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Predicting coastal morphological changes with empirical orthogonal function method

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Abstract

In order to improve the accuracy of prediction when using the empirical orthogonal function (EOF) method, this paper describes a novel approach for two-dimensional (2D) EOF analysis based on extrapolating both the spatial and temporal EOF components for long-term prediction of coastal morphological changes. The approach was investigated with data obtained from a process-based numerical model, COAST2D, which was applied to an idealized study site with a group of shore-parallel breakwaters. The progressive behavior of the spatial and temporal EOF components, related to bathymetric changes over a training period, was demonstrated, and EOF components were extrapolated with combined linear and exponential functions for long-term prediction. The extrapolated EOF components were then used to reconstruct bathymetric changes. The comparison of the reconstructed bathymetric changes with the modeled results from the COAST2D model illustrates that the presented approach can be effective for long-term prediction of coastal morphological changes, and extrapolating both the spatial and temporal EOF components yields better results than extrapolating only the temporal EOF component.

Keywords: EOF method; Coastal morphological change; Long-term prediction; Process-based numerical model; Shore-parallel breakwater

1. Introduction

Coastal defense structures represent an effective measure against coastal erosion. These structures include sea walls, breakwaters, groynes, or a combination of the above, in addition to soft engineering approaches, such as beach nourishment. All of these structures and approaches have advantages and disadvantages, and coastal engineers must study each case and propose the best solution for a particular site. However, the ways these structures affect the shoreline in the long term and their impacts on the adjacent coastal areas are not always clear.

In particular, shore-parallel breakwaters have proven to be an effective way to mitigate coastal erosion. However, the construction of coastal structures can sometimes produce unexpected problems, which may result in an increase of the maintenance cost along that stretch of coast or even generation of new erosion issues beyond the protected areas (Dolphin et al., 2012). Therefore, coastal dynamics should be fully studied during design of these structures.

In past decades, most of the experience regarding shore-parallel breakwaters has been gathered at locations with micro-tides, tidal ranges smaller than 2 m. In these places, design criteria drawn from field experience work accurately, and the effect of coastal defense structures can be assessed with confidence. However, for locations with meso- or macro-tides, field experience regarding shore-parallel breakwaters is limited, and classical design criteria may not always lead to high accuracy (Johnson et al., 2010).

There are different approaches to studying the shoreline evolution behind a group of shore-parallel breakwaters. Process-based numerical models have been widely used in recent decades. They are powerful tools for understanding the hydrodynamics in detail in the areas surrounding breakwaters, but they are less suitable for long-term simulation (De Vriend and

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Ribberink, 1996; Du et al., 2010; Roelvink and Reniers, 2011). Reasons include not only the high running cost of the process-based model for long-term simulation, but also the nonlinearity of coastal processes, causing accumulation of numerical and physical errors (Larson et al., 2003). Other models, such as the one-line model, may be more appropriate for long-term prediction, due to the reduced physical processes in the model. The one-line model is based on the continuity of alongshore sediment transport fluxes and ignores cross-shore sediment transport (Dean and Dalrymple, 2001). Wang and Reeve (2010) presented an application of the one-line model to shore-parallel breakwater schemes at Sea Palling (Norfolk, UK), and showed strong long-term prediction of shoreline changes. If the field data are abundant, data-driven methods such as the empirical orthogonal function (EOF) method can also be used to investigate, in a qualitative manner, the way the shoreline evolves under the wave and tidal conditions (Miller and Dean, 2007). However, as a non-physical process-based approach, the EOF method presents a number of limitations. For instance, the EOF method requires a data set consistently distributed in space and time, which is always impossible to obtain (Fairley et al., 2009). Also, the EOF method provides a means of discussing the behavior of the studied variable within the sampling period, but results cannot, in principle, be simply extrapolated beyond that period. In the work of Horrillo-Caraballo et al. (2014), extrapolation of temporal EOF components was conducted for prediction of shoreline changes beyond the training period, but with limited success. Alvarez and Pan (2014) found that, in order to reconstruct the morphological variables more accurately, both the spatial and temporal EOF components must be extrapolated.

