Efficient Inventory Control for Imperfect Quality Items

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Abstract

In this paper, we present a general EOQ model for items that are subject to inspection for imperfect quality. Each lot that is delivered to the sorting facility undertakes a 100% screening and the percentage of defective items per lot reduces according to a learning curve. The generality of the model is viewed as important both from an academic and practitioner perspective. The mathematical formulation considers arbitrary functions of time that allow the decision maker to assess the consequences of a diverse range of strategies by employing a single inventory model. A rigorous methodology is utilised to show that the solution is a unique and global optimal and a general step-by-step solution procedure is presented for continuous intra-cycle periodic review applications. The value of the temperature history and flow time through the supply chain is also used to determine an efficient policy. Furthermore, coordination mechanisms that may affect the supplier and the retailer are explored to improve inventory control at both echelons. The paper provides illustrative examples that demonstrate the application of the theoretical model in different settings and lead to the generation of interesting managerial insights.

Keywords: Inventory; Imperfect quality; Deterioration; Perishable items; Periodic review.

1. Introduction and background

Since the introduction of the Economic Order Quantity (EOQ) model by Harris (1913), frequent contributions have been made in the literature towards the development of alternative models that overcome the unrealistic assumptions embedded in the EOQ formulation. For example, the assumption related to the perfect quality items is technologically unattainable in most supply chain applications (Cheng, 1991). In contrast, products can be categorised as ‘good quality’, ‘good quality after reworking’, ‘imperfect quality’ and ‘scrap’ (Chan et al., 2003; Pal et al., 2013). In practice, the presence of defective items in raw material or finished products inventories may deeply affect supply chain coordination and, consequently, the product flows among supply chain levels may become unreliable (Roy et al., 2013). In response to this concern, the enhancement of currently available production and inventory order quantity models, that accounts for imperfect items in their mathematical formulation, has become an operational priority in supply chain management (Khan et al., 2011). This enhancement may also include the knowledge transfer between supply chain entities in order to reduce the percentage of defective items.

EOQ models are associated with another implicit assumption that stored items may retain the same utility indefinitely, i.e. they do not lose their value as time goes on. This assumption may be valid for certain items. However, real-life systems analysis suggests that goods are subject to 'obsolescence', 'perishability' and 'deterioration' that have a direct impact on the flow of an item as it moves through the supply chain (Goyal and Giri, 2001; Bakker et al., 2012; Pahl and Voß, 2014). In relation to the perishable items, Amorim et al. (2013) presented a classification of perishable models for items that have explicit characteristics related to their physical status (e.g. by spoilage, decay or depletion) and/or changes in their value as perceived by the customer and/or a risk of future reduced functionality according to specialist's opinion.

The complexity and drivers associated with product waste and loss have been increasingly discussed in the academic literature and include such issues as imperfect quality items (that necessitate an inspection to take place at various supply chain stages to ensure the quality of the product is adequate) (Gunders, 2012). For example, in the food and drink industry, different proportions of food waste are attributed to different stages in the supply chain, from production to handling and storage, processing and packaging, distribution and retail and finally at the household consumption stage. In particular, the fresh meat sector has been identified as the largest producer of waste and accounts overall for 25% of the waste, ahead of fruit and vegetables at 13% (WRAP, 2012a). The waste and spoilage related to inventory decisions represent a large proportion, and it is estimated that around 10% of all perishable goods are spoiled before they reach consumers (Roberti, 2005; Tortola, 2005; Boyer, 2006). WRAP (2012b) published that "5-25% of fruit and vegetable crop might not get through the supply chain to retail customers". For example, in the onion supply chain, losses related to grading account for 9-20%; storage 3-10% and in the packing process they equate to 2-3% loss. The main causes of waste in these examples relate to product specification, product deterioration and reliance on (excessive) storage to cope with fluctuations in (forecasted) demand. The product shelf lifetime also depends on various environmental factors such as the product's temperature history, humidity, transportation and handling (Ketzenberg et al., 2015). Further, increases in the time products are being stored as well as changes in the environment of the storage facilities (e.g. temperature storage and controlled atmosphere storage) may result in an increase (or a decrease) of the deterioration rate of certain commodities. This means that the identification of an appropriate ordering policy is an essential but challenging task.

Appropriate management of perishable inventories, in conjunction with modern technologies, play an important role in monitoring the condition of those goods in different stages of the supply chain. Ketzenberg et al. (2015) emphasised the importance of the value of information generated from using different systems in the decision making process in the grocery industry (that is associated with low net margins). For example, continuous automated inventory control systems are capable of tracking, recording and transmitting relevant information regarding an item as it moves through the network. The deployment of radio-frequency identification (RFID) systems, data loggers and time–temperature integrators and sensors lead to a reduction of product spoilage and economic benefits (Ketzenberg et al., 2015). The potential benefits of RFID for logistics, transportation and warehousing relate to increased supply chain visibility, which in turn increases efficiency, lowers safety stocks, and provides the same or even better customer service level (Gaukler et al., 2007). For further details related to this technology, see Jedermann et al. (2008) and Wessel (2007).

This paper aims to address the quality related issues discussed above when modelling inventories for items that require 100% screening. In particular, a general EOQ model for items with imperfect quality is presented; the solution to the underlying inventory system, if it exists, is shown to be a unique and global optimal. The mathematical formulation considers arbitrary functions of time that allow the decision maker to assess the consequences of a diverse range of strategies by employing a single inventory model. Previously published models in this area are shown to be special cases of our model. The behavior of different conditions (such as using functions for varying demand, screening, defective and deterioration rates, value of information (VOI) and perishable items that are subject to deterioration while in storage) is studied using illustrative examples, and interesting insights are offered to practitioners. This paper intersects the areas of fixed and random items' lifetimes since the assumption that each lot is subjected to a 100% screening will render the (potential) random lifetime of a product deterministic. The focus of this study is on the value and use of technologies such as RFID to capture the time and temperature history (TTH) to model shelf lifetime and not the technologies themselves.

The remainder of the paper is organised as follows: Section 2 discusses the literature related to perishable inventories, value of information (VOI) and models that consider imperfect quality items. Our EOQ model for items with imperfect quality, the assumptions and notations of the inventory system are presented in Section 3. The solution procedures are presented in Section 4, followed, in Section 5, by illustrative examples and special cases that demonstrate the application of the theoretical results in practice. Managerial insights and concluding remarks are provided in Sections 6 and 7 respectively. Finally, supplementary material presented in an electronic companion to this paper offers a proof of the optimality and uniqueness of our solution.

2. Literature review

The academic literature related to inventory control for imperfect quality items is multidisciplinary in nature and, for reviewing / presentation purposes in this paper, is thematically organised around three main streams of research: 1) deterioration, perishability and shelf lifetime constraints; 2) the value of information; and 3) model formulations and related solution techniques that consider imperfect quality items.

2.1. Deterioration, perishability and lifetime constraints

The terms 'deterioration', 'perishability' and 'obsolescence' are used interchangeably in the literature and may often be perceived as ambiguous because they are linked to particular underlying assumptions regarding the physical state/fitness and behaviour of items over time (Teunter and Flapper, 2003; Pahl and Voß, 2010; 2014; Krommyda et al., 2013). In this paper, deterioration refers to the process of decay, damage or spoilage of a product, i.e. the product loses its value of characteristics and can no longer be sold/used for its original purpose (Dave, 1986; Wee, 1993; Shah et al., 2005; Darlington and Rahimifard, 2006; Ferguson and Koenigsberg, 2007; Pei-xin, 2007). In contrast, an item with a fixed lifetime perishes once it exceeds its maximum shelf lifetime and then must be discarded (Ferguson and Ketzenberg, 2005; Olsson, 2009). Obsolescence incurs a partial or a total loss of value of the on hand inventory in such a way that the value for a product continuously decreases with its perceived utility (Joglekar and Lee, 1993; Jain and Silver, 1994; Dohi and Osaki, 1995; Song and Zipkin, 1996; Van Delft and Vial, 1996; Elmaghraby and Keskinocak, 2003; Leung and Ng, 2007). Pahl and Voß (2014) provided an extensive discussion on mathematical modelling and the quality of information underlying deterioration, perishability and lifetime constraints of items. They demonstrate that the combination of such aspects is important in many industries where deterioration effects significantly influence the business outcomes. Nahmias (1975, 1977) introduced the fixed lifetime case and analysed the problem of a random lifetime product managed under periodic review with stationary stochastic demand. He assumed no fixed order cost and backlogged demand and orders perishing in the same sequence that they enter stock (i.e. a First In First Out - FIFO policy). Ketzenberg et al. (2012) extended the work of Nahmias (1977) and addressed the random lifetime as a function of the product's TTH in the supply chain. They allowed for orders to perish out of sequence, to discard inventory that remains good for sale and to sell inventory that may have already perished. In a follow-up study, Ketzenberg et al. (2015) considered a case according to which unsatisfied demand is lost, products may arrive already perished and orders may not perish in sequence by focusing on determining the VOI for integrating the TTH into the order policy. As we attempt to provide a general model that takes into account many possible practical scenarios, the behaviour of these conditions will be discussed through illustrative examples. In this paper, we consider perishable and non-perishable (infinite shelf lifetime) items, which are subject to deterioration while they are in storage. As discussed above, this study intersects two areas of fixed and random lifetimes since the assumption that each lot is subjected to a 100% screening will render a potential random lifetime of a product deterministic.

