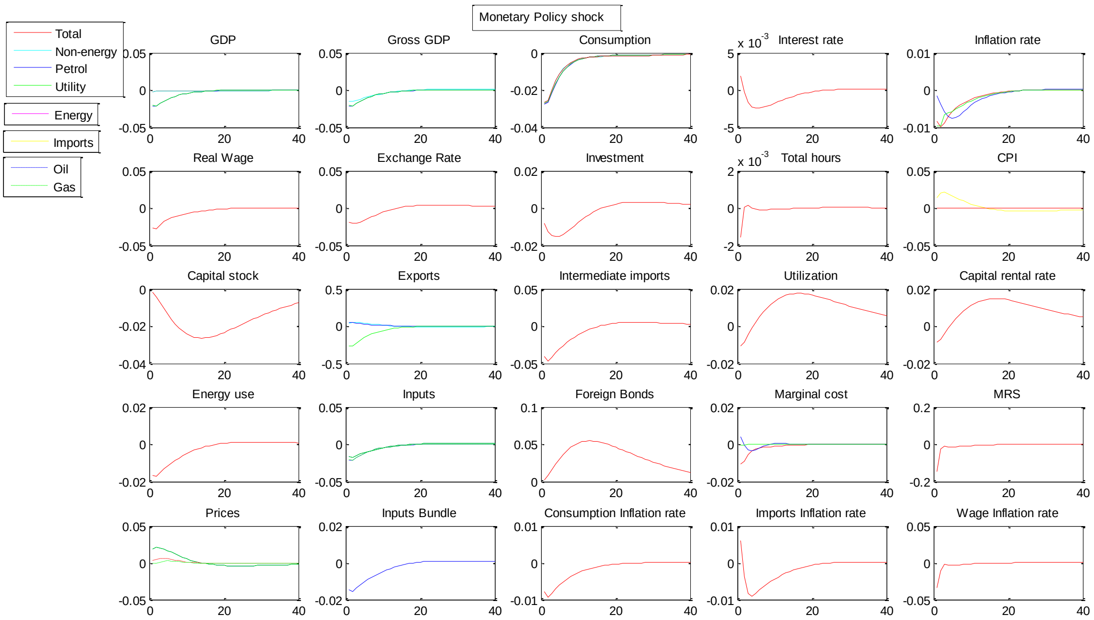


Figure 8 shows the responses of model variables to a positive unit shock in monetary policy. Following this shock, the short-term and real interest rate will rise. Reflecting the role of nominal rigidities, the increase in rates causes a fall in consumption, the output is affected negatively as it falls, as well as an investment. The fall in investment will about twice as that of output. Firms will also reduce employment while the rate of utilization of capital will fall due to lower aggregate demand.  Oil exports rise significantly while foreign demand for gas goes the other way as a result of the shock. Petrol prices after tax fall as a result of the rise in interest rate that shows the maximum response of real variables to the shock is instantaneous. The exchange rate tracks interest rate's pattern; this is because of uncovered interest rate parity (UIP) condition with the initial effect of the shock being an appreciation. The appreciation of exchange rates comes with the increase in interest rate. Domestic sticky prices will lead to rise in exchange rates, and this will consequently reduce the demand for exports. There is a fall in real wages as demand falls, and households are willing to work due to fall in inflation across sectors will returns to steady-state after about three and a half years. The responses here are in line with the empirical study of di Cecio and Nelson (2007), Kamber and Millard (2010) and Christiano et al., (2005).

This model is developed precisely to evaluate the effects of a shock to world energy prices. Therefore, it is most important to study the impact of the shock to world oil prices.

Figure 9        World oil price shock

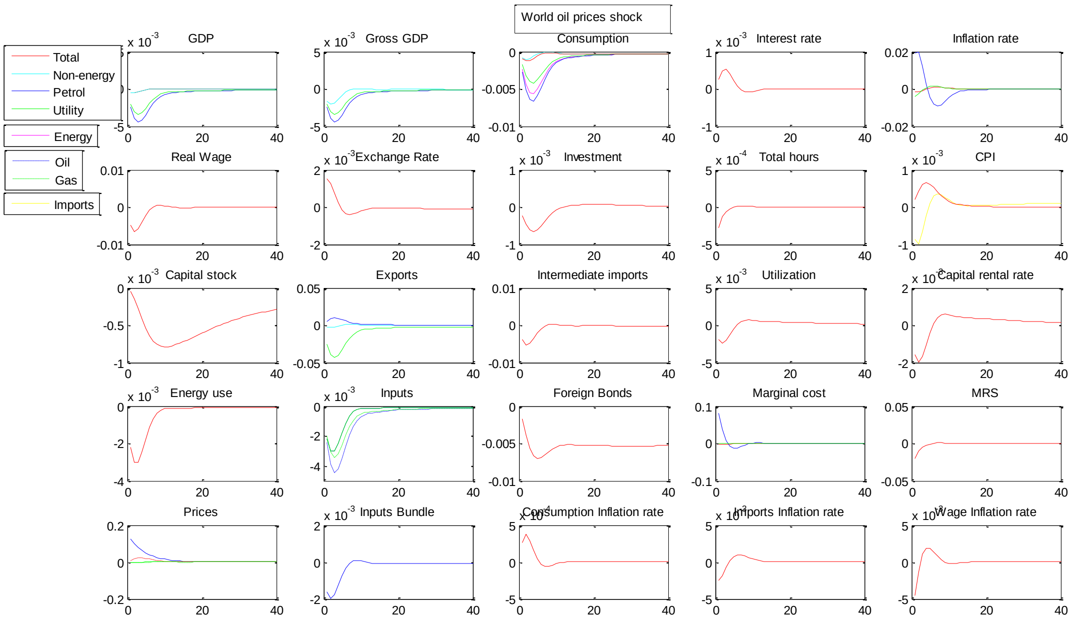


Figure 10        World gas price shock

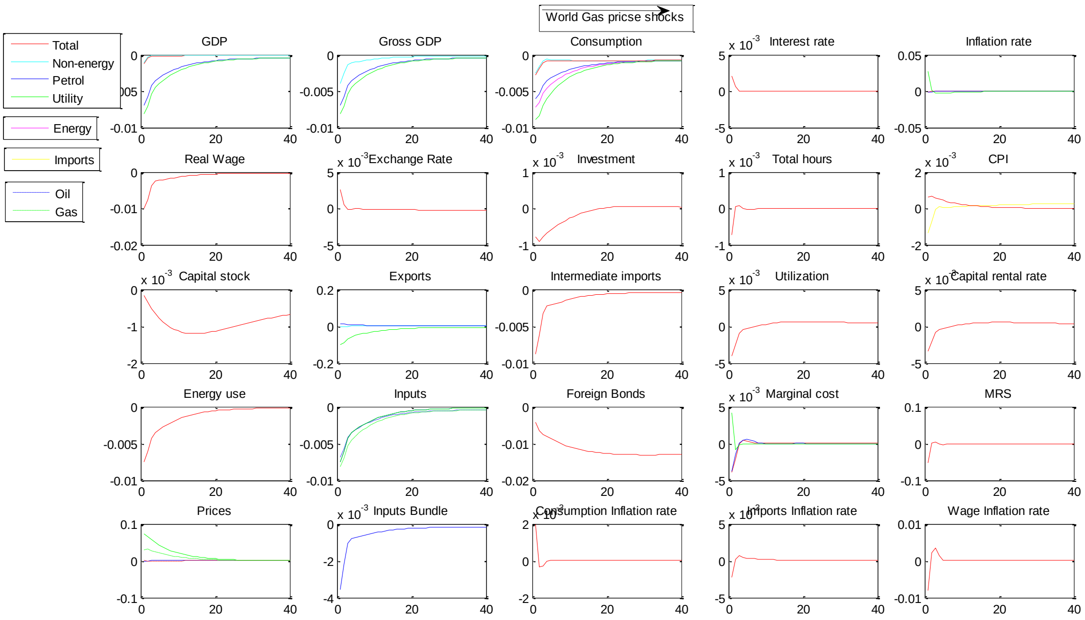
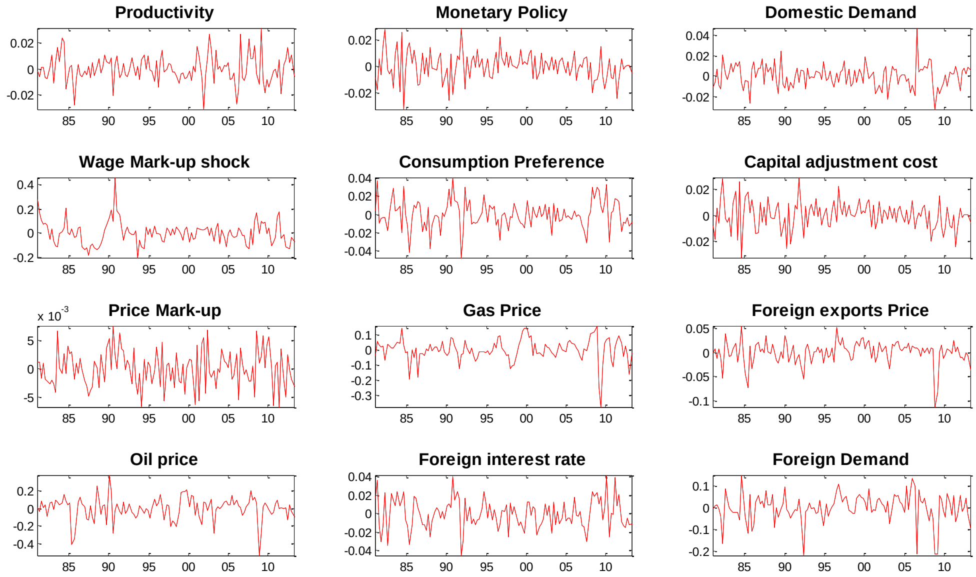


Figure 9 shows the responses of variables to an exogenous increase in the world price of oil of 12.7% (a one standard deviation shock). The effects of the shock have output and consumption falling consumption but converges within five and ten quarters respectively. The effects of this shock are minimal as can is seen in proportion to the shock. Generally, energy price shock is argued to be less effective in DSGE models (Hamilton (2003), and Killian and Vigfusson (2014)). The effects, on falling output, are only a temporary terms of trade shock. As GDP only falls briefly, the UK can borrow against such a temporary fall. This effect comes as exchange rates rise which makes a demand for foreign goods fall, hence a drop in demand for intermediate imports in the model. The marginal cost of producing petrol increases as firms demand less of labour to reduce that, output falls. Inflation is decreasing and then rises above its steady-state as a response to the shock. Labour takes a hit in their real wage for a five quarters following the shock which means there is a slight indication of real wage resistance. Figure 9 shows the responses of real variables to an exogenous increase in the world price of gas of 7.4% (a one standard deviation shock). The effects of this shock are qualitatively similar to those of an oil price shock. The effects on real variables are, again small this is because the shock has low persistence. The impact of the shock have output and consumption falling consumption. Inflation of the sector rises above its steady-state as a response to the shock. Labour takes a hit in their real wage for a few quarters following the shock which means there is a slight indication of real wage resistance.

## 2.10 Accounting of the shocks during the crisis period

Figure 11 and shows the time series for the shocks in the model which include the domestic as well as foreign shocks

Figure 11 Shock's Innovations



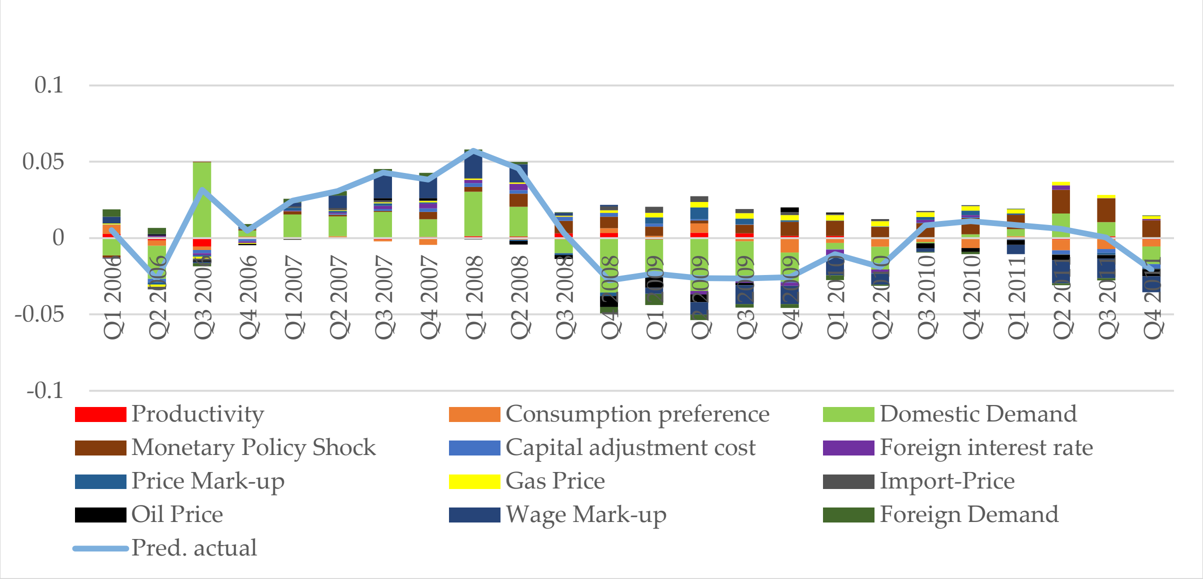
As noted earlier, from the estimation results, one can see that the shocks to energy prices, wage mark-up and foreign demand have been highly volatile over this period. Conversely, monetary policy, productivity and consumption preference shocks have been less volatile. If we pay attention to the recent past, one can say that the world economies have been affected by huge negative shocks to energy prices and foreign demand. The foreign demand shock reflects what happened to world trade during the 2008 and the 2009 calendar year.

### 2.10.1 Shock decomposition during the crisis period

The evaluation of this DSGE model will conclude with the analysis of the macroeconomic aggregates during the financial crisis period. The recent financial crisis of 2008-2009 was caused by an enormous decrease in market lending, that led to a drop in consumer confidence due to financial instability. It led to fall in foreign demand for home goods as a result of the global recession, government's austerity measures by governments that contributed to the initial fall in output. The UK introduced the Quantitative Easing by borrowing more to cover aginst the drop of output. The nominal rate of interest was slashed to 0.5% and with inflation rate of about 2%, the UK, like the rest of the world, was facing negative real rate of interest. Here, I decompose what happened during the crisis period as a result of the shocks, according to the model. By doing this, I show what determines the shocks that have been the main drivers of these variables. Here, I show the crisis period of output, gross (non-energy) GDP, inflation and interest rate.

The crisis period was not caused by a rise in oil price but rather oil prices peaked as the recession was kicking in, and governments were taking austere measures to curb it. Matters were complicated as the rise in oil prices caused cost-push inflation that made central banks reluctant to reduce the interest rate. World oil prices peaked during this period which contributed to lower spending as a result of a reduction in discretionary income. Global oil prices peaked due to high demand from China and India even as Europe, and the US were in a recession.

Figure 12 Shock decomposition of output



For output, figure 12, exogenous government spending (light green) is most dominant, as expected, in the fall of output as also before the crisis where demand contributed to increasing output. Again, changes in oil prices which were similar to world gas prices are expected to be key in the fall of output during this period. From the output chart, one can see that the high oil prices (oil price shock is in black color) of late 2008 noticeably contributed to the fall in GDP. Although gas prices (yellow) are high which shows energy prices were also pushing up on output towards the end of the period.

Looking at the non-energy output, in Figure 13, domestic demand shock dominates while there is a foreign demand (purple) was non-existent due fall in exports as a result of the global recession. It is also no surprising that energy prices are among the shocks that cause its movements. This is because energy is part of the inputs of firms production goods and also because energy was at its peaked over this period.

Looking at monetary policy shocks, it can be explained that the shock was supporting output in during the crisis period due to interest rates cuts by higher than what would have been recommended by the Taylor rule in the model. Evidently, the 'systematic' monetary policy response is contributing to output coming from quantitative easing. These results are consistent with linear models results, e.g. Millard (2011) were he reported little effects of energy prices but high monetary policy shock effects.

Figure 13 Shock decomposition of non-energy gross output

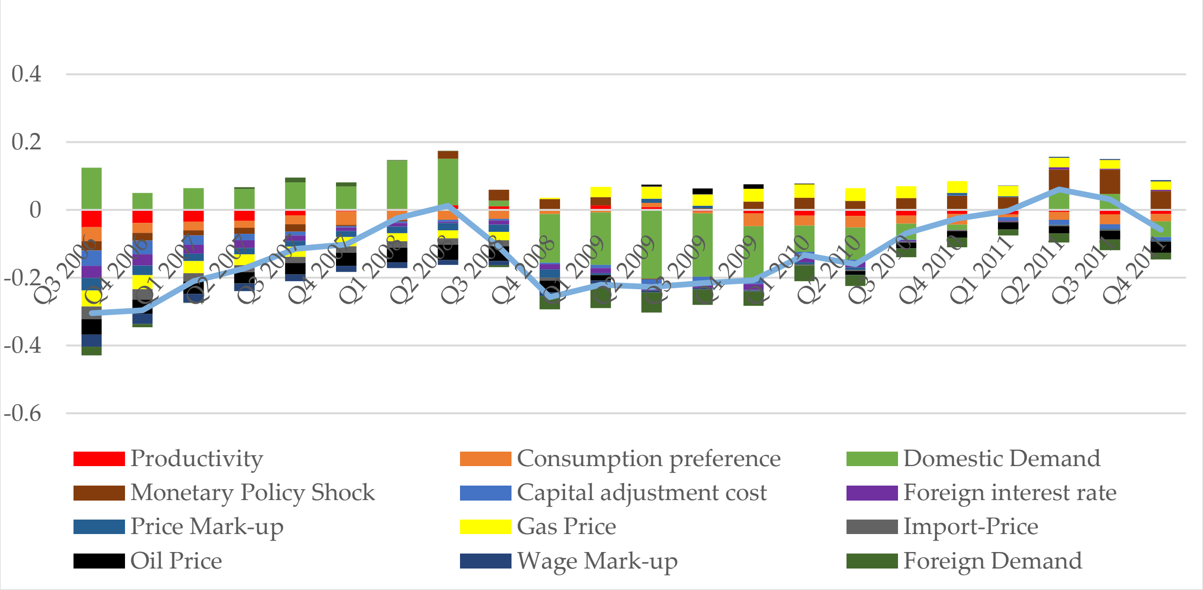
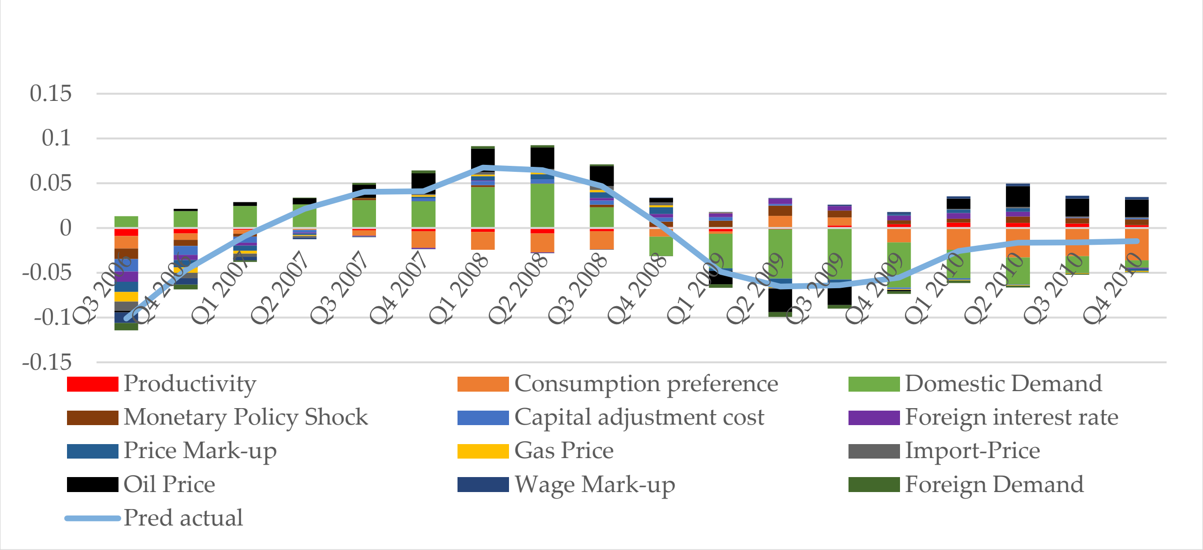
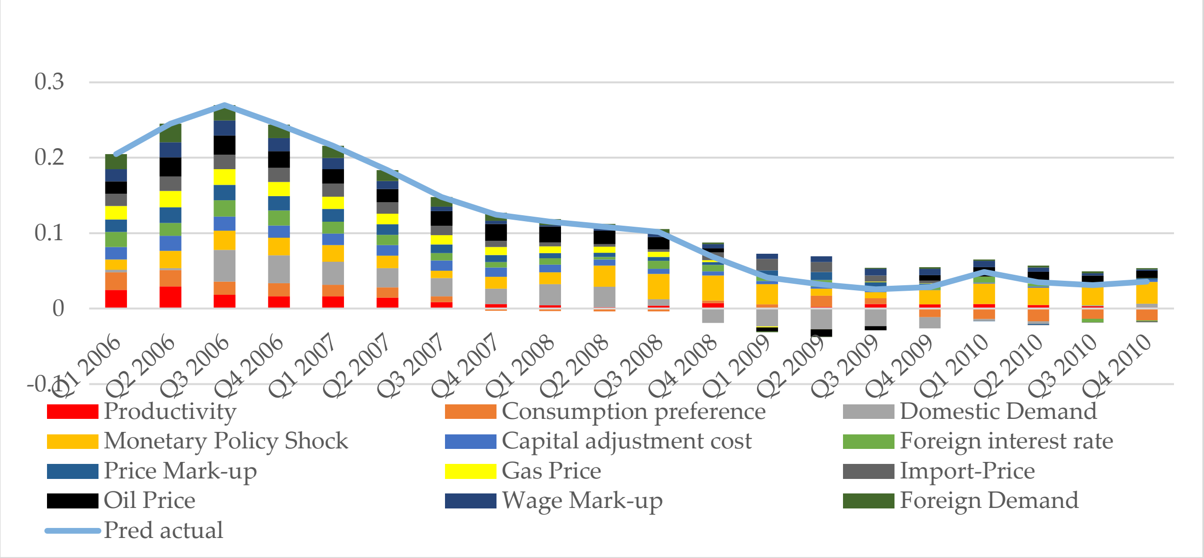


Figure 14 Shock decomposition of real interest rate



For interest rate, figure 14, shows that there is a domination by government exogenous spending shock. This comes from the effects the shock has on the output that pulls down real interest rate from 2008: Q3 as it falls steeply. Also the consumption preference shock that comes in as a result of lower consumer confidence as a result of financial instability and the credit crunch. World oil price shock is visible in 2009 as interest rates were at minimal, and the recession was impacting more.

Figure 15 Shock decomposition of Inflation rate



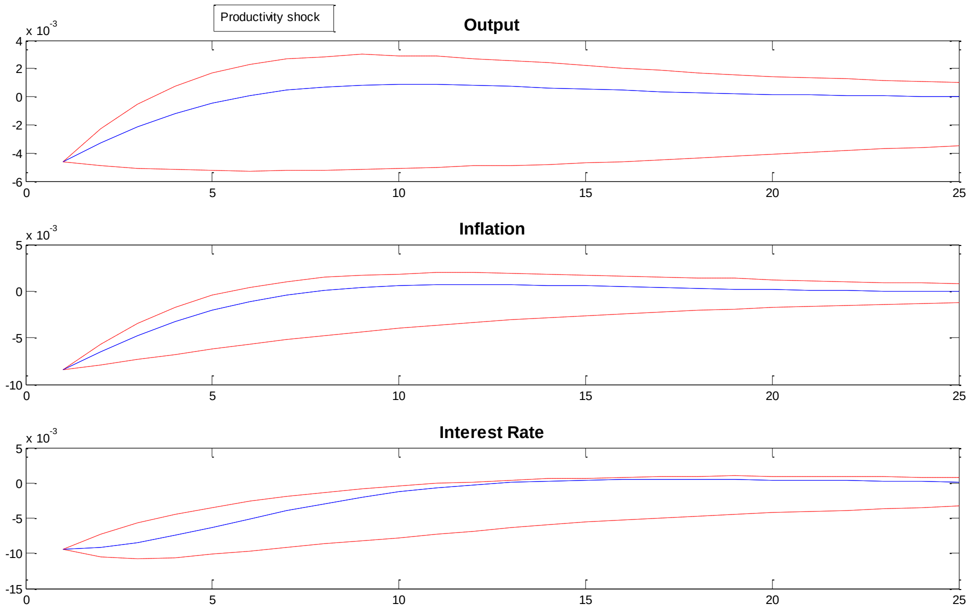
Looking at inflation, Figure 15, the shock decomposition suggests that the monetary policy shock was pushing down substantially on inflation from 2008:Q3 to 2010: Q4 with the domestic shock as well as energy prices also contributing. Contrary to this, positive foreign exports price shock, from 2008: Q4 to 2009: Q4, were contributing to pushing inflation up. The rise in oil prices and gas prices in 2008: Q1, and later in 2009: Q4, gas prices throughout 2008 put pressure on inflation to increase in 2008. Therefore, as oil and gas prices began to drop in 2009, they again moved to reduce inflation.

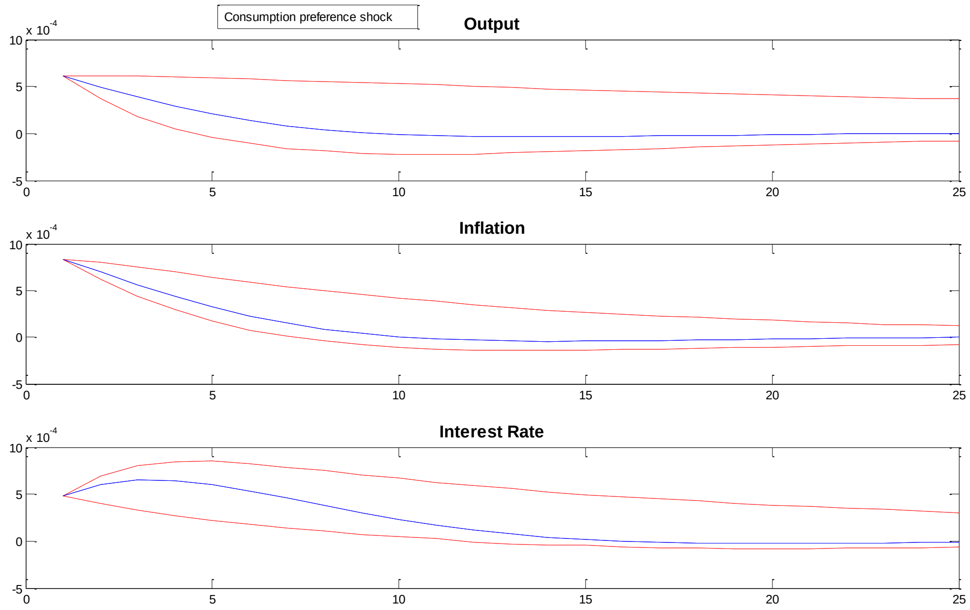
## 2.11 Summary

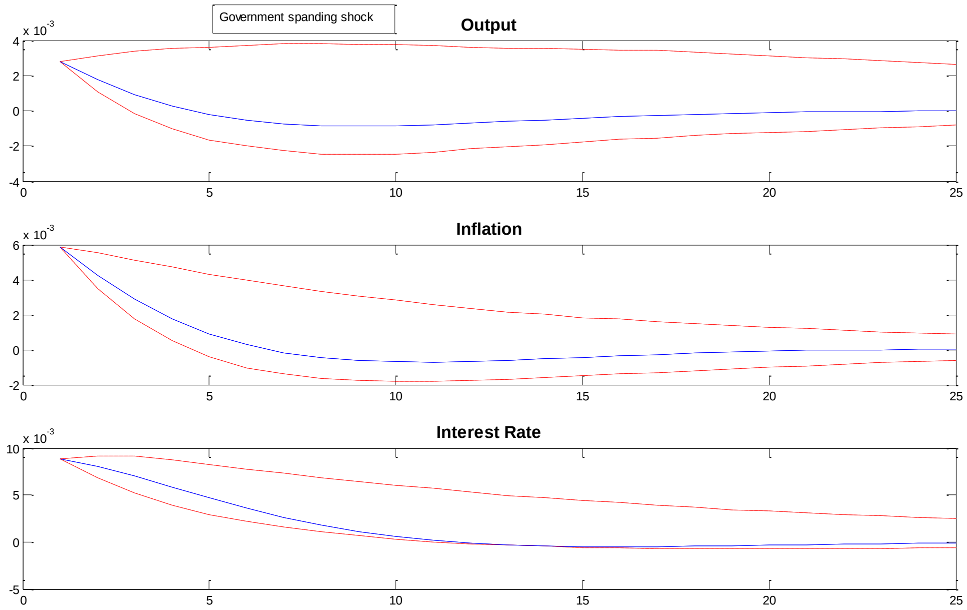
In recent times, the Bayesian estimation method has proven to be an effective tool in improving DSGE models by incorporating prior information about the economy. Nevertheless, it has its shortcomings. I use an effective method of estimation that proves to be the optimum way of evaluating a DSGE model that overcomes most of the problems that are faced by DSGE models. This model is applied quantitatively using an efficient, practical tool on the UK stationary data from 1981: Q1 to 2013: Q1. At first approach, I evaluated the performance of the calibrated model which was found to be poor. It fails to match the data and its variances using this set of parameters. Based on the assessment, I went on to estimate the model using simulated annealing. In matching the data, the shock processes play a key role and the foreign shocks (especially the energy shocks) are estimated to have high persistence. In the application of the model, the study showed how this could be done by evaluating the effects of different shocks on output, inflation and interest rates from the VAR impulse response functions. By decomposition the changes in these variables caused by each of the structural shocks showed that a fall in output during the financial crisis period 2008:Q2 to 2009:Q4 was driven by domestic demand shocks, oil prices shocks and world demand shocks. The effects of the productivity shock were minimal. These same shocks of domestic demand also put downward pressure on inflation since the world demand was less significant in determining the movement of inflation over this period. The model shows that the stationary energy shocks' negative effects on output are only a temporary terms of trade shock as GDP only falls briefly, as the UK can borrow against such a temporary fall.

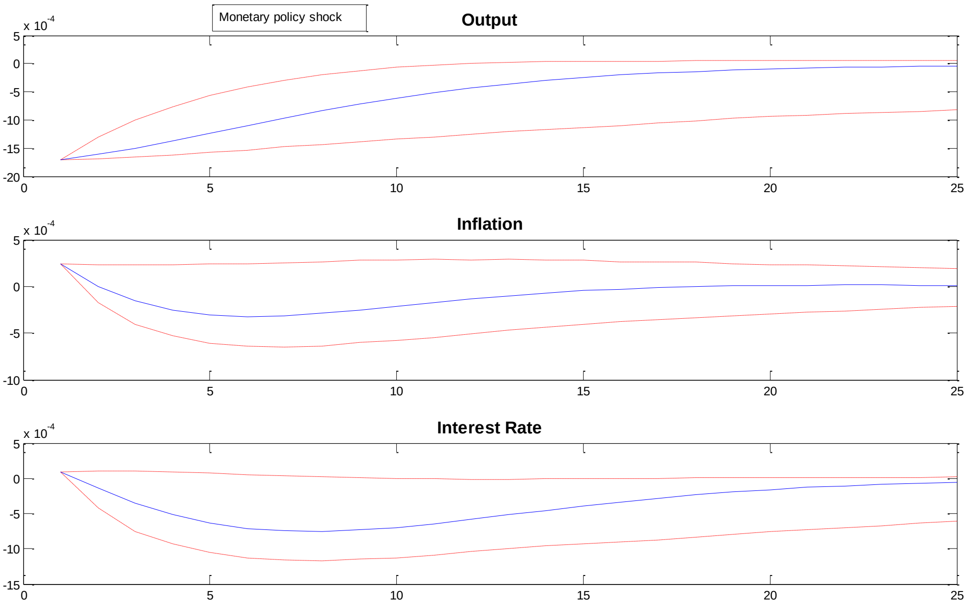
Meenagh, Minford and Wickens (2012) noted that filtering data may distort a DSGE model's dynamic properties in some unknown ways. This could be from the way that the HP-filter alters the lag dynamic structure or generating cycles where none exists. The forward-looking properties of the model are also transformed due to the filter being two-sided. As a result, there could be a serious defect in the DSGE model estimation. The study suggests a promising avenue for future research which is evaluating the model on non-stationary UK data. Several studies have shown that oil prices have proven to be non-stationary, and that requires the model to incorporate non-stationary exogenous variables of energy shocks among others. Also, given the nonstationarity of world energy prices, depicting stationary data for such variables may not be show the true impact of energy prices. The issue of nonstationarity could be a solution to the problem of DSGE models not showing the effects of energy shocks as emphasised by Killian (2008a), Killian and Vigfusson (2014), and Hamilton (2008). All of the firms in this model are assumed to be energy efficient. An extension of this work could also be very interesting by incorporating a non-energy efficient firm (such as services) to the supply side to complement an energy efficient (like manufacturing) firm.

