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Gains from Variety?

Product Differentiation and the Possibility of Losses from Trade under Cournot Oligopoly with Free Entry

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Abstract

In a free-entry Cournot oligopoly model with a quadratic utility function that yields differentiated products, it is shown that there are losses from trade when the trade cost is close to the prohibitive level. Although the total number of varieties increases, there is a reduction in consumer surplus. This occurs because trade leads to an increase in imported varieties where consumer surplus is low due to the high trade cost and a decrease in domestically-produced varieties where consumer surplus is high. This result is in contrast with results from the free-entry Cournot oligopoly models with homogeneous products of Brander and Krugman (1983) and Venables (1985); the monopolistic competition models such as Krugman (1980) and Venables (1987), and heterogeneous firm models such as Melitz (2003) and Melitz and Ottaviano (2008).

Keywords: Gains from Trade, Trade Liberalisation, Free Entry, Cournot Oligopoly, Product Variety.

JEL Classification: F12.

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1. Introduction

An important source of gains from trade identified by the *new* trade theory is the increase in product variety available to consumers. The gains from increased variety were first demonstrated by Krugman (1979) using a monopolistic competition model with a general *love of variety* utility function and by Krugman (1980) using a CES utility function that became standard in the literature. In a monopolistic competition model with trade costs, Venables (1987) showed that there were unambiguously gains from trade due to increased variety. In the *new new* trade theory with heterogeneous firms started by Melitz (2003), there were aggregate productivity gains as well as the gains from increased variety. Melitz and Ottaviano (2008) used a quadratic utility function and found that there were gains from increased variety as well as the aggregate productivity gains. Although, according to Arkolakis et al. (2008) and Arkolakis, Costinot, and Rodríguez-Clare (2012) these new sources of the gains from trade have not affected the magnitude of the estimated gains from trade.

In the literature on international trade under Cournot oligopoly with homogeneous products, Brander and Krugman (1983) and Venables (1985) considered the case when there is free entry and exit of firms. They demonstrated that there were always gains from trade whatever the level of trade costs, due to the pro-competitive effect of international trade. In a Cournot oligopoly model with differentiated products and free entry, but with zero trade costs, Bernhofen (2001) showed that trade increased the number of varieties and had a pro-competitive effect on prices.[1]

[1] Marjit and Mukherjee (2015) consider a Cournot oligopoly model with differentiated products where there is free entry of domestic firms that compete with a single foreign firm only in the domestic market. They show that a reduction in trade costs may lead to a decrease in consumer surplus, but domestic firms do not export to the foreign country and a reduction in the trade cost has no effect on the prices set by domestic firms in their model.

In this note, international trade will be analysed in a free-entry Cournot oligopoly model with a quadratic utility function that yields differentiated products, and it is shown that there are losses from trade when trade costs are close to the prohibitive level.

2. The Cournot Oligopoly Model

Suppose that there are two symmetric countries, a home country labelled as A and a foreign country labelled as B. In each country, there is an imperfectly competitive industry producing differentiated products and a perfectly competitive industry producing a homogeneous good using a constant returns to scale technology. The imperfectly competitive industry consists of identical firms that each have a constant marginal cost c and a fixed cost F. Free entry and exit of firms ensures that profits are equal to zero in equilibrium and, ignoring the integer constraint, this determines the number of firms n in each country. There is also a per-unit trade cost t, which may be a real trade cost, such as a transport cost, or an import tariff that generates revenue for the country. Since the two countries are symmetric and all the firms are identical, the equilibria in the two countries will be symmetric. Therefore, the analysis will derive the equilibrium in the home country then exploit the symmetry between the two countries to work out the equilibrium number of firms and the welfare effects of international trade. The quantity sold by the *i*th oligopolistic firm from country $C = A, B$ in the home country is x_{Ci}, and the price it receives is p_{Ci}. The quantity of the numeraire good sold in the home country is z and its price is normalised at unity, $p_z = 1$. It is assumed that there is a representative consumer in each country with quasi-linear preferences that can be represented by a quadratic utility function, adapted from Vives (1985):

$$u(\mathbf{x}, z) = \alpha \sum_{C=A}^{B} \sum_{i=1}^{n} x_{Ci} - \frac{\beta}{2} \left[\sum_{C=A}^{B} \sum_{i=1}^{n} x_{Ci}^2 + 2\phi \left(\sum_{i=1}^{n} \sum_{j=1}^{n} x_{Ai} x_{Bj} + \sum_{C=A}^{B} \sum_{i \neq j} x_{Ci} x_{Cj} \right) \right] + z \qquad (1)$$