This study attempted to refine and improve the extrapolation method proposed by Alvarez and Pan (2014), in order to improve the accuracy of prediction using the EOF method. A process-based numerical model, COAST2D, was first run for a certain period of time to produce the full morphological evolution within the computational domain (Pan et al., 2005; Du et al., 2010). EOF analysis was performed for several periods of time with a regular time increment within the sampling period. Spatial EOF components were extrapolated by fitting both linear and exponential functions. Temporal EOF components were also extrapolated. The extrapolated EOF components were then used to reconstruct bathymetric changes. The reconstructed bathymetric changes were compared with the modeled results from the COAST2D model to check the accuracy.

2. EOF method

The EOF method can be used to calculate a set of orthogonal functions (eigenvectors), representing both the spatial and temporal components, which can be used to reconstruct the original data set at any point during the studied period. Each component represents a percentage of the total variation of a given variable (Larson et al., 2003). The set of functions obtained is sorted, so the first couple of functions, including the spatial and temporal components, represents the most significant part of the variation of the variable. The EOF method also guarantees that the number of functions is lower than that in other methods. It can provide the spatial and temporal patterns of variation. These features make the EOF method a simple and objective method for analyzing shoreline evolution.

While a detailed description of the method can be found in Jollife (2002), and an example of EOF analysis of coastal morphological changes can be found in Muñoz-Perez et al. (2001), a brief description is provided here. Let $\boldsymbol{F}$ be an $n \times m$ matrix containing the data of n points along a shoreline during m surveys. Each point represents the position of a given variable. The EOF method requires two series of functions, $X(n,i)$ and $T(m,i)$, to describe the spatial and temporal components of the variable, respectively:

$$d(x=n,t=m)=\sum X(n,i)T(m,i) \tag{1}$$

where $d(x=n,t=m)$ is the variable value at point n and time of m surveys, and i represents the number of components/functions considered. To obtain $X(n,i)$ and $T(m,i)$ for component i, the eigenvalue and vector problem in Eq. (2) have to be solved:

$$\begin{cases}(\boldsymbol{A}-\lambda\boldsymbol{I})X=0 & \boldsymbol{A}=\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}\\(\boldsymbol{B}-\lambda\boldsymbol{I})T=0 & \boldsymbol{B}=\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F}\end{cases} \tag{2}$$

where λ represents the eigenvalue for the system, and $\boldsymbol{I}$ is the identity matrix. Therefore, by applying the EOF method to prediction of shoreline changes, for instance, it is possible to reproduce the shoreline behavior within the surveyed period using a reduced number of orthogonal functions. However, in order to obtain satisfactory results of long-term prediction of coastal morphological changes using the EOF method, a certain quantity and high quality of field data are required to obtain the functions $X(n,i)$ and $T(m,i)$ at the desired spatial and temporal resolutions. This, however, may not be always available. As an example, Fairley et al. (2009) applied the EOF method to a shore-parallel breakwater scheme at Sea Palling to gain insights into coastal morphological changes at the site using shoreline positions measured with an Argos (video imaging) system. Results showed that the first two couples of the spatial and temporal EOF components only represent 59% and 16% of the total variation of the coastal morphological changes, respectively. When there are insufficient field data available for the coastal scheme to be built, it is difficult to perform EOF analysis, and results will be inaccurate. Therefore, in this study, a process-based numerical model was run in a well-controlled environment to generate sufficient and accurate data for EOF analysis, through which long-term prediction of bathymetric changes could be achieved.

3. COAST2D model

The COAST2D model is a two-dimensional (2D) depth-averaged hydrodynamic and morphodynamic model, which has been well validated during its development and refinement (Pan et al., 2005; Du et al., 2010). The model consists of a number of fully interactive modules, mainly the following: a wave module to determine the wave period-averaged wave energy or wave height and the wave propagation direction for wave transformation from offshore to nearshore areas; a current module to compute the depth-integrated current velocity and water surface elevation under tidal and wave actions; and a morphological module to compute the sediment transport rates using equilibrium formulas, as well as the resulting bed level changes. In the model, the wave-current and hydrodynamic-morphological interactions are considered. Further information on the governing equations for the model can be found in Pan et al. (2005) and Du et al. (2010). A COAST2D model was set up over a domain containing four shore-parallel breakwaters, as shown in Fig. 1.