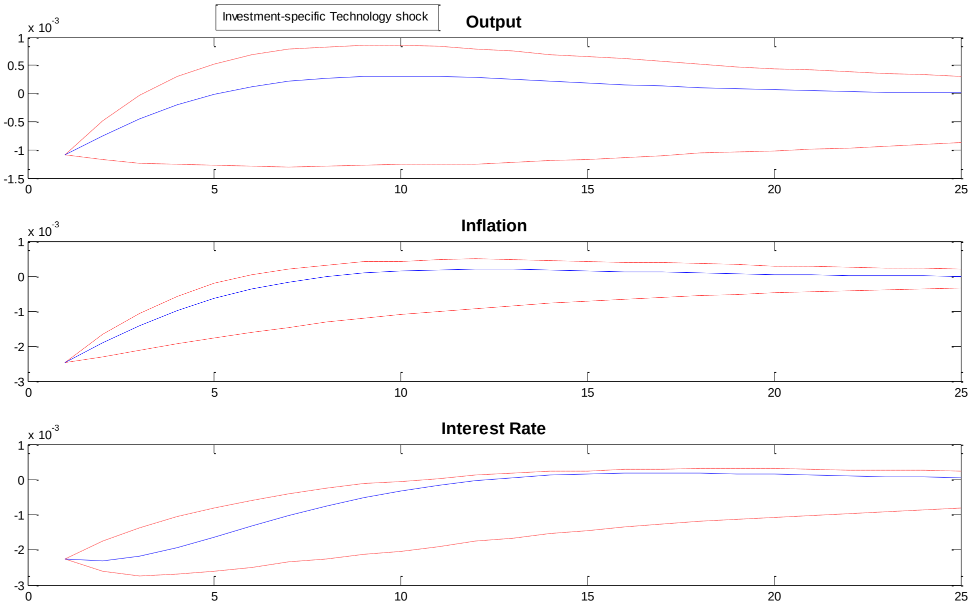
# Appendix 1.1 VAR-Impulse response functions

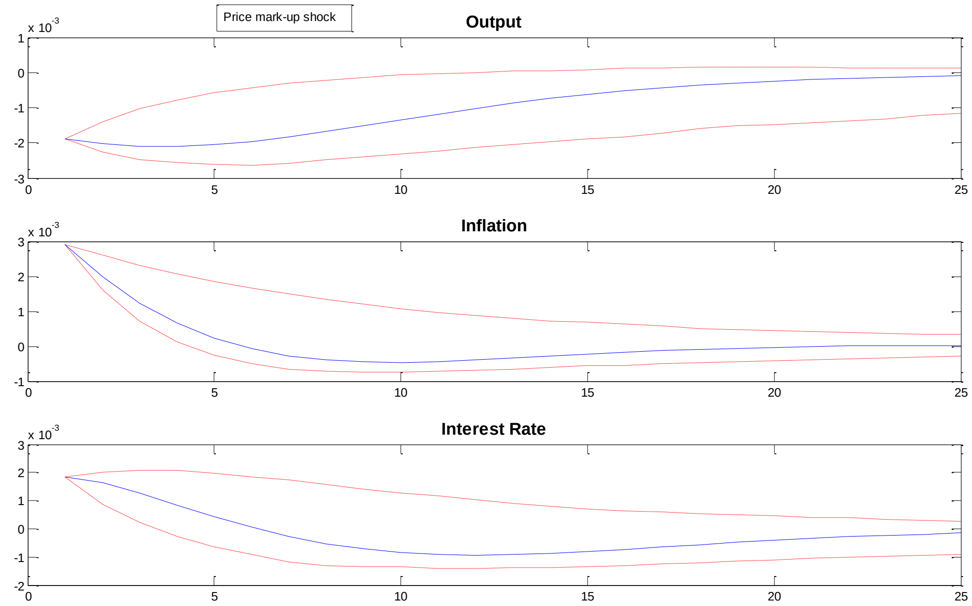


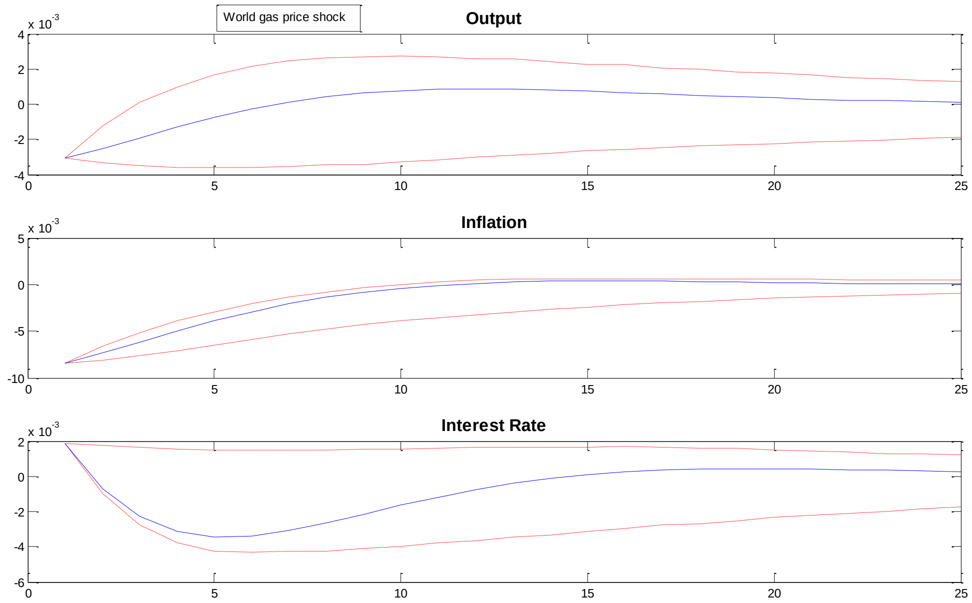


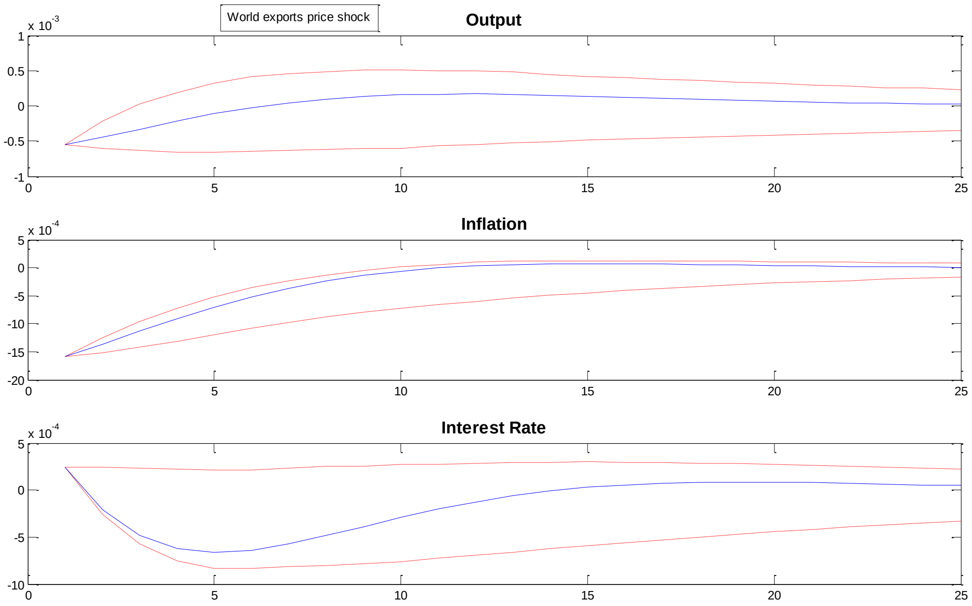

Government spanding shock
Output
x 10^-3
4
2
0
-2
-4
Inflation
x 10^-3
6
4
2
0
-2
Interest Rate
x 10^-3
10
5
0
-5
0
5
10
15
20
25

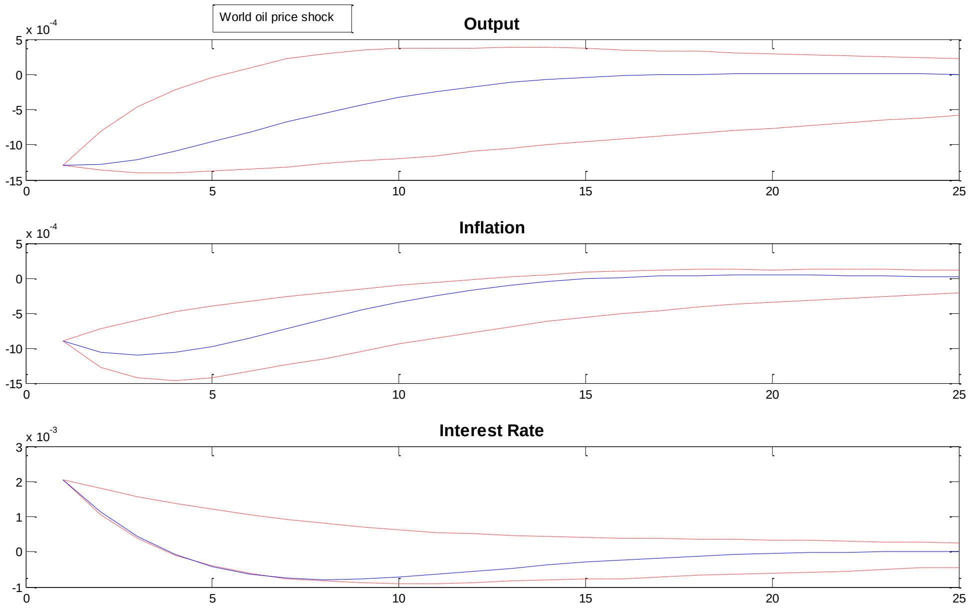

Monetary policy shock
Output
x 10^-4
5
0
-5
-10
-15
-20
Inflation
x 10^-4
5
0
-5
-10
Interest Rate
x 10^-4
5
0
-5
-10
-15
0
5
10
15
20
25

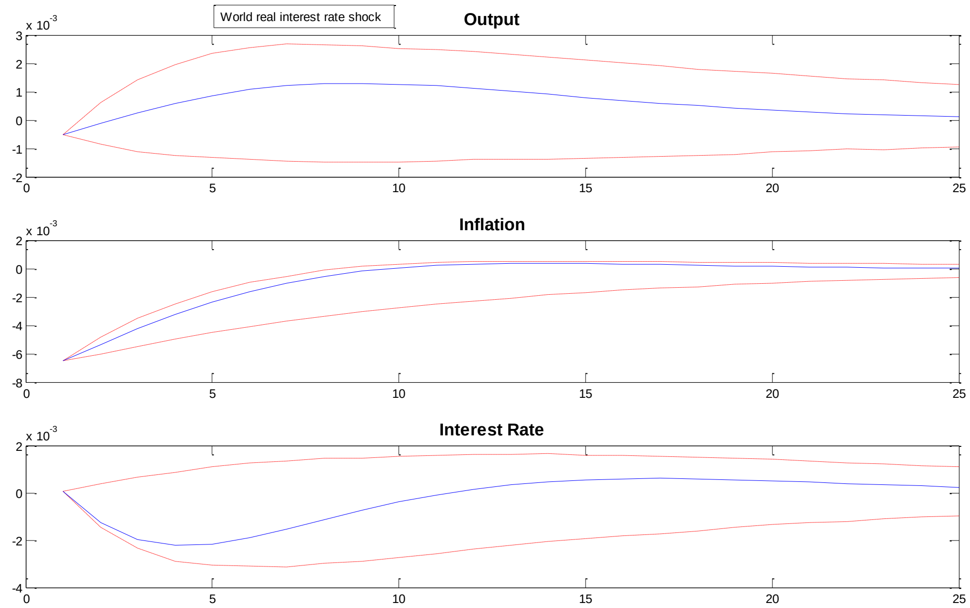

Investment-specific Technology shock
Output
Inflation
Interest Rate
x 10^-3
1
0.5
0
-0.5
-1
-1.5
1
0
-1
-2
-3
0
5
10
15
20
25

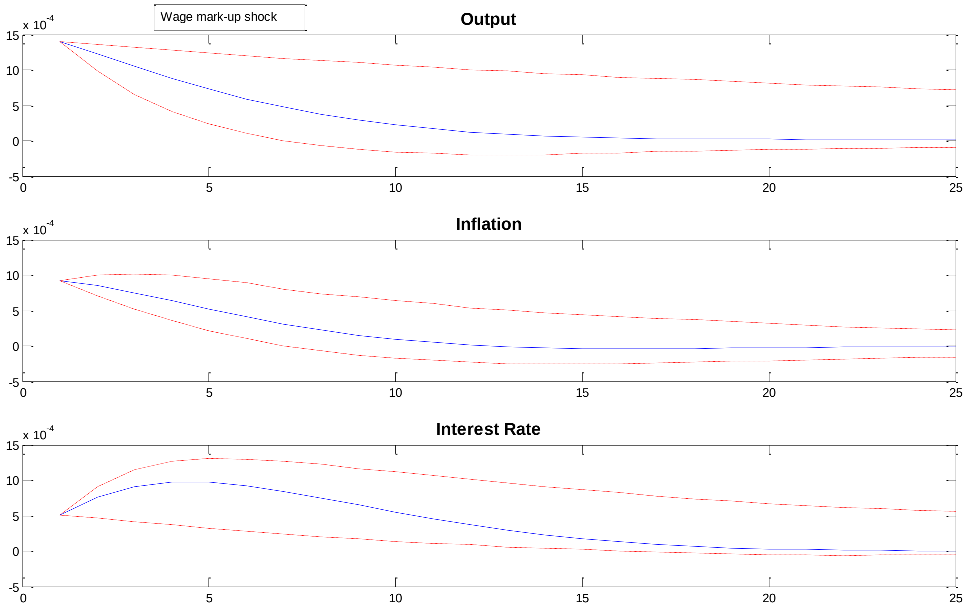

Price mark-up shock
Output
Inflation
Interest Rate
x 10^-3
3
2
1
0
-1
-2
-3
0
5
10
15
20
25

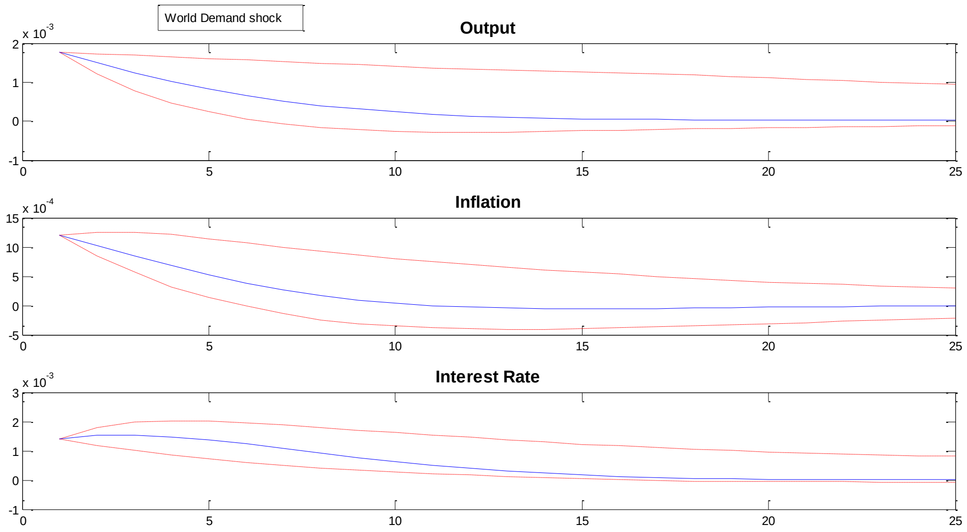

World gas price shock
Output
x 10^-3
4
2
0
-2
-4
Inflation
x 10^-3
5
0
-5
-10
Interest Rate
x 10^-3
2
0
-2
-4
-6
0
5
10
15
20
25


World exports price shock
Output
x 10^-3
1
0.5
0
-0.5
-1
Inflation
x 10^-4
5
0
-5
-10
-15
-20
Interest Rate
x 10^-4
5
0
-5
-10
0
5
10
15
20
25


World oil price shock
Output
x 10^-4
5
0
-5
-10
-15
0
5
10
15
20
25
Inflation
x 10^-4
5
0
-5
-10
-15
0
5
10
15
20
25
Interest Rate
x 10^-3
3
2
1
0
-1
0
5
10
15
20
25


World real interest rate shock
Output
x 10^-3
3
2
1
0
-1
-2
0
5
10
15
20
25
Inflation
x 10^-3
2
0
-2
-4
-6
-8
0
5
10
15
20
25
Interest Rate
x 10^-3
2
0
-2
-4
0
5
10
15
20
25


Wage mark-up shock
Output
x 10^-4
15
10
5
0
-5
0
5
10
15
20
25
Inflation
x 10^-4
15
10
5
0
-5
0
5
10
15
20
25
Interest Rate
x 10^-4
15
10
5
0
-5
0
5
10
15
20
25


World Demand shock
Output
x 10^-3
2
1
0
-1
0
5
10
15
20
25
Inflation
x 10^-4
15
10
5
0
-5
0
5
10
15
20
25
Interest Rate
x 10^-3
3
2
1
0
-1
0
5
10
15
20
25

## Appendix 1.2 Model's Impulse response functions (continued)

Figure 16 World interest rate shock

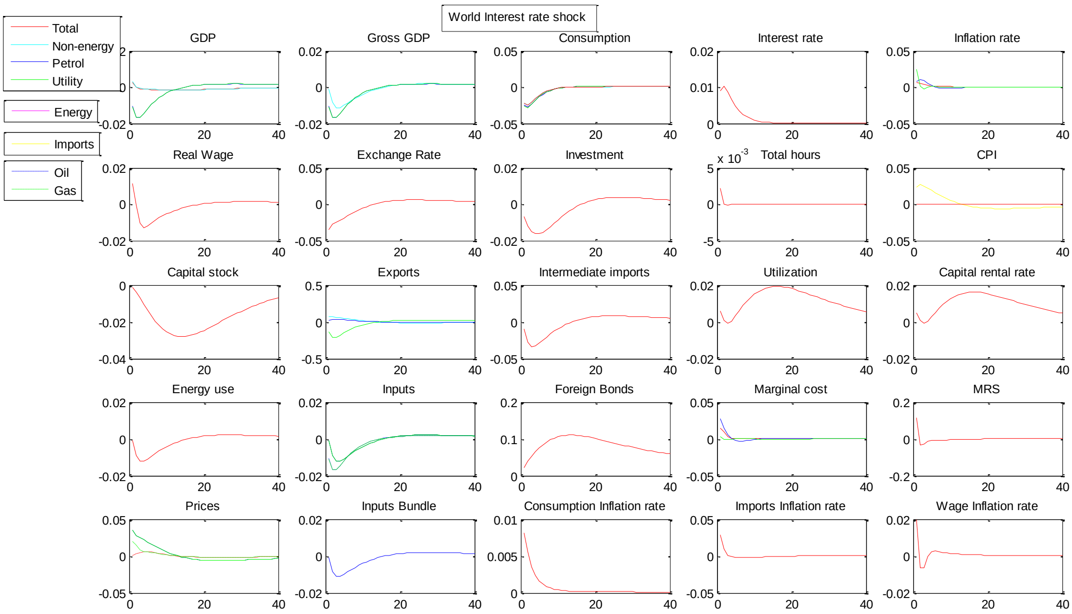


Figure 16 shows the responses of the macroeconomics variables to a 15% (one standard deviation) foreign interest rate shock (one can also view this shock as foreign exchange risk premium shock). This affects both a fall in aggregate consumption and depreciation of sterling. On the turn, output, employment as well as oil and gas prices all rise as export demand rises in response to the fall in the relative price of UK exports. Besides this inflation rises as the increase in sterling import prices leads to a distinct rise in costs, and this leads to a rise in nominal wages as labour try to reduce the fall in real wages.

Figure 17 shows the effects of a foreign demand for UK goods. Foreign demand shock leads to an increase in output, consumption, and total hours of employment and ultimately exports. The rise in relative demand for exports causes an appreciation of the exchange rate that then pushes down on domestic inflation through falling import prices.

Figure 17 World demand shock

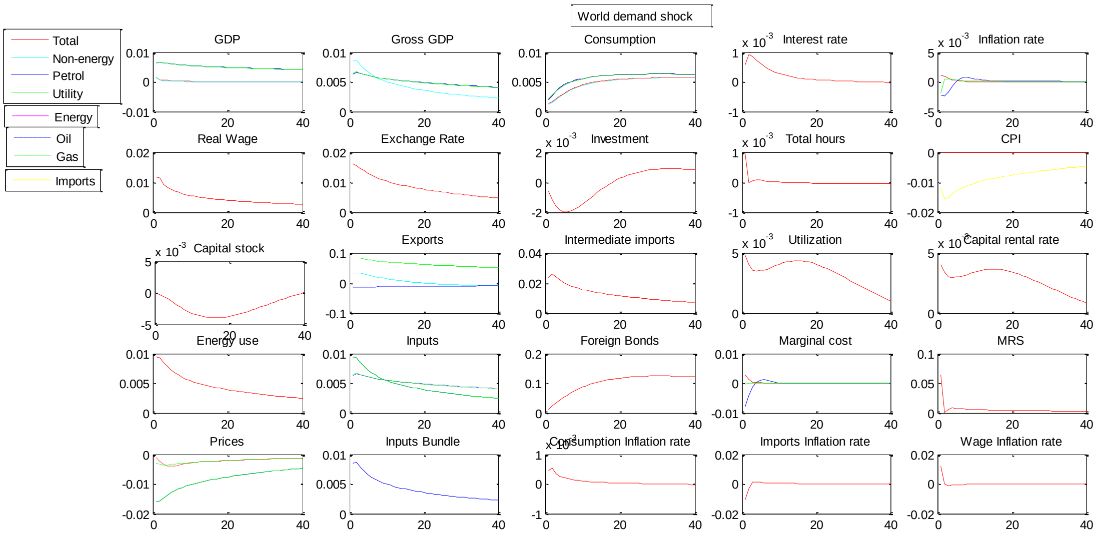


Figure 18 Capital adjustment cost shock

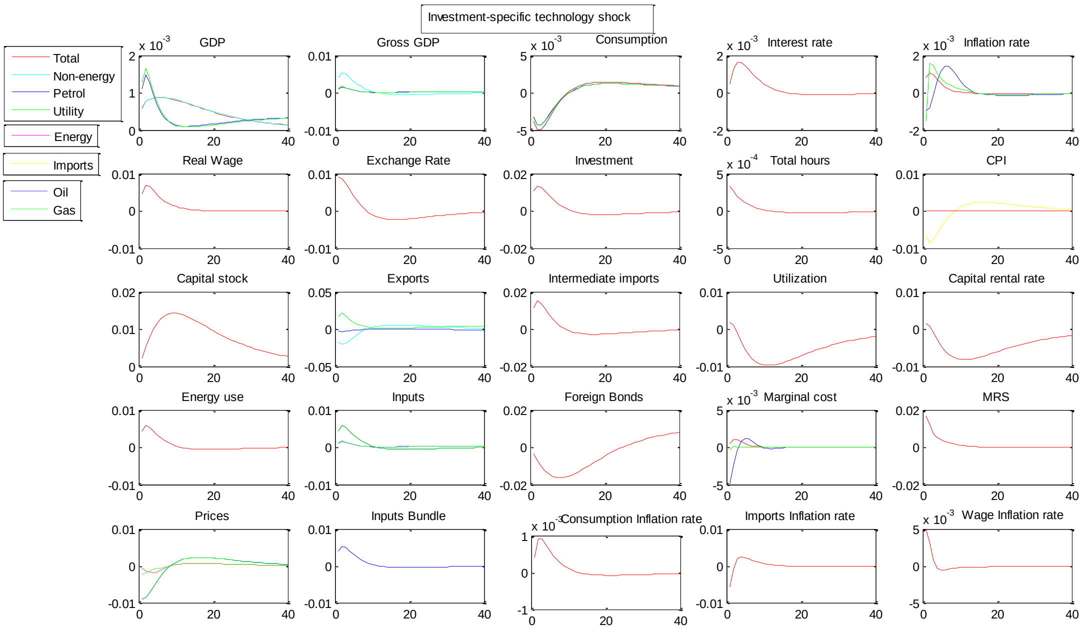


Figure 18 shows a positive shock to capital adjustment cost shock of 2.8% % (one standard deviation) makes investment to rise, this then brings about increase in output and employment but consumption falls by about 0.5% that converges after about quarters. As expected, the rate of real wages increases with this shock coordinated by output while utilization and rental rate of capital continue to fall for about ten quarters (medium term) as these are the variables that push down investment to converge. As factor cost rises with pressure on demand, the inflation rate will increase, and monetary policy will respond by rising interest rate in the economy.

Figure 19 shows the effects of 11% positive wage mark-up shock. Following the shock, households will be keen to supply additional labour at a given wage rate. This shock impacts inflation and aggregate demand positively because real wages are high. The difference is that total hours of employment decreases as firms are not able to pay higher wages. Due to high aggregate demand, consumption inflation rises as well as a rise in investment to boost output. Finally, the exchange rate depreciates due to falling imports prices, and monetary policy will react by cutting nominal rates.

Figure 19 Wage mark-up shock

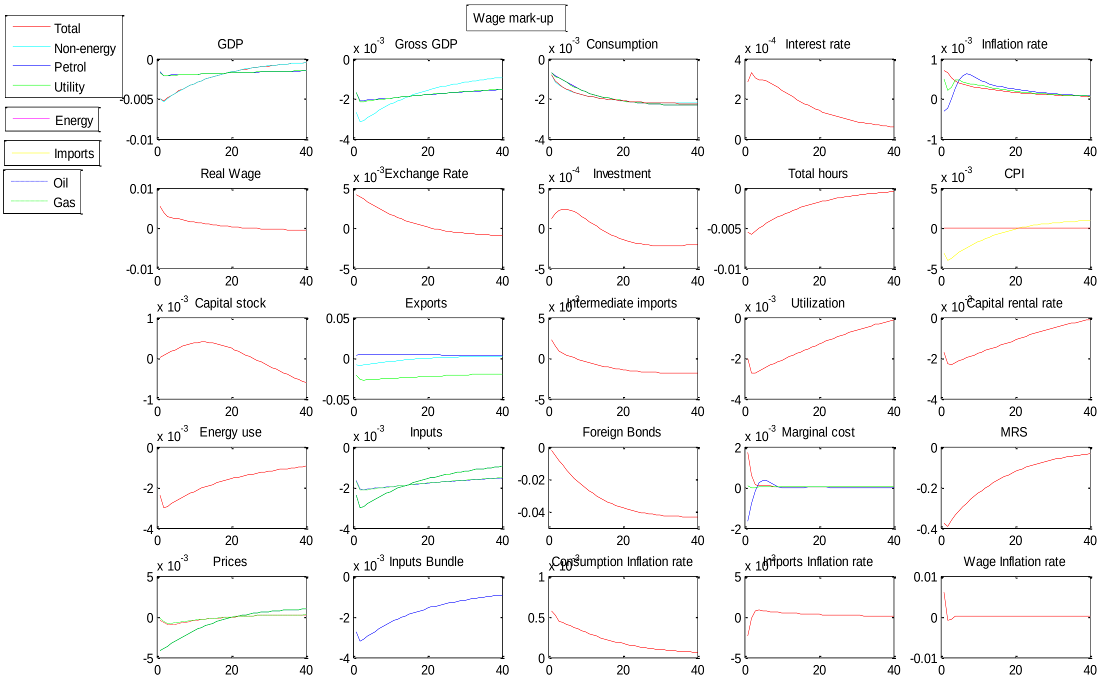


Figure 20 Price mark-up shock

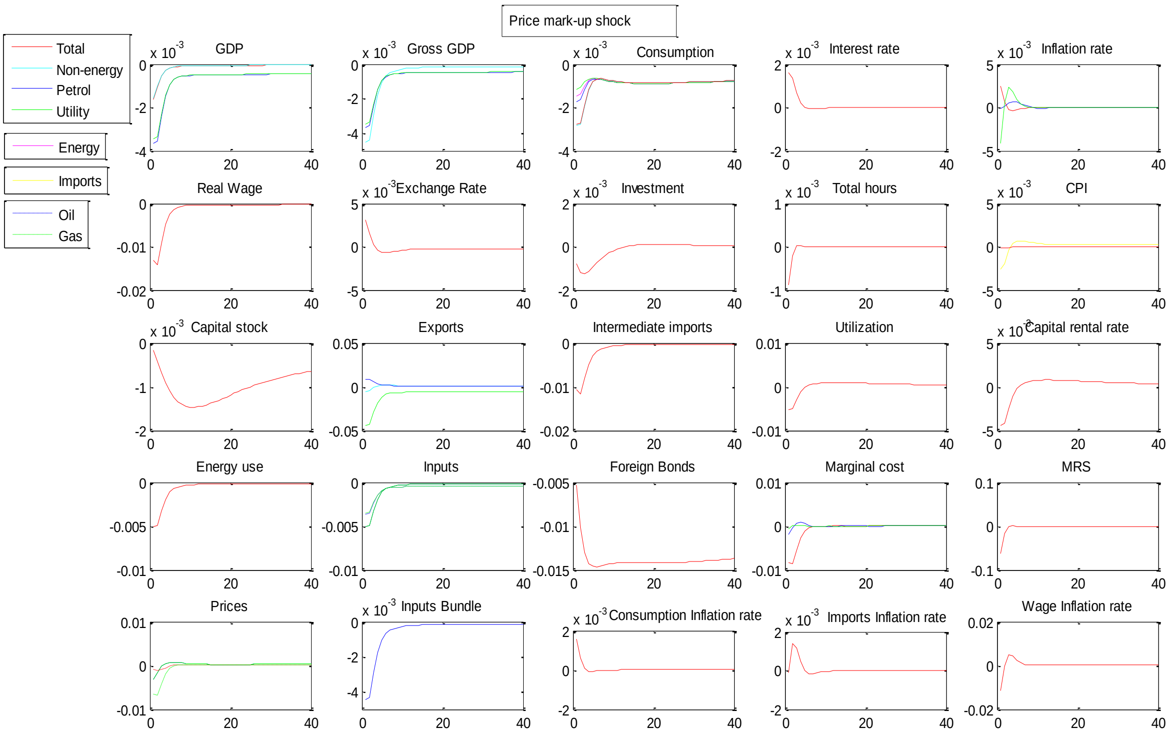
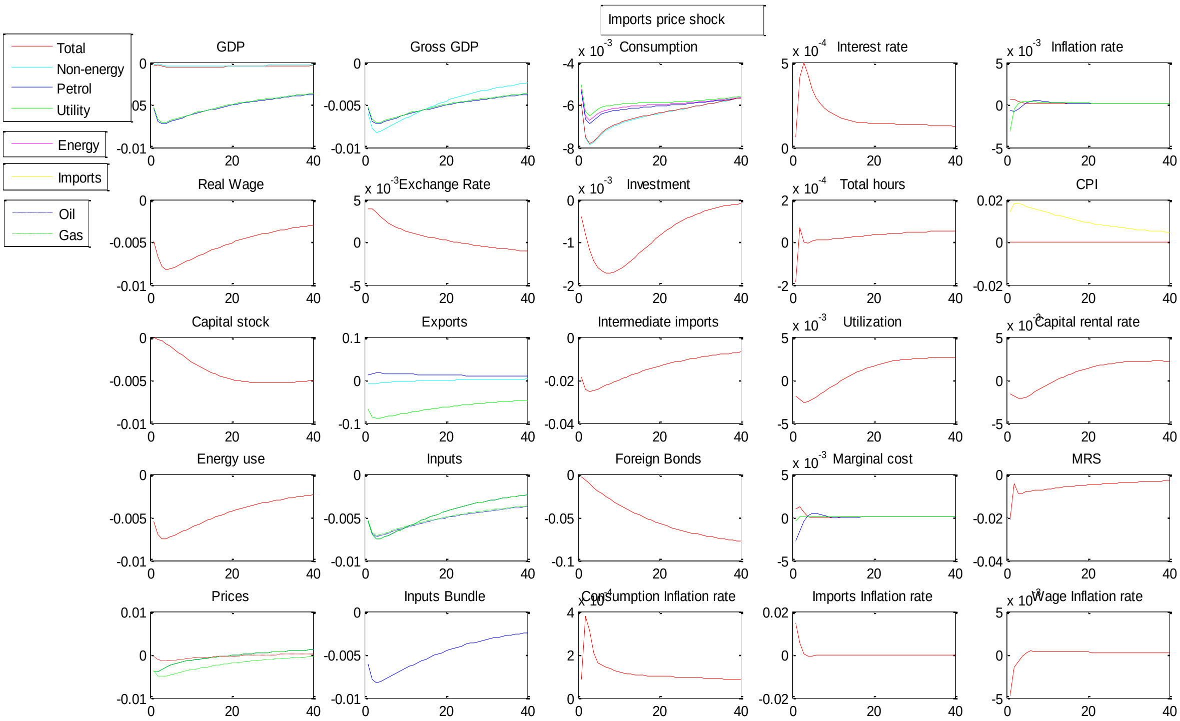


Figure 20 shows the effects of a 0.3% (one standard deviation) price mark-up shock.

The instantaneous response is a fall in producer price inflation as consumer inflation reflects this and a rise interest rate that pushes down investment. Firms respond to reduce their marginal cost by cutting down employment that makes output fall. The households' willingness to work and intention to increase demand pushes down real wages for a quarter before it begins to pick up. As exchange rate appreciates, exports will rise gradually back to its steady-state level.

Figure 21 shows a shock to world export prices. A shock to foreign export will lead to a rise in home import prices that, sequentially, feeds into the home price, employment and real wages. Hence wage inflation rises. Consumption at home falls as output turn out to be more expensive. Lastly, the exchange rate appreciates in response to the increase in demand for the domestic exports.

Figure 21 Import price shock



## Appendix 1.3

## 1.3 The model

### 1.3.1 The Household

There is a continuum of households of unit mass. Households, indexed by $\epsilon(0,1)$ , maximises each of their utility functions defined over consumption *(c),* hours worked *(h)* and real money balances $\frac{MON_t}{P^c}$ . Budget constraint shows how the end of period holdings of nominal government debt$(BG)$, nominal foreign bonds $(BF)$, capital $(k)$ and money $(MON)$ are given by their start of period holdings, plus net income. The net income includes earnings from labour supply (at wage) and capital services $(k_{t-1}z_t$ rented at rate$(W^k)$ to firms plus dividend payments $(DV)$ from firms less expenditures on consumption $(c)$, taxes $(\tau)$ adjustment costs will be discussed and the cost of servicing capital. Depreciation of capital is at a rate of:

$$\delta + \frac{\chi^z}{1+\phi^z}\left[z_t(j)^{1+\phi_z} - z_{ss}^{1+\phi_z}\right]$$

$z$ is the capital utilization rate and $z_{ss}$ represents the steady-state level. The domestic output nominal price is $p$, the nominal prices of consumption and the nominal exchange rate are $c \; and \; s$ respectively.

The domestic economy, here, assumes a costless operation of gas field and oil well that produces exogenous flows of gas and oil denoted as $\bar{G} \; and \; \bar{O}$, respectively. The sale (with prices, $P_t p_t^g \; and \, P_t p_t^o$, respectively) of these resources on the world markets are distributed to consumers. Following Finn (2000), capital utilization decision depends on energy prices. The assumption has it that households must purchase $e_z$ units of energy in the following way:

$$\frac{e_{z,t}(\mathrm{j})}{k_{t-1}(\mathrm{j})} = \frac{\chi_e}{\gamma_e} z_t(j)^{\gamma}$$

for $\chi_e \geq 0 \; and \; \gamma_e > 1$. The equation above can be thought of as a demand curve for energy. This shows that the amount of energy per capital stock unit is related, positively, to the capital utilization rate. Hence, using the stock of capital more intensive will require more energy.

The maximisation problem is therefore given by:

$$max\, E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left[1 - \frac{1}{\sigma^c}\right]^{-1} \left[\frac{c_t(j)}{c_{t-1}^{\psi^{hab}}}\right]^{1-\frac{1}{\sigma^c}} \right] - \frac{\sigma^c (k^h)^{\frac{1}{\sigma^h}}}{\sigma^h} h_{t+r}(j)^{\frac{\sigma^h+1}{\sigma^h}}$$

$$+ \frac{(k^{mon})^{\frac{1}{\sigma^c}}}{1 - \frac{1}{\sigma^c}} \left[\frac{MON_t(j)}{P_t^c}\right]^{1-\frac{1}{\sigma^c}}$$

subject to

$$BG_t(j) + MON_t(j) + \frac{BF_t(j)}{S_t} + P_t \frac{\chi^{bf}}{2} \left[\frac{BF_t(j)}{P_t S_t} - nfa^{ss}\right]^2 + P_t k_t(j) -$$

$$MON_{t-1}(j) - R_{t-1}^{g} BG_{t-1}(j) - R_{t-1}^{f} \frac{BF_{t-1}(j)}{S_t} - P_t \left[1 - \left(\delta + \frac{\chi^z}{1+\phi^z} \left[z_t(j)^{1+\phi_z} -\right.\right.\right.$$

$$\left.\left.\left. z_{ss}^{1+\phi_z}\right]\right)\right] k_{t-1}(j) + P_t \frac{\chi^k}{2k_{t-1}} \left[k_t(j) - (k_{t-1}/k_{t-2})^{\epsilon^k} k_{t-1}(j)\right]^2 -$$

$$W_t^k z_t(j) k_{t-1}(j) - W_t(j) h_t(j) - P_t p_t^o \bar{O}_t - \quad P_t p_t^g \bar{G}_t + p_t^c c_t(j) + P_t p_t^e e_{z,t}(j) -$$

$$DV_t + P_t \tau_t = 0$$

First-order conditions:

Consumption:

$$\left[\frac{c_t(j)}{c_{t-1}^{\psi^{hab}}}\right]^{-\frac{1}{\sigma^c}} \frac{1}{c_{t-1}^{\psi^{hab}}} - p_t^c \lambda_t = 0 \tag{71}$$

Money:

$$(P_t^c)^{\frac{1}{\sigma^c}-1}\left[\frac{MON_t(j)}{\kappa^{mon}}\right]^{-\frac{1}{\sigma^c}} - \lambda_t + \beta E_t \lambda_{t+1} = 0 \tag{72}$$

Government bonds:

$$-\lambda_t + \beta R_t^g E_t \lambda_{t+1} = 0 \tag{73}$$

Foreign bonds:

$$-\frac{\lambda_t}{S_t} - \chi^{bf}\frac{\lambda_t}{S_t}\left[\frac{BF_t(j)}{P_t S_t} - nfa^{ss}\right] + \beta R_t^f E_t \frac{\lambda_{t+1}}{S_{t+1}} = 0 \tag{74}$$

Capital:

$$\begin{aligned} &\lambda_t P_t \left[1 + \chi^k \left(\frac{k_t(j)}{k_{t-1}} - \left(\frac{k_{t-1}}{k_{t-2}}\right)^{\epsilon^k} \frac{k_{t-1}(j)}{k_{t-1}}\right)\right] = \\ &\beta E_t \lambda_{t+1} P_{t+1} \left[\begin{array}{c} 1 - \left(\delta + \frac{\chi^z}{1+\phi^z}\left[z_t(j)^{1+\phi_z} - z_{ss}^{1+\phi_z}\right]\right) \\ +\chi^k \left(\frac{k_{t+1}(j)}{k_t} - \left(\frac{k_t}{k_{t-1}}\right)^{\epsilon^k} \frac{k_t(j)}{k_t}\right)\left(\frac{k_t}{k_{t-1}}\right)^{\epsilon^k} \\ + \frac{W_{t+1}^k}{P_{t+1}} z_{t+1}(j) - p_{t+1}^e \frac{\chi_e}{\gamma_e} z_{t+1}(j)^{\gamma_e} \end{array}\right] \end{aligned} \tag{75}$$

Capital utilization:

$$\begin{aligned} &W_t^k k_{t-1}(j) - \chi^z Z_t(j)^{\phi_z} P_t k_{t-1}(j) - P_t p_t^e \chi_e k_{t-1}(j) z_t(j)^{\gamma_e - 1} \\ &= 0 \end{aligned} \tag{76}$$

where $\lambda$ is the lagrange multiplier in the budget constraint.