2.2. Value of information (VOI)

There is a unanimous agreement among researchers and practitioners on the benefits of information sharing that allows more timely material flows in a supply chain (Kahn, 1987; Metters, 1997; Costantino et al., 2013). Ketzenberg et al. (2007) conducted an extensive literature review of papers that: (1) address VOI in the context of inventory control, (2) provide a numerical study to explore VOI over a set of varying operating characteristics and (3) compare two or more scenarios. In addition, they developed and tested a VOI framework to help identify the determinants of VOI. Common examples are agricultural, food or chemical products that are transported over long distances in refrigerated containers, where temperature variability has a direct impact on product shelf lifetime. Moreover, temperature control is not absolute and may indeed vary for items shipped in the same container (Doyle, 1995; Taoukis et al., 1999; Koutsoumanis et al., 2005).

Accurate shelf lifetime monitoring is a goal of technologies that have been developed to collect and transmit data about the state of a product. For certain items, if temperature departs from a pre-defined range, the items are spoiled and must be discarded (Zacharewicz et al., 2011). Ketzenberg and Ferguson (2008) examined the VOI for a product with fixed lifetime in the context of a serial supply chain. They evaluated the case in which a supplier shares retailer's demand and inventory information as well as the case in which a centralised decision maker collects full information at both echelons. Recently, Ketzenberg et al. (2015) addressed the VOI for inventory replenishment decisions to demonstrate the wide fluctuations in a supply chain's TTH, the applicability and accuracy of using RFID temperature tags to capture the TTH, and the use of TTH to model shelf lifetime. White and Cheong (2012) considered the benefit of observing the quality of a perishable product in a food supply chain that is processed in multiple stages from origin to destination. At each stage, it is presumed essential to decide whether or not to inspect the quality of the product at a certain cost. In this regard, a 100% screening assumption not only guarantees the isolation of defective and/or already perished items, but also classifies the order quantity based on a first-to-expire first-out (FEFO) policy, rather than a FIFO one.

2.3 Imperfect quality items

The classical EOQ has been a widely accepted model for inventory control purposes due to its simple and intuitively appealing mathematical formulation. However, it is true to say that the operation of the model is based on a number of explicitly or implicitly made unrealistic mathematical assumptions that are never actually met in practice (Jaber et al., 2004; Liao et al., 2013). Salameh and Jaber (2000) developed a mathematical model that permits some of the items to drop below the quality requirements, i.e. a random proportion of defective items are assumed for each lot size shipment, with a known probability distribution. The researchers assumed that each lot is subject to a 100% screening, where defective items are kept in the same warehouse until the end of the screening process and then can be sold at a price lower to that of perfect quality items. Huang (2004) developed a model to determine an optimal integrated vendor–buyer inventory policy for flawed items in a just-in-time (JIT) manufacturing environment. Maddah and Jaber (2008) developed a new model that rectifies a flaw in the one presented by Salameh and Jaber (2000) using renewal theory. Jaber et al. (2008) extended it by assuming that the percentage defective per lot reduces according to a learning curve. They examined empirical data from the automotive industry for several learning curve models and the S-shaped logistic learning curve (Jordan, 1958; Carlson, 1973) was found to fit well. Jaggi and Mittal (2011) investigated the effect of deterioration on a retailer's EOQ when the items are of imperfect quality. In that paper, defective items were assumed to be kept in the same warehouse until the end of the screening process. Jaggi et al. (2011) and Sana (2012) presented inventory models, which account for imperfect quality items under the condition of permissible delay in payments. Moussawi-Haidar et al. (2014) extended the work of Jaggi and Mittal (2011) to allow for shortages.

In a real manufacturing environment, the defective items are not usually stored in the same warehouse where the good items are stored. As a result, the holding cost must be different for the good items and the defective ones (e.g. Paknejad et al., 2005). With this consideration in mind, Wahab and Jaber (2010) presented the case where different holding costs for the good and defective items are assumed. They showed that if the system is subject to learning, then the lot size with the same assumed holding costs for the good and defective items is less than the one with differing holding costs. When there is no learning in the system, the lot size with differing holding costs increases with the percentage of defective items. In this section, we have cited only references that are directly relevant to this paper. For more details about the extensions of a modified EOQ model for imperfect quality items, see Khan et al. (2011).

One basic assumption of the above cited contributions is that the demand rate is assumed to be constant and known. A survey of the inventory literature reveals that there is no published work that investigates the model of Wahab and Jaber (2010) for time-varying demand and product deterioration. Product life cycle analysis suggests that a constant demand rate assumption is usually valid in the mature stage of the life cycle of the product. In the growth and/or declining stages, the demand rate can be well approximated by a linear demand function (e.g. Alamri and Balkhi (2007)). Also, one implicit assumption is that the stored items that are screened may retain the same utility indefinitely, i.e. they do not lose their value as time goes on. In fact, the variation of demand and/or product deterioration with time (or due to any other factors) is a quite natural phenomenon.

3. Formulation of the general EOQ model

This paper presents a general EOQ model for items with imperfect quality under varying demand, defective items, a screening process and deterioration rates for an infinite planning horizon. Consequently, the generality of the model goes beyond academic interests to enable inventory managers to establish the optimum order quantities that minimise the total system cost. In the model, each lot is subject to a 100% screening where items that are not conforming to certain quality standards are stored in a different warehouse. Therefore, different holding costs for the good and defective items are considered in the mathematical model. Items deteriorate while they are in storage, with demand, screening and deterioration rates being arbitrary functions of time. The percentage of defective items per lot reduces according to a learning curve. After a 100% screening, imperfect quality items may be sold at a discounted price as a single batch at the end of the screening process or incur a disposal penalty charge. Moreover, a general step-by-step solution procedure is provided for continuous intra-cycle periodic review applications.

3.1 Assumptions and notations

The mathematical model is developed under the following assumptions and notations:

1. A single item is held in stock.
2. The lead-time is negligible and no capacity restrictions are assumed, i.e. any replenishment ordered at the beginning of a cycle arrives just prior to the end of that same cycle.

3. The demand, screening and deterioration rates are arbitrary functions of time denoted by $D(t)$, $x(t)$ and $\delta(t)$ respectively. The percentage defective per lot reduces according to a learning curve denoted by p_j, where j is the cycle index.
4. Shortages are not allowed, i.e. we require that $(1-p_j)x(t) \geq D(t)\ \forall\ t \geq 0$.
5. The following notations are used for the cost parameters:

   c is the unit purchasing cost.

   d is the unit screening cost.

   h_g denotes the holding cost of good items per unit per unit time.

   h_d denotes the holding cost of defective items per unit per unit time.

   k is the ordering cost per cycle.

3.2 The model

At the beginning of each cycle $j(j = 1,2, \dots)$, a lot of size Q_j is delivered, which covers the actual demand and deterioration during both the first phase (screening) and the second phase (non-screening). Each lot is subjected to a 100% screening process at a rate of $x(t)$ that starts at the beginning of the cycle and ceases by time T_{1j}, by which point in time Q_j units have been screened and y_j units have been depleted, which is the summation of demand and deterioration. During this phase, items not conforming to certain quality standards are stored in a different warehouse.

The variation in the inventory level during the first and second phase (please refer to Figure 1) and the variation in the inventory level for the defective items (shaded area) is given by (1), (3) and (4) respectively.

$$\frac{dI_{gj}(t)}{dt} = -D(t) - p_j x(t) - \delta(t) I_{gj}(t), \qquad 0 \leq t < T_{1j} \qquad (1)$$

with the boundary condition $I_{gj}(0) = Q_j$,

where

$$Q_j = \int_0^{T_{1j}} x(u)du. \qquad (2)$$

$$\frac{dI_{gj}(t)}{dt} = -D(t) - \delta(t) I_{gj}(t), \qquad T_{1j} \leq t \leq T_{2j} \qquad (3)$$

with the boundary condition $I_{gj}(T_{2j}) = 0$.

$$\frac{dI_{dj}(t)}{dt} = p_j x(t), \qquad 0 \leq t \leq T_{1j} \qquad (4)$$

with the boundary condition $I_{dj}(0) = 0$.