A mesh was composed of 241 × 111 nodes, and grid cells were 25 m by 15 m in the alongshore and cross-shore directions, respectively. It covered a coastal area of 6025 m along the shore and 1665 m perpendicular to the shore. The offshore water depth was set to 15 m, and the initial beach slope was set to 1:50. The breakwater scheme, which was similar to that at Sea Palling, consisted of four breakwaters (B1 through B4) parallel to the shore, located approximately 200 m from the initial shoreline (denoted with the dashed line in Fig. 1). Each breakwater was 200 m long with a gap of 250 m between them. The crest of the breakwaters was set to 3 m above the mean sea level. The size of the bed sediment was assumed to be 250 μm.

In order to reduce the computation cost of the EOF method applied several times to the 2D domain, EOF analysis was limited to a sub-mesh with 142 × 72 nodes or an area of 3550 m along the shore and 1080 m perpendicular to the shore within the computational domain, centered in the breakwater area, as shown in Fig. 1.

Incident waves, 2 m in height and 6 s in period, were imposed along the offshore boundary. The incident wave direction was 30° from the normal direction of the shoreline. Stationary M2 tides with a tidal range of 3 m were used during the simulation.

4. Methodology

In this study, the COAST2D model was run for 1500 h, producing the full morphological evolution within the computational domain. The results of the first 750 h were used for EOF analysis, and the resulting EOF components were extrapolated to reconstruct bathymetric changes at 1500 h. Then, the reconstructed bathymetric changes were compared with the modeled results from the COAST2D model at 1500 h to check the accuracy. The EOF method was used to obtain bathymetric changes for each computational cell within the red rectangle in Fig. 1.

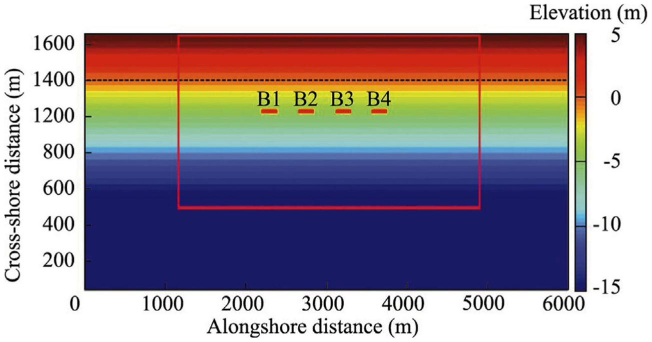
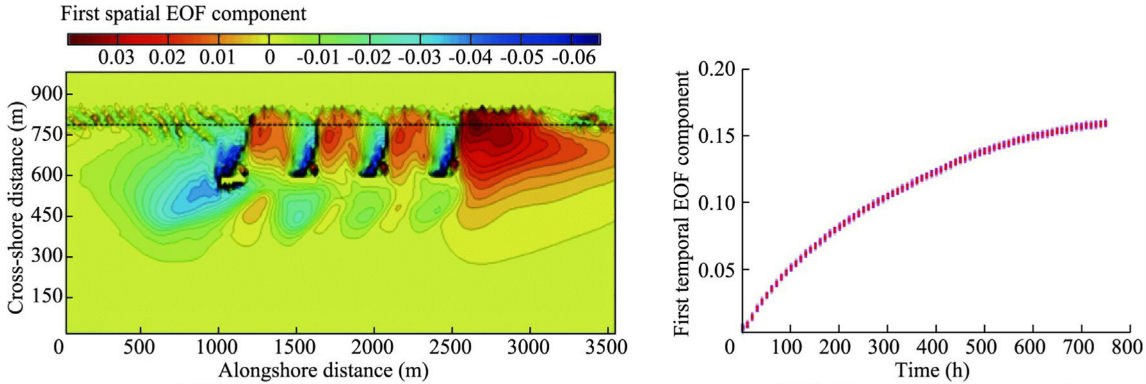


Fig. 1. Computational domain and reduced domain (red rectangle) for EOF analysis.

Fig. 2(a) shows the first spatial EOF component distribution from EOF analysis at 750 h, and Fig. 2(b) shows the first temporal EOF component for the period of 750 h. In this study, the first spatial and temporal EOF components represented 90% of the total variation.

Fig. 2 shows that the first spatial EOF component presents a complex pattern affected by the breakwaters, clearly representing areas of erosion and accretion with positive and negative values, respectively. The first temporal EOF component presents an exponential trend describing the manner of the shoreline evolving from an initially flat beach profile to an equilibrium state under the given conditions. In Alvarez and Pan (2014), the variations of these components were examined, and two approaches were used to extrapolate the EOF components for long-term prediction. In the first approach, only the first temporal component was extrapolated to 1500 h using the exponential function proposed by Alvarez and Pan



(a) First spatial EOF component (b) First temporal EOF component

Fig. 2. First spatial and temporal EOF components for 750-h simulation.