The labour index has the following CES form:

$$W_t \equiv \left[\int_0^1 W_t(j)^{1-\sigma^w} dj\right]^{1-\sigma^w}$$

Each household *j* faces a downward sloping demand curve for its own labour.

$$h_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\sigma^w} h_t$$

Each household sets nominal wages in staggered contracts. Whenever a household *j* has not reset its contract wage since period *t*, then wage rate in *t+r* is adjusted by indexation factor, $\xi^w_{t,t+r}$. i.e.

$$W_{t,t+r}(j) = \xi^w_{t,t+r}\widetilde{W}_t(j)$$

The indexation factor is:

$$\xi^w_{t,t+r} = \begin{cases} 1 & if\ r = 0 \\ (\Pi^{ss})^{1-\epsilon^w}\left(\frac{W_{t+r-1}}{W_{t+r-2}}\right)^{\epsilon^w} \xi^w_{t,t+r-1} & if\ r \geq 1 \end{cases}$$

This expression implies that if a household who has set wages in period *t* does not receive a signal to update its wages at time *t+r* its wage rate is increased in proportion with the weighted average of the steady-state inflation and the lagged nominal wage inflation.

In any period *t* in which household *j* is able to reset its contract wage, it aims to maximize the following:

$$E_t \sum_{r=0}^{\infty} \beta^r (1-\psi^w)^r \Big\{ \Lambda_{t+r} \xi^w_{t,t+r} W_t(j) h_{t+r}(j)$$

$$- (\kappa^h)^{-\frac{1}{\sigma^h}} \frac{\sigma^h}{\sigma^h+1} [h_{t+r}(j)]^{\frac{\sigma^h+1}{\sigma^h}} \Big\}$$

The first order condition:

$$0 = E_t \sum_{r=0}^{\infty} \beta^r (1-\psi^w)^r \, \Lambda_{t+r} \xi^w_{t,t+r} 1 - \sigma^w \left( \frac{\xi^w_{t,t+r} \widetilde{W}_t(j)}{W_{t+r}} \right)^{-\sigma^w} h_{t+r}$$

$$+ E_t \sum_{r=0}^{\infty} \beta^r (1-\psi^w)^r \tag{77}$$

$$- (\kappa^h)^{-\frac{1}{\sigma^h}} \sigma^w \left[ \left( \frac{\xi^w_{t,t+r} \widetilde{W}_t(j)}{W_{t+r}} \right)^{-\sigma^w} h_{t+r} \right]^{\frac{\sigma^h+1}{\sigma^h}} \widetilde{W}_t(j)^{-1}$$

which can be written in terms of optimal real wage rate $\widetilde{w}_t \equiv \widetilde{W}_t / P_t$ as:

$$\widetilde{w}_t^{\frac{1+\sigma^w}{\sigma^h}} = w_t^{\frac{\sigma^w}{\sigma^h}} \frac{(\kappa^h)^{-\frac{1}{\sigma^h}} \sigma^w}{\sigma^w - 1} \frac{\Xi^w_t}{\Theta^w_t} \tag{78}$$

where $$\Xi^w_t = h_t^{\frac{\sigma^h+1}{\sigma^h}} + \beta(1-\psi^w) E_t \left[ \frac{\xi^w_t W_t}{W_{t+1} \Pi_{t+1}} \right]^{-\sigma^w \frac{\sigma^h+1}{\sigma^h}} \Xi^w_{t+1}$$

$$\xi^w_t = (\Pi^{ss})^{1-\epsilon^w} \left( \frac{W_t}{W_{t-1}} \right)^{\epsilon^w}$$

where

and $$= (\Pi^{ss})^{1-\epsilon^w} \left( \frac{w_t}{w_{t-1}} \Pi_t \right)^{\epsilon^w}$$

where $$\Xi^w_t = h_t^{\frac{\sigma^h+1}{\sigma^h}} + \beta(1-\psi^w) E_t \left[ \frac{\xi^w_t W_t}{W_{t+1} \Pi_{t+1}} \right]^{-\sigma^w \frac{\sigma^h+1}{\sigma^h}} \Xi^w_{t+1}$$

where $$\xi^w_t = (\Pi^{ss})^{1-\epsilon^w} \left( \frac{W_t}{W_{t-1}} \right)^{\epsilon^w}$$

and
$$= (\Pi^{ss})^{1-\epsilon^w} \left(\frac{w_t}{w_{t-1}}\Pi_t\right)^{\epsilon^w}$$

where $\Pi_t = \frac{\mathrm{P}_t}{\mathrm{P}_{t-1}}$ i.e. the rate of output price inflation in period *t*, and

$$\Theta_t^w = \Lambda_t h_t + \beta(1-\psi^w)E_t\Pi_{t+1}^{-1}\left[\frac{\xi_t^w W_t}{W_{t+1}\Pi_{t+1}}\right]^{-\sigma^w} \Theta_{t+1}^w$$

The nominal wage index satisfies:

$$\mathrm{W}_t^{1-\sigma^w} \equiv \int_0^1 W_t(j)^{1-\sigma^w} dj = (1-\psi^w)\ (\xi_{t-1}^w W_{t-1})^{1-\sigma^w} + \psi^w \widetilde{W}_t^{\,1-\sigma^w}$$

It assumes here that the final consumption bundle consists of a CES aggregate of domestically produced non-energy goods and energy:

$$c_t \equiv \kappa^c \left[(1-\psi^e)\{(1-\phi^e)c_t^n\}^{1-\frac{1}{\sigma^e}} + \psi^e\{\phi^e c_t^e\}^{1-\frac{1}{\sigma^e}}\right]^{\frac{\sigma^e}{\sigma^e-1}}$$

where consumption of energy is defined in terms of consumption of petrol and utility:

$$c_t^e \equiv \kappa^e \left[(1-\psi^p)\{(1-\phi^p)c_t^u\}^{1-\frac{1}{\sigma^p}} + \psi^p\{\phi^p c_t^p\}^{1-\frac{1}{\sigma^p}}\right]^{\frac{\sigma^p}{\sigma^p-1}}$$

Nominal expenditure on consumption is:

$$P_t^c c_t = P_t c_t^n + P_t p_t^u c_t^u + P_t p_t^p c_t^p$$

Optimal consumption choices imply that the relative demands for consumption goods solve the following problem:

$$max\ \kappa^c \left[(1-\psi^e)\{(1-\phi^e)c_t^n\}^{1-\frac{1}{\sigma^e}} + \psi^e\{\phi^e c_t^e\}^{1-\frac{1}{\sigma^e}}\right]^{\frac{\sigma^e}{\sigma^e-1}}$$
$$- v\left[P_t c_t^n + P_t p_t^u c_t^u + P_t p_t^p c_t^p - Z\right]$$

The first-order conditions:

for $c_t^n$:

$$0 =$$

$$\kappa^c \Big[(1-\psi^e)\{(1-\phi^e)c_t^n\}^{1-\frac{1}{\sigma^e}} + \tag{79}$$

$$\psi^e\{\phi^e c_t^e\}^{1-\frac{1}{\sigma^e}}\Big]^{\frac{\sigma^e}{\sigma^e-1}-1} \text{ x } (-\psi^e)\{(-\phi^e)\}^{1-\frac{1}{\sigma^e}}\{c_t^n\}^{-\frac{1}{\sigma^e}} - vP_t$$

for $c_t^u$:

$$0 = \tag{80}$$

$$\kappa^c \Big[(1-\psi^e)\{(1-\phi^e)c_t^n\}^{1-\frac{1}{\sigma^e}} +$$

$$\psi^e\{\phi^e c_t^e\}^{1-\frac{1}{\sigma^e}}\Big]^{\frac{\sigma^e}{\sigma^e-1}-1} \psi^e\{\phi^e\}^{1-\frac{1}{\sigma^e}}[\![\{c]\!]_t^e\}^{-\frac{1}{\sigma^e}} \text{ x } \kappa^e \Big[(1-\psi^p)\{(1-$$

$$\phi^p)c_t^u\}^{1-\frac{1}{\sigma^p}} + \psi^p\{\phi^p c_t^p\}^{1-\frac{1}{\sigma^p}}\Big]^{\frac{\sigma^p}{\sigma^p-1}} (1-\psi^p)\Big\{(1-\phi^p\ )^{1-\frac{1}{\sigma^e}} c_t^u\Big\}^{-\frac{1}{\sigma^p}} -$$

$$vP_t p_t^u$$

for

$$0 = \tag{81}$$

$c_t^p$:

$$\kappa^c \Big[(1-\psi^e)\{(1-\phi^e)c_t^n\}^{1-\frac{1}{\sigma^e}} +$$

$$\psi^e\{\phi^e c_t^e\}^{1-\frac{1}{\sigma^e}}\Big]^{\frac{\sigma^e}{\sigma^e-1}-1} \psi^e\{\phi^e\}^{1-\frac{1}{\sigma^e}}\{c_t^e\}^{-\frac{1}{\sigma^e}} \text{ x}\kappa^e \Big[(1-\psi^p)\{(1-$$

$$\phi^p)c_t^u\}^{1-\frac{1}{\sigma^p}} + \psi^p\{\phi^p c_t^p\}^{1-\frac{1}{\sigma^p}}\Big]^{\frac{\sigma^p}{\sigma^p-1}} \{\psi^p(\phi^p)^{1-\frac{1}{\sigma^e}} c_t^p\}^{-\frac{1}{\sigma^p}} - vP_t p_t^p$$

which can be represented as

$$\frac{\frac{(1-\psi^e)}{\psi^e}\{\frac{1-\phi^e}{\phi^e}\}^{1-\frac{1}{\sigma^e}}\left\{\frac{c_t^n}{c_t^e}\right\}^{-\frac{1}{\sigma^e}}\left\{\frac{c_t^e}{c_t^u}\right\}^{-\frac{1}{\sigma^p}}}{[\kappa^e]^{1-\frac{1}{\sigma^p}}(1-\psi^p)\{(1-\phi^p)\}^{1-\frac{1}{\sigma^p}}} = \frac{1}{p_t^u}$$

and

$$\frac{1-\psi^p}{\psi^p}\{\frac{1-\phi^p}{\phi^p}\}^{1-\frac{1}{\sigma^p}}\left\{\frac{c_t^u}{c_t^p}\right\}^{-\frac{1}{\sigma^p}} = \frac{p_t^u}{p_t^p}$$

### 1.3.2 The firms

It is assumed here that value-added is produced by combining domestic capital and labour using a CES production function:

$$V_t = \kappa^v \left[ (1-\alpha_v)\{(1-\phi_v)h_t\}^{1-\frac{1}{\sigma^v}} + \alpha_v\{\phi_v K_t^s\}^{1-\frac{1}{\sigma^v}} \right]^{\frac{\sigma^v}{\sigma^v-1}}$$

where $h$ is total hours and $K^s$ represents capital services rented from households. This sector is perfectly competitive so that factor demands are implied by profit maximisation:

$$\max \mathrm{P_t} p_t^{vc} \kappa^v \left[ (1-\alpha_v)\{(1-\phi_v)h_t\}^{1-\frac{1}{\sigma^v}} + \alpha_v\{\phi_v K_t^s\}^{1-\frac{1}{\sigma^v}} \right]^{\frac{\sigma^v}{\sigma^v-1}} - \mathrm{P_t w_t h_t} \qquad (82)$$

$$- \mathrm{P_t w_t^k} K_t^s$$

Giving the first-order condition:

for $h_t$:

$$(1-\alpha_v)\{\kappa^v(1-\phi_v)\}^{1-\frac{1}{\sigma^v}} \left\{\frac{V_t}{h_t}\right\}^{\frac{1}{\sigma^v}} = \frac{\mathrm{w_t}}{p_t^{vc}} \qquad (83)$$

for $K_t^s$:

$$\alpha_v\{\kappa^v\phi_v\}^{1-\frac{1}{\sigma^v}} \left\{\frac{V_t}{K_t^s}\right\}^{\frac{1}{\sigma^v}} = \frac{w_t^k}{p_t^{vc}} \qquad (84)$$

where $P_t^{vc}$ denotes the perfectly competitive price of value added, which can be derived from the zero-profit condition:

$$p_t^{vc} = \mathrm{w_t}\frac{h_t}{V_t} + w_t^k \frac{K_t^s}{V_t} \qquad (85)$$

Final non-energy output is produced by firms operating the following production function:

$$q_t = A_t\left[(1-\alpha_q)\{(1-\phi_q)B_t\}^{1-\frac{1}{\sigma^q}} + \alpha_q\{\phi_q E_t\}^{1-\frac{1}{\sigma^q}}\right]^{\frac{\sigma^q}{\sigma^q-1}} \tag{86}$$

where $q_t$ is final output of non-energy, consisting of a bundle ( $B$ , defined below) that combines value added and imports and 'energy' ($E$). $A$ is denoted as exogenous productivity.

The bundle of value added ($V^N$ ) and imports ($M^N$ ) is a Cobb-Douglas aggregator:

$$B_t = \kappa^B (V_t^N)^{1-\alpha_B} (M_t^N)^{\alpha_B} \tag{87}$$

The energy input is a Leontief bundle of petrol and utilities:

$$e_t = \kappa_N \min\left\{\frac{I_t^p}{\psi_N}, \frac{I_t^u}{1-\psi_N}\right\} \tag{88}$$

where $I^p$ and $I^u$denote intermediate inputs of petrol and utilities. Efficient use of energy inputs implies the following fixed-proportion factor demand conditions:

$$I_t^p = \psi_N e_t \tag{89}$$

$$I_t^u = (1-\psi_N) e_t \tag{90}$$

Nominal dividends are defined as:

$$DV_t^q = P_t^b q_t - P_t p_t^{vc} V_t^N - P_t p_t^m M_t^N - P_t p_t^p I_t^p - P_t p_t^u I_t^u \tag{91}$$

which says that dividends are the difference between the value of output sold (at basic prices $P^b$) and purchases of value added, petrol and utilities (at market prices). Since petrol and utilities are used in fixed proportions to form the energy input, we can write the dividend flow as:

$$DV_t^q = P_t^b q_t - P_t p_t^{vc} V_t^N - P_t p_t^m M_t^N - P_t\left[\psi_N p_t^p + (1 - \psi_N p_t^u)\right] e_t \tag{92}$$

And treats energy as a single input with price

$$P_t\left[\psi_N p_t^p + (1 - \psi_N p_t^u)\right] \tag{93}$$

Firms maximise the discounted flow of dividends net of the costs of adjusting prices:

$$E_t \sum_{t=0}^{\infty} \beta^t \lambda_t \left[ \begin{array}{c} P_t^b(k)\left(\frac{P_t^b(k)}{P_t^b}\right)^{-\eta} q_t - P_t p_t^{vc} V_t^N - P_t p_t^m M_t^N \\ -P_t\left[\psi_N(1 + \tau_t^{vat}) p_t^p + (1 - \psi_N)(1 + \tau_t^u) p_t^u\right] E_t - \frac{\chi^p}{2}\left(\frac{\frac{P_t^b(k)}{P_{t-1}^b(k)}}{(\Pi^{ss})^{1-\epsilon}\left(\frac{P_t^b}{P_{t-1}^b}\right)^{\epsilon}} - 1\right)^2 P_t^b q_t \\ -LM_t\left[\left(\frac{P_t^b(k)}{P_t^b}\right)^{-\eta} q_t - A_t\left[(1 - \alpha_q)\left\{(1 - \phi_q) B_t\right\}^{1 - \frac{1}{\sigma^q}} + \alpha_q\left\{\phi_q E_t\right\}^{1 - \frac{1}{\sigma^q}}\right]^{\frac{\sigma^q}{\sigma^q - 1}}\right] \end{array} \right]$$

subject to (value added) and (materials aggregator):

$$E_t \sum_{t=0}^{\infty} \beta^t \lambda_t \left[ \begin{array}{c} P_t^b(k)\left(\frac{P_t^b(k)}{P_t^b}\right)^{-\eta} q_t - P_t p_t^{vc} V_t^N - P_t p_t^m M_t^N \\ -P_t\left[\psi_N p_t^p + (1 - \psi_N) p_t^u\right] e_t - \frac{\chi^p}{2}\left(\xi_t^p(k)\right)^2 P_t^b q_t \\ -LM_t\left[\left(\frac{P_t^b(k)}{P_t^b}\right)^{-\eta} q_t - A_t\left[(1 - \alpha_q)\left\{(1 - \phi_q) B_t\right\}^{1 - \frac{1}{\sigma^q}} + \alpha_q\left\{\phi_q e_t\right\}^{1 - \frac{1}{\sigma^q}}\right]^{\frac{\sigma^q}{\sigma^q - 1}}\right] \end{array} \right]$$

where

$$\xi_t^p(k) = \frac{\frac{P_t^b(k)}{P_{t-1}^b(k)}}{(\Pi^{ss})^{1-\epsilon}\left(\frac{P_t^b}{P_{t-1}^b}\right)^{\epsilon}} - 1$$

summarises the adjustment cost for prices. The adjustment costs depend on the rate at which firm $k$ adjusts its price ($P_t^b(k)$) relative to a weighted average of trend inflation and lagged aggregate price inflation. This formulation has similar effects to the assumptions about wage stickiness described above. The first-order conditions are:

for $V_t^N$:
$$\frac{P_t p_t^{vc}}{P_t^b} = LM_t A_t^{1-\frac{1}{\sigma^q}}\left(1-\alpha_q\right)\left(1-\phi_q\right)^{1-\frac{1}{\sigma^q}}\left[\frac{q_t}{B_t}\right]^{\frac{1}{\sigma^q}}\left(1-\alpha_B\right)\frac{B_t}{V_t^N} \tag{94}$$

for $M_t^N$:
$$\frac{P_t p_t^{m}}{P_t^b} = LM_t A_t^{1-\frac{1}{\sigma^q}}\left(1-\alpha_q\right)\left(1-\phi_q\right)^{1-\frac{1}{\sigma^q}}\left[\frac{q_t}{B_t}\right]^{\frac{1}{\sigma^q}}\alpha_B\frac{B_t}{M_t^N} \tag{95}$$

for $e_t$:
$$P_t\frac{\psi_N p_t^p + (1-\psi_N)p_t^u}{P_t^b} = LM_t A_t^{1-\frac{1}{\sigma^q}}\alpha_q {\phi_q}^{1-\frac{1}{\sigma^q}}\left[\frac{q_t}{e_t}\right]^{\frac{1}{\sigma^q}} \tag{96}$$

for $q_t$:
$$1-\eta\left(\frac{P_t^b(k)}{P_t^b}\right)^{-\eta} q_t - \chi^p \xi_t^p(k) P_t^b q_t \frac{\xi_t^p(k)+1}{P_t^b(k)} + LM_t\,\eta\left(\frac{P_t^b(k)}{P_t^b}\right)^{-\eta}\frac{q_t}{P_t^b(k)} \tag{97}$$
$$= \beta\chi^p E_t\frac{\lambda_{t+1}}{\lambda_t}\xi_{t+1}^p(k)\,P_{t+1}^b q_{t+1}\frac{\xi_{t+1}^p(k)+1}{P_t^b(k)}$$

Finally, the production of energy goods, assume that the output (petrol) follows a Leontief combination of value added and gas (oil):

$$q_t^u = \min\left\{\frac{I_t^g}{1-\psi^u},\frac{V_t^u}{\psi^u}\right\} \tag{98}$$

and
$$q_t^p = \min\left\{\frac{I_t^o}{1-\psi^{qp}},\frac{V_t^p}{\psi^{qp}}\right\} \tag{99}$$

The factor demands are simple linear functions of production:

$$I_t^g = (1-\psi^u)q_t^u \tag{100}$$

$$V_t^u = \psi^u q_t^u \tag{101}$$

$$I_t^o = (1-\psi^{qp})q_t^p \tag{102}$$

Nominal dividends from utilities production function are:

$$DV_t^u = P_t^{ub} q_t^u - P_t p_t^{vc} V_t^u - P_t p_t^g I_t^g \tag{103}$$

with the given factor demands as:

$$DV_t^u = \left[P_t^{ub} - \psi^u P_t p_t^{vc} - (1-\psi^u) P_t p_t^g\right] q_t^u \tag{104}$$

A monopolistic competition is assumed for the demand schedule for utilities:

$$q_t^u(k) = \left(\frac{P_t^{ub}(k)}{P_t^{ub}}\right)^{-\eta_u} q_t^u \tag{105}$$

Moreover, that utility producers maximise the discounted flow of dividends subject to price adjustment costs:

$$E_t \sum_{t=0}^{\infty} \beta^t \lambda_t \left[ \left[P_t^{ub}(k) - \psi^u P_t p_t^{vc} - (1-\psi^u) P_t p_t^g\right] \left(\frac{P_t^{ub}(k)}{P_t^{ub}}\right)^{-\eta_u} q_t^u \right.$$
$$\left. - \frac{\chi^u}{2} (\xi_t^u(k))^2 P_t^{ub} q_t^u \right]$$

where the adjustment cost is summarized as:

$$\xi_t^u(k) = \frac{\frac{P_t^{ub}(k)}{P_{t-1}^{ub}(k)}}{(\Pi^{ss})^{1-\epsilon_u} \left(\frac{P_{t-1}^{ub}}{P_{t-2}^{ub}}\right)^{\epsilon_u}} - 1$$

The first-order condition for pricing is:

$$\begin{gathered} (1-\eta_u) \left(\frac{P_t^{ub}(k)}{P_t^{ub}}\right)^{-\eta_u} q_t^u - \chi^u \xi_t^u(k) P_t^{ub} q_t^u \frac{\xi_t^u(k)+1}{P_t^{ub}(k)} + \\ \begin{bmatrix} \psi^u P_t p_t^{vc} + \\ (1-\psi^u) P_t p_t^g \end{bmatrix} \eta_u \left(\frac{P_t^{ub}(k)}{P_t^{ub}}\right)^{-\eta_u} \frac{q_t^u}{P_t^{ub}(k)} = -\beta \chi^u E_t \frac{\lambda_{t+1}}{\lambda_t} \xi_{t+1}^u(k) P_{t+1}^{ub} q_{t+1}^u \frac{\xi_{t+1}^u(k)+1}{P_t^{ub}(k)} \end{gathered} \tag{106}$$

nominal dividends from petrol production:

$$DV_t^p = P_t^{pb} q_t^p - P_t p_t^{vc} V_t^p - P_t p_t^o I_t^o \tag{107}$$

$$DV_t^p = \left[P_t^{pb} - \psi^{qp} P_t p_t^{vc} - (1-\psi^{qp}) P_t p_t^o\right] q_t^p \tag{108}$$

which is analogous to the expression for dividends from utilities and again the price earned from petrol production, $P_t^{pb}$ is measured at basic prices. More details on the taxation of petrol are given below. Again assuming monopolistic competition so that the demand schedule for utilities is:

$$q_t^p(k) = \left(\frac{P_t^{pb}(k)}{P_t^{pb}}\right)^{-\eta_p} q_t^p \tag{109}$$

and that utility producers maximise the discounted flow of dividends subject to price adjustment costs:

$$E_t \sum_{t=0}^{\infty} \beta^t \lambda_t \left[ \left[P_t^{pb}(k) - \psi^{qp} P_t p_t^{vc} - (1-\psi^{qp}) P_t p_t^o\right] \left(\frac{P_t^{pb}(k)}{P_t^{pb}}\right)^{-\eta_p} q_t^p \right.$$

$$\left. - \frac{\chi^{pp}}{2} \left(\xi_t^{pp}(k)\right)^2 P_t^{pb} q_t^p \right]$$

where

$$\xi_t^{pp}(k) = \frac{\frac{P_t^{pb}(k)}{P_{t-1}^{pb}(k)}}{(\Pi^{ss})^{1-\epsilon_p} \left(\frac{P_{t-1}^{pb}}{P_{t-2}^{pb}}\right)^{\epsilon_p}} - 1$$

is the summary of the price adjustment costs. The first-order condition for pricing is:

$$(1-\eta_p)\left(\frac{P_t^{pb}(k)}{P_t^{pb}}\right)^{-\eta_p} q_t^p - \chi^{pp} \xi_t^{pp}(k) P_t^{pb} q_t^p \frac{\xi_t^{pp}(k)+1}{P_t^{pb}(k)}$$

$$+\left[\psi^{qp} P_t p_t^{vc} + (1-\psi^{qp}) P_t p_t^o\right] \eta_p \left(\frac{P_t^{pb}(k)}{P_t^{pb}}\right)^{-\eta_p} \frac{q_t^p}{P_t^{pb}(k)} = \tag{110}$$

$$-\beta \chi^{pp} E_t \frac{\lambda_{t+1}}{\lambda_t} \xi_{t+1}^{pp}(k) P_{t+1}^{pb} q_{t+1}^u \frac{\xi_{t+1}^p(k)+1}{P_t^{pb}(k)}$$

Domestic production of oil and gas are given exogenously by *O* and *G* respectively.

### 2.2.3 Rest of the world and exogeneity assumptions

There is an assumption, here, of a downward-sloping export demand function for domestically produced goods. So the demand for domestic non-energy exports is given by:

$$x_t^n = (x_{t-1}^n)^{\psi_x} \left[ \kappa_x \left( \frac{S_t P_t^f}{P_t} \right)^{-\eta_x} y_t^f \right]^{1-\psi_x} \tag{111}$$

where $\psi_x$ captures an assumption that foreign preferences exhibit a form of 'habit formation' similar to that assumed for domestic agents.

Also, there is an infinitely elastic supply of oil (gas) available from the world market at a world *relative* price $P_t^{of}$ $P_t^{gf}$. The prices of oil and gas in domestic currency are given by the law of one price:

$$P_t^o = \frac{P_t^{of} P_t^f}{S_t} \tag{112}$$

$$P_t^g = \frac{P_t^{gf} P_t^f}{S_t} \tag{113}$$

The Import prices follow the assumption of being priced as a mark-up over the world import price that measured in domestic currency. The import prices are subject to Calvo price adjustment costs. The import pricing equation as:

$$\begin{aligned} ln\left(\frac{p_t^m \Pi_t}{p_{t-1}^m}\right) &= \frac{\beta}{1+\beta\varepsilon^{pm}} E_t ln\left(\frac{p_{t+1}^m \Pi_{t+1}}{p_t^m}\right) + \frac{\varepsilon^{pm}}{1+\beta\varepsilon^{pm}} ln\left(\frac{p_{t-1}^m \Pi_{t-1}}{p_{t-2}^m}\right) \\ &+ \frac{[1-\beta(1-\psi^{pm})]\psi^{pm}}{(1+\beta\varepsilon^{pm})(1-\psi^{pm})} ln \frac{p_t^{mf}}{S_t p_t^m} \end{aligned} \tag{114}$$

### 1.3.4 Fiscal and monetary policy

The government's nominal budget constraint is satisfied each period:

$$\begin{aligned} BG_t + MON_t = R^g_{t-1}BG_{t-1} + MON_{t-1} + P_t c^g_t - P_t\tau_t - P_t\tau_t\left[p^b_t q_t + p^{pb}_t q^p_t\right] \\ - (1-\tau_v)P_t d^p_t q^p_t - \tau_u P_t p^{ub}_t q^u_t \end{aligned} \tag{115}$$

with procurement ($c^g_t$ ) exogenous, lump-sum taxes ($\tau$ ) move to satisfy a balanced budget process for government debt:

$$BG_t = 0 \tag{116}$$

Tax revenue includes value added tax on final output which means that the price of output is given by:

$$P_t = (1-\tau_v)p^p_t \tag{117}$$

The revenue from tax on utilities reflects the fact that utilities are taxed at a different rate $\tau_u$ :

$$P_t p^u_t = (1-\tau_u)P_t p^{ub}_t \tag{118}$$

The tax revenue from petrol sales includes duties ($d^p_t$) as well as value added taxes so that the market price for petrol is:

$$P_t p^p_t = (1-\tau_v)P_t(p^{pb}_t + d^p_t) \tag{119}$$

The baseline monetary reaction function says that nominal interest rates respond to deviations of annual consumer price inflation from target:

$$\frac{R^g_t}{R^{gss}} = \left(\frac{R^g_{t-1}}{R^{gss}}\right)^{\theta^{rg}} \left(\frac{\left[P^c_t/P^c_{t-4}\right]^{0.25}}{\Pi^{ss}}\right)^{\theta^{pdot}(1-\theta^{rg})} \tag{120}$$

The baseline reaction function does not include a response to an output gap measure because the precise definition of the output gap may be significant.

### 1.3.5 Aggregation, market clearing and the resource constraint

Total dividends received by households are given by:

$$DV_t = DV_t^q + DV_t^u + DV_t^p \tag{121}$$

Market clearing for value added requires:

$$V_t = V_t^n + V_t^u + V_t^p \tag{122}$$

Market clearing for petrol requires:

$$q_t^p = c_t^p + I_t^p + \psi_N e_{z,t} + X_t^p - M_t^p \tag{123}$$

where for the demand for petrol by households to facilitate capital utilisation is accounted. This is done under the assumption that the energy bundle used by households is the same Leontief bundle used by non-energy producing firms. Net trade in petrol is assumed to be zero ($X_t^p = M_t^p$).

Also, the accounting for household demand for utilities to facilitate capital utilisation implies that market clearing for utilities requires:

$$q_t^u = c_t^u + I_t^u + (1 - \psi_N) e_{z,t} \tag{124}$$

Total demand for oil can be sourced from the domestic well and net trade:

$$I_t^o = \bar{O}_t - X_t^o \tag{125}$$

where $X_t^o$ measures net trade (which has a possibility of being negative).

Similarly, total demand for gas satisfies that:

$$I_t^g = \bar{G}_t - X_t^g \tag{126}$$

Non-oil final production function satisfies demand as:

$$q_t = c_t^g + c_t^n + k_t - \left(1 - \delta - \frac{\chi^z}{1 + \phi^z}\left[z_t^{1+\phi^z} - 1\right]\right) k_{t-1} + x_t^n \tag{127}$$

Substituting the government budget constraint, the expression for dividends (*DV)* and the market clearing conditions into the household budget constraint delivers an aggregate resource constraint describing how the net foreign assets of the economy evolve:

$$\begin{aligned}\frac{BF_t}{s_t} = \left(R_{t-1}^f\right)\frac{BF_{t-1}}{\Pi_t^f s_t} + x_t^n - p_t^m M_t^N + p_t^g X_t^g + p_t^o X_t^o - \frac{\chi^{bf}}{2}\left[\frac{BF_t}{s_t} - nfa^{ss}\right]^2 - \\ \frac{\chi^k}{2k_{t-1}}\left[k_t - \left(\frac{k_{t-1}}{k_{t-2}}\right)^{\varepsilon^k} k_{t-1}\right]^2\end{aligned} \tag{128}$$

# Chapter 2: An evaluation of a two-sector Real Business Cycle (RBC) model of energy in United Kingdom using non-stationary data

## 3.1 Introduction

In this chapter, I evaluate a Real Business Cycle (RBC) model[34] of a small open economy of the United Kingdom. The model includes sector-specific energy efficiency as a factor-augmenting technology while firms demand capital services, labour and energy use in their production function. Most of the real domestic macroeconomic aggregates have not been able to account consistently for the recession like the oil price does in the literature. High oil prices worsen the current account of countries that are net importers of oil, like the United Kingdom, increasing their current account deficits and depreciating their currencies. High oil prices primarily lead to a rise in the demand for money (Mork, 1994), which will consequently affect the real balances. Rising oil prices causes increase the general price level and relative prices, thereby, appreciating the real exchange rate. This situation would be the same if an economy is a net exporter of oil faced with low oil prices.

The UK economy, in this study, is characterized as a two-sector small open economy that produces energy intensive goods and energy-extensive (non-energy) goods/services. The UK is assumed to be a net importer of crude oil (energy) despite being a primary producer of crude oil. This assumption follows reality since the

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[34] The model started O. Oyekola (have similar production functions and preferences) but we took different path under the supervision of Professor Minford.

production of crude oil in the United Kingdom is in decline (Webb, 2013). This, continuous decline of energy resource extraction is likely to particularly affect domestic consumption and the exchange rate. As a result, changes in energy prices will probably affect the real macroeconomic aggregates as well as economic policies.

In this chapter, I present an RBC model with detailed explanation of how it is formed. I then show the calibration of the model and how the data is collected. I follow with the methodology of the model estimation and evaluation where I show a test of fit of the model. I also show the model's IRFs, variance decomposition and shock decomposition of the financial crisis of 2008. The log-linear equations of the model, the VAR-IRFs are in the appendix together with all other outputs obtained from the model evaluation.

## 3.2 The model

In this model, the United Kingdom is characterized as a small open economy and a primary producer of energy (crude oil). It is also assumed to be a net importer of energy which it imports at a world price, $P_O$. The model could be viewed as an augmentation of a model developed by Kim and Loungani (1992), and Finn (1996). It is similar to these model(s) in the way that the domestic country's (UK) economic activities are carried out. The way that the world's economic activities in relation to trade with the United Kingdom are carried out is similar to Backus et al., (1993). The model maintains the assumption of perfect competitive firms in the economy as well as real frictions. This is different from the previous authors' assumption in their models of the absence of real frictions since this is now a standard practice in the literature. There is a continuum of households of unit mass on the demand side while on the supply side there is a continuum of firms of unit mass. There are two sectors in the economy: energy intensive, denoted by $e$, and energy extensive (non-energy), denoted by $n$, intensive. This model assumes a total consumption, $C$, in the economy but the consumption of these goods is done in a similar fashion for both energy good and non-energy good consumption. This model incorporates real rigidities that includes habit formation in household's consumption, investment adjustment costs as well as capital utilization. Domestic absorption comes from households demanding composite good, $D$, that is used for consumption $C$, investment $I$ or as government spending $G_t$. The household also has the choice to hold either domestic bonds or foreign bonds. The production involves three combination of inputs: labour, capital and primary energy (assumed to be crude-oil). The firms produce the aggregate output of $Y$ where each sector produces $Y_e \ and \ Y_n$ of energy intensive and energy extensive output respectively. There is assumed to be immobility of labour and capital across borders while the accumulation of capital is subject to adjustment costs. The goods and energy produced in the UK are traded with the rest of the world which is traded by the household. The households supply differentiated labour, $H$, to each sector of the firms at a given wage rate $W$. They also have the option of investing in two kinds of physical capitals $K_e$ and $K_n$ which are subject to adjustment cost.

### 3.2.1 Household

The household lifetime utility is prescribed as:

$$\mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \tau_t U((C_t - \psi_h C_{t-1}), \xi_{w,t} H_t) \quad (129)$$

where $\mathrm{E}_t$ represents rational expectation of household, and $0 < \beta < 1$ denotes discount factor. $C$ represents aggregate demand by household (nominal consumption) and $H$ denotes the work hours supplied by household with $\psi_h$ representing the degree of habit formation by households. The household lifetime utility differs from Smets and Wouters (2003, 2007) as it includes habit formation as

($\psi_h > 0$). $\tau$ denotes exogenous consumption preference shock and $\xi_w$ denotes exogenous labour supply shock[35].

The Budget constraint, closely follows Harrison, et al., (2011), shows how the end of period holdings of nominal government debt($B$), nominal foreign bonds ($B^f$) and nominal capital ($k$) are given by their start of period holdings, plus net income. The net income includes earnings from labour supply (at wage) and capital services ($Z_t^j K_{t-1}^j$ rented at rate$R_t^j$), for $j = e, n$, to firms less expenditures on consumption ($c$), lump-sum taxes ($T$), adjustment costs will be discussed below as well as the cost of servicing capital. Given that I use world imports prices as the numeraire in the model, the values of the nominal variables are converted to US Dollars and deflated by world manufacturing prices.

$$\frac{B_t}{1+r_t} + \frac{B_t^f}{S_t\left(1+r_t^f\right)} + C_t + T_t + I_t = R_{t-1}B_{t-1} + R_{t-1}^f \frac{B_{t-1}^f}{S_t} + W_t H_t + \Pi_t + R_t^e Z_t^e K_{t-1}^e + R_t^n Z_t^n K_{t-1}^n \tag{130}$$

This gives a clear picture of how household have the option to hold either domestic or Foreign bonds. $r$ denotes domestic interest rate and $r^f$[36] denotes exogenous world interest rates, given that world prices are exogenous. $\Pi$ denotes income profits from firm ownership.

Households decide on what capital stocks $K_{t-1}^j$ to choose as new capital must take one-quarter lag to become effective. The model assumes households have access to technology after decision on which sector to install capital in the previous quarter.

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[35] The shock is assumed to follow a first order autoregressive process with an i.i.d. normal error term: $\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \eta_{\varepsilon,t}$

[36] See footnote 35.

$$K_t^e = \big(1 - \delta(Z_t^e)\big)K_{t-1}^e + \xi_{INV,t}^e I_t^e + \chi_e\left(\frac{K_t^e}{K_{t-1}^j}\right) \tag{131}$$

$$K_t^n = \big(1 - \delta(Z_t^n)\big)K_{t-1}^n + \xi_{INV,t}^n I_t^n + \chi_n\left(\frac{K_t^n}{K_{t-1}^n}\right) \tag{132}$$

where $I^j$ denotes sector-specific gross nominal investment. $\delta(.)$ denotes sector-specific time varying depreciation rate of capital installed: for $0 \leq \delta(.) \leq 1,\ \delta'(.) > 0, and\ \delta''(.) > 0$ . $Z^j$ denotes capital utilization rate of each period's effective capital installation. $\xi_{INV}^j$ denotes sector-specific exogenous investment-specific technology shock[37]. $\chi_j$ denotes adjustment costs which depends on the rate at which each sector adjusts its price, for $\chi_j > 1$. The assumption is consistent with standard DSGE literature.