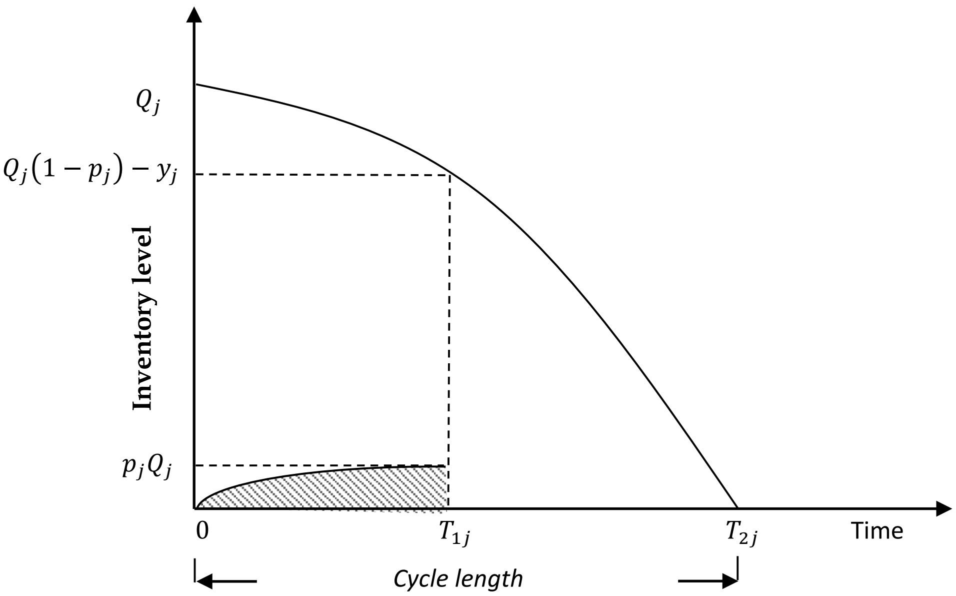

Figure 1. Inventory variation of an Economic Order Quantity (EOQ) model for one cycle.

The solutions of the above differential equations are:

$$I_{gj}(t) = e^{-(g(t)-g(0))}\int_0^{T_{1j}} x(u)du - e^{-g(t)}\int_0^t \big[D(u)+p_jx(u)\big]e^{g(u)}du,\ 0 \le t < T_{1j} \quad (5)$$

$$I_{gj}(t) = e^{-g(t)}\int_t^{T_{2j}} D(u)e^{g(u)}du, \qquad T_{1j} \le t \le T_{2j} \quad (6)$$

$$I_{dj}(t) = \int_0^t p_jx(u)du, \qquad 0 \le t \le T_{1j} \quad (7)$$

respectively, where

$$g(t) = \int \delta(t)\,dt. \quad (8)$$

The per cycle cost components for the given inventory system are as follows:

Total purchasing cost during the cycle $= c\int_0^{T_{1j}} x(u)du$. Note that this cost includes the defective and deterioration costs.

Holding cost $= h_g\big[I_{gj}\big(0,T_{1j}\big) + I_{gj}(T_{1j},T_{2j})\big] + h_dI_{dj}\big(0,T_{1j}\big)$.

Thus, the total cost per unit time of the underlying inventory system during the cycle $\big[0,T_{2j}\big]$, as a function of T_{1j} and T_{2j}, say $Z(T_{1j},T_{2j})$ is given by:

$$Z\big(T_{1j},T_{2j}\big) = \frac{1}{T_{2j}}\Big\{(c+d)\int_0^{T_{1j}} x(u)du + h_g\Big[-G(0)\int_0^{T_{2j}} D(u)e^{g(u)}du + \int_0^{T_{1j}} D(u)G(u)e^{g(u)}du - \int_0^{T_{1j}}[G(0)-G(u)]p_jx(u)e^{g(u)}du + \int_{T_{1j}}^{T_{2j}} D(u)G(u)e^{g(u)}du\Big] + h_d\Big[\int_0^{T_{1j}}\big[T_{1j}-u\big]p_jx(u)du\Big] + k\Big\}, \quad (9)$$

where

$$G(t) = \int e^{-g(t)}\, dt. \tag{10}$$

Our objective is to find T_{1j} and T_{2j} that minimise $Z(T_{1j}, T_{2j})$. However, the variables T_{1j} and T_{2j} are related to each other as follows:

$$0 < T_{1j} < T_{2j}, \tag{11}$$

$$e^{g(0)} \int_0^{T_{1j}} x(u)du = \int_0^{T_{2j}} D(u)e^{g(u)}du + \int_0^{T_{1j}} p_j x(u)e^{g(u)}du. \tag{12}$$

Thus, our goal is to solve the following optimisation problem, which we shall call problem (m)

$$(m) = \begin{Bmatrix} \text{minimise } Z\big(T_{1j}, T_{2j}\big) \text{ given by (9)} \\ \text{subject to (11) and } h_j = 0 \end{Bmatrix},$$

where

$$h_j = e^{g(0)} \int_0^{T_{1j}} x(u)du - \int_0^{T_{1j}} p_j x(u)e^{g(u)}du - \int_0^{T_{2j}} D(u)e^{g(u)}du.$$

It can be noted from Eq. (12), that $T_{1j} = 0 \Longrightarrow T_{2j} = 0$ and $T_{1j} > 0 \Longrightarrow T_{1j} < T_{2j}$. Thus Eq. (12) implies constraint (11). Consequently, if we temporarily ignore the monotony constraint (11) and call the resulting problem as (m_1) then (11) does satisfy any solution of (m_1). Hence (m) and (m_1) are equivalent. Moreover, $T_{1j} > 0 \Longrightarrow$ RHS of $(6) > 0$, i.e. Eq. (12) guarantees that the number of good items is at least equal to the demand during the first phase.

4. Solution procedure

First, we note from (2) that T_{1j} can be determined as a function of Q_j, say

$$T_{1j} = f_{1j}(Q_j). \tag{13}$$

Taking also into account Eq. (12) we find that T_{2j} can be determined as a function of T_{1j}, and thus of Q_j, say

$$T_{2j} = f_{2j}(Q_j). \tag{14}$$

Thus, if we substitute (12)-(14) in (9) then problem (m) will be converted to the following unconstrained problem with the variable Q_j (which we shall call problem (m_2)).

$$W\big(Q_j\big) = \frac{1}{f_{2j}}\Big\{(c+d)\int_0^{f_{1j}} x(u)du + h_g\Big[-G(0)e^{g(0)}\int_0^{f_{1j}} x(u)du + \int_0^{f_{1j}} p_j x(u)G(u)e^{g(u)}du + \int_0^{f_{2j}} D(u)G(u)e^{g(u)}du\Big] + h_d\Big[\int_0^{f_{1j}}[f_{1j} - u]p_j x(u)du\Big] + k\Big\}. \tag{15}$$

Now, the necessary condition for having a minimum for problem (m_2) is

$$\frac{dW}{dQ_j} = 0. \quad (16)$$

To find the solution of (16), let $W = \frac{w}{f_{2j}}$ then

$$\frac{dW}{dQ_j} = \frac{w'_{Q_j} f_{2j} - f'_{2j,Q_j} w}{f_{2j}^2}, \quad (17)$$

where w'_{Q_j} and f'_{2j,Q_j} are the derivatives of w and f_{2j} w.r.t Q_j, respectively. Hence, (16) is equivalent to

$$w'_{Q_j} f_{2j} = f'_{2j,Q_j} w. \quad (18)$$

Also, taking the first derivative of both sides of (12) w.r.t Q_j we obtain

$$e^{g(0)} - p_j e^{g(f_{1j})} = f'_{2j,Q_j} D(f_{2j}) e^{g(f_{2j})}. \quad (19)$$

From which and (13)-(15) we have

$$w'_{Q_j} = (c+d) + h_g \left[\left(G(f_{2j}) - G(0) \right) e^{g(0)} + \left(G(f_{1j}) - G(f_{2j}) \right) p_j e^{g(f_{1j})} \right] + \frac{h_d}{x(f_{1j})} \int_0^{f_{1j}} p_j x(u) du. \quad (20)$$

Also, (18) $\Leftrightarrow$ $W = \frac{w}{f_{2j}} = \frac{w'_{Q_j}}{f'_{2j,Q_j}}$, (21)

where W is given by (15) and w'_{Q_j} is given by (20). Eq. (21) can be used to determine the optimal value of Q_j and its corresponding total minimum cost. Then the optimal values of T_{1j} and T_{2j} can be found from (13) and (14), respectively.

5. Illustrative examples for different settings

In this section we present a number of examples and special cases to illustrate the efficiency of our mathematical model and solution procedures. First we consider scenarios with varying demand, screening, defective and deterioration rates. Then we present special cases for intra-cycle periodic review, perishable products and renewal theory.

5.1 Varying demand, screening, defective, and deterioration rates

In practice, the demand for products relies heavily on price (when price elasticity holds), time and quality (Karmarkar and Pitbladdo, 1997). In addition, increasing (decreasing) demand functions over time with quadratic, linear, exponential and stock-dependent trends is a natural phenomenon (Murdeshwar, 1988; Hariga and Benkherouf, 1994; Datta et al. 1998; Alamri, 2011; Benkherouf et al., 2013). For example, essential commodities and seasonal products may follow steadily increasing quadratic or linear demand functions over time (Mandal and Maiti, 2000). On the other hand, exponentially increasing demand applies to products such as new spare parts, new electronic chips and seasonal goods in which the demand rate is likely to increase very fast with time (Sana, 2010). As such, the mathematical formulation presented in this paper considers arbitrary functions of time, which allows the decision maker to assess the consequences of a diverse range of strategies by employing a single inventory model. It is worth noting that the dominant form of a learning curve implemented by researchers and practitioners alike is either an S-shaped (Jordan, 1958; Carlson, 1973), or a power one as suggested by Wright (1936); please refer to Jaber (2006) for discussion on this issue.