(2014). The extrapolated temporal component, together with the first spatial component at 750 h (without extrapolation), was used to reconstruct shoreline changes at 1500 h. The second approach was to extrapolate both the first spatial and temporal EOF components from 750 h to 1500 h, so that shoreline changes could be properly reconstructed using the extrapolated EOF components at 1500 h. In the latter approach, in order to extrapolate the spatial component at each computational cell, EOF analysis had to be performed progressively, as described in later sections in detail. For the 2D computational domain, EOF analysis involved some extensive computations in analyzing a three-dimensional matrix: two dimensions for the spatial component and one dimension for the temporal component. In order to simplify EOF analysis, a dimension reduction proposed by Reeve et al. (2001) was used.

Generally, before EOF analysis is performed, data are demeaned. When such an operation is done, the EOF components represent variations over the mean value. This approach is commonly employed in studying steady processes or changes around an equilibrium state. However, in this study, changes from an initially flat beach were studied. In this situation, the mean value did not represent an equilibrium state, as the beach continuously evolved during the experiment. Instead of demeaning, the initial profile should be subtracted to study bathymetric changes relative to the initial situation. This technique is useful when, for instance, studying the effects of a singular storm on the shoreline evolution (Muñoz-Perez et al., 2001). When the initial beach profile or the profile before the storm is subtracted, the results of the EOF method represent changes over this reference beach profile. In the present study, the initial bathymetric change was subtracted from the original data set. However, since the initial bathymetric change was zero, the first spatial EOF component corresponded to the average bathymetric change over the period of analysis at each point.

4.1. Temporal EOF component extrapolation

In principle, extrapolating the temporal EOF component is straightforward. It can be done by fitting a function directly using the EOF temporal component data. In this study, the first temporal EOF component manifests clear exponential behavior, as shown in Fig. 2(b), and, therefore, an exponential function was used to fit and extrapolate the first EOF temporal component from 750 h to 1500 h. Other temporal EOF components or other data sets could require other forms of function for fitting and extrapolation.

4.2. Spatial EOF component extrapolation

As previously mentioned, extrapolating the spatial EOF component requires performing EOF analysis progressively over a surveyed period: that is, starting EOF analysis for a certain period of time within the surveyed period and running it again for the period with a regular time increment. In this study, EOF analysis was carried out, at each location, for the first 100 h and repeated with a time increment of 10 h. Over the total period of 750 h, EOF components from EOF analysis of 66 periods could be obtained, corresponding to the bathymetric changes at each location, which is sufficient for determining the fitting function for extrapolating EOF components.

In Alvarez and Pan (2014), exponential growth was used to describe the shoreline change from an initially flat beach under the constant wave and tidal conditions until it reached an equilibrium state. In the present study, the behavior of the bathymetric changes at each cell was more complicated, but, in general, it could be described using an exponential function. However, at some locations, the linear function was found to be more appropriate.

To determine which function, linear or exponential, fits the data better at any particular location, an extensive examination is needed. Different data sets of the last 20, 30, 40, and 50 points of the total 66 points were considered to fit both exponential and linear functions. The resulting functions were then compared with the considered data sets for the same period of 750 h, and correlation coefficients were calculated for each case. In order to ensure that the extrapolation was working effectively, the correlation coefficient was compared with a certain tolerance value, which was set at 0.8. For the final reconstruction, the fitting presenting a higher correlation coefficient was selected, provided that this value was higher than the tolerance value. If none of the fittings provided a correlation coefficient higher than the tolerance value, the spatial EOF component for 750 h was used for reconstruction, meaning no extrapolation was done for this location. This process was repeated for each of the 10224 nodes in the mesh.

Fig. 3 shows an example of a specific location. The percentage of the bathymetric variability explained by the first spatial and temporal components, obtained from EOF analysis of 66 periods, almost remained constant and was always above 90% for the studied period of 750 h. As they explained the majority of the changes occurring in the system, only the first spatial and temporal components were used in this study.