Subject to the budget constraint, the household maximise[38] their expected lifetime utility value with the sequence $\left\{C_t, H_t, I_t, I_t^e, I_t^n, , B_{t+1}^f, Z_t^e, Z_t^n, K_t^e, K_t^n\right\}_{t=0}^{\infty}$ . The first-order condition that solves the consumer's problem are:

$$W_t = -\frac{U_2\big(C_t - \psi_h C_{t-1}, \xi_{w,t} H_t\big)}{U_1\big(C_t - \psi_h C_{t-1}, \xi_{w,t} H_t\big)} \tag{133}$$

$$R_t = \frac{\tau_t U_1\big(C_t - \psi_h C_{t-1}, \xi_{w,t} H_t\big)}{\beta \mathrm{E}_t \tau_{t+1} U_1\big(C_{t+1} - \psi_h C_t, \xi_{w,t+1} H_{t+1}\big)} \tag{134}$$

$$R_t^e = \frac{\delta(Z_t^e)}{\xi_{INV,t}^e} \tag{135}$$

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[37] See footnote 35.

[38] A consolidated budget constraint of the model is shown in the sub-chapter of log-linearized version of the model.

$$R_t^n = \frac{\delta(Z_t^n)}{\xi_{INV,t}^n} \tag{136}$$

$$\left(1 + \chi_e\left(\frac{K_t^e}{K_{t-1}^e}\right)\right) =$$

$$\beta \mathrm{E}_t \frac{\xi_{INV,t}^e}{\xi_{INV,t+1}^e} \frac{\tau_{t+1} U_1(C_{t+1} - \psi_h C_t, \xi_{w,t+1} H_{t+1})}{\tau_t U_1(C_t - \psi_h C_{t-1}, \xi_{w,t} H_t)} \begin{bmatrix} R_{t+1}^e Z_{t+1}^e - \chi_e\left(\frac{K_{t+1}^e}{K_t^e}\right) + \\ 1 - \delta(Z_{t+1}^e) + \chi'_e\left(\frac{K_{t+1}^e}{K_t^e}\right)\frac{K_{t+1}^e}{K_t^e} \end{bmatrix} \tag{137}$$

$$\left(1 + \chi_n\left(\frac{K_t^n}{K_{t-1}^n}\right)\right) =$$

$$\beta \mathrm{E}_t \frac{\xi_{INV,t}^n}{\xi_{INV,t+1}^n} \frac{\tau_{t+1} U_1(C_{t+1} - \psi_h C_t, \xi_{w,t+1} H_{t+1})}{\tau_t U_1(C_t - \psi_h C_{t-1}, \xi_{w,t} H_t)} \begin{bmatrix} R_{t+1}^n Z_{t+1}^n - \chi_n\left(\frac{K_{t+1}^n}{K_t^n}\right) + \\ 1 - (Z_{t+1}^n) + \chi'_n\left(\frac{K_{t+1}^n}{K_t^n}\right)\frac{K_{t+1}^n}{K_t^n} \end{bmatrix} \tag{138}$$

$$-\frac{U_1}{S_t} - \chi_{bf}\frac{U_1}{S_t}\left[\frac{B_t^f}{S_t} - nfa\right] + \beta R_t^f E_t \frac{U_2}{S_{t+1}} = 0 \tag{139}$$

where $U_g$ denotes partial derivative of $U$ with respect to its $g-th$ argument. The equilibrium condition (133) states the marginal rate of substitution between leisure and the aggregate demand (consumption) is equal to the existing wage rate in the economy. Equation (134) gives the equilibrium condition that states the marginal rate of substitution in the intertemporal consumption is equal to the relative price of bonds, which means consumers are indifferent of consumption and saving between today and tomorrow. Equation (135) and (136) states the sector-specific equilibrium condition of marginal cost of user to benefits in the capital utilization. Equation (137) and (138) states the sector-specific equilibrium condition of marginal cost and returns between consumption and investment of the capital stock.

### 3.2.2 Firms

The sectoral outputs' production functions are assumed to be homogeneous-of-degree-one, following Kim and Loungani (1992) that includes primary energy use $E^e$ as input, which differs from the standard neo-classical practice. The representative firm's technology employs a production function which can be characterized as a nested constant-elasticity of substitution (CES) specification of the form:

$$Y_t^e = A_t^e F^e(H_t^e, Z_t^e K_{t-1}^e, O_t^e E_t^e) = D_{d,t}^e + X_t^e \qquad (140)$$

The equation above is the production function of the energy intensive sector with sector-specific endogenous variables and exogenous shocks. $Y^e$ denotes the sector nominal output, measured in the nominal value of the numeraire, world imports price of US Dollars. $F^e(.)$ obeys the standard regularity conditions, $A^e$ denotes the exogenous energy intensive sector productivity shock, $H^e$denotes sector's labour demand, $Z^e K_{-1}^e$ denotes demand for capital services in the sector and $O^e$ denotes the exogenous energy intensive sector energy input efficiency shock.[39]

$D_d^e$, denotes domestic absorption, states that the sectoral output can either be consumed at home or to be exported $X^e$ to satisfy the world demand.

Given the above assumption, and firms in the energy intensive sector are also perfectly competitive, the typical firms maximises the following profit function subject to the budget constraint in equation (140): $max\, \Pi_t = P_t^e Y_t^e - \big(W_t H_t^e +$

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[39] Sector-specific energy efficiency shock is a factor-augmenting technology. This energy efficiency in production, which might capture a switch in the composition of capital towards machines with different energy intensities. Also, see footnote 33.

$R_t^e Z_t^e K_{t-1}^e + P_{O,t} E_t^e\big)$ where $P^e$ and $R^e$denotes the relative price of energy intensive goods and rental rate of capital services for the sector, respectively. World prices are exogenous, hence, I assume energy prices[40], $P_O$, to follow an exogenous process adjusts immediately to their world prices. $E^e$ denotes nominal energy use, in the sector, the value is measured in US Dollars given assumption of the numeraire of world imports.

United Kingdom is a net importer of energy (crude oil, in this study). The energy (non-energy) extensive sector output has a CES production function of homogeneous-of-degree-one with properties similar to the energy intensive sector, denoted by superscript $n$, is

$$Y_t^n = A_t^n F^n(H_t^n, Z_t^n K_{t-1}^n, O_t^n E_t^n) = D_{d,t}^n + X_t^n \tag{141}$$

where the exogenous shocks and endogenous variables are similar to the energy-intensive sector. $Y^n$ denotes the sector nominal output, measured in the nominal value of the numeraire, world imports price of US Dollars. $F^n(.)$ obeys the standard regularity conditions, $A^n$ denotes the exogenous energy extensive sector productivity shock, $H^n$denotes sector's labour demand, $Z^n K_{-1}^n$ denotes demand for capital services in the sector and $O^n$ denotes the exogenous energy extensive sector energy input efficiency shock. $D_d^n$, denotes domestic absorption, states that the energy extensive sector output can either be consumed at home or to be exported $X^n$ to

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[40] Initially, I assumed $P_t^o = \frac{P_t^{of} P_t^f}{S_t}$ as the energy price shock, like in Harrison, et al., (2011). $P_t^{of} P_t^f$ is the assumed world exogenous price but after linearization, the data residual is equal to observed price. See residual plots in previous chapter. I simply assumed world energy price shock to avoid complications in the model and reduce the number of equations. Again, see footnote 35.

satisfy the world demand. The UK has a very high of its exports, services, from this sector.

Given the above assumption, and firms in the energy extensive sector are also perfectly competitive, the typical firms maximises the following profit function subject to the budget constraint in equation (140): $max\,\Pi_t = P_t^n Y_t^n - \left(W_t H_t^n + R_t^n Z_t^n K_{t-1}^n + P_{O,t} E_t^n\right)$ where $P^n$ and $R^n$denotes the relative price of energy extensive goods and rental rate of capital services for the sector, respectively. $P_O$ represents the world price of energy. $E^n$ denotes nominal energy use, in the sector, the value is measured in US Dollars given assumption of the numeraire of world imports.

Given that, the respective demand for labour, capital and energy use in the energy intensive sector is

$$W_t = P_t^e A_t^e F_1^e(H_t^e, Z_t^e K_{t-1}^e, O_t^e E_t^e) \tag{142}$$

$$R_t^e = P_t^e A_t^e F_2^e(H_t^e, Z_t^e K_{t-1}^e, O_t^e E_t^e) \tag{143}$$

and $$Q_t = P_t^e A_t^e O_t^e F_3^e(H_t^e, Z_t^e K_{t-1}^e, O_t^e E_t^e) \tag{144}$$

and the respective demand for labour, capital and energy use in the energy extensive sector is

$$W_t = P_t^n A_t^n F_1^n(H_t^n, Z_t^n K_{t-1}^n, O_t^n E_t^n) \tag{145}$$

$$R_t^n = P_t^n A_t^n F_2^n(H_t^n, Z_t^n K_{t-1}^n, O_t^n E_t^n) \tag{146}$$

and $$Q_t = P_t^n A_t^n O_t^n F_3^n(H_t^n, Z_t^n K_{t-1}^n, O_t^n E_t^n) \tag{147}$$

The first-order condition, of the above two sectors, gives the marginal productivity of each input relative to its marginal cost given the assumption of perfect competitive firms.

### 3.2.3 Government

Following An and Schorfheide (2007) and Justiniano, et al., (2009), the fiscal authorities are assumed to be fully Ricardian, and the following budget constraint does not change over time. This is based on the assumption that the government will continue to adjust taxes and through the monetary authority adjusts interest rates to achieve its policy objective.

$$G_t + B_t = T_t + E_t R_{t+1} B_{t+1} \tag{148}$$

where $G$ represents the exogenous government spending shock following a univariate autoregressive form[41]. The budget deficit of the government is financed by issuing short term bonds to households. Therefore, households can access the domestic bond market where nominal government bonds, that pay a gross interest rate $R_t$, are traded.

### 3.2.4 International Trade

I assume in this model, logically, that the United Kingdom is an open economy. I also assume that consumption, investment and government are composites of United Kingdom's and world's sectoral goods. I can note that by definition:

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[41] See footnote 35.

$$C_t = \Phi_c\left(C_t^{d,e} C_t^{f,n} C_t^{d,e} C_t^{f,n}\right) \tag{149}$$

$$I_t = \Phi_I\left(I_t^{d,e} I_t^{d,n} I_t^{d,e} I_t^{f,n}\right) \tag{150}$$

and $$G_t = \Phi_G\left(G_t^{d,e} G_t^{d,n} G_t^{f,e} G_t^{f,n}\right) \tag{151}$$

for $J = C, I, G,$ where $\Phi_j$ is the Armington aggregator, CES utility function with homothetic preferences assumed to be homogenous-of-degree-one and increasing. For all variables $J$, superscripts $d, e$ denotes demand for domestically produced goods in the energy intensive sector, while superscripts $d, n$ denotes demand for domestically produced goods in the energy extensive sector (non-energy). Superscripts $f, e$ denotes demand for foreign produced goods in the energy intensive sector, while superscripts $f, n$ denotes demand for foreign produced goods in the energy extensive sector (non-energy). In order to maintain focus on the macro-variables, I choose to use aggregate expenditures of variables $j$, and in that way, the total sum of these variables yields the domestic absorption:

$$D_t = C_t + I_t + G_t \tag{152}$$

where $$D_t = \kappa\left(D_t^d, M_t\right) \tag{153}$$

This means $D$ is a composite for the four outputs. The Armington aggregator function here, κ, is assumed to be homogeneous-of-degree-one and increasing in both arguments. $D^d$represents the households' demand of goods produced in the United Kingdom and M denotes the total spending on imported goods. Unlike Bakus et al., (1993) where they assumed two goods in an open economy, this model assumes four produced goods in the world which require some more disaggregation.

Here, $D$ and $M$ are assumed to be a function both sectoral outputs, I can note that by definition:

$$D_t = \Sigma(D_t^e, D_t^n) \tag{154}$$

and $$M_t = \Xi(M_t^e, M_t^n) \tag{155}$$

where the Armington aggregator functions of $\Sigma \; and \; \Xi$ are homogeneous-of-degree-one and increasing in both arguments. $D^e \; and \; D^n$ represents the nominal expenditure on domestic output from the energy intensive and energy extensive (non-energy) sectors by domestic agents, respectively. Similarly, $M^e \; and \; M^n$ represents the nominal expenditure on imports from the energy intensive and energy extensive (non-energy) sectors by households, respectively.

The domestic agents will $min\{P_t^d D_t^d + P_{M,t} M_t - S_t D_t\}$ subject to equations (153) where $P^d$ is the price index of composite goods produced in the United Kingdom while $S$ is the consumer price index of the United Kingdom. $P_M$ is world's price index of composite goods, assumed to be the numeraire in the model. Given that, therefore, $S$ is also the nominal exchange rate variable. The agents have another problem of $min\{P_t^e D_t^e + P_t^n D_t^n - S_t D_t\}$ subject to equation (154) with the assumption of Walras' law that "all markets clear", the energy extensive sector goods market is silent, here, as the law implies the market will clear. The domestic agents will, also, solve the problem of share of imported goods expenditure in the respective sectors by using the budget constraint of equation (155) to solve $min\{P_{M,t}^{e}M_t^e + P_{M,t}^{n}M_t^n - M_t\}$. $P_M^e$ and $P_M^n$ are imports prices in the energy intensive and extensive sectors, respectively. Like energy prices, world prices are exogenous as they adjust to their world prices, therefore, imports prices are treated as exogenous shocks in this model[42]. The first-order conditions are:

$$\frac{1}{S_t} = \kappa_2\left(D_t^d, M_t\right) \tag{156}$$

$$\frac{P_t^e}{S_t} = \Sigma_1(D_t^e, D_t^n) \tag{157}$$

$$P_{M,t}^e = \Xi_1(M_t^e, M_t^n) \tag{158}$$

where $\lambda = \kappa_2, \Sigma_1 \Xi_1$ denotes a partial derivative of $\lambda$ with respect to its $g - th$ argument. Here, one can see that the agents' problem of the world is similar to the domestic economy. This is why the imports function will be used to set-up the world's demand (exports) function:

$$D_t^w = \kappa^w\left(D_t^f, M_t^f\right) \tag{159}$$

Similarly, where the model assumes $D_t^w = C_t^w + I_t^w + G_t^w$ as the aggregate world demand, $D^f$denoted as world's demand for home goods and $M^f$ denotes the total imports in the world's economy which signifies the United Kingdom's nominal exports ($X$). where $\kappa^w$ is homogeneous-of-degree- one and increasing in both its arguments.

The first order condition with respect to exports is:

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[42] See footnote 35.

$$S_t = \kappa_2^w\left(D_t^f, X_t\right) \tag{160}$$

$\kappa_2^w$denotes a partial derivative of $\lambda$ with respect to its $g - th$ argument. Also, exports is a composite defined by:

$$X_t = \Xi^w\left(M_t^{f,e}, M_t^{f,n}\right) = \Xi^w(X_t^e, X_t^n) \tag{161}$$

where $\Xi^w$ are homogeneous-of-degree- one and increasing in both its arguments. The first-order condition with respect to exports of energy intensive sector goods $X^e$ is:

$$\frac{P_t^e}{S_t} = \Xi_1^w(X_t^e, X_t^n) \tag{162}$$

$\Xi_1^w$denotes a partial derivative of $\lambda$ with respect to its $g - th$ argument. And finally, by definition I note the exchange rate as:

$$P_t = \Sigma(P_t^e, P_t^n) \tag{163}$$

### 3.2.5 Aggregation, Market clearing and the resource constraint

The assumption of this two-sector model is to have a total nominal output that produce in the domestic country. The nominal sectoral outputs are measured in US dollars then added to give total output measured[43] in US Dollars with the assumption of the numeraire as the world imports prices, simply given as:

$$Y_t = Y_t^e + Y_t^n \tag{164}$$

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[43] A detailed explanation of data collection and construction is given in the next section 3.3.

where $Y$ is denoted as total nominal output and $Y^e$ $and$ $Y^n$ are the sectoral output of the energy intensive firm and the energy extensive firm, respectively. The aggregate for labour supply and total energy use are:

$$H_t = H_t^e + H_t^n \tag{165}$$

$$E_t = E_t^e + E_t^n \tag{166}$$

Aggregate investment is defined as:

$$I_t = I_t^e + I_t^n \tag{167}$$

where $I^e$ $and$ $I^n$ are sector-specific investment.

Energy intensive sector market clears

$$Y_t^e = D_t^e + X_t^e - M_t^e \tag{168}$$

Final production satisfies demand as:

$$Y_t = C_t + I_t + G_t + X_t - M_t - E_t \tag{169}$$

It means the aggregate resource constraint is describing how the output is absorbed by consumption, investment, governments exogenous spending, net exports and energy use.

The dynamic of the current account equation is given as:

$$B_t^f - \left(1 + R_{t-1}^f\right)B_{t-1}^f = S_t X_t - M_t - P_{O,t}E_t \tag{170}$$

Above denotes aggregate resource constraint describing how the net foreign assets of the economy evolve. The left-hand side shows the changes made in foreign asset holdings within one period lag while the right-hand side states the expenditures of net exports, with imports price assumed to be the numeraire in the model, and primary energy use yielding adjustment of bond wealth.

### 3.2.6 Functional forms

The model evaluation provides avenue to select explicit functional forms for preferences, technologies, time-varying depreciation rates, capital adjustment costs, and the aggregator functions.

Households' utility function takes the form of:

$$\left(\frac{(C_t - \psi_h C_{t-1})^{1-\sigma_c}}{1-\sigma_c} - \xi_{w,t}\frac{H_t^{1+\omega}}{1+\omega}\right) \tag{171}$$

$$-\frac{U_1}{S_t} - \chi_{bf}\frac{U_1}{S_t}\left[\frac{B_t^f}{S_t} - nfa\right] + \beta R_t^f E_t \frac{U_2}{S_{t+1}} = 0 \tag{172}$$

This model assumes that output is produced by combining domestic capital, energy input and labour using a CES production function, For $j = e, n$:

$$Y_t^j = A_t^j \left(H_t^j\right)^{1-\alpha_j}\left(\theta_j\left(Z_t^j K_{t-1}^j\right)^{-v_j} + \left(\left(1-\theta_j\right)\left(O_t^j E_t^j\right)^{-v_j}\right)\right)^{-\frac{\alpha_j}{v_j}} \tag{173}$$

The time-varying rates of depreciation, following Basu and Kimball (1997), For $j = e, n$:

$$\delta U_t^j = \delta_{j0+}\frac{\delta_{j1}\left(U_t^j\right)^{\mu_j}}{\mu_j} \tag{174}$$

The capital adjustment cost functions adopted are:

$$\chi_j\left(\frac{K_t^j}{K_{t-1}^j}\right) = 0.5\chi_j\left(\frac{K_t^j - K_{t-1}^j}{K_{t-1}^j}\right)^2 K_{t-1}^j \tag{175}$$

The aggregator functions of $\Sigma, \Xi, \kappa, \kappa^w and\ \Xi^w$ as given CES are:

$$D_t = \left(\kappa^{\frac{1}{\eta}}\left(D_t^d\right)^{\frac{\eta-1}{\eta}} + (1-\kappa)^{\frac{1}{\eta}}(M_t)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{176}$$

$$D_t = \left(\sigma^{\frac{1}{\varsigma}}(D_t^e)^{\frac{\varsigma-1}{\varsigma}} + (1-\sigma)^{\frac{1}{\varsigma}}(D_t^n)^{\frac{\varsigma-1}{\varsigma}}\right)^{\frac{\varsigma}{\varsigma-1}} \tag{177}$$

$$M_t = \left(\chi^{\frac{1}{\phi}}(M_t^e)^{\frac{\phi-1}{\phi}} + (1-\chi)^{\frac{1}{\phi}}(M_t^n)^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}} \tag{178}$$

$$Y_t^f = \left(\kappa^{\frac{1}{\eta_w}}(D_t^e)^{\frac{\eta_w-1}{\eta_w}} + (1-\kappa)^{\frac{1}{\eta_w}}(D_t^n)^{\frac{\eta_w-1}{\eta_w}}\right)^{\frac{\eta_w}{\eta_w-1}} \tag{179}$$

$$X_t = \left(\chi^{\frac{1}{\phi_w}}(X_t^e)^{\frac{\phi_w-1}{\phi_w}} + (1-\chi)^{\frac{1}{\phi_w}}(X_t^n)^{\frac{\phi_w-1}{\phi_w}}\right)^{\frac{\phi_w}{\phi_w-1}} \tag{180}$$

## 3.3 Data

I assume the world imports prices (world manufacturing prices) as numeraire in the model, for the sake of clarity, the nominal value of data collected in or converted to US Dollars. However, the world manufacturing prices are constructed using the weighted average of some OECD countries (index, 2010=100).

The data for endogenous variables and exogenous forcing processes cover the period 1990Q1 to 2014Q4. I aimed at going further back, but the data availability of some structural variables such as sectoral output and energy use only starts from 1990Q1. Due to this constraint, I can only cover the crisis periods during the great moderation era of the UK and the 2008 financial crisis. I use the three-month Treasury bill rate series, for the interest rate, from Bank of England database (IUQAAJNB). For exchange rate, I use Quarterly Average Effective exchange rate index XUQABK67 from Bank of England. I use DataStream for data collection.

The definition of energy intensive sector as regards to data collection is the combination of industries in the UK that spends over 3% of their production cost on crude oil products. This defintion is similar to the definition of EU 2000 Regulation on Pollution Prevention and Control that define energy intensive sector in terms of energy use. These industries include Agriculture, Production Sector, Construction sector, and finally Transport & Storage from the Services sector. The energy extensive (non-energy) sector is the sector of the economy that use less than 3% of their cost on crude oil products. These include: Services industry that includes

Accommodation & Food Service Activities, Information & Communication, Financial and Insurance Activities, Real Estate Activities, Professional, Scientific & Technical Activities, Administrative and Support Service Activities, Public Administration, Education, Health and Social Work, Arts, Entertainment and Recreation, and Other Service Activities. I use final consumption expenditure of households and NPISHs (ABJQ.Q + HAYE.Q). For total hours of employment, I use ONS series of (YBUS.Q). Real wages I divided UK wages (XPEWF.B) from ONS series by total hours worked (YBUS.Q) and then divided through by consumption deflator where the consumption deflator is (ABJQ.Q + HAYE.Q)/(ABJM.Q + HAYO.Q). The foreign bonds are the UK investment abroad which net acquisition of financial assets are by monetary financial institutions, Central government Local government, Public corporations and other sectors (UKHBNR). Capital utilization rate is represented by Manufacturing sector utilization rate and the corporate sector utilization rate for the energy intensive sector and energy extensive (non-energy) sector, UKCBICAPE and UKXCAPU.R, respectively.

For world data, I used the series of the world import prices (IMPPRCF) index 2010=100, for energy (crude oil, as proxy) prices I collected the world prices of crude oil (WDXWPOB). I deflated the variables by, the numeraire, world's manufacturing price index by using the weighted average of some OECD countries: Canada, Germany, France, Japan, Italy, South Africa and the United States. I seasonally adjusted energy use, world prices and world demand. Likewise, the foreign interest rate is a weighted average of the stated OECD countries. All variables are in per capita basis, this is done by dividing through by a UK working-age population before taking natural logs and all were detrended using Hodrick-Prescott (HP) filter setting the smoothing parameter $\lambda$=1600 I use the ONS quarterly series (UKMGSL.Q) for population.

### 3.4. The error processes

The data is used to estimate the model errors and the properties of errors. The model is augmented with 13 exogenous processes and 3 of these shocks are tested to be non-stationary and are treated as non-stationary and are modelled as ARIMA (1,1,0) processes with a constant. The sectoral productivity shocks can be directly estimated while the world energy price shock is measured with the observed data. Other shocks are tested to be stationary or trend stationary, hence, treated as stationary or trend-stationary ARMA(1,0) processes with a deterministic trend. Figure (22) shows the nonstationary data charts.

The properties of the errors is represented below with the persistence estimated us from AR(1) process and the standard deviation estimated from the errors' innovations[44]. One can see the volatility of energy price is quite high. Foreign shocks have high persistence while investment specific-technology shock possess high persistence and volatility.

$\varepsilon_{ae,t} = \varepsilon_{ae,t-1} + 0.3394(\varepsilon_{ae,t-1} - \varepsilon_{ae,t-2}) + \eta_{ae,t}$, $\sigma_{ae,t}$=0.0259

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[44] The method of estimation is given in the steps of using indirect inference test.

$\varepsilon_{an,t} = \varepsilon_{an,t-1} + 0.1896(\varepsilon_{an,t-1} - \varepsilon_{an,t-2}) + \eta_{an,t}$, $\sigma_{an,t}$=0.0241

$\varepsilon_{b,t} = 0.4367\varepsilon_{b,t-1} + \eta_{b,t}$, $\sigma_{b,t}$=0.0807

$\varepsilon_{g,t} = 0.9894\varepsilon_{g,t-1} + \eta_{g,t}$, $\sigma_{g,t}$=0.0235

$\varepsilon^{e}_{inv,t} = 0.9209\varepsilon^{e}_{inv,t-1} + \eta^{e}_{inv,t}$, $\sigma^{e}_{inv,t}$=0.1689

$\varepsilon^{n}_{inv,t} = 0.8696\varepsilon^{n}_{inv,t-1} + \eta^{n}_{inv,t}$, $\sigma^{n}_{inv,t}$=0.2335

$\varepsilon^{e}_{o,t} = 0.9039\varepsilon^{e}_{o,t-1} + \eta^{e}_{o,t}$, $\sigma^{e}_{o,t}$=0.3071

$\varepsilon^{n}_{o,t} = 0.8954\varepsilon^{n}_{o,t-1} + \eta^{n}_{o,t}$, $\sigma^{n}_{o,t}$=0.3053

$\varepsilon_{pm,t} = 0.9741\varepsilon_{pm,t-1} + \eta_{pm,t}$, $\sigma_{pm,t}$=0.0181

$\varepsilon_{po,t} = \varepsilon_{po,t-1} + 0.2257(\varepsilon_{po,t-1} - \varepsilon_{po,t-1}) + \eta_{po,t}$, $\sigma_{po,t}$=0.1388

$\varepsilon_{rf,t} = 0.9227\varepsilon_{rf,t-1} + \eta_{rf,t}$, $\sigma_{rf,t}$=0.0031

$\varepsilon_{w,t} = 0.8568\varepsilon_{w,t-1} + \eta_{w,t}$, $\sigma_{w,t}$=0.1158

$\varepsilon_{yf,t} = 0.9250\varepsilon_{yf,t-1} + \eta_{yf,t}$, $\sigma_{yf,t}$=0.0430

Figure 22 Unfiltered data of the UK

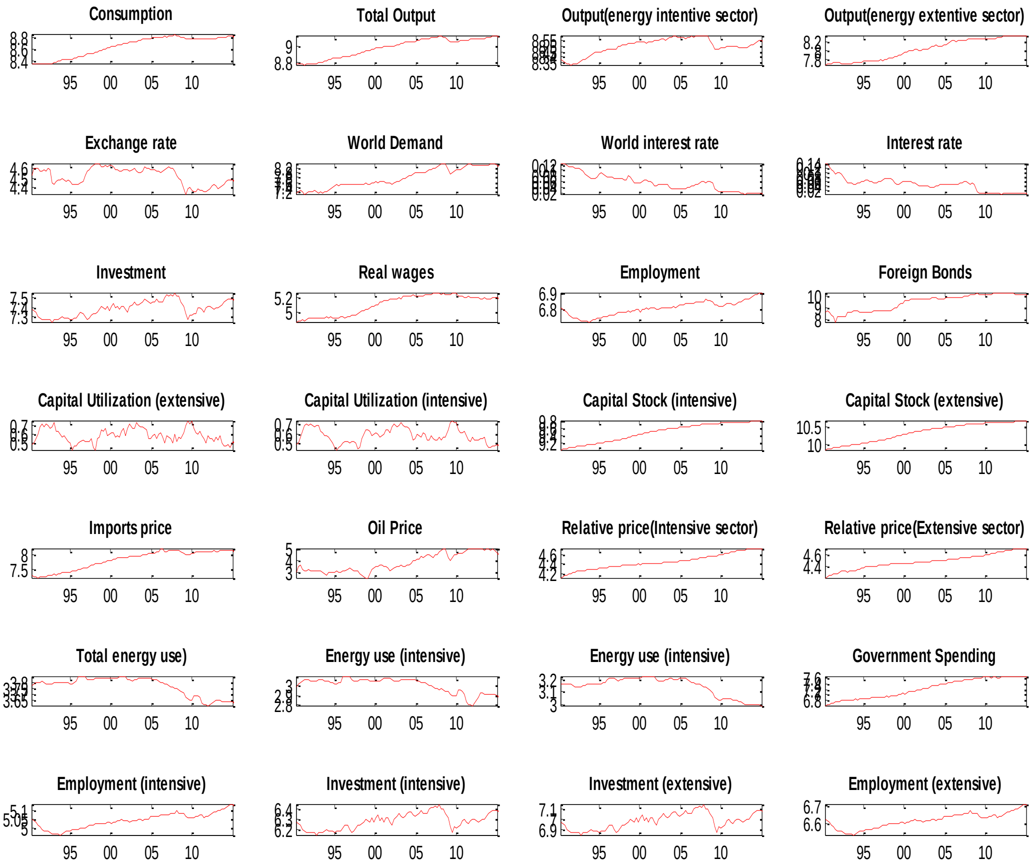

Table 10 Error processes

| **Shock** | **Process** | ***c*** | **trend** | **AR(1)** |
|---|---|---|---|---|
| Productivity (energy-intensive sector)[45] | Non-stationary | | -2.3387 | 0.3394 |
| Productivity shock (energy-extensive sector) | Non-stationary | | -1.0939 | 0.1896 |
| Consumption preference | Stationary | 0.1966 | | 0.4367 |
| Government spending *[46] | Stationary | 0.2082 | | 0.9894 |
| Investment Specific-Tech. shock (non-energy) | Stationary | 0.1082 | | 0.9209 |
| Investment Specific-Tech. shock (non-energy) | Stationary | 0.1045 | | 0.8696 |
| Energy efficiency (energy intensive sector) | T-stationary | | 0.0589 | 0.9039 |
| Energy efficiency shock (non-energy) | T-stationary | | 0.0599* | 0.8954 |
| World exports price | T-stationary | | 0.1013 | 0.9741 |
| Energy price | Non-stationary | | -3.6603 | 0.2257 |

---

[45] Negative numbers come from ADF test while others show result from KPSS test.

[46] * 1% level of significance

| | | | |
|---|---|---|---|
| World interest rate | T-stationary | 0.0904* | 0.9227 |
| Labour supply | T-stationary | 0.2108* | 0.8568 |
| World demand | T-stationary | 0.1587* | 0.9250 |

Following the result above showed the sector-specific productivity shocks and energy price shocks are tested to be nonstationary[47]. The results is concluded following a robust stationary test of KPSS test and ADF test.

---

[47] Thus, I use first-difference in the shock estimation: $\varepsilon_t = \varepsilon_{t-1} + \rho(\varepsilon_{t-1} - \varepsilon_{t-1}) + \eta_t$.

## 3.5 Calibration

As I prepare to evaluate the log-linearized model, I will have to set values for the parameters. I will first split the parameters into two groups. The first group of parameters are the set that are important in deriving the model's steady state. Derived by taking average ratios of the data used in the study covering the period 1990-2014, with little influence on the dynamics properties. These parameters are set to match steady-state values. When I estimate the model, these set of parameters remain unchanged, hence the name fixed parameters. I set the discount factor $\beta$ at 0.96, this means that the model will generate a steady-state annual real interest rate of 4%. The cost shares of between labour and capital services, $\alpha_e \; and \; \alpha_n$, are set to 0.35 and 0.28 for energy intensive sector and energy extensive sector, respectively. This means that steady-state labour share is 65% and 72% in energy intensive sector and energy extensive sector, respectively.

The depreciation rate is set at 0.0125 per quarter which implies 5% annual depreciation on capital. Nonetheless, I had the opportunity to estimate using the model's structural parameters in steady-state as follows: I divided the depreciation rate of capital into two sectors for $j = e, n$. $\delta U^j = \delta_{j0} + \delta_{j1}(\mu_j)^{-1}(U^j)^{\mu_j}$. In setting $\delta_{j1} = 1$ and assuming households optimality conditions with regards to capital utilization rates conditioned on the values for the respective sectors' steady-state real capital rental rate, $\delta_{j1}(U^j)^{\mu_j} = R^j = \frac{1}{\beta} - (1 - \delta U^j)$. Having calibrated $\delta U^j$ using the data, $\delta_{j1}(U^j)^{\mu_j} = 0.0544 \; and \; 0.0606$ for energy intensive sector and energy extensive (non-energy) sector, respectively. To calibrate the elasticity in capital utilization rate $\mu_j$, I augmented the previous result which I assumed the conditioned values of the discount factor and rental rate as $\mu_j = \frac{\delta_{j1}(U^j)^{\mu_j}}{1+\delta_{j1}(U^j)^{\mu_j}-\frac{1}{\beta}}$ =1.404 and 1.1 for energy intensive sector and energy extensive (non-energy) sector, respectively. The cost share parameter between capital services and energy is calibrated using the capital-energy ratio from the sample period and the structural parameter that results in $\theta_j = \frac{1}{1+\frac{Po}{\delta_{j1}(U^j)^{\mu_j}}\left(\frac{e^j}{k^j}\right)^{1+v_e}}$ where $\frac{e^j}{k^j}$ is the steady-state ratio of energy-capital and $Po$ is the steady-state value of energy prices.

The fixed parameters are shown in table 11, below:

Table 11 Fixed parameters

| Parameter | Value | Description |
|---|---|---|
| β | 0.99 | Discount factor |
| $\delta^e$ | 0.0125 | Depreciation rate energy intensive sector |
| $\delta^n$ | 0.0125 | Depreciation rate energy extensive sector |
| $1-\alpha_e$ | 0.65 | Labour share in energy intensive sector |
| $1-\alpha_n$ | 0.72 | Labour share in energy extensive sector |
| $\theta_e$ | 0.9998 | Capital services weight in energy intensive sector |
| $\theta_n$ | 0.9999 | Capital services weight in energy extensive sector |
| $\frac{c}{y}$ | 0.1773 | Share of private consumption in total output |
| $\frac{i}{y}$ | 0.2019 | Ratio of investment to total output |
| $\frac{x}{y}$ | 0.2933 | Share of exports in total output |
| $\frac{m}{y}$ | 0.3126 | Ratio of imports to total output |
| $\frac{m^e}{y^e}$ | 0.0990 | Ratio of imports to output in energy intensive sector |
| $\frac{e}{y}$ | 0.2355 | Share of energy use in total output |
| $\frac{g}{y}$ | 0.1773 | Share of government consumption in total output |

| | | |
|---|---|---|
| $\frac{y^e}{y}$ | 0.6145 | Ratio of energy intensive output to total output |
| $\frac{y^n}{y}$ | 0.3855 | Ratio of energy extensive output to total output |
| $\frac{i^e}{i}$ | 0.3320 | Ratio of investment in energy intensive sector to total investment |
| $\frac{i^n}{i}$ | 0.6680 | Ratio of investment in energy extensive sector to total investment |
| $\frac{e^e}{e}$ | 0.710 | Ratio of energy usage in energy intensive sector to total energy usage |
| $\frac{i^e}{k^e}$ | 0.0420 | Ratio of investment to capital in energy intensive sector |
| $\frac{i^n}{k^n}$ | 0.0362 | Ratio of investment to capital in energy extensive sector |
| $\frac{c}{d}$ | 0.6514 | Share consumption in domestic absorption |
| $\frac{i}{d}$ | 0.1869 | Ratio of investment in domestic absorption |
| $\frac{g}{d}$ | 0.1617 | Share of government consumption in domestic absorption |
| $\frac{p^e}{s}$ | 0.7753 | Ratio of price to exchange rate in energy intensive sector |
| $\frac{p^n}{s}$ | 0.9448 | Ratio of price to exchange rate in energy extensive sector |
| $\frac{e^e}{k^e}$ | 0.0827 | Energy-capital ratio in energy intensive sector |
| $\frac{e^n}{k^n}$ | 0.0289 | Energy-capital ratio in energy extensive sector |
| $\frac{h^e}{h}$ | 0.1710 | Ratio of employment in energy intensive sector to total employment |
| $\frac{h^n}{h}$ | 0.8290 | Ratio of employment in energy extensive sector to total employment |
| $s\frac{x}{b^f}$ | 0.2057 | Ratio of demand for exports to foreign bonds |
| $\frac{m}{b^f}$ | 0.2134 | Ratio of demand for imports to foreign bonds |
| $\frac{e}{b^f}$ | 0.1584 | Ratio of energy demand to foreign bonds |

I set the parameter for the degree of habit formation parameter, at 0.7, to be consistent with standard DSGE models, intertemporal elasticity of substitution to 2 and the Frisch inverse elasticity of labour supply parameter at 3. I choose either to assume the UK has a balanced current account by setting the foreign bonds' adjustment cost to 0 or as a creditor > 0, I chose the latter and set the parameter at 0.25. The elasticity of substitution between capital services and energy use in the respective sectors, $v_e$ *and* $v_n$, are set to 0.7. The value of the capital adjustment cost which is set at 5. This means that cost of capital costs gives incentives for households to change the capital stock. That means, ceteris paribus, a higher capital adjustment cost parameter will decrease the elasticity of the change in capital stock relating to real interest rate.