In this example (example 1), we consider the following functions for varying demand, screening, defective, and deterioration rates:

$$x(t) = at + b, \qquad D(t) = \alpha t + r,$$
$$p_j = \frac{\tau}{\pi + e^{\gamma j}}, \qquad \delta(t) = \frac{l}{z - \beta t}$$

where $b, d, l, \tau, \pi, z > 0$; $\quad a, r, \gamma, \beta, t \geq 0$, and $\beta t < z$.

The parameter "α", represents the rate of change in the demand. The case of $\alpha = 0$ reflects a constant demand rate, when then $D(t) = r \;\; \forall\, t \geq 0$. A similar behaviour is observed for the effect of "a", the rate of change in the screening rate. Note that $\delta(t)$ is an increasing function of time. The case of $\beta = 0$ reflects a constant deterioration rate and $l = 0$ corresponds to the case associated with no deterioration. The percentage defective per lot reduces according to an S-shaped logistic learning curve (Jordan, 1958; Carlson, 1973), where τ and π are model parameters, γ is the learning exponent and j is the cycle index. The case $\gamma = 0$ applies to a constant percentage of defective items per lot.

The problem (m_2) has been coded in *MATLAB* for the above demand, screening, defective, and deterioration rates and solutions were obtained using Eq. (21) for a wide range of the control parameter values. Here, we adopt the values considered in the study by Wahab and Jaber (2010), that are presented in Table 1 below.

Table 1. Input parameters for example 1.

c	d	h_g	h_d	k	a	b	α	r
100	0.5	20	5	3000	1000	100200	500	50000
\$/unit	\$/unit	\$/unit/year	\$/unit/year	\$/cycle	unit/year	unit /year	unit/year	unit /year

l	z	β	τ	π	γ
1	20	25	70.067	819.76	0.7932
unit /year	unit/year	unit /year	unit /year	unit/year	unit /year

The optimal values of $Q_j^*, T_{1j}^*, T_{2j}^*, \omega_j^*$, and the corresponding total minimum cost for 10 successive cycles are obtained and the results are shown in Table 2. In the first cycle, we have taken $p_1 = 0.08524$ resulting in a total number of $Q_1^* = 3550$ units, which is screened by time $T_{11}^* = 0.0354 \cong 13$ days and consumed by time $T_{21}^* = 0.0648 \cong 24$ days. The total minimum cost per year is $W_1^* = \$558{,}546$ and the total minimum cost per cycle is $w_1^* = \$362030$. The amount of defective items is $p_1 Q_1^* = 303$ units and the amount of deteriorated items is $\omega_1^* = 5.4$ units, which is the difference between the actual demand and the amount held in stock at the beginning of the cycle. The amount $p_1 Q_1^*$ may be sold at a salvage price at time T_{11}^* or incur a disposal penalty charge. The tabulated results indicate that all optimal quantities decrease as learning increases except for the amount of deteriorated items that incur a minor increase that can be justified by the slight increase in the cycle length (Table 2). Figure 2 depicts the effect of each additional model parameter on the EOQ and Figure 3 compares the case of having the same holding costs for the good and defective items with that of differing holding costs.

Table 2. Optimal results for varying demand, screening, and deterioration rates with $p_j = \frac{70.067}{819.76 + e^{0.7932 \times j}}$.

j	p_j	f_{1j}^*	f_{2j}^*	Q_j^*	$p_j Q_j^*$	ω_j^*	W_j^*	w_j^*
1	0.08524	0.035424	0.06482	3550	303	5.4	5585464	362030
2	0.08497	0.035419	0.06483	3550	302	5.4	5583830	361980
3	0.08436	0.035407	0.06485	3548	299	5.4	5580142	361850
4	0.08305	0.035380	0.06489	3546	294	5.4	5572240	361580
5	0.08030	0.035324	0.06498	3540	284	5.4	5555724	361020
6	0.07482	0.035212	0.06516	3529	264	5.5	5523107	359900
7	0.06502	0.035013	0.06548	3509	228	5.5	5465734	357890
8	0.05042	0.034715	0.06594	3479	175	5.6	5382467	354900
9	0.03369	0.034376	0.06644	3445	116	5.7	5290159	351490
10	0.01944	0.034088	0.06686	3416	66	5.8	5214030	348600

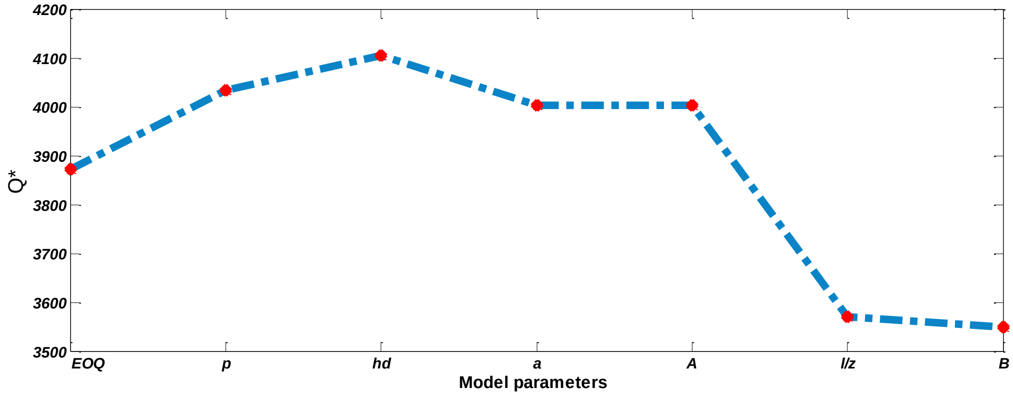

Figure. 2. The effect of model parameters on the Economic Order Quantity (EOQ).

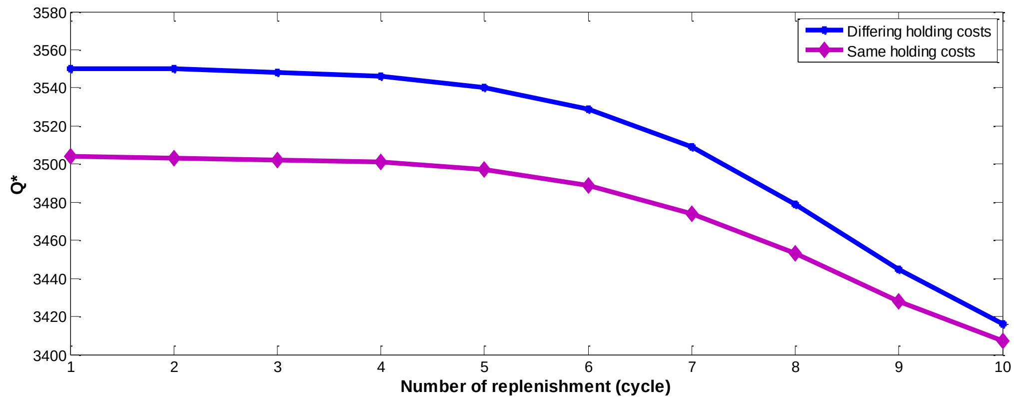

Figure. 3. EOQ with same and differing holding costs when $p_j = \frac{70.067}{819.76 + e^{0.7932 \times j}}$.

The results presented in Table 3 summarise the sensitivity analysis of the optimal order quantity, total minimum cost per unit time and total minimum cost per cycle with respect to all model parameters. The first row represents the original values of the proposed model and the last one yields the values of the EOQ model. As can be seen from the tabulated results the effect of α, significantly influences the optimal order quantity and the total minimum cost per year. Moreover, this effect holds true for the case in which the deterioration rate is assumed to be of a fixed value as well as for the case associated with no deterioration. A comparison between the results obtained in Tables 2 and 3 reveals that the reduction of the optimal order quantity does not imply that the total minimum cost per year decreases; in fact it may increase. Example 1 is replicated for 20 consecutive cycles to compare $p_j = \frac{\tau}{\pi + e^{\gamma j}}$ (Jordan, 1958; Carlson, 1973) with $p_j = \frac{\tau}{\pi+1} j^{-\gamma}$ (Wright, 1936) and the result is shown in Figure 4 for $\tau = 40,\ \pi = 999,\ \gamma = 0.75$. Wright's learning curve leads to smaller quantities in the incipient phase. However, in practice, improvement is slow in this short phase making the S-shaped learning curve an appropriate model to use (Dar-El, 2000).