Fig. 3(c) shows the values of the first spatial EOF component for each of the 66 periods (blue circles) at the specific location (a red circle in Figs. 3(a) and (b)). In this particular case, the last 40 points (green squares) were selected to fit an exponential function in order to find the spatial component beyond the studied period. The spatial EOF component extrapolated at 1500 h is indicated with a black square in Fig. 3(c). The extrapolated value at 1500 h shows a significant improvement over the value at 750 h, and corresponds well with the modeled value obtained directly from the COAST2D model over the 1500-h period, shown as a red square in Fig. 3(c).

Fig. 4 shows another example at a different location, where a linear function for extrapolating the spatial EOF component was obtained using the last 20 points. These two examples clearly show the reasons that a unique function is not adequate for extrapolation and a more exhaustive analysis is required.

4.3. Reconstruction

Once EOF components were extrapolated, they were used to reconstruct bathymetric changes at each cell in the

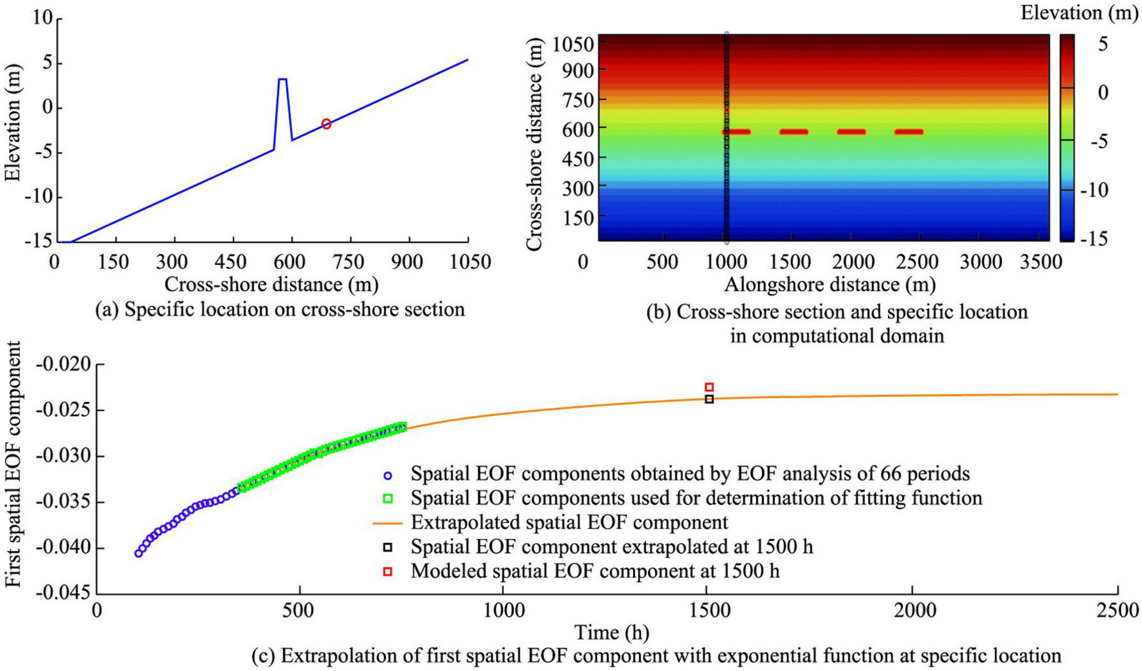


Fig. 3. Temporal evolution of first spatial EOF component at specific location showing exponential behavior.

computational domain based on Eq. (1). Bathymetric changes were predicted at $t = 1500$ h by extrapolating the EOF components obtained from a 750-h simulation, doubling the simulation time due to extrapolation of both the spatial and temporal components.

5. Results and discussion

The extrapolated EOF components were used to reconstruct bathymetric changes for a particular cross-shore section and the whole 2D domain. Fig. 5 shows the bathymetric changes of a particular cross-shore section, with elevation values modeled with the COAST2D model and obtained by the EOF method by extrapolation of the spatial and temporal EOF components. It can be seen that at the lee of the breakwater, the results obtained by extrapolating both the spatial and temporal EOF components are more accurate than those obtained by extrapolating the temporal EOF component only. This methodology was applied to the whole 2D domain and results are discussed below.