where $\alpha > c > 0$, $\beta > 0$ and $\phi \in (0,1]$ is the degree of product substitutability. Note that this utility function exhibits the same *love of variety* effect as the CES utility function. Utility maximisation by the representative consumer yields the inverse demand facing the *i*th oligopolistic firm from each of the two countries in the home country:

$$\begin{aligned} p_{Ai} &= \alpha - \beta\left[x_{Ai} + \phi\left(\sum_{j\neq i} x_{Aj} + \sum_{j=1}^{n} x_{Bj}\right)\right] \\ p_{Bi} &= \alpha - \beta\left[x_{Bi} + \phi\left(\sum_{j\neq i} x_{Bj} + \sum_{j=1}^{n} x_{Aj}\right)\right] \end{aligned} \tag{2}$$

Since the utility function is quasi-linear, consumer surplus is a valid measure of consumer welfare that in the home country is given by:

$$CS = u - \sum_{C=A}^{B}\sum_{i=1}^{n} p_{Ci}x_{Ci} - z = \frac{\beta}{2}\left[\sum_{C=A}^{B}\sum_{i=1}^{n} x_{Ci}^2 + 2\phi\left(\sum_{i=1}^{n}\sum_{j=1}^{n} x_{Ai}x_{Bj} + \sum_{C=A}^{B}\sum_{i\neq j} x_{Ci}x_{Cj}\right)\right] \tag{3}$$

Since the foreign oligopolistic firms face the per-unit trade cost t when they supply the market in the home country, the gross profits in the home country of the *i*th firm from each of the two countries are:

$$\pi_{Ai} = (p_{Ai} - c)x_{Ai} \qquad\qquad \pi_{Bi} = (p_{Bi} - c - t)x_{Bi} \tag{4}$$

The gross profits of the firms in the market of the foreign country are defined analogously except that the home firms now face the trade cost.

3. The Free-Entry Cournot Equilibrium under Autarky

Under autarky, there is no trade with the foreign country so the home country only consumes the goods produced by the home firms. Solving for the symmetric Cournot-Nash equilibrium, taking the number of firms as given, yields the output, price and gross profits of each firm:

$$x_A = \frac{\alpha - c}{\beta\left(2+(n-1)\phi\right)} \qquad p_A = c + \frac{(\alpha - c)}{\left(2+(n-1)\phi\right)} \qquad \pi_A = \frac{(\alpha - c)^2}{\beta\left(2+(n-1)\phi\right)^2} \tag{5}$$

Setting net profits $\Pi_A = \pi_A - F = 0$ then solving for the equilibrium number of firms under autarky, while ignoring the integer constraint, yields:

$$n_A^N = \frac{\Omega}{\phi\sqrt{\beta F}} \qquad \text{where} \quad \Omega \equiv (\alpha - c) - (2-\phi)\sqrt{\beta F} > 0 \tag{6}$$

Assuming that $\Omega > 0$ or $F < (\alpha - c)^2 / \beta(2-\phi)^2$ then substituting (6) into (5) yields the free-entry Cournot-Nash equilibrium output and price under autarky:

$$x_A^N = \sqrt{\frac{F}{\beta}} \qquad p_A^N = c + \sqrt{\beta F} \tag{7}$$

Welfare of the home country under autarky is given by the sum of consumer surplus plus the profits of the home firms, but profits are zero in equilibrium, therefore using (3), (6) and (7) yields autarky welfare:

$$W_A^N = CS_A^N = \frac{\beta}{2}\left[n_A^N\left(x_A^N\right)^2 + \phi n_A^N\left(n_A^N - 1\right)\left(x_A^N\right)^2\right] = \frac{\Omega}{2\phi\beta}\left(\alpha - c - \sqrt{\beta F}\right) \tag{8}$$

4. The Free-Entry Cournot Equilibrium with International Trade

If the two countries open up to international trade multilaterally then firms in the home country can export their products to the foreign country and consumers in the home country can buy imported products from the foreign firms. The inverse demand functions facing the firm are given by (2) and the gross profits of the firms are given by (4) in the market of the home country. Hence, the first-order conditions for a Cournot-Nash equilibrium with international trade in the home country are:

$$\frac{\partial \pi_{Ai}}{\partial x_{Ai}} = \alpha - 2\beta x_{Ai} - \phi\beta\left(\sum_{j\neq i} x_{Aj} + \sum_{j=1}^{n} x_{Bj}\right) - c = 0$$

$$\frac{\partial \pi_{Bi}}{\partial x_{Bi}} = \alpha - 2\beta x_{Bi} - \phi\beta\left(\sum_{j=1}^{n} x_{Aj} + \sum_{j\neq i} x_{Bj}\right) - c - t = 0 \tag{9}$$