The parameters governing foreign trade are assumed to follow the standard DSGE literature.

Table 12 Parameters to be estimated

| Parameter | Parameters explaining: | Value |
|---|---|---:|
| $\omega$ | Frisch elasticity of labour supply | 0.33 |
| $\psi_h$ | Habit formation in consumption | 0.7 |
| $\sigma_c$ | Intertemporal Elasticity of substitution | 2 |
| $\eta$ | Elasticity of demand for imports | 1.5 |
| $\eta_w$ | Elasticity of demand for exports | 1.5 |
| $\mu_e$ | Elasticity in capital utilization rate; energy-intensive sector | 1.404 |
| $\mu_n$ | Elasticity in capital utilization rate; energy-extensive sector | 1.1 |
| $v_e$ | Elasticity of substitution between energy and capital in energy-intensive production | 0.7 |
| $v_n$ | Elasticity of substitution between energy and capital in energy-extensive production | 0.7 |
| $\varsigma$ | Elasticity of substitution between consumption of energy-intensive and energy-extensive goods | 1 |
| $\chi_e$ | Cost parameter: capital stock in energy intensive sector | 5 |
| $\chi_n$ | Cost parameter: capital stock in energy-extensive sector | 5 |
| $\phi$ | Elasticity of demand for imports of energy-intensive goods | 0.6145 |
| $\phi_w$ | Elasticity of demand for exports of energy-intensive goods | 0.5 |
| $\epsilon$ | Share of energy intensive goods | 0.5 |

|  |  |  |
|---|---|---|
| $\delta_{e1}z^{\mu_e}$ | Cost of capital utilization in energy intensive sector | 0.0544 |
| $\delta_{n1}z^{\mu_n}$ | Cost of capital utilization in energy-extensive sector | 0.0606 |
| $\chi_{BF}$ | Cost of adjusting portfolio of foreign bonds | 0.25 |

$\varsigma$ that denotes the elasticity of substitution between consumption of the sectoral goods is set to unity, the elasticity of demand for imports $\eta$ is set at 1.5 which I did the same for the rest of the world equation $\eta_w$ as I assume the world has the same agent's problem as the UK. The elasticity of demand for imports of energy intensive goods is set at 0.4. All values of shares and ratio are consistent with the DSGE model of the United Kingdom literature.

## 3.6 Methodology

In this part, I take this model to the UK nonstationary data. In standard practice, there are conventional tools used to understand how a simulated DSGE model works such as Variance decomposition and Impulse response functions that I will show in this study. I will also add the VAR-impulse response functions[48] in assessing the fit of the estimated model. I will also be accounting for the crisis period with the model's shock decomposition. All these comes following the model estimation method which I use the powerful simulated annealing algorithm. I will use an approach of sampling variability of the simulated data to match the actual data using indirect inference testing. This is in contrast to indirect inference estimation. I will show the difference in the same section.

### 3.6.1 Model evaluation by indirect inference test

Indirect inference test method of model evaluation offers a classical econometrics inferential structure for assessing calibrated models Le, Meenagh, Minford and Wickens (2012). This method is used to judge partially or fully estimated models while maintaining the fundamental ideas utilized in the evaluation of early RBC models of comparing data generated moments from the model simulation by the actual data. Instead of using moments to compare with no distributions, this method provides a simple model (auxiliary model) that includes the conditional mean of the

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[48] Authors like Christiano, et al., (2005) evaluated their model of the US exclusively on the fit to the structural shock

distribution which one can compare the features of the model estimated from actual and simulated data. This, indirect inference test, the method on structural DSGE models, although different, has similar features in the widely used indirect estimation method. The primary feature of this similarity is utilization of the auxiliary model in addition to the structural macroeconomic model. The estimation by indirect inference chooses the parameters of the DSGE model in a way that the simulated model generates estimates of the auxiliary model that is similar to those obtained from the data.

An account of inferential problem is as follows: using Canova (2005) notations designed for indirect inference estimation, where $y_t$ is defined as $m \times 1$ vector observed data ($t = 1, \dots, T$) and $x_t(\theta)$ is a $m \times 1$ vector of simulated (time series) data with the number of observations $S$ which is generated from the structural model, $\theta$ is a $k \times 1$ vector of the model's structural parameters. The assumption here is that $y_t$ and $x_t(\theta)$ are stationary and ergodic. Then set $S = T$ with the requirement of the actual dal data sample being regarded as the expected imitation from the population of the samples that have been bootstrapped by the data. The auxiliary model is assumed as $f(y_t, \alpha)$, with $\alpha$ as the vector of descriptors. From the given null hypothesis $H_0\colon \theta = \theta_0$, the auxiliary model then becomes $f$[$x$_$t$ ($\theta$), ($\theta$_0 )]= as $f[y_t, \alpha]$. The test of the null hypothesis is by a q × 1 vector of a continuous function $g(\alpha)$. Therefore, under the null hypothesis, one is going to have $g(\alpha) = g(\alpha(\theta_0))$. The estimator for $\alpha$ using the actual data is $a_T$ while the estimator for $(\theta_0)$ based on simulated data is $a_S(\theta_0)$. This gives us $g(a_T)$ and $g(a_S(\theta_0))$. We then get the mean of the bootstraps as: $\overline{g(\alpha(\theta_0))} = \frac{1}{N}\sum_{k=1}^{N} g_k(\alpha(\theta_0))$. From here, we get the Wald statistic ($WS$) by using the bootstrapped distribution of $g(a_S)$- $\overline{g(a_S(\theta_0))}$. This is then defined as:

$$WS = g(a_T)\text{-}\ \overline{g(a_S(\theta_0))}'W(\theta_0)^{-1}(\theta_0)g(a_T)\text{-}\ \overline{g(a_S(\theta_0))} \tag{181}$$

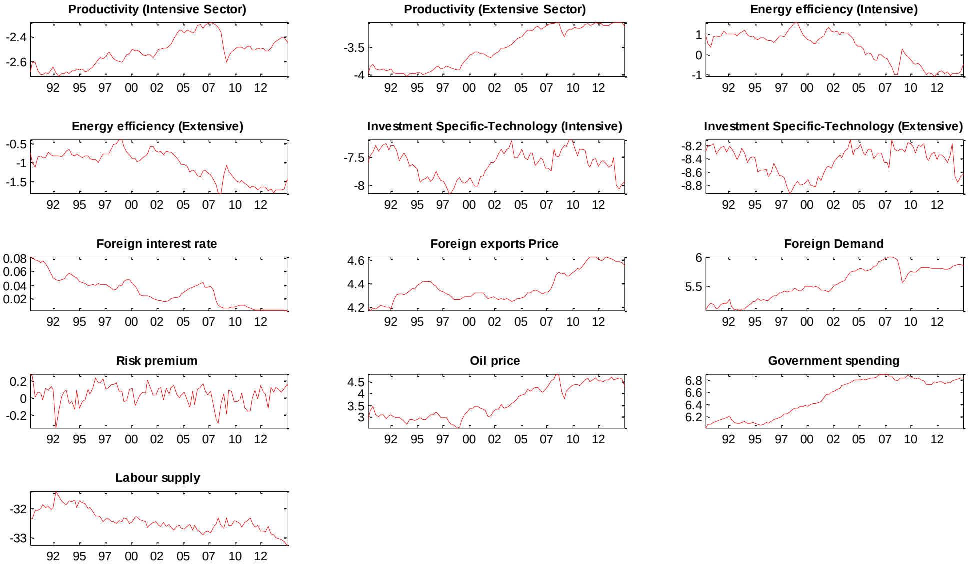
where $W(\theta_0)^{-1}$ is the variance-covariance of the bootstrapped distribution of $g(a_T)$- $\overline{g(a_S(\theta_0))}$. Furthermore, $W(\theta_0)$ is obtained from the asymptotic distribution of $g(a_T)$- $\overline{g(a_S(\theta_0))}$ and then the asymptotic distribution of the Wald statistic would then be chi-squared. Unlike the above, with an indirect inference test one will obtain an empirical distribution of the Wald statistic bootstrap using a bootstrap method through defining $g(\alpha)$ as a vector consisting of the VAR coefficients and the variances of the data or the disturbances of the VAR model.

Following the work of Meenagh, Minford and Wickens (2012), I will show how the Wald test by bootstrap is conducted:

*Step 1: Estimating the errors of the structural model based on observed data and* $\theta_0$.

The number of exogenous shocks must be equal to or less than the endogenous variables in the DSGE model. The structural residuals $\varepsilon_t$ are estimated from the DSGE model $x_t(\theta_0)$, given the stated values of $\theta_0$ and the actual observed data. There is an assumption the errors will be normally distributed and will follow AR(1) process. If a structural equation contains no expectation, the residuals may be backed out of the equation and the observed data. If the equation includes some expectations on some variables then there will be estimation for the expected variables. In this case, I carry this out using McCallum (1976) and Wickens (1982) a robust instrumental variables method with lagged endogenous observed data as the instruments. This is more or less an auxiliary model VAR.

Figure 23 Shocks estimated residuals



*Step 2: Deriving the simulated data*

In this model, like many DSGE models, the structural shocks are assumed to be autoregressive processes rather than being serially independent. OLS is used to estimate the innovations from the residuals[49]. The innovations are repeatedly drawn by time vector to preserve any simultaneity between the shocks, and then solving the model by *dynare*. I then go on to obtain *N* bootstrapped simulations by repeating the drawing of the sample independently. *N=1000*.

---

[49] The coefficients of the residuals from the OLS estimation are the model's persistence.

*Step 3: Compute the Wald Statistic*

The auxiliary model is then estimated, a VAR(1), on the bootstrap sample and the actual data to obtain the estimates[50], of the distribution of the observed data and the VAR coefficients, $a_T$ and $a_S$ of the vector $\alpha$. I am able to obtain the covariance matrix $W(\theta_0)$ of the distribution $(a_S)$- $\overline{(a_S(\theta_0))}$ through estimating the auxiliary VAR on the 1000 bootstrapped simulations of $a_S(\theta_0)$ while the covariance of the simulated variables from the bootstrap samples were obtained. This shows the variations in the data sampling as implied by the model from the result set of $a_k$ vectors ( $k = 1, \ldots . N$), thus the estimate of $W(\theta_0)$ will be:

$$\frac{1}{N}\sum_{k=1}^{N}(a_k - \overline{a_k})'(a_k - \overline{a_k}) \tag{182}$$

where $\overline{a_k} = \frac{1}{N}\sum_{k=1}^{N} a_k$. From here, the Wald statistic is calculated for the data sample and then the bootstrap distribution of the Wald from the 1000 samples of the bootstrap is estimated.

A combination of output ($y$) and real exchange rate ($s$) were chosen as the auxiliary model of VAR, for the evaluation to fit the model although other combinations were used, this set was used in the estimation as the variables in the VAR auxiliary model. This auxiliary model allows for joint distribution testing, with the null hypothesis as the structural macroeconomic model is the data generating mechanism.

---

[50] Actual and simulated data variances have been included in the estimates to determine the model's dynamics and volatility.

### 3.6.2 Using Non-Stationary Data

As stated earlier in the literature review, filtering observed data will distort the dynamic properties of the model in several ways that one cannot tell. It also changes the forward-looking properties of the structural model as the filtering method is two-sided. Since the DSGE model is supposed to mimic the activities of the economy, like in this open economy model if world prices of oil data are distorted, the imperfections will be huge. In a model like this, where the expectation structure and impulse response functions are critical, a filtered data will be a flaw in the study. It is common knowledge that the data generated by a DSGE model on most occasions proved to be non-stationary as generated by the model structure or due to incorporation of non-stationary exogenous variables, which are unobservable, such as the productivity shocks or world oil prices function which is an observed variable. Therefore, the linearized model's solution will be denoted by a vector error correction model (VECM), this will allow the model to have higher number of endogenous variables than cointegrating vectors if there are unobservable non-stationary variables. With this, there will be non-stationary errors in the long-run structural model. Given that, this will show the estimated model as a VECM where the non-stationary errors will be represented as observable variables and the unrestricted version of the VECM will be used as the auxiliary model. This method includes the non-stationary errors estimated from the structural model in the auxiliary model as the auxiliary model is required to have key variables for cointegration that will allow the VECM to be stationary and error correction. One should also remember that the auxiliary model is partly conditioned by the structural model that is also null hypothesis $H_0$, therefore, the construction of the VECM came through the null hypothesis. A non-rejection is far from certain under this condition of data generated VECM because the DSGE structural model picks a range of parameters which could be inconsistent with the DSGE structural model. Rather, the objective of the null hypothesis constraint is to make sure the VECM obtains cointegration under the null hypothesis which is also the assumption of the errors.

A test for cointegration is not carried out because of all non-stationary errors are treated as valid cointegrating variables and without cointegration a DSGE model will not have a solution which means there will be no simulation and that will be impossible to have the Wald test. Therefore, the indirect inference carried out here will impose cointegration and will test the simulation performance of the DSGE model at the latter stage of the work.

### 3.6.3 The auxiliary equation

A linearized DSGE model can be written as:

$$A(L)y_t = BE_t y_{t+1} + C(L)x_t + D(L)e_t \tag{183}$$

where $y_t$ are the number, $p$, of endogenous variables and $x_t$ are the number, $q$, of exogenous variables that are driven by the assumed equation:

$$\Delta x_t = a(L)x_{t-1} + d + c(L)\epsilon_t \tag{184}$$

As stated earlier, based on using non-stationary data, the exogenous variables can have observed and unobserved variables such as the world oil prices shock and productivity shocks. The errors $e_t \; and \; \epsilon_t$ are i.i.d. variables each with a zero mean. $L$ symbolises the lag operator where $\pi_{t-s} = L^s \pi_t$ and $A(L), B(L) \ldots$ are polynomial functions each with its root outside the unit circle. Therefore, the solution for $y_t$, where it follows $y_t \; and \; x_t$ are non-stationary, will be:

$$y_t = G(L)y_{t-1} + H(L)x_t + f + M(L)e_t + M(L)\epsilon_t \tag{185}$$

where polynomial functions each with its root outside the unit circle. As $y_t \; and \; x_t$ are non-stationary, a $p$ cointegration relation will have the solution as:

$$y_t = [I - G(1)]^{-1}[H(1)x_t + f] \tag{186}$$

$$= \Pi x_t + g \tag{187}$$

and a long-run solution of the model will be:

$$\bar{y}_t = \Pi \bar{x}_t + g \tag{188}$$

$$\bar{y}_t = [1 - a(1)]^{-1}[dt + c(1)\xi_t] \tag{189}$$

$$\xi_t = \Sigma_{i=0}^{t-1} \epsilon_{t-s} \tag{190}$$

In the long-run solution, $x_t$, defined as $\bar{x}_t = \bar{x}_t^D + \bar{x}_t^S$ will have a deterministic trend represented as $\bar{x}_t^D = [1 - a(1)]^{-1} dt$ and a stochastic trend represented as $\bar{x}_t^S = [1 - a(1)]^{-1} c(1)\xi_t$.

One can now re-write the solution for $y_t$ as the VECM

$$\Delta y_t = -[I - G(1)](y_{t-1} - \Pi\bar{x}_{t-1}) + P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + M(L)e_t + N(L)\epsilon_t \tag{191}$$

$$= -[I - G(1)](y_{t-1} - \Pi\bar{x}_{t-1}) + P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + \omega_t \tag{192}$$

$$\omega_t = M(L)e_t + N(L)\epsilon_t \tag{193}$$

The disturbance of $\omega_t$ is assumed to be a mixed moving average process which means that the VECM may be estimated by the VARX

$$\Delta y_t = K(y_{t-1} - \Pi x_{t-1}) + R(L)\Delta y_{t-1} + S(L)\Delta x_t + g + \zeta_t \tag{194}$$

where $\zeta_t$ is an iid process with a zero mean as $\bar{x}_t = \bar{x}_{t-1} + [1 - a(1)]^{-1}[d + \epsilon_t]$ and Finally, the VECM can be written as

$$\Delta y_t = K(y_{t-1} - \bar{y}_{t-1}) + \Pi(x_{t-1} - \bar{x}_{t-1}) + R(L)\Delta y_{t-1} + S(L)\Delta x_t + h + \zeta_t \tag{195}$$

The latter two equation can be used as the auxiliary model, but equation (195) shows the difference between the effects of the trend elements in $x$ and temporary deviations it has from the trend. The estimation of (195) is done by OLS because it is straight forward and efficient, I chose to use it in this study.

### 3.6.4 Assessing the estimated model fit and other results

In this section, we will examine how the model fits the data. This comes following the model estimation by simulated annealing algorithm. The model parameter are consistent with related literature. Table 15 shows the values of the estimated parameters.

Table 13 Estimated parameters

| Parameter | Description | Value |
|---|---|---:|
| $\omega$ | Frisch elasticity of labour supply | 4.8112 |
| $\psi_h$ | Habit formation in consumption | 0.8318 |
| $\sigma_c$ | Intertemporal elasticity of substitution | 1.1688 |
| $\eta$ | Elasticity of demand for imports | 3.2899 |
| $\eta_w$ | Elasticity of demand for exports | 2.1813 |
| $\mu_e$ | Elasticity in capital utilization rate; energy intensive sector | 1.6856 |
| $\mu_n$ | Elasticity in capital utilization rate; energy extensive sector | 1.0858 |
| $v_e$ | Elasticity of substitution between energy and capital in energy intensive production | 1.8880 |
| $v_n$ | Elasticity of substitution between energy and capital in energy extensive production | 2.873 |
| $\varsigma$ | Elasticity of substitution between consumption of energy intensive and energy extensive goods | 0.595 |
| $\chi_e$ | Cost parameter: capital stock in energy intensive sector | 78.1 |
| $\chi_n$ | Cost parameter: capital stock in energy extensive sector | 49.5 |
| $\phi$ | Elasticity of demand for imports of energy intensive goods | 0.4506 |
| $\phi_w$ | Elasticity of demand for exports of energy intensive goods | 0.5310 |
| $\epsilon$ | Share of energy intensive goods | 0.4750 |
| $\delta_{e1}z^{\mu_e}$ | Cost of capital utilization in energy intensive sector | 0.0171 |
| $\delta_{n1}z^{\mu_n}$ | Cost of capital utilization in energy extensive sector | 0.0022 |
| $\chi_{BF}$ | Cost of adjusting portfolio of foreign bonds | 0.7548 |

| **Shock (*j*)** | **Persistenc** | **Volatility $\sigma_j$** |
|---|---|---|

|  | **e $\boldsymbol{\rho_j}$** |  |
|---|---:|---:|
| Productivity (energy intensive sector) | 0.3297 | 0.0280 |
| Productivity (energy extensive sector) | 0.2628 | 0.0370 |
| Consumption preference | 0.4362 | 0.1001 |
| Government spending | 0.9894 | 0.0235 |
| Investment Specific-Technology (energy intensive sector) | 0.9008 | 0.1104 |
| Investment Specific-Technology (energy extensive sector) | 0.8639 | 0.1031 |
| Energy efficiency (energy intensive sector) | 0.9059 | 0.1769 |
| Energy efficiency shock (energy extensive sector) | 0.8917 | 0.1007 |
| World exports price | 0.9741 | 0.0181 |
| Energy price | 0.2257 | 0.1388 |
| World interest rate | 0.9227 | 0.0031 |
| Labour supply | 0.7741 | 0.1299 |
| World demand | 0.9250 | 0.0430 |

The value of the habit persistence parameter is similar to the assumed value reported by Smets and Wouters (2003) to be between 0.5 and less than 1 and also close to Boldrin, Christiano and Fisher (2001) where they argued the ability of a standardized DSGE model to account for the equity premium among other points. The elasticity of labour supply is consistent with Chadha *et al.,* (2001), as we have a similar utility function. The shock persistence and volatility follow an AR(1) process for the stationary shocks and ARIMA(1,1,0) process for the non-stationary shocks. The energy efficiency shock in the energy intensive sector has a high persistence and volatile rate. However, Government spending has the highest persistence and low volatility while the volatility of energy price shock is high with low persistence.

I use a wide range set of variable set combinations in the model testing with the aggregate output ($y$) remaining a constant in each of these sets. I finally used with

GDP ($Y$) and consumption ($C$), being that the model is a study of a UK open economy with world's prices and foreign bonds included in the model. The model is tested with asset prices, i.e. exchange rate ($S$), which it proves to get close to the data. However, as is common in DSGE models for data to reject the model, the calibrated model is rejected. I show how the simulated behaviour of the model matches the simulated behaviour of the data, following the model estimation. It shows the model have the ability to match the behaviour of the set $Y$ and $C$ as well as a few other sub-sets gets very close to the data. In the results for the subset of $y$ and $c$, given that I use VECM(1) for the auxiliary model, $\alpha$ contains 4 elements and 2 actual data variances. This amount of descriptors is able to provide a condition for the structural model to match the data.

Table 14 VECM results and summary

| | 95% lower | 95% upper | Actual | IN/OUT |
|---|---|---|---|---|
| $A_y^y$ | 0.267471 | 0.879874 | 0.684021 | IN |
| $A_y^c$ | -0.142090 | 0.117258 | 0.040889 | IN |
| $A_c^y$ | -0.400267 | 0.208570 | -0.062125 | IN |
| $A_c^c$ | 0.642383 | 0.926467 | 0.820774 | IN |
| $\sigma_y^2$ | 0.000166 | 0.000426 | 0.000218 | IN |
| $\sigma_c^2$ | 0.000174 | 0.000458 | 0.000237 | IN |
| Summary of results | Wald percentile | Normalised t-statistic | p-value | |
| Dynamic | 55.2 | 0.023 | 0.448 | |
| Volatility | 41 | 0.014 | 0.59 | |
| Overall | 73.3 | 0.482 | 0.267 | |

The first column in table 14 shows coefficients of the VECM, that characterises the dynamic relationships in the data, the middle part shows data variances (that represents the volatility in the data) and the lower part shows each aspect's Wald percentiles as well as the combination of both(overall). The second and third columns shows the DSGE model's implied 95% bounds, the fourth column shows the observed data values while the fifth column tells whether the values of the actual data are inside the 95% bounds or out. Generally, the estimated model has a non-rejection and it fits the data very well.

Table 15 shows the test representing GDP, asset prices and consumption where one can see the relationship of real exchange rate on its lag is what causes the rejection of the overall test while the variances are jointly accepted.

Table 15 VECM results and summary

| | 95% lower | 95% upper | Actual | IN/OUT |
|---|---|---|---|---|
| $A_y^y$ | 0.276583 | 0.854594 | 0.690920 | IN |
| $A_y^s$ | -0.232373 | 0.210150 | 0.008705 | IN |
| $A_y^c$ | -0.148588 | 0.127998 | 0.032067 | IN |
| $A_s^y$ | -0.587111 | 0.024414 | -0.341698 | IN |
| $A_s^s$ | -0.029008 | 0.489667 | 0.842244 | OUT |
| $A_s^c$ | -0.001516 | 0.259051 | 0.083090 | IN |
| $A_c^y$ | -0.393847 | 0.228210 | -0.022303 | IN |
| $A_c^s$ | -0.182722 | 0.256981 | 0.050248 | IN |
| $A_c^c$ | 0.640687 | 0.929797 | 0.769845 | IN |
| $\sigma_y^2$ | 0.000171 | 0.000404 | 0.000218 | IN |
| $\sigma_s^2$ | 0.000242 | 0.000515 | 0.000518 | OUT |
| $\sigma_c^2$ | 0.000171 | 0.000449 | 0.000233 | IN |

| Summary | Wald % | Normalised t-statistic |
|---|---|---|
| Dynamic | 100 | 3.78 |
| Volatility | 90.8 | 1.26 |
| Overall | 99.9 | 3.88 |

Table 16 shows that GDP is well explained together with consumption and the results of the subset shows that other real variables are within the bounds of the model.

Table 16 Summary of VECM for various variable subsets

| Output + other variables | Wald percentile | Normalised t-statistic |
|---|---|---|
| GDP, Wage rate (volatility) | 99.9 | 3.943 |
| GDP, Real interest rate (overall) | 99.8 | 3.427 |
| GDP, Investment (volatility) | 99.0 | 2.5 |

Generally, I can say that the tests implies that this model performs very well in its context, as a DSGE model, as it can explain GDP and consumption among the real variables.

### 3.7 VAR Impulse response functions (VAR-IRFs)

Standard practice allows to evaluate a model exclusively on the DSGE model fit and the VAR, authors like Christiano, et al., (2005) evaluated the DSGE model of the US exclusively on the fit to the structural shock IRFs. Following Le et al., (2012), where the model estimation is based on passing the Wald test using VAR(1), the process generates 95 percent confidence limits for implied VAR responses that simply includes the data-based VAR responses to the structural shocks for the variables in the auxiliary model, output, exchange rate and aggregate consumption. I show the VAR IRFs of the 13 shocks in the appendix. The red lines indicate 95 percent confidence intervals about the point estimates. Overall, the auxiliary model falls within the 95 percent boundary. The response is identified in a similar assumption of the real aggregate output, aggregate demand and real exchange rate evolve in this DSGE model. The behaviour of these endogenous variables displays the fit of the DSGE model. The VAR-IRFs, in appendix 2, simply shows the fit of the model with the data. More analysis follows when I discuss the impulse responses of the model.

## 3.8 A Stochastic Variance Decomposition

Table 17 shows the variance decomposition for all 29 variables in the model with respect to contribution of the 13 shocks in the model following the model estimation.

Table 17 Variance decomposition

| | **Productivity in energy intensive sector** | **Productivity in energy extensive sector** | **Energy price** | **Invest. specific-Technology in energy intensive sector** | **Invest. specific-Technology in energy extensive sector** | **Government spending** | **Consumption preference** |
|---|---|---|---|---|---|---|---|
| GDP | 7.7 | 30.0 | 56.0 | 0.7 | 0.1 | 0.4 | 0.3 |
| Consumption | 14.0 | 39.1 | 45.1 | 0.0 | 0.0 | 0.8 | 0.1 |
| Foreign Bonds | 12.1 | 54.9 | 24.5 | 1.7 | 0.2 | 0.0 | 0.3 |
| Interest rate | 5.7 | 31.8 | 10.3 | 8.0 | 0.4 | 0.1 | 5.5 |
| Exchange rate | 10.2 | 48.4 | 20.2 | 3.9 | 0.3 | 0.0 | 1.5 |
| Wage rate | 16.6 | 15.6 | 56.5 | 0.2 | 0.0 | 0.0 | 0.7 |
| Investment | 1.0 | 12.6 | 61.8 | 2.7 | 0.9 | 0.3 | 1.8 |
| Total Hours | 11.3 | 43.3 | 36.4 | 0.1 | 0.0 | 1.3 | 1.4 |
| Total Energy use | 0.2 | 2.4 | 91.9 | 0.8 | 0.0 | 0.0 | 0.0 |
| Domestic Absorption | 9.0 | 32.1 | 54.5 | 0.4 | 0.1 | 0.0 | 0.0 |
| Total Exports | 9.1 | 43.1 | 18.0 | 3.5 | 0.2 | 0.0 | 1.3 |
| Total Imports | 3.7 | 14.6 | 43.0 | 3.0 | 0.1 | 0.0 | 1.8 |
| **Energy intensive sector** | | | | | | | |
| GDP | 30.8 | 19.2 | 42.4 | 2.2 | 0.1 | 0.0 | 0.2 |
| Investment | 0.7 | 15.1 | 65.3 | 3.4 | 0.0 | 0.1 | 0.8 |
| Employment | 12.6 | 55.0 | 16.0 | 0.1 | 0.3 | 0.1 | 1.3 |
| Energy use | 0.2 | 5.8 | 85.6 | 1.4 | 0.0 | 0.0 | 0.0 |
| Capital stock | 0.4 | 6.5 | 29.9 | 55.8 | 0.0 | 0.5 | 1.2 |
| Capital utilisation | 0.4 | 13.1 | 62.7 | 14.4 | 0.0 | 0.0 | 0.0 |
| Price of goods | 68.1 | 18.3 | 1.2 | 6.0 | 0.0 | 0.0 | 0.4 |
| Domestic Absorption | 23.6 | 18.0 | 51.0 | 0.9 | 0.0 | 0.0 | 0.0 |
| Exports | 39.4 | 14.0 | 9.8 | 7.1 | 0.2 | 0.0 | 1.6 |
| Imports | 3.6 | 14.3 | 42.1 | 3.0 | 0.1 | 0.0 | 1.7 |
| **Energy extensive sector** | | | | | | | |
| GDP | 1.1 | 36.4 | 56.5 | 0.0 | 0.2 | 1.9 | 0.4 |
| Investment | 2.3 | 9.2 | 23.8 | 2.3 | 17.3 | 1.5 | 8.9 |
| Employment | 3.4 | 57.2 | 35.7 | 0.1 | 0.0 | 1.0 | 0.5 |
| Energy use | 0.3 | 1.0 | 84.5 | 0.0 | 0.1 | 0.1 | 0.0 |
| Capital stock | 0.2 | 0.2 | 1.0 | 0.0 | 82.3 | 1.2 | 3.0 |
| Capital utilisation | 0.6 | 2.1 | 94.2 | 0.0 | 1.5 | 0.2 | 0.0 |
| Price of goods | 7.7 | 67.2 | 17.9 | 0.1 | 0.2 | 0.0 | 0.3 |

Variance decomposition (continued)

| | **Energy efficiency in energy intensive sector** | **Energy efficiency in energy extensive sector** | **Foreign interest rate** | **Foreign demand** | **Foreign exports price** | **Labour supply** |
|---|---|---|---|---|---|---|
| Output | 2.7 | 0.2 | 0.0 | 0.2 | 0.0 | 1.6 |
| Consumption | 0.4 | 0.1 | 0.0 | 0.1 | 0.0 | 0.4 |
| Foreign Bonds | 0.0 | 1.9 | 0.2 | 2.3 | 0.0 | 1.9 |
| Interest rate | 3.5 | 9.9 | 0.6 | 12.6 | 0.0 | 11.4 |
| Exchange rate | 0.7 | 4.6 | 0.0 | 5.8 | 0.0 | 4.3 |
| Wage rate | 5.8 | 0.5 | 0.0 | 2.2 | 0.0 | 2.0 |
| Investment | 12.1 | 0.8 | 0.0 | 2.6 | 0.0 | 3.5 |
| Employment | 0.1 | 0.2 | 0.0 | 0.1 | 0.0 | 5.8 |
| Energy use | 3.1 | 1.3 | 0.0 | 0.2 | 0.0 | 0.1 |
| Domestic Absorption | 2.3 | 0.2 | 0.0 | 0.5 | 0.0 | 1.0 |
| Exports | 0.6 | 4.1 | 0.0 | 16.1 | 0.0 | 3.9 |
| Imports | 10.0 | 5.7 | 0.1 | 13.7 | 0.0 | 4.4 |
| **Energy intensive sector** | | | | | | |
| Output | 2.7 | 0.4 | 0.0 | 0.4 | 0.1 | 1.6 |
| Investment | 10.7 | 0.4 | 0.0 | 1.6 | 0.0 | 1.8 |
| Employment | 2.7 | 2.0 | 0.0 | 0.6 | 0.4 | 9.1 |
| Energy use | 6.7 | 0.0 | 0.0 | 0.2 | 0.0 | 0.1 |
| Capital stock | 2.8 | 0.2 | 0.0 | 0.6 | 0.0 | 2.1 |
| Capital utilisation | 8.5 | 0.1 | 0.0 | 0.5 | 0.0 | 0.2 |
| Price of goods | 2.4 | 0.3 | 0.0 | 2.0 | 0.0 | 1.2 |
| Domestic Absorption | 4.6 | 0.2 | 0.0 | 0.6 | 0.0 | 1.1 |
| Exports | 0.1 | 3.7 | 0.0 | 19.5 | 0.0 | 4.6 |
| Imports | 9.7 | 5.6 | 0.1 | 13.4 | 2.1 | 4.3 |
| **Energy extensive sector** | | | | | | |
| Output | 1.9 | 0.1 | 0.0 | 0.1 | 0.1 | 1.3 |
| Investment | 11.8 | 3.1 | 0.2 | 7.1 | 0.0 | 12.6 |
| Employment | 0.1 | 0.6 | 0.0 | 0.1 | 0.0 | 1.4 |
| Energy use | 0.5 | 13.5 | 0.0 | 0.1 | 0.0 | 0.0 |
| Capital stock | 5.0 | 0.4 | 0.0 | 1.4 | 0.0 | 5.2 |
| Capital utilisation | 0.9 | 0.3 | 0.0 | 0.2 | 0.0 | 0.0 |
| Price of goods | 2.6 | 2.0 | 0.0 | 1.0 | 0.0 | 0.9 |

The analysis of variance decomposition is one of the outstanding features of this model compared to other studies. This is because of the dominant role that the energy price shock plays as it explains the variance of real macroeconomic aggregates of the model. The energy price shock, with its high volatility rate, explains 56% of the GDP variance in the model. This shock also explains 45% of consumption variation, 24% of foreign assets variance, and 20% of asset prices (real exchange rate). The energy price shock much to domestic sectoral prices as it explains 18% of variations in the non-energy-intensive sector. One can see that energy prices can explain rise or fall in domestic prices. Also, 10% of domestic interest rates and 56% of wage rate and about 62% of total investment in the economy. It also explains 36% of total employment in the economy, 54% of total domestic absorption of the economy and with no surprise it explains 92% of total energy use in both sectors of the economy. The shock has effects on the variance of total exports with 18% contribution while it dominates as it explains 43% of total imports. Comparing with related literature, authors like Bjornland (2000)[51] as well as Jimenez-Rodriguez and Sanchez (2004)[52] finds the oil price shock explains 9% of the variability in the GDP in the UK.

One can also see how sector-specific productivity shocks play important roles in explaining variance of key variables in the model. The contribution of each sector productivity shock shows first two columns of the table. One can see the dominance in sectoral GDP as well as total GDP. The productivity shock in energy intensive sector has a higher dominance than its counterpart in energy intensive sector when explaining aggregates except for real wage rates where the latter shows marginally higher contribution. The sectoral productivity shocks play a dominant role in

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[51] Bjornland (2000) looked at variance decomposition for countries in the euro area that includes Germany, Norway and the United Kingdom.

[52] Jimenez-Rodriguez and Sanchez (2004) find empirical evidence for some OECD.

explaining the prices of goods in each sector where each contributes about 67% of its sector's price variability. The shocks explain 14% and 39% of economy's total consumption as well as 9% and 32% of domestic absorption, respectively. The shocks dominate in the variability of foreign bonds with 12% and 54% while it explains 10% and 48% of the assets prices. UK Exports is dominated by these shocks as it contributes 9% and 43% to its variations while the demand for foreign goods (imports) shows the significant contribution of 3% and 14%, respectively. Overall, one can say the sectoral productivity shocks have played a vital role in explaining the key variable's variations in the model.

The domestic demand shocks which comprises of the high persistent the government spending shock, high persistent sectoral investment-specific technology shocks and consumption preference shocks dominates the sectoral capital stocks while the shocks jointly shows significant contribution to the variations in interest rates, sectoral GDP, total investments, exports and imports. The sector-specific energy efficiency shocks show significant contribution in investments, capital stock, utilization rate, energy use, employment and wage rate. The contribution of latter three can explain the relationship this shock has between these variables in terms of employment and use of energy in firms. Given the relationship to world energy prices, these shocks show significant effects on world demand with over 15% combined contribution on imports and a significant 4% on total exports.

The world demand shocks of foreign interest rate, foreign exports prices and foreign demand show considerable contribution in explaining foreign bonds, foreign asset prices (real exchange rate), wage rate and total investment. This combination of shocks explains 12% of the domestic interest rate that is considered to be excellent.

These findings, especially in energy prices, are significant to this study as it opposes Hamilton (2009), and Killian & Vigfusson (2014). They argued that energy prices shocks be thought to be less effective in DSGE models.

## 3.9 Impulse response functions (IRFs)

The impulse responses of the model result from the 13 exogenous shocks, given each shock's persistence and volatility of the estimated model. The charts of the IRFs in the appendix outlines the model's 29 endogenous variables' responses given a change to each shock. Given that the sectoral productivity shocks and energy prices shock are treated as non-stationary, the shock responses will show a permanent effect on some of the variables instead of the conventional temporary effects shocks have on variables. The two sectors have the same production function that follows a similar linearized equation. The responses follow the same pattern despite having different values of shocks. The x-axis, in each IRF figure, refers to 'quarters' since the shocks are presumed to occur in the first quarter.