Table 3. Sensitivity analysis for the general model.

a	b	α	r	p_j	h_g	h_d	l	z	β	f_{2j}^*	Q_j^*	ω_j^*	W_j^*	w_j^*
1000	100200	500	50000	0.08524	20	5	1	20	25	0.06482	3550	5.4	5585464	362030
1000	100200	500	50000	0.08524	20	20	1	20	25	0.06397	3504	5.2	5586696	357370
1000	100200	500	50000	0.08524	20	5	1	40	25	0.06874	3762	3	5580407	383600
1000	100200	500	50000	0.08524	20	5	1	10	25	0.05827	3196	9	5595130	326040
1000	100200	500	50000	0.08524	20	5	1	20	50	0.06440	3528	5.5	5585745	359730
1000	100200	500	50000	0.08524	20	5	1	20	10	0.06505	3563	5.4	5585305	363310
1000	100200	500	50000	0.08524	20	5	1	20	0	0.06519	3571	5.6	5585202	364110
0	100200	500	50000	0.08524	20	5	1	20	25	0.06482	3550	5.4	5585464	362030
-1000	100200	500	50000	0.08524	20	5	1	20	25	0.06482	3550	5.4	5585464	362030
1000	100200	0	50000	0.08524	20	5	1	20	25	0.06612	3621	5.6	5583646	369190
1000	100200	-500	50000	0.08524	20	5	1	20	25	0.06751	3695	5.9	5581790	376800
0	100200	0	50000	0.08524	20	5	1	20	25	0.06612	3621	5.6	5583646	369190
0	100200	0	50000	0.08524	20	5	0	20	25	0.07511	4105*	0	5573127	418590
0	100200	0	50000	0.08524	20	20	0	20	25	0.07380	4034*	0	5574546	411390
0	100200	0	50000	0.00	20	20	0	20	25	0.07746	3873	0	5102460	395230

* The order quantity as in Wahab and Jaber (2010).

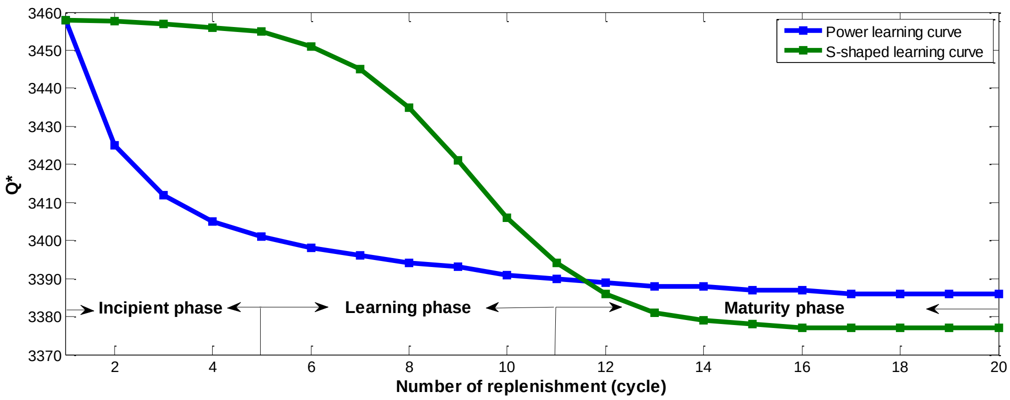

Figure. 4. A comparison of the optimal lot sizes for $p_j = \frac{40}{999 + e^{0.75 \times j}}$ and $p_j = \frac{40}{999+1} j^{-0.75}$.

As illustrated above, the dis-location of good and defective items together with other forms of varying demand, screening, defective and deterioration rates may be incorporated to allow managers to assess the consequences of a diverse range of strategies. The result obtained using the S-shaped learning curve coincides with the behaviour observed in many industrial situations. The proposed model is not limited to the above contributions; its formulation may trigger other applications as shown in section 5.2 below.

5.2 Special cases

5.2.1 Intra-cycle periodic review

It is often desirable to adjust input parameters to be responsive to a new policy due to acquired new knowledge. Such adjustment may occur due to the dynamic nature of demand, screening and deterioration rates or as a result of price fluctuations. Therefore, the periodic review is also beneficial for the purpose of illustrating what happens if the decision maker deviates from the optimal solution to assess the consequences of such a deviation. In this section, we present a step-by-step solution procedure to determine the optimal policy for intra-cycle periodic review applications.

For each periodic review:

1. Reset the new input parameters and obtain the optimal values using Eq. (21).
2. The optimal quantity that needs to be added to the on hand inventory for the next replenishment is given by

$$Q_{rj} = Q_j - I_{gj-1}(t_{rj}), \tag{22}$$

where t_{rj} is the time up to the periodic review.

From Eq. (22) we distinguish two cases.

Case 1: $0 \le t_{rj} < T_{1j-1}$.

Considering Eqs. (5)-(7) and (22) we have

$$Q_{rj} = \int_0^{T_{1j}} x(u)du - e^{-\left(g(t_{rj})-g(0)\right)} \int_0^{T_{1j-1}} x(u)du + e^{-g(t_{rj})} \int_0^{t_{rj}} \left[D(u) + p_j x(u)\right] e^{g(u)} du,$$

$$0 \le t_{rj} < T_{1j-1} \tag{23}$$

from which the number of units to be screened is given by

$$q_{rj} = Q_{rj} + \int_{t_{rj}}^{T_{1j-1}} x(u)du. \qquad 0 \le t_{rj} < T_{1j-1} \tag{24}$$

Note that the time T_{qj}, by which q_{rj} units are screened can be readily determined by $q_{rj} = \int_0^{T_{qj}} x(u)du$, where $q_{rj} \ge Q_{rj}$ and $s_{rj} = \int_0^{t_{rj}} x(u)du$.

Thus, the total cost per unit time of the underlying inventory system during the periodic review is adjusted as:

$$W(Q_j) = \frac{1}{f_{2j}}\Big\{cQ_{rj} + dq_{rj} + h_g\Big[-G(0)e^{g(0)}\int_0^{f_{qj}} x(u)du + \int_0^{f_{qj}} p_j x(u)G(u)e^{g(u)}du + \int_0^{f_{2j}} D(u)G(u)e^{g(u)}du\Big] + h_d\Big[f_{qj}\Big(\int_0^{f_{rj}} px(u)du + \int_0^{f_{qj}} px(u)du\Big) - \int_0^{f_{qj}} up_j x(u)du\Big] + k\Big\}. \tag{25}$$

It is worth noting here that the amount of defective items held up to the periodic review may be sold at a salvage price at time t_{rj}. In this case, we can set $t_{rj} = 0$ (without loss of generality) in Eq. (25) or it can be kept as is in Eq. (25) up to time T_{qj} by which the screening process ceases. Moreover, in the extreme case $t_{rj} = 0 \implies T_{qj} = T_{1j-1}$, then the LHS of (23) is equal to zero (recall (24), then the optimal values resulted from solving Eq. (21) constitute the optimal policy for the decision maker. Alternatively, Q_{rj} is to be substituted by q_{rj} in Eq. (25).

Case 2: $T_{1j-1} \le t_{rj} \le T_{2j-1}$.

$$Q_{rj} = \int_0^{T_{1j}} x(u)du - e^{-g(t_{rj})}\int_{t_{rj}}^{T_{2j-1}} D(u)e^{g(u)}du, \qquad T_{1j-1} \le t_{rj} \le T_{2j-1} \tag{26}$$

Note that $Q_{rj} = q_{rj}$, i.e. the items ordered to fulfil the demand, defects and deterioration during the planning horizon are the only ones that need to be screened (recall that the on hand inventory has been already screened).

Thus, the total cost per unit time of the underlying inventory system during the periodic review is adjusted to:

$$W(Q_j) = \frac{1}{f_{2j}}\Big\{(c+d)Q_{rj} + h_g\Big[-G(0)e^{g(0)}\int_0^{f_{qj}} x(u)du + \int_0^{f_{qj}} p_j x(u)G(u)e^{g(u)}du + \int_0^{f_{2j}} D(u)G(u)e^{g(u)}du\Big] + h_d\Big[f_{qj}\Big(\int_0^{f_{rj}} px(u)du + \int_0^{f_{qj}} px(u)du\Big) - \int_0^{f_{qj}} up_j x(u)du\Big] + k\Big\}. \tag{27}$$

From Eq. (27), the extreme case $t_{rj} = T_{2j-1} \Longrightarrow T_{qj} = T_{1j}$ (recall (26)).

Remark 1

The above suggested procedure is valid for $t_{rj} \in [0, T_{2j-1}]$ *as well as for the generalised models and the proposed idea can be further extended to be implemented in inventory* *mathematical modelling. Note that the structure of the model allows for both continuous and discrete periodic review.*

Let us now assume that the decision maker would like to change the current status within the fifth cycle. Here, we consider the same set of values as in the previous example (Example 1) except that a different demand rate is assumed, $r = 45000$, and the coordination regarding the on hand inventory for the fifth batch of defective items has been made, i.e. $p_6 = 0.07482$ is to be implemented. The optimal values of $Q_6^*, Q_{r6}^*, q_{r6}^*, T_{16}^*, T_{26}^*, T_{q6}^*$, and the total minimum cost are obtained for a given periodic review time say $t_{r6} = 0.0137 = 5$ days . In this periodic cycle, we have taken $p_6 = 0.07482$ resulting in a total number of $Q_6^* \cong 3348$ units, which is screened by time $T_{16}^* = 0.0334 \cong 12$ days and consumed by time $T_{26}^* = 0.0687 \cong 25$ days. The optimal quantity that needs to be added to the on hand inventory is $Q_{r6}^* = 617$ units. The number of units that need to be screened is $q_{r6}^* \cong 2784$ units, which is being done by time $T_{q6}^* = 0.0278 \cong 10$ days, by which point in time the total amount of defective items is accumulated. The total minimum cost per year is $W_{r6}^* = \$877640$ and the total minimum cost in this periodic cycle is $w_{r6}^* = \$60270$. The amount of defective items is $p_6 q_{r6}^* = 208$ units. Note that this amount is to be added to the previous defective items that have been accumulated during time $t_{r6} = 0.0137$, i.e. $p_5 s_{r6} = 103$ units, where both quantities constitute the total amount of defective items.