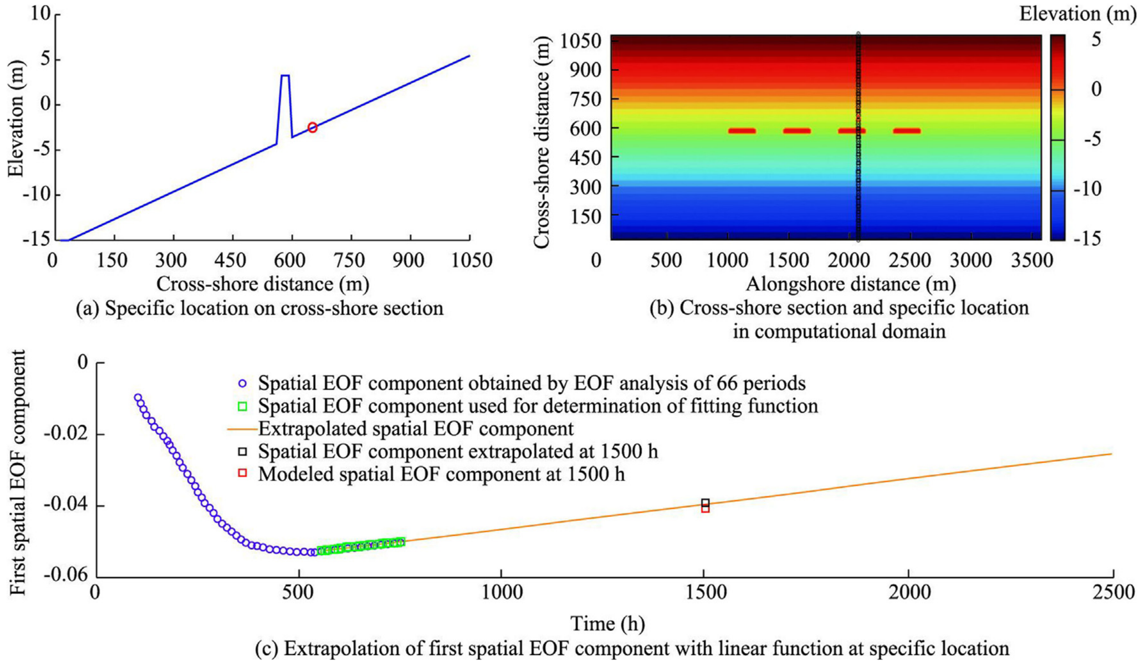


Fig. 4. Temporal evolution of first spatial EOF component at specific location showing linear behavior.

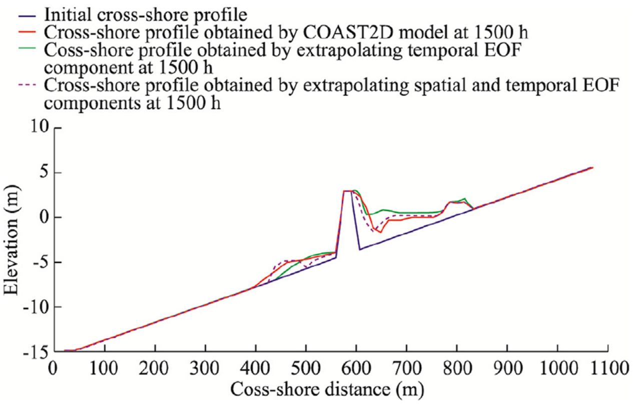
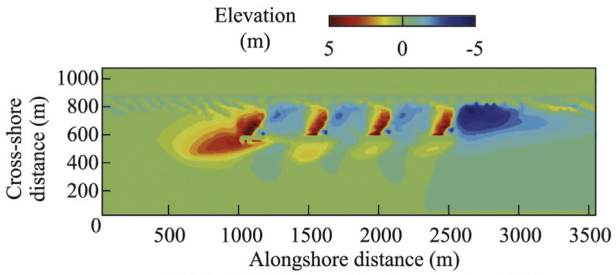
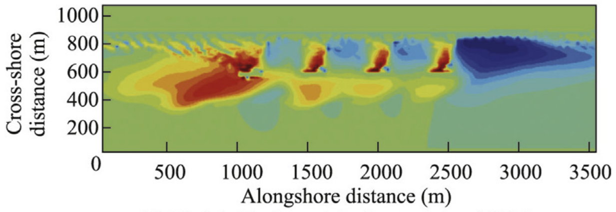


Fig. 5. Bathymetric changes reconstructed for particular cross-shore section.