Figure 24 Productivity shock (Energy intensive sector)

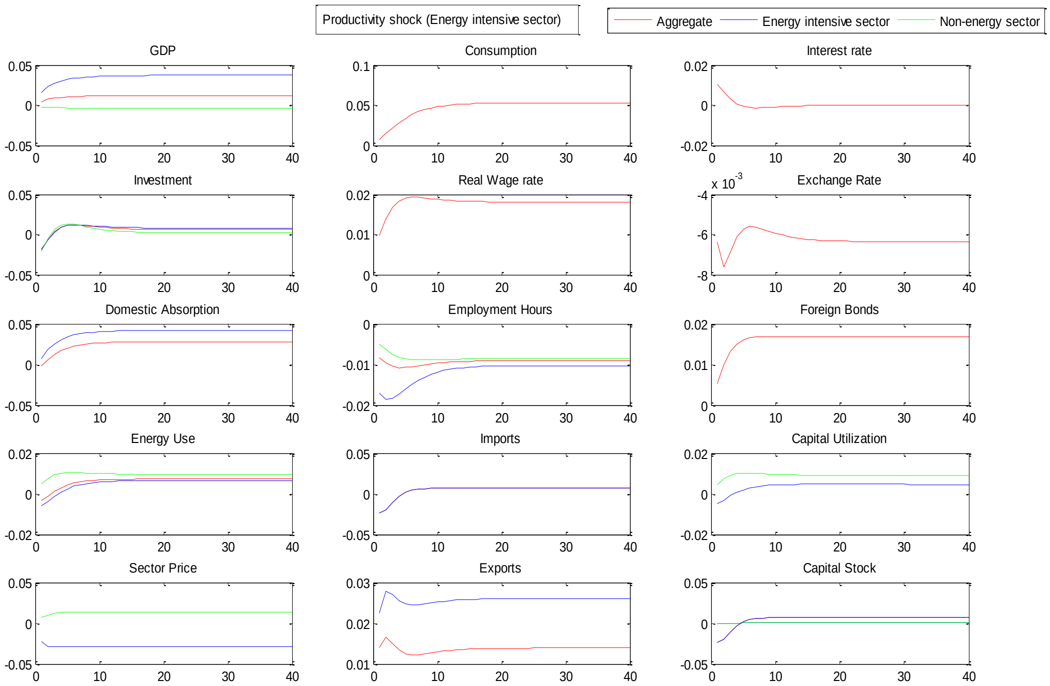


Figure 24 shows the effects of a 2.8% productivity shock (one standard deviation) of the energy intensive sector with an increase in sector output of almost 5% for a given labour and capital which leads to higher output supplied by about 5%. Due to the slow adjustment of prices, firms' demand for labour and capital utilisation falls which then reduces the marginal cost of production for firms. The rise in output increases welfare in the economy. The real wages will increase  which will have a 'knock-on' effect on employment. As households become richer (through the wealth effect of lower prices of output), they consume more and have more leisure than work since income increases and domestic absorption increases. The rise in productivity allows households to set their wages higher due to increasing productivity. This increase also comes at the expense of high energy use, short-term decreasing capital stock and its utilization rate. Monetary policy reacts to the output gap with regards to productivity by raising domestic interest rates in the short-run. This drives down investment in a similar fashion, but the effects of the latter will be permanent. The impact of high output will push down domestic prices since goods in the UK are produced at lower prices which makes the exchange rate appreciate relative to domestic prices. The latter, being foreign asset prices, causes foreign bond investment to rise. The demand for UK goods rises due to lower prices and the UK households' demand for foreign goods initially declines in the first few quarters then rises to have a permanent positive effect.

Figure 25 Productivity shock (Non-energy intensive sector)

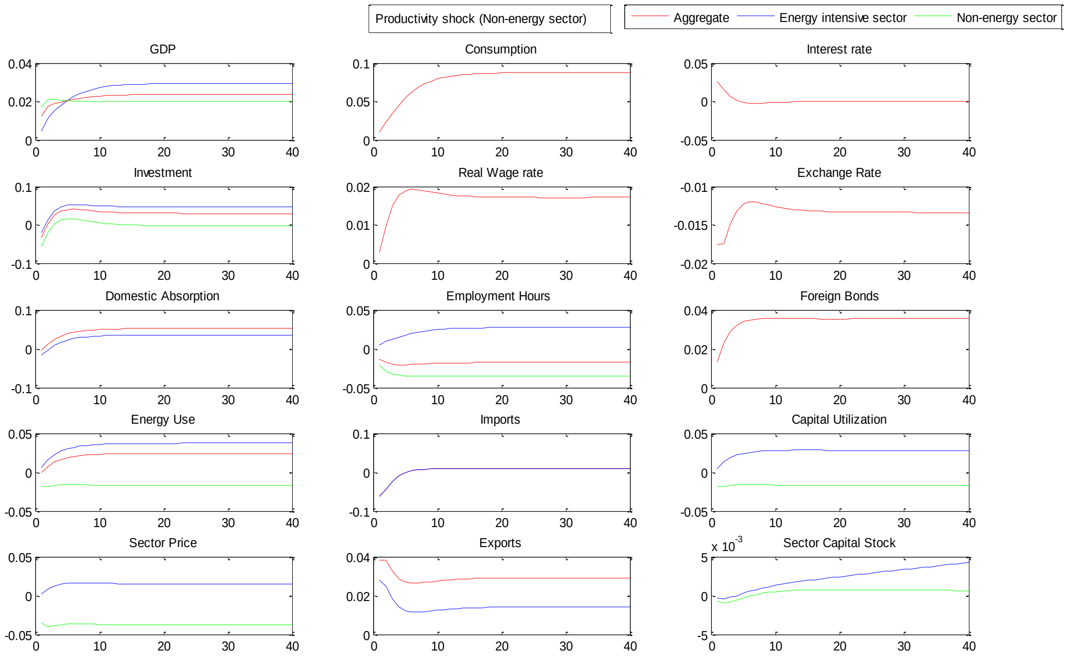
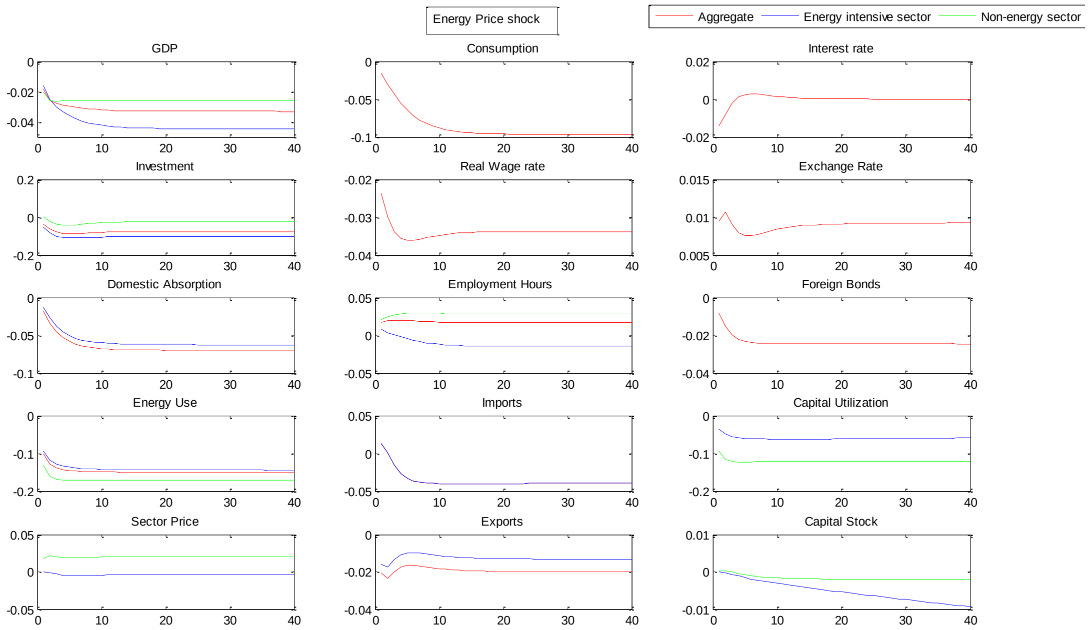


Figure 25 shows that the non-energy intensive sector responses are qualitatively similar to the energy-intensive sector. A positive productivity shock of 3.7% (one standard deviation) in the energy-extensive sector tends to increase output by about 2% as compared to a higher percentage in the other sector. The rise in output increases real wages that have a knock-on effect on employment since as households become richer, they consume more. Conversely, consumption rises due to an increase in welfare by about 10% as against 5% in the other sector. The demand for foreign bonds increases as the assets prices are cheaper than in the other sector. Lastly, the capital utilization rate in this sector remained positive in the short-run because more capital is used here.

Figure 26 Energy price shock

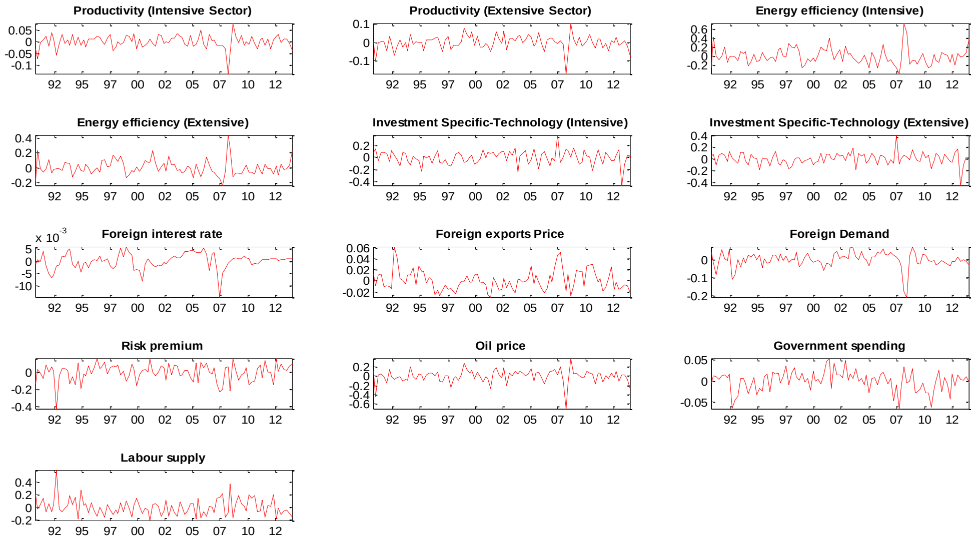


A positive energy price shock will likely send the economy into a recession as empirical evidence suggests. The effect of nonstationary energy price shock on declining output means that the terms of trade decline permanently when energy price changes as it is non-stationary and there is no other way to balance the current account than to reduce absorption- because there is no way to substitute away from energy by enough to eliminate the problem. Therefore, as shown in Figure 26, a positive world energy price shock of 14% (one standard deviation) will have permanent effects on the real macroeconomic aggregates. The impact of this shock is mostly in the energy intensive sector due to the energy intensity in that sector. There is a fall in output of over 4% while the aggregate output falls by about 3%. As output declines, the economy faces a welfare loss thereby causing the aggregate demand to fall as income is reduced. Firms' demand for inputs will decline as energy use, and capital utilization falls. As revenue declines, households choose to work more than to have leisure. Therefore, the employment is skewed to the non-energy sector as households reduce their real wages to gain employment. The firms' marginal cost will decline in this case. Monetary policy will react to this shock by decreasing interest rates to finance borrowing and investment in the economy as the exchange rate appreciates. The lack of income and high asset prices reduces the demand for foreign bonds.

## 3.10 Accounting for shocks during the crisis period

The time series of the 13 shocks in the model is shown in Figure 27. From the estimation results, one can see that the shocks to sector investment-specific technology, sector energy efficiency and labour supply have been highly volatile over the sample period, 1990Q2-2014Q4. Conversely, foreign export prices, foreign interest rates, sector productivity and government spending shocks have low volatility.

Figure 27 Shock's innovations



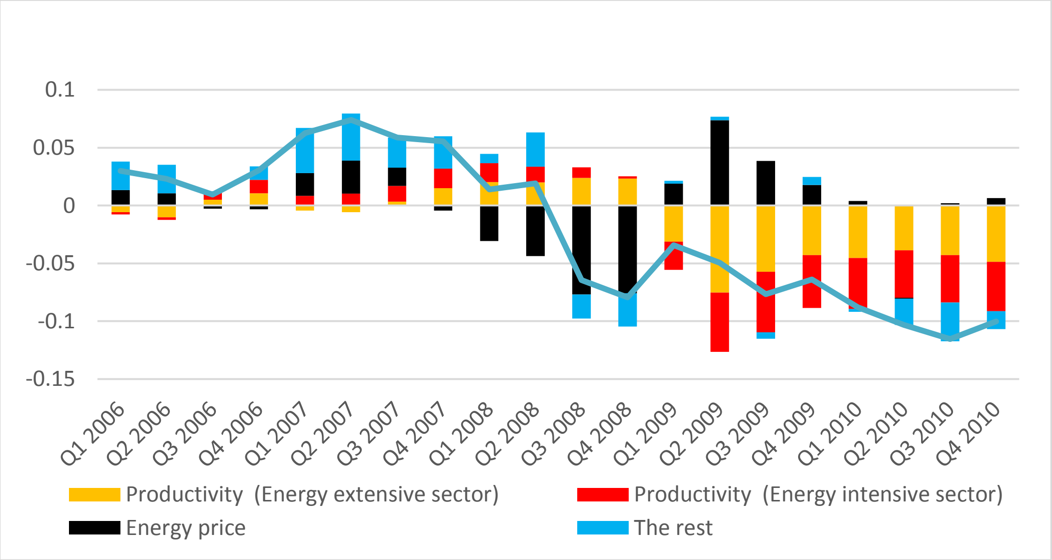
Looking at the recent past, the world economies have been affected by huge negative shocks to energy prices and this has affected world demand. These shocks reflect what happened to world trade during the 2008 and the 2009 calendar year. For example, looking at the consumer preference shock (risk premium shock), it is possible to observe the loss of consumer confidence due to credit rationing in that period. Government spending shock reflects the quantitative easing during the same period followed by the austerity measures of the 2010 political regime.

### 3.10.1 Shock Decomposition for the Crisis Period

The decomposition of shocks gives the timeline for the crisis period. Looking at the observed output data, it is clear that the energy intensive sector took a larger hit during the crisis period. I analyse the sector's output as well as the aggregate output (GDP), demand and real exchange rate. I report the contribution of three dominant shocks in each variable and combine the other ten shocks as 'the rest'.

The aggregate output variable (GDP) in Figure 28 shows energy prices to be a significant determinant of movements of aggregate output. High world energy prices during the crisis period, 2008 especially, significantly reduced aggregate output as can be seen with the energy price shock. Towards the end of the sample period, energy prices have dropped which caused the GDP to increase. Furthermore, the reduced productivity occurred due to low demand for inputs in the energy-extensive sector which helped in increasing similar demand in the energy intensive sector as firms substituted towards energy input. The rest, representing other shocks, shows how labour supply, consumption preference (risk premium) and world demand were significant shocks that contributed to pulling down UK output as observed in the effects of the 2008 financial crisis.

Figure 28 Shock decomposition of aggregate Output



The downward movement in the output of the energy intensive sector is driven by energy prices shock during the crisis period despite some effort for the productivity shocks to push up the output as shown in Figure 29. As the crisis deepened in 2008Q4, one can see how other shocks contributed to the loss of output as well as the energy-intensive sector productivity shock that dominates the sample period.

The low demand for inputs contributed to negative productivity. The rest of the shock also contributed including labour supply, exogenous government spending, foreign interest rates with regards to appreciating exchange rates and world demand. Towards the end of the period, one can see how lower energy prices helped push up the sectoral output.

Figure 29 Shock decomposition of energy intensive sector output

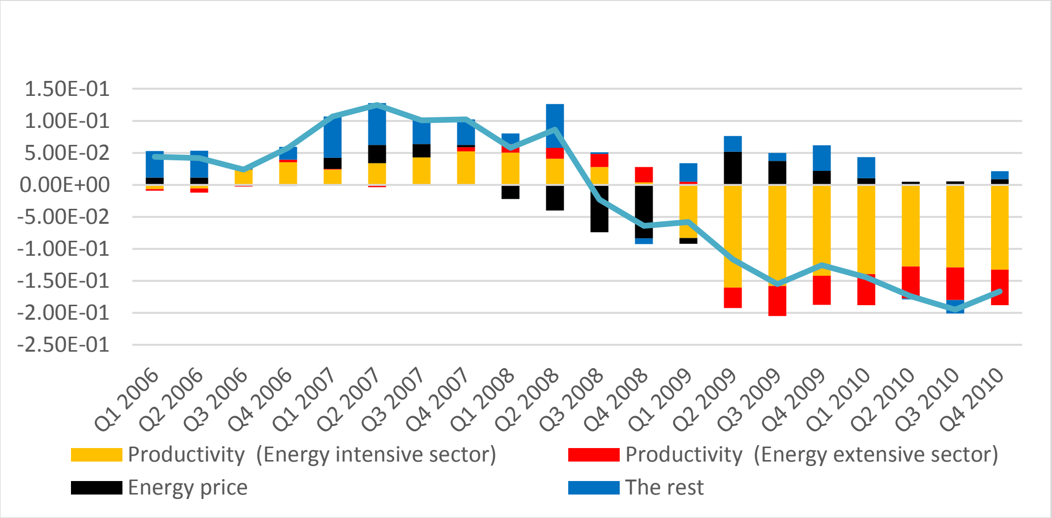


Figure 30 shows that the decomposition of movements in the output of the non-energy intensive sector can be strongly attributed to the energy prices shock. Energy intensive sector productivity shock has contributed slightly to the movement due to firm's input substitution from energy as world prices of energy were rising, therefore, reducing energy use. As the financial crisis kicked in, in 2008Q3, other shocks played important roles in pushing down output. These factors included: credit rationing to firms and households, corporation closures, labour supply shock and the depreciation of the exchange rate in the previous quarter. This made UK exports less attractive. However, as can be observed from the data, the decline of output in the non-energy-intensive sector did not last as long as the energy-intensive sector. One of the main reasons was due to lower world energy prices that increased output. The appreciation of exchange rate in the sector contributed to higher demand for UK services, since the data shows that the services sector contributed to over 70% of UK exports.

Figure 30 Shock decomposition of non-energy intensive sector output

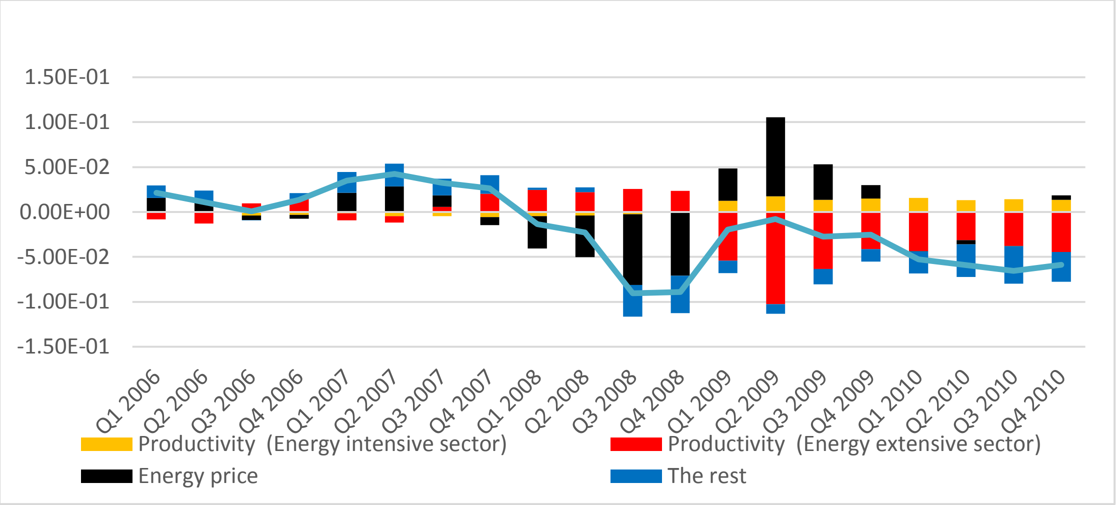
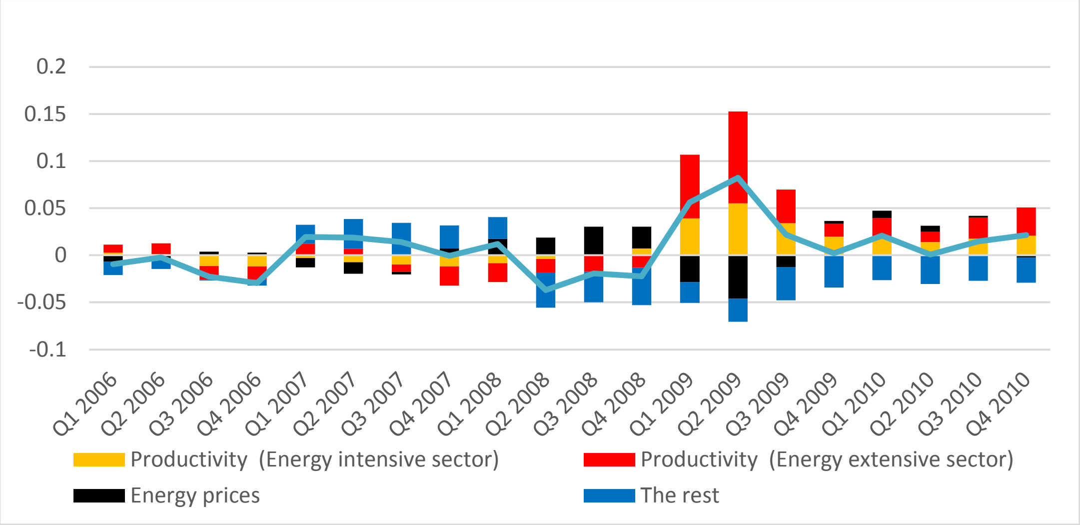


Figure 31 Shock decomposition of real exchange rate



The decomposition movement of the real exchange rate, as shown in Figure 31, one can observe that the energy price shock was pushing down the dollar substantially from between 2008:Q1-2008Q4. There was empirical evidence that oil prices and real exchange rates have an inverse relationship. There was a weaker US dollar when the US housing market went bust in December 2007, and then energy prices peaked at over $100 during that quarter and the real exchange rate depreciated. The non-energy intensive sector productivity shock contributed in moving the real exchange as corporate firms in the sector traded with foreign currency as well as the UK exports that are dominated by the output of this sector. The energy intensive sector played a significant role in the movement of the exchange rate as can be seen in 2009Q1 when world energy prices dropped. Other shocks that contributed to the real exchange rate movement in the crisis period include world demand as UK products were competitive in the world market since domestic prices fell in the UK.

Figure 32 Shock decomposition of aggregate consumption

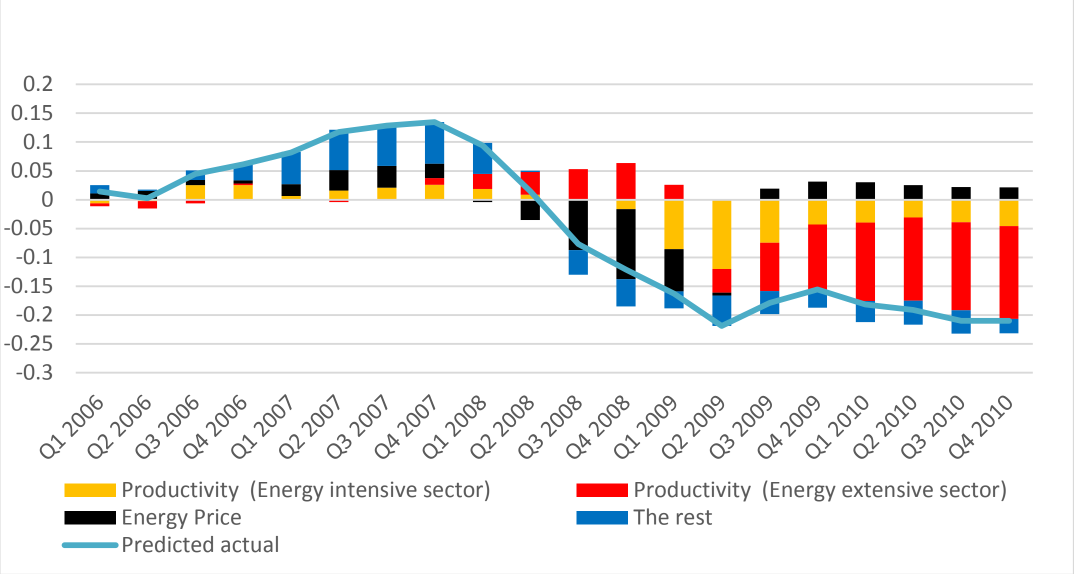


Figure 32 shows the decomposition in aggregate consumption and that the energy prices shock drives the decline. Non-stationary energy price shock causes the terms of trade decline permanently and there is no other way to balance the current account than to reduce absorption- because there is no way to substitute away from energy by enough to eliminate the problem. The effects that the shock has on output, as a result of lower income, pulls down aggregate consumption. From 2008:Q3, it can be seen that other shocks (the rest) contribute to driving it down further. Since demand falls steeply, this impact can be linked to low consumer confidence due to financial instability, low employment and the credit crunch. The productivity shock in the energy intensive sector pushed down the demand in 2009Q2. As a result, aggregate demand reached its lowest point, and the recession was having more of an impact. However, as world energy prices fell, confidence began to build although there was another mild recession that followed.

### 3.11 Summary

The idea of developing a two-sector open model with energy intensive firms has proven to be successful with this model. The use of three inputs of employment, capital service and energy in the production function of each sector shows a real business cycle scenario where the reality of such firms exists. The availability of stratified quarterly data has also helped in this case. This study contributes to two important features of the data, specifically, the cyclical behaviour of real aggregate output and asset prices with the response to energy prices and productivity shocks. I found that the effect of nonstationary energy price shock on declining output means that the terms of trade decline permanently when oil price falls as it is non-stationary and there is no other way to balance the current account than to reduce absorption-because there is no way to substitute away from energy by enough to eliminate the problem. The study also demonstrates linear models, in the context of a standard dynamic stochastic general equilibrium model, can show the real effects of energy prices. The results are due to the concept of treating an observed non-stationary shock as a non-stationary shock. The treatment of the energy price shock, as well as unobserved productivity shocks in the energy intensive sector and non-energy intensive sector provided a pathway for this study. This study contrasts with an earlier study with an assumed oil price shock in a linear approach that had no impacts on real economic activities.

The estimation of the model parameters is vital for achieving an adequate general picture of the economy's dynamics in reaction to volatile world energy prices. Meenagh, Minford and Wickens (2012) noted that filtering data may distort a DSGE model's dynamic properties in unknown ways. This could be due to the way that the HP-filter alters the lag dynamic structure or generating cycles where none exists. The forward-looking properties of the model is also transformed due to the filter being two-sided. As a result, there could be a serious defect in the DSGE model estimation. I applied the model quantitatively using an efficient, practical tool, indirect inference testing, to estimate a DSGE model using the UK stationary data from 1990: Q1 to 2014: Q4. At first approach I evaluated the performance of the calibrated model which in matching the UK data and found it to be poor, that is the model fails to match the data and its variances using this set of parameters. In matching the data, the shock processes play a key role and energy shocks are estimated to have a high persistence. The model application shows the evaluation effects of different shocks on real output, real exchange rate and real aggregate demand from the VAR impulse response functions. The model also proved to a fit assessment by evaluation with VAR impulse response functions. By decomposition, the variability in the real macroeconomic aggregates shows that the fall in output during the financial crisis period between 2008: Q2 to 2009: Q4 was driven by the energy price shock and sectoral productivity shocks.

Avenues for further research would be to add a monetary policy equation into the model to see the effects of contemporaneous feedback. This is because the standard assumptions in the empirical study assume that oil (energy) prices are predetermined with respect to real output in the economy and feedback is not generated from the real domestic aggregate variables. Given the current economic volatility, it will also be important to see how DSGE models will behave on monthly data.

## Appendix 2.1 Agent's Maximisation problems with Consolidated Budget Constraint

### Household

Household's maximise their lifetime utility value given the consolidated budget constraint:

$$\max_{\left\{C_t,H_t,I_t,I_t^e,I_t^n,,B_{t+1}^f,Z_t^e,Z_t^n,K_t^e,K_t^n\right\}_{t=0}^{\infty}} \left[\sum_{t=0}^{\infty} \beta^t \tau_t \left(\frac{(C_t-\psi_h C_{t-1})^{1-\epsilon}}{1-\epsilon} - \xi_{w,t}\frac{H_t^{1+\omega}}{1+\omega}\right)\right] \tag{196}$$

s.t. (197)

$$B_t + \frac{B_t^f}{S_t} + \frac{\chi_{bf}}{2}\left[\frac{B_t^f}{S_t\left(1+r_t^f\right)} - nfa^{ss}\right]^2 + C_t + T_t + \frac{K_t^e}{\xi_t^e} + \frac{K_t^n}{\xi_t^n}$$

$$+ 0.5\chi_e\left(\frac{K_t^e-K_{t-1}^e}{K_{t-1}^e}\right)^2\frac{K_{t-1}^e}{\xi_t^e} + 0.5\chi_n\left(\frac{K_t^n-K_{t-1}^n}{K_{t-1}^n}\right)^2\frac{K_{t-1}^n}{\xi_t^n}$$

$$= R_{t-1}B_{t-1} - R_{t-1}^f\frac{B_{t-1}^f}{S_t} + W_tH_t + \Pi_t +$$

$$\left(R_t^eZ_t^e + \frac{1-\delta_{e1}-\delta_e\mu_e^{-1}(Z_t^e)^{\mu_e}}{\xi_t^e}\right)K_{t-1}^e + \left(R_t^nZ_t^n + \frac{1-\delta_{n1}-\delta_n\mu_n^{-1}(Z_t^n)^{\mu_n}}{\xi_t^n}\right)K_{t-1}^n$$

### Firm

The Firm produces final good $Y_t$ derived from combining $Y_t^e$ and $Y_t^n$. The production of the latter requires labour, capital services and energy use. Firms' production technology is defined by a nested CES function, Cobb-Douglas production function with constant returns to scale. The maximisation problem of the firms with respect to consolidated budget constraints above is prescribed as:

$$\max_{(H_t^e, H_t^n, Z_t^e K_{t-1}^e, Z_t^n K_{t-1}^n, E_t^e, E_t^n)_{t=0}^{\infty}} \Pi_t = P_t^e Y_t^e + P_t^n Y_t^n - \big(W_t H_t^e + W_t H_t^n + R_t^e Z_t^e K_{t-1}^e + R_t^n Z_t^n K_{t-1}^n + P_{O,t} E_t^e + P_{E,t} E_t^n\big) \tag{198}$$

s.t
$$Y_t^e = A_t^e (H_t^e)^{1-\alpha_e} \Big(\theta_e (Z_t^e K_{t-1}^e)^{-v_e} + \big((1-\theta_e)(O_t^e E_t^e)^{-v_e}\big)\Big)^{-\frac{\alpha_e}{v_e}} \tag{199}$$

$$Y_t^n = A_t^n (H_t^n)^{1-\alpha_n} \Big(\theta_n (Z_t^n K_{t-1}^n)^{-v_n} + \big((1-\theta_n)(O_t^n E_t^n)^{-v_n}\big)\Big)^{-\frac{\alpha_n}{v_n}} \tag{200}$$

The Government spending is given by:

$$G_t + B_t = T_t + \mathrm{E}_t R_{t+1} + B_{t+1} \tag{201}$$

**Foreign sector: Trade with rest of the world**

The domestic agent's problem of trade with the foreign economy given the consolidated budget constraint will be:

$$\max_{\{D_t^e, M_t, M_t^e\}_{t=0}^{\infty}} \{D_t - P_t^e D_t^e + P_t^n D_t^n - P_{M,t} M_t - P_{M,t}^e M_t^e - P_{M,t}^n M_t^n\} \tag{202}$$

s.t.
$$D_t = \left(\sigma^{\frac{1}{\varsigma}} (D_t^e)^{\frac{\varsigma-1}{\varsigma}} + (1-\sigma)^{\frac{1}{\varsigma}} (D_t^n)^{\frac{\varsigma-1}{\varsigma}}\right)^{\frac{\varsigma}{\varsigma-1}} \tag{203}$$

$$D_t = \left(\kappa^{\frac{1}{\eta}} (D_t^e)^{\frac{\eta-1}{\eta}} + (1-\kappa)^{\frac{1}{\eta}} (M_t)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

$$M_t = \left(\chi^{\frac{1}{\phi}} (M_t^e)^{\frac{\phi-1}{\phi}} + (1-\chi)^{\frac{1}{\phi}} (M_t^n)^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}$$

**Rest of the World economy**

The domestic agent's problem of the foreign demand given the consolidated budget constraint will be

$$\max_{\{X_t, X_t^e\}_{t=0}^{\infty}} \{Y_t^f - P_t^f D_t^f - S_t X_t + -P_t^e X_t^e - P_t^n X_t^n\} \tag{204}$$

s.t. $$Y_t^f = \left(\kappa^{\frac{1}{\eta_w}}(D_t^e)^{\frac{\eta_w-1}{\eta_w}} + (1-\kappa)^{\frac{1}{\eta_w}}(D_t^n)^{\frac{\eta_w-1}{\eta_w}}\right)^{\frac{\eta_w}{\eta_w-1}} \tag{205}$$

$$X_t = \left(\chi^{\frac{1}{\phi_w}}(X_t^e)^{\frac{\phi_w-1}{\phi_w}} + (1-\chi)^{\frac{1}{\phi_w}}(X_t^n)^{\frac{\phi_w-1}{\phi_w}}\right)^{\frac{\phi_w}{\phi_w-1}}$$

**Market clearing**

The aggregate for employment, energy use, domestic absorption in the energy intensive sector is:

$$H_t = H_t^e + H_t^n \tag{206}$$

$$E_t = E_t^e + E_t^n \tag{207}$$

$$Y_t^e = D_t^e + X_t^e - M_t^e \tag{208}$$

The final production of output in the economy satisfies:

$$Y_t = C_t + I_t + G_t + X_t - M_t - E_t \tag{209}$$

and finally substituting into the market clearing condition, the government's budget constraint into the households budget constraint gives an aggregate resource constraint of how the economy's net foreign assets evolve:

$$\frac{B_t^f}{s_t} - \left(R_{t-1}^f\right)\frac{B_{t-1}^f}{\Pi_t^f s_t} = S_t X_t - M_t - P_{O,t}E_t - \frac{\chi^{bf}}{2}\left[\frac{BF_t}{s_t} - nfa^{ss}\right]^2 \tag{210}$$

## Appendix 2.2 Account for model variables

In all, this model takes a set of 29 log-linearized, see appendix, endogenous variables shown in table. The model of linear equations is driven by 13 exogenous shocks:

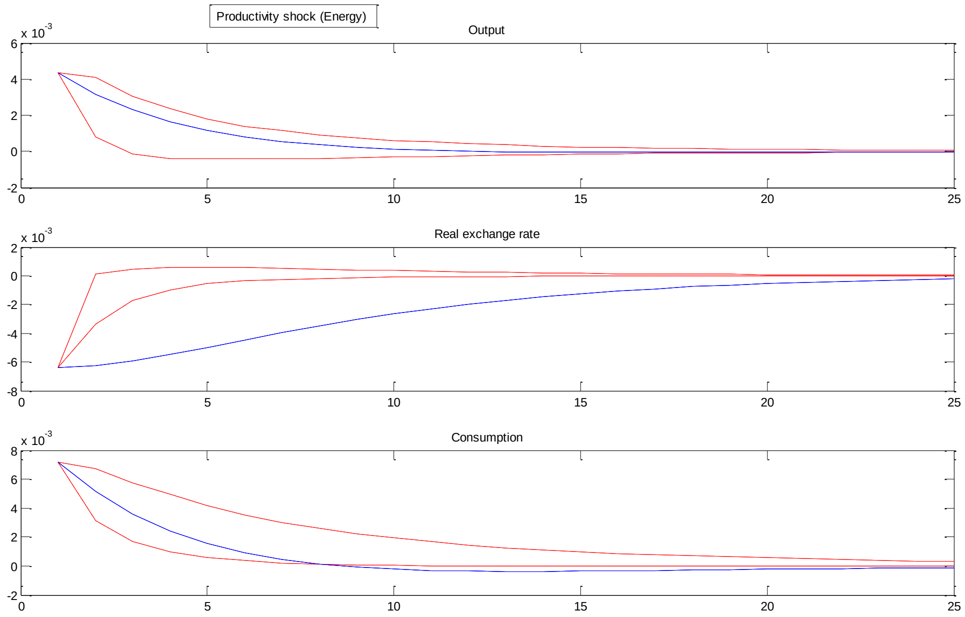
Table 18 List of endogenous variables

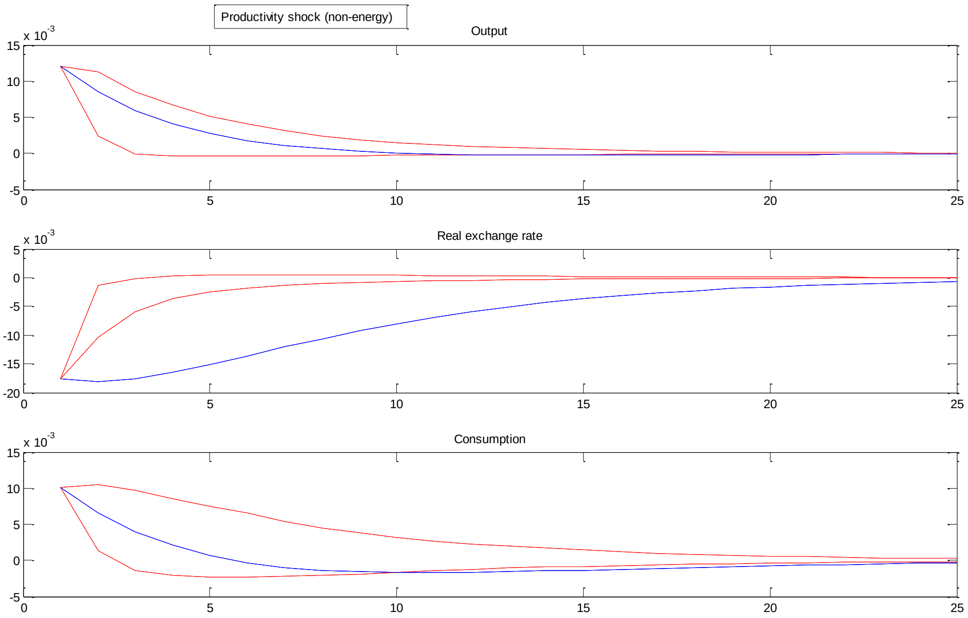
| **Endogenous variable** | | |
|---|---|---|
| **Aggregate** | **Energy extensive sector** | **Energy intensive sector** |
| Output $y_t$ | Output $y_t^e$ | Output $y_t^n$ |
| Consumption $c_t$ | Investment $i_t^e$ | Investment $i_t^n$ |
| Foreign Bonds | Employment $h_t^e$ | Employment $h_t^n$ |
| Interest rate $r_t$ | Energy use $e_t^e$ | Energy use $e_t^n$ |
| Exchange rate $s_t$ | Capital stock $k_t^e$ | Capital stock $k_t^n$ |
| Wage rate $w_t$ | Capital utilisation $z_t^e$ | Capital utilisation $z_t^n$ |
| Investment $i_t$ | Price of goods $p_t^e$ | Price of goods $p_t^n$ |
| Total Hours $h_t$ | Domestic Absorption $d_t^e$ | |
| Total Energy use $e_t$ | Exports $x_t^e$ | |
| Domestic Absorption $d_t$ | Imports $m_t^e$ | |
| Total Exports $x_t$ | | |
| Total Imports $m_t$ | | |