Since all the home firms are symmetric they will all produce the same quantity in the Cournot-Nash equilibrium, $x_{Ai} = x_{Aj} = x_A$, receive the same price, $p_{Ai} = p_{Aj} = p_A$, and earn the same profits $\pi_{Ai} = \pi_{Aj} = \pi_A$ in the market of the home country. Similarly, since all the foreign firms are symmetric: $x_{Bi} = x_{Bj} = x_B$, $p_{Bi} = p_{Bj} = p_B$ and $\pi_{Bi} = \pi_{Bj} = \pi_B$ in the market of the home country. Also, symmetry implies that the number of firms in each country will be the same in the two countries in equilibrium. Solving for the Cournot-Nash equilibrium, taking the number of firms as given, yields the output of each firm:

$$x_A = \frac{(2-\phi)(\alpha - c) + n\phi t}{\beta\left[(2-\phi)(2+(2n-1)\phi)\right]} \qquad x_B = \frac{(2-\phi)(\alpha - c) - (2+(n-1)\phi)t}{\beta\left[(2-\phi)(2+(2n-1)\phi)\right]} \tag{10}$$

The total net profits of a home firm are equal to the sum of gross profits in the two countries minus the fixed cost, $\Pi_A = \pi_A + \pi_A^* - F$, where π_A^* is the gross profits of a home firm from exporting to the foreign country and, by symmetry, this is equal to the gross profits of a foreign firm from exporting to the home country, $\pi_A^* = \pi_B$. Therefore, the total net profits of a home firm are:

$$\Pi_A = \frac{1}{2\beta}\left[\frac{\left(2(\alpha - c) - t\right)^2}{\left(2+(2n-1)\phi\right)^2} + \frac{t^2}{(2-\phi)^2}\right] - F \tag{11}$$

Setting $\Pi_A = 0$ and solving for the equilibrium number of firms, which is the same in both countries, while ignoring the integer constraint, yields:

$$n^T = \frac{(2-\phi)\left[\left(2(\alpha-c)-t\right)\sqrt{\Phi}-\Phi\right]}{2\Phi\phi} \qquad \text{where} \quad \Phi \equiv 2(2-\phi)^2 \beta F - t^2 > 0 \tag{12}$$

Therefore, substituting (12) into (10) yields the free-entry Cournot-Nash equilibrium outputs with international trade:

$$x_A^T = \frac{\sqrt{\Phi}+t}{2\beta(2-\phi)} \qquad x_B^T = \frac{\sqrt{\Phi}-t}{2\beta(2-\phi)} \tag{13}$$

Note that the trade cost will be prohibitive so that there will be no trade, $x_B^T = 0$, if $t \geq \sqrt{\Phi} > 0$, and hence, using the definition of Φ from (12), the prohibitive trade cost is $\bar{t} \equiv (2-\phi)\sqrt{\beta F}$.

5. Welfare Effects of International Trade

The welfare effects of international trade can now be analysed by comparing the equilibrium with international trade with the equilibrium under autarky. Using symmetry together with (12) and (13) in (3), the consumer surplus of the home country with international trade is:

$$\begin{aligned} CS_A^T(t) &= \frac{\beta}{2}\left[n^T\left(x_A^T\right)^2 + n^T\left(x_B^T\right)^2 + \phi\left(2\left(n^T\right)^2 x_A^T x_B^T + n^T\left(n^T-1\right)\left(\left(x_A^T\right)^2 + \left(x_B^T\right)^2\right)\right)\right] \\ &= \frac{1}{8\Phi\phi\beta}\left[\left(2(\alpha-c)-t\right)\sqrt{\Phi}-\Phi\right]\left[\left(2(\alpha-c)-t\right)\sqrt{\Phi}-\Phi+2(1-\phi)(2-\phi)\beta F\right] \end{aligned} \tag{14}$$

If a fraction μ of the trade cost is a tariff, then the tariff revenue of the home country is $R_A^T(t,\mu) = \mu t n^T x_B^T$. Since profits are equal to zero in equilibrium, the welfare of the home country is the sum of consumer surplus and tariff revenue:

$$W_A^T(t,\mu) = CS_A^T(t) + R_A^T(t,\mu) = CS_A^T + \frac{\mu t\left(\sqrt{\Phi}-t\right)}{4\Phi\phi\beta}\left[\left(2(\alpha-c)-t\right)\sqrt{\Phi}-\Phi\right] \tag{15}$$