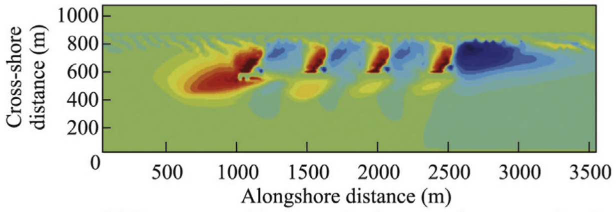
Figs. 6(a) and (b) show bathymetric changes modeled by the COAST2D model at 750 h and 1500 h, respectively. The results of the first 750 h were used to perform EOF analysis and extrapolate the EOF components. The results obtained from the COAST2D model at $t = 1500$ h were compared with the results of bathymetric changes reconstructed with the extrapolated EOF components. Fig. 6(c) shows the reconstructed bathymetric changes using the temporal EOF component extrapolated to 1500 h and the spatial EOF component at 750 h. Fig. 6(d) shows the reconstructed bathymetric changes using both the spatial and temporal EOF components extrapolated to 1500 h.



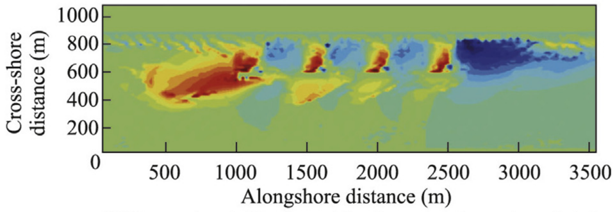
(a) Modeled bathymetric changes at t =750 h



(b) Modeled bathymetric changes at t =1500 h



(c) Reconstructed bathymetric changes using extrapolated temporal EOF component at t =1500 h



(d) Reconstructed bathymetric changes using extrapolated spatial and temporal EOF components at t =1500 h

Fig. 6. Modeled and reconstructed bathymetric changes.

From Figs. 6(a) and (b), it is clear that the accretion area is extended in the updrift area of breakwater B1 on the left side of the domain, and the area of erosion is extended in the downdrift area of breakwater B4 on the right side of the domain. It is also shown that the offshore sandbars in front of the breakwaters grow seaward. Additional erosion is found in the embayment of the breakwaters, but in a near-equilibrium state during those periods.

The bathymetric changes reconstructed only with the extrapolated temporal component, shown in Fig. 6(c), are under-represented, as the time-varying aspects of the spatial EOF component were not considered. The bathymetric changes reconstructed with both the extrapolated spatial and temporal EOF components, shown in Fig. 6(d), indicate a better agreement with those shown in Fig. 6(b). This clearly demonstrates that extrapolating the spatial EOF component improves the accuracy of prediction, as more realistic accretion/erosion areas are reproduced in terms of their shapes as well as magnitudes. For example, in Fig. 6(d), a large area of deposition can be found in the updrift area on the left side of breakwater B1 and an extended erosion area can be found in the downdrift area on the right side of breakwater B4. Also, Fig. 6(d) shows less erosion in the embayment at 1500 h than at 750 h, as the modeled results suggest. However, these results are absent from Fig. 6(c). Finally, the sandbars in front of the breakwaters are better represented in Fig. 6(d) than in Fig. 6(c), where the accretion areas have developed, nearly coming in contact with each other, as suggested by the modeled results at 1500 h.

6. Conclusions

A novel methodology for extrapolating EOF components with combined linear and exponential functions has been presented. Results show an overall improvement of prediction when both the spatial and temporal EOF components are extrapolated. This methodology can be effectively used for long-term morphological prediction in coastal areas, especially in areas protected by offshore defense structures, such as breakwaters. This approach could be further improved by using other types of functions to extrapolate EOF components. Only the first EOF spatial and temporal components were used in this study, although including more EOF components could further improve the results, depending on the complexity of the situation. In this study, the hydrodynamic conditions were well controlled with constant incident waves (height, period, and direction), producing very consistent morphological changes, and leading to a high percentage of the variability represented by the first spatial and temporal EOF components. When applying this methodology to field data, a higher percentage of the variability is expected to represent by more EOF components. For field studies, three or four couples of EOF components are commonly suggested, in order to represent over 90% of the coastal morphological changes. For these cases, the methodology presented in this paper can be readily applied.

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