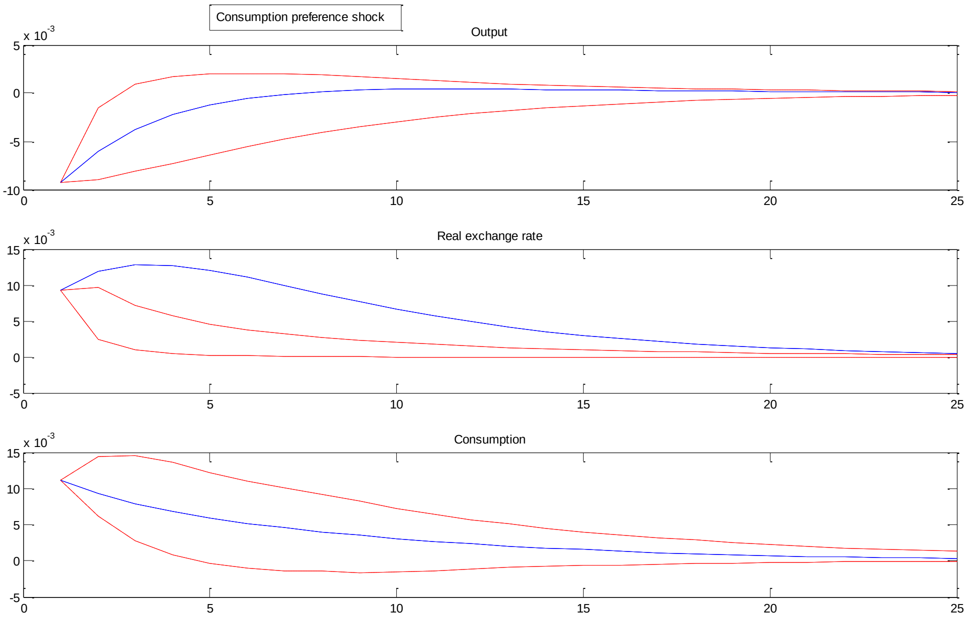
Table 19 List of exogenous shocks

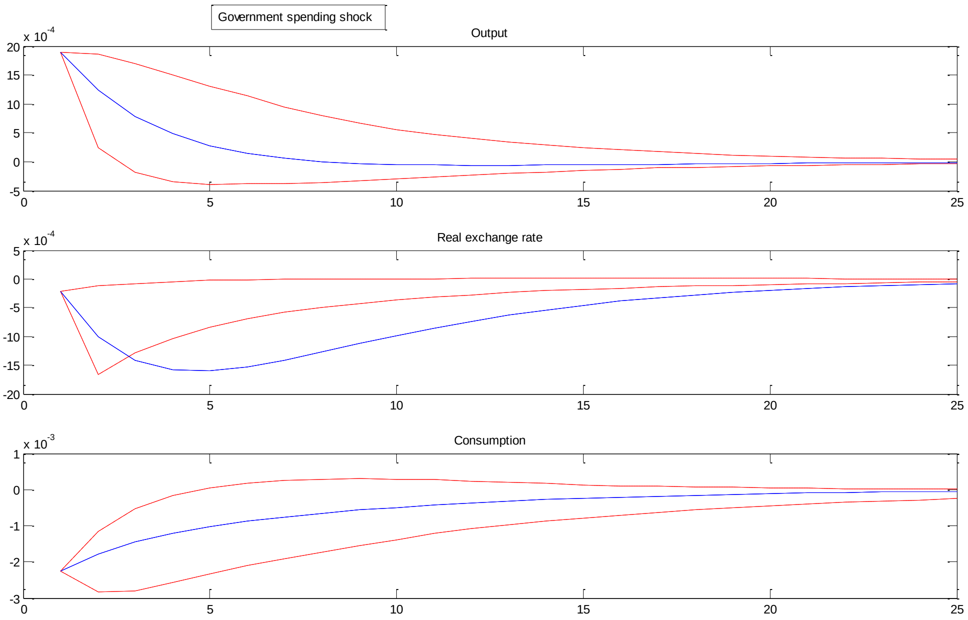
| **Shock** | |
|---|---|
| Productivity (energy-intensive sector) | $\varepsilon_{ae,t}$ |
| Productivity shock (energy-extensive sector) | $\varepsilon_{an,t}$ |
| Consumption preference | $\varepsilon_{b,t}$ |
| Government spending | $\varepsilon_{g,t}$ |
| Investment Specific-Technology shock (energy intensive sector) | $\varepsilon_{inv,t}^{e}$ |
| Investment Specific-Technology shock (energy extensive sector) | $\varepsilon_{inv,t}^{n}$ |
| Energy efficiency (energy intensive sector) | $\varepsilon_{o,t}^{e}$ |
| Energy efficiency shock (energy-extensive sector) | $\varepsilon_{o,t}^{n}$ |
| World exports price | $\varepsilon_{pm,t}$ |
| Energy price | $\varepsilon_{po,t}$ |
| World interest rate | $\varepsilon_{rf,t}$ |
| Labour supply | $\varepsilon_{w,t}$ |
| World demand | $\varepsilon_{yf,t}$ |

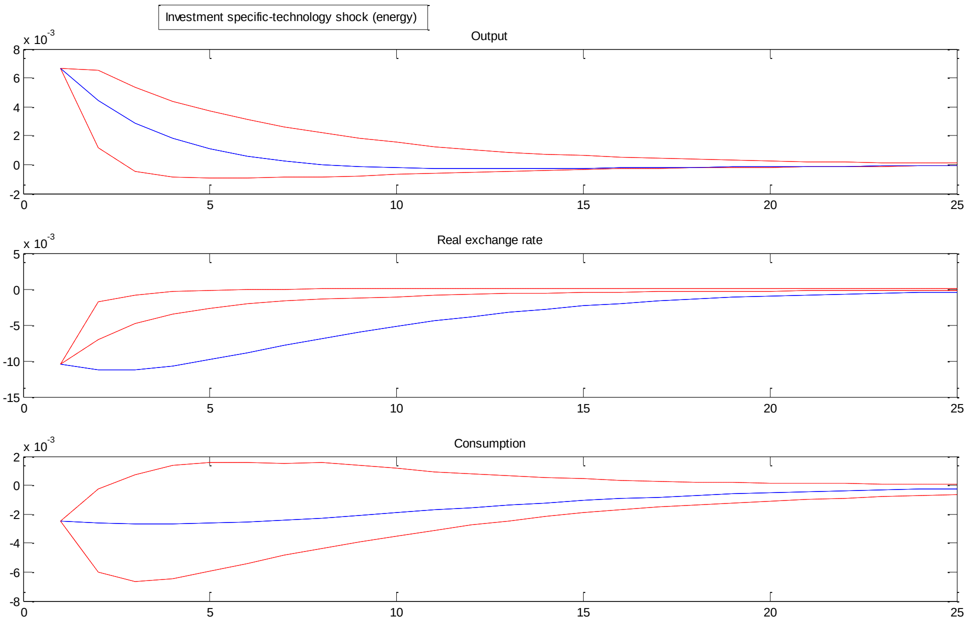
## Appendix 2.3 VAR-Impulse response functions

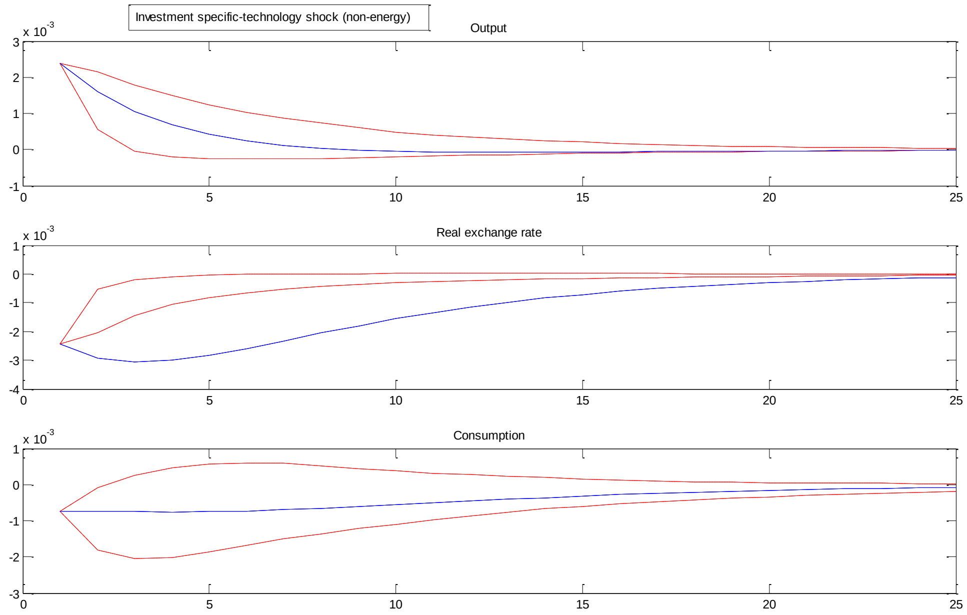


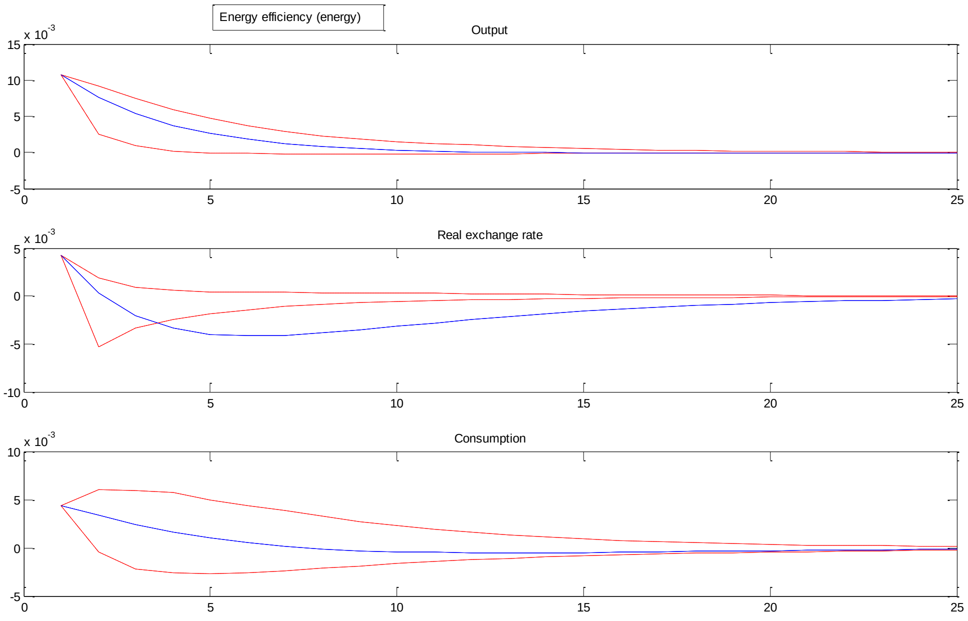


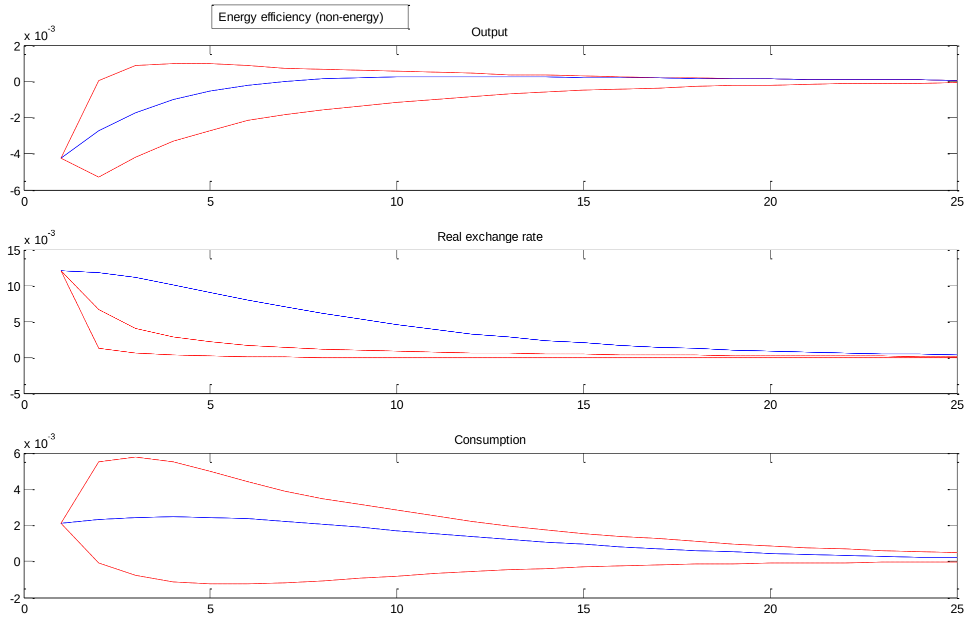

Consumption preference shock
Output
x 10^-3
5
0
-5
-10
Real exchange rate
x 10^-3
15
10
5
0
-5
Consumption
x 10^-3
15
10
5
0
-5
0
5
10
15
20
25

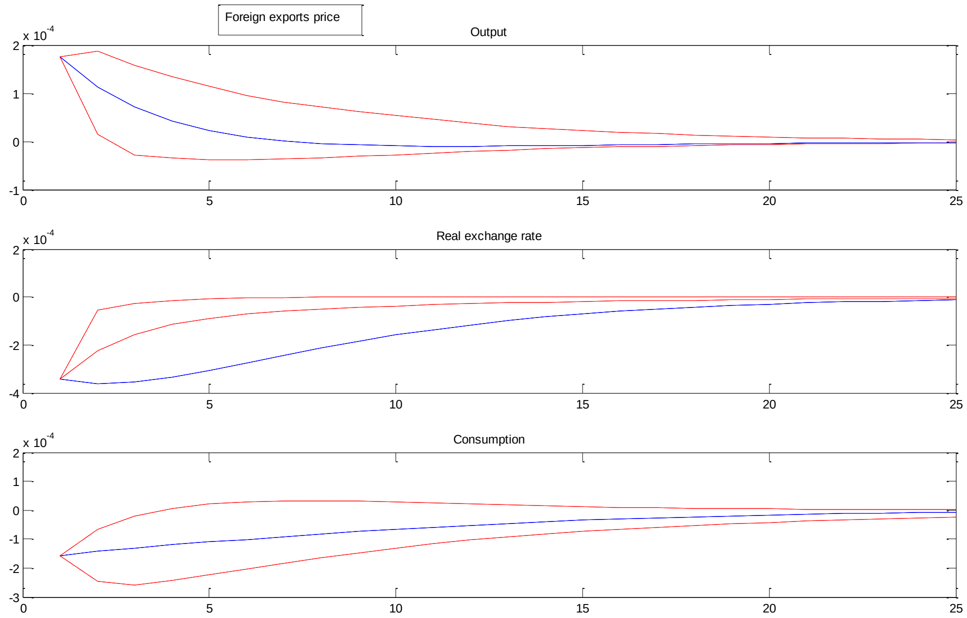

Government spending shock
Output
x 10^-4
20
15
10
5
0
-5
Real exchange rate
x 10^-4
5
0
-5
-10
-15
-20
Consumption
x 10^-3
1
0
-1
-2
-3
0
5
10
15
20
25

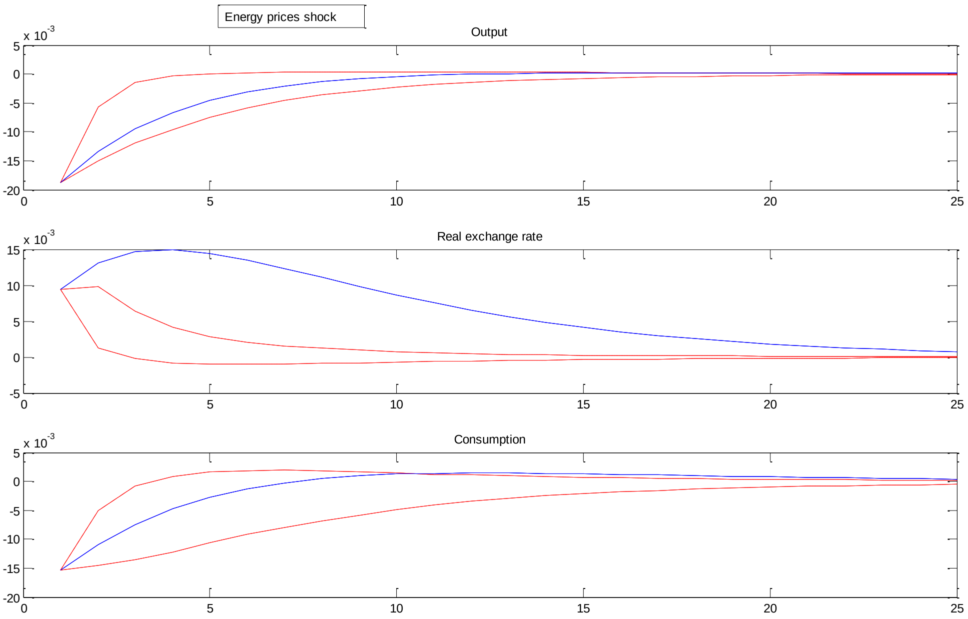

Investment specific-technology shock (energy)
Output
x 10^-3
8
6
4
2
0
-2
Real exchange rate
x 10^-3
5
0
-5
-10
-15
Consumption
x 10^-3
2
0
-2
-4
-6
-8
0
5
10
15
20
25

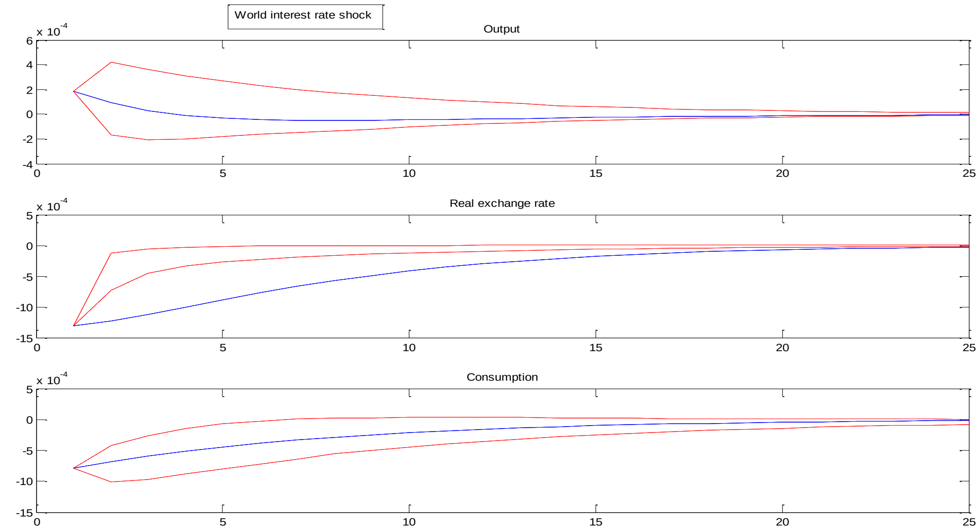

Investment specific-technology shock (non-energy)
Output
x 10^-3
3
2
1
0
-1
Real exchange rate
x 10^-3
1
0
-1
-2
-3
-4
Consumption
x 10^-3
1
0
-1
-2
-3
0
5
10
15
20
25

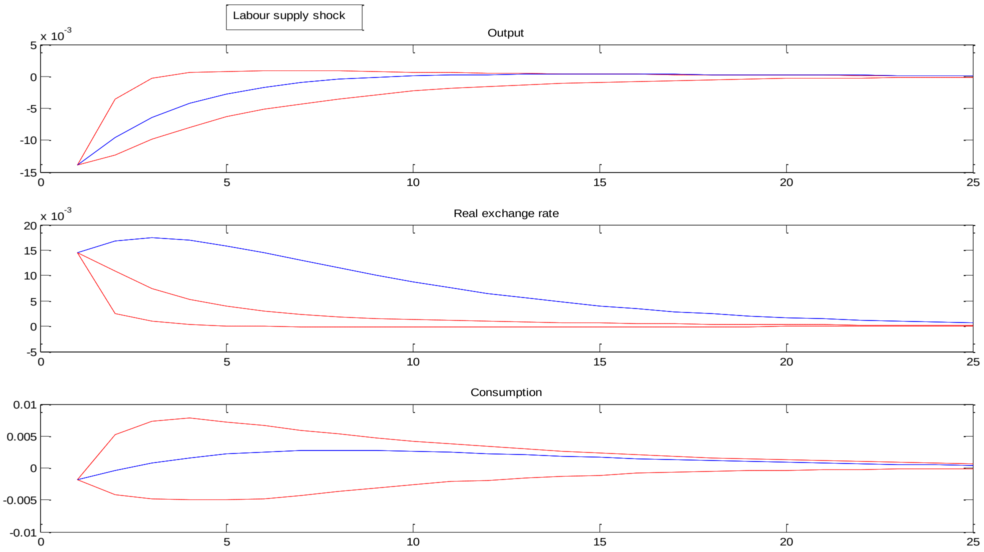

Energy efficiency (energy)
Output
x 10^-3
15
10
5
0
-5
0
5
10
15
20
25
Real exchange rate
x 10^-3
5
0
-5
-10
0
5
10
15
20
25
Consumption
x 10^-3
10
5
0
-5
0
5
10
15
20
25

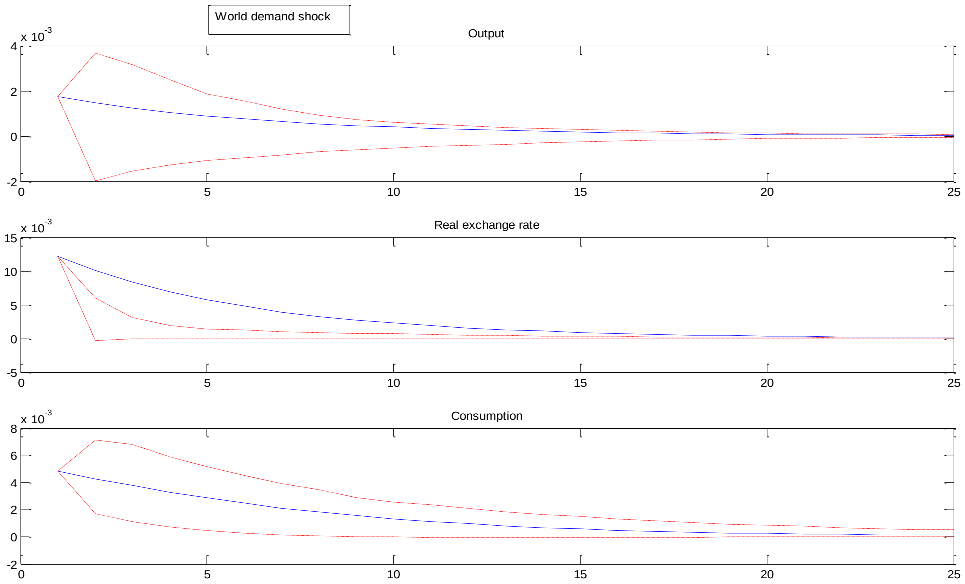

Energy efficiency (non-energy)
Output
x 10^-3
2
0
-2
-4
-6
0
5
10
15
20
25
Real exchange rate
x 10^-3
15
10
5
0
-5
0
5
10
15
20
25
Consumption
x 10^-3
6
4
2
0
-2
0
5
10
15
20
25


Foreign exports price
Output
x 10$^{-4}$
2
1
0
-1
0
5
10
15
20
25
Real exchange rate
x 10$^{-4}$
2
0
-2
-4
0
5
10
15
20
25
Consumption
x 10$^{-4}$
2
1
0
-1
-2
-3
0
5
10
15
20
25


Energy prices shock
Output
x 10$^{-3}$
5
0
-5
-10
-15
-20
0
5
10
15
20
25
Real exchange rate
x 10$^{-3}$
15
10
5
0
-5
0
5
10
15
20
25
Consumption
x 10$^{-3}$
5
0
-5
-10
-15
-20
0
5
10
15
20
25


World interest rate shock
Output
x 10^-4
6
4
2
0
-2
-4
0
5
10
15
20
25
Real exchange rate
x 10^-4
5
0
-5
-10
-15
0
5
10
15
20
25
Consumption
x 10^-4
5
0
-5
-10
-15
0
5
10
15
20
25


Labour supply shock
Output
x 10^-3
5
0
-5
-10
-15
0
5
10
15
20
25
Real exchange rate
x 10^-3
20
15
10
5
0
-5
0
5
10
15
20
25
Consumption
0.01
0.005
0
-0.005
-0.01
0
5
10
15
20
25


World demand shock
Output
x 10^-3
4
2
0
-2
0
5
10
15
20
25
Real exchange rate
x 10^-3
15
10
5
0
-5
0
5
10
15
20
25
Consumption
x 10^-3
8
6
4
2
0
-2
0
5
10
15
20
25

## Appendix 2.4 Model's Impulse response functions (continued)

Figure 33 Consumption preference shock

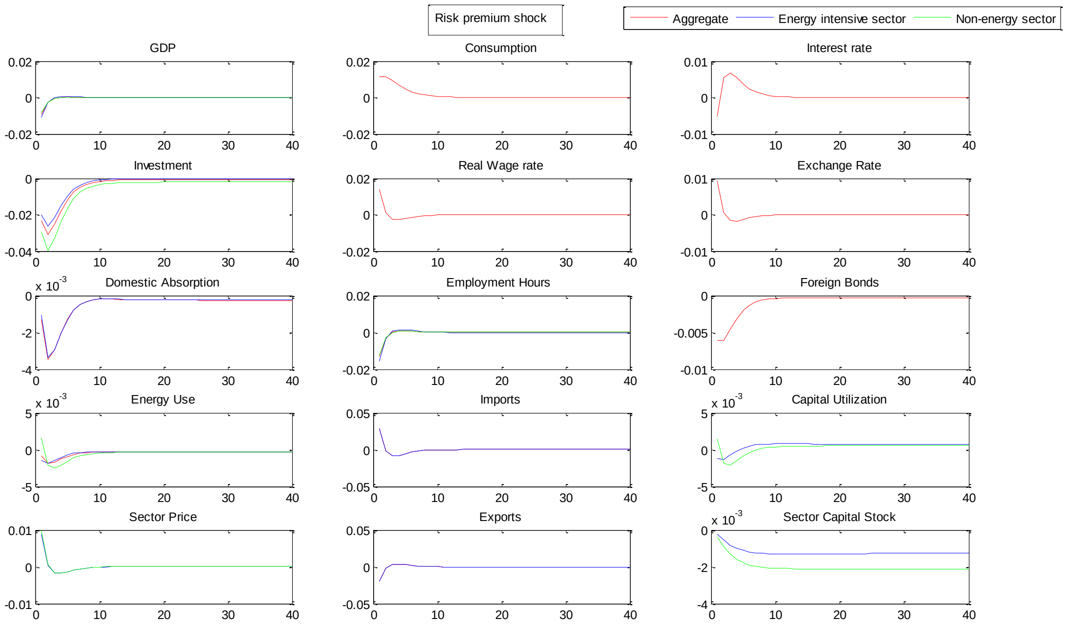
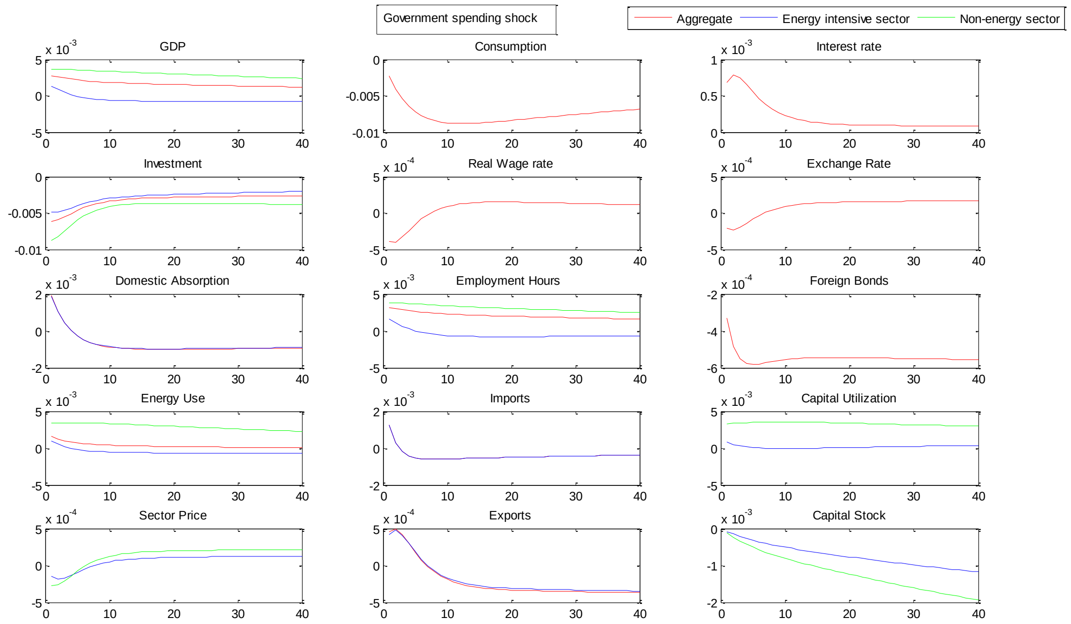


Figure 33 shows the effects of a 10% positive consumption shock (one standard deviation) increases the utility UK households will derive from each unit of goods consumed. Consumption will increase, and this raises goods prices and decreases employment as household choose more leisure than work that decreases output. The rise in wage rate reflects households' willingness to work less and firms strategy to attract more labour to meet rising demand. As output decreases, drop in domestic absorption is inevitable, hence lower exports. To meet the increasing demand, the output must rise. Therefore, firms will increase capital utilization and energy use in the energy-extensive sector. The increase in demand for foreign goods reflects on the high domestic prices and aggregate demand in the UK. The lower demand for foreign bonds is as a result of households' choice of consumption than investment.

Figure 34 Government spending shock



The effects of a positive government spending shock (one standard deviation) kicks in with a decline in consumption that reflects the 'crowd-out' impact in the economy (that is, an increase in government spending is funded by lump-sum taxes on households). Figure 34 shows a positive exogenous government spending shock creates a welfare loss in the economy. The response of output and welfare multiplier are in opposite direction. The net effects on aggregate demand will be positive as output increases. The monetary policy responds to increasing output by raising the interest rate. This will make firms raise their capital utilization as well as their employment as they face the higher domestic demand, as can be seen by the rise in domestic absorption. Capital rental rates will rise, domestic interest rates, but this shock will have little effects on wages as fall in consumption makes household willing to work more hours thereby offsetting the increase in wages. Firms will initially rise exports as consumption decreases with lower prices and then gradually decreases exports as prices will increase as aggregate demand increases. Demand for imports initially rises due to falling imports prices and this is why imports begin to fall. There is pressure on domestic prices as it falls in line with real exchange rates.

Figure 35 Labour supply shock

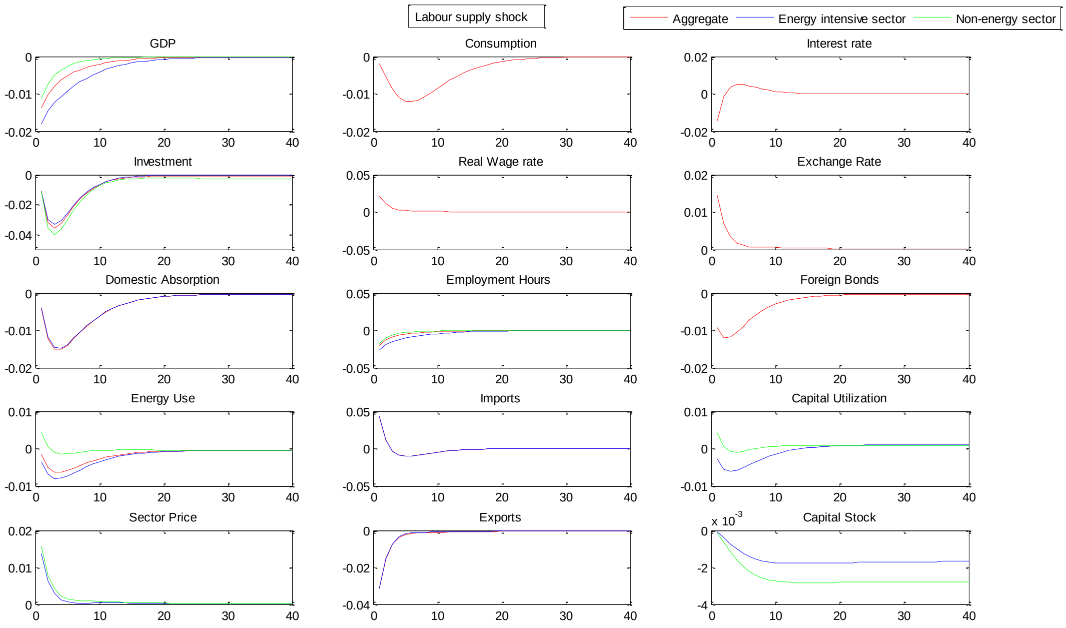


Figure 35 shows a positive labour supply shock leads to the willingness of households to supply more labour to the firms at a given wage rate. Given that, employment will increase, and real wages will fall as households gain disutility from raising labour supply. As output declines, as a result of lower demand for inputs, such as energy use and capital utilization and employment, due to higher production costs. Exports decline as a consequence therefore creating negative net exports as imports increase with the lower domestic output. The monetary policy will respond by cutting domestic interest rates to increase the falling investment.