5.2.2 Perishable products

In real life settings, a large number of perishable items encounter spoilage and deterioration that occur out of sequence. This can be attributed to random lifetimes that are associated with the time elapsing for the items to flow through the supply chain. Packaged foods, seafood, fruit, baked goods, milk, cheese, processed meet, pharmaceutical and blood products, etc. would be examples of such items (Lashgari et al., 2016). To show that our model can be easily responsive to manage such perishable items, consider the amount ordered $\underline{Q_j} = \left(q_{mj}, q_{m-1j}, \dots, q_{0j}\right)$ where q_{ij} is the number of units with $i(i = 0,1, \dots, m)$ useful periods of shelf lifetime. The special case of shelf lifetime equal to zero refers to newly replenished items that have arrived already perished or items not satisfying certain quality standards (defective items). It is worth noting here that the assumption that each lot is subject to a 100% screening underpins such classification, where $p_j Q_j = q_{0j}$. It is often the case that the system is credited so that no outdating costs apply for this quantity. However, the potential interest exists so as to reduce the presence of both defective and already perished items in subsequent replenishments. Therefore, coordination can be made between inter-related entities from which we can set $p_{j+1} = \xi\left(\frac{q_{0j}}{Q_j}\right)$ without loss of generality. This is so, since such an assumption seems realistic given that any information gained from previous replenishments can be incorporated to enhance the subsequent delivery. Now, let ω_{ij} denote the quantity of the on hand inventory of shelf lifetime i that perishes by the end of period i. Thus, we have

$$\omega_{ij} = \begin{cases} q_{ij} - \left[D_{ij} - \left(\sum_{x=1}^{i-1} q_{xj} - \sum_{x=1}^{i-1} \omega_{xj} - \sum_{x=1}^{i} d_{xj}\right)\right], & D_{ij} < \left(\sum_{x=1}^{i} q_{xj} - \sum_{x=1}^{i-1} \omega_{xj} - \sum_{x=1}^{i} d_{xj}\right) \\ 0 & \text{otherwise,} \end{cases}$$

where D_{ij} is the actual demand observed up to the periodic review i, and d_{ij} is the number of items of shelf lifetime i that deteriorate while on storage. Hence, $\sum_{i=1}^{m} \omega_{ij}$ denotes the total sum of inventory that perishes in cycle j, excluding any replenished items that have arrived already perished, and $\sum_{y=i}^{m} d_{yj}$ refers to the total sum of deteriorated items in period i, i.e. an item may not retain the same utility throughout its shelf lifetime. Therefore, the two amounts that need to be discarded in each periodic review i are ω_{ij} and $\sum_{y=i}^{m} d_{yj}$.

Assuming an automated inventory control system, the observation of D_{ij} seems realistic since all items are tracked. Thus, the information gained so far, collectively, constitutes a means by which the input parameters can be known and then may or may not be adjusted. Note that $\underline{Q_{ij}} = \left(q_{mj}, q_{m-1j}, \dots, q_{0j}\right)$ and the amounts ω_{ij} and $\sum_{y=i}^{m} d_{yj}$ are known and that D_{ij} is fulfilled based on a FEFO policy. Then we have

$$I_{gij}(t_{ij}) = \begin{cases} Q_j - q_{0sj} - D_{ij} - \sum_{x=1}^{i} \omega_{xj} - \sum_{x=1}^{i} \sum_{y=x}^{m} d_{yj}, & 0 \le t_{ij} < T_{1j}, \\ (1-p_j)Q_j - D_{ij} - \sum_{x=1}^{i} \omega_{xj} - \sum_{x=1}^{i} \sum_{y=x}^{m} d_{yj}, & T_{1j} \le t_{ij} \le T_{2j}, \end{cases} \quad (32)$$

$$\Leftrightarrow I_{gij}(t_{ij}) = \begin{cases} \left(q_{m-ij}, q_{m-i-1j}, \dots, q_{1j}, q_{0rj}\right), & 0 \le t_{ij} < T_{1j}, \\ \left(q_{m-ij}, q_{m-i-1j}, \dots, q_{1j}\right), & T_{1j} \le t_{ij} \le T_{2j}, \end{cases}$$

where $q_{0rj} = \int_{t_{ij}}^{T_{1j}} p_j x(u)du$, $q_{0sj} + q_{0rj} = p_j Q_j$, and

$$\underline{Q_{ij}} = \underline{Q_{j+1}} - \underline{I_{gij}\big(t_{ij} + \Delta\big)}. \tag{33}$$

The necessary condition to place an order is given by

$$I_{gij}\big(t_{ij}\big) \le \big(D_{ij} + \omega_{ij} + \textstyle\sum_{y=i}^{m} d_{yj} - D_{i-1j}\big)\Delta, \tag{34}$$

with a lead-time $\Delta\big(\Delta \le T_{2j} - t_{ij}\big)$, the initial amount $D_{0j} = 0$ and t_{ij} being the time up to the periodic review. If condition (34) holds true for periodic review i, then Eq. (33) calculates the next optimal replenishment quantity that needs to be added to the on hand inventory (given by (32)). In Eq. (34), the quantity $\big(D_{ij} + \omega_{ij} + \sum_{y=i}^{m} d_{yj} - D_{i-1j}\big)$ is taken as an approximation for the behaviour of inventory fluctuation during the lead-time Δ. Note that if $\underline{Q_{ij}} = \underline{Q_{j+1}}$, then we may assume that unsatisfied demand is lost. On the other hand, if demand is fulfilled based on a record of known quantity, then the unsatisfied demand $D_{lj} = \big|I_{gij}\big(t_{ij} + \Delta\big)\big|$ is known and consequently any relevant cost may apply. In this case, $(1-\varphi)\big|I_{gij}\big(t_{ij} + \Delta\big)\big|$ forms the lost sales quantity with a fraction $\varphi(0 \le \varphi \le 1)$ being backordered, i.e. $I_{gij}\big(t_{ij} + \Delta\big) < 0$.

We now introduce a third example where we consider the values summarised in Table 4 below for an item with a maximum shelf life-time $m = 5$, i.e. $i \in [0,5]$.

Table 4. Input parameters for Example 3.

c	d	h_g	h_d	k	a	b	α	r
10	0.2	0.4	0.1	300	500	20000	50	700
\$/unit	\$/unit	\$/unit/week	\$/unit/week	\$/cycle	unit/week	unit/week	unit/week	unit/week
l	z	β	τ	π	γ	m	Δ	
1	20	2	1	49	0	5	1	
unit/week	unit/week	unit/week	unit/week	unit/week	unit/week	days	days	

The optimal values of $Q_j^*, T_{1j}^*, T_{2j}^*, \omega_j^*$, and the corresponding total minimum cost is obtained. The system parameters specified in Table 4 yield a lot size of $Q_1^* \cong 518$ units, which is screened by time $T_{11}^* = 0.026 \cong 0.18$ days and consumed by time $T_{21}^* = 0.695 \cong 4.8$ days. The total minimum cost per week is $W_1^* = \$\,8136$ and the total minimum cost per cycle is $w_1^* = \$5650$. The amount of defective items is $p_1 Q_1^* = 10.4$ units and the amount of outdated (spoiled) and /or deteriorated items is $\omega_1^* = 9$ units. If it is beneficial to operate on a discrete cycle length of a complete period, then $W_1^* = \min\left[W_1 = \frac{w}{0.5714}, W_1 = \frac{w}{0.7143}\right]$. The optimal order quantity in this case is $Q_1^* = 533$ units, which is given by $W_1^* = \frac{w_1^*}{0.7143} = \$8136$ per week to satisfy the demand, defects and deterioration for 5 days. This quantity is screened by time $T_{11}^* = 0.027 \cong 0.19$ days and the total minimum cost per cycle is $w_1^* = \$5812$. The amount of defective items is $p_1 Q_1^* = 10.6$ units and the amount of outdated (spoiled) and/or deteriorated items is $\omega_1^* = 9.6$ units.