Consider the case of totally free trade analysed by Bernhofen (2001) where the trade cost is equal to zero, $t = 0$, so there is no tariff revenue.[2] It is straightforward to show that a move from autarky to free trade will increase welfare:

$$W_A^T(0,\mu) = \frac{1}{4\beta\phi}\left(\sqrt{2}(\alpha - c) - \sqrt{\beta F}\right)\left(\sqrt{2}(\alpha - c) - \sqrt{\beta F}(2-\phi)\right) > W_A^N \tag{16}$$

Now consider the case when the trade cost is positive and, in particular, the case when the trade cost is close to the prohibitive level. Welfare with international trade (relative to welfare under autarky) is plotted in figure one as a function of the trade cost (for the parameter values: $\alpha = 50$, $\beta = 1$, $\phi = 1/3$, $c = 10$, $F = 20$, and $\mu = 0, 1/5, 1$). Obviously, when the trade cost is prohibitive, $t \geq \overline{t}$, the equilibrium in each country will be exactly the same as under autarky and therefore welfare will be exactly the same as under autarky, $W_A^T(\overline{t},\mu) = W_A^N$. A reduction in the trade cost evaluated at the prohibitive level, $t = \overline{t}$, will lead to a reduction in the number of firms in both countries since, using (12), $\partial n^T/\partial t = \Omega/\phi(2-\phi)\beta F > 0$. The effect on the welfare of the home country is obtained by differentiating (15) and evaluating at the prohibitive trade cost:

$$\left.\frac{\partial W_A^T}{\partial t}\right|_{t=\overline{t}} = \frac{\Omega}{2\phi\beta(2-\phi)}\left(1-\phi-2(2-\phi)\mu\right) \tag{17}$$

A reduction in trade costs will decrease (increase) welfare if this derivative is positive (negative), which will be the case if the fraction of the trade cost that is a tariff $\mu < (>)\, \mu^* \equiv (1-\phi)/2(2-\phi)$. Note that μ^* is decreasing in the degree of product

[2] Proposition 3 of Bernhofen (2001) does not explicitly prove that there are gains from trade, but shows that trade increases the number of varieties, reduces price-cost margins and increases the output of each firm.

substitutability, ϕ, from $\mu^* = 1/4$ when the products are independent, $\phi = 0$, to $\mu^* = 0$ when the products are homogeneous, $\phi = 1$. These results lead to the following proposition:

Proposition: *When products are differentiated, $\phi \in (0,1)$, and the trade cost is close to the prohibitive level, there are losses (gains) from trade for both countries if $\mu < (>) \mu^*$.*

Consider the case when trade costs are real and there is no tariff revenue, $\mu = 0$, then (17) is strictly positive if products are differentiated, $\phi \in (0,1)$ and equal to zero, as in Venables (1985), if products are homogeneous, $\phi = 1$. Therefore, with differentiated products, welfare is upward-sloping at the prohibitive trade cost as shown in figure one and there will be a range of values for the trade cost, $t \in [\underline{t}, \overline{t}]$, where welfare with international trade is lower than welfare under autarky so there are losses from trade. The explanation is that the increased competition with international trade reduces the profits of the home firms and leads to the exit of some firms. As a result, consumers have fewer domestically-produced varieties, and the imported varieties provide little consumer surplus as consumption of these varieties is infinitesimally small. Since the model is symmetric, both countries will lose from trade if the trade cost is close to the prohibitive level. The critical values for the trade cost $\underline{t}$ and $\overline{t}$ are plotted in figure two as functions of the degree of product substitutability, ϕ, for the same parameters as in figure one, and it can be seen that the region where there are losses from trade becomes larger as the products become more differentiated.

Now consider the case when the trade cost is an import tariff that generates revenue for the government, $\mu = 1$, then (17) is always strictly negative and there are always gains from trade as shown in figure one. In this case, the loss of consumer surplus when the trade cost is reduced is outweighed by the gain from the tariff revenue. When only a fraction of the trade cost is a tariff, the gain in tariff revenue outweighs the loss of consumer surplus if $\mu \geq \mu^*$ and there will always be gains from trade as shown in figure one.