Figure 36 Investment specific-technology shock (Energy intensive sector)

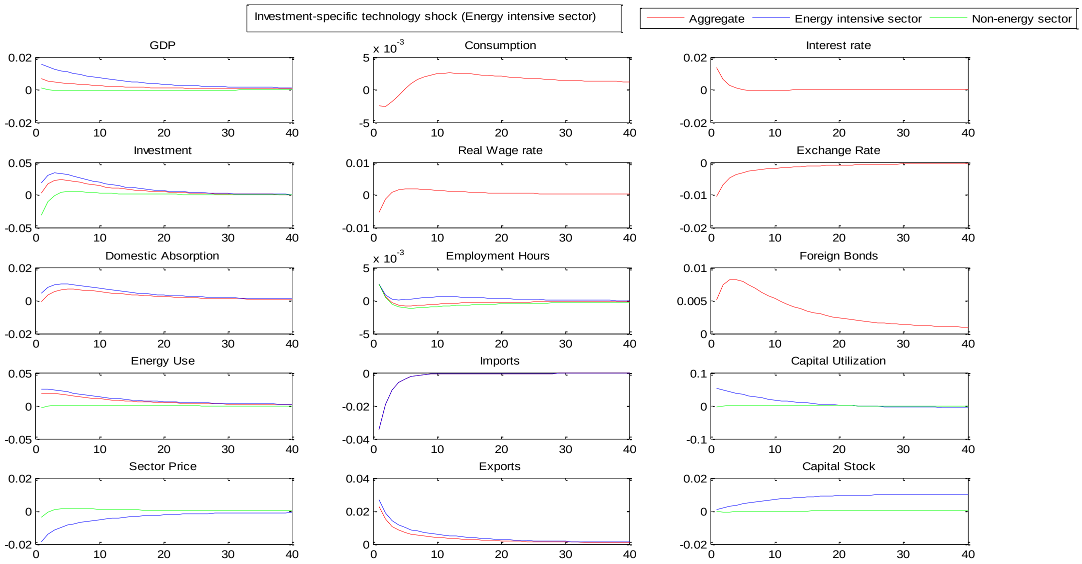
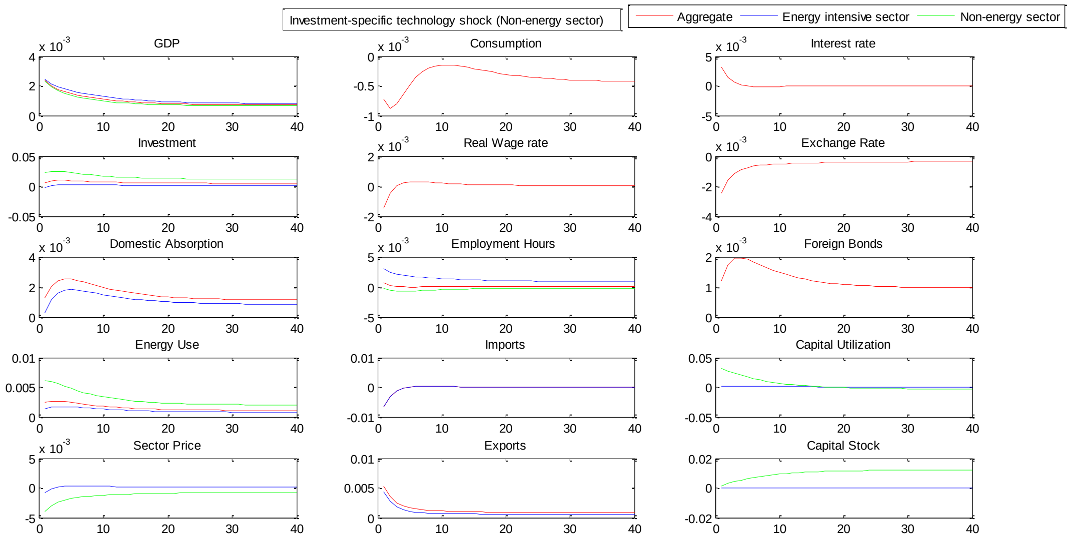


Figure 37 Investment specific-technology shock (Non-energy intensive sector)



A positive investment-specific technology shock (one standard deviation) in the energy intensive sector drives up investment as shown in figure 36. There will be a rise in output as firms raise employment and capital utilization to increase capital. Monetary policy responds by raising the interest rate to bring investment back to its steady-state. The analysis is qualitatively similar to the energy intensive sector shock, figure 37 shows a positive investment-specific technology shock in the energy-extensive sector brings similar response as it pushes up investment. This leads to an increase in output and employment as capital utilization will rise to raise the capital stock in the firms while monetary policy responds by raising the interest rate to bring investment back to its steady-state.

Figure 38 Energy efficiency shock (Energy intensive sector)

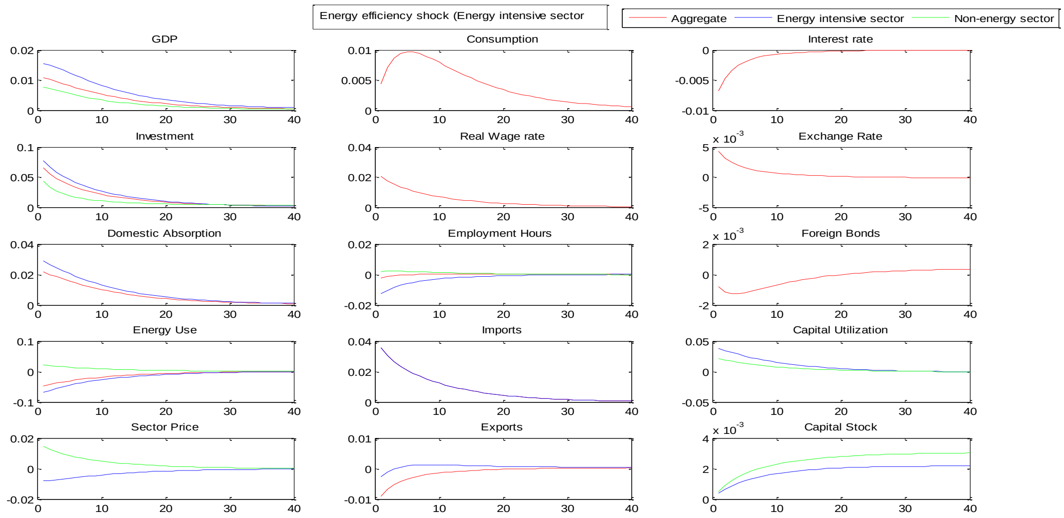


Figure 39 Energy efficiency shock (Non-energy intensive sector)

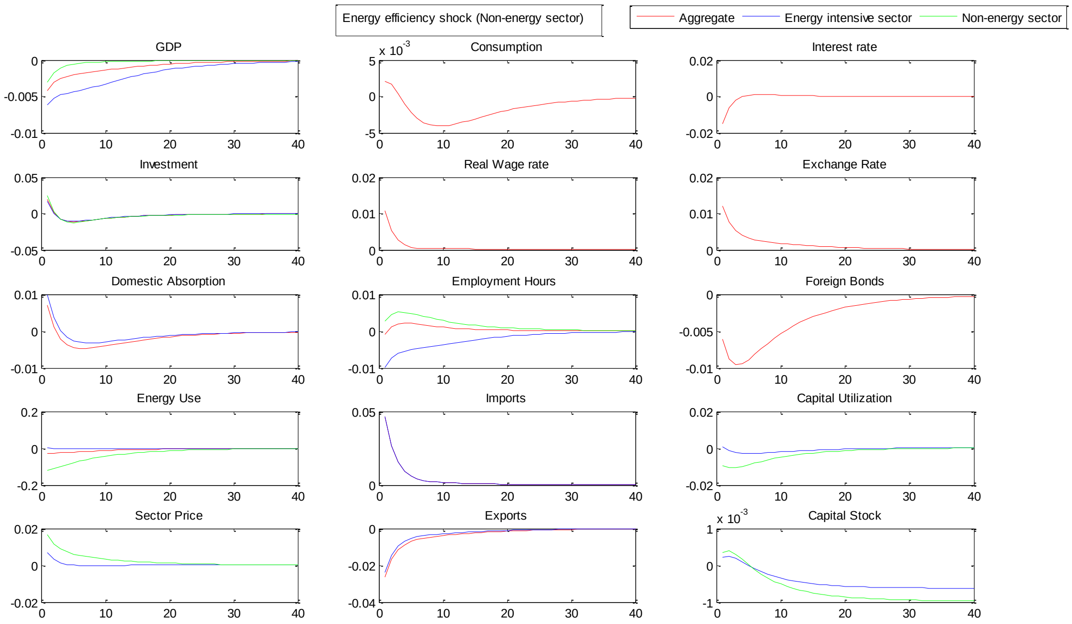


Figure 38 shows a positive energy efficiency shock (one standard deviation) in the energy intensive sector will reduce labour demand and increase output with the efficient use of energy. Thus, a lower marginal cost of production, hence the sectoral price of energy intensive goods are lower. The depreciating home price will appreciate exchange rate that will make foreign bonds less attractive. Energy efficiency shock will reduce labour hours as it Capital utilization will be increased to increase capital as investment increases as monetary policy decreases domestic interest rates. As employment decreases, firms will be pushed to increase wages. The aggregate consumption of household will increase due to higher output comes with lower prices of goods and above all, lower interest rates, hence savings is discouraged. From figure 39, unlike the energy intensive sector, a positive to energy efficiency shock in the non-energy sector will result in lower use of energy, high labour demand and therefore, lower output. As a consequence of the latter, real wages will increase which will raise the marginal cost of production, hence the price. Aggregate demand rises in the short-run as firms begin to reduce demand for labour and push output up, exports fall significantly due to rising domestic prices. The monetary policy will move domestic interest rate down to raise investment in the sector, and this will quickly recover and get both variables back to its steady-states values.

A positive imports price shock (one standard deviation) has some different responses from each sector with regards to reaction to output, employment and capital utilization is shown in figure 40. The shock tends to lower both sectors' producer price that makes imports less attractive. Thus, marginally increasing the aggregate production. The rise of total output comes from higher output from the energy intensive sector with increasing employment and capital utilisation in the sector. Investment increases immediately after the shock in a similar fashion as output does. Households will choose to consume less to work more hours as wages drop as the non-energy-intensive sector is reducing productivity. Monetary policy will respond to output gap measure by raising the interest rate that will increase exports. In analytical terms, one can say the price level dynamics, and the exchange rate are very similar which is due to the household's demand for foreign bonds.

Figure 40 Imports price shock

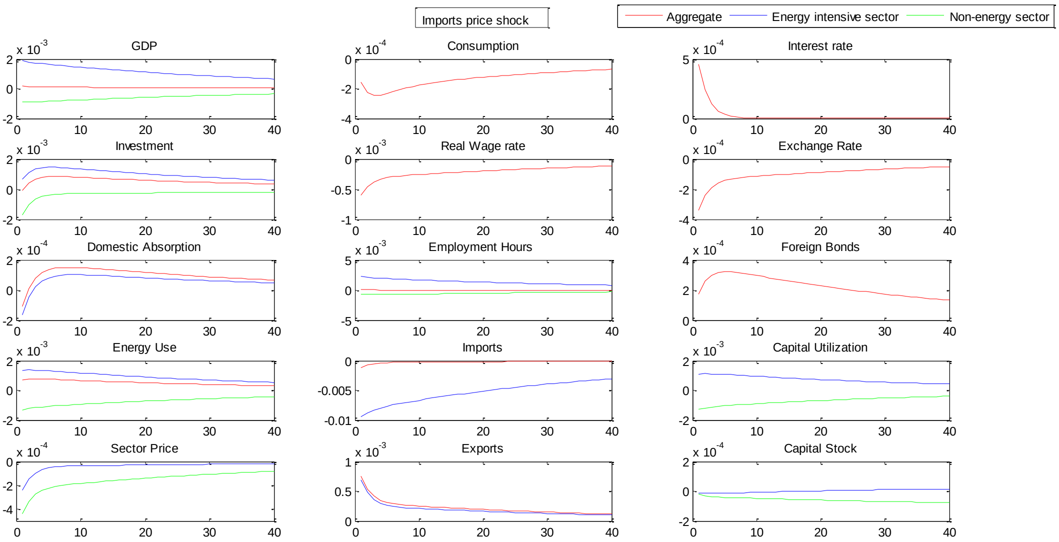


Figure 41    World interest rate shock

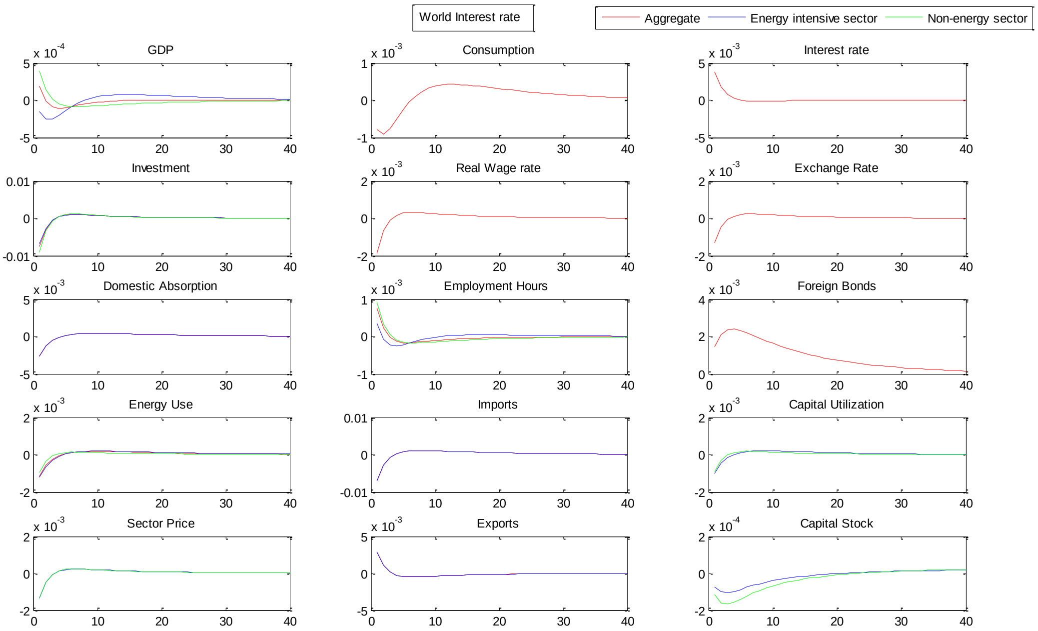


Figure 41 shows a positive world interest rates shock (one can also view this shock as foreign exchange consumption preference shock) leads to high aggregate output in the UK. The impact of this shock leads to a fall in aggregate consumption and depreciation of sterling. As aggregate output rise, employment and world energy prices all rise as demand for exports rises in response to the fall in the relative price of UK exports. The increase in world energy prices is what makes energy intensive sector output to drop marginally in the short-run along with energy use in both sectors. The rise in sterling import prices leads to a distinct rise in costs, and this leads to a rise in nominal wages as labour try to reduce the fall in real wages. There will be an appreciation of exchange rate that makes households demand for foreign bonds increase. The exchange rate tracks the uncovered interest rate parity (UIP) condition with the initial effect of the shock being an appreciation. The responses here are in line with the empirical study of di Cecio and Nelson (2007), Kamber and Millard (2010) and Christiano et al., (2005).

Figure 42 World demand shock

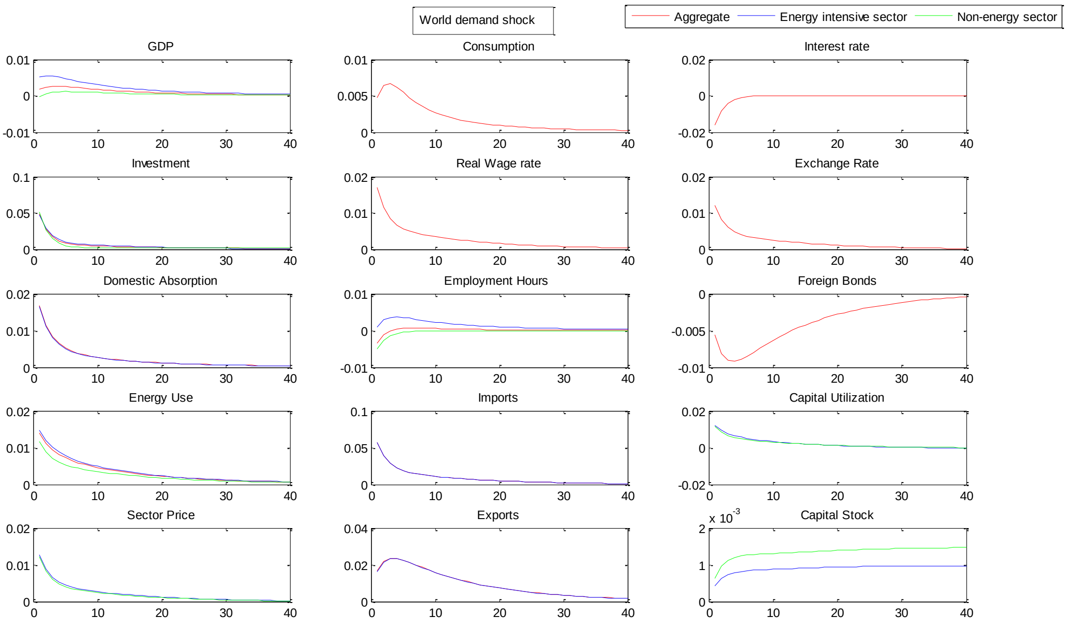


Figure 42 shows the impulse responses of the model's variables given a positive world demand shock (one standard deviation) lead to an increase in aggregate output, and one can see the positive response come from both sectors. A positive shock quickly raises investment in the economy as aggregate demand increases to meet the increasing world demand. The rise in output combines with increasing wage rates for households. It also leads to an increase in demand for productivity inputs, that is, higher energy use, higher capital utilization, imports of intermediate goods and labour in the energy-intensive sector. There is a trade-off of higher aggregate energy use to aggregate employment that reflects in the non-energy sector as this reduces the demand for labour. The increase in world demand includes appreciating asset prices that make foreign bonds less attractive. The world demand shock is expected to rise world commodity prices such as energy prices with diminishing effects on UK output.

## Appendix 2.5 Log Linearized Model

### 2.5.1 Household

The model prescribes households to consume the two final goods as they supply differentiated labour to two firms. Dynamics of consumption follows from Euler equation as:

$$\hat{c}_t = \frac{1}{1+\psi_h} E_t \hat{c}_{t+1} + \frac{1}{1+\psi_h} \hat{c}_{t-1} + \frac{1-\psi_h}{\sigma_c(1+\psi_h)} \left( \varepsilon_{b,t} - E_t \varepsilon_{b,t+1} - \hat{r}_t \right) \tag{211}$$

where $\psi_h$, $(0 < \psi_h < 1)$, represents external habit formation and $\sigma_c$ denotes the parameter that explains intertemporal elasticity in consumption $\frac{1}{\sigma_c - 1}$. Households are also assumed to own the sector-specific capital stock and make decisions about capital accumulation and utilisation. The equation for sector-specific capital accumulation shows lagged capital due to the assumption of capital adjustment costs:

$$k_t^e = \frac{\frac{\sigma_c}{1-\psi_h}(c_t - \psi_h c_{t-1}) - \frac{\sigma_c}{1-\psi_h}(c_{t+1} - \psi_h c_t) + \varepsilon_{b,t+1} - \varepsilon_{b,t} - \varepsilon_{inv,t+1}^e + \varepsilon_{inv,t}^e}{\chi_e(1+\beta)} + \frac{\beta \delta_{e1} z^{\mu_e} (\mu_e - 1) z_{t+1}^e + \chi_e(\beta k_{t+1}^e + k_{t-1}^e)}{\chi_e(1+\beta)} \tag{212}$$

$$k_t^n = \frac{\frac{\sigma_c}{1-\psi_h}(c_t - \psi_h c_{t-1}) - \frac{\sigma_c}{1-\psi_h}(c_{t+1} - \psi_h c_t) + \varepsilon_{b,t+1} - \varepsilon_{b,t} - \varepsilon_{inv,t+1}^n + \varepsilon_{inv,t}^n}{\chi_n(1+\beta)} + \frac{\beta \delta_{n1} z^{\mu_n} (\mu_n - 1) z_{t+1}^n + \chi_n(\beta k_{t+1}^n + k_{t-1}^n)}{\chi_n(1+\beta)} \tag{213}$$

where $\chi_j$ $(for\, j = e, n)$. The model assumes sector-specific capital adjustment costs as a function of the lagged change in capital stock that leads to persistent changes of the capital stock from its steady-state. The combined equation also states that sector-specific capital adjustment costs allow for households to vary the capital stock slowly. Thus, as the cost of capital adjustment parameter increases, the elasticity of the change in capital stock will increase with respect to real interest rate and rental rates.

The dynamics of investment is given by:

$$i_t^e = \frac{k_t^e - (1-\delta^e)k_{t-1}^e}{\frac{i^e}{k^e}} + \frac{\delta_{e1} z^{\mu_e} z_t^e}{\frac{i^e}{k^e}} - \varepsilon_{inv,t}^e \tag{214}$$

$$i_t^n = \frac{k_t^n - (1-\delta^n)k_{t-1}^n}{\frac{i^n}{k^n}} + \frac{\delta_{n1} z^{\mu_n} z_t^n}{\frac{i^n}{k^n}} - \varepsilon_{inv,t}^n \tag{215}$$

$\delta_{j1} z^{\mu_j}$ denotes the cost parameter that governs the investment sensitivity to changes in the sector-specific capital utilisation rate. $\frac{i^j}{k^j}$ denotes the steady-state ratio of investment to capital. Ceteris paribus, a high capital utilisation rate will deplete the capital stock carried over to the next quarter which makes investment to increase so as to keep the physical capital stock at its equilibrium.

Capital is effectively used in sector-specific production that depends on the intensity of capital utilisation. Also, it is assumed that the sector-specific capital utilisation decision depends on the price of energy, following Finn (2000). the dynamics of utilization of capital is given as:

$$z_t^e = \frac{p_t^e + y_t^e + \varepsilon_{inv,t}^e + \left( \frac{v_e}{1 + \frac{1-\theta_e}{\theta_e}\left(\frac{e^e}{k^e}\right)^{-v_e}} - v_e - 1 \right) k_{t-1}^e + \frac{v_e}{1 + \frac{\theta_e}{1-\theta_e}\left(\frac{e^e}{k^e}\right)^{v_e}} \left(\varepsilon_{o,t}^e + e_t^e\right)}{\mu_e + v_e - \frac{v_e}{1 + \frac{1-\theta_e}{\theta_e}\left(\frac{e^e}{k^e}\right)^{-v_e}}} \tag{216}$$

$$z_t^n = \frac{p_t^n + y_t^n + \varepsilon_{inv,t}^n + \left( \frac{v_n}{1 + \frac{1-\theta_n}{\theta_n}\left(\frac{e^n}{k^n}\right)^{-v_n}} - v_n - 1 \right) k_{t-1}^n + \frac{v_n}{1 + \frac{\theta_n}{1-\theta_n}\left(\frac{e^n}{k^n}\right)^{v_n}} \left(\varepsilon_{o,t}^n + e_t^n\right)}{\mu_n + v_n - \frac{v_n}{1 + \frac{1-\theta_n}{\theta_n}\left(\frac{e^n}{k^n}\right)^{-v_n}}} \tag{217}$$

The households assume to have an option of holding either foreign or domestic bonds, as trade in foreign bonds incurs quadratic costs. This results in the UIP condition:

$$r_t = \varepsilon_{rf,t} - E_t s_{t+1} - s_t - \chi_{bf} b_{f,t} \tag{218}$$

The model assumes household to be a monopoly supplier of differentiated labor.

$$w_t = \omega h_t + \frac{\sigma_c}{1-\psi_h}\left(c_t - \psi_h c_{t-1}\right) + \varepsilon_{w,t} \tag{219}$$

where $\omega$ denotes Frisch inverse elasticity of labour supply. Therefore, households will set real wage as the marginal rate of substitution between consumption and leisure.

### 2.5.2 The Firm

Production is assumed to be divided into two sectors of energy intensive producing firm and non-energy intensive producing firm. The representative firms in respective sectors have the following production functions:

$$y_t^e = \varepsilon_{a,t}^e + (1-\alpha_e)h_t^e + \frac{\alpha_e}{1+\frac{1-\theta_e}{\theta_e}\left(\frac{e^e}{k^e}\right)^{-v_e}}(z_t^e + k_{t-1}^e) + \frac{\alpha_e}{1+\frac{\theta_e}{1-\theta_e}\left(\frac{e^e}{k^e}\right)^{v_e}}\left(\varepsilon_{o,t}^e + e_t^e\right) \tag{220}$$

$$y_t^n = \varepsilon_{a,t}^n + (1-\alpha_n)h_t^n + \frac{\alpha_n}{1+\frac{1-\theta_n}{\theta_n}\left(\frac{e^n}{k^n}\right)^{-v_n}}(z_t^e + k_{t-1}^e) + \frac{\alpha_n}{1+\frac{\theta_n}{1-\theta_n}\left(\frac{e^n}{k^n}\right)^{v_n}}\left(\varepsilon_{o,t}^n + e_t^n\right) \tag{221}$$

where $1-\alpha_j$ denotes sector-specific cost share of labour, $\theta_j$ represents the cost share parameter between capital services and energy. $v_j$ is equal to $(1-s)/s$ where $s$ represents sector-specific elasticity of substitution between energy and capital in production, for $j = e, n$. Each sector requires labour, capital services and energy use in production. The demand curve for sector-specific labour and energy is:

$$h_t^e = p_t^e + y_t^e - w_t \tag{222}$$

$$h_t^n = p_t^n + y_t^n - w_t \tag{223}$$

$$e_t^e = \frac{y_t^e + p_t^e + p_{o,t} + \left(\frac{v_e}{1+\frac{1-\theta_e}{\theta_e}\left(\frac{e^e}{k^e}\right)^{-v_e}} - v_e\right)(k_{t-1}^e + z_t^e) + \frac{v_e}{1+\frac{\theta_e}{1-\theta_e}\left(\frac{e^e}{k^e}\right)^{v_e}}(\varepsilon_{o,t}^e)}{v_e + 1 - \frac{v_e}{1+\frac{\theta_e}{1-\theta_e}\left(\frac{e^e}{k^e}\right)^{v_e}}} \tag{224}$$

$$e_t^n = \frac{y_t^n + p_t^n + p_{o,t} + \left(\frac{v_n}{1+\frac{1-\theta_n}{\theta_n}\left(\frac{e^n}{k^n}\right)^{-v_n}} - v_n\right)(k_{t-1}^n + z_t^n) + \frac{v_n}{1+\frac{\theta_n}{1-\theta_n}\left(\frac{e^n}{k^n}\right)^{v_n}}(\varepsilon_{o,t}^n)}{v_n + 1 - \frac{v_n}{1+\frac{\theta_n}{1-\theta_n}\left(\frac{e^n}{k^n}\right)^{v_n}}} \tag{225}$$

### 2.5.3 Foreign sector: Trade with rest of the world

The dynamics of the asset prices, sector-specific prices, domestic absorption, imports expenditure and domestic exports is given as:

$$m_t = m_t^e + \phi\varepsilon_{pm,t} \tag{226}$$

$$x_t = \varepsilon_{yf,t} - \eta_w s_t \tag{227}$$

$$s_t = \frac{m_t - d_t}{\eta} \tag{228}$$

$$d_t^e = \varsigma(s_t - p_t^e) + d_t \tag{229}$$

$$s_t = \epsilon\left(\frac{p^e}{s}\right)^{1-\varsigma} p_t^e + (1-\epsilon)\left(\frac{p^n}{s}\right)^{1-\varsigma} p_t^n \tag{230}$$

$$p_t^e = s_t - \frac{x_t^e - x_t}{\phi_w} \tag{231}$$

where $\varsigma > 0$ is denoted as the parameter in elasticity of substitution between consumption of the sectoral goods, $\eta \geq 1$ represents elasticity of demand for imports and $\phi > 0$ is denoted as elasticity of demand for imports of energy intensive goods and $\eta_w \geq 1$ represents elasticity of demand for imports and $\phi_w > 0$ represents elasticity of demand for imports of energy intensive goods. As world demand increases with $\eta_w$, domestic exports will rise. Conversely exports decreases as real exchange rate appreciates.

### 2.5.4 Aggregation, Market Clearing

$$y_t = \frac{y^e}{y} y_t^e + \frac{y^n}{y} y_t^n \tag{232}$$

$$i_t = \frac{i^e}{i} i_t^e + \frac{i^n}{i} i_t^n \tag{233}$$

$$h_t = \frac{h^e}{h} h_t^e + \frac{h^n}{h} h_t^n \tag{234}$$

$$e_t = \frac{e^e}{e} e_t^e + \frac{e^n}{e} e_t^n \tag{235}$$

The current account is given as:

$$E_t b_t^f = \frac{1}{\beta} b_{t-1}^f + s\frac{x}{y}(x_t) - \frac{m}{y}\frac{e}{y}\left(\varepsilon_{pm,t} + m_t\right) - \frac{e}{y}\left(\varepsilon_{po,t} + e_t\right) \tag{236}$$

The equilibrium in the market goods is given as:

$$y_t = \frac{c}{y} c_t + \frac{i}{y} i_t + \frac{x}{y} x_t - \frac{m}{y} m_t - \frac{e}{y} e_t - \frac{e}{y} \varepsilon_{po,t} + \frac{g}{y} \varepsilon_{g,t} \tag{237}$$

Aggregate domestic absorption and energy-intensive output is given as:

$$d_t = \frac{c}{d} c_t + \frac{i}{d} i_t + \frac{g}{d} \varepsilon_{g,t} \tag{238}$$

$$y_t^e = \frac{d^e}{y^e} d_t^e + \frac{x^e}{y^e} x_t^e - \frac{m^e}{y^e} m_t^e \tag{239}$$

### 2.5.5 The exogenous shock processes

The exogenous shocks follow an AR(1) process

$$\varepsilon_{ae,t} = \rho_{ae}\varepsilon_{ae,t-1} + \eta_{ae,t} \tag{240}$$

$$\varepsilon_{an,t} = \rho_{an}\varepsilon_{an,t-1} + \eta_{an,t} \tag{241}$$

$$\varepsilon_{b,t} = \rho_{b}\varepsilon_{b,t-1} + \eta_{b,t} \tag{242}$$

$$\varepsilon_{g,t} = \rho_{g}\varepsilon_{g,t-1} + \eta_{g,t} \tag{243}$$

$$\varepsilon_{inv,t}^e = \rho_{inv,t}^e\varepsilon_{inv,t-1}^e + \eta_{inv,t}^e \tag{244}$$

$$\varepsilon_{inv,t}^n = \rho_{inv,t}^n\varepsilon_{inv,t-1}^n + \eta_{inv,t}^n \tag{245}$$

$$\varepsilon_{o,t}^e = \rho_{o,t}^e\varepsilon_{o,t-1}^e + \eta_{o,t}^e \tag{246}$$

$$\varepsilon_{o,t}^n = \rho_{o,t}^n\varepsilon_{o,t-1}^n + \eta_{o,t}^n \tag{247}$$

$$\varepsilon_{pm,t} = \rho_{pm}\varepsilon_{pm,t-1} + \eta_{pm,t} \tag{248}$$

$$\varepsilon_{po,t} = \rho_{po}\varepsilon_{po,t-1} + \eta_{po,t} \tag{249}$$

$$\varepsilon_{rf,t} = \rho_{rf}\varepsilon_{rf,t-1} + \eta_{rf,t} \tag{250}$$

$$\varepsilon_{w,t} = \rho_{w}\varepsilon_{w,t-1} + \eta_{w,t} \tag{251}$$

$$\varepsilon_{yf,t} = \rho_{yf}\varepsilon_{yf,t-1} + \eta_{yf,t} \tag{252}$$

where $\eta' s$ are all assumed to be i.i.d. normal processes with standard deviations estimated.

## 4.0 Summary of results, Policy Implications and Conclusion.

I have estimated a single sector DSGE model of energy, inflation and monetary policy (NKPC model) with stationary shocks, and a RBC two-sector model of energy intensive sector and non-energy intensive sector with non-stationary shocks. I gave an account of why the energy price shock reduces GDP in the RBC two-sector model. I concluded that it must be that the terms of trade decline permanently when oil price falls as it is non-stationary and there is no other way to balance the current account than to reduce absorption- because there is no way to substitute away from energy by enough to eliminate the problem. In NKPC model with stationary shocks this is only a temporary terms of trade shock and so GDP only falls briefly- the UK can borrow against such a temporary fall.

I use an effective method of estimation that proves to be the optimum way of evaluation by overcoming most of the problems faced by DSGE models such as identification. I follow a unique method of evaluating a DSGE model that efficiently fits the actual data of the United Kingdom. The current lower oil prices have increased the discretionary income in the United Kingdom after years of a real wage squeeze. Lower prices will be welcomed by the Bank of England to delay a rise in interest rates. I have developed and estimated a small-open economy DSGE model of energy for the United Kingdom. The model includes the features that are now standard in the literature that builds on Kim and Loungani (1992), Finn (1996), and Smets and Wouters (2003). It also includes features that are considered to make it vital for the analysis of a small open economy like the United Kingdom. The UK economy operates as a net-importer of energy commodities to satisfy its domestic demand from the energy-intensive sector. Estimation of the model proceeds in stages. First, I evaluate a calibrated version of the stochastic models with twelve and thirteen types of structural shocks, respectively. I then estimate its parameters on UK data using the powerful simulated annealing algorithm. This follows a closely related work of Le et al., (2010, 2012) on stationary data and nonstationary data, respectively. Finally, I reassess the efficiency and adequacy of each estimated model.

The approach to first assess the fit of the calibrated model before the model estimation creates a better understanding of the model in a way in which the parameters can and cannot help it to fit the data. The decision of using nonstationary data was vital to the fit of the model. Stationary data tells a different story about the economic forces governing the dynamics of the data as compared to nonstationary data. The treatment of sectoral productivity shocks and energy price shock shows the effects despite the models having key distinctive features. Using the calibration approach allowed me to treat the model as not true. In the initial model evaluation, the performance of the calibrated model in matching UK data, I find that both models perform poorly. The models fail to match both the patterns of variability within the frequency of the selected variables in the auxiliary models. Changes to parameter values may increase the fit, such as the capital adjustment cost that is assumed to be over 200 while some authors assume its value to be set as low as 1. I find that such changes alone cannot give a good result that is a match between the model and the data.

For the model with NKPC and monetary policy shock: The assessment of the ways that the dynamics of the model fail to match those of the data lead to the augmentation of four additional shocks to consumption and hours worked as preference shocks in the household utility function, to real wages in the form of a wage mark-up, to investment as a shock to the cost of capital adjustment, and as a mark-up shock to inflation before estimating it by Indirect inference testing. I find that price stickiness and nominal wage rigidities are preferred by the data and still significant for matching the UK data when combined with standard real rigidities, such as habit formation and capital adjustment costs. The assessment sees a low significant evidence for effects on lagged inflation on indexation (wage-setting and price setting). This is one of the reasons that mark-up and monetary policy equations were ignored in the next model. The foreign shock processes play a key role in matching the UK data, especially the energy price shocks and world demand shocks. The former are estimated to be highly persistent and had effects on the CPI inflation rate as well as GDP. The effects are consistent with a few authors that studied the effects of energy shocks in the UK and Euro areas. Productivity shock had little effect on output. This is due to the model's settings of the firms' production function that assumes that only gross-output has a shock to its productivity.

The investigation above has focussed on evaluating the fit of the model over the period between 1981 Q1 and 2013 Q1. Assessing the calibrated model, I find that it poorly matches the dynamics of the UK data and that the fit of the real output, real interest rate and inflation rate are the best subset fitting variables. Thus, the model suggests several promising future research avenues that motivated a two-sector DSGE model. The fit of the output and inflation series also improves after estimation, while the fit of the nominal interest rate did not change much. These results are assumed to reflect the misspecification present in the monetary policy rule. The model assumes that the short-term interest rate is set by the policymaker following a simple Taylor-type reaction function. Following regime shifts that are evident in the UK monetary policy, a suitable extension of the model would be to evaluate the model's fit when one can account for the different policy regimes.

In the two-sector model, I followed a similar methodology with a few exceptions where I used a VECM (1) as my auxiliary model as compared to VAR(1). The model initially assumed exogenous processes of 13 exogenous shocks and, therefore, required no augmentation to include shocks. In matching the data, I assumed the same approach as stated earlier. In the first instance, I calibrated the adjustment cost parameter value to be far lower than the former model. The calibrated model was rejected by the data as expected, thus, I went on to estimation. The analysis of the model has focussed on evaluating the fit of the model over the period between 1981 Q1 and 2013 Q1. In assessing the model fit, I am able to prove the effects of energy price shock on real output, real exchange rate and real aggregate demand from the VAR impulse response functions. I am also able to show that the simulated data dynamics as well as its variances fit within the 95% boundary of the actual data. By decomposition, the variability in the real macroeconomic aggregates shows that the fall in output during the financial crisis period between 2008: Q2 to 2009: Q4 was driven by an energy price shock and sectoral productivity shocks. However, in assesing the former models, the shock processes play a key role and the foreign shocks, energy shocks especially, are estimated to have high persistence. In the application of the model, the study showed how this could be done by evaluating the effects of different shocks on output, inflation and interest rate as can be seen from the VAR impulse response functions. In addition, by decomposition, the changes in these variables caused by each of the structural shocks showed that a fall in output during the financial crisis period between 2008:Q2 to 2009:Q4 was driven by domestic demand shocks (consumption preference, government spending and capital adjustment cost), oil prices shock and world demand shock. I showed how the estimated model can create additional input to the policymaker’s choice of models through the economic shocks’ effects of the macroeconomic variables.

Finally, I can conclude that overall, the models to have good accuracy and power following the indirect inference test method of estimation that includes Monte Carlo simulations. Avenues for further research would include adding a monetary policy equation into the model to see the effects of contemporaneous feedback. This is because the standard assumptions in the empirical study assume that oil (energy) prices are predetermined with respect to real output in the economy and feedback is not generated from the real domestic aggregate variables. A suitable extension would be to evaluate the model's fit when one can account for the different policy regimes. Given the current economic volatility, it will also be important to see how DSGE models will behave on monthly data.

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