Suppose that after a 100% screening the lot size is classified based on a FEFO policy and is found to be on the set $\underline{Q_1} = (120,114,134,91,67,7)$, which corresponds to a 5 day policy, i.e. $Q_1^* = 533$ units. Now, let us assume that at the end of the first day the relevant information gathered indicates that $D_{11} = 63$, $\omega_{11} = 3$ and $\sum_{y=1}^{5} d_{y1} = 1 + 0 + 1 + 0 + 0 = 2$, then $I_{gr1}(0.1429) = 526 - 63 - 3 - 2 = 458$ units. The necessary condition to place an order is $I_{gr1}(t_{11}) \leq \left(D_{11} + \omega_{11} + \sum_{y=1}^{5} d_{y1} - D_{01}\right)(T_{21} - t_{11})$, but $I_{gr1}(0.1429) > 68(4)$ and consequently we do not place an order. Suppose that after the third day we have:

$D_{21} = 151$, $D_{31} = 276$, $\omega_{31} = 7$, $\sum_{i=1}^{3} \omega_{i1} = 12$, $\sum_{y=3}^{5} d_{y1} = 0 + 1 + 0 = 1$ and $\sum_{x=1}^{3} \sum_{y=x}^{5} d_{y1} = 1 + 1 + 2 + 2 + 1 = 7$, then $I_{g41}(0.429) = 526 - 276 - 12 - 7 = 231$ units and $I_{gr1}(0.429) < 134(2)$.

Thus, an order must be placed in which Eqs. (21) and (33) can be used to obtain the optimal replenishment quantity that takes into account a suitable adjustment to avoid lost sales. As such, approximations for the demand and deteriorating rates, say $\hat{r} = \widehat{D_j} = \frac{\sum_{i=1}^{m}(D_{ij} - D_{i-1j})}{m}$, $\hat{\alpha} = \max\left[0, \alpha\left(\frac{Q_{j+1}}{Q_j}\right)\right]$, and $\hat{z} = z\frac{\sum_{i=1}^{m} d_{ij} + \sum_{i=1}^{m} \omega_{ij}}{\omega_j^*}$ may be employed. Note that if a record is kept for the actual demand requested, then the unsatisfied demand is given by D_{lj}, where φD_{lj} is backordered and the rest $(1 - \varphi) D_{lj}$ is lost.

Finally, let us assume that VOI such as TTH of an item as it moves through a supply chain is transferable within that supply chain. In this case, the remaining shelf lifetime can be readily calculated. For example, Bremner (1984) and Ronsivalli and Charm (1975) developed a shelf lifetime model for fresh fish that links the spoilage rate to a given temperature. Let $℃_y$ and t_y denote respectively, the temperature and time elapsed of an item in a supply chain entity y, then the remaining shelf lifetime is given by

$$L = M - s(℃_a)t_a - s(℃_b)t_b,$$

where

$$s(℃_y) = (0.1℃_y + 1)^2.$$

If this VOI is available to the next supply chain entity x, then a significant reduction in the cost per cycle can be achieved (Ketzenberg et al., 2015). In our model, the VOI can be perceived at external and/or internal domains of coordination. At the domain of external coordination, this model addresses the VOI to capture a safe remaining shelf lifetime and acknowledges the potential impact of transporting and handling of a product at both external and internal levels. Hence, the reflection of the VOI can result in a reduction of the percentage of imperfect items that may arrive already perished and/or defective, i.e. $f_{2j} \leq L,\ \delta(t) = \frac{l}{z(1+\gamma)-\beta t},\ p_j = \frac{\tau}{\pi + e^{\gamma j}},\ \gamma = \frac{\phi}{M}$ and $\phi = [s(℃_a)t_a + s(℃_b)t_b]$.

To illustrate this, consider the same set of values as in the previous example (Example 3) and let $M = m + t_a + t_b, ℃_a = 3, t_a = 2, ℃_b = 0$ and $t_b = 2$, then $L = 9 - 3.38 - 2 = 3.62 \cong 4$ days. If this information is available, then the optimal quantity with shelf lifetime $i \in [0,4]$ is $Q_1^* = 420$ units, which is consumed in 4 days. The total minimum cost per week is $W_1^* = \$\,8108$ and the total minimum cost per cycle is $w_1^* = \$\,4633$. The amount of defective items is $p_1 Q_1^* = 8.3$ units and the amount of outdated (spoiled) and/or deteriorated items is $\omega_1^* = 3.7$ units. Thus, with the VOI, a reduction of $c_s = \$1460/$ year can be achieved for this single item.

Remark 2

The proposed model is viable for the case in which items are classified based on their quality, size, appearance, freshness, etc. In this case, a distinct selling price s_i may be linked to its corresponding quantity q_{ij}, i.e. $\underline{S} = (s_m, s_{m-1}, \dots, s_0)$ is applied for the set $\underline{Q_j} = (q_{mj}, q_{m-1j}, \dots, q_{0j})$. Further, it is still applicable if an item loses partially its value based on *its perceived actuality (obsolescence). Here ω_{ij} can be kept on store at a discounted price and $s(°\mathrm{C}_y)$ is based on the shelf lifetime model suitable for the item ordered.*

5.2.3 Renewal theory

When it comes to defective items, according to the academic literature, a random proportion of such items is usually assumed with a known probability distribution. Hence, from (25) we have

$$E(W) = \frac{E(w)}{E(f_{2j})} = \frac{E\left(w'_{Q_j}\right)}{E\left(f'_{2j,Q_j}\right)}, \tag{35}$$

where $(1 - E[p_j])x(t) > D(t)$.

Eqs. (2), (18), (22), (23) and (27) can be used to find Eq. (35).

If $\delta(t) = \frac{l}{z} = \theta, D(t) = r,$ and $x(t) = b$, then we have

$E[f_{2j}] = \frac{\log\left(\frac{E[K]\theta}{r}\right)}{\theta}, E[K] = \left(bf_{1j} - \frac{E[p_j]e^{\theta f_{1j}b}}{\theta}\right) + \frac{E[p_j]b}{\theta} + \frac{r}{\theta}.$

In this paper, we have introduced the assumption that defective items are stored in a different warehouse. This assumption relaxes the behaviour of the inventory level that is presented by Jaggi and Mittal (2011) and Moussawi-Haidar et al. (2014). This is because not every defective item can be sold at a salvage price; rather defective items may encounter a disposal cost. For comparison purposes, Table 5 presents the input parameters of the example used by Jaggi and Mittal (2011). Their model leads to $Q^{JM} = 1283$ units, which is larger than the optimal quantity obtained using Eq. (21) in our paper, i.e. $Q^* = 1167$ units.

Table 5. Input parameters for comparison examples for renewal theory ($p \sim U[0, \ell]$, E $[p] = \int_0^\ell pf(p)\,dp = \frac{\ell}{2}$).

c	d	h	k	r	b	l	z	ℓ
25	0.25	5	100	50000	175200	1	10	0.04
\$/unit	\$/unit	\$/unit/year	\$/cycle	unit/year	unit/year	unit/year	unit/year	

Similarly, Moussawi-Haidar et al. (2014) use the same set of values from Jaggi and Mittal (2011) except from the following parameters: $d = \$0.5$, $z = 20$, where $Q^{M\ et\ al} = 1280$ units, which is greater than our optimal $Q^* = 1278$ units. In both papers, the objective is to maximise the total profit per unit time, where $s = \$50$ $(v = \$20)$ is the selling price for a good (defective) item. Therefore, the maximum total profit per year, is set equals to:

$TPU^*(Q) = \frac{s(Q^*(1-p)-\omega^*)+vpQ^*}{T_2^*} - W^* = \frac{50\times(1166.8\times0.98-1.3)+20\times(1166.8)\times0.02}{0.0228} - 1297016 = \$1223400 < TPU(Q)^{JM} = \$1224183.$

On the other hand, when $d = \$0.5$, $z = 20$, the corresponding maximum total profit per year is reduced to $TPU(Q)^{M\ et\ al} = \$1211414 \cong TPU^*(Q) = \$1211415.$

Although the difference in the order quantities is negligible between the two compared papers, our model produces larger quantity when the deterioration rate decreases, which supports the findings presented by Moussawi-Haidar et al. (2014).

Now, if $\delta(t) = 0$, then $\frac{E(w)}{(\alpha E[f_{2j}]+r)E[f_{2j}]} = \frac{E\left(w'_{Q_j}\right)}{(1-E[p_j])}$, $E[f_{2j}] = \frac{-r+\left(r^2+2\alpha Q_j(1-E[p_j])\right)^{\frac{1}{2}}}{\alpha}$.

For simplicity, let $D(t) = r, x(t) = b,$ and $\delta(t) = 0,$ then Eq. (35) reduces to the model of Wahab and Jaber (2010) as follows: $Q_j^* = \sqrt{\frac{2rk}{h_g E\left[(1-p_j)^2\right]+\frac{rE[p_j]}{b}[h_g+h_d]}}$.

For $h_g = h_d = h$, it reduces to the work of Jaber et al. (2008) and with $p_j = p$ it yields the optimal order quantity presented by Salameh and Jaber (2000) and (Maddah and Jaber (2008) as follows: $Q_j^* = \sqrt{\frac{2rk}{hE\left[(1-p_j)^2\right]+\frac{2hrE[p_j]}{b}}}$. Finally, if $p_j = 0$, then, $Q_j^* = \sqrt{\frac{2rk}{h}} = \mathrm{EOQ}.$

6. Implications and managerial insights

In this section we emphasise the financial implications and managerial aspects of our work and we offer a number of cases to illustrate the efficiency of our mathematical model.