6. Conclusions

This note has demonstrated the possibility of losses from trade in a free-entry Cournot oligopoly model with differentiated products when the trade cost is close to the prohibitive level despite an increase in the total number of varieties available to consumers. The gains from the availability of imported varieties are outweighed by the loss of domestically-produced varieties as consumption of each of the imported varieties is infinitesimally small when the trade cost is close to the prohibitive level. Since the model is symmetric, both countries will lose from trade if the trade cost is close to the prohibitive level.

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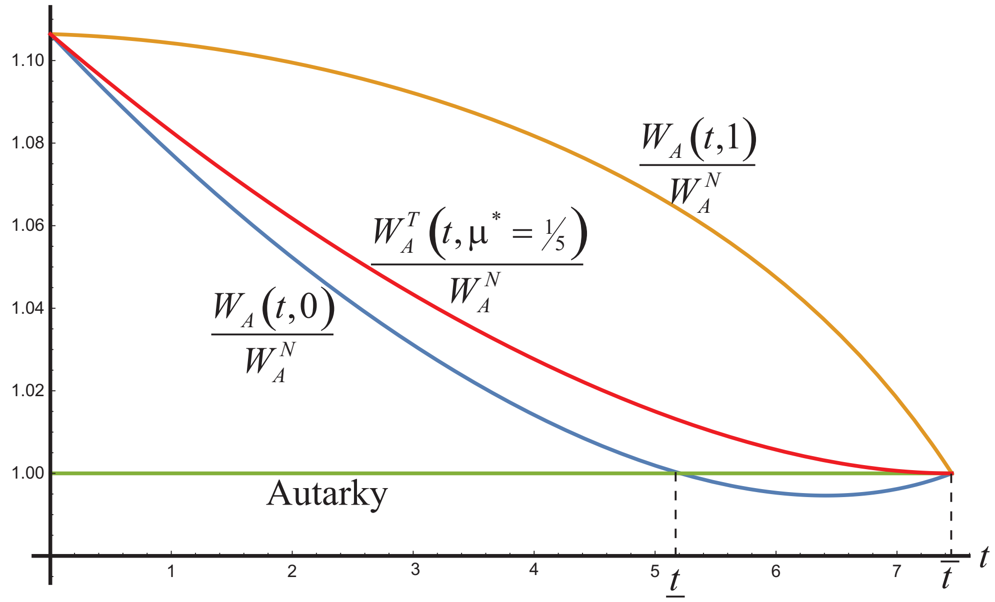

Figure 1: The Welfare Effects of International Trade $\phi=1/3$

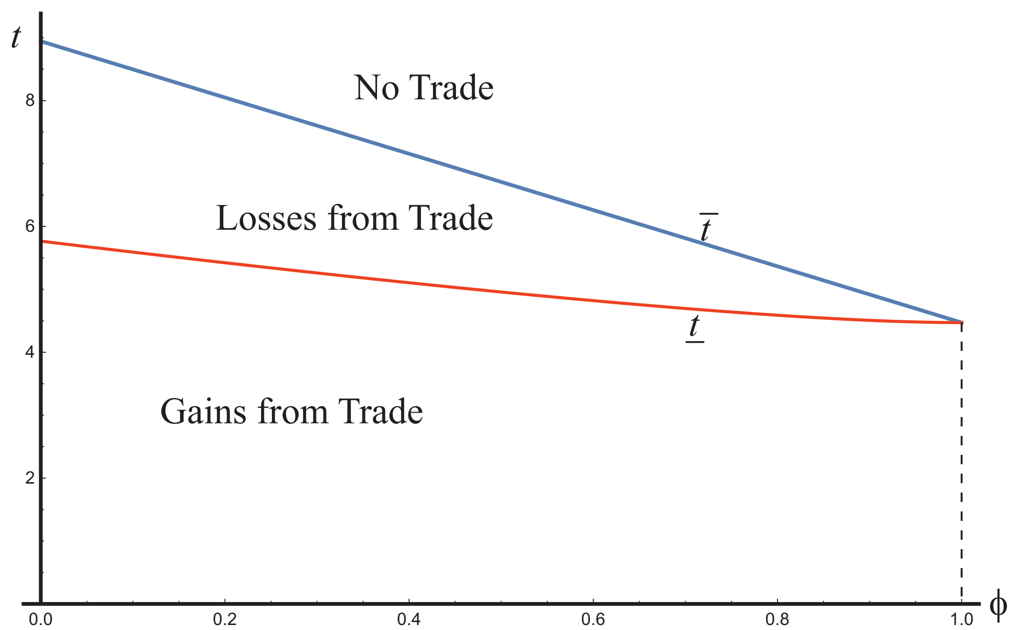

Figure 2: Critical Trade Costs versus Product Substitutability