6.1. Model overview

The general model developed in this paper reflects a number of practical concerns with regards to product quality related issues and may assist operations managers to respond to many real world challenges/opportunities for inventory improvements. Those opportunities include poor supplier service levels (imperfect items received from suppliers), potential dis-location of good and defective items (different warehouses for the good and defective items), tracking the quality of perishable products in a supply chain and transfer of knowledge from one inventory cycle to another. Furthermore, it provides a general procedure for continuous intra-cycle periodic reviews so as to adjust and control flows of raw materials, component parts and finished goods to maintain sustainable competitive advantage. This formulation could also potentially be of some value to software manufactures since it forms a generalised model inclusive of many existing ones.

6.2. Coordination mechanisms

The generic nature of the model explores various coordination mechanisms that may improve inventory management as shown below.

Let $\dot{D}(t) = (q - c_1 e^{-c_d g})D(t) > 0$, where $\dot{D}(t)$ is the demand based on an acceptance quality level $q(q_{min} \leq q \leq 1)$ and a discount rate $g(0 \leq g \leq 1)$ for a cut-price c_d offered by the supplier for a single purchased item and c_1 is a positive parameter. The case of $c_1 = 0$ implies that $\dot{D}(t) = qD(t)$, and the case of $c_1 = 0$ and $q = 1$ reflects an independent demand function, where $\dot{D}(t) = D(t)\ \forall t \geq 0$. Any item that does not satisfy the minimum acceptance quality level q_{min} is considered to be a defective item. This function may apply for a demand-driven pricing model assumed by the supplier for which a unit purchasing price $\dot{c} = ce$, where e can take the from $e = q - c_d g > 0$ or $e = q - \frac{c_d g}{q} > 0$. The case $q = 1$ applies for a discounted purchasing price, where $e = 1 - c_d g$, and the case of $g = 0$ and $q = 1$ reflects an independent purchasing cost, where $\dot{c} = c$. Note that $\dot{D}(t)$ increases (decreases) as the acceptance quality level and/or discount rate increases (decreases). Such a contract unifies three managerial decisions strategies that govern both the supplier and the retailer, i.e. the acceptance quality level, unit discount rate and unit purchasing price. Moreover, it encourages the supplier to invest in quality innovation to maintain sustainable product quality levels that may reduce defects per shipment in order to maximise its discounted stream of net revenue. Further, for the case of $e = q - c_d g$, or $e = q - \frac{c_d g}{q}$, the supplier would reap the benefit of improving quality levels by increasing the purchase price, while simultaneously the retailer would incur an additional charge payable to the supplier in order to receive better quality items.

Another demand function that can be increased by improving the quality level and decreased by increasing the unit purchasing price may be implemented, where $\ddot{D}(t) = D(t) - c_1\ddot{c}e^{-c_2 q} > 0$ and c_2 is a positive parameter (Vörös, 2002). A similar demand function that depends on price and quality may take the form $\dddot{D}(t) = D(t) - c_1\dddot{c} + c_2 q$, where $\dddot{D}(t)$ increasers with an acceptance quality level $q(q_{min} \leq q \leq 1)$ and decreases with a unit purchasing price $\dddot{c}$ (Chenavaz, 2012).

For all of the above scenarios, a unit purchasing price $\dot{c} = \ddot{c} = \dddot{c} = cq$ can be incorporated as well. In the last two demand functions, the case of $c_1 = c_2 = 0$ reflects an independent arbitrary demand function.

In a decentralised coordinated scenario, where the supplier and the retailer cooperate in order to render the total minimum (maximum) cost (revenue) closer to that associated with a centralised one, a selling price for the retailer say $c_3 s e^{-c_4 s}$ can be assumed with $s \geq \dot{c}$ and c_3 and c_4 being positive parameters (Smith and Achabal, 1998; Roy et al., 2015). It is worth noting here, that c_4 must be chosen such that $\frac{1}{c_4} > \dot{c}$.

6.3. Stochastic parameters

It is often the case that input parameters are randomly distributed. The versatile nature of our model accommodates such randomness. Let D_j be a random variable of the demand that is predetermined according to the information gained by the supplier due to its coordination as an output of the j^{th} inspection process. Suppose that $D_j \sim U\left[\mu_j - \sqrt{3}\sigma_j, \mu_j + \sqrt{3}\sigma_j\right]$. It is clear that $E(D_j) = \mu_j = D(t) = r$ (Modaka et al., in press). Similarly, $E(x_j) = x_j = x(t) = b$, and $E(\delta_j) = \delta_j = \delta(t) = \frac{l}{z}$, which is the case provided in section 5.2.3. Note that D_j and hence the actual yield of may vary from one cycle to another (e.g. the parameters are nonstationary).

If q is assumed to be a random variable with known mean and variance, then the yield at the supplier site would be represented by a random draw from a quality distribution. If this is the case, then the yield is simultaneously influenced by internal and external randomness.

6.4. A 100% inspection and sampling test

There is no doubt that many products require inspection, so as to guarantee an appropriate service to the customers. In addition, such inspection is essential to update the Information System records with good items that are actually available in stock in order to satisfy demand. Further, when raw materials are required in a production setting, their ordering policy depends on the production batch size of the products that require such raw materials. Therefore, the presence of defective items in raw materials has a direct impact on the production batch size. Moreover, there exists a plethora of factors that may force supply chain management to initiate both an inspection process and periodic review to enhance productivity, improve profitability, meet total product demand and avoid the tarnished reputation associated with product recalls (Klassen and Vereecke 2012). A 100% inspection may eliminate the return service cost caused by defective items. It can be used in real-life settings where the impact of letting through defective items could be severe. Different types of inspection can occur including seal inspection, outer case label inspection or damaged carton inspection. The service cost may include goodwill cost, transportation cost, and re-processing cost, etc. and that may affect all supply chain members. The assumption that each lot undergoes a 100% inspection implicitly applies to any smaller amount of the lot. For example, let ϵ be a fraction of the amount ordered representing a random sample size drawn from the batch. It is clear that $\epsilon x(t)$ can also be implemented in the model.

6.5. Further implications

If safety issues exist in keeping defective items in store, then the model formulation allows for an immediate disposal of defective items, i.e. $h_d = 0$.

In practice, the actual consumption period is random and, consequently, $t_{rj}\left(T_{1j-1} \le t_{rj} \le T_{2j-1}\right)$ can be used to represent the actual cycle length. If $I_{gj-1}\left(t_{rj}\right) > 0$, then the subsequent replenishment is cycle dependent, where Eq. (27) can be used to derive the optimal lot size.

7. Conclusion

In this paper, a general Economic Order Quantity model for items with imperfect quality was presented. Each lot is subjected to a 100% screening and items not conforming to certain quality standards are stored in a separate facility with different holding costs of the good and defective items being considered. The obtained numerical results reflect the learning effects incorporated in the proposed model. The presence of product deterioration and varying demand and screening rates significantly impact on the optimal order quantity.

The paper presents illustrative examples and special cases to support application of the model and solution procedures in different realistic situations. The proposed solution procedure to determine the optimal policy for continuous intra-cycle periodic review takes into account different inventory fluctuations during the planning horizon. We observe the effect of changing all model parameters and find that a reduction in the optimal order size does not necessarily lead to a lower total minimum cost per unit time.

This study intersects the areas of fixed and random lifetimes of perishable products, where unsatisfied demand may or may not be lost, products may arrive already perished, and orders may not perish in sequence. The accuracy of RFID temperature tags that capture the TTH, and the use of that TTH data are adopted to model a shelf lifetime of an item.

The generality of our model stems from the fact that the demand, screening, and product deterioration rates are arbitrary functions of time. The proposed model unifies and extends the academic literature related to imperfect quality items, which is quite diverse in nature. Practical examples that are published in the literature for generalised models are used to demonstrate that the solution quality is the same as in published sources or in some cases produces better results, i.e. the validity and realistic qualities of the general model are ascertained. The versatile nature of our model and the fact that it may accommodate many real world concerns has been emphasised, where the results obtained are compatible with the behaviour observed in many real-life settings. A mathematical proof is presented, which shows that the solution to the underlying inventory system, if it exists, is unique and global optimal. Coordination mechanisms that may affect the supplier and the retailer are also explored to improve inventory management at both echelons. To the best of our knowledge, this appears to be the first time that such a general EOQ model is formulated, investigated, and numerically verified.

Based on the findings of this paper, several interesting extensions are possible such as considering that the screening rate follows learning and forgetting curves with allowed shortages and the risk of failure during screening (Type I and Type II errors). Also, it seems plausible to assess the formulation of a two warehouse system (due to the capacity limitations of the owned warehouse), where a comparison between Last-In-First-Out (LIFO) and First-In-First-Out (FIFO) dispatching policies that are governed by a fixed shelf life time may be implemented. In addition, considering different supplier trade credit practices such as a permissible delay in payment and formulation of an EPQ model in which product quality levels depend on an instantaneous cost of investing in product innovation are interesting lines of further inquiry in this area. All of the above suggested next steps of research can be addressed for finite or infinite planning horizons.

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